Project: Image Completion and Inpainting

Week 1: Image Inpainting

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Optimization and Inference Techniques for Computer Vision

Master in Computer Vision *Barcelona*

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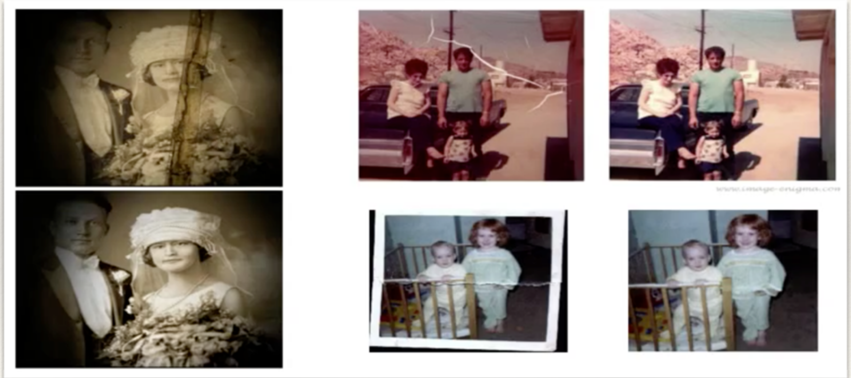
What is Inpainting?

Detail of "Cornelia, Mother of the Gracchi" by J. Suvee (Louvre). Courtesy of Emile-Male "The Restorer's Handbook of easel painting".

What is Inpainting?



What is Inpainting?

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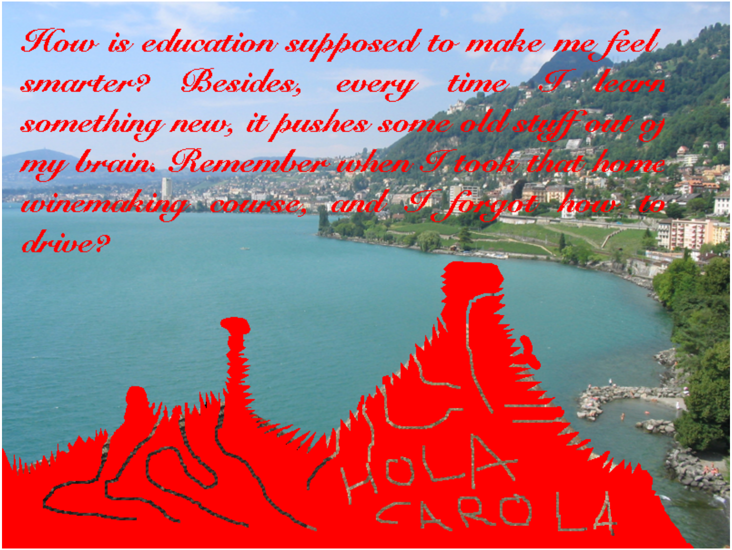
What is not Inpainting?

Goal


How is education supposed to make me feel smarter? Besides, every time I learn something new, it pushes some old stuff out of my brain. Remember when I took that home winemaking course, and I forgot how to drive?
HOLA
CAROLA

Inpainting using Laplace's Equation

Let $f : \Omega \to \mathbb{R}$ be a given grayscale image and let $D \subset \Omega$ be an open set representing the region to be inpainted.

It is supposed that f is known in $\Omega \backslash D := \{x \in \Omega : x \notin D\}$

The inpainting solution u can be found as the minimum of

$$\begin{cases} \displaystyle\arg\min_{u \in W^{1,2}(\Omega)} \int_D |\nabla u(x)|^2 dx, \\ u|_{\partial D} = f \end{cases}$$

and satisfies the Laplace's equation

$$\begin{cases} \Delta u & = & 0 & \text{in } D, \\ u & = & f & \text{in } \partial D \end{cases}$$

where Δ denotes the Laplacian and $u = f$ in $\Omega \backslash D$

The equation is completed with homogeneous Neumann boundary conditions at the boundary of the image.

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Inpainting using Laplace's Equation

Laplacian of u

$$\Delta u = \mathrm{div}(\nabla u)$$

Spatial Gradient

Forward differences

$$(\nabla u)_{i,j} = (u_{x_{i,j}},\ u_{y_{i,j}})$$

with

$$u_{x_{i,j}} = \frac{u_{i+1,j} - u_{i,j}}{h_i} \qquad u_{y_{i,j}} = \frac{u_{i,j+1} - u_{i,j}}{h_j}$$

Divergence

Backward Differences. Let $\bar{v} = (v_x, v_y)$ be a vector

$$\mathrm{div}(\bar{v})_{i,j} = \frac{(v_{x_{i,j}} - v_{x_{i-1,j}})}{h_i} + \frac{(v_{y_{i,j}} - v_{y_{i,j-1}})}{h_j}$$

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Mandatory

- Build the matrix A and vector b for the algebraic system of equations obtained after the given discretization of the operators.
- Test with the given images. There is one image (image6.tif) that missed the 99% of information

- Proof that the solution of the Laplace's equation is a solution of the problem

$$\begin{cases} \displaystyle\arg\min_{u\in W^{1,2}(\Omega)} \int_D |\nabla u(x)|^2 dx, \\ u|_{\partial D} = f \end{cases}$$

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