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Cointegration and Threshold Adjustment

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This article proposes an extension to the Engle–Granger testing strategy by permitting asymmetry in the adjustment toward equilibrium in two different ways. We demonstrate that our test has good power and size properties over the Engle–Granger test when there are asymmetric departures from equilibrium. We consider an application—namely, whether there exists cointegration among interest rates for instruments with different maturities. This issue has been widely tested with mixed results. We argue that either cautious policy, or possibly opportunistic behavior on the part of the Federal Reserve implies that an equilibrium relationship between short- and long-term interest rates exists but that adjustments from disequilibrium are asymmetric in nature. Empirical tests using U.S. yields confirm the asymmetric nature of error correction among interest rates of different maturities.

KEY WORDS: Asymmetric adjustment; Monte Carlo; Nonlinear autoregression.

An important development in the recent time series literature is the examination of nonlinear adjustment mechanisms. Much of the impetus for this interest stems from a large number of studies showing that key macroeconomic variables such as real gross domestic product, unemployment, and industrial production display asymmetric adjustment over the course of the business cycle. For example, Neftci (1984), Falk (1986), DeLong and Summers (1986), Teräsvirta and Anderson (1992), Sichel (1993), Beaudry and Koop (1993), Ramsey and Rothman (1996), and Bradley and Jensen (1997) all supported various forms of asymmetric adjustment in one or more of these variables.

A natural extension to these univariate findings is to examine the possibility of nonlinear adjustment in a multivariate context. Toward that end, Granger and Lee (1989) found that U.S. sales, production, and inventories display asymmetric error correction toward a long-run multicointegrating relationship. Siklos and Granger (1997) showed that the strength of the interest parity relation changes over time, while both Balke and Fomby (1997) and Enders and Granger (1998) examined the relationship between short-term and long-term interest rates in an asymmetric framework.

This article introduces and develops an explicit test for cointegration with asymmetric error correction. In particular, we generalize the Enders and Granger (1998) threshold autoregressive (TAR) and momentum-TAR (M-TAR) tests for unit roots to a multivariate context. The basic TAR model, developed by Tong (1983), allows the degree of autoregressive decay to depend on the state of the variable of interest. The M-TAR model, used by Enders and Granger (1998) and Caner and Hansen (1998), allows a variable to display differing amounts of autoregressive decay depending on whether it is increasing or decreasing. This is in contrast to the Engle–Granger (1987) and Johansen (1996) tests that implicitly assume a linear adjustment mechanism. The distinction is important since Pippenger and Goering (1993), Balke and Fomby (1997), and Enders and Granger (1998) showed that tests for unit roots and cointegration all have low power in the presence of asymmetric adjustment. In particular, our M-TAR

modification of the Engle–Granger (1987) testing strategy has good power and size properties relative to the alternative assumption of symmetric adjustment. The Engle–Granger test emerges as a special case of our testing procedure.

The article is organized as follows. Section 1 describes a class of models with asymmetric adjustment toward a long-run cointegrating relationship. Section 2 develops a testing methodology and analyzes the power of the two tests. Section 3 illustrates the appropriate use of the tests using U.S. short-term and long-term interest rates. It is shown that an M-TAR adjustment mechanism best describes the behavior of the interest rates. Section 4 contains our concluding remarks.

1. ASYMMETRIC TIME SERIES MODELS

Standard models of cointegrated variables assume linearity and symmetric adjustment. Consider the simple linear relationship used as the basis for the many cointegration tests:

$$\Delta x_t = \pi x_{t-1} + v_t, \quad (1)$$

where x_t is an $(n \times 1)$ vector of random variables all integrated of degree 1, π is an $(n \times n)$ matrix, and v_t is an $(n \times 1)$ vector of the normally distributed disturbances v_{it} that may be contemporaneously correlated.

For example, the methodologies developed by Johansen (1996) and Stock and Watson (1988) entail the estimation of π and determining its rank. Equation (1) can be modified in many different ways including the introduction of deterministic regressors, the addition of lagged changes in Δx_t , and allowing the components of x_t to be integrated of various orders. Nevertheless, if $\text{rank}(\pi) \neq 0$, the implicit assumption is that the system exhibits symmetric adjustment around $x_t = 0$ in that for any $x_t \neq 0$, Δx_{t+1} always equals πx_t .

Similarly, the alternative hypothesis in the Engle and Granger (1987) test assumes symmetric adjustment. In the simplest case, the two-step methodology entails using ordinary least squares (OLS) to estimate the long-run equilibrium relationship as

$$x_{1t} = \beta_0 + \beta_2 x_{2t} + \beta_3 x_{3t} + \cdots + \beta_n x_{nt} + \mu_t, \quad (2)$$

where x_{it} are the individual $I(1)$ components of x_t , β_i are the estimated parameters, and μ_t is the disturbance term that may be serially correlated.

The second step focuses on the OLS estimate of ρ in the regression equation

$$\Delta\mu_t = \rho\mu_{t-1} + \varepsilon_t, \quad (3)$$

where ε_t is a white-noise disturbance and the residuals from (2) are used to estimate (3).

Rejecting the null hypothesis of no cointegration (i.e., accepting the alternative hypothesis $-2 < \rho < 0$) implies that the residuals in (2) are stationary with mean 0. As such, (2) is an attractor such that its pull is strictly proportional to the absolute value of μ_t . The Granger representation theorem guarantees that, if $\rho \neq 0$, (2) and (3) jointly imply the existence of an error-correction representation of the variables in the form

$$\Delta x_{it} = \alpha_i(x_{1t-1} - \beta_0 - \beta_2 x_{2t-1} - \cdots - \beta_n x_{nt-1}) + \cdots + v_{it}. \quad (4)$$

The point is that these cointegration tests and their extensions are misspecified if adjustment is asymmetric. Consider therefore an alternative specification of the error-correction model, called the threshold autoregressive (TAR) model, such that (3) can be written as

$$\Delta\mu_t = I_t \rho_1 \mu_{t-1} + (1 - I_t) \rho_2 \mu_{t-1} + \varepsilon_t, \quad (5)$$

where I_t is the Heaviside indicator function such that

$$I_t = \begin{cases} 1 & \text{if } \mu_{t-1} \geq \tau \\ 0 & \text{if } \mu_{t-1} < \tau \end{cases} \quad (6)$$

and τ = the value of the threshold and $\{\varepsilon_t\}$ is a sequence of zero-mean, constant-variance iid random variables, such that ε_t is independent of μ_j , $j < t$, and τ is the value of the threshold.

Petrucelli and Woolford (1984) showed that the necessary and sufficient conditions for the stationarity of $\{\mu_t\}$ is $\rho_1 < 0$, $\rho_2 < 0$ and $(1 + \rho_1)(1 + \rho_2) < 1$ for any value of τ . If these conditions are met, $\mu_t = 0$ can be considered the long-run equilibrium value of the system in the sense that $x_{1t} = \beta_0 + \beta_2 x_{2t} + \beta_3 x_{3t} + \cdots + \beta_n x_{nt}$. Since adjustment is symmetric if $\rho_1 = \rho_2$, the Engle–Granger test is a special case of (5) and (6). Moreover, Tong (1983, 1990) showed that the least squares estimates of ρ_1 and ρ_2 have an asymptotic multivariate normal distribution.

In general, the value of τ is unknown and needs to be estimated along with the values of ρ_1 and ρ_2 . However, in a number of economic applications it seems natural to set $\tau = 0$

so that the cointegrating vector coincides with the attractor. In such circumstances, adjustment is $\rho_1 \mu_{t-1}$ if μ_{t-1} is above its long-run equilibrium value and $\rho_2 \mu_{t-1}$ if μ_{t-1} is below long-run equilibrium.

Equations (2), (5), and (6) are consistent with a wide variety of error-correcting models. Given the existence of a single cointegrating vector in the form of (2), the error-correcting model for any variable x_{it} can be written in the form

$$\Delta x_{it} = \rho_{1,i} I_t \mu_{t-1} + \rho_{2,i} (1 - I_t) \mu_{t-1} + \cdots + v_{it}, \quad (7)$$

where $\rho_{1,i}$ and $\rho_{2,i}$ are the speed of adjustment coefficients of Δx_{it} . Since the speeds of adjustment can differ for each of the Δx_{it} , there is no requirement that $\rho_{1,1} = \rho_{1,2}$ or $\rho_{2,1} = \rho_{2,2}$.

Figure 1 shows the time paths of two $I(1)$ variables—say x_{1t} and x_{2t} —exhibiting threshold cointegration. For simplicity, the cointegrating vector is such that the system is in long-run equilibrium whenever $x_{1t} = x_{2t}$. Next, two sets of 500 normally distributed and serially uncorrelated pseudorandom numbers with standard deviations equal to unity were drawn to represent the $\{v_{1t}\}$ and $\{v_{2t}\}$ sequences. Setting $\rho_{1,1} = -\rho_{1,2} = -.05$, $\rho_{2,1} = -\rho_{2,2} = -.25$, $\tau = 0$, and initializing the initial values of the sequences equal to 0, the next 500 values of $\{x_{1t}\}$ and $\{x_{2t}\}$ were generated as in (7). Notice that the variables do not wander “too far” from each other in that positive and negative departures from long-run equilibrium are eventually eliminated. On inspection, it is clear that positive discrepancies persist for substantially longer periods than negative ones.

Further insight into the asymmetric nature of the adjustment process can be obtained using the specific numerical values for $\rho_{i,j}$ and subtracting Δx_{2t} from Δx_{1t} so that (5) becomes

$$\Delta\mu_t = -.1 I_t (x_{1t-1} - x_{2t-1}) - .5 (1 - I_t) (x_{1t-1} - x_{2t-1}) + v_{1t} - v_{2t}. \quad (8)$$

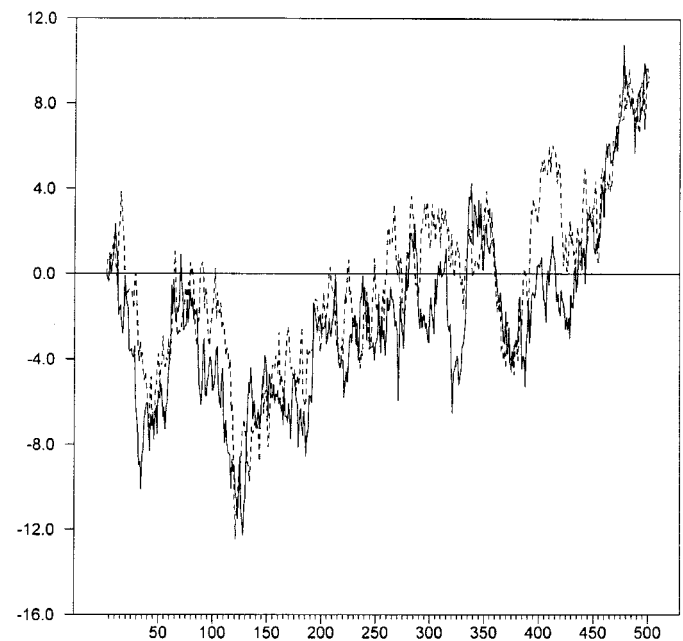


Figure 1. Threshold Cointegration: —, $x(1)$; - -, $x(2)$.

Notice that the line $x_{1t} - x_{2t} = 0$ is a more powerful attractor for negative values of the $\{\mu_{t-1}\}$ sequence than for positive values. On average, 90% of a positive discrepancy persists from one period to the next while only 50% of a negative discrepancy persists. As such, near random-walk behavior occurs for positive values of $\{\mu_t\}$, whereas there is rapid convergence when $\{\mu_t\}$ is negative. Clearly, the magnitudes of the ρ_i can be reversed. For example, policy makers might be more tolerant of falling interest rates and/or exchange rates than rising ones.

There are two important ways to modify the basic threshold cointegration model:

1. *Higher-order Processes*: Equation (5) may not be sufficient to capture the dynamic adjustment of $\Delta\mu_t$ toward its long-run equilibrium value. However, in working with any time series model, it is important to ensure that the errors approximate a white-noise process. A convenient specification is to augment (5) with lagged changes in the $\{\mu_t\}$ sequence such that it becomes the p th-order process

$$\Delta\mu_t = I_t \rho_1 \mu_{t-1} + (1 - I_t) \rho_2 \mu_{t-1} + \sum_{i=1}^{p-1} \gamma_i \Delta\mu_{t-i} + \varepsilon_t. \quad (9)$$

It is possible to allow $\Delta\mu_t$ to display asymmetric adjustment to its lagged changes. For example, the magnitude of each γ_i could depend on whether $\Delta\mu_{t-1}$ was positive or negative. Moreover, as discussed by Granger and Teräsvirta (1993), the values of ρ_1 and ρ_2 can be allowed to adjust smoothly over time. We do not consider these extensions in this article.

In addition to its simplicity, an important feature of (9) is that it retains its equivalence to the Engle–Granger specification when $\rho_1 = \rho_2$. In (9), various model-selection criteria [such as the Akaike information criterion (AIC) or the Bayesian information criterion (BIC)] can be used to determine the appropriate lag length. Alternatively, various tests for white noise, such as those discussed by Tong (1983) or Granger and Teräsvirta (1993), can be used for diagnostic checking. Lukkonen, Saikkonen, and Teräsvirta (1988) showed that the usual asymptotic theory cannot be applied to derive ordinary Lagrange multiplier tests for nonlinearity. Eitrheim and Teräsvirta (1996) suggested, via Monte Carlo simulations, that the Ljung–Box test for residual autocorrelation does not follow the χ^2 asymptotic distribution in nonlinear time series models. To ensure that there is no more than a single unit root, all the values of r satisfying the inverse characteristic equation $1 - \gamma_1 r - \gamma_2 r^2 + \dots + \gamma_{p-1} r^{p-1} = 0$ must lie outside the unit circle.

2. *Alternative Adjustment Specifications*: In (6), the Heaviside indicator depends on the level of μ_{t-1} . Enders and Granger (1998) and Caner and Hansen (1998) suggested an alternative such that the threshold depends on the previous period's change in μ_{t-1} . Consider an alternative rule for setting the Heaviside indicator according to

$$M_t = \begin{cases} 1 & \text{if } \Delta\mu_{t-1} \geq \tau \\ 0 & \text{if } \Delta\mu_{t-1} < \tau. \end{cases} \quad (10)$$

Models constructed using (2), (5), and (10) are called momentum-threshold autoregressive (M-TAR) models in that

the $\{\mu_t\}$ series exhibits more “momentum” in one direction than the other. Note that it is possible to use the Heaviside indicator of (10) in a dynamic model augmented by lagged changes in $\Delta\mu_t$.

M-TAR adjustment can be especially useful when policy makers are viewed as attempting to smooth out any large changes in a series. For example, the Federal Reserve might take strong measures to offset shocks to the term structure relationship if such shocks are deemed to indicate *increases*, but not decreases, in inflationary expectations. Similarly, with a managed float, the exchange-rate authority may want to mitigate large changes in the exchange rate without attempting to influence the long-run level of the rate.

The two series shown in Figure 2 were constructed using the identical values of $\rho_{i,j}$ and the same two sets of 500 pseudorandom numbers used to construct Figure 1. The sole difference is that the M-TAR sequence is constructed using (10) instead of (6). Although $x_{1t} - x_{2t} = 0$ remains the attractor, the attraction is more powerful for negative values of μ_{t-1} than for positive values. Comparing Figures 1 and 2, it is clear that the overall time paths follow each other reasonably well. Figure 3 shows the deviations from long-run equilibrium derived from Figures 1 and 2. Note that the positive discrepancies from long-run equilibrium are shorter lived in the M-TAR model than in the TAR model. In the M-TAR model, a negative realization of $\Delta\mu_t$ decays at a 50% rate and a positive realization decays at a 10% rate. In the TAR model, decay occurs at a 10% rate for as long as the discrepancy from long-run equilibrium is positive and at a 50% rate for as long as the discrepancy is negative. A second major difference concerns the nature of the spikes in that decreases are sharper and more pronounced in the M-TAR model. This pattern would be reversed if $|\rho_1| > |\rho_2|$. Intuitively, with $|\rho_1| < |\rho_2|$, the M-TAR model exhibits little decay for positive $\Delta\mu_{t-1}$ but substantial decay for negative μ_{t-1} . In a sense, in the M-TAR model increases tend to persist but decreases tend to revert quickly toward the attractor.

2. TESTING FOR COINTEGRATION WITH TAR AND M-TAR ADJUSTMENT

If the various $\{x_{it}\}$ are not cointegrated, there is no threshold τ and the value of ρ_1 and/or ρ_2 is equal to 0. In such circumstances, Andrews and Ploberger (1994) and Hansen (1996) showed that inference is difficult since the nuisance parameters are not identified under the null hypothesis. In this section we describe two Monte Carlo experiments that can be used to test the null hypothesis of no cointegration against the alternative of cointegration with threshold (either TAR or M-TAR) adjustment. In the first test, the value of τ is set equal to 0, and in the second test, τ is unknown.

Case 1: τ Equals 0

To conduct a Monte Carlo experiment that can be used, 50,000 random-walk processes of the following form were generated:

$$x_{1t} = x_{1,t-1} + v_{1t}, \quad t = 1, \dots, T, \quad (11)$$

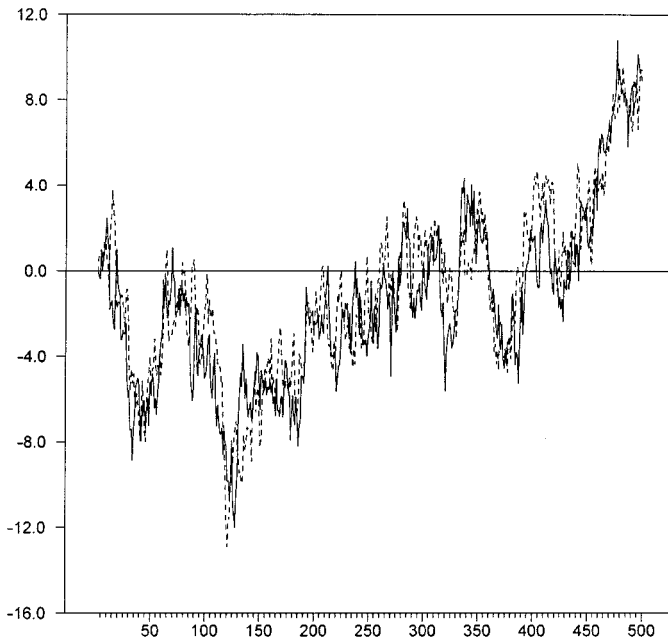


Figure 2. Momentum Cointegration: —, $x(1)$; ---, $x(2)$.

and

$$x_{2t} = x_{2t-1} + v_{2t}, \quad t = 1, \dots, T. \quad (12)$$

For $T = 50, 100, 250$, and 500 , two sets of T normally distributed and uncorrelated pseudorandom numbers with standard deviation equal to unity were drawn to represent the $\{v_{1t}\}$ and $\{v_{2t}\}$ sequences. Randomizing the initial values of x_{1t} and x_{2t} , the next T values of each were generated using (11) and (12). For each of the 50,000 series, the TAR model given by (2), (5), and (6) was estimated setting $\tau = 0$. For each estimated equation, we recorded the two t statistics for the null

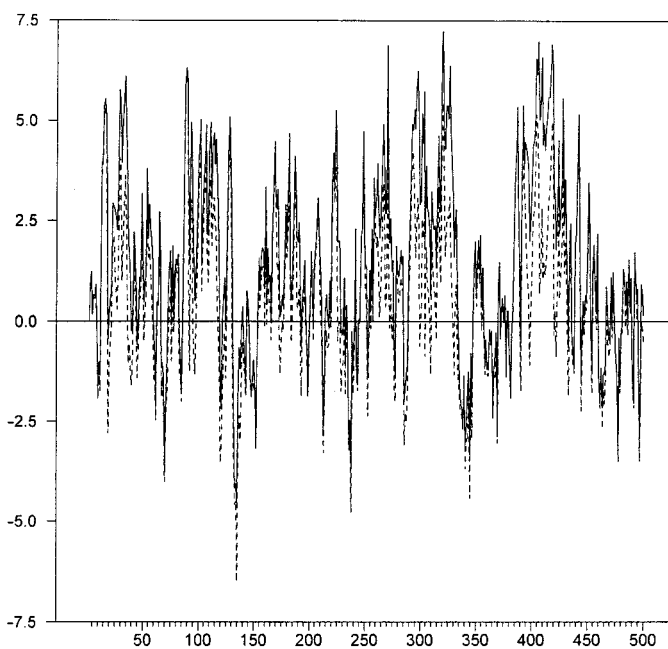


Figure 3. The Error-Correction Terms: —, $x(1)$; ---, $x(2)$.

hypotheses $\rho_1 = 0$ and $\rho_2 = 0$ along with the F statistic for the joint hypothesis $\rho_1 = \rho_2 = 0$. The largest of the individual t statistics is called t -Max, the smallest is called t -Min, and the F statistic is called Φ . For example, if the two t statistics are -2.5 and -1.5 , t -Min is -2.5 and t -Max is -1.5 . Recall that the necessary conditions for convergence are for ρ_1 and ρ_2 to be negative. Thus, the t -Max statistic is a direct test of these conditions. The Φ statistic can lead to a rejection of the null hypothesis $\rho_1 = \rho_2 = 0$ when only one of the values is negative. However, as will be shown, the Φ statistic is quite useful because it can have substantially more power than the t -Max statistic. Nevertheless, the Φ statistic should be used only in those cases in which the point estimates for ρ_1 and ρ_2 imply convergence. The t -Min statistic was found to have very little power and is not reported here.

Panel A of Table 1 reports the critical values of Φ and Panel A of Table 2 reports the critical values of t -Max. For $T = 100$, Panel A of Table 1 shows that the Φ statistic for the null hypothesis $\rho_1 = \rho_2 = 0$ exceeded 5.98 in approximately 5% of the 50,000 trials and Panel A of Table 2 shows that the largest of the two t statistics was more negative than -2.11 in approximately 5% of the trials. These statistics can be used as critical values to test the null hypothesis of a unit-root process against the alternative of a TAR model.

Suppose that the process used to generate the data shown in Figure 1 was unknown. Using realizations 201–300 of the TAR series shown in the figure, the estimated model and t statistics are

$$x_{1t} = -1.34 + .498 x_{2t} + \mu_t \quad (13)$$

(−5.88) (5.06)

and

$$\Delta \mu_t = -.226 I_t \mu_{t-1} - .354(1 - I_t) \mu_{t-1} + \varepsilon_t. \quad (14)$$

(−2.48) (−3.33)

The F statistic for the null hypothesis $\rho_1 = \rho_2 = 0$ is 8.63. As shown in Panel A of Table 1, such a value will occur in less than 1% of the trials when the data-generating process is a random walk (the 1% critical value is 8.24). Similarly, the largest of the t statistics equals -2.48 . Panel A of Table 2 reports the 95% and 99% critical values as -2.11 and -2.55 , respectively. As such, the Φ statistic rejects the null hypothesis of no cointegration at the 1% significance level while the t -Max statistic rejects the null hypothesis at the 5%, but not the 1%, significance level.

The distributions of Φ and t -Max depend on sample size and the number of variables included in the cointegration relationship. As in the Engle–Granger test, the critical values also depend on the nature of the dynamic adjustment process [i.e., the value of ρ and the magnitude of the γ_i in Eq. (9)]. Panel A of Tables 1 and 2 reports the critical values of Φ and t -Max for sample sizes of 50, 100, 250, and 500 and for various assumptions concerning the adjustment process. In generating the data for lag lengths of 1 and 4, (11) and (12) were modified such that

$$x_{1t} = x_{1t-1} + \delta_1 \Delta x_{1t-j} + v_{1t}, \quad t = 1, \dots, T; j = 1, 4 \quad (11')$$

Table 1. The Distribution of Φ

Obs.	No lagged changes			One lagged change			Four lagged changes		
	90%	95%	99%	90%	95%	99%	90%	95%	99%
<i>Panel A: TAR adjustment</i>									
50	5.09	6.20	8.78	5.08	6.18	8.67	5.22	6.33	9.05
100	5.01	5.98	8.24	4.99	6.01	8.30	5.20	6.28	8.82
250	4.94	5.91	8.08	4.92	5.87	8.04	5.23	6.35	8.94
500	4.91	5.85	7.89	4.88	5.79	7.81	5.21	6.33	9.09
<i>Panel B: M-TAR adjustment</i>									
50	5.59	6.73	9.50	5.56	6.67	9.32	5.32	6.39	8.89
100	5.45	6.51	8.78	5.47	6.51	8.85	5.20	6.20	8.46
250	5.38	6.42	8.61	5.36	6.38	8.62	5.13	6.12	8.26
500	5.36	6.35	8.43	5.32	6.28	8.40	5.06	6.05	8.31

and

$$x_{2t} = x_{2t-1} + \delta_2 \Delta x_{2t-j} + v_{2t}, \quad t = 1, \dots, T; j = 1, 4. \quad (12')$$

The tables report results using the values $\delta_1 = \delta_2 = .6$, and $Ev_{1t}v_{2t} = 0$. The reported critical values tend to be quite conservative since the magnitude of the Φ statistic and the absolute value of the t -Max statistic increase with $|\delta_1 - \delta_2|$. The critical values are reasonably insensitive to the level of serial correlation in $\{x_{it}\}$. For example, as shown in Tables 1 and 2, with 100 observations and one lagged change such that $\delta_1 = \delta_2 = .6$, the t -Max statistic is -2.14 , and the Φ statistic is 6.01 . Respectively, for $\delta_1 = \delta_2 = .1$ and $\delta_1 = \delta_2 = .8$, the t -Max statistic is -2.14 and -2.12 , while the Φ statistics are 5.95 and 6.05 . However, for $\delta_1 = .1$ and $\delta_2 = .6$, the critical values for t -Max and Φ are -2.10 and 5.72 , respectively. Finally, for $\delta_1 = .0$ and $\delta_2 = .8$, the critical values for t -Max and Φ are -2.09 and 5.69 , respectively. A table of the various critical values is available from the authors on request.

The Monte Carlo experiment was repeated for an M-TAR model using the indicator function given by (10). The corresponding test statistics—called $\Phi(M)$ and t -Max(M)—are reported in Panel B of Tables 1 and 2.

To use the statistics, perform the following three steps:

Step 1: Regress one of the variables on a constant and the other variable(s) and save the residuals in the sequence $\{\hat{\mu}_t\}$. Next, depending on the type of asymmetry under consideration, set the indicator function I_t according to (6) or M_t according to (10), using $\tau = 0$. Estimate a regression equation in the form of (5) and record the larger of the t statistics for the null hypothesis $\rho_i = 0$ along with the F statistic for the null hypothesis $\rho_1 = \rho_2 = 0$. Compare these sample statistics with the appropriate critical values shown in Tables 1 or 2.

Step 2: If the alternative hypothesis of stationarity is accepted, it is possible to test for symmetric adjustment (i.e., $\rho_1 = \rho_2$). This issue is difficult because regression residuals (i.e., μ_{t-1}) rather than the actual errors need to be utilized in any such test. Enders and Falk (1999) and Hansen (1997) considered issues of inference in TAR models. When the value of the threshold is known, Enders and Falk (1999) stated that bootstrap t intervals and classic t intervals work well enough to be recommended in practice.

Step 3: Diagnostic checking of the residuals should be undertaken to ascertain whether the $\hat{\epsilon}_t$ series can reasonably be characterized by a white-noise process. If the residuals are

Table 2. The Distribution of t -Max

Obs.	No lagged changes			One lagged change			Four lagged changes		
	90%	95%	99%	90%	95%	99%	90%	95%	99%
<i>Panel A: TAR Adjustment</i>									
50	-1.89	-2.12	-2.58	-1.92	-2.16	-2.64	-1.89	-2.14	-2.69
100	-1.90	-2.11	-2.55	-1.91	-2.14	-2.57	-1.76	-1.98	-2.43
250	-1.90	-2.12	-2.53	-1.90	-2.12	-2.53	-1.71	-1.91	-2.34
500	-1.89	-2.11	-2.52	-1.90	-2.10	-2.51	-1.69	-1.89	-2.29
<i>Panel B: M-TAR Adjustment</i>									
50	-1.79	-2.04	-2.53	-1.82	-2.07	-2.57	-1.84	-2.10	-2.67
100	-1.77	-2.02	-2.47	-1.79	-2.03	-2.49	-1.77	-2.00	-2.51
250	-1.76	-1.99	-2.45	-1.77	-2.00	-2.44	-1.76	-1.99	-2.45
500	-1.76	-1.98	-2.41	-1.75	-1.99	-2.42	-1.75	-1.98	-2.42

correlated, return to Step 2 and reestimate the model in the form

$$\Delta\hat{\mu}_t = I_t\rho_1\hat{\mu}_{t-1} + (1 - I_t)\rho_2\hat{\mu}_{t-1} + \gamma_1\Delta\hat{\mu}_{t-1} + \dots + \gamma_p\Delta\hat{\mu}_{t-p} + \varepsilon_t \quad (15)$$

for the TAR model. For the M-TAR case, replace I_t with M_t , as specified in (10). Lag lengths can be determined by an analysis of the regression residuals and/or using a number of widely used model-selection criteria such as the AIC or BIC.

Power Tests. Since unit-root tests suffer from low power, it is of interest to compare the power of the Φ , t -Max, $\Phi(M)$, and t -Max(M) test statistics to the power of the more traditional Engle–Granger test. Toward this end, two sets of 100 normally distributed random numbers were drawn to represent the $\{v_{1t}\}$ and $\{v_{2t}\}$ sequences. For various values of ρ_1 and ρ_2 , these random numbers were used to generate the basic two-variable TAR model given by (2), (5), and (6) for $T = 100$. Following Steps 1 and 2, x_{1t} was regressed on x_{2t} and a constant, and an equation in the form of (5) and (6) was estimated setting the indicator function I_t using the value $\tau = 0$. This process was replicated 5,000 times, and the percentage of instances in which the Φ and t -Max tests correctly rejected the null hypothesis of no cointegration is reported in Table 3 for test sizes of 10%, 5%, and 1%. For comparison purposes, the Engle–Granger method (assuming symmetric adjustment) was applied to the same data. Thus, x_{1t} was regressed on x_{2t} and a constant and the residuals were used to estimate an equation in the form of (3). The estimated value of ρ was compared to the critical values reported by Engle and Granger (1987).

The overwhelming impression is that the power of the Engle–Granger test usually exceeds that of the Φ and t -Max statistics at the 10% and 5% significance levels. For example, if the true adjustment parameters are $\rho_1 = -.50$ and $\rho_2 = -.15$, at the 10% significance level the Φ and t -Max statistics correctly identified the model as stationary in 52.2% and 53.3% of the trials, respectively. However, for the same sized test, the Engle–Granger correctly identified the model as stationary in 57% of the trials. Restricting the size to 1%

induces a slight improvement in the relative performance of both the Φ and t -Max statistics. For these same values of ρ_1 and ρ_2 , at the 1% level the Φ , t -Max, and Engle–Granger statistics correctly indicated that the series are cointegrated in 11.7%, 12.2%, and 10.6% of the cases, respectively. This pattern carries over to the other values of ρ_1 and ρ_2 reported in Table 3. The disappointingly low power of the Φ and t -Max statistics may seem surprising since the true data-generating process displays asymmetric adjustment to long-run equilibrium. The explanation lies in the fact that the TAR model entails the estimation of an additional coefficient with a consequent loss of power. For the degree of asymmetry shown in the table, the gain in power resulting from estimating the correctly specified model does not outweigh the loss from the additional coefficient. Note also that the power of the t -Max statistic is usually superior to that of the Φ statistic when the size of the test is 1% and there is a reasonable amount of asymmetry. As such, Balke and Fomby's (1997) recommendation seems appropriate: In the presence of threshold adjustment, use the Engle–Granger test to determine whether the variables are cointegrated and, if nonlinearity is suspected, estimate a nonlinear adjustment process.

The situation is quite different for the M-TAR model. The identical random numbers used for the preceding power tests were used to generate the basic two-variable M-TAR model given by (2), (5), and (10) for $T = 100$. Inspection of Table 4 shows the following:

1. If adjustment is nearly symmetric (such that $\rho_1 \approx \rho_2$), the power of the Engle–Granger test exceeds that of the $\Phi(M)$ and t -Max(M) statistics. When adjustment is nearly symmetric, the assumption of asymmetric adjustment entails the needless estimation of an additional coefficient with a consequent loss of power.

2. For a reasonable range of asymmetry, the power of the $\Phi(M)$ test exceeds that of the Engle–Granger test. For example, if $\rho_1 = -.025$ and $\rho_2 = -.20$, the $\Phi(M)$ test correctly rejects the null hypothesis of no cointegration 26% more often than the Engle–Granger test at the 5% level and more than twice as often at the 1% level.

Table 3. Power Functions for the TAR Model

ρ_1	ρ_2	The Φ statistic			The t -Max statistic			The Engle–Granger test		
		10%	5%	1%	10%	5%	1%	10%	5%	1%
-.025	-.500	.173	.094	.020	.196	.101	.022	.199	.102	.017
	-.100	.240	.143	.032	.261	.153	.038	.270	.152	.028
	-.150	.299	.186	.054	.305	.189	.053	.333	.190	.047
	-.200	.349	.223	.070	.352	.228	.073	.382	.233	.062
	-.250	.385	.257	.086	.364	.240	.077	.411	.261	.072
-.050	-.100	.405	.251	.070	.429	.280	.083	.570	.274	.063
	-.150	.522	.357	.117	.533	.368	.122	.570	.375	.106
	-.200	.610	.436	.175	.582	.421	.169	.646	.453	.153
	-.250	.682	.516	.228	.637	.473	.205	.718	.526	.202
-.100	-.150	.854	.712	.363	.840	.709	.372	.881	.736	.340
	-.200	.923	.823	.492	.889	.783	.487	.942	.839	.466
	-.250	.904	.789	.613	.922	.835	.551	.965	.898	.575

NOTE: Each entry is the percentage of instances in which the null hypothesis of no cointegration was correctly rejected. For the test sizes of 10%, 5%, and 1%, the statistic with the largest percentage correct is highlighted in boldface.

Table 4. Power Functions for the M-TAR Model

ρ_1	ρ_2	The Φ statistic			The t -Max statistic			The Engle-Granger test		
		10%	5%	1%	10%	5%	1%	10%	5%	1%
-.025	-.500	.192	.102	.024	.184	.094	.019	.217	.110	.017
	-.100	.407	.259	.076	.255	.148	.041	.395	.225	.049
	-.150	.674	.502	.223	.249	.179	.060	.596	.387	.107
	-.200	.868	.753	.459	.297	.196	.073	.786	.597	.219
	-.250	.956	.896	.686	.271	.190	.078	.894	.751	.361
-.050	-.100	.476	.303	.094	.373	.234	.069	.512	.322	.072
	-.150	.743	.574	.257	.408	.291	.113	.730	.532	.177
	-.200	.902	.794	.495	.414	.299	.135	.869	.719	.313
	-.250	.973	.927	.729	.422	.321	.149	.958	.875	.509
-.100	-.150	.862	.724	.379	.704	.565	.284	.910	.778	.357
	-.200	.961	.883	.619	.714	.595	.352	.971	.899	.560
	-.250	.993	.966	.825	.703	.587	.364	.992	.968	.748

NOTE: Each entry is the percentage of instances in which the null hypothesis of no cointegration was correctly rejected. For the test sizes of 10%, 5%, and 1%, the statistic with the largest percentage correct is highlighted in boldface.

3. The power of the t -Max(M) test is always less than that of the $\Phi(M)$ and/or the Engle-Granger test. Thus, in spite of its intuitive appeal, we cannot recommend using the t -Max(M) test.

4. The $\Phi(M)$ and Engle-Granger tests using M-TAR adjustment have at least as much power as those for the corresponding TAR model. The t -Max(M) test has less power than the corresponding t -Max statistic.

Case 2: τ Is Unknown

In many applications, there is no a priori reason to expect the threshold to coincide with the attractor. In such circumstances, it is necessary to estimate the value of τ along with the values of ρ_1 and ρ_2 . A second Monte Carlo study was undertaken to develop a test for a cointegration when the value of τ is unknown. Chan (1993) showed that searching over the potential threshold values so as to minimize the sum of squared errors from the fitted model yields a superconsistent estimate of the threshold.

The second experiment was similar to the first in that, for each of the 50,000 $\{x_{1t}\}$ and $\{x_{2t}\}$ series, we estimate a long-run equilibrium relationship in the form of (2) and saved the residuals as $\{\hat{\mu}_t\}$. However, to utilize Chan's methodology, the estimated residual series was sorted in ascending order

and called $\mu_1^\tau < \mu_2^\tau < \dots < \mu_T^\tau$, where T denotes the number of usable observations. The largest and smallest 15% of the $\{\mu_i^\tau\}$ values were discarded and each of the remaining 70% of the values were considered as possible thresholds. For each of these possible thresholds, we estimated an equation in the form of (5) and (6). The estimated threshold yielding the lowest residual sum of squares was deemed to be the appropriate estimate of the threshold. Using this threshold value, we called the analog of the Φ and t -Max statistics the Φ^* and t -Max* statistics. We performed the same analysis augmenting (5) with one and four lagged changes of the $\{\mu_t\}$ sequence. The various Φ^* and t -Max* statistics are reported in Panel A of Tables 5 and 6. We next performed the identical experiment using the momentum model so that the potential thresholds are $\Delta\mu_1^\tau < \Delta\mu_2^\tau < \dots < \Delta\mu_T^\tau$. The $\Phi^*(M)$ and the t -Max*(M) are shown in Panel B of Tables 5 and 6.

Inference concerning the individual values of ρ_1 and ρ_2 , and the restriction $\rho_1 = \rho_2$, is problematic when the true value of the threshold τ is unknown. The property of asymptotic multivariate normality has not been established for this case. In discussing the difficulty of establishing the distribution of the parameter estimates, Chan and Tong (1989) conjectured that utilizing a consistent estimate should establish the asymptotic normality of the coefficients. Moreover, Enders and Falk

Table 5. The Distribution of Φ^*

Obs.	No lagged changes			One lagged change			Four lagged changes		
	90%	95%	99%	90%	95%	99%	90%	95%	99%
Panel A: TAR adjustment									
50	6.05	7.24	9.90	6.20	7.31	10.00	6.79	8.05	11.05
100	5.95	6.95	9.27	6.02	7.08	9.51	6.35	7.41	9.88
250	5.93	6.93	9.15	5.92	6.93	9.18	6.44	7.56	10.18
Panel B: M-TAR adjustment									
50	5.97	7.12	9.96	5.99	7.14	9.93	5.99	7.11	9.79
100	5.73	6.78	9.14	5.76	6.86	9.29	5.52	6.56	8.91
250	5.58	6.62	8.82	5.57	6.63	8.84	5.32	6.32	8.47

Table 6. The Distribution of t -Max*

Obs.	No lagged changes			One lagged change			Four lagged changes		
	90%	95%	99%	90%	95%	99%	90%	95%	99%
Panel A: TAR adjustment									
50	-1.62	-1.89	-2.43	-1.70	-1.97	-2.59	-1.78	-2.05	-2.65
100	-1.61	-1.85	-2.35	-1.65	-1.90	-2.39	-1.66	-1.92	-2.44
250	-1.59	-1.84	-2.31	-1.61	-1.86	-2.33	-1.52	-1.73	-2.30
Panel B: M-TAR adjustment									
50	-1.65	-1.92	-2.44	-1.71	-1.98	-2.51	-1.72	-2.01	-2.60
100	-1.65	-1.90	-2.37	-1.67	-1.94	-2.44	-1.65	-1.90	-2.42
250	-1.66	-1.90	-2.36	-1.67	-1.91	-2.37	-1.66	-1.90	-2.37

(1999) found that the inversion of the bootstrap distribution for the likelihood ratio statistic provides reasonably good coverage in small samples.

3. APPLICATION: THE TERM STRUCTURE OF U.S. INTEREST RATES

We obtained monthly values of the federal funds rate and the 10-year yield on federal government securities for the sample 1964:01–1998:12. The data are averages of daily figures and were obtained from FRED (at the Federal Reserve Bank of St. Louis at www.stls.frb.org/fred/data/irates.html).

We chose the federal funds rate because it is the Federal Reserve's main instrument of monetary policy, as well as being a widely analyzed series in empirical finance. Figure 4 shows the time path of the two interest-rate series. It is generally agreed (see Stock and Watson 1988) that interest-rates series are $I(1)$ variables that should be cointegrated.

The Johansen (1996) procedure is not able to detect a long-run equilibrium relationship between these two interest

rates at conventional significance levels. In terms of Equation (1), we let the vector x_t consist of the logarithms of the federal funds and 10-year rates and included an intercept in the potential cointegrating vector. The multivariate AIC selected a model using two-lagged changes of $\{\Delta x_t\}$ and the multivariate BIC selected a model with only a one-lagged change. If we use one lag of $\{\Delta x_t\}$, the $\lambda_{\text{trace}}(0)$ and $\lambda_{\text{max}}(0, 1)$ statistics are 10.58 and 8.67, respectively. Instead, if we use the two-lag specification, the $\lambda_{\text{trace}}(0)$ and $\lambda_{\text{max}}(0, 1)$ statistics are 12.83 and 9.46, respectively. Since the respective 90% critical values for these two test statistics are 15.66 and 12.91, it seems possible to conclude that the rates are not cointegrated.

Following the Engle–Granger methodology, the estimated long-run equilibrium relationship (with t statistics in parentheses) is

$$r_{ft} = -1.313 + 1.513r_{10t} + \hat{\mu}_t, \quad (-10.802) \quad (27.251) \quad (16)$$

where r_{ft} and r_{10} are the logarithmic values of the federal funds and 10-year yield, respectively.

Next, we used the residuals of (16) to estimate a model of the form

$$\Delta \hat{\mu}_t = \rho_1 \hat{\mu}_{t-1} + \sum_{i=1}^p \gamma_i \Delta \hat{\mu}_{t-i} + \varepsilon_t. \quad (17)$$

As shown in Table 7, the model using two-lagged changes seems to be appropriate. In absolute value, both values of γ_i have t statistics exceeding 2.0. The key point to note is that the t statistic for the coefficient of $\hat{\mu}_{t-1}$ is only -2.858 . At conventional significance levels, the Engle–Granger test indicates that the two interest-rate series are not cointegrated.

Next, we estimated the residuals of (16) in the form of the TAR model using the threshold $\tau = 0$. As shown in the third column of Table 7, the point estimates for $\rho_1 = -.085$ and $\rho_2 = -.020$ suggest convergence. However, the sample value of $\Phi = 4.32$ is less than the 10% critical value (approximately 4.92) reported in Table 1. Moreover, the larger of the two t values (equal to -1.582) exceeds the 10% critical value for t -Max. Hence, at conventional significance levels, one cannot reject the null hypothesis of no cointegration.

The fourth column of Table 7 reports results using an M-TAR model. As in the previous two cases, diagnostic

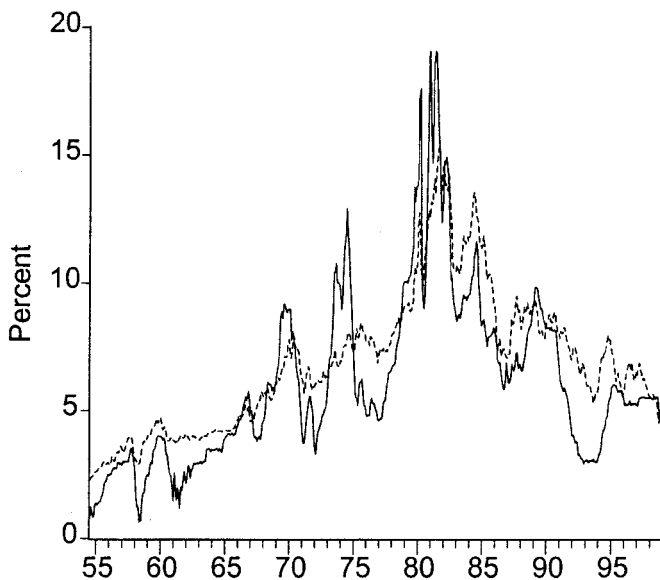


Figure 4. Interest-Rate Data: —, Federal Funds Rate; - - -, 10-Year Yield (both monthly).

Table 7. Estimates of the Interest-Rate Differential (sample: 1964:01–1998:12, monthly)

	Engle–Granger	Threshold	Momentum	Momentum-consistent
ρ_1^a	-.068 (-2.858)	-.085 (-2.522)	-.021 (-.628)	-.020 (-.680)
ρ_2^a	NA	-.020 (-1.582)	-.117 (-3.526)	-.141 (-3.842)
γ_1^a	.188 (-2.782)	.190 (2.787)	.183 (2.730)	.186 (2.790)
γ_2^a	-.149 (-2.197)	-.147 (-2.153)	-.161 (-2.376)	-.155 (-2.312)
AIC ^b	11.74	13.24	9.285	7.022
Φ^c	NA	4.32	6.363	7.548
$\rho_1 = \rho_2^d$	NA	.495 (.482)	4.418 (.037)	6.698 (.010)

^a Entries are estimated value of ρ_i (or γ_i) with the t statistic in parentheses.

^b The AIC is calculated as $T \log(SSR) + 2n$, where T = number of usable observations, SSR = sum of squared residuals, and n = number of regressors.

^c Entries in this row are the sample values of φ , $\varphi(M)$, or $\varphi^*(M)$.

^d Entries in this row are the sample F statistic for the null hypothesis that the adjustment coefficients are equal. Significance levels are in parentheses below.

statistics suggest a model with two lagged changes of $\{\Delta\hat{\mu}_t\}$. The sample value of the Φ statistic = 6.363 indicates that the null hypothesis $\rho_1 = \rho_2 = 0$ can be rejected near the 5% level. Note that the point estimates for ρ_1 and ρ_2 suggest substantially faster convergence for negative than for positive discrepancies from long-run equilibrium. Since the largest of the two t statistics is only $-.628$, we cannot reject the null of no cointegration using the t -Max(M) test. This is not surprising given the low power of the test. Since interest rates are cointegrated, the null hypothesis of symmetric adjustment (i.e., $\rho_1 = \rho_2$) can be tested using a standard F distribution. The sample value of $F = 4.418$ has a p value of .037 so that one can reject the null hypothesis of symmetric adjustment.

Next, we use Chan's (1993) method to find the consistent estimate of the threshold. When we searched over the possible thresholds lying in the middle 70% of the arranged values of $\{\Delta\hat{\mu}_t\}$, we found that a threshold of $-.0261$ results in the smallest residual sum of squares. As reported in the last column of Table 7, the M-TAR model using the consistent estimate of the threshold (with t statistics in parentheses) is

$$\Delta\hat{\mu}_t = -.0201 M_t \hat{\mu}_{t-1} - .141 (1 - M_t) \hat{\mu}_{t-1} + .186 \Delta\hat{\mu}_{t-1} - .155 \Delta\hat{\mu}_{t-2} + \varepsilon_t, \quad (18)$$

(-.680) (-3.842) (2.790) (-2.312)

where

$$M_t = \begin{cases} 1 & \text{if } \Delta\mu_{t-1} \geq -.0261 \\ 0 & \text{if } \Delta\mu_{t-1} < -.0261 \end{cases}. \quad (19)$$

Now, the point estimates of ρ_1 and ρ_2 suggest convergence such that the speed of adjustment is more rapid for negative than for positive discrepancies from $\tau = -.0261$. The sample value $\Phi(M)^*$ is 7.548 and the largest of the two t statistics for the ρ_i equals $-.680$. Hence, the $\Phi(M)^*$ statistic allows us to soundly reject the null hypothesis of no cointegration at the 5% level, while the F test for symmetric adjustment can just be rejected at the 1% significance level. Hence, (18) strongly

suggests that the two interest rates are cointegrated and that the adjustment mechanism is asymmetric. Based on the AIC, the M-TAR model with the consistent estimate of the threshold fits the data substantially better than the other models. Hence, the estimates suggest that discrepancies from long-term equilibrium resulting from decreases in the federal funds rate or increases in the long rate (such that $\Delta\mu_{t-1} < -.0261$) are eliminated relatively quickly, whereas other changes display a large amount of persistence. This result is consistent with asymmetric federal reserve policy.

The positive finding of cointegration with M-TAR adjustment justifies estimation of the following error-correction model (with t statistics in parentheses):

$$\Delta r_{ft} = -.0020 \quad -.0347 M_t \hat{\mu}_{t-1} - .0274 (1 - M_t) \hat{\mu}_{t-1} + A_{11}(L) \Delta r_{ft-1} + A_{12}(L) \Delta r_{10t-1} + \varepsilon_{1t}, \quad (20)$$

(-.504) (-1.418) (-.922)

$F_{11} = .00 \quad F_{12} = .00$

and

$$\Delta r_{10t} = -.0003 \quad -.0084 M_t \hat{\mu}_{t-1} + .0648 (1 - M_t) \hat{\mu}_{t-1} + A_{21}(L) \Delta r_{ft-1} + A_{22}(L) \Delta r_{10t-1} + \varepsilon_{2t}, \quad (21)$$

(-.104) (-.491) (3.121)

$F_{21} = .85 \quad F_{22} = .00$

where $\hat{\mu}_{t-1}$ is obtained from (16), the Heaviside indicator is set in accord with (19), $A_{ij}(L)$ are first-order polynomials in the lag operator L , and $F_{ij} = p$ value for the null hypothesis that both coefficients of A_{ij} are equal to 0.

The t statistics for the error-correction terms indicate that the federal funds rate is weakly exogenous but that the long rate adjusts to deviations from long-run equilibrium if $\hat{\mu}_{t-1} < -.0261$. Notice that the 10-year rate adjusts in the "wrong" direction for positive values of $\Delta\hat{\mu}_{t-1}$ (although the value of the t statistic is only $-.491$). The F statistics indicate that the long rate is not Granger-caused by the short rate but that

lagged changes in both rates affect movements in the short rate. A concern is that the estimates of the $\rho_{i,j}$ are somewhat sensitive to the sample period. Moreover, since Hansen (1997) and Enders and Falk (1999) showed that OLS estimates of the speed of adjustment terms have poor small-sample properties, we next consider the model using the 1954:7–1997:4 sample period. The estimated adjustment equation [i.e., the analog of Eq. (18)] is similar to that previously reported:

$$\begin{aligned}\Delta\hat{\mu}_t = & -.0135 M_t \hat{\mu}_{t-1} - .2384 (1 - M_t) \hat{\mu}_{t-1} \\ & (-.694) \quad (-8.298) \\ & + .0102 \Delta \hat{\mu}_{t-1} - .0527 \Delta \hat{\mu}_{t-2} \\ & (.2326) \quad (1.249) \\ & + .1618 \Delta \hat{\mu}_{t-2} + \varepsilon_t, \quad (18') \\ & (3.861)\end{aligned}$$

where τ is estimated to be $-.0383$, the sample value of $\Phi(M)^*$ is 34.56 , and the F statistic for the null hypothesis $\rho_1 = \rho_2$ is 42.859 with a p value of nearly 0 .

The hypotheses $\rho_1 = \rho_2 = 0$ and $\rho_1 = \rho_2$ are both soundly rejected. Again the model suggests that negative discrepancies from long-run equilibrium (such that $\Delta\mu_t < -.0383$) are eliminated rather quickly but that others are allowed to persist. However, the error-correction model using the entire sample does have a different interpretation. Consider

$$\begin{aligned}\Delta r_{ft} = & -.0029 - .0088 M_t \hat{\mu}_{t-1} - .2266 (1 - M_t) \hat{\mu}_{t-1} \\ & (-.733) \quad (-.464) \quad (-8.232) \\ & + A_{11}(L) \Delta r_{ft-1} + A_{12}(L) \Delta r_{10t-1} + \varepsilon_{1t} \quad (20') \\ & F_{11} = .00 \quad F_{12} = .00\end{aligned}$$

and

$$\begin{aligned}\Delta r_{10t} = & .0013 + .0085 M_t \hat{\mu}_{t-1} - .0016 (1 - M_t) \hat{\mu}_{t-1} \\ & (.887) \quad (1.265) \quad (-.167) \\ & + A_{21}(L) \Delta r_{ft-1} + A_{22}(L) \Delta r_{10t-1} + \varepsilon_{2t} \quad (21') \\ & F_{21} = .16 \quad F_{22} = .00\end{aligned}$$

For the longer sample period, the 10-year rate, but not the federal funds rate, appears to be weakly exogenous. Instead, an increase in inflationary expectations and the long rate such that $\Delta\mu_{t-1} < -.0383$ induces an increase in the federal funds rate. The F statistics still indicate that the long rate is not Granger-caused by the short rate but that lagged changes in both rates affect movements in the short rate.

In contrast, if we assume symmetric adjustment, the error-correction model over the oft-studied 1979:10–1997:4 sample period, which begins following changes in Federal Reserve operating procedures, is

$$\begin{aligned}\Delta r_{ft} = & -.0021 - .0318 \hat{\mu}_{t-1} + A_{11}(L) \Delta r_{ft-1} \\ & (-.547) \quad (-1.678) \quad F_{11} = .00 \\ & + A_{12}(L) \Delta r_{10t-1} + \varepsilon_{1t} \quad (22) \\ & F_{12} = .00\end{aligned}$$

and

$$\begin{aligned}\Delta r_{10t} = & -.0016 + .021 \hat{\mu}_{t-1} + A_{21}(L) \Delta r_{ft-1} \\ & (-.580) \quad (1.569) \quad F_{21} = .83 \\ & + A_{22}(L) \Delta r_{10t-1} + \varepsilon_{2t} \quad (23) \\ & F_{22} = .00\end{aligned}$$

Except for the error-correction terms, the coefficient estimates in (22) and (23) are all similar to those in (20) and (21). The key difference is that the symmetric adjustment assumption implies that there is no convergence toward the long-run equilibrium; both error-correction terms are not significant at conventional levels. Moreover, in spite of the fact that the M-TAR model contains an additional two coefficients, the multivariate AIC selects the M-TAR model. The multivariate AIC equals $-2,555.89$ for the M-TAR model of (20) and (21) and equals $-2,551.83$ for the system given by (22) and (23).

4. CONCLUSIONS

This article developed a generalization of the Engle–Granger (1987) procedure that allows for either threshold autoregressive (TAR) or momentum-TAR (M-TAR) adjustment toward a cointegrating vector. The power of the test for TAR adjustment is poor compared to that of the Engle–Granger test. However, for a plausible range of the adjustment parameters, the power of the M-TAR test can be many times that of the Engle–Granger test.

We illustrate the tests using short-term and long-term interest rates. The Engle–Granger and TAR tests indicated that the federal funds rate and the 10-year yield on government bonds are *not* cointegrated. However, models that permit M-TAR adjustment indicate that the two rates are cointegrated.

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