Homework assignment 2:
Due date: Saturday, September 12, 2020 at 11:59pm

1. Prove that $f(n) = 10n^4 + 2n^2 + 3$ is *O(n⁴)*, provide the appropriate C and k constants.
2. Prove that $f(n) = 2n^2 - n \log n + 3\log n$ is $O(n^2)$, provide the appropriate C and k constants.
3. Prove that $f(n) = 2n^4 \log n^4 - n^2 + 3\log n$ is $O(n^4 \log n)$, provide the appropriate C and k constants.
4. Prove or disprove

   $$f(n) = 5n^3 - n + 3$$

   :

   a. $f(n) = O(n^2)$
   b. $f(n) = \Omega(n)$
   c. $f(n) = \Theta(n^3)$
   d. $f(n) = \omega(n)$
   e. $f(n) = o(n^2)$

   Provide the appropriate C and k constants if possible (for parts a,b,c).
5. Prove that $(n+5)^{100} = \Theta(n^{100})$.
6. Prove transitivity of big-O: if *f(n) = O(g(n))*, and *g(n) = O(h(n))*, then *f(n) = O(h(n)).*
7. Prove that *f(n)=O(g(n))* iff *g(n)= Ω(f(n)).*
8. Compare the growth of:
   a. $f(n) = n$ and $g(n) = n^{1+\sin n}$.
   b. $f(n) = \sqrt{n}$ and $g(n) = \mathrm{n} + \sin(\mathrm{n})$
   c. $f(n) = n$ and $g(n) = \mathrm{n} * |\sin(\mathrm{n})|$
9. Prove or disprove: $2^{n+1}=O(2^n)$.
10. Prove or disprove: $2^{2n}=o(2^n)$.
11. Prove that if $lim_{\longrightarrow\infty} \dfrac{f(n)}{g(n)} = C$, for some constant C>0, then *f(n)= Θ(g(n)).*

Hint: $lim_{\longrightarrow\infty} \dfrac{f(n)}{g(n)} = C$ means that for every ϵ>0, there exists k>=0 such that, for all n>=k,

$$\left| \frac{f(n)}{g(n)} - C \right| < \varepsilon$$