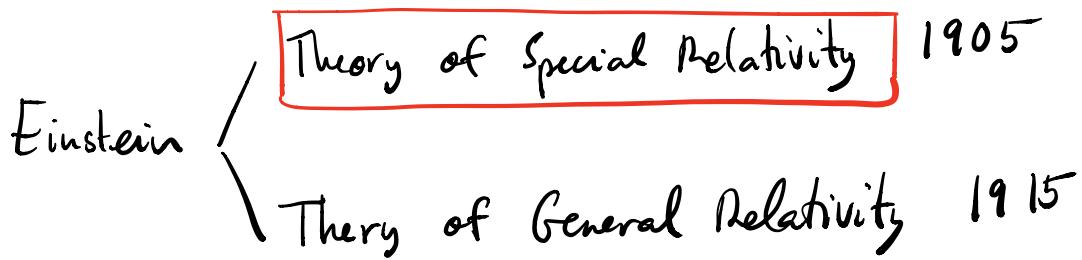
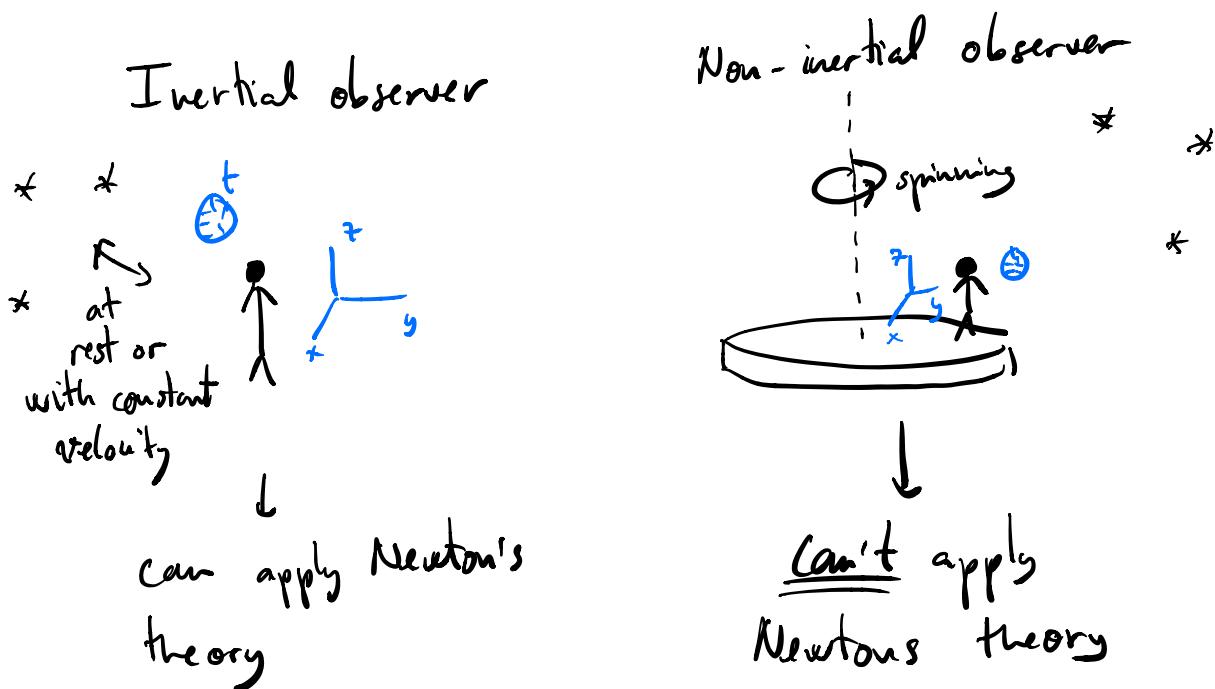


Lecture 11: Einstein's theory of Special Relativity



- Corrects Newton's theory of Classical Mechanics, which was the scientific paradigm until Einstein came along.

Recall that Newton's theory applied only to inertial observers / frames of reference.



- Einstein's goal: Find the laws of nature that can be used by all observers / F.O.R (not just inertial)

e.g. $\vec{F} = m\vec{a}$ or $\vec{F}_{\text{gravity}} = -G \frac{Mm}{|\vec{s}|^3} \vec{s}$
 can only be used with success by inertial observers.

e.g. $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^2} T_{\mu\nu}$ this formula that describes gravity in Einstein's theory of General Relativity can be applied by all observers / F.O.R.

However, before arriving to this last formula, Einstein had to correct Newton's theory even in the absence of gravity.

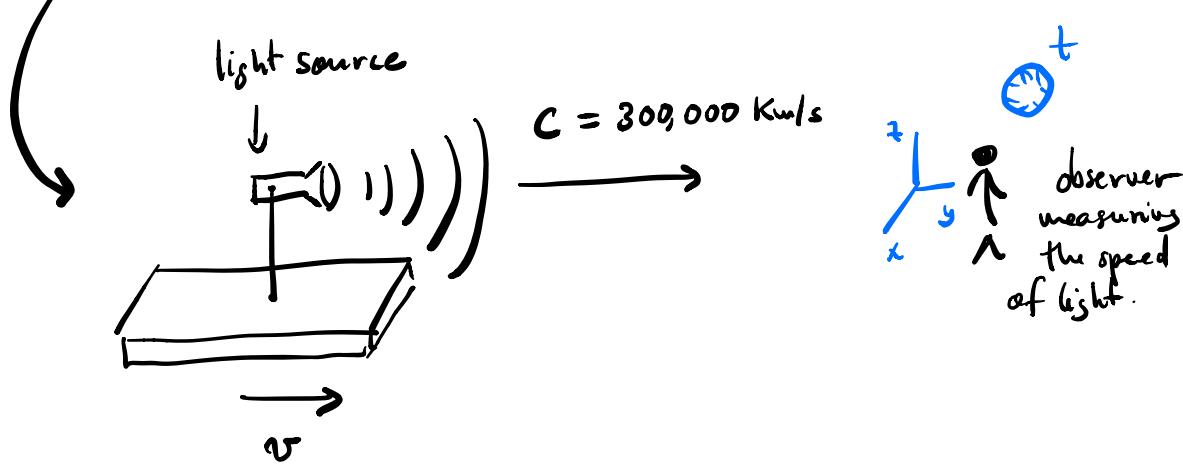


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Theory of Special Relativity (Einstein 1905)

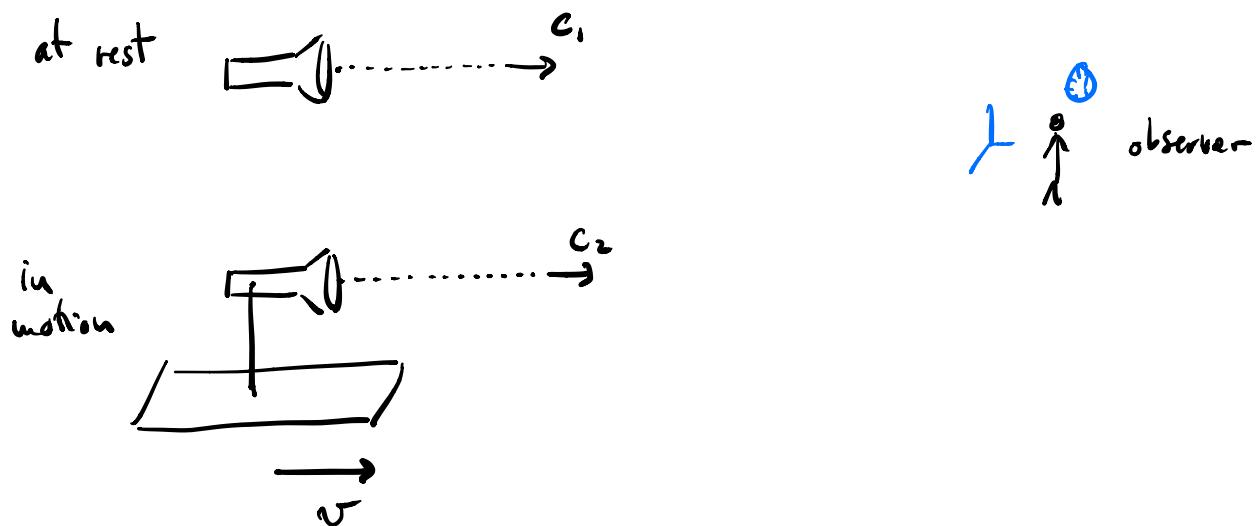
① Principle of relativity: the laws of physics are the same for all inertial frames of reference.

② Principle of invariance of the speed of light: the speed of light is independent of the state of motion of the source emitting it.



Symbol for the speed of light: $c \approx 3 \times 10^8 \text{ m/s}$ in vacuum.

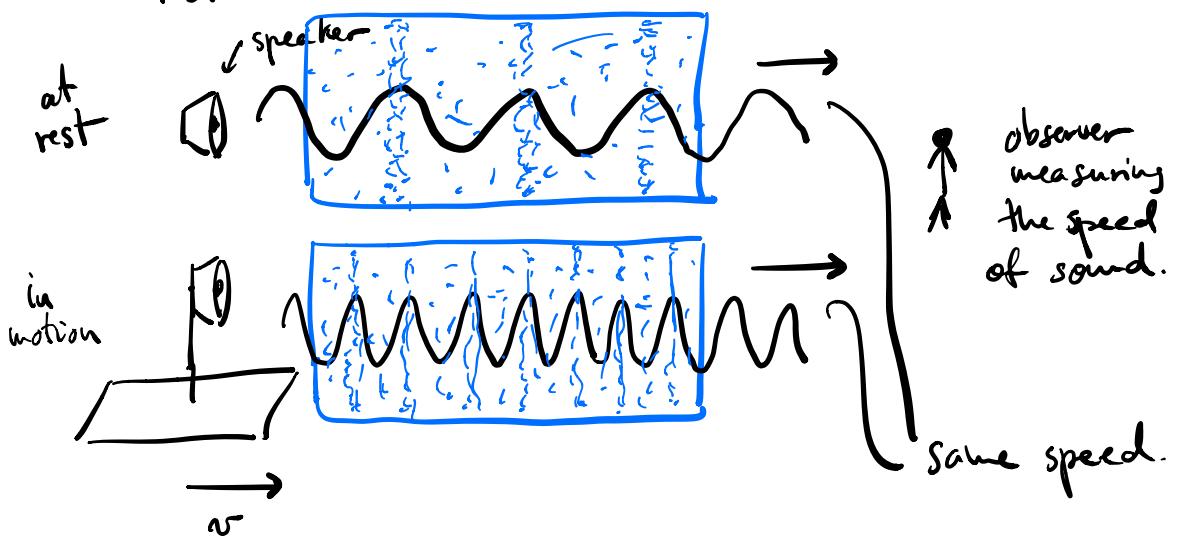
- If light was made of "usual" particles we would expect its speed to depend on the motion of the source emitting it



If light was a particle we expect $c_2 > c_1$. However this doesn't happen and we observe $c_2 = c_1$. Therefore light doesn't seem to behave like a particle.

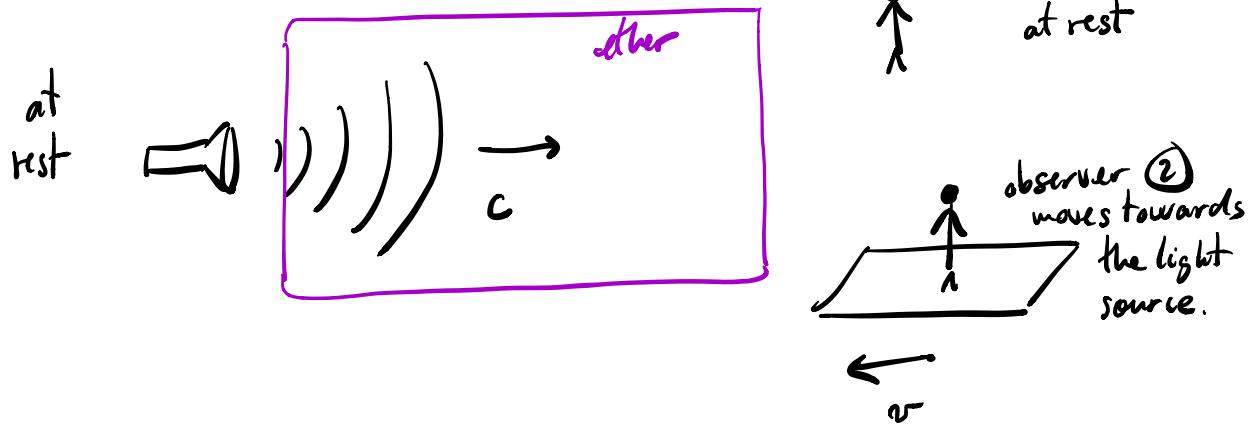
\Rightarrow It looks as if light is behaving like a wave!

For a sound we have



- This made physicist think that light was a wave.
- But waves need a medium (e.g. air for sound) to propagate.
- Physicists thought that there should be a medium that allows "light waves" to propagate just as sound waves propagate in air.
⇒ they called that medium the "ether".

Next experiment we can do

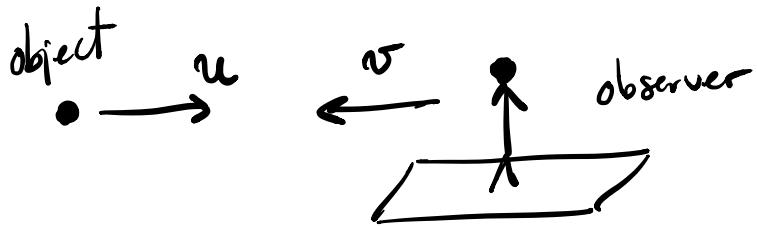


- If light behaved like sound (like a wave), then observer ② would measure an speed of light greater than observer ①.
- However this does not happen to light, as evidenced by experiment (Michelson-Morley experiment)
- This result looks inconsistent with the first postulate but it is not. We are making an assumption that it is not correct. We assumed that the velocity of light with

respect to observer ② is the simple addition $c + v$.

- Einstein claims that this way of adding velocities is incorrect!

He provides a different formula:

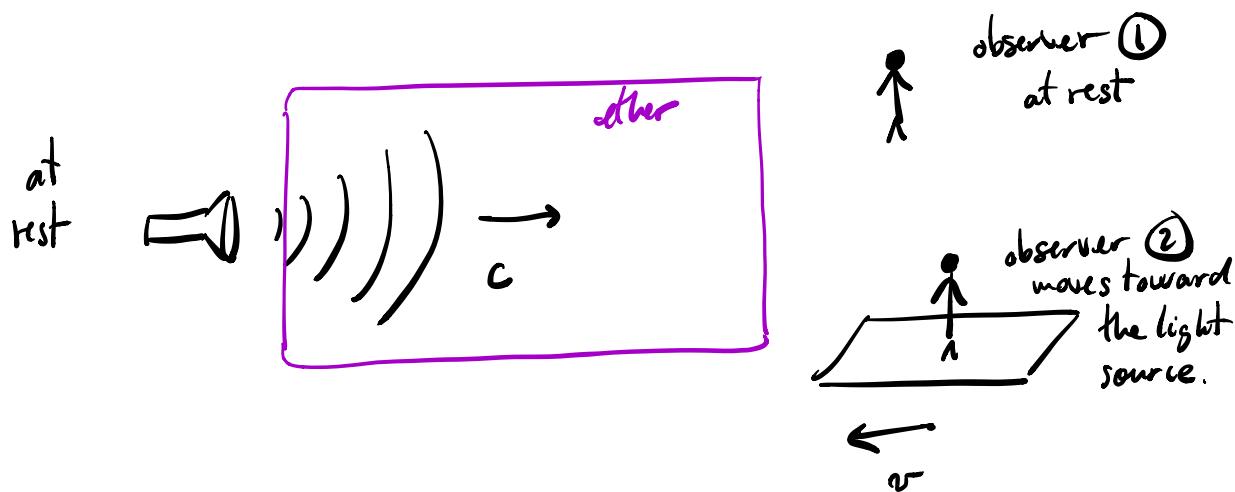


Let's call u' the velocity of the object with respect to the observer (perceived by the observer). Then $u' = u + v$ is incorrect.

The correct formula is

$$u' = \frac{u + v}{1 + \frac{u \cdot v}{c^2}}$$

Applying this formula to the last experiment:



Observer ①:

$$v = 0 \quad u' = \frac{u+v}{1+\frac{u \cdot v}{c^2}} = \frac{c+0}{1+\frac{0 \cdot 0}{c^2}} = \frac{c}{1} = c$$

$$u = c \quad u = c$$

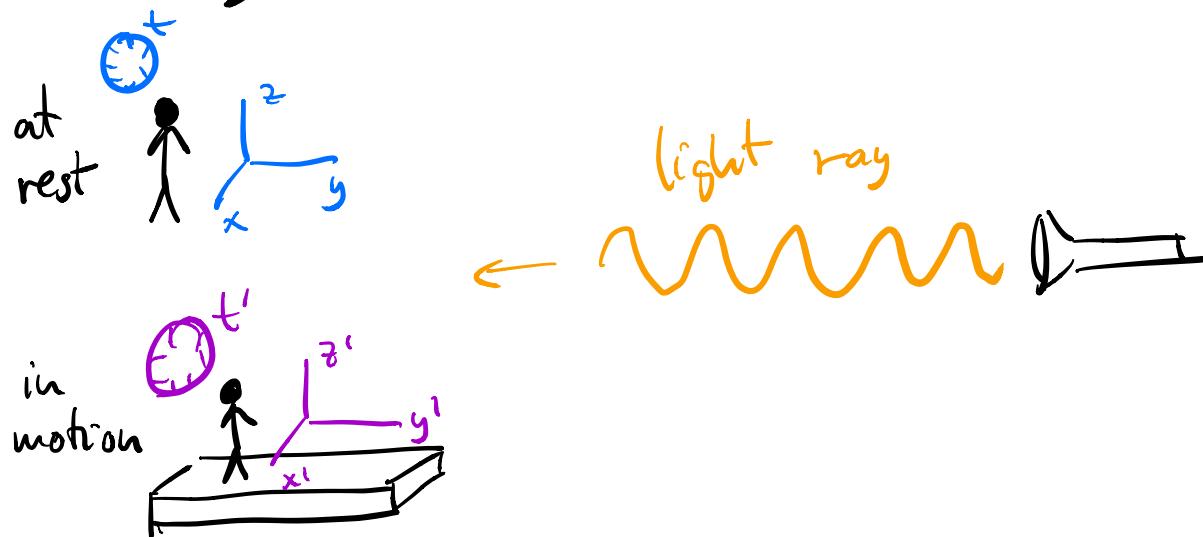
Observer ②:

$$v \neq 0. \quad u' = \frac{u+v}{1+\frac{u \cdot v}{c^2}} = \frac{c+v}{1+\frac{c \cdot v}{c^2}} = \frac{c+v}{1+\frac{v}{c}}$$

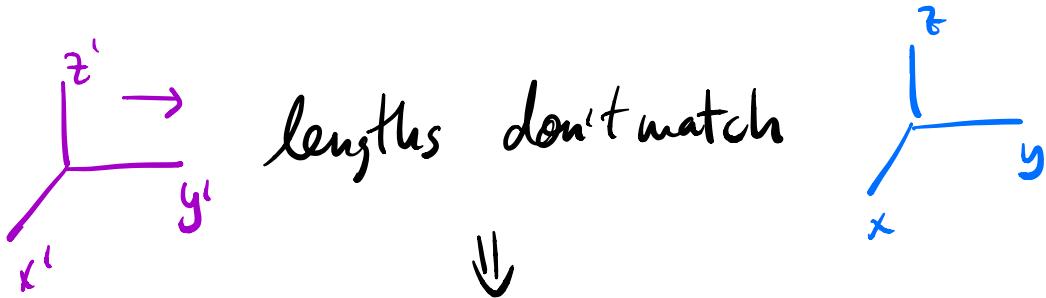
$$u = c \quad = \frac{c(c+v)}{c(1+\frac{v}{c})} = \frac{c(c+v)}{\cancel{c+v}} = c$$

We always get c no matter how the observer moves!

Where does Einstein's formula for adding velocities comes from?



t' runs slower than t
↓
"time dilation"



"length contraction"

These two effects combine such that both observers measure the same speed of light.

Einstein's theory of Special Relativity has proven to work to very high accuracy until this day. For example this theory is needed at particle colliders where particles travel at speeds close to the speed of light.

e.g LHC

