

**0.1** Let  $G = (V, E)$  be an undirected graph. By the degree of a vertex  $v \in V$ , we mean the number of edges in  $E$  that are incident to  $v$ . For each of the following conditions on the graph  $G$ , is the condition satisfied only by dense graphs, only by sparse graphs, or by both some sparse and some dense graphs? As usual,  $n = |V|$  denotes the number of vertices. Assume that  $n$  is large (say, at least 10,000)

1. At least one vertex of  $G$  has degree at most 10.
2. Every vertex of  $G$  has degree at most 10.
3. At least one vertex of  $G$  has degree  $n - 1$ .
4. Every vertex of  $G$  has degree  $n - 1$ .

- (1) both sparse and dense
- (2) sparse
- (3) both sparse and dense
- (4) dense

**0.2** Consider an undirected graph  $G = (V, E)$  that is represented as an adjacency matrix. Given a vertex  $v \in V$ , how many operations are required to identify the edges incident to  $v$ ?

1.  $\Theta(1)$
2.  $\Theta(k)$
3.  $\Theta(n)$
4.  $\Theta(m)$

(3) We are required to check the columns of the vertex  $v$  to determine all vertices adjacent to  $v$ . Since we have an  $N \times N$  matrix this  $\Theta(n)$ .

**0.3** Consider a directed graph  $G = (V, E)$  represented with adjacency lists, with each vertex storing an array of its outgoing edges. Given a vertex  $v \in V$ , how many operations are required to identify the incoming edges of  $v$ ?

1.  $\Theta(1)$
2.  $\Theta(k)$
3.  $\Theta(n)$
4.  $\Theta(m)$

(4) For the adjacent list representation we have a list of size  $m$  where  $m$  denotes the number of edges. We iterate through the list and count the number of times that vertex  $v$  is an right endpoint of an edge. This gives us an order of  $m$  and hence  $\Theta(m)$ .