1. State the definition of big-O, and use it to prove that $2^{n+1} = O(2^n)$.

$$f(n) = O(g(n)) \iff \exists c > 0, k \geq 0\, s.t\, f(n) \leq c \cdot g(n), n \geq k$$

$$\begin{aligned} 2^{n+1} &\leq c \cdot 2^n \\ 2 \cdot 2^n &\leq c \cdot 2^n \\ 2 &\leq c \end{aligned}$$

$c = 3$ and $k = 1$ are sufficient

2. If $f(x)$ and $g(x)$ are differentiable at x and $g(x) \neq 0$, then the quotient rule rule of differentiation

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$$

Use this rule to compute the deriative of $\frac{(3 \cdot x+5)}{(7 \cdot x^2+x+11)}$.

$$\frac{3 \cdot (7 \cdot x^2 + x + 11) - (3 \cdot x + 5) \cdot (14 \cdot x + 1)}{(7 \cdot x^2 + x + 11)^2}$$

3. When using mathematical induction to prove that $1+2+\ldots+n = \frac{n \cdot (n+1)}{2}$

1. State the inductive assumption of the proof.

   $P(k)$ holds for some positive integer k

2. State what needs to be proved in the inductive step.

   $P(k+1)$

3. Prove the inductive step.

$$
\begin{aligned}
\sum_{i=1}^{n+1} i &= \sum_{i=1}^{n} i + (n+1) \\
&= \frac{n \cdot (n+1)}{2} + (n+1) \\
&= \frac{n \cdot (n+1) + 2 \cdot (n+1)}{2} \\
&= \frac{(n+1) \cdot (n+2)}{2}
\end{aligned}
$$