1 Compute the values for

1. $\sum_{i=-1}^{4} 3$

$$\begin{aligned} \sum_{i=-1}^{4} 3 &= 3+3+3+3+3+3 \\ &= 3 \cdot 6 \\ &= 18 \end{aligned}$$

2. $\sum_{i=1}^{5} (\frac{1}{3})^i$

$$\begin{aligned} \sum_{i=1}^{5} (\frac{1}{3})^i &= (\frac{1}{3})^1 + (\frac{1}{3})^2 + (\frac{1}{3})^3 + (\frac{1}{3})^4 + (\frac{1}{3})^5 \\ &= \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243} \\ &= \frac{121}{243} \end{aligned}$$

3. $\sum_{i=1}^{n} 3$

$$\begin{aligned} \sum_{i=1}^{n} 3 &= 3 \cdot \sum_{i=1}^{n} 1 \\ &= \frac{3 \cdot n \cdot (n+1)}{2} \end{aligned}$$

4. $\sum_{i=-3}^{n} 3$

$$\begin{aligned} \sum_{i=-3}^{n} 3 &= \sum_{i=-3}^{0} 3 + \sum_{i=1}^{n} 3 \\ &= 3 \cdot 4 + \frac{3 \cdot n \cdot (n+1)}{2} \\ &= \frac{3 \cdot n^2 + 3 \cdot n + 24}{2} \end{aligned}$$

5. $\sum_{k=0}^{n} 2^k + \sum_{k=5}^{n} 2^k$

$$
\begin{aligned}
\sum_{k=0}^{n} 2^k + \sum_{k=5}^{n} 2^k &= 2^{n+1} - 1 + \sum_{k=5}^{n} 2^k \\
&= 2^{n+1} - 1 + \sum_{k=0}^{n} 2^k - \sum_{k=0}^{4} 2^k \\
&= 2^{n+1} - 1 + 2^{n+1} - 1 - (2^5 - 1) \\
&= 2 \cdot 2^{n+1} - 2 - (31) \\
&= 2 \cdot 2^{n+1} - 33 \\
&= 2^{n+2} - 33
\end{aligned}
$$

6. $\sum_{i=0}^{n}(\frac{2}{3})^i + \sum_{i=-4}^{n}(\frac{2}{3})^i$

$$
\begin{aligned}
\sum_{i=0}^{n}(\frac{2}{3})^i + \sum_{i=-4}^{n}(\frac{2}{3})^i &= \sum_{i=0}^{n}(\frac{2}{3})^i + \sum_{i=-4}^{-1}(\frac{2}{3})^i + \sum_{i=0}^{n}(\frac{2}{3})^i \\
&= 2 \cdot \sum_{i=0}^{n}(\frac{2}{3})^i + \sum_{i=-4}^{-1}(\frac{2}{3})^i \\
&= 2 \cdot \frac{1}{1-\frac{2}{3}} + \sum_{i=-4}^{-1}(\frac{2}{3})^i \\
&= \frac{2}{\frac{1}{3}} + \sum_{i=-4}^{-1}(\frac{2}{3})^i \\
&= 6 + (\frac{2}{3})^{-4} + (\frac{2}{3})^{-3} + (\frac{2}{3})^{-2} + (\frac{2}{3})^{-1} \\
&= \frac{291}{16}
\end{aligned}
$$

7. $\sum_{i=1}^{n}(i^3 + 2 \cdot i^2 - i + 1)$

$$
\begin{aligned}
\sum_{i=1}^{n}(i^3 + 2 \cdot i^2 - i + 1) &= \sum_{i=1}^{n} i^3 + \sum_{i=1}^{n} 2 \cdot i^2 - \sum_{i=1}^{n} i + \sum_{i=1}^{n} 1 \\
&= (\frac{n(n+1)}{2})^2 + 2 \cdot \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + n \\
&= (\frac{n(n+1)}{2})^2 + \frac{n(n+1)(2n+1)}{3} - \frac{n(n+1)}{2} + n
\end{aligned}
$$

8. $\sum_{i=5}^{n}(-4 \cdot i + \frac{i}{5})$

$$
\begin{aligned}
\sum_{i=5}^{n}(-4\cdot i+\frac{i}{5}) &= \sum_{i=5}^{n}-4\cdot i+\sum_{i=5}^{n}\frac{i}{5} \\
&= -4\cdot\sum_{i=5}^{n} i+\frac{1}{5}\cdot\sum_{i=5}^{n} i \\
&= -4\cdot\sum_{i=1}^{n} i+\frac{1}{5}\cdot\sum_{i=1}^{n} i+4\cdot\sum_{i=1}^{4} i-\frac{1}{5}\cdot\sum_{i=1}^{4} i \\
&= -4\cdot\frac{n(n+1)}{2}+\frac{1}{5}\cdot\frac{n(n+1)}{2}+4\cdot\frac{4(5)}{2}-\frac{1}{5}\cdot\frac{4(5)}{2} \\
&= -4\cdot\frac{n(n+1)}{2}+\frac{n(n+1)}{10}+40-2 \\
&= -4\cdot\frac{n(n+1)}{2}+\frac{n(n+1)}{10}+38
\end{aligned}
$$

9. $\sum_{j=0}^{k}\sum_{i=1}^{j}(i-j^2-2)$

$$
\begin{aligned}
\sum_{j=0}^{k}\sum_{i=1}^{j}(i-j^2-2) &= \sum_{j=0}^{k}\sum_{i=1}^{j} i-\sum_{j=0}^{k}\sum_{i=1}^{j} j^2-\sum_{j=0}^{k}\sum_{i=1}^{j} 2 \\
&= \sum_{j=0}^{k}\frac{j(j+1)}{2}-\sum_{j=0}^{k} j^3-\sum_{j=0}^{k} 2\cdot j \\
&= \sum_{j=0}^{k}\frac{j^2+j}{2}-\sum_{j=0}^{k} j^3-\sum_{j=0}^{k} 2\cdot j \\
&= \sum_{j=0}^{k}\frac{j^2}{2}+\sum_{j=0}^{k}\frac{j}{2}-\sum_{j=0}^{k} j^3-\sum_{j=0}^{k} 2\cdot j \\
&= \frac{1}{2}\cdot\sum_{j=0}^{k} j^2+\frac{1}{2}\cdot\sum_{j=0}^{k} j-\sum_{j=0}^{k} j^3-2\cdot\sum_{j=0}^{k} j \\
&= \frac{n(n+1)(2n+1)}{12}+\frac{n(n+1)}{4}-(\frac{n(n+1)}{2})^2-n(n+1)
\end{aligned}
$$

10. $\sum_{j=1}^{m}\sum_{k=1}^{j}(3\cdot+k-3\cdot j+i)$

11. $\sum_{l=-4}^{n}\sum_{j=1}^{k}\sum_{i=1}^{j}(i-4)$

2. Calculate the answer

1. $log_4 x = 5 \rightarrow x = ?$

$$\begin{aligned} log_4 x &= 5 \\ x &= 4^5 \end{aligned}$$

2. $log_3 y = 4 \rightarrow y = ?$

$$\begin{aligned} log_3 y &= 4 \\ y &= 3^4 \end{aligned}$$

3. $x = 7^2 \rightarrow log_7 x = ?$

$$\begin{aligned} x &= 7^2 \\ log_7 x &= log_7 7^2 \\ log_7 x &= 2 \cdot log_7 7 \\ log_7 x &= 2 \end{aligned}$$

4. $x = 32 \rightarrow log_2 x = ?$

$$\begin{aligned} x &= 32 \\ log_2 x &= log_2 32 \\ log_2 x &= 5 \end{aligned}$$

5. $2^{log5} + 4^{log6} - 27^{log_3 5}$
6. $9^{log_3 2} - 25^{log_5 4} - 36^{log_6 7} + 8^{log_8 6}$
7. $log(4^5 \times 8^3) - log(16 - 8) + log(\frac{2^{10}}{4 \times 3^2})$
8. $log(3^2 \times 64^3) - log(\frac{2^{10} \times 128^3}{9 \times 8^2})$
9. $loglog16$
10. $log16 \times log16$
11. $log^2 16$
12. $log_2 log_5 625 - log_3 log_4 2^{3^9} + log^4 2^5 - \frac{log^2(4^3 \times 3^5)}{log_5 125}$
13. $loglog_8 log256 + log^5(3^2) \times 4^{log7}$
14. $log_6 x = 5 \rightarrow log_x 6 = ?$
15. $log_y x = 10 \rightarrow log_x y = ?$
16. $log_4 32 - log_8^2 4$
17. $log_4 8 + log_9 27 - log_{25}^2 125 - log_8^3 16 + log_4 log256$

3. Compute the deriative of

1. $-5 \cdot x^3 + 2 \cdot x - 1$
2. $3 \cdot x^4 - 2\sqrt{x} + x^{\frac{1}{2}} - 6x^{-\frac{2}{3}} - 5$
3. $x \cdot \sqrt{x} + \sqrt{\sqrt{x}}$
4. $logx - x^2 lnx + lnx^4$
5. $ln^3(x\sqrt{2x-3}) + \sqrt{lnx^2}$
6. $\frac{\sqrt[4]{x+5} - lnx}{(x-1)^3}$

4. Determine the limit of

1. $lim_{x\to\infty} \frac{3x+2}{-5x-6}$
2. $lim_{x\to\infty} (\frac{1}{x} + 3)$
3. $lim_{x\to\infty} \frac{x^3+x-\sqrt{3x}}{\sqrt{x}}$
4. $lim_{x\to\infty} \frac{x^3+x-\sqrt{3x}}{5 \cdot x^{2.25} \cdot \sqrt{\sqrt{x}}}$
5. $lim_{x\to\infty} \frac{x^{0.1}-\sqrt{3}}{\sqrt{\sqrt{x}}}$
6. $lim_{x\to\infty} \frac{x^x}{2^x}$
7. $lim_{x\to\infty} \frac{x^x}{x(2^x)}$
8. $lim_{x\to\infty} \frac{\sqrt{2}^{log^4 x^3}}{log(2 \cdot x+7)}$
9. $lim_{x\to\infty} \frac{x+1}{\frac{3 \cdot x^{lnx}}{2x^2}}$
10. $lim_{x\to\infty} \frac{\sqrt{2}^{logx^3}}{log^{lnx}(2x)}$

5. Compute the exact values for

1. $\int_1^n (2 \cdot x^4 + 5\sqrt{x})$
2. $\int_1^n (x^4 - 3 \cdot x^2 + \frac{1}{x} - \frac{1}{x^2})dx$
3. $\int_1^n (\frac{3}{\sqrt{x}} + lnx + e^x)dx$
4. $\int_1^n x \cdot sinxdx$

6. Use mathematical induction to prove that

$$1 + 2 + \ldots + n \quad = \quad \frac{n(n+1)}{2}$$

Proof. $1 + 2 + \ldots + n = \frac{n(n+1)}{2}$

Base case $n = 1$: If $n = 1$ then the left hand side and the right hand size is $1 = 1 = \frac{1(2)}{2}$.

Inductive hypothesis: Suppose the theorem holds for all values of n up to some $k, k \geq 1$.

Inductive step: let $n = k + 1$ then our left hand side is

$$\begin{aligned}
\sum_{i=1}^{k+1} i &= \sum_{i=1}^{k} i + (k+1) \\
&= \frac{k(k+1)}{2} + (k+1) \\
&= \frac{k(k+1) + 2 \cdot k + 2}{2} \\
&= \frac{(k+1) \cdot (k+2)}{2}
\end{aligned}$$

which is equal to our right hand side. By the principle of mathematical induction, the theorem holds for all integers $n \geq 1$. □

7. Use mathematical induction to prove that

$$1 + 2^2 + \ldots + n^2 \quad = \quad \frac{n(n+1)(2n+1)}{6}$$

$1 + 2 + \ldots + n = \frac{n(n+1)}{2}$

Proof. Base case $n = 1$: If $n = 1$ then the left hand side and the right hand size is $1^2 = 1 = \frac{1(2)(3)}{6}$.

Inductive hypothesis: Suppose the theorem holds for all values of n up to some $k, k \geq 1$.

Inductive step: let $n = k + 1$ then our left hand side is

$$
\begin{aligned}
\sum_{i=1}^{k+1} i^2 &= \sum_{i=1}^{k} i^2 + (k+1)^2 \\
&= \frac{k \cdot (k+1) \cdot (2 \cdot k + 1)}{6} + (k+1)^2 \\
&= \frac{k \cdot (k+1) \cdot (2 \cdot k + 1) + 6 \cdot (k+1)^2}{6} \\
&= \frac{(6 \cdot (k+1) + k \cdot (2 \cdot k + 1)) \cdot (k+1)}{6} \\
&= \frac{(6 \cdot k + 6 + 2 \cdot k^2 + k) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k^2 + 7 \cdot k + 6) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k^2 + 4 \cdot k + 3 \cdot k + 6) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k \cdot (k+2) + 3 \cdot (k+2)) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k + 3) \cdot (k+2) \cdot (k+1)}{6}
\end{aligned}
$$

which is equal to our right hand side. By the principle of mathematical induction, the theorem holds for all integers $n \geq 1$. $\square$