Algorithme 1 Binary Search

prodedure(A,key) :
$left := 0, right := len(A) - 1$
$while\, left \neq right$:
  $mid := \lfloor \frac{left+right}{2} \rfloor$
  $if\; A[mid] < key$:
      $left := mid + 1$
  $else\, if\; A[mid] > key$:
      $right := mid - 1$
    $else : return\, mid$
return -1

1. $T(n) = T(\frac{n}{2}) + O(1)$ which evaluates to $T(n) = O(log_2(n))$.

2. A machine takes 10^{-8} seconds to run a single line on a machine. What is the largest input size an algorithm with the specified time complexity can run in 1 second.

1. $T(n) = n$

$$\begin{aligned} n \cdot 10^{-8} &= 1 \\ n &= 10^8 \end{aligned}$$

2. $T(n) = log_2(n)$

$$\begin{aligned} log_2(n) \cdot 10^{-8} &= 1 \\ log_2(n) &= 10^8 \\ n &= 2^{10^8} \end{aligned}$$

3. $T(n) = n^2$

$$\begin{aligned} n^2 \cdot 10^{-8} &= 1 \\ n &= 10^4 \end{aligned}$$

4. $T(n) = n^3$

$$n = 10^{\frac{8}{k}}$$

5. $T(n) = 2^n$

$$\begin{aligned} 2^n \cdot 10^{-8} &= 1 \\ 2^n &= 10^8 \\ n &= 8 \cdot log_2(n) \end{aligned}$$

3. Calculate the running time of the below algorithm for all the specified array sizes

$$for(i = 1 \qquad ; i < n; \qquad i++$$
$$binary\_search(a[i])$$

1. $len(a) = n$

$$\begin{aligned} \sum_{i=1}^{n} T(n) &= \sum_{i=1}^{n} O(log_2(n)) \\ &= n \cdot log_2(n) \end{aligned}$$

2. $len(a) = log_2(n)$

$$\begin{aligned} \sum_{i=1}^{n} T(n) &= \sum_{i=1}^{n} O(log_2(log_2(n)) \\ &= n \cdot log_2(n) \end{aligned}$$

3. $len(a) = n \cdot log_2(n)$

$$\begin{aligned} \sum_{i=1}^{n} T(n) &= \sum_{i=1}^{n} log_2(n \cdot log_2(n)) \\ &= \sum_{i=1}^{n} log_2(n) + \sum_{i=1}^{n} log_2(log_2(n)) \\ &= O(n \cdot log_2(n)) \end{aligned}$$

4. $len(a) = n^2$

$$\begin{aligned} \sum_{i=1}^{n} T(n) &= \sum_{i=1}^{n} log_2(n^2) \\ &= O(n \cdot log_2(n)) \end{aligned}$$

5. $len(a) = 2^n$

$$\begin{aligned}\sum_{i=1}^{n} T(n) &= \sum_{i=1}^{n} log_2(2^n) \\ &= \sum_{i=1}^{n} n \\ &= O(n^2)\end{aligned}$$

4. Calculate the run time of the following algorithm

$$\begin{aligned} for(i=0; \quad i < n^3; \quad & i++) \\ if\, i < n^2: \quad & binary\_search(a[i], i) \\ else: \quad & bubble\_sort(a)\end{aligned}$$

$$\sum_{i=1}^{n^2-1} O(log_2(n)) + \sum_{i=n^2}^{n^3} O(n^2) \quad = \quad O(n^5)$$