

1 Compute the values for

1. $\sum_{i=-1}^4 3$

$$\begin{aligned}\sum_{i=-1}^4 3 &= 3 \cdot (4 + 1 + 1) \\ &= 3 \cdot 6 \\ &= 18\end{aligned}$$

2. $\sum_{i=1}^5 (\frac{1}{3})^i$

$$\begin{aligned}\sum_{i=1}^5 (\frac{1}{3})^i &= \sum_{i=0}^5 (\frac{1}{3})^i - \sum_{i=0}^0 (\frac{1}{3})^i \\ &= \frac{(\frac{1}{3})^6 - 1}{\frac{1}{3} - 1} - 1\end{aligned}$$

3. $\sum_{i=1}^n 3$

$$\begin{aligned}\sum_{i=1}^n 3 &= 3 \cdot \sum_{i=1}^n 1 \\ &= 3 \cdot n\end{aligned}$$

4. $\sum_{i=-3}^n 3$

$$\sum_{i=-3}^n 3 = 3 \cdot (n + 4)$$

5. $\sum_{k=0}^n 2^k + \sum_{k=5}^n 2^k$

$$\begin{aligned}
\sum_{k=0}^n 2^k + \sum_{k=5}^n 2^k &= \sum_{k=0}^n 2^k + \sum_{k=0}^n 2^k - \sum_{k=0}^4 2^k \\
&= 2 \sum_{k=0}^n 2^k - \sum_{k=0}^4 2^k \\
&= 2 \cdot \frac{2^{n+1} - 1}{2 - 1} - \frac{2^5 - 1}{2 - 1}
\end{aligned}$$

6. $\sum_{i=0}^n (\frac{2}{3})^i + \sum_{i=-4}^n (\frac{2}{3})^i$

$$\begin{aligned}
\sum_{i=0}^n (\frac{2}{3})^i + \sum_{i=-4}^n (\frac{2}{3})^i &= \frac{(\frac{2}{3})^{n+1} - 1}{\frac{2}{3} - 1} + \sum_{i=-4}^{-1} (\frac{2}{3})^i + \sum_{i=0}^n (\frac{2}{3})^i \\
&= \frac{(\frac{2}{3})^{n+1} - 1}{\frac{2}{3} - 1} + \sum_{i=-4}^{-1} (\frac{2}{3})^i + \frac{(\frac{2}{3})^{n+1} - 1}{\frac{2}{3} - 1} \\
&= \frac{(\frac{2}{3})^{n+1} - 1}{\frac{2}{3} - 1} + \sum_{i=1}^4 (\frac{3}{2})^i + \frac{(\frac{2}{3})^{n+1} - 1}{\frac{2}{3} - 1} \\
&= \frac{(\frac{2}{3})^{n+1} - 1}{\frac{2}{3} - 1} + \frac{(\frac{3}{2})^5 - 1}{(\frac{3}{2}) - 1} + \frac{(\frac{2}{3})^{n+1} - 1}{\frac{2}{3} - 1}
\end{aligned}$$

7. $\sum_{i=1}^n (i^3 + 2 \cdot i^2 - i + 1)$

$$\begin{aligned}
\sum_{i=1}^n (i^3 + 2 \cdot i^2 - i + 1) &= \sum_{i=1}^n i^3 + 2 \cdot \sum_{i=1}^n i^2 - \sum_{i=1}^n i + \sum_{i=1}^n 1 \\
&= (\frac{n \cdot (n+1)}{2})^2 + 2 \cdot \frac{(n) \cdot (n+1) \cdot (2n+1)}{6} - \frac{n \cdot (n+1)}{2} + n
\end{aligned}$$

8. $\sum_{i=5}^n (-4 \cdot i + \frac{i}{5})$

$$\begin{aligned}
\sum_{i=5}^n (-4 \cdot i + \frac{i}{5}) &= -4 \sum_{i=5}^n i + \frac{1}{5} \sum_{i=5}^n i \\
&= -4 \sum_{i=0}^n i + \frac{1}{5} \sum_{i=0}^n i + 4 \sum_{i=0}^4 i - \frac{1}{5} \sum_{i=0}^4 i \\
&= -4 \cdot \frac{n(n+1)}{2} + \frac{1}{5} \cdot \frac{n(n+1)}{2} + 4 \cdot \frac{4 \cdot 5}{2} - \frac{1}{5} \cdot \frac{4 \cdot 5}{2}
\end{aligned}$$

$$9. \sum_{j=0}^k \sum_{i=1}^j (i - j^2 - 2)$$

$$\begin{aligned}
\sum_{j=0}^k \sum_{i=1}^j (i - j^2 - 2) &= \sum_{j=0}^k \sum_{i=1}^j i - \sum_{j=0}^k \sum_{i=1}^j j^2 - \sum_{j=0}^k \sum_{i=1}^j 2 \\
&= \sum_{j=0}^k \frac{j(j+1)}{2} - \sum_{j=0}^k j^3 - \sum_{j=0}^k 2 \cdot j \\
&= \sum_{j=0}^k \frac{j^2}{2} + \sum_{j=0}^k \frac{j}{2} - (\frac{k \cdot (k+1)}{2})^2 - 2 \cdot \frac{k \cdot (k+1)}{2} \\
&= \frac{1}{2} \cdot \frac{(k) \cdot (k+1) \cdot (2k+1)}{6} + \frac{1}{2} \cdot \frac{k \cdot (k+1)}{2} - (\frac{k \cdot (k+1)}{2})^2 - 2 \cdot \frac{k \cdot (k+1)}{2}
\end{aligned}$$

$$10. \sum_{j=1}^m \sum_{k=1}^j (3 \cdot C + k - 3 \cdot j + i)$$

$$\begin{aligned}
\sum_{j=1}^m \sum_{k=1}^j (3 \cdot C + k - 3 \cdot j + i) &= \sum_{j=1}^m \sum_{k=1}^j (3 \cdot C + k - 3 \cdot j + i) \\
&= \sum_{j=1}^m \sum_{k=1}^j 3 \cdot C + \sum_{j=1}^m \sum_{k=1}^j k - 3 \sum_{j=1}^m \sum_{k=1}^j j + \sum_{j=1}^m \sum_{k=1}^j i \\
&= \sum_{j=1}^m 3 \cdot C \cdot j + \sum_{j=1}^m \frac{j(j+1)}{2} - 3 \sum_{j=1}^m j^2 + \sum_{j=1}^m i \cdot j \\
&= 3 \cdot C \cdot \frac{m \cdot (m+1)}{2} + \sum_{j=1}^m \frac{j^2}{2} + \sum_{j=1}^m \frac{j}{2} - 3 \cdot \frac{(m)(m+1)(2m+1)}{6} \\
&\quad + i \cdot \frac{(m)(m+1)}{2} \\
&= 3 \cdot C \frac{m \cdot (m+1)}{2} + \frac{1}{2} \cdot \frac{m(m+1)(2m+1)}{6} + \frac{1}{2} \cdot \frac{(m)(m+1)}{2} \\
&\quad - 3 \cdot \frac{(m)(m+1)(2m+1)}{6} + i \cdot \frac{(m)(m+1)}{2}
\end{aligned}$$

11. $\sum_{l=-4}^n \sum_{j=1}^k \sum_{i=1}^j (i-4)$

$$\begin{aligned}
\sum_{l=-4}^n \sum_{j=1}^k \sum_{i=1}^j (i-4) &= \sum_{l=-4}^n \sum_{j=1}^k \sum_{i=1}^j i - 4 \sum_{l=-4}^n \sum_{j=1}^k \sum_{i=1}^j 1 \\
&= \sum_{l=-4}^n \sum_{j=1}^k \frac{j(j+1)}{2} - \sum_{l=-4}^n \sum_{j=1}^k j \\
&= \sum_{l=-4}^n \sum_{j=1}^k \frac{j^2}{2} + \sum_{l=-4}^n \sum_{j=1}^k \frac{j}{2} - \sum_{l=-4}^n \frac{k(k+1)}{2} \\
&= \sum_{l=-4}^n \frac{(k)(k+1)(2k+1)}{12} + \sum_{l=-4}^n \frac{(k)(k+1)}{4} - \sum_{l=-4}^n \frac{k(k+1)}{2} \\
&= (n+5) \frac{(k)(k+1)(2k+1)}{12} + (n+5) \frac{(k)(k+1)}{4} - (n+5) \frac{(k)(k+1)}{2}
\end{aligned}$$

2. Calculate the answer

1. $\log_4 x = 5 \rightarrow x = ?$

$$\begin{aligned}
\log_4 x &= 5 \\
x &= 4^5
\end{aligned}$$

$$2. \log_3 y = 4 \rightarrow y = ?$$

$$\begin{aligned} \log_3 y &= 4 \\ y &= 3^4 \end{aligned}$$

$$3. x = 7^2 \rightarrow \log_7 x = ?$$

$$\begin{aligned} x &= 7^2 \\ \log_7 x &= \log_7 7^2 \\ \log_7 x &= 2 \cdot \log_7 7 \\ \log_7 x &= 2 \end{aligned}$$

$$4. x = 32 \rightarrow \log_2 x = ?$$

$$\begin{aligned} x &= 32 \\ \log_2 x &= \log_2 32 \\ \log_2 x &= 5 \end{aligned}$$

$$5. 2^{\log_2 5} + 4^{\log_2 6} - 27^{\log_3 5}$$

$$2^{\log_2 5} + 4^{\log_2 6} - 27^{\log_3 5} = 5 + 36 - 5^3$$

$$6. 9^{\log_3 2} - 25^{\log_5 4} - 36^{\log_6 7} + 8^{\log_8 6}$$

$$\begin{aligned} 9^{\log_3 2} - 25^{\log_5 4} - 36^{\log_6 7} + 8^{\log_8 6} &= 3^{\log_3(2^2)} - 5^{\log_5(4^2)} - 6^{\log_6 7^2 + 6} \\ &= 4 - 16 - 49 + 6 \end{aligned}$$

$$7. \log(4^5 \times 8^3) - \log(16 - 8) + \log\left(\frac{2^{10}}{4 \times 3^2}\right)$$

$$\begin{aligned}
\log_2(4^5 \times 8^3) - \log_2(16 - 8) + \log_2\left(\frac{2^{10}}{4 \times 3^2}\right) &= \log_2(4^5) + \log_2(8^3) - \log_2(8) + \log_2(2^{10}) - \log_2(4 \times 3^2) \\
&= 5 \cdot \log_2(4) + 3 \cdot \log_2(8) - \log_2(8) + 10 \cdot \log_2(2) - \log_2(4) \\
&= 5 \cdot 2 + 3 \cdot 3 - 3 + 10 - 2 - 2 \cdot 1.5 \\
&= 21
\end{aligned}$$

$$8. \log(3^2 \times 64^3) - \log\left(\frac{2^{10} \times 128^3}{9 \times 8^2}\right)$$

$$\begin{aligned}
\log_2(3^2 \times 64^3) - \log_2\left(\frac{2^{10} \times 128^3}{9 \times 8^2}\right) &= \log_2(3^2) + \log_2(64^3) - \log_2(2^{10} \times 128^3) + \log_2(9) + \log_2 8^2 \\
&= 2 \cdot \log_2(3) + 3 \cdot \log_2(64) - 10 \cdot \log_2 2 - 3 \cdot \log_2 128 + 2 \cdot \log_2 3 + 2 \cdot \log_2 8 \\
&= 2 \cdot 1.5 + 3 \cdot 6 - 10 - 3 \cdot 7 + 2 \cdot 1.5 + 2 \cdot 3 \\
&= -1
\end{aligned}$$

$$9. \log \log 16$$

$$\begin{aligned}
\log_2 \log_2 16 &= \log_2 4 \\
&= 2
\end{aligned}$$

$$10. \log 16 \times \log 16$$

$$\begin{aligned}
\log_2 16 \times \log_2 16 &= 4 \times 4 \\
&= 16
\end{aligned}$$

$$11. \log^2 16$$

$$\begin{aligned}
\log_2^2 16 &= 4^2 \\
&= 16
\end{aligned}$$

$$12. \log_2 \log_5 625 - \log_3 \log_4 2^{3^9} + \log^4 2^5 - \frac{\log^2(4^3 \times 3^5)}{\log_5 125}$$

$$\begin{aligned}
\log_2 \log_5 625 - \log_3 \log_4 2^{3^9} + \log_2^4 2^5 - \frac{\log^2(4^3 \times 3^5)}{\log_5 125} &= \log_2 4 - \log_3(3^9 \cdot \log_4 2) + \log_2^4 2^5 - \frac{\log_2^2(4^3 \times 3^5)}{\log_5 125} \\
&= 2 - 9 \log_3\left(\frac{3}{2}\right) + (5 \cdot \log_2(2))^4 - \frac{(\log_2(4^3 \times 3^5))^2}{3} \\
&= 2 - 9 + 9 \cdot \log_3\left(\frac{1}{2}\right) + 5^4 - \frac{(3 \cdot 2 + 5 \cdot \log_2(3))^2}{3} \\
&= 2 - 9 + 9 \cdot \frac{-1}{1.5} + 5^4 - \frac{(3 \cdot 2 + 5 \cdot (1.5)^2)}{3}
\end{aligned}$$

13. $\log \log_8 \log 256 + \log^5(3^2) \times 4^{\log 7}$

$$\begin{aligned}
\log_2 \log_8 \log_2 256 + \log_2^5(3^2) \cdot 4^{\log_2 7} &= \log_2 \log_8 8 + \log_2^5(3^2) \cdot 4^{\log_2 7} \\
&= \log_2 1 + \log_2^5(3^2) \cdot 4^{\log_2 7} \\
&= \log_2^5(3^2) \cdot 4^{\log_2 7} \\
&= (2 \cdot 1.5)^5 \cdot 2^{\log_2 49} \\
&= (2 \cdot 1.5)^5 \cdot 49 \\
&= 11907
\end{aligned}$$

14. $\log_6 x = 5 \rightarrow \log_x 6 = ?$

$$\begin{aligned}
\log_6 x &= 5 \\
x &= 6^5 \\
\log_x x &= 5 \cdot \log_x 6 \\
1 &= 5 \cdot \log_x 6 \\
\frac{1}{5} &= \log_x 6
\end{aligned}$$

15. $\log_y x = 10 \rightarrow \log_x y = ?$

$$\begin{aligned}
\log_y x &= 10 \\
x &= y^{10} \\
\log_x x &= 10 \cdot \log_x y \\
\frac{1}{10} &= \log_x y
\end{aligned}$$

16. $\log_4 32 - \log_8^2 4$

$$\begin{aligned}\log_4 32 - \log_8^2 4 &= \log_4(32) - (\log_8 4)^2 \\ &= \frac{5}{2} - \left(\frac{\log_2 4}{\log_2 8}\right)^2 \\ &= \frac{5}{2} - \left(\frac{2}{3}\right)^2 \\ &= \frac{5}{2} - \frac{4}{9}\end{aligned}$$

17. $\log_4 8 + \log_9 27 - \log_{25}^2 125 - \log_8^3 16 + \log_4 \log_2 256$

$$\begin{aligned}\log_4 8 + \log_9 27 - \log_{25}^2 125 - \log_8^3 16 + \log_4 \log_2 256 &= \frac{3}{2} + \frac{3}{2} - \left(\frac{3}{2}\right)^2 - \left(\frac{4}{3}\right)^3 + \log_4(8) \\ &= \frac{3}{2} + \frac{3}{2} - \left(\frac{9}{4}\right) - \left(\frac{64}{27}\right) + \frac{3}{2}\end{aligned}$$

3. Compute the derivative of

1. $-5 \cdot x^3 + 2 \cdot x - 1$

$$\frac{d}{dx}(-5 \cdot x^3 + 2 \cdot x - 1) = -15 \cdot x^2 + 2$$

2. $3 \cdot x^4 - 2\sqrt{x} + x^{\frac{1}{2}} - 6x^{-\frac{2}{3}} - 5$

$$\begin{aligned}\frac{d}{dx}(3 \cdot x^4 - 2\sqrt{x} + x^{\frac{1}{2}} - 6x^{-\frac{2}{3}} - 5) &= 12 \cdot x^3 - x^{-\frac{1}{2}} + \frac{1}{2} \cdot x^{-\frac{1}{2}} + 4 \cdot x^{-\frac{5}{3}} \\ &= 12 \cdot x^3 - \frac{1}{2}x^{-\frac{1}{2}} + 4 \cdot x^{-\frac{5}{3}}\end{aligned}$$

3. $x \cdot \sqrt{x} + \sqrt{\sqrt{x}}$

$$\frac{d}{dx}(x^{\frac{3}{2}} + x^{\frac{1}{4}}) = \frac{3}{2}x^{\frac{1}{2}} + \frac{1}{4} \cdot x^{-\frac{3}{4}}$$

4. $\log x - x^2 \ln x + \ln x^4$

$$\frac{d}{dx}(\log_2 x - x^2 \cdot \ln x + \ln x^4) = \frac{1}{x \cdot \ln 2} - 2 \cdot x \cdot \ln x - x + \frac{4}{x}$$

5. $\ln^3(x\sqrt{2x-3}) + \sqrt{\ln x^2}$

$$\begin{aligned} \frac{d}{dx}(\ln^3(x \cdot \sqrt{2x-3}) + \sqrt{\ln x^2}) &= \frac{3 \cdot (\ln(x\sqrt{2x-3}))^2 \cdot (\sqrt{2x-3} + x \cdot (2 \cdot x - 3)^{-\frac{1}{2}})}{x\sqrt{2x-3}} \\ &+ \frac{1}{x \cdot \sqrt{\ln(x^2)}} \end{aligned}$$

6. $\frac{\sqrt[4]{x+5} - \ln x}{(x-1)^3}$

$$\frac{d}{dx}\left(\frac{(x+5)^{\frac{1}{4}} - \ln x}{(x-1)^3}\right) = \frac{(x-1)^3 \cdot \left(\frac{1}{4} \cdot (x+5)^{-\frac{3}{4}} - \frac{1}{x}\right) - 3 \cdot ((x+5)^{\frac{1}{4}} - \ln x) \cdot (x-1)^2}{(x-1)^6}$$

4. Determine the limit of

1. $\lim_{x \rightarrow \infty} \frac{3x+2}{-5x-6}$

$$\lim_{x \rightarrow \infty} \frac{3x+2}{-5x-6} = -\frac{3}{5}$$

2. $\lim_{x \rightarrow \infty} \left(\frac{1}{x} + 3\right)$

$$\begin{aligned} \lim_{x \rightarrow \infty} \left(\frac{1}{x} + 3\right) &= \lim_{x \rightarrow \infty} \frac{1}{x} + \lim_{x \rightarrow \infty} 3 \\ &= \lim_{x \rightarrow \infty} \overset{0}{\cancel{\frac{1}{x}}} + \lim_{x \rightarrow \infty} 3 \\ &= 3 \end{aligned}$$

3. $\lim_{x \rightarrow \infty} \frac{3 \cdot x \cdot \log_2(x) + 2}{\sqrt{x^3} + 7 \cdot x}$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{3 \cdot x \cdot \log_2(x) + 2}{\sqrt{x^3} + 7 \cdot x} &= \lim_{x \rightarrow \infty} \frac{3 \cdot x \cdot \log_2(x)}{7 \cdot x} \\ &= \lim_{x \rightarrow \infty} \frac{3 \cdot \log_2(x)}{7} \\ &= \infty \end{aligned}$$

$$4. \lim_{x \rightarrow \infty} \frac{x^3 + x - \sqrt{3x}}{\sqrt{x}}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x^3 + x - \sqrt{3x}}{\sqrt{x}} &= \lim_{x \rightarrow \infty} \frac{x^3}{\sqrt{x}} + \lim_{x \rightarrow \infty} \frac{x}{\sqrt{x}} - \lim_{x \rightarrow \infty} \frac{\sqrt{3x}}{\sqrt{x}} \\ &= \infty \end{aligned}$$

$$5. \lim_{x \rightarrow \infty} \frac{x^3 + x - \sqrt{3x}}{5 \cdot x^{2.25} \cdot \sqrt{\sqrt{x}}}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x^3 + x - \sqrt{3x}}{5 \cdot x^{2.25} \cdot x^{.25}} &= \lim_{x \rightarrow \infty} \frac{x^3 + x - \sqrt{x}}{5 \cdot x^3} \\ &= \lim_{x \rightarrow \infty} \frac{x^3}{5 \cdot x^3} + \lim_{x \rightarrow \infty} \frac{x}{\cancel{5 \cdot x^3}}^0 - \lim_{x \rightarrow \infty} \frac{\sqrt{x}}{\cancel{5 \cdot x^3}}^0 \\ &= \frac{1}{5} \end{aligned}$$

$$6. \lim_{x \rightarrow \infty} \frac{x^{0.1} - \sqrt{3}}{\sqrt{\sqrt{x}}}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x^{0.1} - \sqrt{3}}{x^{0.25}} &= \lim_{x \rightarrow \infty} \frac{x^{0.1}}{x^{0.25}} - \frac{\sqrt{3}}{x^{0.25}} \\ &= \lim_{x \rightarrow \infty} \frac{1}{x^{0.15}} - \frac{\sqrt{3}}{x^{0.25}} \\ &= 0 \end{aligned}$$

$$7. \lim_{x \rightarrow \infty} \frac{x^x}{2^x}$$

$$\begin{aligned} \log(\lim_{x \rightarrow \infty} \frac{x^x}{2^x}) &= \lim_{x \rightarrow \infty} \log(\frac{x^x}{2^x}) \\ &= \lim_{x \rightarrow \infty} \log(x^x) - \lim_{x \rightarrow \infty} \log(2^x) \\ &= \lim_{x \rightarrow \infty} x \log(x) - \lim_{x \rightarrow \infty} x \log(2) \\ &= \infty \\ e^{\ln(\lim_{x \rightarrow \infty} \frac{x^x}{2^x})} &= e^\infty \\ &= \infty \end{aligned}$$

$$8. \lim_{x \rightarrow \infty} \frac{x^x}{x(2^x)}$$

$$\begin{aligned}
\log(\lim_{x \rightarrow \infty} \frac{x^x}{x(2^x)}) &= \lim_{x \rightarrow \infty} \log(\frac{x^x}{x \cdot 2^x}) \\
&= \lim_{x \rightarrow \infty} \log(x^x) - \lim_{x \rightarrow \infty} \log(x \cdot 2^x) \\
&= \lim_{x \rightarrow \infty} x \log(x) - \lim_{x \rightarrow \infty} \log(x) - \lim_{x \rightarrow \infty} \log(2^x) \\
&= \lim_{x \rightarrow \infty} x \log(x) - \lim_{x \rightarrow \infty} \log(x) - \lim_{x \rightarrow \infty} x \log(2) \\
&= \infty \\
e^{\ln(\lim_{x \rightarrow \infty} \frac{x^x}{x(2^x)})} &= e^\infty \\
&= \infty
\end{aligned}$$

$$9. \lim_{x \rightarrow \infty} \frac{\log_2(x)^{\log_2(x)}}{x^{\frac{1}{5}}}$$

$$\begin{aligned}
\lim_{x \rightarrow \infty} \log_2\left(\frac{\log_2(x)^{\log_2(x)}}{x^{\frac{1}{5}}}\right) &= \lim_{x \rightarrow \infty} \log_2(x) \cdot \log_2(\log_2(x)) - \frac{1}{5} \cdot \log_2(x) \\
&= \lim_{x \rightarrow \infty} \infty \\
2^{\lim_{x \rightarrow \infty} \log_2\left(\frac{\log_2(x)^{\log_2(x)}}{x^{\frac{1}{5}}}\right)} &= 2^\infty \\
&= \infty
\end{aligned}$$

$$10. \lim_{x \rightarrow \infty} \frac{\sqrt{2}^{\log^4 x^3}}{\log(2 \cdot x + 7)}$$

$$\begin{aligned}
\lim_{x \rightarrow \infty} \frac{2^{\frac{1}{2}(3 \log(x))^4}}{\log(2 \cdot x + 7)} &= \lim_{x \rightarrow \infty} \frac{2^{\frac{81}{4}(\log(x))^4}}{\log(2 \cdot x + 7)} \\
&= \infty
\end{aligned}$$

$$11. \lim_{x \rightarrow \infty} \frac{x+1}{\frac{3 \cdot x^{\ln x}}{2 \cdot x^2}}$$

$$\begin{aligned}
\lim_{x \rightarrow \infty} \frac{x+1}{\frac{3 \cdot x^{\ln x}}{2 \cdot x^2}} &= \lim_{x \rightarrow \infty} \frac{x+1}{6 \cdot x^2 \cdot x^{\ln x}} \\
&= 0
\end{aligned}$$

$$12. \lim_{x \rightarrow \infty} \frac{\sqrt{2}^{\log x^3}}{\log^{\ln x}(2x)}$$

$$\begin{aligned}
\lim_{x \rightarrow \infty} \frac{\sqrt{2}^{\log x^3}}{\log^{\ln x}(2x)} &= \lim_{x \rightarrow \infty} \frac{2^{\frac{1}{2} \cdot \log x^3}}{\log^{\ln x}(2x)} \\
&= \lim_{x \rightarrow \infty} \frac{2^{\log x^{\frac{3}{2}}}}{\log^{\ln x}(2x)} \\
&= \lim_{x \rightarrow \infty} \frac{x^{\frac{3}{2}}}{\log^{\ln x}(2x)} \\
\lim_{x \rightarrow \infty} \log\left(\frac{x^{\frac{3}{2}}}{\log^{\ln x}(2x)}\right) &= \lim_{x \rightarrow \infty} \frac{3}{2} \log(x) - \log(x) \log(\log(2x)) \\
&= \lim_{x \rightarrow \infty} \frac{3}{2} \log(x) - \log(x) (\log(1 + \log(x))) \\
&= -\infty \\
2^{\log(\lim_{x \rightarrow \infty} \frac{x^{\frac{3}{2}}}{\log^{\ln x}(2x)})} &= 2^{-\infty} \\
&= 0
\end{aligned}$$

5. Compute the exact values for

1. $\int_1^n (2 \cdot x^4 + 5\sqrt{x}) \cdot dx$

$$\begin{aligned}
\int_1^n (2 \cdot x^4 + 5\sqrt{x}) \cdot dx &= 2 \int_1^n x^4 dx + 5 \int_1^n \sqrt{x} dx \\
&= 2 \cdot \left(\frac{x^5}{5} \Big|_1^n\right) + 5 \left(\frac{2}{3} x^{\frac{3}{2}} \Big|_1^n\right) \\
&= 2 \cdot \left(\frac{n^5}{5} - \frac{1}{5}\right) + 5 \left(\frac{2}{3} n^{\frac{3}{2}} - \frac{2}{3}\right) \\
&= \frac{2 \cdot n^5}{5} - \frac{2}{5} + \frac{10}{3} n^{\frac{3}{2}} - \frac{10}{3}
\end{aligned}$$

2. $\int_1^n (x^4 - 3 \cdot x^2 + \frac{1}{x} - \frac{1}{x^2}) dx$

$$\begin{aligned}
\int_1^n (x^4 - 3 \cdot x^2 + \frac{1}{x} - \frac{1}{x^2}) dx &= \int_1^n x^4 dx - 3 \int_1^n x^2 dx + \int_1^n \frac{1}{x} dx - \int_1^n \frac{1}{x^2} dx \\
&= \frac{x^5}{5} \Big|_1^n - 3 \cdot \frac{x^3}{3} \Big|_1^n + \ln x \Big|_1^n + \frac{1}{x} \Big|_1^n \\
&= \left(\frac{n^5}{5} - \frac{1}{5}\right) - n^3 + 1 + \ln n + \frac{1}{n} - 1 \\
&= \frac{n^5}{5} - n^3 + \ln n + \frac{1}{n} - \frac{1}{5}
\end{aligned}$$

3. $\int_1^n (\frac{3}{\sqrt{x}} + \ln x + e^x) dx$

$$\begin{aligned}
\int_1^n \frac{3}{\sqrt{x}} + \ln x + e^x dx &= \int_1^n \frac{3}{\sqrt{x}} dx + \int_1^n \ln x dx + \int_1^n e^x dx \\
&= 6\sqrt{x}|_1^n + (x \ln x - x)|_1^n + e^n - e \\
&= 6\sqrt{n} - 6 + (n \ln n - n + 1) + e^n - e \\
&= 6\sqrt{n} + n \ln n - n - 5 + e^n - e
\end{aligned}$$

4. $\int_1^n x \cdot \sin x dx$

$$\begin{aligned}
\int_1^n x \cdot \sin x dx &= -x \cos x|_1^n + \int_1^n \cos x dx \\
&= -n \cos n + \cos(1) + \sin(n) - \sin(1)
\end{aligned}$$

6. Use mathematical induction to prove that

$$1 + 2 + \dots + n = \frac{n(n+1)}{2}$$

Proof. $1 + 2 + \dots + n = \frac{n(n+1)}{2}$

Base case $n = 1$: If $n = 1$ then the left hand side and the right hand side is $1 = 1 = \frac{1(2)}{2}$.

Inductive hypothesis: Suppose the theorem holds for all values of n up to some $k, k \geq 1$.

Inductive step: let $n = k + 1$ then our left hand side is

$$\begin{aligned}
\sum_{i=1}^{k+1} i &= \sum_{i=1}^k i + (k+1) \\
&= \frac{k(k+1)}{2} + (k+1) \\
&= \frac{k(k+1) + 2 \cdot k + 2}{2} \\
&= \frac{(k+1) \cdot (k+2)}{2}
\end{aligned}$$

which is equal to our right hand side. By the principle of mathematical induction, the theorem holds for all integers $n \geq 1$. \square

7. Use mathematical induction to prove that

$$1 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1 + 2 + \dots + n = \frac{n(n+1)}{2}$$

Proof. Base case $n = 1$: If $n = 1$ then the left hand side and the right hand size is $1^2 = 1 = \frac{1(2)(3)}{6}$.

Inductive hypothesis: Suppose the theorem holds for all values of n up to some $k, k \geq 1$.

Inductive step: let $n = k + 1$ then our left hand side is

$$\begin{aligned}
\sum_{i=1}^{k+1} i^2 &= \sum_{i=1}^k i^2 + (k+1)^2 \\
&= \frac{k \cdot (k+1) \cdot (2 \cdot k + 1)}{6} + (k+1)^2 \\
&= \frac{k \cdot (k+1) \cdot (2 \cdot k + 1) + 6 \cdot (k+1)^2}{6} \\
&= \frac{(6 \cdot (k+1) + k \cdot (2 \cdot k + 1)) \cdot (k+1)}{6} \\
&= \frac{(6 \cdot k + 6 + 2 \cdot k^2 + k) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k^2 + 7 \cdot k + 6) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k^2 + 4 \cdot k + 3 \cdot k + 6) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k \cdot (k+2) + 3 \cdot (k+2)) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k + 3) \cdot (k+2) \cdot (k+1)}{6}
\end{aligned}$$

which is equal to our right hand side. By the principle of mathematical induction, the theorem holds for all integers $n \geq 1$. \square