

Insertion Sort

$a = [4, 2, 6, 8,]$

4 2 6 8

1st pass

2 4 6 8

2nd pass

2 4 6 8

3rd pass

2 4 6 8

4th pass

1 2 4 6 8

Final array

Best-case:

$a = [2, 3, 6, 8]$

2 3 6 8

$n-1$ comparisons

2 3 6 8

$\Omega(n)$

2 3 6 8

worst case:

$$a = [8, 6, 4, 1]$$

8 6 4 1

1st phase

8 8 4 1

2nd phase

4 6 8 1

3rd phase

1 4 6 8

Array sorted

of inversions in general

$$I(a) = O(n^2)$$

$$T(n) = 1 + 2 + 3 + \dots + n-1$$
$$= \sum_{i=1}^{n-1} i = O(n^2)$$

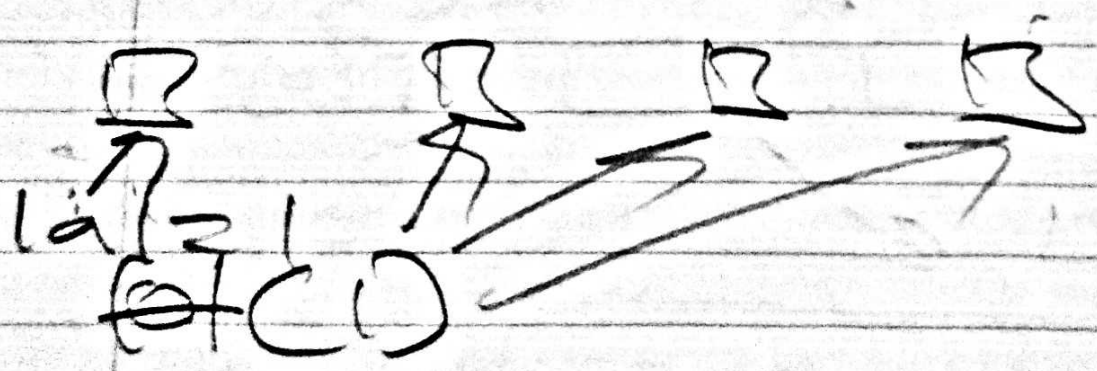
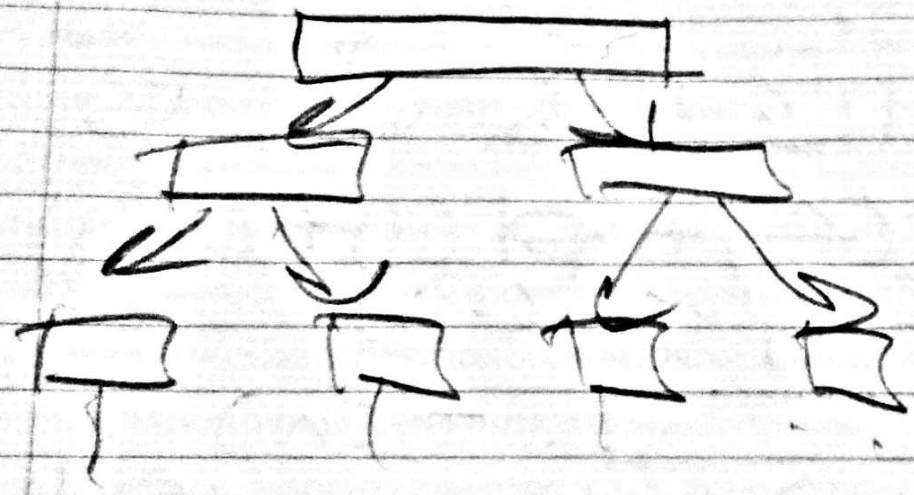
avg case analysis is the same

DIVIDE & CONQUER

MERGE-SORT

→ [] $|a| = 10,000$

→ [] $\xrightarrow{\text{sort}}$ $|a| = 1$



1 2 3 4 5 8 10 12

$a = [8, 2, 4, 10, 12, 5]$

1 2 3 4 5

3 5 10 12

[8 2 4]

[10 12 5]

2 8 4
[8 2] [4]

10 12 5
[10 12] [5]

[8] [2] [4] [10] [12] [5]

$$T(n) = 2T\left(\frac{n}{2}\right) + \Theta(1) + O(n)$$

$$= O(n \log_2 n)$$

Base case
 $\Theta(1)$

[] $T(n)$

[] [] $T\left(\frac{n}{2}\right)$

[]

combined the answers

<u>STEP</u>	<u>input size</u>	<u>tree cost</u>
0	n	Cn
1	$\frac{n}{2}$	Cn
2	$\frac{n}{4}$	Cn
...
k	$\frac{n}{2^k}$	Cn

$$\sum_{i=0}^k Cn = Cn(k+1)$$

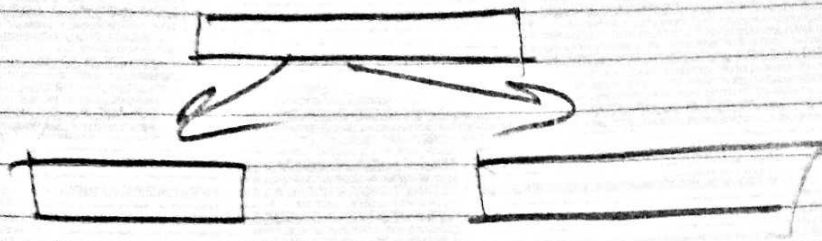
$$\Theta(Cn \log_2 Cn)$$

$$1 = \frac{n}{2^k}$$

$$\log_2 Cn = k$$

QUICK SORT

$a = [5, 2, 8, 9, 4, 0, 6, 10, 7]$



smaller M BIGGER
new array

$M(2, 7, 8) \rightarrow 7$

pivot = 1st element / last element / median
or three

pivot = 1st

1 | 5 | 8 | 9

$$T(n) = T(n-1) + O(n) \\ = O(n^2)$$

median as
new array /

version #

① Pivot = M (5, 4, 7) = 5

② swap (Pivot, last) = 5

a = [7, 2, 9, 8, 3, 6, 10, 5]

2 arrays
left & right

[<] M [>]

[2 9 8 3 6 10 5]

↑ ↑ × × × × ×
× ✓ × × × × ✓

[2 4 5 8 7 6 10 9]

[0 2 4] 5 [8 7 6 10 9]

sort by hand

$$T(n) = 2T(\frac{n}{2}) + O(n)$$

$$= O(n \log n)$$