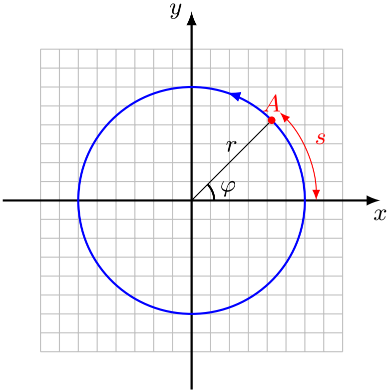
Rotational motion of a point and its angular characteristics. Kinematics of a rigid body

Goals for Lecture

- To determine what is angular position
- To determine what is angular displacement
- To determine what is angular velocity
- To determine what is angular acceleration
- To describe the rotational motion under constant angular acceleration
- To understand what is relations between angular and linear quantities
- To describe the circular motion of a particle

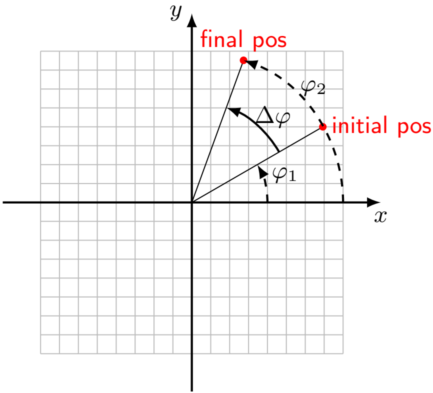
Angle and Radian

1. φ can be defined as the arc length s along a circle divided by the radius r:
2. The angular position, measured in radians, is the angle of rotation of the object with respect to a reference position.
3. The angular position of point A at this point in time is equal to φ. In order to uniquely define this position, we have assume that an the angular position is measured with respect to the x axis.



Angle and Angular Displacement

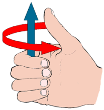
Angular displacement $\Delta\varphi = \varphi_2 - \varphi_1$

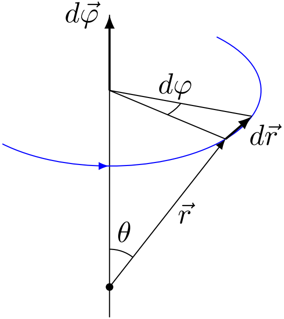


Angular Displacement

Angular Displacement is VECTOR!

Suppose a particle, while rotating about an axis, accomplishes an infinitesimal rotation during the time interval dt. We shall describe the corresponding rotation angle by the vector $d\vec{\varphi}$ whose modulus is equal to the rotation angle and whose direction coincides with the axis, with the rotation direction obeying the right-hand screw rule with respect to the direction of the vector $d\vec{\varphi}$.





Angular Displacement

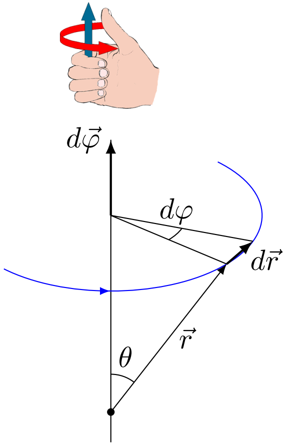
Angular Displacement is VECTOR!

Then the linear displacement of the end point of the radius vector $\vec{r}$ is associated with the rotation angle $d\varphi$ by the relation:

$$|d\vec{\varphi}| = r \sin\theta d\varphi$$

or in a vector form

$$\boxed{d\vec{r} = d\vec{\varphi} \times \vec{r}}$$

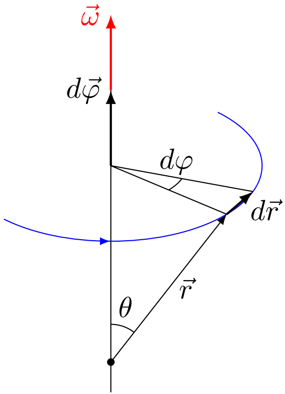


Angular Velocity

The angular velocity vector $\vec{\omega}$ is defined as

$$\boxed{\vec{\omega} = \frac{d\vec{\varphi}}{dt}}$$

where dt is the time interval during which a body performs the rotation $d\vec{\varphi}$. The vector $\vec{\omega}$ to is axial and its direction coincides with that of the vector $d\vec{\varphi}$.



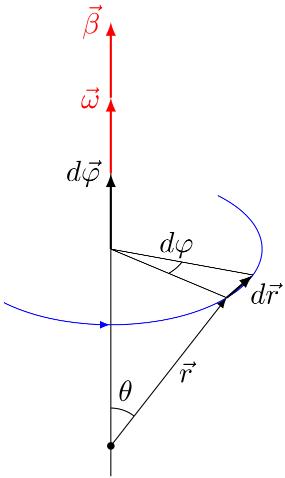
Angular Acceleration

The time variation of the vector $\vec{\omega}$ is defined by the angular acceleration vector $\vec{\beta}$

$$\boxed{\vec{\beta} = \frac{d\vec{\omega}}{dt}}.$$

The direction of the vector $\vec{\beta}$ coincides with the direction of $d\vec{\omega}$, the increment of the vector $\vec{\omega}$. Both vectors, $\vec{\beta}$ and $\vec{\omega}$, are axial.
If $\vec{\omega}$ grows up with respect to time, then

$$\vec{\beta} \uparrow\uparrow \vec{\omega}$$



Angular Acceleration

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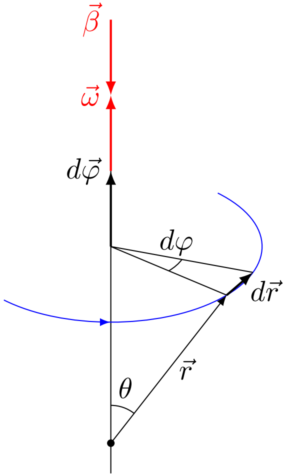
The direction of the vector $\vec{\beta}$ coincides with the direction of $d\vec{\omega}$, the increment of the vector $\vec{\omega}$. Both vectors, $\vec{\beta}$ and $\vec{\omega}$, are axial.
If $\vec{\omega}$ grows down with respect to time, then

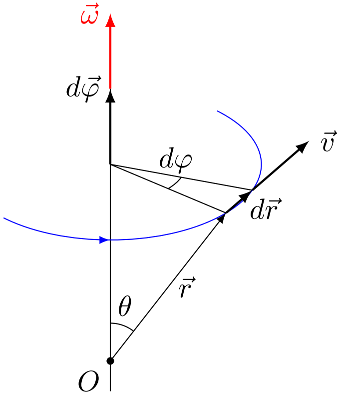
$$\vec{\beta} \downarrow\uparrow \vec{\omega}$$



Relationship between linear and angular quantities

Let us find the velocity $\vec{v}$ of an arbitrary point A of a solid rotating about a stationary axis at an angular velocity $\vec{\omega}$. Let the location of the point A relative to some point O of the rotation axis be defined by the radius vector $\vec{r}$. Dividing both sides of $d\vec{r} = d\vec{\varphi} \times \vec{r}$ by the corresponding time interval! dt and taking $\frac{d\vec{r}}{dt} = \vec{v}$ and $\frac{d\vec{\varphi}}{dt} = \vec{\omega}$, we obtain:

$$\boxed{\vec{v} = d\vec{\omega} \times \vec{r}}$$



Relationship between linear and angular quantities

$$\boxed{\vec{v} = d\vec{\omega} \times \vec{r}}$$

Having differentiated this with respect to time, we find the acceleration of the point A:

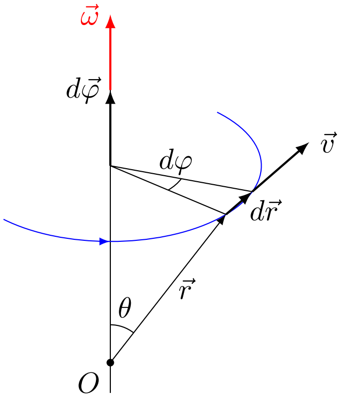
$$\boxed{\vec{a} = \vec{\beta} \times \vec{r} + \vec{\omega} \times [\vec{\omega} \times \vec{r}]}$$

The vector $\vec{\omega} \times [\vec{\omega} \times \vec{r}]$ is the normal acceleration. The moduli of these vectors are

$$a_\tau = \beta\rho \quad a_n = \omega^2\rho.$$

whence the modulus of the total acceleration:

$$a = \sqrt{a_\tau^2 + a_n^2}.$$



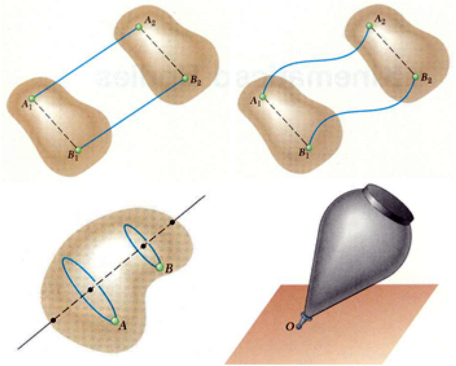
What is Rigid Bodies?

$\boxed{\text{Rigid body}}$ is a solid body in which deformation is zero or so small it can be neglected. The distance between any two given points on a rigid body remains constant in time regardless of external forces exerted on it.

Kinematics of Rigid Bodies

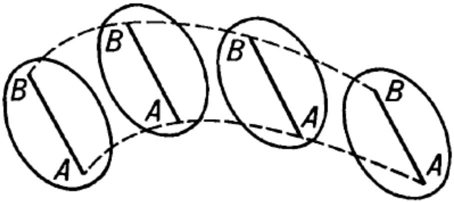
Classification of rigid body motions

1. translation:
   1. rectilinear translation
   2. curvilinear translation
2. rotation about a fixed axis
3. general plane motion
4. motion about a fixed point
5. general motion



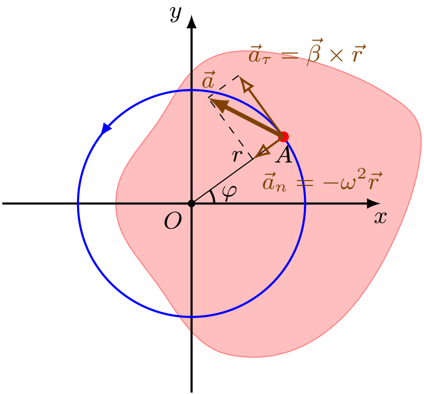
Translational motion

Purely translational motion occurs when every particle of the body has the same instantaneous velocity as every other particle; then the path traced out by any particle is exactly parallel to the path traced out by every other particle in the body. Under translational motion, the change in the position of a rigid body is specified completely by three coordinates such as x, y, and z giving the displacement of any point, such as the center of mass, fixed to the rigid body.



Rotation about fixed axis

Rotation about fixed axis means if every points in the body moves in a circle about a single line. This line is called the axis of rotation. Then the radius vectors from the axis to all points undergo the same angular displacement in the same time. The axis of rotation need not go through the body. In general, any rotation can be specified completely by the three angular displacements with respect to the rectangular coordinate axes x, y, and z.



Planar Motion of Rigid Body

In this kind of motion each point of a body moves in a plane which is parallel to a certain stationary (in a given reference frame) plane.