

1. Compute  $\sum_{j=5}^{200} 6$

$$\begin{aligned}\sum_{j=5}^{200} 6 &= 6 \cdot (200 - 5 + 1) \\ &= 6 \cdot (196)\end{aligned}$$

2. Compute  $\sum_{p=-20}^{100} k$

$$\begin{aligned}\sum_{p=-20}^{100} k &= k \cdot (100 + 20 + 1) \\ &= k \cdot (121)\end{aligned}$$

3. Compute  $\sum_{k=-5}^{100} 8^k$

$$\begin{aligned}\sum_{k=-5}^{100} 8^k &= \sum_{k=-5}^{-1} 8^k + \sum_{k=0}^{100} 8^k \\ &= \sum_{k=1}^5 \left(\frac{1}{8}\right)^k + \frac{8^{101} - 1}{8 - 1} \\ &= \frac{\left(\frac{1}{8}\right)^k - 1}{\left(\frac{1}{8}\right) - 1} - 1 + \frac{8^{101} - 1}{8 - 1}\end{aligned}$$

4. Compute  $\sum_{j=1}^n \sum_{i=1}^j (2 \cdot i + 3)$

$$\begin{aligned}\sum_{j=1}^n \sum_{i=1}^j (2 \cdot i + 3) &= \sum_{j=1}^n \sum_{i=1}^j 2 \cdot i + \sum_{j=1}^n \sum_{i=1}^j 3 \\ &= \sum_{j=1}^n j^2 + \sum_{j=1}^n j + \sum_{j=1}^n 3 \cdot j \\ &= \frac{(n)(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} + 3 \cdot \frac{n(n+1)}{2}\end{aligned}$$

5. Show  $\log_b(x^a) = a \log_b(x)$

$$\begin{aligned}\log_b x &= c \\ b^c &= x \\ b^{c \cdot a} &= x^a \\ c \cdot a &= \log_b(x^a) \\ c \cdot \log_b(x) &= \log_b(x^a)\end{aligned}$$

6. Show  $\log_b(x \cdot y) = \log_b(x) + \log_b(y)$

$$\begin{aligned}
 \log_b(x) &= p \\
 \log_b(y) &= s \\
 x &= b^p \\
 y &= b^s \\
 x \cdot y &= b^{p \cdot s} \\
 x \cdot y &= b^{p+s} \\
 \log_b(x \cdot y) &= p + s \\
 \log_b(x \cdot y) &= \log_b(x) + \log_b(y)
 \end{aligned}$$

7. Show  $\log_b(\frac{x}{y}) = \log_b(x) - \log_b(y)$

$$\begin{aligned}
 \log_b(x) &= c \\
 \log_b(y) &= k \\
 x &= b^c \\
 y &= b^k \\
 \frac{x}{y} &= \frac{b^c}{b^k} \\
 \frac{x}{y} &= b^{c-k} \\
 \log_b(\frac{x}{y}) &= c - k \\
 \log_b(\frac{x}{y}) &= \log_b(x) - \log_b(y)
 \end{aligned}$$

8. Show  $b^{\log_b x} = x$

$$\begin{aligned}
 \log_b x &= c \\
 x &= b^c \\
 x &= b^{\log_b x}
 \end{aligned}$$

9. Show  $y^{\log_b x} = x^{\log_b y}$

$$\begin{aligned}
 \log_b(x) &= k \iff x = b^k \\
 \log_b(y) &= c \iff y = b^c \\
 x^{\log_b(y)} &= b^{k \cdot \log_b(y)} \\
 x^{\log_b(y)} &= b^{\log_b(y^k)} \\
 x^{\log_b(y)} &= y^k \\
 x^{\log_b(y)} &= y^{\log_b(x)}
 \end{aligned}$$

10. Show  $\log_b(x) = \frac{1}{\log_x(b)}$

$$\begin{aligned}
 \log_b(x) &= c \\
 \log_x(b) &= k \\
 x &= b^c \\
 b &= x^k \\
 x \cdot b &= b^c \cdot x^k \\
 \log_b(x) + 1 &= c + k \cdot \log_b(x) \\
 \log_b(x) + 1 &= \log_b(x) + \log_x(b) \cdot \log_b(x) \\
 1 &= \log_x(b) \cdot \log_b(x) \\
 \frac{1}{\log_x(b)} &= \log_b(x)
 \end{aligned}$$

11. Show  $\log_b(x) = \frac{\log_a(x)}{\log_a(b)}$

$$\begin{aligned}
 y &= \log_b(x) \\
 z &= \log_a(x) \\
 z &= \log_a(b^y) \\
 z &= y \cdot \log_a(b) \\
 z &= \log_b(x) \cdot \log_a(b) \\
 \log_a(x) &= \log_b(x) \cdot \log_a(b) \\
 \frac{\log_a(x)}{\log_a(b)} &= \log_b(x)
 \end{aligned}$$

12. Compute  $\log_2(2^2 \times 3^4)$

$$\begin{aligned}
 \log_2(2^2) + \log_2(3^4) &= 2 \cdot \log_2(2) + 4 \cdot \log_2(3) \\
 &= 2 + 4 \cdot (1.5)
 \end{aligned}$$

13. Compute  $\log_2\left(\frac{64 \times 128 \times 3}{1024 \times 2^{15}}\right)$

$$\begin{aligned}
 \log_2\left(\frac{64 \times 128 \times 3}{1024 \times 2^{15}}\right) &= \log_2(64 \times 128 \times 3) - \log_2(1024 \times 2^{15}) \\
 &= \log_2(64) + \log_2(128) + \log_2(3) - \log_2(1024) - 15 \cdot \log_2(2) \\
 &= 6 + 7 + (1.5) - 10 - 15
 \end{aligned}$$

14. Compute  $\log_2(\log_2(16))$

$$\begin{aligned}\log_2(\log_2(16)) &= \log_2(4) \\ &= 2\end{aligned}$$

15. Compute  $(\log_2(8))^4$

$$(\log_2(8))^4 = (3)^4$$

16. Compute  $(\log_2^4 128^2)$

$$\begin{aligned}(\log_2(128^2))^4 &= (2 \cdot \log_2(128))^4 \\ &= 2^4 \cdot 7^4 \\ &= (2 \cdot 7)^4 \\ &= 14^4\end{aligned}$$

17. Compute  $5^{\log_5(15)} + 3^{\log_3(8)}$

$$5^{\log_5(5)} + 3^{\log_3(3)} = 5 + 3$$

18. Compute  $8^{\log_2(6)}$

$$\begin{aligned}8^{\log_2(6)} &= 2^{\log_2(6^3)} \\ &= 6^3\end{aligned}$$

19. Compute  $4^{\log_2(\log_2(5))} + 6^{\log_6(2)}$

$$2^{\log_2(\log_2(5)^2)} + 2 = \log_2(5)^2 + 2$$

20. Compute  $\log_2((\frac{\log_4(8)}{\log_3(27)})^2)$

$$\begin{aligned}\log_2((\frac{\log_4(8)}{\log_3(27)})^2) &= \log_2((\frac{3}{2})^2) \\ &= \log_2((\frac{1}{2})^2) \\ &= \log_2(\frac{1}{4}) \\ &= -2\end{aligned}$$

21. Compute the derivative of  $(2x^5 + 3x^9 + \ln x)$

$$\frac{d}{dx}(2x^5 + 3x^9 + \ln x) = 10x^4 + 27x^8 + \frac{1}{x}$$

22. Compute the derivative of  $(\ln^2(x) + x \cdot (x+1)^2)$

$$\frac{d}{dx}(\ln^2(x) + x \cdot (x+1)^2) = 2 \cdot \ln(x) \cdot \frac{1}{x} + (x+1)^2 + 2 \cdot x \cdot (x+1)$$

23. Compute the derivative of  $\frac{\sqrt{5x+2}}{x \cdot \sin(x)}$

$$\frac{d}{dx}\left(\frac{\sqrt{5 \cdot x + 2}}{x \cdot \sin(x)}\right) = \frac{\frac{5}{2} \cdot (5 \cdot x + 2)^{-\frac{1}{2}}(x \cdot \sin(x)) - (5 \cdot x + 2)^{\frac{1}{2}}(\sin(x) + x \cdot \cos(x))}{x^2 \cdot \sin^2(x)}$$

24. Compute the derivative of  $\sqrt{\frac{\ln^2 x}{5 \cdot \log_2 x^2 + 2}}$

$$\begin{aligned} \frac{d}{dx}\left(\sqrt{\frac{\ln^2 x}{10 \cdot \frac{\ln x}{\ln(2)} + 2}}\right) &= \frac{1}{2} \cdot \left(\frac{\ln^2(x)}{10 \cdot \frac{\ln(x)}{\ln(2)} + 2}\right)^{-\frac{1}{2}} \\ &\quad \cdot \frac{(2 \cdot \ln(x) \cdot \frac{1}{x}) \cdot (10 \cdot \frac{\ln(x)}{\ln(2)} + 2) - (\ln^2(x) \cdot (10 \cdot \frac{\ln(x)}{\ln(2)} + 2))}{(10 \cdot \frac{\ln(x)}{\ln(2)} + 2)^2} \end{aligned}$$

25. Compute the derivative of  $\sqrt{\sqrt{x}} \cdot (\ln^2(x^2 + 5))$

$$\begin{aligned} \frac{d}{dx}(\sqrt{\sqrt{x}} \cdot (\ln^2(x^2 + 5))) &= \frac{1}{4}x^{-\frac{3}{4}} \cdot (\ln^2(x^2 + 5)) \\ &\quad + x^{\frac{1}{4}} \cdot (2\ln(x^2 + 5)) \cdot \frac{1}{x^2 + 5} \cdot 2 \cdot x \end{aligned}$$