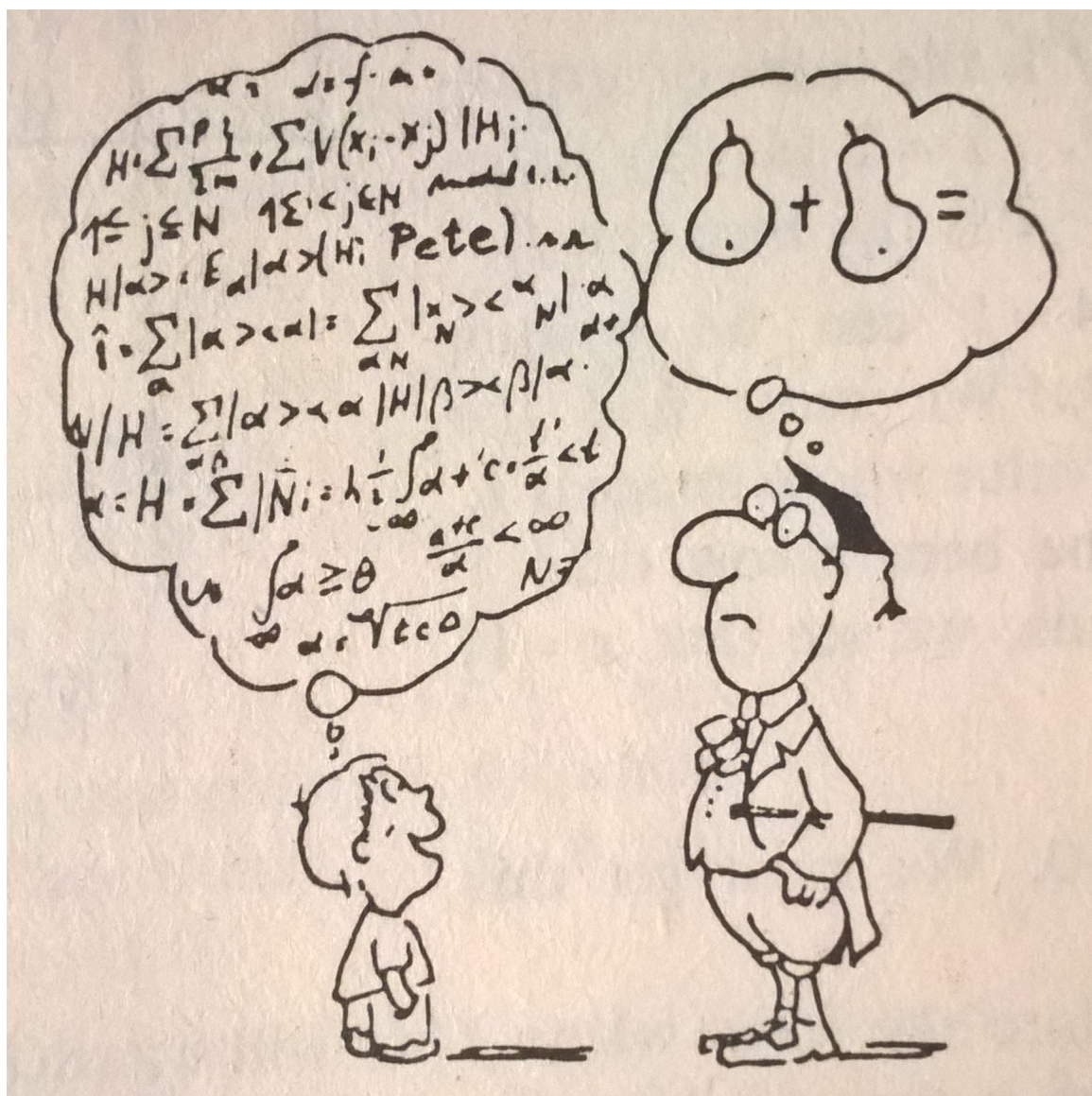


MATH BASICS



1 Definition

1.1 Exponents

1.

$$a^n = a \times a \times a \times a \dots n \text{ factors} \quad (n \in \mathbb{N}, a \in \mathbb{R})$$

2.

$$a^{-m} = \frac{1}{a^m} \quad (m \in \mathbb{Z}^+, a \in \mathbb{R}, a \neq 0)$$

and

$$\frac{1}{a^{-m}} = a^m$$

3.

$$a^0 = 1 \quad (a \in \mathbb{R}, a \neq 0)$$

1.2 Rational Exponents:

1.

$$\sqrt[n]{a} = r \quad (a > 0, n \in \mathbb{N}, n \geq 2, r > 0), \iff r^n = a$$

2.

$$a^{\frac{1}{n}} = \sqrt[n]{a}; \quad (a > 0, n \geq 2, n \in \mathbb{N})$$

3.

$$a^{\frac{-1}{n}} = \sqrt[n]{a^{-1}}; \quad (a > 0, n > 0, n \in \mathbb{N})$$

4.

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}; \quad (a > 0; m, n \in \mathbb{Z}, n \geq 2)$$

2 Law

2.1 Exponents

1.

$$a^m \times a^n = a^{m+n} \quad (m, n \in \mathbb{N})$$

$$a^m \times a^n = a^{m+n} \quad (m, n \in \mathbb{Z}; a \neq 0, \text{ if } m \text{ or } n < 0)$$

2.

$$\frac{a^m}{a^n} = a^{m-n} \quad (m, n \in \mathbb{Z}; a \in \mathbb{R}; a \neq 0)$$

3.

$$(ab)^m = a^m b^m \quad (m \in \mathbb{Z})$$

4.

$$(a^m)^n = a^{mn} \quad (m, n \in \mathbb{Z})$$

2.2 Rational Exponents

1.

$$a^r \times a^t = a^{r+t} \quad (a > 0; r, t \in \mathbb{Q})$$

2.

$$\frac{a^r}{a^t} = a^{r-t} \quad (a > 0; r, t \in \mathbb{Q})$$

3.

$$(a^t)^r = a^{tr} \quad (a > 0, t, r \in \mathbb{Q})$$

4.

$$(ab)^t = a^t b^t; \quad \left(\frac{a}{b}\right)^t = \frac{a^t}{b^t}; \quad (a, b > 0, t \in \mathbb{Q})$$

and

$$a^t b^t = (ab)^t \quad \text{and} \quad \frac{a^t}{b^t} = \left(\frac{a}{b}\right)^t$$

2.3 Distributive law

$$a(b + c) = ab + ac$$

$$\begin{aligned}(a + b)(c + d) &= (a + b)c + (a + b)d \\ &= ac + bc + ad + bd\end{aligned}$$

$$A^2 - B^2 = (A - B)(A + B)$$

2.4 Commutative law

$$ab = ba$$

3 Properties

3.1 Addition

$$0 + a = a$$

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b} \quad (b \neq 0)$$

$$\frac{a}{b} - \frac{c}{b} = \frac{a - c}{b} \quad (b \neq 0)$$

3.2 Multiplication

$$0 \times a = 0$$

$$\frac{0}{a} = 0 \times \frac{1}{a} = 0$$

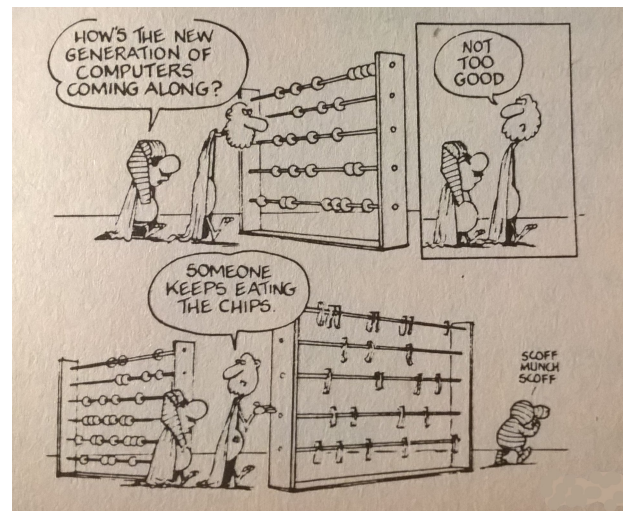
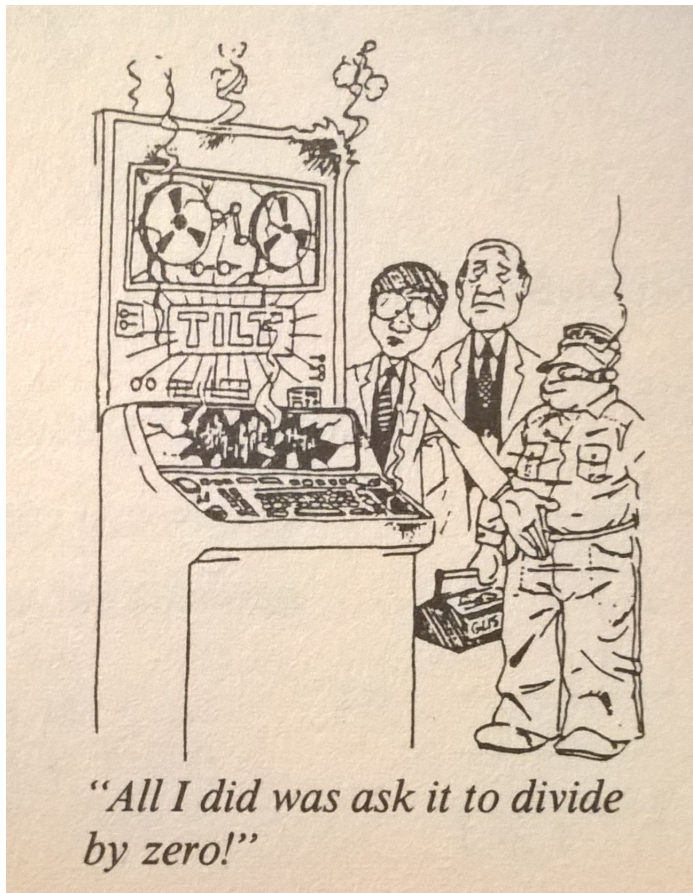
$$1 \times a = a$$

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \quad (b \neq 0; d \neq 0)$$

3.3 Division

$$\frac{a}{0} = \textit{undefined}$$

$$\frac{p}{q} \div \frac{r}{s} = \frac{p}{q} \times \frac{s}{r} = \frac{ps}{qr} \quad (q \neq 0; r \neq 0; s \neq 0)$$



4 Examples



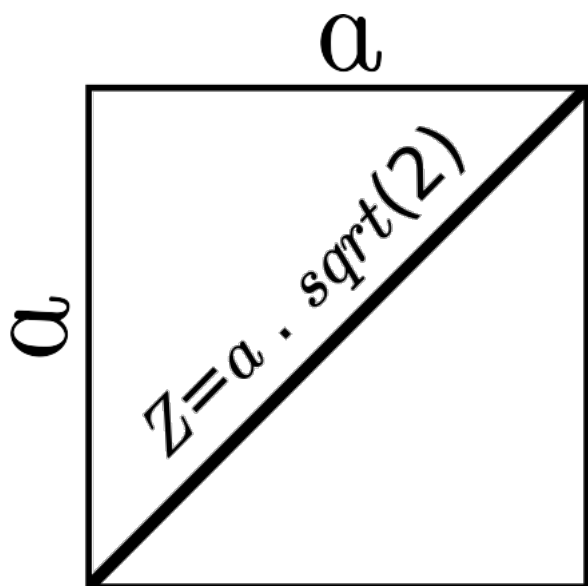
$$\sqrt{a^2} = a \quad (a > 0) \quad \sqrt{a^n} = a^{\frac{n}{2}}$$

$$\sqrt{\frac{1}{a}} = \frac{1}{\sqrt{a}}$$

$$a^{\frac{m}{1}} = a^m \quad \frac{a^m}{a^n} = a^{m-n}$$

$$\frac{a \angle \alpha^\circ \ b \angle \beta^\circ}{c \angle \gamma^\circ} = \frac{a \times b}{c} \angle (\alpha^\circ + \beta^\circ - \gamma^\circ)$$

$$\text{If } a \cdot b = 0, \text{ then } a = 0 \text{ or } b = 0$$



Square Property

$$Z = \sqrt{a^2 + a^2}$$

$$Z = \sqrt{2} a^2$$

$$Z = \sqrt{2} \quad \sqrt{a^2}$$

$$Z = \sqrt{2} \quad a$$

$$a \angle \alpha + b \angle \beta = \sqrt{(a \sin \alpha + b \sin \beta)^2 + (a \cos \alpha + b \cos \beta)^2}$$

and

$$\angle \arctan\left(\frac{(a \sin \alpha + b \sin \beta)}{(a \cos \alpha + b \cos \beta)}\right)$$

$$\lim_{s \rightarrow 0} GH(s) = \lim_{s \rightarrow 0} 7 \frac{4 + 3s}{5 + 2s + 6s^2}$$

$$= \lim_{s \rightarrow 0} 7 \frac{4(1 + \frac{3}{4}s)}{5(1 + \frac{2}{5}s + \frac{6}{5}s^2)}$$

$$= 7 \frac{4}{5}$$

$$\begin{aligned} y'(x) &= e^{4x+5} \\ &= 4 e^{4x+5} \end{aligned}$$

Derivatives made easy.

$$y = \cos^3(\sin(x^2+x))$$

$$y' = x^3 \cdot \cos(x) \cdot \sin(x) \cdot (x^2+x)'$$

$$\left| \begin{array}{l} x = \cos(\sin(x^2+x)) \\ x = \sin(x^2+x) \\ x = x^2+x \end{array} \right.$$

$$= 3x^2 \cdot (-1) \cdot \sin(x) \cdot \cos(x) \cdot (2x+1)$$

$$\left| \begin{array}{l} x = \cos(\sin(x^2+x)) \\ x = \sin(x^2+x) \\ x = x^2+x \end{array} \right.$$

$$= 3x^2 \cos(\sin(x^2+x)) \cdot (-1) \cdot \sin(\sin(x^2+x)) \cdot \cos(x^2+x) \cdot (2x+1)$$

$$= -(6x+3) \cdot \cos^2(\sin(x^2+x)) \cdot \sin(\sin(x^2+x)) \cdot \cos(x^2+x)$$

$$y = \frac{(x+1)^3}{x^{3/2}}$$

$$y' = \frac{(x+1)^3 \cdot x^{3/2} - x^{3/2} \cdot (x+1)^3}{x^3}$$

$$= \frac{3(x+1)^2 \cdot x^{3/2} - \frac{3}{2} x^{1/2} \cdot (x+1)^3}{x^3} \quad \text{HCF}$$

$$= \frac{[3(x+1)^2 \cdot x - \frac{3}{2}(x+1)^3] x^{1/2}}{x^3}$$

$$= \frac{3(x+1)^2 \cdot x - \frac{3}{2}(x+1)^3}{x^{5/2}} \quad \text{HCF}$$

$$= \frac{(x+1)^2 \cdot (3x - \frac{3}{2}(x+1))}{x^{5/2}}$$

$$= \frac{(x+1)^2 \cdot (3x - \frac{3}{2}x - \frac{3}{2})}{x^{5/2}}$$

$$= \frac{(x+1)^2 \cdot (\frac{3}{2}x - \frac{3}{2})}{x^{5/2}}$$

$$= \frac{(x+1)^2 \cdot \frac{3}{2}(x-1)}{x^{5/2}}$$

$$= \frac{3}{2} \cdot \frac{(x+1)^2 \cdot (x-1)}{x^{5/2}}$$

$$\text{L.O.} \quad \frac{3}{2} - \frac{2}{2} = \frac{1}{2}$$

$$x^{3/2} = x + x^{1/2}$$

$$\frac{1}{2} - \frac{6}{2} = -\frac{5}{2}$$

$$\frac{6}{2} - \frac{3}{2} = \frac{3}{2}$$

$$\int x = \frac{x^2}{2}$$

$$\int_{-1}^1 \sqrt{\frac{1+x}{1-x}} = \pi$$

$$e = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!} + \dots$$

5 Methods

HCF - highest common factor (pôr variável em evidência)

Factorisation

LCD or LCM - Lowest common denominator or lowest common multiple

