Quick Select (Randomized Find Statistics)

Tuesday, October 20, 2020 5:00 PM

Reminder: HW6 & lab3 are due this Sunday.

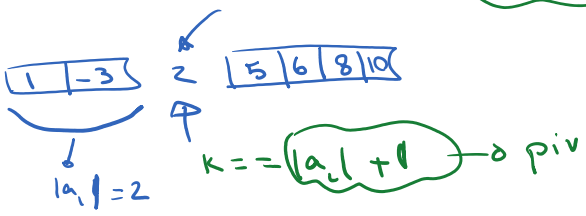
Example: Find the k^{th} least element.

$a = [5, 2, 1, 6, 8, 10, -3]$

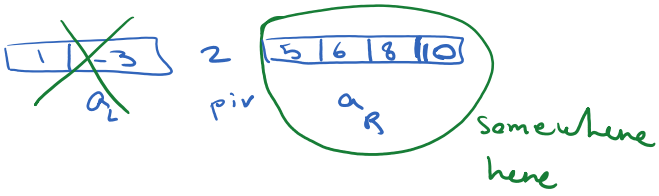
$T(n) = 1T(\frac{n}{2}) + cn$

$k =$ 3rd

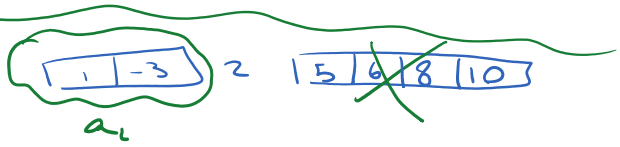
piv = 2



$k =$ 5th



$k =$ 2nd

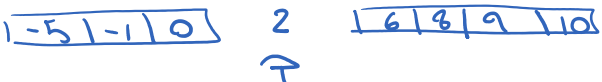


Example:

$a = [2, 6, 8, -5, -1, 0, 9, 10]$

$k = 4^{th}$

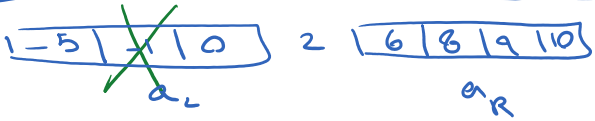
piv = 2 (1st element)



$i \Rightarrow$ print: 4^{th} Least nr is 2

$k =$ 7th

piv = 2



update k? ← 6 8 9 10

$k = k - (|a_L| + 1)$

$k = 3$

$k = k - (|a_L| + 1)$

$k = 3$

piv = 6 | ∅ | 6 | 8 | 9 | 10 |

$k = 2$ | 8 | 9 | 10 |

piv = 8 | ∅ | 8 | 9 | 10 |

$k = 1$ | 9 | 10 |

piv = 9 | ∅ | 9 | 10 |

⇒ 7th least nr 9

$a = [2, 6, 8, -5, -1, 0, 9, 10]$

K = 2nd

piv = 2

| -5 | -1 | 0 | (= a_L) | 2 | 6 | 8 | 9 | 10 | (= a_R)

$k = 2$ | -5 | -1 | 0 |

piv = -5

| ∅ | -5 | -1 | 0 |

(a_L = ∅, piv = -5, a_R = -1 | 0)

$k = 1$ | -1 | 0 |

piv = -1

| ∅ | -1 | 0 |

answer!

⇒ -1 is the 2nd least nr

Avg-case

$T(n) = T(\frac{n}{2}) + cn$ ← amt work to partition a to a_L and a_R

$\hookrightarrow T(n) = O(?)$

step	size	tree
0	n	cn
1	$\frac{n}{2}$	$\frac{cn}{2}$
2	$\frac{n}{2^2}$	$\frac{cn}{2^2}$
⋮	⋮	⋮
k	$\frac{n}{2^k} = 1$	$\Theta(1)$

$\hookrightarrow n = 2^k$

$k = \log n$

$\sum_{i=0}^{k-1} \frac{cn}{2^i} + \Theta(1) =$

$= n \frac{\frac{1}{2}^k - 1}{\frac{1}{2} - 1} + \Theta(1)$

$\approx n + \Theta(1) = O(n)$

Avg-case
Best-case

Quick-select(a, K, ~~)

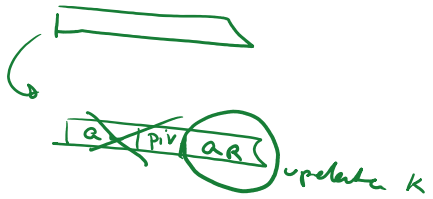
piv_idx = partition(a, ~~)

if

else if

else

end



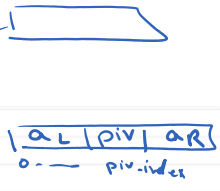
Vivian

```
Quick Select(arr, l, r, k)        // k: LHS, r: RHS
  pivIndex = partition(arr, L, r)

  if(pivIndex+1 == k)             // if pivIndex == k
      return arr[pivIndex]

  else if (index+1 > k)           // left subarr
      return QuickSelect(arr, L, pivIndex-1, k)

  else
      return QuickSelect(arr, pivIndex+1, r, k - pivIndex-1)   // right subarr
```



Lab 3 part B

ex = [1, 0, 5, 10, 2, 8, 6]

:?

(n³) worst of the worst
But why? :)
HW6 first

ex = [1, 0, 5, 10, 2, 8, 6]

print the max k nrs

k = 3 ⟹ output = [10 6 8]

n-th, n-2 th, n-1 th

bad idea :(

```
itr = 0
while itr < k
    print quick_select(a, a.len - itr)
    itr++
end
```

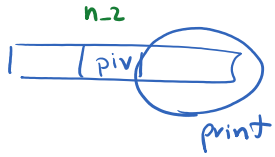
:?

$T(n) = O(n^3)$ worst & the
But why? :)
look at ltw6 first

$T(n) = O(n^2)$
if best-case & or quick-select
Avg-case
and (k = n)
input

Code

Kristinann :)




```
maxfunc(a, k)
    index = |a| - k                          Θ(1)
    pivindex = quickselect(a, start, end, index)   O(n)
    for i = pivindex : |a|                   O(n)
        print a[i]
    end
end
```

⟹ $T(n) = O(n) + O(n) + \Theta(1) = O(n)$

avg-case
Best-case