Диференціальні операції

$\operatorname{div} \vec{E}$

$$\operatorname{div} \vec{E} = \vec{\nabla} \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\vec{E} = \vec{E}_m e^{i(\omega t - k_x x - k_y y - k_z z)},$$

$$E_x = E_{x_m} e^{i(\omega t - k_x x - k_y y - k_z z)}, \quad E_y = E_{y_m} e^{i(\omega t - k_x x - k_y y - k_z z)}, \quad E_z = E_{z_m} e^{i(\omega t - k_x x - k_y y - k_z z)}$$

$$\frac{\partial E_x}{\partial x} = \frac{\partial}{\partial x} E_{x_m} e^{i(\omega t - k_x x - k_y y - k_z z)} =$$

$$= E_{x_m} e^{i(\omega t - k_x x - k_y y - k_z z)} \frac{\partial}{\partial x} \left(i(\omega t - k_x x - k_y y - k_z z) \right) = -i k_x E_x$$

$$\frac{\partial E_y}{\partial y} = -i k_y E_y, \quad \frac{\partial E_z}{\partial z} = -i k_z E_z.$$

$$\operatorname{div} \vec{E} = \vec{\nabla} \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = -i k_x E_x - i k_y E_y - i k_z E_z = -i \vec{k} \cdot \vec{E}$$

$$\vec{\nabla} \cdot \vec{E} = -i \vec{k} \cdot \vec{E}.$$

Диференціальні операції

rot $\vec{E}$

$$\operatorname{rot}\vec{E} = \vec{\nabla}\times\vec{E} = \left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)\vec{e}_x + \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\vec{e}_y + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\vec{e}_z$$

$$\vec{E} = \vec{E}_m e^{i(\omega t - k_x x - k_y y - k_z z)},$$

$$E_x = E_{x_m} e^{i(\omega t - k_x x - k_y y - k_z z)}, \quad E_y = E_{y_m} e^{i(\omega t - k_x x - k_y y - k_z z)}, \quad E_z = E_{z_m} e^{i(\omega t - k_x x - k_y y - k_z z)}$$

$$\frac{\partial E_z}{\partial y} = \frac{\partial}{\partial y} E_{x_m} e^{i(\omega t - k_x x - k_y y - k_z z)} =$$

$$= E_{z_m} e^{i(\omega t - k_x x - k_y y - k_z z)} \frac{\partial}{\partial y}\left(i(\omega t - k_x x - k_y y - k_z z)\right) = -ik_y E_z$$

$$\operatorname{rot}\vec{E} = \left(-ik_y E_z + ik_z E_y\right)\vec{e}_x + \left(-ik_z E_x + ik_x E_z\right)\vec{e}_y + \left(-ik_x E_y + ik_y E_x\right)\vec{e}_z = -i\vec{k}\times\vec{E}.$$

$$\operatorname{rot}\vec{E} = -i\vec{k}\times\vec{E}.$$

$$\vec{\nabla} \to -i\vec{k}.$$

Диференціальні операції

$\frac{\partial \vec{E}}{\partial t}$

$$\frac{\partial \vec{E}}{\partial t} = \frac{\partial E_x}{\partial t}\vec{e}_x + \frac{\partial E_y}{\partial t}\vec{e}_y + \frac{\partial E_z}{\partial t}\vec{e}_z.$$

$$\vec{E} = \vec{E}_m e^{i(\omega t - k_x x - k_y y - k_z z)},$$

$$E_x = E_{x_m} e^{i(\omega t - k_x x - k_y y - k_z z)}, \quad E_y = E_{y_m} e^{i(\omega t - k_x x - k_y y - k_z z)}, \quad E_z = E_{z_m} e^{i(\omega t - k_x x - k_y y - k_z z)}$$

$$\frac{\partial E_x}{\partial t} = E_{x_m} e^{i(\omega t - k_x x - k_y y - k_z z)} \frac{\partial}{\partial t}\left(i(\omega t - k_x x - k_y y - k_z z)\right) = i\omega E_x$$

$$\frac{\partial \vec{E}}{\partial t} = \frac{\partial E_x}{\partial t}\vec{e}_x + \frac{\partial E_y}{\partial t}\vec{e}_y + \frac{\partial E_z}{\partial t}\vec{e}_z = i\omega E_x \vec{e}_x + i\omega E_y \vec{e}_y + i\omega E_z \vec{e}_z = i\omega \vec{E}.$$

$$\frac{\partial}{\partial t} \to i\omega.$$