MATE 6551: Tarea 1

Due on October 8, 2025

Prof. Iván Cardona , C41, October 8, 2025

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Problem 0

If X is a topological space homeomorphic to D^n, then every continuous $f : X \to X$ has a fixed point.

Demo:

MEP

Problem 1

Let $f \in \operatorname{Hom}(A, B)$ be a morphism in a category $\mathcal{C}$. If f has an inverse g and a right inverse h, then $g = h$.

Demo:

MEP

Problem 2

Let G be a group and let $\mathcal{C}$ be the one-object category it defines: $\operatorname{obj} \mathcal{C} = \{*\}, \operatorname{Hom}(*, *) = G$, and composition is a group operation. If H is a normal subgroup of G, define $x \sim y$ to mean $xy^{-1} \in H$. Show that $\sim$ is a congruence on $\mathcal{C}$ and that $[*, *] = G/H$ in the corresponding quotient category.

Demo:

MEP

Problem 3

Let $x_0, x_1 \in X$ and let $f_i : X \to X$ for $i \in \{0, 1\}$ denote the constant map at x_i. Prove that $f_0 \simeq f_1$ if and only if there is a continuous $F : I \to X$ with $F(0) = x_0$ and $F(1) = x_1$.

Demo:

MEP

Problem 4

(i) Give an example of a continuous image of a contractible space that is not contractible.

(ii) Show that a retract of a contractible space is contractible.

Demo:

MEP