1. Consider the Bayes net shown in Figure 1. Write answers with a scale of 4, i.e., 01.234.

1. Calculate the value of $P(b, i, \neg m, g, j)$.

$$
\begin{aligned}
P(b, i, \neg m, g, i) &= P(b) \cdot P(\neg m) \cdot P(i|b, \neg m) \cdot P(g|b, i, \neg m) \cdot P(j|g) \\
&= (.9) \cdot (.8) \cdot (.5) \cdot (.8) \cdot (.8) \\
&= .2304
\end{aligned}
$$

2. Calculate the value of $\vec{P}\,(J|b, i, m)$.

$$
\begin{aligned}
\vec{P}\,(J|b, i, m) &= \alpha \cdot \vec{P}\,(J, b, i, m) \\
&= \alpha \cdot \sum_{g\prime} \vec{P}\,(J|g\prime) \cdot P(b) \cdot P(i|b, m) \cdot P(m) \cdot P(g\prime|i, b, m) \\
&= \beta \cdot \sum_{g\prime} \vec{P}\,(J|g\prime) \cdot P(g\prime|i, b, m) \\
&= \beta \cdot \begin{bmatrix} P(j|g) \cdot P(g|i, b, m) + P(j|\neg g) \cdot P(\neg g|i, b, m), \\ P(\neg j|g) \cdot P(g|i, b, m) + P(\neg j|\neg g) \cdot P(\neg g|i, b, m) \end{bmatrix} \\
&= \beta \cdot \big[(.8) \cdot (.9) + (.1) \cdot (.1), (.2) \cdot (.9) + (.9) \cdot (.1), (.2) \cdot (.9) + (.9) \cdot (.1)\big] \\
&= \beta \cdot \big[.73, .27\big] \\
&= \big[.73, .27\big]
\end{aligned}
$$

3. Calculate the value of $\vec{P}\,(J|\neg b, \neg i, m)$.

$$
\begin{aligned}
\vec{P}\,(J|\neg b, \neg i, m) &= \alpha \cdot \vec{P}\,(J, \neg b, \neg i, m) \\
&= \alpha \cdot \sum_{g\prime} \vec{P}\,(J|g\prime) \cdot P(g\prime|\neg b, \neg i, m) \cdot P(\neg b) \cdot P(\neg m) \cdot P(i|\neg b, \neg m) \\
&= \beta \cdot \sum_{g\prime} \vec{P}\,(J|g\prime) \cdot P(g\prime|\neg b, \neg i, m) \\
&= \beta \cdot \begin{bmatrix} P(j|g) \cdot P(g|\neg b, \neg i, m) + P(j|\neg g) \cdot P(\neg g|\neg b, \neg i, m) \\ P(\neg j|g) \cdot P(g|\neg b, \neg i, m) + P(\neg j|\neg g) \cdot P(\neg g|\neg b, \neg i, m) \end{bmatrix} \\
&= \beta \cdot \begin{bmatrix} (.8) \cdot (.0) + (.1) \cdot (1) \\ (.2) \cdot 0 + (.9) \cdot (1) \end{bmatrix} \\
&= \beta \cdot \begin{bmatrix} .1 \\ .9 \end{bmatrix} \\
&= \big[.1, .9\big]
\end{aligned}
$$

2. Suppose we observe an unending sequence of days on which the umbrella appears. As the days go by, the probability of rain on the current day increases toward a fixed point, we expect that $\vec{P}(R_t|u_{1:t}) = \vec{P}(R_{t-1}|u_{1:t-1}) = \left[\rho, 1-\rho\right]$. Find ρ.

$$\begin{aligned}
\left[\rho, 1-\rho\right] &= \alpha \cdot \left[0.7, 0.3\right] \cdot (\left[0.9, 0.1\right] \cdot \rho + \left[0.2, 0.8\right] \cdot (1-p)) \\
&= \alpha \cdot \left[0.7, 0.3\right] \cdot (\left[0.9, 0.1\right] \cdot \rho + \left[0.2 - (0.2) \cdot \rho, 0.8 - (0.8) \cdot \rho\right] \\
&= \alpha \cdot \left[0.7, 0.3\right] \cdot (\left[0.2 + 0.7 \cdot \rho, 0.8 - 0.7 \cdot \rho\right]) \\
&= \alpha \cdot \left[0.49 \cdot \rho + .14, 0.24 - 0.21 \cdot \rho\right] \\
&= \frac{1}{0.28 \cdot \rho + .38} \cdot \left[0.49 \cdot \rho + .14, 0.24 - 0.21 \cdot \rho\right]
\end{aligned}$$

$$\begin{aligned}
\rho \cdot (0.28 \cdot \rho + .38) &= .49 \cdot \rho + .14 \\
0.28 \cdot \rho^2 + .38 \cdot \rho &= .49 \cdot \rho + .14 \\
0.28 \cdot \rho^2 - .11 \cdot \rho - .14 &= 0
\end{aligned}$$

$$\vec{P}(R_t|u_{1:t}) = \left[0.9304, 0.0696\right]$$

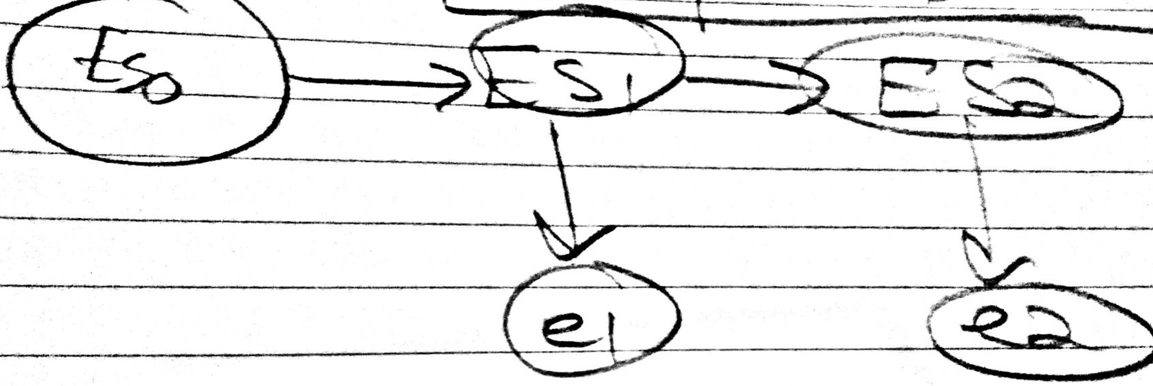
3. A professor wants to know if students are getting enough sleep. Each day, tthe professor observes whether they have no red eyes. The professor has the following domain theory:

- The prior probability of getting enough sleep with no observations, is 0.7
- The probability of getting enough sleep on night t is 0.8 given that the student got enough sleep the previous night, and 0.3 if not.
- The probability of having red eyes is 0.2 if the student got enough sleep, and 0.7 if not.

1. Formulate this information as a hidden Markov model. Give a Bayesian network and conditional distributions

$P(Es_0) = 0.7$

ES_{t-1}	$P(ES_t \mid ES_{t-1})$
t	.8
f	.3



ES_t	$P(e_t \mid ES_t)$
t	.2
f	.7

2. Consider the following evidences, and compute $\overrightarrow{P}(ES_2|\hat{e}_{1:2})$ with a scale of 4

   - $\overrightarrow{e}_1 = red\,eyes$
   - $\overrightarrow{e}_2 = not\,red\,eyes$

$$P(ES_0) \quad = \quad \left[0.7, 0.3\right]$$

$$\begin{aligned} P(ES_1) &= \sum_{es_0} P(ES_1|es_0) \cdot P(es_0) \\ &= \begin{bmatrix} P(es_1|es_0) \cdot P(es_0) + P(es_1|\neg es_0) \cdot P(\neg es_0) \\ P(\neg es_1|es_0) \cdot P(es_0) + P(\neg es_1|\neg es_0) \cdot P(\neg es_0) \end{bmatrix}^T \\ &= \begin{bmatrix} (.8) \cdot (.7) + (.3) \cdot (.3) \\ 1 - .65 \end{bmatrix}^T \\ &= \left[.65, .35\right] \end{aligned}$$

$$\begin{aligned} P(ES_1|e_1) &= \alpha \cdot P(ES_1, e_1) \\ &= \alpha \cdot P(ES_1) \cdot P(e_1|ES_1) \\ &= \alpha \cdot \left[.65, .35\right] \otimes \left[.2, .7\right] \\ &= \left[.3467, .6533\right] \end{aligned}$$

$$\begin{aligned} P(ES_2|e_1) &= \sum_{es_1} P(ES_2|es_1) \cdot P(es_1|e_1) \\ &= \left[.4733, .5267\right] \end{aligned}$$

$$\begin{aligned} P(ES_2|e_1, \neg e_2) &= \alpha \cdot P(ES_2, e_1, \neg e_2) \\ &= \alpha \cdot P(ES_2|e_1) \cdot P(\neg e_2|ES_2) \\ &= \alpha \cdot \left[.4733, .5267\right] \otimes \left[.8, .3\right] \\ &= \left[.7056, 0.2944\right] \end{aligned}$$