University of Michigan
Econ 502: Applied Macroeconomics
Winter 2024
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Problem set 3
Due February 19, 2024, 6pm

1. Consider a two-period model with income endowments. Suppose the utility function for each period is given by

$$u(C) = \frac{C^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} \; for \; \gamma \neq 1, \quad \text{and } u(C) = lnC \; for \; \gamma = 1.$$

The budget constraint is

$$C_2 = (1+r)(W_1 - C_1) + W_2$$

   a. Find the Euler equation for the general case in this example (for $\gamma \neq 1$). The discount rate for utility is ρ, introduced just as in class.
   b. How does C_1 depend on r in case when
      i. $\gamma \neq 1$ and $W_2 = 0$?
      ii. $\gamma = 1$ and $W_2 = 0$?
      iii. $\gamma = 1$ and $W_2 > 0$?
   c. Explain intuition for why the dependence in (b) is not the same in all cases

2. Consider a two-period closed-economy model with production. There is only one generation in the model that lives for the two periods. Economy starts with endowment of capital K_1 and produces according to production function $Y_t = F(K_t)$, where t=1, 2. First period production is then split between consumption and investment. Capital stock fully depreciates between periods (depreciation rate is 100%), so next period's capital is equal to this period's investment. There are no methods of saving aside from investing into capital. The utility function is $u(C_t) = \ln C_t$, the discount rate is ρ, introduced just as in class.

   a. Write down the budget constraint, that is, second-period consumption C_2 as a function of K_1 and C_1.
   b. Derive the Euler equation for this economy. Try to make the Euler equation look as close as possible to the one obtained in class. It should link consumptions C_1 and C_2 to each other.
   c. Normally, the Euler equation contains the interest rate. Does the one obtained in (b) contain an interest rate? Why does it or does it not? Explain intuition.
   d. Now suppose that $F(K) = K^{1/2}$, $K_1=1$, and ρ=0. Find exact values of C_1 and C_2.

3. Consider a two-period open economy with endowment income as in class, with slight modification: economy produces a constant amount of output *Y* every period, but its international price P_t varies between periods. The price of the consumption good is normalized to 1 throughout. So think about this economy as a country that produces only oil, which it sells in the international market at volatile price, and then buys consumption goods from abroad. The price P_t is then called the *terms of trade* – price of export relative to the price of import.

The budget constraint is then $C_1 + \frac{1}{1+r} C_2 = P_1 Y + \frac{1}{1+r} P_2 Y$, where r is once again the exogenous world interest rate.

The utility function is logarithmic: u(c) = lnC, the discount factor is ρ, which in general is not equal to r.

a. Derive the expression for the current account in the first period as a function of exogenous variables. Can you tell if it is positive or negative? Explain.
b. Suppose that a country experiences a temporary improvement of the terms of trade – the price P_1 goes up, while P_2 is not expected to increase. What happens to the current account in this situation? Provide intuition.
c. Suppose now the shock is expected to be permanent: P_1 and P_2 go up together by the same amount. What happens to the current account? Again provide intuition.
d. Finally, instead of considering a terms of trade shock, consider an increase in the interest rate r. How would it affect the current account and why? Show on the graph with indifference curves.

4. Read the blog post by Michael Baltensperger, Why China's current account balance approaches zero (bruegel.org), that summarizes the (pre-covid) views on the development of China's current account. Explain what the possible "structural" reasons are for Chinese current account to go down. Do you think the Covid recession altered these trends in any way?