

1. What kind of growth of the function $f(n) = n^{\frac{1}{\log(n)}}$ have? Explain your answer. Your answer should be as accurate as possible. For example, if the growth was exponential, the super polynomial would not be the most accurate answer.

$$\begin{aligned} \log(\lim_{n \rightarrow \infty} \frac{n^{\frac{1}{\log(n)}}}{1}) &= \lim_{n \rightarrow \infty} 1 \\ 2^1 &= 2 \\ n^{\frac{1}{\log(n)}} &= \Theta(1) \end{aligned}$$

2. Use the integral theorem to establish that $1+2^{10}+3^{10}+\dots+n^{10} = \Theta(n^{11})$

$$\begin{aligned} \sum_{i=1}^n i^{10} &= \Theta(\int_1^n x^{10} dx) \\ &= \Theta(n^{11}) \end{aligned}$$

3. Consider the following code

```
sum = 0;
for(i=0; i < n; i++)
    for(j=0; j < i*i; j++)
        for(k=0; k < j; sum++)
            sum++;
```

Give an evaluation of the summation expression

$$\begin{aligned} \sum_{i=0}^{n-1} \sum_{j=0}^{i^2} \sum_{k=0}^{j-1} 1 &= \sum_{i=0}^{n-1} \sum_{j=0}^{i^2} j \\ &= \sum_{i=0}^{n-1} \frac{(i^2+1)(i^2)}{2} \\ &= \sum_{i=0}^{n-1} \frac{i^4}{2} + \sum_{i=0}^{n-1} \frac{i^2}{2} \\ &= \sum_{i=0}^{n-1} O(n^4) \\ &= O(n^5) \end{aligned}$$