

**1.** Find the

1.  $2^{nd}$  least element
2.  $4^{th}$  least element

Using the random find statistics algorithm (Quickselect)

- 2,-1,3,8,9,19,6,35,17,20
- 10,11,12,13,14,15,16,17,18

①a Part 1

2 - 1 3 8 9 0 1 9 6 3 5 1 7 2 0

$n=2$   
 $P_{\text{Wt}} = 2$

2 0 - 1 3 8 9 0 1 9 6 3 5 1 7 2 0  
↑ ↑ ↑ ↑ ↑ ↑  
X V V V V V

snap 0 and 2 0

0 - 1 3 8 9 2 0 1 9 6 3 5 1 7 2 0  
↑ ↑ ↑ ↑ ↑ ↑  
V V X V V V  
↑ STOP

0 - 1 2 8 9 2 0 1 9 6 3 5 1 3

QS

? K - 9  
P<sub>Wt</sub> = 2

0 - 1

k=2

pNot = 0

-1 0



Step: PreOrder

0 is the 2<sup>nd</sup> left element

1a part 2

10 11 12 13 14 15 16 17 18

$k=2$

pivot = 10

18 11 12 13 14 15 16 17 10  
↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑  
X ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓

Stop partition

10 (11 12 13 14 15 16 17 18)

Q ↴

$k=1$  11 12 13 14 15 16 17 18

pivot = 11

18 12 13 14 15 16 17 11  
↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑  
X ✓ ✓ ✓ ✓ ✓ ✓ ✓

11 12 13 14 15 16 17 18

11 is the 2nd  
least element

⑬ part 2

2 - 1 3 8 9 0 1 9 6 3 5 1 2 0 6

~~Prob = 2~~  
~~k = 4~~

20 - 1 3 8 9 0 1 9 6 3 5 1 2 0  
↑                    ↑ ↑ ↑ ↑  
X                    X ✓ ✓ ✓ ✓ ✓

swap 20 and 0

0 - 1 3 8 9 2 0 1 9 6 3 5 1 2 0  
↑                    ↑ ↑ ↑ ↑ ↑  
✓                    ✓ ✓ ✓ ✓ ✓

Stop partition

0 - 1 2 8 9 2 0 1 9 6 3 5 1 2 3

as

8 9 20 19 6 35 17 3

$$k=1$$
$$\text{Pivot} = 8$$

3 9 20 19 6 35 17 8  
↑ ↑ ↑ ↑ ↑ ↑ ↑  
✓ ✗ ✗ ✗ ✗ ✗ ✗

3 6 20 19 9 35 17 8  
↑ ↑ ↑ ↑ ↑ ↑  
✓ ✗ ✗ ✗ ✗ ✗

3 6 8 19 9 35 17 20

QS

$$k=1$$

$$\text{Pivot} = 3$$

6 3  
↑  
↑

3 ↲

3 is 4th least element

①b) Part 2

10 11 12 13 14 15 16 17 18

$$P_{\text{fail}} = 10 \\ k = 4$$

18 11 12 13 14 15 16 17 10  
↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑  
X ✓ ✓ ✓ ✓ ✓ ✓ ✓ ✓

Stop position

10 (11 12 13 14 15 16 17 18)

↖  
QS

11 12 13 14 15 16 17 18

$$P_{\text{fail}} = 16 \\ k = 3$$

18 12 13 14 15 16 17 11  
↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑  
X ✓ ✓ ✓ ✓ ✓ ✓ ✓

Stop position

11 (12 13 14 15 16 17 18)

↖  
QS

12 13 14 15 16 17 18

$$P_{\text{rot}} = 12 \\ k = 2$$

18 13 14 15 16 17 12  
↑ ↑ ↑ ↑ ↑ ↑ ↑  
X ✓ ✓ ✓ ✓ ✓

Stop part 91 Party

12 13 14 15 16 17 18  
↓  
as

P<sub>rot</sub> = 13 13 14 15 16 17 18  
k = 1

18 14 15 16 17 13  
↑ ↑ ↑ ↑ ↑ ↑  
X ✓ ✓ ✓ ✓ ✓

↑  
Stop part 91 Party

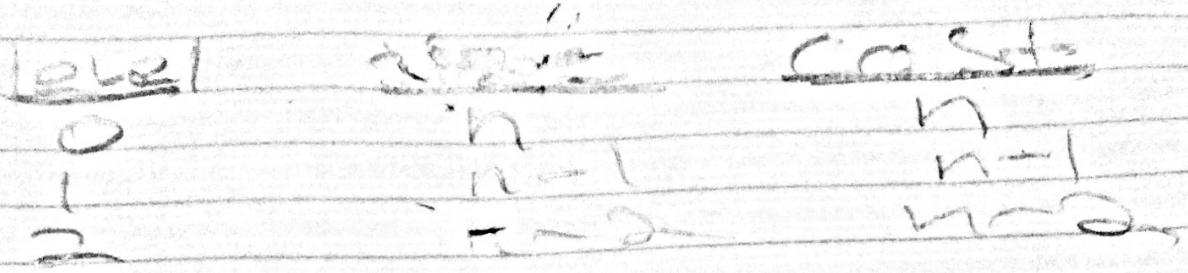
13 14 15 16 17 18

13 is the 4th least  
error

**2.** Calculate the worst-case running time of Quickselect.

An example of the worst-case of Quickselect is looking for the  $n^{th}$  least element in a sorted array with a length of  $n$ . The Quickselect would select the leftmost valid index then perform the partition and if the index is not the  $n^{th}$  element would result in needing to do a recursive call on a subproblem of one less than the original size. The recurrence is of the form  $T(n) = T(n-1) + O(n)$  and evaluates to  $T(n) = O(n^2)$ .

$$T(n) = T(n-1) + O(n)$$



K            1            i

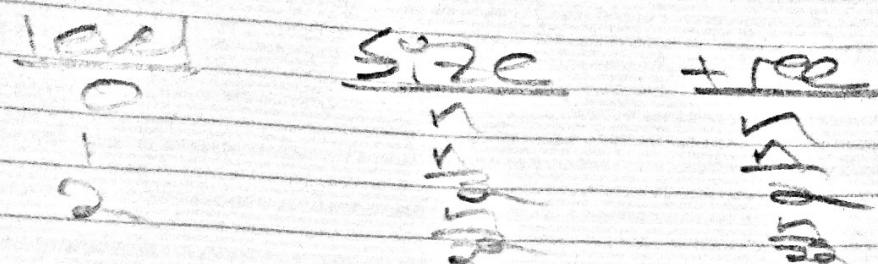
$$\sum_{i=1}^k i = \frac{n(n+1)}{2} = O(n^2)$$

$$T(n) = O(n^2)$$

- 3.** Calculate the average-case running time of Quickselect algorithm. Explain.

In the average case the pivot splits the subarrays roughly in half and if it is not the  $k^{th}$  least element will recurse on one of the subarrays. The recurrence is of the form  $T(n) = T(\frac{n}{2}) + O(n)$ . This evaluates to  $O(n)$ .

$$T(n) = T\left(\frac{n}{2}\right) + O(n)$$



$$k = \log_2(n)$$

$$\sum_{i=0}^{k-1} \frac{n}{2^i} + 2^k =$$

$$n \sum_{i=0}^{k-1} \left(\frac{1}{2}\right)^i + \Theta(\log_2(n)) =$$

$$n \cancel{\Theta(1)} + n =$$

$O(n)$

$$T(n) = O(n)$$

4. Explain an algorithm to return the max  $k$  numbers from an unsorted array.

Assume  $A$  is an array. Alter the Quickselect to return the index of the  $k^{th}$  smallest value as opposed to the value itself. Compute the  $k^{th}$  largest index by using the Quickselect with the value  $\text{len}(A) - k + 1$ . The elements in the right-subarray of the index are the remaining max numbers to be returned.

5. Suppose a student wrote the below code for the Maximum Subsequence Sum problem. What is the running time of this algorithm?

$$\begin{aligned}
 \sum_{i=0}^{n-1} \sum_{j=i}^{n-1} \sum_{k=i}^j c &= \sum_{i=0}^{n-1} \sum_{j=i}^{n-1} c \cdot (j - i + 1) \\
 &= \sum_{i=0}^{n-1} \sum_{j=i}^{n-1} c \cdot j - \sum_{i=0}^{n-1} \sum_{j=i}^{n-1} \cdot i + \sum_{i=0}^{n-1} \sum_{j=i}^{n-1} c \\
 &\leq \sum_{i=0}^{n-1} \sum_{j=i}^{n-1} c \cdot j + \sum_{i=0}^{n-1} \sum_{j=i}^{n-1} c \\
 &= \sum_{i=0}^{n-1} c \cdot \left( \sum_{j=1}^{n-1} j - \sum_{j=1}^{i-1} j \right) + \sum_{i=0}^{n-1} c \cdot (n - 1 - i + 1) \\
 &= \sum_{i=0}^{n-1} c \cdot \left( \frac{n \cdot (n - 1)}{2} - \frac{i \cdot (i - 1)}{2} \right) + \sum_{i=0}^{n-1} c \cdot (n - i) \\
 &\leq \sum_{i=0}^{n-1} c \cdot \frac{n \cdot (n - 1)}{2} + \sum_{i=0}^{n-1} c \cdot n \\
 &= c \cdot \frac{n \cdot n \cdot (n - 1)}{2} + c \cdot n \cdot (n - 1 + 1) \\
 &= O(n^3) + O(n^2) \\
 &= O(n^3)
 \end{aligned}$$

- 6.** Suppose another student who is a better programmer wrote the below code for the Maximum Subsequence Sum Problem. What is the running time of this algorithm?

$$\begin{aligned}
 \sum_{i=0}^{n-1} \sum_{j=i}^{n-1} c &= \sum_{i=0}^{n-1} c \cdot (n - i) \\
 &= c \cdot \sum_{i=0}^{n-1} n - c \cdot \sum_{i=0}^{n-1} i \\
 &= c \cdot n \cdot n - c \cdot \frac{(n-1)n}{2} \\
 &= c \cdot \frac{n^2}{2} - c \cdot \frac{n}{2} \\
 &= \Theta(n^2)
 \end{aligned}$$

- 7.** Suppose another student decides to solve the Maximum Subsequence problem using the divide and conquer technique. what is the running time of this algorithm.

The recurrence of the algorithm will be of the form  $T(n) = 2 \cdot T(\frac{n}{2}) + O(n)$ , this evaluates to  $O(n \cdot \log(n))$ .

$$P(n) = \Omega\left(\frac{n^2}{\alpha}\right) + O(n)$$

~~total size tree had for sun~~

K = log<sub>2</sub>(n)

$$\sum_{i=0}^k n = n(k+1)$$

$$= n(\log_2(n) + 1)$$

$$= O(n \log_2(n))$$

$$T(n) = O(n \log_2(n))$$

- 8.** Suppose an extraordinary student decides to solve the Maximum Subsequence Sum problem. using the below code. What is the running time of this algorithm.

$$\begin{aligned}\sum_{i=0}^{n-1} c &= c \cdot (n - 1 + 1) \\ &= c \cdot n \\ &= \Theta(n)\end{aligned}$$

- 9.** Find the MSS of the below array using solutions for Q.7 and Q.8:

1. 7, 2, -14, 38, 52, -37, 4, 12, -4, 6, 3, 2
2. 3, 6, -20, 11, -15, 26, -43, 10, -14, 27, 0, 39

Q7 a 90

7	2	-14	38	52	-37	41	2	-4	6	3	2
48	47	39	53	15	-37	4	16	12	18	21	23

7	2	-14	38	52	-37	4	12	-4	6	3	2
4	12	-4	6	3	2	16	11	6	3	2	

7	2	38	52	4	12	6	5
13	2	38	52	14	12	16	32
2	12	52	-37	12	12	12	12
2	12	-14	14	52	12	12	12

MSSL =

MSS 45 90

Q8 a

7 2 -14 38 52 -37 4 12 -4 6 3 2

Sum 9 38 38 90 53 57 69 65 71 24 26  
MSS 9 9 38 90 90 90 90 90 90 90 90

MSS PS 90

Q7 B

66

$$\begin{array}{r} \cancel{3 \mid 61 - 20 \mid 11 \mid -15 \mid 26 \mid -43 \mid 10 \mid -14 \mid 27 \mid 0 \mid 39 \mid} \\ \cancel{1 \mid 1 \mid 9 \mid -20 \mid 26 \mid 11 \mid -15 \mid 26 \mid -43 \mid 10 \mid -14 \mid 27 \mid 0 \mid 39 \mid} \\ \cancel{-3 \mid 0 \mid -5 \mid 13 \mid -20 \mid 11 \mid -15 \mid 26 \mid -43 \mid 10 \mid -14 \mid 27 \mid 0 \mid 39 \mid} \\ \hline \cancel{9 \mid 6 \mid -20 \mid 11 \mid 26 \mid 10 \mid 66 \mid 0 \mid 39 \mid} \\ \cancel{3 \mid 6 \mid -20 \mid 11 \mid 26 \mid -43 \mid 10 \mid -14 \mid 27 \mid 0 \mid 39 \mid} \\ \cancel{3 \mid 6 \mid -20 \mid 11 \mid 26 \mid -43 \mid 10 \mid -14 \mid 27 \mid 0 \mid 39 \mid} \\ \cancel{3 \mid 6 \mid -20 \mid 11 \mid 26 \mid -43 \mid 10 \mid -14 \mid 27 \mid 0 \mid 39 \mid} \\ \cancel{6 \mid 6 \mid -20 \mid 11 \mid 26 \mid -43 \mid 10 \mid -14 \mid 27 \mid 0 \mid 39 \mid} \\ \cancel{6 \mid 6 \mid -20 \mid 11 \mid 26 \mid -43 \mid 10 \mid -14 \mid 27 \mid 0 \mid 39 \mid} \end{array}$$

66 9SS MSS

Q8 Part B

1 3 6 - 2 0 1 1 - 1 5 1 2 6 - 4 3 1 0 - 1 4 2 2 1 3 9

Sum: 3 9 0 1 1 0 2 6 0 1 0 0 2 7 2 3 6 6  
MSS: 3 9 9 1 1 1 1 2 6 2 6 2 6 2 6 2 6 2 6 2 6 2 6 2 6 6 6

The MSS is 6 6