Graphs

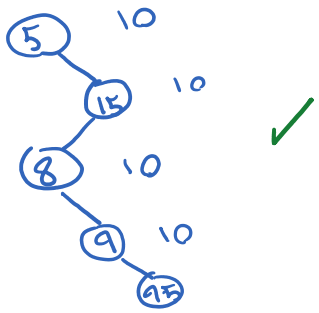
Thursday, November 19, 2020 5:00 PM

Reminder: HW 8 and lab 6 are due this Sunday. → Q5/7/9/10

Q5

key = 10

seq = 5/15/8/9/9.5



✓

seq2 = 5/2/6/4/7/...

✗

Review from L25

Graphs

$G = (V, E, \zeta_w)$

V: set of vertices

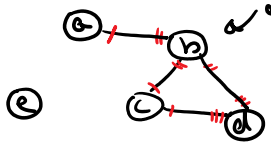
E: set of edges

$V = \{a, b, c, d, e\}$

$E = \{\{a,b\}\{b,c\}\{c,d\}\{b,d\}\}$

order = $|V| = 5$

size = $|E| = 4$

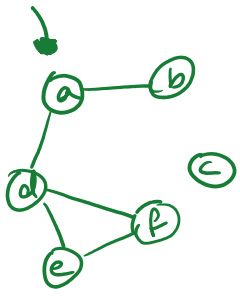


deg(b) = 3

deg(v) = nr of edges adjacent to v.

vertex	deg(v)
a	1
b	3
c	2
d	2
e	0
	8

undirected graph: $\sum_{i=1}^{|V|} deg(v_i) = 2|E|$



$|V| = 6$ / $|E| = 5$

$E = \{\{a,b\}\{a,d\}\{de\}\{df\}\{fe\}\}$

E = [[a,b] [a,d] [d,c] [d,f] [f,e]]

V = { a b c d e f }

O(E)

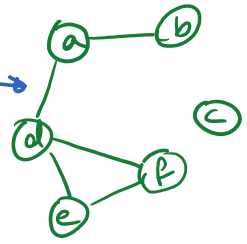
Adjacency matrix:

$|Adj| = |V| \times |V|$

Adj = symmetric

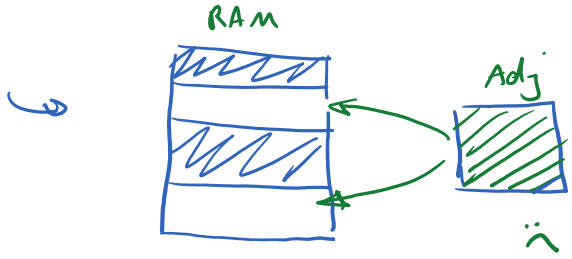
$Adj = Adj^T$

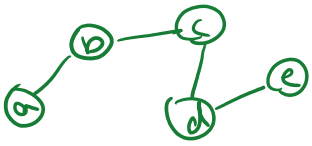
	a	b	c	d	e	f	deg(v_i)
a	0	1	0	1	0	0	2
b	1	0	0	0	0	0	1
c	0	0	0	0	0	0	0
d	1	0	0	0	1	1	3
e	0	0	0	1	0	1	2
f	0	0	0	1	1	0	2
	2	1	0	3	2	2	10 = 2|E|



Q: Are a and c are connected? $\Theta(1)$

Q: What are the neighbors of d? $\Theta(V)$





a.adj = {b}

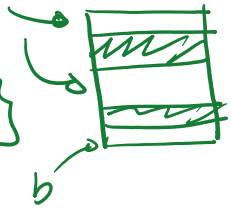
c.adj = {b, d}

object

node.adj = { neighbors }

node.key = name

node.parent

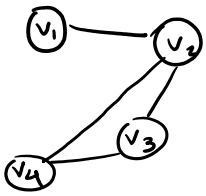


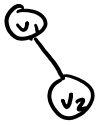
Q: to check if b and c are connected? $O(deg(c)) = O(|adj[c]|)$

$= O(|adj(c)|)$

c.adj = [b → d → ∅] ←

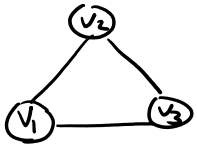
Max nr of edges:- → complete graph





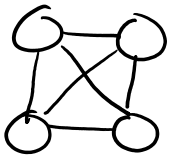
$|E| \geq 1$

$deg(v_2) = deg(v_1) = 1$



$|E| = 3$

$deg(v_i) = 2$



$|E| = 6$

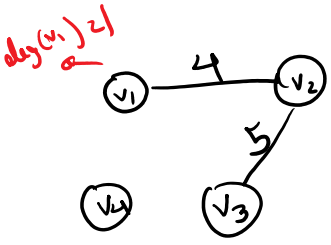
$deg(v_i) = 3$

$\Rightarrow |V| = n \Rightarrow deg(v_i) = |V| - 1$

$\sum_{i=1}^{|V|} deg(v_i) = 2|E| \Rightarrow \sum_{i=1}^{|V|} (|V|-1) = |V|(|V|-1) = 2|E|$

(total nodes · deg(vi))

$\Rightarrow |E| = \frac{|V|(|V|-1)}{2}$ ← max nr of edges

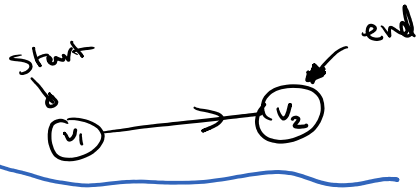


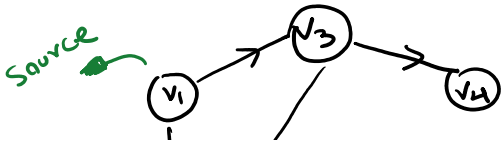
$\Rightarrow$ Adj =

	v_1	v_2	v_3	v_4
v_1	0	4	0	0
v_2	4	0	5	0
v_3	0	5	0	0
v_4	0	0	0	0

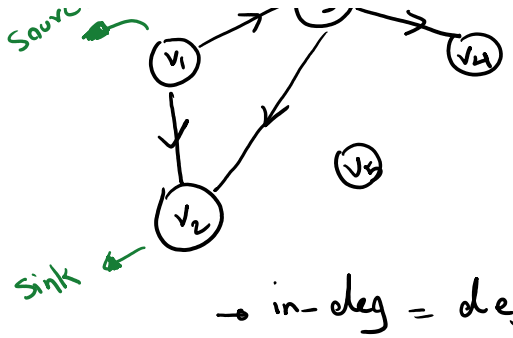
deg = Count non-zero elements

Directed Graphs





$|V| = 5$



$|V| = 5$

$|E| = 4$

- in-deg = $deg^-(v_i)$
- out-deg = $deg^+(v_i)$

	in-deg	out-deg
v_1	0	2
v_2	2	0
v_3	1	2
v_4	1	0
v_5	0	0
	4	4

Adj

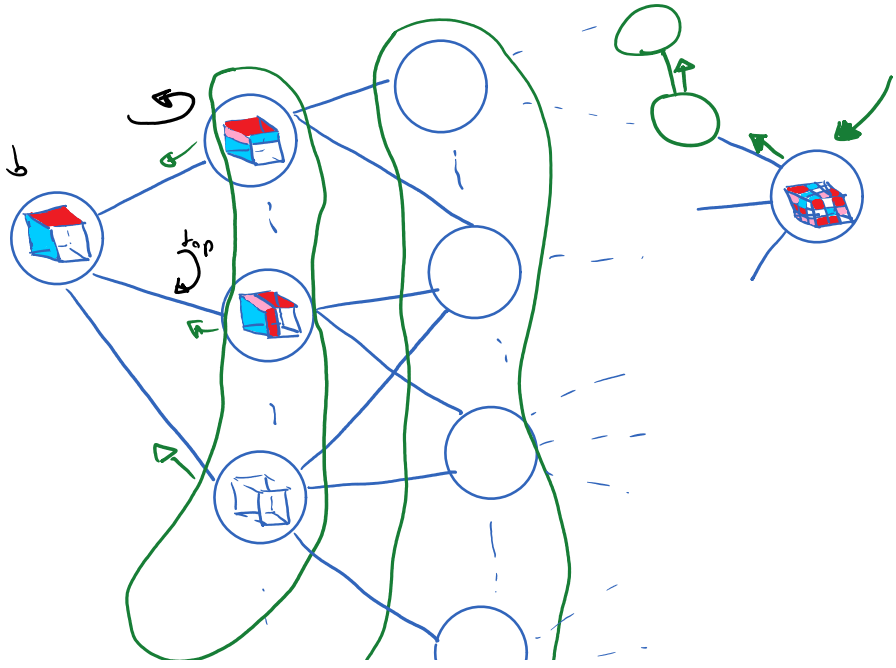
start →	v_1	v_2	v_3	v_4	v_5	out-deg
v_1	0	1	1	0	0	2
v_2	0	0	0	0	0	0
v_3	0	1	0	1	0	2
v_4	0	0	0	0	0	0
v_5	0	0	0	0	0	0
	0	2	1	1	0	

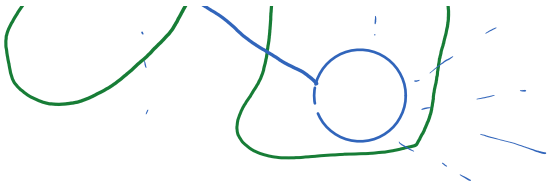
in-deg

$$\sum_{i=1}^{|V|} deg^-(v_i) = \sum_{i=1}^{|V|} deg^+(v_i) = |E|$$

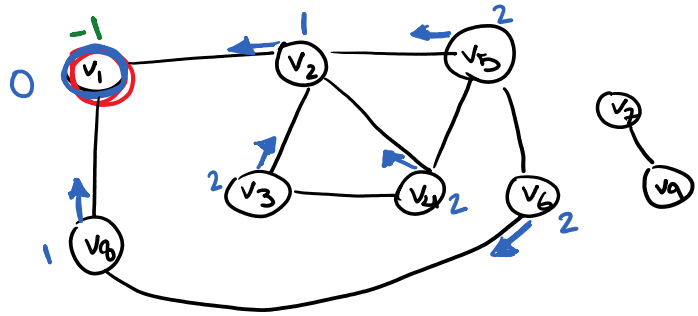
Breadth First Search (BFS)

- min dist from one vertex to all the reachable vertices
- solve Rubik's cube
- ~~—~~

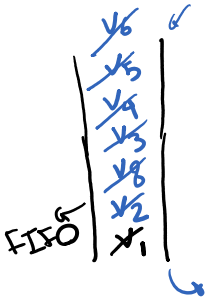




Example:



$v_2.adj = \{v_1, v_3, v_4, v_5\}$



order of pop →

$v_1 / v_2 / v_8 / v_3 / v_4$

$v_1 \rightsquigarrow v_4$ dist = 2

v_4 v_2 v_1 ←

Example: