Problem 8.2 Consider a data set with three data points in $\mathbb{R}^2$

$$X = \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ -2 & 0 \end{bmatrix} \qquad y = \begin{bmatrix} -1 \\ -1 \\ +1 \end{bmatrix}$$

Manually solve the optimization problem:

$$\begin{aligned} \underset{b,w}{minimize} : \quad & \frac{1}{2} w^T w \\ subject\,to : \quad & y_n(w^T x_n + b) \geq 1 \quad (n = 1, \dots, N) \end{aligned}$$

to get the optimal hyperplane $(b^*, \mathbf{w}^*)$ and its margin.

We get three inequalities

$$\begin{aligned} -b &\geq 1 \quad (i) \\ w_2 - b &\geq 1 \quad (ii) \\ -2w_1 + b &\geq 1 \quad (iii) \end{aligned}$$

Combining (i) and (iii) gives us

$$w_1 \leq -1$$

Combining (i) and (ii) gives us

$$w_2 \geq 0$$

We thus have

$$(b^* = -1, w_1^* = -1, w_2^* = 0)$$

The hyperplane is given by the following:

$$\begin{aligned} g(x) &= sign(-1x_1 - 1) \\ margin : \quad \frac{1}{\|w^*\|} &= \frac{1}{1} = 1 \end{aligned}$$