NAME:

Today's worksheet consists of a few pencil and paper calculations and some theoretical questions. Don't rush over the calculations, make sure you break down each calculation in all the necessary steps. Show your work flow in a clean and accurate style.

1. As we saw in the last lecture, before Newton came up with his theory, the german astronomer Johannes Kepler had developed a theory (1619) to explain the motion of the planets in the solar system. State the three postulates of his theory and make a drawing of the orbit of a planet around the Sun according to Kepler. Locate the sun, the planet and the semi-major axis in your drawing.

2. The third postulate tells us that the square of the orbital period T is directly proportional to the cube of the semi-major axis a of the ellipse. That is

$$T^2 = ka^3 \tag{1}$$

where k is a constant (sometimes we also write $T^2 \propto a^3$). Therefore k is a number that doesn't change (it is the same for all planets in the solar system).
In this and the remaining exercises we will consider the orbits to be approximately circular, that means a is approximately the distance from the planet to the Sun. Knowing that the orbital period of the Earth around the Sun is approximately 365 days and that the distance between the Earth and the Sun is about 150 million kilometers, calculate k. Use scientific notation for the final result and write down its units.

3. Now that you know the value of k you can use it to calculate the orbital period of different planets if you know their distance to the Sun, and vice-versa. For example, take Mercury, the distance a to the Sun is 58 million kilometers. Using Kepler’s formula and the value of k that you got calculate the orbital period of Mercury.

4. With Newton’s new theory at hand (1687), one can derive all three of Kepler’s laws. That means **they give up their status as postulates in Kepler’s theory and become predictions of Newton’s theory**. In fact, Newton was able to tell us where the value of k was coming from. He found that

$$k = \frac{4\pi^2}{GM} \tag{2}$$

where M is the mass of the Sun and G is the gravitational constant that has the value $G = 6.674 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$. Calculate the mass of the Sun (in kilograms) using this formula and using your previous result for k. Give your result in scientific notation.

5. Kepler’s third law, together with Newton’s addition (formula in exercise 4), is a perfect example of how in a theory of motion there are **three fundamental quantities** entering the game. What are they and where do they appear in the formula?

$$T^2 = \frac{4\pi^2}{GM} a^3 \tag{3}$$

6. In the case of the system composed by the Earth and the Sun, of those three quantities, which one do you think is the easiest to measure **experimentally** (i.e by means of some observations)? Explain why.