Genetic algorithms

- 4-state approach (8 queens problem)
  - Better 'gene' has more 'chance' to survive.

A B C D

r = rand( )

A	24748552	24 → $\frac{24}{24+23+20+11} = 0.31$
B	32752411	23 → 0.29
C	24415124	20 → 0.26
D	32543213	11 → 0.14

4-State

For each state, select a gene based on fitness

19

19

Genetic algorithms

- 4-state approach
  - Better 'gene' has more 'chance' to survive.

uniform random

# of None attacking pairs

		Fitness		Selection
A	24748552	24	31%	32752411
B	32752411	23	29%	24748552
C	24415124	20	26%	32752411
D	32543213	11	14%	24415124

Fitness Selection Pairs

$\frac{24}{24+23+20+11} = 0.31$ (for each state select gene)

r = rand( ) $0 \le r < 1$

0 0.31 0.6 0.86 1

if r < 0.31 ⇒ select A

elif r < 0.6 ⇒ select B

elif r < 0.86 ⇒ select C

else ⇒ select D

$0.86 \le r < 1$

20

20

Genetic algorithms

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randomly choose

	Fitness		Selection	Cross-Over
24748552	24	31%	32752411	32748552
32752411	23	29%	24748552	24752411
24415124	20	26%	32752411	32752124
32543213	11	14%	24415124	24415411

randomly

Fitness Selection Pairs Cross-Over

21

21

Genetic algorithms

- 4-state approach
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r = randint(0,9)

$0 \le r \le 8$

	Fitness		Selection	Cross-Over	Mutation
24748552	24	31%	32752411	32748552	32748152
32752411	23	29%	24748552	24752411	24752411
24415124	20	26%	32752411	32752124	32252124
32543213	11	14%	24415124	24415411	24415417

8 queens

Fitness Selection Pairs Cross-Over Mutation

$\binom{8}{2} = \frac{8\cdot 7}{2} = 28$ ⇐ optimal fit. value

randomly chosen

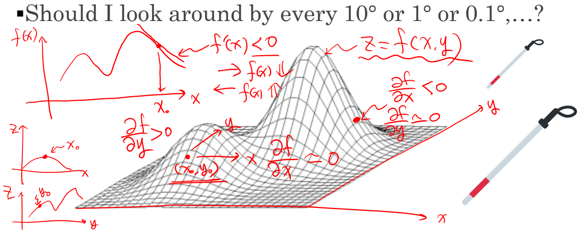
current state

22

22

State space

- Assume that we can choose the cane.
  - Which length would be the best?
  - Should I look around by every 10° or 1° or 0.1°,…?



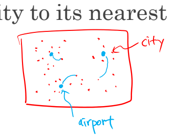
23

23

Continuous state spaces

- Suppose we want to place new three airports
  - Goal; the sum of distances from each city to its nearest airport is minimized (squared)
  - 6-D state space defined by

(x_1, y_1) location of airport 1, (x_2, y_2) airport 2, (x_3, y_3) airport 3



- C_i be the set of cities whose closest airport is airport i
- The objective function
  - $f(x_1, y_1, x_2, y_2, x_3, y_3) = \sum_{i=1}^{3} \sum_{c \in C_i} (x_i - x_c)^2 + (y_i - y_c)^2$

objective function

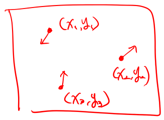
C_1, C_2, C_3

24

24

Continuous state spaces

- How can we define a local search space in 6-D state space?
  - $\vec{z} = (x_1, y_1, x_2, y_2, x_3, y_3)$
  - $\vec{z}' = ?$



Continuous state spaces

- Gradient gives the magnitude and direction of the **steepest** slope.
  - $f(x_1, y_1, x_2, y_2, x_3, y_3) = \sum_{i=1}^{3} \sum_{c \in C_i} (x_i - x_c)^2 + (y_i - y_c)^2$
  - $\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3} \right)$ $\quad \frac{\partial f}{\partial x_2} = 2 \sum_{c \in C_2} (x_2 - x_c)$
  - $\frac{\partial f}{\partial x_1} = 2 \sum_{c \in C_1} (x_1 - x_c)$

$f = \sum_{c \in C_1} \left[(x_1 - x_c)^2 + \cancel{(y_1 - y_c)^2} \right] + \cancel{\sum_{c \in C_2} \left[(x_2 - x_c)^2 + (y_2 - y_c)^2 \right]} + \cancel{\sum_{c \in C_3} \left[(x_3 - x_c)^2 + (y_3 - y_c)^2 \right]}$

$\frac{\partial f}{\partial x_1} = \sum_{c \in C_1} \frac{\partial}{\partial x_1} (x_1 - x_c)^2 = \sum_{c \in C_1} 2(x_1 - x_c) = 2 \sum_{c \in C_1} (x_1 - x_c)$

Continuous state spaces

- Gradient gives the magnitude and direction of the **steepest** slope.
  - $\vec{z}' \leftarrow \vec{z} - \alpha \nabla f(\vec{z})$ where α is a small constant

Current state