## Sergio Villamaring assignment 2

### A few notes before I start solving the problem

Recall the the original formulation of the problem resulted in the similar equation without harvest by normalizing the population by the max carrying capacity (this is the reason of the factor. Since we are now working with the parameter , then I will decide on appropriate values for this parameter based on this normalization, that is if we pick then it means we are harvesting half of the total carrying capacity. This means that if we were to pick values bigger than 1 there will probably be issues as the rate of harvest would exceed the maximum sustainable population. This will come up later in the calculations. Additionally negative numbers would instead add to the population, and since this is a “harvest” equation, those would be meaningless for the purpose of this study. We therefore will consider and will explore a single value of the parameter unless there’s something in the calculations that suggest there would be a different behaviour.

### Equilibrium points

As requested on the previous feedback I’ll expand a little on the definition of equilibrium points. Equilibrium points of a differential equations are constant functions that solve a given differential equation. This is important because this implies that every 1 and higher order derivative with respect to time/dependent variable is 0, which simplifies the differential equation significantly (in our case it reduces it to an algebraic problem) and more over study of DE has shown that other solutions, while not constant, commonly converge or diverge from this equilibrium solutions. This in practice means that one is able to understand intuitively the behavior of solutions without necessarily having to solve the equation.

Now on to the practice. Once more, finding the equilibrium points implies solving the equation

From where we can solve the quadratic equation to find the equilibrium solutions

This creates a disjunction for values for which we have imaginary roots that can potentially point to a wave-like behavior. We therefore need to consider the three cases for , , .

### Stability

Again, I’ll expand a bit more as requested. Stability analysis relevance was partially mentioned before when talking about equilibrium points. The specific role of stability analysis is to determine the behaviour of equilibrium points and whether they are attracting or stable, divergent or unstable, or neither of those (more classifications exist outside our scope). Stable points are solutions to which other “nearby” solutions converge, and unstable ones are ones for which they diverge. Other behaviour may involve the a mix of these two or periodicity of some kind.

Now let’s proceed to see if the real solutions are stable or unstable using again the local linearization method around the equilibrium points. We first find the derivative with respect to the solution variable of the function

Namely

Now we evaluate on the equilibrium points found before

From where we conclude that the positive root is an unstable fix point and the negative root is a stable fix point. In the particular case when the two roots merge at , in which case on has the separable equation

Which can be solved as

We therefore focus our attention on the remaining two cases where for .

### Numerical solutions and graphs

#### Real roots

Let’s see some solutions for for a fix value of , say . In this case we have a divergent solution at and a convergent solution at

A diagram of a solution

Description automatically generated

Now for the requested values we put black lines at the equilibrium solutions. For we have

A diagram of a solution

Description automatically generated with medium confidence

And for we have

A graph of solutions and solutions

Description automatically generated

On the later case we can see how the solution has convergent behaviour for values and divergent for values . This system is particularly bad as the only equilibrium solution if perturbed will most likely converge to 0.

Finally, for the case in which we can see from the plot below that there’s no stable solution and all solutions converge to zero.

A diagram of a solution

Description automatically generated

### Potential plots

Setting up the potential equation

And so solving for we get

The plots for values of are shown below

A graph of solutions and solutions

Description automatically generated

The potential function, if understood on the provided material as the “ball un a cup” diagram shows that there’s a non zero stable equilibrium around 0.8 for as the “ball” won’t roll out in the vicinity. For the case of we see that there’s a single horizontal spot at but overall the system tends to go down to 0. Finally for we see that there’s an incline that would push the system towards 0 no matter what values are used.