Lecture 2 (Review)

Thursday, August 27, 2020 5:00 PM

Series

① $$\sum_{j=5}^{200} 6 = \underset{j=5}{6} + \underset{j=6}{6} + \sim + \underset{j=200}{6} = \boxed{6(200-5+1)}$$

② $$\sum_{P=-20}^{100} K = K(100-(-20)+1) = K(121)$$

③ $$\sum_{K=-5}^{100} 8^K = 8^{-5} + 8^{-4} + \sim + 8^{-1} + \boxed{8^0 + \sim + 8^{100}}$$

$$= \frac{1}{8}^5 + \frac{1}{8}^4 + \sim + \frac{1}{8}^1 + \sum_{K=0}^{100} 8^K$$

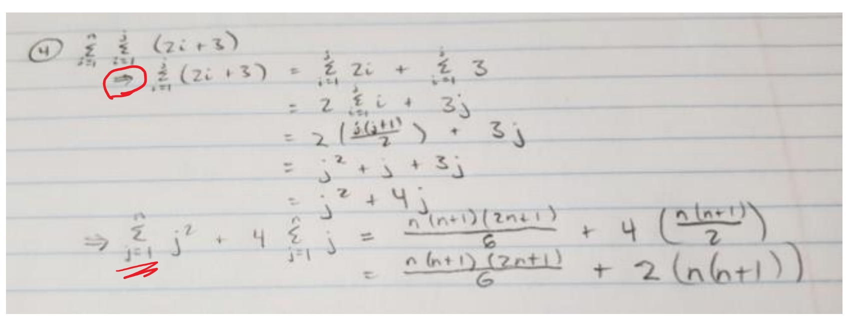
$$= \sum_{i=1}^{5} \frac{1}{8}^i + \frac{8^{101}-1}{8-1}$$

$$= \underbrace{\sum_{i=1}^{5} \frac{1}{8}^i + \frac{1}{8}^0} - \frac{1}{8}^0 + 〃$$

$$= \sum_{i=0}^{5} \frac{1}{8}^i - 1 + 〃 = \boxed{\frac{\frac{1}{8}^6 - 1}{\frac{1}{8} - 1} - 1 + 〃}$$

Seliner

④ $$\sum_{j=1}^{n} \sum_{i=1}^{j} (2i+3) =$$



④ $\sum_{j=1}^{n} \sum_{i=1}^{j} (2i+3)$

$\Rightarrow \sum_{i=1}^{j} (2i+3) = \sum_{i=1}^{j} 2i + \sum_{i=1}^{j} 3$

$= 2 \sum_{i=1}^{j} i + 3j$

$= 2\left(\frac{j(j+1)}{2}\right) + 3j$

$= j^2 + j + 3j$

$= j^2 + 4j$

$\Rightarrow \sum_{j=1}^{n} j^2 + 4 \sum_{j=1}^{n} j = \frac{n(n+1)(2n+1)}{6} + 4\left(\frac{n(n+1)}{2}\right)$

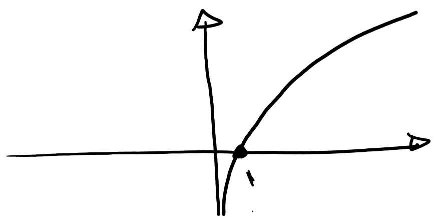
$= \frac{n(n+1)(2n+1)}{6} + 2(n(n+1))$

Logs

Def: $\log_b x = n \Rightarrow b^n = x$

Rules b

1. $\log_b(x^a) = a\log_b(x)$
2. $\log_b(xy) = \log_b(x) + \log_b(y)$
3. $\log_b(\frac{x}{y}) = \log_b(x) - \log_b(y)$
4. $b^{\log_b x} = x$
5. $y^{\log_b x} = x^{\log_b y}$
6. $\log_b(x) = \dfrac{1}{\log_x(b)}$
7. $\log_b(x) = \dfrac{\log_a(x)}{\log_a(b)}$



$\Leftarrow \log_b 1 = 0$

$\log 0 = \text{UND}$

$\log(-1) = \text{UND}$

$\log x = \infty$

$x \to +\infty$

$$\Rightarrow \boxed{\log_b a = c \Leftrightarrow b^c = a}$$

$\log 4 = 2$ / $\log 8 = 3$

$\log 128 = 7$ / $\log_3 81 = 4$ / $\log_3 234 \doteq 5$

$\log_2 2^{20} = ? \Leftrightarrow 2^{?} = 2^{20} \Rightarrow \boxed{? = 20}$

① $\log_b x^a = a\log_b x$

$$\boxed{\log_b x = c \Leftrightarrow b^c = x}$$

RS $\Rightarrow \log_b x = c \Leftrightarrow b^c = x$

$\xrightarrow{\wedge a} (b^c)^a = x^a \Leftarrow$

$\Rightarrow b^{ca} = x^a$

$\log_b \quad \log_b b^{ca} = \log x^a$

$\Rightarrow \quad b = \ldots$

$\log_b \Rightarrow \quad \log_b b^{ca} = \log_b x^a$

$\Rightarrow ca = \log_b x^a$

$(\log_b x) a = \log_b x^a$

$\Rightarrow \quad a \log_b x = \log_b x^a$

② $\log_b xy = \log_b x + \log_b y$

RS $\Rightarrow \quad \underbrace{\log_b x = p \Leftrightarrow b^p = x}_{\text{Def}} \quad / \quad \underbrace{\log_b y = s \Leftrightarrow b^s = y}_{\text{Def}}$

$\Rightarrow \quad x = b^p, \; y = b^s \quad \Rightarrow xy = b^p b^s$

Note: $b^p b^s = b^{p+s}$

$(b^p)^s = b^{ps}$

$\Rightarrow xy = b^{p+s}$

$\log_b \Rightarrow \quad \log_b xy = \log_b b^{p+s} = p+s$

$\Rightarrow \log_b xy = p+s$

✓ $\log_b xy = \log_b x + \log_b y$

Example:

(1) $\log(2^2 \times 3^4) = \log 2^2 + \log 3^4 = 2\log 2 + 4\log 3$

$= \boxed{2(1) + 4(1.5)}$ $\log 3 \sim 1.5$

(2) $\log\left(\dfrac{64 \times 128 \times 3}{1024 \times 2^{15}}\right) = \log(64 \times 128 \times 3) - \log(1024 \times 2^{15})$

$= \log 64 + \log 128 + \log 3 - \log 1024 - \log 2^{15}$

$= 6 + 7 + 1.5 - 10 - 15$

(3) $\log \underbrace{\log 16} = \log 4 = 2$

(4) $\log^4 8 = \log 8 \times \log 8 \times \log 8 \times \log 8 = (\log 8)^4$

$= \boxed{3^4 = 81}$

(5) $\log 8^4 = 4\log 8 = 4 \times 3 = \boxed{12}$

(6) $\log^4 128^2 = (2\log 128)^4 = 2^4 (\log 128)^4$

$= 2^4 \times 7^4 = (2 \times 7)^4$

$\boxed{= 14^4}$

(7) $5^{\log_5 15} + 3^{\log_3 8} = \boxed{15 + 8}$

$\log x^a = a\log x$

(8) $8^{\log 6} = (2^3)^{\log 6} = 2^{3\log 6} = 2^{\log 6^3} = \boxed{6^3}$

$\log\log 5$ $\log_2 2$ $(2)^{\log \cancel{\log 5}}$ $\log(\log 5)^2$

⑨ $4^{\log\log 5} + 6^{\log_6 2} = (2^2)^{\log(\log 5)} + 2 = 2^{\log(\log 5)^2} + 2$

$= \boxed{\log^2 5 + 2}$

⑩ $\log\left(\frac{\log_4 8}{\log_3 27}\right)^2 = \log\left(\frac{\frac{\log 8}{\log 4}}{3}\right)^2$

$= \log\left(\frac{\frac{\cancel{3}}{2}}{\cancel{3}}\right)^2$

$= \log\left(\frac{1}{2}\right)^2 = \log\frac{1}{4} = \log 2^{-2} = \boxed{-2}$

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Derivative:

$\boxed{(x^p)' = p x^{p-1}}$

① $(2x^5 + 3x^4 + \ln x)' = 10x^4 + 12x^3 + \frac{1}{x}$

② $(\ln^2 x + x(x+1)^2)' = \boxed{2\ln x \left(\frac{1}{x}\right) + 1(x+1)^2 + 2(x+1)x}$

$(PS)' = P'S + S'P$

$(u^p)' = p u^{p-1} u'$

③ $\left(\sqrt{5x+2}\right)'$ $\frac{\left(\sqrt{5x+2}\right)' x\sin x - (x\sin x)'\sqrt{5x+2}}{}$

$$③\quad \left(\frac{\sqrt{5x+2}}{x\sin x}\right)' = \frac{(\sqrt{5x+2})'\,x\sin x - (x\sin x)'\sqrt{5x+2}}{(x\sin x)^2}$$

$$\left(\frac{P}{S}\right)' = \frac{P'S - S'P}{S^2}$$

$$= \frac{\frac{1}{2}(5x+2)^{-\frac{1}{2}}(5)\,x\sin x - (\sin x + x\cos x)\sqrt{5x+2}}{(x\sin x)^2}$$

$$④\quad \left(\sqrt{\frac{\ln^2 x}{5\log x^2+2}}\right)' = \left(P^{\frac{1}{2}}\right)'$$

P

$$⑤\quad \left(\sqrt{\sqrt{x}}\left(\ln^2(x^2+5)\right)\right)' = (PS)' =$$

$P \qquad S$