Lecture 5 (Growth of functions)

Tuesday, September 8, 2020 5:00 PM

(Reminder: HW2 is due this Saturday)

Example: prove or disprove: $6n\log n + 2n + \sqrt{n} = \Omega(n\log^2 n)$

$f(n) = \Omega(g(n)) \iff \exists c > 0, \exists k \geq 0$ s.t. $f(n) \geq c\,g(n) \quad \forall n \geq k$

$\Rightarrow \dfrac{6n\log n + 2n + \sqrt{n}}{n\log^2 n} \overset{?}{\geq} \dfrac{c\,\cancel{n\log^2 n}}{\cancel{n\log^2 n}}$

$\Rightarrow \lim_{n\to\infty} \dfrac{6n\log n + 2n + \sqrt{n}}{n\log^2 n} \geq c \quad \Rightarrow 0 \geq c \quad \therefore X$

Example: $f(n) = 4n^3 + n\log n \qquad g(n) = n^2\log n$

$f(n) \overset{?}{=} O(g(n)) \iff \exists c > 0, \exists k \geq 0$ s.t. $f(n) \leq c\,g(n) \quad \forall n \geq k$

$\Rightarrow 4n^3 + n\log n \overset{?}{\leq} c\,n^2\log n \quad \Rightarrow \dfrac{4n^3 + n\log n}{n^2\log n} \leq c \Rightarrow \infty \leq c \quad \therefore X$

Little_Oh: $f(n) = o(g(n)) \iff \lim_{n\to\infty} \dfrac{f(n)}{g(n)} = 0$

Little_Omega: $f(n) = \omega(g(n)) \iff \lim_{n\to\infty} \dfrac{f(n)}{g(n)} = \infty$

Example: Compare the growth of $f(n)$ and $g(n)$.

① $f(n) = 8n\log^2 n + \sqrt{n} + 2$
$g(n) = n^2 + n\log^5 n$

$\Rightarrow \lim_{n\to\infty} \dfrac{f(n)}{g(n)} = \lim_{n\to\infty} \dfrac{8n\log^2 n + \sqrt{n} + 2}{n^2 + n\log^5 n} = 0$

$\Rightarrow \boxed{f(n) = o(g(n))}$

(2) $f(n) = \sqrt{n} + \log^{10} n + \sqrt{n}\,\log n$

$g(n) = \sqrt{\sqrt{n}} + \log\log n + 6$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{\sqrt{n}\,\log n}{\sqrt{\sqrt{n}}} = \lim_{n\to\infty} \frac{\sqrt{\sqrt{n}}\,\log n}{1} = \infty \Leftrightarrow \boxed{f(n) = \omega(g(n))}$$

$f(n) = ?\,(g(n))$

$f(n)$	$O\ \Omega\ \Theta$ / $o\ \omega$	$g(n)$	
$n^2\log n$ $5n^2\log n + \underbrace{2^{\log n}}_{n} + \sqrt{n}$	$O\ \Omega$ Θ	$6n^2\log n^3 + n^2$	$18n^2\log n$
$\underbrace{2^{\log n^5}}_{n^5} + \underbrace{2^{\log\log^2 n}}_{\log^2 n} + \frac{4^{\log n^3}}{n^6}$	O o	$n^5\log n + n^{6.1}$	$\lim_{n\to\infty} \frac{n^6}{n^{6.1}} = 0$
$\log\log n + \log^2 n + \log n^{10}$	O o	$\underbrace{\log\log^{10} n}_{10\log\log n} + \sqrt{\log n} + \frac{2^{\log\log^3 n}}{\log^3 n}$ $\lim \frac{f(n)}{g(n)} = 0$	
$n\log n + n^{1.01} + \sqrt{n}$	ω Ω	$\underbrace{\sqrt{n}\,\log^3 n}_{\cancel{n^{.5}}\log^3 n} + \log^{10} n^5 + n^{.56}$	$\cancel{n^{.5}}\, n^{.06}$ ✓

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n^{1.01}}{n^{.56}} = \infty$$

$\Rightarrow f(n) \geq c\,g(n)$ ✓

$f(n) = o(g(n)) \Rightarrow f(n) = O(g(n))$

$f(n) = \omega(g(n)) \Rightarrow f(n) = \Omega(g(n))$

$f(n) = o(g(n)) \Rightarrow f(n) = ?\ (g(n))$

Θ or o

$f(n) = \Omega(g(n)) \Rightarrow f(n) = ?\ g(n))$

IDK because Θ or ω

① $\lim \frac{f}{g} = 0 \Rightarrow f(n) = \underset{o}{O}(g(n))$

② $\lim \frac{f}{g} = \infty \Rightarrow f(n) = \underset{\omega}{\Omega}(g(n))$

③ $\lim \frac{f}{g} =$ non zero nr $\Rightarrow f(n) = \Theta(g(n))$
e.g. 10