1. Prove or disprove: $6 \cdot n \cdot log_2(n) + 2 \cdot n + \sqrt{n}$ is $\Omega(n \cdot log^2(n))$.

$$\begin{aligned} c \cdot n \cdot log^2(n) &\leq 6 \cdot n \cdot log_2(n) + 2 \cdot n + \sqrt{n} \\ c &\leq \frac{6 \cdot n \cdot log_2(n) + 2 \cdot n + \sqrt{n}}{n \cdot log^2(n)} \\ c &\leq 0 \end{aligned}$$

which is a contradiction hence $6 \cdot n \cdot log_2(n) + 2 \cdot n + \sqrt{n} \neq \Omega(n \cdot log^2(n))$.

2. Prove or disprove $4n^3 + n \cdot log_2(n)$ is $\Omega(n^2 \cdot log_2(n))$.

$$\begin{aligned} c \cdot n^2 \cdot log_2(n) &\leq 4 \cdot n^3 + n \cdot log_2(n) \\ c &\leq \frac{4 \cdot n^3 + n \cdot log_2(n)}{n^2 \cdot log_2(n)} \\ c &\leq \infty \end{aligned}$$

which is a contradiction hence $4 \cdot n^3 + n \cdot log_2(n) \neq \Omega(n^2 \cdot log_2(n))$.

3. Compare the growth of $f(n)$ and $g(n)$ where

$$\begin{aligned} f(n) &= 8 \cdot n \cdot log^2(n) + \sqrt{n} + 2 \\ g(n) &= n^2 + n \cdot log^5(n) \end{aligned}$$

$$\begin{aligned} lim_{n\to\infty} \frac{8 \cdot n \cdot log^2(n) + \sqrt{n} + 2}{n^2 + n \cdot log^5(n)} &= lim_{n\to\infty} \frac{8 \cdot n \cdot log^2(n)}{n^2} \\ &= 0 \end{aligned}$$

hence by the definition of little-o notation, $f(n) = o(g(n))$.

4. Compare the growth of $f(n)$ and $g(n)$ where

$$\begin{aligned} f(n) &= \sqrt{n} + log^{10}(n) + \sqrt{n} \cdot log(n) \\ g(n) &= \sqrt{\sqrt{n}} + log(log(n)) + 6 \end{aligned}$$

$$\begin{aligned} lim_{n\to\infty} \frac{f(n)}{g(n)} &= lim_{n\to\infty} \frac{\sqrt{n} + log^{10}(n) + \sqrt{n} \cdot log(n)}{\sqrt{\sqrt{n}} + log(log(n)) + 6} \\ &= lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{\sqrt{n}}} = \infty \end{aligned}$$

which by the definition of little-ω notation, $f(n) = \omega(f(n))$.

5. Compare the growth of $f(n)$ and $g(n)$ where

$$\begin{aligned} f(n) &= 5 \cdot n^2 \cdot log(n) + 2^{log(n)} + \sqrt{n} \\ g(n) &= 6 \cdot n^2 \cdot log(n^3) + n^2 \end{aligned}$$

$$\begin{aligned} lim_{n\to\infty} \frac{f(n)}{g(n)} &= \frac{5 \cdot n^2 \cdot log(n) + n + \sqrt{n}}{18 \cdot n^2 \cdot log(n) + n^2} \\ &= \frac{5}{18} \end{aligned}$$

hence $f(n) = \Theta(g(n))$. This then means that $f(n)$ is both $O(g(n))$ and $\Omega(g(n))$.

6. Compare the growth of $f(n)$ and $g(n)$ where

$$\begin{aligned} f(n) &= 2^{log(n^5)} + 2^{log(log^2(n))} + 4^{log(n^3)} \\ g(n) &= n^5 \cdot log(n) + n^{6.1} \end{aligned}$$

$$\begin{aligned} lim_{n\to\infty} \frac{f(n)}{g(n)} &= lim_{n\to\infty} \frac{n^5 + log^2(n) + n^6}{n^5 \cdot log(n) + n^{6.1}} \\ &= lim_{n\to\infty} \frac{n^6}{n^{6.1}} \\ &= 0 \end{aligned}$$

hence $f(n) = o(g(n))$, this implies $f(n) = O(g(n))$.

7. Compare the growth of $f(n)$ and $g(n)$ where

$$\begin{aligned} f(n) &= n \cdot log(n) + n^{1.01} + \sqrt{n} \\ g(n) &= \sqrt{n} \cdot log^3(n) + log^{10} n^5 + n^{.56} \end{aligned}$$

$$\begin{aligned} lim_{n\to\infty} \frac{f(n)}{g(n)} &= lim_{n\to\infty} \frac{n \cdot log(n) + n^{1.01} + \sqrt{n}}{\sqrt{n} \cdot log^3(n) + log^{10}(n^5) + n^{.56}} \\ &= lim_{n\to\infty} \frac{n^{1.01}}{n^{.56}} \\ &= \infty \end{aligned}$$

hence $f(n) = \omega(g(n))$. this also implies $f(n) = \Omega(g(n))$.