# Mate 6551

## Tarea 1

0.4. If $X$ is a topological space homeomorphic to $D^n$, then every continuous $f\colon X \to X$ has a fixed point.

0.7. Let $f \in \mathrm{Hom}(A, B)$ be a morphism in a category $\mathscr{C}$. If $f$ has a left inverse $g$ ($g \in \mathrm{Hom}(B, A)$ and $g \circ f = 1_A$) and a right inverse $h$ ($h \in \mathrm{Hom}(B, A)$ and $f \circ h = 1_B$), then $g = h$.

0.14. Let $G$ be a group and let $\mathscr{C}$ be the one-object category it defines (Exercise 0.10 applies because every group is a monoid): obj $\mathscr{C} = \{*\}$, $\mathrm{Hom}(*, *) = G$, and composition is the group operation. If $H$ is a normal subgroup of $G$, define $x \sim y$ to mean $xy^{-1} \in H$. Show that $\sim$ is a congruence on $\mathscr{C}$ and that $[*, *] = G/H$ in the corresponding quotient category.

1.1. Let $x_0, x_1 \in X$ and let $f_i\colon X \to X$ for $i = 0, 1$ denote the constant map at $x_i$. Prove that $f_0 \simeq f_1$ if and only if there is a continuous $F\colon \mathbf{I} \to X$ with $F(0) = x_0$ and $F(1) = x_1$.

1.8. (i) Give an example of a continuous image of a contractible space that is not contractible.

(ii) Show that a retract of a contractible space is contractible.