$$\operatorname{rot}\left[\vec{A}\times\vec{B}\right]=\left(\vec{B}\cdot\vec{\nabla}\right)\vec{A}-\left(\vec{A}\cdot\vec{\nabla}\right)\vec{B}+\vec{A}\operatorname{div}\vec{B}-\vec{B}\operatorname{div}\vec{A}$$

$$\vec{B}=\operatorname{rot}\left(\vec{p}_m\times\frac{\vec{r}}{r^3}\right)=-\left(\vec{p}_m\cdot\vec{\nabla}\right)\frac{\vec{r}}{r^3}=\frac{3\left(\vec{p}_m\cdot\vec{r}\right)\vec{r}}{r^5}-\frac{\vec{p}_m}{r^3}.$$

$\vec{A}=\vec{P_m}$ $\vec{B}=\frac{\vec{r}}{r^3}$ $\vec{P_m}=\text{const}$ $\operatorname{div}\vec{E}=\operatorname{div}\frac{q\vec{r}}{r^3}=0\ (\vec{r}\neq 0)$

$$\vec{B}=\operatorname{rot}\left(\vec{p}_m\times\frac{\vec{r}}{r^3}\right)=\cancel{\left(\frac{\vec{r}}{r^3}\cdot\vec{\nabla}\right)\vec{P}_m}^{\,0}-\left(\vec{P}_m\cdot\vec{\nabla}\right)\frac{\vec{r}}{r^3}+\vec{P}_m\cancel{\operatorname{div}\frac{\vec{r}}{r^3}}^{\,0}-\cancel{\frac{\vec{r}}{r^3}\operatorname{div}\vec{P}_m}^{\,0}$$

$$\vec{B}=-\left(\vec{P}_m\cdot\vec{\nabla}\right)\frac{\vec{r}}{r^3}=-\frac{\left(\vec{P}_m\cdot\vec{\nabla}\right)\vec{r}}{r^3}-\vec{r}\left(\vec{P}_m\cdot\vec{\nabla}\right)\frac{1}{r^3}$$

$$\left(P_x\frac{\partial}{\partial x}+P_y\frac{\partial}{\partial y}+P_z\frac{\partial}{\partial z}\right)\left(x\vec{i}+y\vec{j}+z\vec{k}\right)=P_x\vec{i}+P_y\vec{j}+P_z\vec{k}=\vec{P}_m$$

$$\left(P_x\frac{\partial}{\partial x}+P_y\frac{\partial}{\partial y}+P_z\frac{\partial}{\partial z}\right)\frac{1}{(x^2+y^2+z^2)^{3/2}}=-\frac{3}{r^5}\left(P_x x+P_y y+P_z z\right)=-\frac{3(\vec{P}_m\cdot\vec{r})}{r^5}$$

$$\vec{B}=\frac{3(\vec{p}_m\cdot\vec{r})\vec{r}}{r^5}-\frac{\vec{P}_m}{r^3}$$