

1. $\lim_{x \rightarrow \infty} 2 \cdot x + 5$

$$\begin{aligned} \lim_{x \rightarrow \infty} 2 \cdot x + 5 &= \lim_{x \rightarrow \infty} 2 \cdot x + \lim_{x \rightarrow \infty} 5 \\ &= \infty \end{aligned}$$

2. $\lim_{x \rightarrow \infty} \log x^5$

$$\lim_{x \rightarrow \infty} 5 \log x = \infty$$

3. $\lim_{x \rightarrow \infty} \log x$

$$\lim_{x \rightarrow -\infty} \log x = \text{undefined}$$

4. $\lim_{x \rightarrow 0^+} \log x$

$$\lim_{x \rightarrow 0} \log x = -\infty$$

5. $\lim_{x \rightarrow \infty} \frac{6 \cdot x + 2}{x^3 + 5 \cdot x}$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{6 \cdot x + 2}{x^3 + 5 \cdot x} &= \frac{6}{\infty} \\ &= 0 \end{aligned}$$

6. $\lim_{x \rightarrow \infty} \frac{x^2 + 5\sqrt{x}}{\log x}$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x^2 + 5\sqrt{x}}{\log x} &= \lim_{x \rightarrow \infty} \frac{x^2}{\log x} \\ &= \infty \end{aligned}$$

7. $\lim_{x \rightarrow \infty} \frac{x^{\frac{3}{2}} + x^2 \log x}{4x^2 (\log x)^2 + x \sqrt{\log x}}$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{x^{\frac{3}{2}} + x^2 \log x}{4x^2 (\log x)^2} &= \frac{1}{4 \cdot \log(x)} \\ &= 0 \end{aligned}$$

$$8. \lim_{x \rightarrow \infty} \frac{\sqrt{x^5} + \log^2 x}{x^{0.1} + \log^{7000} x}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\sqrt{\log x^5} + \log^2(x)}{x^{0.1} + \log^{7000} x} &= \frac{\log^2(x)}{x^{0.1}} \\ &= 0 \end{aligned}$$

$$9. \lim_{x \rightarrow \infty} \frac{\log \log x + \sqrt{\sqrt{\log x}}}{10 \log x}$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt{\sqrt{\log x}}}{10 \log x} = 0$$

$$10. \lim_{x \rightarrow \infty} \frac{2^{\log x^5} + 8^{\log x^{\frac{1}{3}}}}{2^{\log \log^2 x} + 6x}$$

$$\lim_{x \rightarrow \infty} \frac{x^5 + x}{\log^2 x + 6x} = \infty$$

$$11. \lim_{x \rightarrow \infty} \frac{x^{\log x}}{2^x}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} 2^{\log \frac{x^{\log x}}{2^x}} &= \lim_{x \rightarrow \infty} 2^{\log x \log x - x} \\ &= \lim_{x \rightarrow \infty} 2^{-x} \\ &= 0 \end{aligned}$$

$$12. \lim_{x \rightarrow \infty} \frac{\log x^{\log x}}{2^{x^2}}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} 2^{\log x \log \log x - x^2} &= \lim_{x \rightarrow \infty} 2^{-x^2} \\ &= 0 \end{aligned}$$

$$13. \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$$

$$\begin{aligned} \lim_{x \rightarrow \infty} e^{x \ln(1 + \frac{1}{x})} &= \lim_{x \rightarrow \infty} e^{\frac{\ln(1 + \frac{1}{x})}{\frac{1}{x}}} \\ &= \lim_{x \rightarrow \infty} e^{\frac{\frac{1}{1 + \frac{1}{x}} \cdot -\frac{1}{x^2}}{-\frac{1}{x^2}}} \\ &= \lim_{x \rightarrow \infty} e^{\frac{1}{1 + \frac{1}{x}}} \\ &= e \end{aligned}$$

$$14. \lim_{x \rightarrow \infty} \frac{\log x \log^5 x}{x^{x^x}}$$

$$\lim_{x \rightarrow \infty} e^{\ln\left(\frac{\log x \log^5 x}{x^{x^x}}\right)} = 0$$