

The running time of the algorithm models a recurrence of the form  $T(n) = 2 \cdot T(\frac{n}{2}) + O(n \cdot \log(n))$ . The reason being that we have two subproblems of size  $\frac{n}{2}$ . The combining portion is  $O(n \cdot \log(n))$  because the bottleneck is in doing two quicksorts, one for the left and another for the right half of the subarrays. Setting up a table and solving the system.

level	size of a problem	cost of a node	# nodes	row sum
0	$n$	$n \cdot \log(n)$	1	$n \cdot \log(n)$
1	$\frac{n}{2}$	$\frac{n}{2} \cdot \log(\frac{n}{2})$	2	$n \cdot \log(\frac{n}{2})$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$i$	$\frac{n}{2^i}$	$\frac{n}{2^i} \cdot \log(\frac{n}{2^i})$	$2^i$	$n \cdot \log(\frac{n}{2^i})$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$k$	1	1	$2^k$	$2^k$

$$\begin{aligned} \frac{n}{2^k} &= 1 \\ k &= \log_2(n) \end{aligned}$$

$$\begin{aligned} \sum_{i=0}^{k-1} n \cdot \log(\frac{n}{2^i}) + 2^k &= n \cdot \sum_{i=0}^{k-1} \log(\frac{n}{2^i}) + 2^k \\ &= n \cdot \sum_{i=0}^{k-1} (\log(n) - \log(2^i)) + 2^{\log_2(n)} \\ &= n \cdot \sum_{i=0}^{k-1} \log(n) - n \cdot \sum_{i=0}^{k-1} \log(2^i) + n \\ &= n \cdot \log(n) \cdot (k - 1 - 0 + 1) - n \cdot \sum_{i=0}^{k-1} i \cdot \log(2) + n \\ &= n \cdot \log(n) \cdot \log(n) - n \cdot \frac{(k-1) \cdot (k)}{2} + n \\ &= n \cdot \log^2(n) - n \cdot \frac{(\log(n) - 1) \cdot (\log(n))}{2} + n \\ &= \Theta(n \cdot \log^2(n)) \end{aligned}$$

Thus the algorithm runtime is  $\Theta(n \cdot \log^2(n))$ .