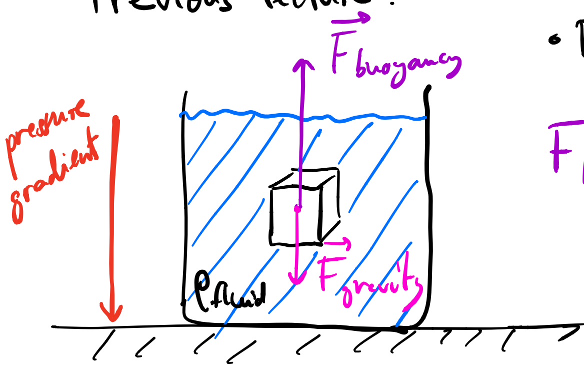
Lecture 10: Fluids: Bernoulli's principle

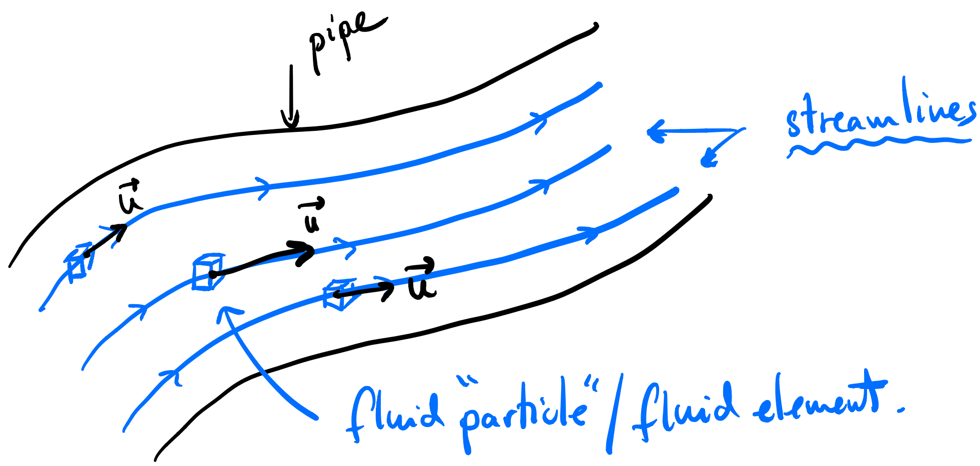
Previous lecture:



• Buoyancy principle

$$F_{buoyancy} = \rho_{fluid} \cdot V_{displaced} \cdot \underset{9.8\ m/s^2}{g}$$

Today we complicate things a bit more.



Quantities that describe a fluid:

- Density ρ ("rho") - Fluid velocity $\vec{u}$
- Pressure P
- Viscosity ν (greek letter "nu")

Applying Newton's 2nd law to the fluid particles we arrive at the equation that governs the motion of a fluid.

$\vec{F} = m\vec{a}$
for each fluid particle
$\longrightarrow$
$$\boxed{\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\vec{u} = -\vec{\nabla} p + \nu \nabla^2 \vec{u}}$$

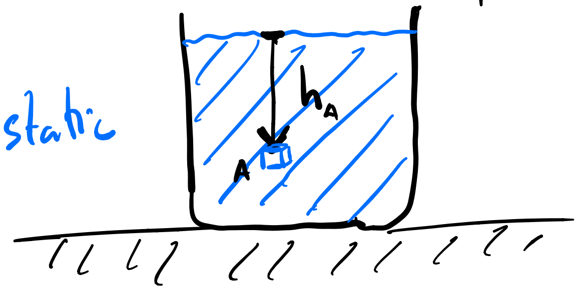
<u>Navier - Stokes equation</u> ~ 1822

- Navier-Stokes (NS) equation is very hard to solve.
  $\Rightarrow$ Most times we need computer simulations to do so.

- However, under special circumstances, simple relations can be found between some fluid quantities such as pressure, velocity, density, ...

Example:

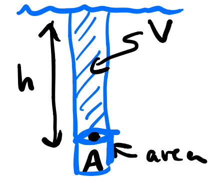
Consider an static fluid under the influence of gravity static



pressure $p_A = \dfrac{\text{weight}}{\text{area}} = \dfrac{mg}{\text{area}}$

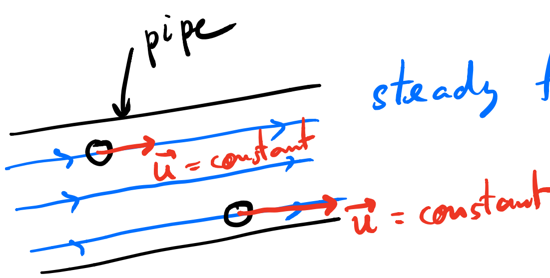
$$= \frac{\rho V g}{\text{area}} = \rho h g$$

$$\Rightarrow \boxed{p = \rho h g}$$



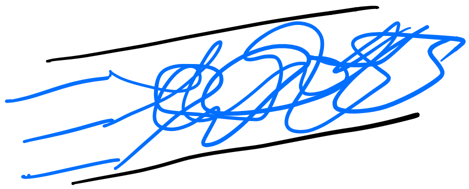
Fluids can be classified as follows:

- $\boxed{\text{Steady}}$: At a given point the velocity is constant.
- Turbulent: "chaotic" flow. The fluid changes speed and direction at a given point.



steady flow

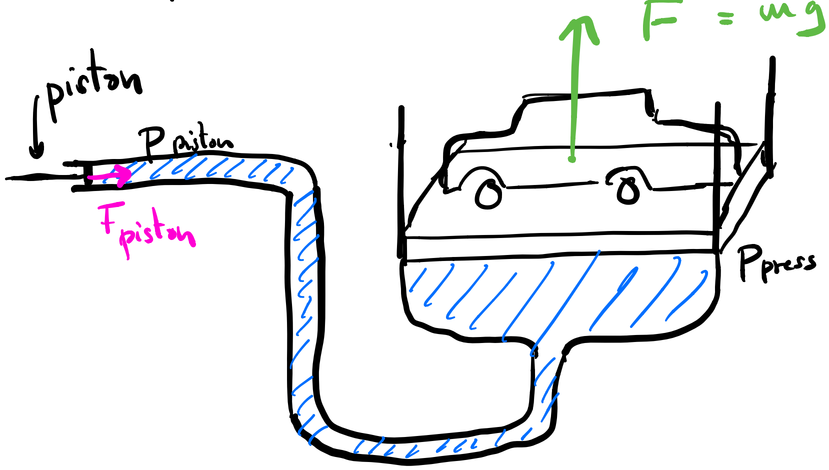
turbulent flow.



Compressible: It reacts to changes in pressure by increasing or decreasing its density

$\boxed{\text{Incompressible}}$: It doesn't. (like water)

Applications: Hydraulic press.



$$P_{piston} = P_{press}$$

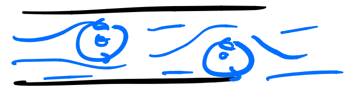
$$\frac{F_{piston}}{A_{piston}} \qquad \frac{F_{press}}{A_{press}}$$

$$\Rightarrow F_{piston} = \frac{A_{piston}}{A_{press}} F_{press}$$

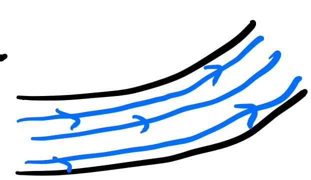
Viscous: pours slowly (e.g honey)

$\boxed{\text{Non-viscous}}$: pours easily (e.g water)

Rotational: swirls



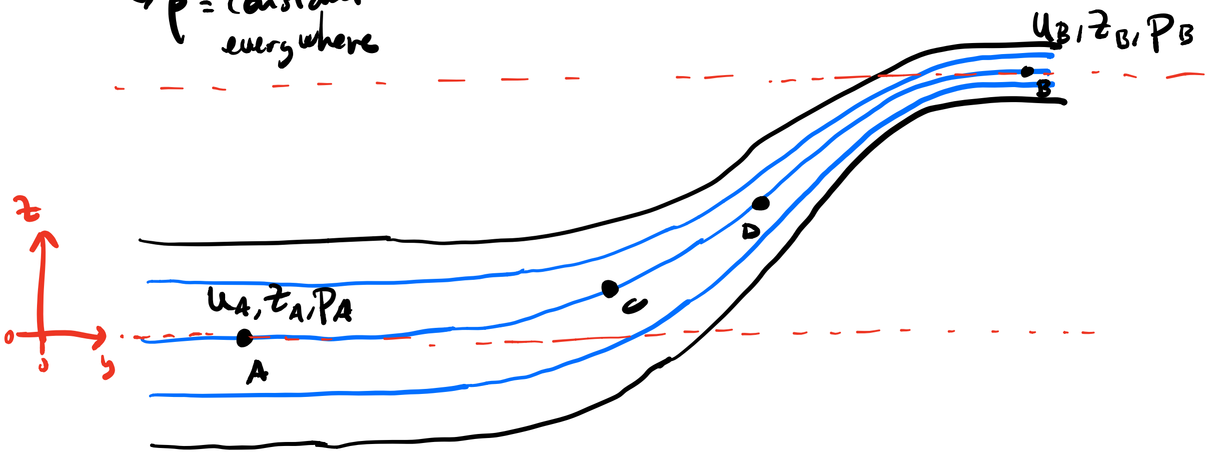
$\boxed{\text{Irrotational}}$: streamlines do not rotate



- Bernoulli's principle applies to steady, incompressible, non-viscous, irrotational fluids.

$\hookrightarrow \rho =$ constant everywhere

$\nu = 0$



- <u>Bernoulli's principle</u>: "An increase in the speed of a fluid occurs simultaneously with a decrease in pressure or a decrease in the fluid's potential energy (height).

As an equation:

pressure

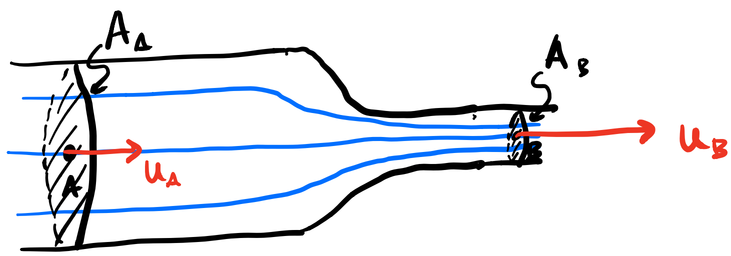
$$\rho \frac{u^2}{2} + \rho \cdot g \cdot z + P = \text{constant along the streamline.}$$

density

This is equivalent to:

$$\boxed{\rho\frac{u_A^2}{2}+\rho g z_A+p_A=\rho\frac{u_B^2}{2}+\rho g z_B+p_B}$$

- **Continuity equation:** $u_A A_A = u_B A_B$ (cross sectional area)

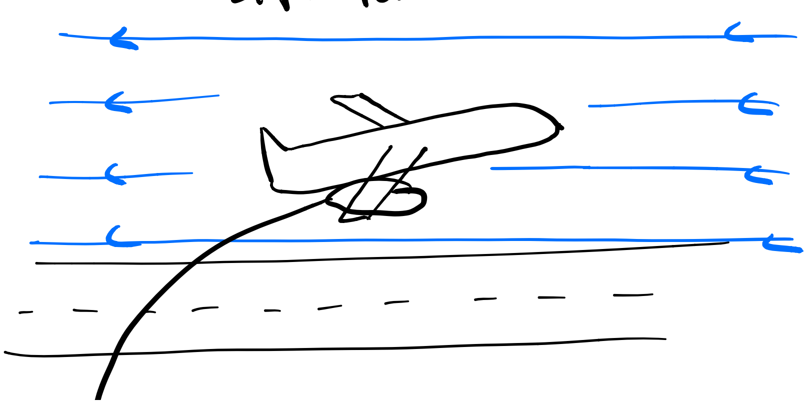


In this case $\rho\frac{u_A^2}{2}+\cancel{\rho g z_A}+P_A=\rho\frac{u_B^2}{2}+\cancel{\rho g z_B}+P_B$

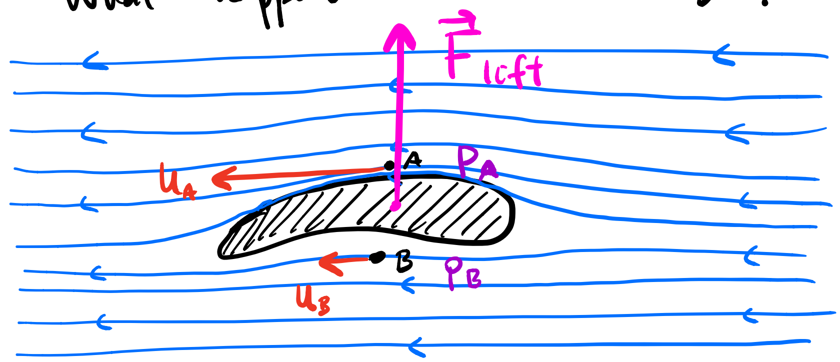
$$\rho\frac{u_A^2}{2}+P_A=\rho\frac{u_B^2}{2}+P_B$$

so if $u_A<u_B \Rightarrow P_A>P_B$

- Applications of Bernoulli's principle.
  - Lift force on an airfoil.



What happens around the wings?



"denser" streamlines mean higher speed.

$u_A > u_B \Rightarrow P_A < P_B \Rightarrow \vec{F}_{lift}$.

- Baseball throw.