

1) Write down the three-by-three matrix with ones on the diagonal and zeros elsewhere.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2) Write down the three-by-four matrix with ones on the diagonal and zeros elsewhere

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

3) Write down the four-by-three matrix with ones on the diagonal and zeros elsewhere

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$4) A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & -2 & 1 \\ 2 & -4 & -2 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, D = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix},$$

$$E = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Compute if defined

$$B - 2A: \begin{bmatrix} 4 & -2 & 1 \\ 2 & -4 & -2 \end{bmatrix} - \begin{bmatrix} -4 & -2 & 2 \\ -2 & 2 & -2 \end{bmatrix} = \begin{bmatrix} 0 & -4 & -1 \\ 0 & -2 & -4 \end{bmatrix}$$

$3C - E$: undefined

AC : undefined

$$CD: \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 10 & 11 \end{bmatrix}$$

$$CB: \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & -2 & 1 \\ 2 & -4 & -2 \end{bmatrix} = \begin{bmatrix} 8 & -10 & -3 \\ 10 & -8 & 0 \end{bmatrix}$$

$$5) \text{ Let } A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}, C = \begin{bmatrix} 4 & 3 \\ 0 & 2 \end{bmatrix}. \text{ Verify that } AB = AC \text{ and}$$

yet $B \neq C$.

$$A \cdot B = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 8 & 14 \end{bmatrix}$$

$$A \cdot C = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & 3 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 8 & 14 \end{bmatrix}$$

$$B \neq C$$

$$6) \text{ Let } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix} \text{ and } D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}. \text{ Compute } AD \text{ and } DA.$$

$$A \cdot D = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 2 & 6 & 12 \\ 2 & 9 & 16 \end{bmatrix}$$

$$D \cdot A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 2 \\ 3 & 6 & 9 \\ 4 & 12 & 16 \end{bmatrix}$$

7) Prove the associative law for matrix multiplication. That is, let A be an $m - by - n$ matrix, B an $n - by - p$ matrix, and C a $p - by - q$ matrix. Then prove that $A(BC) = (AB)C$.

$$A(BC)_{ij} = \sum_{k=1}^n a_{ik} \cdot (bc)_{kj} = \sum_{k=1}^n a_{ik} \sum_{r=1}^p b_{kr} \cdot c_{rk} = \sum_{r=1}^p \sum_{k=1}^n a_{ik} \cdot b_{kr} \cdot c_{rk} = \sum_{r=1}^p (AB)_{ir} C_{rj} = [(AB)C]_{ij}$$

8) Let $A = \begin{bmatrix} -1 & 2 \\ 4 & -8 \end{bmatrix}$. Construct a two-by-two matrix B such that AB is the zero matrix. Use two different nonzero columns for B .

$$\begin{bmatrix} -1 & 2 \\ 4 & -8 \end{bmatrix} \cdot \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 2 \\ 4 & -8 \end{bmatrix} \cdot \begin{bmatrix} 4 & 4 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

9) Verify that $\begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} \cdot \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix} = \begin{bmatrix} a_1 \cdot b_1 & 0 \\ 0 & a_2 \cdot b_2 \end{bmatrix}$. Prove that in general that the product of the two diagonal matrices is a diagonal matrix, with elements given by the product of the diagonal matrix

$$\begin{bmatrix} a_1 & 0 \\ 0 & a_2 \end{bmatrix} \cdot \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix} = \begin{bmatrix} a_1 \cdot b_1 & 0 \\ 0 & a_2 \cdot b_2 \end{bmatrix}$$

In general

$$C_{ij} = \sum_{k=1}^n a_{ik} \cdot b_{kj} \text{ where the sum equals to zero when } i \neq j$$

10) Verify that $\begin{bmatrix} a_1 & a_2 \\ 0 & a_3 \end{bmatrix} \cdot \begin{bmatrix} b_1 & b_2 \\ 0 & b_3 \end{bmatrix} = \begin{bmatrix} a_1 \cdot b_1 & a_1 \cdot b_2 + a_2 \cdot b_3 \\ 0 & a_3 \cdot b_3 \end{bmatrix}$. Prove in general that the product of two upper triangular matrices is an upper triangular matrix, with the diagonal elements of the product given by the product of the diagonal elements.

$$\begin{bmatrix} a_1 & a_2 \\ 0 & a_3 \end{bmatrix} \cdot \begin{bmatrix} b_1 & b_2 \\ 0 & b_3 \end{bmatrix} = \begin{bmatrix} a_1 \cdot b_1 & a_1 \cdot b_2 + a_2 \cdot b_3 \\ 0 & a_3 \cdot b_3 \end{bmatrix}$$

$$C_{ij} = \sum_{k=1}^n a_{ik} \cdot b_{kj} \text{ where the sum equals to zero when } i > j$$

11) Identify the two-by-two matrix with matrix elements $a_{ij} = i - j$.

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$12) \text{ The matrix product } \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$$

13) Prove that $(AB)^T = B^T A^T$.

$$((AB)_{ij})^T = (\sum_{k=1}^n a_{ik} \cdot b_{kj})^T = \sum_{k=1}^n a_{k1} \cdot b_{jk} = B^T A^T$$