Candidacy Exam
Fall 2020

The exam has 6 questions. You must answer 4 of them correctly in order to pass. If you attempt more than 4, your best 4 answers will be kept. This is closed book. You have 4 hours to take the exam.

1. The leading operator in the SM that produces an anomalous magnetic moment for a muon is a dimension-5 operator

$$\mathcal{O} = \frac{1}{\Lambda}\bar{\mu}\,\sigma_{\alpha\beta}\,\mu\,F^{\alpha\beta}.$$

(a) Explain why, from symmetry considerations alone, this is effectively a dimension-6 operator from the perspective of new physics. In so doing, explain what replaces $1/\Lambda$ in the coefficient of $\mathcal{O}$.

(b) Draw the leading QED and electroweak diagrams that contribute to this process in the SM, giving for each their naive degree of divergence. What is the overall degree of divergence for these diagrams, when summed?

2. Show all of the diagrams that contribute to

$$p\bar{p} \to t\bar{t}$$

at parton level (both quark and gluon initial states). Explain the steps that are needed to carry over the calculation to proton level.

3. The Fermi theory for the weak interactions is not renormalizable. Why is that so, and what does it imply about the theory? In particular, under what circumstances can the Fermi theory be used, and where does it fail? In what way does it fail?

4.) Imagine we extend the SM by adding a 'squark', a complex scalar $\tilde{U}$ that's a color triplet, hypercharge $+2/3$, and $SU(2)_w$ singlet. Using $\tilde{U}$ and it's complex conjugate $\tilde{U}^*$: i.) write down the $SU(3)_c \otimes SU(2)_w \otimes U(1)_Y$ representation for $\tilde{U}$, ii.) What's the covariant derivative for $\tilde{U}$? iii.) What renormalizable terms can we write down (kinetic terms + interactions) using $\tilde{U}$ and $\tilde{U}^*$ alone? iv.) What renormalizable interactions can we write that involve $\tilde{U}$ or $\tilde{U}^*$ and combinations of SM matter? v.) Describe in words or diagrams the interaction terms present in parts iii.) and iv.).

5.) At the LHC there are several different ways to produce W bosons, including production in association with jets $pp \to W^\pm + j$, production in association with Higgses $pp \to W^\pm H$ and decay from a hypothetical heavy scalar $pp \to \phi \to W^+W^-$ (which we'll treat as a heavy Higgs for the purpose of how it couples to SM matter). Discuss the differences among these three production methods. Be sure to include some information about the relative rates.

6. In QFT we say that couplings 'run' with energy. i.) Explain how this arises as a consequence of renormalization. ii.) Consider a theory consisting of a single massive Dirac fermion ψ charged under a $U(1)$ gauge group and interacting with a massive, neutral real scalar ϕ:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + i\bar{\psi}(\not{D} - m_\psi)\psi + \frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}M_\phi^2\phi^2 - y\,\bar{\psi}\psi\phi \tag{1}$$

where $D_\mu = \partial_\mu + i\,g\,A_\mu$. Draw the diagrams that contribute to the running of the gauge coupling g. iii.) What are the possible outcomes for running? Meaning if we extrapolate couplings to arbitrarily high or low energies, what types of phenomena can occur?