Lecture 12: Einstein's theory of General Relativity.

Einstein's
- Theory of Special Relativity 1905
- $\boxed{\text{Theory of General Relativity}}$ 1915

Note: Gravitational waves are a prediction of the theory of General Relativity. Predicted by Einstein in 1916.

Recap of Special Relativity 1905:

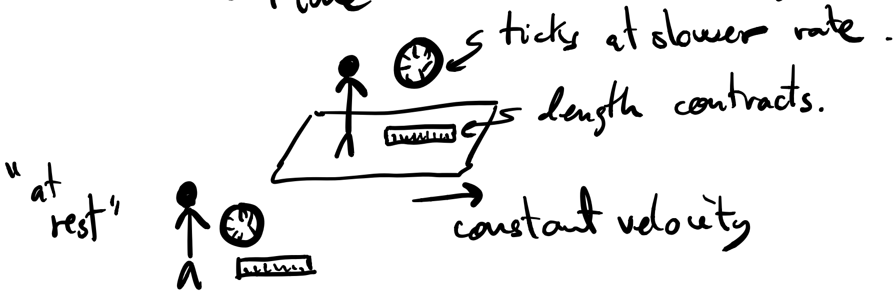
Postulates:

① Laws of physics are the same for all <u>inertial</u> observers.

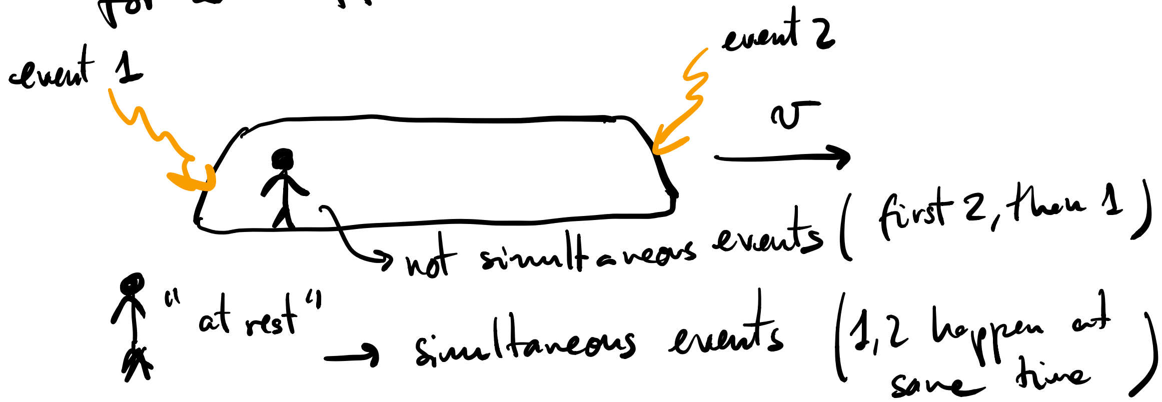
② Speed of light is constant and does <u>not</u> depend on the state of motion of the source.

Consequences of accepting these two postulates:

- Measurement of space ant time by observers moving at different constant velocities differ when compared.
  - Time dilates and lengths contract.



- Two events that are simultaneous for an observer "at rest", are **not** simultaneous for a different observer moving at some speed.



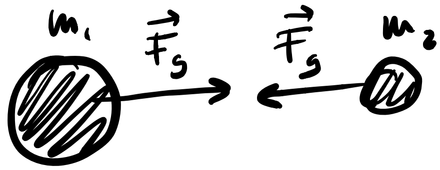
Recall that Einstein's goal was to come up with equations that <u>any observer</u> irrespective of their state of motion can apply.

In other words, he wanted to <u>upgrade</u> the principle of relativity (postulate ①) to a **principle of general relativity**:

"The laws of physics are the same for all observers / frames of reference."

Recall Newton's theory for gravity:

For Newton gravity is a force:



$$\vec{F}_g = -G \frac{m_1 m_2}{|\vec{s}|^3} \vec{s}$$

To predict how they move we use the 2nd law:

$$m_1 \vec{a}_1 = \vec{F}_{g,1} \quad , \quad m_2 \vec{a}_2 = \vec{F}_{g,2}$$

$\downarrow$ calculus $\qquad\qquad$ $\downarrow$ calculus

$\vec{v}_1 = \dots$ $\qquad\qquad$ $\vec{v}_2 = \dots$

$\vec{x}_1 = \dots$ $\qquad\qquad$ $\vec{x}_2 = \dots$

However, the above equations only work for inertial observers / F.O.R.

Einstein wasn't happy about that!

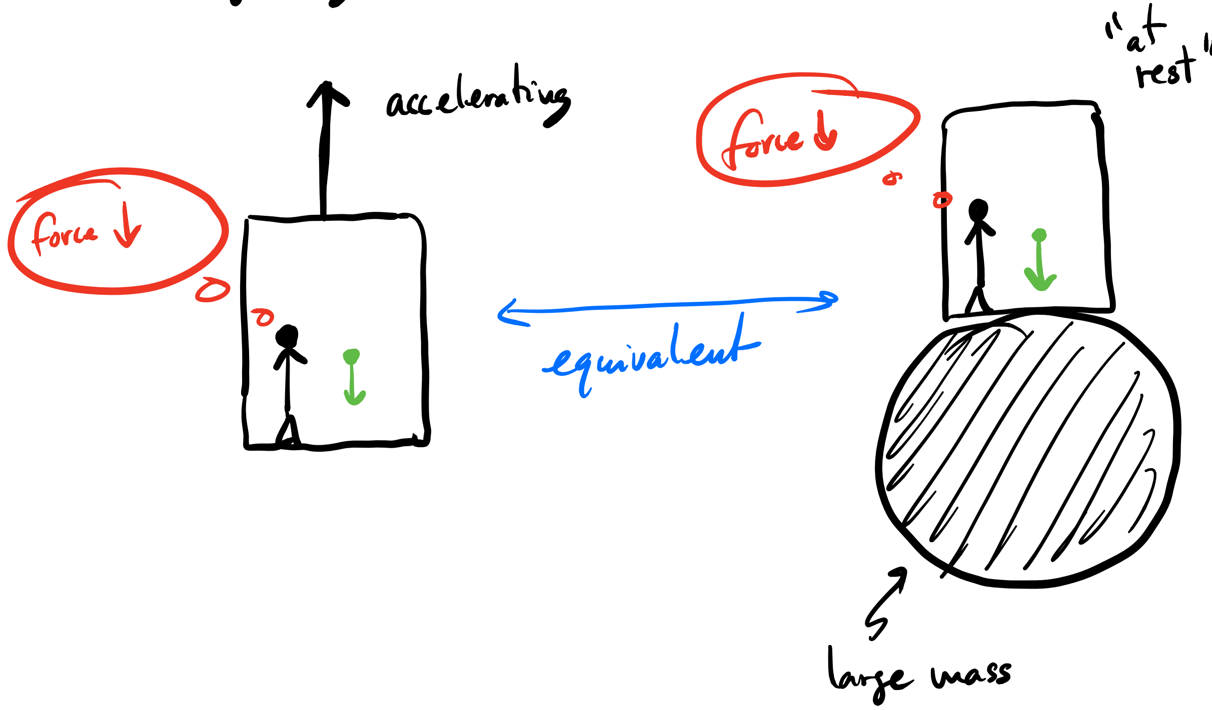
$\Rightarrow$ Einstein's theory of General Relativity solves that.

Einstein's theory of General Relativity.

Starting point / idea:

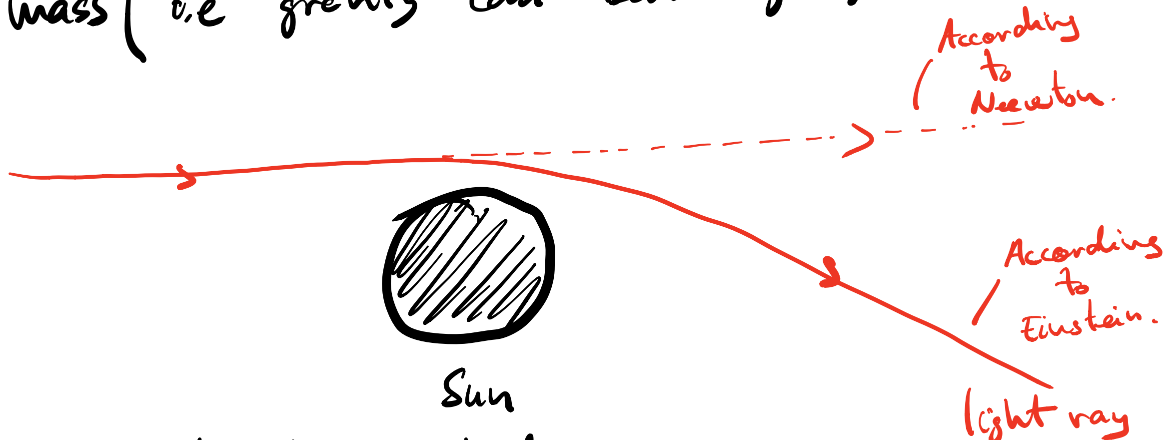
Einstein's equivalence principle:

"It is impossible to distinguish between being a non-inertial observer (accelerated) and being an inertial observer in the presence of gravity".

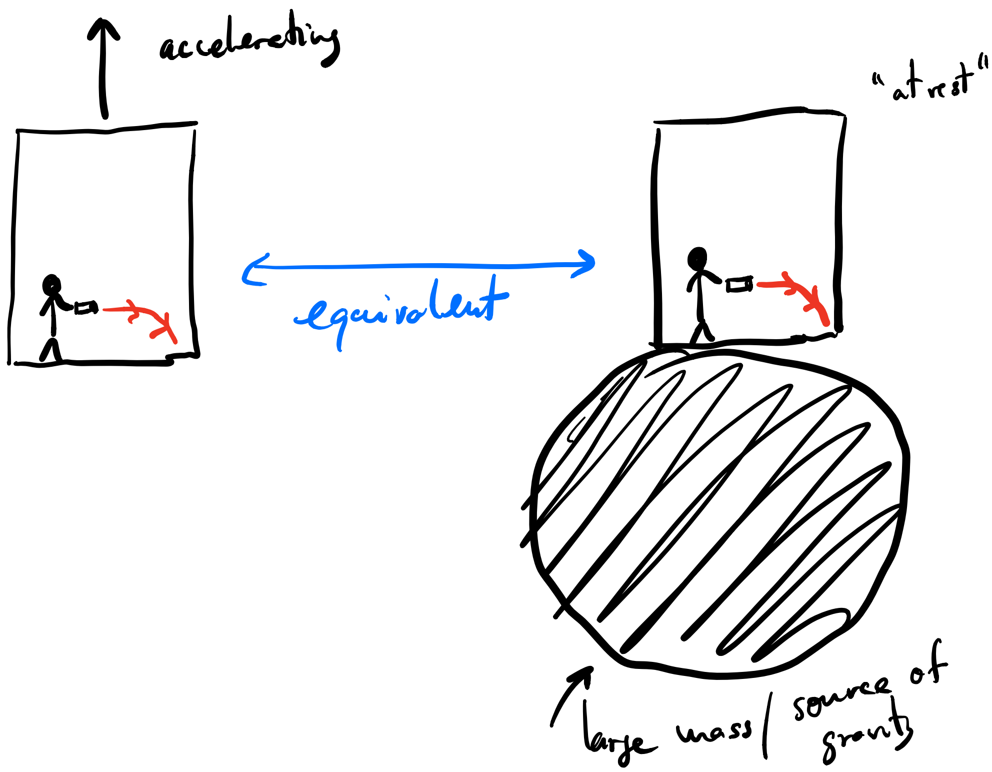


From the equivalence principle one can predict:

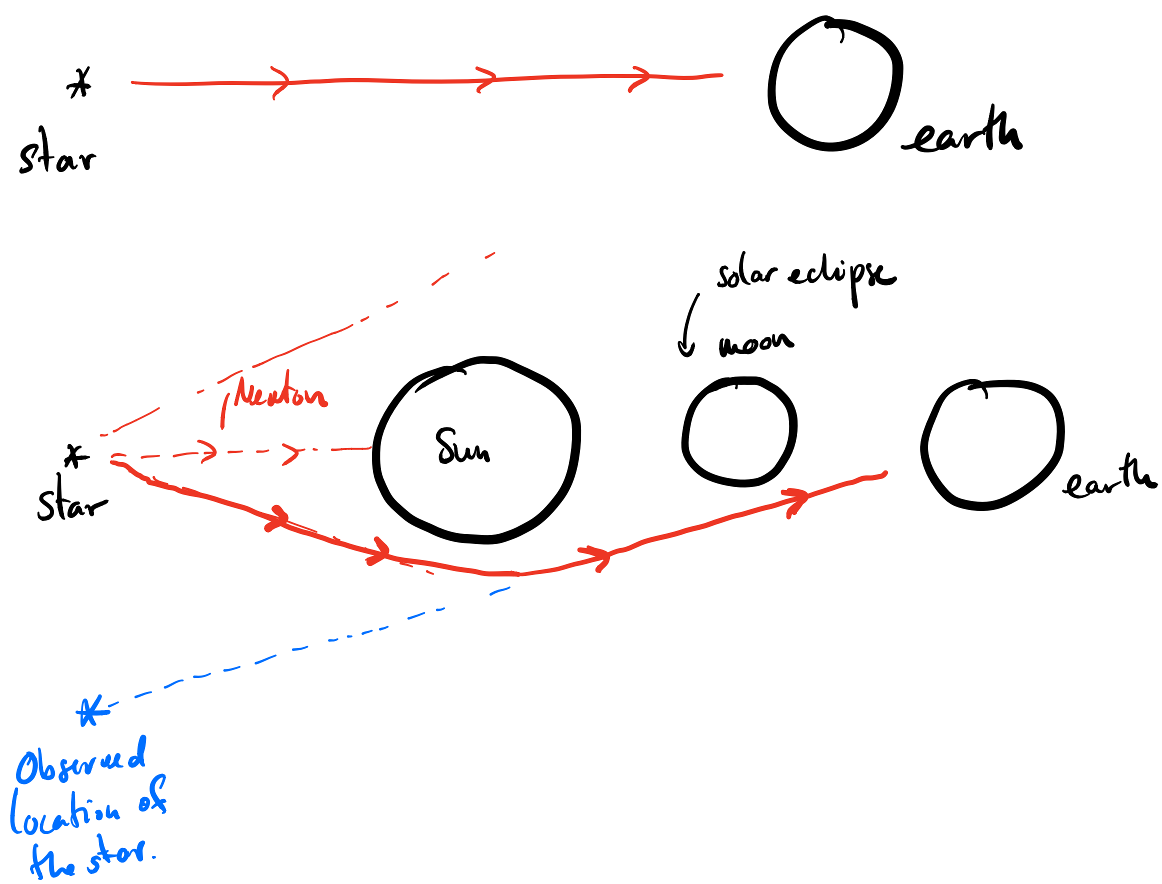
- Light bending when passing near a large mass (i.e gravity can bend light)



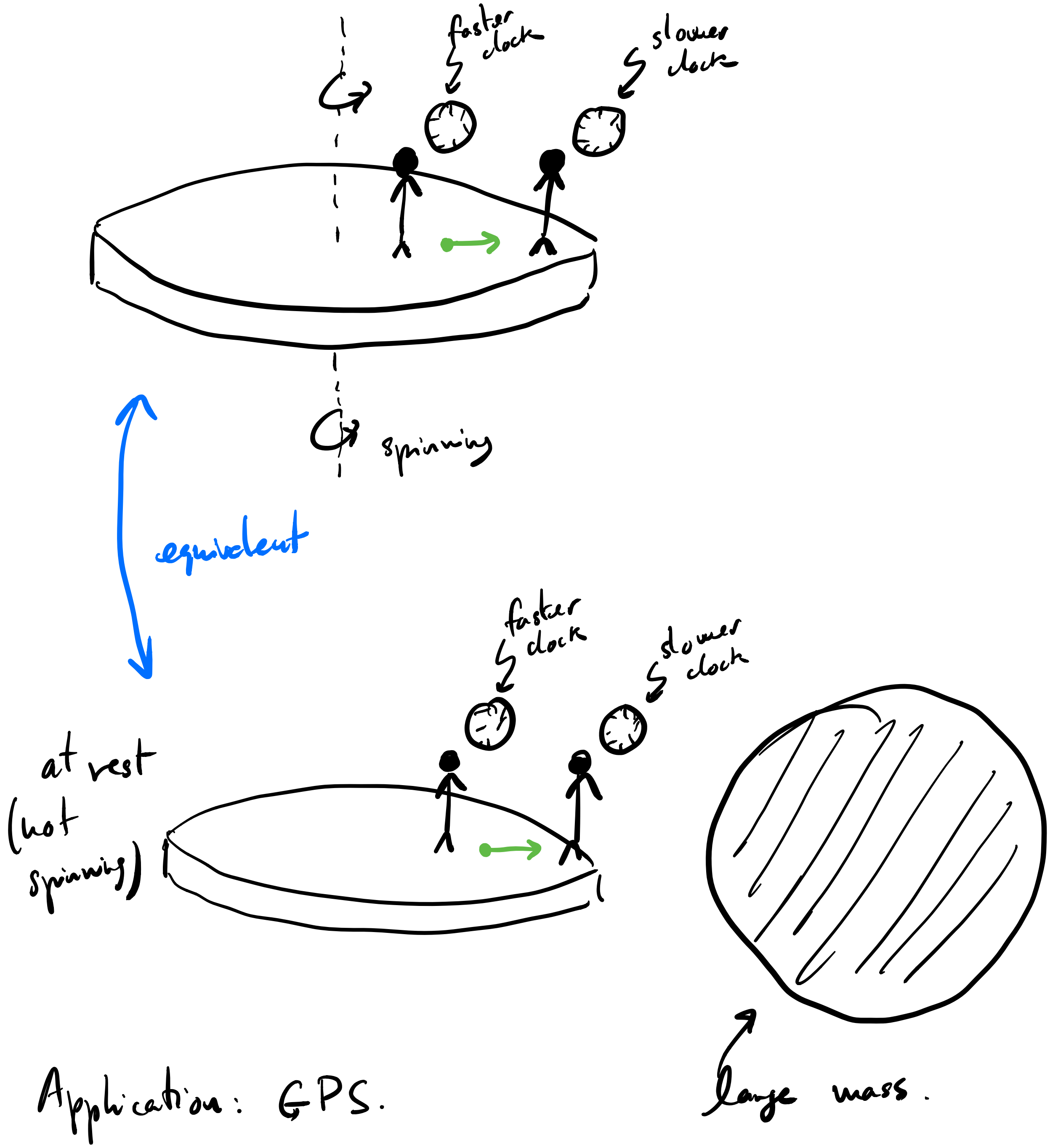
Proof that light should bend:



This prediction was tested in 1919 during a solar eclipse.



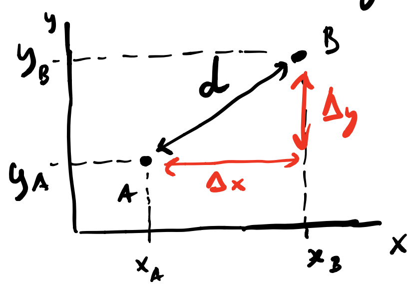
- Time dilation by gravity.



Application: GPS.

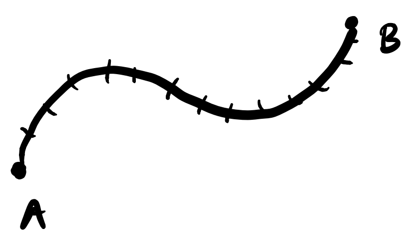
- General Relativity as a geometric theory (modern view).

- Recall Eucliden geometry

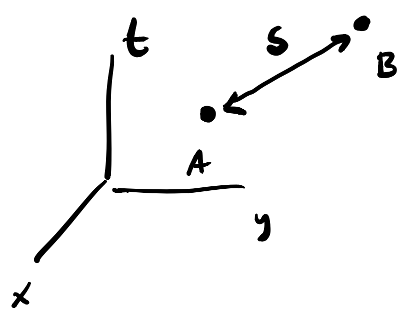


$$d^2 = (x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_B)^2$$

$$= \Delta x^2 + \Delta y^2 + \Delta z^2$$



- Special relativity adds time into geometry.



$$\boxed{s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c\Delta t^2}$$

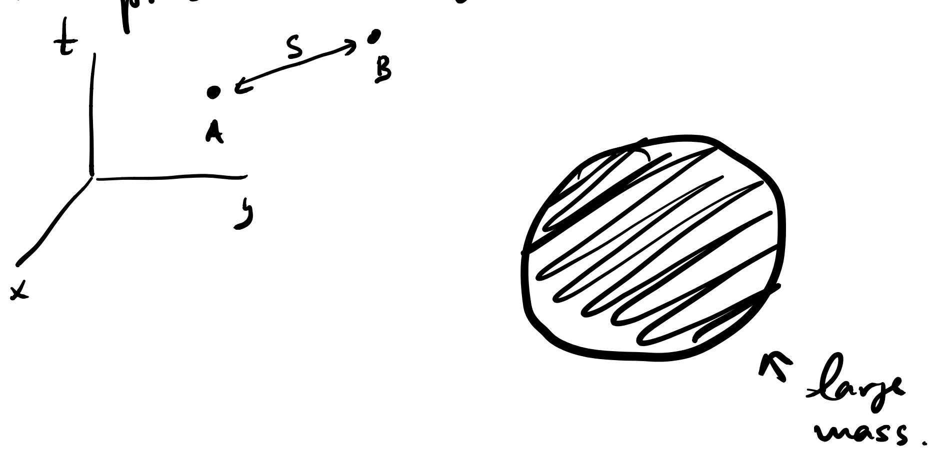
↑

Space-time distance between two event.

In Special Relativity all inertial observers agree in the space-time distance between any pair of events.

Theory of General Relativity adds gravity into geometry.

In the presence of gravity



$$s^2 = g_{xx}\,\Delta x^2 + g_{yy}\,\Delta y^2 + g_{zz}\,\Delta z^2 - g_{tt}\, c\Delta t^2$$

These numbers are affected by gravity!!

These numbers are encoded in what we call the "metric" of spacetime.

$$g_{\mu\nu} = \begin{pmatrix} g_{tt} & g_{tx} & g_{ty} & g_{tz} \\ g_{xt} & g_{xx} & g_{xy} & g_{xz} \\ g_{yt} & g_{yx} & g_{zz} & g_{yt} \\ g_{zt} & g_{zx} & g_{zy} & g_{tt} \end{pmatrix} \leftarrow 16 \text{ numbers.}$$

So the goal becomes finding what $g_{\mu\nu}$ is at different points in space time.

Einstein's master equation:

$$\underbrace{R_{\mu\nu} - \tfrac{1}{2} R\, g_{\mu\nu}}_{\text{metric}} = \overbrace{\frac{8\pi G}{c^2}}^{\text{constant}} \underbrace{T_{\mu\nu}}_{\substack{\text{content of} \\ \text{mass and energy} \\ \text{of space time.}}}$$

The spacetime distance is the same no matter what observer / F.O.R we chose!

Postulate of General Relativity:

- Objects move following geodesics in spacetime.

shortest path between 2 points.