

SOVEREIGN RISK AND INTERNATIONAL RESERVE MANAGEMENT

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Abstract

Emerging market economies hold substantial amounts of international reserves. An open question is how a government should optimally manage these reserves when facing a debt crisis. The main contribution of this paper is to show that the answer depends on the nature of the crisis. I build a model of sovereign debt and default in which multiple equilibria are possible. In the model, government borrowing costs can increase for two reasons: a productivity shock that leads to a transitory contraction in output or a shock to lenders' beliefs that increases the possibility of a rollover crisis in the next period. The optimal reserve management policy for the government is to run down reserves in response to the first type of shock, and to accumulate reserves in response to the second. This is because reserves both help the government smooth spending when borrowing costs are high *and* reduce the effect that lenders' beliefs have on the possibility of a rollover crisis. I fit the model to match Argentine data and find that lenders' beliefs about the possibility of a rollover crisis are important for understanding the behavior of international reserves during the 2018 crisis.

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1 Introduction

International reserves are a key policy instrument that can be used by emerging market economies facing a sovereign debt crisis. Despite their importance, there is no consensus on how international reserves should be optimally managed. For example, in the summer of 2018 borrowing costs for the Argentine government increased substantially.¹ Interest rate spreads eventually exceeded 1,000 basis points.² During this episode, the central bank first ran down its reserves and provided financing to the government, but then partially rolled back assistance even while high spreads persisted. The latter response is inconsistent with existing economic theory that deems international reserves as insurance.

The common view in the literature is that international reserves should be accumulated in good times and used when the economy is weak and borrowing costs are high. However, this policy recommendation abstracts from the effect that lenders' beliefs may have on default risk. Critically, both bad economic fundamentals *and* lenders' pessimistic beliefs can lead to higher borrowing costs for the government. In this paper, I develop a model where international reserves can also limit the exposure of the economy to the effects of lenders' beliefs.

In my model, the optimal reserve management policy during a crisis depends on whether the source of default risk is fundamental or non-fundamental. Fundamental risk is associated with bad economic fundamentals that make it challenging for the government to service its debt, such as low tax revenues and high levels of debt. Non-fundamental risk is instead associated with the possibility of a rollover crisis caused by lenders' self-fulfilling beliefs. Drawing on reserves helps smooth government spending and avoids the need to borrow when interest rates are high, but also exposes the economy more to non-fundamental risk. The higher the level of international reserves, the less susceptible the economy is to the effects that these beliefs may have on borrowing costs for the government. The main contribution of this paper is to show that if fundamental default risk increases, the government should run down reserves, whereas if non-fundamental risk increases, the government should accumulate reserves.

¹Argentina was excluded from financial markets after the government defaulted on its debt in 2001. A group of creditors held out from restructuring their claims and opted for international litigation. The Argentine government eventually settled with the holdouts during 2016 and regained access to international financial markets.

²Spreads intend to capture how risky a country is relative to a benchmark. In the data, spreads are the difference between the implied interest rate on Argentine government bonds denominated in US dollars and the US Treasury benchmark. In the model, spreads are the difference between the implied interest rate on government bonds and the risk-free rate.

I analyze the mechanisms behind this result in a quantitative general equilibrium model of sovereign default in the style considered in [Eaton and Gersovitz \(1981\)](#), [Aguiar and Gopinath \(2006\)](#), and [Arellano \(2008\)](#). The economy faces fundamental risk in the form of a stochastic output process. The government can issue long-term debt and also hold a short-term risk-free asset that represents international reserves.³ Debt is purchased by foreign investors and the price of bonds endogenously depends on the probability that the government defaults. Importantly, the government can strategically default on its debt.

In my model, the timing of events opens up the possibility for a self-fulfilling rollover crisis. As in [Cole and Kehoe \(2000\)](#), multiplicity of equilibria can emerge by having the government issue debt before deciding whether to default. On the one hand, if lenders expect the government to default, they shut down access to debt financing. Maturing debt might be so costly to service through tax revenues and reserves that the government opts to default, fulfilling the lenders' expectations. On the other hand, if lenders expect the government to repay, they are willing to purchase new debt. Under these circumstances, the government would not default on its debt, confirming the lenders' beliefs. I show that by accumulating international reserves, the government is able to limit the exposure of the economy to the possibility of a self-fulfilling crisis. If enough reserves are held, the government has sufficient resources to avoid defaulting regardless of lenders' beliefs, eliminating the bad equilibrium. Hence, reserves contribute towards avoiding future crises.

It is important that the government has access to long-term as opposed to short-term debt. With long-term debt, the two financial instruments are not as close substitutes and their interaction creates an interesting portfolio problem for the government. Consider the trade in which the government issues additional long-term debt to purchase international reserves. Given that the debt is not entirely due in the next period, the reserves can be held in the meantime, providing insurance to the government. The liquidity provided by reserves can be used in case of future bad times and when borrowing costs are high. In contrast, if debt were short-term, any acquired reserves would need to be entirely used in the next period to service the additional debt, providing no further insurance to the government.⁴

³Long-term debt is introduced in the form of bonds with a geometrically decaying coupon. Maturity management has interesting implication in sovereign default models and is outside the scope of this paper. In [Bocola and Dovis \(2018\)](#) the authors develop a model with no international reserves where the object of interest is debt maturity.

⁴I provide a formal proof of this argument in Appendix B.1. [Alfaro and Kanczuk \(2009\)](#) first documented the result that with short-term debt the government has few incentives to hold reserves. In a model with only fundamental default risk [Bianchi, Hatchondo, and Martinez \(2018\)](#) also highlight the relevance

To analyze its quantitative properties, I calibrate the model to match data from Argentina. I use the correlation between spreads and changes in international reserve as well as the correlation between reserves and output to indirectly infer the importance of non-fundamental default risk. I then analyze the average optimal response in simulations of the model. A rough policy rule from this exercise indicates that when output falls by 1 percent, international reserves should on average fall by 1.6 percent. However, this simple rule is not informed by the possibility of a self-fulfilling crisis. Simulations show that the ratio of international reserves to maturing debt is a good indicator of whether the economy is in a state where the government is exposed to the non-fundamental source of default risk. This result resembles the Guidotti–Greenspan rule used by practitioners, which states that a country’s stock of international reserves should be equal to its short-term external debt.⁵ In my setting, if I condition on the ratio of international reserves to maturing debt being less than one, when the government faces a drop in output it optimally chooses to increase reserves.

Finally, I estimate the source of default risk faced by Argentina during the most recent crisis. In taking the model to the data, I assume that the government was aware of the tradeoffs in using reserves during a crisis. The discussion between the government of Argentina and the International Monetary Fund (IMF) supports this assumption. For instance, in its guidance notes on assessing reserve adequacy, the IMF explicitly mentions rollover risk as a potential threat during emerging market crisis episodes. I apply a particle filter to the model to find the structural shocks that best fit the observed behavior of output, interest rate spreads, and international reserves in Argentina during the 2017–18 period. My estimates suggest that non-fundamental risk is important for understanding the observed behavior of reserves in Argentina during this period. The observed low levels of output in Argentina, translates to a significant increase in the fundamental risk of default. This called for the government to use reserves to smooth government spending. However, towards the end of 2018 the Argentine government in fact increased its stock of reserves even while high spreads persisted. Through the eyes of the model, this is rationalized with an increase in the probability of a self-fulfilling crisis. Absent this component, the central bank would have further used reserves.

Related Literature. I combine building blocks from two strands of the literature to study how the government should manage international reserves during a crisis. First, my paper relates to models of sovereign debt and default that incorporate the possibility for

of this maturity difference.

⁵The Guidotti–Greenspan rule is a traditional rule of thumb that practitioners use to guide adequacy levels of international reserves.

the government to hold reserves. Second, the paper builds on the literature that studies multiplicity of equilibria in models of sovereign debt. More generally my paper is also related to an extensive literature on the precautionary aspects of international reserves and papers that study reserve management in a context with no default.

Since [Eaton and Gersovitz \(1981\)](#), a rich body of quantitative literature on sovereign debt has developed in recent years. The most representative works are [Aguiar and Gopinath \(2006\)](#) and [Arellano \(2008\)](#). [Aguiar and Amador \(2014\)](#) provide an extensive survey of this literature. Recent papers have introduced the possibility for the government to hold a reserve asset. [Alfaro and Kanczuk \(2009\)](#) and [Bianchi, Hatchondo, and Martinez \(2018\)](#) introduce this in an otherwise-standard model of sovereign default. I see my work as complementary to the latter. Rather than studying the fundamental role of reserves, I ask the question of how optimal reserve policy interacts with multiple sources of default risk. For this purpose, I enrich the quantitative benchmark model of sovereign default not only with long-term debt and a reserve asset but also with the possibility that multiple equilibria can arise.

Second, my paper is related to the literature on multiplicity of equilibria in models of sovereign debt. This issue has received a great deal of attention in the literature.⁶ In the spirit of [Diamond and Dybvig \(1983\)](#), the seminal work by [Cole and Kehoe \(2000\)](#) finds that a government lacking commitment to repay its outstanding liabilities can lead to a self-fulfilling rollover crisis. More recent literature adopting this as a workhorse model includes [Conesa and Kehoe \(2012\)](#), [Aguiar et al. \(2013\)](#), and [Bocola and Dovis \(2018\)](#). In this paper I adopt the timing convention of [Cole and Kehoe \(2000\)](#) that enables the benchmark model of sovereign debt to feature multiple equilibria. To resolve the indeterminacy, I follow the selection mechanism developed in [Bocola and Dovis \(2018\)](#). In my model, by allowing for reserve accumulation, the government is able to endogenously control the exposure of the economy to the sunspot equilibrium. If enough reserves are held, the government is able to completely shut down the multiplicity.

A closely related paper is [Hernández \(2018\)](#). In a model of sovereign default with international reserves, the author documents that by allowing for the possibility of rollover crises to occur, a quantitative version of the model delivers a higher average level of re-

⁶In the benchmark model of [Eaton and Gersovitz \(1981\)](#), [Auclert and Rognlie \(2016\)](#) and [Aguiar and Amador \(2019\)](#) show that with short-term debt, the model has a unique equilibrium. However, with long-term debt multiple equilibria can arise. [Calvo \(1988\)](#) explores the role of expectations in sudden stop episodes. [Aguiar and Amador \(2018\)](#) establish that creditor beliefs regarding future borrowing can lead to multiple equilibria with different paths for debt accumulation. [Lorenzoni and Werning \(2018\)](#) show how fear of a future default can lead to higher interest rates, which in turn lead to an accumulation of debt that eventually proves investors' fears right. In [Aguiar et al. \(2017\)](#), a government in crisis might face the possibility of bond auctions that lead to fire-sale prices.

serves. Relative to previous work, his model is closer to achieving the observed levels of reserves in emerging market economies, while at the same time accounting for key business cycle facts. The main contribution of my paper is instead normative. In a similar model I study how the government should optimally manage its reserves when facing a debt crisis. A critical difference is that in my model the non-fundamental source of default risk varies. I model the probability that lenders coordinate on a self-fulfilling crisis in the next period if indeterminacy arises as a stochastic process. This allows me to study the optimal response of the government to an increase in non-fundamental risk. On that account, a key contribution of my paper is its finding that the cost of borrowing is not a sufficient statistic to guide international reserve management policy. By allowing for both fundamental and non-fundamental sources of default risk, my model can rationalize why in times of crisis a central bank would rather accumulate than deplete its stock of international reserves.

My paper is also related to an extensive literature on the precautionary aspects of international reserves in models of sovereign debt. See for example Obstfeld, Shambaugh, and Taylor (2010), Jeanne and Korinek (2010), De Gregorio (2010), Calvo, Izquierdo, and Loo-Kung (2013), Jeanne (2016), Jeanne and Sandri (2016), Bocola and Lorenzoni (2018), and Arce, Bengui, and Bianchi (2019). Relative to this strand of the literature, while allowing for the possibility of default, I focus on when is the right time to use these reserves during a crisis. Similar to my paper, in Bocola and Lorenzoni (2018) a lender of last resort can eliminate the multiplicity of equilibria by holding international reserves.⁷ In the context of a financially closed economy, Jeanne and Sandri (2016) find that when output is low, the optimal international reserve management policy is to use reserves to smooth out consumption.⁸ In my paper this policy recommendation is only optimal when the source of default risk is fundamental. Moreover, it is in stark contrast to the optimal reserve management policy if the source of default risk were non-fundamental.

Debt maturity management plays a similar precautionary role in models of sovereign default. Short-term debt opens the door to rollover risk and the possibility that lenders' beliefs lead to self-fulfilling crises. Sachs, Tornell, and Velasco (1996), Feldstein (1999), Cole and Kehoe (2000), and Bocola and Dovis (2018) suggest countries lengthen their maturity structure to address this problem. Interesting interactions between maturity management and international reserve policy during a crisis can exist. A longer maturity structure reduces the probability of a rollover crisis. Furthermore, it relaxes the need for

⁷In their setting, multiplicity of equilibria is associated with households' dollarization.

⁸The authors study the impact of both export income and nontraded output shocks. After a fall in either, the government chooses to run down reserves.

additional reserve accumulation, freeing resources that can be used to smooth consumption. My model is silent in this dimension and keeps maturity fixed. While debt maturity may play a significant role in advanced economies, Lorenzoni, Broner, and Schmukler (2013) find that for emerging market economies the risk term premium dominates the benefits of long maturity management in a crisis. This makes international reserves even more relevant during an emerging market economy debt crisis.

In emerging market economies the exchange rate has an important role for price stability, financial conditions, and external competitiveness (Carstens, 2019). Despite that, exchange rate management motives for international reserve policy are outside the scope of this paper.⁹ The central mechanism in my paper is the distinct implications that different sources of default risk have for reserve management policy. Going forward, the setting in my paper provides an ideal context for future literature to study exchange rate management as an additional force that drives the government's reserve management policy.¹⁰ Interesting questions arise on the interaction of sovereign risk with other policy objectives.

Layout. The paper proceeds as follows. Section 2 presents the model. Section 3 describes the mechanism underlying the main forces of the model. In Section 4 I turn to the quantitative analysis of the model. I go over the optimal international reserve management policy in Section 5. In Section 6, I use the quantitative model to study the 2018 Argentine crisis. Section 7 concludes.

2 Model

I start with the workhorse model of sovereign default in the spirit of Arellano (2008) in which the government of a small open economy receives a stochastic stream of income. The government borrows from risk neutral competitive creditors in international markets, and can default on its debt at any time. The model is enriched with long-term debt

⁹ Exchange rates are often considered in practice an important component of monetary policy in emerging market economies. Calvo and Reinhart (2002) assert that emerging market economies actively use monetary policy to avoid sharp depreciations. Christiano, Gust, and Roldos (2004) and Braggion, Christiano, and Roldos (2009) argue that balance sheet effects of exchange rate fluctuations are important for optimal monetary policy.

¹⁰ For a recent treatment of international reserves and exchange rate management, see Amador et al. (2019), Cavallino (2019), and Fanelli and Straub (2017). These papers generally abstract from government default. In a context of foreign currency debt, Burnside, Eichenbaum, and Rebelo (2004) explore the role played by government guarantees as a root of self-fulfilling banking and currency crises. In a policy context, Burnside, Eichenbaum, and Rebelo (2003) look into currency crises and fiscal policy sustainability. Relative to these last set of papers, in my model default by the government is strategic.

and the possibility for the government to positively hold a risk-free asset. I interpret this asset as international reserves and the consolidated government as both the fiscal and monetary authority. Another key modification is in the timing assumption of default. In the benchmark model of sovereign default, a government must decide whether to default before issuing new debt. Instead I follow [Cole and Kehoe \(2000\)](#) and have the government decide whether to default after issuing new debt. This timing of events allows for a multiplicity of equilibria to arise. If lenders believe that a default is imminent, then they will offer a price of debt consistent with those beliefs. Being shut out from financial markets might in fact lead the government to default, validating the lenders' beliefs. I employ the mechanism developed in [Bocola and Dovis \(2018\)](#) to select between potential equilibria when multiplicity arises. Conditional on the economy being in the indeterminacy region, a sunspot process determines the probability with which lenders shut down access to borrowing markets.

2.1 Environment

The model is that of a small open economy populated by a government and international investors. Time is discrete and indexed by $t = 0, 1, 2, \dots$. The government receives tax revenues τy_t in every period and decides the path of spending g_t . The government has limited commitment to repay its debt à la [Eaton and Gersovitz \(1981\)](#). In this framework, the government chooses in each period between repaying its debt or defaulting and incurring a default cost. The government issues long-term debt b_{t+1} in the form of bonds with a geometrically decaying coupon ι . That is, every period a fraction ι of outstanding debt becomes due. The government also has access to international reserves that can be positively held. Reserves are short-term and yield interest at the risk-free rate. Each period the government also sets the level of international reserves f_{t+1} . When the government chooses to repay its debt, the budget constraint for the government is

$$g_t + \iota b_t \leq \tau y_t + f_t(1 + r) - f_{t+1} + q_t(b_{t+1} - (1 - \iota)b_t), \quad (1)$$

where q_t is the price of government debt. The government values a stochastic stream of spending $\{g_t\}_{t=0}^\infty$ according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(g_t), \quad (2)$$

where $\beta \in (0, 1)$ is the subjective discount factor of the government and $u(\cdot)$ is the per-period utility function, which is assumed to satisfy the standard regularity conditions.

The timing of events within the period follows Cole and Kehoe (2000). First, the government issues new debt and lenders choose the price of newly issued debt. Only after these steps does the government decide to default ($\delta_t = 1$) or not ($\delta_t = 0$). If the government defaults, it is excluded from debt markets and suffers a one-time default cost φ . In this section the default cost remains a flexible parameter. Later in Section 4 I parametrize the cost to match certain features of the data. International reserves held by central banks have sovereign immunity and thus cannot be seized by creditors. Therefore, upon default the government retains control of international reserves. Investors that held government debt do not receive any repayment.¹¹ When the government chooses to default its budget constraint is given by

$$g_t \leq \tau y_t + f_t(1+r) - f_{t+1}. \quad (3)$$

Creditors are deep-pocketed, risk-neutral, competitive, international investors that have access to an international credit market in which they can borrow or lend at a constant international risk-free rate r . They have perfect information regarding the economy and price bonds such that in expectation they break even.

2.2 Recursive equilibrium

I consider a recursive Markov equilibrium. Let $s = (y, b, f, \pi', \zeta)$ be the state of the economy in the present period and s' be the state of the economy in the next period. The state of the world is $s_t \in S$ and follows a Markov process with transition probability $\mu(\cdot|s_{t-1})$. The fundamental component of the state is given by output, y ; outstanding debt carried from last period, b ; and the level of international reserves, f . The non-fundamental component is given by π' and ζ . These two states are used by agents as a coordination device when indeterminacy arises. It is convenient to postpone any additional discussion of the non-fundamental component to Section 2.3 where I look into multiplicity and equilibria selection. The problem for the government that has not defaulted yet can be written as

$$\begin{aligned} V(s) &= \max_{b', f', g, \delta \in \{0, 1\}} (1 - \delta)\{u(g) + \beta \mathbb{E}_{s'|s}[V(s')]\} + \delta\{V^d(s)\}, \\ \text{s.t. } & g + \iota b \leq \tau y + f(1+r) - f' + q(s, b', f')[b' - (1 - \iota)b], \end{aligned} \quad (4)$$

where the constraint is the resource constraint. The risk-free rate is r and $q(s, b', f')$ is the price of a defaultable long-term bond given the state s , the choice of international reserves

¹¹For tractability and computational purposes this assumption is often used in the sovereign default literature. The model implications would be the same if I considered a positive recovery rate. For an example of a sovereign default model with positive debt recovery, see Yue (2010).

is f' , and government debt b' . $V^d(\mathbf{s})$ denotes the value for the government conditional on a default.

The problem for the government if it has chosen to default can be written recursively as

$$\begin{aligned} V^d(\mathbf{s}) &= \max_{f', g} u(g) - \varphi + \beta \mathbb{E}_{\mathbf{s}'|\mathbf{s}}[V(y', 0, f', \pi')], \\ \text{s.t. } g &\leq \tau y + f(1+r) - f', \end{aligned} \tag{5}$$

where the risk-free rate is r and φ is a default cost that the government faces if it decides to default.

International creditors' no-arbitrage conditions require that

$$q(\mathbf{s}, b', f') = (1 - \delta(\mathbf{s})) \mathbb{E} \left\{ \frac{(1 - \delta(\mathbf{s}')) (\iota + (1 - \iota) q(\mathbf{s}', b'', f''))}{1 + r} \right\}, \tag{6}$$

where b'' and f'' are the optimal debt and reserve choices given the state $\mathbf{s}' = (y', b', f', \pi'', \zeta')$.

The timing of events adopted in this paper shows up in Equation (6). As is the norm in standard models of sovereign default, the pricing schedule of debt consistent with lenders' no-arbitrage condition depends on the future default decision, $\delta(\mathbf{s}')$. However, under the Cole and Kehoe (2000) timing assumption, lenders set the price of debt prior to the government's decision of whether to default on its debt. For this reason, now the pricing condition must consider in addition the possibility of a default in the present period, $\delta(\mathbf{s})$.

Equilibrium. A recursive Markov equilibrium is the value functions for the government $\{V, V^d\}$, the associated decision rules $\{\delta, b', f', g\}$, and a pricing function q such that $\{V, \delta, b', g', f'\}$ are a solution to the government problem (4), $\{V^d, f'\}$ are a solution to the government problem on default (5), and q satisfies the lenders' no-arbitrage condition (6).

2.3 Multiplicity and equilibria selection

To discuss the possibility of multiple equilibria, it is convenient to first consider a government that is committed to repaying its debt in the current period. Such a government would face the price of debt consistent with Equation (6) and $\delta = 0$ in the present period. Following the literature, I refer to this price as the fundamental price of debt,

$$\tilde{q}(\mathbf{s}, b', f') = \mathbb{E} \left\{ \frac{(1 - \delta(\mathbf{s}')) (\iota + (1 - \iota) q(\mathbf{s}', b'', f''))}{1 + r} \right\}. \tag{7}$$

With this tool in hand, I now turn to the discussion on multiplicity of equilibria and their selection. The possibility of multiple equilibria arises in some states. In such a state, if lenders expect debt to be repaid, the government is able to obtain funding from the market at positive prices and therefore repays its debt:

$$\max_{b', f'} u(\tau y + f(1+r) - f' - \iota b + \tilde{q}(\mathbf{s}, b', f')(b' - (1-\iota)b)) + \beta \mathbb{E}_{\mathbf{s}'|\mathbf{s}}[V(\mathbf{s}')] \geq V^d(\mathbf{s}). \quad (8)$$

However, if lenders expect a default, the government faces a price of zero for new debt issuance and instead chooses to default:

$$V^d(\mathbf{s}) > \max_{b' \leq (1-\iota)b, f'} u(\tau y + f(1+r) - f' - \iota b + \tilde{q}(\mathbf{s}, b', f')(b' - (1-\iota)b)) + \beta \mathbb{E}_{\mathbf{s}'|\mathbf{s}}[V(\mathbf{s}')]. \quad (9)$$

Conditions (8) and (9) are useful to partition the state-space into three regions. If Condition (8) does not hold, then the government defaults even if lenders expect repayment. Following the literature, I refer to this region of the state-space as the **default zone**.¹² In this region, defaults are fundamental. That is, given the economy's fundamentals (output, debt and reserves), the government chooses to default on its debt. If Condition (9) does not hold, even if the government is offered the worst possible price for its debt, it will choose to roll over its nonmaturing debt rather than default. This region in the state-space is referred to as the **safe zone**. Condition (9) allows the government to buy back part of its debt at the worst possible price for the government. As shown in Appendix B.3, if the government is to buy back debt, the worst possible price they could face, consistent with the lenders' breakeven condition, is the fundamental price. Thus Condition (9) accurately characterizes the limit of the safe zone.

If both equations hold, then indeterminacy arises. In line with the literature, I refer to this region of the state-space as the **crisis zone**. The non-fundamental component of the state-space is characterized as in Bocola and Dovis (2018) by π_{t+1} and ζ_t . Conditional on being in a crisis zone at $t+1$, lenders coordinate on the bad equilibrium with probability π . In this case, lenders offer the government a price of zero for new debt. Upon this realization, the government opts for a default. With complementary probability $(1-\pi)$, lenders extend credit at a positive price. In that instance, the government is able to access financial markets and opts to honor its debt obligations. Conditional on being in a crisis zone at time t , the sunspot state is determined by $\zeta \in \{0, 1\}$. If $\zeta = 1$, there is a self-fulfilling crisis, and the government defaults. If $\zeta = 0$, lenders offer the government access to financial markets, and there is no default.

¹²Bocola and Dovis (2018) were the first to introduce this notation.

3 International reserves and sovereign risk

Before turning to the quantitative analysis of the model, it is convenient to discuss what motivates the government to hold international reserves. For illustration purposes, I assume that the value functions and the price of government bonds is differentiable. Section 3.1 shows in a model with only fundamental risk what motivates a government to hold reserves. This section describes the use of reserves as an insurance mechanism. Section 3.2 then describes how, in a model with non-fundamental risk, international reserves can be used to shape the crisis region and reduce the economy's exposure to a self-fulfilling crisis.

3.1 Reserves and fundamental risk

To understand the key forces behind reserves management, I first consider the common setting in the literature without the possibility of a self-fulfilling crisis. In the model developed in Section 2, this holds if the government chooses whether to default on its debt before the portfolio choice is made. This results in a model indistinguishable from the one proposed by [Bianchi, Hatchondo, and Martinez \(2018\)](#). Therefore, in what follows I present an identical argument to the one in their paper to show why, even in the absence of non-fundamental risk, the government holds a positive amount of reserves. Namely,

$$V(\mathbf{s}) = \max_{\delta \in \{0,1\}} (1 - \delta)V^{rp}(\mathbf{s}) + \delta V^d(\mathbf{s}), \quad (10)$$

where $V^{rp}(\cdot)$ is the value function of a government that has chosen not to default in the current period,

$$\begin{aligned} V^{rp}(\mathbf{s}) &= \max_{b', f', g} u(g) + \beta \mathbb{E}_{\mathbf{s}'|\mathbf{s}}[V(\mathbf{s}')], \\ \text{s.t. } g + \iota b &\leq \tau y + f(1+r) - f' + q(\mathbf{s}, b', f')[b' - (1 - \iota)b], \end{aligned} \quad (11)$$

where the constraint is the resource constraint in the economy.

Consistent with this timing assumption, the price of debt at which lenders break even is

$$q(\mathbf{s}, b', f') = \mathbb{E} \left\{ \frac{(1 - \delta(\mathbf{s}')) (\iota + (1 - \iota) q(\mathbf{s}', b'', f''))}{1 + r} \right\}. \quad (12)$$

In this setting, consider a state of the economy \mathbf{s} and a target for government spending \bar{g} . Let $x = \{\mathbf{s}, \bar{g}\}$ be the vector of the initial state and the spending target. The combination

of debt and reserves (b', f') that delivers such a spending target is given by

$$f' = \tau y + f(1 + r) - \iota b - \bar{g} + q(\mathbf{s}, b', f')(b' - (1 - \iota)b). \quad (13)$$

The amount of reserves that the government can accumulate with newly issued bonds depends on how the bond price varies with the portfolio choice. I define $\tilde{f}(b', x)$ as the amount of reserves that can be purchased when the government borrows b' , while satisfying Equation (13).

Current utility is fixed by current government spending, so the optimal portfolio needs to maximize the expected continuation value for the government. Assuming that the government optimally chooses government spending, debt, and reserves from the next period onwards, the optimal portfolio solves:¹³

$$\begin{aligned} & \max_{b' \geq 0} \mathbb{E}_{\mathbf{s}'|\mathbf{s}}[V(y', b', \tilde{f}(b', x))], \\ & \text{s.t.} \quad \tilde{f}(b', x) \geq 0. \end{aligned} \quad (14)$$

Totally differentiating (14) with respect to b' and using the envelope conditions yields

$$\frac{d\mathbb{E}_{\mathbf{s}'|\mathbf{s}}V(y', b', f', \pi'')}{db'} = \underbrace{\frac{\partial \tilde{f}}{\partial b'} \mathbb{E}_{\mathbf{s}'|\mathbf{s}}[u_g(g')(1 + r)]}_{\text{Benefit from buying reserves}} - \underbrace{\iota \mathbb{E}_{\mathbf{s}'|\mathbf{s}}[u_g(g')(1 - \delta')]}_{\text{Cost of coupon payment}} - \underbrace{(1 - \iota) \mathbb{E}_{\mathbf{s}'|\mathbf{s}}[u_g(g')q'(1 - \delta')]}_{\text{Cost of carrying long-term debt}}, \quad (15)$$

where $u_g(\cdot)$ is the marginal utility of government spending and ' refer to next-period variables.

At an interior optimum, the government equates this expression to zero. That is, the government equates the benefit of accumulating $\partial \tilde{f} / \partial b'$ additional reserves, at the cost incurred by financing them through debt. In Appendix B.2 I derive the analytic expression for $\partial \tilde{f} / \partial b'$. The key insight is that a government issuing debt can accumulate more reserves the more favorable the changes in the bond price in response to the increase in debt ($\partial q / \partial b'$) and reserves ($\partial q / \partial f'$). In [Bianchi, Hatchondo, and Martinez \(2018\)](#) the authors note that long-term debt plays an important role, as the net benefit is decreasing in the price of debt tomorrow q' . This can be better appreciated by reordering Equation (15) across default and repayment states. In Equation (16) I do this. It shows that in repay-

¹³Without the possibility of multiple equilibria, the probability of a self-fulfilling crisis next period, π'' , is not relevant. Thus, in this setting $\mathbf{s} = \{y, b, f\}$.

ment states the net benefit is decreasing in the price of debt next period. This has the effect of transferring resources from repayment states with high q' to repayment states with low q' . Moreover, in this setting states with a low price of debt are states where output is low and thus are states with high marginal utility of government spending. The opposite is true for states with high q' . The government thus accumulates reserves to insure against the risk of facing high borrowing costs during weak economic conditions.

$$\frac{d\mathbb{E}_{s'|s}V(y', b', f')}{db'} = \mathbb{E}_{s'|s} \underbrace{\left[\frac{\partial \tilde{f}}{\partial b'}(1+r)u_g(g')\delta' + \right]}_{\text{Default states}} + \underbrace{\left[\frac{\partial \tilde{f}}{\partial b'}(1+r) - \iota - (1-\iota)q' \right] u_g(g')(1-\delta')}_{\text{Repayment states}}. \quad (16)$$

The setup above illustrates the classic reserve management motive brought forward in the literature. Reserves are accumulated in good times when the cost of carrying reserves is low (i.e., when the price of debt is high) and provide insurance for the government when the price of debt is low (i.e., when borrowing costs are high). Whenever output and the price of debt are low, the government finds it challenging to issue new bonds and service its debt, so it uses the stock of international reserves as its rainy-day fund.

3.2 Reserves management with non-fundamental risk

Now I turn to my setting in Section 2 where there is a possibility of a self-fulfilling crisis occurring. A government that faces the fundamental price for debt solves a similar portfolio problem as the one described in Equation (14). In this setting it is also true that the government holds reserves to insure against states in which borrowing becomes prohibitively costly. However, the government must additionally consider the effect that reserves have on the possibility of a self-fulfilling crisis tomorrow. It is important to note that the government can influence the risk of facing a self-fulfilling crisis in the future. Consider the fundamental price of debt described in Equation (7). After some algebra (see Appendix B.4), default risk can be decomposed into the following two components,

$$P_t(s_{t+1} \in \mathcal{S}^{def}) + P_t(s_{t+1} \in \mathcal{S}^{crisis}) \times \pi_{t+1}, \quad (17)$$

where $P_t(s_{t+1} \in \mathcal{S}^{def})$ is the probability that the economy is in the default zone in the next period, and $P_t(s_{t+1} \in \mathcal{S}^{crisis})$ is the probability that the economy is in the crisis zone

in the next period. By managing its portfolio, the government can alter the boundaries of the crisis zone defined by Conditions (8) and (9), affecting with this the probability that the economy is exposed to non-fundamental risk tomorrow (i.e., $P_t(s_{t+1} \in \mathcal{S}^{crisis})$). The government responds to an increase in π_{t+1} by taking actions that reduce the risk of being in the crisis zone at $t + 1$. This can be achieved by increasing the stock of international reserves.

To understand why increasing reserves in the present period reduces the exposure of the government to a rollover crisis tomorrow, consider a government in the crisis zone at $t + 1$ (i.e., $s_{t+1} \in \mathcal{S}^{crisis}$). In this region, if the government could borrow, it would choose to issue additional debt. This is clear from the fact that as Condition (8) holds, if the government could issue debt, it would choose not to default. However, it defaults when net issuance cannot be positive. Hence, the maximum of the right side of Condition (9) in the crisis zone is attained for a portfolio with positive debt issuance. I can therefore write Condition (9) under the assumption of no debt buybacks. This can be written as

$$\begin{aligned} & \max_{f''} u(\tau y' + f'(1+r) - f'') - \varphi + \beta \mathbb{E}_{s''|s'}[V(y'', 0, f'', \pi'')] \\ & > \max_{f''} u(\tau y' + f'(1+r) - f'' - \iota b') + \beta \mathbb{E}_{s'|s}[V(y'', (1-\iota)b', f'', \pi'')], \end{aligned} \tag{18}$$

where x' and x'' refer to the state variable x at times $t + 1$ and $t + 2$, respectively. The left side of Condition (18) is the problem for the government on default, while the right side is the problem for the government in the event that lenders coordinate in not extending additional credit. As long as this condition holds, the government is in the crisis zone. Both sides of Condition (18) are increasing in the choice of international reserves today, f' . However, it is unambiguous that the marginal utility of government spending is higher on the right side. Thus, by increasing reserves in the current period, the government reduces the probability that the economy is in the crisis zone next period, $P_t(s_{t+1} \in \mathcal{S}^{crisis})$. First note that for the same choice of f'' in period $t + 1$, under no default (right side), the government is able to spend $\iota b'$ less resources than in default. Second, following the previous discussion in Section 3.1, if the government chooses to default, then it has lower incentives to accumulate reserves at $t + 1$, since debt at $t + 2$ would be zero. These two effects go in the direction of having a higher marginal utility of government spending when the government decides not to default and repay maturing debt. Thus, when π' increases, the government has an incentive to accumulate reserves in order to shrink the crisis region, \mathcal{S}^{crisis} .

From the previous discussion, two important observation arise in this setting. First, if in the crisis zone next period, the change in the bond price at time t in response to an

increase in reserves, $(\partial q / \partial f')$, is large. This fact maps to the optimal portfolio problem developed in the previous section. It indicates that the government can accumulate more reserves with newly issued bonds (i.e., $\partial \tilde{f} / \partial b'$ is high). Second, the fact that in a self-fulfilling crisis state, if the government could borrow, it would issue additional debt. Consider a variation in which the government increases reserves by ε at time t while keeping constant the level of government spending. To achieve this, the government must issue additional debt by ε/q . As long as $q > \varepsilon/(1+r)$, this has the effect of increasing the amount of resources available at time $t+1$.¹⁴ As the government would choose to issue additional debt if it were possible, this means that the marginal utility of government spending at $t+1$ is higher than the expected marginal loss in the future value due to higher debt. There is the potential for the government to increase international reserves at time t by issuing additional debt. This has the benefit of reducing the exposure to non-fundamental risk without sacrificing present government spending. However, the effect that this portfolio trade choice has on the crisis zone is ambiguous. On the one hand, from the discussion above, the increase in reserves has the unambiguous effect of reducing the crisis zone. On the other hand, additional debt reduces the right side of Condition (18) with no effect on the left side. Therefore, although the aggregate effect on the right is positive, one must compare the benefit of this trade during a self-fulfilling crisis with the gains from additional reserves in default.

The previous discussion provides insights on the optimal response of a government to default risk. To summarize, when there is only fundamental risk, if an economy is hit with a bad output realization, fundamental risk increases. The increase in fundamental risk is accompanied by higher sovereign borrowing costs that create incentives for the government to deplete some of its stock of international reserves. In contrast, when there is only non-fundamental risk, if the probability of a self-fulfilling crisis increases, the government has additional incentives to accumulate international reserves to escape the danger of a non-fundamental default. The next two sections present a comprehensive analysis with the numerical solution to the quantitative model outlined in Section 2.

4 Quantitative analysis

In this section I parametrize, calibrate, and numerically solve the model developed in Section 2. Once the model is solved, it is possible to study the optimal reserve management

¹⁴For any interesting state, this places no restriction. The assumption implies that the spread on government bonds must be less than 1,246.4 percent.

response during a crisis. In Section 4.1 I parametrize the model by introducing functional forms for the relevant objects of the model. In Section 4.2 I calibrate the model to match features of the Argentine economy. In Section 4.3 I outline the computational strategy for solving the model numerically. Section 4.4 shows how the model fits key moments of the data. In Section 4.5 I explore, in the quantitative model, the interaction between default risk and international reserves.

4.1 Parametrization

I assume the per-period utility function is a variation of the constant relative risk aversion form:

$$u(g_t) = \frac{(g_t - \bar{g})^{1-\sigma}}{1-\sigma}, \quad (19)$$

where σ is the coefficient of the relative risk aversion parameter and \bar{g} is a minimum level of spending. I interpret this as the fraction of spending that is difficult for the government to reduce. [Bocola and Dovis \(2018\)](#), [Bocola, Bornstein, and Dovis \(2019\)](#) and [Bornstein \(2018\)](#) have shown that this specification of the utility function helps the model match the cyclicalities of government debt in the data. In this setting, the parameter \bar{g} also provides more incentives to the government to insure against a rollover crisis.

The output process, y_t , follows a Gaussian AR(1) process in logs. Namely,

$$\log(y_t) = (1 - \rho_y)\mu_y + \rho \log(y_{t-1}) + \sigma_y \varepsilon_{y,t}. \quad (20)$$

Tax revenues are assumed to be a constant fraction τ of output and thus equal to τy_t .

Upon default the government is excluded from debt markets for that period and suffers a one-time utility loss. Default costs are widely used in the quantitative literature of sovereign default models to match certain features of the data. A utility loss is often used to capture various default costs that arise upon default and could be related to reputation, bailouts, sanctions, or financial frictions, among others. A microfoundation of default costs is outside the scope of this paper.¹⁵ Another common specification in the literature is to consider an output cost. Typically quantitative models of sovereign default rely on high output costs to achieve the desired levels of debt observed in the data. However, in this setting overstating the drop in output upon default has implications for reserve management policy. This is true since international reserves could be used to smooth out government spending in those states. In Equation (16) an output cost would show up via a higher marginal utility of government spending in a default. There

¹⁵For a microfoundation of default costs, see [Bocola \(2016\)](#), [Hébert and Schreger \(2017\)](#), and [Pérez \(2018\)](#).

is no reason to suspect that there is a discontinuity in the marginal utility of consumption across the default decision. Similarly, an output cost would show up in Equation (18) as lowering the amount of resources available in default. Hence for low levels of debt one would have to compare the coupon payment to the output loss. [Borensztein and Panizza \(2008\)](#) estimate small negative effect on growth of 1.2 percent. A cost of this magnitude in the model would result too low to achieve the level of debt to GDP observed in the data. The focus of this paper is the role of reserves in insuring against default risk. Therefore, by utilizing a utility cost I abstract from the role of reserves in smoothing counterfactual output default costs. I follow [Chatterjee and Eyigunor \(2012\)](#) to parametrize the default cost as a convex function of output,

$$\varphi(y_t) = \max\{0, d_0 + d_1 y_t\}. \quad (21)$$

This specification displays two important properties. First, it allows for a larger cost of default in good times relative to low-output states.¹⁶ Second, it is tractable, as the loss is only dependent on one state variable (y) and is constant across level of sovereign debt and international reserves.

Finally, I parametrize the probability of lenders coordinating on the self-fulfilling equilibria in the next period conditional on being in the crisis zone. I follow [Bocola and Dovis \(2018\)](#) and assume that in each period the probability is independent and identically distributed according to

$$\pi_{t+1} = \frac{\exp(\tilde{\pi}_{t+1})}{1 + \exp(\tilde{\pi}_{t+1})}, \quad (22)$$

$$\tilde{\pi}_{t+1} = \pi^* + \sigma_\pi \varepsilon_{\pi,t+1}, \quad (23)$$

where the shocks ε_π are assumed to be Gaussian.

4.2 Data and calibration

Recent economic history in Argentina has been marked by debt crises and default. Most recently the government of Argentina defaulted on over 100 billion US dollars in external sovereign debt in 2001.¹⁷ This recent default episode explains some features of the ob-

¹⁶This happens for positive values of d_1 .

¹⁷In 2001, Argentina suspended all debt payments and started negotiations with bondholders to restructure their obligations. Argentina's unwillingness to renegotiate in good faith has been amply commented upon, and restructuring was a long and tortuous process. After the case made it through the courts in New York, a final settlement was reached with the last group of creditors in 2016. Until that time, Argentina remained excluded from international financial markets. In [Hébert and Schreger \(2017\)](#) the authors provide

served government debt, international reserves and lack of data on government spreads in recent years. Figure 1 shows the trajectory for output, government debt, international reserves, and interest rate spreads on government bonds.

External debt was on a downward trajectory until the Argentine government regained access to international markets on 2016. In Argentina, the central bank has historically been used to finance the government. This explains the difference between international reserves and net international reserves. From 2008 to 2016 the government relied heavily on central bank financing. This had a negative impact on the stock of international reserves, with net international reserves being negative in December 2015. It was not until 2016, under a new government administration, that international reserves started on an upward trajectory. As the government regained access to financial markets in 2016, government spreads remained stable throughout the period.

The quantitative model outlined in Section 2 and Section 4.1 contains 13 structural parameters: the discount factor, β ; the parameter, σ , that measures the degree of relative risk aversion; the minimum government spending threshold \bar{g} ; the mean, μ_y , persistence, ρ , and volatility, σ_y , of output; the tax rate, τ ; the parameters that control the utility loss on default, d_0 and d_1 ; the parameter that controls maturity of government debt, ι ; the risk-free rate, r ; and, finally, the two parameters that control the sunspot coordination mechanism, π^* and σ_π .¹⁸

One period is assumed to be one quarter. Consistent with most of the literature, I set the degree of relative risk aversion, σ , to 2. Some observable parameters are set to match features of the economy of Argentina.¹⁹ Persistence and volatility of output are estimated from Argentina's quarterly gross domestic product. Persistence, ρ , is set to 0.915 and volatility, σ_y , to 0.018. Government revenues are on average 31 percent of output, so τ is set to 0.31. I normalize tax revenues to equal one at the mean. For this purpose μ_y is set to 1.171. Wages and nondiscretionary transfers represent 74.9 percent of revenue for the government. Consistent with this fact, \bar{g} is set to 0.749. Likewise, the parameter ι is set to match features of Argentina's external debt. It is set to the ratio of maturing long- and medium-term debt, plus short-term debt of the last period, to outstanding debt. This ratio was equal to 0.093 at the end of 2017, the year before the most recent crisis. The risk-free interest rate, r , is set to 1 percent to match that of the United States.

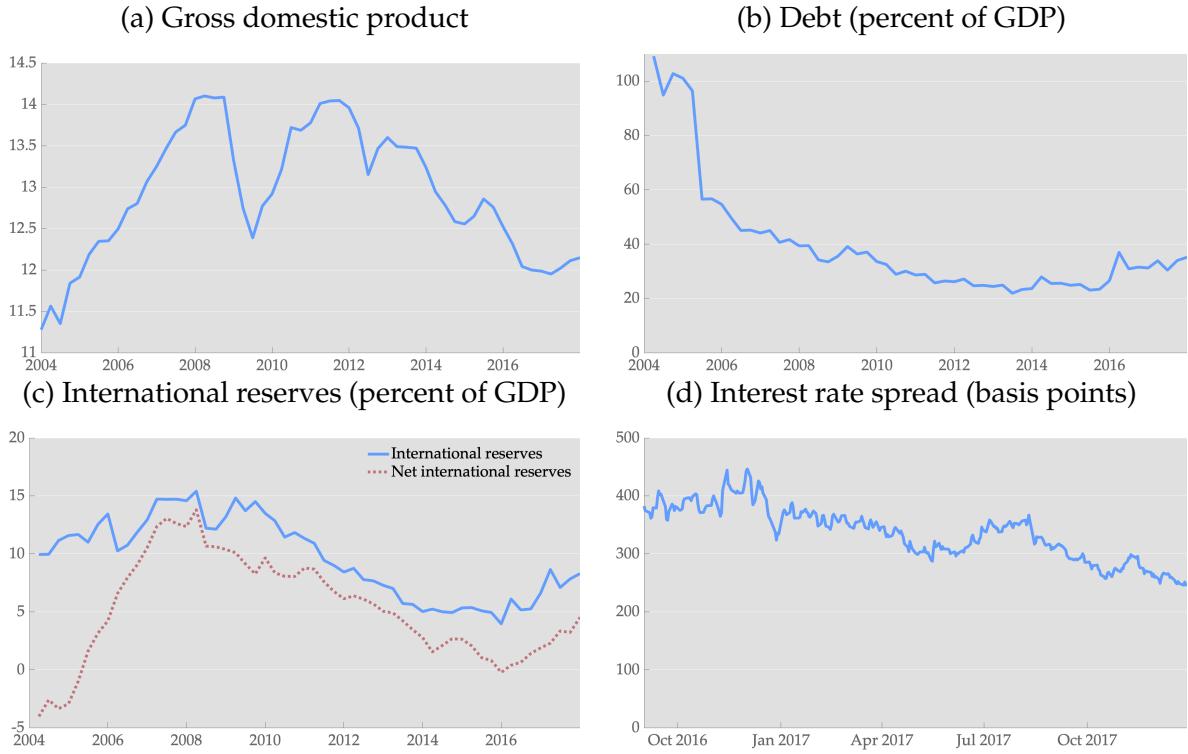
The remaining five parameters (β , d_0 , d_1 , π^* , and σ_π) are calibrated using a method of simulated moments to match desired features of the Argentine economy. In particular,

a detailed account of this process. See Section 6 for a complete account of recent macroeconomic history in Argentina.

¹⁸Maturity measured as average life of a bond is equal to the inverse of ι .

¹⁹The source for the data is detailed in Appendix A.

Figure 1: Output, debt, international reserves, and interest rate spreads for Argentina



Notes: Gross domestic product is detrended log gross domestic product. Debt is the ratio of external debt to gross domestic product for Argentina. International reserves are those held at the central bank. Net international reserves consider short-term foreign currency liabilities of the central bank consistent with the technical appendix to the memorandum agreed between the International Monetary Fund and the government of Argentina in 2018. Spreads are interest-rate differentials between government five-year US dollar-denominated bonds and the US Treasury benchmark for that maturity. Argentina regained access to international financial markets in 2016. See Appendix A for a complete description of the data.

the discount factor and the utility loss parameters are chosen to match in simulations the average level of external debt and international reserves to gross domestic product, as well as the average spread on government bonds observed in the data. A constant in the sovereign default literature is the use of low values for the discount factor. However, the per-period utility specification allows for discount factors more in line with the macroeconomic literature.²⁰ The parameters that control the sunspot probability are difficult to map into observable data. From the discussion in Section 3 a relevant observation can be made. When the economy is weak and interest rate spreads are high for fundamental reasons, the government uses reserves to smooth out spending. However, when spreads are high for non-fundamental reasons, the government accumulates reserves to limit the exposure of the economy to the multiplicity of equilibria. Thus, I use the correlation be-

²⁰This was previously documented by [Bocola and Dovis \(2018\)](#).

tween spreads and international reserve accumulation as well as the correlation between output and international reserves to indirectly infer the importance of the sunspot. Table 1 reports the choice of parameters for the model.

Table 1: Parameter values

Parameter	Notation	Value	Source
Government discount factor	β	0.960	Method of simulated moments
Risk aversion	σ	2.000	Standard in macroeconomics
Government spending coefficient	\bar{g}	0.749	To match nondiscretionary transfers
Risk-free interest rate	r	0.010	Risk-free rate in the US
Output AR coefficient	ρ	0.915	To match persistence of output
Output variance coefficient	σ_y	0.018	To match output volatility
Average output level	μ_y	1.171	Normalization
Tax rate	τ	0.310	To match tax revenue to GDP
Utility loss from default	d_0	46.000	Method of simulated moments
	d_1	36.800	Method of simulated moments
Debt maturity	ι	0.093	To match average debt maturity
Average sunspot probability	π^*	-5.250	Method of simulated moments
Volatility of sunspot probability	σ_π	1.156	Method of simulated moments

Notes: This table reports the choice of parameters for the model. The choices for parameter values either follow standard parameter values used in the macroeconomics literature or are selected to match certain moments in the data. The source for the data is detailed in Appendix A. A method of simulated moments is used to calibrate some parameters of the model to match the data for Argentina. Details on this process can be found in Appendix C.2.

4.3 Solution strategy and computational algorithm

To solve the model numerically, I define three value functions for the government. The value function of the government conditional on not defaulting, $V^{nd}(\cdot)$, is defined as the left side of Condition (8). The value function conditional on lenders shutting down access to debt markets, $V^{ss}(\cdot)$, is the right side of Condition (9) under no debt buybacks. The value function on default, $V^d(\cdot)$, is consistent with the problem of the government that chooses to default. The assumption of a complete debt wipeout upon default and the shock structure for π' simplifies the state-space on default to two variables, y and f .

$$V^{nd}(y, b, f, \pi') = \max_{b', f'} u(\tau y + f(1+r) - f' - \iota b + \tilde{q}(\mathbf{s}, b', f')(b' - (1-\iota)b)) + \beta \mathbb{E}_{\mathbf{s}'|\mathbf{s}}[V(\mathbf{s}')], \quad (24)$$

$$V^{ss}(y, b, f, \pi') = \max_{f'} u(\tau y + f(1+r) - f' - \iota b) + \beta \mathbb{E}_{\mathbf{s}'|\mathbf{s}}[V(y', (1-\iota)b, f', \pi'')], \quad (25)$$

$$V^d(y, f) = \max_{f'} u(\tau y + f(1+r) - f') - \varphi(y) + \beta \mathbb{E}_{\mathbf{s}'|\mathbf{s}}[V(y', 0, f', \pi'')], \quad (26)$$

where $\mathbf{s}' = \{y', b', f', \pi''\}$. The numerical solution of the model consists in approximating the value functions $\{V^{nd}, V^{ss}, V^d\}$ as well as the price function \tilde{q} ,

$$\tilde{q}(y, \pi', b', f') = \mathbb{E}_{\mathbf{s}'|\mathbf{s}} \left\{ \frac{(1 - \delta(\mathbf{s}')) (\iota + (1 - \iota) q(\mathbf{s}', b'', f''))}{1 + r} \right\}. \quad (27)$$

The value functions are approximated using a combination of linear interpolation and a Chebyshev approximation. In the dimension of debt I do a linear interpolation. For each level of debt I then approximate the value function on the remaining dimensions using a Chebyshev approximation. A known feature in quantitative models of sovereign default is the sharp nonlinearity in the pricing schedule of debt. For this reason the price function is approximated over a finer grid.

The solution algorithm iterates on the value functions and pricing schedule until the maximum difference between iterations is small enough. In computing the value functions and price, expectations are approximated using a Gauss–Hermite quadrature. The numerical solution is $\{\gamma^{nd}, \gamma^{ss}, \gamma^d, \hat{q}\}$, where for a realization of the state-space, (y, b, f, π') , and its corresponding vector collecting the Chebyshev's polynomials, $\mathbf{T}(y, f, \pi')$, V^x is approximated as

$$V^x(y, b, f, \pi') = \gamma_b^{x'} \mathbf{T}(y, f, \pi'). \quad (28)$$

In Appendix C.1, I provide in full detail the computational approach and solution algorithm.

4.4 Model Fit

First I examine some basic properties of the calibrated model. Table 2 shows the capability of the model to match certain moments in the data. The quantitative model is able to match the average debt-to-output ratio of the Argentine economy. In the period between the first quarter of 2004 and the last quarter of 2017, external debt in Argentina averaged 39.98 percent of GDP. Meanwhile the average debt-to-output ratio over 10,000 simulations of the model is 41.92 percent. It has been documented that quantitative models of sovereign default have a hard time achieving high levels of interest rate spreads for government bonds. Given its history of recurrent defaults, Argentina is an emerging market economy that features high costs for government debt. Having non-fundamental default risk in the model contributes to higher levels of interest rate spreads on government debt. Since Argentina regained access to international financial markets in 2016 and up to the end of 2017, the average spread on five-year US dollar–denominated government bonds averaged 334 basis points. The average interest rate spread on government debt from

simulating the model is 204 basis points. The model overstates the ratio of average international reserves to gross domestic product. Net international reserves to GDP averaged 5.24 percent over the same time period, while in simulations the model achieves an average of 9.54 percent. During that period gross international reserves held at the central bank in Argentina averaged 9.65 percent. Argentina is an outlier in a group of similar emerging market economies. In a panel of similar emerging market countries, [Bianchi, Hatchondo, and Martinez \(2018\)](#) find an average level of debt of 42 percent of output, similar to that of Argentina. However, they document that international reserves represent on average 16 percent of GDP, almost three times those of Argentina. The authors find that on average these countries have spreads of 224 basis points. The model matches this data well.

With respect to the calibration of the sunspot mechanism, the model does a good job in matching the behavior of international reserves observed in the data. The correlation of net international reserves and output was 0.765 for Argentina during the sample period. In simulations of the model the correlation between these two variables is 0.655. Finally, the correlation between the change in international reserves and the spread on government bonds was -0.207 in the data, while in the model this correlation averaged -0.297 across simulations.

Table 2: Calibration targets

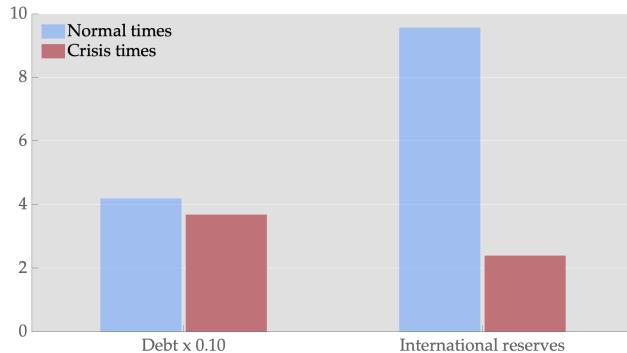
Statistic	Data	Model
Average debt to GDP ratio	39.98%	41.92 %
Average reserves to GDP ratio	5.24%	9.54%
Average spread	334 bp	204 bp
Correl. reserves and GDP	0.765	0.655
Correl. %Δreserves and spreads	-0.207	-0.297

Notes: This table presents statistics for targeted moments. Total external debt-to-output ratio is considered for the 2004–17 period. For international reserves I consider net international reserves as defined in the technical appendix to the memorandum agreed between the International Monetary Fund and the government of Argentina over the same time period. Spreads are interest rate differentials between government five-year US dollar-denominated bonds and the US Treasury benchmark for that maturity for the 2016–17 period. Moments coming from the model are computed as the average moment over 10,000 simulations of 40 periods each. For details on the data see Appendix A. For details on the procedure for simulations see Appendix C.2.

In Figure 2 I compare the portfolio composition conditional on the economy being in

the crisis zone. In the simulations, the economy is in the crisis region approximately 2.94 percent of the time. The main difference in the target moments does not come from the ratio of debt to output. During normal times, debt, as a percent of gross domestic product, is 41.88 percent. In comparison, if I condition on the economy being in the crisis zone, the ratio is 36.81 percent on average. The fact that stands out is that when the economy is in the crisis zone, it has a considerably lower level of international reserves to gross domestic product. During normal times the average is 9.56 percent, in contrast to 2.39 percent for an economy in the crisis region. This represents approximately 25 percent the level in normal times for 88 percent the level of debt. The low level of reserves exposes the government to the possibility of a self-fulfilling crisis.²¹ The higher likelihood of the economy experiencing a self-fulfilling crisis is reflected in a considerably higher cost of funding for the government.

Figure 2: Targeted moments in normal and crisis times



Notes: This figure presents the mean ratio of debt and reserves to GDP conditional on whether the state-space is consistent with crisis times. An economy is considered to be out of normal times when its observed fundamentals (output, debt, and international reserves) situate the government in the crisis zone. Moments are computed over 10,000 simulations of 40 periods each. For details on the procedure for simulations see Appendix C.2.

4.5 Default risk, international reserves, and the crisis zone

In this section I discuss the main results from the quantitative model. I postpone the discussion on optimal international reserve management to first study in the quantitative

²¹This fits well with empirical evidence. Perhaps the most common example of this is the Mexican crisis of 1994. Before the country defaulted on its debt, the central bank had depleted all its reserves, exposing the economy to the possibility of a self-fulfilling rollover crisis. For a treatment of Mexico's crisis see [Calvo and Mendoza \(1996\)](#) and [Cole and Kehoe \(2000\)](#). Another benchmark example is the European debt crisis in 2011. Member countries of the Eurosystem renounced control over the reserve policy in favor of the European Central Bank. This exposed European countries to the possibility of lenders' expectations triggering a self-fulfilling crisis.

model the interaction between the crisis zone and reserves.

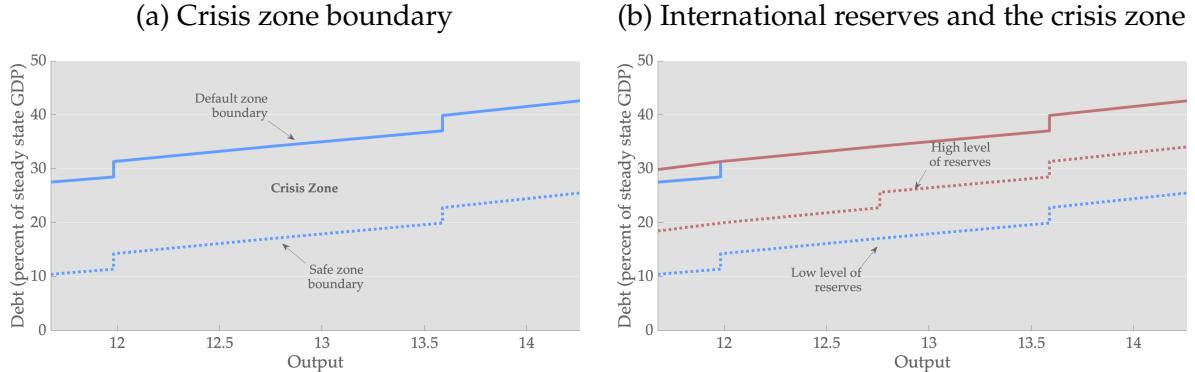
Figure 3 shows a graphical representation of the interaction between international reserves and the crisis zone. In Figure 3a I show the boundary conditions that define the relevant regions in the state-space. For a given level of international reserves, the dotted line shows the boundary condition for the safe zone obtained from Condition (9). Intuitively, for lower values of debt, even if investors shut down the government's access to debt markets, the government will choose not to default. When debt levels are low, the government can keep a satisfactory level of government spending, as servicing maturing debt requires the government to spend a relatively modest amount of tax revenues. As output increases, the amount of debt that the government can hold safely also increases. The intuition for this result comes from the fact that if the government has a higher level of tax revenues, it can better tolerate paying for maturing debt when there is no access to borrowing.²² The solid line in Figure 3a shows the boundary condition for the default zone consistent with Condition (8). At this point it is useful to remember that this condition considers the tradeoff for the government to default under the fundamental price of debt. Thus, this boundary is identical to the default boundary in standard models of sovereign default. Consistent with the data, the government is able to hold more debt when output is high. The crisis zone, shown as the darker grey area, is delimited by these two bounds.

In Figure 3b I consider the effect that an increase in the level of international reserves has on the crisis zone. The red lines represent the boundary condition for a higher level of reserves relative to the blue lines. An increase in the stock of international reserves held by the government shrinks (from below) the region in which the economy is exposed to the possibility of a self-fulfilling crisis. This confirms in the quantitative solution of the model the theoretical discussion set forth in Section 3.2. An increase in the stock of international reserves has the unambiguous effect of shifting up the boundary between the safe zone and the crisis zone. Note that Figure 3b shows an indistinguishable effect of international reserves on the boundary for the default zone. This might mislead one to conclude that international reserves have no effect on the default zone boundary. International reserves can also shift the default boundary up, allowing the government to hold more debt at a given output level. Appendix Figures D.1 and D.2 show how the boundary conditions change as I condition on higher levels of international reserves.

Figure 4 shows how negative output shocks (fundamental risk) affect the price of government debt. In panel 4a I show the price of debt conditional on the level of international reserves. In contrast, panel 4b shows the price of debt as a function of reserves conditional

²²In the model, tax revenues are a constant fraction τ of output.

Figure 3: International reserves and the crisis zone

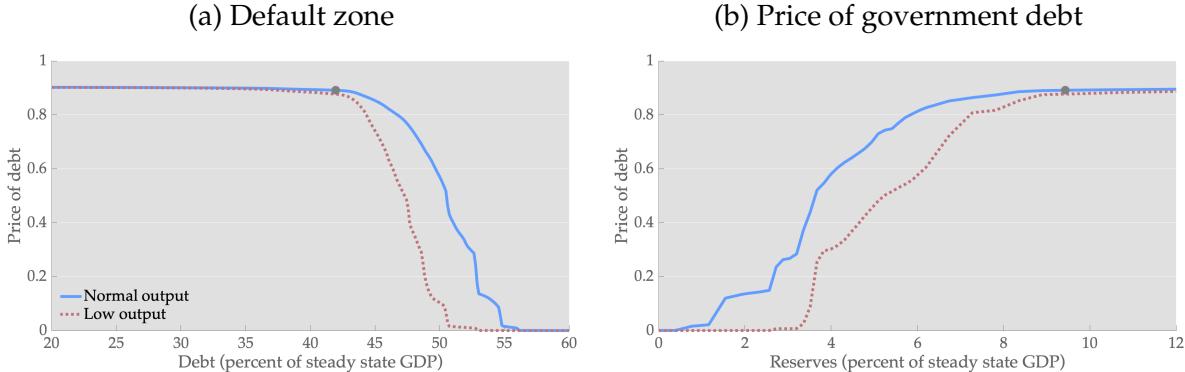


Notes: This figure shows the boundary conditions for the default zone (solid lines) and safe zone (dotted lines) for a given level of international reserves that are consistent with Conditions (8) and (9). The shaded region in the left panel highlights the crisis region. The blue line corresponds to $f = 0.0$, while the red line corresponds to $f = 0.1$.

on a given level of government debt. The blue solid line shows the pricing schedule in the steady-state level of output and a zero probability of a self-fulfilling crisis. The red dotted line shows the same boundary for a lower level of output and the same zero probability. An economy that is in the mean of the ergodic distribution is at the grey point. Figure 4a shows how for the same choice of reserves, the pricing schedule shifts left, making debt more costly. Moreover, Figure 4b shows that increasing the level of reserves has a very limited effect on the price of debt. Going back to the discussion in Section 3, these two figures illustrate the sensitivity of the pricing schedule to the portfolio choice. It shows that, after the output shock, both $\partial q / \partial b'$ and $\partial q / \partial f'$ are low, with the former potentially being considerably negative. This suggests very little benefit from issuing additional debt to increase the stock of reserves.

Similarly, in Figure 5 I show how an increase in the probability of a self-fulfilling crisis impacts the economy through the price of government securities. The blue solid line shows the pricing schedule at an output level consistent with the mean of the ergodic distribution conditional on the economy being in the crisis zone. The probability of a self-fulfilling crisis is set to the steady-state level π^* . The red dotted line shows the same objects but for a higher probability of a self-fulfilling crisis. An economy at the mean of the ergodic distribution conditional on the economy being in the crisis region is at the grey point. Panel 5a in the left shows the price of government securities for different levels of debt while keeping the level of international reserves fixed. The price of debt, rather than shifting left, is now depressed for levels of debt that, conditional on the level of international reserves, would put the economy in the crisis zone. In this scenario, the

Figure 4: The effects of an output shock on the economy



Notes: The figure on the left shows the pricing schedule of government debt for a given level of output and international reserves. The reserve level corresponds to the mean of the ergodic distribution, $f = 1.24$ (9.59 percent of steady-state GDP). The probability of a self-fulfilling crisis tomorrow is set to zero ($\pi_{t+1} = 0$). The figure on the right shows the pricing schedule of government debt for a given level of output and debt. The level of debt corresponds to the mean of the ergodic distribution, $b = 5.41$ (41.94 percent of steady-state GDP). For both figures, the blue lines correspond to $y = 12.903$, while the red lines correspond to the lower level of output $y = 12.34$. The grey point represents the allocation at the mean of the ergodic distribution.

intuition from panel 5b is very insightful. After the shock (red dotted line) there are big returns from accumulating international reserves, even at the expense of higher debt prices if such a policy were to be accompanied by an increase in debt. Mapping this to the discussion in Section 3 is equivalent to saying that $\partial q / \partial f'$ is high, suggesting big benefits from issuing additional debt to finance an increase in reserves.

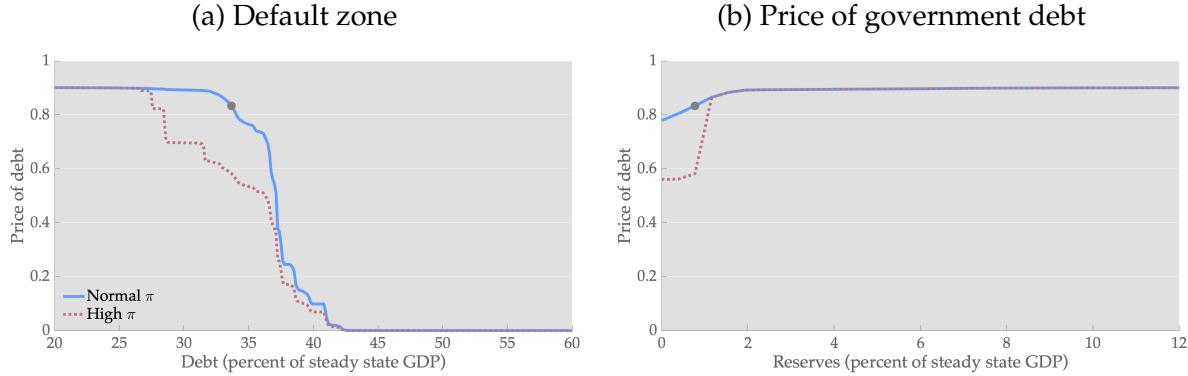
5 International reserve management policy

In this section I turn to the main theoretical contribution of the paper, the optimal international reserve management policy during a sovereign default crisis. In Section 5.1 I study how a commonly used rule of thumb by practitioners, the Guidotti–Greenspan rule, shows up in the model simulations. In Section 5.2, I analyze the average response of international reserves to changes in output. Finally, in Section 5.3 I show the optimal international reserve management policy for the government. This last section illustrates the relevance of the source of default risk for reserves management during a crisis.

5.1 Self-fulfilling crises and the Guidotti–Greenspan rule

The discussion in Section 3 highlighted that a government exposed to a self-fulfilling crisis would act differently in choosing its stock of international reserves. In this context,

Figure 5: The effects of a shock on the probability of a self-fulfilling crisis



Notes: The figure on the left shows the pricing schedule of government debt for a given level of output and international reserves. The reserve level corresponds to the mean of the ergodic distribution conditional on the economy being in the crisis zone, $f = 0.21$ (1.60 percent of steady-state GDP). The figure on the right shows the pricing schedule of government debt for a given level of output and debt. The level of debt corresponds to the mean of the ergodic distribution conditional on the economy being in the crisis region, $b = 4.35$ (33.74 percent of steady-state GDP). For both figures, the output is set at $y = 11.72$. The blue line is consistent with a probability of a self-fulfilling crisis tomorrow equal to that of the steady state ($\pi_{t+1} = \pi^*$). The red line is consistent with an increased probability of a self-fulfilling crisis tomorrow $\pi_{t+1} = \pi^* + 3\sigma_\pi$. The grey point represents the allocation at the mean of the ergodic distribution conditional on the economy being in the crisis region.

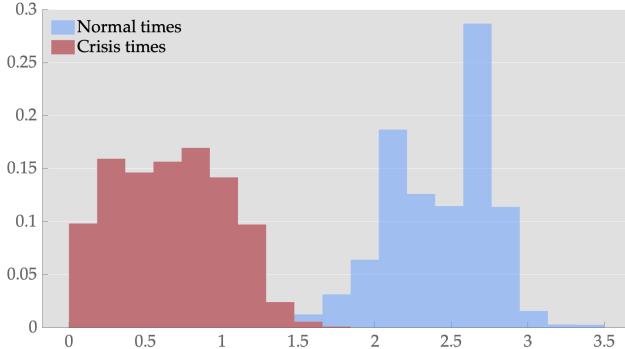
a relevant rule to explore is the Guidotti–Greenspan rule. This traditional rule of thumb is often used by policymakers to guide adequacy levels of international reserves.²³ It states that a country's stock of international reserves should be equal to its short-term external debt. In the calibrated model, debt maturity is set to 2.7 years. This calibration implies that 9.3 percent of the stock of debt matures each quarter.²⁴ At the simulation mean level of debt (41.92 percent of GDP), a government has maturing debt each quarter on the order of 3.90 percent of GDP. The average level of reserves to GDP in the model is 9.54 percent. This suggests that the level of international reserves held by the government is higher than the amount of short-term (maturing) debt.

Further exploring the issue yields an important observation. Figure 6 shows the ratio of international reserves over outstanding maturing debt (i.e., f/ι_b). A ratio of one is consistent with the above interpretation of the Guidotti–Greenspan rule. It is convenient to remember that a government is only exposed to the sunspot equilibrium when the state situates the economy in the crisis region. Simulations from the model highlight that this mostly occurs when the ratio of international reserves over outstanding maturing debt is less than 1.5. This suggests that the Guidotti–Greenspan rule is a good rule of thumb to rule out self-fulfilling crises in the model. This result is informative for studying the

²³See [Jeanne and Rancière \(2006\)](#) for further details on rules of thumb and the Guidotti–Greenspan rule.

²⁴All debt in the model is external.

Figure 6: The Guidotti–Greenspan rule in model simulations



Notes: This figure presents the ratio of reserves to maturing debt (i.e., $f/\tau b$) conditional on whether the state-space is consistent with crisis times. An economy is considered to be out of normal times when its observed fundamentals (output, debt, and international reserves) situate the government in the crisis zone. Each histogram is normalized to probabilities over the relevant observations. The figure is computed from 10,000 simulations of 40 periods each. The economy is in the crisis region 2.94 percent of the time. For details on the procedure for simulations see Appendix C.2.

average behavior of international reserves in the model simulations next.

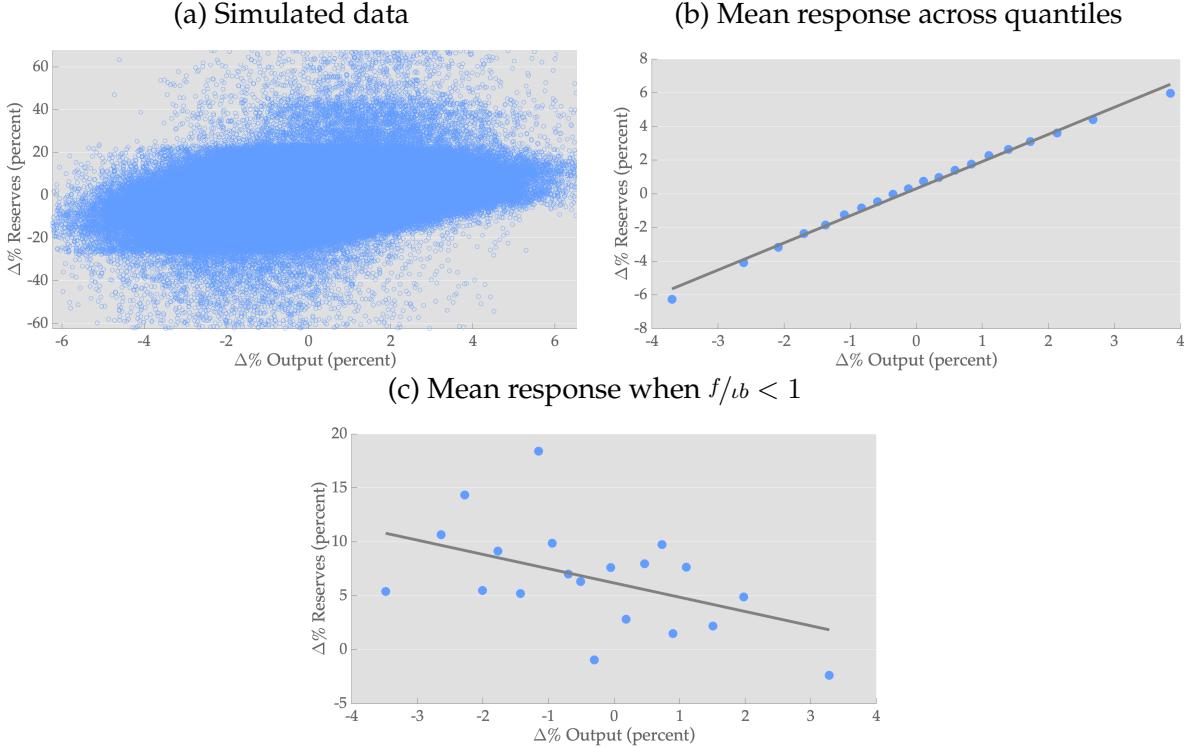
5.2 Quantitative analysis of international reserves management

To illustrate when is a good time for the government to use reserves, in this Section I characterize the average behavior of reserve management in the model. The simulation results suggest that a drop in output should be accompanied by the government running down reserves. Panel 7a shows a scatter plot of the optimal reserves response by the government to changes in output in the model simulations. Panel 7b aggregates the data by quantile of output variation. Each dot represents the mean response within the quantile. The figure suggests that it is possible to prescribe a rough policy recommendation: a 1 percent drop in output should be matched with a decline in the stock of reserves of 1.6 percent. This is in line with the common view in the literature that reserves should be used to smooth out government spending. However, it completely disregards the possibility of the economy being exposed to the possibility of a self-fulfilling crisis.

I use the information conveyed by the Guidotti–Greenspan rule to condition on whether the economy is exposed to the possibility of a self-fulfilling crisis. Panel 7c shows that, when the ratio of international reserves to maturing debt is less than 1 in the simulations, the government increases its stock of international reserves. In addition, the more exposed the economy is given the drop in output, the more reserves the government chooses to accumulate. Since crises are costly for the government, this result illustrates the importance of the non-fundamental source of default risk in determining the best

policy response.

Figure 7: International reserves' response to a change in output



Notes: Figure 7a comes from simulating the model 10,000 times for 40 periods. It shows the percent change in output on the horizontal axis and the percent change in international reserves on the vertical axis. Figure 7b aggregates observations in quantiles with respect to the percent change in output. Each dot represents the mean response within that quantile. The black line represents the best linear fit to all observations. The line is given by $\Delta\%f = 0.003 + 1.609\Delta\%y$. Figure 7c shows a similar plot conditioning on the ratio $f/\lambda b$ being less than 1. In this case the best linear fit is given by $\Delta\%f = 0.062 - 1.323\Delta\%y$.

5.3 Optimal international reserve management policy

To discuss optimal reserve management policy during a sovereign default crisis I perform a quantitative exercise. In order to isolate each source of default risk, the exercise is done in two parts. First I consider an increase in fundamental risk. To highlight only the effects of fundamental default risk, I consider a drop in output while keeping the probability of a self-fulfilling crisis at zero (i.e., $\pi_{t+1} = 0$ for all t). Second, I consider an increase in the probability that, conditional on the economy being in the crisis zone, lenders coordinate on the bad equilibrium in which there is a self-fulfilling crisis. To make the exercise above comparable, I consider that the shock to π_{t+1} decays at the same rate as the output shock.²⁵

²⁵The parametrization of the model has π being independent and identically distributed.

Figure 8 shows the impulse response functions of interest rate spreads, international reserves, debt, and government spending. The values reported for interest rates correspond to basis points changes. For international reserves, debt, and government spending, the values correspond to percent change relative to the mean of the ergodic distribution. It is important to highlight that models of sovereign debt are known to exhibit considerable nonlinearities. As an example, the price of debt in Figures 4 and 5 features sharp nonlinearities. For this reason, impulse response functions are computed as the mean response over 10,000 simulations. For the fundamental risk exercise, simulations are started sampling from the ergodic distribution. To have a meaningful impact on the economy from a change in the sunspot probability, simulations for the non-fundamental risk exercise are started by sampling from the ergodic distribution conditional on the economy being in the crisis zone.²⁶ In Appendix C.2 I describe the procedure to initialize the impulse response functions. The procedure is slightly different from what is usually done in the literature.²⁷ Given the nonlinearities in these types of models, I consider my approach to yield a more satisfactory result of the object of interest.

First I study a shock to the economy's fundamentals. The impulse response functions shown in blue solid lines in Figure 8 correspond to a one-standard-deviation drop in output. This is equivalent to a drop in output of approximately 4.36 percent. Figure 8a shows (blue solid lines) that when the economy's fundamentals deteriorate the interest rate spread on government securities increases. This reflects the previously discussed fact that when the economy is weak the risk of a default by the government increases. The drop in output (and thus in tax revenues) constrains government spending. In a model with no frictions a government would like to smooth consumption through debt or drawing on its savings (international reserves). In this model, weak economic conditions are associated with a high cost of borrowing (high spreads), and thus, as shown in Figure 8b, the government uses reserves to attenuate the fall in government spending. In fact Figure 8c shows that debt becomes so costly that the government's response is also to reduce its choice for debt.

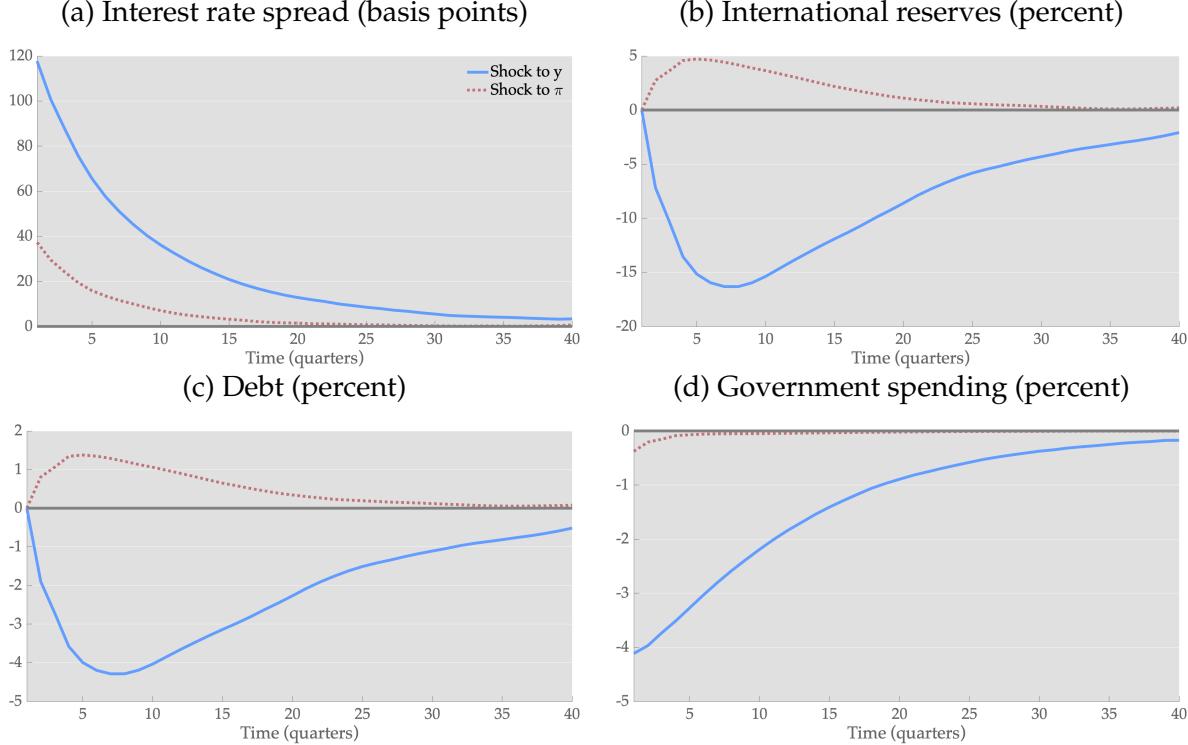
This result deserves further discussion. Upon the shock, the government faces an output drop. All things constant, this maps into a drop in consumption and thus a higher marginal utility of consumption. The government has two ways to finance additional resources: issuing additional debt or depleting part of its stock of international reserves. As Figure 4a illustrates, issuing additional debt comes with a high cost, in particular after

²⁶Figure 2 shows relevant moments for the ergodic distribution and the ergodic distribution conditional on the economy being in the crisis zone.

²⁷In the literature, impulse response functions are traditionally initialized at the mean of the ergodic distribution. This has no qualitative difference in the results showed in this section.

the weak output has been realized. Further, Figure 4b shows that increasing the level of reserves has little to no effect on the price of debt. Finally, it is important to note that the probability of a self-fulfilling crisis occurring in the next period is set to zero in the exercise. All these considered, the government finds it optimal to finance any additional fiscal needs by depleting its stock of reserves. In this sense, reserves are a government rainy-day fund used during economic downturns.

Figure 8: Optimal international reserve management policy



Notes: This figure shows the impulse response functions (IRFs) of interest rate spreads, international reserves, debt, and government spending. Interest rate values correspond to basis point changes. International reserves, debt, and government spending values correspond to percent change relative to the mean value of the ergodic distribution. The blue solid line reports the response to a one-standard-deviation decline in output. The red dotted line reports the response to a three-standard-deviation increase in the probability of a self-fulfilling crisis. It assumes that the non-fundamental shock decays at a rate of 0.915. The IRF to output, y , is initialized by drawing the state variables from the ergodic distribution and considers $\pi_{t+1} = 0$ at all periods in the simulations. The IRF to π_{t+1} is initialized by drawing the state variables from the ergodic distribution conditional on the government being in the crisis zone. IRFs are the average across 10,000 simulations. For details on the procedure to for simulations and IRFs see Appendix C.2.

I now turn to the second part of the exercise when the economy experiences an increase in π_{t+1} . The impulse response functions shown in red dotted lines in Figure 8 correspond to a three-standard-deviation increase in the probability of lenders not extending credit tomorrow if the economy is in the crisis zone. This is equivalent to an increase of 0.139 in such a probability. When non-fundamental risk increases, the risk of a govern-

ment default increases as spreads in Figure 8a show (red dotted line). The government's optimal response to this shock is to increase its stock of international reserves (Figure 8b). As explained above, an increase in the level of international reserves has the desired effect of shrinking the crisis zone. By shrinking the crisis zone, the government is able to contain the effects of an increase in the probability of a self-fulfilling crisis. As previously discussed, the increase in π_{t+1} has no effect on the boundary of the crisis zone. Figure 8c shows that the government is able to mostly finance the increase in international reserves by issuing additional debt with little cost to government spending (see Figure 8d)

In contrast to the first exercise, Figure 5b shows that, faced with an increase in the probability of a self-fulfilling crisis, in what pertains to the price of debt, the government has high gains from increasing the stock of international reserves. It is convenient to remember that the government starts from a state drawn from the ergodic distribution conditional on the economy being in the crisis zone. This implies that the government is in a position where the economy is potentially exposed to a multiplicity of equilibria arising from lenders' expectations. By increasing its stock of reserves the government is able to limit the exposure of the economy to the bad equilibrium. Moreover, to contain the cost to consumption from having to increase the level of international reserves, the government finds it optimal to partially finance the increase in reserves by issuing additional debt. This is consistent with the discussion in Section 3.2 and highlights the importance of having long-term debt in the model. With short-term debt this trade would not provide any additional insurance to the government. The reason for this is that all additional debt would mature next period, providing no additional liquidity insurance for the government.

To summarize, the results obtained in this section corroborate the discussion in Section 3. Optimal international reserves management during a sovereign debt crisis depends on whether the source of default risk is fundamental or non-fundamental. During weak economic conditions, when fundamental risk is high, debt becomes too costly, and drawing on reserves helps smooth government spending. When spreads increase because of an increase in non-fundamental risk, it is optimal for the government to accumulate reserves. Increasing reserves reduces the exposure of the economy to a self-fulfilling crisis.

6 The 2018 Argentine crisis

Since the two sources of default risk have opposite implications for reserve management, the model provides an ideal setup to attempt to quantify the source of default risk that a

government is facing. In taking the model to the data one must assume that policymakers are aware of the different tradeoffs that using reserves have during a crisis. In what follows I illustrate that the discussion between the government of Argentina and the IMF seems to indicate that the relevant stakeholders were aware of this situation. A clear example of this is the Guidance Note on the Assessment of Reserve Adequacy prepared by the IMF to guide adequacy levels of international reserves. The guidelines make explicit that close attention should be paid to potential rollover risk arising from high external short-term debt. In constructing their reserve adequacy metric, the component that receives more weight (30 percent) by the IMF is precisely the ratio of short-term debt to reserves. Granted this, I use the model to illustrate the self-fulfilling component of the recent debt crisis in Argentina. Before turning to the quantitative exercise, it is convenient to review the recent macroeconomic history of Argentina. In Section 6.1 I review the macroeconomic history of Argentina since the 2001 default and up to 2017. The most recent debt crisis, which occurred in 2018, is reviewed in Section 6.2. Once this has been settled, the quantitative exercise is put forward in Section 6.3.

6.1 Prelude to the crisis

Buera and Nicolini (2018), Calvo (2018), and Powell (2018) do an outstanding job in reviewing and commenting on the recent macroeconomic history of Argentina. Between 1960 and 2017 Argentina experienced several balance-of-payments crises and two defaults on government debt.²⁸ To put things in the correct dimension, income per capita in Argentina in 2003 was roughly at the same level that it was thirty years earlier.

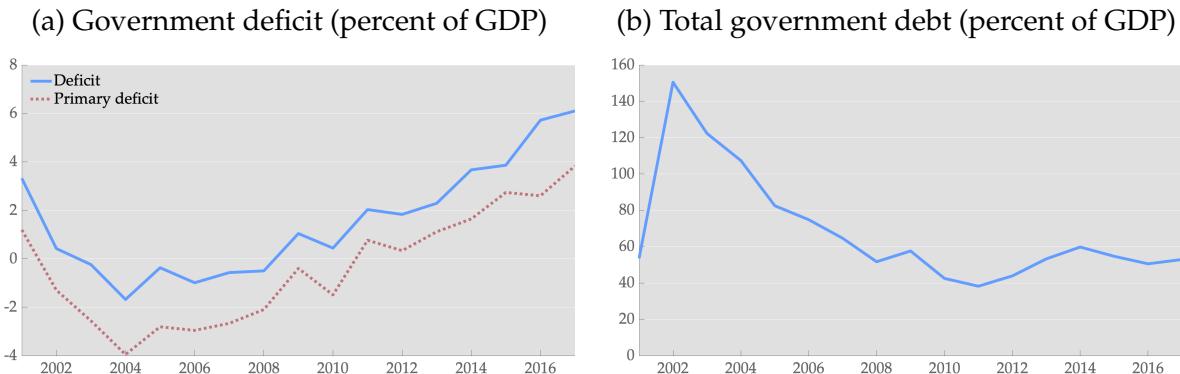
To study the 2018 debt crisis in Argentina, it is convenient to go back and study Argentina's macroeconomic history starting with the sovereign default of 2001. That year, Argentina suspended all debt payments and defaulted on over 100 billion US dollars in external sovereign debt. During the long restructuring process Argentina remained excluded from international financial markets. In 2005 Argentina reached a first agreement with most of its creditors on the terms for restructuring its debt obligations.²⁹ Another exchange was made in 2010. At that time, more than 90 percent of the original liabili-

²⁸The technical default of 2014 is not considered a default, as Argentina did not willingly stop servicing its debt obligations. After all the legal impediments were cleared, Argentina continued honoring all debt claims. Hébert and Schreger (2017) use this episode in an effort to quantify the costs of default.

²⁹An exchange was made for 37 cents on the dollar. A country cannot command bondholders to accept the exchange, and some bondholders opted to hold out on the exchange. Sovereign bonds were issued with unanimity requirements similar to those outlined by the Trust Indenture Act of 1939. It was not until 2003 that a country (Mexico) first issued a bond with a collective action clause. Díaz de León (2016) provides a historical account of the contractual innovation that led Mexico to include collective action clauses in its sovereign bond issuances.

ties had been restructured. Holdouts, mostly hedge funds, decided to embark in a legal strategy and demanded they be paid in full.³⁰ The case took several years to work its way through the legal system, and a final ruling was made in favor of holdout creditors, which technically led to a new default by Argentina in 2014. It was not until 2016 that Argentina reached an agreement with holdout creditors. Once settled, Argentina regained access to international financial markets.

Figure 9: Government deficit and total debt



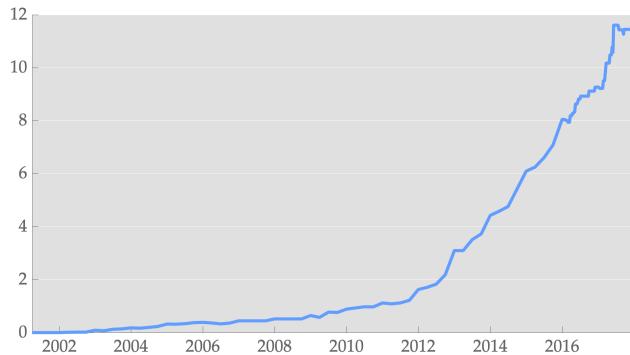
Notes: Government deficit is the ratio of government deficit to gross domestic product. Positive (negative) values are consistent with a government deficit (surplus). Debt is the ratio of total government debt to gross domestic product for Argentina. The source of the data is the Becker Friedman Institute's Monetary and Fiscal History of Latin America project. The data are complemented with the budget and debt reports of the Ministry of Finance of Argentina.

Figure 9 shows that the recovery from the crisis and default of 2001 was accompanied by a level of fiscal discipline that was almost unheard of in Argentina's history: the government banked six consecutive years with a fiscal surplus. However, things changed during and after the global financial crisis as the commodities boom faded away. The surpluses turned into consistent government deficits. By 2013, total government debt was at the level it had been prior to the 2001 default. In 2013 the government was running a 2 percent fiscal deficit, and by 2017 the deficit reached 6 percent of gross domestic product. As the fiscal imbalance continued, debt grew alongside it. Since the government remained excluded from international financial markets, the government relied heavily on the central bank as a source of financing, to the detriment of international reserves. Figure 10 shows direct financing from the central bank to the central government as reflected in the bank's balance sheet. The central government using the central bank as an additional source of financing exerted additional pressure on the central bank's balance sheet.

³⁰Holdout creditors collectively held approximately seven percent of defaulted bonds. For further detail of this case see [Hébert and Schreger \(2017\)](#), [Armella \(2017\)](#), and court documents for *Republic of Argentina v. NML Capital*.

This partly explains the wedge between international reserves at the central bank and those net of short-term foreign currency liabilities. Figure 11c shows that, by the end of 2016, international reserves net of short-term foreign currency liabilities held by the central bank were negative. Not all funding came from the central bank, and domestic debt was also issued during this period. In 2016 the Argentine government finally reached an agreement with holdout creditors. With this settlement the government regained access to international credit markets.

Figure 10: Central bank financing to the central government (billion US dollars)



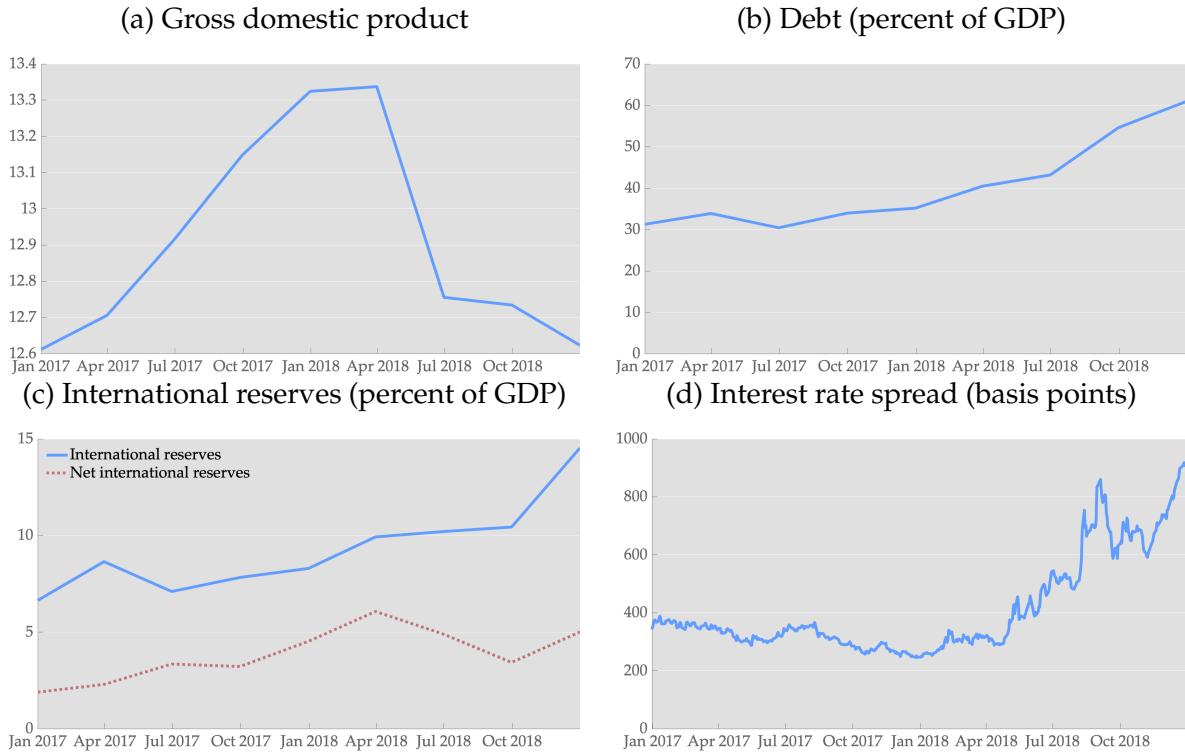
Notes: Direct central bank financing to the central government is reported in the bank's balance sheet as *temporary advances to the Argentine government*. Data reflect the cumulative balance up to the reported date. Data are obtained from the weekly balance report of the Banco Central de la República Argentina. The balance, in Argentine pesos, is converted to US dollars at the exchange rate set in the IMF program of 41.25 pesos per dollar.

6.2 The 2018 debt crisis

In December 2015 a new administration assumed office in Argentina. One of the first actions undertaken by the new government was to outline a broad agenda of economic reforms. Currency controls were eliminated, and a settlement with holdout creditors was achieved. As Argentina regained access to financial markets, there was investor appetite for its sovereign securities, and financing pressure was eased from the central bank. To put things into perspective, in June 2017 investors oversubscribed by more than three and a half times a bond maturing in 100 years. In fact, during 2017 the governor of the central bank of Argentina set forth an ambitious target for international reserve accumulation to achieve a ratio of reserves to gross domestic product of more than 15 percent. The main reason expressed by the central bank for this policy was to have enough firepower in case of an adverse scenario in the future, highlighting the precautionary motive that governments have in accumulating reserves. On the fiscal side, the primary deficit continued

in 2016 and 2017 and was increasingly financed with external debt. These events set the scenario for what came in 2018. Figure 11 shows how, after two years of stable spreads on government bonds (Figure 11d) and a policy of international reserves accumulation (Figure 11c), things changed abruptly during the summer of 2018. Spreads increased substantially, reaching levels of more than 1,000 basis points.

Figure 11: Output, debt, international reserves, and interest rate spreads for Argentina



Notes: Gross domestic product is detrended log gross domestic product. Debt is the ratio of external debt to gross domestic product for Argentina. International reserves are those held at the central bank. Net international reserves consider short-term foreign currency liabilities of the central bank consistent with the technical appendix to the memorandum agreed between the International Monetary Fund and the government of Argentina in 2018. See Appendix A for a complete description of the data.

As borrowing became prohibitive at such a high cost, international reserve management by the central bank was central to the government's policy response. The central bank provided direct financing to the government at the peak of the crisis during the summer of 2018. Then, as part of an agreement with the IMF, the central bank decided to partially rollback its support to the government during the last quarter of 2018. This agreement was part of an aid package approved by the IMF in June 2018. The Technical Memorandum of Understanding agreed between Argentina and the IMF sets out the specific objectives to which the Argentine government committed to achieve in order to access IMF aid funds. The document states that the central bank is to "discontinue all di-

rect or indirect central bank financing of the government and reduce the credit exposure of the central bank to the government." Furthermore, as part of the full independence that the government committed to grant the central bank, it had to confirm that international reserves should only serve to implement monetary policy. The aid package was approved by the IMF in June 2018. In addition to stopping all direct financing to the central government, the Argentine government also agreed to increase the level of net international reserves.³¹ This evidence is suggestive of stakeholders perceiving an increase in non-fundamental risk during the second half of 2018. Through the eyes of the model, the relevant decision makers are not only the Argentine officials, but also the IMF through its Stand-By Arrangement.

6.3 The quantitative role of self-fulfilling expectations

In this section I proceed to use the calibrated model to measure the non-fundamental component of the increase in interest rate spreads observed in Argentina during 2018.³² The model provides an ideal setup to attempt to illustrate the source of default risk that the Argentine government was facing.³³ As the discussion so far conveyed, both fundamental and non-fundamental sources of sovereign default risk are able to explain an increase in government spreads. However, the two sources of default risk have opposite implications for the optimal behavior of international reserve management. Faced with an increase in fundamental risk, as apparent from the output realization portrayed in Figure 11a, in the model the government would want to use reserves to smooth government spending. Conversely, if the source of risk is non-fundamental, and the crisis is instead a self-fulfilling crisis led by investors' expectations, then the government would ideally choose to protect and increase its stock of reserves. I capitalize on this insight and employ a particle filter in order to extract from the model the sequence of structural shocks that best account for the behavior of output, reserves, and interest rate spreads. With this in hand, I then perform a counterfactual exercise by simulating the model under the extracted fundamental shocks, while setting the probability of a self-fulfilling crisis to zero.

³¹Net international reserves in the Technical Memorandum are defined as official international reserves minus gross official liabilities with maturities of under one year. See Appendix A for more details.

³²For the quantitative model used in this section I introduce an adjustment cost on international reserves and debt. The purpose of this addition is to limit the government from increasing international reserves too fast. Parametrically the adjustment cost is quadratic and enters the value function as $-\varphi_b(b' - b)^2$ and $-\varphi_f(f' - f)^2$. The calibrated values for this parameter are $\varphi_b = 0.25$, $\varphi_f = 0.50$. All other parameters in the model remain unchanged.

³³In this section Argentine government refers to the agents making the policy decisions in Argentina. Thus, at a later stage in 2018 the IMF plays an important role through the stand-by arrangement agreed with Argentina.

I view the difference between these two paths as the contribution of non-fundamental risk.

To highlight the filtering exercise, the model can be rewritten as the nonlinear state-space system

$$\mathbf{x}_t = \mathbf{g}(\mathbf{s}_t) + \boldsymbol{\eta}_t, \quad (29)$$

$$\mathbf{s}_t = \mathbf{f}(\mathbf{s}_{t-1}, \boldsymbol{\varepsilon}_t), \quad (30)$$

where the state vector is $\mathbf{s}_t = [y_t, b_t, f_t, \pi_{t+1}]$, $\boldsymbol{\varepsilon}_t = [\varepsilon_{y,t}, \varepsilon_{\pi,t+1}, \zeta_t]$ is the vector of structural shocks, and \mathbf{x}_t is a vector of observable variables. The vector $\boldsymbol{\eta}_t$ contains uncorrelated Gaussian errors to capture deviations between the data and the model's outcome. The functions $\mathbf{g}(\cdot)$ and $\mathbf{f}(\cdot)$ are the model solution obtained in Section 4.

As the observable in \mathbf{x}_t I consider output, reserves-to-GDP ratio, interest rate spreads on government bonds, and whether the government defaults on its debt. To extract the structural shocks I employ the particle filter in the above state-space model.³⁴ Since no default was observed in the data up to 2018, the measurement errors $\eta_{\delta,t}$ are set to zero. This has the effect of having all default decisions in the model coincide with the ones in the data. That is, only nondefault choices coming from the model are going to be consistent with the data. This leaves $\varepsilon_{y,t}$ and $\varepsilon_{\pi,t+1}$ as the unobserved structural shocks. The variance of the measurement errors associated with international reserves and interest rate spreads is set to 5 percent of their respective sample variance. The variance of the measurement error to output is set to 3.5 percent.³⁵

Figure 12 shows the main results from this quantitative exercise. The model is able to replicate the observed increase in the interest rate spread of government bonds observed in Argentina during the summer of 2018. Prior to the crisis, the model has a difficult time matching the level of interest rate spreads observed. For a level of debt of close to 40 percent as the one observed in Argentina, the model cannot rationalize spreads being so high given the relatively good level of output. It is important to remember where the Argentine economy was coming from. The economy has defaulted eight times in the past and has spent close to 30 percent of the last 100 years in recession. Given this context, spreads for Argentina are higher than those of other comparable emerging market economies.

During 2018 the Argentine economy experienced a contraction in output, an increase in the ratio of external debt to GDP, and a an increase in interest rate spreads. The quan-

³⁴The particle filter algorithm followed is detailed in Appendix C.3.

³⁵In Appendix C.4 I conduct a sensitivity analysis with respect to the choice of these parameters.

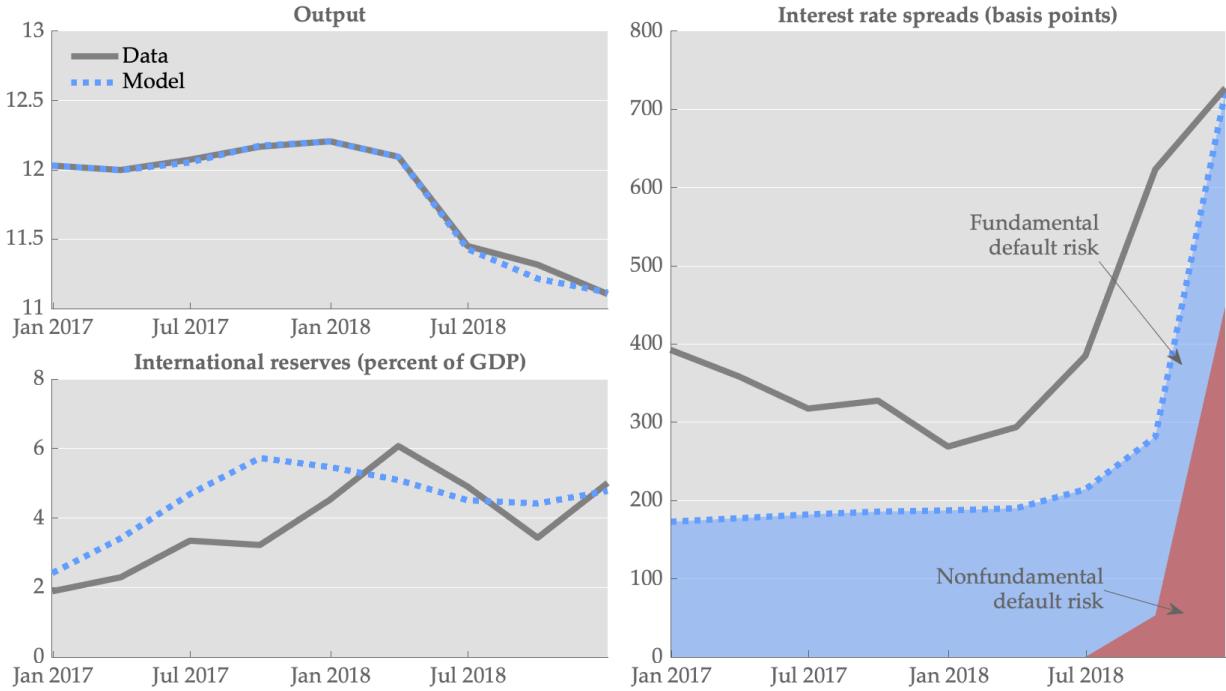
titative exercise shows that in order to rationalize the observed data, both fundamental and non-fundamental sources of risk account for a fraction of the observed increase in government spreads. During the first quarter of 2018 there was little to no change in the yield of Argentine government securities. For the next three quarters the model mirrors the increase in observed interest rate spreads. In the second and third quarters of 2018, the economy experienced a contraction in output, higher interest rate spreads, and a depletion of international reserves. All consistent with an increase in the fundamental source of default risk. During the fourth quarter the contraction in output and the increase in spreads continued. However, there was an increase in the stock of international reserves, consistent with an increase in non-fundamental default risk.

Results from the quantitative exercise suggest that during the second and third quarters, more than 80 percent of the increase in interest rate spreads can be explained from higher fundamental default risk. During the third quarter non-fundamental risk accounted for 18.93 percent of spreads. All consistent with an increase in the fundamental source of default risk. During the fourth quarter the reverse is true, where non-fundamental default risk accounts for two-thirds of the increase in spreads. Relative to the average observed during 2017, fundamental default risk contributed on average 60 basis points to the increase in interest rate spreads, while non-fundamental risk contributed on average 167 basis points.

To further understand the forces in the model, Figure 13 compares the quantitative path of international reserves with the counterfactual path of reserves if the sunspot shock is set to zero. During 2018 output and international reserves decreased in Argentina. Under the optimal reserve management framework developed in Section 4, this suggests that the increase in the fundamental source of default risk was the driver of the increase in default risk. However, an important highlight is the fact that absent non-fundamental risk, international reserves would have been even lower. That is particularly true during the last quarter of 2018. The increase in reserves observed during that quarter is consistent with an increase in non-fundamental risk. This increase is in part a result of the agreement reached with the IMF. Thus, the result is suggestive of the IMF and Argentine officials perceiving non-fundamental risk as an important component to be factored into policy decisions.

The previous exercise takes seriously the assumption that the government of Argentina is following an international reserve management policy consistent with the model's optimality behavior. The existence of an exchange rate management channel or any other policy objective challenges the previous result. Another observation is the fact that part of the increase in international reserves came as a constraint agreed upon in the support

Figure 12: Contribution of non-fundamental default risk

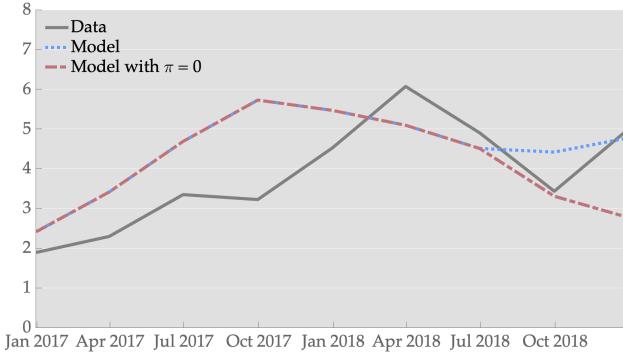


Notes: This figure presents the trajectory of normalized output, the ratio of international reserves to gross domestic product, and the interest rate spread on government bonds for the 2017–18 period. The black solid line represents the data for Argentina. The blue dotted line represents the best fit of the model obtained by applying a particle filter to the model. Non-fundamental default risk is equal to the difference between the model's best fit and a counterfactual exercise where the probability of a self-fulfilling crisis is set to zero.

package that the IMF provided to Argentina. Through the eyes of the model, the relevant decision makers are not only the Argentine officials, but also the IMF through its Stand-By Arrangement. The support plan for Argentina considered an increase in international reserves in part financed with a loan from the IMF. This result illustrates the tradeoffs that policymakers in Argentina were facing and is consistent with a perceived increase in non-fundamental default risk.

I conduct an additional quantitative exercise. For this second exercise, in implementing the particle filter, I do not consider international reserves as an observable. In Figure 14 I compare the trajectories of international reserves and non-fundamental default risk for both filtering exercises. The blue dotted line shows the trajectory for this variable from the previous exercise. The red line shows the trajectory for international reserves and non-fundamental risk from applying the particle filter to the model and dropping international reserves from the set of observables. Up until the first half of 2018 both exercises yield similar trajectories. There is no non-fundamental risk, and international reserve management policy is guided by the fundamental state of the economy. However, for the

Figure 13: Non-fundamental default risk and international reserves



Notes: This figure shows the trajectory for the ratio of international reserves to gross domestic product for the 2017–18 period. The black solid line represents the data for Argentina. The blue dotted line shows the best fit of the model obtained by applying the particle filter. The red dashed line shows the counterfactual path for international reserves if the probability of a self-fulfilling crisis is set to zero throughout the period.

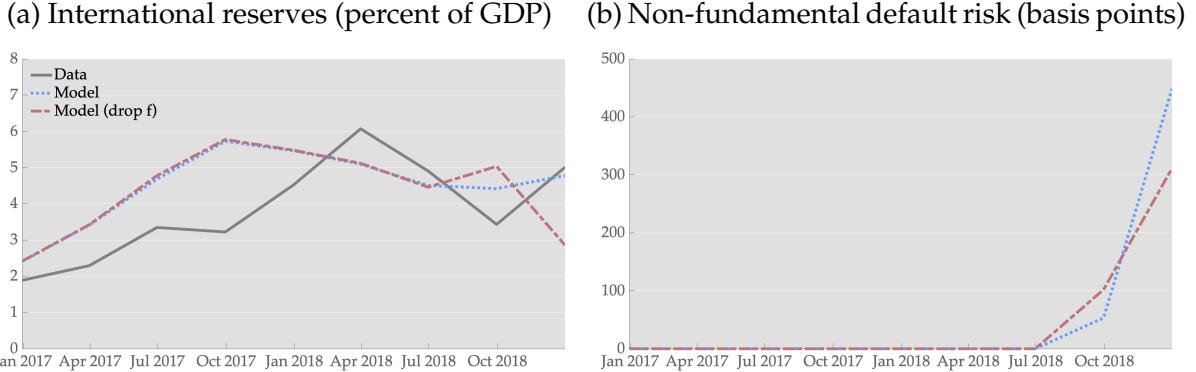
second half of 2018 the trajectories for international reserves differ. Not conditioning on observable reserves (red dashed line) yields a counterfactual path for reserves. During the third quarter of 2018, to match the increase in government interest rate spreads, without the discipline of observed reserves there is a larger increase in non-fundamental default risk. This leads to the counterfactual observation that international reserves increase during this quarter rather than decline. Finally, during the fourth quarter of 2018 the reverse is true. Not conditioning on the observed increase in reserves leads the model to assign a lower probability of a self-fulfilling crisis. This has the effect of the government following a fundamental risk policy and instead depleting additional reserves.

This exercise contributes towards learning about the international reserve management policy followed by Argentina and later guided by the standby agreement with the International Monetary Fund. In general, it is possible to say that the policy of international reserve management followed in Argentina during the 2018 crisis was consistent with the forces suggested by the model.

7 Conclusion

In this paper I provided some insight into how the government should manage its stock of international reserves during a sovereign debt crisis. I showed that interest rate spreads are not sufficient to guide international reserve management. The reason for this is that the source of default risk is important in determining the optimal reserve management policy. When output is low and the government finds it challenging to service its debt, fundamental default risk is high. In such environment, the government's optimal behav-

Figure 14: Evaluating Argentina's reserve management policy



Notes: This figure shows the trajectory for the ratio of international reserves to gross domestic product and non-fundamental default risk for the 2017–18 period. The black solid line represents the data for Argentina. Non-fundamental default risk is unobservable in the data. The blue dotted line shows the best fit of the model obtained by applying the particle filter. The red dashed line shows the best fit of the model obtained by applying the particle filter and dropping international reserves from the set of observables. Non-fundamental default risk is the difference between the model's predicted interest rate spreads and the counterfactual spreads if the probability of a self-fulfilling crisis is set to zero throughout the period.

ior is to lower reserves in order to smooth government spending. When there is a high probability of a self-fulfilling crisis next period, non-fundamental default risk is high. In response the government should increase its stock of reserves to reduce the economy's exposure to the possibility of multiple equilibria.

The central mechanism is the interaction between reserves and the possibility of multiple equilibria in some regions of the state-space. By accumulating reserves the government is able to shut down the possibility of self-fulfilling crises. Given this understanding, when non-fundamental risk is high, the government has additional incentives to increase its stock of international reserves to limit the economy's exposure. On the contrary, when non-fundamental risk is low, reserves serve as a rainy-day fund for the government. During weak economic conditions, additional borrowing becomes prohibitively costly for the government. In this scenario reserves provide funds for the government to smooth government spending intertemporally.

In an empirical application I used the model to illustrate the source of default risk that Argentina was facing during the 2018 debt crisis. Fundamental risk contributed predominantly to the increase in interest rate spreads on government debt during the first three quarters of 2018. The findings also point out that in order for the model to rationalize the observed behavior of international reserves, non-fundamental risk must have played an important role in the second half of 2018.

While the model shows how reserve policy should respond to different sources of de-

fault risk, it does not address other policy objectives governments could address using international reserves. In particular, exchange rate management is often considered in practice an important component of monetary policy in emerging market economies. In these countries, the exchange rate has an important role for price stability, financial conditions, and external competitiveness (Carstens, 2019). Taking account of the exchange rate seems instrumental in providing a comprehensive view of international reserve management. In future work I plan to incorporate the role of the exchange rate in the domestic economy.

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Appendix

A Data appendix

Real gross domestic product (GDP) data is obtained from OECD Quarterly National Accounts. Values correspond to volume estimates in the GDP expenditure approach, volume with reference year 2010. Data are obtained for the period 2004:Q1–2018:Q4. I linearly detrend the log series for this data.

International reserves data are obtained from two different sources. For the quarterly data on international reserves, data are obtained from the International Monetary Fund. Data are provided in million US dollars in the international financial statistics data set under International Reserves and Liquidity, Reserves, Official Reserve Assets. Daily data on international reserves of Argentina are provided by the Banco Central de La República Argentina (Argentina's central bank). Data are provided as Reservas Internacionales del BCRA.

Net international reserves data for Argentina are obtained from the weekly balance reports of the Banco Central de La República Argentina (Argentina's central bank). Data are provided as Reservas Internacionales del BCRA. Following the definition of net international reserves outlined in the Memorandum of Understanding agreed between the IMF and Argentina, I use the Summary Balance of Assets and Liabilities Series to construct a weekly series for the level of net international reserves. To aggregate the series quarterly I average between the weeks of the quarter.

External debt data are obtained from the World Bank Quarterly External Debt Statistics (SDDS). Detailed data are obtained from the quarterly report of public debt prepared by the Finance Ministry in Argentina.

Consumer price index data are converted, when pertinent, into 2010 real US dollars using the quarter average of the Monthly CPI Seasonally Adjusted obtained from the Federal Reserve Economic Data (FRED).

Interest rate spreads on government bonds are obtained from the Bloomberg Terminal. For Argentina I consider the yield on a generic five-year maturity bond in US dollars. The spread is the difference in yield from the US Treasury bill benchmark of the same matu-

rity.

Debt maturity information is obtained from the reports of the debt management office. In fitting the model to the data it is important to capture the rollover risk that Argentina was facing. For this purpose I consider the value of maturing long- and medium-term debt plus short-term debt from the last period over the stock of outstanding debt. Data are gathered from the quarterly report of public debt prepared by the Finance Ministry in Argentina.

Tax rate and nondiscretionary spending data for Argentina are obtained from the federal government Finance Ministry public account report. I consider the base year to be 2016.

B Mathematical appendix

B.1 Short-term debt and international reserves

In this section I show the intuition for the need to have long-term debt in models of sovereign default with international reserves. To understand the interaction of international reserves with short-term debt, it is convenient to look at a simple two-period model. Consider an economy in which the government only faces fundamental risk.

$$\begin{aligned} & \max_{b_2, f_2, \delta_2} u(g_1) + \beta \mathbb{E}[\delta_2 u(g_2) + (1 - \delta_2) \underline{V}_2] \\ \text{s.t. } & g_1 \leq \tau y_1 - b_1 + f_1(1 + r) - f_2 + qb_2 \\ & g_2 \leq \tau y_2 - b_2 + f_2(1 + r) \end{aligned} \tag{B.1}$$

Proposition 1. *In the two-period economy described above, with positive fundamental default risk the government holds no reserves.*

Proof. By way of contradiction, suppose $\{b_2, f_2\}$ is an equilibrium outcome with $f_2 > 0$ and $b_2 > 0$. Consider the following variation: decrease f_2 by ε and decrease b_2 by ε/q so that g_1 remains unchanged at the original price. The variation can replicate the consumption pattern $\{g_1, g_2\}$ prescribed by the original allocation. In period 2, reserves plus its return decrease by $\varepsilon(1 + r)$, while debt repayment decreases by ε/q , under the assumption that there is positive fundamental risk $q < (1 + r)^{-1}$. Thus debt repayment decreases by more than the loss in reserves since $\varepsilon(1 + r) < \varepsilon/q$. This results in a contradiction

of g_2 being optimal, proving that the original allocation cannot be an equilibrium allocation. \square

B.2 Optimal portfolio problem

Consider a state of the economy s and a target for government spending \bar{g} . Let $x = \{s, \bar{g}\}$ be the vector of the initial state and the spending target. The combination of debt and reserves (b', f') that delivers such a spending target is given by

$$f' = \tau y + f(1 + r) - \iota b - \bar{g} + q(s, b', f')(b' - (1 - \iota)b). \quad (\text{B.2})$$

The amount of reserves that the government can accumulate with newly issued bonds depends on how the bond price varies with the portfolio choice. I define $\tilde{f}(b', x)$ as the amount of reserves that can be purchased when the government borrows b' , while satisfying Equation (B.2). By the implicit function theorem, an extra unit of debt in the current period enables the government to purchase

$$\frac{\partial \tilde{f}(b', x)}{\partial b'} = \frac{q(s, b', \tilde{f}(b', x)) + \frac{\partial q(s, b', \tilde{f}(b', x))}{\partial b'}(b' - (1 - \iota)b)}{1 - \frac{\partial q(s, b', \tilde{f}(b', x))}{\partial f'}(b' - (1 - \iota)b)} \quad (\text{B.3})$$

additional reserves without changing current government spending. The key insight from Equation (B.3) is that the more favorable the changes in the bond price in response to the increase in debt ($\partial q/\partial b'$) and reserves ($\partial q/\partial f'$), the more reserves a government issuing debt can accumulate. The changes in the bond price, in turn, depend on how today's choices affect future incentives to default.

B.3 Crisis zone

In this section, I show that Conditions (8) and (9) are enough to define the crisis zone along the equilibrium path. That is, imposing the fundamental price along the equilibrium path is enough to characterize the crisis region. This proof follows the one in Online Appendix A to [Bocola and Dovis \(2018\)](#).

Denote by \mathcal{S}^{\max} the collection of states for which the government defaults if lenders choose the worst possible price for the government that is consistent with the lenders' no-arbitrage condition (this condition is defined by Equation (6)). Proposition 2 characterizes such a set.

Proposition 2. Given $V(\mathbf{s})$ and $q(\mathbf{s}, b', f')$, $\mathbf{s} \in \mathcal{S}^{\max}$ if and only if

$$V^d(\mathbf{s}) > \max_{b' \leq (1-\iota)b, f'} u(\tau y + f(1+r) - f' - \iota b + q(\mathbf{s}, b', f')(b' - (1-\iota)b)) + \beta \mathbb{E}_{\mathbf{s}'|\mathbf{s}}[V(\mathbf{s}')]. \quad (\text{B.4})$$

Proof. If Equation (B.4) does not hold, then the government can do better than default by purchasing back part of its outstanding stock of debt. Thus, in such a state \mathbf{s} the government does not default on its debt and $\mathbf{s} \notin \mathcal{S}^{\max}$. The highest possible price $q(\mathbf{s}, b', f')$ in Equation (B.4) is the fundamental pricing function $\tilde{q}(\mathbf{s}, b', f')$ defined by Equation (7). A lower price would only increase the righthand side of Equation (B.4) since the government is only allowed to repurchase debt, which makes $b' - (1-\iota)b \leq 0$. Thus imposing the fundamental pricing function is without loss of generality. This proves necessity.

For sufficiency, note that $\mathbf{s} \in \mathcal{S}^{\max}$ if for all b' such that $b' - (1-\iota)b' \geq 0$ then

$$\max_{f'} u(\tau y + f(1+r) - f' - \iota b + \beta \mathbb{E}_{\mathbf{s}'|\mathbf{s}}[V(\mathbf{s}')]) < V^d(\mathbf{s}), \quad (\text{B.5})$$

and for all b' such that $b' - (1-\iota)b' < 0$ then

$$\max_{b', f'} u(\tau y + f(1+r) - f' - \iota b + \tilde{q}(\mathbf{s}, b', f')(b' - (1-\iota)b)) + \beta \mathbb{E}_{\mathbf{s}'|\mathbf{s}}[V(\mathbf{s}')]) < V^d(\mathbf{s}). \quad (\text{B.6})$$

In Equation (B.5) I use the fact that when the government issues additional debt the worst price for the government is zero. Likewise, in Equation (B.6), when the government purchases back part of its debt, the worst price for the government is the fundamental price. If these two conditions are satisfied (Equations (B.5) and (B.6)), then it is rational for lenders to expect a default, and it is in turn optimal for the government to default. Further, as $\mathbb{E}_{\mathbf{s}'|\mathbf{s}}[V(\mathbf{s}')]$ is decreasing in b' , it is sufficient to check Equation (B.5) at $b' = (1-\iota)b$ when net issuances equal zero. Combining this simpler version of Equation (B.5) with Equation (B.6) implies that $\mathbf{s} \in \mathcal{S}^{\max}$. \square

I can then define the crisis zone as $\mathcal{S}^{\text{crisis}} = \mathcal{S}^{\max} \setminus \mathcal{S}^{\text{funda}}$, where $\mathcal{S}^{\text{funda}}$ is the collection of states for which the government defaults if lenders choose the fundamental price.

B.4 Price decomposition

In this section I provide a step-by-step decomposition of government spreads. The starting point is the fundamental price of debt. This is the price of debt that a government that is committed to repaying its debt in the current period would face. Such a government

would face the price of debt consistent with Equation (7):

$$\tilde{q}_t = \mathbb{E} \left\{ \frac{\delta_{t+1} (\iota + (1 - \iota) q_{t+1})}{1 + r} \right\}. \quad (\text{B.7})$$

For a given price of debt tomorrow, q_{t+1} , the expectation operator acts only over the decision to default tomorrow. Thus,

$$\begin{aligned} \tilde{q}_t(1 + r) &= P_t(\text{No default at } t+1)(\iota + (1 - \iota)q_{t+1}) \\ &= (1 - P_t(\mathbf{s}_{t+1} \in \mathcal{S}^{\text{def}})) - P_t(\mathbf{s}_{t+1} \in \mathcal{S}^{\text{crisis}}) \times \pi_{t+1}(\iota + (1 - \iota)q_{t+1}) \end{aligned} \quad (\text{B.8})$$

where the last line follows from the definition of the default and crisis regions in Section 2.3. Consider the analogue equation for a bond that faces no default risk tomorrow:

$$q_t^{rf}(1 + r) = \iota + (1 - \iota)q_{t+1} \quad (\text{B.9})$$

taking the difference of the fundamental price from a risk-free bond yields the spread decomposition showed in Section 3.

$$\frac{(q_t^{rf} - q_t^*)(1 + r)}{\iota + (1 - \iota)q_{t+1}} = P_t(\mathbf{s}_{t+1} \in \mathcal{S}^{\text{def}}) - P_t(\mathbf{s}_{t+1} \in \mathcal{S}^{\text{crisis}}) \times \pi_{t+1} \quad (\text{B.10})$$

C Computational appendix

C.1 Solution algorithm

To solve the model numerically I define three relevant value functions for the government: the value function of the government conditional on not defaulting, $V^{nd}(\cdot)$; the value function conditional on lenders shutting down access to debt markets, $V^{ss}(\cdot)$; and the value function on default. The assumption of a complete debt wipeout upon default and the independent and identically distributed shock structure for π' simplifies the state-space on default to two variables, y and f .

$$V^{nd}(y, b, f, \pi') = \max_{b', f'} u(\tau y + f(1 + r) - f' - \iota b + \tilde{q}(\mathbf{s}, b', f')(b' - (1 - \iota)b)) + \beta \mathbb{E}_{\mathbf{s}'|\mathbf{s}}[V(\mathbf{s}')], \quad (\text{C.1})$$

$$V^{ss}(y, b, f, \pi') = \max_{f'} u(\tau y + f(1 + r) - f' - \iota b) + \beta \mathbb{E}_{\mathbf{s}'|\mathbf{s}}[V(y', (1 - \iota)b, f', \pi'')], \quad (\text{C.2})$$

$$V^d(y, f) = \max_{f'} u(\tau y + f(1 + r) - f') - \varphi(y) + \beta \mathbb{E}_{\mathbf{s}'|\mathbf{s}}[V(y', 0, f', \pi'')]. \quad (\text{C.3})$$

where $\mathbf{s}' = \{y', b', f', \pi''\}$. The numerical solution of the model consists in approximating the value functions $\{V^{nd}, V^{ss}, V^d\}$ as well as the price function \tilde{q} .

$$\tilde{q}(y, \pi', b', f') = \mathbb{E}_{\mathbf{s}'|\mathbf{s}} \left\{ \frac{(1 - \delta(\mathbf{s}')) (\iota + (1 - \iota) q(\mathbf{s}', b'', f''))}{1 + r} \right\}. \quad (\text{C.4})$$

The value functions are approximated using a combination of linear interpolation and a Chebyshev approximation. A known feature in quantitative models of sovereign default is the sharp nonlinearity in the pricing schedule of debt. For this reason the price function is approximated over a finer grid. Specifically, $V^x(\cdot)$ is approximated as follows:

$$V^x(y, b, f, \pi') = \gamma_b^{x'} \mathbf{T}(y, f, \pi'), \quad (\text{C.5})$$

where (y, b, f, π') is a realization of the state variables, γ_b^x is a vector of coefficients for each different value of b , and $\mathbf{T}(\cdot)$ is a vector collecting Chebyshev's polynomials. The numerical solution is $\{\gamma^{nd}, \gamma^{ss}, \gamma^d, \hat{q}\}$ which is obtained via value function iteration.

Before solving I select the bounds and number of grid points for the four state variables in \mathbf{s} . For output, I consider seven points located in the Chebyshev's nodes in an interval of plus and minus three-and-a-half standard deviations. Label this grid as \mathcal{Y} . For debt, I consider 41 equally spaced points in the interval $[0, 14.7]$, \mathcal{B} . The upper bound of this interval is consistent with a debt-to-output ratio of 114 percent at the steady-state level of output. For international reserves, I set 11 points at the Chebyshev's nodes in the $[0, 2.1]$ interval, \mathcal{F} . Finally, for the probability that lenders coordinate on the bad equilibrium, I set five points at the Chebyshev's nodes for an interval of minus three and plus four standard deviations, π . The state-space grid \mathbf{S} is constructed using tensor multiplication of these nodes, $\mathbf{S} = \mathcal{Y} \times \mathcal{B} \times \mathcal{F} \times \pi$. These grid points, together with the Chabyshev's polynomials, are used in the approximation of the value functions using a collocation method. The pricing schedule is approximated over a finer grid. I set 51 equally spaced points in the y dimension and 11 equally spaced points in the π dimension. The policy dimensions in the price schedule are on the same grid as the policy choice for the government. On the b' dimension I set 401 points in the $[0, 14.70]$ interval. For the interval $[0, 3)$ I assign 15 equally spaced points. In the interval $[3, 8.5]$ I set 336 equally spaced points. Finally, the remaining 50 points are equally spaced in the $(8.5, 14.7]$ interval. For the f' dimension I set 51 points in the $[0, 2.1]$ interval. In the interval $[0, 0.25]$ I set 6 equally spaced points. In $(0.25, 0.8)$ I equally spread 27 points. The remaining 18 points are equally spaced in the interval $[0.8, 2.1]$. Table C.1 shows the value for the relevant computational parameters.

Table C.1: Computational parameters for the solution algorithm

Variable or Parameter	Min	Max	Nodes/Points
Output, \mathcal{Z}	$-3.5 \frac{\sigma_u}{\sqrt{1-\rho_y^2}}$	$3.5 \frac{\sigma_u}{\sqrt{1-\rho_y^2}}$	7
Debt, \mathcal{B}	0.0	14.7	41
International reserves, \mathcal{F}	0.0	2.1	11
Non-fundamental prob., Π	$\pi^* - 3\sigma_\pi$	$\pi^* + 4\sigma_\pi$	5
Debt choice, $\bar{\mathcal{B}}$	0.0	14.7	401
Reserves choice, \mathcal{F}'	0.0	2.1	51
Number of grid points in z dimension of q			51
Number of grid points in π' dimension of q			11
Order of Gauss–Hermite quadrature			7×7

Notes: This table shows the choice of computational parameters used in the solution of the model following the algorithm outlines in the current section. As a collocation method is used, the order of the Chebyshev polynomial is equal to the number of nodes. For notation purposes I define z as the natural logarithm of tax revenues.

The solution algorithm follows the following steps:

1. Initialize the algorithm with an initial guess for the value functions $\{\hat{V}_0^{nd}, \hat{V}_0^{ss}$ and $\hat{V}_0^d\}$ as well as the price schedule $\{\hat{q}_0\}$.
2. Let $\{\hat{V}_n^{nd}, \hat{V}_n^{ss}$ and $\hat{V}_n^d\}$ be the value functions at iteration n . Given these values use the projection method to update the value of $\{\gamma^{nd}, \gamma^{ss}$ and $\gamma^d\}$ used to approximate the value function.
3. Update the value functions using their definitions. To compute expectations use Gauss–Hermite quadrature.
4. For each element in the pricing grid compute the pricing schedule using the new value function to determine the governments default decision. I use a Gauss–Hermite quadrature to compute expectations.
 - (a) The step requires the interim computation of the price of debt tomorrow. I use a linear interpolation to compute both the policy choice of b and f tomorrow (i.e., b'' and f'').
 - (b) I then linearly interpolate the price for the relevant space and choices computed above (i.e., $\hat{q}_{n-1}(z', \pi', b'', f'')$).
5. Check for convergence in the value functions and pricing schedule using the norm

of choice. If the value functions and pricing schedule have converged, then a solution has been achieved. If not, set $n = n + 1$ and repeat from Step 2.

For convergence of the value functions I consider the sup norm in logs. Convergence is achieved at a level of 0.01633 for the value function of repayment, 0.00929 for the value function in the sunspot state, and 0.00019 for the value function of default. For convergence of the price schedule I consider the square norm. A convergence of 0.08507 is achieved in this dimension.

A relevant issue arises in certain regions in the grid space. For high values of debt b it could be that there exists no value of b' such that the government is able to repay its debt and have a level of consumption above the subsistence level \bar{g} . That is, for certain values in the grid space there is an empty budget set. I take the following stance on this issue. First, in such a state the government must choose to default on its debt obligations. Second, the imputed value for the value function at those states is equal to either the minimum value of repayment conditional on the problem being well defined, the minimum value in the sunspot state conditional on the problem being well defined, or the minimum value of default. This region of the state-space is never visited in model simulations, as the government would, well before reaching that state, have defaulted on its debt obligations.

C.2 Simulation algorithm and impulse response functions

To simulate the model I consider the solution to the model given by $\{\gamma^{nd}, \gamma^{ss}, \gamma^d, \hat{q}\}$ as well as the policy functions $\{b'_{rp}(\cdot), f'_{rp}(\cdot), f'_d(\cdot)\}$. The policy functions are defined over the state-space grid, and thus are a function of $\{z, b, f, \pi'\}$, where z is the natural logarithm of y . Before simulating the model I set the number of simulations N_{sim} as well as the length period of each simulation N_T . In what follows, I index the number of simulation with i , the time period with t , and the value of variable x in simulation i at time t as $x(t, i)$. To simulate the model I then follow the following algorithm:

1. Initialize the state variables. That is, set a value for $\{z(0, i), b(1, i), f(1, i), \pi'(0, i)\}$. Set $indefault(0, i) = 0$ to indicate the economy was not in default in the previous period. Finally set $t = 1$.
2. Draw innovations for output, $\varepsilon_y(t, i)$, and the probability of a self-fulfilling crisis tomorrow, $\varepsilon_\pi(t, i)$, from standard normal distributions. Update z and π following

their law of motions:

$$z(t, i) = \rho_y z(t - 1, i) + \sigma_y \varepsilon_y(t, i) \quad (\text{C.6})$$

$$\pi'(t, i) = \pi^* + \sigma_p i \varepsilon_\pi(t, i) \quad (\text{C.7})$$

3. Use $\{\gamma^{nd}, \gamma^{ss}, \gamma^d\}$ to compute the value function $\{v^{rp}(t, i), v^{ss}(t, i), v^d(t, i)\}$ at the state given by $\{z(t, i), b(t, i), f(t, i), \pi'(t, i)\}$.
4. Determine the default decision of the government.
 - (a) If $v^d(t, i) > v^{rp}(t, i)$ then the government defaults. Set $\delta(t, i) = 1$ and $indfault(t, i) = 1$. Move to step 9.
 - (b) Otherwise, if $v^d(t, i) > v^{ss}(t, i)$ then the government is in the crisis region. In this case I draw a random number from the uniform distribution in the unit interval. If the number is less than $\pi'(t - 1, i)$ then lenders coordinate on the bad equilibrium and the government defaults. Set $\delta(t, i) = 1$ and $indfault(t, i) = 1$. Move to step 8.
 - (c) Otherwise, the government does not default and instead chooses to honor its outstanding liabilities. Set $\delta(t, i) = 0$ and $indfault(t, i) = 0$. Move to the next step.
5. Using $\{b'_{rp}(\cdot), f'_{rp}(\cdot)\}$ linearly interpolate the policy decisions of the government at the state given by $\{z(t, i), b(t, i), f(t, i), \pi'(t, i)\}$. Set $b(t + 1, i)$ and $f(t + 1, i)$ as the interpolated values.
6. Using $\{\hat{q}\}$ I linearly interpolate the price of government debt at $\{z(t, i), \pi'(t, i), b(t + 1, i), f(t + 1, i)\}$ to compute $q(t, i)$.
7. Government spending is computed from the resource constraint following:

$$g(t, i) = y(t, i) + (1.0 + r)f(t, i) + q(t, i)(b(t + 1, i) - \iota b(t, i)) - \iota b(t, i) - f(t + 1, i). \quad (\text{C.8})$$

If $t < N_T$ I set $t = t + 1$ and return to step 2. Otherwise I finish the simulation procedure.

8. When the economy is in default I linearly interpolate the policy decision of the government using $\{f'_d(\cdot)\}$ at the state $\{z(t, i), f(t, i)\}$. Government spending is computed

from the relevant resource constraint:

$$g(t, i) = y(t, i) + (1.0 + r)f(t, i) - f(t + 1, i). \quad (\text{C.9})$$

If $t < N_T$ I set $t = t + 1$ and return to step 2. Otherwise I finish the simulation procedure.

In computing the moments from simulated data, I only consider periods when the government is not in default. Moreover, to avoid unintended effects coming from the state in which simulations are initialized, I drop the first 100 periods. In the paper, interest rate spreads are shown annualized. To compute spreads in the simulations I consider the following definition:

$$\text{spread}(t, i) = (1 + (\iota/q(t, i) - \iota))^4 - (1 + r)^4. \quad (\text{C.10})$$

Impulse response functions. Models of sovereign default are nonlinear. For this reason I compute impulse response functions following [Koop, Pesaran, and Potter \(1996\)](#). In the literature, simulations for impulse response functions are often initialized at the mean of the ergodic distribution. Rather than following this procedure, I sample with replacement from the ergodic distribution to initialize all simulations used to compute the response. I consider this to be a more accurate procedure to capture what is intended from impulse response functions. That is, instead of capturing $f(\mathbb{E}[x])$, the impulse response function should capture $\mathbb{E}[f(x)]$, with x being distributed as in the ergodic distribution. Following [Koop, Pesaran, and Potter \(1996\)](#) the impulse response function is then computed as the average difference between the simulation with and without the shock. Further, to best capture the nonlinearities in the government's decision, I deviate slightly from the previous algorithm in simulating the model for the impulse response functions. In Step 6, rather than linearly interpolating the policy decisions of the government, I fully compute the optimization problem. This is computationally costly. However, it is important to fully capture the optimal response of the government to an increase in sovereign risk. In this setting, the continuation value is computed using a Gauss–Hermite quadrature, and the price of government debt is linearly interpolated.

C.3 Particle filter algorithm

For the filtering procedure I follow [Bocola and Dovis \(2018\)](#). The model can be rewritten as the nonlinear state-space-system:

$$\mathbf{x}_t = \mathbf{g}(\mathbf{s}_t) + \boldsymbol{\eta}_t, \quad (\text{C.11})$$

$$\mathbf{s}_t = \mathbf{f}(\mathbf{s}_{t-1}, \boldsymbol{\varepsilon}_t), \quad (\text{C.12})$$

where the state vector is $\mathbf{s}_t = [y_t, b_t, f_t, \pi_{t+1}]$, $\boldsymbol{\varepsilon}_t = [\varepsilon_{y,t}, \varepsilon_{\pi,t+1}, \zeta_t]$ is the vector of structural shocks, and \mathbf{x}_t is a vector of observable variables. The vector $\boldsymbol{\eta}_t$ contains uncorrelated Gaussian errors in order to capture deviations between the data and the model's outcome. The functions $\mathbf{g}(\cdot)$ and $\mathbf{f}(\cdot)$ are the model solution obtained in Section 4.

The particle filter is used to approximate $p(\mathbf{s}_t | \mathbf{x}^t)$, where \mathbf{x}^t is the history of realizations up to period t (i.e., $\mathbf{x}^t = [x_1, \dots, x_t]$). The approximation is done via the particle-weight pairs $\{\mathbf{s}_t^i, \tilde{w}_t^i\}$ for $i = 1, \dots, N$. The algorithm used is the following:

1. Initialize the filter with $\{\mathbf{s}_0^i\}$ for all i . Set $t = 1$.
2. Given \mathbf{s}_{t-1} , for each $i = 1, \dots, N$, obtain a realization for the state vector $\mathbf{s}_{t|t-1}^i$ from simulating the model in one period. (For this step follow the algorithm outlined in Appendix C.2.)
3. To each particle $\mathbf{s}_{t|t-1}^i$ assign the weight:

$$w_t^i = p(\mathbf{x}_t | \mathbf{s}_{t|t-1}^i) \tilde{w}_{t-1}^i. \quad (\text{C.13})$$

The weights are rescaled so that they add up to one and denoted $\{\tilde{w}_t^i\}$.

4. Resample N values from the state vector $\{\mathbf{s}_{t|t-1}\}$ with replacement using the distribution given by $\{\tilde{w}_t^i\}$, and denote these draws as $\{\mathbf{s}_t\}$. If $t > T$, finish. Otherwise, set $t = t + 1$ and repeat from step 2.

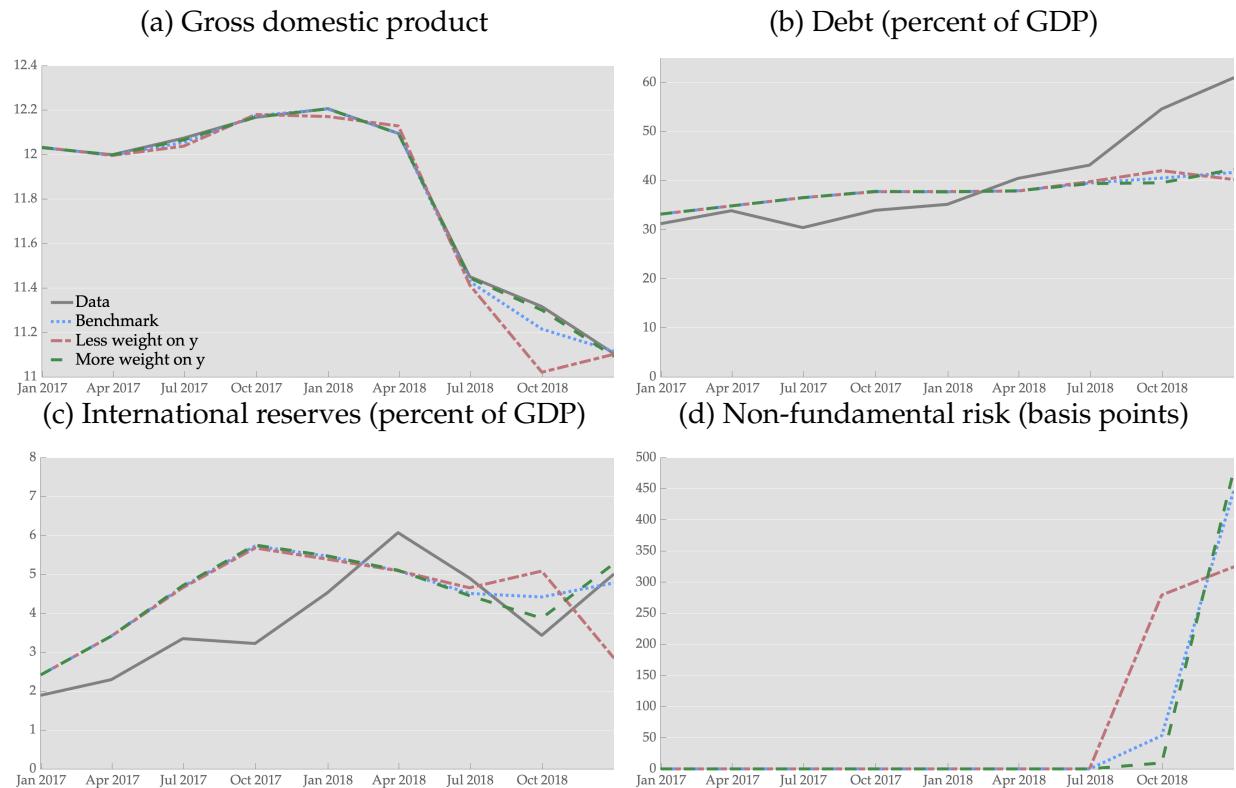
C.4 Counterfactual particle filer calibrations

Applying the particle filter to the model is sensitive to the weight that I assign to the output process. This has implications for the composition of default risk as well as the trajectories for external debt and international reserves. In this section I compare the trajectory of output, debt, international reserves, and non-fundamental default risk under different specifications for the variance of the measurement error in the output process. This has precisely the effect of assigning a larger or smaller weight to output in the particle filter exercise.

In Figure C.1 I show the sensitivity of the sovereign default risk decomposition to the weight that the particle filter assigns to output. Before the start of the crisis the trajectory of all variables of interest is very similar. However, once the crisis started during the summer of 2018, if I place too little weight on output, the particle filter uses that shock as

a residual to match interest rate spreads. The blue line shows my preferred specification. Assigning less weight to output (in red dotted lines in the figure) results in a larger counterfactual drop in output during the third quarter of 2018. Interestingly, this has the effect of exposing the economy to a higher possibility of multiple equilibria. Non-fundamental default risk then acts as a residual to match the observed higher interest rate spreads. The higher non-fundamental risk has the counterfactual implication of the government wanting to increase the level of reserves during the third quarter of 2018. In contrast, assigning too much weight to output relative to international reserves and interest rate spreads has the effect of delaying the contribution of non-fundamental risk fully to the last quarter of 2018.

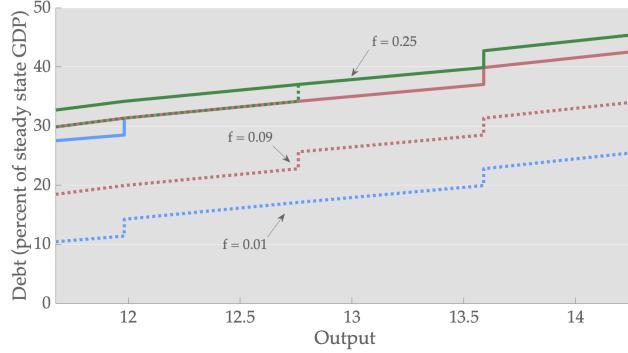
Figure C.1: Counterfactual particle filter specifications



Notes: This figure shows the trajectory for output, the ratio of debt and international reserves to gross domestic product, and non-fundamental default risk for the 2017–18 period. The black solid line represents the data for Argentina. The blue dotted line shows the best fit of the model obtained by applying the particle filter under the benchmark specification. The benchmark specification sets the variance of the measurement error on output to 3.5 percent. The red dotted-dashed line and the green dashed line show the results of having the particle filter assign lower (variance on measurement error equal to 5 percent) and higher (variance on measurement error equal to 1.5 percent) weight respectively to the output process.

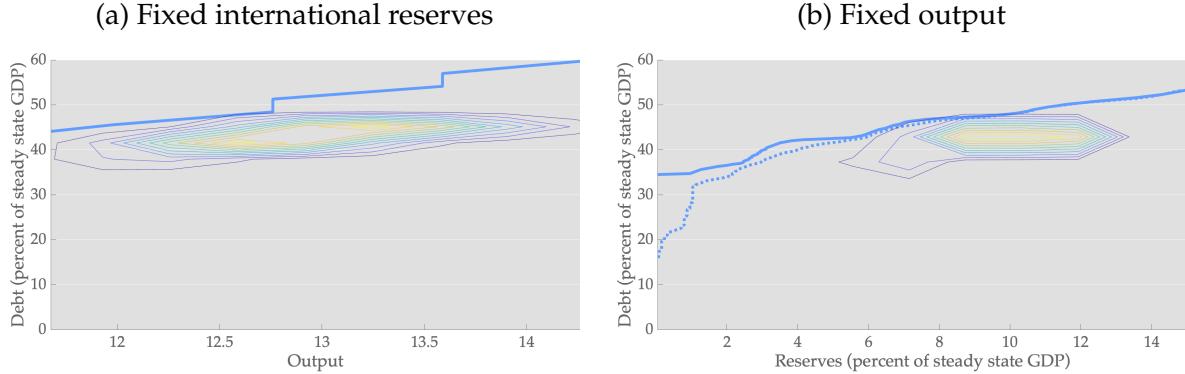
D Additional figures

Figure D.1: Crisis zone and international reserves



Notes: This figure shows the boundary conditions for the default zone and safe zone for a given level of international reserves that are consistent with Conditions (8) and (9). The blue line corresponds to $f = 0.01$, the red line corresponds to $f = 0.09$, and the green line corresponds to $f = 0.25$.

Figure D.2: Crisis zone at the ergodic distribution



Notes: This figure shows the boundary conditions for the default zone and safe zone consistent with Conditions (8) and (9). In the left panel, international reserves are considered at the mean of the ergodic distribution $f = 1.238$ that corresponds to 9.59 percent of steady-state GDP. In the right panel, output is considered at its steady-state level, $y = 12.903$. The ergodic distribution is over 10,000 simulations of 40 periods each. For details on the procedure for simulations see Appendix C.2.