$$\vec{E} = \frac{q\vec{r}_+}{r_+^3} - \frac{q\vec{r}_-}{r_-^3}$$

$e \ll r$

$(1+x)^n \approx 1 + nx$

$\vec{p} = q\vec{e}$

$$\vec{r}_- = \vec{r} + \frac{\vec{e}}{2} \Rightarrow r_-^2 = r^2 + \vec{r}\vec{e} + \cancel{\frac{e^2}{2}} \Rightarrow r_-^2 = r^2\left(1 + \frac{\vec{r}\vec{e}}{r^2}\right)^{3/2} \Rightarrow$$

$$\Rightarrow r_-^3 = r^3\left(1 + \frac{\vec{r}\vec{e}}{r^2}\right)^{3/2} \Rightarrow$$

$$\Rightarrow \boxed{\frac{1}{r_-^3} = \frac{1}{r^3}\left(1 - \frac{3}{2}\frac{\vec{r}\vec{e}}{r^2}\right)}$$

$$\vec{r}_+ = \vec{r} - \frac{\vec{e}}{2} \Rightarrow$$

$$\boxed{\frac{1}{r_+^3} = \frac{1}{r^3}\left(1 + \frac{3}{2}\frac{\vec{r}\vec{e}}{r^2}\right)}$$

$$\vec{E} = \frac{q(\vec{r} - \frac{\vec{e}}{2})}{r^3}\left(1 + \frac{3}{2}\frac{\vec{r}\vec{e}}{r^2}\right) - \frac{q(\vec{r} + \frac{\vec{e}}{2})}{r^3}\left(1 - \frac{3}{2}\frac{\vec{r}\vec{e}}{r^2}\right) =$$

$$= \frac{q(\vec{r} - \vec{e}/2)}{r^3} + \frac{3}{2}\frac{(\vec{p}\cdot\vec{r})}{r^5}(\vec{r} - \vec{e}/2) - \frac{q(\vec{r} + \vec{e}/2)}{r^3} + \frac{3}{2}\frac{(\vec{p}\cdot\vec{r})}{r^5}(\vec{r} + \vec{e}/2)$$

$$= \frac{3(\vec{p}\cdot\vec{r})\vec{r}}{r^5} - \frac{\vec{p}}{r^3}$$