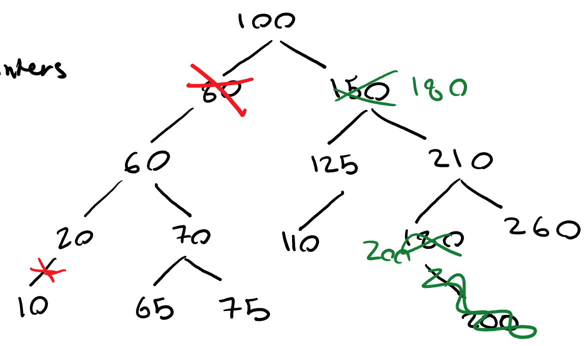
BST / AVL Trees

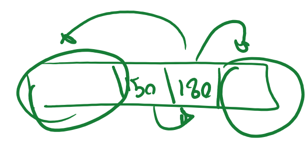
Tuesday, November 10, 2020   5:00 PM

*Reminder: Exam 2 is this Thursday.

Deleting a vertex: $O(h)$

① Deleting a leaf: Simply delete & update the pointers → $\Theta(1)$





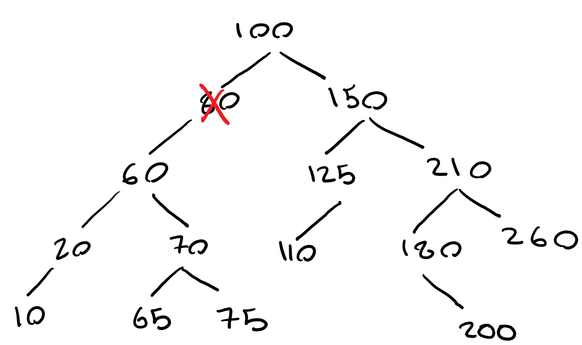
② Deleting a node with one child: → $\Theta(1)$

replace the node with its child

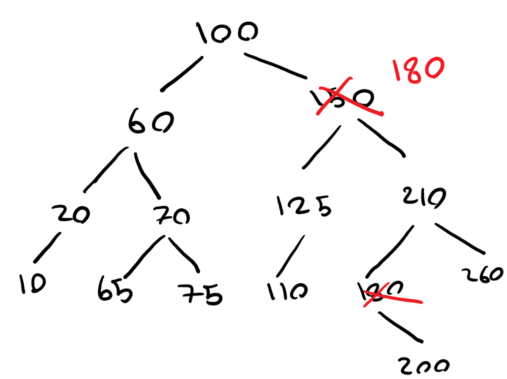
③ Deleting a node with two children: → $O(h)$

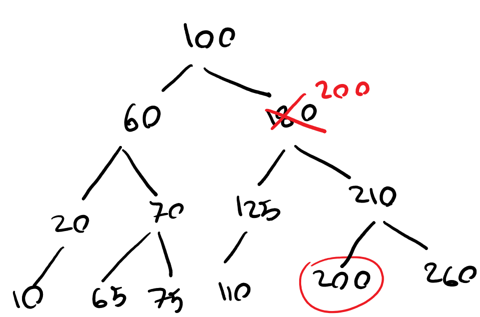
find and replace the node with its successor

and I will remove the successor. (step 1 or step 2)

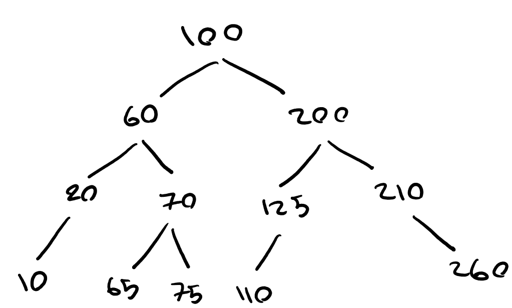


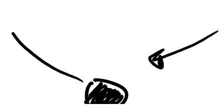
⇒

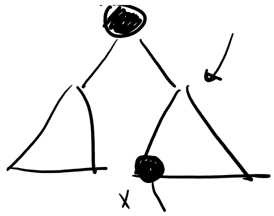




⇒



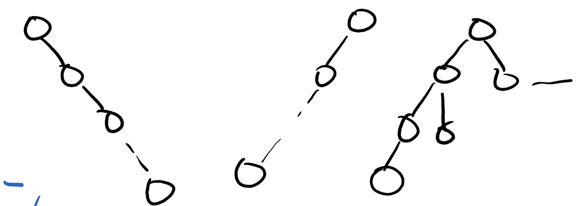




AVL Tree

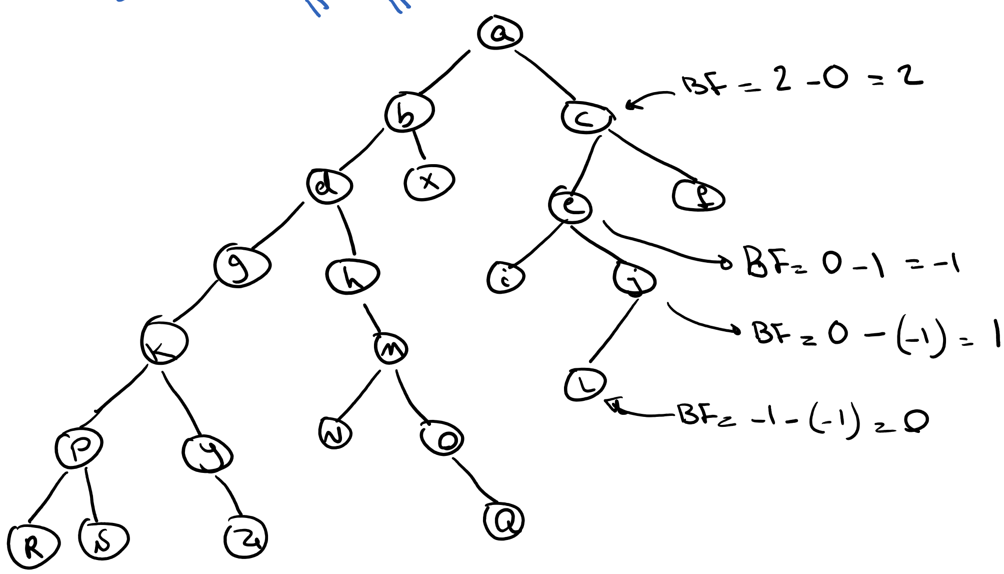
n elements $\Rightarrow \log n \leq h < n$

They are balanced BST.

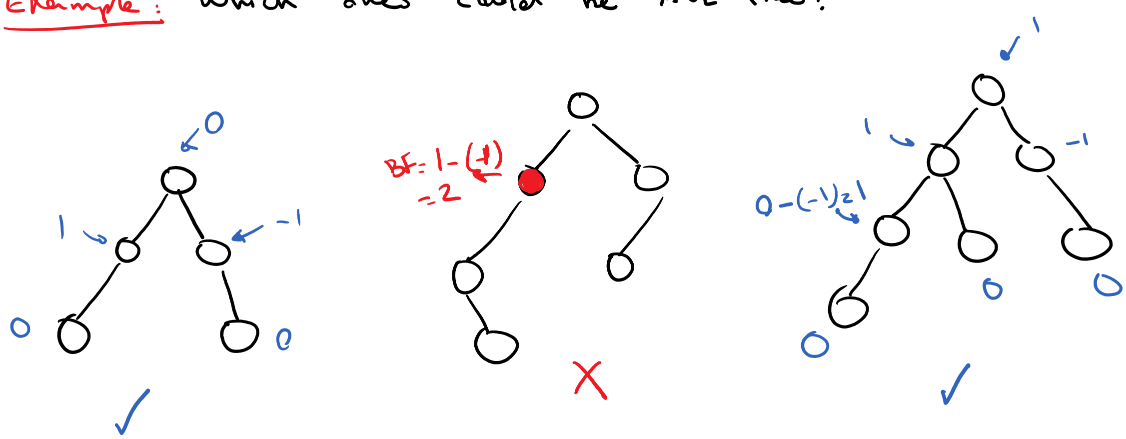


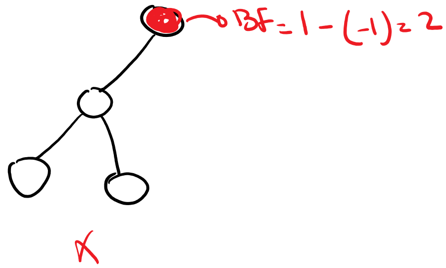
* $\underbrace{\text{balance\_factor}}_{BF} = h(\text{left child}) - h(\text{right child})$

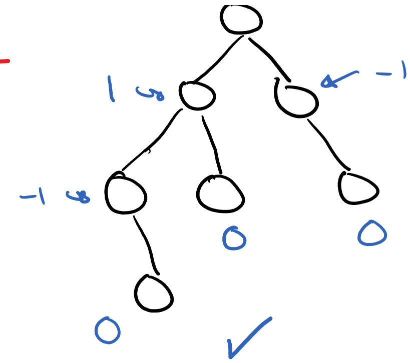
BST $\Rightarrow$ AVL $-1 \leq BF \leq 1$ for each vertex



Example: Which ones could be AVL tree?







Theorem: The height of AVL trees are $O(\log n)$ (having n nodes)

→ $N(h)$: min nr of nodes in an AVL tree having height h

$N(0) = 1$

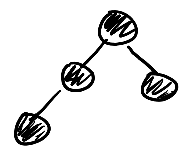
$h = 1$

$N(1) = 2$

$n = 2$ or 3

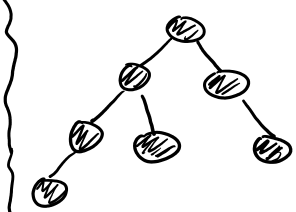
$h = 2$

$N(2) = 4$



$n = 4$ or 5 or 6 or 7

$N(3) = 7$



$$\boxed{* N(h) = N(h-1) + N(h-2) + 1}$$

$N(3) = N(2) + N(1) + 1 = 4 + 2 + 1 = 7$ ✓

$N(2) = N(1) + N(0) + 1 = 2 + 1 + 1 = 4$ ✓

looks fib equation ⇒ $f_n = f_{n-1} + f_{n-2}$ $f_0 = 0$, $f_1 = 0$

Dynamic Program

```
fib(n)
  if n == 0 || n == 1
     return 0
  else
     return fib(n-1) + fib(n-2)
  end
end
```

$\Rightarrow T(n) = T(n-1) + T(n-2) + \Theta(1)$

$$1+2+2^2+2^3+\cdots+2^{n-1}$$

$$= \sum_{i=0}^{n-1} 2^i = \frac{2^n-1}{2-1} = 2^n$$

fib

$$T(n) = \underset{\Omega}{O}(2^n) \Rightarrow \boxed{T(n) = \Omega(2^n) \Leftarrow f(n) \geq c2^n}$$

$$f_n = f_{n-1} + f_{n-2} + 1 \Rightarrow f_n \geq c2^n$$

$$N(h) = N(h-1) + N(h-2) + 1 \Rightarrow N(h) \geq c2^h$$

$$\Rightarrow \log(N(h)) \geq \log c + \log 2^h$$

$$\log(N(h)) \geq \log c + h$$

$$\log(N(h)) - \log c \geq h$$

$$h \leq \log N(h) - \log c \leq \log N(h)$$

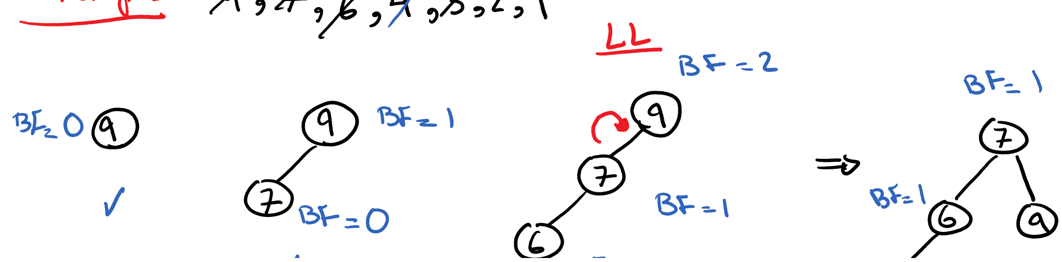
$$h \leq \log N(h) \Rightarrow h \leq \log n \Rightarrow \boxed{h = O(\log n)}$$

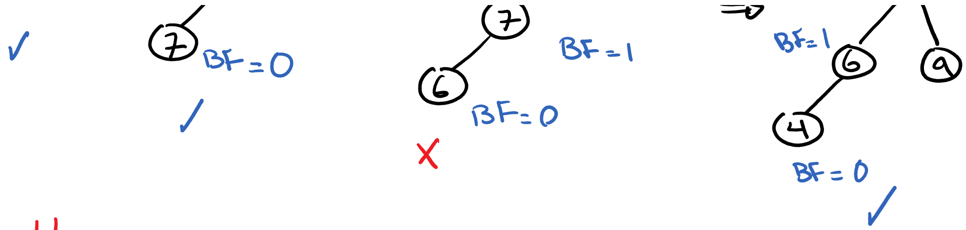
:D

$N(h) \leq n$

Insertion:

Example: ~~9~~, ~~7~~, ~~6~~, ~~4~~, ~~3~~, 2, 1

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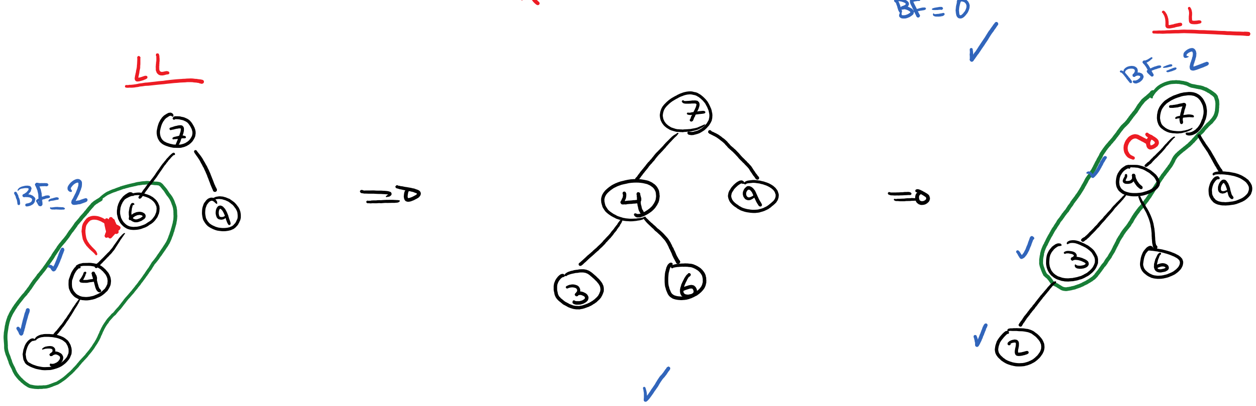
BF=1
BF=0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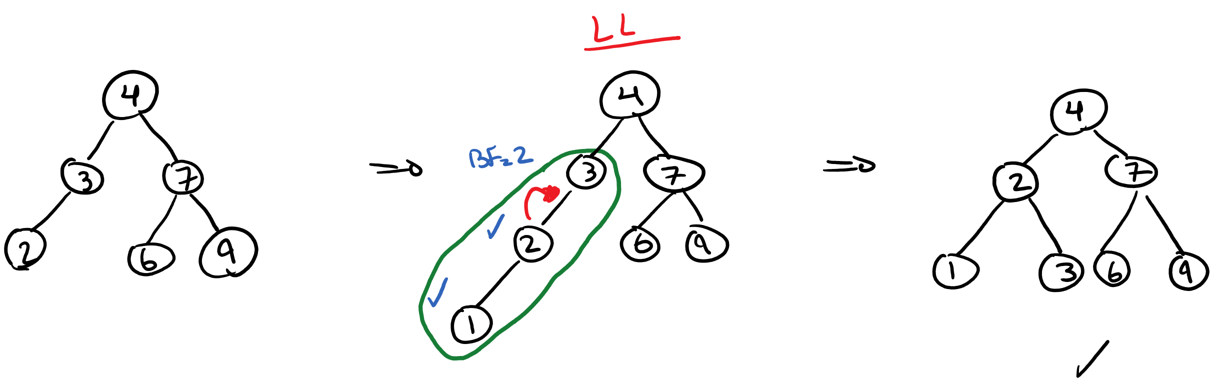
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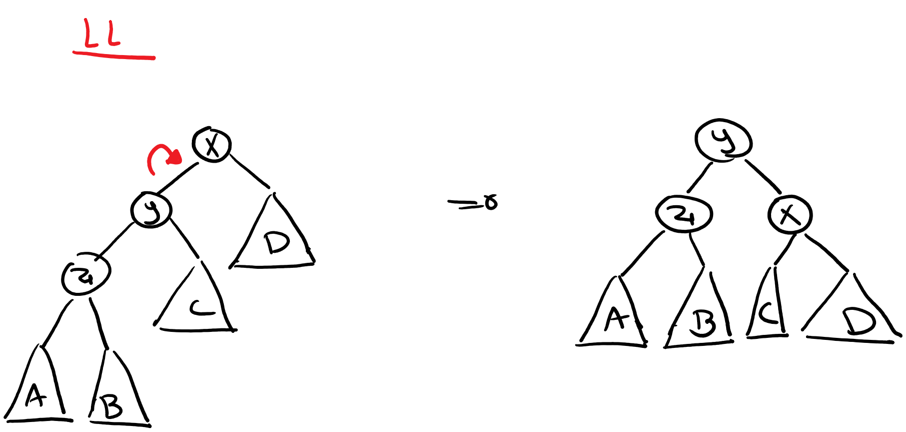
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