

# Units, Physical Quantities, and Vectors

# Goals for Lecture

- To learn three fundamental quantities of physics and the units to measure them
- To keep track of significant figures in calculations
- To understand vectors and scalars and how to add vectors graphically
- To determine vector components and how to use them in calculations
- To understand unit vectors and how to use them with components to describe vectors
- To learn two ways of multiplying vectors

# What is Physics

Physics is an experimental science in which physicists seek patterns that relate the phenomena of nature.

- The patterns are called **physical theories**.
- A very well established or widely used theory is called a **physical law** or **principle**.

# Standards and units

- Length, time, and mass are three fundamental quantities of physics.
- The International System (SI for *Système International*) is the most widely used system of units.
- In SI units, length is measured in *meters*, time in *seconds*, and mass in *kilograms*.

# Unit prefixes

The Prefixes Used with SI Units			
Prefix	Symbol	Meaning	Scientific Notation
<i>exa-</i>	E	1,000,000,000,000,000,000	$10^{18}$
<i>peta-</i>	P	1,000,000,000,000,000	$10^{15}$
<i>tera-</i>	T	1,000,000,000,000	$10^{12}$
<i>giga-</i>	G	1,000,000,000	$10^9$
<i>mega-</i>	M	1,000,000	$10^6$
<i>kilo-</i>	k	1,000	$10^3$
<i>hecto-</i>	h	100	$10^2$
<i>deka-</i>	da	10	$10^1$
—	—	1	$10^0$
<i>deci-</i>	d	0.1	$10^{-1}$
<i>centi-</i>	c	0.01	$10^{-2}$
<i>milli-</i>	m	0.001	$10^{-3}$
<i>micro-</i>	$\mu$	0.000 001	$10^{-6}$
<i>nano-</i>	n	0.000 000 001	$10^{-9}$
<i>pico-</i>	p	0.000 000 000 001	$10^{-12}$
<i>femto-</i>	f	0.000 000 000 000 001	$10^{-15}$
<i>atto-</i>	a	0.000 000 000 000 000 001	$10^{-18}$

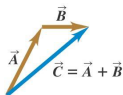
# Vectors and scalars

- A scalar quantity can be described by a *single number*.
- A vector quantity has both a *magnitude* and a *direction in space*.
- A vector quantity is represented in italic type with an arrow over it:  $\vec{A}$ .
- The magnitude of  $\vec{A}$  is written as  $A$  or  $|\vec{A}|$ .

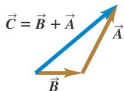
# Adding two vectors

Two vectors may be added graphically using either the **parallelogram method** or the **head-to-tail method**.

(a) We can add two vectors by placing them head to tail.



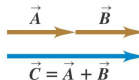
(b) Adding them in reverse order gives the same result.



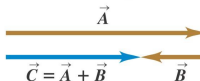
(c) We can also add them by constructing a parallelogram.



(a) The sum of two parallel vectors



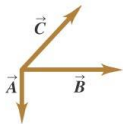
(b) The sum of two antiparallel vectors



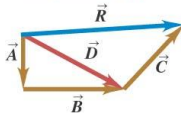
# Adding more than two vectors graphically

To add several vectors, use the head-to-tail method.

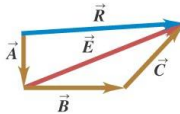
(a) To find the sum of these three vectors ...



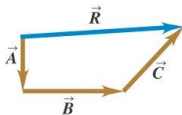
(b) we could add  $\vec{A}$  and  $\vec{B}$  to get  $\vec{D}$  and then add  $\vec{C}$  to  $\vec{D}$  to get the final sum (resultant)  $\vec{R}$ , ...



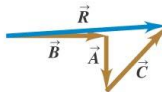
(c) or we could add  $\vec{B}$  and  $\vec{C}$  to get  $\vec{E}$  and then add  $\vec{A}$  to  $\vec{E}$  to get  $\vec{R}$ , ...



(d) or we could add  $\vec{A}$ ,  $\vec{B}$ , and  $\vec{C}$  to get  $\vec{R}$  directly, ...



(e) or we could add  $\vec{A}$ ,  $\vec{B}$ , and  $\vec{C}$  in any other order and still get  $\vec{R}$ .





# Subtracting vectors

Subtracting  $\vec{B}$  from  $\vec{A}$  ...

... is equivalent to adding  $-\vec{B}$  to  $\vec{A}$ .

The diagram illustrates the equivalence between subtracting vector  $\vec{B}$  from vector  $\vec{A}$  and adding the negative vector  $-\vec{B}$  to  $\vec{A}$ . On the left, vector  $\vec{A}$  is represented by a horizontal arrow pointing left, and vector  $\vec{B}$  is a diagonal arrow pointing down and to the right. A minus sign is between them. On the right, vector  $\vec{A}$  is the same horizontal arrow, and vector  $-\vec{B}$  is a diagonal arrow pointing up and to the right. An equals sign is between the two expressions.

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

$$\vec{A} + (-\vec{B}) = \vec{A} - \vec{B}$$

The diagram shows two equivalent ways to find the resultant vector  $\vec{A} - \vec{B}$  using the triangle rule. On the left, vector  $\vec{A}$  is horizontal (pointing left) and vector  $-\vec{B}$  is diagonal (pointing up and right). A third vector,  $\vec{A} - \vec{B}$ , connects the tail of  $\vec{A}$  to the head of  $-\vec{B}$ . On the right, vector  $\vec{A}$  is horizontal (pointing left) and vector  $\vec{B}$  is diagonal (pointing down and right). A third vector,  $\vec{A} - \vec{B}$ , connects the tail of  $\vec{A}$  to the tail of  $\vec{B}$ . An equals sign is between the two diagrams.

$$\vec{A} + (-\vec{B}) = \vec{A} - \vec{B}$$

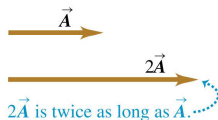
With  $\vec{A}$  and  $-\vec{B}$  head to tail,  $\vec{A} - \vec{B}$  is the vector from the tail of  $\vec{A}$  to the head of  $-\vec{B}$ .

With  $\vec{A}$  and  $\vec{B}$  head to head,  $\vec{A} - \vec{B}$  is the vector from the tail of  $\vec{A}$  to the tail of  $\vec{B}$ .

# Multiplying a vector by a scalar

If  $c$  is a scalar, the product  $c\vec{A}$  has magnitude  $cA$ .

(a) Multiplying a vector by a positive scalar changes the magnitude (length) of the vector, but not its direction.

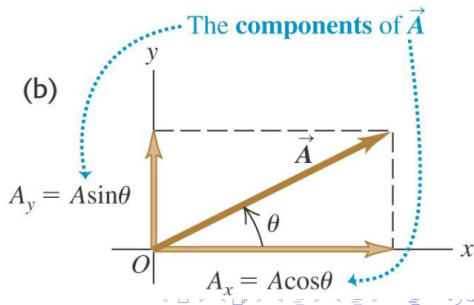
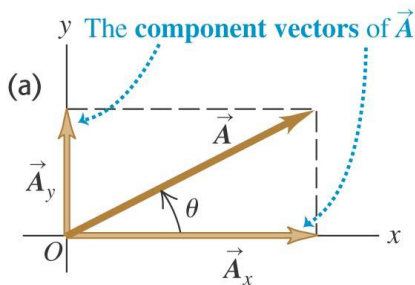


(b) Multiplying a vector by a negative scalar changes its magnitude and reverses its direction.



# Components of a vector

- 1 Adding vectors graphically provides limited accuracy. Vector components provide a general method for adding vectors.
- 2 Any vector can be represented by an  $x$ -component  $A_x$  and a  $y$ -component  $A_y$ .
- 3 Use trigonometry to find the components of a vector:  $A_x = A \cos \theta$  and  $A_y = A \sin \theta$ , where  $\theta$  is measured from the  $+x$ -axis toward the  $+y$ -axis.

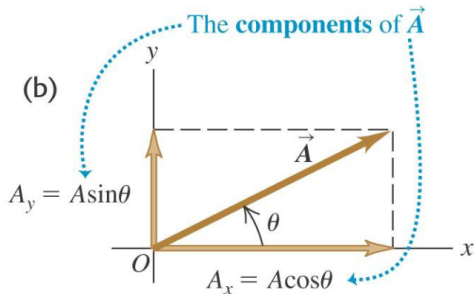
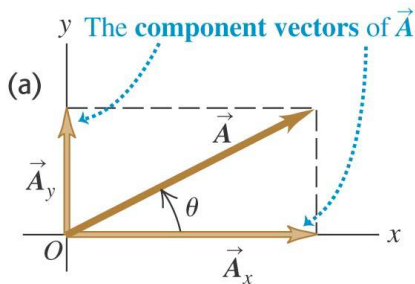


# Magnitude and components of the vector

We can use the components of a vector to find its magnitude and direction:

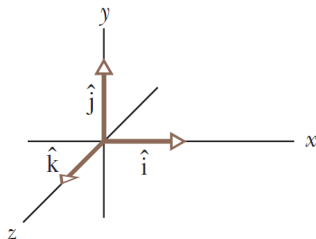
$$A = \sqrt{A_x^2 + A_y^2}$$

$$\tan \theta = \frac{A_x}{A_y}$$



# Unit vectors

- 1 A unit vector has a magnitude of 1 with no units.
- 2 The unit vector  $\vec{i}$  points in the  $+x$ -direction,  $\vec{j}$  points in the  $+y$ -direction,  $\vec{k}$  points in the  $+z$ -direction.
- 3 Any vector can be expressed in terms of its components as



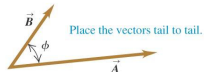
$$\vec{A} = A_x \vec{i} + A_y \vec{j} + A_z \vec{k}$$

# The scalar product

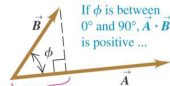
The scalar (or dot) product of two vectors  $\vec{a}$  and  $\vec{b}$  is written as  $\vec{a} \cdot \vec{b}$  and is the scalar quantity given by:

$$\vec{a} \cdot \vec{b} = ab \cos \varphi$$

(a)



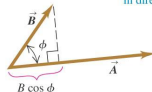
(a)



... because  $B \cos \phi > 0$ .

(b)

$\vec{A} \cdot \vec{B}$  equals  $A(B \cos \phi)$ .  
(Magnitude of  $\vec{A}$ ) times (Component of  $\vec{B}$  in direction of  $\vec{A}$ )



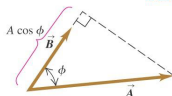
(b)

If  $\phi$  is between  $90^\circ$  and  $180^\circ$ ,  $\vec{A} \cdot \vec{B}$  is negative ...

... because  $B \cos \phi < 0$ .

(c)

$\vec{A} \cdot \vec{B}$  also equals  $B(A \cos \phi)$ .  
(Magnitude of  $\vec{B}$ ) times (Component of  $\vec{A}$  in direction of  $\vec{B}$ )



(c)

If  $\phi = 90^\circ$ ,  $\vec{A} \cdot \vec{B} = 0$  because  $\vec{B}$  has zero component in the direction of  $\vec{A}$ .

$\phi = 90^\circ$

# The scalar product via components

In terms of components the scalar product can be calculated as:

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

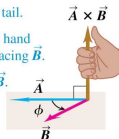
# The vector product

The vector (or cross) product of two vectors  $\vec{a}$  and  $\vec{b}$  is written as  $\vec{a} \times \vec{b}$  and is the vector whose magnitude is given by:

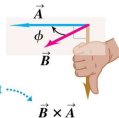
$$|\vec{a} \times \vec{b}| = ab \sin \varphi$$

(a) Using the right-hand rule to find the direction of  $\vec{A} \times \vec{B}$

- ① Place  $\vec{A}$  and  $\vec{B}$  tail to tail.
- ② Point fingers of right hand along  $\vec{A}$ , with palm facing  $\vec{B}$ .
- ③ Curl fingers toward  $\vec{B}$ .
- ④ Thumb points in direction of  $\vec{A} \times \vec{B}$ .



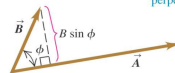
(b)  $\vec{B} \times \vec{A} = -\vec{A} \times \vec{B}$  (the vector product is anticommutative)



Same magnitude but opposite direction

(a)

(Magnitude of  $\vec{A} \times \vec{B}$  equals  $A(B \sin \phi)$ .  
(Magnitude of  $\vec{A}$ ) times (Component of  $\vec{B}$  perpendicular to  $\vec{A}$ )



(b)

(Magnitude of  $\vec{A} \times \vec{B}$  also equals  $B(A \sin \phi)$ .  
(Magnitude of  $\vec{B}$ ) times (Component of  $\vec{A}$  perpendicular to  $\vec{B}$ )

