1 Compute the values for

1. $\sum_{i=-1}^{4} 3$

$$\begin{aligned} \sum_{i=-1}^{4} 3 &= 3 \cdot (4+1+1) \\ &= 3 \cdot 6 \\ &= 18 \end{aligned}$$

2. $\sum_{i=1}^{5} (\frac{1}{3})^i$

$$\begin{aligned} \sum_{i=1}^{5} (\frac{1}{3})^i &= \sum_{i=0}^{5} (\frac{1}{3})^i - \sum_{i=0}^{0} (\frac{1}{3})^i \\ &= \frac{(\frac{1}{3})^6 - 1}{\frac{1}{3} - 1} - 1 \end{aligned}$$

3. $\sum_{i=1}^{n} 3$

$$\begin{aligned} \sum_{i=1}^{n} 3 &= 3 \cdot \sum_{i=1}^{n} 1 \\ &= 3 \cdot n \end{aligned}$$

4. $\sum_{i=-3}^{n} 3$

$$\sum_{i=-3}^{n} 3 = 3 \cdot (n+4)$$

5. $\sum_{k=0}^{n} 2^k + \sum_{k=5}^{n} 2^k$

$$
\begin{aligned}
\sum_{k=0}^{n} 2^k + \sum_{k=5}^{n} 2^k &= \sum_{k=0}^{n} 2^k + \sum_{k=0}^{n} 2^k - \sum_{k=0}^{4} 2^k \\
&= 2\sum_{k=0}^{n} 2^k - \sum_{k=0}^{4} 2^k \\
&= 2 \cdot \frac{2^{n+1}-1}{2-1} - \frac{2^5-1}{2-1}
\end{aligned}
$$

6. $\sum_{i=0}^{n}(\frac{2}{3})^i + \sum_{i=-4}^{n}(\frac{2}{3})^i$

$$
\begin{aligned}
\sum_{i=0}^{n}(\frac{2}{3})^i + \sum_{i=-4}^{n}(\frac{2}{3})^i &= \frac{(\frac{2}{3})^{n+1}-1}{\frac{2}{3}-1} + \sum_{i=-4}^{-1}(\frac{2}{3})^i + \sum_{i=0}^{n}(\frac{2}{3})^i \\
&= \frac{(\frac{2}{3})^{n+1}-1}{\frac{2}{3}-1} + \sum_{i=-4}^{-1}(\frac{2}{3})^i + \frac{(\frac{2}{3})^{n+1}-1}{\frac{2}{3}-1} \\
&= \frac{(\frac{2}{3})^{n+1}-1}{\frac{2}{3}-1} + \sum_{i=1}^{4}(\frac{3}{2})^i + \frac{(\frac{2}{3})^{n+1}-1}{\frac{2}{3}-1} \\
&= \frac{(\frac{2}{3})^{n+1}-1}{\frac{2}{3}-1} + \frac{(\frac{3}{2})^{5}-1}{(\frac{3}{2})-1} + \frac{(\frac{2}{3})^{n+1}-1}{\frac{2}{3}-1}
\end{aligned}
$$

7. $\sum_{i=1}^{n}(i^3 + 2 \cdot i^2 - i + 1)$

$$
\begin{aligned}
\sum_{i=1}^{n}(i^3 + 2 \cdot i^2 - i + 1) &= \sum_{i=1}^{n} i^3 + 2 \cdot \sum_{i=1}^{n} i^2 - \sum_{i=1}^{n} i + \sum_{i=1}^{n} 1 \\
&= (\frac{n \cdot (n+1)}{2})^2 + 2 \cdot \frac{(n) \cdot (n+1) \cdot (2n+1)}{6} - \frac{n \cdot (n+1)}{2} + n
\end{aligned}
$$

8. $\sum_{i=5}^{n}(-4 \cdot i + \frac{i}{5})$

$$
\begin{aligned}
\sum_{i=5}^{n}(-4\cdot i+\frac{i}{5}) &= -4\sum_{i=5}^{n} i+\frac{1}{5}\sum_{i=5}^{n} i \\
&= -4\sum_{i=0}^{n} i+\frac{1}{5}\sum_{i=0}^{n} i+4\sum_{i=0}^{4} i-\frac{1}{5}\sum_{i=0}^{4} i \\
&= -4\cdot\frac{n(n+1)}{2}+\frac{1}{5}\cdot\frac{n(n+1)}{2}+4\cdot\frac{4\cdot 5}{2}-\frac{1}{5}\cdot\frac{4\cdot 5}{2}
\end{aligned}
$$

9. $\sum_{j=0}^{k}\sum_{i=1}^{j}(i-j^2-2)$

$$
\begin{aligned}
\sum_{j=0}^{k}\sum_{i=1}^{j}(i-j^2-2) &= \sum_{j=0}^{k}\sum_{i=1}^{j} i-\sum_{j=0}^{k}\sum_{i=1}^{j} j^2-\sum_{j=0}^{k}\sum_{i=1}^{j} 2 \\
&= \sum_{j=0}^{k}\frac{j(j+1)}{2}-\sum_{j=0}^{k} j^3-\sum_{j=0}^{k} 2\cdot j \\
&= \sum_{j=0}^{k}\frac{j^2}{2}+\sum_{j=0}^{k}\frac{j}{2}-(\frac{k\cdot(k+1)}{2})^2-2\cdot\frac{k\cdot(k+1)}{2} \\
&= \frac{1}{2}\cdot\frac{(k)\cdot(k+1)\cdot(2k+1)}{6}+\frac{1}{2}\cdot\frac{k\cdot(k+1)}{2}-(\frac{k\cdot(k+1)}{2})^2-2\cdot\frac{k\cdot(k+1)}{2}
\end{aligned}
$$

10. $\sum_{j=1}^{m}\sum_{k=1}^{j}(3\cdot C+k-3\cdot j+i)$

$$
\begin{aligned}
\sum_{j=1}^{m}\sum_{k=1}^{j}(3\cdot C+k-3\cdot j+i) &= \sum_{j=1}^{m}\sum_{k=1}^{j}(3\cdot C+k-3\cdot j+i)\\
&= \sum_{j=1}^{m}\sum_{k=1}^{j}3\cdot C+\sum_{j=1}^{m}\sum_{k=1}^{j}k-3\sum_{j=1}^{m}\sum_{k=1}^{j}j+\sum_{j=1}^{m}\sum_{k=1}^{j}i\\
&= \sum_{j=1}^{m}3\cdot C\cdot j+\sum_{j=1}^{m}\frac{j(j+1)}{2}-3\sum_{j=1}^{m}j^2+\sum_{j=1}^{m}i\cdot j\\
&= 3\cdot C\cdot\frac{m\cdot(m+1)}{2}+\sum_{j=1}^{m}\frac{j^2}{2}+\sum_{j=1}^{m}\frac{j}{2}-3\cdot\frac{(m)(m+1)(2m+1)}{6}\\
&+ i\cdot\frac{(m)(m+1)}{2}\\
&= 3\cdot C\frac{m\cdot(m+1)}{2}+\frac{1}{2}\cdot\frac{m(m+1)(2m+1)}{6}+\frac{1}{2}\cdot\frac{(m)(m+1)}{2}\\
&- 3\cdot\frac{(m)(m+1)(2m+1)}{6}+i\cdot\frac{(m)(m+1)}{2}
\end{aligned}
$$

11. $\sum_{l=-4}^{n}\sum_{j=1}^{k}\sum_{i=1}^{j}(i-4)$

$$
\begin{aligned}
\sum_{l=-4}^{n}\sum_{j=1}^{k}\sum_{i=1}^{j}(i-4) &= \sum_{l=-4}^{n}\sum_{j=1}^{k}\sum_{i=1}^{j}i-4\sum_{l=-4}^{n}\sum_{j=1}^{k}\sum_{i=1}^{j}1\\
&= \sum_{l=-4}^{n}\sum_{j=1}^{k}\frac{j(j+1)}{2}-\sum_{l=-4}^{n}\sum_{j=1}^{k}j\\
&= \sum_{l=-4}^{n}\sum_{j=1}^{k}\frac{j^2}{2}+\sum_{l=-4}^{n}\sum_{j=1}^{k}\frac{j}{2}-\sum_{l=-4}^{n}\frac{k(k+1)}{2}\\
&= \sum_{l=-4}^{n}\frac{(k)(k+1)(2k+1)}{12}+\sum_{l=-4}^{n}\frac{(k)(k+1)}{4}-\sum_{l=-4}^{n}\frac{k(k+1)}{2}\\
&= (n+5)\frac{(k)(k+1)(2k+1)}{12}+(n+5)\frac{(k)(k+1)}{4}-(n+5)\frac{(k)(k+1)}{2}
\end{aligned}
$$

2. Calculate the answer

1. $log_4 x=5\rightarrow x=?$

$$
\begin{aligned}
log_4 x &= 5\\
x &= 4^5
\end{aligned}
$$

2. $log_3 y = 4 \rightarrow y =?$

$$\begin{aligned} log_3 y &= 4 \\ y &= 3^4 \end{aligned}$$

3. $x = 7^2 \rightarrow log_7 x =?$

$$\begin{aligned} x &= 7^2 \\ log_7 x &= log_7 7^2 \\ log_7 x &= 2 \cdot log_7 7 \\ log_7 x &= 2 \end{aligned}$$

4. $x = 32 \rightarrow log_2 x =?$

$$\begin{aligned} x &= 32 \\ log_2 x &= log_2 32 \\ log_2 x &= 5 \end{aligned}$$

5. $2^{log 5} + 4^{log 6} - 27^{log_3 5}$

$$2^{log_2 5} + 4^{log_2 6} - 27^{log_3 5} = 5 + 36 - 5^3$$

6. $9^{log_3 2} - 25^{log_5 4} - 36^{log_6 7} + 8^{log_8 6}$

$$\begin{aligned} 9^{log_3 2} - 25^{log_5 4} - 36^{log_6 7} + 8^{log_8 6} &= 3^{log_3 (2^2)} - 5^{log_5 (4^2)} - 6^{log_6 7^2 + 6} \\ &= 4 - 16 - 49 + 6 \end{aligned}$$

7. $log(4^5 \times 8^3) - log(16 - 8) + log(\frac{2^{10}}{4 \times 3^2})$

$$
\begin{aligned}
log_2(4^5 \times 8^3) - log_2(16-8) + log_2(\frac{2^{10}}{4 \times 3^2}) &= log_2(4^5) + log_2(8^3) - log_2(8) + log_2(2^{10}) - log_2(4 \times 3^2) \\
&= 5 \cdot log_2(4) + 3 \cdot log_2(8) - log_2(8) + 10 \cdot log_2(2) - log_2(4) \\
&= 5 \cdot 2 + 3 \cdot 3 - 3 + 10 - 2 - 2 \cdot 1.5 \\
&= 21
\end{aligned}
$$

8. $log(3^2 \times 64^3) - log(\frac{2^{10} \times 128^3}{9 \times 8^2})$

$$
\begin{aligned}
log_2(3^2 \times 64^3) - log_2(\frac{2^{10} \times 128^3}{9 \times 8^2}) &= log_2(3^2) + log_2(64^3) - log_2(2^{10} \times 128^3) + log_2(9) + log_2 8^2 \\
&= 2 \cdot log_2(3) + 3 \cdot log_2(64) - 10 \cdot log_2 2 - 3 \cdot log_2 128 + 2 \cdot log_2 3 + 2 \cdot \\
&= 2 \cdot 1.5 + 3 \cdot 6 - 10 - 3 \cdot 7 + 2 \cdot 1.5 + 2 \cdot 3 \\
&= -1
\end{aligned}
$$

9. $loglog16$

$$
\begin{aligned}
log_2 log_2 16 &= log_2 4 \\
&= 2
\end{aligned}
$$

10. $log16 \times log16$

$$
\begin{aligned}
log_2 16 \times log_2 16 &= 4 \times 4 \\
&= 16
\end{aligned}
$$

11. $log^2 16$

$$
\begin{aligned}
log_2^2 16 &= 4^2 \\
&= 16
\end{aligned}
$$

12. $log_2 log_5 625 - log_3 log_4 2^{3^9} + log^4 2^5 - \frac{log^2(4^3 \times 3^5)}{log_5 125}$

$$
\begin{aligned}
log_2 log_5 625 - log_3 log_4 2^{3^9} + log_2^4 2^5 - \frac{log^2(4^3 \times 3^5)}{log_5 125} &= log_2 4 - log_3(3^9 \cdot log_4 2) + log_2^4 2^5 - \frac{log_2^2(4^3 \times 3^5)}{log_5 125} \\
&= 2 - 9log_3(\frac{3}{2}) + (5 \cdot log_2(2))^4 - \frac{(log_2(4^3 \times 3^5)^2}{3} \\
&= 2 - 9 + 9 \cdot log_3(\frac{1}{2}) + 5^4 - \frac{(3 \cdot 2 + 5 \cdot log_2(3))^2}{3} \\
&= 2 - 9 + 9 \cdot \frac{-1}{1.5} + 5^4 - \frac{(3 \cdot 2 + 5 \cdot (1.5)^2}{3}
\end{aligned}
$$

13. $loglog_8 log256 + log^5(3^2) \times 4^{log7}$

$$
\begin{aligned}
log_2 log_8 log_2 256 + log_2^5(3^2) \cdot 4^{log_2 7} &= log_2 log_8 8 + log_2^5(3^2) \cdot 4^{log_2 7} \\
&= log_2 1 + log_2^5(3^2) \cdot 4^{log_2 7} \\
&= log_2^5(3^2) \cdot 4^{log_2 7} \\
&= (2 \cdot 1.5)^5 \cdot 2^{log_2 49} \\
&= (2 \cdot 1.5)^5 \cdot 49 \\
&= 11907
\end{aligned}
$$

14. $log_6 x = 5 \rightarrow log_x 6 = ?$

$$
\begin{aligned}
log_6 x &= 5 \\
x &= 6^5 \\
log_x x &= 5 \cdot log_x 6 \\
1 &= 5 \cdot log_x 6 \\
\frac{1}{5} &= log_x 6
\end{aligned}
$$

15. $log_y x = 10 \rightarrow log_x y = ?$

$$
\begin{aligned}
log_y x &= 10 \\
x &= y^{10} \\
log_x x &= 10 \cdot log_x y \\
\frac{1}{10} &= log_x y
\end{aligned}
$$

16. $log_4 32 - log_8^2 4$

$$\begin{aligned} log_4 32 - log_8^2 4 &= log_4(32) - (log_8 4)^2 \\ &= \frac{5}{2} - (\frac{log_2 4}{log_2 8})^2 \\ &= \frac{5}{2} - (\frac{2}{3})^2 \\ &= \frac{5}{2} - \frac{4}{9} \end{aligned}$$

17. $log_4 8 + log_9 27 - log_{25}^2 125 - log_8^3 16 + log_4 log_2 256$

$$\begin{aligned} log_4 8 + log_9 27 - log_{25}^2 125 - log_8^3 16 + log_4 log_2 256 &= \frac{3}{2} + \frac{3}{2} - (\frac{3}{2})^2 - (\frac{4}{3})^3 + log_4(8) \\ &= \frac{3}{2} + \frac{3}{2} - (\frac{9}{4}) - (\frac{64}{27}) + \frac{3}{2} \end{aligned}$$

3. Compute the deriative of

1. $-5 \cdot x^3 + 2 \cdot x - 1$

$$\frac{d}{dx}(-5 \cdot x^3 + 2 \cdot x - 1) = -15 \cdot x^2 + 2$$

2. $3 \cdot x^4 - 2\sqrt{x} + x^{\frac{1}{2}} - 6x^{-\frac{2}{3}} - 5$

$$\begin{aligned} \frac{d}{dx}(3 \cdot x^4 - 2\sqrt{x} + x^{\frac{1}{2}} - 6x^{-\frac{2}{3}} - 5) &= 12 \cdot x^3 - x^{-\frac{1}{2}} + \frac{1}{2} \cdot x^{-\frac{1}{2}} + 4 \cdot x^{-\frac{5}{3}} \\ &= 12 \cdot x^3 - \frac{1}{2}x^{-\frac{1}{2}} + 4 \cdot x^{-\frac{5}{3}} \end{aligned}$$

3. $x \cdot \sqrt{x} + \sqrt{\sqrt{x}}$

$$\frac{d}{dx}(x^{\frac{3}{2}} + x^{\frac{1}{4}}) = \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{4} \cdot x^{-\frac{3}{4}}$$

4. $logx - x^2 lnx + lnx^4$

$$\frac{d}{dx}(log_2 x - x^2 \cdot lnx + lnx^4) \quad = \quad \frac{1}{x \cdot ln2} - 2 \cdot x \cdot lnx - x + \frac{4}{x}$$

5. $ln^3(x\sqrt{2x-3}) + \sqrt{lnx^2}$

$$\begin{aligned}\frac{d}{dx}(ln^3(x \cdot \sqrt{2x-3}) + \sqrt{lnx^2}) \quad &= \quad \frac{3 \cdot (ln(x\sqrt{2x-3}))^2 \cdot (\sqrt{2x-3} + x \cdot (2 \cdot x - 3)^{-\frac{1}{2}}}{x\sqrt{2x-3}} \\ &+ \quad \frac{1}{x \cdot \sqrt{ln(x^2)}}\end{aligned}$$

6. $\frac{\sqrt[4]{x+5} - lnx}{(x-1)^3}$

$$\frac{d}{dx}\left(\frac{(x+5)^{\frac{1}{4}} - lnx}{(x-1)^3}\right) \quad = \quad \frac{(x-1)^3 \cdot (\frac{1}{4} \cdot (x+5)^{-\frac{3}{4}} - \frac{1}{x}) - 3 \cdot ((x+5)^{\frac{1}{4}} - lnx) \cdot (x-1)^2}{(x-1)^6}$$

4. Determine the limit of

1. $lim_{x\to\infty} \frac{3x+2}{-5x-6}$

$$lim_{x\to\infty} \frac{3x+2}{-5x-6} \quad = \quad -\frac{3}{5}$$

2. $lim_{x\to\infty}(\frac{1}{x} + 3)$

$$\begin{aligned}lim_{x\to\infty}(\frac{1}{x} + 3) \quad &= \quad lim_{x\to\infty} \frac{1}{x} + lim_{x\to\infty} 3 \\ &= \quad lim_{x\to\infty} \cancelto{0}{\frac{1}{x}} + lim_{x\to\infty} 3 \\ &= \quad 3\end{aligned}$$

3. $lim_{x\to\infty} \frac{x^3 + x - \sqrt{3x}}{\sqrt{x}}$

$$\begin{aligned}lim_{x\to\infty} \frac{x^3 + x - \sqrt{3x}}{\sqrt{x}} \quad &= \quad lim_{x\to\infty} \frac{x^3}{\sqrt{x}} + lim_{x\to\infty} \frac{x}{\sqrt{x}} - lim_{x\to\infty} \frac{\sqrt{3x}}{\sqrt{x}} \\ &= \quad \infty\end{aligned}$$

4. $lim_{x\to\infty}\frac{x^3+x-\sqrt{3x}}{5\cdot x^{2.25}\cdot\sqrt{\sqrt{x}}}$

$$\begin{aligned}
lim_{x\to\infty}\frac{x^3+x-\sqrt{3x}}{5\cdot x^{2.25}x^{.25}} &= lim_{x\to\infty}\frac{x^3+x-\sqrt{x}}{5\cdot x^3} \\
&= lim_{x\to\infty}\frac{x^3}{5\cdot x^3}+lim_{x\to\infty}\cancelto{0}{\frac{x}{5\cdot x^3}}-lim_{x\to\infty}\cancelto{0}{\frac{\sqrt{x}}{5\cdot x^3}} \\
&= \frac{1}{5}
\end{aligned}$$

5. $lim_{x\to\infty}\frac{x^{0.1}-\sqrt{3}}{\sqrt{\sqrt{x}}}$

$$\begin{aligned}
lim_{x\to\infty}\frac{x^{0.1}-\sqrt{3}}{x^{0.25}} &= lim_{x\to\infty}\frac{x^{0.1}}{x^{0.25}}-\frac{\sqrt{3}}{x^{0.25}} \\
&= lim_{x\to\infty}\frac{1}{x^{0.15}}-\frac{\sqrt{3}}{x^{0.25}} \\
&= 0
\end{aligned}$$

6. $lim_{x\to\infty}\frac{x^x}{2^x}$

$$\begin{aligned}
log(lim_{x\to\infty}\frac{x^x}{2^x}) &= lim_{x\to\infty}log(\frac{x^x}{2^x}) \\
&= lim_{x\to\infty}log(x^x)-lim_{x\to\infty}log(2^x) \\
&= lim_{x\to\infty}xlog(x)-lim_{x\to\infty}xlog(2) \\
&= \infty \\
e^{ln(lim_{x\to\infty}\frac{x^x}{2^x})} &= e^{\infty} \\
&= \infty
\end{aligned}$$

7. $lim_{x\to\infty}\frac{x^x}{x(2^x)}$

$$\begin{aligned}
log(lim_{x\to\infty}\frac{x^x}{x(2^x)}) &= lim_{x\to\infty}log(\frac{x^x}{x\cdot 2^x}) \\
&= lim_{x\to\infty}log(x^x)-lim_{x\to\infty}log(x\cdot 2^x) \\
&= lim_{x\to\infty}xlog(x)-lim_{x\to\infty}log(x)-lim_{x\to\infty}log(2^x) \\
&= lim_{x\to\infty}xlog(x)-lim_{x\to\infty}log(x)-lim_{x\to\infty}xlog(2) \\
&= \infty \\
e^{ln(lim_{x\to\infty}\frac{x^x}{x(2^x)}} &= e^{\infty} \\
&= \infty
\end{aligned}$$

8. $lim_{x \to \infty} \frac{\sqrt{2}^{log^4 x^3}}{log(2 \cdot x+7)}$

$$\begin{aligned} lim_{x \to \infty} \frac{2^{\frac{1}{2}(3log(x))^4}}{log(2 \cdot x + 7)} &= lim_{x \to \infty} \frac{2^{\frac{81}{4}(log(x))^4}}{log(2 \cdot x + 7)} \\ &= \infty \end{aligned}$$

9. $lim_{x \to \infty} \frac{x+1}{\frac{3 \cdot x^{lnx}}{2x^2}}$

$$\begin{aligned} lim_{x \to \infty} \frac{x+1}{\frac{3 \cdot x^{lnx}}{2 \cdot x^2}} &= lim_{x \to \infty} \frac{x+1}{6 \cdot x^2 \cdot x^{lnx}} \\ &= 0 \end{aligned}$$

10. $lim_{x \to \infty} \frac{\sqrt{2}^{logx^3}}{log^{lnx}(2x)}$

$$\begin{aligned} lim_{x \to \infty} \frac{\sqrt{2}^{logx^3}}{log^{lnx}(2x)} &= lim_{x \to \infty} \frac{2^{\frac{1}{2} \cdot logx^3}}{log^{lnx}(2x)} \\ &= lim_{x \to \infty} \frac{2^{logx^{\frac{3}{2}}}}{log^{lnx}(2x)} \\ &= lim_{x \to \infty} \frac{x^{\frac{3}{2}}}{log^{lnx}(2x)} \\ lim_{x \to \infty} log(\frac{x^{\frac{3}{2}}}{log^{lnx}(2x)}) &= lim_{x \to \infty} \frac{3}{2} log(x) - log(x) log(log(2x)) \\ &= lim_{x \to \infty} \frac{3}{2} log(x) - log(x)(log(1 + log(x))) \\ &= -\infty \\ 2^{log(lim_{x \to \infty} \frac{x^{\frac{3}{2}}}{log^{lnx}(2x)})} &= 2^{-\infty} \\ &= 0 \end{aligned}$$

5. Compute the exact values for

1. $\int_1^n (2 \cdot x^4 + 5\sqrt{x}) \cdot dx$

$$\begin{aligned}\int_1^n (2\cdot x^4+5\sqrt{x}\quad)\cdot dx = &\quad 2\int_1^n x^4dx + 5\int_1^n \sqrt{x}dx \\ = &\quad 2\cdot(\frac{x^5}{5}|_1^n)+5(\frac{2}{3}x^{\frac{3}{2}}|_1^n) \\ = &\quad 2\cdot(\frac{n^5}{5}-\frac{1}{5})+5(\frac{2}{3}n^{\frac{3}{2}}-\frac{2}{3}) \\ = &\quad \frac{2\cdot n^5}{5}-\frac{2}{5}+\frac{10}{3}n^{\frac{3}{2}}-\frac{10}{3}\end{aligned}$$

2. $\int_1^n (x^4 - 3\cdot x^2 + \frac{1}{x} - \frac{1}{x^2})dx$

$$\begin{aligned}\int_1^n (x^4-3\cdot x^2+\frac{1}{x}-\frac{1}{x^2}dx &= \int_1^n x^4dx - 3\int_1^n x^2dx + \int_1^n \frac{1}{x}dx - \int_1^n \frac{1}{x^2}dx \\ &= \frac{x^5}{5}|_1^n - 3\cdot\frac{x^3}{3}|_1^n + lnx|_1^n + \frac{1}{x}|_1^n \\ &= (\frac{n^5}{5}-\frac{1}{5}) - n^3 + 1 + lnn + \frac{1}{n} - 1 \\ &= \frac{n^5}{5} - n^3 + lnn + \frac{1}{n} - \frac{1}{5}\end{aligned}$$

3. $\int_1^n (\frac{3}{\sqrt{x}} + lnx + e^x)dx$

$$\begin{aligned}\int_1^n \frac{3}{\sqrt{x}} + lnx + e^x dx &= \int_1^n \frac{3}{\sqrt{x}}dx + \int_1^n lnxdx + \int_1^n e^x dx \\ &= 6\sqrt{x}|_1^n + (xlnx - x|_1^n) + e^n - e \\ &= 6\sqrt{n} - 6 + (nlnn - n + 1) + e^n - e \\ &= 6\sqrt{n} + nlnn - n - 5 + e^n - e\end{aligned}$$

4. $\int_1^n x\cdot sinxdx$

$$\begin{aligned}\int_1^n x\cdot sinxdx &= -xcosx|_1^n + \int_1^n cosxdx \\ &= -ncosn + cos(1) + sin(n) - sin(1)\end{aligned}$$

6. Use mathematical induction to prove that

$$1+2+\ldots+n = \frac{n(n+1)}{2}$$

Proof. $1 + 2 + \ldots + n = \frac{n(n+1)}{2}$

Base case $n = 1$: If $n = 1$ then the left hand side and the right hand size is $1 = 1 = \frac{1(2)}{2}$.

Inductive hypothesis: Suppose the theorem holds for all values of n up to some $k, k \geq 1$.

Inductive step: let $n = k + 1$ then our left hand side is

$$\begin{aligned}
\sum_{i=1}^{k+1} i &= \sum_{i=1}^{k} i + (k+1) \\
&= \frac{k(k+1)}{2} + (k+1) \\
&= \frac{k(k+1) + 2 \cdot k + 2}{2} \\
&= \frac{(k+1) \cdot (k+2)}{2}
\end{aligned}$$

which is equal to our right hand side. By the principle of mathematical induction, the theorem holds for all integers $n \geq 1$. $\square$

7. Use mathematical induction to prove that

$$1 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$1 + 2 + \ldots + n = \frac{n(n+1)}{2}$

Proof. Base case $n = 1$: If $n = 1$ then the left hand side and the right hand size is $1^2 = 1 = \frac{1(2)(3)}{6}$.

Inductive hypothesis: Suppose the theorem holds for all values of n up to some $k, k \geq 1$.

Inductive step: let $n = k + 1$ then our left hand side is

$$
\begin{aligned}
\sum_{i=1}^{k+1} i^2 &= \sum_{i=1}^{k} i^2 + (k+1)^2 \\
&= \frac{k \cdot (k+1) \cdot (2 \cdot k + 1)}{6} + (k+1)^2 \\
&= \frac{k \cdot (k+1) \cdot (2 \cdot k + 1) + 6 \cdot (k+1)^2}{6} \\
&= \frac{(6 \cdot (k+1) + k \cdot (2 \cdot k + 1)) \cdot (k+1)}{6} \\
&= \frac{(6 \cdot k + 6 + 2 \cdot k^2 + k) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k^2 + 7 \cdot k + 6) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k^2 + 4 \cdot k + 3 \cdot k + 6) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k \cdot (k+2) + 3 \cdot (k+2)) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k + 3) \cdot (k+2) \cdot (k+1)}{6}
\end{aligned}
$$

which is equal to our right hand side. By the principle of mathematical induction, the theorem holds for all integers $n \geq 1$. $\square$