

Spring 2022

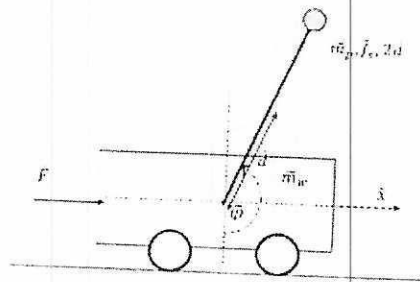
HW 1

EE 493 and EE 5234

Each problem has 50 points

Problem 1 Single-link manipulator robot – The Inverted Pendulum (IV)

Given: Consider an inverted pendulum shown here.



Dynamic model of a simplified version of the pendulum is given below:

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{g \sin(x_1) - a m l x_2^2 \sin(2x_1)/2 - a \cos(x_1)u}{4l/3 - a m l \cos^2(x_1)}\end{aligned}$$

Where $x_1 = \theta$, (angular position), $x_2 = d\theta/dt$ (angular velocity) are state variables and control variable, u is force f acting on the base of the pendulum's platform.

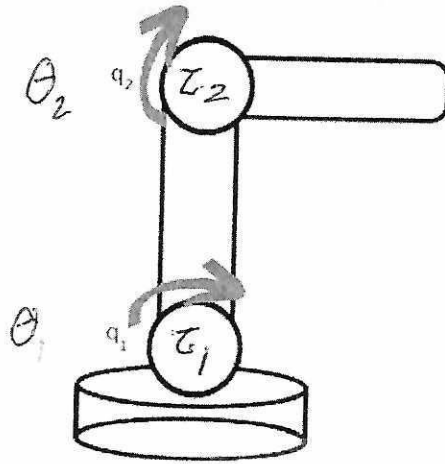
IV system variables are: Mass of pendulum, $m = 2$ kg, Cart mass, $M = 8$ Kg, $a = 1/(m + M)$, Gravitational acceleration, $g = 9.8$ m/sec², and IV length $l = 0.5$ m.

Required:

- Using Matlab, simulate the IV system with open loop control for a force, $u = 1\text{ Newton}$. Plot responses of the two state variables vs time for up to 10 seconds.
- Use Matlab to design a PD controller $u = K_p\theta + K_d(d\theta/dt)$, where gains K_p and K_d are unknown gains. Use trial and error approach to find the controller that will give a settling time of 5 seconds.

Problem 2 Two-link manipulator (TLM)

Given: A two-link planar robot is shown below



Dynamic model of this manipulator is given below:

$$\begin{aligned}\tau_1 &= [a_1 + a_2 \cos \theta_2] \ddot{\theta}_1 + [a_3 + \frac{a_2}{2} \cos \theta_2] \ddot{\theta}_2 + a_4 \cos \theta_1 \\ &\quad - (a_2 \sin \theta_2)(\dot{\theta}_1 \dot{\theta}_2 + \frac{\dot{\theta}_2^2}{2}) + a_5 \cos(\theta_1 + \theta_2) \\ \tau_2 &= [a_3 + \frac{a_2}{2} \cos \theta_2] \ddot{\theta}_1 + a_3 \ddot{\theta}_2 + (a_2 \sin \theta_2) \frac{\dot{\theta}_1^2}{2} + a_5 \cos(\theta_1 + \theta_2)\end{aligned}$$

Where $a_1 = 3.82$, $a_2 = 2.12$, $a_3 = 0.71$, $a_4 = 81.82$.

Required: Use Matlab to simulate this system given torques inputs,

- a) Write the state equation for this robot as:

$$dx/dt = f(x,u)$$

Where state vector $x = [x_1, x_2, x_3, x_4] = [\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2]$ and $u = [\tau_1 \ \tau_2]$.

- b) Use two joint torque values of 2 newton each and simulate this nonlinear model of the 2-link manipulator robot. Use the following initial conditions: $[\theta_1, \theta_2] = [0 \ 0]$, $[\dot{\theta}_1, \dot{\theta}_2] = [30^\circ \ 90^\circ]$, and $[\theta_1, \theta_2] = [90^\circ \ 30^\circ]$. Use a 10 seconds simulation time, but you can change that. Plot $[\theta_1, \theta_2]$ vs time and plot θ_1 vs. θ_2 (horizontal vs vertical axis).
- c) Discuss your results and try to write a visualization of what is physically going on?

Hint: See pages Reference 2 , book by Behar, et al., pp. 324-325.

