Lecture 3 (Review) (Reminder: HW 1 is due this Saturday)

Tuesday, September 1, 2020 5:00 PM

④ $\left(\sqrt{\frac{\ln^2 x}{5\log x^2+2}}\right)' = \left(p^{1/2}\right)' = \frac{1}{2}p^{-1/2}\, p'$

where $p = \frac{\ln^2 x}{5\log x^2+2}$

$= \frac{1}{2}\left(\frac{\ln^2 x}{5\log x^2+2}\right)^{-1/2}\left(\frac{\ln^2 x}{5\log x^2+2}\right)'$

$= \quad " \quad " \quad \left(\frac{2\ln x\,(1/x)\,(5\log x^2+2) - \left(10\frac{1}{(\ln 2)x}\right)(\ln^2 x)}{(5\log x^2+2)^2}\right)$

⑤ $\left(\sqrt{\sqrt{x}}\left(\ln^2(x^2+5)\right)\right)' = (PS)' = P'S + S'P$

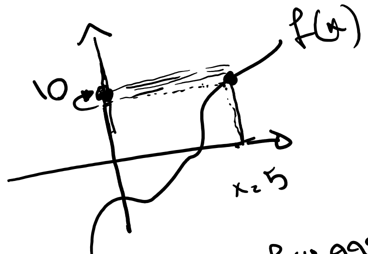
$P = \sqrt{\sqrt{x}} = x^{\frac{1}{4}}$, $S = \ln^2(x^2+5)$

$= \left(\frac{1}{4}x^{-3/4}\right)\left(\ln^2(x^2+5)\right) + \left(2\ln(x^2+5)\left(\ln(x^2+5)\right)'\right)\left(\sqrt{\sqrt{x}}\right)$

$= \quad " \quad " \quad + \left(\quad " \quad " \quad \frac{2x}{x^2+5}\right)\left(\sqrt{\sqrt{x}}\right)$

Limits

$\lim_{x\to 5^-} f(x) = 10$

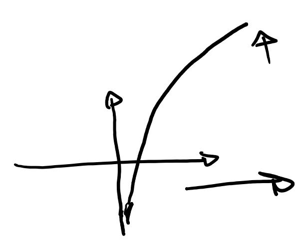


$f(x) = f(4.9999\ldots9) = 9.\ldots9$

Examples:

① $\lim_{x\to+\infty} 2x+5 = +\infty$

② $\lim_{x\to+\infty} \log x^5 = \lim_{x\to\infty} 5\log x = +\infty$



③ $\lim_{x \to -\infty} \log x = \text{UND}$

④ $\lim_{x \to 0^+} \log x = -\infty$

⑤ $\lim_{x \to \infty} \frac{6x+2}{x^3+5x} = \frac{\infty}{\infty}$ or $\frac{0}{0}$ L'Hopital

$\boxed{0 \times \infty}$ ⟹ change to $\frac{0}{0}$ or $\frac{\infty}{\infty}$

$\lim_{x \to \infty} fg = 0 \times \infty = 0.1 \times 10^{20} = 10^{19} \to \infty$

$0.00\ldots1 \times 100$ = close to 0

$= 0.0001 \times 10^4 = 1 \to nr$

∞
0
nr

In limits

$\lim_{x \to \infty} f(x) \Rightarrow$ ① $\frac{\infty}{\infty}$ or $\frac{0}{0}$ ⟹ L'Hopital rule

② $0 \times \infty$ change it to ⟹ $\frac{\infty}{\infty}$ or $\frac{0}{0}$ ⟹ L'Hopital rule

$$\lim_{x \to \infty} \frac{6x+2}{x^3+5x} \overset{\text{L'H}}{=} \lim_{x \to \infty} \frac{6}{3x^2+5} = 0$$

another way $\Rightarrow \lim\limits_{x \to \infty} \dfrac{6x + \cancel{2}}{x^3 + \cancel{5x}} = \lim\limits_{x \to \infty} \dfrac{6x}{x^3} = 0$

Growth	Terminology
O(1)	constant growth
O(log n)	logarithmic growth
O(n)	linear growth
~~O(n log n)~~	~~log-linear growth~~
O(n^b)	polynomial growth
$\Omega(n^k)$ for every k	superpolynomial growth
$\Omega(b^n)$ for some $b > 1$	exponential growth

$f \gg x^{500\ldots0}$

(6) $\lim\limits_{x \to \infty} \dfrac{x^2 + 5\sqrt{x}}{\log x} = \lim\limits_{x \to \infty} \dfrac{x^2}{\log x} = +\infty$

(7) $\lim\limits_{x \to \infty} \dfrac{x\sqrt{x} + x^2 \log x}{x^2 \log^2 x^2 + x\sqrt{\log x}} = \lim\limits_{x \to \infty} \dfrac{\cancel{x^2} \log x}{4\cancel{x^2} \log^2 x} = \lim\limits_{x \to \infty} \dfrac{\log x}{4 \log^2 x} = 0$

$\log^2 x^2 = (\log x^2)^2 = (2 \log x)^2 = 4 \log^2 x$

(8) $\lim\limits_{x \to \infty} \dfrac{\sqrt{\log x^5} + \log^2 x}{x^{0.1} + \log^{7000} x} \approx \lim\limits_{x \to \infty} \dfrac{(\log x)^{1/2} + (\log x)^2}{x^{.1}} = \lim\limits_{x \to \infty} \dfrac{(\log x)^2}{x^{.1}} = \boxed{0}$

(9) $\lim\limits_{x\to\infty} \dfrac{\log \log x + \sqrt{\log x}}{\log x^{10}} = \lim\limits_{x\to\infty} \dfrac{\log p + p^{1/4}}{10 \log x} = \lim\limits_{x\to\infty} \dfrac{(\log x)^{1/4}}{10 \log x} = \boxed{0}$

(10) $\lim\limits_{x\to\infty} \dfrac{2^{\log x^5} + 8^{\log x^{\frac{1}{3}}}}{2^{\log \log^2 x} + 6x} = \lim\limits_{x\to\infty} \dfrac{x^5 + x}{\log^2 x + 6x} = \lim\limits_{x\to\infty} \dfrac{x^5}{6x} = \boxed{+\infty}$

(11) $\lim\limits_{x\to\infty} \dfrac{x^{\log x}}{2^x}$ (call this p) $\Rightarrow \lim\limits_{x\to\infty} \log p$

$= \lim\limits_{x\to\infty} \log \dfrac{x^{\log x}}{2^x}$

$= \lim\limits_{x\to\infty} \log\left(x^{\log x}\right) - \log\left(2^x\right)$

$= \lim\limits_{x\to\infty} \log x \left(\log x\right) - x \log 2$

$= \lim\limits_{x\to\infty} \left(\cancel{\log^2 x} - x\right) = \boxed{-\infty}$

$\Rightarrow \lim \log p = -\infty \Rightarrow \lim p = 2^{-\infty} = \left(\frac{1}{2}\right)^{\infty} = \boxed{0}$

$$\Rightarrow \lim \log p = -\infty \xRightarrow{\text{log def}} \lim p = 2^{-\infty} = \left(\frac{1}{2}\right)^{\infty} = \boxed{0}$$

$$\boxed{\text{log rules: } \log_b a = c \iff a = b^c}$$

$$\lim_{x \to \infty} \frac{f}{g} \longrightarrow \lim \log \frac{f}{g}$$

$$\longrightarrow \lim \cancel{\frac{\log f}{\log g}} \Longleftarrow \text{WRONG X}$$

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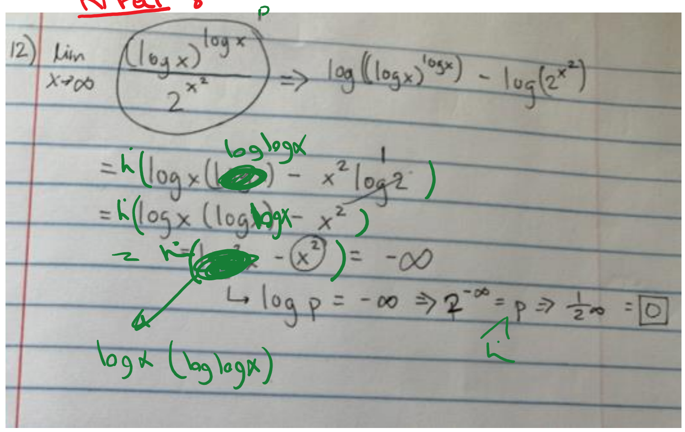
(12) $$\lim_{x \to \infty} \frac{(\log x)^{\log x}}{2^{(x^2)}} =$$

Rifat:



12) $\lim_{x\to\infty} \frac{(\log x)^{\log x}}{2^{x^2}}$ (p) $\Rightarrow \log\left((\log x)^{\log x}\right) - \log\left(2^{x^2}\right)$

$= \lim\left(\log x (\log\log x) - x^2 \log 2\right)$

$= \lim\left(\log x (\log\log x - x^2)\right)$

$= \lim\left(\log x (\log\log x) - (x^2)\right) = -\infty$

$\hookrightarrow \log p = -\infty \Rightarrow 2^{-\infty} = p \Rightarrow \frac{1}{2^\infty} = \boxed{0}$

$\log x (\log \log x)$

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(13) $$\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x \Rightarrow$$

Note 1: $\log(a+b) \neq \log a + \log b$

Note 2: $0 \times \infty \Rightarrow \frac{0}{0}$ or $\frac{\infty}{\infty}$

Note 3: $2 = \frac{1}{\frac{1}{2}}$

$$\lim \underbrace{\left(1 + \frac{1}{x}\right)^x}_{p} \Rightarrow \lim \ln p = \lim \ln \left(1 + \frac{1}{x}\right)^x$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x} \Rightarrow \lim_{x\to\infty} \ln p = \lim_{x\to\infty} \ln\left(1+\frac{1}{x}\right)^{x}$$

$$= \lim_{x\to\infty} \overset{\infty \times 0}{x \ln\left(1+\frac{1}{x}\right)} = \lim_{x\to\infty} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} = \frac{0}{0}$$

$$\overset{L.H}{=} \lim_{x\to\infty} \frac{\frac{\left(1+\frac{1}{x}\right)'}{1+\frac{1}{x}}}{-\frac{1}{x^2}} = \lim_{x\to\infty} \frac{\frac{\cancel{-\frac{1}{x^2}}}{1+\frac{1}{x}}}{\cancel{-\frac{1}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{1+\frac{1}{x}}}{1} = \lim_{x\to\infty} \frac{1}{1+\underbrace{\frac{1}{x}}_{\to 0}} = \boxed{1}$$

$$\Rightarrow \lim \ln(p) = \lim \operatorname{Ln}\left(1+\frac{1}{x}\right)^{x} = 1$$

$\log_e p = 1$ $\quad \Rightarrow \lim p = e^1 = \boxed{e}$:D

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(14) $$\lim_{x\to\infty} \frac{(\log x)^{\log^5 x}}{x^{x^x}} \Rightarrow \lim_{x\to\infty}\left(\log^5 x \log\log x - \underbrace{x^x \log x}\right)$$

$$= -\infty \Rightarrow \lim \log p = -\infty$$

$$\boxed{\Rightarrow \lim p = 0}\ \checkmark$$