3.5 ANILLOS DE POLINOMIOS. IDEALES MAXIMALES EN K[x]

$$\operatorname{mcd}(f,g)=d=\frac{r_n}{cp(r_n)}=\frac{a_n}{cp(r_n)}\cdot f+\frac{b_n}{cp(r_n)}\cdot g$$

$$\lambda=\frac{a_n}{cp(r_n)}\ ,\quad \mu=\frac{b_n}{cp(r_n)}$$

Ej:

$n=46 \quad m=32 \in \mathbb{Z}$

$n\to$	46		$(1,0)$	
$m\to$	32		$(0,1)$	
	14	1	$(1,-1)$	$46-32=14$
	4	2	$(-2,3)$	$(-2)46+(3)32=4$
	2	3	$(7,-10)$	$(7)46+(-10)32=2$
	0	2		

$\boxed{46}=32\cdot 1+14$

$\boxed{32}=14\cdot 2+4$

$\boxed{14}=4\cdot 3+2$

$\boxed{4}=\boxed{2}\,2+\boxed{0}$

$\to 2=\operatorname{mcd}(x,y)$

Ej:

$f=x^4+2x^3+x^2-x-1,\quad g=x^2+2x+2\in\mathbb{Q}[x]$

$x^4+2x^3+x^2-x-1$		$(1,0)$
x^2+2x+2		$(0,1)$
$x+1$	x^2-1	$(1,-x^2+1)$
1	$x+1$	$(-x-1,x^3+x^2-x)$
0	$x+1$	

$\to (x+1)(x^3+x^2-1)$

$$\begin{array}{r|rrrrr} & 1 & 2 & 1 & -1 & -1\\ -1 & & -1 & -1 & 0 & 1\\ \hline & 1 & 1 & 0 & -1 & \\ +1 & & & & & \\ \hline \end{array}$$

$1=(-x-1)(x^4+2x^3+x^2-x-1)+(x^3+x^2-x)(x^2+2x+2)$

$(1,0)-(x^2-1)(0,1)=(1,-x^2-1)$

$(0,1)-(x+1)(1,-x^2-1)=(-x-1,x^3+x+x^2+1)$

EJEMPLO

$f = x^5 + 4x^4 - 4x^3 + 6x^2 + 5x$, $g = x^3 + 4x^2 - 4x + 5 \in \mathbb{Q}[x]$

$x^5+4x^4-4x^3+6x^2+5x$		$(1,0)$
x^3+4x^2-4x+5		$(0,1)$
x^2+5x (resto)	x^2 (cociente)	$(1,-x^2)$
$x+5$ (resto	$x-1$ (cociente)	$(-x+1, x^3-x^2+1)$
0	x	

$$x+5 = (-x+1)(x^5+4x^4-4x^3+6x^2+5x)+(x^3-x^2+1)(x^3+4x^2-4x+5)$$

EJEMPLO

$f = x^5 + 2x^3 + x$, $g = x^4 + x^3 + 2x^2 + x + 1 \in \mathbb{Q}[x]$

x^5+2x^3+x		$(1,0)$
$x^4+x^3+2x^2+x+1$		$(0,1)$
x^3+x^2+x+1	$x-1$	$(1,(-x)$
x^2+1	x	$(-x, 1-x+x^2)$
0	$x+1$	

$$x^2+1 = (-x)(x^5+2x^3+x)+(1-x+x^2)(x^4+x^3+2x^2+x+1) \text{ en } \mathbb{Q}[x]$$

1. Estudia unidades anillos:

a) $(\mathbb{Z}[x], +, \cdot)$

$U(\mathbb{Z}[x]) = \{1, -1\}$

b) $(\mathbb{R}[x], +, \cdot)$

$U(\mathbb{R}[x]) = \{f \in \mathbb{R}[x] : gr(f) = 0\} = \{a \in \mathbb{R} : a \neq 0\}$

polinomios constantes distintos del cero)

c) $(\mathbb{Z}_{11}[x], +_{11}, \cdot_{11})$

$U(\mathbb{Z}_{11}[x]) = \{1, 2, 3, .. 10\} = \{f \in \mathbb{Z}_{11}[x] : gr(f) = 0\}$

2. Anillo $(\mathbb{Z}_4[x], +_4, \cdot_4)$, polinomio $g = 2x+1$ es una unidad.

$(2x+1)(2x+1) = 4x^2+4x+1 \xrightarrow{\text{en } \mathbb{Z}_4} = 1$

Si $\mathbb{Z}_4$ no es D.I. $\Rightarrow$ grado $(g \cdot g) \neq$ grado (g) + grado (g)

4.1. CUERPOS DE FRACCIONES Y EXTENSIONES DE CUERPOS

1. Obtener el cuerpo de fracciones de los anillos:

c) $\mathbb{Z}[\sqrt{3}i] = \{a + b\sqrt{3}i : a, b \in \mathbb{Z}\}$

$\mathbb{Z}[\sqrt{3}i]$, $\mathbb{Z} \subset \mathbb{Z}[i]$

$\mathbb{Q} \subset cq(\mathbb{Z}[\sqrt{3}i]) \subset \mathbb{Q}([\sqrt{3}i])$

$x^2+3 \in \mathbb{Q}[x]$ tiene como raíz a $\alpha = \sqrt{3}i \Rightarrow$

$\mathbb{Q}[\sqrt{3}i] = \mathbb{Q}(\sqrt{3}i) \Rightarrow cq(\mathbb{Z}[\sqrt{3}i]) = \mathbb{Q}[\sqrt{3}i]$

2. Considera el dominio de integridad $\mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$

a) Dar cuerpo de fracciones $\mathbb{Z}[\sqrt{2}]$

$(a+b\sqrt{2})(c+d\sqrt{2}) = 0$ spg. que $a + b\sqrt{2} \neq 0$ $\left\{ \begin{array}{l} ¿c + d\sqrt{2} = 0? \\ ¿c = d = 0? \end{array} \right.$

$\left. \begin{array}{l} ac + 2bd = 0 \\ ad + bc = 0 \end{array} \right\} a + b\sqrt{2} \neq 0 \Rightarrow \left\{ \begin{array}{l} a \neq 0 \\ ó \\ b \neq 0 \end{array} \right.$

- Si $a \neq 0 \Rightarrow c = -2\frac{b}{a}d = ad - 2\frac{b^2}{a}d = 0 \Rightarrow (a - 2\frac{b^2}{a})d = 0$

$\Rightarrow \left(\frac{a^2-2b^2}{a}\right)d = 0 \Rightarrow \frac{1}{a}(a^2-2b^2)\underset{\overset{\uparrow}{\mathbb{Q} \text{ es DI}}}{d} = 0 \Rightarrow \left\{ \begin{array}{l} a^2 - 2b^2 = 0 \\ ó \\ d = 0 \end{array} \right\} \Rightarrow$

$\Rightarrow \frac{a^2}{b^2} = 2 \Rightarrow \frac{a}{b} = \sqrt{2} \in \mathbb{Q}$!!! $\Rightarrow d = 0 \Rightarrow c = \frac{-2b}{a} \cdot d = 0 \Rightarrow c + d\sqrt{2} = 0$

- Si $b \neq 0 \Rightarrow d = -\frac{1}{2}\frac{a}{b}c \Rightarrow -\frac{1}{2}\frac{a^2}{b}c + bc = 0 \Rightarrow c\left(\frac{-1}{2}\frac{a^2}{b} + b\right) = 0$

$\Rightarrow \frac{1}{b}c\left(\frac{2b^2-a^2}{2}\right) = 0$

OTRA FORMA $\Rightarrow \left\{ \begin{array}{l} \mathbb{Z}[\sqrt{2}] \subseteq \mathbb{Q}[\sqrt{2}] \text{ que es cuerpo} \\ \mathbb{Z}[\sqrt{2}] \text{ es DI. pero no es cuerpo} \end{array} \right.$

b) Dem. en $\mathbb{Z}[\sqrt{2}]$ hay ∞ unidades

$(a+b\sqrt{2})^{-1} = \frac{a-b\sqrt{2}}{a^2-2b^2} = \frac{a}{a^2-2b^2} - \frac{b}{a^2-2b^2}\sqrt{2} \left\{ \begin{array}{l} \in \mathbb{Q}[\sqrt{2}] \\ \notin \mathbb{Z}[\sqrt{2}] \text{ en general} \end{array} \right.$

Toma $1 + \sqrt{2}$

$(1+\sqrt{2})(\sqrt{2}-1) = 1 \Rightarrow (1+\sqrt{2}) = \alpha$ es una unidad de $\mathbb{Z}[\sqrt{2}]$

$(1+\sqrt{2})^n(\sqrt{2}-1)^n = 1 \Rightarrow \alpha^n$ es unidad $\forall n \in \mathbb{N} \Rightarrow$ hay ∞.

Por inducción $\longleftarrow$ $(\alpha^i \neq \alpha^j \; \forall i \neq j)$

1. $\alpha^0 \neq \alpha^1$, $\alpha^2 = 3 + 2\sqrt{2} \neq \alpha \neq \alpha^0 = 1$

2. Spg cierto $i, j < n$ se verifica $\alpha^i \neq \alpha^j$, $i \neq j$

Sea $k < n$ ¿$\alpha^k \neq \alpha^n$?

Si $\alpha^k = \alpha^n \Rightarrow \underset{\overset{\uparrow}{n > n-k > 0}}{\alpha^{n-k}} = 1 = \alpha^0$

3. Sea $\mathbb{Z}[\sqrt{21}] = \{a+b\sqrt{21}\}$ F cuerpo fracc. Dem x^2-x-5 irred.

$\hookrightarrow (\mathbb{Z}[\sqrt{21}])[x]$ no en $F[x]$

$x^2-x-5=0 \Rightarrow$

$\alpha = \frac{1\pm\sqrt{21}}{2} \Rightarrow \alpha_1 = \frac{1}{2}+\frac{1}{2}\sqrt{21}$, $\alpha_2 = \frac{1}{2}-\frac{1}{2}\sqrt{21}$

$x^2-x-5 = (x-\alpha_1)(x-\alpha_2)$ $\quad \alpha_1,\alpha_2 \notin \mathbb{Z}[\sqrt{21}] \leftarrow$ xq $1/2 \notin \mathbb{Z}$

$\alpha_1,\alpha_2 \in \mathbb{Q}[\sqrt{21}]$ cuerpo de fracc. de $\mathbb{Z}[\sqrt{21}]$

4. Dem $\mathbb{Q}(4-i) = \mathbb{Q}(1+i)$

$$\left.\begin{array}{l} 4-i = -(1+i)+5 \in \mathbb{Q}(1+i) \\ 1+i = -(4-i)+5 \in \mathbb{Q}(4-i) \end{array}\right\} \Rightarrow \left.\begin{array}{l} \mathbb{Q}(4-i) \subset \mathbb{Q}(1+i) \\ \mathbb{Q}(1+i) \subset \mathbb{Q}(4-i) \end{array}\right\} \Rightarrow \mathbb{Q}(1+i) = \mathbb{Q}(4-i)$$

<u>Otra forma</u>

El mínimo cuerpo que contiene a $\left\{\begin{array}{l} \mathbb{Q}(1+i) \text{ es } \mathbb{Q}(i) \\ \mathbb{Q}(4-i) \longrightarrow \end{array}\right.$

5. EXTENSIÓN SIMPLE DE K $\Rightarrow F = K(\alpha)$. Dem:

a) $\mathbb{Q}(\sqrt{2},\sqrt{3})$

spg $\alpha = \sqrt{2}+\sqrt{3}$, ¿$\mathbb{Q}(\alpha) = \mathbb{Q}(\sqrt{2},\sqrt{3})$?

$\sqrt{2}+\sqrt{3} \in \mathbb{Q}(\sqrt{2},\sqrt{3}) \to \mathbb{Q}(\alpha) \subseteq \mathbb{Q}(\sqrt{2},\sqrt{3})$ ¿$\mathbb{Q}(\alpha) \supseteq \mathbb{Q}(\sqrt{2},\sqrt{3})$? $\Rightarrow$

$\Rightarrow \left\{\begin{array}{l} \sqrt{2} \in \mathbb{Q}(\alpha)\ ? \\ \sqrt{3} \in \mathbb{Q}(\alpha)\ ? \end{array}\right.$

$\alpha \in \mathbb{Q}(\alpha) \Rightarrow \alpha^2 = 5+2\sqrt{6} \in \mathbb{Q}(\alpha) \Rightarrow \sqrt{6} \in \mathbb{Q} \Rightarrow \sqrt{6}\,\alpha \in \mathbb{Q}(\alpha) \Rightarrow$

$\Rightarrow 2\sqrt{3}+3\sqrt{2} \in \mathbb{Q}(\alpha)$, $\alpha = \sqrt{3}+\sqrt{2} \in \mathbb{Q}(\alpha)$

$2\alpha - (2\sqrt{3}+3\sqrt{2}) \in \mathbb{Q}(\alpha) \Rightarrow \sqrt{2} \in \mathbb{Q}(\alpha)$

$\alpha - \sqrt{2} \in \mathbb{Q}(\alpha) \Rightarrow \sqrt{3} \in \mathbb{Q}(\alpha)$

b) $\mathbb{Q}(\sqrt{2}, i)$

$\alpha = \sqrt{2}+i$ $\quad \mathbb{Q}(\alpha) = \mathbb{Q}(\sqrt{2},i)$; $\mathbb{Q}(\sqrt{2}+i) \subseteq \mathbb{Q}(\sqrt{2},i)$

¿$\sqrt{2} \in \mathbb{Q}(\sqrt{2}+i)$? $\quad \alpha = \sqrt{2}+i \in \mathbb{Q}(\sqrt{2}+i)$

¿$i \in \mathbb{Q}(\sqrt{2}+i)$? $\quad \alpha^2 = 1+2\sqrt{2}i \in \mathbb{Q}(\sqrt{2}+i)$

$\frac{\alpha^2-1}{2} \in \mathbb{Q}(\sqrt{2}+i) \Rightarrow \sqrt{2}i \in \mathbb{Q}(\sqrt{2}+i)$

si está α y $\sqrt{2}i \Rightarrow$ su producto también

$$\left.\begin{array}{l} \sqrt{2}i\cdot\alpha = 2i-\sqrt{2} \in \mathbb{Q}(\sqrt{2}+i) \\ \alpha = i+\sqrt{2} \in \mathbb{Q}(\sqrt{2}+i) \end{array}\right\}$$ si tengo esos dos elementos $\Rightarrow$ $\Rightarrow$ tiene que estar su suma

$\Rightarrow \alpha+\sqrt{2}i\alpha = 3i \in \mathbb{Q}(\sqrt{2}+i)$

$\Rightarrow \frac{\alpha+\sqrt{2}i\alpha}{3} = i \in \mathbb{Q}(\sqrt{2}+i)$

$\Rightarrow$ tiene que estar $\alpha - i = \sqrt{2} \in \mathbb{Q}(\sqrt{2}+i)$

6. $h = x^3 - 6x - 3 \in \mathbb{Q}[x]$

a) Mínimo cuerpo $\mathbb{K}$, extensión de $\mathbb{Q}$, $h \in \mathbb{K}[x]$ raíz $\alpha \in \mathbb{K}$

Es irreducible por Eisenstein, $p = 3$

$\mathbb{Q}[x]/(x^3-6x-3)$ es el mín. cuerpo en el cual h tiene 1 raíz.

$\alpha = [x](x^3-6x-3)$

Como h es irreducible $\Rightarrow$ es maximal el ideal generado por él

$\mathbb{Q}[x]/(x^3-6x-3)$ es cuerpo $\Leftrightarrow$ h es maximal

$\mathbb{Q}(\alpha) = \mathbb{Q}[x]/(x^3-6x-3)$

1. $\underline{\alpha^3 - 6\alpha}$ $\begin{cases} \mathbb{Q}(\alpha) \not\cong \mathbb{Q} \Rightarrow B = \{1, \alpha, \alpha^2\} \\ \alpha^3 - 6\alpha - 3 = 0 \Rightarrow \alpha^3 - 6\alpha = 3 \end{cases}$

2. $\underline{\alpha^4 - 6\alpha^2 - 1}$ $\begin{cases} \alpha^3 - 6\alpha - 3 = 0 \Rightarrow \alpha^3 - 6\alpha = 3 \Rightarrow \alpha^4 - 6\alpha^2 = 3\alpha \Rightarrow \\ \Rightarrow \alpha^4 - 6\alpha^2 - 1 = 3\alpha - 1 \end{cases}$

También $\dfrac{x^4 - 6x^2 - 1}{x^3 - 6x - 3} = \underset{\text{resto}}{3x - 1} + \text{cociente}$

3. $\underline{(\alpha^2 - 2\alpha - 2)^{-1}}$

$\text{mcd}(x^2 - 2x - 2, \underset{\text{irreduc.}}{x^3 - 6x - 3}) = 1$

x^3-6x-3		$(1,0)$
x^2-2x-2		$(0,1)$
1	$x+2$	$(1,-x-2)$

$1 = 1(x^3 - 6x - 3) + (-x-2)(x^2-2x-2)$

$(\alpha^2 - 2\alpha - 2)^{-1} = (-\alpha - 2)$ //

$(-\alpha - 2)(\alpha^2 - 2\alpha - 2) = 1$ para comprobarlo.

7. $h = x^3 - 2x^2 - 2x + 2 \in \mathbb{Q}[x]$

a) Mínimo cuerpo K, extensión $\mathbb{Q}$, $h \in K[\alpha]$

$\mathbb{Q}(\alpha) = \mathbb{Q}[x]/(x^3-2x^2-2x+2)$ xq es irre.

$\alpha = [x](x^3-2x^2-2x+2)$

b) Calcula en $\mathbb{Q}(\alpha)$:

Dividir $h / \alpha^3 - 6\alpha$ da el resultado en el resto.

1. $\underline{\alpha^3 - 6\alpha}$ $\begin{cases} \alpha^3 - 2\alpha^2 - 2\alpha + 2 = 0 \\ \alpha^3 - 2\alpha = 2\alpha^2 - 2 \\ \alpha^3 - 6\alpha = 2\alpha^2 - 4\alpha - 2 \end{cases}$

2. $\underline{\alpha^4 - 6\alpha^2 - 1}$ $\begin{cases} \alpha^3 = 2\alpha^2 + 2\alpha - 2 \\ \alpha^4 = 2\alpha^3 + 2\alpha^2 - 2\alpha = 4\alpha^2 - 4\alpha - 4 + 2\alpha^2 - 2\alpha = 6\alpha^2 + 2\alpha - 4 \\ \Rightarrow \alpha^4 - 6\alpha^2 - 1 = 2\alpha - 5 / \end{cases}$

3. $(\alpha^2 - 2\alpha - 2)^{-1}$ $\begin{cases} \alpha^3 - 2\alpha^2 - 2\alpha + 2 = 0 \Rightarrow \alpha(\alpha^2 - 2\alpha - 2) = -2 \Rightarrow \\ -1/2\, \alpha(\alpha^2 - 2\alpha - 2) = 1 \Rightarrow (\alpha^2 - 2\alpha - 2)^{-1} = -\frac{1}{2}\alpha \end{cases}$

4.2 ELEMENTOS ALGEBRAICOS Y TRASCENDENTES

1. Base y grado de extensión:

a) $\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{2})$

$x^2-2 \in \mathbb{Q}[x]$ es irreducible en $\mathbb{Q}[x] \Rightarrow$ el polinomio mínimo de $\alpha = \sqrt{2}$ en $\mathbb{Q}[x]$ $\begin{cases} [\mathbb{Q}(\sqrt{2}):\mathbb{Q}] = 2 \\ B = \{1, \sqrt{2}\} \end{cases}$

Una base siempre es $B = \{1, \alpha, \alpha^2, \dots\}$ y nos paramos cuando podamos construir ese elemento como c.l. de la base

b) $\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{2}, \sqrt{3})$

$\mathbb{Q}(\sqrt{2}, \sqrt{3}) = \mathbb{Q}(\sqrt{2}+\sqrt{3})$

$B = \{1, \alpha, \dots \alpha^n, \dots\}$

$[\mathbb{Q}(\sqrt{2}, \sqrt{3}) : \mathbb{Q}] = 4$

$$\left.\begin{array}{c} (\mathbb{Q}(\sqrt{2}))(\sqrt{3}) \\ 2 \uparrow \\ \mathbb{Q}(\sqrt{2}) \\ 2 \uparrow \\ \mathbb{Q} \end{array}\right\} \rightarrow 4$$

$B_1 = \{1, \sqrt{2}, +\sqrt{3}, (\sqrt{2}+\sqrt{3})^2, (\sqrt{2}+\sqrt{3})^3\}$

$B_2 = \{1, \sqrt{2}, \sqrt{3}, \sqrt{6}\}$

c) $\mathbb{Q} \subseteq \mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5})$

$$\left.\begin{array}{c} ((\mathbb{Q}(\sqrt{2})(\sqrt{3}))(\sqrt{5}) \\ 2 \uparrow \\ (\mathbb{Q}(\sqrt{2}))(\sqrt{3}) \\ 2 \uparrow \\ \mathbb{Q}(\sqrt{2}) \\ 2 \uparrow \\ \mathbb{Q} \end{array}\right\} \rightarrow 2^3 \text{ grado de extensión}$$

$[\mathbb{Q}(\sqrt{2}, \sqrt{3}, \sqrt{5}) : \mathbb{Q}] = 8$

$B = \{1, \sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{10}, \sqrt{15}, \sqrt{30}\}$

d) $\mathbb{Q} \subseteq \mathbb{Q}(\sqrt[3]{2}, \sqrt{2})$

$\mathbb{Q} \subseteq \mathbb{Q}(\sqrt[3]{2}, \sqrt{2})$ $(a+b\sqrt{2})(a'+b'\sqrt{2})(\sqrt[3]{2}) + (a''+b''\sqrt{2})\sqrt[3]{2}$

$$\left.\begin{array}{c} (\mathbb{Q}(\sqrt{2}))\sqrt[3]{2} \\ \uparrow \\ \mathbb{Q}(\sqrt{2}) \longrightarrow a+b\sqrt{2} \\ \uparrow \\ \mathbb{Q} \end{array}\right\}$$

$[\mathbb{Q}(\sqrt{2}, \sqrt[3]{2}) : \mathbb{Q}] = 6$

$B = \{1, 2^{1/6}, 2^{2/6}, 2^{3/6}, 2^{4/6}, 2^{5/6}\}$

2. Polinomio mínimo y grado extensión.

a) P.m. $\alpha = \sqrt[3]{7}$ sobre $\mathbb{Q}$. G. ex: $[\mathbb{Q}(\sqrt[3]{7}):\mathbb{Q}]$

$\alpha = \sqrt[3]{7} \Rightarrow \alpha^3 - 7 = 0 \Rightarrow \alpha =$ raíz $x^3 - 7 \in \mathbb{Q}[x]$ (Pol. Mín)

es irreducible, asegura que es el mínimo.

$[\mathbb{Q}(\sqrt[3]{7}):\mathbb{Q}] = 3$ // (G E)

b) P.m $\alpha = \sqrt[4]{2}$ sobre $\mathbb{Q}(\sqrt{2})$. G ext: $[\mathbb{Q}(\sqrt[4]{2}):\mathbb{Q}(\sqrt{2})]$

$\alpha^2 = \sqrt{2} \Rightarrow \alpha^2 - \sqrt{2} = 0 \Rightarrow \alpha$ es raíz de $x^2 - \sqrt{2} \in \mathbb{Q}(\sqrt{2})$ (PM)

Irreducible en $\mathbb{Q}(\sqrt{2}) \Rightarrow$ No tiene raíz y tiene grado 2.

$[\mathbb{Q}(\sqrt[4]{2}):\mathbb{Q}(\sqrt{2})] = 2$ (G E)

c) P.m $\alpha = \sqrt{3}$ sobre $\mathbb{R}$. G.ext: $[\mathbb{R}(\sqrt{3}):\mathbb{R}]$

$\alpha = \sqrt{3} \Rightarrow \alpha - \sqrt{3} = 0 \Rightarrow \alpha$ es raíz de $x - \sqrt{3} \in \mathbb{R}[x]$ (PM)

Grado de extensión es 1

d) PM $\alpha = \sqrt[6]{5}$ sobre $\mathbb{Q}$. G ext: $[\mathbb{Q}(\sqrt{5}, \sqrt[6]{5}):\mathbb{Q}]$

$\alpha = \sqrt[6]{5} \Rightarrow \alpha^6 = 5 \Rightarrow \alpha^6 - 5 = 0 \Rightarrow \alpha$ raíz de $x^6 - 5 \in \mathbb{Q}[x]$ (PM)

Irreducible para $p=5$

$[\mathbb{Q}(\sqrt{5}, \sqrt[6]{5}):\mathbb{Q}] = 6$
$[\mathbb{Q}(\sqrt[6]{5}):\mathbb{Q}] = 6$ } Grado extensión

e) PM $\alpha = \sqrt{1+\sqrt{3}}$ sobre $\mathbb{Q}$. G ext: $[\mathbb{Q}(\sqrt{1+\sqrt{3}}):\mathbb{Q}]$

$\alpha = \sqrt{1+\sqrt{3}} \Rightarrow \alpha^2 = 1 + \sqrt{3} \Rightarrow \alpha^2 - 1 = \sqrt{3} \Rightarrow (\alpha^2-1)^2 = 3 \Rightarrow (\alpha^2-1)^2 - 3 = 0 \Rightarrow$

$\Rightarrow \alpha^4 - 2\alpha^2 - 2 = 0 \Rightarrow \alpha$ raíz $x^4 - 2x^2 - 2 \in \mathbb{Q}[x]$ Irreducible $p=2$ (PM)

$[\mathbb{Q}(\sqrt{1+\sqrt{3}}):\mathbb{Q}] = 4$ G.ext.

3. Pol. mínimo de α:

a) Pol mín $\alpha = \pi + ei$ sobre $\mathbb{R}$

$\alpha = \pi + ei \Rightarrow \alpha - \pi = ei \Rightarrow (\alpha - \pi)^2 = -e^2 \Rightarrow \alpha^2 - 2\pi\alpha + \pi^2 + e^2 = 0$

$\hookrightarrow \alpha$ es raíz de $x^2 - 2\pi x + \pi^2 + e^2 = 0 \in \mathbb{R}[x]$

Irreducible pues es de grado 2 y $b^2 - 4ac < 0$

b) Pol mín $\alpha = \frac{1+\sqrt{-7}}{2}$ sobre $\mathbb{Q}$

$\alpha = \frac{1+\sqrt{-7}}{2} \Rightarrow 2\alpha = 1 + \sqrt{-7} \Rightarrow 2\alpha - 1 = \sqrt{-7} \Rightarrow (2\alpha-1)^2 = -7 \Rightarrow$

$\Rightarrow 4\alpha^2 - 4\alpha + 1 = -7 \Rightarrow 4\alpha^2 - 4\alpha + 8 = 0 \Rightarrow \alpha^2 - \alpha + 2 = 0$

α es raíz de $x^2 - x + 2 = 0 \in \mathbb{Q}[x]$

Es irreducible de grado 2, posibles raíces en $\mathbb{Q}$ $\{-1, 1, -2, 2\}$

c.) PM de $\alpha = \sqrt{2+\sqrt{2}}$ sobre $\mathbb{Q}$

$\alpha = \sqrt{2+\sqrt{2}} \Rightarrow \alpha^2 = 2+\sqrt{2} \Rightarrow \alpha^2 - 2 = \sqrt{2} \Rightarrow (\alpha^2-2)^2 = 2 \Rightarrow x^4 - 4\alpha^2 + 4 = 2 \Rightarrow$

$\Rightarrow \alpha^4 - 4\alpha^2 + 4 - 2 = 0 \Rightarrow \alpha^4 - 4\alpha^2 + 2 = 0$

α es raíz de $x^4 - 4x^2 + 2 = 0 \in \mathbb{Q}[x]$

Irreducible x Eisenstein

d.) PM de $\alpha = \sqrt{2} + \sqrt{6}$ sobre $\mathbb{Q}$

$\alpha = \sqrt{2} + \sqrt{6} \Rightarrow \alpha - \sqrt{2} = \sqrt{6} \Rightarrow (\alpha - \sqrt{2})^2 = 6 \Rightarrow \alpha^2 - 2\sqrt{2}\alpha + 2 = 6 \Rightarrow$

$\Rightarrow \alpha^2 - 2\sqrt{2}\alpha - 4 = 0$

¿? $\to (\alpha^2 - 4)^2 = 8\alpha^2 \Rightarrow \alpha^4 - 8\alpha^2 + 16 - 8\alpha^2 = 0 \Rightarrow$

$\Rightarrow \alpha^4 - 16\alpha^2 + 16 = 0$ Aplicación para ver irreducible.

Base $\mathbb{Q}(\sqrt{2}+\sqrt{6}) = \{1, \sqrt{2}, \sqrt{3}, \sqrt{6}\}$

$\boxed{g(x) = \alpha x = (\sqrt{2}+\sqrt{6})x}$ $\quad g(\sqrt{2}) = 2 + 2\sqrt{3} = \begin{pmatrix} 2 \\ 0 \\ 2 \\ 0 \end{pmatrix}$

$g(1) = \sqrt{2} + \sqrt{6} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$ $\quad g(\sqrt{3}) = \sqrt{6} + 3\sqrt{2} = \begin{pmatrix} 0 \\ 3 \\ 0 \\ 1 \end{pmatrix}$

$M = \begin{pmatrix} 0 & 2 & 0 & 6 \\ 1 & 0 & 3 & 0 \\ 0 & 2 & 0 & 2 \\ 1 & 0 & 1 & 0 \end{pmatrix} \Rightarrow \det(M) = \lambda^4 - 16\lambda^2 + 16 \Rightarrow$ irreducible

e.) PM de $\alpha = \sqrt[3]{2} + \sqrt[3]{4}$ sobre $\mathbb{Q}$

$B\mathbb{Q}(\alpha) = \{1, \alpha, \dots \alpha^n\}$

$B'\mathbb{Q}(\alpha) = \{1, \sqrt[3]{2}, \sqrt[3]{4}\}$

$\alpha = \sqrt[3]{2} + \sqrt[3]{4} \Rightarrow \alpha^2 = \sqrt[3]{4} + 2\sqrt[3]{2} + 4 \Rightarrow \alpha^3 = 2 + 2\sqrt[3]{4} + 4\sqrt[3]{2} + 2\sqrt[3]{2} + 4 + 4\sqrt[3]{4} =$

$= 6 + 6\sqrt[3]{2} + 6\sqrt[3]{4} \longrightarrow 1 + \sqrt[3]{2} + \sqrt[3]{4}$

$g: \mathbb{Q}(\alpha) \longrightarrow \mathbb{Q}(\alpha)$
$\quad x \longrightarrow \alpha x$

$g(1) = \sqrt[3]{2} + \sqrt[3]{4} \longrightarrow \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

$g(\sqrt[3]{2}) = \sqrt[3]{4} + 2 \longrightarrow \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

$g(\sqrt[3]{4}) = 2 + 2\sqrt[3]{2} \longrightarrow \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$

$\Rightarrow \begin{pmatrix} 0 & 2 & 2 \\ 1 & 0 & 2 \\ 1 & 1 & 0 \end{pmatrix} = A$ Matriz de la aplicación lineal con base B'

Calculo det $\begin{vmatrix} -\lambda & 2 & 2 \\ 1 & -\lambda & 2 \\ 1 & 1 & -\lambda \end{vmatrix} = -\lambda^3 + 6 + 6\lambda \Rightarrow x^3 - 6x - 6$ pol mín.

f.) $\alpha = \sqrt[3]{9} + \sqrt[3]{3} + 1$ sobre $\mathbb{Q}$

$B = \{1, \sqrt[3]{3}, \sqrt[3]{9}\}$

$\mathbb{Q}(\sqrt[3]{3})$
$\uparrow 3$
$\mathbb{Q}$

$g(1) = 1 + \sqrt[3]{3} + \sqrt[3]{9} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$g(\sqrt[3]{3}) = \sqrt[3]{3} + \sqrt[3]{9} + 3 = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ $\quad A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}$

$g(\sqrt[3]{9}) = \sqrt[3]{9} + 3 + 3\sqrt[3]{3} = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}$

$g(x) = (\sqrt[3]{9} + \sqrt[3]{3} + 1)x$

$g(x) = \alpha x$

$\begin{vmatrix} 1-\lambda & 3 & 3 \\ 1 & 1-\lambda & 3 \\ 1 & 1 & 1-\lambda \end{vmatrix} = \dots (1-\lambda)^2(-2-2\lambda) - \lambda(2+\lambda)(1-\lambda) - 3(-2-2\lambda)$

4. $h = x^3 - x - 2 \in \mathbb{Q}[x]$ y $a \in \mathbb{Q}[x]/(h)$ raíz de h.
Base $\mathbb{Q}(\alpha)$ sobre $\mathbb{Q}$, expresar $\frac{\alpha+1}{\alpha-1} \in \mathbb{Q}[x]/(h)$

α raíz de $x^3 - x - 2$

$x^3 - x - 2 \Rightarrow$ irreducible xq grado 3 y raíz forma $\frac{p}{q}$ tq $\left| \begin{array}{l} p \mid 2 \text{ (term ind)} \\ q \mid 1 \text{ (coef de } x^3) \end{array} \right.$

Posibles raíces $\{\pm 1, \pm 2\}$

h pol mín de $a \Rightarrow [\mathbb{Q}(\alpha) : \mathbb{Q}] = 3$

$B = \{1, \alpha, \alpha^2\}$

$\frac{a+1}{a-1} = (\alpha+1)(a-1)^{-1}$ Inverso de $(\alpha - 1)$
$\hookrightarrow a^2 + a + 1$

$(\alpha - 1)^{-1} = \frac{1}{2}(\alpha^2 + \alpha)$

$x^3 - x - 2$		
$x - 1$		
-2	$x^2 + x$	$(1, -x^2 - x)$

$$\begin{array}{l|l} x^3 - x - 2 & x - 1 \\ -x^3 + x^2 & x^2 + x \\ \hline x^2 - x - 2 & \\ -x^2 \; x & \\ \hline \boxed{-2} & \end{array}$$

$-2 = (x^3 - x - 2) \cdot 1 + (x-1)(-x^2 - x)$

$1 = (x^3 - x - 2)(\frac{-1}{2}) + (x-1)(-x^2 - x)(-1/2)$ inv. de $x-1$

5. $h = x^4 - 6x + 3 \in \mathbb{Q}[x]$ y $a \in \mathbb{Q}[x]/(h)$ raíz de h.
$(a^2-1)(a^2+2) \in \mathbb{Q}(\alpha)/(h)$ c.e base.

h es pol mínimo x Eisenstein, luego la base es $B = \{1, a, a^2, a^3\}$

Opera en base $(\alpha^2 - 1)(a^2 + 2) \in \mathbb{Q}[x]/(h)$

$\alpha^4 + 2a^2 - a^2 + 2 \doteq \alpha^4 = 6a - 3$ (de ip en h) $\Rightarrow$

$(\alpha^2 - 1)(\alpha^2 + 2) = 6a - 3 + 2\alpha^2 - \alpha^2 - 2 = 6a - 3 + a^2 - 2 = 6a + \alpha^2 - 5 =$

$= \boxed{a^2 + 6a - 5}$ resto de div. $\frac{x^4 - 6x + 3}{x^4 - x^2 - 2}$

6. $\exists$ relación igualdad o cont entre $\mathbb{Q}(\sqrt{3})$ y $\mathbb{Q}(\sqrt{3 + \sqrt{3}})$

$[\mathbb{Q}(\sqrt{3}) : \mathbb{Q}] = 2$

$\alpha = \sqrt{3 + \sqrt{3}} \Rightarrow (\alpha^2 - 3)^2 = 3 \Rightarrow \alpha^4 - 6a^2 + 6 = 0 \Rightarrow x^4 - 6x^2 + 6$ pol. x Eisenstein

$[\mathbb{Q}(\sqrt{3 + \sqrt{3}} : \mathbb{Q}) = 4$

No relación pero $\mathbb{Q}(\sqrt{3}) \subsetneq \mathbb{Q}(\sqrt{3 + \sqrt{3}})$

xq $\alpha^2 = 3 + \sqrt{3} \in \mathbb{Q}(\sqrt{3 + \sqrt{3}}) \Rightarrow \sqrt{3} \in \mathbb{Q}(\sqrt{3 + \sqrt{3}})$

4.3. CUERPOS FINITOS

1. Dem anillo $(F,+,\cdot)$ es cuerpo, orden grupo aditivo y dat res.

a.) $F_4 = \mathbb{Z}_2[x]/(x^2+x+1)$

Si $h = x^2+x+1$ es pol min $\Rightarrow F_4$ es cuerpo.

Es irreducible en $\mathbb{Z}_2 \Rightarrow$ es pol mínimo $\Rightarrow F_4$ es cuerpo

Orden del grupo

$B = \{1, \alpha\}$ con coeficientes en $\mathbb{Z}_2$: $a1 + b\alpha$, $a,b \in \mathbb{Z}_2$

$|F_4| = 2.2 = 4 = 2^{2}$ base $\mathbb{Z}_2$ $\qquad \underbrace{0,1}_{2}\ \underbrace{0,1}_{2}$ elementos

1. Lo para diferente $\Rightarrow \underline{x^3(x+1)+x+1} \Rightarrow x^4+x^3+x+1$

$x^2+x+1=0 \Rightarrow x^2 = x+1$

$x^3 = x^2+x = 1 \Rightarrow x^4 = x$

$x^4+x^3+x+1 = x+1+x+1 = 0$

2. $\underline{x^2(x+1)+1} = x^3+x^2+1 = 1+x+1+1 = x+1$

3. $\underline{x^3(x+1)+x^2+1} = x^4+x^3+x^2+1 = \cancel{x}+\cancel{1}-\cancel{x}+1+1 = 1$

b.) $F_9 = \mathbb{Z}_3[x]/(x^2+x+2)$

$h = x^2+x+2$ pol min en $\mathbb{Z}_3$ (prob $\{0,1,2\}$) $\Rightarrow F_9$ es cuerpo

Orden del grupo $|F_9| = 9 = 3^2$, $\overline{a}^{3}1 + \overline{b}^{3}\alpha$

$B = \{1, \alpha\}$ xq $\alpha^2 = -\alpha-2$ (c Q)

1. $\underline{(x+2)(2x+2)+x+1} = 2x^2+2x+4x+4+x+1 = 2x^2+\cancel{2x}+\cancel{x}+1+x+1 =$

$= 2x^2+x+2$

$x^2 = -x-2$

$2(-x-2)+x+2 = -2x-4+x+2 = -x-2 = 2x+1$

2. $\underline{x^{-1}-(2x+2)^{-1}} = 1/x - \dfrac{1}{2x+2} = x+1-2x$

Inverso de x y $2x+2$

x^2+x+2		$(1,0)$
x		$(0,1)$
2	$x+1$	$(1,-x-1)$

$1 = x(x+1)$

$\boxed{x^{-1} = x+1}$

$(2x+2)^{-1} = 2x$

A The best I could hope for was to be at the bottom, but even that honour has to be earned.

B So I knew that everyone at the training session that night would have cheered with good-natured delight if I had done that.

C I had been attracted to castelling because I had been told that it requires almost no skill or co-ordination.

D Within seconds I had assisted in the formation of a three-tier tower without really noticing what was happening.

E It's not easy being the bottom man of a human pyramid.

F Each casteller is wound into a large strip of material worn around the waist to support the back and to help the other castellers grip when they climb.

G The group I had joined in Figueres, near the French border, is very much a second-division outfit.

TEST 2

$\cancel{\alpha^2-1=(\alpha^2-\alpha+1)}$

$\alpha^2-1 \;\; \underline{\alpha^2-\alpha+1}$ $\quad \boxed{\alpha-1} \;\; 1.$

$\alpha^2-\alpha+1 \;\; \underline{\alpha-1}$ $\quad \underline{-\alpha^2/\alpha} \;\; \alpha \quad 1$

$\alpha^2-1=(\alpha^2-\alpha+1)1+(\alpha-1)$

$\alpha^2-\alpha+1=(\alpha-1)\cdot\alpha+1$

$\alpha-1=1\cdot(\alpha-1)+0$

$1=(\alpha^2-\alpha+1)-\alpha(1(\alpha-1))$

$\cancel{0=1\cdot(\alpha-1)}$

$\alpha-1\$=$

c.) $\mathbb{F}_{16} = \mathbb{Z}_2[x]/(s)$ siendo $s = x^4+x^3+1 \in \mathbb{Z}_2[x]$

$(x^2+x+1)(x^2+x+1) = x^4+x^2+1$

L polinomio mínimo en $\mathbb{Z}_2[x] \Rightarrow \mathbb{F}_{16}$ es cuerpo

el orden de $\mathbb{F}_{16} \longrightarrow |\mathbb{F}_{16}| = 2^4 = 16$

$B = \{1, \alpha, \alpha^2, \alpha^3\} \Rightarrow$ No divisible por x^2+x+1 que es el único polinomio irreducible de grado 2 en $\mathbb{Z}_2[x]$

1. $[x^2+x+1]^{-1}_{(s)} \Rightarrow (x^2+x+1)^{-1}$

x^4+x^3+1		$(1,0)$
x^2+x+1		$(0,1)$
x	x^2+1	$(1, 1+x^2)$
1	$x+1$	$(x+1, 1+(1+x^2)(x+1))$

$1+x+1+x^3+x^2 = x^3+x^2+x$

$\Rightarrow 1 = (x^4+x^3+1)(x+1) + (x^3+x^2+x)\cdot(x^2+x+1)$

$(x^2+x+1)^{-1} = \underline{x^3+x^2+x}$

d.) $\mathbb{F}_{25} = \mathbb{Z}_5[x]/(x^2+x+2)$

:

$(2x+3)^{-1}$

x^2+x+2		$(1,0)$
$2x+3$		$(0,1)$
-1	$3x+1$	$(1, 2x-1)$

$\longrightarrow -1 = x^2+x+2+(2x-1)(2x+3)$

$(2x+3)^{-1} = (3x+1)$

2. Dem $K = x^3+x^2+1 \in \mathbb{Z}_2[x]$ es irreducible en $\mathbb{Z}_2[x]$ orden 8.

$1 = x^0$
$x = x$
$x^2 = x^2$
$x^3 = x^2+1$
$x^4 = x^3+x = x^2+1+x$
$x^5 = x+1$
$x^6 = x^2+1$
$x^7 = 1$

$\left.\begin{array}{l} |K_8^*| = 7 \text{ elementos} \\ |K_8| = 8 \text{ elementos} \end{array}\right\}$ Sí es primitivo.

3. $\mathbb{F}_9 = \mathbb{Z}_3[x]/(x^2+2x+2)$

a.) $(\mathbb{F}_9^*, \cdot)$ cíclico?. Gen unid $\mathbb{F}_9$? Pol x^2+2x+2 irred prim?

$1 = x^0$
$x = x$
$x^2 = -2x-2 = x+1$
$x^3 = x(x+1) = x^2+x = x+1+1 = 2x+1$
$x^4 = (x+1)^2 = x^2+2x+1 = x+1+2x+1 = 2$
$x^5 = x^3+2x^2+x = 2x+1+2x+2+x = 2x$
$x^6 = 2x^2 = 2x+2$
$x^7 = 2x^2+2x = 2x+2+2x = x+2$
$x^8 = (x^4)^2 = 2^2 = 4 = 1$

b.) $1, \; x+1, \; 2, \; 2x+2$

$1 \downarrow 1, 2$; $x+1 \downarrow x$; $2 \downarrow x+1, 2x+2$; $2x+2 \downarrow$

¿elem $\mathbb{F}_9$ adm raíz cuadrada en $\mathbb{F}_9$?

2P 2018

1. V o F.

a) 2 cuerpos $(K_1, +_1, \cdot_1)$ y $(K_2, +_2, \cdot_2)$ verifica el anillo producto $K_1 \times K_2$ cuerpo.

$\boxed{F}$ K_1 cuerpo y K_2 cuerpo pero $K_1 \times K_2$ no tiene inverso

$a \neq 0_{K_1} \Rightarrow (a, 0_{K_2}) \neq (0_{K_1}, 0_{K_2})$ $\qquad 1_{K_1 \times K_2} = (1_{K_1}, 1_{K_2})$

b) $\mathbb{Z}[x]$ es ideal de $\mathbb{Q}[x]$

$\boxed{F}$ $x+1 \in \mathbb{Z}[x]$, $\frac{1}{2} \in \mathbb{Q}[x]$ pero $\frac{1}{2}x + \frac{1}{2} \notin \mathbb{Z}[x]$

c) $f: \mathbb{Z}_{11} \longrightarrow \mathbb{Z}_{11}$ dada $f(x) = x^{11}$, homomorfismo de anillos.

$\boxed{V}$ $f(xy) = (xy)^{11} = x^{11}y^{11} = f(x)f(y)$

$f(x+y) = f(x) + f(y)$

d) Polinomio x^2+1 irreducible en $\mathbb{Z}_{17}$

$\boxed{F}$ $x^2+1 = x^2 - 17x + 52 = (x-4)(x-13)$ en $\mathbb{Z}_{17}[x]$

e) $\mathbb{Q}[x]/(x^2-1)$ cuerpo.

$\boxed{F}$

f) $\sqrt[8]{2} \in \mathbb{Q}[\sqrt[12]{2}]$ $\qquad \sqrt[6]{2} \quad \sqrt[4]{2} \quad \sqrt[3]{2} \quad \sqrt[12]{2^5} \quad \sqrt{2} \quad \sqrt[12]{2^7} \quad \sqrt[3]{2^2} \quad \sqrt[4]{2^3} \quad \sqrt[6]{2^5} \quad \sqrt[12]{2^{11}}$

$\boxed{F}$ $B_{\mathbb{Q}}(\sqrt[12]{2}) = \{1, \sqrt[12]{2}, \sqrt[12]{2^2}, \sqrt[12]{2^3}, \sqrt[12]{2^4}, \sqrt[12]{2^5}, \sqrt[12]{2^6}, \sqrt[12]{2^7}, \sqrt[12]{2^8}, \sqrt[12]{2^9}, \sqrt[12]{2^{10}}, \sqrt[12]{2^{11}}\}$

$\sqrt[8]{2}$ no sale en la base

2. $R = \{aI + bM : a, b \in \mathbb{Z}\}$ donde $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ y $M = \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}$

a) Dem $(R, +, \cdot)$ anillo con identidad $(+, \cdot)$

R subanillo de $\mathbb{Z}_{2\times 2}$

b) R es conmutativo y división ? Anillo

R es conmutativo | R no división

$xy = acI + (ad + bc + 2bd)M = yx$ | $(2I - M)M = 0$

c) Unidades de $R \Rightarrow 2I$, $I - M$?

$2I$ no unidad de R

$I - M$ unidad en R: $(I-M)^2 = I$

d) Dem. $J = \{3aI + 3bM : a, b \in \mathbb{Z}\}$ ideal de R

e) Div. de cero en R/J: $[2I - M]_J$, $[M^2]_J$

$M^2(2I - M) = 2M(2I - M) = 0$

f) J ideal maximal de R?

J no es ideal maximal xq R/J tiene divisores de cero.

$\boxed{\text{P2 2018}}$

1. Anillo? → Sí: conmutativo, identidad, división, cuerpo?

$S=\{a+b\sqrt{2}+c\sqrt{3} : a,b,c\in\mathbb{Z}\}$

No es anillo xq $\sqrt{2}$ y $\sqrt{3}\in S$ pero $\underbrace{\sqrt{6}}_{}$ $\notin \mathbb{Z}$, con $\sqrt{2}\cdot\sqrt{3}\to\sqrt{6}$

2. Describir unidades y los divisores de cero del anillo $(\mathbb{Z}\times\mathbb{Q}\times\mathbb{Z},+,\cdot)$

Unidades: $U=\{(a,b,c)\in\mathbb{Z}\times\mathbb{Q}\times\mathbb{Z} : (a,c)\in\{1,-1\},\ b\in\mathbb{Q}^*\}$

Divisores del cero: $C=\{(a,b,c)\in\mathbb{Z}\times\mathbb{Q}\times\mathbb{Z} : (\boxed{a}\boxed{b}\boxed{c})\neq(000)$ y $\boxed{abc}=0\}$

3. Cuerpo? $(\{0,2,4,6,8\},+_{10},\cdot_{10})$

$(\{0,2,4,6,8\},+_{10})$ es abeliano xq subgrupo de $(\mathbb{Z}_{10},+_{10})$

$\cdot_{10}$ ⇒ conmutativa, asociativa.

Tabla

$\cdot_{10}$	2	4	6	8
2	4	8	2	6
4	8	6	4	2
6	2	4	6	8
8	6	2	8	4

anillo con identidad y de división

No es cuerpo.

4. Características del anillo $(\mathbb{Z}_6\times\mathbb{Z}_{15},+,\cdot)$

$\text{char}(\mathbb{Z}_6\times\mathbb{Z}_{15})=\text{mcm}(6,15)=30$

5. Sea $(R,+,\cdot)$ un anillo y sea $a\in R$. Dem $N(a)=\{x\in R: xa=0_R\}$ subanillo y estudia si es un ideal.

$N(a)=\{x\in R : xa=0_R\}\neq\emptyset$ xq $0_R\in N(a)$

Si $x,y\in N(a)\Rightarrow xa=0_R,\ ya=0_R$ y x tanto $(x-y)a=0_R\Rightarrow x-y\in N(a)$ y

$x(ya)=$