1 Compute the values for

1. $\sum_{i=-1}^{4} 3$

$$
\begin{aligned}
\sum_{i=-1}^{4} 3 &= 3 \cdot (4 + 1 + 1) \\
&= 3 \cdot 6 \\
&= 18
\end{aligned}
$$

2. $\sum_{i=1}^{5} \left(\frac{1}{3}\right)^i$

$$
\begin{aligned}
\sum_{i=1}^{5} \left(\frac{1}{3}\right)^i &= \sum_{i=0}^{5} \left(\frac{1}{3}\right)^i - \sum_{i=0}^{0} \left(\frac{1}{3}\right)^i \\
&= \frac{\left(\frac{1}{3}\right)^6 - 1}{\frac{1}{3} - 1} - 1
\end{aligned}
$$

3. $\sum_{i=1}^{n} 3$

$$
\begin{aligned}
\sum_{i=1}^{n} 3 &= 3 \cdot \sum_{i=1}^{n} 1 \\
&= 3 \cdot n
\end{aligned}
$$

4. $\sum_{i=-3}^{n} 3$

$$
\sum_{i=-3}^{n} 3 = 3 \cdot (n + 4)
$$

5. $\sum_{k=0}^{n} 2^k + \sum_{k=5}^{n} 2^k$

$$\sum_{k=0}^{n} 2^k + \sum_{k=5}^{n} 2^k = \sum_{k=0}^{n} 2^k + \sum_{k=0}^{n} 2^k - \sum_{k=0}^{4} 2^k$$

$$= 2 \sum_{k=0}^{n} 2^k - \sum_{k=0}^{4} 2^k$$

$$= 2 \cdot \frac{2^{n+1} - 1}{2 - 1} - \frac{2^5 - 1}{2 - 1}$$

6. $\sum_{i=0}^{n} \left(\frac{2}{3}\right)^i + \sum_{i=-4}^{n} \left(\frac{2}{3}\right)^i$

$$\sum_{i=0}^{n} \left(\frac{2}{3}\right)^i + \sum_{i=-4}^{n} \left(\frac{2}{3}\right)^i = \frac{\left(\frac{2}{3}\right)^{n+1} - 1}{\frac{2}{3} - 1} + \sum_{i=-4}^{-1} \left(\frac{2}{3}\right)^i + \sum_{i=0}^{n} \left(\frac{2}{3}\right)^i$$

$$= \frac{\left(\frac{2}{3}\right)^{n+1} - 1}{\frac{2}{3} - 1} + \sum_{i=-4}^{-1} \left(\frac{2}{3}\right)^i + \frac{\left(\frac{2}{3}\right)^{n+1} - 1}{\frac{2}{3} - 1}$$

$$= \frac{\left(\frac{2}{3}\right)^{n+1} - 1}{\frac{2}{3} - 1} + \sum_{i=1}^{4} \left(\frac{3}{2}\right)^i + \frac{\left(\frac{2}{3}\right)^{n+1} - 1}{\frac{2}{3} - 1}$$

$$= \frac{\left(\frac{2}{3}\right)^{n+1} - 1}{\frac{2}{3} - 1} + \frac{\left(\frac{3}{2}\right)^5 - 1}{\left(\frac{3}{2}\right) - 1} + \frac{\left(\frac{2}{3}\right)^{n+1} - 1}{\frac{2}{3} - 1}$$

7. $\sum_{i=1}^{n} \left(i^3 + 2 \cdot i^2 - i + 1\right)$

$$\sum_{i=1}^{n} \left(i^3 + 2 \cdot i^2 - i + 1\right) = \sum_{i=1}^{n} i^3 + 2 \cdot \sum_{i=1}^{n} i^2 - \sum_{i=1}^{n} i + \sum_{i=1}^{n} 1$$

$$= \left(\frac{n \cdot (n+1)}{2}\right)^2 + 2 \cdot \frac{(n) \cdot (n+1) \cdot (2n+1)}{6} - \frac{n \cdot (n+1)}{2} + n$$

8. $\sum_{i=5}^{n} \left(-4 \cdot i + \frac{i}{5}\right)$

2

$$\sum_{i=5}^{n}(-4 \cdot i + \frac{i}{5}) = -4\sum_{i=5}^{n} i + \frac{1}{5}\sum_{i=5}^{n} i$$

$$= -4\sum_{i=0}^{n} i + \frac{1}{5}\sum_{i=0}^{n} i + 4\sum_{i=0}^{4} i - \frac{1}{5}\sum_{i=0}^{4} i$$

$$= -4 \cdot \frac{n(n+1)}{2} + \frac{1}{5} \cdot \frac{n(n+1)}{2} + 4 \cdot \frac{4 \cdot 5}{2} - \frac{1}{5} \cdot \frac{4 \cdot 5}{2}$$

9. $\sum_{j=0}^{k}\sum_{i=1}^{j}(i - j^2 - 2)$

$$\sum_{j=0}^{k}\sum_{i=1}^{j}(i - j^2 - 2) = \sum_{j=0}^{k}\sum_{i=1}^{j} i - \sum_{j=0}^{k}\sum_{i=1}^{j} j^2 - \sum_{j=0}^{k}\sum_{i=1}^{j} 2$$

$$= \sum_{j=0}^{k}\frac{j(j+1)}{2} - \sum_{j=0}^{k} j^3 - \sum_{j=0}^{k} 2 \cdot j$$

$$= \sum_{j=0}^{k}\frac{j^2}{2} + \sum_{j=0}^{k}\frac{j}{2} - (\frac{k \cdot (k+1)}{2})^2 - 2 \cdot \frac{k \cdot (k+1)}{2}$$

$$= \frac{1}{2} \cdot \frac{(k) \cdot (k+1) \cdot (2k+1)}{6} + \frac{1}{2} \cdot \frac{k \cdot (k+1)}{2} - (\frac{k \cdot (k+1)}{2})^2 - 2 \cdot \frac{k \cdot (k+1)}{2}$$

10. $\sum_{j=1}^{m}\sum_{k=1}^{j}(3 \cdot C + k - 3 \cdot j + i)$

$$\sum_{j=1}^{m}\sum_{k=1}^{j}(3\cdot C + k - 3\cdot j + i) \;=\; \sum_{j=1}^{m}\sum_{k=1}^{j}(3\cdot C + k - 3\cdot j + i)$$

$$= \sum_{j=1}^{m}\sum_{k=1}^{j}3\cdot C + \sum_{j=1}^{m}\sum_{k=1}^{j}k - 3\sum_{j=1}^{m}\sum_{k=1}^{j}j + \sum_{j=1}^{m}\sum_{k=1}^{j}i$$

$$= \sum_{j=1}^{m}3\cdot C\cdot j + \sum_{j=1}^{m}\frac{j(j+1)}{2} - 3\sum_{j=1}^{m}j^2 + \sum_{j=1}^{m}i\cdot j$$

$$= 3\cdot C\cdot\frac{m\cdot(m+1)}{2} + \sum_{j=1}^{m}\frac{j^2}{2} + \sum_{j=1}^{m}\frac{j}{2} - 3\cdot\frac{(m)(m+1)(2m+1)}{6}$$

$$+ \; i\cdot\frac{(m)(m+1)}{2}$$

$$= 3\cdot C\frac{m\cdot(m+1)}{2} + \frac{1}{2}\cdot\frac{m(m+1)(2m+1)}{6} + \frac{1}{2}\cdot\frac{(m)(m+1)}{2}$$

$$- \; 3\cdot\frac{(m)(m+1)(2m+1)}{6} + i\cdot\frac{(m)(m+1)}{2}$$

11. $\sum_{l=-4}^{n}\sum_{j=1}^{k}\sum_{i=1}^{j}(i-4)$

$$\sum_{l=-4}^{n}\sum_{j=1}^{k}\sum_{i=1}^{j}(i-4) \;=\; \sum_{l=-4}^{n}\sum_{j=1}^{k}\sum_{i=1}^{j}i - 4\sum_{l=-4}^{n}\sum_{j=1}^{k}\sum_{i=1}^{j}1$$

$$= \sum_{l=-4}^{n}\sum_{j=1}^{k}\frac{j(j+1)}{2} - \sum_{l=-4}^{n}\sum_{j=1}^{k}j$$

$$= \sum_{l=-4}^{n}\sum_{j=1}^{k}\frac{j^2}{2} + \sum_{l=-4}^{n}\sum_{j=1}^{k}\frac{j}{2} - \sum_{l=-4}^{n}\frac{k(k+1)}{2}$$

$$= \sum_{l=-4}^{n}\frac{(k)(k+1)(2k+1)}{12} + \sum_{l=-4}^{n}\frac{(k)(k+1)}{4} - \sum_{l=-4}^{n}\frac{k(k+1)}{2}$$

$$= (n+5)\frac{(k)(k+1)(2k+1)}{12} + (n+5)\frac{(k)(k+1)}{4} - (n+5)\frac{(k)(k+1)}{2}$$

2. Calculate the answer

1. $log_4 x = 5 \to x =?$

$$log_4 x \;=\; 5$$
$$x \;=\; 4^5$$

2. $log_3 y = 4 \rightarrow y = ?$

$$log_3 y = 4$$
$$y = 3^4$$

3. $x = 7^2 \rightarrow log_7 x = ?$

$$x = 7^2$$
$$log_7 x = log_7 7^2$$
$$log_7 x = 2 \cdot log_7 7$$
$$log_7 x = 2$$

4. $x = 32 \rightarrow log_2 x = ?$

$$x = 32$$
$$log_2 x = log_2 32$$
$$log_2 x = 5$$

5. $2^{log5} + 4^{log6} - 27^{log_3 5}$

$$2^{log_2 5} + 4^{log_2 6} - 27^{log_3 5} = 5 + 36 - 5^3$$

6. $9^{log_3 2} - 25^{log_5 4} - 36^{log_6 7} + 8^{log_8 6}$

$$9^{log_3 2} - 25^{log_5 4} - 36^{log_6 7} + 8^{log_8 6} = 3^{log_3(2^2)} - 5^{log_5(4^2)} - 6^{log_6 7^2 + 6}$$
$$= 4 - 16 - 49 + 6$$

7. $log(4^5 \times 8^3) - log(16 - 8) + log(\frac{2^{10}}{4 \times 3^2})$

$$log_2(4^5 \times 8^3) - log_2(16 - 8) + log_2(\frac{2^{10}}{4 \times 3^2}) = log_2(4^5) + log_2(8^3) - log_2(8) + log_2(2^{10}) - log_2(4 \times 3^2)$$
$$= 5 \cdot log_2(4) + 3 \cdot log_2(8) - log_2(8) + 10 \cdot log_2(2) - log_2(4)$$
$$= 5 \cdot 2 + 3 \cdot 3 - 3 + 10 - 2 - 2 \cdot 1.5$$
$$= 21$$

8. $log(3^2 \times 64^3) - log(\frac{2^{10} \times 128^3}{9 \times 8^2})$

$$log_2(3^2 \times 64^3) - log_2(\frac{2^{10} \times 128^3}{9 \times 8^2}) = log_2(3^2) + log_2(64^3) - log_2(2^{10} \times 128^3) + log_2(9) + log_2 8^2$$
$$= 2 \cdot log_2(3) + 3 \cdot log_2(64) - 10 \cdot log_2 2 - 3 \cdot log_2 128 + 2 \cdot log_2 3 + 2 \cdot$$
$$= 2 \cdot 1.5 + 3 \cdot 6 - 10 - 3 \cdot 7 + 2 \cdot 1.5 + 2 \cdot 3$$
$$= -1$$

9. $log\,log\,16$

$$log_2 log_2 16 = log_2 4$$
$$= 2$$

10. $log\,16 \times log\,16$

$$log_2 16 \times log_2 16 = 4 \times 4$$
$$= 16$$

11. $log^2 16$

$$log_2^2 16 = 4^2$$
$$= 16$$

12. $log_2 log_5 625 - log_3 log_4 2^{3^9} + log^4 2 5 - \frac{log^2(4^3 \times 3^5)}{log_5 125}$

$$log_2 log_5 625 - log_3 log_4 2^{3^9} + log_2^4 2^5 - \frac{log^2(4^3 \times 3^5)}{log_5 125} = log_2 4 - log_3(3^9 \cdot log_4 2) + log_2^4 2^5 - \frac{log_2^2(4^3 \times 3^5)}{log_5 125}$$

$$= 2 - 9 log_3(\frac{3}{2}) + (5 \cdot log_2(2))^4 - \frac{(log_2(4^3 \times 3^5))^2}{3}$$

$$= 2 - 9 + 9 \cdot log_3(\frac{1}{2}) + 5^4 - \frac{(3 \cdot 2 + 5 \cdot log_2(3))^2}{3}$$

$$= 2 - 9 + 9 \cdot \frac{-1}{1.5} + 5^4 - \frac{(3 \cdot 2 + 5 \cdot (1.5)^2}{3}$$

13. $loglog_8 log256 + log^5(3^2) \times 4^{log7}$

$$log_2 log_8 log_2 256 + log_2^5(3^2) \cdot 4^{log_2 7} = log_2 log_8 8 + log_2^5(3^2) \cdot 4^{log_2 7}$$

$$= log_2 1 + log_2^5(3^2) \cdot 4^{log_2 7}$$

$$= log_2^5(3^2) \cdot 4^{log_2 7}$$

$$= (2 \cdot 1.5)^5 \cdot 2^{log_2 49}$$

$$= (2 \cdot 1.5)^5 \cdot 49$$

$$= 11907$$

14. $log_6 x = 5 \rightarrow log_x 6 = ?$

$$\begin{aligned} log_6 x &= 5 \\ x &= 6^5 \\ log_x x &= 5 \cdot log_x 6 \\ 1 &= 5 \cdot log_x 6 \\ \frac{1}{5} &= log_x 6 \end{aligned}$$

15. $log_y x = 10 \rightarrow log_x y = ?$

$$\begin{aligned} log_y x &= 10 \\ x &= y^{10} \\ log_x x &= 10 \cdot log_x y \\ \frac{1}{10} &= log_x y \end{aligned}$$

16. $log_4 32 - log_8^2 4$

$$
\begin{aligned}
log_4 32 - log_8^2 4 &= log_4(32) - (log_8 4)^2 \\
&= \frac{5}{2} - \left(\frac{log_2 4}{log_2 8}\right)^2 \\
&= \frac{5}{2} - \left(\frac{2}{3}\right)^2 \\
&= \frac{5}{2} - \frac{4}{9}
\end{aligned}
$$

17. $log_4 8 + log_9 27 - log_{25}^2 125 - log_8^3 16 + log_4 log_2 256$

$$
\begin{aligned}
log_4 8 + log_9 27 - log_{25}^2 125 - log_8^3 16 + log_4 log_2 256 &= \frac{3}{2} + \frac{3}{2} - \left(\frac{3}{2}\right)^2 - \left(\frac{4}{3}\right)^3 + log_4(8) \\
&= \frac{3}{2} + \frac{3}{2} - \left(\frac{9}{4}\right) - \left(\frac{64}{27}\right) + \frac{3}{2}
\end{aligned}
$$

3. Compute the deriative of

1. $-5 \cdot x^3 + 2 \cdot x - 1$

$$
\frac{d}{dx}(-5 \cdot x^3 + 2 \cdot x - 1) = -15 \cdot x^2 + 2
$$

2. $3 \cdot x^4 - 2\sqrt{x} + x^{\frac{1}{2}} - 6x^{-\frac{2}{3}} - 5$

$$
\begin{aligned}
\frac{d}{dx}(3 \cdot x^4 - 2\sqrt{x} + x^{\frac{1}{2}} - 6x^{-\frac{2}{3}} - 5) &= 12 \cdot x^3 - x^{-\frac{1}{2}} + \frac{1}{2} \cdot x^{-\frac{1}{2}} + 4 \cdot x^{-\frac{5}{3}} \\
&= 12 \cdot x^3 - \frac{1}{2}x^{-\frac{1}{2}} + 4 \cdot x^{-\frac{5}{3}}
\end{aligned}
$$

3. $x \cdot \sqrt{x} + \sqrt{\sqrt{x}}$

$$
\frac{d}{dx}(x^{\frac{3}{2}} + x^{\frac{1}{4}}) = \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{4} \cdot x^{-\frac{3}{4}}
$$

4. $logx - x^2 lnx + lnx^4$

$$\frac{d}{dx}(log_2 x - x^2 \cdot lnx + lnx^4) = \frac{1}{x \cdot ln2} - 2 \cdot x \cdot lnx - x + \frac{4}{x}$$

5. $ln^3(x\sqrt{2x-3}) + \sqrt{lnx^2}$

$$\frac{d}{dx}(ln^3(x \cdot \sqrt{2x-3}) + \sqrt{lnx^2}) = \frac{3 \cdot (ln(x\sqrt{2x-3}))^2 \cdot (\sqrt{2x-3} + x \cdot (2 \cdot x - 3)^{-\frac{1}{2}}}{x\sqrt{2x-3}}$$

$$+ \quad \frac{1}{x \cdot \sqrt{ln(x^2)}}$$

6. $\frac{\sqrt[4]{x+5}-lnx}{(x-1)^3}$

$$\frac{d}{dx}\left(\frac{(x+5)^{\frac{1}{4}} - lnx}{(x-1)^3}\right) = \frac{(x-1)^3 \cdot (\frac{1}{4} \cdot (x+5)^{-\frac{3}{4}} - \frac{1}{x}) - 3 \cdot ((x+5)^{\frac{1}{4}} - lnx) \cdot (x-1)^2}{(x-1)^6}$$

4. Determine the limit of

1. $lim_{x\to\infty}\frac{3x+2}{-5x-6}$

$$lim_{x\to\infty}\frac{3x+2}{-5x-6} = -\frac{3}{5}$$

2. $lim_{x\to\infty}(\frac{1}{x} + 3)$

$$lim_{x\to\infty}(\frac{1}{x} + 3) = lim_{x\to\infty}\frac{1}{x} + lim_{x\to\infty}3$$

$$= lim_{x\to\infty}\frac{1}{x}^{0} + lim_{x\to\infty}3$$

$$= 3$$

3. $lim_{x\to\infty}\frac{3\cdot x\cdot log_2(x)+2}{\sqrt{x^3}+7\cdot x}$

$$lim_{x\to\infty}\frac{3 \cdot x \cdot log_2(x) + 2}{\sqrt{x^3} + 7 \cdot x} = lim_{x\to\infty}\frac{3 \cdot x \cdot log_2(x)}{7 \cdot x}$$

$$= lim_{x\to\infty}\frac{3 \cdot log_2(x)}{7}$$

$$= \infty$$

9

4. $lim_{x\to\infty} \frac{x^3+x-\sqrt{3x}}{\sqrt{x}}$

$$lim_{x\to\infty} \frac{x^3+x-\sqrt{3x}}{\sqrt{x}} \quad = \quad lim_{x\to\infty}\frac{x^3}{\sqrt{x}} + lim_{x\to\infty}\frac{x}{\sqrt{x}} - lim_{x\to\infty}\frac{\sqrt{3x}}{\sqrt{x}}$$
$$= \quad \infty$$

5. $lim_{x\to\infty} \frac{x^3+x-\sqrt{3x}}{5\cdot x^{2.25}\cdot\sqrt{\sqrt{x}}}$

$$lim_{x\to\infty} \frac{x^3+x-\sqrt{3x}}{5\cdot x^{2.25}x^{.25}} \quad = \quad lim_{x\to\infty}\frac{x^3+x-\sqrt{x}}{5\cdot x^3}$$
$$= \quad lim_{x\to\infty}\frac{x^3}{5\cdot x^3} + lim_{x\to\infty}\frac{x}{5\cdot x^3}\nearrow^{0} - lim_{x\to\infty}\frac{\sqrt{x}}{5\cdot x^3}\nearrow^{0}$$
$$= \quad \frac{1}{5}$$

6. $lim_{x\to\infty} \frac{x^{0.1}-\sqrt{3}}{\sqrt{\sqrt{x}}}$

$$lim_{x\to\infty} \frac{x^{0.1}-\sqrt{3}}{x^{0.25}} \quad = \quad lim_{x\to\infty}\frac{x^{0.1}}{x^{0.25}} - \frac{\sqrt{3}}{x^{0.25}}$$
$$= \quad lim_{x\to\infty}\frac{1}{x^{0.15}} - \frac{\sqrt{3}}{x^{0.25}}$$
$$= \quad 0$$

7. $lim_{x\to\infty} \frac{x^x}{2^x}$

$$log(lim_{x\to\infty} \frac{x^x}{2^x}) \quad = \quad lim_{x\to\infty}log(\frac{x^x}{2^x})$$
$$= \quad lim_{x\to\infty}log(x^x) - lim_{x\to\infty}log(2^x)$$
$$= \quad lim_{x\to\infty}xlog(x) - lim_{x\to\infty}xlog(2)$$
$$= \quad \infty$$
$$e^{ln(lim_{x\to\infty}\frac{x^x}{2^x})} \quad = \quad e^{\infty}$$
$$= \quad \infty$$

8. $lim_{x\to\infty} \frac{x^x}{x(2^x)}$

$$
\begin{aligned}
log(lim_{x\to\infty}\frac{x^x}{x(2^x)}) &= lim_{x\to\infty}log(\frac{x^x}{x\cdot 2^x}) \\
&= lim_{x\to\infty}log(x^x) - lim_{x\to\infty}log(x\cdot 2^x) \\
&= lim_{x\to\infty}xlog(x) - lim_{x\to\infty}log(x) - lim_{x\to\infty}log(2^x) \\
&= lim_{x\to\infty}xlog(x) - lim_{x\to\infty}log(x) - lim_{x\to\infty}xlog(2) \\
&= \infty \\
e^{ln(lim_{x\to\infty}\frac{x^x}{x(2^x)})} &= e^{\infty} \\
&= \infty
\end{aligned}
$$

9. $lim_{x\to\infty}\dfrac{log_2(x)^{log_2(x)}}{x^{\frac{1}{5}}}$

$$
\begin{aligned}
lim_{x\to\infty}log_2(\frac{log_2(x)^{log_2(x)}}{x^{\frac{1}{5}}}) &= lim_{x\to\infty}log_2(x)\cdot log_2(log_2(x)) - \frac{1}{5}\cdot log_2(x) \\
&= lim_{x\to\infty}\infty \\
2^{lim_{x\to\infty}log_2(\frac{log_2(x)^{log_2(x)}}{x^{\frac{1}{5}}})} &= 2^{\infty} \\
&= \infty
\end{aligned}
$$

10. $lim_{x\to\infty}\dfrac{\sqrt{2}^{log^4 x^3}}{log(2\cdot x+7)}$

$$
\begin{aligned}
lim_{x\to\infty}\frac{2^{\frac{1}{2}(3log(x))^4}}{log(2\cdot x + 7)} &= lim_{x\to\infty}\frac{2^{\frac{81}{4}(log(x))^4}}{log(2\cdot x + 7)} \\
&= \infty
\end{aligned}
$$

11. $lim_{x\to\infty}\dfrac{x+1}{\frac{3\cdot x^{lnx}}{2x^2}}$

$$
\begin{aligned}
lim_{x\to\infty}\frac{x+1}{\frac{3\cdot x^{lnx}}{2\cdot x^2}} &= lim_{x\to\infty}\frac{x+1}{6\cdot x^2\cdot x^{lnx}} \\
&= 0
\end{aligned}
$$

12. $lim_{x\to\infty}\dfrac{\sqrt{2}^{logx^3}}{log^{lnx}(2x)}$

$$\lim_{x\to\infty}\frac{\sqrt{2}^{log x^3}}{log^{lnx}(2x)} = \lim_{x\to\infty}\frac{2^{\frac{1}{2}\cdot log x^3}}{log^{lnx}(2x)}$$

$$= \lim_{x\to\infty}\frac{2^{log x^{\frac{3}{2}}}}{log^{lnx}(2x)}$$

$$= \lim_{x\to\infty}\frac{x^{\frac{3}{2}}}{log^{lnx}(2x)}$$

$$\lim_{x\to\infty}log(\frac{x^{\frac{3}{2}}}{log^{lnx}(2x)}) = \lim_{x\to\infty}\frac{3}{2}log(x) - log(x)log(log(2x))$$

$$= \lim_{x\to\infty}\frac{3}{2}log(x) - log(x)(log(1+log(x)))$$

$$= -\infty$$

$$2^{log(\lim_{x\to\infty}\frac{x^{\frac{3}{2}}}{log^{lnx}(2x)})} = 2^{-\infty}$$

$$= 0$$

5. Compute the exact values for

1. $\int_1^n (2\cdot x^4 + 5\sqrt{x})\cdot dx$

$$\int_1^n (2\cdot x^4 + 5\sqrt{x}\ )\cdot dx = 2\int_1^n x^4 dx + 5\int_1^n \sqrt{x}dx$$

$$= 2\cdot(\frac{x^5}{5}|_1^n) + 5(\frac{2}{3}x^{\frac{3}{2}}|_1^n)$$

$$= 2\cdot(\frac{n^5}{5} - \frac{1}{5}) + 5(\frac{2}{3}n^{\frac{3}{2}} - \frac{2}{3})$$

$$= \frac{2\cdot n^5}{5} - \frac{2}{5} + \frac{10}{3}n^{\frac{3}{2}} - \frac{10}{3}$$

2. $\int_1^n (x^4 - 3\cdot x^2 + \frac{1}{x} - \frac{1}{x^2})dx$

$$\int_1^n (x^4 - 3\cdot x^2 + \frac{1}{x} - \frac{1}{x^2}dx = \int_1^n x^4 dx - 3\int_1^n x^2 dx + \int_1^n \frac{1}{x}dx - \int_1^n \frac{1}{x^2}dx$$

$$= \frac{x^5}{5}|_1^n - 3\cdot\frac{x^3}{3}|_1^n + lnx|_1^n + \frac{1}{x}|_1^n$$

$$= (\frac{n^5}{5} - \frac{1}{5}) - n^3 + 1 + lnn + \frac{1}{n} - 1$$

$$= \frac{n^5}{5} - n^3 + lnn + \frac{1}{n} - \frac{1}{5}$$

3. $\int_1^n (\frac{3}{\sqrt{x}} + lnx + e^x)dx$

$$\int_1^n \frac{3}{\sqrt{x}} + lnx + e^x dx = \int_1^n \frac{3}{\sqrt{x}} dx + \int_1^n lnx dx + \int_1^n e^x dx$$
$$= 6\sqrt{x}|_1^n + (xlnx - x|_1^n) + e^n - e$$
$$= 6\sqrt{n} - 6 + (nlnn - n + 1) + e^n - e$$
$$= 6\sqrt{n} + nlnn - n - 5 + e^n - e$$

4. $\int_1^n x \cdot e^x dx$

$$\int_1^n x \cdot e^x dx = x \cdot e^x|_1^n - \int_1^n e^x dx$$
$$= n \cdot e^n - e - e^n + e$$
$$= n \cdot e^n - e^n$$

5. $\int_1^n (x \cdot ln(x) - 4 \cdot ln(x)) dx$

$$\int_1^n (x \cdot ln(x) - 4 \cdot ln(x)) dx = \int_1^n x \cdot ln(x) dx - 4 \int_1^n ln(x) dx$$
$$= \frac{x^2 \cdot ln(x)}{2}|_1^n - \int_1^n dx - 4(x \cdot ln(x)|_1^n - \int_1^n x dx)$$
$$= \frac{x^2 \cdot ln(x)}{2}| - n|_1^n - 4(x \cdot ln(x)|_1^n - \frac{x^2}{2}|_1^n)$$
$$= \frac{n^2 \cdot ln(n)}{2} - n + 1 - 4 \cdot n \cdot ln(x) + 2 \cdot n^2 - 2$$

6. $\int_1^n x \cdot sinx dx$

$$\int_1^n x \cdot sinx dx = -xcosx|_1^n + \int_1^n cosx dx$$
$$= -ncosn + cos(1) + sin(n) - sin(1)$$

6. Use mathematical induction to prove that

$$1 + 2 + \ldots + n = \frac{n(n+1)}{2}$$

Proof. $1 + 2 + \ldots + n = \frac{n(n+1)}{2}$
   Base case $n = 1$: If $n = 1$ then the left hand side and the right hand size is
$1 = 1 = \frac{1(2)}{2}$.

Inductive hypothesis: Suppose the theorem holds for all values of n up to some $k, k \geq 1$.

Inductive step: let $n = k + 1$ then our left hand side is

$$
\begin{aligned}
\sum_{i=1}^{k+1} i &= \sum_{i=1}^{k} i + (k+1) \\
&= \frac{k(k+1)}{2} + (k+1) \\
&= \frac{k(k+1) + 2 \cdot k + 2}{2} \\
&= \frac{(k+1) \cdot (k+2)}{2}
\end{aligned}
$$

which is equal to our right hand side. By the principle of mathematical induction, the theorem holds for all integers $n \geq 1$. $\square$

7. Use mathematical induction to prove that

$$
1 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}
$$

$1 + 2 + \ldots + n = \frac{n(n+1)}{2}$

Proof. Base case $n = 1$: If $n = 1$ then the left hand side and the right hand size is $1^2 = 1 = \frac{1(2)(3)}{6}$.

Inductive hypothesis: Suppose the theorem holds for all values of n up to some $k, k \geq 1$.

Inductive step: let $n = k + 1$ then our left hand side is

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$$\sum_{i=1}^{k+1} i^2 = \sum_{i=1}^{k} i^2 + (k+1)^2$$

$$= \frac{k \cdot (k+1) \cdot (2 \cdot k + 1)}{6} + (k+1)^2$$

$$= \frac{k \cdot (k+1) \cdot (2 \cdot k + 1) + 6 \cdot (k+1)^2}{6}$$

$$= \frac{(6 \cdot (k+1) + k \cdot (2 \cdot k + 1)) \cdot (k+1)}{6}$$

$$= \frac{(6 \cdot k + 6 + 2 \cdot k^2 + k) \cdot (k+1)}{6}$$

$$= \frac{(2 \cdot k^2 + 7 \cdot k + 6) \cdot (k+1)}{6}$$

$$= \frac{(2 \cdot k^2 + 4 \cdot k + 3 \cdot k + 6) \cdot (k+1)}{6}$$

$$= \frac{(2 \cdot k \cdot (k+2) + 3 \cdot (k+2)) \cdot (k+1)}{6}$$

$$= \frac{(2 \cdot k + 3) \cdot (k+2) \cdot (k+1)}{6}$$

which is equal to our right hand side. By the principle of mathematical induction, the theorem holds for all integers $n \geq 1$. $\square$