Characterizing the Performance of a Fan

Thermal-Fluids Laboratory: ME 4251-502

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Abstract

The purpose of this experiment was to document the performance of a fan by measuring its head, airflow, and other coefficients and computations, both dimensional and non-dimensional. Ultimately, the experiment aimed to observe trends as the fan speed increased in both hertz (Hz) and revolutions per minute (RPM). The experiment involved turning on the fan at a given speed and then measuring the change in pressure at the fan and the laminar flow element (LFE), the output temperature in degrees Fahrenheit (°F), and the output power of the fan in Watts (W), based on calculations.

The experiment involved running the fan at three increasing speeds, and it was found that as the fan speed increased, so did the airflow. As a result, the heat decreased exponentially, power increased linearly, and pump/fan efficiency increased until it reached a global maximum, after which it promptly reduced. The fan behaved according to pre-established efficiency laws, in which increasing the input power of the system led to a continuous increase in efficiency at an exponentially decreasing rate, as stated by the IEC (International Electrotechnical Commission) and NEMA (National Electrical Manufacturers Association) [Figure 8: Efficiency as a function of the capacity coefficient].

The experiment also determined that the candidate operating point, where the flow rate (Q) is 0.1 $\frac{m^3}{s}$ and the head (H) of 116m 'of air' can be performed with our fan between 20 to 60 Hz (1200 to 3600 RPM), specifically at 30.2 Hz (1811 RPM), using the Bernoulli Head Equation [Table 7]. The fan in the laboratory proved to be more than suitable for producing the head and flow rate of the candidate operating point.

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Symbols

β	Diameter ratio, the inner diameter of the thin orifice divided by the inner diameter of the pipe ($\frac{d}{D}$)
Bhp	the brake horsepower of the fan, in watts [W] and kilowatts [kW]
C_H	The head coefficient, $C_H = \frac{gH}{n^2D^2}$
C_P	The power coefficient, $C_P = \frac{Bhp}{\rho n^3 D^5}$
C_Q	The capacity coefficient, $C_P = \frac{Q}{nD^3}$
D	Impeller diameter: 0.503m
ΔP_{fan}	The pressure difference of the fan measured with & without flow, $[inH_2O]$
ΔP_{LFE}	The pressure difference of the Laminar Flow Element (LFE) with an inclined manometer with & without flow, $[inH_2O]$
Δp	The conversion of ΔP_{fan} which is in inH_2O to Pa
H	The head of the fan
γ	Specific weight of air: 12 $\frac{N}{m^3}$
g	Gravity, 9.81 $\frac{m}{s^2}$
η	Fan speed, $[\frac{rev}{s}]$
P_o	The atmospheric pressure of the laboratory, 26.884 $inHg$
Q	The air flow rate
Q_{LFE}	The air flow rate through the Laminar Flow Element (LFE)
ρ	The density of dry air, 1.2754 $\frac{kg}{m^3}$
T_o	The atmospheric temperature of the laboratory, 68 ^oF
T_f	The temperature output that the LabVIEW program displayed from the fan
u_Q	The measured uncertainty of the airflow
V_2	The calculated velocity of the fan, [m/s], [Hz], [RPS] or [RPM]

Introduction

Fans are devices that create a flow of air using rotating blades or impellers. They are widely used in industrial applications to prevent stagnant or stale air, such as in factories, warehouses, and large buildings. Therefore, understanding the appropriate size and speed of the fan is crucial for the specific application. This experiment establishes the relationship between metrics, such as fan speed and airflow, to help determine the appropriate fan for a given application. To make informed decisions, institutions like the IEC and NEMA provide guidelines to ensure the efficiency of fans, which is crucial for the capacity coefficient.

Plots of dimensional and non-dimensional values of the fan are attached to a laminar flow element within the thermal fluid's lab room to accurately detail the fan's performance. These plots include *Head vs. Flow Rate*, *Brake vs. Flow Rate*, and *Efficiency vs. Flow* [Dimensional Plots] as well as *Head Coefficient vs. Capacity Coefficient, Power Coefficient vs. Capacity Coefficient*, and *Efficiency vs. Capacity Coefficient* [Non-dimensional plots]. Several of the values are computed by Bernoulli's equation for Head $H = \frac{\Delta p}{\gamma} + \frac{V_2^2}{2g}$ and non-dimensional values obtained through $\frac{gH}{n^2D^2} = g_1\left(\frac{Q}{nD^3}\right)$, $\frac{Bhp}{\rho n^3 D^5} = g_2\left(\frac{Q}{nD^3}\right)$, and $\eta = g_3\left(\frac{Q}{nD^3}\right)$. One leading theory behind fan performance attempted to be replicated and proven is that the returning efficiency of a fan continues to rise at higher frequencies, however, with an exponential decline. The objective is to prove that the fan used works within proper scaling laws by examining graphical trends within the data and testing a candidate operating point.

Methods

Equipment Utilized

The methodology of conducting this lab relies on the Variable Power Supply, VPS, to power the fan motor at different prescribed frequencies to allow proper documentation of the observed values of temperature (pre and post laminar flow element, LFE), pressure (pre and post LFE), and the brake horsepower in watts, recorded by the LabVIEW system onto the computer. This allows for the observers to calculate the head loss, via the U-tube Manometers used to acquire quantitative values of pressure differences pre and post element, while quantifying that this fan setup would be able to perform the required task within the confines and requirements as mentioned in the lab handout.

Schematics

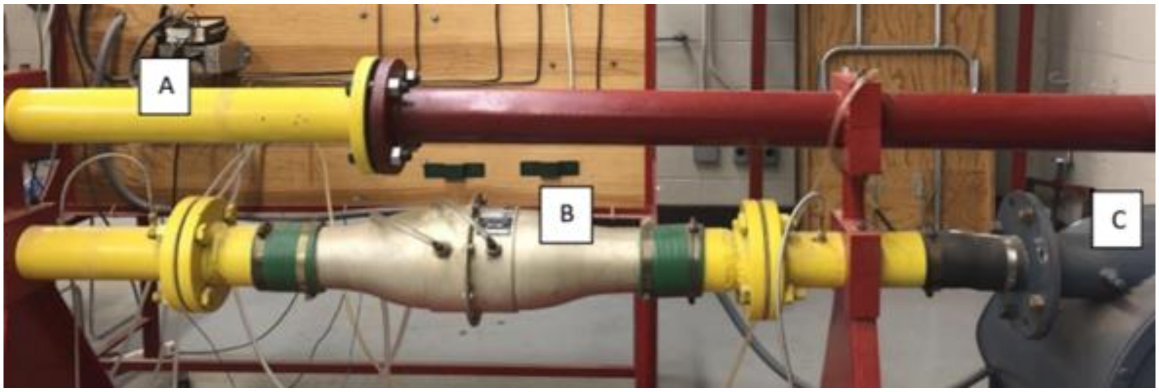

Figure 1: Piping System [A] with the Laminar Flow Element (LFE) [B] and the Fan [C]

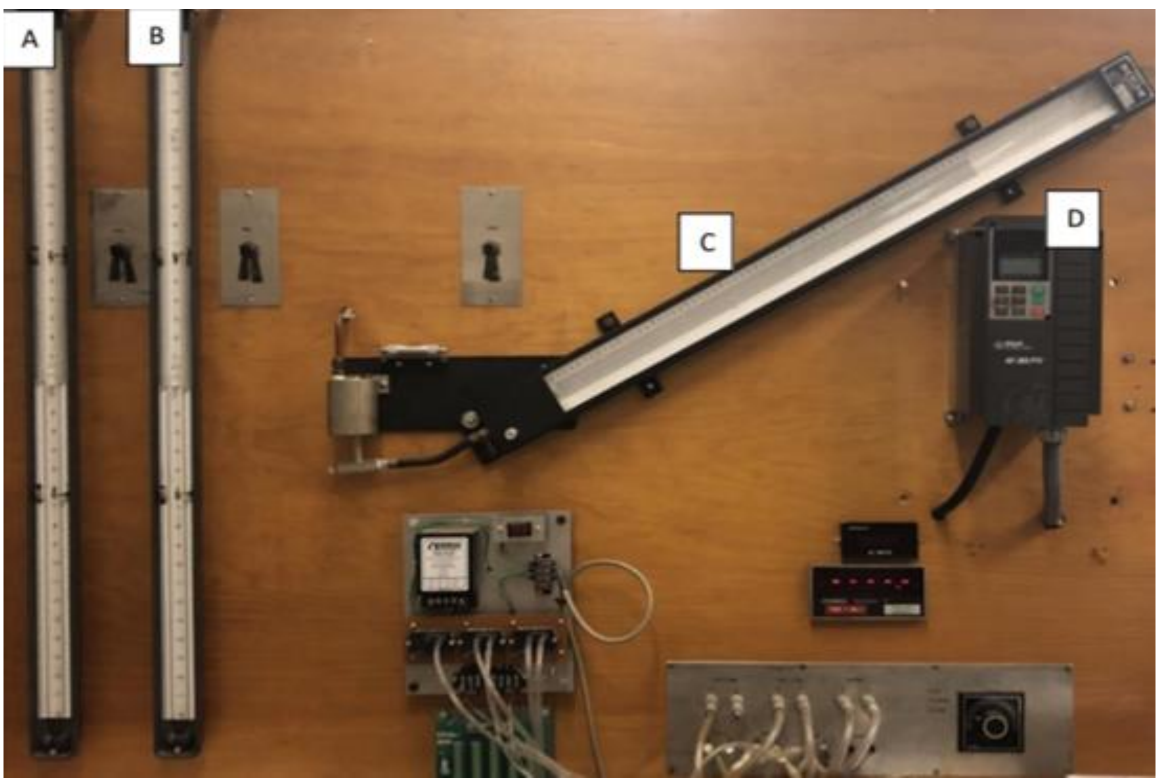

Figure 2: Manometer for fan [A], Orifice Plate (not used) [B], LFE [C], and Variable Power Supply [D]

Narrative Description

In this lab, we will investigate how different speeds affect the performance of a fan. The performance will be measured using various metrics such as brake horsepower, head, and efficiency. A fan that blows air through the piping generates the airflow. The airflow will first pass through a temperature and pressure sensor, followed by the laminar flow element, and then another pressure and temperature sensor. U-tube water-based manometers are connected to each pressure sensor to measure the pressure differences (Δp) between the fan impeller and LFE. The experiment will be repeated at different fan speeds by adjusting the fan speed using the Variable Power Supply. The LabVIEW program will document the data obtained from each trial.

Results & Discussion

Raw Measured Data

Table 1: Barometer

Pressure (P_o) [inHg]	**Temperature** (T_o) [$°F$]
26.884	68

Table 2: Pipe Inner Diameter

	Diameter [in]	**Diameter** [m]
D	3.124	0.079
	3.122	0.079
	3.125	0.079
	3.124	0.079
D_{avg}	3.124	0.079
D_{std}	$1.26 \cdot 10^{-3}$	$2.77 \cdot 10^{-5}$

Table 3: Impeller Diameter

Impeller Diameter [m]	0.503

Data Runs

Table 4: Results for Fan at 30 RPS

Frequency (Input) [Hz]	30	**Speed** (Measured) [RPS]	30
ΔP_{fan} $[inH_2O]$	ΔP_{LFE} $[inH_2O]$	T_3 [$°F$]	Power [kW]
3.250	2.680	74.7617	0.330
4.350	2.140	74.2796	0.290

5.300	1.620	73.9591	0.240
5.850	1.070	73.9330	0.190
7.250	0.540	73.8228	0.140
7.250	0.00	73.3550	0.110

Table 5: Results for Fan at 42 RPS

Frequency (Input) [Hz]	42	**Speed** (Measured) [RPS]	42
$\Delta P_{fan}\ [inH_2O]$	$\Delta P_{LFE}\ [inH_2O]$	$T_3\ [°F]$	Power [kW]
5.50	4.14	76.45080	0.880
8.20	3.31	78.09903	0.740
10.050	2.47	78.30740	0.620
11.40	1.66	78.92000	0.500
12.350	0.82	78.75610	0.340
13.350	0.00	78.00360	0.250

Table 6: Results for Fan at 54 RPS

Frequency (Input) [Hz]	54	**Speed** (Measured) [RPS]	54
$\Delta P_{fan}\ [inH_2O]$	$\Delta P_{LFE}\ [inH_2O]$	$T_3\ [°F]$	Power [kW]
8.150	5.780	84.80500	1.660
13.00	4.610	85.12060	1.370
16.40	3.470	85.43230	1.180
18.90	2.310	85.58930	0.960
20.40	1.16	85.25730	0.680
20.45	0.00	83.70390	0.470

Table 7: Candidate Operating Point

Flow rate [m^3/s]	**Head** [m of air]
0.1	116

1.

In order to plot head (in m of air) as a function of flow rate (m^3/s), recall that the flow rate through the laminar flow element, Q_{LFE}, can be mathematically expressed as:

$$Q_{LFE} \ = \ \beta \Delta P_{LFE}$$

Where the manufacturer gives β to be $\frac{400 \ SCFM}{8 \ inH_2O} \ at \ 70^o F \ and \ 29.92 \ inHg$. Ergo, the flow rate is calculated by multiplying that beta factor of $50\frac{SCFM}{inH_2O}$ to the given ΔP_{LFE} data that was recorded in the experiment for each corresponding value of frequency and speed.

Similarly, head, or the relationship between the energy in an incompressible fluid to the height of an equivalent static column of that fluid, is denoted by the equation:

$$H \ = \ \frac{\Delta p}{\gamma} + \frac{V_2^2}{2g}$$

Also, recall, that the velocity of the fan's air was given in revolutions per second (rps), in intervals of 30, 42, 54 rps, respectively—thus, to keep the SI units copacetic, the conversion between rps and m/s must be done. This conversion is done by taking the fan's impeller radius, converted from inches to meters, and taking the product between that and the converted revolution ($2\pi r_{impeller}$). All in all, the plot for head as a function of flow rate is as follows:

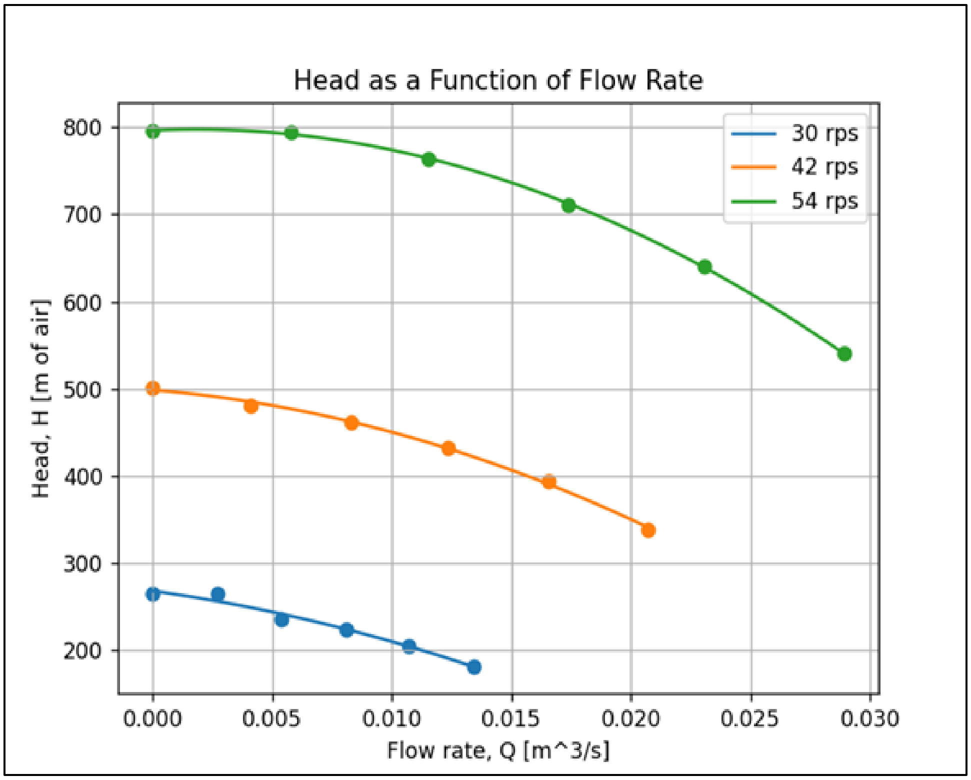

Figure 3: Head, H, as a function of Flow Rate, Q

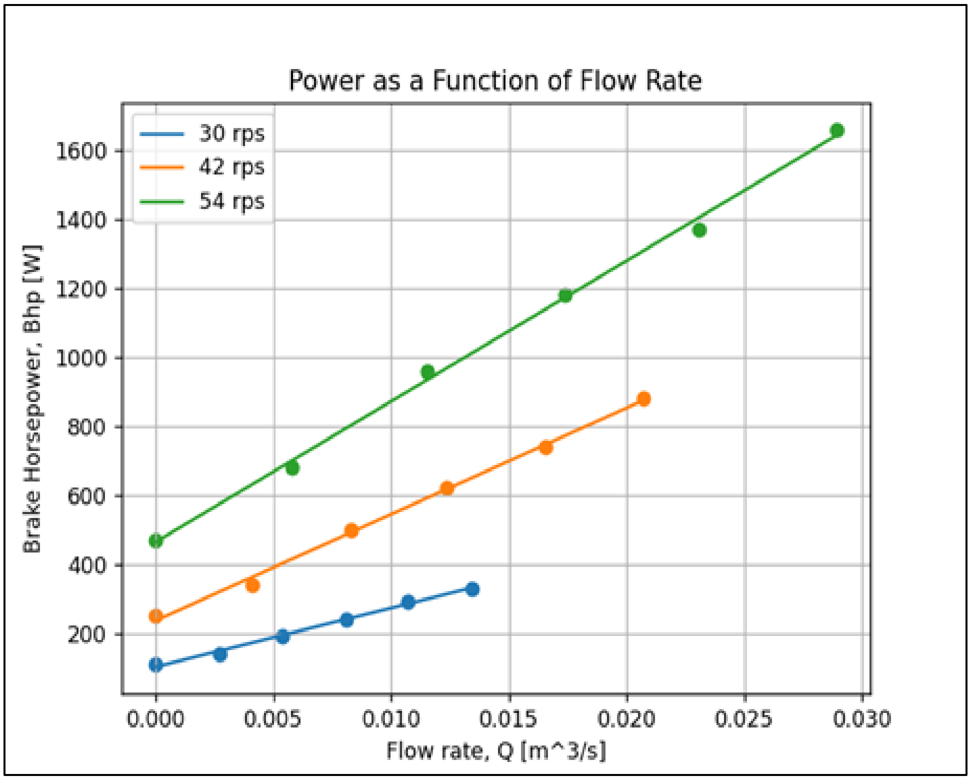

Figure 4: Brake Horsepower, Bhp, as a function of Flow Rate, Q

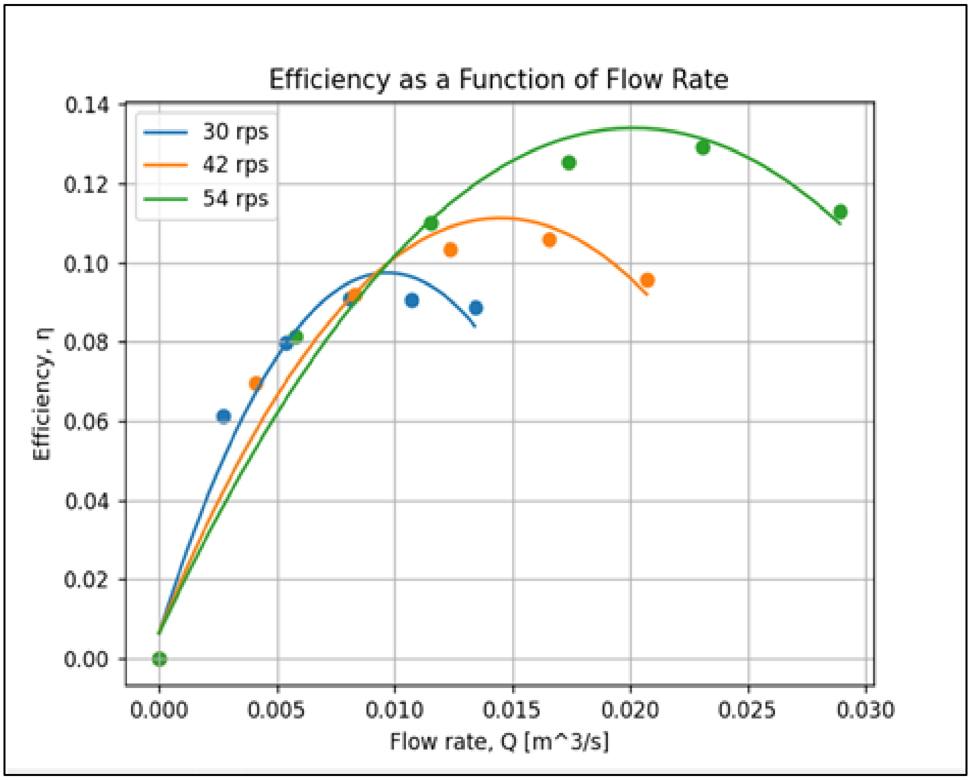

Figure 5: Efficiency, η, as a function of Flow Rate, Q

2.

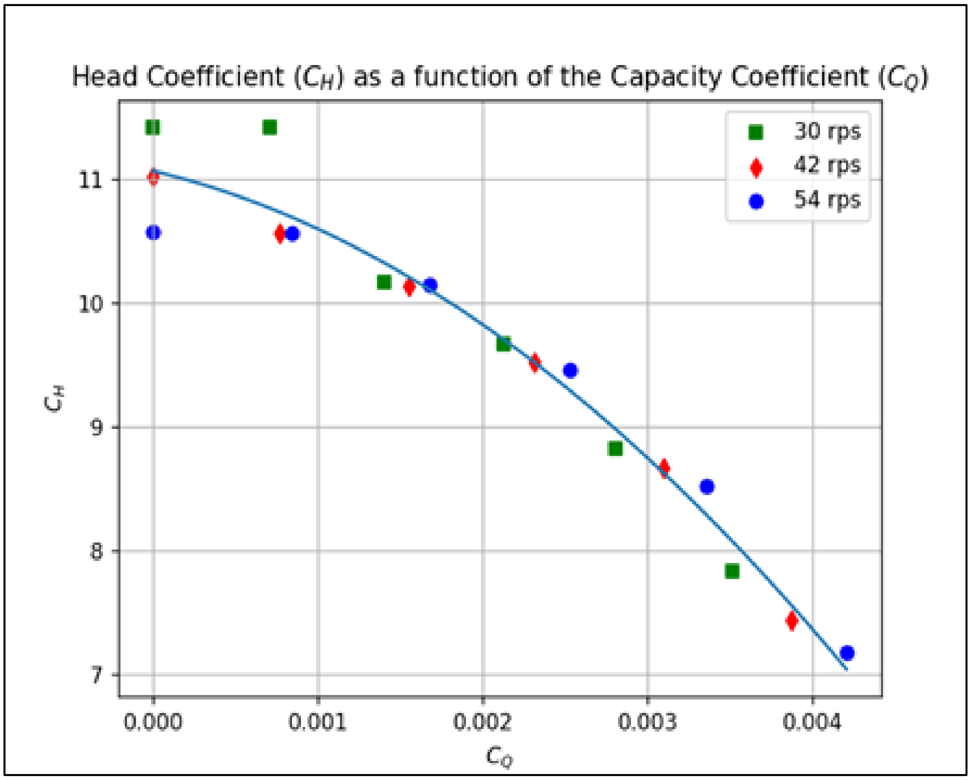

Figure 6: Head Coefficient as a function of the Capacity Coefficient

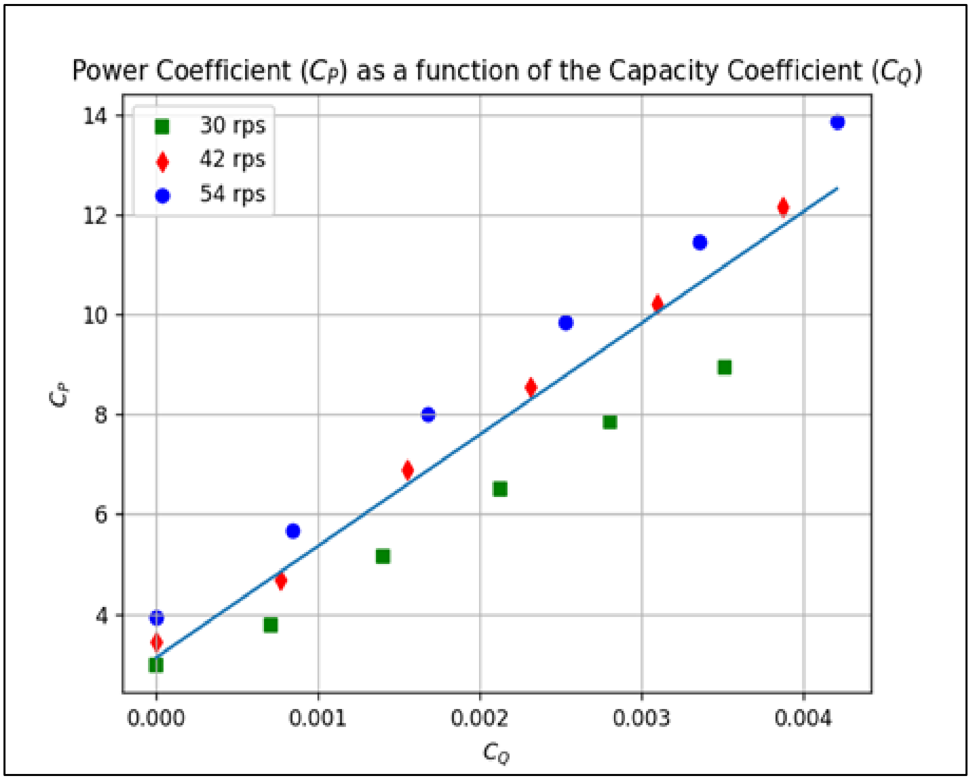

Figure 7: Power Coefficient as a function of the Capacity Coefficient

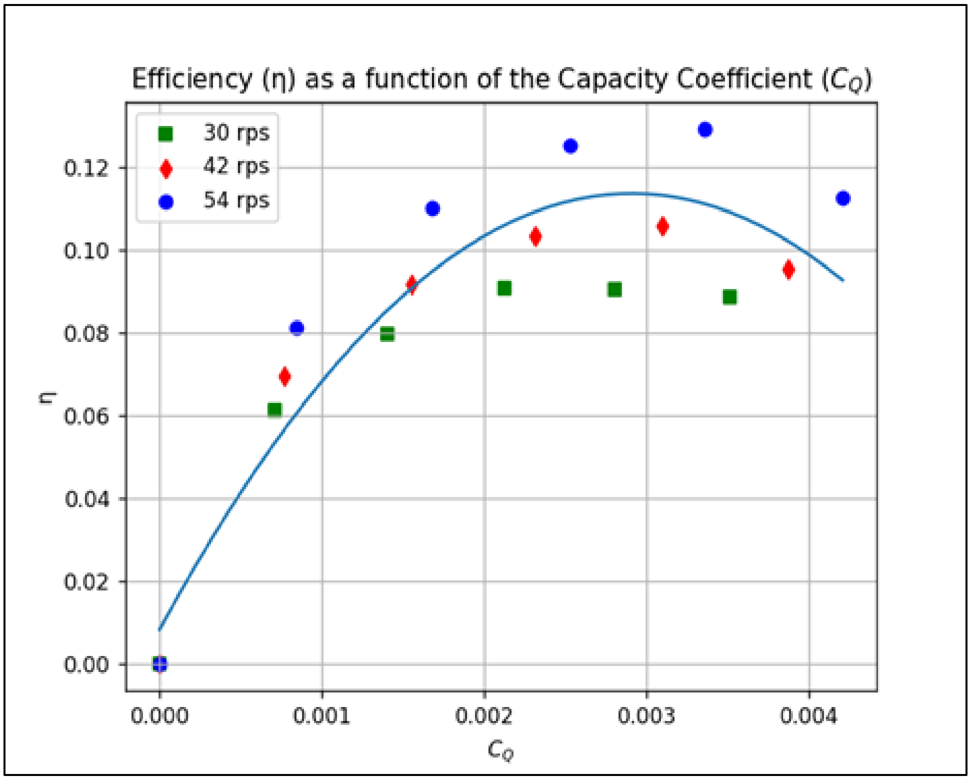

Figure 8: Efficiency as a function of the Capacity Coefficient

After the production of the graphs, there are many conclusions to be drawn from these results. Firstly, the head coefficient decreases exponentially as the capacity coefficient increases—meaning that as the efficiency at allowing a fluid (air, in this case) to flow increases, then the air height (head) generated by the fan given certain operating conditions decreases more and more rapidly. In fact, thanks to the Python script done for these computations, the function can be modeled by the equation: $f(x) = (-1.52 \cdot 10^5)x^2 - 315.3x + 11.06$, where for every unit of the capacity coefficient the head coefficient decreases by $3.04 \cdot 10^5$ units $\left(\frac{dC_H}{dC_Q}(f(x)) = (-3.04 \cdot 10^5)x - 315.3\right)$.

As for the plot for the power coefficient, the deductions to be made are quite straightforward. It can be observed that the correlation between the flow coefficient and the power coefficient are positively linear. This seems rational—for a user's fan to output more air, the fan will require more power in order to do so. Nonetheless, the function for the line of best fit is as follows:

$f(x) = 2232x + 3.13.$

Lastly, efficiency as a function of the capacity coefficient is quite interesting because the line of best fit is of a quadratic power, but it differs quantitatively and qualitatively compared to head and capacity coefficients' relationship. Essentially, a sound conclusion based on the plot is: efficiency increases as the flow increases up until a certain point—that can be deemed as peak efficiency—and then efficiency promptly decreases in magnitude as flow continues to increase. Since the quadratic curve that best aligns with the data is modeled by $f(x) = (-1.25 \cdot 10^4)x^2 + 72.51x + 0.008$, it can be calculated that the peak efficiency (η) is found to be 0.114%, occurring at a C_Q of 0.0029.

3.

For the experiment, a given candidate operating point with a specific flow rate and head was provided. The specs of the candidate operating point were $0.1\frac{m^3}{s}$ and 116m of air of flow rate and head, respectively. Using the data obtained from the experiment, it shall be determined whether the fan can achieve these metrics within any speed between 1200 and 3600 RPM—and, if based on other performance metrics *if* is suitable for the fan to be operating at such a point. Recall that 1 Hz = 60 rpm, and so the fan speeds between 1200 and 3600 RPM are comparable to speeds of 20 and 60 Hz. Given the head equation, one can substitute and find that:

$$116m = \frac{\Delta p}{12\,\frac{N}{m^3}} + \frac{V_2^2}{2 \cdot 9.81\,\frac{m}{s^2}}$$

$$116m = \frac{\frac{Q}{\beta = 50\,\frac{SCFM}{inH_2O}}}{12\,\frac{N}{m^3}} + \frac{V_2^2}{2 \cdot 9.81\,\frac{m}{s^2}}$$

$$V_2 = \sqrt{2275.92 - 8.1384Q}$$

After algebraic manipulation (and some conversion), it is determined that the fan speed must operate at a speed of $47.7\frac{m}{s}$, or 1811 RPM, or approximately 30.2 Hz. Thus, in conclusion, not only is the fan operating within the bounds of 1200 to 3600 RPM (or 20 to 60 Hz), but the value of the fan speed is relatively sustainable and perhaps could be deemed to be on the lower end of fan speed.

4.

Uncertainty analysis aims at pinpointing the variability of the output, which is the airflow for this case, due to the variability of the input—similarly, the inputs are preliminarily outlined as the change in pressure of the LFE and the temperature from the fan. The airflow, a function of those two metrics, is defined as:

$$Q = func(\Delta P_{LFE}, T_f)$$

After expanding the equation for the error of propagation for uncertainty analysis, the equation then becomes:

$$u_Q = Q\sqrt{\left(\frac{\delta Q}{\delta \Delta P_{LFE}} u_{\Delta P_{LFE}}\right)^2 + \left(\frac{\delta Q}{\delta T_f} u_{T_f}\right)^2}$$

Ultimately,

Table 8: Uncertainty Analysis

Frequency [Hz]	Δp **[Pa]**	**Q** $[\frac{m^3}{s}]$
30	808.600	$1.340 \cdot 10^{-2} \pm 9.3 \cdot 10^{-5}$
	1082.280	$1.070 \cdot 10^{-2} \pm 9.3 \cdot 10^{-5}$
	1318.640	$8.10 \cdot 10^{-3} \pm 9.4 \cdot 10^{-5}$
	1455.480	$5.350 \cdot 10^{-3} \pm 9.4 \cdot 10^{-5}$
	1803.800	$2.70 \cdot 10^{-3} \pm 9.4 \cdot 10^{-5}$
	1803.800	$0.00 \pm 9.5 \cdot 10^{-5}$
42	1368.400	$2.070 \cdot 10^{-2} \pm 9.1 \cdot 10^{-5}$
	2040.160	$1.650 \cdot 10^{-2} \pm 8.9 \cdot 10^{-5}$
	2500.440	$1.240 \cdot 10^{-2} \pm 8.9 \cdot 10^{-5}$
	2836.320	$8.30 \cdot 10^{-3} \pm 8.8 \cdot 10^{-5}$
	3072.680	$4.10 \pm 8.8 \cdot 10^{-5}$
	3321.480	$0.00 \pm 8.9 \cdot 10^{-5}$
54	2027.720	$2.890 \cdot 10^{-2} \pm 8.2 \cdot 10^{-5}$
	3234.400	$2.310 \cdot 10^{-2} \pm 8.2 \cdot 10^{-5}$
	4080.320	$1.740 \cdot 10^{-2} \pm 8.1 \cdot 10^{-5}$
	4702.320	$1.120 \cdot 10^{-2} \pm 8.1 \cdot 10^{-5}$
	5075.520	$5.80 \cdot 10^{-3} \pm 8.1 \cdot 10^{-5}$
	5087.960	$0.00 \pm 8.3 \cdot 10^{-5}$

Conclusion

In conclusion, this experiment demonstrated some principles and bases governing scaling laws for pumps and fans (affinity laws). The website Science Direct explicitly states, "If pump speed is changed, the flow rate is directly proportional to the speed, whereas the head is directly proportional to the square of the speed. As with diameter change, the BHP is proportional to the cube of the impeller speed." Furthermore, the plots from above indeed resemble many pump curves found online, whether for airflow vs. head or airflow vs. brake horsepower.

First, what can be observed is that for each relatively large fan speed, the starting metric, whether it was brake horsepower, head, or efficiency, was more significant than the last, lower fan speed (i.e., the efficiency for 54 RPS is larger than the one for 30 RPS). Second, it can be noted that the head, as a function of airflow, decreases exponentially where the faster the fan's impeller rotates, the steeper the head loss the system experiences. Similarly, power seemed to be the clearest trend to be observed. Naturally, if one wants to achieve more airflow or have a faster fan speed, the fan requires more power: and, again, the faster the impeller rotates, the more rapid the change in watts for SCFM is.

Lastly, efficiency details one of the most complex relationships between the other metrics vs. airflow. As mentioned in the report, efficiency experiences a quadratic relationship with airflow but differs in that the function has a global maximum. Meaning, that for whatever speed the fan operates at, as that speed and toleration of flow (as in, the more that is allowed to flow) increases, so does the fan's efficiency. Recall that *efficiency* is the useful power output divided by the total power consumed. Ultimately, efficiency increases up to a certain point, that maximum where the exact change in efficiency for airflow/capacity coefficient is zero ($\frac{d\eta}{dQ}$ or $\frac{d\eta}{dC_Q} = 0$). Then, after that point, the efficiency decreases to a point and at a rate comparable to the portion of the function before the maximum occurs. Knowing that this relation's parent function is $-x^2$, anyone can substitute values into previously stated equations to find the most optimal and suitable point (the global maxima).

On that same note, every line of best fit (via linear or polynomial regression) can be found throughout the report—and if not—in the appendices section. Ergo, given the parameters of the laboratory conditions and the fan specs, any other person wishing to replicate this experiment can substitute variables into said equations to find the appropriate metrics.

References

Bleier, Greene, Hernandez-Acosta, Taylor. *Determining the Discharge Coefficient of an Orifice Plate*. Texas Tech University, 2022.

ScienceDirect Topics. *Affinity Laws – an Overview*. ScienceDirect, 2018. https://www.sciencedirect.com/topics/engineering/affinity-law

White, Frank M. *Fluid Mechanics*. McGraw-Hill Engineering, 2011.

Appendices

Appendix A – Equations

Equation A.1: Q_{LFE}

$$Q_{LFE} = \beta \Delta P_{LFE}$$

$$Q_{LFE} = \frac{400\ SCFM}{8\ inH_2O} \cdot \Delta P_{LFE}$$

Equation A.2: H

$$H = \frac{\Delta p}{\gamma} + \frac{V_2^2}{2g}$$

$$H = \frac{3.250\ inH_2O \cdot \frac{248.8\ Pa}{1\ inH_2O}}{12\ \frac{N}{m^3}} + \frac{\left(30rps \cdot 2\pi r_{impeller}\right)^2}{2 \cdot 9.81\ \frac{m}{s^2}}$$

Equation A.3: Bhp

$$Bhp = Power \cdot 1000 = Bhp\ in\ W$$

Equation A.4: η

$$\eta = \frac{\gamma Q H}{Bhp}$$

$$\eta = \frac{12 \frac{N}{m^3} \cdot 0.0134 \frac{m^3}{s} \cdot 181.929\ m}{330\ W}$$

Equation A.5: C_H

$$C_H = \frac{gH}{n^2D^2}$$

$$C_H = \frac{9.81\frac{m}{s^2} \cdot 181.929m}{30\frac{rev^2}{s} \cdot 0.503m^2}$$

Equation A.6: C_Q

$$C_Q = \frac{Q}{nD^3}$$

$$C_Q = \frac{0.0134\ \frac{m^3}{s}}{30\frac{rev^2}{s} \cdot 0.503m^3}$$

Equation A.7: C_P

$$C_P = \frac{Bhp}{\rho n^3 D^5}$$

$$C_P = \frac{330W}{1.2754\frac{kg}{m^3} \cdot 30\ \frac{rev^3}{s} \cdot 0.503^5}$$

Equation A.8: u_Q

$$u_Q = Q\sqrt{\left(\frac{\delta Q}{\delta \Delta P_{LFE}} u_{\Delta P_{LFE}}\right)^2 + \left(\frac{\delta Q}{\delta T_f} u_{T_f}\right)^2}$$

Appendix B – Python Codes

Code B.1: Head as a Function of Air Flow

```
# Importing Libraries
import numpy as np
import matplotlib.pyplot as plt

def fit_and_plot(Q_actual, Head, speed_label):
    Q_actual = np.array(Q_actual)
    Head = np.array(Head)

    model = np.poly1d(np.polyfit(Q_actual, Head, degree_of_fit))
    polyline = np.linspace(0, Q_actual[0], num_points)

    ax.scatter(Q_actual, Head)
    ax.plot(polyline, model(polyline), label=speed_label)

    print(f"Model for {speed_label}: {model}")
    return model

degree_of_fit = 2
num_points = 100

# Fan speed data
fan_speeds = [
    ([0.0134, 0.0107, 0.0081, 0.00535, 0.0027, 0], [181.92899, 204.73565, 224.43232,
235.83565, 264.86232, 264.86232], '30 rps'),
    ([0.0207, 0.01655, 0.01235, 0.0083, 0.0041, 0], [338.54322, 394.52322, 432.87989,
460.86989, 480.56655, 501.29989], '42 rps'),
    ([0.0289, 0.02305, 0.01735, 0.01155, 0.0058, 0], [540.10476, 640.66142, 711.15476,
762.98809, 794.08809, 795.12476], '54 rps')
]

fig, ax = plt.subplots()

for Q_actual, Head, speed_label in fan_speeds:
    fit_and_plot(Q_actual, Head, speed_label)

# Overall Plot Settings
ax.set_xlabel('Flow rate, Q [m$^3$/s]')
ax.set_ylabel('Head, H [m of air]')
ax.set_title('Head as a Function of Flow Rate')
ax.legend(loc='upper right')
ax.grid(True)
plt.show()
```

Code B.2: Brake Horsepower as a Function of Air Flow

```
# Importing Libraries
import numpy as np
import matplotlib.pyplot as plt

def fit_and_plot(Q_actual, Bhp, speed_label):
    model = np.poly1d(np.polyfit(Q_actual, Bhp, degree_of_fit))
    polyline = np.linspace(0, Q_actual[0], num_points)

    plt.scatter(Q_actual, Bhp)
    plt.plot(polyline, model(polyline), label=speed_label)

    print(f"Model for {speed_label}: {model}")
    return model

degree_of_fit = 1
num_points = 100

# Fan speed data
fan_speeds = [
    ([0.0134, 0.0107, 0.0081, 0.00535, 0.0027, 0], [330, 290, 240, 190, 140, 110], '30
rps'),
    ([0.0207, 0.01655, 0.01235, 0.0083, 0.0041, 0], [880, 740, 620, 500, 340, 250], '42
rps'),
    ([0.0289, 0.02305, 0.01735, 0.01155, 0.0058, 0], [1660, 1370, 1180, 960, 680, 470], '54
rps')
]

fig, ax = plt.subplots()

for Q_actual, Bhp, speed_label in fan_speeds:
    fit_and_plot(Q_actual, Bhp, speed_label)

# Overall Plot Settings
ax.set_xlabel('Flow rate, Q [m$^3$/s]')
ax.set_ylabel('Brake Horsepower, Bhp [W]')
ax.set_title('Power as a Function of Flow Rate')
ax.legend(loc='upper left')
ax.grid(True)
plt.show()
```

Code B.3: Efficiency as a Function of Air Flow

```
# Importing Libraries
import numpy as np
import matplotlib.pyplot as plt

def fit_and_plot(Q_actual, efficiency, speed_label):
    model = np.poly1d(np.polyfit(Q_actual, efficiency, degree_of_fit))
    polyline = np.linspace(0, Q_actual[0], num_points)

    plt.scatter(Q_actual, efficiency)
    plt.plot(polyline, model(polyline), label=speed_label)

    print(f"Model for {speed_label}: {model}")
    return model

degree_of_fit = 2
num_points = 100

# Fan speed data
fan_speeds = [
    ([0.0134, 0.0107, 0.0081, 0.00535, 0.0027, 0], [0.088649034, 0.090648476, 0.09089509,
0.079687626, 0.061296708, 0], '30 rps'),
    ([0.0207, 0.01655, 0.01235, 0.0083, 0.0041, 0], [0.095561518, 0.105881502, 0.103472257,
0.091805281, 0.069540807, 0], '42 rps'),
    ([0.0289, 0.02305, 0.01735, 0.01155, 0.0058, 0], [0.112836343, 0.129348139,
0.125476628, 0.110156406, 0.081277252, 0], '54 rps')
]

fig, ax = plt.subplots()

for Q_actual, efficiency, speed_label in fan_speeds:
    fit_and_plot(Q_actual, efficiency, speed_label)

# Overall Plot Settings
ax.set_xlabel('Flow rate, Q [m$^3$/s]')
ax.set_ylabel('Efficiency, η')
ax.set_title('Efficiency as a Function of Flow Rate')
ax.legend(loc='upper left')
ax.grid(True)
plt.show()
```

Code B.4: Head Coefficient as a Function of the Capacity Coefficient

```
# Importing Libraries
import numpy as np
import matplotlib.pyplot as plt

def plot_fan_speed(ax, C_of_Q, C_of_H, label, color, marker):
    ax.scatter(C_of_Q, C_of_H, color=color, marker=marker, label=label)

# Fan speed data
fan_speeds = [
    (30, [0.003509778, 0.002802584, 0.002121582, 0.00140129, 0.000707194, 0],
         [7.837768449, 8.820313207, 9.668874589, 10.16014697, 11.41065848, 11.41065848]),
    (42, [0.003872729, 0.003096312, 0.002310541, 0.001552833, 0.000767062, 0],
         [7.441295952, 8.671755529, 9.514848201, 10.13007799, 10.56301747, 11.01874324]),
    (54, [0.00420533, 0.003354078, 0.002524653, 0.001680677, 0.000843976, 0],
         [7.18163818, 8.518715081, 9.456047342, 10.14526224, 10.55879118, 10.57257548])
]

# Combine data into 1D vectors
CofQ = np.concatenate([C_of_Q for _, C_of_Q, _ in fan_speeds])
CofH = np.concatenate([C_of_H for _, _, C_of_H in fan_speeds])

# Overall plot settings
fig, ax = plt.subplots()
ax.set_xlabel('$C_{Q}$')
ax.set_ylabel('$C_{H}$')
ax.set_title('Head Coefficient ($C_{H}$) as a function of the Capacity Coefficient
($C_{Q}$)')
ax.grid(True)

# Plot each fan speed
for speed, C_of_Q, C_of_H in fan_speeds:
    color = "green" if speed == 30 else ("red" if speed == 42 else "blue")
    marker = "s" if speed == 30 else ("d" if speed == 42 else "o")
    label = f"{speed} rps"
    plot_fan_speed(ax, C_of_Q, C_of_H, label, color, marker)


# Fit a second degree polynomial to the data
model = np.poly1d(np.polyfit(CofQ, CofH, 2))
# Plot the curve fit
polyline = np.linspace(0, max(CofQ))
ax.plot(polyline, model(polyline), label="Curve Fit")

ax.legend(loc='upper right')
plt.show()
```

Code B.5: Power Coefficient as a Function of Air Flow

```
# Importing Libraries
import numpy as np
import matplotlib.pyplot as plt

def plot_fan_speed(ax, C_of_Q, C_of_P, label, color, marker):
    ax.scatter(C_of_Q, C_of_P, color=color, marker=marker, label=label)

# Fan speed data
fan_speeds = [
    (30, [0.003509778, 0.002802584, 0.002121582, 0.001401292, 0.000707194, 0],
         [8.928635473, 7.846376628, 6.493553071, 5.140729515, 3.787905958, 2.976211824]),
    (42, [0.003872729, 0.003096312, 0.002310541, 0.001552833, 0.000767062, 0],
         [12.14780336, 10.21519828, 8.558679643, 6.902161003, 4.693469482, 3.451080501]),
    (54, [0.00420533, 0.003354078, 0.002524653, 0.001680677, 0.000843976, 0],
         [13.86226607, 11.44054489, 9.85389998, 8.016732187, 5.678518632, 3.924858466])
]

# Combine data into 1D vectors
CofQ = np.concatenate([C_of_Q for _, C_of_Q, _ in fan_speeds])
CofP = np.concatenate([C_of_P for _, _, C_of_P in fan_speeds])

# Overall plot settings
fig, ax = plt.subplots()
ax.set_xlabel('$C_{Q}$')
ax.set_ylabel('$C_{P}$')
ax.set_title('Power Coefficient ($C_{P}$) as a function of the Capacity Coefficient
($C_{Q}$)')
ax.grid(True)

# Plot each fan speed
for speed, C_of_Q, C_of_P in fan_speeds:
    color = "green" if speed == 30 else ("red" if speed == 42 else "blue")
    marker = "s" if speed == 30 else ("d" if speed == 42 else "o")
    label = f"{speed} rps"
    plot_fan_speed(ax, C_of_Q, C_of_P, label, color, marker)

# Fit a first degree polynomial to the data
model = np.poly1d(np.polyfit(CofQ, CofP, 1))
# Plot the curve fit
polyline = np.linspace(0, max(CofQ), num=100)
ax.plot(polyline, model(polyline), label="Curve Fit")

ax.legend(loc='upper left')
plt.show()
```

Code B.6: Efficiency as a Function of Capacity Coefficient

```
# Importing Libraries
import numpy as np
import matplotlib.pyplot as plt

def plot_fan_speed(ax, C_of_Q, n_values, label, color, marker):
    ax.scatter(C_of_Q, n_values, color=color, marker=marker, label=label)

# Fan speed data
fan_speeds = [
    (30, [0.003509778, 0.002802584, 0.002121582, 0.001401292, 0.000707194, 0],
         [0.088649034, 0.090648476, 0.09089509, 0.079687626, 0.061296708, 0]),
    (42, [0.003872729, 0.003096312, 0.002310541, 0.001552833, 0.000767062, 0],
         [0.095561518, 0.105881502, 0.103472257, 0.091805281, 0.069540807, 0]),
    (54, [0.00420533, 0.003354078, 0.002524653, 0.001680677, 0.000843976, 0],
         [0.112836343, 0.129348139, 0.125476628, 0.110156406, 0.081277252, 0])
]

# Combine data into 1D vectors
CofQ = np.concatenate([C_of_Q for _, C_of_Q, _ in fan_speeds])
n = np.concatenate([n_values for _, _, n_values in fan_speeds])

# Overall plot settings
fig, ax = plt.subplots()
ax.set_xlabel('$C_{Q}$')
ax.set_ylabel('η')
ax.set_title('Efficiency (η) as a function of the Capacity Coefficient ($C_{Q}$)')
ax.grid(True)

# Plot each fan speed
for speed, C_of_Q, n_values in fan_speeds:
    color = "green" if speed == 30 else ("red" if speed == 42 else "blue")
    marker = "s" if speed == 30 else ("d" if speed == 42 else "o")
    label = f"{speed} rps"
    plot_fan_speed(ax, C_of_Q, n_values, label, color, marker)

# Fit a second degree polynomial to the data
model = np.poly1d(np.polyfit(CofQ, n, 2))

# Plot the curve fit
polyline = np.linspace(0, max(CofQ), num=100)
ax.plot(polyline, model(polyline), label="Curve Fit")

# Display the legend
ax.legend(loc='upper left')
plt.show()
```