1 Let S denote the set of outcomes for which the sum of two standard dice equals 7. What is the probability of the event S?

1. $\frac{1}{36}$
2. $\frac{1}{12}$
3. $\frac{1}{6}$
4. $\frac{1}{2}$

(3) is correct. We have $\frac{6}{36} = \frac{1}{6}$ probability of getting a roll that totals 7.

2 Let S denote the event that the chosen pivot element in the outermost call to *QuickSort* is an approximate median. What is the probability of the event S?

1. $\frac{1}{n}$
2. $\frac{1}{4}$
3. $\frac{1}{2}$
4. $\frac{3}{4}$

(3) is correct we can imagine dividing the entries in the array into four groups. the smallest $\frac{n}{4}$ elements, the next smallest $\frac{n}{4}$ elements, the next smallest $\frac{n}{4}$ elements and finally the last $\frac{n}{4}$ elements which are the largest. $Pr(S) = |S| \cdot \frac{1}{n} = \frac{\frac{n}{2}}{n} = \frac{1}{2}$.

3 What is the expectation of the sum of two dice?

1. 6.5
2. 7
3. 7.5
4. 8

(2) is the correct answer.Let X be the expected sum of the two dice and X_1 be the expected sum of the first dice and X_2 be the expected sum of the second dice. Then by linearity of expectation $E[X] = E[X_1] + E[X_2] = \frac{7}{2} + \frac{7}{2} = 7$.

4 Which of the following is closest to the expectation of the size of the subarray passed to the first recursive call in the *QuickSort?*

1. $\frac{n}{4}$
2. $\frac{n}{3}$
3. $\frac{n}{2}$
4. $\frac{3 \cdot n}{4}$

$E[X] = \frac{1}{n} \cdot \sum_{i=0}^{n-1} i = \frac{1}{n} \cdot \frac{n \cdot (n-1)}{2} = \frac{n-1}{2}$.

5 Consider a group of k people. Assume that each person's birthday is drawn uniformly at random from the 365 possibilities. What is the smallest value of k such that the expected number of pairs of distinct people with the same birthday is at least one?

1. 20
2. 23
3. 27
4. 28
5. 366

(4) is correct. Let X_{ij} with $i < j$ be the random variable that is equal to 1 if i and j has the same birthday and 0 otherwise.$E[Y] = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} E[X_{ij}] = \frac{1}{365} \cdot \binom{k}{2} = \frac{1}{365} \cdot \frac{k \cdot (k-1)}{2} = \frac{k \cdot (k-1)}{730}$. (4) is the smallest value that is greater than or equal to 1.