

1 Use Θ notation to express the statement

$$4n^6 \leq 17n^6 - 45n^3 + 2n + 8 \leq 30n^6, n \geq 3$$

Let $A = 4$, $B = 30$ and $k = 3$ then the statement translates to

$$An^6 \leq 17n^6 - 45n^3 + 2n + 8 \leq Bn^6, n \geq k$$

hence by the definition of Θ notation $17n^6 - 45n^3 + 2n + 8$ is $\Theta(n^6)$.

2 Use Ω notation to express the statement

1. Use Ω notation to express the statement

$$\frac{11}{4}n^2 \leq 3 \cdot \left(\left\lfloor \frac{n}{4} \right\rfloor\right)^2 + 5n^2, n \geq 2$$

Let $A = \frac{11}{4}$ and $k = 2$ then $An^2 \leq 3 \cdot \left(\left\lfloor \frac{n}{4} \right\rfloor\right)^2 + 5n^2, n \geq 2$ then the statement translates to

$$An^2 \leq 3 \cdot \left(\left\lfloor \frac{n}{4} \right\rfloor\right)^2 + 5n^2, n \geq k$$

which by the definition of Ω notation, $3 \cdot \left(\left\lfloor \frac{n}{4} \right\rfloor\right)^2 + 5n^2$ is $\Omega(n^2)$.

2. Use O notation to express the statement

$$0 \leq 3 \cdot \left(\left\lfloor \frac{n}{4} \right\rfloor\right)^2 + 5n^2 \leq 6n^2, n \geq 1$$

Let $A = 6$ and $k = 1$ then the statement translates to

$$0 \leq 3 \cdot \left(\left\lfloor \frac{n}{4} \right\rfloor\right)^2 + 5n^2 \leq An^2, n \geq k$$

which by the definition of O notation, $3 \cdot \left(\left\lfloor \frac{n}{4} \right\rfloor\right)^2 + 5n^2$ is $O(n^2)$.

3. Justify the statement: $3 \cdot \left(\left\lfloor \frac{n}{4} \right\rfloor\right)^2 + 5n^2$ is $\Theta(n^2)$.

Let $A = \frac{11}{4}$, $B = 6$ and $k = 2$ then $A \cdot n^2 \leq 3 \cdot \left(\left\lfloor \frac{n}{4} \right\rfloor\right)^2 + 5n^2 \leq Bn^2, n \geq k$
which by the definition of Θ notation, $3 \cdot \left(\left\lfloor \frac{n}{4} \right\rfloor\right)^2 + 5n^2$ is $\Theta(n^2)$.

3. Given the function $15n^3 + 11n^2 + 9$

1. Show that the function is $\Omega(n^3)$.

$$15n^3 \leq 15n^3 + 11n^2 + 9, n \geq 1$$

Let $A = 15$ and $k = 1$ then the statements translates to $An^3 \leq 15n^3 + 11n^2 + 9, n \geq k$ which by the definition of Ω notation, $15n^3 + 11n^2 + 9$ is $\Omega(n^3)$.

2. Show that the function is $O(n^3)$.