**Candidacy Exam - Dec. 10, 2021**

There are eight questions. Answer five of them as completely as you can. Each question should be answered on a different sheet of paper as the grading is distributed among the HEP faculty. The exam is closed book and no laptops, cellphones etc should be used except for a standard calculator. A Formula sheet is attached. The time allowed is 4 hours.

1. CMS and ATLAS use the anti-$k_T$ jet clustering algorithm to cluster particle flow (PF) candidates into jets.

   (a) Explain the main concept behind particle flow reconstruction. In particular, list the different types of physics objects reconstructed using PF and explain which detectors subsystems are used in the reconstruction of each. Here we're asking about the objects (i.e. different categories of physical particles) that will serve as inputs to the jet clustering.

   (b) Give a brief description of how the anti-$k_T$ jet clustering algorithm works. In particular, you should address the following points: How does the algorithm decide whether a particular PF candidate is included in the jet? What is $k_T$, and why is the algorithm called "anti-$k_T$"?

   (c) Pile-up (PU) collisions can contribute in an unwanted way to jet reconstruction. List at least two approaches basic approaches to mitigate the impacts of PU on jet reconstruction. Describe briefly how each approach you list works.

2. In the Standard Model, neutrinos are massless, but measurements in neutrino oscillations have demonstrated that neutrinos definitely have a mass.

   (a) Briefly explain the mechanism by which quarks and leptons acquire a mass in the Standard Model. Specifically, give the form of the term added to the Lagrangian that results in the particles acquiring a mass.

   (b) Why can't neutrinos in the Standard Model simply acquire a mass the same way that quarks and leptons can?

   (c) Give at least one hypothetical modification to the Standard Model that would allow neutrinos to acquire a mass.

   (d) For the modification(s) to the Standard Model that you listed above, what experimental evidence (aside from the non-zero neutrino mass) would lend support to that hypothesis.

3. Consider two distributions, each with three bins:

$$A = (A_1, A_2, A_3) \tag{1}$$
$$B = (B_1, B_2, B_3) \tag{2}$$

(a) Write the 6x6 correlation matrix for all six bins given above under the following scenarios:

  i. The uncertainties of all bins are uncorrelated will all other bins.
  ii. The uncertainties of all bins are fully correlated with all other bins.
  iii. Half of the uncertainty in each bin is fully correlated with all other bins and half is uncorrelated with all other bins.
  iv. The uncertainties of the contents of A are fully correlated with each other, but uncorrelated with the contents of B. The uncertainties of the contents of B are fully correlated with each other.
  v. The uncertainties on $A_1$ and $A_3$ are fully anticorrelated, $B_1$ and $B_3$ are fully anticorrelated, and all other bins are uncorrelated.

(b) Consider the scenario where the distributions have the following contents and uncertainties:

$$A = (50 \pm 10, 50 \pm 10, 50 \pm 10) \tag{3}$$
$$B = (30 \pm 10, 50 \pm 10, 70 \pm 10) \tag{4}$$

and compute the bin contents and uncertainty on those bin contents of the following distributions:

$$\begin{aligned} A/B &= (A_1/B_1, A_2/B_2, A_3/B_3) \\ A + B &= (A_1 + B_1, A_2 + B_2, A_3 + B_3) \end{aligned}$$

for the following two scenarios:

  i. The uncertainties of all bins are uncorrelated with all other bins.
  ii. The uncertainties of all bins are fully correlated with all other bins.

(c) Compute the chi square comparing A/B to a horizontal line at 1.0 using each of the two values for A/B you computed above. When computing the chi-square, you can ignore bin-to-bin correlations.

(d) Suppose we were trying to assess whether A and B were consistent with each other, using the chi square values computed in part c. Does the assumption we make about the correlation of uncertainties alter the conclusion here? Why or why not?

4. An important and useful signal in the collider experiment is the following:

$$J/\psi \to \mu^+\mu^-$$

(a) Write down a Feynman diagram for this decay.

(b) Describe how the generic collider detector (like CMS or ATLAS) consisting of a tracker, electromagnetic and hadronic calorimeter, and muon system, identifies and distinguishes a muon from other elementary particles.

(c) A $J/\psi$ is produced in the above generic detector with a total energy of 20 GeV. In the center-of-mass system of the $J/\psi$, the two muons decay back-to-back, perpendicular to the direction of the $J/\psi$ . Find the angle between the two muons in the detector.

(d) Muons are also produced in the decays of charged pions. Assuming that the average distance from the interaction point to the first layer of muon chambers is 3.0 meters, estimate the probability that a pion with total energy of 5 GeV will decay to a muon before reaching the muon system. (Assume that pions decay to muons 100% of the time) How might at least some of these muons be identified as coming from pion decay?

Useful constants:

$$M_{J/\psi} = 3.10 \text{ GeV/c}^2 \qquad M_\mu = 106 \text{ MeV/c}^2$$
$$M_\pi = 140 \text{ MeV/c}^2 \qquad \tau_{\pi\to\mu\nu} = 2.6 \times 10^{-8} \text{ sec}$$

5. Discuss the behavior of the famous $R$ ratio:

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$R$ is the ratio of the inclusive production cross section for hadrons to that of muon pairs in $e^+e^-$ collisions. Graph $R$ versus the center of mass energy of the $e^+e^-$ collisions from a center of mass energy of 200 MeV to 110 GeV. Annotate your graph, explaining any changes in $R$ with energy. Include any resonances that are important markers of hadron production, and explain why they are located where they are in center of mass energy. Don't worry about the magnitude of $R$ on these resonances, but merely indicate the resonance positions.

6. Calorimetry and the Higgs Boson:

   (a) Explain how an electromagnetic shower is formed from a high energy ($> 1GeV$) photon or electron in an electromagnetic calorimeter.

   (b) The CMS electromagnetic calorimeter was designed to detect $H \to \gamma\gamma$. Describe the technology used and the important design considerations that led to the choice of that technology.

   (c) The resolution of the EM calorimeter is given by

   $$(\frac{\Delta E}{E})^2 = (\frac{A}{\sqrt{E}})^2 + (\frac{B}{E})^2 + C^2 \tag{5}$$

   Describe qualitatively what determines the value of the three constants A,B and C

   (d) How is the CMS electromagnetic calorimeter calibrated?

   (e) Derive an expression for the reconstructed mass of the Higgs boson candidate in terms of the energy of the two photons and the opening angle between them and use this expression to explain what determines the mass resolution of the candidate.

7. The dominant decay modes of the $\eta$ meson are as follows:

   - $\eta \to \gamma\gamma$ (39%)
   - $\eta \to 3\pi$ (56%)
   - $\eta \to \pi\pi\gamma$ (5%)

   It is classified as a "stable particle", so none of these is purely a strong interaction. This seems odd since the mass of the $\eta$ is $549\,\mathrm{MeV/c^2}$ and has plenty of energy to decay strongly into $2\pi$ and $3\pi$.

   (a) Explain why the $2\pi$ decay is forbidden for strong and electromagnetic interactions.

   (b) Explain why the $3\pi$ decay is forbidden as a strong interaction but allowed as an electromagnetic decay.

8. With increasingly large $t\bar{t}$ samples being collected at the LHC, it is interesting to study the production of a top quark pair in association with other particles. The table below shows the cross sections for inclusive $t\bar{t}$ production as well as for a variety of associated production modes:

| Process | 8 TeV | 13 TeV | Ratio |
|---|---|---|---|
| $t\bar{t}H$ | 133 fb | 507 fb | 3.8 |
| $t\bar{t}Z$ | 206 fb | 840 fb | 4.1 |
| $t\bar{t}W$ | 232 fb | 570 fb | 2.5 |
| $t\bar{t}$ | 246 pb | 816 pb | 3.3 |

Table 1: Cross sections for various top pair production processes at 8 and 13 TeV.

(a) Draw a representative Feynman diagram for each production. (You can leave top quarks, and $W$, $Z$, and $H$ bosons in the final state; no need to draw the those decays as part of the diagram.)

(b) Why does the $t\bar{t}W$ cross section grow noticeably less than the others?

(c) How do each of these four processes manifest (or not) themselves in the following final states? Explain how each process can contribute the objects listed in the signature, making sure to explain a particular object may be misidentified. If objects have to be missed for the process to match a particular signature, please note that as well. If the process cannot produce the given final state, please note that as well.

   i. One charged lepton, missing transverse momentum, and multiple jets, of which four are $b$-quark jets.

   ii. Three charged leptons, missing transverse momentum, and multiple jets, or which two are $b$-quark jets.

   iii. Six charged leptons, missing transverse momentum, and two $b$-quark jets.