Intro to Quantitative methods

Chuvakin Sergey

«School of Advanced Studies»

November 18, 2020

Plan

- Quantitative vs Qualitative
- Calculus and Linear Algebra basics
- Probability theory basics

Quantitative vs Qualitative

1. type of data
   - Surveys
   - Polls
   - Internal statistics
2. type of question
   - What is the relation?
   - What is a difference?
   - How one affects on other?
3. methods
   - Statistical tests
   - Regression models (the most important one)
   - Machine Learning (e.g. cluster analysis)

Calculus and Linear Algebra

The most important terms:

1. vectors
2. vector operations
3. matrix
4. matrix operations

Vectors

Vector - is element of some vector space. It could be Euclidean, Hilbert, Topological and many others.

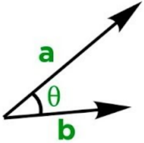
1. Begins
2. Ends
3. *Direction*
4. magnitude

Notation: $\overline{AB}$ or $\overline{a}$

Vector Operations

1. Addition
2. Subtraction
3. Scalar multiplication
4. Dot product

The Vector Dot Product



$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$$

$$\cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$$

Vector Operations

$$\text{similarity} = \cos(\theta) = \frac{\mathbf{A} \cdot \mathbf{B}}{\|\mathbf{A}\| \|\mathbf{B}\|} = \frac{\sum\limits_{i=1}^{n} A_i B_i}{\sqrt{\sum\limits_{i=1}^{n} A_i^2} \sqrt{\sum\limits_{i=1}^{n} B_i^2}},$$

Matrix

Matrix - set of vectors

1. Same as vectors
2. + transposing
3. Notation (Roman Catholichs)
4. Dot product

Matices types

Matrix - set of vectors

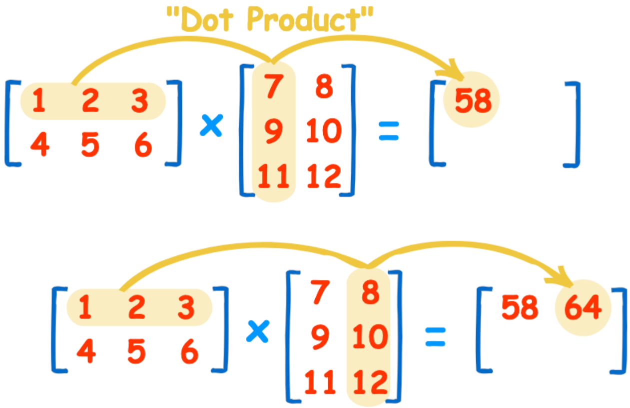
1. Row Matrix (aka vector)
2. Column Matrix (aka vector)
3. Square Matrix
4. Rectangular Matrix
5. Diagonal matrix
6. Zero or Null Matrix
7. Unit or Identity Matrix

Dot Product

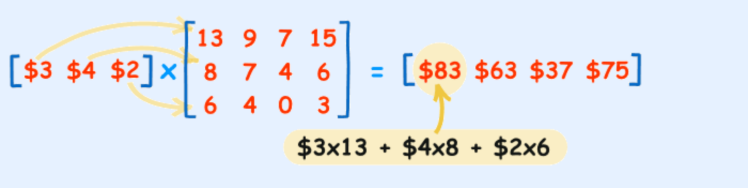
$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} a_{11}b_{11} + \cdots + a_{1n}b_{n1} & a_{11}b_{12} + \cdots + a_{1n}b_{n2} & \cdots & a_{11}b_{1p} + \cdots + a_{1n}b_{np} \\ a_{21}b_{11} + \cdots + a_{2n}b_{n1} & a_{21}b_{12} + \cdots + a_{2n}b_{n2} & \cdots & a_{21}b_{1p} + \cdots + a_{2n}b_{np} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}b_{11} + \cdots + a_{mn}b_{n1} & a_{m1}b_{12} + \cdots + a_{mn}b_{n2} & \cdots & a_{m1}b_{1p} + \cdots + a_{mn}b_{np} \end{pmatrix}$$

Dot Product



Dot Product



Dot Product

So ... multiplying a **1×3** by a **3×1** gets a **1×1** result:

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 1\times4+2\times5+3\times6 \end{bmatrix} = \begin{bmatrix} 32 \end{bmatrix}$$

But multiplying a **3×1** by a **1×3** gets a **3×3** result:

$$\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 4\times1 & 4\times2 & 4\times3 \\ 5\times1 & 5\times2 & 5\times3 \\ 6\times1 & 6\times2 & 6\times3 \end{bmatrix} = \begin{bmatrix} 4 & 8 & 12 \\ 5 & 10 & 15 \\ 6 & 12 & 18 \end{bmatrix}$$

Probability basics

The most important terms:

1. Probability
2. Independent Events
3. Dependent Events
4. Conditional
5. Bayes

Notation: $P(A)$ or $P(A|B)$

In general:

$$\text{Probability of an event happening} = \frac{\text{Number of ways it can happen}}{\text{Total number of outcomes}}$$

Conditional Probability

$$P(A \mid B) = P(A \cap B) \,/\, P(B)$$

Independent Events

Events A and B are independent iff

$$P(\mathrm{A} \cap \mathrm{B}) = P(\mathrm{A}) \cdot P(\mathrm{B})$$

Rule of Addition

$$P(\mathrm{A} \cup \mathrm{B}) = P(\mathrm{A}) + P(\mathrm{B}) - P(\mathrm{A} \cap \mathrm{B})$$

Bayes Formula

$$P(\mathrm{A} \mid \mathrm{B}) = P(\mathrm{B} \mid \mathrm{A}) \cdot P(\mathrm{A}) \,/\, P(\mathrm{B})$$