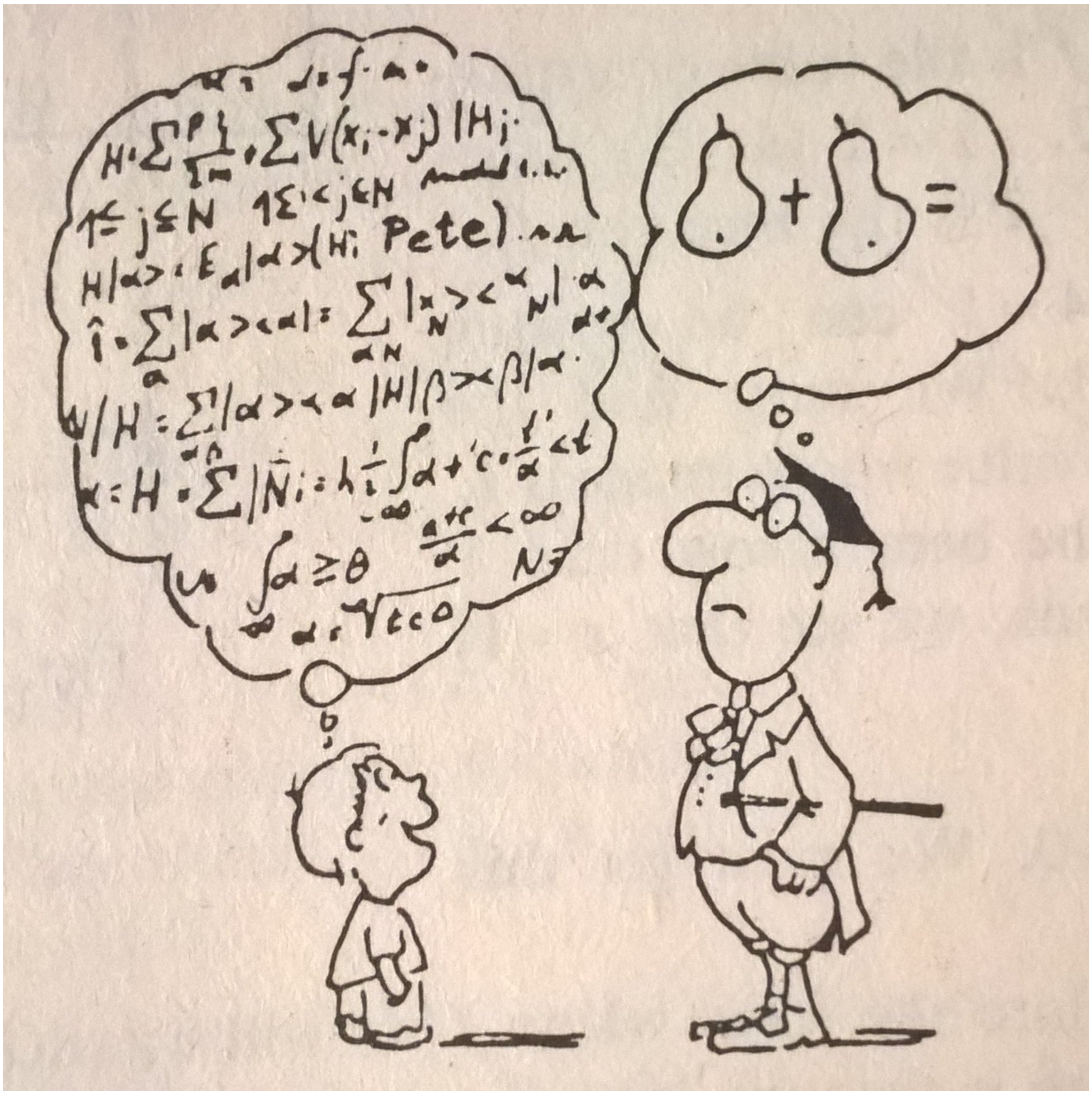
MATH BASICS



1 Definition

1.1 Exponents

1.

$$a^n = a \times a \times a \times a \; .... \; n \quad factors \qquad (n \in \mathbb{N}, \; a \in \mathbb{R})$$

2.

$$a^{-m} = \frac{1}{a^m} \qquad (m \in \mathbb{Z}^+, \; a \in \mathbb{R}, \; a \neq 0)$$

and

$$\frac{1}{a^{-m}} = a^m$$

3.

$$a^0 = 1 \qquad (a \in \mathbb{R}, \; a \neq 0)$$

1.2 Rational Exponents:

1.

$$\sqrt[n]{a} = r \qquad (a > 0, \; n \in \mathbb{N}, \; n \geqslant 2, \; r > 0), \iff r^n = a$$

2.

$$a^{\frac{1}{n}} = \sqrt[n]{a}; \qquad (a > 0, \; n \geqslant 2, \; n \in \mathbb{N})$$

3.

$$a^{\frac{-1}{n}} = \sqrt[n]{a^{-1}}; \qquad (a > 0, \; n > 0, \; n \in \mathbb{N})$$

4.

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}; \qquad (a > 0; \; m, \; n \in \mathbb{Z}, \; n \geqslant 2)$$

2 Law

2.1 Exponents

1.

$$a^m \times a^n = a^{m+n} \qquad (m,\ n \in \mathbb{N})$$
$$a^m \times a^n = a^{m+n} \qquad (m,\ n \in \mathbb{Z};\ a \neq 0,\ if \quad m \quad or \quad n < 0)$$

2.

$$\frac{a^m}{a^n} = a^{m-n} \qquad (m,\ n \in \mathbb{Z};\ a \in \mathbb{R};\ a \neq 0)$$

3.

$$(ab)^m = a^m\, b^m \qquad (m \in \mathbb{Z})$$

4.

$$(a^m)^n = a^{mn} \qquad (m,\ n \in \mathbb{Z})$$

2.2 Rational Exponents

1.

$$a^r \times a^t = a^{r+t} \qquad (a > 0; \quad r,\ t \in \mathbb{Q})$$

2.

$$\frac{a^r}{a^t} = a^{r-t} \qquad (a > 0; \quad r,\ t \in \mathbb{Q})$$

3.

$$(a^t)^r = a^{tr} \qquad (a > 0, \quad t,\ r \in \mathbb{Q})$$

4.

$$(ab)^t = a^t\, b^t\ ; \quad \left(\frac{a}{b}\right)^t = \frac{a^t}{b^t}\ ; \qquad (a,\ b > 0, \quad t \in \mathbb{Q})$$

and

$$a^t\, b^t = (ab)^t \qquad and \qquad \frac{a^t}{b^t} = \left(\frac{a}{b}\right)^t$$

2.3 Distributive law

$$a(b + c) = ab + ac$$

$$\begin{aligned}(a + b)(c + d) &= (a + b)c + (a + b)d \\ &= ac + bc + ad + bd\end{aligned}$$

$$A^2 - B^2 = (A - B)(A + B)$$

2.4 Commutative law

$$ab = ba$$

3 Properties

3.1 Addition

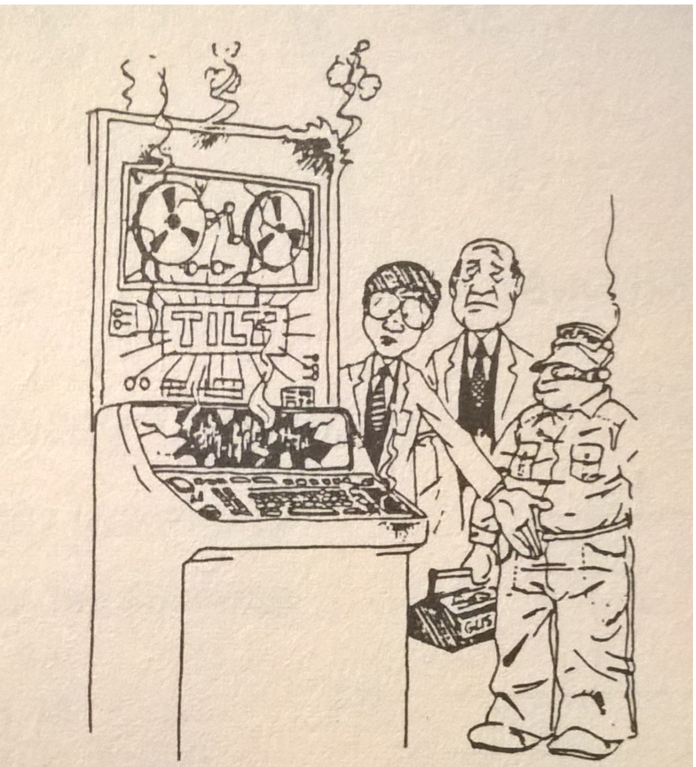
$$0 + a = a$$
$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b} \qquad (b \neq 0)$$
$$\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b} \qquad (b \neq 0)$$

3.2 Multiplication

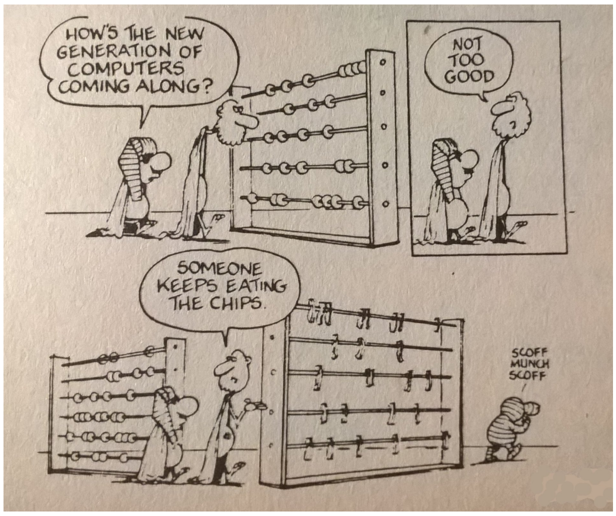
$$0 \times a = 0$$
$$\frac{0}{a} = 0 \times \frac{1}{a} = 0$$
$$1 \times a = a$$
$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \qquad (b \neq 0;\ d \neq 0)$$

3.3 Division

$$\frac{a}{0} = undefined$$
$$\frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \times \frac{s}{r} = \frac{ps}{qr} \qquad (q \neq 0;\ r \neq 0;\ s \neq 0)$$



"All I did was ask it to divide by zero!"



4 Examples

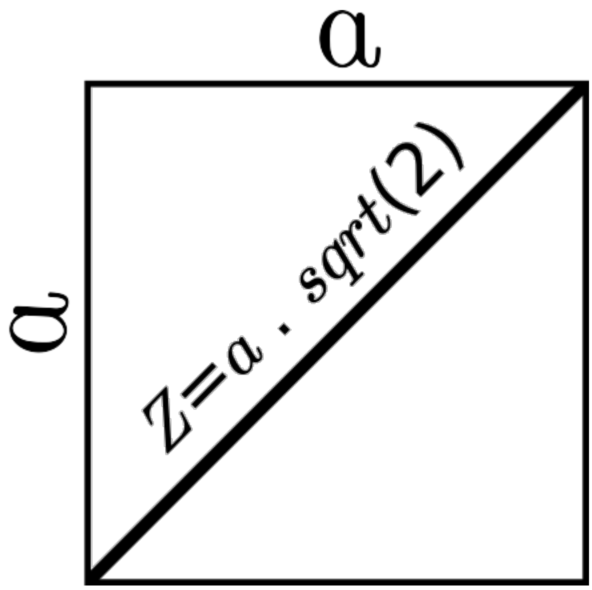
$$\sqrt{a^2} = a \qquad (a > 0) \qquad\qquad \sqrt{a^n} = a^{\frac{n}{2}}$$

$$\sqrt{\frac{1}{a}} = \frac{1}{\sqrt{a}}$$

$$a^{\frac{m}{1}} = a^m \qquad\qquad \frac{a^m}{a^n} = a^{m-n}$$

$$\frac{a \angle\alpha^\circ \; b \angle\beta^\circ}{c \angle\gamma^\circ} = \frac{a \times b}{c} \quad \angle\left(\alpha^\circ + \beta^\circ - \gamma^\circ\right)$$

$$If \quad a \, . \, b = 0, \quad then \quad a = 0 \quad or \quad b = 0$$



Square Property

$$Z = \sqrt{a^2 \ + \ a^2}$$
$$Z = \sqrt{2 \, a^2}$$
$$Z = \sqrt{2} \quad \sqrt{a^2}$$
$$Z = \sqrt{2} \quad a$$

$$a \angle\alpha \ + \ b \angle\beta \ = \ \sqrt{(a \ \sin\alpha \ + \ b \ \sin\beta)^2 \ + \ (a \ \cos\alpha \ + \ b \ \cos\beta)^2}$$

and

$$\angle \arctan(\frac{(a \ \sin\alpha \ + \ b \ \sin\beta)}{(a \ \cos\alpha \ + \ b \ \cos\beta)})$$

$$lim_{s \to 0} \, GH(s) = lim_{s \to 0} \quad 7 \, \frac{4 + 3s}{5 + 2s + 6s^2}$$

$$= lim_{s \to 0} \quad 7 \, \frac{4(1 + \frac{3}{4}s)}{5(1 + \frac{2}{5}s + \frac{6}{5}s^2)}$$

$$= 7 \, \frac{4}{5}$$

$$y'(x) = \mathrm{e}^{4x+5}$$
$$= 4 \, \mathrm{e}^{4x+5}$$

Derivatives made easy.

$$y = \cos^3(\operatorname{sen}(x^2+x))$$

$$y' = \left.(x^3)'\right|_{x=\cos(\operatorname{sen}(x^2+x))} \cdot \left.\cos(x)'\right|_{x=\operatorname{sen}(x^2+x)} \cdot \left.\operatorname{sen}(x)'\right|_{x=x^2+x} \cdot (x^2+x)'$$

$$= \left.3x^2\right|_{x=\cos(\operatorname{sen}(x^2+x))} \cdot \left.(-1)\cdot\operatorname{sen}(x)\right|_{x=\operatorname{sen}(x^2+x)} \cdot \left.\cos(x)\right|_{x=x^2+x} \cdot (2x+1)$$

$$= 3\cos^2(\operatorname{sen}(x^2+x)) \cdot (-1) \cdot \operatorname{sen}(\operatorname{sen}(x^2+x)) \cdot \cos(x^2+x) \cdot (2x+1)$$

$$= -(6x+3) \cdot \cos^2(\operatorname{sen}(x^2+x)) \cdot \operatorname{sen}(\operatorname{sen}(x^2+x)) \cdot \cos(x^2+x)$$

5 Methods

HCF - highest common factor (pôr variável em evidência)
CF - Common factors
Factorisation
LCD or LCM - Lowest common denominator or lowest common multiple