

1. Prove that  $f(n) = 10 \cdot n^4 + 2 \cdot n^2 + 3$  is  $O(n^4)$ , provide the appropriate  $C$  and  $k$  constants.

$$f(n) = O(g(n)) \iff \exists c > 0, \exists k \geq 0 \text{ s.t. } f(n) \leq c \cdot g(n) \forall n \geq k$$

$$\begin{aligned} 10 \cdot n^4 + 2 \cdot n^2 + 3 &\leq 10 \cdot n^4 + 2 \cdot n^4 + 3 \\ &\leq 10 \cdot n^4 + 2 \cdot n^4 + 3 \cdot n^4 \\ &= 15 \cdot n^4, n \geq 1 \end{aligned}$$

Let  $c = 15$  and  $k = 1$  then the statement translates to

$$f(n) \leq c \cdot g(n), n \geq k$$

By the definition of  $O$  notation  $f(n) = O(g(n))$

2. Prove that  $f(n) = 2 \cdot n^2 - n \cdot \log_2(n) + 3 \cdot \log_2(n)$  is  $O(n^2)$ , provide the appropriate  $C$  and  $k$  constants.

$$f(n) = O(g(n)) \iff \exists c > 0, \exists k \geq 0 \text{ s.t. } f(n) \leq c \cdot g(n) \forall n \geq k$$

$$\begin{aligned} 2 \cdot n^2 - n \cdot \log_2(n) + 3 \cdot \log_2(n) &\leq 2 \cdot n^2 + n^2 + 3 \cdot \log_2(n) \\ &\leq 2 \cdot n^2 + n^2 + 3 \cdot n^2 \\ &= 6 \cdot n^2, n \geq 1 \end{aligned}$$

Let  $c = 6$  and  $k = 1$ , then the statement translates to

$$f(n) \leq c \cdot g(n), n \geq k$$

which by the definition of  $O$  notation  $f(n) = O(g(n))$ .

3. Prove that  $f(n) = 2 \cdot n^4 \cdot \log_2(n^4) - n^2 + 3 \cdot \log_2(n)$  is  $O(n^4 \log_2(n))$ , provide the appropriate  $C$  and  $k$  constants.

$$f(n) = O(g(n)) \iff \exists c > 0, \exists k \geq 0 \text{ s.t. } f(n) \leq c \cdot g(n) \forall n \geq k$$

$$\begin{aligned}
2 \cdot n^4 \cdot \log_2(n^4) - n^2 + 3 \cdot \log_2(n) &= 8 \cdot n^4 \cdot \log_2(n) - n^2 + 3 \cdot \log_2(n) \\
&\leq 8 \cdot n^4 \cdot \log_2(n) + n^4 \cdot \log_2(n) + 3 \cdot \log_2(n) \\
&\leq 8 \cdot n^4 \cdot \log_2(n) + n^4 \cdot \log_2(n) + 3 \cdot n \cdot \log_2(n) \\
&= 12 \cdot n^4 \cdot \log_2(n), n \geq 1
\end{aligned}$$

Let  $c = 12$  and  $k = 1$  then the statement translates to

$$2 \cdot n^4 \cdot \log_2(n^4) - n^2 + 3 \cdot \log_2(n) \leq c \cdot n^4 \cdot \log_2(n), n \geq 1$$

which by the definition  $O$  notation  $f(n) = O(g(n))$ .

4. Prove or disprove  $f(n) = 5 \cdot n^3 - n + 3$

1.  $f(n) = O(n^2)$
2.  $f(n) = \Omega(n)$
3.  $f(n) = \Theta(n^3)$
4.  $f(n) = \omega(n)$
5.  $f(n) = o(n^2)$

Provide the appropriate  $C$  and  $k$  constants if possible

5. Prove that  $(n + 5)^{100} = \Theta(n^{100})$

$$f(n) = \Theta(g(n)) \iff f(n) = O(g(n)) \text{ and } f(n) = \Omega(g(n))$$

1. Showing  $(n + 5)^{100} = O(n^{100})$

$$f(n) = O(g(n)) \iff \exists c > 0, \exists k \geq 0 \text{ s.t } f(n) \leq c \cdot g(n) \forall n \geq k$$

$$\begin{aligned}
(n + 5)^{100} &= \sum_{i=0}^{100} \binom{100}{i} \cdot n^{100-i} \cdot 5^i \\
&\leq \sum_{i=0}^{100} \binom{100}{i} \cdot n^{100}, \geq 1
\end{aligned}$$

Let  $c = \sum_{i=0}^{100} \binom{100}{i}$  and  $k = 1$  then the statements translates to

$$f(n) \leq c \cdot g(n), n \geq k$$

which by the definition of  $O$  notation  $f(n) = O(g(n))$ .

2. Showing  $(n + 5)^{100} = \Omega(n^{100})$

$$f(n) = \Omega(g(n)) \iff \exists c > 0, \exists k \geq 0 \text{ s.t } f(n) \geq c \cdot g(n) \forall n \geq k$$

$$\begin{aligned} (n + 5)^{100} &= \sum_{i=0}^{100} \binom{100}{i} \cdot n^{100-i} \cdot 5^i \\ &\geq n^{100} \end{aligned}$$

Let  $c = 1$  and  $k = 1$  then the statement translates to

$$f(n) \geq c \cdot g(n), n \geq k$$

which by the definition of  $\Omega$  notation  $f(n) = \Omega(g(n))$ .