Graphics and Imaging Lab

Geometry
Informática Gráfica
Grado en Ingeniería Informática

Goal

- Apply theoretical knowledge about geometry.
- Design and implement basic geometrical structures that will be used for later assignments.

1 Problem statement

Recent advances on interplanetary travel have reached **Faster Than Light (FTL)** speeds. Spaceship FTL hyperdrives are gradually reaching three times the speed of light ($3c$), while wormhole-based trips (star gates and space portals) have reach a peak $10c$ without compromising their stability. Gates and portals are built in connected pairs, and that connection can never be changed. Once a gate is destroyed, its connected counterpart is rendered useless. This complicates logistics and the star gate infrastructure gets really expensive.

You work for a company named FTL Dynamics, settled in Spain, that invests a lot of money and resources on research and development (this fact should give you a hint that this is science fiction). Researchers have developed a new FTL technology, named *quantum catapult* that has the potential to reach $20c$ and a single station can be connected to any other station in the universe. This is a great technological breakthrough, and when industrially produced it will become a new dawn on interplanetary travel.

While the marketing department is looking for a better commercial name, you are in charge of designing and implementing the control software of the device. This software has to provide:

- The outgoing direction from the launching station (from the point of view of the launching station).
- The incoming direction at the receiving station (from the point of view of the receiving station).

This is not trivial, because the connection happens through the Universe and can only be measured from the *Universe Coordinate System* (UCS), while each station works at their own local coordinate system.

2 Planets

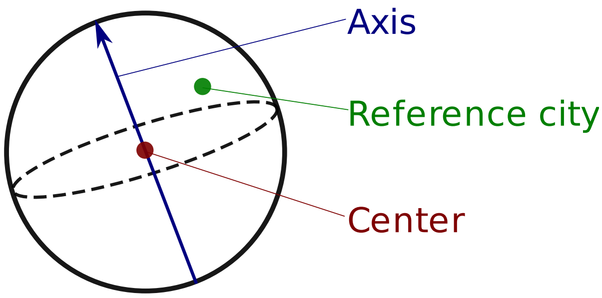

Figure 1: *Elements that define a planet: its **center**, its **axis** that connects the south pole with the north pole and the **reference city** from which the azimuth (longitude) is measured.*

Planets are modeled as perfect spheres. They are defined by:

- **Center**: A point in space, measured from the Universe Coordinate System (UCS).
- **Axis**: The direction that connects the South Pole with the North Pole of the planet. It's modulus should therefore be twice the radius of the planet.
- **Reference city**: The position in space for the reference city for the planet, from which the azimuth (longitude) is measured, measured from the UCS. On Earth, this city is Greenwich. The distance between this reference city and the planet's center is the radius of the planet.

When asked for a planet, the user will introduce these three vectors. The system must double check that the radius defined by the axis and by the distance between the center and the reference city is the same (maximum error of 10^{-6}). A representative diagram with the three vectors can be found at Figure 1.

3 Planetary station

A planetary station are positioned at a specific location on the surface on the planet, defined by:

- **Inclination (θ)**: The angle with respect to the planetary axis (that connects the south pole to the north pole), similar to the Earth's latitude but measured from the axis instead of the equator. It is measured in radians within the range $(0, \pi)$.
- **Azimuth (ϕ)**: The angle around the globe with respect to a specific $0-$meridian, similar to the Earth's longitude but the reference meridian is obviously not Greenwich because Greenwich is a city on Earth. It is measured in radians within the range $(-\pi, \pi]$.

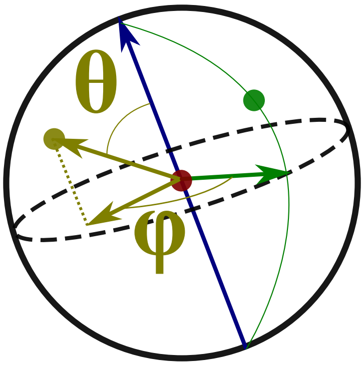

Figure 2: *Positioning of an element (the station) on the surface of the planet in spherical coordinates: the inclination (θ) and the azimuth (ϕ).*

From the inclination and the azimuth of a station, it is possible to calculate the UCS position (see Figure 2) as well as its local coordinate system. Specifically, the following information can be deduced:

- The **position** of the station in the Universe Coordinate System (UCS).
- The **surface normal** of the planet at that position (modulus is 1).
- The **longitude tangent direction** which is the direction tangent to the surface of the planet and perpendicular to the planet's axis (modulus is 1). This tangent follows the positive variation of the azimuth (and no variation of the inclination).
- The **latitude tangent direction** which is the direction tangent to the surface of the planet and perpendicular to the other tangent direction (modulus is 1). This tangent follows the negative variation of the inclination (and no variation of the azimuth).

The coordinate system of the station is defined by the longitude tangent direction as the $\mathbf{i}$ vector (first axis), the latitude tangent direction as the $\mathbf{j}$ vector (second axis) and the surface normal as the $\mathbf{k}$ vector (third axis). All three directions are linearly independent (perpendicular). These coordinates system can be seen from the global (UCS) point of view in Figure 3 and from the local (per-station) point of view in Figure 4.

When asked for a planetary station, the user will first introduce the planet (the three vectors) followed by the inclination and azimuth of the station. The system will then compute the position of the station and the corresponding coordinate system.

4 Interplanetary connection

Given two stations the connection in the Universe Coordinate System is the direction from launching station's position with the receiving station's position (see Figure 3). However, for

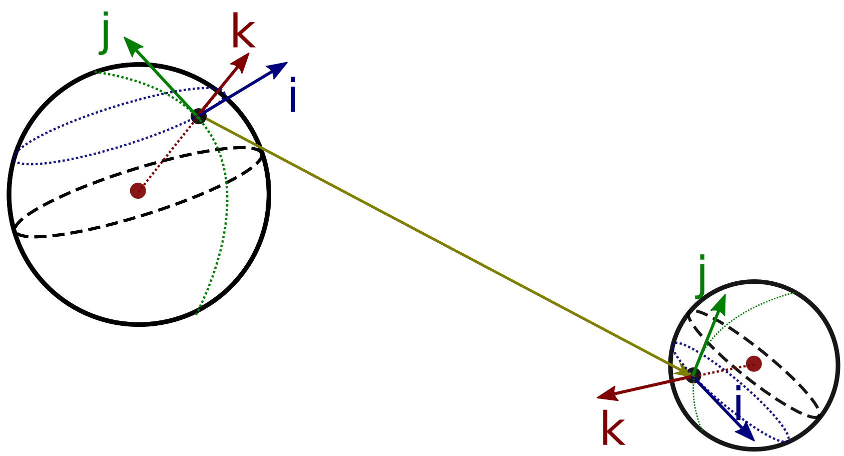

Figure 3: *Connection between two stations, including the local coordinate systems of both stations displayed represented as three colored vectors (per station).*

the transport to work properly, each station needs the connection on its specific coordinate system, as seen at ground level on each planet (see Figure 4).

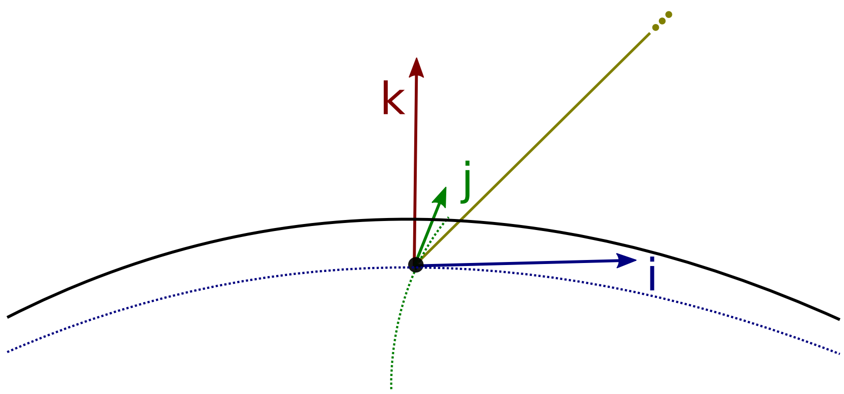

Figure 4: *Representation of the connection as seen from the local coordinate system of a single station.*

There is a downside, though. The quantum catapult requires between 1 and 2 seconds to reach its peak $20c$ speed. At that peak speed, the transported matter is completely out of phase at its very own specific frequency and can never collide with other matter, such as existing planets or stars, or even other transported out-of-phase matter. However, before that happens, the matter has a very high risk of in-phase collision. This is specially dangerous when the quantum catapult launcher is pointing towards the inside of its own planet, so as a safety mechanism the launcher will never work if pointing inwards.

The same happens on the other end: the quantum catapult receiver requires between 1 and 2 seconds to dampen the speed, and in those moments the transported matter may collide. Therefore the quantum receiver will never work if the matter reaches the station from within its own planet.

The full prototype will ask for two stations and then will print on screen the connection between them from the two local coordinate systems of the two stations. It will also give a warning if the trajectory of the matter traverses any of the two planets.

Submission

This assignment does not have to be submitted for grading. However, it is advised that it is finished by the recommended deadline in order to balance the workload.