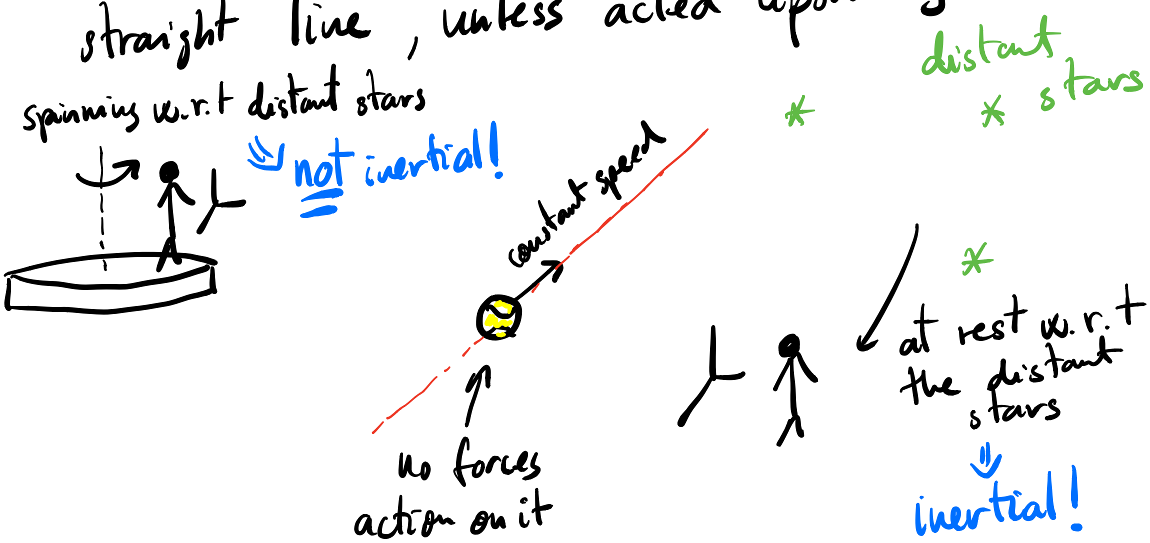
Lecture 5: Mathematical tools and their use in Newton's theory.

- Recap from previous lecture:

There are 3 fundamental laws in Newton's theory. First one is: due to Galileo

- <u>1st Newton law ("law of inertia"):</u>

<u>For an inertial observer</u> (or inertial frame of reference) an object either remains at rest or continues to move at constant speed in a straight line, unless acted upon by a force.



- Mathematical tools:

  - In physical science, in particular in Newton's theory, we use symbols to represent physical quantities.

    For quantities that have units of length, time, mass (or combinations) we use latin letters.

    e.g.

    x : coordinate in the x-direction

    m : mass of the object

    t : time coordinate

    $\vec{v}$: velocity of the object

    $\vdots$

    etc

Angles are represented by greek latters

$\alpha, \beta, \gamma, \delta, \ldots$

- **Equation**: statement that two things are equal. We will use them to describe relationships between physical quantities.

  Comment: the units should also be equal.

Two types of quantities: scalar and vector.

- **Scalar quantity**: a quantity represented by just one number.

  e.g. m (mass), t (time), E (energy), $|\vec{v}|$ (speed), ...

- **Vector quantities**: a quantity that is described by more than one number (they have "direction"). Each number we call a component of the vector in each direction of the frame of reference.

In Newton's theory the quantities that characterize the "<u>state of motion</u>" of an object are the vector quantities:

- position vector $\vec{x}$
- velocity vector $\vec{v}$
- acceleration vector $\vec{a}$

* Vectors will alway be represented with an arrow on top of the letter. We write their components inside a parenthesis separated by comas

y-component of $\vec{v}$

e.g $\vec{v} = (v_x, v_y, v_z)$ z-component of $\vec{v}$.

x-component of $\vec{v}$

e.g $\vec{a} = (a_x, a_y, a_z)$

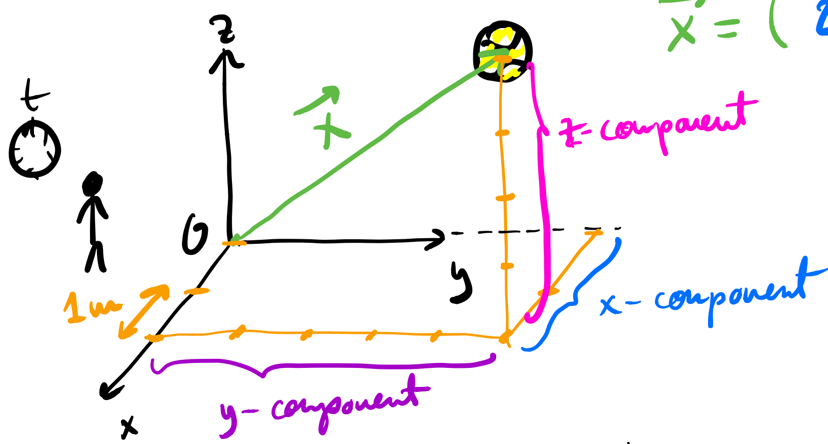
e.g $\vec{x} = (x, y, z)$

- <u>Position vector</u>

Symbol: $\vec{x} = (x, y, z)$

Given a frame of ref. (FOR), the components of the position vector are the spatial coordinates of the object w.r.t this FOR.

$\vec{x} = (2, 5, 4)$ m



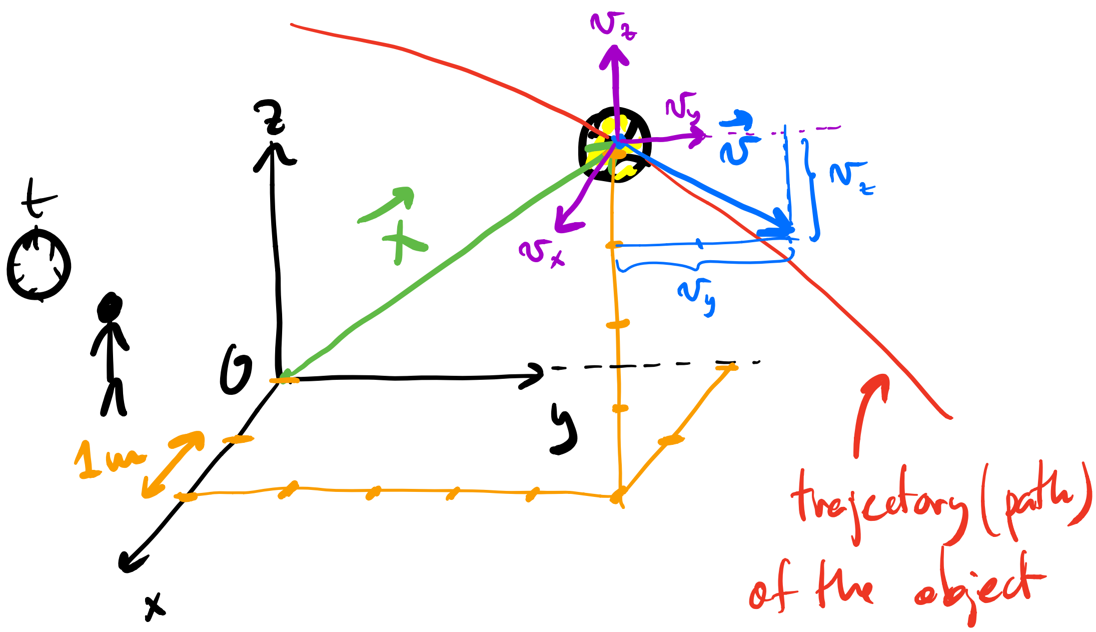
Note: when we draw the arrow corresponding to $\vec{x}$, it starts at the origin of the FOR and ends (tip of the arrow) at the position of the object.

- Velocity vector

Symbol: $\vec{v} = (v_x, v_y, v_z)$

Given a FOR, the components of the velocity vector correspond to the "speed in each direction" of the FOR.
It is also the instantaneous rate of change of the position in time

$$\vec{v} = (0, 2, -1)\ \frac{m}{s}$$

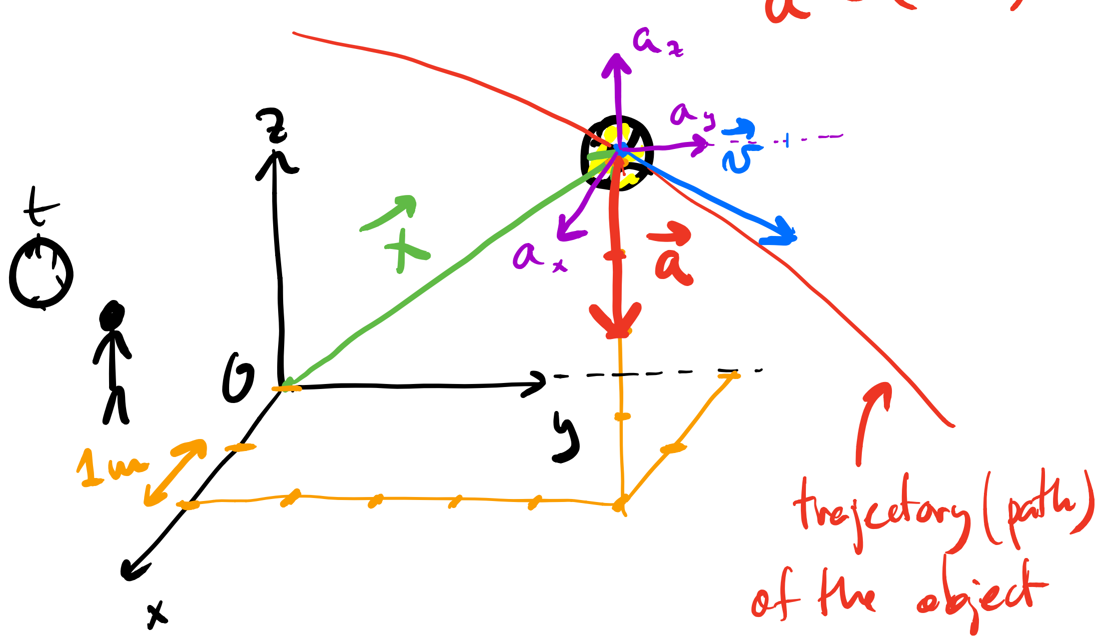


- Acceleration vector

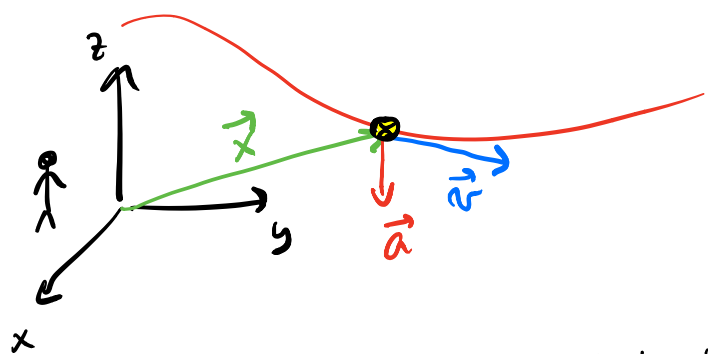
Symbol: $\vec{a} = (a_x, a_y, a_z)$

Given a FOR the components of the acceleration vector correspond to the "acceleration in each direction" of the FOR. It is also the instantaneous rate of change of the velocity in time.

$\vec{a} = (0, 0, -2) \frac{m}{s^2}$



$\vec{x}, \vec{v}, \vec{a}$ characterize the motion.



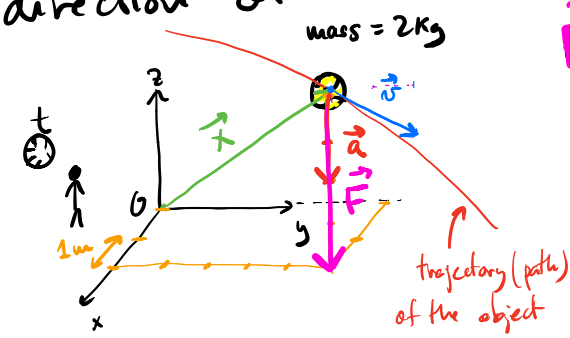
In order to predict what these are we need to be given the force/s acting on the object.

- **Force vector**

Symbol: $\vec{F} = (F_x, F_y, F_z)$

Given a FOR, the components of the force correspond to the "force in each direction of the FOR".

$\vec{F} = (0, 0, -4) \frac{Kg\,m}{s^2}$

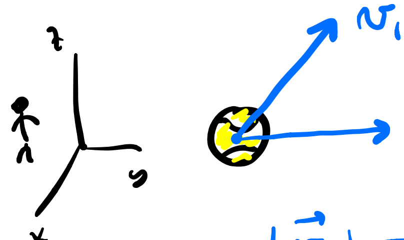


- **Magnitude of a vector**: It is the result of adding the square of each component of the vector and taking the square root of that. The magnitude is the "length of the arrow". It is an scalar quantity.

Example:

- **Speed**: Magnitude of the velocity vector
  Symbol $|\vec{v}|$, $|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$

Important: the speed $|\vec{v}|$ is <u>not</u> the same as the velocity $\vec{v} = (v_x, v_y, v_z)$



$\vec{v}_1 = (0, 3, 4)$ m/s

$\vec{v}_2 = (0, 5, 0)$ m/s

$|\vec{v}_1| = \sqrt{3^2 + 4^2} = 5$ m/s

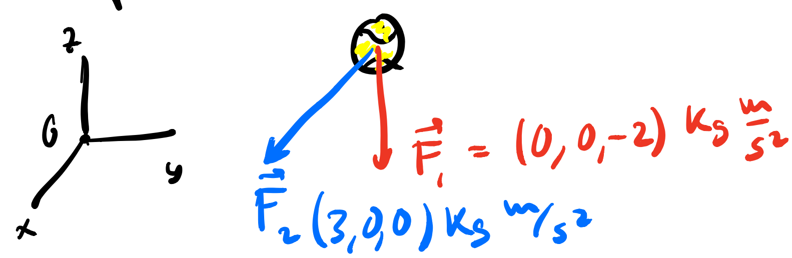
$|\vec{v}_2| = \sqrt{5^2} = 5$ m/

Comment: velocity $\vec{v}$ encodes more information (e.g direction) than speed $|\vec{v}|$.

- Operations with vectors

- **Addition of vectors:** two or more vectors expressed w.r.t the same FOR can be added by adding their components. The result is another vector. Vectors can only be added if they have the same units

Example



$$\vec{F}_{tot} = \vec{F}_1 + \vec{F}_2 = (0, 0, -2) + (3, 0, 0)$$

$$= (3, 0, -2)\ \frac{kg\,m}{s^2}$$

- **Scalar times a vector:** we multiply each of the components of the vector by the scalar. The result is another vector. The resulting units are the product of the original units

Example: $\vec{F} = m\vec{a}$ (Newton's 2nd law)

$m = 5\,Kg$

$\vec{a} = (2, 0, 3)\ m/s^2$

$\vec{F} = m \cdot \vec{a} = 5 \cdot (2, 0, 3) = (10, 0, 15)\ Kg\frac{m}{s^2}$

- <u>Function</u>: Represents a relationship between some "input" quantities (independent variables) and an "output" quantity (dependent variable).
  - If the output is an scalar quantity we call it a <u>scalar function</u>

e.g $m = 5t + 3t^2 + \sqrt{t}$

sometimes we write $m(t) = 5t + 3t^2 + \sqrt{t}$ to specify which is the input quantity.

- If the output quantity is a vector we call it a vector function

$$\text{e.g}\quad \underbrace{\vec{X}}_{\text{output}} = (x, y, z) = (0, \underbrace{5t, -2t^2+t}_{\text{input}})$$

or $\vec{x}(t) = (0, 5t, -2t^2 + t)$

- Trajectory of an object corresponds to the function $\vec{x}(t)$.

The goal of Newton's theory will be finding what $\vec{x}(t)$ is for the object under consideration.