Exam 1.
Name:
Student ID:

Part C (You need to answer at least **6** questions completely to pass Part C**)**

1- Compute:

a) $\sum_{i=0}^{n}\sum_{k=0}^{i} 2^k$

b) $4^{log5} + log^2 8^3$

2- Use L'Hopital's rule to determine the limit of:

$$lim_{x \to \infty} \frac{xlnx^5 + 4}{(3x+1)^2}$$

3- What is the definition of *f(n)= Ω(g(n))*.

4- What is the growth of the below function: **(What is the most accurate answer?)**

$$f(n) = n2^{logn} + \sqrt{n} + 5nlog^3 n + log^2 n^n$$

a) Θ(n^2)
b) Θ ($nlog^3 n$)
c) Θ ($\sqrt{n}$)
d) Θ ($n^2 log^2 n$)
e) Neither!

5- What is the growth of the below function: **(What is the most accurate answer?)**

$$f(n) = 6logn + 4log^4 log^3 n + 3logn^2 + log^2 n$$

a) Θ($logn$)
b) Θ ($logn^2$)
c) Θ ($log^4 log^3 n$)
d) Θ ($log^2 n$)
e) Neither!

6- Suppose a machine on average takes 10^{-6} seconds to execute a single algorithm step. When does the machine finish executing the below code when n = 1000?

```
for(i=1; i <= n; i++)
    counting_sort(a); //a.length == n, max(a) = logn
```

7- Sort the below numbers using insertion sort: (**Show the work**)

1, 7, 2, 0, 4, 5, 3

Part B (You need to answer at least **3** questions completely to pass Part B**)**

8- Prove that $f(n) = 2nlogn^2 - 6log^2 n + \sqrt{n}$ is *O(nlogn),* provide the appropriate *C* and *k* constants.

9- Compare the growth of $f(n) = 4^{2^{logn}}$ and $g(n) = 2^{n+logn}$

10- Prove transitivity of big-O: if *f(n) = O(g(n))*, and *g(n) = O(h(n)),* then *f(n) = O(h(n)).*

11- What is the growth of $n^2 + 2n^2 + 3n^2 + 4n^2 + \cdots + n^5$?

Part A (You need to answer at least **2** questions completely to pass Part A**)**

12- Prove

$$\forall k > 0, \varepsilon > 0 \Rightarrow \log^k n = o(n^{\varepsilon})$$

13- Use the definition of big-Θ to prove that *f(n)+g(n) =* O(*max(f(n), g(n))*).

14- Prove or disprove:

$$(n!^{logn}) = \omega(n^{2^{logn}})$$

15- Given a sorted array of n distinct numbers where the range of the numbers are between *0* to *m* and *m > n* (*m* is given by user). Find the smallest missing number.

(**Example**:

Example 1: *input: a = [0, 1, 3, 6, 8, 9], m = 10* ➔ *Output: 2*

Example 2: *input: a = [0, 2, 5, 7, 11], m = 15* ➔ *Output: 1*