1. Compare the growth of $n^{log_2^4(n)}$

$$\begin{aligned}
log_2(lim_{n\to\infty}\frac{n^{log_2^4(n)}}{2^n}) &= lim_{n\to\infty}log_2^4(n)\cdot log_2(n) - n \\
&= lim_{n\to\infty}log_2^5(n) - n \\
&= -\infty \\
2^{-\infty} &= 0
\end{aligned}$$

$$\begin{aligned}
log_2(lim_{n\to\infty}\frac{n^{log_2(n)}}{n^k}) &= lim_{n\to\infty}log_2^4(n)\cdot log_2(n) - k\cdot log_2(n) \\
&= lim_{n\to\infty}log_2^5(n) - k\cdot log_2(n) \\
&= \infty \\
2^{\infty} &= \infty
\end{aligned}$$

We conclude $n^{log_2^4(n)} = \omega(n^k)$.

2. Compare the growth of $n^{log_2(n!)}$

$$\begin{aligned}
log_2(lim_{n\to\infty}\frac{n^{log_2(n!)}}{2^n}) &= lim_{n\to\infty}log_2(n!)\cdot log_2(n) - n \\
&= lim_{n\to\infty}\Theta(n\cdot log_2^2(n)) - n \\
&= \infty \\
2^{\infty} &= \infty
\end{aligned}$$

We conclude $n^{log_2(n!)} = \omega(2^n)$

3. Compare the growth of $\Theta(n\cdot(2^{log_2(\sqrt{n})}))$

$$\begin{aligned}
n\cdot 2^{log_2(\sqrt{n})} &= n\cdot\sqrt{n} \\
&= \Theta(n\cdot\sqrt{n})
\end{aligned}$$

4. Compare the growth of $\Theta(log_2(n)^{log_2(n)})$

$$\begin{aligned}
log_2(lim_{n\to\infty}\frac{log_2(n)^{log_2(n)}}{2^n}) &= lim_{n\to\infty}log_2(n)\cdot log_2(log_2(n)) - n \\
&= -\infty \\
2^{-\infty} &= 0
\end{aligned}$$

$$
\begin{aligned}
log_2(lim_{n\to\infty}\frac{log_2(n)^{log_2(n)}}{n^k}) &= lim_{n\to\infty}log_2(n)\cdot log_2(log_2(n)) - k\cdot log_2(n) \\
&= \infty \\
2^{\infty} &= \infty
\end{aligned}
$$

Hence we conclude $log_2(n)^{log_2(n)} = \omega(n^k)$.

5. Compare the growth of $16^{log_2(n)}$

$$
\begin{aligned}
16^{log_2(n)} &= 2^{4\cdot log_2(n)} \\
&= 2^{log_2(n^4)} \\
&= n^4 \\
&= \Theta(n^4)
\end{aligned}
$$

6. Compare the growth of $log_2((n^2)!)$

$$
\begin{aligned}
\sum_{i=1}^{n} log_2(i) &= \Theta(a\cdot log_2(a)) \\
&= \Theta(n^2\cdot log_2(n))
\end{aligned}
$$

7. Compare the growth of $(log_2^3(n))!$

$$
\begin{aligned}
lim_{n\to\infty}\frac{log_2^3(n)!}{2^n} &= lim_{n\to\infty}\Theta(log_2^3(n)\cdot log_2(log_2(n))) - n \\
&= -\infty \\
2^{-\infty} &= 0
\end{aligned}
$$

8. Compare the growth of $log_2(n^{log_2(n!)})$

$$
\begin{aligned}
log_2(n^{log_2(n!)}) &= log_2(n!)\cdot log_2(n) \\
&= \Theta(n\cdot log_2^2(n))
\end{aligned}
$$