Lecture 7: Applications of Newton's 2nd law.

Review from lecture 6:

- Newton's 2nd law: $m\vec{a} = \vec{F}_{total}$
- We want to deduce the trajectory $\vec{x}(t)$

If the force $\vec{F}(t)$ is known then:

$$\vec{F}(t) \xrightarrow{\text{2nd law}} \vec{a}(t) \xrightarrow[\text{integration}]{\text{if } \vec{v}_0 \text{ is known}} \vec{v}(t) \xrightarrow[\text{integration}]{\text{if } \vec{x}_0 \text{ is known}} \vec{x}(t)$$

- We applied this prescription to the case of a "free particle": $\vec{F}_{total} = (0,0,0)$ and we deduced the following kinematic equations:

$$\boxed{\begin{aligned} \vec{a} &= (0,0,0) \\ \vec{v} &= \vec{v}_0 \\ \vec{x} &= \vec{x}_0 + \vec{v}_0 t \end{aligned}}$$

$\updownarrow$ equivalent to write

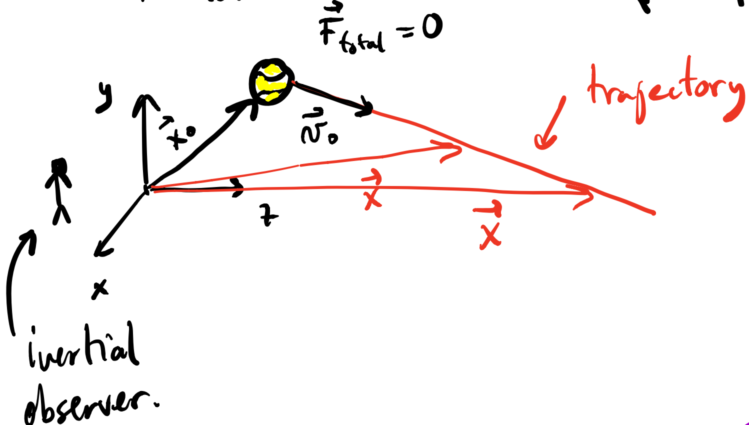
$$\boxed{\begin{aligned} (a_x, a_y, a_z) &= (0,0,0) \\ (v_x, v_y, v_z) &= (v_{0x}, v_{0y}, v_{0z}) \\ (x,y,z) &= (x_0, y_0, z_0) + (v_{0x}, v_{0y}, v_{0z}) \cdot t \end{aligned}}$$

$\uparrow$

$\downarrow$ equivalent to

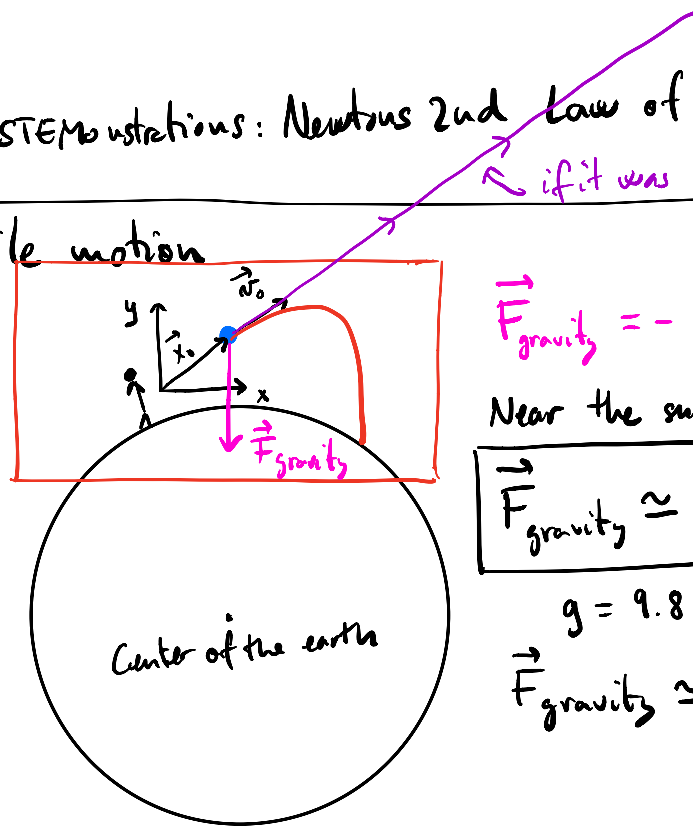
$$\boxed{\begin{array}{l} a_x = 0, \quad a_y = 0, \quad a_z = 0 \\ v_x = v_{0x}, \quad v_y = v_{0y}, \quad v_z = v_{0z} \\ x = x_0 + v_{0x} t, \quad y = y_0 + v_{0y} t, \quad z = z_0 + v_{0z} t \end{array}}$$

Pictorial view of the free particle



- STEMonstrations: Newtons 2nd Law of Motion. (Youtube)

- Projectile motion



$$\vec{F}_{gravity} = -G \frac{M \cdot m}{|\vec{s}|^3} \vec{s}$$

Near the surface of the earth

$$\boxed{\vec{F}_{gravity} \simeq m \cdot (0, -9.8)}$$

$g = 9.8 \, m/s^2$

$$\vec{F}_{gravity} \simeq m \cdot (0, -g)$$

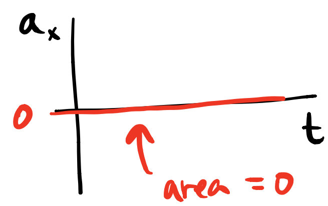
- Next step: apply 2nd law $m\vec{a} = \vec{F}_{total}$

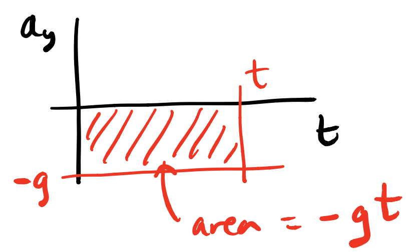
$$m\vec{a} = \vec{F}_{gravity} \Rightarrow \vec{a} = \frac{\vec{F}_{gravity}}{m}$$

$$\vec{a} = \frac{\cancel{m}\cdot(0,-g)}{\cancel{m}} = (0,-g)\ m/s^2$$

- Next step: get $\vec{v}$

$$\vec{v} = \vec{v}_0 + \int \vec{a}\,dt = \vec{v}_0 + \int (\overset{a_x}{0}, \overset{a_y}{-g})\,dt = (*)$$



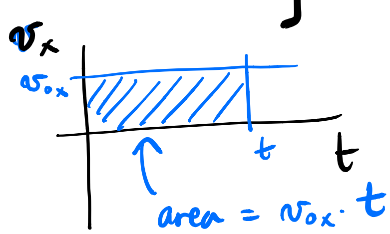


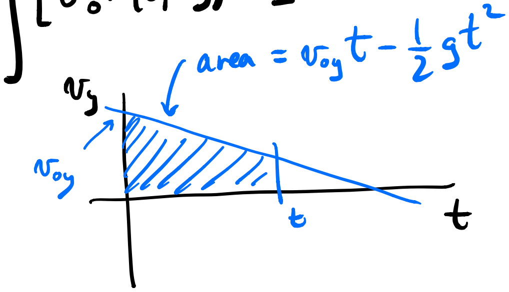
$$(*) = \vec{v}_0 + (0,-gt) = \vec{v}_0 + (0,-g)\cdot t$$

or equivalently $\boxed{v_x = v_{0x}}$ $\boxed{v_y = v_{0y} - gt}$

- Next step: get $\vec{x}$

$$\vec{x} = \vec{x}_0 + \int \vec{v}\,dt = \vec{x}_0 + \int \left[\vec{v}_0 + (0,-g)\cdot t\right] dt = (*)$$





$$(*) = \vec{x}_0 + \vec{v}_0 t - \frac{1}{2}(0,g)t^2$$

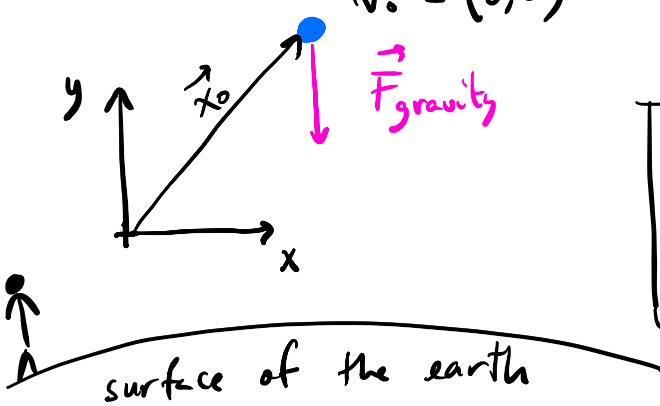
or equivalently $\boxed{x = x_0 + v_{0x} t}$ $\boxed{y = y_0 + v_{0y} t - \frac{1}{2} g t^2}$

So the kinematic equations of a projectile

$$\boxed{\begin{aligned} \vec{a} &= (0,-g) \\ \vec{v} &= \vec{v}_0 + (0,-g)\cdot t \\ \vec{x} &= \vec{x}_0 + \vec{v}_0 t - \frac{1}{2}(0,g)t^2 \end{aligned}}$$

- Free fall motion (= projectile motion with $\vec{v}_0 = (0,0)$)

$\vec{v}_0 = (0,0)$



$\Downarrow$

equations become

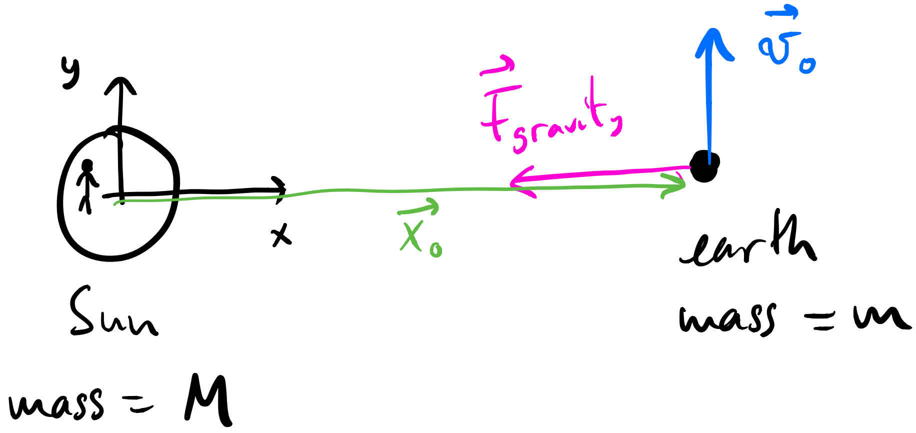
$$\boxed{\begin{aligned} \vec{a} &= (0,-g) \\ \vec{v} &= (0,-g)\cdot t \\ \vec{x} &= \vec{x}_0 - \frac{1}{2}(0,g)t^2 \end{aligned}}$$

Motion only in y-direction so:

$$\boxed{\begin{aligned} a_y &= -g \\ v_y &= -gt \qquad y = y_0 - \frac{1}{2} g t^2 \end{aligned}}$$

- Brian Cox vacuum video (Youtube)

- Earth's orbital motion.



we need to solve the 2nd law

$$m\vec{a} = \vec{F}_{gravity}$$

$$\vec{F}_{gravity} = -G\frac{M \cdot m}{|\vec{s}|^3}\vec{s} = -G\frac{M \cdot m}{|\vec{x}|}\vec{x}$$

$$\vec{F} \rightarrow \vec{a} \longrightarrow \vec{v} \longrightarrow \vec{x}$$ doesn't work here!

$$\begin{tikzcd} \vec{F}(\vec{x}) \arrow[r] & \vec{a} \arrow[d] \\ \vec{x} \arrow[u] & \vec{v} \arrow[l] \end{tikzcd}$$

when this happens we need to solve a system of <u>differential equations</u>.

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solving it EarthOrbit.nb