

1. Give an example of a permutation p of the numbers $1 - 6$ that has the following property: if the first and last numbers of p are swapped, then the resulting permutation will have exactly one less inversion than p . Verify your answer by counting the number of inversions for each permutation.

$$\begin{aligned} p &= [2, 3, 4, 5, 6, 1] \\ p' &= [1, 3, 4, 5, 6, 2] \\ \text{inv}(p) &= 5 \\ \text{inv}(p') &= 4 \end{aligned}$$

2. Given the recurrence $T(n) = 3 \cdot T(\frac{n}{3}) + n$, provide its master equation. Use it to determine the order of growth of $T(n)$.

$$T(n) = \Theta(n \cdot \log(n))$$

3. Suppose an implementation of the Quicksort algorithm uses the first array element as the partitioning pivot. Explain why an array that is already sorted will produce the worst-case running time for this implementation. Include in your explanation a summation expression that represents the worst-case running time, and explain how you derived it.

$$\begin{aligned} T(n) &= T(n-1) + O(n) \\ &= O(n^2) \end{aligned}$$