

Algorithmic Redistricting Problem for Washington using Integer Programming

Introduction

The purpose of this assignment is to define a fair and equitable redistricting plan for the state of Washington. Redistricting refers to the process of redrawing the boundaries of electoral districts within a region or country. The primary purpose of redistricting is to ensure that each district has roughly equal population sizes to uphold the principle of "one person, one vote" and to provide fair and equal representation to all citizens. In this paper, we delve into the intricate problem of redistricting using integer programming, focusing on defining fairness, setting constraints, and implementing population balance and compactness measures. The chosen state for this analysis, which is Washington, has 10 legislative districts and 39 counties.

Data Sources

We utilized data from reputable sources such as the 2020 US Census of Population, the World Population Review for county demographics, and the US Census Bureau's county adjacency information. Since these are trusted sources by the official US Census website, we are very confident in their data.

Objective Function and Constraints

We aim to determine the target population that our 10 districts should have when redistricting. Two Washington counties (Pierce and Snohomish) have large enough populations that they can each be automatically assigned one individual district, and one county (King) has a large enough population to be assigned three districts of its own. The remaining 5 districts have a target population of 572,411.

The objective function uses LpSum to calculate the sum of a list of linear expressions in an array of binary decision variables. Model constraints include allocating 100% of the population from each county, ensuring each county is assigned exactly one district, adjacency constraints for contiguous districts, and population size constraints that keep each district within 10% of the target population. The primary decision variables use the format "Y_countyNumber_districtNumber", which are binary variables that indicate if a given county has been assigned to a given district.

For one-person one-vote, we created constraints that set an upper and lower bound on the final district populations. This results in districts that are roughly equal in population.

To define geographically compact districts, we used county adjacency data to create constraints on which sets of counties result in acceptable districts.

However, the group faced challenges trying to incorporate the adjacency constraints into the model. The adjacency constraints and population size constraints were developed separately, and we were unable to correctly combine them in `integer_programming.py`, resulting in some districts that are non-contiguous. Therefore, further investigation into this must be considered to determine more accurate electoral districting for the state of Washington.

Programming Implementation

The linear programming problem was implemented using Python's PuLP library. The code and output listings are available in our GitHub repository dedicated to this assignment (<https://github.com/sergiovalentini11/msds460-assignment6>). The following are files contained within the repository with descriptions of how they were obtained:

001.Data Bases: Contains various .csv files containing Washington county census data.

Importing_Data_Web_Adjacency.ipynb: Utilizes BeautifulSoup to extract county adjacency data, which was saved in `washington_counties_adjacency.csv`

Importing_Data_Web_Population_Extractor.ipynb: Utilizes BeautifulSoup to extract county census data, which was saved in `washington_census_data.csv`

`integer_programming_constraints_done.ipynb`: Creates and implements constraints for ensuring districts contain only adjacent counties

`integer_programming.py`: The primary code file uses PuLP to create and solve the integer programming problem. This file attempts to incorporate the adjacency constraints from `integer_programming_constraints_done.ipynb` into its model.

Actual Districts.pdf: The 2021 Washington State Congressional District Map, sourced from <https://www.redistricting.wa.gov/district-maps-handouts>

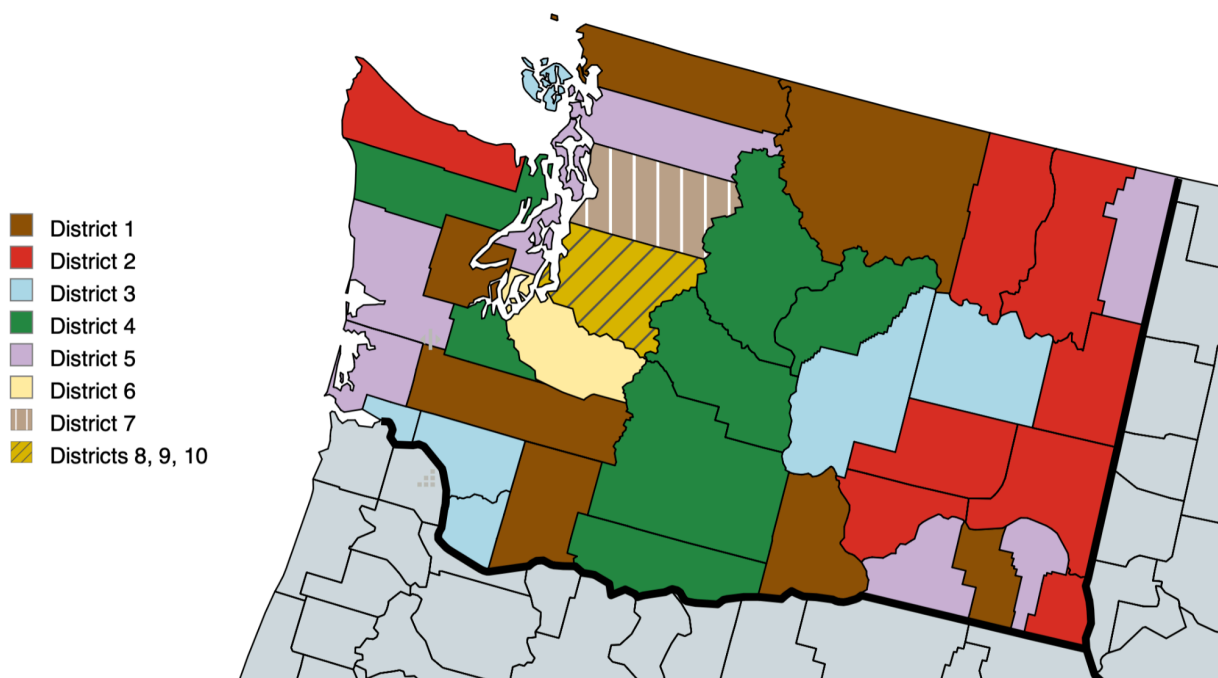
Redistricted Counties.png: the districts resulting from the `integer_programming.py` model output

output.txt: output of `integer_programming.py`

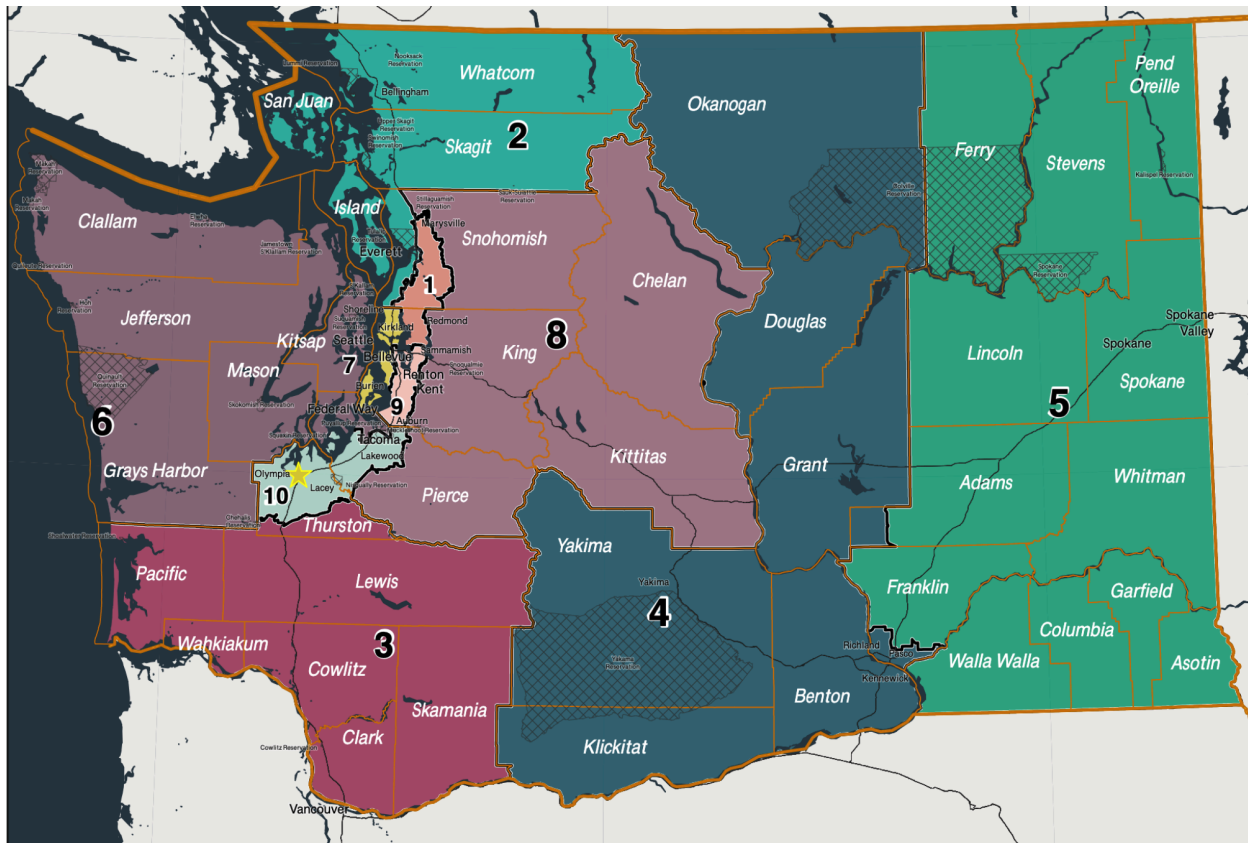
Maps and Discussion

A color-coded map showcasing our algorithmic/optimal redistricting was created, as seen below, highlighting each of the 10 new Congressional districts. A comparison with existing redistricting plans, such as those from the Districtr utility, revealed variations in district shapes and demographics. Evaluating fairness and equity requires considering multiple factors beyond mathematical optimization, including legal frameworks, community input, and historical context.

Ultimately, we will not be able to submit our redistricting plan to the state's legislature because our plan lacks contiguous districts. Before we submit our plan for review, we will need more time to correctly incorporate the adjacency constraints into the model and produce legally eligible districts.



The actual district for the state of Washington is shown below.



Conclusion

Our optimal redistricting plan for Washington aimed at achieving population balance and adjacent districts. The plan balances district populations while considering geographic compactness. As the code was being written, we realized we need 36 counties to fill 5 districts, which gives us 180 decision variables.

After running the integer programming model, the solution to this problem for the state of Washington is that the minimum number of counties is 36, and the below table shows how the redistricting for these 36 counties across 5 districts looked like:

	Total_Population	Total_Population_White_Alone	state_county	Name	Latitude	Longitude	District
36	421551	354720	53063	Spokane	47.5909	-117.278	2
5	385835	300790	53011	Clark	45.7466	-122.519	3
27	231089	175784	53067	Thurston	46.8646	-122.77	4
16	219552	169927	53035	Kitsap	47.6477	-122.641	5
30	184371	97808	53077	Yakima	46.5436	-120.756	4
29	182483	145835	53073	Whatcom	48.8787	-121.972	1
2	152644	113123	53005	Benton	46.3166	-119.502	1
35	101710	79544	53057	Skagit	48.4242	-121.711	5
7	85596	72143	53015	Cowlitz	46.1746	-122.775	3
14	71175	57361	53029	Island	48.1976	-122.58	5
12	70900	43781	53025	Grant	47.1981	-119.373	3
10	66302	33500	53021	Franklin	46.4937	-118.867	2
4	64588	53896	53009	Clallam	48.1134	-123.799	2
18	64000	54326	53041	Lewis	46.6195	-122.456	1
3	61405	45417	53007	Chelan	47.9445	-120.675	4
13	60530	49109	53027	GraysHarb	46.9954	-123.701	5
33	53265	43108	53045	Mason	47.4251	-123.195	1
28	49597	37430	53071	WallaWalla	46.237	-118.586	5
38	40521	31026	53075	Whitman	46.8363	-117.589	1
26	36608	31669	53065	Stevens	48.4558	-117.906	5
17	36115	30252	53037	Kittitas	47.175	-120.932	4
20	32978	22925	53047	Okanogan	48.5897	-119.652	1
8	32199	22276	53017	Douglas	47.7791	-119.748	4
31	28951	25524	53031	Jefferson	47.7425	-123.304	4
21	19358	16338	53049	Pacific	46.5181	-123.827	5
32	18296	15080	53039	Klickitat	45.8888	-120.971	4
1	17729	16099	53003	Asotin	46.146	-117.209	2
23	15258	13439	53055	SanJuan	48.6118	-122.96	3
0	13630	6621	53001	Adams	46.9272	-118.511	2
34	10878	9729	53051	PendOreill	48.4359	-117.317	5
24	9798	8531	53059	Skamania	45.9429	-121.896	1
19	8582	7782	53043	Lincoln	47.5321	-118.47	3
9	5831	4352	53019	Ferry	48.4718	-118.497	2
37	3655	3297	53069	Wahkiakur	46.3259	-123.495	3
6	3277	2896	53013	Columbia	46.2775	-117.814	1
11	1799	1661	53023	Garfield	46.5186	-117.528	5

The next table below summarizes the total population distributed across 5 districts:

District	Total Population
1	538966
2	589631
3	569826
4	592426
5	571207

The result is a roughly equivalent distribution in population among the 5 districts, as the population in each district ranges from 538,000 to 593,000, which fits into the allowed 10% variance from the target population.

The 5 automatically assigned district populations for the excluded counties are as follows:

District 6: 639,797 (Snohomish County)

District 7: 709,366 (Pierce County)

District 8, 9, 10: 1,813,470 (King County)

The redistricting problem, approached through integer programming, provides a structured methodology for fair and equitable electoral districting. Redistricting remains a complex yet vital component of democratic governance, requiring ongoing refinement and public discourse to ensure representative democracy's integrity. During the integer programming process, the group faced challenges with incorporating all the adjacency constraints and therefore, further investigation into this must be considered to determine more accurate electoral districting for the state of Washington. The current solution would need more work before being submitted to the state's legislature, the governor, or the courts.