

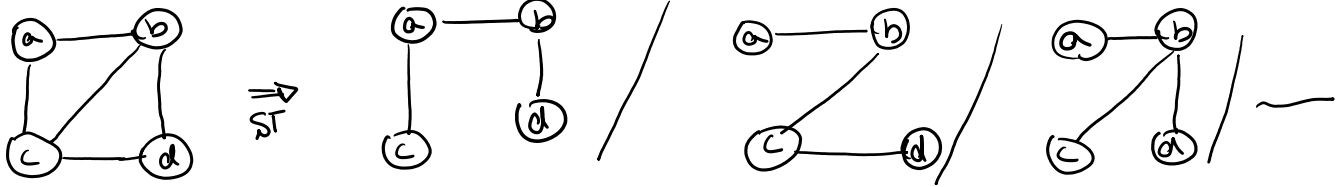
Graphs (Kruskal / Prim)

Tuesday, December 8, 2020 5:00 PM

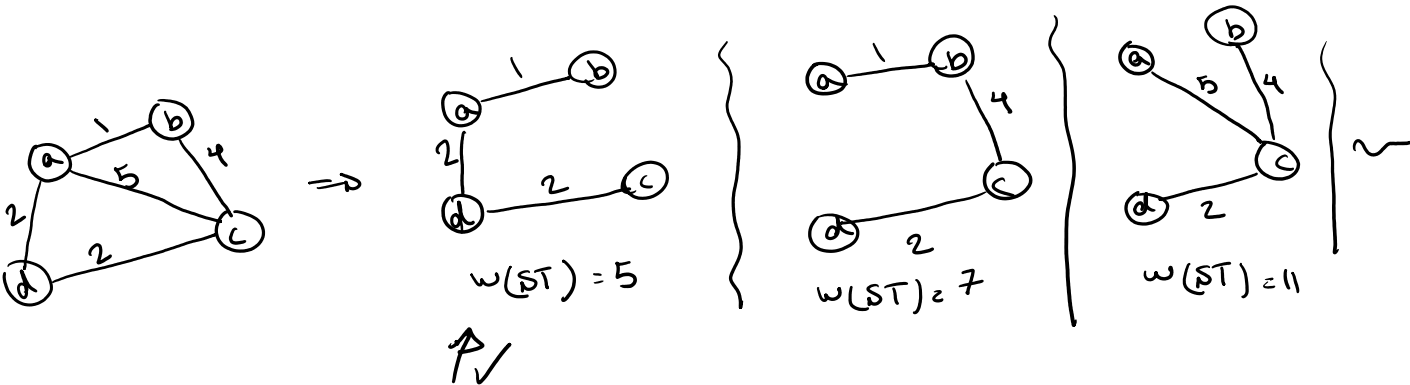
Reminder: *Final is this Thursday.

(Dynamic Programming / Greedy Algorithm / NP) * Hw 9 is due This Thursday → Monday Dec 14
Graphs

Spanning Tree: A tree you create from a "connected undirected graph."



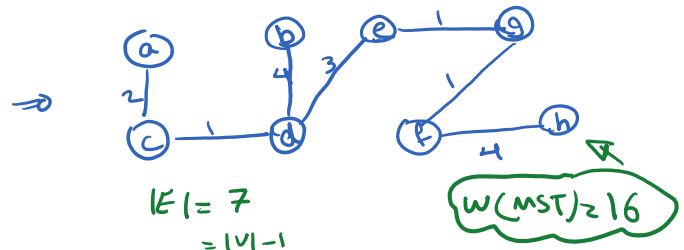
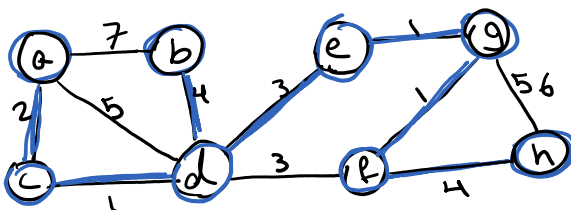
Minimum Spanning Tree (MST)



$$v-1 \leq E \leq \frac{v(v-1)}{2} \Rightarrow E = O(v^2)$$

Kruskal is $O(E \log E) = O(E \log v^2) = O(E \log v)$
 Greedy

$|V| = 8$



Sort
 $O(E \log E)$

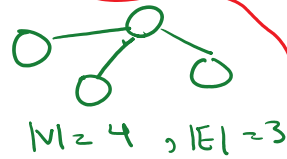
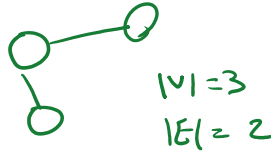
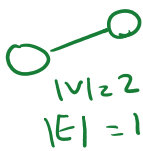
cd	eg	gf	ac	de	df	bd	th	ad	ab	gh
1	1	1	2	3	3	4	4	5	7	56
✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

1 | 1 | 1 | 2 | 3 | 3 | 4 | 4 | 5 | 7 | 56

✓ ✓ ✓ ✓ X ✓ ✓ ✓

STOP!

Note 1 Tree



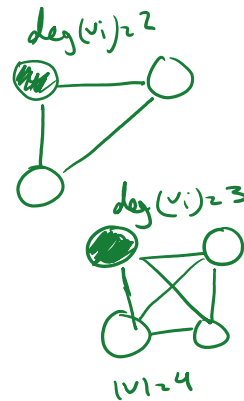
$\Rightarrow |E| = |V| - 1$

Note 2 undirected "connected" graph

$|V| - 1 \leq |E| \leq \frac{|V|(|V| - 1)}{2}$

Tree complete graph

$|E| = O(|V|^2)$



$\sum \deg(v_i) = 2|E|$

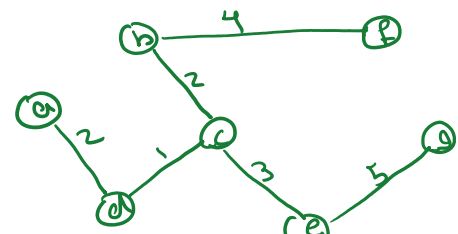
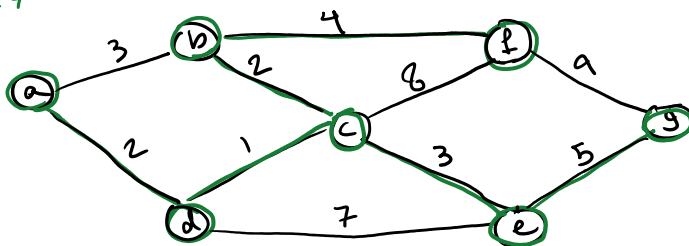
$\sum_{i=1}^{|V|} (|V| - 1) = 2|E|$

$|V|(|V| - 1) = 2|E|$

$\Rightarrow |E| = \frac{|V|(|V| - 1)}{2}$

Example

$|V| = 7$



$w(MST) = 17$

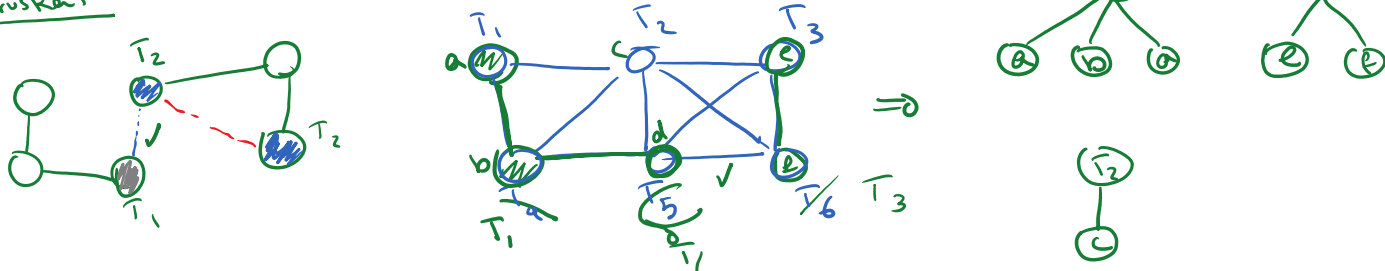
cd	ad	bc	ab	ce	bf	eg	de	cf	fg
1	2	2	3	3	4	5	7	6	9
✓	✓	✓	X	✓	✓	✓	↑		

stop!

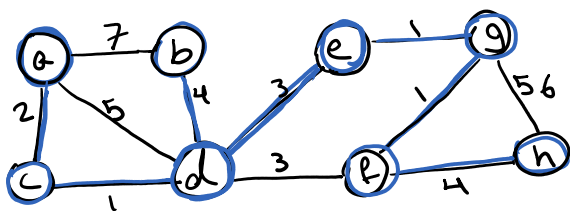
Kruskal



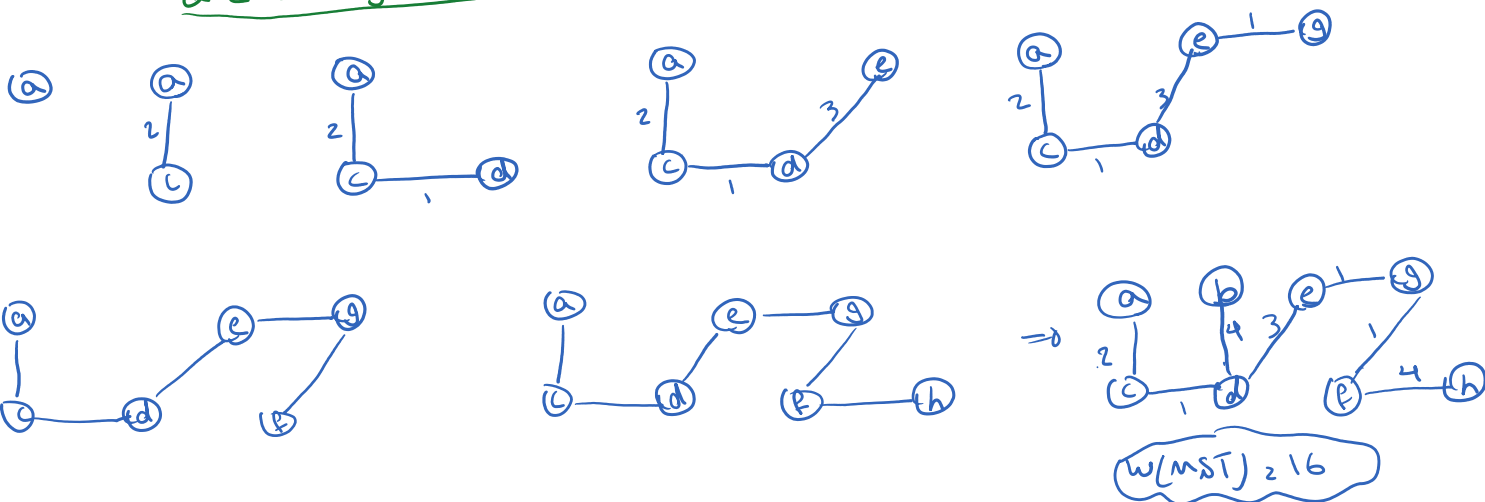
Kruskal



Prim : (Greedy)

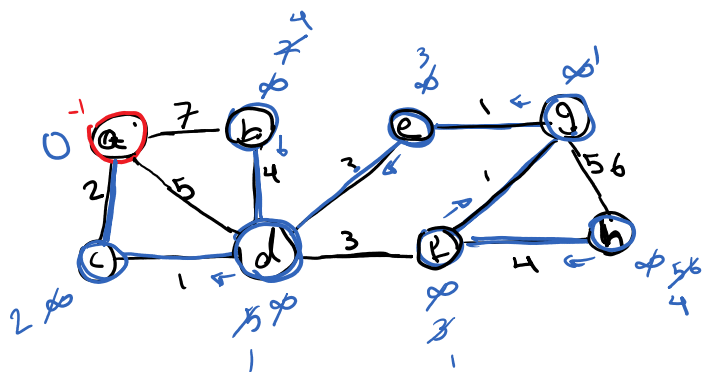


a c d e g f h b



One more time

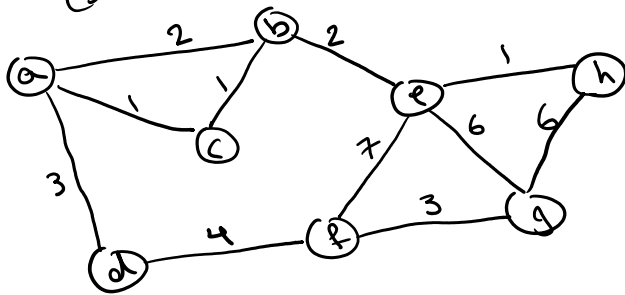
→ Cost 9



Example:

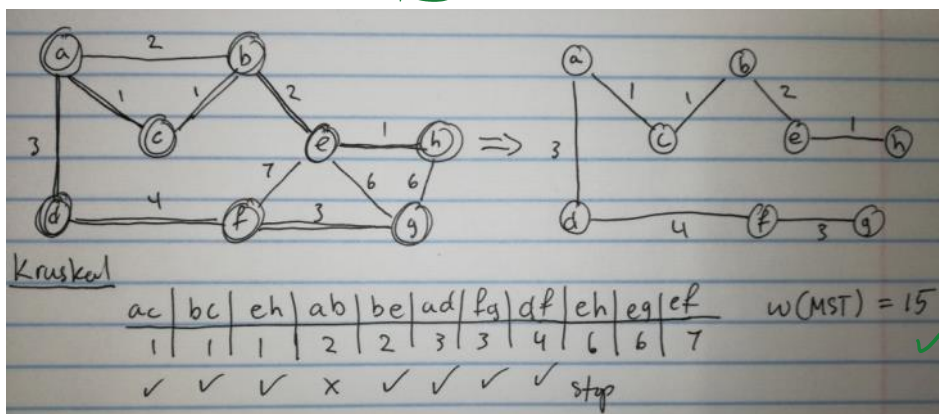
- ① Kruskal
- ② Prim
- ③ code for Prim
- ④ time complexity

$$|V|-1 \leq |E| \leq \frac{|V|(|V|-1)}{2}$$



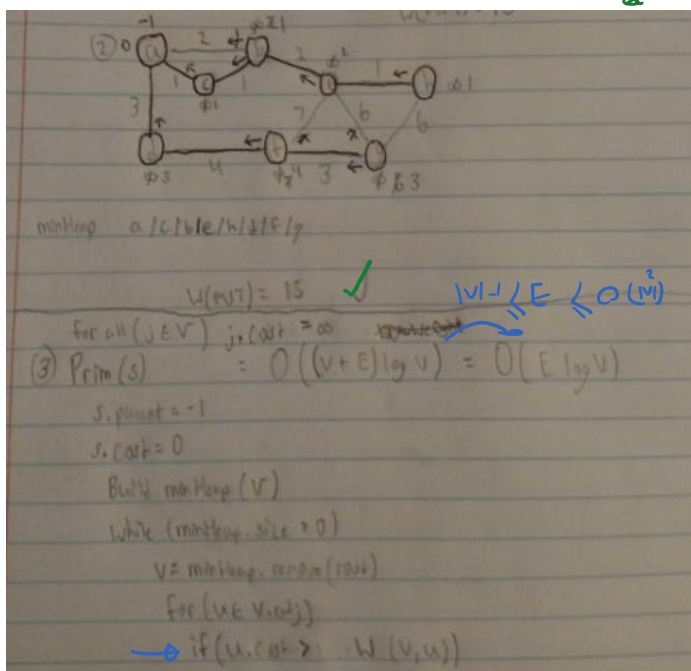
Khai

Kruskal



Prim

Mathew



```
for (u < v; u++)  
    if (u.cost > W(v, u))  
        u.cost = W(v, u)  
    u.parent = v  
    minHeapify(u)
```