

1. Compute

$$1. \sum_{i=-10}^n \left(\frac{1}{2}\right)^i + \sum_{i=200}^{n^2} (3)^i$$

$$\begin{aligned} \sum_{i=-10}^n \left(\frac{1}{2}\right)^i + \sum_{i=200}^{n^2} (3)^i &= \sum_{i=-10}^{-1} \left(\frac{1}{2}\right)^i + \sum_{i=0}^n \left(\frac{1}{2}\right)^i + \sum_{i=0}^{n^2} (3^i) - \sum_{i=0}^{199} (3^i) \\ &= \sum_{i=-10}^0 \left(\frac{1}{2}\right)^i - \sum_{i=0}^0 \left(\frac{1}{2}\right)^i + \sum_{i=0}^{n^2} (3^i) - \sum_{i=0}^{199} (3^i) \\ &= -\sum_{i=0}^{10} \left(\frac{1}{2}\right)^i - 1 + \frac{3^{n^2+1} - 1}{3 - 1} - \frac{3^{200} - 1}{3 - 1} \\ &= -\frac{\left(\frac{1}{2}\right)^{11} - 1}{\frac{1}{2} - 1} - 1 + \frac{3^{n^2+1} - 1}{3 - 1} - \frac{3^{200} - 1}{3 - 1} \end{aligned}$$

$$2. 7^{\log_2 \log_2 4} + \log_3 \log_2^2 8$$

$$\begin{aligned} 7^{\log_2 \log_2 4} + \log_3 \log_2^2 8 &= 7^{\log_2 2} + \log_3 9 \\ &= 7 + 2 \\ &= 9 \end{aligned}$$

2. Use L'Hopital's rule to determine the limit of

$$\lim_{x \rightarrow \infty} \frac{x \ln x^2 + 3x}{\sqrt{4x^2 - 1}}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x \ln x^2 + 3x}{\sqrt{4x^2 - 1}} &= \lim_{x \rightarrow \infty} \frac{2 \cdot x \cdot \ln x + 3 \cdot x}{\sqrt{4 \cdot x^2 - 1}} \\ &= \lim_{x \rightarrow \infty} \frac{2 \cdot \ln x + 3}{\frac{1}{2}(4 \cdot x^2 - 1)^{-\frac{1}{2}}} \cdot 8 \cdot x \\ &= \infty \end{aligned}$$

3. What is the growth of the below function

$$f(n) = 8^{\log_2 n} + \sqrt{n^6 \log_2 n} + n \log_2^8 n + \log_2 n^{2^{\log_2 n}}$$

$$1. \Theta(n^3)$$

2. $\Theta(n^3 \log_2 n)$
3. $\Theta(n^3 \sqrt{\log_2 n})$
4. $\Theta(n \log_2 n)$
5. Neither!

$$\begin{aligned}
 f(n) &= 8^{\log_2 n} + \sqrt{n^6 \log_2 n} + n \cdot \log_2^8 n + \log_2 n^{2^{\log_2 n}} \\
 &= 2^{\log_2 n^3} + n^3 \cdot \sqrt{\log_2 n} + n \cdot \log_2^8 n + n \cdot \log_2 n \\
 &= n^3 + n^3 \cdot \sqrt{\log_2 n} + n \cdot \log_2^8 n + n \cdot \log_2 n
 \end{aligned}$$

Which is $\Theta(n^3 \sqrt{\log_2 n})$.

4. What is the growth of the below function.

1. $\Theta(\log_2 n)$
2. $\Theta(\log_2 \log_2^6 n)$
3. $\Theta(\log_2 n^{10})$
4. $\Theta(\log_2^2 n)$
5. Neither!

$$\begin{aligned}
 f(n) &= 2^{\log_2 \log_2 n} + 3 \log_2 \log_2^6 n + 5 \log_2^2 n + \log_2 n^{10} \\
 &= \log_2 n + 3 \cdot \log_2(\log_2^6 n) + 5 \cdot \log_2^2 n + 10 \cdot \log_2 n
 \end{aligned}$$

which is $\Theta(\log_2^2 n)$.