

Regression basics

dark horse of statistics

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Outline

- ▶ Why do we need it?
- ▶ Applications
- ▶ Formal representation
- ▶ Intuition behind
- ▶ Estimators of regression
- ▶ Optimizations for regression
- ▶ Assumptions (Gauss-Markov)

Why do we need it?

- ▶ *Forecast* future
- ▶ *Explain* present and the past

Applications

- ▶ Social Sciences (in broad sense)
- ▶ Economics
- ▶ Natural Sciences (originally came from Physics)
- ▶ Business
- ▶ Medicine (Pharmacology)
- ▶ Urban Studies
- ▶ Criminology
- ▶ etc

Formal representation

Things in «real» abstract world:

$$\hat{y} = \beta X \quad (1)$$

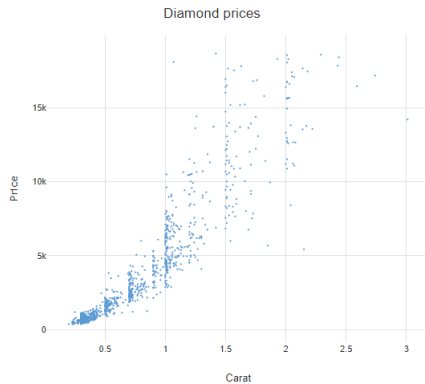
Things we are doing in statistics:

$$\hat{y} = \beta X + \hat{\epsilon} \quad (2)$$

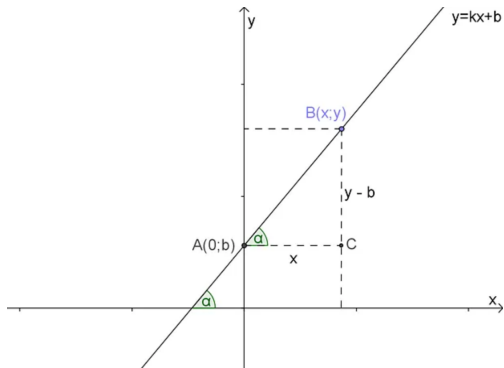
More common representation:

$$\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \dots + \beta_n x_n + \hat{\epsilon} \quad (3)$$

Intuition behind

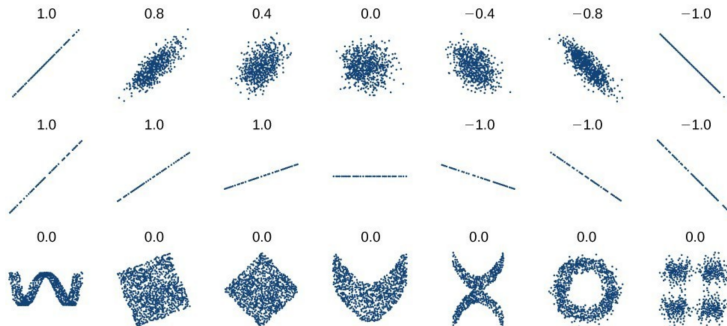


Intuition behind



Intuition behind

What if we have smth like this?



Intuition behind

Once we decided that our data can be fitted by line model, let's select β coefs. Let's follow [here](#).

Estimators

Estimation method (aka LOSS functions) - equation that help to find coefs. The most common techniques:

- ▶ OLS: $\beta = \Sigma(\hat{y} - y)^2 \rightarrow \min$
- ▶ MLE: $\theta = P(X|\theta) \rightarrow \max$
- ▶ Other purpose specified (Ridge, Quantile, PLS, Bayes Models)

Estimators

When and what

What should I use for my model?

- ▶ OLS (and ols related): only continuous variables
- ▶ MLE (and mle related): all other variables

How it works inside

- ▶ Analytical solution $\beta = (X^T X)^{-1} X^T y$
- ▶ (Stochastic) Gradient Descent (related variations)
- ▶ MCMC (for Bayes equations)

Gauss-Markov Assumptions

- ▶ Linearity of data
- ▶ Sample should be *randomly* selected for population
- ▶ X matrix should not be correlated within
- ▶ X matrix should not be correlated with error
- ▶ Variance of error should be constant