Dynamics

Goals for Lecture

- To introduce the concept of force.
- To understand the types of interactions in modern physics.
- To describe the types of forces in mechanics.
- Newtonian laws.
- To analize the equation of particle motion and many-particle system.
- To understand the Momentum Conservation Law.

Newtonian Mechanics

The relation between a force and the acceleration it causes was first understood by Isaac Newton (1642 –1727). The study of that relation, as Newton presented it, is called Newtonian mechanics.

Newtonian Mechanics

Newtonian mechanics apply if:

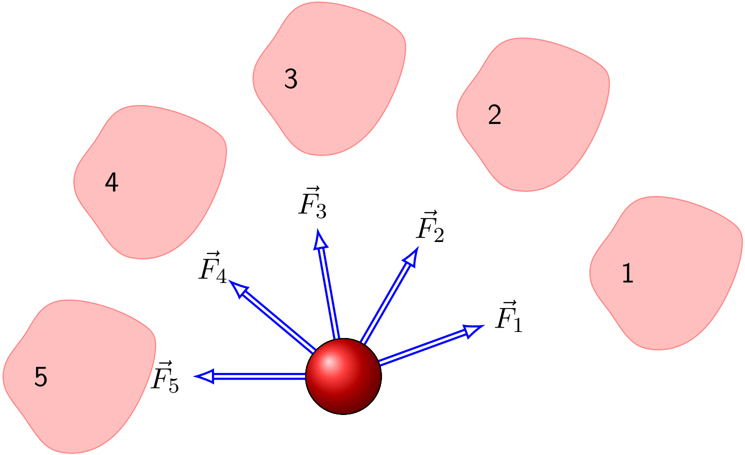
1. the velocities of the interacting bodies are very small compared to speed of light $v \ll c = 299792458$ m/s;
2. the interacting bodies are very large then the scale of atomic structure; in an atom)

Newtonian mechanics still, it is a very important special case because it applies to the motion of objects ranging in size from the very small (almost on the scale of atomic structure) to astronomical (galaxies and clusters of galaxies).

What is FORCE?

Physical bodyes interact with each other!

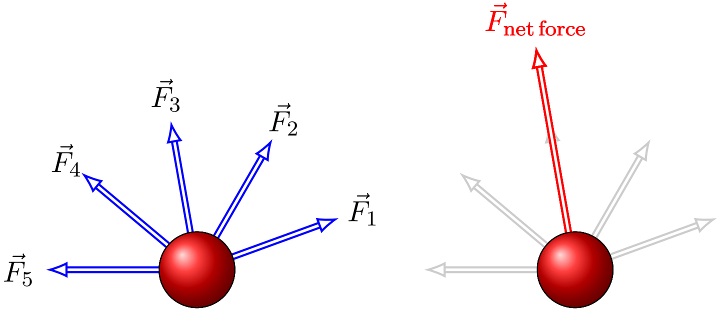
A force is an characteristics of interaction between two physical bodyes involving a push or a pull.



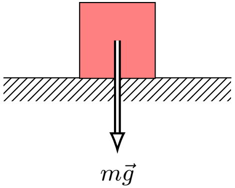
What is FORCE?

The sum of forces acting on a body is called the $\boxed{\text{total force}}$ or the $\boxed{\text{net force}}$. The net force is a single force that replaces the effect of the original forces on the body:

$$\vec{F}_{\text{net force}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5$$

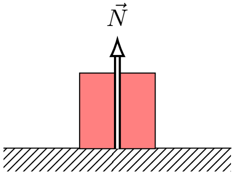


Forces in mechanics



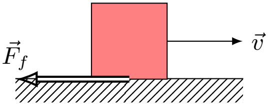
Force of gravity ($m\vec{g}$)

The force of gravity acting on an object due to its mass. An object's weight is directed down, toward the center of the gravitating body; like the Earth or moon, for example.



Normal ($\vec{N}$)

The force between two solids in contact that prevents them from occupying the same space. The normal force is directed perpendicular to the surface. A «normal» in mathematics is a line perpendicular to a planar curve or surface; thus the name «normal force»



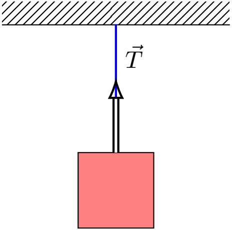
Friction ($\vec{F}_f$)

The force between solids in contact that resists their sliding across one another. Friction is directed opposite the direction of relative motion or the intended direction of motion of either of the surfaces.

Forces in mechanics

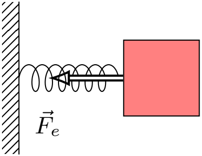
Tension ($\vec{T}$)

The force exerted by an object being pulled upon from opposite ends like a string, rope, cable, chain, etc. Tension is directed along the axis of the object. (Although normally associated with solids, liquids and gases can also be said exert tension in some circumstances.)



Elasticity ($\vec{F}_e$)

The force exerted by an object under deformation (typically tension or compression) that will return to its original shape when released like a spring or rubber band. Elasticity, like tension, is directed along an axis (although there are exceptions to this rule).



Fundamental forces

All the forces in the universe can be explained in terms of the following four fundamental interactions.

1. Gravity

   The interaction between objects due to their mass. Weight is a synonym for the force of gravity.

2. Electromagnetism

   The interaction between objects due to their charge. All the forces discussed above are electromagnetic in origin except weight.

3. Strong Nuclear Interaction

   The interaction between subatomic particles with "color" (an abstract quantity that has nothing to do with human vision). This is the force that holds protons and neutrons together in the nucleus and holds quarks together in the protons and neutrons. It cannot be felt outside of the nucleus.

4. Weak Nuclear Interaction

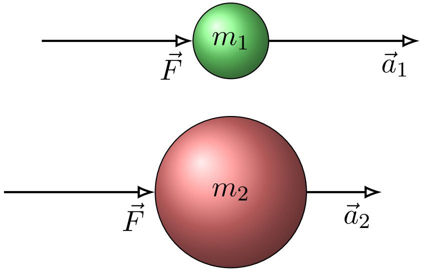
   The interaction between subatomic particles with "flavor" (an abstract quantity that has nothing to do with human taste). This force, which is many times weaker than the strong nuclear interaction, is involved in certain forms of radioactive decay.

What is MASS?

Experience shows that every body «resists» any effort to change its velocity, both in magnitude and direction. This property expressing the degree of unsusceptibility of a body to any change in its velocity is called inertness. Different bodies reveal this property in different degrees.
A measure of inertness is provided by the quantity called mass. A body possessing a greater mass is more inert, and vice versa.
The basic SI unit of mass is the kilogram (kg).

The notion of mass m by defining the ratio of masses of two different bodies via the inverse ratio of accelerations imparted to them by equal forces:

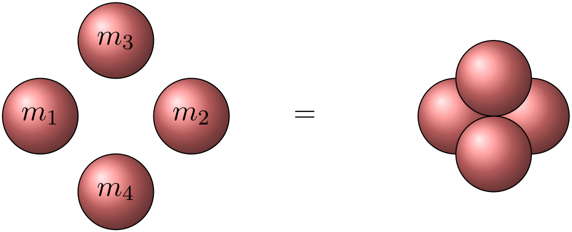
$$\frac{m_1}{m_2} = \frac{a_2}{a_1}.$$



Properties of MASS

1. mass is an $\boxed{\text{additive quantity}}$, i.e. the mass of a composite body is equal to the sum of the masses of its constituents;

$$m_1 + m_2 + m_3 + m_4 = m$$



2. the mass of a body proper is a constant quantity, remaining invariable in the process of motion.

Linear Momentum

In Newtonian mechanics, linear momentum, translational momentum, or simply momentum is the product of the mass and velocity of an object. It is a three-dimensional vector quantity, possessing a magnitude and a direction. If m is an object's mass and v is the velocity (also a vector), then the momentum is

$$\vec{p} = m\vec{v}$$

In SI units, it is measured in kilogram meters per second (kg·m/s).

Inertial frame of reference

Newton First Law

There is a reference frame in which acceleration of a mass point arises solely due to its interaction with other bodies. Then a **free mass point experiencing no action from any other bodies moves rectilinearly and uniformly, relative to such a frame**, or, in other words, due to inertia. Such a reference frame is called inertial.

The statement of the existence of inertial reference frames formulates the content of the **first law of classical mechanics**, the law of inertia of Galileo and Newton.

Newton Second Law

Product of the mass of a particle by its acceleration is equal to the net force acting on it:

$$\boxed{m\vec{a} = \vec{F}}$$

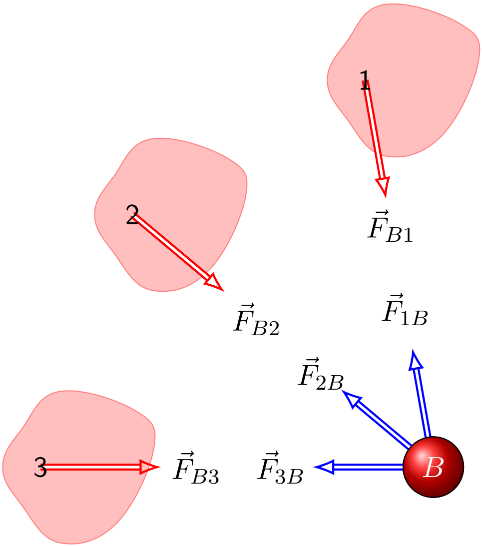
This equation is referred to as the motion equation of a particle.

Newton Third Law

Two mass points act on each other with forces which are always equal in magnitude and oppositely directed along a straight line connecting these points:

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

This implies that interaction forces always appear in pairs. The two forces are applied to different mass points; besides, they are the forces of the same nature.



Examples

Body diagram of an an object in inclined plane

Newton Second Low for the body m_1:

$$m_1\vec{a} = m_1\vec{g} + \vec{N} + \vec{F}_f$$

Projections

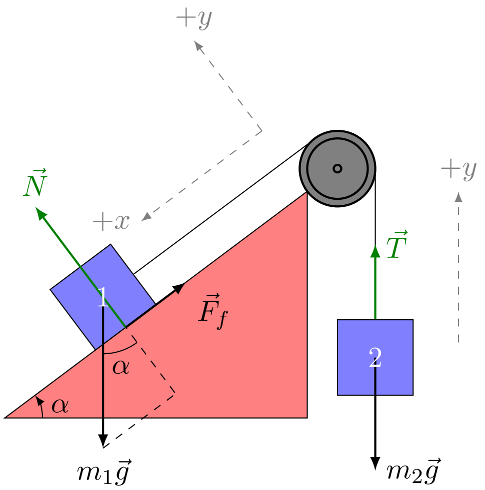
$$OX: m_1 a = m_1 g \sin\alpha - F_f$$
$$OY: 0 = N - m_1 g \cos\alpha$$

Newton Second Low for the body m_2:

$$m_2\vec{a} = m_2\vec{g} + \vec{T}$$

Projections

$$OY: m_2 a = T - m_2 g$$

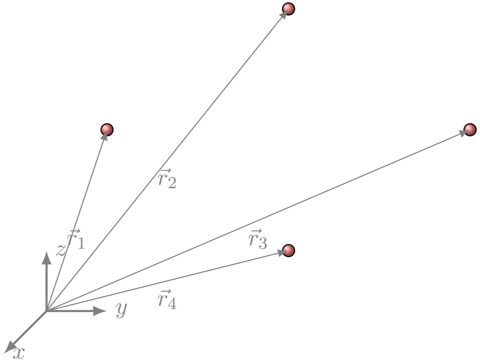


The system of particles

Consider now a system of N particles of masses $m_1, m_2, \ldots, m_N$. The total mass is

$$M = m_1 + m_2 + \ldots + m_N$$

Each particle can be represented by its location $\vec{r}_i$,



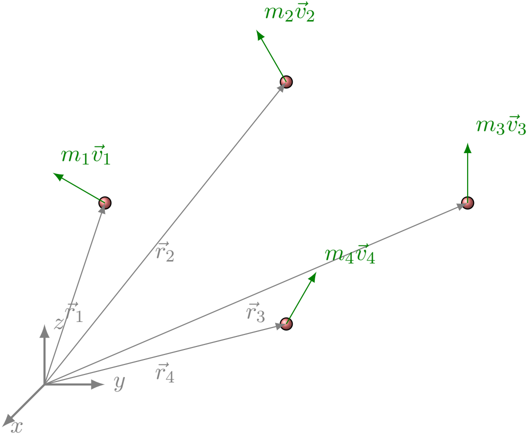
The system of particles

Consider now a system of N particles of masses $m_1, m_2, \ldots, m_N$. The total mass is

$$M = m_1 + m_2 + \ldots + m_N$$

Each particle can be represented by its location $\vec{r}_i$, velocity $\vec{v}_i$ and linear momentum $m_i\vec{v}_i$. The system as a whole has a total linear momentum $\vec{P}$, which is defined to be the vector sum of the individual particle linear momenta:

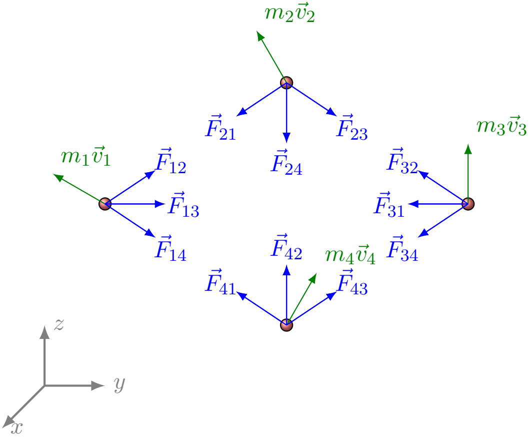
$$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + \ldots + m_N\vec{v}_N$$



The system of particles

Let's extend our system to two interacting objects, for example the cart and the spring. The forces between the spring and cart are now internal forces. Both objects, the cart and the spring, experience these internal forces, which by Newton's Third Law are equal in magnitude and applied in opposite directions. So when we sum up the internal forces for the whole system, they cancel. Thus the sum of all the internal forces is always zero,

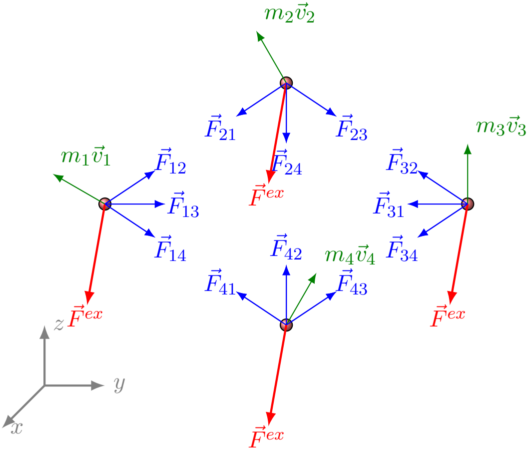
$$\sum \vec{F}_{ij} = 0$$



The system of particles

External forces are acting on our system $\vec{F}^{ext}$; the gravitational force, the contact force between the inclined plane and the cart, and also a new external force, the force between the spring and the force sensor. The force acting on the system is the sum of the internal and the external forces. However, as we have shown, the internal forces cancel, so we have the total force acting on system equal to:

$$\vec{F} = \sum \vec{F}^{ext}$$



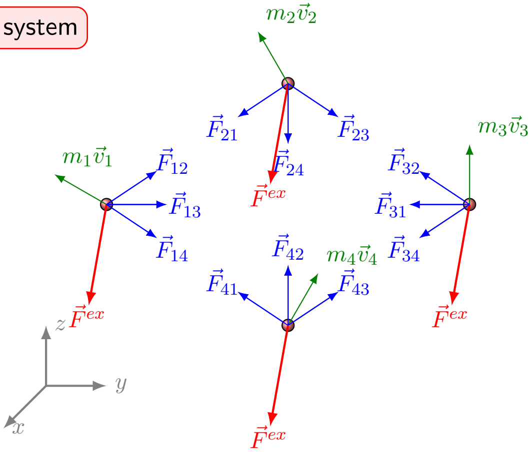
The system of particles

The Law of Particle System Motion

The time derivative of the momentum of a system is equal to the vector sum of all external forces acting on the particles of the system:

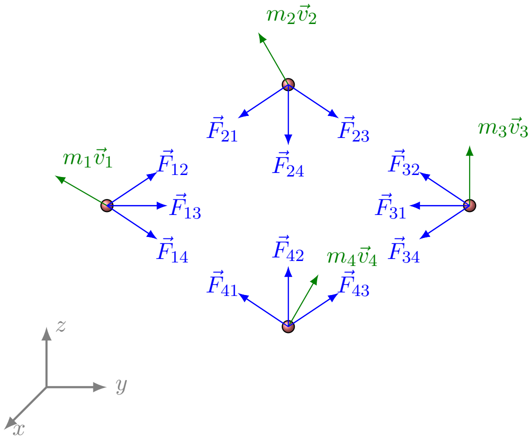
$$\boxed{\frac{d\vec{P}}{dt} = \vec{F}^{ext}.}$$

In accordance with this equation the momentum of a system may vary only due to external forces. Internal forces cannot change the momentum, of a system.



Closed system is a physical system that doesn't exchange any matter with its surroundings, and isn't subject to any net force whose source is external to the system:

$$\boxed{\vec{F}^{ext} = 0.}$$



The system of particles

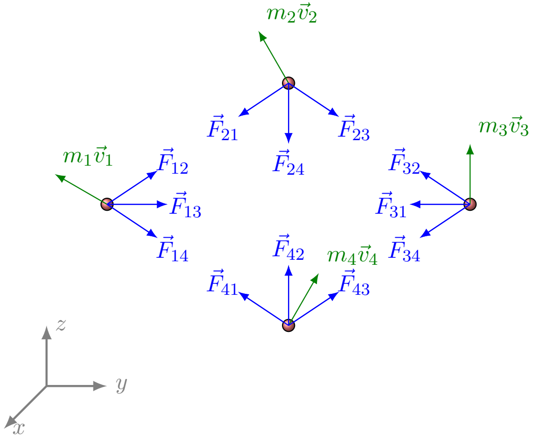
The Law of Linear Momentum Conservation

From the Law of Particle System Motion

$$\boxed{\frac{d\vec{P}}{dt} = 0.}$$

follows the law of momentum conservation: in an inertial reference frame the momentum of a closed system of particles remains constant, i.e. does not change in the course of time:

$$\boxed{\vec{P} = \text{const.}}$$



The low of Linear Momentum Conservation

Sometimes in a non-closed system it is not the momentum $\vec{P}$ itself that remains constant, but its P_x projection on a certain x direction. This happens when the projection of the resultant $\vec{F}^{ext}$ of the external forces on the x direction is equal to zero, i.e. the vector $\vec{F}^{ext}$ is perpendicular to that direction, for example, if $F_x^{ext} = 0$ whence it follows that if $\boxed{P_x = \mathrm{const}}$.

For example, when a system moves in a uniform field of gravity, the projection of its momentum on any horizontal direction remains constant whatever happens inside the system.