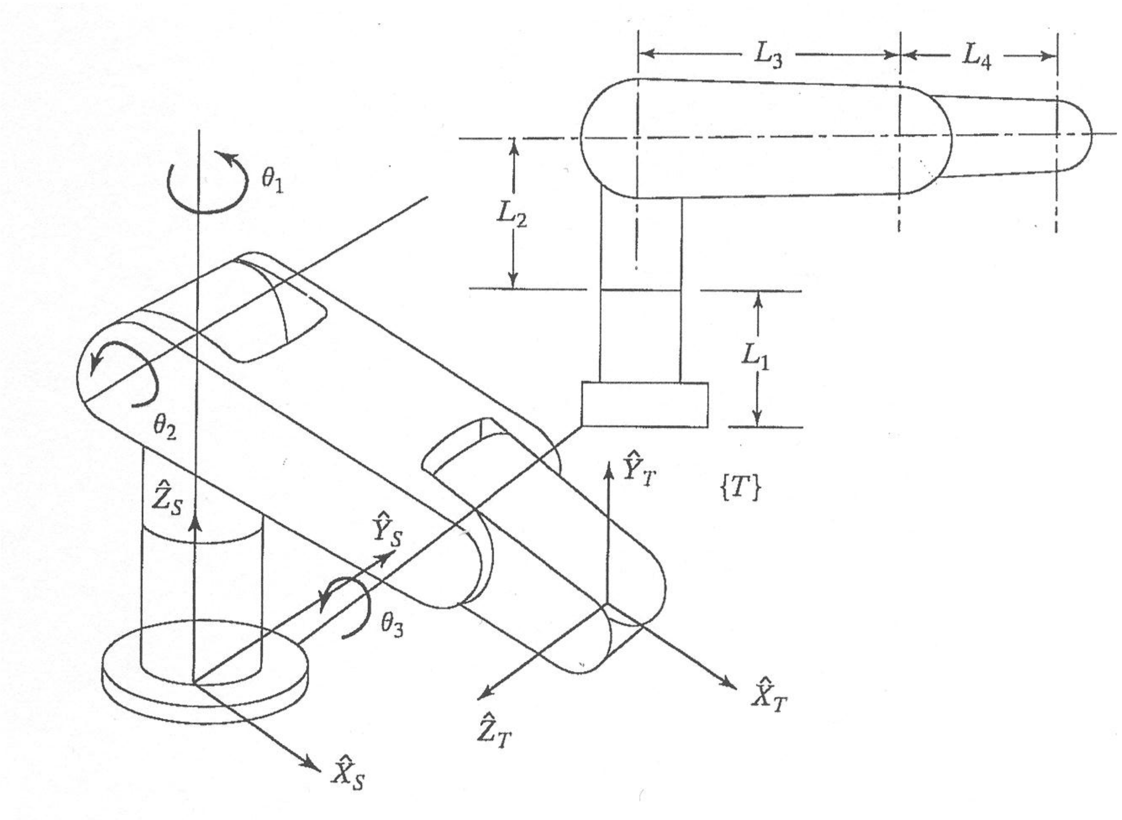
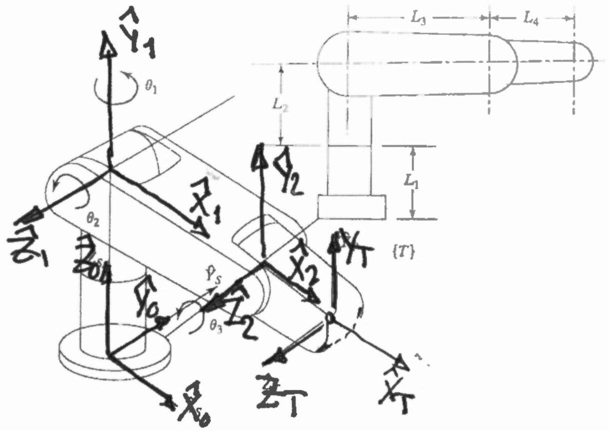
Robòtica Exercise

Given the following 3R robot



where L_1=4, L_2=3, L_3=2, and L_4=1.

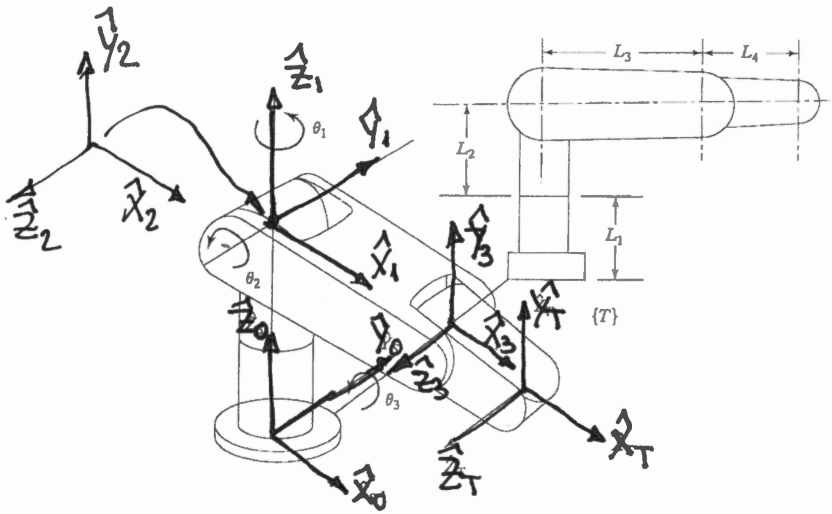
1) Derive the DH parameters and the neighbouring homogeneous transformation matrices ${}^{i-1}_{i}T$, for i=1,2,3, as functions of the joint angles.
2) Implement the forward kinematics, that is, ${}^{0}_{3}T(\theta_1, \theta_2, \theta_3)$.
3) Calculate the result for the following joint angles: (0, 0, 0), (0, π/2, 0), and (0, π/2, π/6).
4) Build the robot using the obtained DH parameters and compare the results using the forward kinematics associated method with that obtained multipliying the neighboring matrices.



Standard frame attachment

Joint/Link i	θ_i	d_i	a_i	α_i
1	θ_1	$L_1 + L_2$	0	$\pi/2$
2	θ_2	0	L_3	0
3	θ_3	0	L_4	0

Standard DH Table



Modified frame attachment

Joint/Link i	θ_i	d_i	a_{i-1}	α_{i-1}
1	θ_1	$L_1 + L_2$	0	0
2	θ_2	0	0	$\pi/2$
3	θ_3	0	L_3	0
4	0	0	L_4	0

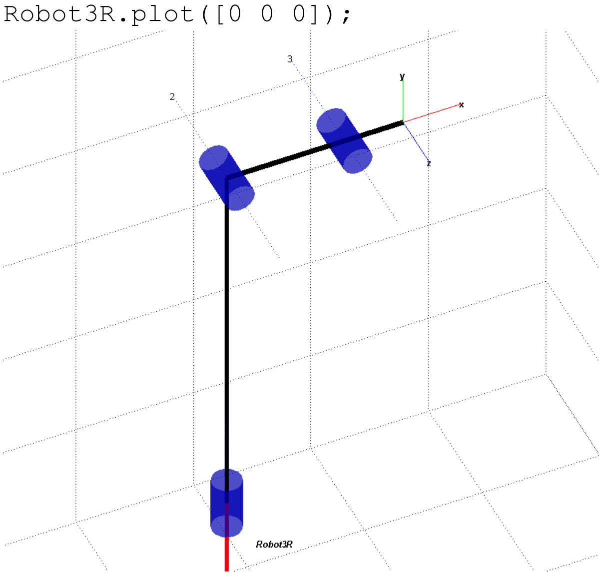
Modified DH Table

```
%%% First we clean the workspace

clear;
clc;

%%% We define the constants

L1 = 4;
L2 = 3;
L3 = 2;
L4 = 1;
%%% We introduce three symbols denoting the joint angles
a1 = sym('a1');
a2 = sym('a2');
a3 = sym('a3');
T01 = trotz(a1)*transl(0, 0, L1+L2)*trotx(pi/2);
T12 = trotz(a2)*transl(L3, 0, 0);
T23 = trotz(a3)*transl(L4, 0, 0);
T03 = T01*T12*T23;
T1 = subs(T03, {a1, a2, a3}, {0, 0, 0});
T2 = subs(T03, {a1, a2, a3}, {0, pi/2, 0});
T3 = subs(T03, {a1, a2, a3}, {0, pi/2, pi/6});
%% We build the robot using DH parameters
L(1) = Link([0 L1+L2    0  pi/2]);
L(2) = Link([0      0  L3     0]);
L(3) = Link([0      0  L4     0]);
Robot3R = SerialLink(L);
%%% Now we plot the robot to check if it has been correctly built
Robot3R.name = 'Robot3R';
Robot3R.plot([0 0 0]);
```




```
TT1 = Robot3R.fkine([0 0 0]);
TT2 = Robot3R.fkine([0 pi/2 0]);
TT3 = Robot3R.fkine([0 pi/2 pi/6]);

%%% It can be checked that Ti is identical to TTi
```

Do the same with the Modified DH Paramenters

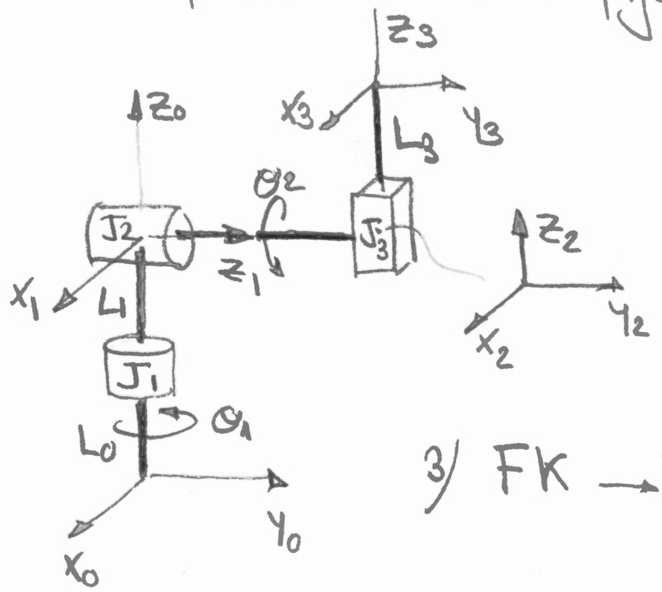
Problem #1 - Kinematics

1. What are the robot morphology of the crane on the truck given in the next figure.
2. Make a sketch of the links and joints that the crane use, to position its end effector.
3. Derive its DH parameters (modified or standards).
4. Derive de Forward Kinematic equation and demonstrate that for a known parameters and variables the function will work.

Problem #1

1.- RRP also RRPP (spherical manipulator)

2.- A posible robot configuration



DH parameter standard

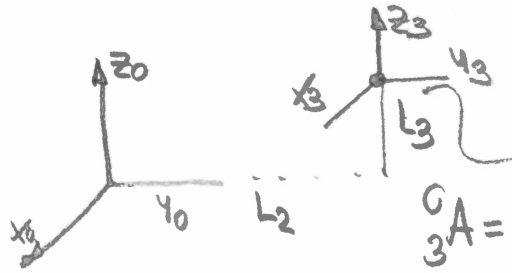
	a_i	α_i	d_i	θ_i
L_1	0	$-\frac{\pi}{2}$	L_0+L_1	θ_1
L_2	0	$\frac{\pi}{2}$	L_2	θ_2
L_3	0	0	L_3	0

3/ FK $\longrightarrow$ $${}^{i-1}_{\ i}A = \begin{pmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

using RTB $\quad {}^{i-1}_{\ i}A = trotz(\theta_i)\cdot transl_z(d_i)\cdot transl_x(a_i)\, trot_x(\alpha$

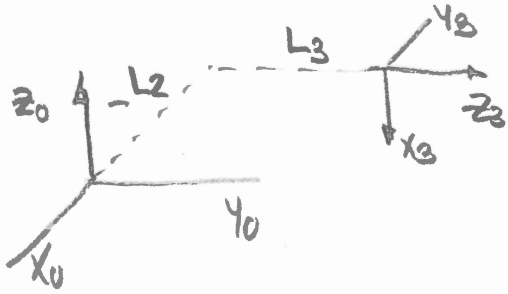
$${}^0_1A = \underbrace{\begin{pmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & d_0+d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\substack{trot_z(\theta_1)\cdot trot_x(\alpha_1) \\ \text{asume } \{0\} \text{ and } \{1\} \\ \text{origen coincident}}} ; \quad {}^1_2A = \underbrace{\begin{pmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}} ; \quad {}^2_3A = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\substack{tranl_z(d_3) \\ L_3}}$$

if $\theta_1=\theta_2=0$ & $d_3=L_3$



The HT $${}^0_3A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & L_2 \\ 0 & 0 & 1 & L_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

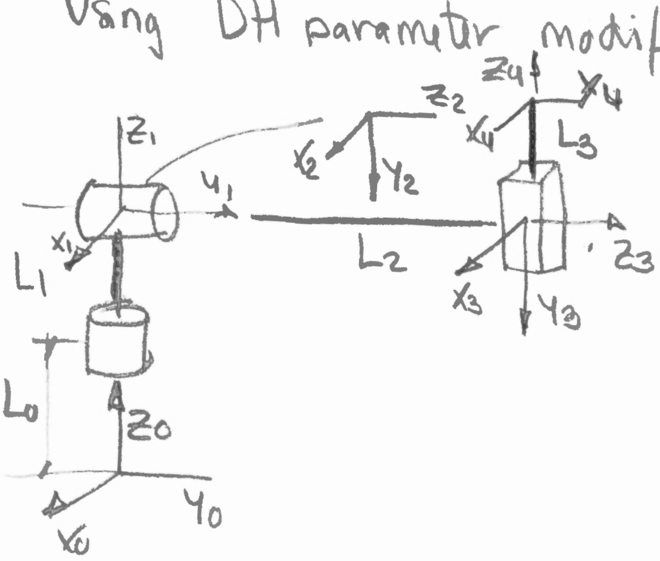
$${}^0_3A = {}^0_1A\,{}^1_2A\,{}^2_3A \underset{\substack{\theta_1=\theta_2=0 \\ \& \\ d_3=L_3}}{=} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & L_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} =$$ idem



$${}^0_3A = \ldots\ldots =$$
if $\theta_1=\theta_2=\frac{\pi}{2}$ & $d_3=L_3$

The HT $${}^0_3A = \begin{pmatrix} 0 & 0 & 0 & -L_2 \\ 0 & -1 & 1 & L_3 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Using DH parameter modified



DH Table

	α_{i-1}	a_{i-1}	d_i	θ_i
${}^0_1A$	0	0	L_0+L_1	θ_1
${}^1_2A$	$-\frac{\pi}{2}$	0	0	θ_2
${}^2_3A$	0	0	L_2	0
${}^3_4A$	$\frac{\pi}{2}$	0	L_3	0

if $\theta_1 = \theta_2 = 0$

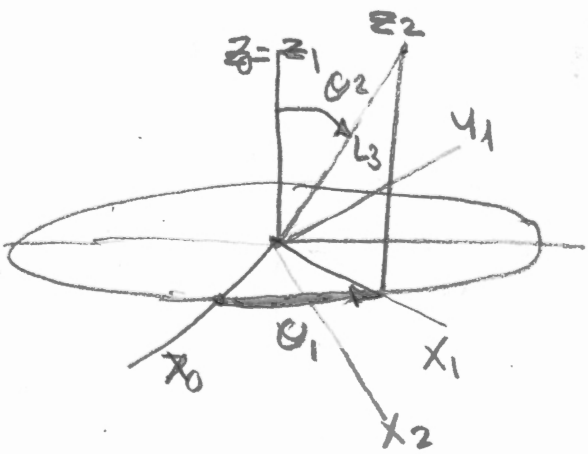
$${}^{i-1}_{\ \ i}A = trot_x(\alpha_{i-1}) \cdot transl_x(a_{i-1})\, trot_z(\theta_i)\, transl(d_i)$$

$${}^0_1A = transl_z(0,0,L_0+L_1)$$
$${}^1_2A = trot\,x\left(-\frac{\pi}{2}\right)$$
$${}^1_3A = transl_z(0,0,L_2)$$
$${}^3_4A = trot\,x\left(\frac{\pi}{2}\right) \cdot transl(0,0,L_3)$$

% will get the same result that using DH standard.

Perhaps the simples spherical manipulator: all the origen are consider

$L_0 = L_1 = L_2 = 0$



$${}^0_2R = trot_{z_0}(\theta_1) \cdot trot_{y_1}(\theta_2)$$

$${}^0_2P = {}^0_2R \; P_2 = {}^0_2R \cdot \begin{pmatrix} 0 \\ 0 \\ L_3 \end{pmatrix}$$

$${}^0_2R = \begin{pmatrix} C_1 & -S_1 & 0 \\ S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_2 & 0 & S_2 \\ 0 & 1 & 0 \\ -S_2 & 0 & C_2 \end{pmatrix} = \begin{pmatrix} C_1C_2 & -S_1 & C_1S_2 \\ S_1C_2 & C_1 & S_1S_2 \\ -S_2 & 0 & C_2 \end{pmatrix}$$

$${}^0_2P = L_3 \begin{pmatrix} C_1S_2 \\ S_1S_2 \\ C_2 \end{pmatrix}; \quad \text{if } \theta_1 = \theta_2 = 0 \quad {}^0_2P = \begin{pmatrix} 0 \\ 0 \\ L_3 \end{pmatrix} L_3$$

$$\text{if } \theta_1 = \theta_2 = \frac{\pi}{2} \quad {}^0_2P = L_3 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$