

1 Compute the values for

1.  $\sum_{i=-1}^4 3$

$$\begin{aligned}\sum_{i=-1}^4 3 &= 3 + 3 + 3 + 3 + 3 + 3 \\ &= 3 \cdot 6 \\ &= 18\end{aligned}$$

2.  $\sum_{i=1}^5 (\frac{1}{3})^i$

$$\begin{aligned}\sum_{i=1}^5 (\frac{1}{3})^i &= (\frac{1}{3})^1 + (\frac{1}{3})^2 + (\frac{1}{3})^3 + (\frac{1}{3})^4 + (\frac{1}{3})^5 \\ &= \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243} \\ &= \frac{121}{243}\end{aligned}$$

3.  $\sum_{i=1}^n 3$

$$\begin{aligned}\sum_{i=1}^n 3 &= 3 \cdot \sum_{i=1}^n 1 \\ &= \frac{3 \cdot n \cdot (n+1)}{2}\end{aligned}$$

4.  $\sum_{i=-3}^n 3$

$$\begin{aligned}\sum_{i=-3}^n 3 &= \sum_{i=-3}^0 3 + \sum_{i=1}^n 3 \\ &= 3 \cdot 4 + \frac{3 \cdot n \cdot (n+1)}{2} \\ &= \frac{3 \cdot n^2 + 3 \cdot n + 24}{2}\end{aligned}$$

5.  $\sum_{k=0}^n 2^k + \sum_{k=5}^n 2^k$

$$\begin{aligned}
\sum_{k=0}^n 2^k + \sum_{k=5}^n 2^k &= 2^{n+1} - 1 + \sum_{k=5}^n 2^k \\
&= 2^{n+1} - 1 + \sum_{k=0}^n 2^k - \sum_{k=0}^4 2^k \\
&= 2^{n+1} - 1 + 2^{n+1} - 1 - (2^5 - 1) \\
&= 2 \cdot 2^{n+1} - 2 - (31) \\
&= 2 \cdot 2^{n+1} - 33 \\
&= 2^{n+2} - 33
\end{aligned}$$

$$6. \sum_{i=0}^n \left(\frac{2}{3}\right)^i + \sum_{i=-4}^n \left(\frac{2}{3}\right)^i$$

$$\begin{aligned}
\sum_{i=0}^n \left(\frac{2}{3}\right)^i + \sum_{i=-4}^n \left(\frac{2}{3}\right)^i &= \sum_{i=0}^n \left(\frac{2}{3}\right)^i + \sum_{i=-4}^{-1} \left(\frac{2}{3}\right)^i + \sum_{i=0}^n \left(\frac{2}{3}\right)^i \\
&= 2 \cdot \sum_{i=0}^n \left(\frac{2}{3}\right)^i + \sum_{i=-4}^{-1} \left(\frac{2}{3}\right)^i \\
&= 2 \cdot \frac{1}{1 - \frac{2}{3}} + \sum_{i=-4}^{-1} \left(\frac{2}{3}\right)^i \\
&= \frac{2}{\frac{1}{3}} + \sum_{i=-4}^{-1} \left(\frac{2}{3}\right)^i \\
&= 6 + \left(\frac{2}{3}\right)^{-4} + \left(\frac{2}{3}\right)^{-3} + \left(\frac{2}{3}\right)^{-2} + \left(\frac{2}{3}\right)^{-1} \\
&= \frac{291}{16}
\end{aligned}$$

$$7. \sum_{i=1}^n (i^3 + 2 \cdot i^2 - i + 1)$$

$$\begin{aligned}
\sum_{i=1}^n (i^3 + 2 \cdot i^2 - i + 1) &= \sum_{i=1}^n i^3 + \sum_{i=1}^n 2 \cdot i^2 - \sum_{i=1}^n i + \sum_{i=1}^n 1 \\
&= \left(\frac{n(n+1)}{2}\right)^2 + 2 \cdot \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + n \\
&= \left(\frac{n(n+1)}{2}\right)^2 + \frac{n(n+1)(2n+1)}{3} - \frac{n(n+1)}{2} + n
\end{aligned}$$

$$8. \sum_{i=5}^n (-4 \cdot i + \frac{i}{5})$$

$$\begin{aligned}
\sum_{i=5}^n (-4 \cdot i + \frac{i}{5}) &= \sum_{i=5}^n -4 \cdot i + \sum_{i=5}^n \frac{i}{5} \\
&= -4 \cdot \sum_{i=5}^n i + \frac{1}{5} \cdot \sum_{i=5}^n i \\
&= -4 \cdot \sum_{i=1}^n i + \frac{1}{5} \cdot \sum_{i=1}^n i + 4 \cdot \sum_{i=1}^4 i - \frac{1}{5} \cdot \sum_{i=1}^4 i \\
&= -4 \cdot \frac{n(n+1)}{2} + \frac{1}{5} \cdot \frac{n(n+1)}{2} + 4 \cdot \frac{4(5)}{2} - \frac{1}{5} \cdot \frac{4(5)}{2} \\
&= -4 \cdot \frac{n(n+1)}{2} + \frac{n(n+1)}{10} + 40 - 2 \\
&= -4 \cdot \frac{n(n+1)}{2} + \frac{n(n+1)}{10} + 38
\end{aligned}$$

9.  $\sum_{j=0}^k \sum_{i=1}^j (i - j^2 - 2)$

10.  $\sum_{j=1}^m \sum_{k=1}^j (3 \cdot k - 3 \cdot j + i)$

11.  $\sum_{l=-4}^n \sum_{j=1}^k \sum_{i=1}^j (i - 4)$

2. Calculate the answer

1.  $\log_4 x = 5 \rightarrow x = ?$

$$\begin{aligned}
\log_4 x &= 5 \\
x &= 4^5
\end{aligned}$$

1.  $\log_3 y = 4 \rightarrow y = ?$

$$\begin{aligned}
\log_3 y &= 4 \\
y &= 3^4
\end{aligned}$$

1.  $x = 7^2 \rightarrow \log_7 x = ?$

$$\begin{aligned}
x &= 7^2 \\
\log_7 x &= \log_7 7^2 \\
\log_7 x &= 2 \cdot \log_7 7 \\
\log_7 x &= 2
\end{aligned}$$

2.  $x = 32 \rightarrow \log_2 x = ?$

$$\begin{aligned} x &= 32 \\ \log_2 x &= \log_2 32 \\ \log_2 x &= 5 \end{aligned}$$

3.  $2^{\log 5} + 4^{\log 6} - 27^{\log 3^5}$

4.  $9^{\log_3 2} - 25^{\log_5 4} - 36^{\log_6 7} + 8^{\log_8 6}$

5.  $\log(4^5 \times 8^3) - \log(16 - 8) + \log\left(\frac{2^{10}}{4 \times 3^2}\right)$

6.  $\log(3^2 \times 64^3) - \log\left(\frac{2^{10} \times 128^3}{9 \times 8^2}\right)$

7.  $\log \log 16$

8.  $\log 16 \times \log 16$

9.  $\log^2 16$

10.  $\log_2 \log_5 625 - \log_3 \log_4 2^{3^9} + \log^4 2^5 - \frac{\log^2(4^3 \times 3^5)}{\log_5 125}$

11.  $\log \log_8 \log 256 + \log^5(3^2) \times 4^{\log 7}$

12.  $\log_6 x = 5 \rightarrow \log_x 6 = ?$

13.  $\log_y x = 10 \rightarrow \log_x y = ?$

14.  $\log_4 32 - \log_8^2 4$

15.  $\log_4 8 + \log_9 27 - \log_{25}^2 125 - \log_8^3 16 + \log_4 \log 256$

### 3. Compute the derivative of

1.  $-5 \cdot x^3 + 2 \cdot x - 1$

2.  $3 \cdot x^4 - 2\sqrt{x} + x^{\frac{1}{2}} - 6x^{-\frac{2}{3}} - 5$

3.  $x \cdot \sqrt{x} + \sqrt{\sqrt{x}}$

4.  $\log x - x^2 \ln x + \ln x^4$

5.  $\ln^3(x\sqrt{2x-3}) + \sqrt{\ln x^2}$

6.  $\frac{\sqrt[4]{x+5} - \ln x}{(x-1)^3}$

4. Determine the limit of

1.  $\lim_{x \rightarrow \infty} \frac{3x+2}{-5x-6}$
2.  $\lim_{x \rightarrow \infty} (\frac{1}{x} + 3)$
3.  $\lim_{x \rightarrow \infty} \frac{x^3+x-\sqrt{3x}}{\sqrt{x}}$
4.  $\lim_{x \rightarrow \infty} \frac{x^3+x-\sqrt{3x}}{5 \cdot x^2 \cdot 25 \cdot \sqrt{\sqrt{x}}}$
5.  $\lim_{x \rightarrow \infty} \frac{x^{0.1}-\sqrt{3}}{\sqrt{\sqrt{x}}}$
6.  $\lim_{x \rightarrow \infty} \frac{x^x}{2^x}$
7.  $\lim_{x \rightarrow \infty} \frac{x^x}{x(2^x)}$
8.  $\lim_{x \rightarrow \infty} \frac{\sqrt{2}^{\log^4 x^3}}{\log(2 \cdot x + 7)}$
9.  $\lim_{x \rightarrow \infty} \frac{x+1}{\frac{3 \cdot x \ln x}{2x^2}}$
10.  $\lim_{x \rightarrow \infty} \frac{\sqrt{2}^{\log x^3}}{\log^{\ln x}(2x)}$

5. Compute the exact values for

1.  $\int_1^n (2 \cdot x^4 + 5\sqrt{x})$
2.  $\int_1^n (x^4 - 3 \cdot x^2 + \frac{1}{x} - \frac{1}{x^2}) dx$
3.  $\int_1^n (\frac{3}{\sqrt{x}} + \ln x + e^x) dx$
4.  $\int_1^n x \cdot \sin x dx$

6. Use mathematical induction to prove that

$$1 + 2 + \dots + n = \frac{n(n+1)}{2}$$

*Proof.*  $1 + 2 + \dots + n = \frac{n(n+1)}{2}$

Base case  $n = 1$ : If  $n = 1$  then the left hand side and the right hand size is  $1 = 1 = \frac{1(2)}{2}$ .

Inductive hypothesis: Suppose the theorem holds for all values of  $n$  up to some  $k, k \geq 1$ .

Inductive step: let  $n = k + 1$  then our left hand side is

$$\begin{aligned}
\sum_{i=1}^{k+1} i &= \sum_{i=1}^k i + (k+1) \\
&= \frac{k(k+1)}{2} + (k+1) \\
&= \frac{k(k+1) + 2 \cdot k + 2}{2} \\
&= \frac{(k+1) \cdot (k+2)}{2}
\end{aligned}$$

which is equal to our right hand side. By the principle of mathematical induction, the theorem holds for all integers  $n \geq 1$ .  $\square$

**7.** Use mathematical induction to prove that

$$1 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1 + 2 + \dots + n = \frac{n(n+1)}{2}$$

*Proof.* Base case  $n = 1$ : If  $n = 1$  then the left hand side and the right hand size is  $1^2 = 1 = \frac{1(2)(3)}{6}$ .

Inductive hypothesis: Suppose the theorem holds for all values of  $n$  up to some  $k, k \geq 1$ .

Inductive step: let  $n = k + 1$  then our left hand side is

$$\begin{aligned}
\sum_{i=1}^{k+1} i^2 &= \sum_{i=1}^k i^2 + (k+1)^2 \\
&= \frac{k \cdot (k+1) \cdot (2 \cdot k + 1)}{6} + (k+1)^2 \\
&= \frac{k \cdot (k+1) \cdot (2 \cdot k + 1) + 6 \cdot (k+1)^2}{6} \\
&= \frac{(6 \cdot (k+1) + k \cdot (2 \cdot k + 1)) \cdot (k+1)}{6} \\
&= \frac{(6 \cdot k + 6 + 2 \cdot k^2 + k) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k^2 + 7 \cdot k + 6) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k^2 + 4 \cdot k + 3 \cdot k + 6) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k \cdot (k+2) + 3 \cdot (k+2)) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k + 3) \cdot (k+2) \cdot (k+1)}{6}
\end{aligned}$$

which is equal to our right hand side. By the principle of mathematical induction, the theorem holds for all integers  $n \geq 1$ .  $\square$