Lecture 1 (Review)

Tuesday, August 25, 2020 5:00 PM

Series

(1) $1+2+4+8+16+32 = \sum_{i=0}^{5} 2^i$

(2) $\frac{1}{3} - 1 + 3 - 9 + 27 = \sum_{p=-1}^{3} (-1)^{p-1} 3^p$

(3) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \sum_{k=1}^{4} \left(\frac{1}{2}\right)^k$

(4) $\log 1 + 2\log 2 + 3\log 3 + \cdots + n\log n = \sum_{j=1}^{n} j \log j$

*(5) $1 + 2 + 3 + \cdots + n = \sum_{i=1}^{n} i = \boxed{\frac{n(n+1)}{2}}$

(5) $0 + 1 + 2 + 3 + \cdots + n = \sum_{i=0}^{n} i = \frac{n(n+1)}{2}$

*(6) $1 + 2^2 + 3^2 + \cdots + n^2 = \sum_{i=1}^{n} i^2 = \boxed{\frac{n(n+1)(2n+1)}{6}}$

*(7) $1 + 2^3 + 3^3 + \cdots + n^3 = \sum_{i=1}^{n} i^3 = \boxed{\left(\frac{n(n+1)}{2}\right)^2}$

*(8) $\underbrace{2^0}_{=1} + 2^1 + 2^2 + \cdots + 2^n = \sum_{i=0}^{n} 2^i = \boxed{\frac{2^{n+1} - 1}{2 - 1}}$ $\Leftarrow$ $\cancel{\sum i}$

(10) $1 + 2 + 3 + \cdots + k^2 = \sum_{i=1}^{k^2} i = \frac{k^2(k^2+1)}{2}$

(11) $1 + 2^2 + 3^2 + \cdots + p^5 = \sum_{i=1}^{p^{2.5}} i^2 \Rightarrow i^2 = p^5$

$1 = i^2 \Rightarrow i = 1$

$2^2 = i^2$

$p^5 = i^2$

$\underset{i=10}{\Rightarrow}$ $i^2 = 10^2$

(12) $\sum_{i=-5}^{k} i^3 = (-5)^3 + (-4)^3 + \cdots (-1)^3 + 0^3 + 1^3 + \cdots + (k)^3 =$

$\Rightarrow (-5)^3 + \cdots (-1)^3 + \sum_{i=0}^{k} i^3$

$= -(5)^3 + \cdots - (1)^3 + \left(\frac{k(k+1)}{2}\right)^2$

$= -(5^3 + 4^3 + \cdots + 1^3) \ldots$

$$= -\left(5^3 + 4^3 + \sim + 1^3\right) + \text{″}$$

$$= -\sum_{i=1}^{5} i^3 + \text{″}$$

$$= -\left(\frac{5(5+1)}{2}\right)^2 + \text{″}$$

(13) $$\sum_{i=10}^{m} 5^i = \underbrace{5^{10} + 5^{11} + 5^{12} + \sim + 5^m + \underbrace{5^0 + \sim + 5^9}}\ \underbrace{- 5^0 - 5^1 - \sim - 5^9}$$

$$= \sum_{i=0}^{m} 5^i - \underbrace{\left(5^0 + 5^1 + \sim + 5^9\right)}_{\sum_{i=0}^{9} 5^i} = \frac{5^{m+1}-1}{5-1} - \frac{5^{10}-1}{5-1}$$

(14) $$\sum_{j=0}^{n} \sum_{i=1}^{j} 6 \Rightarrow \sum_{i=1}^{j} \boxed{6} = \underset{i=1}{\boxed{6}} + \underset{i=2}{\boxed{6}} + \sim + \underset{i=j}{\boxed{6}} = 6j$$

$$\Rightarrow \sum_{j=0}^{n} 6j = 6 \sum_{j=0}^{n} j = \boxed{6\,\frac{n(n+1)}{2}}$$

(15) $$\sum_{k=-5}^{p} \sum_{i=1}^{j} 4 =$$

$$\Rightarrow \sum_{i=1}^{5} 4p = 20p \qquad \sum_{i=a}^{b} 6 = 6(b-a+1)$$

$$\Rightarrow \sum_{i=0}^{5} 4 = 4(6)$$

$$\sum_{i=-3}^{5} 4 = 4\left(5-(-3)+1\right)$$

$$\Rightarrow \sum_{i=1}^{j} 4 = 4j$$

$$\Rightarrow \sum_{k=-5}^{p} 4j = 4j + 4j + \sim + 4j$$

$$= 4j(p+5+1) = \boxed{4j(p+6)}$$

(16) $$\sum_{k=-10}^{200} \left(2k^3 + 8\right) = \sum_{k=-10}^{200} 2k^3 + \sum_{k=-10}^{200} 8$$

$$= 2\sum^{200} k^3 + 8\left(\overset{211}{\cancel{200+10}}+1\right)$$

$$= 2 \sum_{K=-10} K^3 + 8(200+10+1)$$

$$= 2\left((-10)^3 + (-9)^3 + \cdots + 0^3 + 1^3 + \cdots + 200^3\right) + 8(211)$$

$$= 2\left((-10)^3 - 9^3 - \cdots - 1^3 + \sum_{i=0}^{200} i^3\right) + 8(211)$$

$$= 2\left(-\sum_{i=1}^{10} i^3 + \sum_{i=0}^{200} i^3\right) + 8(211)$$

$$= 2\left(-\left(\frac{10(11)}{2}\right)^2 + \left(\frac{200(201)}{2}\right)^2\right) + 8(211)$$