

Experimental Graduate Qualifying Exam 2014

9 January 2014

Answer 5 questions. Formula/Data sheet is attached.

1. QED

- (a) Draw the first order Feynman diagram(s) and write down the matrix elements in terms of the incoming p_1, p_2 and outgoing p_3, p_4 four momenta of the following QED process (see attached sheet for Feynman Rules).

i. $e^- \mu^- \rightarrow e^- \mu^-$

ii. $e^- e^+ \rightarrow \mu^- \mu^+$

iii. $e^- e^- \rightarrow e^- e^-$

iv. $e^+ e^- \rightarrow e^+ e^-$

- (b) In unpolarized electron-positron collisions the cross-section for $e^+ e^- \rightarrow \mu^+ \mu^-$ is given by

$$\sigma = \frac{4\pi\alpha^2}{3s^2} \quad (1)$$

where $\alpha = 1/137$ in natural units ($\hbar = c = 1$) and s is the center of mass energy in GeV. Explain qualitatively the steps needed to derive the expression for σ starting from the expression for the matrix element. Note you do not need to calculate anything but instead just outline the procedure to derive a cross-section from a matrix element.

- (c) Calculate the event rate (events/second) for $e^- e^+ \rightarrow \mu^- \mu^+$ at an unpolarized $e^+ e^-$ machine operating at a luminosity $L = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ and $s = 10.56 \text{ GeV}$. ($\hbar = 6.6 \times 10^{-34} \text{ Js}, c = 3 \times 10^8 \text{ ms}^{-1}$)

(d) Draw the second order Feynman diagrams for $e^-e^+ \rightarrow \mu^-\mu^+$.

2. Allowed Decays

Which of the following processes are allowed. If allowed, state which kind of interaction is involved (strong, electromagnetic or weak). If not allowed, give the reason or reasons why not.

- (a) $\Lambda^0 \rightarrow pe^-\bar{\nu}_e$
- (b) $p \rightarrow \mu^+\nu_\mu$
- (c) $p + p \rightarrow ppp\bar{p}$
- (d) $\pi^0 \rightarrow \gamma\gamma\gamma$
- (e) $\psi \rightarrow D^+D^-$
- (f) $\tau^+ \rightarrow \rho^+\bar{\nu}_\tau$
- (g) $p \rightarrow ne^+\nu_e$
- (h) $\rho^0 \rightarrow \pi^0\pi^0$
- (i) $H^0 \rightarrow W^+W^-$
- (j) $t \rightarrow W^+b$

3. Taus

- (a) Draw the Feynman diagram for the weak decay $\tau^- \rightarrow \mu^-\bar{\nu}_\mu\nu_\tau$ and write down the matrix element in terms of the feynman rules (see attached sheet).
- (b) The width is given by

$$\Gamma(\tau^- \rightarrow \mu^-\bar{\nu}_\mu\nu_\tau) = \frac{G_F^2 m_\tau^5}{192\pi^3}$$

By considering the modifications necessary to the feynman diagram for different processes modify the expression for $\Gamma(\tau^- \rightarrow \mu^-\bar{\nu}_\mu\nu_\tau)$ to give expressions for the other possible tau decays (assuming you can neglect all final state masses).

$$\Gamma(\tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau) =$$

$$\Gamma(\tau^- \rightarrow \bar{u}d\nu_\tau) =$$

$$\Gamma(\tau^- \rightarrow \bar{u}s\nu_\tau) =$$

- (c) Calculate the Branching fraction (fraction of all decays) for
 - i. $Br(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)$
 - ii. $Br(\tau^- \rightarrow \bar{u}s\nu_\tau)$
 - (d) Describe how tau decays are identified in CMS and what are the principal backgrounds
4. This question explores π^0 decay and its implications for calorimetry design and electron identification.
- (a) Derive an expression for the observed energy E of a γ -ray from the decay of a neutral pion, $\pi^0 \rightarrow \gamma\gamma$, in terms of the π^0 mass m , the velocity βc of the pion, and the angle of emission of the photon θ in the center-of-mass frame.
 - (b) Show/argue that the distribution in θ is isotropic, and that the energy distribution of the γ -rays will be flat, extending from $E(1 - \beta)/2$ to $E(1 + \beta)/2$.
 - (c) The CMS barrel Ecal crystals have dimensions $2.2 \text{ cm} \times 2.2 \text{ cm}$ at their front face, and are approximately 1.5 meters from the interaction point. For the “best case scenario” for π^0 /photon separation (*i.e.*, the photons from the π^0 decay are emitted perpendicular to the pion line of flight), estimate the maximum π^0 energy such that a neutral pion traveling at $\eta = 0$ can be distinguished from a photon.
 - (d) The depressingly low value you just calculated in (c) can be helped by the construction of a pre-shower detector, if you are willing to take the hit in extra scattering material in front of your Ecal (like CMS did in the Endcap). Explain the design principles of a pre-shower detector, and given reasonable assumptions, an estimate of the new upper limit on the π^0 momentum where good π^0 /photon separation is possible.

5. Jets and Particle Flow

CMS uses the anti- k_T jet clustering algorithm to cluster particle flow (PF) candidates into jets.

- (a) Give a brief overview of PF reconstruction. In particular, list the different types of objects reconstructed using PF and explain which detectors subsystems are used in the reconstruction of each.
 - (b) Give a brief description of how the anti- k_T jet clustering algorithm works. In particular, you should address the following points: How does the algorithm decide whether a particular PF candidate is included in the jet? What is k_T and why is the algorithm called “anti- k_T ”?
 - (c) Pile-up (PU) collisions can contribute in an unwanted way to jet reconstruction. List at least two approaches that CMS uses to mitigate the impacts of PU on jet reconstruction. Describe briefly how each approach you list works.
 - (d) A $t\bar{t}$ event produced in the so-called “lepton+jets” final state which includes the following particles: $\ell^\pm \nu b\bar{b}q\bar{q}'$. Ideally this signature should produce four jets (one for each of the four quarks, including the two bottom quarks). However, in a real experiment, the actual number of reconstructed jets varies, with most events having 3–5 jets. Explain what factors might contribute to reconstructing a number of jets different from the four expected.
 - (e) List at least two ways in which the jet reconstruction approach taken by ATLAS differs from that used by CMS.
6. Statistics plays an increasingly important role in the interpretation of particle physics results, especially searches for new particles like the Higgs boson or supersymmetry.
- (a) Define the following commonly used statistical terms
 - i. p -value
 - ii. Nuisance parameter
 - iii. Profiling
 - iv. Test statistic
 - v. MVA

- (b) A typical analysis approach on CMS begins with the construction of a binned likelihood function for a particular reconstructed distribution (say a reconstructed invariant mass or an MVA output). Write an expression for a binned likelihood for a distribution with N bins. You may assume there is a single signal process of interest (s) as well as one relevant background (b). The amount of signal should be characterized by a signal strength parameter μ which is a multiplicative factor that scales the up or down the expected amount of signal. You should also write the signal and background expectation as functions of a set of arbitrary nuisance parameters (θ_i). For simplicity, assume that Gaussian constraints are applied to each nuisance parameter. You may find the following statistical functions helpful:

$$p(n; \nu) = \frac{\nu^n e^{-\nu}}{n!} \quad (Poisson)$$

$$p(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (Gaussian)$$

7. Muons and ψ decays

($M_{J/\psi}=3.10$ GeV. $M_\mu=106$ MeV. $M_\pi=140$ MeV. $\tau_{\pi\rightarrow\mu\nu} = 2.6 \times 10^{-8}$ seconds.)

- (a) An important and useful signal in many collider experiments is the following:

$$J/\psi \rightarrow \mu^+ \mu^-$$

Write down a Feynman diagram for this decay.

- (b) Describe how the CMS detector is used to identify and distinguish a muon from other elementary particles.
- (c) A J/ψ is produced in a collider experiment with a total energy of 20 GeV. In the center-of-mass system of the J/ψ , the two muons decay back-to-back, perpendicular to the direction of the J/ψ . Find the angle between the two muons in the detector.
- (d) Muons are also produced in the decays of charged pions. Assuming that the average distance from the interaction point to the first

layer of muon chambers in this detector is 5.0 meters, estimate the probability that a pion with total energy of 5 GeV will decay to a muon before reaching the muon system. (Assume that the pions decay to muons 100% of the time). How might at least some of these muons be identified as coming from pion decay?

8. Tracking and Trigger

- (a) Explain how the CMS trigger system works. Include in your answer a description of why CMS needs a trigger, how the different level triggers work, which subdetectors are used in the trigger at each level, approximately what event rates are at the output of each trigger level.
- (b) Compare and contrast the signatures for photons, electrons, and muons in the different levels of the trigger. What is the earliest level of the trigger at which each of these can be distinguished from the others?
- (c) Currently, CMS does not include any tracking information in the L1 part of the trigger. What advantages would CMS gain by including tracking at level 1?

Feynman Rules

External Particles

For each spin $\frac{1}{2}$ fermion (f), antifermion (\bar{f}) or photon (γ)

	f	\bar{f}	γ
IN	$u(p, s)$	$\bar{v}(p, s)$	$\epsilon_\mu(k, \lambda)$
OUT	$\bar{u}(p, s)$	$v(p, s)$	$\epsilon_\mu^*(k, \lambda)$

where p = momenta s = spin

Propagators

Spin 0 $\text{---} \overset{p}{\text{---}} \text{---}$ $\frac{i}{p^2 - m^2}$

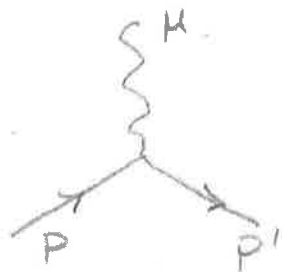
spin $\frac{1}{2}$ $\text{---} \overset{p}{\text{---}} \text{---}$ $\frac{i}{\not{p} - m}$

photon $\text{---} \overset{q}{\text{---}} \text{---}$ $-\frac{i g^{\mu\nu}}{q^2}$

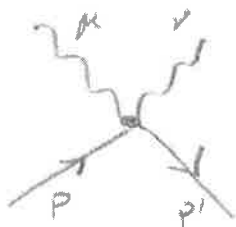
Massive Vector Boson $\text{---} \overset{q}{\text{---}} \text{---}$ $\frac{i \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{M^2} \right)}{q^2 - M^2}$

QED Vertices

Spin 0

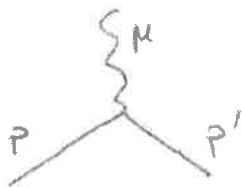


$$-ie(p+p')_\mu$$



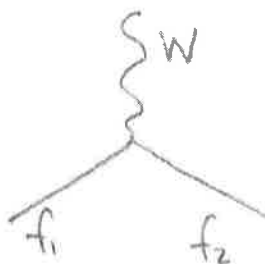
$$2ie^2 g_{\mu\nu}$$

Spin $\frac{1}{2}$



$$-ie\delta_\mu$$

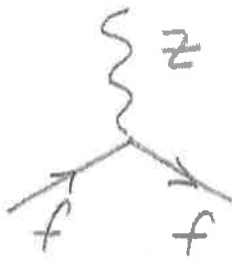
Charged Current Vertices



f_1	f_2	Factor
ν_e	e^-	$\frac{-ig}{\sqrt{2}} \gamma_\mu \frac{(1-\gamma_5)}{2}$
q_1	q_2	$\frac{-ig}{\sqrt{2}} V_{q_1 q_2} \gamma_\mu \frac{(1-\gamma_5)}{2}$

($V_{q_1 q_2}$ is CKM Matrix for quarks)

Neutral Current Vertex



$$\frac{-ig}{2\cos\Theta_W} \gamma_\mu [g_V - g_A \gamma_5]$$

CKM Matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.97 & 0.23 & 0.004 \\ 0.23 & 0.97 & 0.04 \\ 0.009 & 0.04 & 1.00 \end{pmatrix}$$

VECTOR MESONS (Spin 1)

Meson	Quark content	Charge	Mass	Lifetime
ρ	$u\bar{d}, d\bar{u}, (u\bar{u} - d\bar{d})/\sqrt{2}$	+1, -1, 0	770	0.4×10^{-23}
K^*	$u\bar{s}, s\bar{u}, d\bar{s}, s\bar{d}$	+1, -1, 0, 0	892	1×10^{-23}
ω	$(u\bar{u} + d\bar{d})/\sqrt{2}$	0	783	7×10^{-23}
ϕ	$s\bar{s}$	0	1020	20×10^{-23}
J/ψ	$c\bar{c}$	0	3097	1×10^{-20}
D^*	$c\bar{d}, d\bar{c}, c\bar{u}, u\bar{c}$	+1, -1, 0, 0	2010	$>1 \times 10^{-22}$
T	$b\bar{b}$	0	9460	2×10^{-20}

BARYONS (Spin $\frac{1}{2}$)

Baryon	Quark content	Charge	Mass	Lifetime
$N \begin{pmatrix} p \\ n \end{pmatrix}$	uud	+1	938.280	∞
	udd	0	939.573	900
Λ	uds	0	1115.6	2.63×10^{-10}
Σ^+	uus	+1	1189.4	0.80×10^{-10}
Σ^0	uds	0	1192.5	6×10^{-20}
Σ^-	dds	-1	1197.3	1.48×10^{-10}
Ξ^0	uss	0	1314.9	2.90×10^{-10}
Ξ^-	dss	-1	1321.3	1.64×10^{-10}
Λ_c^+	udc	+1	2281	2×10^{-13}

BARYONS (Spin $\frac{3}{2}$)

Baryon	Quark content	Charge	Mass	Lifetime
Δ	uuu, uud, udd, ddd	+2, +1, 0, -1	1232	0.6×10^{-23}
Σ^*	uus, uds, dds	+1, 0, -1	1385	2×10^{-23}
Ξ^*	uss, dss	0, -1	1533	7×10^{-23}
Ω^-	sss	-1	1672	0.82×10^{-10}

PSEUDOSCALAR MESONS (Spin 0)

Meson	Quark content	Charge	Mass	Lifetime
π^+	$u\bar{d}, d\bar{u}$	+1, -1	139.569	2.60×10^{-8}
π^0	$(u\bar{u} - d\bar{d})/\sqrt{2}$	0	134.964	8.7×10^{-17}
K^+	$u\bar{s}, s\bar{u}$	+1, -1	493.67	1.24×10^{-8}
K^0, \bar{K}^0	$d\bar{s}, s\bar{d}$	0, 0	497.72	$\begin{cases} K_S^0 0.892 \times 10^{-10} \\ K_L^0 5.18 \times 10^{-8} \end{cases}$
η	$(u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$	0	548.8	7×10^{-19}
η'	$(u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$	0	957.6	3×10^{-21}
D^+	$c\bar{d}, d\bar{c}$	+1, -1	1869	9×10^{-13}
D^0, \bar{D}^0	$c\bar{u}, u\bar{c}$	0, 0	1865	4×10^{-13}
F^+ (now D_s^+)	$c\bar{s}, s\bar{c}$	+1, -1	1971	3×10^{-13}