1. When using mathematical induction to prove that $1+2+\ldots+n = \frac{n\cdot(n+1)}{2}$, state the inductive assumption that is used in the inductive step. Also, state what is to be prove in the inductive step.

Assume $P(n)$ is true and show it must the case that $P(n+1)$ is also true.

2. If $f(n) \leq g(n) + 50$ and $g(n) \geq 1$ for $n \geq 1$. Then use the definition of big$-O$ to prove that $f(n) = O(g(n))$.

$$\begin{aligned} f(n) &\leq g(n) + 50 \\ &\leq g(n) + 50 \cdot g(n) \\ &= g(n) \cdot (1 + 50) \\ &= 51 \cdot g(n) \end{aligned}$$

which by the definition of big-O notation, $f(n) = O(g(n))$.

3. Prove that $(n+a)^b = \Theta(n^b)$, for all real a and $b > 0$.

$$\begin{aligned} (n+a)^b &= \sum_{i=0}^{b} \binom{b}{i} \cdot n^{b-i} \cdot a^i \\ &\leq \sum_{i=0}^{b} \binom{b}{i} \cdot n^b \end{aligned}$$

which by the definition of O notation, $(n+a)^b = O(n^b)$.

$$\begin{aligned} (n+a)^b &= \sum_{i=0}^{b} \binom{b}{i} \cdot n^{b-i} \cdot a^i \\ &\geq n^b \end{aligned}$$

which by the definition of Ω notation, $(n+a)^b = \Omega(n^b)$.