

1. Consider the Bayes net shown in Figure 1. Write answers with a scale of 4, i.e., 01.234.

1. Calculate the value of $P(b, i, \neg m, g, j)$.

$$\begin{aligned} P(b, i, \neg m, g, j) &= P(b) \cdot P(\neg m) \cdot P(i|b, \neg m) \cdot P(g|b, i, \neg m) \cdot P(j|g) \\ &= (.9) \cdot (.8) \cdot (.5) \cdot (.8) \cdot (.8) \\ &= .2304 \end{aligned}$$

2. Calculate the value of $\vec{P}(J|b, i, m)$.

$$\begin{aligned} \vec{P}(J|b, i, m) &= \alpha \cdot \vec{P}(J, b, i, m) \\ &= \alpha \cdot \sum_{g'} \vec{P}(J|g') \cdot P(b) \cdot P(i|b, m) \cdot P(m) \cdot P(g'|i, b, m) \\ &= \beta \cdot \sum_{g'} \vec{P}(J|g') \cdot P(g'|i, b, m) \\ &= \beta \cdot \left[\begin{array}{l} P(j|g) \cdot P(g|i, b, m) + P(j|\neg g) \cdot P(\neg g|i, b, m), \\ P(\neg j|g) \cdot P(g|i, b, m) + P(\neg j|\neg g) \cdot P(\neg g|i, b, m) \end{array} \right] \\ &= \beta \cdot [(.8) \cdot (.9) + (.1) \cdot (.1), (.2) \cdot (.9) + (.9) \cdot (.1), (.2) \cdot (.9) + (.9) \cdot (.1)] \\ &= \beta \cdot [.73, .27] \\ &= [.73, .27] \end{aligned}$$

3. Calculate the value of $\vec{P}(J|\neg b, \neg i, m)$.

$$\begin{aligned} \vec{P}(J|\neg b, \neg i, m) &= \alpha \cdot \vec{P}(J, \neg b, \neg i, m) \\ &= \alpha \cdot \sum_{g'} \vec{P}(J|g') \cdot P(g'|\neg b, \neg i, m) \cdot P(\neg b) \cdot P(\neg m) \cdot P(i|\neg b, \neg m) \\ &= \beta \cdot \sum_{g'} \vec{P}(J|g') \cdot P(g'|\neg b, \neg i, m) \\ &= \beta \cdot \left[\begin{array}{l} P(j|g) \cdot P(g|\neg b, \neg i, m) + P(j|\neg g) \cdot P(\neg g|\neg b, \neg i, m) \\ P(\neg j|g) \cdot P(g|\neg b, \neg i, m) + P(\neg j|\neg g) \cdot P(\neg g|\neg b, \neg i, m) \end{array} \right] \\ &= \beta \cdot \left[\begin{array}{l} (.8) \cdot (.0) + (.1) \cdot (1) \\ (.2) \cdot 0 + (.9) \cdot (1) \end{array} \right] \\ &= \beta \cdot \left[\begin{array}{l} .1 \\ .9 \end{array} \right] \\ &= [.1, .9] \end{aligned}$$

3. A professor wants to know if students are getting enough sleep. Each day, the professor observes whether they have no red eyes. The professor has the following domain theory:

- The prior probability of getting enough sleep with no observations, is 0.7
- The probability of getting enough sleep on night t is 0.8 given that the student got enough sleep the previous night, and 0.3 if not.
- The probability of having red eyes is 0.2 if the student got enough sleep, and 0.7 if not.

1. Formulate this information as a hidden Markov model. Give a Bayesian network and conditional distributions

2. Consider the following evidences, and compute $\vec{P}(ES_2|\hat{e}_{1:2})$ with a scale of 4

- $\vec{e}_1 = \text{red eyes}$
- $\vec{e}_2 = \text{not red eyes}$

$$P(ES_0) = [0.7, 0.3]$$

$$\begin{aligned} P(ES_1) &= \sum_{es_0} P(ES_1|es_0) \cdot P(es_0) \\ &= [.65, .35] \end{aligned}$$

$$\begin{aligned} P(ES_1|e_1) &= \alpha \cdot P(ES_1, e_1) \\ &= \alpha \cdot P(ES_1) \cdot P(e_1|ES_1) \\ &= [0.3466, 0.6534] \end{aligned}$$

$$\begin{aligned} P(ES_2|e_1) &= \sum_{es_1} P(ES_2|es_1) \cdot P(es_1|e_1) \\ &= [.4733, .5267] \end{aligned}$$

$$\begin{aligned}
P(ES_2|e_1, \neg e_2) &= \alpha \cdot P(ES_2, e_1, \neg e_2) \\
&= \alpha \cdot P(ES_2|e_1) \cdot P(\neg e_2|ES_2) \\
&= \alpha \cdot [.4733, .5267] \otimes [.8, .3] \\
&= [.7056, 0.2944]
\end{aligned}$$