**1.** $\int 6dx$

$$\int 6dx = 6x + c$$

**2.** $\int (7x+2)dx$

$$\begin{aligned}\int (7\cdot x+2)dx &= \int 7\cdot xdx + \int 2\cdot dx\\ &= \frac{7\cdot x^2}{2} + 2\cdot x\end{aligned}$$

**3.** $\int (\frac{5}{x} + x^{\frac{1}{2}} + 8\cdot x^2 + sin(x))dx$

$$\int \frac{5}{x}dx + \int x^{\frac{1}{2}}dx + \int 8\cdot x^2dx + \int sin(x)dx = 5\cdot ln(x) + \frac{2}{3}\cdot x^{\frac{3}{2}} + \frac{8}{3}\cdot x^3 - cos(x)$$

**4.** $\int (x\cdot\sqrt{x} - \frac{1}{x\cdot ln(2)} + e^x)dx$

$$\int x^{\frac{3}{2}} - \frac{1}{x\cdot ln(2)} + e^x dx = \frac{2}{5}\cdot x^{\frac{5}{2}} - \frac{ln(x)}{ln(2)} + e^x$$

**5.** $\int x\cdot e^x dx$

$$\int x\cdot e^x dx = x\cdot e^x - e^x$$

**6.** $\int x\cdot ln(x)dx$

$$\begin{aligned}\int x\cdot ln(x)dx &= \frac{x^2}{2}\cdot ln(x) - \frac{1}{2}\cdot\int xdx\\ &= \frac{x^2}{2}\cdot ln(x) - \frac{1}{4}\cdot x^2\end{aligned}$$

**7.** $\int ln(x)dx$

$$\int ln(x)dx \quad = \quad x \cdot ln(x) - x$$

**8.** Prove or disprove $f(n) = O(g(n))$

$$\begin{aligned} f(n) &= 2 \cdot n \cdot log_2(n) + 6n - 10 \\ g(n) &= n^2 \end{aligned}$$

$$\begin{aligned} f(n) = O(g(n)) \quad &\iff \quad \exists c > 0, \exists k \geq 0 \, s.t \, f(n) \leq c \cdot g(n) \, \forall n \geq k \\ 2 \cdot n \cdot log_2(n) + 6n - 10 \quad &\leq \quad 2 \cdot n^2 + 6n - 10 \\ &\leq \quad 2 \cdot n^2 + 6 \cdot n^2 - 10 \\ &\leq \quad 2 \cdot n^2 + 6 \cdot n^2 + 10 \cdot n^2 \\ &= \quad 18 \cdot n^2, n \geq 1 \end{aligned}$$

Thus by the definition of $O$ notation $f(n) = O(g(n))$

**9.** Prove or disprove $f(n) = \Omega(g(n))$

$$\begin{aligned} f(n) &= 2 \cdot n \cdot log_2(n) + 6 \cdot n - 10 \\ g(n) &= n^2 \end{aligned}$$

$$f(n) = \Omega(g(n)) \quad \iff \quad \exists c > 0, \exists k \geq 0 \, s.t \, f(n) \geq c \cdot g(n) \, \forall n \geq k$$

$$\begin{aligned} 2 \cdot n \cdot log_2(n) + 6 \cdot n - 10 \quad &\geq \quad c \cdot n^2 \\ \frac{2 \cdot n \cdot log_2(n)}{n^2} + \frac{6 \cdot n}{n^2} - \frac{10}{n^2} \quad &\geq \quad c \\ lim_{n \to \infty} \cancelto{0}{\frac{2 \cdot n \cdot log_2(n)}{n^2}} + lim_{n \to \infty} \cancelto{0}{\frac{6 \cdot n}{n^2}} - lim_{n \to \infty} \cancelto{0}{\frac{10}{n^2}} \quad &\geq \quad c \\ 0 \quad &\geq \quad c \end{aligned}$$

which is a contradiction thus $f(n) \neq \Omega(g(n))$.

**10.** Prove or disprove $f(n) = O(g(n))$

$$\begin{aligned} f(n) &= n^2 \cdot \sqrt{n} + 2 \cdot n + 4 \cdot log_2(n) \\ g(n) &= n^3 \end{aligned}$$

$$f(n) = O(g(n)) \iff \exists c > 0, \exists k \geq 0 \, s.t \, f(n) \leq c \cdot g(n), \forall n \geq k$$

$$\begin{aligned} n^{\frac{5}{2}} + 2 \cdot n + 4 \cdot log_2(n) &\leq n^3 + 2 \cdot n + 4 \cdot log_2(n) \\ &\leq n^3 + 2 \cdot n^3 + 4 \cdot log_2(n) \\ &\leq n^3 + 2 \cdot n^3 + 4 \cdot n^3 \\ &= 7 \cdot n^3, n \geq 1 \end{aligned}$$

Hence by the definition of $O$ notation $f(n) = O(g(n))$