

1. Consider the  $101 \times 3$  world shown in Figure 1. In the start state the agent has a choice of two deterministic actions, Up or Down, but in the other states the agent has one deterministic actions, Right. Assuming a discounted reward function.

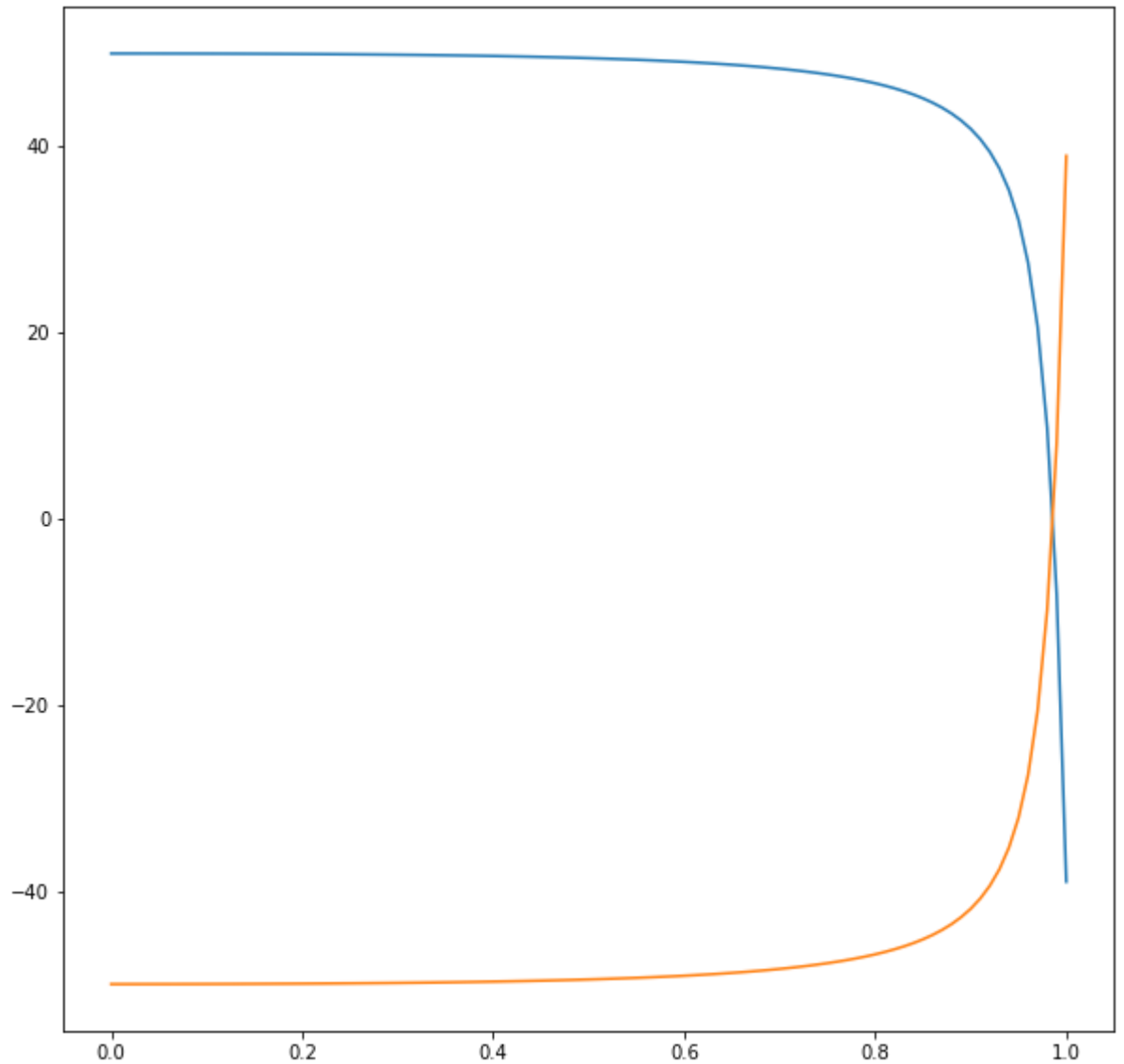
1. Compute the utility of each action as a function of  $\gamma$ .

$$\begin{aligned}
 U(Up, Right, \dots, Right) &= 50 - \sum_{i=1}^{100} \gamma^i + 10 \cdot \gamma^{101} \\
 U(Down, Right \dots, Right) &= -50 + \sum_{i=1}^{100} \gamma^i - 10 \cdot \gamma^{101}
 \end{aligned}$$

setting the equations to solve for  $\gamma$ .

$$\begin{aligned}
 U(Up, Right, \dots, Right) &= U(Down, Right, \dots, Right) \\
 50 - \sum_{i=1}^{100} \gamma^i + 10 \cdot \gamma^{101} &= -50 + \sum_{i=1}^{100} \gamma^i - 10 \cdot \gamma^{101} \\
 100 &= 2 \cdot \sum_{i=1}^{100} \gamma^i - 2 \cdot \gamma^{101} \\
 \gamma &= .986
 \end{aligned}$$

2. Draw the utility of each action for the range  $0 \leq \gamma < 1$  using Matlab or your familiar numerical analysis software.



1. For  $\gamma = \frac{1}{2}$ , which action is recommended? Why?

We should go up. The utility is larger.

**2.** Consider the following data set comprised of three binary input attributes ( $A_1$ ,  $A_2$  and  $A_3$ ) and one binary output:

1. Compute  $Gain(A_1)$ .  
 $.9709 - \frac{4}{5} \cdot B(\frac{1}{3}) - \frac{1}{5} \cdot B(0) = 0.1709$

2. Compute  $Gain(A_2)$ .

$$B(\frac{2}{5}) - \frac{3}{5} \cdot B(\frac{2}{3}) = 0.420$$

3. Compute  $Gain(A_3)$ .

$$B(\frac{2}{5}) - \frac{2}{5} \cdot B(\frac{1}{2}) - \frac{3}{5} \cdot B(\frac{1}{3}) = .0200$$

**3.** Consider the  $XOR$  function of three binary input attributes ( $A_1, A_2$  and  $A_3$ ). Which produces the value 1 if and only if an odd number of the three input attributes has value 1. Draw a minimal-sized decision tree for the three-input  $XOR$  function.

