Term Premia in Bonds: A Trading Strategy and Risk Analysis Perspective

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Abstract

Term premia, representing the excess return on long-term bonds over short-term bonds, is widely studied in asset pricing and macroeconomic research. This report examines the historical existence of term premium through a trading strategy where a long-term bond is financed by rolling short-term bonds. Using zero-coupon bond data from 1962 to 2025, we analyse the strategy's performance over varying time periods, incorporating margin requirements and early termination to reflect real-world trading constraints. To assess the statistical significance of the observed excess returns, we employ Value at Risk and Expected Shortfall methodologies and fit a Gaussian mixture model to approximate the return distribution. Our results suggest that while term premium can generate positive excess returns, strategy performance is very sensitive to market conditions. In particular, periods with interest rate spikes can lead to excessive short positions and forced liquidations, causing significant drawdowns and large losses. These findings highlight the importance of dynamic risk controls and provide valuable implications for fixed-income investment.

1 Introduction

Term premium is the extra yield that investors expect when they choose to invest in longer-term bonds rather than in short-term bonds. This premium compensates investors for the risks associated with longer-term investments, such as interest rate fluctuations and inflation uncertainty.

As an example, if the yield curve experiences a parallel upward shift, the price of a long-term bond drops more than that of a short-term bond due to its longer modified duration, which measures the percentage change of a bond's price given a change in yield. This creates greater uncertainty for investors, who might face a bigger loss if they need to sell the bond before it matures. Therefore, investors require a higher return on long-dated bonds to compensate for this risk. In addition, the flexibility of short-dated bonds are desirable to investors, as the principal payment is guaranteed at maturity. As a result, term premium arises from both the additional risk of interest rate shifts and the demand for a fixed cash flow in the short term.

There are several literature that discuss the existence and implications of term premium [6], [1]. In this case, we focus on exploring trading strategies, attempting to obtain statistically significant excess returns.

2 Methodology

2.1 US Treasury Yield Curve

The dataset used in this study is sourced from a paper, published by the Federal Reserve, titled "The U.S. Treasury Yield Curve: 1961 to the Present"[3], which constructs the yield curve for every trading day from 1961 onwards. Despite the U.S. Treasury market being one of the largest and most liquid markets, securities are not issued for every possible maturity, and so a continuous yield curve must be estimated to obtain yields for all maturities. For this purpose, the Svensson model [7], introduced in 1994, is employed. This model extends the Nelson-Siegel [5] framework by incorporating an additional curvature term, capturing both the initial short-term interest rate hump and then the second long-term hump due to convexity, which pulls down the yields on longer-term securities. Specifically, at time t, the continuously compounded zero-coupon yield for tenor T takes the form

$$y_t(T) = \beta_0 + \beta_1 \frac{1 - e^{-T/\tau_1}}{T/\tau_1} + \beta_2 \left(\frac{1 - e^{-T/\tau_1}}{T/\tau_1} - e^{-T/\tau_1} \right) + \beta_3 \left(\frac{1 - e^{-T/\tau_2}}{T/\tau_2} - e^{-T/\tau_2} \right).$$

Here, β_0 represents the long-term interest rate level, while β_1 determines the short-term slope of the yield curve. Parameters β_2 and β_3 control the curvature, with β_3 specifically capturing the convexity associated with a second hump. As we will now see, once the Svensson parameters are estimated for a given day, they can be used to derive zero-coupon prices for different maturities.

2.2 Data Preprocessing

We use the provided dataset [3], which contains daily data on the continuously compounded zero-coupon yield curve from 1962 to 2025. From this curve, we compute the zero-coupon bond prices using

$$P(t, T) = \exp(-(T - t) y_t(T - t)).$$

For bonds with times to maturity that do not correspond exactly to a traded bond maturity, market prices are estimated using linear interpolation of the prices of the bonds with closest available tenors on that date. Specifically, the price of a 7-year bond with 4.5 years left to maturity is estimated using the 4-year and 5-year bonds at that time. As a result, we obtain an estimated price process of a bond issued on any particular date (Figure 1).

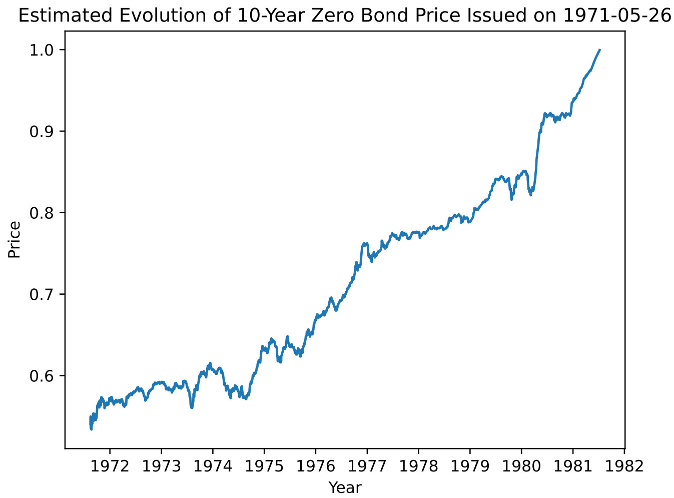

Figure 1: Estimated price process of zero-coupon bond

Although this could produce inaccurate estimates and we acknowledge that liquidity (on-the-run compared to off-the-run issues) could result in drastically different bond prices, all the results in this project are produced using this interpolation estimation. The methodology and results we present can easily be replicated with daily market data on individual bond issues.

2.3 Trading Strategies

In general, a term premia strategy consists of buying a long-term bond and holding it until maturity, financing this purchase by selling a short-term bond and rolling the short position until the long-maturity bond expires.

In this section, we discuss the different adaptions of term premia strategies applied in our analysis, and consider their strengths and limitations. It is important to note that, throughout our analysis, interest rates are assumed to be non-negative.

2.3.1 Naive Strategy

Before exploring other more complex and realistic strategies, we begin with a naive strategy, whereby a long-maturity zero-coupon bond is purchased and held until maturity, and this is financed by shorting a 1-year zero-coupon bond and rolling it until the long-maturity bond expires.

More formally, we fix a start date t and a long maturity date T, and purchase a zero-coupon bond for price $P(t,T)$, (where $P(a,b)$ denotes the price of a zero-coupon bond at time a with maturity b), therefore fixing the long-maturity bond's tenor at $\tau = T - t$. This purchase is financed by selling x_t units of a 1-year zero-coupon bond, such that $x_t P(t, t+1) = P(t,T)$. After 1 year, when the initial 1-year bond expires, we are obligated to pay x_t and we finance this by selling x_{t+1} units of a new 1-year bond, such that $x_{t+1}P(t+1, t+2) = x_t$. This continues until the expiry of the long-maturity bond, which gives the following iterative equation

$$x_{t+i}P(t+i, t+i+1) = x_{t+i-1}, \quad \text{for } i = 1, \ldots, \tau - 1.$$

At time T, we receive a payoff of 1 from our long position in the long-maturity bond, but we are also obligated to pay x_{T-1} (where x_{T-1} is the units of the 1 year bond shorted between $T-1$ and T). The annualised return of this τ-year strategy starting at time t is then computed as

$$r_t = \left(1 + \frac{1 - x_{t+\tau-1}}{C_t}\right)^{1/\tau} - 1$$

where C_t is the initial capital deployed.

By considering a single run of the strategy from time t to T, we observe several practical issues. Firstly, once negative returns are incurred at any time during the lifetime of the strategy, it becomes impossible to recover the losses or achieve profitability. As a result, continuing to hold the long-maturity bond is suboptimal and would guarantee a loss even far into the future. To see this, note that $x_{t+i} \geq x_{t+i-1}$ for $i = 1, \ldots, \tau - 1$, since, from our recursive equation, $\frac{x_{t+i-1}}{x_{t+i}} = P(t+i, t+i-1) \leq 1$ due to the assumption that interest rates are always non-negative. Therefore, once x_{t+i} reaches 1 for some $i < \tau$, it is guaranteed that our bond obligation at time-T will be larger than 1, and so the strategy's return will be negative. As a result, once our short position reaches 1 at any time in (t, T), it is rational to discontinue the strategy as potential future returns can only worsen.

Additionally, the unrestricted ability to short bonds in this strategy theoretically exposes an investor to unlimited losses. As will be shown later in our analysis, running the strategy until maturity during periods with sudden interest rates jumps (such as the 1980s) can result in enormous losses, shown by the large terminal short positions. In reality, traders cannot take short positions without strict risk controls and this motivates the inclusion of margin requirements in our strategies going forward, consistent with standard industry practice.

2.3.2 Early Termination Strategy

To improve the realism of the naive strategy, we introduce early termination, addressing the irrationality of continuing the strategy once the short position exceeds 1. In such cases, the strategy ends before the long-term bond's maturity, and the payoff received is then the current market price of the long-term bond minus the short position.

Furthermore, we assume a margin requirement of 120% of the short position, which ensures that sufficient funds are always available to close our positions unless the short position increases by 20% in a day (or a similar decrease in the long position). This decision is motivated by the short-sale requirements discussed in [2]. To test this strategy, we first have to determine C_t, the initial capital deployed for one run of the strategy, controlling the trade-off between the returns of the strategy and the likelihood of early termination (which almost always results in a loss). This quantity is estimated every month by simulating the strategy on preceding data.

More specifically, given a start date T_1 and end date $T_2 = T_1 + \tau$, the capital deployed for one single run of the strategy for a given month is determined as follows. We first retrieve daily historical prices for the long and short-term bonds over the preceding τ years from the start of current month. For each date t within the τ-year period, we simulate the naive strategy and compute the theoretical cash margin requirement

$$M_t = \alpha \times V_t^{\text{short}} - V_t^{\text{long}}$$

where V_t^{short} and V_t^{long} are the market values of the short and long position respectively, and α represents the margin requirement factor (e.g. 1.2 for a 120% margin). We then set $C_{T_1} = \max\ M_t$, where the maximum is taken over the τ-year window preceding T_1. This approach allows us to predict the initial capital that should provide a sufficient capital buffer in order to hold the strategy until maturity.

However, to prevent this from being too small to open the short position, particularly if it is computed during a period of stable interest rates, we take the maximum over the previous quantity and the initial cash margin required. We also introduce a margin multiplier $m > 1$ and analyse the impact of this parameter on the trade-off between early termination and returns.

$$\begin{aligned} C_{T_1} &= m * \max(\max\ M_t, \alpha \times V_{T_1}^{short} - V_{T_1}^{long}) \\ &= m * \max(\max\ M_t, (\alpha - 1)P(T_1, T_2)) \end{aligned}$$

An early termination check is performed every two days throughout the strategy (instsead of daily for lower computational time). On any given day t, early termination occurs if either of the following conditions is met:

1. The cash margin required exceeds the cash on hand, i.e.

$$\alpha V_t^{\text{short}} - V_t^{\text{long}} > C_{T_1},$$

2. The current short position exceeds 1 unit

If early termination occurs at time t, the strategy is liquidated and the annualised return for the strategy starting at T_1 is computed based on the realised value at termination, given by

$$r_{T_1} = \left(1 + \frac{V_t^{long} - V_t^{short}}{C_{T_1}}\right)^{1/(t-T_1)} - 1 \tag{1}$$

This framework introduces realistic risk management constraints, preventing excessive leveraging and ensuring sufficient funding throughout the strategy's execution.

2.3.3 Overnight Cash Growth

We also consider the returns excess of the risk-free rate by using a mechanism that is more closely aligned with the practical implementation of such a strategy. Often, the cash deployed is much larger than the cash margin requirement, especially when the margin multiplier m is large. This means that while we are calculating returns on a much larger base C_{T_1}, the extra cash on hand do not experience any growth and will cause our strategy to underperform. Therefore, we allow any extra cash to grow at the overnight rate. We use $C(T_1, T_1)$ to denote the initial cash instead of C_{T_1}, and $C(T_1, s)$ to denote the cash process for $T_1 \leq s \leq T_2$. Then, $C(T_1, \cdot)$ evolves according to the following equation,

$$C(T_1, s + \Delta t) = \frac{C(T_1, s) - (\alpha V_s^{short} - V_s^{long})}{P(s, s + \Delta t)} + (\alpha V_s^{short} - V_s^{long})$$

where Δt is taken to be 1 day and $P(s, s+\Delta t)$ is estimated using linear interpolation as before. This strategy has the corresponding wealth process

$$W_s = C(T_1, s) + V_s^{long} - V_s^{short}, \quad s \in [T_1, T_2]. \tag{2}$$

The returns are given by

$$r_{T_1} = \left(\frac{W_t}{C(T_1, T_1)} \right)^{1/(t-T_1)} - 1$$

and the excess returns (over the overnight rate) are given by

$$r_{T_1} = \left(\frac{W_t}{C(T_1, T_1)} \right)^{1/(t-T_1)} - \left(\prod_{k=0}^{\frac{t}{\Delta t}-1} \frac{1}{P(T_1 + k\Delta t, T_1 + (k+1)\Delta t)} \right)^{1/(t-T_1)}$$

where t is the termination time.

2.3.4 Variations in the Strategy

We conclude by briefly testing variations of the final strategy, such as entering the same positions as before but unwinding the positions after one year. In this case, the short position does not roll forward, and the returns is solely determined by the price of the long bond after a year.

Lastly, we consider a 10-year strategy, in which we roll two-year bonds instead of one-year in the short position, and evaluate the impact this has on term premium.

3 Results

3.1 Naive Strategy Returns

The naive strategy is run with 2, 3, 5, 7, 10, 15, 20 and 30 year tenors. Whilst this strategy exhibits non-negative average returns over the majority of these tenors, its primary limitation is its infeasibility. During periods of high interest rates (such as the 1970s), the naive strategy performs poorly and large negative returns are realised. Figure 2 displays the returns of the 15-year naive strategy and its final short position, if run every day from the 1970s to 2010s. We see that for the runs of the naive strategy during the 1970s, the terminal short position exceeds 1. This is clearly impractical, as without an additional cash buffer it would be impossible to close this position at the long-term bond's maturity. Therefore, we focus on analysing the results of more realistic strategies.

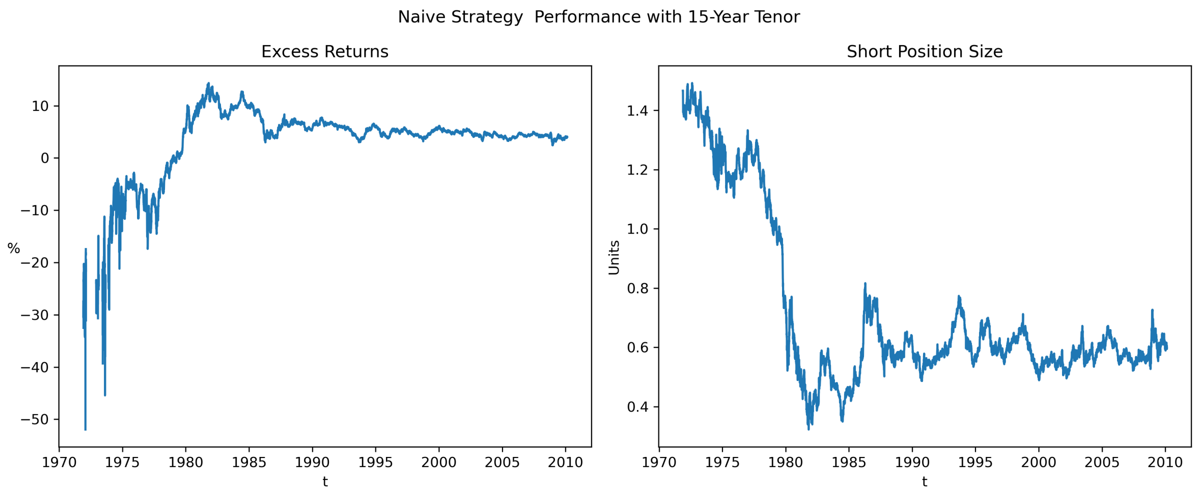

Figure 2: Excess returns for naive strategy for 15-year tenor

3.2 Early Termination Returns

We now analyse returns for the more realistic strategy (which incorporates early termination and margin requirements), whilst varying the margin multiplier m. The returns of the 5,7,10 and 15-year strategy are shown in Figure 3 for $m = 2.5$ and in Figure 4 for $m = 5$. It can be observed that a smaller multiplier has two main effects. Firstly, for the tenors 10 and 15, extreme losses (annualised loss of 60-100%) occur much more frequently. This can be attributed to the early termination of the strategies, as shown in Figures 5 and 6. This is much more pronounced for the runs that terminate before the one-year mark. These figures illustrate how the strategy consistently terminates earlier for a lower multiplier, which is expected. However, for longer tenors, the difference becomes notably more significant, especially during the 1980s. On the other hand, when the multiplier increases, the positive returns in Equation 1 are reduced. Hence, it is important to select a correct multiplier to manage the trade-off between the likelihood of early termination, and the magnitude of profits when positive returns are realised.

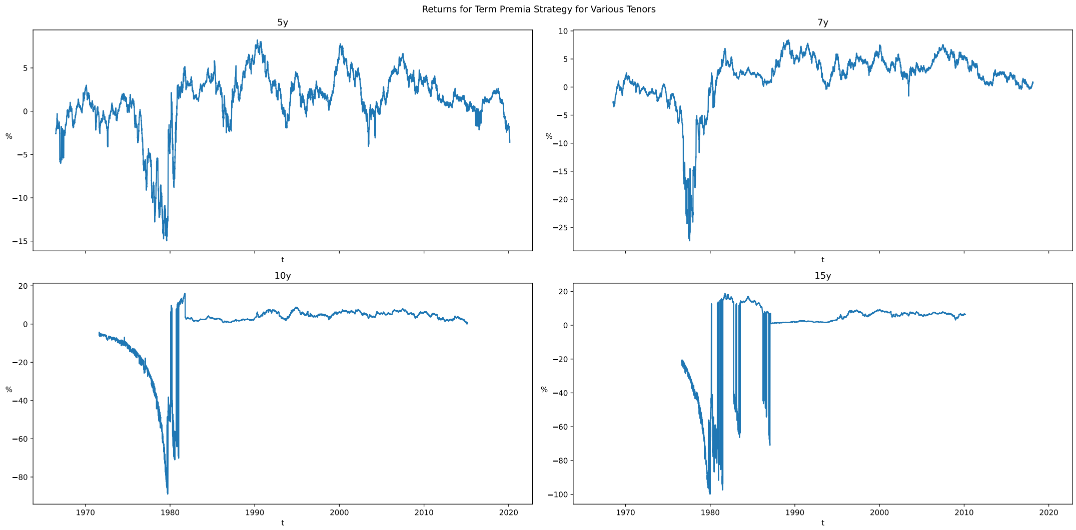

Figure 3: Returns when running the strategy on the full dataset with $m = 2.5$.

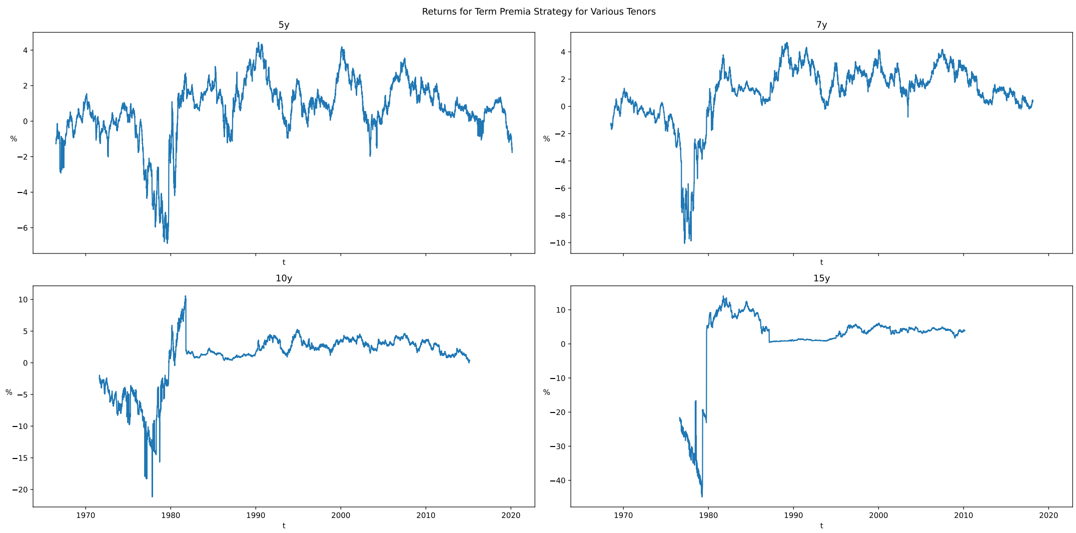

Figure 4: Returns when running the strategy on the full dataset with $m = 5$.

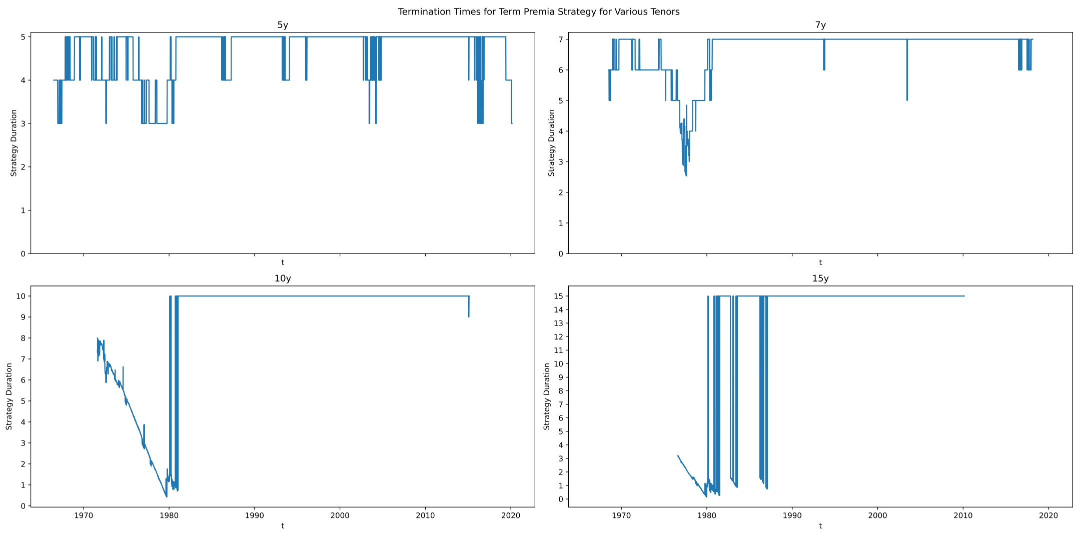

Figure 5: Termination times when running the strategy on the full dataset with $m = 2.5$.

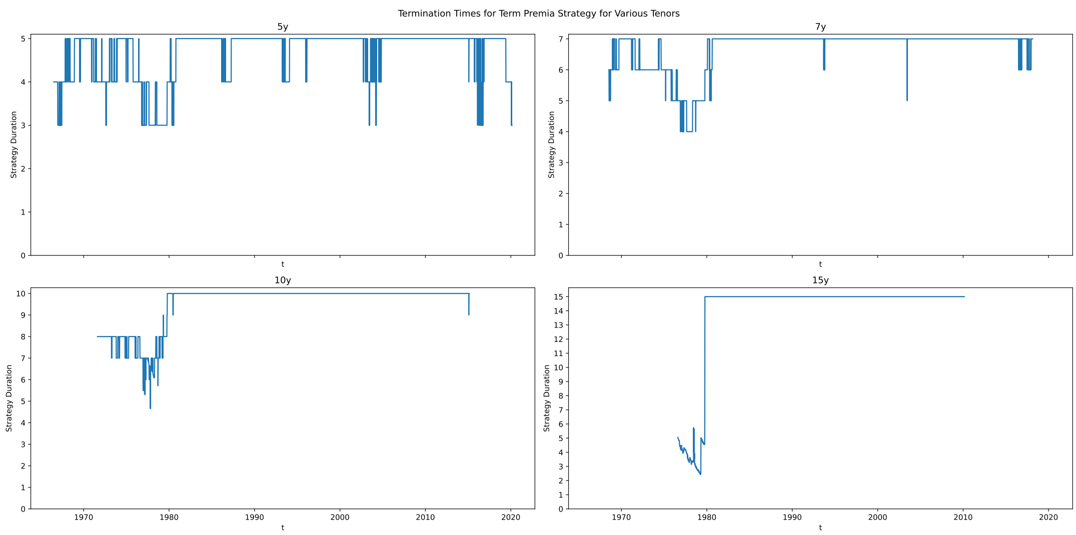

Figure 6: Termination times when running the strategy on the full dataset with $m = 5$.

To assess the statistical significance of the returns, we attempt to fit a distribution to the empirical results. By examining the histograms of the returns, we observe that the returns are not normal and seem to be clustered. In particular, we generally observe three clusters, suggesting a Gaussian Mixture with three components. We follow the algorithm outlined in [4] to estimate the parameters using a quasi-Bayesian approach. The CDF of a Gaussian mixture with three components, given weights $(w_i)_{i=1}^3$, means $(\mu_i)_{i=1}^3$, and standard deviations $(\sigma_i)_{i=1}^3$, is:

$$F(x) = \sum_{i=1}^{3} w_i \Phi\left(\frac{x - \mu_i}{\sigma_i}\right)$$

where $\Phi(z)$ is the CDF of the standard normal distribution and $w_3 = 1 - w_1 - w_2$.
It is of particular interest to assess risk by measuring the Value at Risk (VaR) and Expected Shortfall (ES) at different confidence levels. The VaR at confidence level α is the value x_α such

that $F(-x_\alpha) = 1 - \alpha$, and is solved numerically. By convention, positive values for VaR and ES represent losses.

The ES at confidence level α, is computed as

$$\mathrm{ES}_\alpha = -\frac{1}{1-\alpha}\int_{-\infty}^{-x_\alpha} t f(t)\,dt = -\frac{1}{1-\alpha}\sum_{i=1}^{3} w_i \int_{-\infty}^{-x_\alpha} \frac{t}{\sigma_i}\phi(t;\mu_i,\sigma_i)\,dt.$$

For each component, we use the substitution $z = \frac{t-\mu_i}{\sigma_i}$, so $t = \mu_i + \sigma_i z$, $dt = \sigma_i\,dz$, and the upper limit becomes $\frac{-x_\alpha-\mu_i}{\sigma_i}$:

$$\begin{aligned}\int_{-\infty}^{-x_\alpha} \frac{t}{\sigma_i}\phi(t;\mu_i,\sigma_i)\,dt &= \int_{-\infty}^{\frac{-x_\alpha-\mu_i}{\sigma_i}} (\mu_i + \sigma_i z)\phi(z)\,dz \\ &= \mu_i \int_{-\infty}^{\frac{-x_\alpha-\mu_i}{\sigma_i}} \phi(z)\,dz + \sigma_i \int_{-\infty}^{\frac{-x_\alpha-\mu_i}{\sigma_i}} z\phi(z)\,dz.\end{aligned}$$

The first integral is $\mu_i\Phi\left(\frac{-x_\alpha-\mu_i}{\sigma_i}\right)$, and the second is $-\sigma_i\phi\left(\frac{-x_\alpha-\mu_i}{\sigma_i}\right)$, since $\int_{-\infty}^{a} z\phi(z)\,dz = -\phi(a)$. Thus, the ES can be computed using the following formula.

$$\mathrm{ES}_\alpha = -\frac{1}{1-\alpha}\sum_{i=1}^{3} w_i\left[\mu_i\Phi\left(\frac{-x_\alpha-\mu_i}{\sigma_i}\right) - \sigma_i\phi\left(\frac{-x_\alpha-\mu_i}{\sigma_i}\right)\right].$$

where $\phi(z)$ is the standard normal PDF.

Figure 7 shows the model's fit to the returns using the strategy with $m = 2.5$ for $T = 10$. Each line represents a component of the Gaussian mixture, with an additional line for the total distribution. While the fit is not perfect, it is evident that one of the components captures the positive returns (around 4.5%), which represents the majority of the returns. Another component accurately models the left tail of the distribution, facilitating VaR and ES analysis, which is centered at around -50%. The final component, with a much smaller weight, represents the returns around -8%. To assess the risk exposure of the strategy, Figure 8 plots the analytical and theoretical (using the Gaussian mixture fit) VaR and ES for different values of α. The analytical values for $\alpha = 0.95, 0.99$ are presented in Table 1. As noted earlier, the quality of the fit to the left tail of the distribution is evident, as demonstrated by the close alignment between the empirical and analytical values.

Figure 9 shows another fit of the model for the same tenor but using $m = 5$. For the curious reader, the rest of the model fits and VaR/ES graphs for the rest of the tenors with $m = 2.5, 5$ are left in Appendix A.

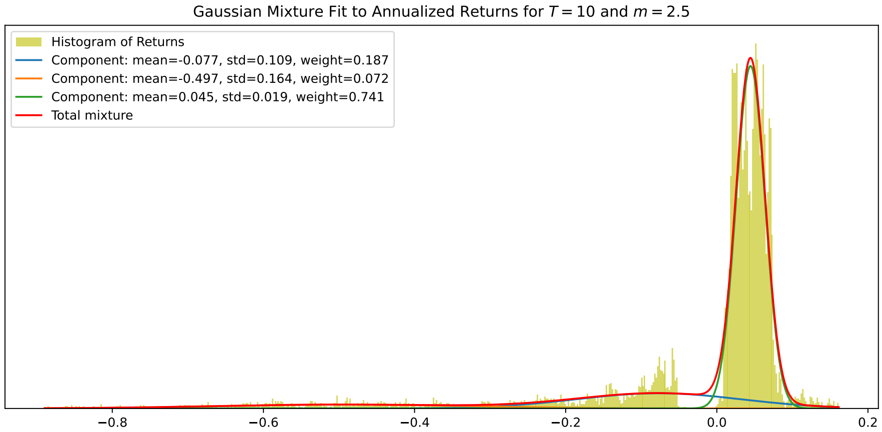

Figure 7: Histogram and Gaussian mixture fit of returns with $m = 2.5$ and $T = 10$.

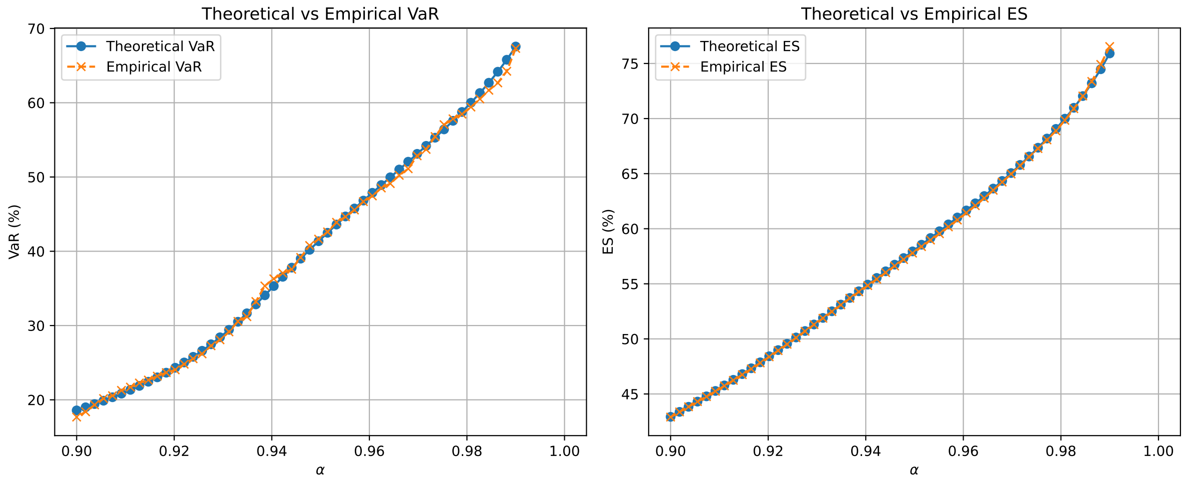

Figure 8: Theoretical and Empirical VaR and ES on the full dataset with $m = 2.5$ and $T = 10$.

	VaR		ES	
Tenor \ α	95%	99%	95%	99%
5	5.98%	12.2%	9.88%	14.4%
7	6.16%	20.5%	14.5%	23.7%
10	41.6%	67.6%	58.1%	75.9%
15	63.2%	84.6%	76.4%	93.5%

Table 1: Analytical VaR and ES of returns with $m = 2.5$.

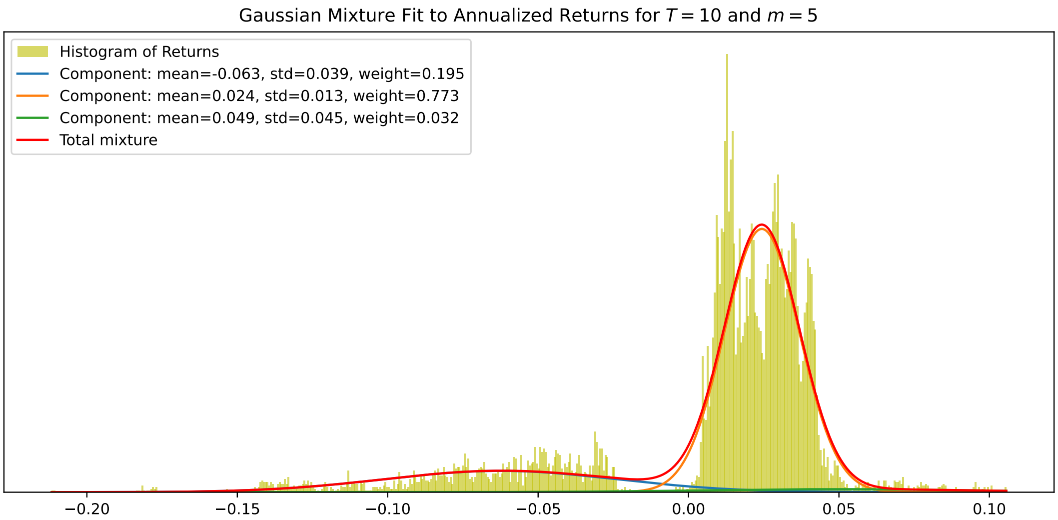

Figure 9: Histogram and Gaussian mixture fit of returns with $m = 5$ and $T = 10$.

We show the means and standard deviations of the 5, 7, 10 and 15-year strategy returns for different multipliers in Figure 10, as well as the Sharpe ratios in Figure 11. The returns are heavily biased due to the left tail of the distribution, which is much longer and heavier than the right one. As a result, even for the highest multiplier, which shortens the left tail, the Sharpe ratio is still under 0.5.

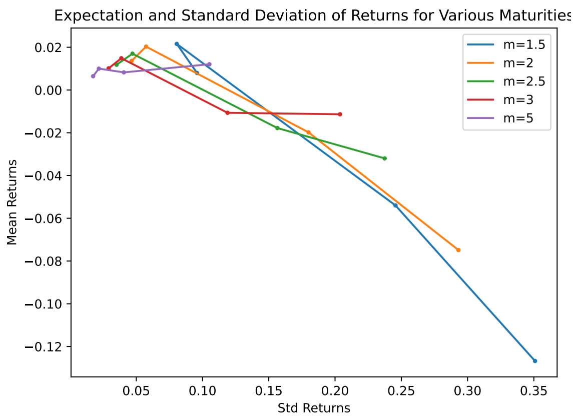

Figure 10: Means and stds of returns on the full dataset.

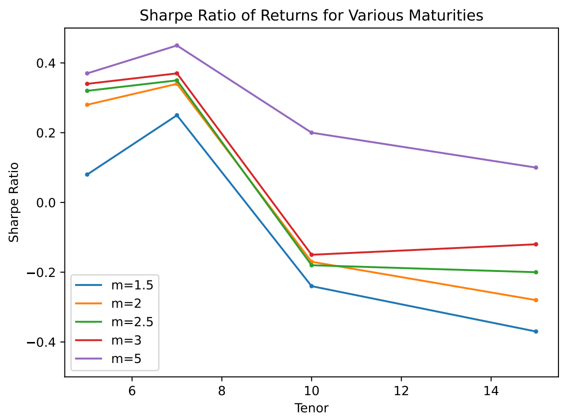

Figure 11: Sharpe ratios of returns on the full dataset.

We observe that most of the extreme negative returns occur during the 70s and 80s, when rates were much higher than recent years and even spiked to 15%. This causes the VaR and ES to reach extreme values, and are noticeably worse for longer tenors. As a result, this might not be an accurate representation of the strategy performance to be expected today. Therefore, in the next subsection, we analyse the returns for runs of our strategy from 1987 onwards.

3.3 1987 Onwards

The strategy performs much better from 1987 onwards, as shown below in Figure 12 and 13. Note that the Sharpe ratio here is calculated simply as the mean divided by the standard deviation of returns without accounting for the risk-free return, which will be analysed in the next subsection. We observe that $m = 2.5$ maximises the Sharpe ratio, which addresses the aforementioned trade-off between early termination and returns. When m increases further, the decrease in expected returns is evident, while the improvement in standard deviation is much less noticeable, as extra cash has a marginal impact on the termination ratio. Therefore, we fix $m = 2.5$ for the rest of the analysis.

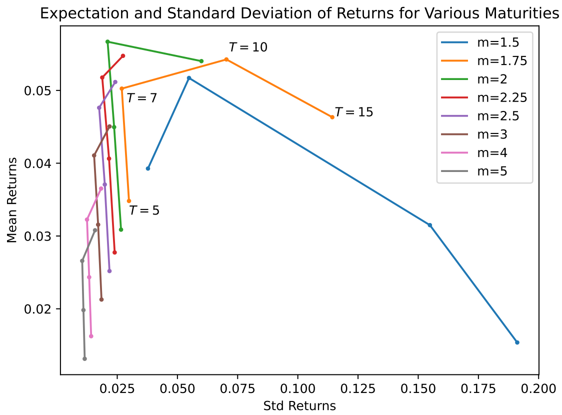

Figure 12: Means and standard deviations of returns after 1987.

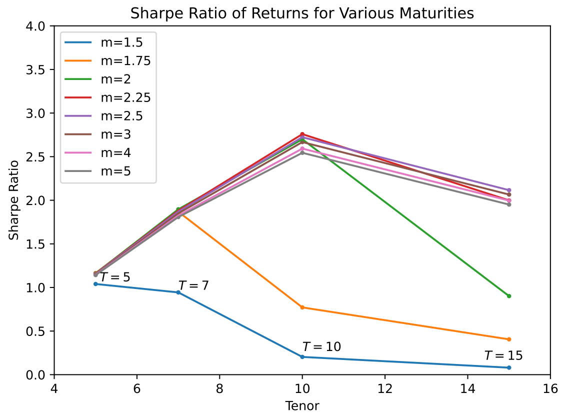

Figure 13: Sharpe ratios (not excess of risk-free) of returns after 1987.

Unlike the full set of returns, the returns from 1987 onwards are mostly positive and deviates from Gaussian, resulting in a much worse fit using a Gaussian mixture (Figure 14). Therefore, we display the empirical VaR and ES (Table 2) instead of their analytical counterparts.

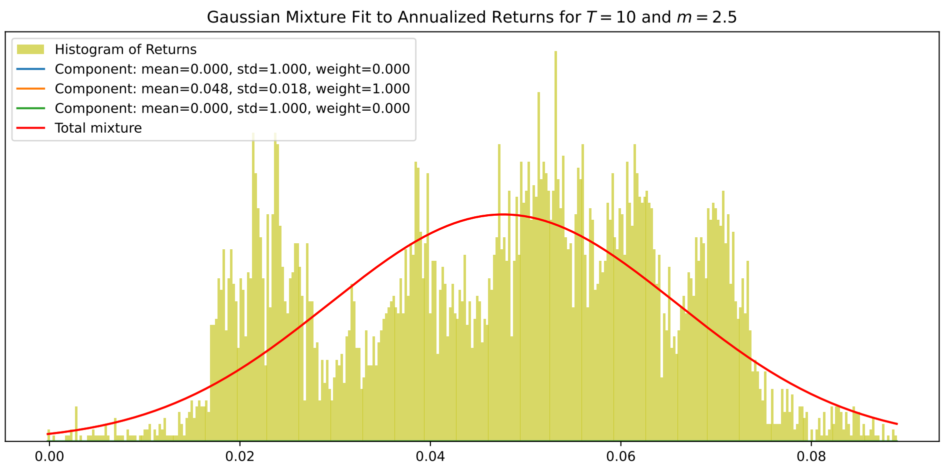

Figure 14: Histogram and Gaussian mixture fit to returns after 1987.

	VaR		ES	
Tenor \ α	95%	99%	95%	99%
5	0.68%	2.08%	1.69%	2.76%
7	-0.41%	0.14%	-0.05%	0.25%
10	-1.91%	-1.38%	-1.56%	-0.80%
15	-1.71%	-1.58%	-1.63%	-1.54%

Table 2: Empirical VaR and ES of returns after 1987.

Although we observe that three out of four 95% VaR and two out of four of the 99% VaR are negative (the strategy very rarely yields negative returns), this does not take the risk-free rate into account. When compared to the risk-free rate of growth, this strategy greatly underperforms. This motivates the next subsection, which considers overnight cash growth and returns excess of the risk-free rate, which is computed by rolling the overnight rate daily until the strategy terminates.

3.4 Overnight Growth and Excess Returns

In this subsection, we compare the excess returns for the strategy with and without overnight cash growth for $m = 2.5$ (Figure 15). We observe that the difference is the largest during the start of this time period, which is unsurprising, as the average one-year yield was around 6% during the 90s.

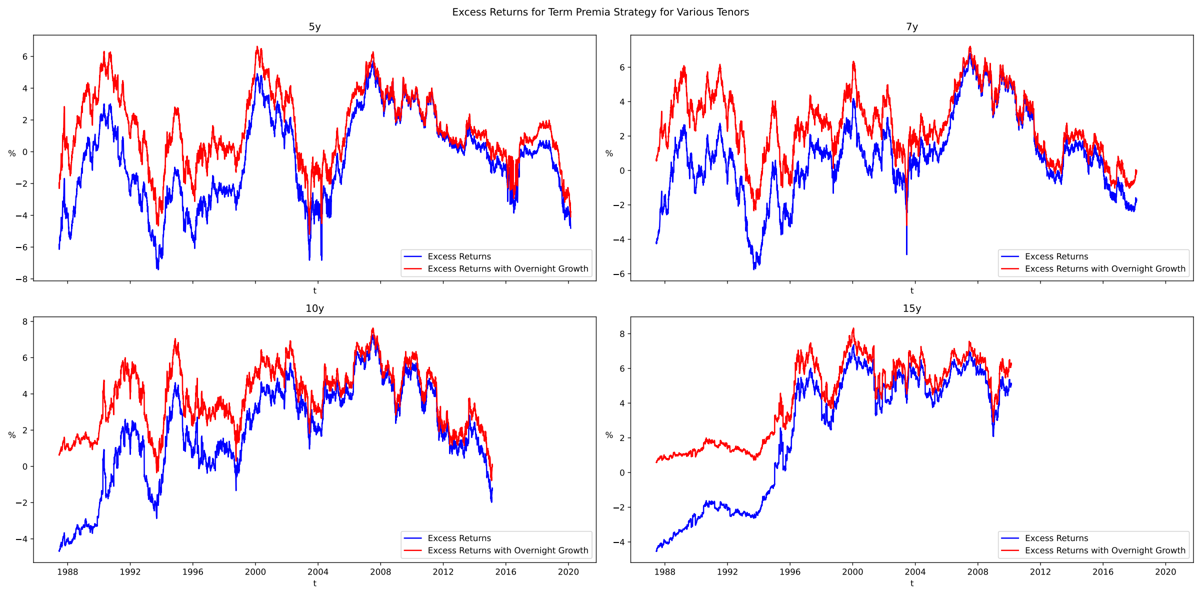

Figure 15: Excess Returns of Strategy With and Without Overnight Cash Growth ($m = 2.5$)

When considering excess returns above the risk-free rate, the strategy without overnight cash growth yields unimpressive results, as extra cash on hand do not experience any growth. Over this simulation window, the 5-year strategy had negative mean excess returns, while the 7, 10 and 15-year strategies had a mean excess return of 0.80%, 1.92% and 2.35% respectively. With overnight growth, the strategies have much higher excess returns (1.40%, 2.62%, 3.79% and 4.29% respectively), with the 10-year strategy having the highest Sharpe ratio, as shown in Figure 16 and Figure 17. In practice, this facilitates choosing a more conservative value for m to prevent early termination, while not wasting idle cash allocated to the strategy.

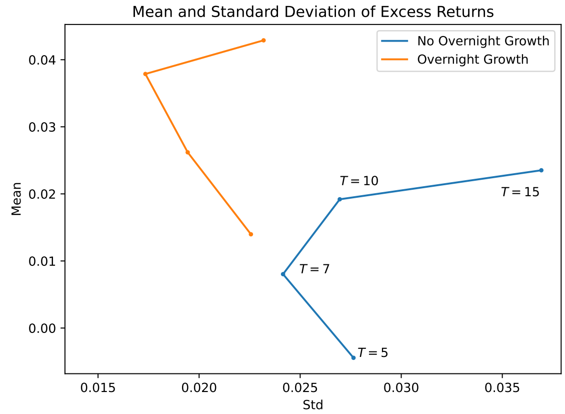

Figure 16: Means and Stds of Excess Returns With and Without Overnight Cash Growth

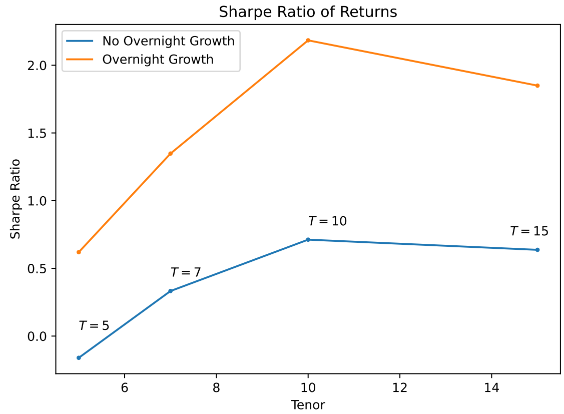

Figure 17: Sharpe Ratios With and Without Overnight Cash Growth

Another metric that we use to evaluate the strategy is the maximum drawdown of the wealth process defined in Equation 2. The graph on the left of Figure 18 shows the maximum drawdown of each run of the 15-year strategy, and the graph on the right shows the wealth process corresponding to the run with the worst drawdown, indicated by the grey line. We observe that the maximum drawdown occurs early in the strategy, which is likely due to the higher sensitivity of the long-term bond to interest rate changes, as it has a much longer duration.

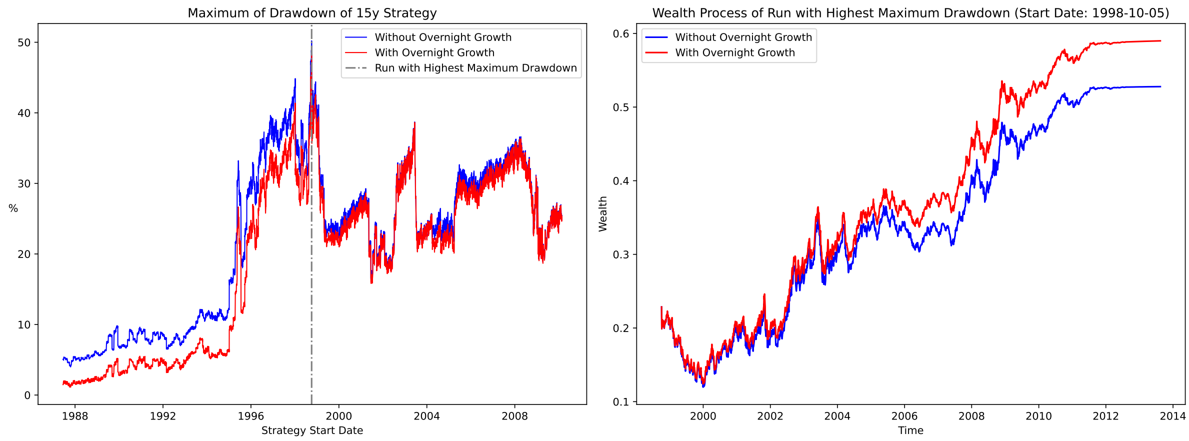

Figure 18: Maximum drawdown of every run and the wealth process of the worst run

It is common for hedge funds and portfolio managers to have limitations on the maximum drawdown of their allocation, and in our framework, this could be achieved by increasing the multiplier m. As a potential improvement, a time-varying multiplier could also be considered, providing a larger buffer for the early stages where the wealth process experiences the biggest fluctuations in general.

For completeness, we include the empirical VaR and ES values for the returns with overnight growth in Table 3. Note that the 15-year strategy achieves a 5% return even at 99% VaR.

	VaR		ES	
Tenor \ α	95%	99%	95%	99%
5	-0.83%	0.81%	0.30%	1.40%
7	-1.40%	-0.95%	-1.09%	-0.66%
10	-2.97%	-2.27%	-2.49%	-1.93%
15	-5.39%	-5.04%	-5.12%	-4.71%

Table 3: Empirical VaR and ES of returns with overnight growth.

3.5 Unwinding Strategy After One Year

In this section, we show the results of the strategy in which we unwound our positions after one year. Unsurprisingly, the variance of the strategy increases, because the returns are solely dependent on the movement of the long-bond price, which are particularly sensitive to interest rate changes near inception. Figure 19 shows the excess returns of the current strategy compared to the previous strategy, both including overnight growth. Although the means of the excess returns have increased, the high variance makes this strategy infeasible.

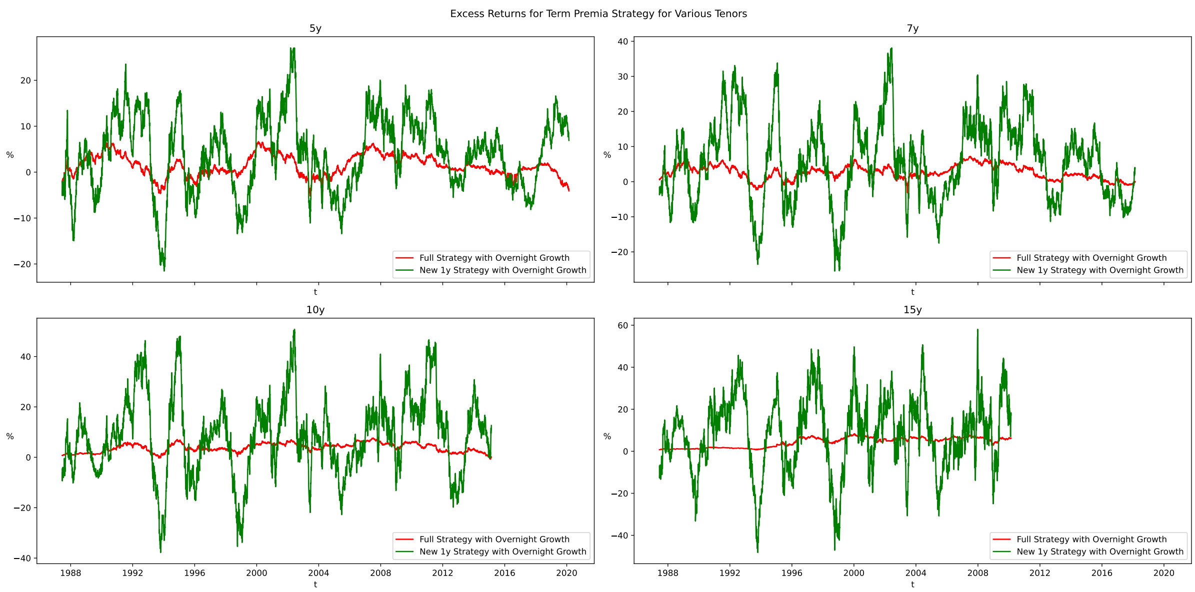

Figure 19: Excess returns of strategy unwound after one year and previous strategy.

In this case, the returns are fitted with ease using the Gaussian mixture model (Figure 20), where the three components fit the returns with high accuracy (Figure 21), and will facilitate effective risk management. For the interested reader, the model fits and VaR/ES graphs for the other tenors are left in Appendix A.

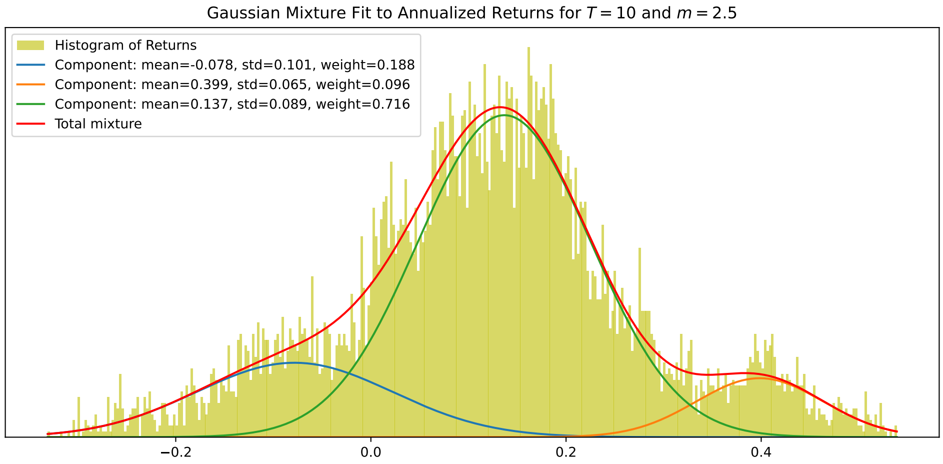

Figure 20: Histogram and Gaussian mixture fit of excess returns of strategy unwound after one year with $T = 10$.

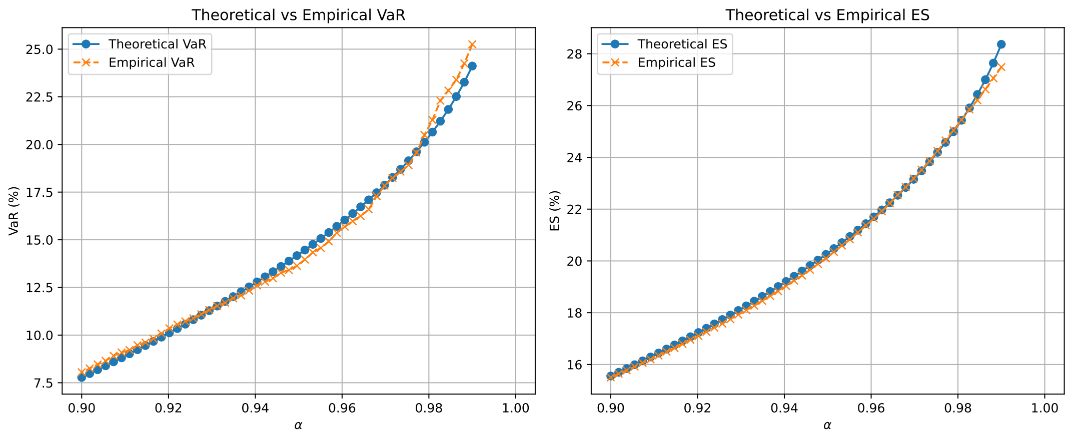

Figure 21: Theoretical and empirical VaR and ES of strategy unwound after one year with $T = 10$.

From Figure 22, we observe that while the mean excess return reaches 8% for some tenors, the standard deviation approximately increases by a factor of 5, which causes the Sharpe ratios to be much lower than the full strategy (Figure 23).

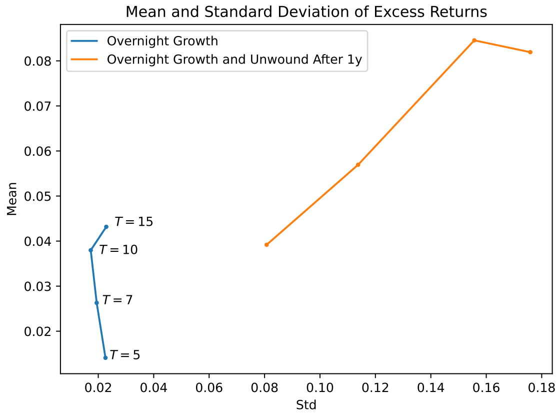

Figure 22: Means and stds of excess returns unwound after 1y.

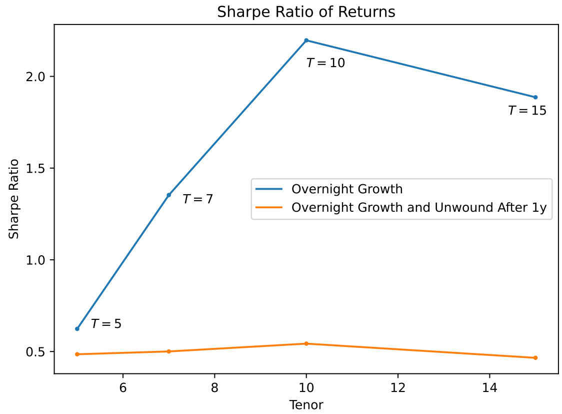

Figure 23: Sharpe Ratios of returns unwound after 1y.

When analysing the maximum drawdown of the wealth processes, we observe in the graph on the left of Figure 24 that the maximum drawdown of the one-year strategy is higher on average for the earlier parts of the dataset, while it is slightly lower for the later parts. Although the strategy only runs for one year, it offers little to no improvement in terms of maximum wealth drawdown. We also note that the highest maximum drawdown is attained by the same run as before, as shown on the left of Figure 24

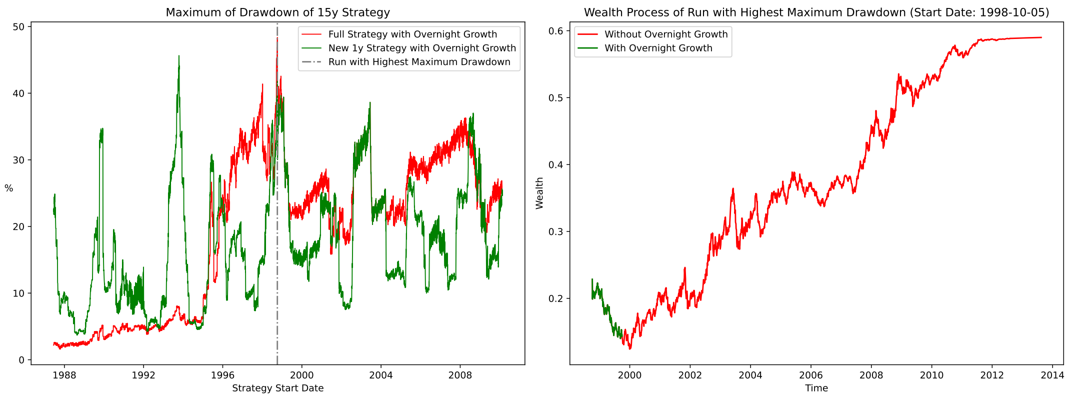

Figure 24: Maximum drawdown of every 15-year run unwound after one year and the wealth process of the worst run

3.6 Two-Year Maturity Short Bonds

In this section, we analyse the returns for a variation of the 10-year strategy, where two-year bonds are shorted rather than one-year bonds. The time series of returns are displayed in Figure 25, and the empirical distributions are shown in 26.

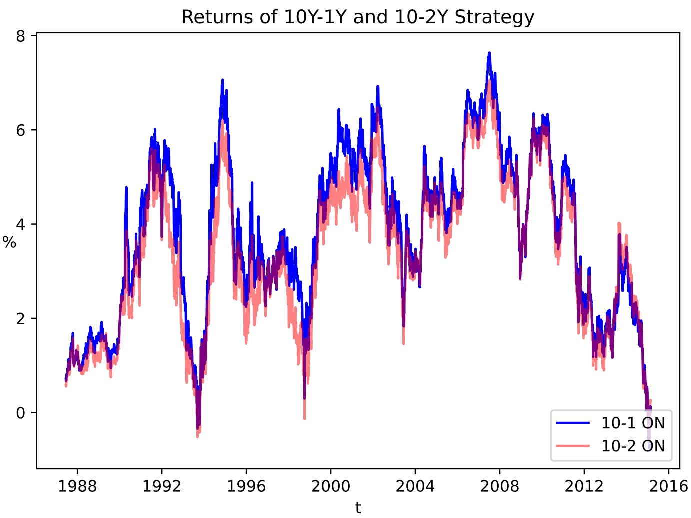

Figure 25: Percentage returns using 1-year versus 2-year short bonds with overnight growth.

As expected, the returns of these strategies are still highly correlated, due to the fact that the long-end exposure (the 10-year bond) is the same in both. Additionally, the proximity of the short bond maturities (1-year and 2-year) relative to the long bond's tenor means that both strategies react similarly to interest rate changes. We observe that, whilst this strategy still delivers positive average returns, it consistently underperforms the standard strategy in which we short 1-year bonds. This underperformance is evident in the return distributions shown in Figure 26. This aligns with our intuition that term premia strategies benefit from the steepness of the yield curve, and hence shorting a longer maturity bond (such as two years) reduces the spread between the long and short yields, diminishing our expected returns. For this reason, we do not experiment with strategies shorting bonds of even longer maturities.

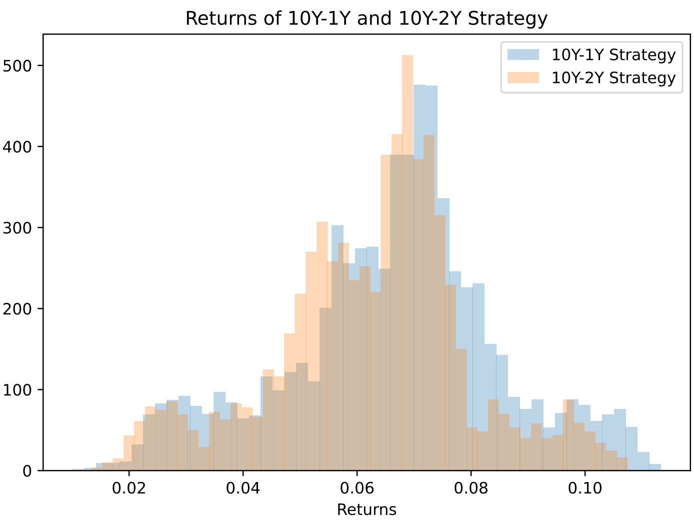

Figure 26: Histogram of returns using 1-year versus 2-year tenor maturity bonds

4 Conclusion

In this report, we examined the existence and impact of term premium through a trading strategy that purchases long-maturity bonds financed by rolling short positions in short-maturity bonds. Our analysis incorporates realistic constraints, such as margin requirements, early termination, and overnight cash growth, to better replicate market conditions. The historical simulations indicate that while the strategy has the potential to yield positive excess returns, its performance is highly dependent on market conditions, particularly during periods of elevated interest rate volatility. Specifically, we observed that the strategy demonstrated significant sensitivity to changes in interest rates, especially during the sharp interest rate fluctuations of the 1980s. When the strategy is run from 1987 onwards (avoiding the earlier period of sudden, large interest rates spikes), we observe strong performance, as evidenced by the expected excess returns and Sharpe ratios presented.

Despite these seemingly positive findings, limitations remain. Transaction costs and liquidity constraints, which would affect real-world implementation are not accounted for, and bond price interpolation introduces inaccuracies. Additionally, we observe that the strategy carries the most risk (in terms of maximum drawdown) near the inception of the strategy. Therefore, it could be beneficial to explore dynamic capital allocation, which provides a bigger capital buffer early on, and is reallocated to other alternatives later in order to achieve a higher portfolio return. Future research might also investigate hedging strategies to mitigate tail risk.

Furthermore, the use of a Gaussian mixture model to fit the return distribution proved beneficial in capturing the nature of the returns, allowing for a more accurate representation of the risks involved in the strategy. By modelling the returns as a mixture of three distributions, we were able to capture both the positive excess returns and the extreme left-tail risks, thus providing a more robust understanding of potential losses.

Ultimately, our results indicate that based on historical bond prices, potential for impressive returns from term premium exists. However, practical challenges such as liquidity constraints, transaction costs, and pricing estimation errors must be addressed for real-world implementation. The strategy's performance is also highly sensitive to market conditions, so future research should focus on refining risk management strategies and integrating advanced techniques such as machine learning to dynamically optimize the margin multiplier based on the yield curve regime, enhancing the adaptability and resilience of such strategies in future market conditions.

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A Gaussian Mixture and VaR/ES plots

A.1 Full dataset returns

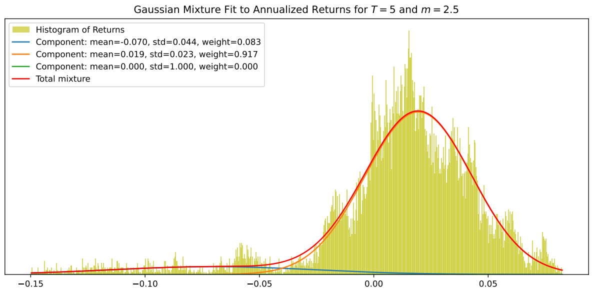

Figure 27: Histogram and Gaussian mixture fit of returns with $m = 2.5$ and $T = 5$.

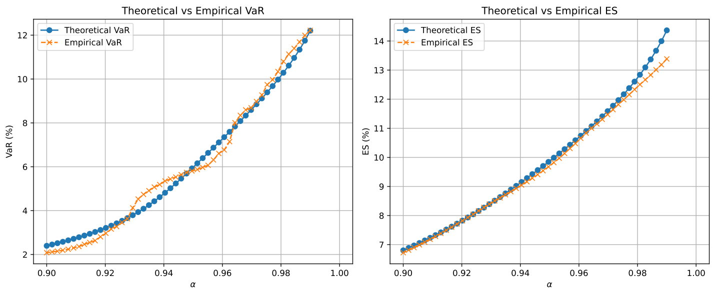

Figure 28: Theoretical and Empirical VaR and ES on the full dataset with $m = 2.5$ and $T = 5$.

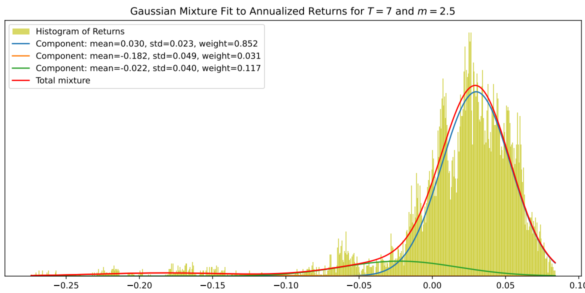

Figure 29: Histogram and Gaussian mixture fit of returns with $m = 2.5$ and $T = 7$.

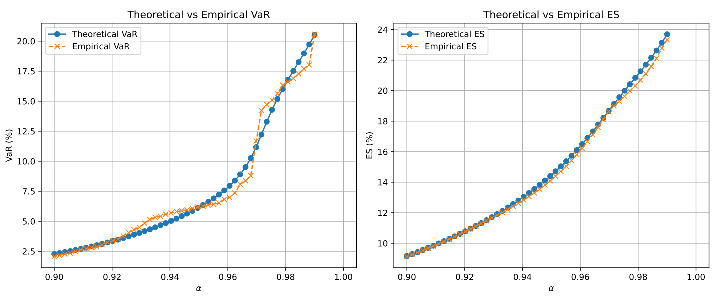

Figure 30: Theoretical and Empirical VaR and ES on the full dataset with $m = 2.5$ and $T = 7$.

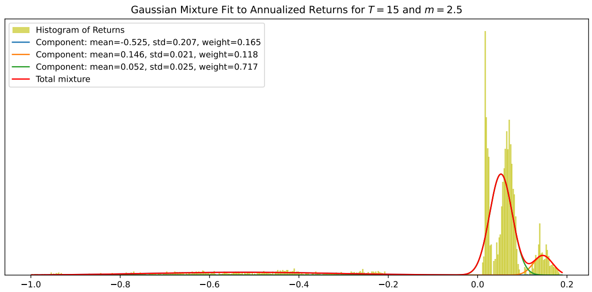

Figure 31: Histogram and Gaussian mixture fit of returns with $m = 2.5$ and $T = 15$.

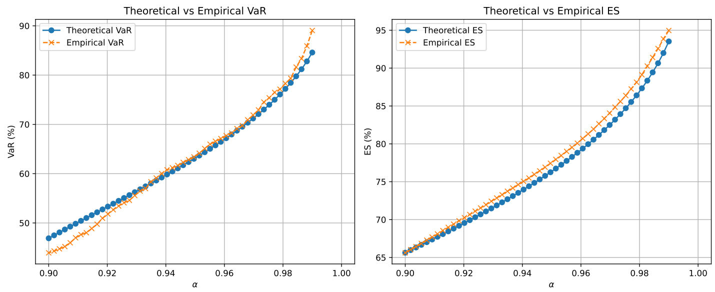

Figure 32: Theoretical and Empirical VaR and ES on the full dataset with $m = 2.5$ and $T = 15$.

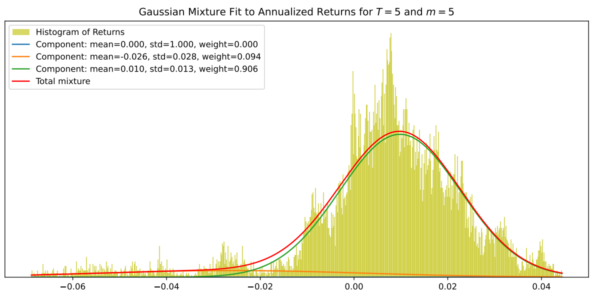

Figure 33: Histogram and Gaussian mixture fit of returns with $m = 5$ and $T = 5$.

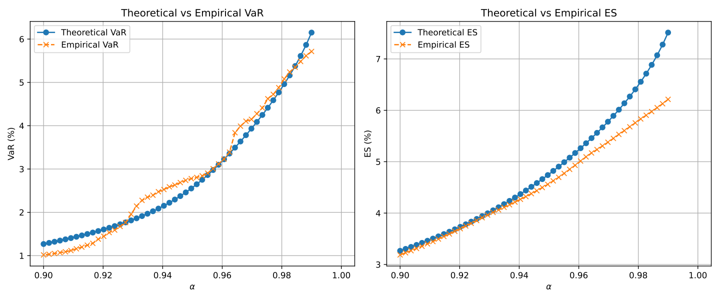

Figure 34: Theoretical and Empirical VaR and ES on the full dataset with $m = 5$ and $T = 5$.

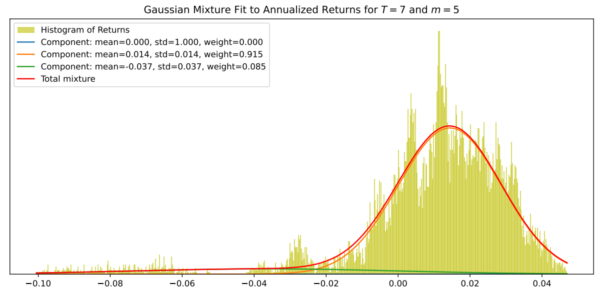

Figure 35: Histogram and Gaussian mixture fit of returns with $m = 5$ and $T = 7$.

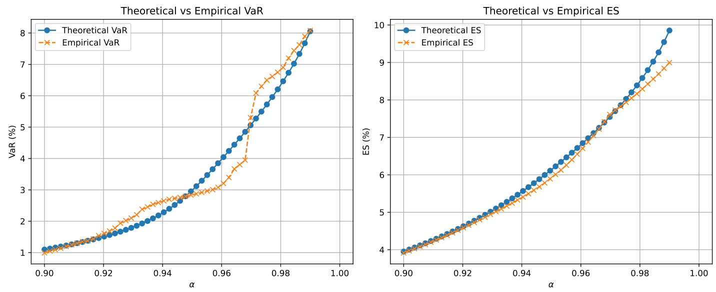

Figure 36: Theoretical and Empirical VaR and ES on the full dataset with $m = 5$ and $T = 7$.

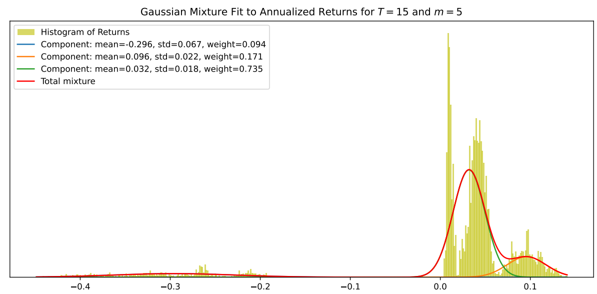

Figure 37: Histogram and Gaussian mixture fit of returns with $m = 5$ and $T = 15$.

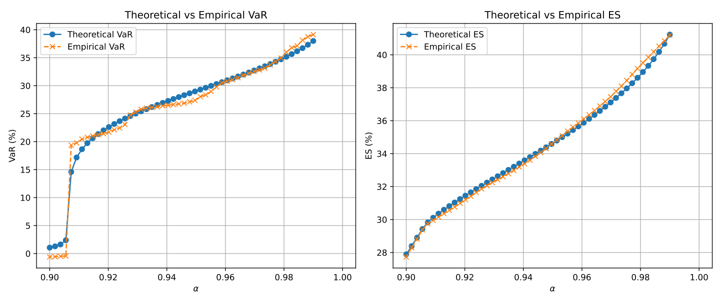

Figure 38: Theoretical and Empirical VaR and ES on the full dataset with $m = 5$ and $T = 15$.

A.2 Unwound After One Year Excess Returns

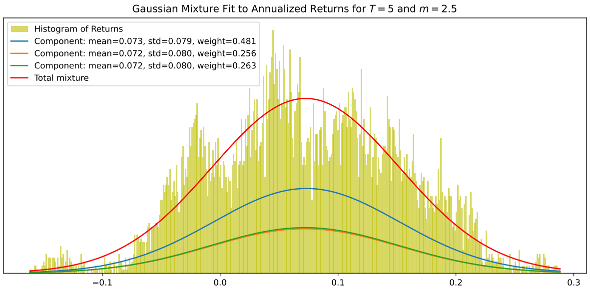

Figure 39: Histogram and Gaussian mixture fit of excess returns after 1y unwound with $m = 2.5$ and $T = 5$.

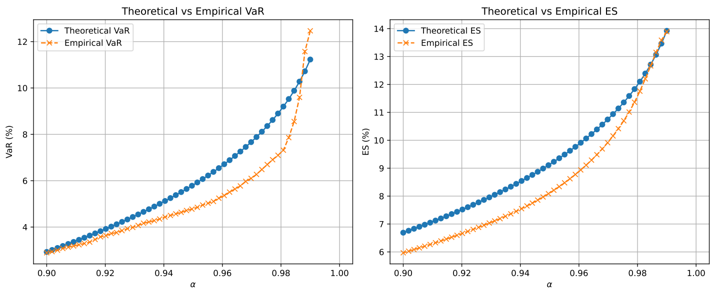

Figure 40: Theoretical and Empirical VaR and ES after 1y unwound with $m = 2.5$ and $T = 5$.

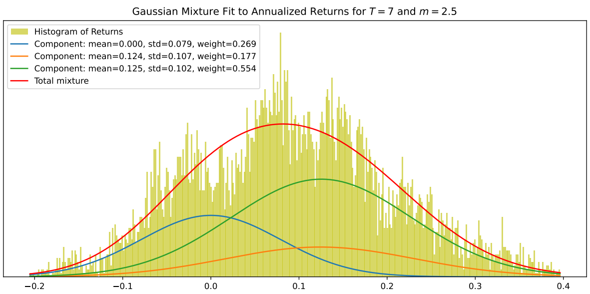

Figure 41: Histogram and Gaussian mixture fit of excess returns after 1y unwound with $m = 2.5$ and $T = 5$.

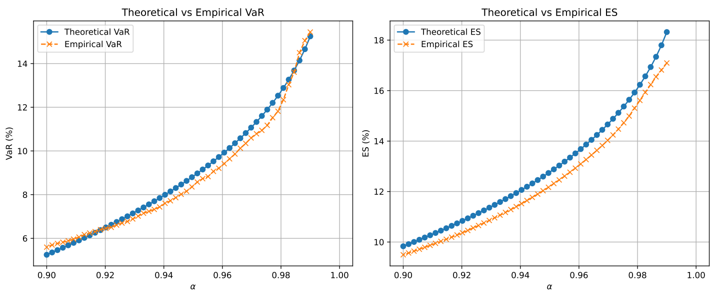

Figure 42: Theoretical and Empirical VaR and ES after 1y unwound with $m = 2.5$ and $T = 5$.

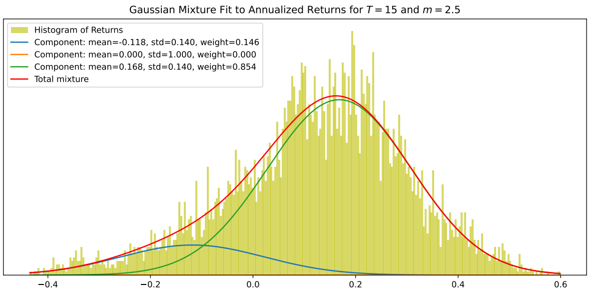

Figure 43: Histogram and Gaussian mixture fit of excess returns after 1y unwound with $m = 2.5$ and $T = 5$.

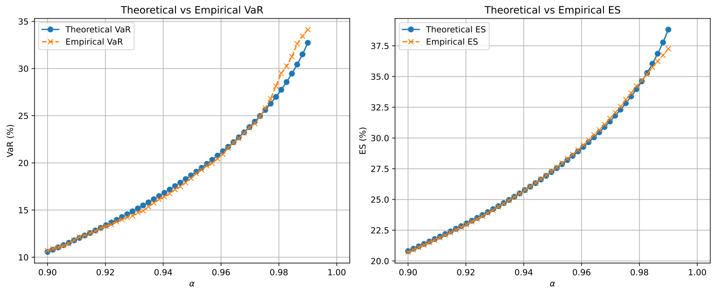

Figure 44: Theoretical and Empirical VaR and ES after 1y unwound with $m = 2.5$ and $T = 5$.