Práctica 4: Generación de Números Aleatorios

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Casos Continuos

Ejercicio 1

Hallar los generadores de números aleatorios para:

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{2} & 0 \leq x < 1 \\ \frac{x}{2} & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

Solución en clases:

TEMA

TEMA

FECHA Día / Mes / Año

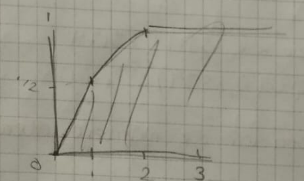
Caso Continuo

Ejercicio 1

Hallar los generadores de números aleatorios

$$F(x) = \begin{cases} 0 \; ; & x < 0 \\ \frac{x^2}{2} \; ; & 0 \leq x < 1 \\ \frac{x}{2} \; ; & 1 \leq x < 2 \\ 1 \; ; & x \geq 2 \end{cases}$$

① Graficar FDP



② Hallar los generadores

$R_1 = \frac{x^2}{2}$; $R_2 = \frac{x}{2}$

$$F(x) = \begin{cases} 0 \; ; & x < 0 \\ \frac{x^2}{2} \; ; & 0 \leq x < 1 \Rightarrow R_1 \\ \frac{x}{2} \; ; & 1 \leq x < 2 \Rightarrow R_2 \\ 1 \; ; & x \geq 2 \end{cases}$$

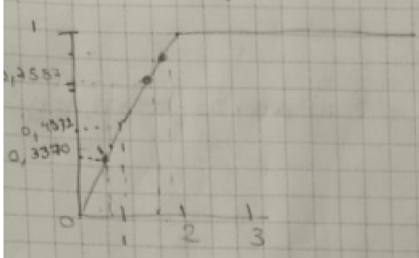
③ Despejar 'x'

$x = \sqrt{2R_1} \Rightarrow 0 \leq R_1 < \frac{1}{2}$

$x = 2R_2 \Rightarrow \frac{1}{2} \leq R_2 \leq 1$

FECHA Día / Mes / Año

Comprobar gráficamente



Para R_1

$R_1 = 0,4537 \Rightarrow x_1 = \sqrt{2(0,4537)} = 0,9325$

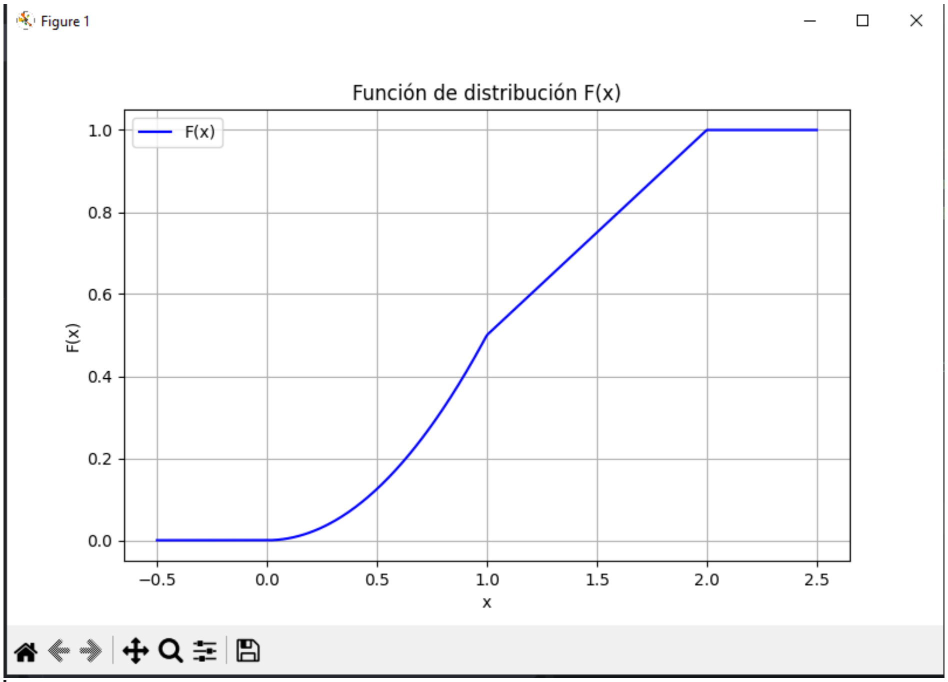
$R_1 = 0,3370 \Rightarrow x_2 = \sqrt{2(0,3370)} = 0,8209$

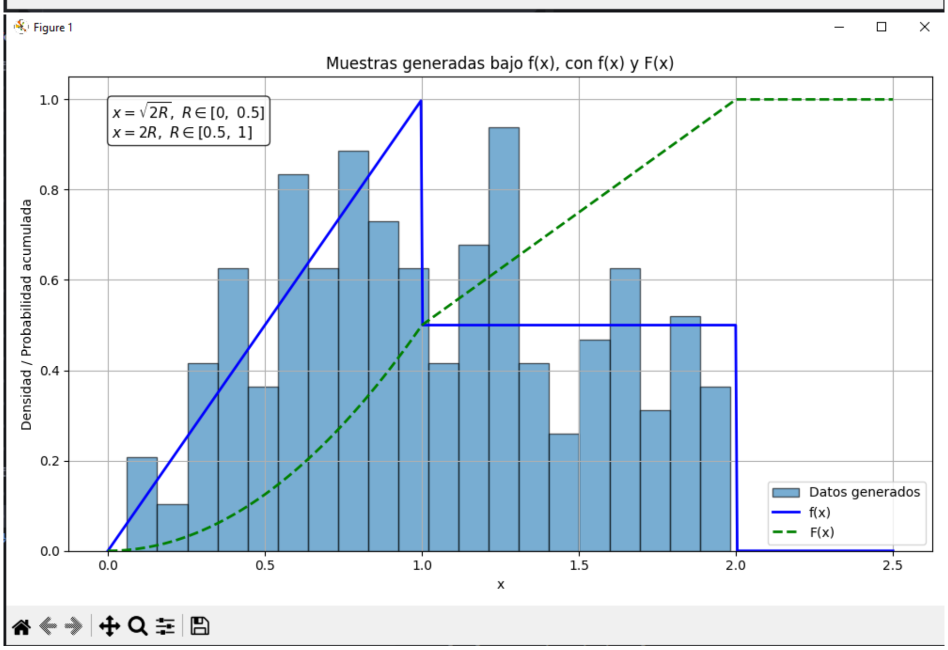
Para R_2

$R_2 = 0,7587 \Rightarrow x_3 = 2(0,7587) = 1,5174$

$R_2 = 0,8804 \Rightarrow x_4 = 2(0,8804) = 1,7608$

Resultado del Script: El script se encuentra en **continuo_1.py**





Ejercicio 2

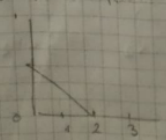
Solución en clases:

Ej 2) Hallar el generador de números aleatorios

$$f(x) = \begin{cases} -\frac{x}{2} + 1 \; ; & 0 \le x < 2 \\ 0 \; ; & \text{en o.c.} \end{cases}$$

Sol)

① Graficar $f(x)$



② Verificar que es una fdp

$$\int_0^2 -\frac{x}{2} + 1 \, dx = 1$$

$$\int_0^2 -\frac{x}{2} + 1 \, dx = \int_0^2 -\frac{x}{2} \, dx + \int_0^2 1 \, dx$$

$$= \left[-\frac{x^2}{4} + x \right]_0^2 = \left[\left(-\frac{4}{4} + 2 \right) - \left(-\frac{0}{4} + 0 \right) \right]$$

$$= \left(-1 + 2 \right) = \underline{1}$$

③ Armar la FDP

i) Para $x < 0$

$$\int_{-\infty}^{x} f(x) \, dx = 0$$

ii) Para $0 \le x < 2$

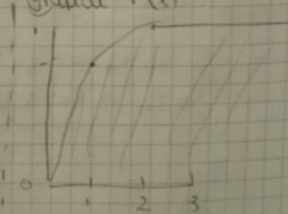
$$\int_{-\infty}^{0} f(x) dx + \int_0^x -\frac{x}{2} + 1 \, dx = \left(-\frac{x^2}{4} + x \right) \Big|_0^x = \left(-\frac{x^2}{4} + x \right)$$

iii) Para $x > 2$

$$\int_{-\infty}^{0} f(x) dx + \int_0^2 -\frac{x}{2} + 1 \, dx = 1$$

$$F(x) = \begin{cases} 0 \; ; & x < 0 \\ -\frac{x^2}{4} + x \; ; & 0 \le x < 2 \\ 1 \; ; & x \ge 2 \end{cases}$$

Gráfica $F(x)$



④ Hallar los generadores y comprobar

$$F(x) = \begin{cases} 0 & , x < 0 \\ -\frac{x^2}{4} + x & , 0 \leq x < 2 \\ 1 & ; x \geq 2 \end{cases} \Rightarrow R_1$$

$$R_1 = -\frac{x^2}{2} + x$$

* Despejar x

$$R_1 = -\frac{x^2}{4} + x \Rightarrow 4R_1 = -x^2 + 4x$$

$$x^2 - 4x + 4R_1 \Rightarrow a = 1 \quad b = -4 \quad c = 4R_1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = \frac{4 \pm \sqrt{16 - 4(4R_1)}}{2}$$

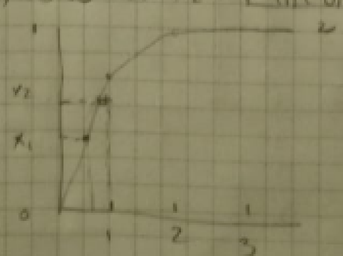
$$x = \frac{4 \pm 4\sqrt{1 - R_1}}{2}, \quad 0 \leq R_1 \leq 1$$

* Escogemos "−" porque "+" siempre da $x \geq 2$

Comprobar

$$R_1 = 0,3345 \Rightarrow x_1 = \frac{4 - 4\sqrt{1 - 0,3345}}{2} = 0,3684$$

$$R_1 = 0,6045 \Rightarrow x_2 = \frac{4 - 4\sqrt{1 - 0,6045}}{2} = 0,74222$$



Resultado del Script: El script se encuentra en **continuo_2.py**

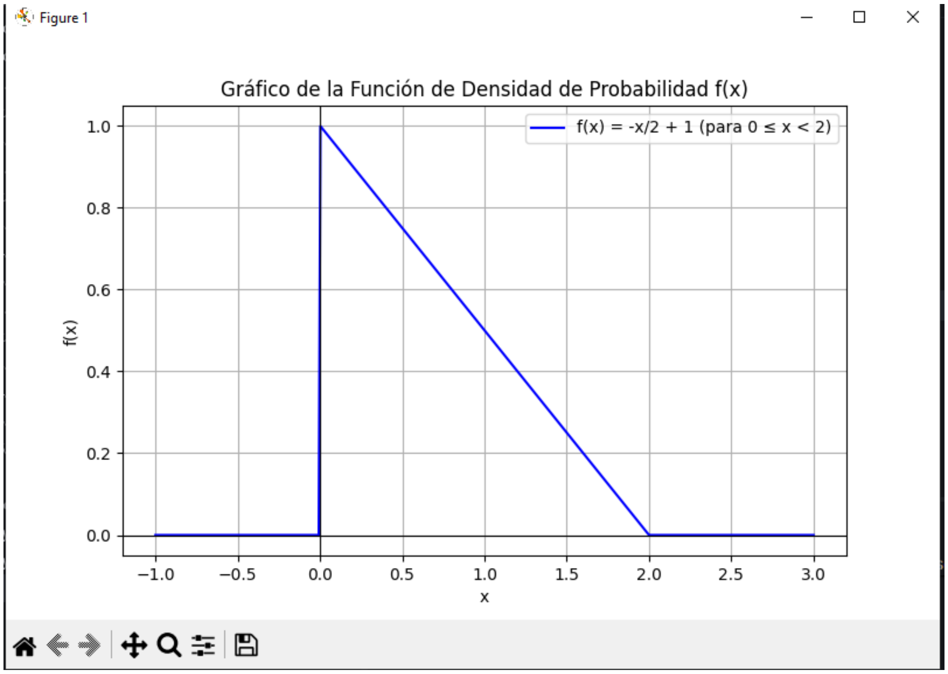


Figure 1
Gráfico de la Función de Densidad de Probabilidad f(x)
f(x) = -x/2 + 1 (para 0 ≤ x < 2)
f(x)
x

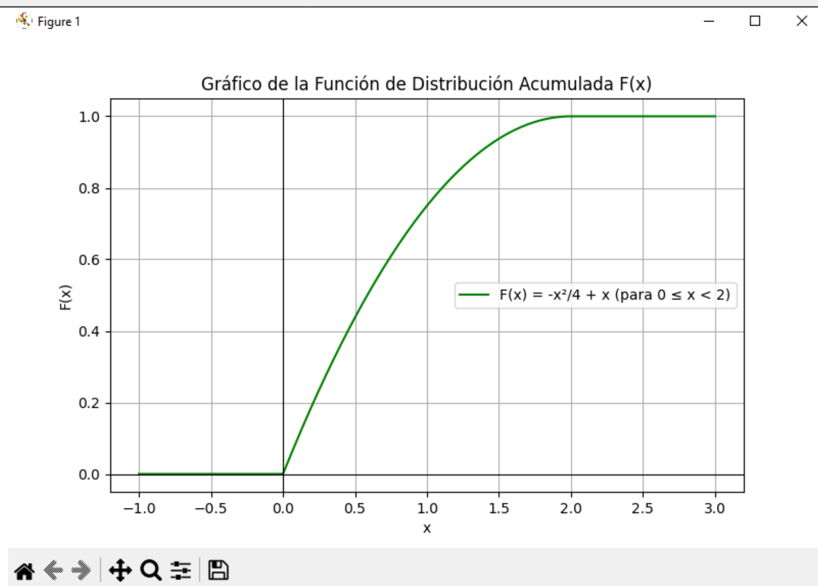


Figure 1
Gráfico de la Función de Distribución Acumulada F(x)
F(x) = -x²/4 + x (para 0 ≤ x < 2)
F(x)
x

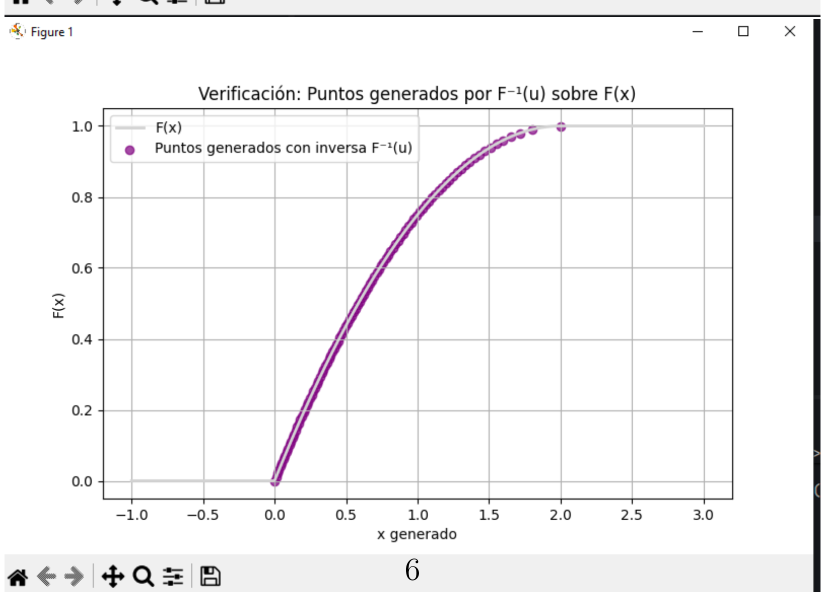


Figure 1
Verificación: Puntos generados por F⁻¹(u) sobre F(x)
F(x)
Puntos generados con inversa F⁻¹(u)
F(x)
x generado

Ejercicio 3

Distribución de probabilidad:

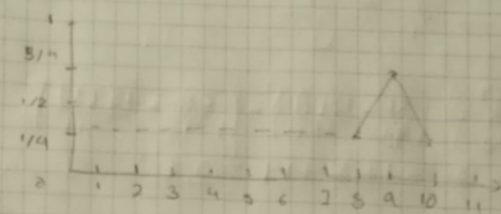
$$f(x) = \begin{cases} 0 & x < 8 \\ \frac{x}{2} - \frac{15}{4} & 8 \leq x < 9 \\ -\frac{x}{2} + \frac{21}{4} & 9 \leq x < 10 \\ 1 & x \geq 10 \end{cases}$$

Solución en clases:

FECHA

3) Determinar el generador de nros aleatorios

Sea:



① Hallar $f(x)$

a) Para $(8, 1/4)$ $(9, 3/4)$

$$m = \frac{(0{,}75 - 0{,}25)}{9 - 8} = \frac{1}{2}$$

$$y - y_1 = m(x - x_1) \Rightarrow y = \frac{x}{2} - \frac{16}{4} + \frac{1}{4} \Rightarrow \boxed{y = \frac{x}{2} - \frac{15}{4}}$$

b) Para $(9, 3/4)$ $(10, 1/4)$

$$m = \frac{(0{,}25 - 0{,}75)}{10 - 9} = -\frac{1}{2}$$

$$y - y_1 = m(x - x_1) \Rightarrow y = -\frac{x}{2} + \frac{9}{2} + \frac{3}{4} \Rightarrow \boxed{f(x) = -\frac{x}{2} + \frac{21}{4}}$$

$$f(x) = \begin{cases} 0 \;; & x < 8 \\ \frac{x}{2} - \frac{15}{4} \;; & 8 \leq x < 9 \\ -\frac{x}{2} + \frac{21}{4} \;; & 9 \leq x < 10 \\ 0 \;; & x \geq 10 \end{cases}$$

FECHA

② Verificar que es FDP

$$F(x) = \underbrace{\int_8^9 \frac{x}{2} + \frac{13}{4}dx}_{t} + \underbrace{\int_9^{10} -\frac{x}{2} + \frac{21}{4}dx}_{U} = 1$$

$$t = \left(\frac{x^2}{4} - \frac{15}{4}x\right)\Big|_8^9 = \left(\frac{81}{4} - \frac{15(9)}{4}\right) - \left(\frac{64}{4} - \frac{13(8)}{4}\right)$$

$$= -13{,}5 + 14 = 0{,}5$$

$$U = \left(-\frac{x^2}{4} + \frac{81}{4}x\right)\Big|_9^{10} = \left(-\frac{100}{4} + \frac{21(10)}{4}\right) - \left(-\frac{81}{4} + \frac{21(9)}{4}\right)$$

$$= 27{,}5 - 27 = 0{,}5$$

$$\int_8^9 \frac{x}{2} - \frac{15}{4}dx + \int_9^{10} -\frac{x}{2} + \frac{21}{4}dx = 1$$ Si es una FDP

③ Hallar la FDP

i) Para $x < 8$

$$\int_{-\infty}^{8} f(x)\,dx = 0$$

ii) Para $8 \leq x < 9$

$$\cancel{\int_{-\infty}^{8} f(x)\,dx} + \int_8^x \frac{x}{2} - \frac{15}{4}dx \Rightarrow \left(\frac{x^2}{4} - \frac{13}{4}x\right)\Big|_8^x$$

$$= \left(\frac{x^2}{4} - \frac{13}{4}x\right) - \left(\frac{64}{4} - \frac{15(8)}{4}\right)$$

$$= \frac{x^2}{4} - \frac{15x}{4} + 14$$

iii) Para $9 \leq x < 10$

$$\int_{-\infty}^{8} \cancel{f(x)dx}^{\,0} + \underbrace{\int_{8}^{9} \frac{x}{2} + \frac{15}{4} dx}_{0,5} + \int_{9}^{x} -\frac{x}{2} + \frac{21}{4} dx$$

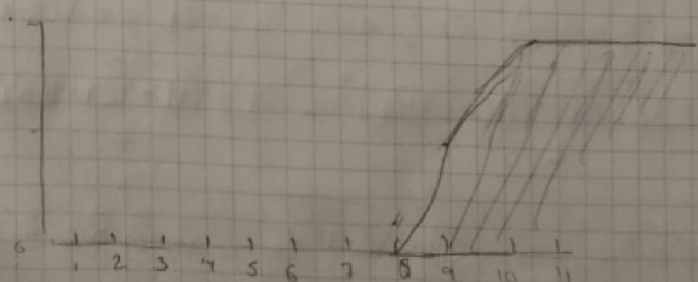
$$= 0,5 + \left(-\frac{x^2}{4} + \frac{21}{4} x \right) \Big|_{9}^{x} = -\frac{x^2}{4} + \frac{21}{4} x - (27)$$

$$= -\frac{x^2}{4} + \frac{21}{4} x - 26,5 \Rightarrow -\frac{x^2}{4} + \frac{21}{4} x - \frac{53}{2}$$

iv) Para $x \geq 10$

$$\int_{-\infty}^{8} \cancel{f(x)dx}^{\,0} + \underbrace{\int_{8}^{9} \frac{x}{2} + \frac{15}{4} dx}_{0,5} + \underbrace{\int_{9}^{10} -\frac{x}{2} + \frac{21}{4} dx}_{0,5} = 1$$

$$F(x) = \begin{cases} 0 & ; \quad x < 8 \\ \frac{x^2}{4} - \frac{15}{4} x + 14 & ; \quad 8 \leq x < 9 \\ -\frac{x^2}{4} + \frac{21}{4} x - \frac{53}{2} & ; \quad 9 \leq x < 10 \\ 1 & ; \quad x \geq 10 \end{cases}$$



4) Hallar los generadores y comprobar

$$F(x) = \begin{cases} 0 & ; \; x < 8 \\ \frac{x^2}{4} - \frac{15}{4}x + 14 & ; \; 8 \leq x < 9 \quad \Rightarrow R_1 \\ -\frac{x^2}{4} + \frac{21}{4} - 26{,}5 & ; \; 9 \leq x < 10 \quad \Rightarrow R_2 \\ 1 & ; \; x \geq 10 \end{cases}$$

Despejar x

$$R_1 = \frac{x^2}{4} - \frac{15}{4}x + 14 \quad // \cdot 4$$

$$4R_1 = x^2 - 15x + 56 \Rightarrow x^2 - 15x + (56 - 4R_1) = 0$$

$a = 1$ $b = -15$ $c = (56 - 4R_1)$

$$x = \frac{-b \pm \sqrt{b^2 - 4(ac)}}{2a} = \frac{+15 \pm \sqrt{15^2 - 4(56 - 4R_1)}}{2}$$

$$x = \frac{15 \pm \sqrt{1 + 16R_1}}{2} \qquad 0 \leq R_1 < 0{,}5$$

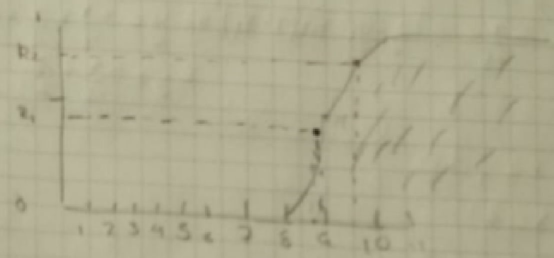
$$R_2 = -\frac{x^2}{4} + \frac{21}{4} - 26{,}5 \quad // \cdot 4 \Rightarrow 4R_2 = -x^2 + 21x - 106$$

$x^2 - 21x + (106 + 4R_2) = 0$; $a = 1$ $b = -21$ $c = 106 + 4R_2$

$$x = \frac{+21 \pm \sqrt{(-21)^2 - 4(106 + 4R_2)}}{2} = \frac{21 \pm \sqrt{441 - 424 - 16R_2}}{2}$$

$$x = \frac{21 \pm \sqrt{17 - 16R_2}}{2} = \frac{21 - \sqrt{17 - 16R_2}}{2} \qquad 0{,}5 \leq R_2 \leq 1$$

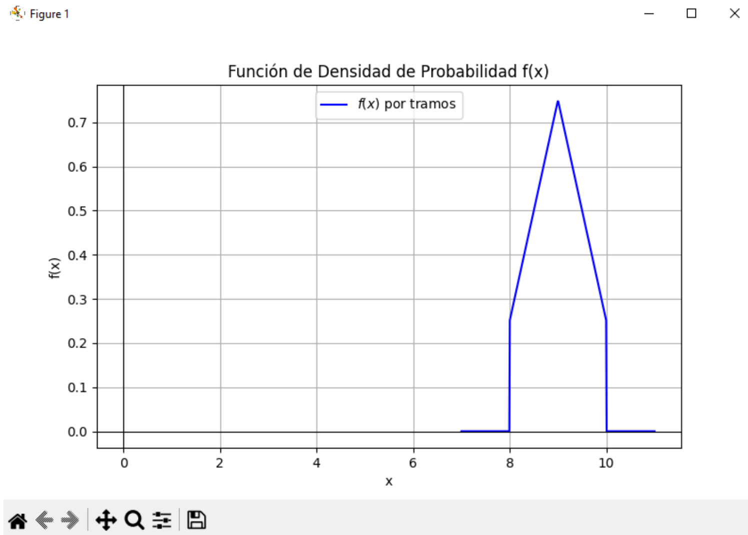
Comprobar

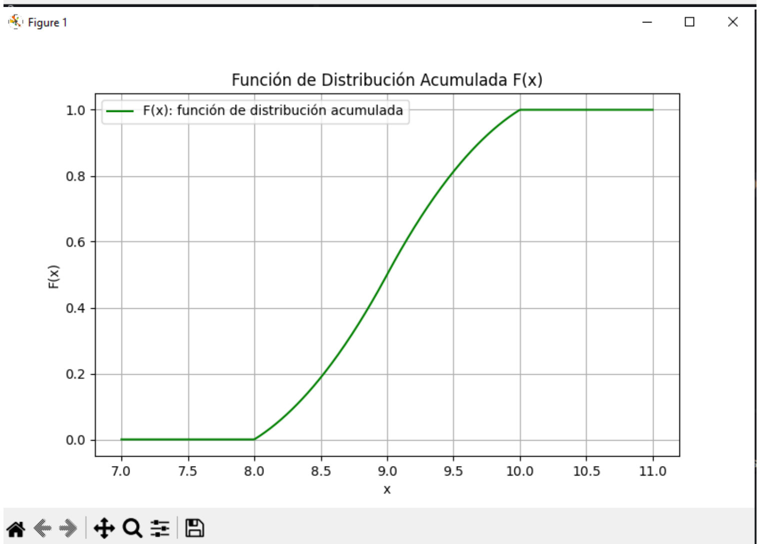


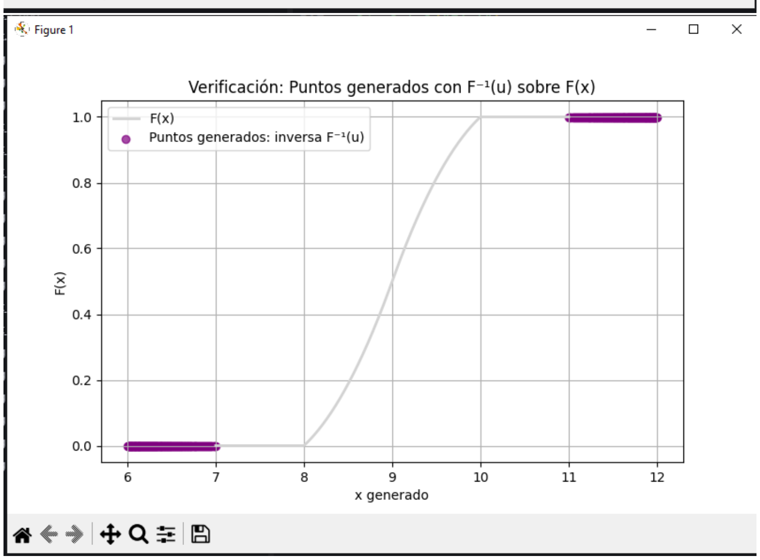
$R_1 = 0,4587 \Rightarrow x_1 = \dfrac{15 + \sqrt{1 + 16(0,4587)}}{2} = 8,94389$

$R_2 = 0,7801 \Rightarrow x_2 = \dfrac{21 - \sqrt{17 - 16(0,7801)}}{2} = 9,4371$

Resultado del Script: El script se encuentra en **continuo_3.py**







Ejercicio 4

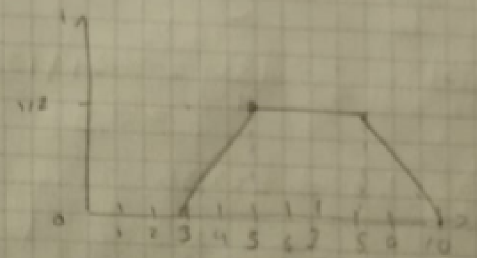
$$f(x) = \begin{cases} 0 & x < 3 \\ \frac{x}{4} + \frac{3}{4} & 3 \leq x < 5 \\ \frac{1}{2} & 5 \leq x < 8 \\ -\frac{x}{4} + \frac{5}{2} & 8 \leq x < 10 \end{cases}$$

Solución en clases:

Ejercicio 4)

Determinar los generadores de nros aleatorios

Sea



① Hallar $f(x)$ y verificar si es una fdp

a) Para $(3,0)$ y $(5, 1/2)$

$m: \frac{0,5 - 0}{5 - 3} = \frac{1}{4}$; $y - y_1 = m(x - x_1) \Rightarrow y = \frac{x}{4} - \frac{3}{4} + 0$

b) Para $(5, 1/2)$ y $(8, 1/2)$

$m: \frac{1/2 - 1/2}{8 - 5} = 0$; $y - y_1 = \cancel{m(x - x_1)}^{0} \Rightarrow y = \frac{1}{2}$

ⓒ Para $(8, \frac{1}{2})$ y $(10, 0)$

$m = \frac{(0 - \frac{1}{2})}{10 - 8} = -\frac{1}{4}$; $y - y_1 = m(x - x_1) \Rightarrow y = -\frac{x}{4} + \frac{8}{4} + 0{,}25$

$= y = -\frac{x}{4} + \boxed{\frac{5}{2}}$

* Comprobar si es fdp

$$f(x) = \begin{cases} 0 \; ; & x < 3 \\ \frac{x}{4} - \frac{3}{4} \; ; & 3 \leq x < 5 \\ \frac{1}{2} \; ; & 5 \leq x < 8 \\ -\frac{x}{4} + \frac{5}{2} \; ; & 8 \leq x < 10 \\ 1 \; ; & x \geq 10 \end{cases}$$

$$\cancel{\int_{-\infty}^{3} f(x)dx}^{\,0} + \underbrace{\int_{3}^{5} \frac{x}{4} - \frac{3}{4}\, dx}_{a} + \underbrace{\int_{5}^{8} \frac{1}{2}\, dx}_{b} + \underbrace{\int_{8}^{10} -\frac{x}{4} + \frac{5}{2}\, dx}_{c} = 1$$

$a = \left(\frac{x^2}{8} - \frac{3x}{4}\right)\Big|_3^5 = \left(\frac{25}{8} - \frac{3(5)}{4}\right) - \left(\frac{9}{8} - \frac{3(3)}{4}\right) = -0{,}625 + 1{,}125$

$\boxed{a = 0{,}5}$

$b = \left(\frac{x}{2}\right)\Big|_5^8 = \left(\frac{8}{2} - \frac{5}{2}\right) = \frac{3}{2} = 1.5 \Rightarrow \boxed{b = 1{,}5}$

$c = \left(-\frac{x^2}{8} + \frac{5x}{2}\right)\Big|_8^{10} = \left(-\frac{100}{8} + \frac{50}{2}\right) - \left(-\frac{64}{8} + \frac{40}{2}\right) = 12{,}5 - 12 = 0{,}5$

$$\cancel{\int_{-\infty}^{3} f(x)dx}^{\,0} + \underbrace{\int_{3}^{5} \frac{x}{4} - \frac{3}{4}\, dx}_{0{,}5} + \underbrace{\int_{5}^{8} \frac{1}{2}\, dx}_{1{,}5} + \underbrace{\int_{8}^{10} -\frac{x}{4} + \frac{5}{2}\, dx}_{0{,}5} = 2{,}5$$

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Como no es $=1$ usamos K

$$K\left(\frac{5}{2}\right) = 1 \Rightarrow K = \frac{2}{5}$$

Nueva $f(x)$ multiplicada por $K\left(\frac{2}{5}\right)$

$$f(x) = \begin{cases} \frac{x}{10} - \frac{3}{10} \ ; & 3 \leq x < 5 \\ \frac{1}{5} \ ; & 5 \leq x < 8 \\ -\frac{x}{10} + 1 \ ; & 8 \leq x < 10 \end{cases}$$

2 Hallamos $F(x)$

i) Para $x < 3$

$$\int_{-\infty}^{3} \cancel{f(x)dx}^{\,0}$$

ii) Para $3 \leq x < 5$

$$\int_{-\infty}^{3} \cancel{f(x)dx}^{\,0} + \int_{3}^{x} \frac{x}{10} + \frac{3}{10}\,dx = \left(\frac{x^2}{20} - \frac{6x}{20}\right)\Bigg|_{3}^{x}$$

$$= \left(\frac{x^2}{20} - \frac{6x}{20}\right) - \left(\frac{9}{20} - \frac{18}{20}\right) = \frac{x^2}{20} - \frac{6x}{20} + \frac{9}{20}$$

iii) Para $5 \leq x < 8$

$$\int_{-\infty}^{3} \cancel{f(x)dx}^{\,0} + \int_{3}^{5} \frac{x}{10} + \frac{3}{10}\,dx + \int_{5}^{x} \frac{1}{5}\,dx$$

$$= \left(\frac{x^2}{20} - \frac{6}{20}x\right)\Bigg|_{3}^{5} = \left(\frac{25}{20} - \frac{30}{20}\right) - \left(\frac{9}{20} - \frac{18}{20}\right) = 0{,}2 \Rightarrow \frac{1}{5}$$

$$= -\frac{1}{4} + \frac{9}{20}$$

$$\int_5^x \frac{1}{2}dx = \left(\frac{x}{2}\right)\Big|_5^x = \left(\frac{x}{2}\right) - \left(\frac{5}{2}\right) - \left(\frac{1}{5}\right)$$

$$= \frac{x}{5} - \frac{4}{5}$$

iii) Para $8 \le x < 10$

$$0 + \left(\frac{1}{5}\right) + \underbrace{\int_5^8 \frac{1}{2}dx}_{a} + \int_8^x -\frac{x}{4} + \frac{5}{2}\,dy$$

$$a = \left(\frac{x}{2}\right)\Big|_5^8 = \left(\frac{8}{2}\right) - \left(\frac{5}{2}\right) = \boxed{\frac{3}{2}}$$

$$\int_8^x -\frac{x}{4} + \frac{5}{2}dx = \left(-\frac{x^2}{8} + \frac{5}{2}x\right)\Big|_8^x = \left(-\frac{x^2}{8} + \frac{5x}{2}\right) - \left(\frac{-64}{8} + \frac{40}{2}\right)$$

$$= -\frac{x^2}{8} + \frac{5}{2} + \frac{1}{5} + \frac{3}{2} - 12 = \frac{-2x^2}{20} + x - 4$$

v) Para $x \ge 10$

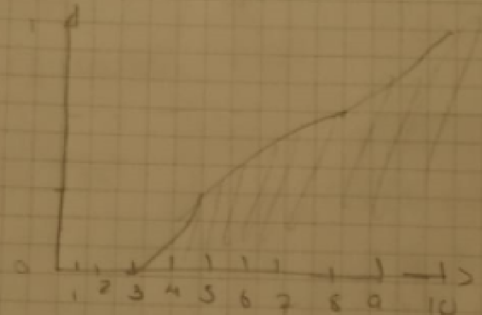
$$0 + 0{,}2 + \frac{3}{2} + \int_8^{10} -\frac{x}{4} + \frac{5}{2}dx = 1$$

$$\int_8^{10} -\frac{x}{4} + \frac{5}{2}\,dy = \left(-\frac{x^2}{8} + \frac{5}{2}x\right)\Big|_8^{10} = \left(-\frac{100}{8} + \frac{50}{2}\right) - \left(-\frac{64}{8} + \frac{40}{20}\right)$$

$$= 12{,}5 - 12 = 0{,}5$$

$$= 0 + \frac{1}{5} + \frac{3}{5} + \frac{1}{5} = 1$$

$$F(x) = \begin{cases} 0 \; ; & x < 3 \\ \frac{x^2}{20} - \frac{6x}{20} + \frac{9}{20} \; ; & 3 \leq x < 5 \\ \frac{x}{5} - \frac{4}{5} \; ; & 3 \leq x < 8 \\ -\frac{x^2}{20} + x - 4 \; ; & 8 \leq x < 10 \\ 1 \; ; & x \geq 10 \end{cases}$$



③ Hallar los generadores

$$F(x) = \begin{cases} \frac{x^2}{20} - \frac{6x}{20} + \frac{9}{20} \; ; & 3 \leq x < 5 \Rightarrow R_1 \\ \frac{x}{5} - \frac{4}{5} \; ; & 5 \leq x < 8 \Rightarrow R_2 \\ -\frac{x^2}{20} + x - 4 \; ; & 8 \leq x < 10 \Rightarrow R_3 \end{cases}$$

Despejar x

$$R_1 = \frac{x^2}{20} - \frac{6x}{20} + \frac{9}{20} \Rightarrow 20R_1 = x^2 - 6x + 9$$

$$x^2 - 6x + (9 - 20R_1) = \; x = \frac{6 \pm \sqrt{36 - 4(9 - 20R_1)}}{2a}$$

$$x = \frac{6 \pm \sqrt{80R_1}}{2} \Rightarrow x = \frac{6 + \sqrt{80R_1}}{2} \; , \; 0 \leq R_1 < 0{,}20$$

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$R_2 = \frac{x}{5} - \frac{4}{5} \Rightarrow x = 5R_2 + 4 \quad ; \quad 0{,}20 \le R_2 < 0{,}8$

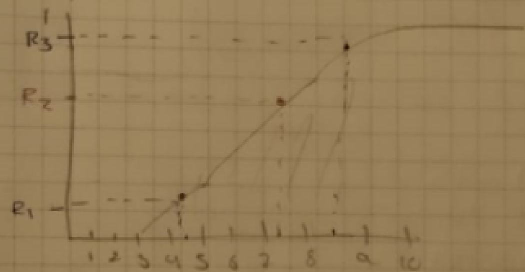
$R_3 = -\frac{x^2}{20} + x - 4 \Rightarrow 20R_3 = -x^2 + 20x - 80$

$x^2 - 20x + (80 + 20R_3) \Rightarrow x = \frac{20 \pm \sqrt{400 - 4(80 + 20R_3)}}{2}$

$a = 1 \quad b = -20 \quad c = 80 + 20R_3$

$x = \frac{20 \pm \sqrt{80 - 80R_3}}{2} = x = \frac{20 - \sqrt{80 - 80R_3}}{2} \; ; \; 0{,}8 <$

Comprobar

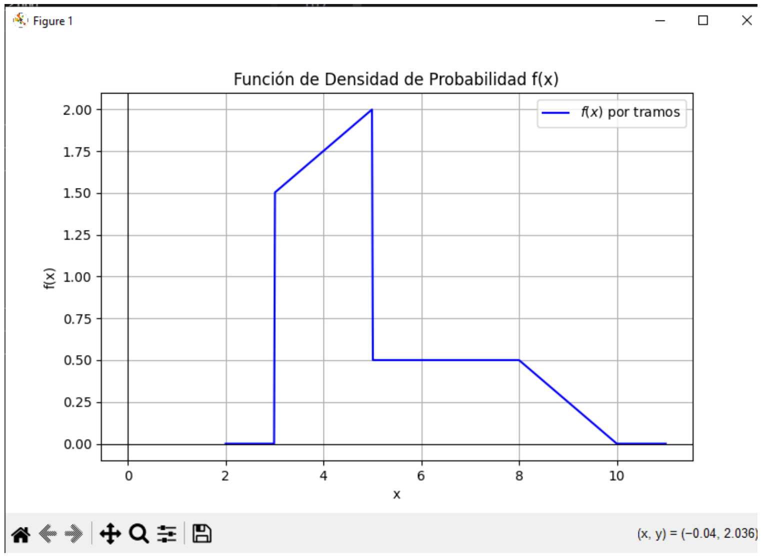


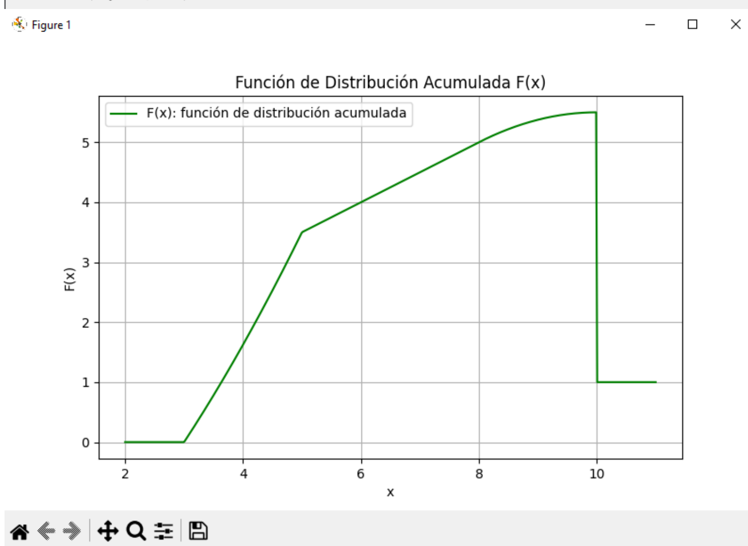
$R_1 = 0{,}1260 \Rightarrow x = \frac{6 + \sqrt{80(0{,}1260)}}{2} = 4{,}5924$

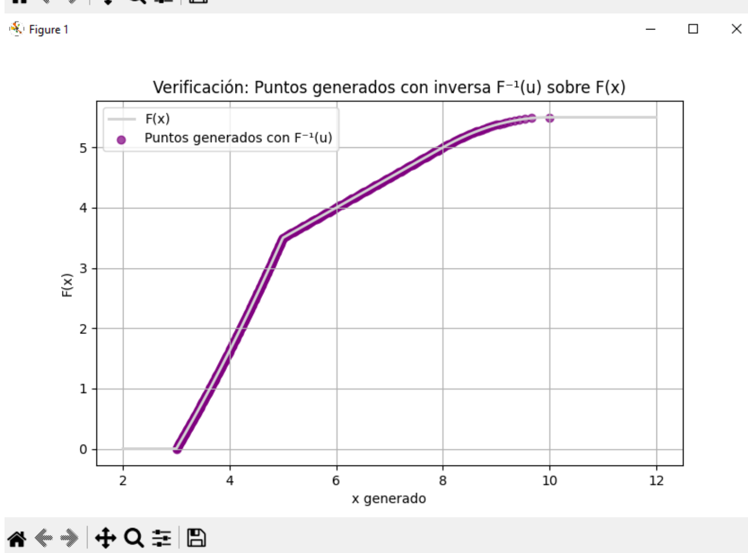
$R_2 = 0{,}6574 \Rightarrow x_2 = 5(0{,}6574) + 4 = 7{,}287$

$R_3 = 0{,}9015 \Rightarrow x_3 = \frac{20 - \sqrt{80 - 80(0{,}9015)}}{2} = 8{,}5964$

Resultado del Script: El script se encuentra en **continuo_4.py**





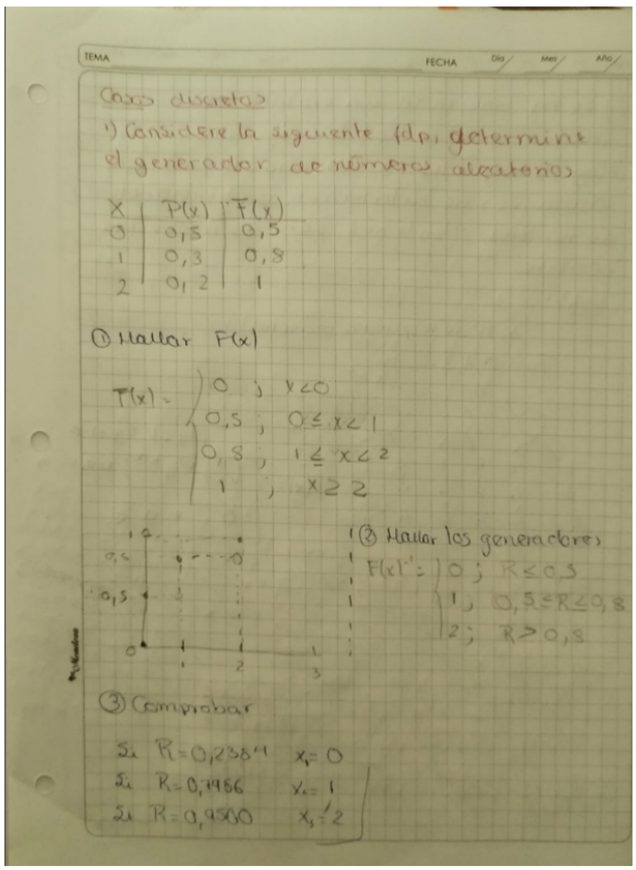


Casos Discretos

Ejercicio 5

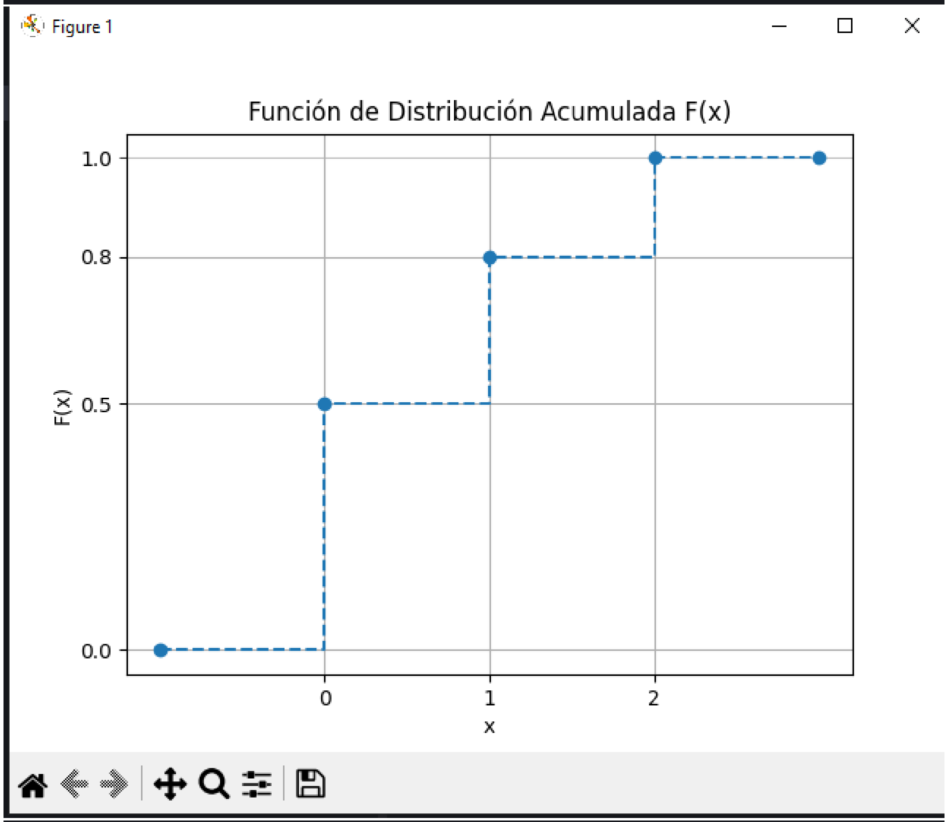
x	$P(x)$	$F(x)$
0	0.5	0.5
1	0.3	0.8
2	0.2	1

Solución en clases:



Resultado del Script: El script se encuentra en **discreto_1.py**

```
PS D:\I - 2025\Modelado\modelado\segundo parcial\practica\scripts> python .\discreto_1.py
Tabla de P(x) y F(x):
x       P(x)     F(x)
0       0.5      0.5
1       0.3      0.8
2       0.2      1.0
```




```
Comprobación de valores R:
Si R = 0.2384, entonces x = 0
Si R = 0.7956, entonces x = 1
Si R = 0.95, entonces x = 2
```

Ejercicio 6

Solución en clases:

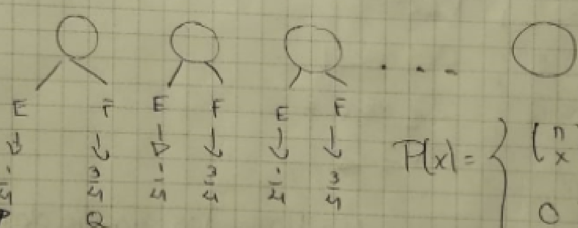
TEMA FECHA Día Mes Año

2) Se ha elaborado un examen de selección multiple con 10 preguntas. Hay 4 respuestas posibles pero solo 1 respuesta correcta. Se pide determinar un generador de numeros aleatorios para las respuestas correctas

Sol)

X: n° de respuestas correctas

R_x: 0, 1, 2 10



$p + q = 1 \Rightarrow q = 1 - p$

$$P(x) = \begin{cases} \binom{n}{x} p^x q^{n-x} & ; x = 0, 1 \dots \\ 0 & ; \text{e.o.c.} \end{cases}$$

$$P_x(x) = \begin{cases} \binom{10}{x} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{10-x} & ; x = 0, 1, 2 \dots n \\ 0 & ; \text{e.o.c.} \end{cases}$$

Para cada x

① $P(x=0) = \binom{10}{0} \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{10-0} = 0,0563$

② $P(x=1) = \binom{10}{1} \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^9 = 0,18771$

③ $P(x=2) = \binom{10}{2} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^8 = 0,28157$

TEMA FECHA Día / Mes / Año

(4) $P(x=3) = \binom{10}{3}\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^7 = 0{,}25028$

(5) $P(x=4) = \binom{10}{4}\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^6 = 0{,}14599$

(6) $P(x=5) = \binom{10}{5}\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^5 = 0{,}05839$

(7) $P(x=6) = \binom{10}{6}\left(\frac{1}{4}\right)^6\left(\frac{3}{4}\right)^4 = 0{,}01622$

(8) $P(x=7) = \binom{10}{7}\left(\frac{1}{4}\right)^7\left(\frac{3}{4}\right)^3 = 0{,}00309$

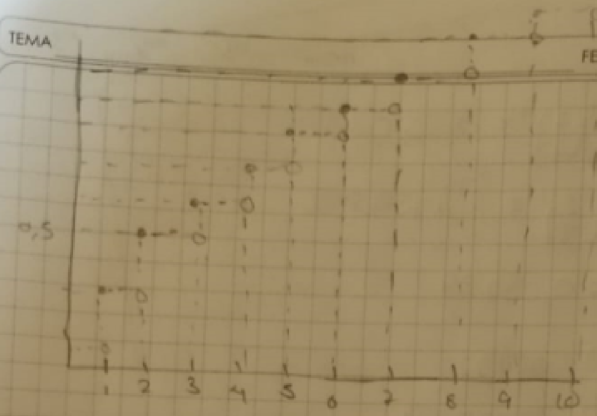
(9) $P(x=8) = \binom{10}{8}\left(\frac{1}{4}\right)^8\left(\frac{3}{4}\right)^2 = 0{,}00039$

(10) $P(x=9) = \binom{10}{9}\left(\frac{1}{4}\right)^9\left(\frac{3}{4}\right)^1 = 2{,}56102 \times 10^{-5}$

(11) $P(x=10) = \binom{10}{10}\left(\frac{1}{4}\right)^{10}\left(\frac{3}{4}\right)^0 = 9{,}53674 \times 10^{-7}$

x	P(x)	F(x)
0	0,0563	0,0563
1	0,15771	0,24401
2	0,28152	0,52558
3	0,25028	0,77586
4	0,14599	0,92185
5	0,05839	0,98024
6	0,01622	0,99646
7	0,00309	0,99957
8	0,00039	0,9994
9	0,000026102	0,999969
10	0,00000095367	
	1	1

$$F(x) = \begin{cases} 0 & ; x < 0 \\ 0{,}563 & ; 0 \le x < 1 \\ 0{,}24401 & ; 1 \le x < 2 \\ 0{,}52558 & ; 2 \le x < 3 \\ 0{,}77586 & ; 3 \le x < 4 \\ 0{,}92185 & ; 4 \le x < 5 \\ 0{,}98024 & ; 5 \le x < 6 \\ 0{,}99646 & ; 6 \le x < 7 \\ 0{,}99953 & ; 7 \le x < 8 \\ 0{,}9994 & ; 8 \le x < 9 \\ 0{,}999969 & ; 9 \le x < 10 \\ 1 & ; x \ge 10 \end{cases}$$



Generadores

$$F(x)^{-1} = \begin{cases} 0, & R < 0{,}0053 \\ 1; & 0{,}0053 \leq R < 0{,}24401 \\ 2; & 0{,}24401 \leq R < 0{,}52558 \\ 3; & 0{,}52558 \leq R < 0{,}7758 \\ 4, & 0{,}77506 \leq R < 0{,}92185 \\ 5; & 0{,}92185 \leq R < 0{,}98024 \\ 6; & 0{,}98024 \leq R < 0{,}99646 \\ 7; & 0{,}99646 \leq R < 0{,}99955 \\ 8; & 0{,}99955 \leq R < 0{,}99994 \\ 9, & 0{,}99994 \leq R < 0{,}999969 \\ 10 & 0{,}999969 \leq R < 1 \end{cases}$$

Probar:

Si $R = 0{,}0043 \Rightarrow x = 0$ | Si $R = 0{,}98024 \Rightarrow x = 6$

Si $R = 0{,}1345 \Rightarrow x = 1$ | Si $R \geq 0{,}99646 \Rightarrow x = 7$

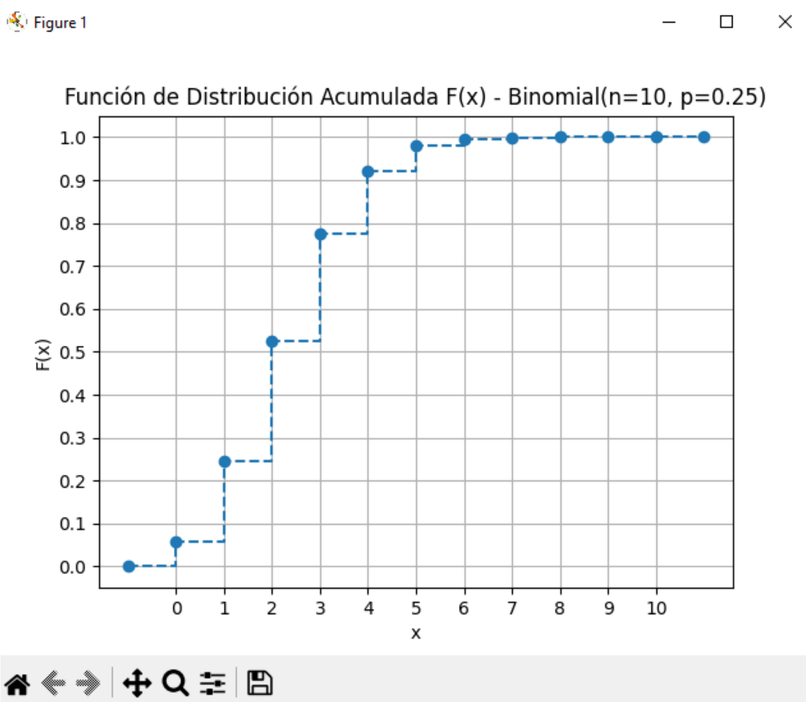
Si $R = 0{,}5559 \Rightarrow x = 2$ | Si $R = 0{,}99936 \Rightarrow x = 8$

Si $R = 0{,}8201 \Rightarrow x = 4$ | Si $R = 0{,}99994 \Rightarrow x = 9$

Si $R = 0{,}9800 \Rightarrow x = 5$ | Si $R = 0{,}999967 \Rightarrow x = 10$

Resultado del Script: El script se encuentra en **discreto_2.py**

```
Tabla de P(x) y F(x):
x         P(x)              F(x)
0         0.056314          0.056314
1         0.187712          0.244025
2         0.281568          0.525593
3         0.250282          0.775875
4         0.145998          0.921873
5         0.058399          0.980272
6         0.016222          0.996494
7         0.003090          0.999584
8         0.000386          0.999970
9         0.000029          0.999999
```




```
Comprobación de valores R:
Si R = 0.05, entonces x = 0
Si R = 0.3, entonces x = 2
Si R = 0.72, entonces x = 3
Si R = 0.99, entonces x = 6
```

Ejercicio 7

Solución en clases:

Ejercicio 3

Se pide generar números aleatorios de la siguiente función de probabilidad

$$f(x) = \begin{cases} \dfrac{e^{-5}\, 5^{x}}{x!} & ; x = 0, 1, 2, \ldots n \\ 0 & ; e.o.c \end{cases}$$

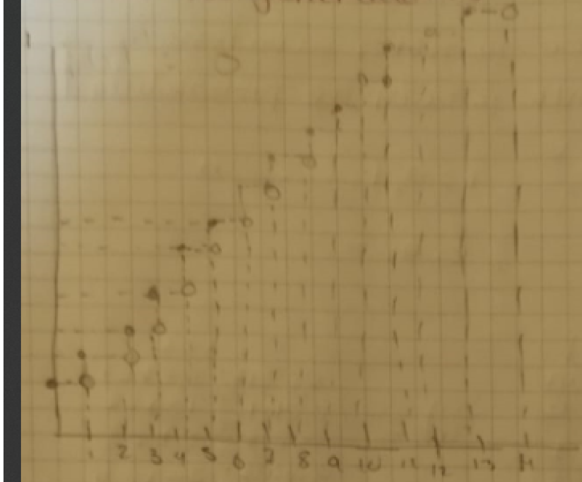
* Por aproximación

x	P(x)	F(x)
0	0,00647	0,00647
1	0,03369	0,04043
2	0,08422	0,2465
3	0,14037	0,26503
4	0,17547	0,44049
5	0,17547	0,61596
6	0,14622	0,76218
7	0,1044	0,86663
8	0,06528	0,93191
9	0,03627	0,96817
10	0,01813	0,9863
11	0,00824	0,99455
12	0,00343	0,99768
13	0,00132	0,9993
14	0,00047	0,99977
15	0,00016	0,99993

$$F(x) = \begin{cases} 0 & ; x < 0 \\ 0{,}00647 & ; 0 \le x < 1 \\ 0{,}04043 & ; 1 \le x < 2 \\ 0{,}12465 & ; 2 \le x < 3 \\ 0{,}26503 & ; 3 \le x < 4 \\ 0{,}44049 & ; 4 \le x < 5 \\ 0{,}61596 & ; 5 \le x < 6 \\ 0{,}76218 & ; 6 \le x < 7 \\ 0{,}86663 & ; 7 \le x < 8 \\ 0{,}93191 & ; 8 \le x < 9 \\ 0{,}96817 & ; 9 \le x < 10 \\ 0{,}9863 & ; 10 \le x < 11 \\ 0{,}99455 & ; 11 \le x < 12 \\ 0{,}99768 & ; 12 \le x < 13 \\ 0{,}9993 & ; 13 \le x < 14 \\ 0{,}99977 & ; 14 \le x < 15 \\ 0{,}99993 & ; 15 \le x < 16 \\ 1 & ; x \ge 16 \end{cases}$$

FECHA

Hallar los generadores

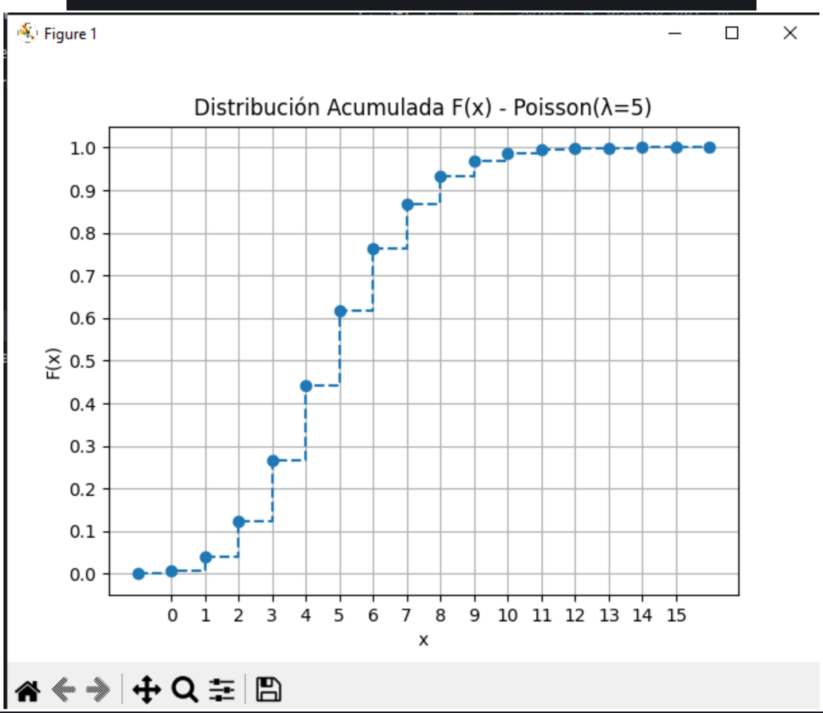


$$F(x) = \begin{cases} 0, & 0 \le R < 0{,}00674 \\ 1, & 0{,}00674 \le R < 0{,}04043 \\ 2, & 0{,}04043 \le R < 0{,}12465 \\ 3, & 0{,}12465 \le R < 0{,}26503 \\ 4, & 0{,}26503 \le R < 0{,}44049 \\ 5, & 0{,}44049 \le R < 0{,}61596 \\ 6, & 0{,}61596 \le R < 0{,}76218 \\ 7, & 0{,}76218 \le R < 0{,}86663 \\ 8, & 0{,}86663 \le R < 0{,}93191 \\ 9, & 0{,}93191 \le R < 0{,}96817 \\ 10, & 0{,}96817 \le R < 0{,}9863 \\ 11, & 0{,}9863 \le R < 0{,}9945 \\ \ldots & \end{cases}$$

Resultado del Script: El script se encuentra en **discreto_3.py**

Tabla de P(x) y F(x):

x	P(x)	F(x)
0	0.006738	0.006738
1	0.033690	0.040428
2	0.084224	0.124652
3	0.140374	0.265026
4	0.175467	0.440493
5	0.175467	0.615961
6	0.146223	0.762183
7	0.104445	0.866628
8	0.065278	0.931906
9	0.036266	0.968172
10	0.018133	0.986305
11	0.008242	0.994547
12	0.003434	0.997981
13	0.001321	0.999302
14	0.000472	0.999774
15	0.000157	0.999931



Comprobación de valores R:
Si R = 0.006, entonces x = 0
Si R = 0.25, entonces x = 3
Si R = 0.52, entonces x = 5
Si R = 0.985, entonces x = 10

Ejercicio 8 (Geométrica)

Solución en clases:

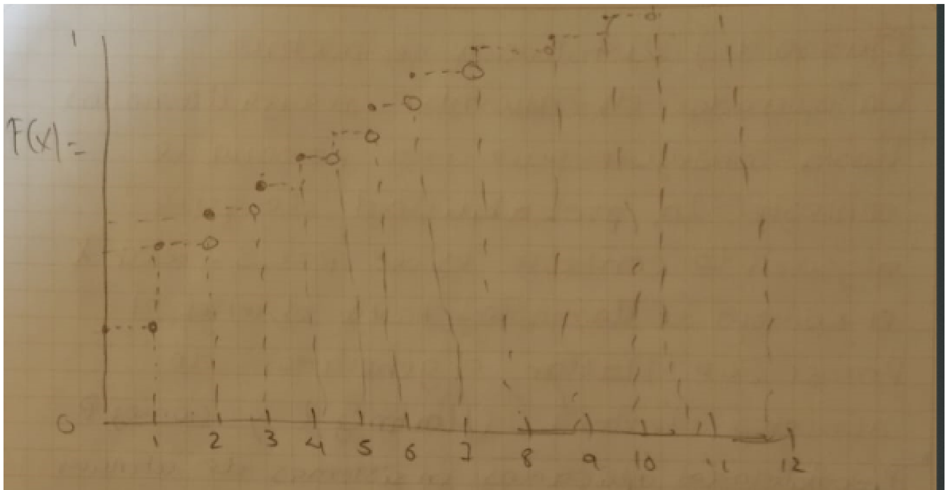
Ejercicio 4) Distribución geométrica

Un operador de callcenter realiza llamadas hasta conseguir que una persona le conteste. La probabilidad de que alguien le conteste es de $p = 0,3$. Sea X el número de llamadas hasta obtener el primer éxito. Hallar el generador de números aleatorios (Martínez L y Gómez P. Probabilidades aplicadas en sistemas de atención telefónica".)

$$P(X=x) = (1-p)^{(x-1)} p \qquad F(x) = 1-(1-p)^{x}$$

X	P(x)	F(x)
1	0,3	0,3
2	0,21	0,51
3	0,147	0,657
4	0,1029	0,7599
5	0,07203	0,83193
6	0,05042	0,88235
7	0,003529	0,91763
8	0,02471	0,94235
9	0,01729	0,9565
10	0,0121	0,97175
11	0,00847	0,98023
12	0,00593	0,98616

$$F(x) = \begin{cases} 1; & 0 \le x < 0,3 \\ 2; & 0,3 \le x < 0,51 \\ 3; & 0,51 \le x < 0,652 \\ 4; & 0,652 \le x < 0,7599 \\ 5; & 0,7599 \le x < 0,83193 \\ 6; & 0,83 \le x < 0,88 \\ 7; & 0,88 \le x < 0,91763 \end{cases}$$



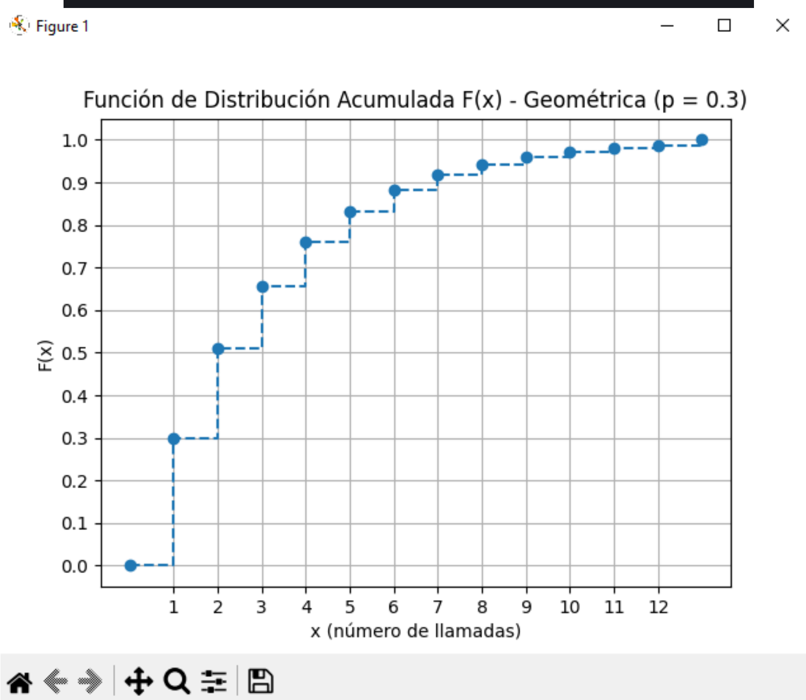
Generadores

$$F(x)^{-1} = \begin{cases} 1; & 0 \leq R < 0,3 \\ 2; & 0,3 \leq R < 0,51 \\ 3; & 0,51 \leq R < 0,657 \\ 4, & 0,657 \leq R < 0,7599 \\ 5; & 0,7599 \leq R < 0,83193 \\ 6; & 0,83193 \leq R < 0,88235 \\ 7; & 0,88,235 \leq R \quad 0,083193 \\ \vdots & \end{cases}$$

Resultado del Script: El script se encuentra en **geometrico.py**

Tabla de P(x) y F(x):

x	P(x)	F(x)
1	0.300000	0.300000
2	0.210000	0.510000
3	0.147000	0.657000
4	0.102900	0.759900
5	0.072030	0.831930
6	0.050421	0.882351
7	0.035295	0.917646
8	0.024706	0.942352
9	0.017294	0.959646
10	0.012106	0.971752
11	0.008474	0.980227
12	0.005932	0.986159




```
Comprobación de valores R:
Si R = 0.2, entonces x = 1
Si R = 0.51, entonces x = 3
Si R = 0.7599, entonces x = 5
Si R = 0.85, entonces x = 6
```

Ejercicio 9 (Hipergeométrica)

Solución en clases:

Ejercicio 5 (Distribución hipergeométrica)

Una urna contiene 10 bolas rojas y 15 bolas azules, un total de 25. Se extraen 5 bolas al azar, sin reemplazo. Hallar el generador de números aleatorios.

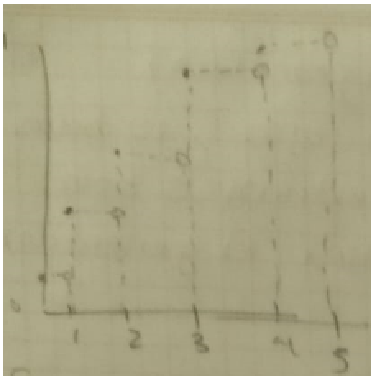
<u>Datos</u>

$N=25 \quad n=5 \quad K=10 \quad X=2$

$$P(X=x)=\frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

X	P(x)	F(x)
0	0,05652	0,05652
1	0,25692	0,31344
2	0,38538	0,69881
3	0,23715	0,93597
4	0,05929	0,99526
5	0,00474	1

$$F^{-1}(x)=\begin{cases} 0 \; ; \; x<0 \\ 1 \; ; \; 0 \leq x < 0{,}05632 \\ 2 \; ; \; 0{,}05632 \leq x < 0{,}3134 \\ 3 \; ; \; 0{,}3134 \leq x < 0{,}6980 \\ 4 \; ; \; 0{,}69881 \leq x < 0{,}935 \\ 5 \; ; \; 0{,}935 \leq x < 0{,}9952 \\ 6 \; ; \; x \geq 0{,}9952 \end{cases}$$

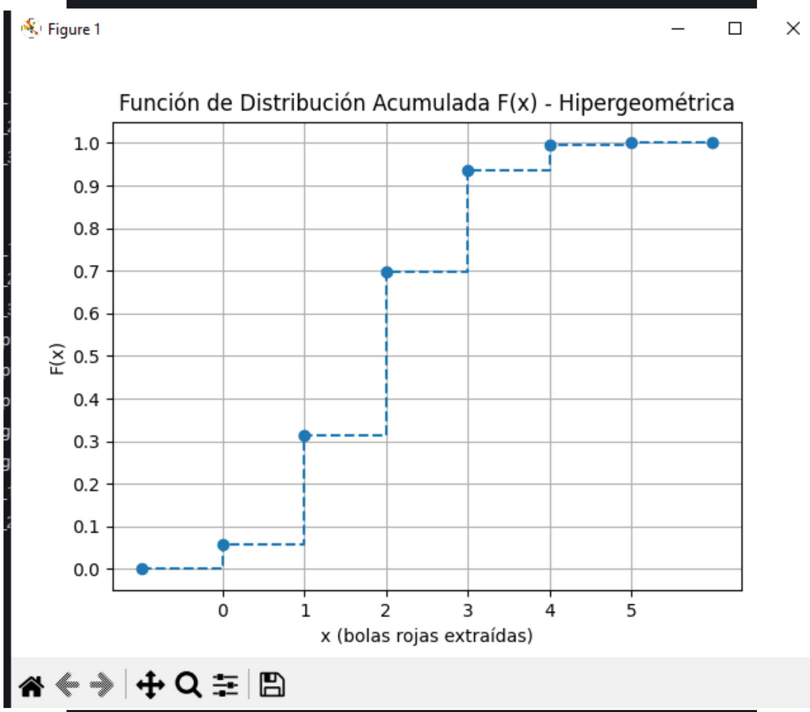


Generadores

$$F(x)^{-1} = \begin{cases} 0; & 0 < R \leq 0{,}0562 \\ 1; & 0{,}562 \leq R < 0{,}313 \\ 2; & 0{,}313 \leq R < 0{,}0{,}6988 \\ 3; & 0{,}6988 \leq R < 0{,}935 \\ 4; & 0{,}935 \leq R < 0{,}995 \\ 5; & 0{,}995 \leq R < 1 \end{cases}$$

Resultado del Script: El script se encuentra en **hipergeometrico.py**

```
Tabla de P(x) y F(x):
x        P(x)              F(x)
0        0.056522          0.056522
1        0.256917          0.313439
2        0.385375          0.698814
3        0.237154          0.935968
4        0.059289          0.995257
5        0.004743          1.000000
```




```
Comprobación de valores R:
Si R = 0.02, entonces x = 0
Si R = 0.3, entonces x = 1
Si R = 0.65, entonces x = 2
Si R = 0.94, entonces x = 4
Si R = 0.97, entonces x = 4
```