

1 Consider a problem instance that has three jobs with $l_1 = 1, l_2 = 2$ and $l_3 = 3$, and suppose they are scheduled in this order. What are the completion times of the three jobs in this schedule?

1. 1, 2 and 3
2. 3, 5, and 6
3. 1, 3, and 6
4. 1, 4, and 6

(3) is the correct answer.

$$\sum_{i=1}^1 l_i = 1$$

$$\sum_{i=1}^2 l_i = 3$$

$$\sum_{i=1}^3 l_i = 6$$

2 (1) If all jobs lengths are identical, should we schedule smaller or larger-weight jobs earlier? (2) If all job weights are identical, should we schedule shorter or longer jobs earlier?

1. larger/shorter
2. smaller/shorter
3. larger/longer
4. smaller/longer

(1) is correct. Our goal is to find the schedule σ such that it minimizes the completion time, that is $\min_{\sigma} \sum_{j=1}^n w_j \cdot C_j(\sigma)$. We see that in the case that all job lengths are identical and we should schedule larger jobs first, the reason being the completion time of any job gets worst as time progresses so we should do the highest cost jobs first. In the case that all the weights are the same should do the shortest job first since we minimize the waiting for any other jobs to start.

3 What is the sum of weighted completion times in the schedules output by the *GreedyDiff* and *GreedyRatio* algorithms, respectively?

1. 22 and 23
2. 23 and 22
3. 17 and 17
4. 17 and 11

. (2) is correct. To see why we look at the data set supplied, that is we have a job j_1 with length $l_1 = 5$ and weight $w_1 = 3$ respectively and j_2 with length $l_2 = 2$ and weight $w_2 = 1$ respectively. We see that the first job j_1 has a bigger ratio than j_2 ($\frac{3}{5} > \frac{1}{2}$) while the second job has a bigger difference than job one ($-1 > -2$). We compute both weighted completion times. The weighted completion time of the greedy algorithm based on ratios is $3 \cdot 5 + 1 \cdot 7 = 22$, while the completion time of the greedy algorithm based on differences is $2 + 7 \cdot 3 = 23$.

4 Prove that every schedule $\hat{\sigma}$ different from the greedy schedule σ has at least one consecutive inversion.

We proof the contrapositive that is we prove that if a schedule $\hat{\sigma}$ has no consecutive inversion then it must be the same as the greedy schedule σ . If we have no schedule with consecutive inversion then that means every job after it is atleast bigger by 1. Since we have n jobs and the max-possible index is n there cannot be a jump of 2 or more between indices of consecutive jobs. Hence the jobs will be schedule as $j_1 < j_2 < \dots < j_n$ which is exactly the same as the greedy algorithm.

5 What effect does the exchange have on the completion item of:
(i) a job other than i or j ; (ii) the job i ; and (iii) the job j ? where $i > j$

1. (i) Not enough information to answer; (ii) goes up; (iii) goes down.
2. (i) Not enough information to answer; (ii) goes down; (iii) goes up.
3. (i) unaffected; (ii) goes up; (iii) goes down.
4. (i) unaffected; (ii) goes down; (iii) goes up.

(3) is correct. Any other job before i and j is unaffected since their completion times don't depend on either i or j . jobs after i and j will also not be affected because regardless of which pairing is chosen all other jobs afterwards will have to wait for both jobs to be completed. i cost must go up because now it has to wait for j to complete before it can be processed and j cost must go down because it no longer has to wait for i to complete.

6 An *inversion* in a schedule is a pair k, m of jobs with $k < m$ and m processed before k . Suppose σ_1 is a schedule with a consecutive inversion i, j with $i > j$, and obtain σ_2 from σ_1 by reversing the order of i and j . How does the number of inversion in σ_2 compare to that in σ_1 ?

1. σ_2 has one fewer inversion than σ_1 .
2. σ_2 has the same number of inversion as σ_1 .
3. σ_2 has one more inversion than σ_1 .
4. None of the other answers are correct.

(1) is the correct answer. (2) and (3) are incorrect because if we fix an inversion we would expect to have one fewer inversion than before. Hence σ_2 must have one fewer inversion than σ_1 .