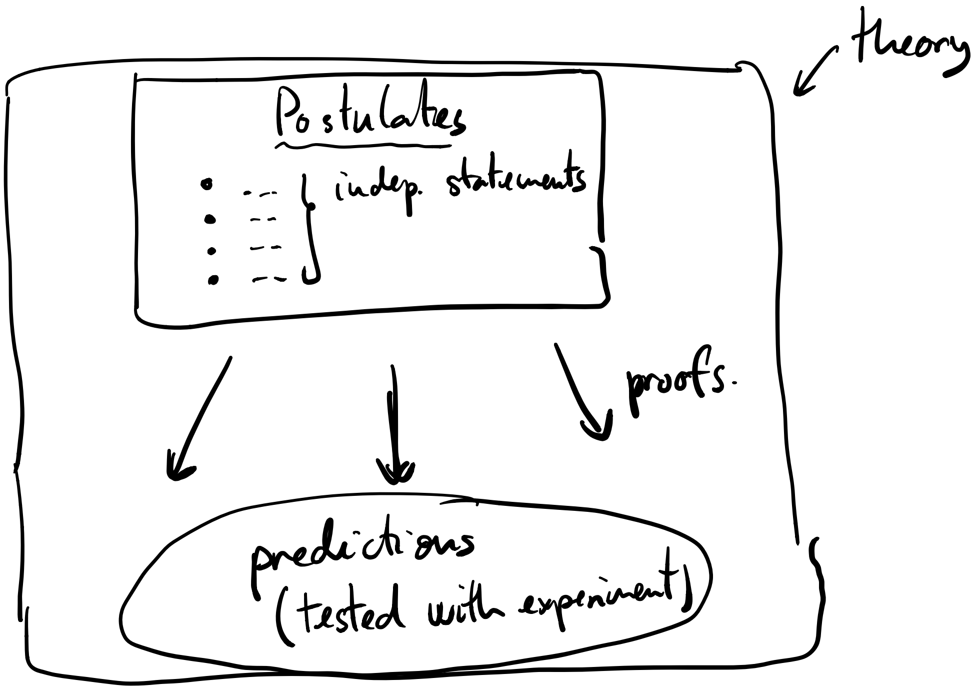
Lecture 2: Structure of a theory.

Example of a theory: Euclidean geometry.

- Theory: Set of fundamental laws (aka postulates) together wit a set of logical and mathematical rules allowing us to make predictions (aka theorems or propositions in math) about the behaviour of a physical system.
  - Postulates or fund. laws: can't be logically derived (or proved). They are the "building blocks" of the theory.
  - Proofs: method that makes use of logical and mathematical rules to deduce predictions (or theorems) about the physical system.
  - Predictions: They are the necessary consequence of the postulates.



- Example of a proof in the theory of arithmetics.

  - Postulate: An even number can always be written as an integer times 2.

    Proof

  - Prediction: Adding two even numbers gives us another even number.

even

integer

Proof: $a = x \cdot 2$

$$\underset{\text{even}}{b} = \underset{\text{integer}}{y} \cdot 2$$

Now,

$$a + b \underset{\text{logic}}{=} x \cdot 2 + y \cdot 2 \underset{\text{logic}}{=} (x + \underbrace{y}_{\text{integer}}) \cdot 2$$

$\Rightarrow$ <u>a+b is an even number</u>

Example of a theory:

- $\boxed{\text{Euclidean geometry}}$

Author: Euclid (greek)

13 books: "Elements"

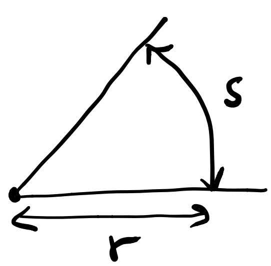
Year: 300 B.C

System that it describes: static (not in motion) physical objects.

It describes their geometric properties (relations b/w lengths, angles, etc)

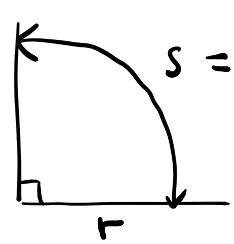
Basic quantity : lenght (and angles)

- <u>Angle</u>: defined as the ratio of the length of a circular arc to its radius. (When expressed as this ratio, the unit is the radian)



$$\text{angle} = \frac{s}{r} \text{ radians}$$

e.g



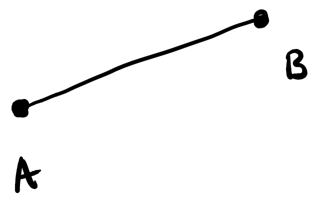
$$s = \frac{2\pi r}{4}$$

$$\text{angle} = \frac{s}{r} = \frac{\frac{2\pi \cancel{r}}{4}}{\cancel{r}} = \frac{\pi}{2}$$

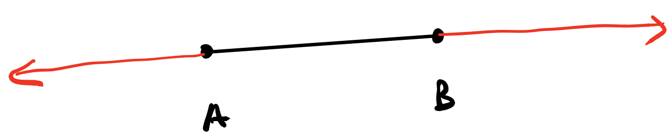
radians

Postulates of Euclidean geometry

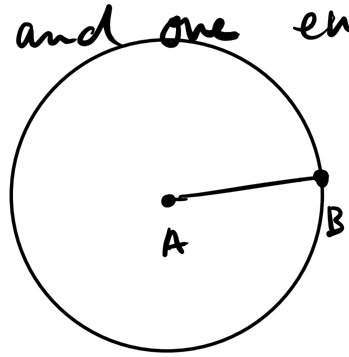
1) A straight line segment can be drawn joining any two points.



2) Any straight line segment can be extended indefinitely in a straight line.

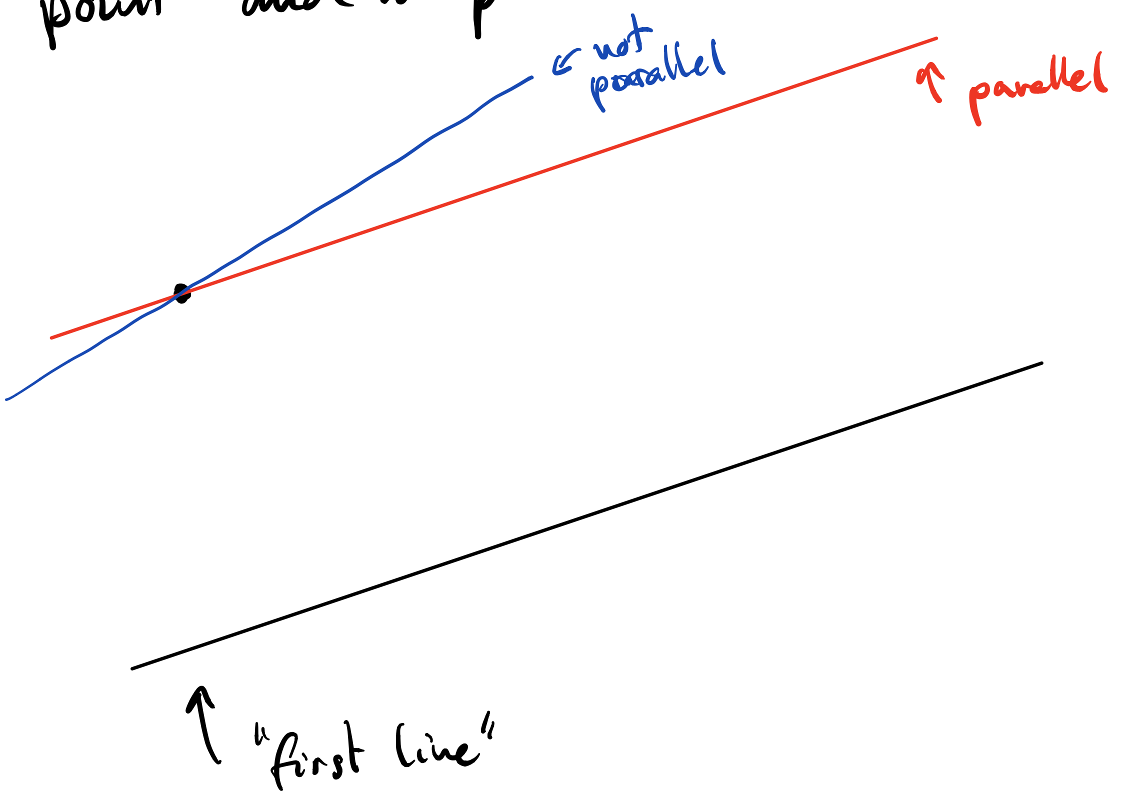


3) Given any straight line segment, a circle can be drawn having the segment as radius and one end point as center.



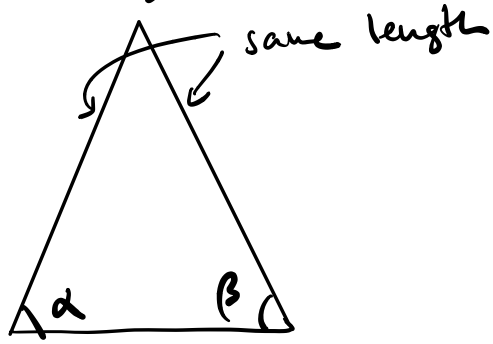
4) All right angles are equal to one another.

5) Given any straight line and a point not on it, there exist <u>one and only one</u> straight line which passes through that point and is parallel to the first line.



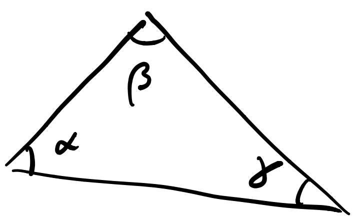
Given this set of postulates let's see some of the most important predictions that the theory makes.

- **Bridge of asses theorem**: In an isosceles triangle $\alpha = \beta$.

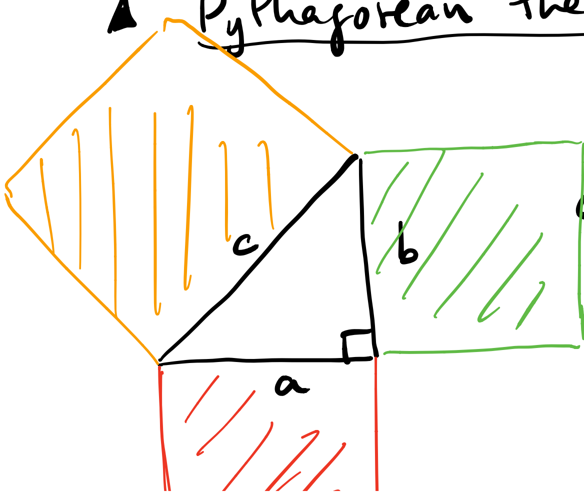


- **Triangle angle sum theorem**.

$$\alpha + \beta + \gamma = 180^\circ$$



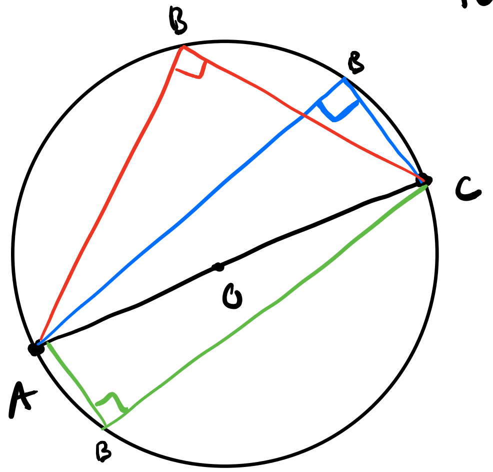
- **Pythagorean theorem**:



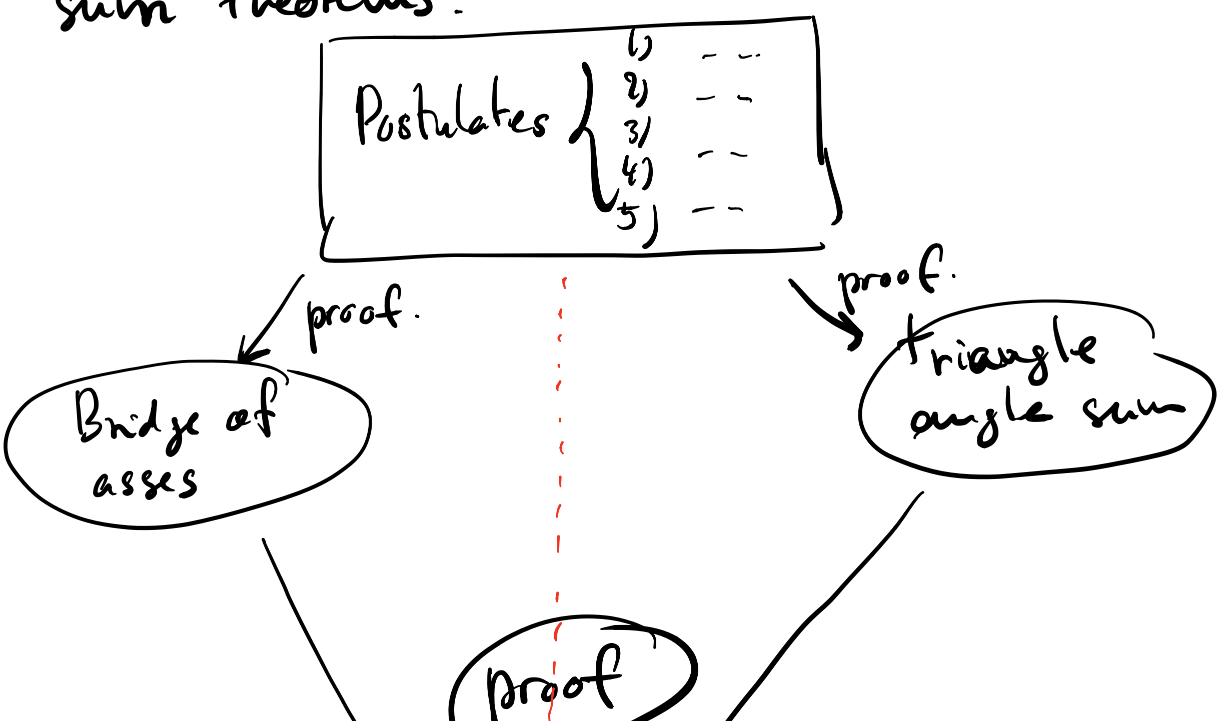
$$c^2 = a^2 + b^2$$

Thales' theorem: If AC is a diameter, then the angle at B is a right angle

↳ 90°



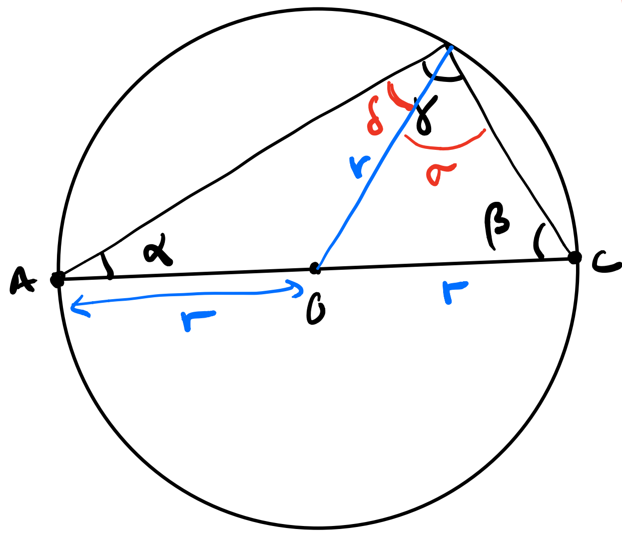
Exercise: Prove Thales' theorem using the bridge of asses and triangle angle sum theorems.



Thales' theorem

Proof:

Need to arrive to
$\gamma = 90^\circ$



* From the triangle angle sum theorem we can say

$$\alpha + \beta + \gamma = 180^\circ$$

* From the bridge of asses theorem we can say

$$\alpha = \delta$$
$$\beta = \sigma$$

$$\gamma = \delta + \sigma$$

Put info together $\begin{cases} \alpha + \beta + \gamma = 180^\circ \\ \alpha = \delta \\ \beta = \sigma \\ \gamma = \delta + \sigma \end{cases}$

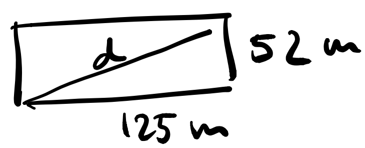
$$\underbrace{\gamma = \delta + \sigma = \alpha + \beta = 180^\circ - \gamma}$$

$$\gamma = 180^\circ - \gamma$$

"implies"

$$\Rightarrow \quad 2\gamma = 180^\circ$$

$$\Rightarrow \quad \boxed{\gamma = \frac{180^\circ}{2} = 90^\circ}$$



a) $d = \cancel{125^2 + 52^2}$

b) $\boxed{d = \sqrt{125^2 + 52^2}} \longleftrightarrow \boxed{d^2 = 52^2 + 125^2}$

$d = \sqrt{52^2 + 125^2}$

$\neq$

$\sqrt{52^2} + \sqrt{125^2}$

c) $d = \cancel{\sqrt{125 + 52}}$

d) $d = \cancel{\sqrt{125^2} + \sqrt{52^2}}$

e) $d = \cancel{\sqrt{(125 + 52)^2}}$