1 Compute the values for

1. $\sum_{i=-1}^{4} 3$

$$\begin{aligned} \sum_{i=-1}^{4} 3 &= 3 \cdot (4+1+1) \\ &= 3 \cdot 6 \\ &= 18 \end{aligned}$$

2. $\sum_{i=1}^{5} (\frac{1}{3})^i$

$$\begin{aligned} \sum_{i=1}^{5} (\frac{1}{3})^i &= \sum_{i=0}^{5} (\frac{1}{3})^i - \sum_{i=0}^{0} (\frac{1}{3})^i \\ &= \frac{(\frac{1}{3})^6 - 1}{\frac{1}{3} - 1} - 1 \end{aligned}$$

3. $\sum_{i=1}^{n} 3$

$$\begin{aligned} \sum_{i=1}^{n} 3 &= 3 \cdot \sum_{i=1}^{n} 1 \\ &= 3 \cdot n \end{aligned}$$

4. $\sum_{i=-3}^{n} 3$

$$\sum_{i=-3}^{n} 3 = 3 \cdot (n+4)$$

5. $\sum_{k=0}^{n} 2^k + \sum_{k=5}^{n} 2^k$

$$\begin{aligned}
\sum_{k=0}^{n} 2^k + \sum_{k=5}^{n} 2^k &= \sum_{k=0}^{n} 2^k + \sum_{k=0}^{n} 2^k - \sum_{k=0}^{4} 2^k \\
&= 2\sum_{k=0}^{n} 2^k - \sum_{k=0}^{4} 2^k \\
&= 2 \cdot \frac{2^{n+1}-1}{2-1} - \frac{2^5-1}{2-1}
\end{aligned}$$

6. $\sum_{i=0}^{n}(\frac{2}{3})^i + \sum_{i=-4}^{n}(\frac{2}{3})^i$

$$\begin{aligned}
\sum_{i=0}^{n}(\frac{2}{3})^i + \sum_{i=-4}^{n}(\frac{2}{3})^i &= \frac{(\frac{2}{3})^{n+1}-1}{\frac{2}{3}-1} + \sum_{i=-4}^{-1}(\frac{2}{3})^i \\
&= \frac{(\frac{2}{3})^{n+1}-1}{\frac{2}{3}-1} + \sum_{i=-4}^{0}(\frac{2}{3})^i - \sum_{i=0}^{0}(\frac{2}{3})^i \\
&= \frac{(\frac{2}{3})^{n+1}-1}{\frac{2}{3}-1} - \sum_{i=0}^{4}(\frac{2}{3})^i - \sum_{i=0}^{0}(\frac{2}{3})^i \\
&= \frac{(\frac{2}{3})^{n+1}-1}{\frac{2}{3}-1} - \frac{(\frac{2}{3})^{5}-1}{\frac{2}{3}-1} - 1
\end{aligned}$$

7. $\sum_{i=1}^{n}(i^3 + 2 \cdot i^2 - i + 1)$

$$\begin{aligned}
\sum_{i=1}^{n}(i^3 + 2 \cdot i^2 - i + 1) &= \sum_{i=1}^{n} i^3 + 2 \cdot \sum_{i=1}^{n} i^2 - \sum_{i=1}^{n} i + \sum_{i=1}^{n} 1 \\
&= (\frac{n \cdot (n+1)}{2})^2 + 2 \cdot \frac{(n) \cdot (n+1) \cdot (2n+1)}{6} - \frac{n \cdot (n+1)}{2} + n
\end{aligned}$$

8. $\sum_{i=5}^{n}(-4 \cdot i + \frac{i}{5})$

$$\begin{aligned}
\sum_{i=5}^{n}(-4\cdot i+\frac{i}{5}) &= -4\sum_{i=5}^{n} i+\frac{1}{5}\sum_{i=5}^{n} i \\
&= -4\sum_{i=0}^{n} i+\frac{1}{5}\sum_{i=0}^{n} i+4\sum_{i=0}^{4} i-\frac{1}{5}\sum_{i=0}^{4} i \\
&= -4\cdot\frac{n(n+1)}{2}+\frac{1}{5}\cdot\frac{n(n+1)}{2}+4\cdot\frac{4\cdot 5}{2}-\frac{1}{5}\cdot\frac{4\cdot 5}{2}
\end{aligned}$$

9. $\sum_{j=0}^{k}\sum_{i=1}^{j}(i-j^2-2)$

$$\begin{aligned}
\sum_{j=0}^{k}\sum_{i=1}^{j}(i-j^2-2) &= \sum_{j=0}^{k}\sum_{i=1}^{j} i-\sum_{j=0}^{k}\sum_{i=1}^{j} j^2-\sum_{j=0}^{k}\sum_{i=1}^{j} 2 \\
&= \sum_{j=0}^{k}\frac{j(j+1)}{2}-\sum_{j=0}^{k} j^3-\sum_{j=0}^{k} 2\cdot j \\
&= \sum_{j=0}^{k}\frac{j^2}{2}+\sum_{j=0}^{k}\frac{j}{2}-(\frac{k\cdot(k+1)}{2})^2-2\cdot\frac{k\cdot(k+1)}{2} \\
&= \frac{1}{2}\cdot\frac{(k)\cdot(k+1)\cdot(2k+1)}{6}+\frac{1}{2}\cdot\frac{k\cdot(k+1)}{2}-(\frac{k\cdot(k+1)}{2})^2-2\cdot\frac{k\cdot(k+1)}{2}
\end{aligned}$$

10. $\sum_{j=1}^{m}\sum_{k=1}^{j}(3\cdot C+k-3\cdot j+i)$

$$
\begin{aligned}
\sum_{j=1}^{m}\sum_{k=1}^{j}(3\cdot C+k-3\cdot j+i) &= \sum_{j=1}^{m}\sum_{k=1}^{j}(3\cdot C+k-3\cdot j+i)\\
&= \sum_{j=1}^{m}\sum_{k=1}^{j}3\cdot C+\sum_{j=1}^{m}\sum_{k=1}^{j}k-3\sum_{j=1}^{m}\sum_{k=1}^{j}j+\sum_{j=1}^{m}\sum_{k=1}^{j}i\\
&= \sum_{j=1}^{m}3\cdot C\cdot j+\sum_{j=1}^{m}\frac{j(j+1)}{2}-3\sum_{j=1}^{m}j^2+\sum_{j=1}^{m}i\cdot j\\
&= 3\cdot C\cdot\frac{m\cdot(m+1)}{2}+\sum_{j=1}^{m}\frac{j^2}{2}+\sum_{j=1}^{m}\frac{j}{2}-3\cdot\frac{(m)(m+1)(2m+1)}{6}\\
&+ i\cdot\frac{(m)(m+1)}{2}\\
&= 3\cdot C\frac{m\cdot(m+1)}{2}+\frac{1}{2}\cdot\frac{m(m+1)(2m+1)}{6}+\frac{1}{2}\cdot\frac{(m)(m+1)}{2}\\
&- 3\cdot\frac{(m)(m+1)(2m+1)}{6}+i\cdot\frac{(m)(m+1)}{2}
\end{aligned}
$$

11. $\sum_{l=-4}^{n}\sum_{j=1}^{k}\sum_{i=1}^{j}(i-4)$

$$
\begin{aligned}
\sum_{l=-4}^{n}\sum_{j=1}^{k}\sum_{i=1}^{j}(i-4) &= \sum_{l=-4}^{n}\sum_{j=1}^{k}\sum_{i=1}^{j}i-4\sum_{l=-4}^{n}\sum_{j=1}^{k}\sum_{i=1}^{j}1\\
&= \sum_{l=-4}^{n}\sum_{j=1}^{k}\frac{j(j+1)}{2}-\sum_{l=-4}^{n}\sum_{j=1}^{k}j\\
&= \sum_{l=-4}^{n}\sum_{j=1}^{k}\frac{j^2}{2}+\sum_{l=-4}^{n}\sum_{j=1}^{k}\frac{j}{2}-\sum_{l=-4}^{n}\frac{k(k+1)}{2}\\
&= \sum_{l=-4}^{n}\frac{(k)(k+1)(2k+1)}{12}+\sum_{l=-4}^{n}\frac{(k)(k+1)}{4}-\sum_{l=-4}^{n}\frac{k(k+1)}{2}\\
&= (n+5)\frac{(k)(k+1)(2k+1)}{12}+(n+5)\frac{(k)(k+1)}{4}-(n+5)\frac{(k)(k+1)}{2}
\end{aligned}
$$

2. Calculate the answer

1. $log_4 x = 5 \rightarrow x = ?$

$$
\begin{aligned}
log_4 x &= 5\\
x &= 4^5
\end{aligned}
$$

2. $log_3 y = 4 \rightarrow y =?$

$$\begin{aligned} log_3 y &= 4 \\ y &= 3^4 \end{aligned}$$

3. $x = 7^2 \rightarrow log_7 x =?$

$$\begin{aligned} x &= 7^2 \\ log_7 x &= log_7 7^2 \\ log_7 x &= 2 \cdot log_7 7 \\ log_7 x &= 2 \end{aligned}$$

4. $x = 32 \rightarrow log_2 x =?$

$$\begin{aligned} x &= 32 \\ log_2 x &= log_2 32 \\ log_2 x &= 5 \end{aligned}$$

5. $2^{log 5} + 4^{log 6} - 27^{log_3 5}$

$$\begin{aligned} 2^{log_2 5} + 4^{log_2 6} - 27^{log_3 5} &= 5 + 2^{2^{log_2 6}} - 3^{3^{log_3 5}} \\ &= 5 + 2^{log_2 6^2} - 3^{log_3 5^3} \\ &= 5 + 6^2 - 5^3 \\ &= 5 + 36 - 125 \\ &= -84 \end{aligned}$$

6. $9^{log_3 2} - 25^{log_5 4} - 36^{log_6 7} + 8^{log_8 6}$

$$\begin{aligned} 9^{log_3 2} - 25^{log_5 4} - 36^{log_6 7} + 8^{log_8 6} &= 3^{3^{log_3 2}} - 5^{2^{log_2 4}} - 6^{2^{log_6 7}} + 6 \\ &= 3^{log_3 8} - 5^{log_2 16} - 6^{log_6 49} + 6 \\ &= 8 - 5^4 - 49 + 6 \\ &= 660 \end{aligned}$$

7. $log(4^5 \times 8^3) - log(16 - 8) + log(\frac{2^{10}}{4 \times 3^2})$

$$\begin{aligned} log_2(4^5 \times 8^3) - log_2(16-8) + log_2(\frac{2^{10}}{4 \times 3^2}) &= log_2(4^5) + log_2(8^3) - log_2(8) + log_2(2^{10}) - log_2(4 \times 3^2) \\ &= 5 \cdot log_2(4) + 3 \cdot log_2(8) - log_2(8) + 10 \cdot log_2(2) - log_2(4) \\ &= 5 \cdot 2 + 3 \cdot 3 - 3 + 10 - 2 - 2 \cdot 1.5 \\ &= 21 \end{aligned}$$

8. $log(3^2 \times 64^3) - log(\frac{2^{10} \times 128^3}{9 \times 8^2})$

$$\begin{aligned} log_2(3^2 \times 64^3) - log_2(\frac{2^{10} \times 128^3}{9 \times 8^2}) &= log_2(3^2) + log_2(64^3) - log_2(2^{10} \times 128^3) + log_2(9) + log_2 8^2 \\ &= 2 \cdot log_2(3) + 3 \cdot log_2(64) - 10 \cdot log_2 2 - 3 \cdot log_2 128 + 2 \cdot log_2 3 + 2 \cdot \\ &= 2 \cdot 1.5 + 3 \cdot 6 - 10 - 3 \cdot 7 + 2 \cdot 1.5 + 2 \cdot 3 \\ &= -1 \end{aligned}$$

9. $loglog16$

$$\begin{aligned} log_2 log_2 16 &= log_2 4 \\ &= 2 \end{aligned}$$

10. $log16 \times log16$

$$\begin{aligned} log_2 16 \times log_2 16 &= 4 \times 4 \\ &= 16 \end{aligned}$$

11. $log^2 16$

$$\begin{aligned} log_2^2 16 &= 4^2 \\ &= 16 \end{aligned}$$

12. $log_2 log_5 625 - log_3 log_4 2^{3^9} + log^4 2^5 - \frac{log^2(4^3 \times 3^5)}{log_5 125}$

$$\begin{aligned} log_2 log_5 625 - log_3 log_4 2^{3^9} + log_2^4 2^5 - \frac{log^2(4^3 \times 3^5)}{log_5 125} &= log_2 4 - log_3(27 \cdot log_4 2) + log_2^4 2^5 - \frac{log_2^2(4^3 \times 3^5)}{log_5 125} \\ &= 2 - log_3(\frac{27}{2}) + 5^4 - \frac{log_2^2(4^3) + log_2(3^5)}{2} \\ &= 2 - log_3(27) + log_3(2) + 5^4 - \frac{(2 \cdot 3)^2 + 5 \cdot log_2 3}{2} \\ &= 2 - 3 + 1.5 + 625 - \frac{36 + 5 \cdot 1.5}{2} \\ &= 603.75 \end{aligned}$$

13. $loglog_8 log256 + log^5(3^2) \times 4^{log7}$

$$\begin{aligned}
log_2 log_8 log_2 256 + log_2^5(3^2) \cdot 4^{log_2 7} &= log_2 log_8 8 + log_2^5(3^2) \cdot 4^{log_2 7} \\
&= log_2 1 + log_2^5(3^2) \cdot 4^{log_2 7} \\
&= log_2^5(3^2) \cdot 4^{log_2 7} \\
&= (2 \cdot 1.5)^5 \cdot 2^{log_2 49} \\
&= (2 \cdot 1.5)^5 \cdot 49 \\
&= 11907
\end{aligned}$$

14. $log_6 x = 5 \rightarrow log_x 6 =?$

$$\begin{aligned}
log_6 x &= 5 \\
x &= 6^5 \\
log_x x &= 5 \cdot log_x 6 \\
1 &= 5 \cdot log_x 6 \\
\frac{1}{5} &= log_x 6
\end{aligned}$$

15. $log_y x = 10 \rightarrow log_x y =?$

$$\begin{aligned}
log_y x &= 10 \\
x &= y^{10} \\
log_x x &= 10 \cdot log_x y \\
\frac{1}{10} &= log_x y
\end{aligned}$$

16. $log_4 32 - log_8^2 4$

$$\begin{aligned}
log_4 32 - log_8^2 4 &= log_4 2 \cdot 16 - log_8^2 4 \\
&= log_4 2 + log_4 16 - log_8^2 4 \\
&= \frac{1}{2} + 2 - (\frac{1}{2})^2 \\
&= \frac{1}{2} + 2 - \frac{1}{4} \\
&= 2.25
\end{aligned}$$

17. $log_4 8 + log_9 27 - log_{25}^2 125 - log_8^3 16 + log_4 log_2 256$

$$\begin{aligned} log_4 8 + log_9 27 - log_{25}^2 125 - log_8^3 16 + log_4 log_2 256 &= log_4 4 \cdot 2 + log_9 9 \cdot 3 - log_{25}^2 25 \cdot 5 - log_8^3 8 \cdot 2 + log_4 l \\ &= log_4 4 + log_4 2 + log_9 9 + log_9 3 - log_{25}^2 25 - log_{25}^2 5 \\ &- log_8^3 8 + log_8^3 2 + log_4 log_2 256 \\ &= 1 + \frac{1}{2} + 1 + \frac{1}{2} - 1 - \frac{1}{4} - 1 + \frac{1}{27} + 2 \\ &= \frac{301}{108} \end{aligned}$$

3. Compute the deriative of

1. $-5 \cdot x^3 + 2 \cdot x - 1$

$$\frac{d}{dx}(-5 \cdot x^3 + 2 \cdot x - 1) = -15 \cdot x^2 + 2$$

2. $3 \cdot x^4 - 2\sqrt{x} + x^{\frac{1}{2}} - 6x^{-\frac{2}{3}} - 5$

$$\begin{aligned} \frac{d}{dx}(3 \cdot x^4 - 2\sqrt{x} + x^{\frac{1}{2}} - 6x^{-\frac{2}{3}} - 5) &= 12 \cdot x^3 - x^{-\frac{1}{2}} + \frac{1}{2} \cdot x^{-\frac{1}{2}} + 4 \cdot x^{-\frac{5}{3}} \\ &= 12 \cdot x^3 - \frac{1}{2} x^{-\frac{1}{2}} + 4 \cdot x^{-\frac{5}{3}} \end{aligned}$$

3. $x \cdot \sqrt{x} + \sqrt{\sqrt{x}}$

$$\frac{d}{dx}(x^{\frac{3}{2}} + x^{\frac{1}{4}}) = \frac{3}{2} x^{\frac{1}{2}} + \frac{1}{4} \cdot x^{-\frac{3}{4}}$$

4. $log x - x^2 ln x + ln x^4$

$$\frac{d}{dx}(log_2 x - x^2 \cdot ln x + ln x^4) = \frac{1}{x \cdot ln 2} - 2 \cdot x \cdot ln x - x + \frac{4}{x}$$

5. $ln^3(x\sqrt{2x-3}) + \sqrt{ln x^2}$

$$\begin{aligned} \frac{d}{dx}(ln^3(x \cdot \sqrt{2x-3}) + \sqrt{ln x^2}) &= \frac{3 \cdot (ln(x\sqrt{2x-3}))^2 \cdot (\sqrt{2x-3} + x \cdot (2 \cdot x - 3)^{-\frac{1}{2}}}{x\sqrt{2x-3}} \\ &+ \frac{1}{x \cdot \sqrt{ln(x^2)}} \end{aligned}$$

6. $\frac{\sqrt[4]{x+5}-lnx}{(x-1)^3}$

$$\frac{d}{dx}(\frac{(x+5)^{\frac{1}{4}}-lnx}{(x-1)^3}) = \frac{(x-1)^3\cdot(\frac{1}{4}\cdot(x+5)^{-\frac{3}{4}}-\frac{1}{x})-3\cdot((x+5)^{\frac{1}{4}}-lnx)\cdot(x-1)^2}{(x-1)^6}$$

4. Determine the limit of

1. $lim_{x\to\infty}\frac{3x+2}{-5x-6}$

$$lim_{x\to\infty}\frac{3x+2}{-5x-6} = -\frac{3}{5}$$

2. $lim_{x\to\infty}(\frac{1}{x}+3)$

$$\begin{aligned} lim_{x\to\infty}(\frac{1}{x}+3) &= lim_{x\to\infty}\frac{1}{x}+lim_{x\to\infty}3 \\ &= lim_{x\to\infty}3 \\ &= 3 \end{aligned}$$

3. $lim_{x\to\infty}\frac{x^3+x-\sqrt{3x}}{\sqrt{x}}$

$$\begin{aligned} lim_{x\to\infty}\frac{x^3+x-\sqrt{3x}}{\sqrt{x}} &= lim_{x\to\infty}\frac{x^3}{\sqrt{x}}+lim_{x\to\infty}\frac{x}{\sqrt{x}}-lim_{x\to\infty}\frac{\sqrt{3x}}{\sqrt{x}} \\ &= \infty \end{aligned}$$

4. $lim_{x\to\infty}\frac{x^3+x-\sqrt{3x}}{5\cdot x^{2.25}\cdot\sqrt{\sqrt{x}}}$

$$\begin{aligned} lim_{x\to\infty}\frac{x^3+x-\sqrt{3x}}{5\cdot x^{2.25}x^{.25}} &= lim_{x\to\infty}\frac{x^3+x-\sqrt{x}}{5\cdot x^3} \\ &= lim_{x\to\infty}\frac{x^3}{5\cdot x^3}+lim_{x\to\infty}\frac{x}{5\cdot x^3}-lim_{x\to\infty}\frac{\sqrt{x}}{5\cdot x^3} \\ &= \frac{1}{5} \end{aligned}$$

5. $lim_{x\to\infty}\frac{x^{0.1}-\sqrt{3}}{\sqrt{\sqrt{x}}}$

$$\begin{aligned} lim_{x\to\infty}\frac{x^{0.1}-\sqrt{3}}{x^{0.25}} &= lim_{x\to\infty}\frac{x^{0.1}}{x^{0.25}}-\frac{\sqrt{3}}{x^{0.25}} \\ &= lim_{x\to\infty}\frac{1}{x^{0.15}}-\frac{\sqrt{3}}{x^{0.25}} \\ &= 0 \end{aligned}$$

6. $lim_{x\to\infty}\frac{x^x}{2^x}$

$$\begin{aligned} ln(lim_{x\to\infty}\frac{x^x}{2^x}) &= lim_{x\to\infty}ln(\frac{x^x}{2^x}) \\ &= lim_{x\to\infty}ln(x^x) - lim_{x\to\infty}ln(2^x) \\ &= lim_{x\to\infty}xln(x) - lim_{x\to\infty}xln(2) \\ &= \infty \\ e^{ln(lim_{x\to\infty}\frac{x^x}{2^x})} &= e^{\infty} \\ &= \infty \end{aligned}$$

7. $lim_{x\to\infty}\frac{x^x}{x(2^x)}$

$$\begin{aligned} ln(lim_{x\to\infty}\frac{x^x}{x(2^x)}) &= lim_{x\to\infty}ln(\frac{x^x}{x\cdot 2^x}) \\ &= lim_{x\to\infty}ln(x^x) - lim_{x\to\infty}ln(x\cdot 2^x) \\ &= lim_{x\to\infty}xln(x) - lim_{x\to\infty}ln(x) - lim_{x\to\infty}ln(2^x) \\ &= lim_{x\to\infty}xln(x) - lim_{x\to\infty}ln(x) - lim_{x\to\infty}xln(2) \\ &= \infty \\ e^{ln(lim_{x\to\infty}\frac{x^x}{x(2^x)}} &= e^{\infty} \\ &= \infty \end{aligned}$$

8. $lim_{x\to\infty}\frac{\sqrt{2}^{log^4 x^3}}{log(2\cdot x+7)}$

9. $lim_{x\to\infty}\frac{x+1}{\frac{3\cdot x^{lnx}}{2x^2}}$

10. $lim_{x\to\infty}\frac{\sqrt{2}^{logx^3}}{log^{lnx}(2x)}$

5. Compute the exact values for

1. $\int_1^n(2\cdot x^4+5\sqrt{x})\cdot dx$

$$\begin{aligned} \int_1^n(2\cdot x^4+5\sqrt{x} \quad)\cdot dx = &\quad 2\int_1^n x^4dx + 5\int_1^n \sqrt{x}dx \\ = &\quad 2\cdot(\frac{x^5}{5}|_1^n) + 5(\frac{2}{3}x^{\frac{3}{2}}|_1^n) \\ = &\quad 2\cdot(\frac{n^5}{5}-\frac{1}{5}) + 5(\frac{2}{3}n^{\frac{3}{2}}-\frac{2}{3}) \\ = &\quad \frac{2\cdot n^5}{5} - \frac{2}{5} + \frac{10}{3}n^{\frac{3}{2}} - \frac{10}{3} \end{aligned}$$

2. $\int_1^n (x^4 - 3\cdot x^2 + \frac{1}{x} - \frac{1}{x^2})dx$

$$\begin{aligned}
\int_1^n (x^4 - 3\cdot x^2 + \frac{1}{x} - \frac{1}{x^2}dx &= \int_1^n x^4 dx - 3\int_1^n x^2 dx + \int_1^n \frac{1}{x}dx - \int_1^n \frac{1}{x^2}dx \\
&= \frac{x^5}{5}|_1^n - 3\cdot\frac{x^3}{3}|_1^n + lnx|_1^n + \frac{1}{x}|_1^n \\
&= (\frac{n^5}{5} - \frac{1}{5}) - n^3 + 1 + lnn + \frac{1}{n} - 1 \\
&= \frac{n^5}{5} - n^3 + lnn + \frac{1}{n} - \frac{1}{5}
\end{aligned}$$

3. $\int_1^n (\frac{3}{\sqrt{x}} + lnx + e^x)dx$

$$\begin{aligned}
\int_1^n \frac{3}{\sqrt{x}} + lnx + e^x dx &= \int_1^n \frac{3}{\sqrt{x}}dx + \int_1^n lnxdx + \int_1^n e^x dx \\
&= 6\sqrt{x}|_1^n + (xlnx - x|_1^n) + e^n - e \\
&= 6\sqrt{n} - 6 + (nlnn - n + 1) + e^n - e \\
&= 6\sqrt{n} + nlnn - n - 5 + e^n - e
\end{aligned}$$

4. $\int_1^n x\cdot sinxdx$

$$\begin{aligned}
\int_1^n x\cdot sinxdx &= -xcosx|_1^n + \int_1^n cosxdx \\
&= -ncosn + cos(1) + sin(n) - sin(1)
\end{aligned}$$

6. Use mathematical induction to prove that

$$1 + 2 + \ldots + n = \frac{n(n+1)}{2}$$

Proof. $1 + 2 + \ldots + n = \frac{n(n+1)}{2}$

Base case $n = 1$: If $n = 1$ then the left hand side and the right hand size is $1 = 1 = \frac{1(2)}{2}$.

Inductive hypothesis: Suppose the theorem holds for all values of n up to some $k, k \geq 1$.

Inductive step: let $n = k + 1$ then our left hand side is

$$
\begin{aligned}
\sum_{i=1}^{k+1} i &= \sum_{i=1}^{k} i + (k+1) \\
&= \frac{k(k+1)}{2} + (k+1) \\
&= \frac{k(k+1) + 2 \cdot k + 2}{2} \\
&= \frac{(k+1) \cdot (k+2)}{2}
\end{aligned}
$$

which is equal to our right hand side. By the principle of mathematical induction, the theorem holds for all integers $n \geq 1$. $\square$

7. Use mathematical induction to prove that

$$1 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$1 + 2 + \ldots + n = \frac{n(n+1)}{2}$

Proof. Base case $n = 1$: If $n = 1$ then the left hand side and the right hand size is $1^2 = 1 = \frac{1(2)(3)}{6}$.

Inductive hypothesis: Suppose the theorem holds for all values of n up to some $k, k \geq 1$.

Inductive step: let $n = k + 1$ then our left hand side is

$$
\begin{aligned}
\sum_{i=1}^{k+1} i^2 &= \sum_{i=1}^{k} i^2 + (k+1)^2 \\
&= \frac{k \cdot (k+1) \cdot (2 \cdot k + 1)}{6} + (k+1)^2 \\
&= \frac{k \cdot (k+1) \cdot (2 \cdot k + 1) + 6 \cdot (k+1)^2}{6} \\
&= \frac{(6 \cdot (k+1) + k \cdot (2 \cdot k + 1)) \cdot (k+1)}{6} \\
&= \frac{(6 \cdot k + 6 + 2 \cdot k^2 + k) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k^2 + 7 \cdot k + 6) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k^2 + 4 \cdot k + 3 \cdot k + 6) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k \cdot (k+2) + 3 \cdot (k+2)) \cdot (k+1)}{6} \\
&= \frac{(2 \cdot k + 3) \cdot (k+2) \cdot (k+1)}{6}
\end{aligned}
$$

which is equal to our right hand side. By the principle of mathematical induction, the theorem holds for all integers $n \geq 1$. $\square$