2. The graph Q_n, $n \geq 1$, has vertex set equal to the set of all binary strings of length n. Moreover, two vertices are adjacent iff they differ in at most one bit place. For example, in Q_3, 000 is adjacent to 010, but not to 011. Draw Q_1, Q_2 and Q_3. Show that Q_3 has a Hamilton cycle.

3. Provide formulas for both the order and size of Q_n. Explain.

There are a total of 2^n bitstrings of length n, hence the order is 2^n. For any bitstring there are n different for a bitstring to differ in one bit, hence the degree of any vertex is n.

$$\begin{aligned} \sum_{v \in V(Q_n)} deg(v) &= 2 \cdot |E(Q_n)| \\ \sum_{v \in V(Q_n)} n &= 2 \cdot |E(Q_n)| \\ 2^n \cdot n &= 2 \cdot |E(Q_n)| \\ 2^{n-1} \cdot n &= |E(Q_n)| \end{aligned}$$

Thus the size of the graph Q_n is $2^{n-1} \cdot n$.

7. What is the running time of

1. Breadth-first search
2. Depth-first search,

as a function of $|V|$ and $|E|$, if the input graph is represented by an adjacency matrix, instead of an adjaceny list?