$$\Phi(1,2) = 1/\sqrt{2}\,[\varphi_1(1)\varphi_2(2) - \varphi_1(2)\varphi_2(1)]$$

$$\hat{H} = \hat{h}(1) + \hat{h}(2) + \frac{1}{r_{12}} \qquad E = \left\langle \Phi(1,2) \left| \hat{H} \right| \Phi(1,2) \right\rangle$$

$$\begin{aligned}
E = \frac{1}{2}\left\langle \varphi_1(1)\varphi_2(2) - \varphi_1(2)\varphi_2(1) \left| \hat{h}(1) + \hat{h}(2) + \frac{1}{r_{12}} \right| \varphi_1(1)\varphi_2(2) - \varphi_1(2)\varphi_2(1) \right\rangle = \\
= \frac{1}{2}\left\langle \varphi_1(1)\varphi_2(2) - \varphi_1(2)\varphi_2(1) \left| \hat{h}(1) \right| \varphi_1(1)\varphi_2(2) - \varphi_1(2)\varphi_2(1) \right\rangle + \\
+ \frac{1}{2}\left\langle \varphi_1(1)\varphi_2(2) - \varphi_1(2)\varphi_2(1) \left| \hat{h}(2) \right| \varphi_1(1)\varphi_2(2) - \varphi_1(2)\varphi_2(1) \right\rangle + \\
+ \frac{1}{2}\left\langle \varphi_1(1)\varphi_2(2) - \varphi_1(2)\varphi_2(1) \left| \frac{1}{r_{12}} \right| \varphi_1(1)\varphi_2(2) - \varphi_1(2)\varphi_2(1) \right\rangle =
\end{aligned}$$

$$\begin{aligned}
= \frac{1}{2}\left\langle \varphi_1(1)\varphi_2(2) \left| \hat{h}(1) \right| \varphi_1(1)\varphi_2(2) \right\rangle + \frac{1}{2}\left\langle \varphi_2(1)\varphi_1(2) \left| \hat{h}(1) \right| \varphi_2(1)\varphi_1(2) \right\rangle - \\
- \frac{1}{2}\left\langle \varphi_1(1)\varphi_2(2) \left| \hat{h}(1) \right| \varphi_2(1)\varphi_1(2) \right\rangle - \frac{1}{2}\left\langle \varphi_2(1)\varphi_1(2) \left| \hat{h}(1) \right| \varphi_1(1)\varphi_2(2) \right\rangle +
\end{aligned}$$

$$\begin{aligned}
+ \frac{1}{2}\left\langle \varphi_1(1)\varphi_2(2) \left| \hat{h}(2) \right| \varphi_1(1)\varphi_2(2) \right\rangle + \frac{1}{2}\left\langle \varphi_2(1)\varphi_1(2) \left| \hat{h}(2) \right| \varphi_2(1)\varphi_1(2) \right\rangle - \\
- \frac{1}{2}\left\langle \varphi_1(1)\varphi_2(2) \left| \hat{h}(2) \right| \varphi_2(1)\varphi_1(2) \right\rangle - \frac{1}{2}\left\langle \varphi_2(1)\varphi_1(2) \left| \hat{h}(2) \right| \varphi_1(1)\varphi_2(2) \right\rangle +
\end{aligned}$$

$$\begin{aligned}
+ \frac{1}{2}\left\langle \varphi_1(1)\varphi_2(2) \left| \frac{1}{r_{12}} \right| \varphi_1(1)\varphi_2(2) \right\rangle + \frac{1}{2}\left\langle \varphi_2(1)\varphi_1(2) \left| \frac{1}{r_{12}} \right| \varphi_2(1)\varphi_1(2) \right\rangle - \\
- \frac{1}{2}\left\langle \varphi_1(1)\varphi_2(2) \left| \frac{1}{r_{12}} \right| \varphi_2(1)\varphi_1(2) \right\rangle - \frac{1}{2}\left\langle \varphi_2(1)\varphi_1(2) \left| \frac{1}{r_{12}} \right| \varphi_1(1)\varphi_2(2) \right\rangle =
\end{aligned}$$

$$\begin{aligned}
= \frac{1}{2}\left\langle \varphi_1(1) \left| \hat{h}(1) \right| \varphi_1(1) \right\rangle + \frac{1}{2}\left\langle \varphi_2(1) \left| \hat{h}(1) \right| \varphi_2(1) \right\rangle + \\
+ \frac{1}{2}\left\langle \varphi_1(2) \left| \hat{h}(2) \right| \varphi_1(2) \right\rangle + \frac{1}{2}\left\langle \varphi_2(2) \left| \hat{h}(2) \right| \varphi_2(2) \right\rangle +
\end{aligned}$$

$$\begin{aligned}
+ \frac{1}{2}\left\langle \varphi_1(1)\varphi_2(2) \left| \frac{1}{r_{12}} \right| \varphi_1(1)\varphi_2(2) \right\rangle + \frac{1}{2}\left\langle \varphi_2(1)\varphi_1(2) \left| \frac{1}{r_{12}} \right| \varphi_2(1)\varphi_1(2) \right\rangle - \\
- \frac{1}{2}\left\langle \varphi_1(1)\varphi_2(2) \left| \frac{1}{r_{12}} \right| \varphi_2(1)\varphi_1(2) \right\rangle - \frac{1}{2}\left\langle \varphi_2(1)\varphi_1(2) \left| \frac{1}{r_{12}} \right| \varphi_1(1)\varphi_2(2) \right\rangle =
\end{aligned}$$

$$= \left\langle \mathbf{1} \left| \hat{h} \right| \mathbf{1} \right\rangle + \left\langle \mathbf{2} \left| \hat{h} \right| \mathbf{2} \right\rangle + \frac{1}{2}\left\langle \mathbf{12} \left| \frac{1}{r_{12}} \right| \mathbf{12} \right\rangle + \frac{1}{2}\left\langle \mathbf{21} \left| \frac{1}{r_{12}} \right| \mathbf{21} \right\rangle - \frac{1}{2}\left\langle \mathbf{12} \left| \frac{1}{r_{12}} \right| \mathbf{21} \right\rangle - \frac{1}{2}\left\langle \mathbf{21} \left| \frac{1}{r_{12}} \right| \mathbf{12} \right\rangle$$

$$E = \varepsilon_1 + \varepsilon_2 + J - K = \sum_{i=1}^{2} \varepsilon_i + \frac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2}(J_{ij} - K_{ij}), \quad J_{ii} = K_{ii} = 0$$

$$F = E - \sum_{i=1}^{2}\sum_{j=1}^{2}(\varepsilon_{ij}\langle\varphi_i|\varphi_j\rangle - \delta_{ij})$$

$$\begin{aligned}\delta F = &\langle\delta\mathbf{1}|\hat{h}|\mathbf{1}\rangle + \langle\delta\mathbf{2}|\hat{h}|\mathbf{2}\rangle + \\ &+\frac{1}{2}\left\langle\delta\mathbf{1}\mathbf{2}\left|\frac{1}{r_{12}}\right|\mathbf{1}\mathbf{2}\right\rangle + \frac{1}{2}\left\langle\mathbf{1}\delta\mathbf{2}\left|\frac{1}{r_{12}}\right|\mathbf{1}\mathbf{2}\right\rangle + \frac{1}{2}\left\langle\delta\mathbf{2}\mathbf{1}\left|\frac{1}{r_{12}}\right|\mathbf{2}\mathbf{1}\right\rangle + \frac{1}{2}\left\langle\mathbf{2}\delta\mathbf{1}\left|\frac{1}{r_{12}}\right|\mathbf{2}\mathbf{1}\right\rangle - \\ &-\frac{1}{2}\left\langle\delta\mathbf{1}\mathbf{2}\left|\frac{1}{r_{12}}\right|\mathbf{2}\mathbf{1}\right\rangle - \frac{1}{2}\left\langle\delta\mathbf{2}\mathbf{1}\left|\frac{1}{r_{12}}\right|\mathbf{1}\mathbf{2}\right\rangle - \frac{1}{2}\left\langle\mathbf{1}\delta\mathbf{2}\left|\frac{1}{r_{12}}\right|\mathbf{2}\mathbf{1}\right\rangle - \frac{1}{2}\left\langle\mathbf{2}\delta\mathbf{1}\left|\frac{1}{r_{12}}\right|\mathbf{1}\mathbf{2}\right\rangle - \\ &-\varepsilon_{11}\langle\delta\mathbf{1}|\mathbf{1}\rangle - \varepsilon_{12}\langle\delta\mathbf{1}|\mathbf{2}\rangle - \varepsilon_{21}\langle\delta\mathbf{2}|\mathbf{1}\rangle - \varepsilon_{22}\langle\delta\mathbf{2}|\mathbf{2}\rangle + \\ &+\text{c. c.} = 0\end{aligned}$$

$$\left(\hat{h} + \frac{1}{2}\left\langle .\mathbf{2}\left|\frac{1}{r_{12}}\right|.\mathbf{2}\right\rangle + \frac{1}{2}\left\langle\mathbf{2}.\left|\frac{1}{r_{12}}\right|\mathbf{2}.\right\rangle - \frac{1}{2}\left\langle .\mathbf{2}\left|\frac{1}{r_{12}}\right|\mathbf{2}.\right\rangle - \frac{1}{2}\left\langle\mathbf{2}.\left|\frac{1}{r_{12}}\right|.\mathbf{2}\right\rangle\right)|\mathbf{1}\rangle = \varepsilon_{11}|\mathbf{1}\rangle + \varepsilon_{12}|\mathbf{2}\rangle$$

$$\left(\hat{h} + \frac{1}{2}\left\langle .\mathbf{1}\left|\frac{1}{r_{12}}\right|.\mathbf{1}\right\rangle + \frac{1}{2}\left\langle\mathbf{1}.\left|\frac{1}{r_{12}}\right|\mathbf{1}.\right\rangle - \frac{1}{2}\left\langle .\mathbf{1}\left|\frac{1}{r_{12}}\right|\mathbf{1}.\right\rangle - \frac{1}{2}\left\langle\mathbf{1}.\left|\frac{1}{r_{12}}\right|.\mathbf{1}\right\rangle\right)|\mathbf{2}\rangle = \varepsilon_{21}|\mathbf{1}\rangle + \varepsilon_{22}|\mathbf{2}\rangle$$

Canonical HF-Equations

$$\left(\hat{h} + \left\langle .\mathbf{2}\left|\frac{1}{r_{12}}\right|.\mathbf{2}\right\rangle - \left\langle .\mathbf{2}\left|\frac{1}{r_{12}}\right|\mathbf{2}.\right\rangle\right)|\mathbf{1}\rangle = \varepsilon_{11}|\mathbf{1}\rangle$$

$$\left(\hat{h} + \left\langle .\mathbf{1}\left|\frac{1}{r_{12}}\right|.\mathbf{1}\right\rangle - \left\langle .\mathbf{1}\left|\frac{1}{r_{12}}\right|\mathbf{1}.\right\rangle\right)|\mathbf{2}\rangle = \varepsilon_{22}|\mathbf{2}\rangle$$

Columb operator

$$\hat{J}_{\mathbf{2}}|\mathbf{1}\rangle = \left\langle .\mathbf{2}\left|\frac{1}{r_{12}}\right|.\mathbf{2}\right\rangle|\mathbf{1}\rangle = \int\frac{.\varphi_{\mathbf{2}}(2).\varphi_{\mathbf{2}}(2)}{r_{12}}d(2)\cdot\varphi_{\mathbf{1}}(1) = \int\frac{.\varphi_{\mathbf{2}}(2)\varphi_{\mathbf{1}}(1)\varphi_{\mathbf{2}}(2)}{r_{12}}d(2)$$

Columb integral

$$\langle\mathbf{1}|\hat{J}_{\mathbf{2}}|\mathbf{1}\rangle = \langle\mathbf{1}|\left\langle .\mathbf{2}\left|\frac{1}{r_{12}}\right|.\mathbf{2}\right\rangle|\mathbf{1}\rangle = \iint\frac{\varphi_{\mathbf{1}}(1)\varphi_{\mathbf{2}}(2)\varphi_{\mathbf{1}}(1)\varphi_{\mathbf{2}}(2)}{r_{12}}d(2)d(1)$$

Exchange operator

$$\hat{K}_{\mathbf{2}}|\mathbf{1}\rangle = \left\langle .\mathbf{2}\left|\frac{1}{r_{12}}\right|\mathbf{2}.\right\rangle|\mathbf{1}\rangle = \int\frac{.\varphi_{\mathbf{2}}(2)\varphi_{\mathbf{2}}(1).}{r_{12}}d(2)\cdot\varphi_{\mathbf{1}}(1) = \int\frac{.\varphi_{\mathbf{2}}(2)\varphi_{\mathbf{2}}(1)\varphi_{\mathbf{1}}(2)}{r_{12}}d(2)$$

Exchange integral

$$\langle\mathbf{1}|\hat{K}_{\mathbf{2}}|\mathbf{1}\rangle = \langle\mathbf{1}|\left\langle .\mathbf{2}\left|\frac{1}{r_{12}}\right|\mathbf{2}.\right\rangle|\mathbf{1}\rangle = \iint\frac{\varphi_{\mathbf{1}}(1)\varphi_{\mathbf{2}}(2)\varphi_{\mathbf{2}}(1)\varphi_{\mathbf{1}}(2)}{r_{12}}d(2)d(1)$$