Lecture 6: the 2nd law of Newton's theory of classical mechanics

- Recap from Lecture 5:

- Mathematical tools:
  - Equation
  - Scalar quantities: described by a single number (e.g time t, mass m, ...)
  - Vector quantities: described by more than one number (e.g position $\vec{x}$, velocity $\vec{v}$, ...)

- Operations with vectors
  - Multiplication of a scalar times a vector

    e.g $m \cdot \vec{a} = m \cdot (a_x, a_y, a_z) = (m a_x, m a_y, m a_z)$

  - Addition of vectors

    e.g $\vec{F}_1 + \vec{F}_2 = (F_{1x}, F_{1y}, F_{1z}) + (F_{2x}, F_{2y}, F_{2z})$

    $= (F_{1x} + F_{2x}, F_{1y} + F_{2y}, F_{1z} + F_{2z})$

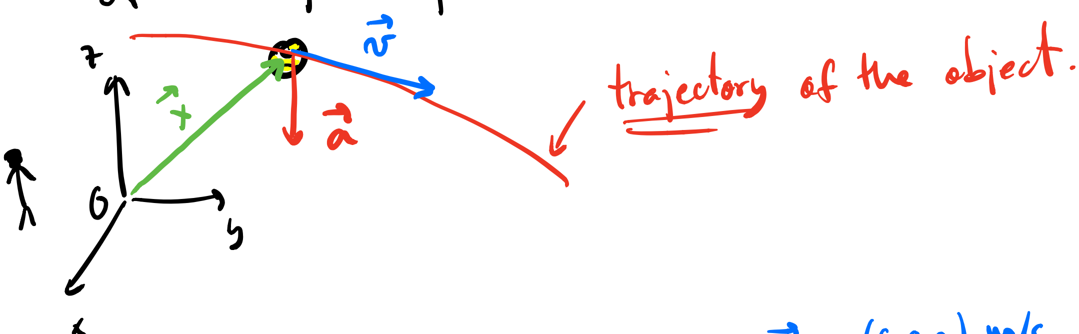
- Magnitude of a vector

e.g $\vec{v}$, magnitude is $|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$

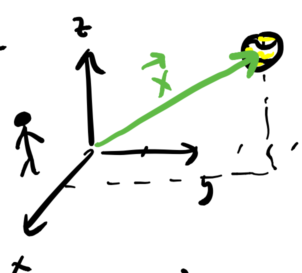
(it is also the "length" of the arrow)

- The **speed** is the magnitude of the velocity vector $\vec{v}$.

- Vectors that allow us to describe the motion of an object: position $\vec{x}$, velocity $\vec{v}$, acceleration $\vec{a}$.



e.g object at rest



$\vec{v} = (0,0,0)$ m/s

$\vec{a} = (0,0,0)$ m/s²

$\vec{x}(t) = (1, 4, 3)$ m

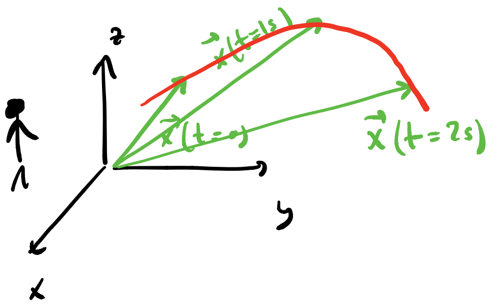
$\vec{x}(t) = (5t, 1+t^2, 3\sqrt{t})$

- The trajectory of an object corresponds to the function $\vec{x}(t)$

$\vec{x}(t)$ ← trajectory of the particle

output quantity

input quantity



$\vec{x}(t) = ???$

what goes here depends on the system we want to describe. Newton's theory gives us a prescription to deduce it.

- How to find $\vec{x}(t)$: Newton's 2nd law.

- Newton's 2nd law: with respect to an inertial observer (inertial FOR), the acceleration $\vec{a}$ of a mass m is related to the force through the following equation

$$m\vec{a} = \vec{F}_{total}$$

- Once $\vec{a}$ is known, as well as the initial data $\vec{x}_0$ and $\vec{v}_0$ of the particle, the trajectory $\vec{x}(t)$ is uniquely determined through a mathematical prescription called integration.

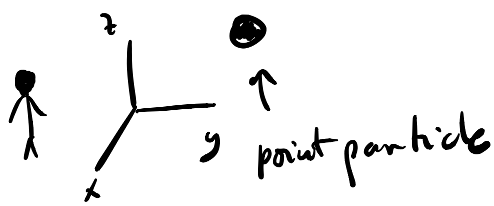
$$\vec{F}_{total} \xrightarrow{\text{2nd law}} \vec{a} \xrightarrow[\text{integration}]{\text{if } \vec{v}_0 \text{ is known}} \vec{v} \xrightarrow[\text{integration}]{\text{if } \vec{x}_0 \text{ is known}} \vec{x}$$

* Questions to ask before starting to solve for the motion of an object.

  - What is the object/s whose motion we want to predict. Point particle? Rigid object? Fluids?...
  - Where is the observer sitting with respect to the object? Pick an inertial observer!
  - What is the initial data $\vec{x}_0$ and $\vec{v}_0$ w.r.t the observer.
  - What forces act on the object?
    $\vec{F}_{total} = \dots$

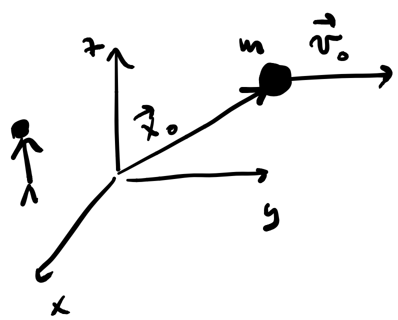
- What is the mass of the object?

Example: a "free particle".



$\vec{F}_{total} = (0,0,0)\ kg\frac{m}{s^2}$ or N

- Initial data (data at $t=0$)



$\vec{x}_0 = (0,3,2)\ m$

$\vec{v}_0 = (0,2,0)\ m/s$

$m = 2\ kg$

- Next step: apply Newton's 2nd law $m\vec{a} = \vec{F}_{total}$ to find $\vec{a}$.

$$m\vec{a} = \vec{F}_{total} \Rightarrow \vec{a} = \frac{\vec{F}_{total}}{m}$$

$$\vec{a} = \frac{(0,0,0)}{2} = \frac{1}{2}\cdot(0,0,0) = (0,0,0)\ m/s^2$$

- Next step: find $\vec{v}$ given $\vec{v}_0$

$$\boxed{\vec{v} = \vec{v}_0 + \underbrace{\int}_{\text{integral}} \vec{a}\, dt} = \vec{v}_0 + \int (0,0,0)\, dt = \vec{v}_0$$

$$= (0,2,0)\, \tfrac{m}{s}$$

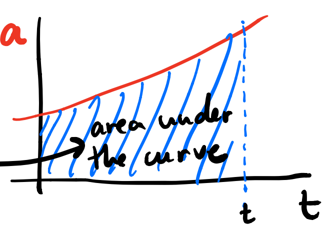
- Next step: find $\vec{x}$ given $\vec{x}_0$

$$\boxed{\vec{x} = \vec{x}_0 + \underbrace{\int}_{\text{integral}} \vec{v}\, dt} = \vec{x}_0 + \int (0,2,0)\, dt$$

$$= \vec{x}_0 + (0,2,0)\cdot t = (0,3,2) + (0,2,0)\cdot t$$

How to solve integrals?

$\int a\, dt$

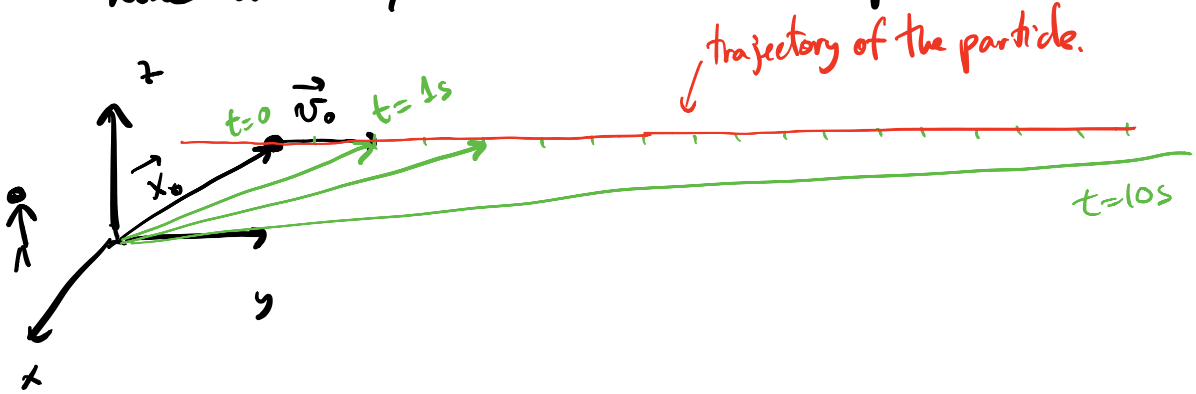


$\int v\, dt$

In conclusion, we found:

$$\boxed{\begin{aligned} \vec{x} &= \vec{x}_0 + \vec{v}_0 t = (0,3,2) + (0,2,0)\cdot t = (0, 3+2t, 2) \text{ m} \\ \vec{v} &= \vec{v}_0 = (0,2,0) \text{ m/s} \\ \vec{a} &= (0,0,0) \text{ m/s}^2 \end{aligned}}$$

Kinematic equations of the free particle



At $t = 0$ s: $\vec{x} = (0, 3+2\cdot 0, 2) = (0,3,2)$ m

At $t = 1$ s: $\vec{x} = (0, 3+2\cdot 1, 2) = (0,5,2)$ m

At $t = 2$ s: $\vec{x} = (0, 3+2\cdot 2, 2) = (0,7,2)$ m

At $t = 10$ s: $\vec{x} = (0, 3+2\cdot 10, 2) = (0,23,2)$ m