

## STAGGERED PRICES IN A UTILITY-MAXIMIZING FRAMEWORK

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We develop a model of staggered prices along the lines of Phelps (1978) and Taylor (1979, 1980), but utilizing an analytically more tractable price-setting technology. 'Demands' are derived from utility maximization assuming Sidrauski-Brock infinitely-lived families. We show that the nature of the equilibrium path can be found out on the basis of essentially graphical techniques. Furthermore, we demonstrate the usefulness of the model by analyzing the welfare implications of monetary and fiscal policy, and by showing that despite the price level being a predetermined variable, a policy of pegging the nominal interest rate will lead to the existence of a continuum of equilibria.

### 1. Introduction

In this paper we analyze the macroeconomic implications of assuming that (1) nominal individual prices are not subject to continuous revisions, and (2) price revisions are non-synchronous. In order to capture this phenomenon in a simple manner we assume that each price-setter (or firm) is allowed to change his price whenever a random signal is 'lit-up'. We assume that the probability that the signal will be emitted in the next  $h$  periods follows a geometric distribution, is independent of the moment it was emitted in the past, and is also (stochastically) independent across firms.

These assumptions generate, at any given point in time, a non-degenerate distribution of prices of different 'vintages'. In a period analysis only a fraction of firms will receive the signal, the fraction converging to zero as the period shrinks towards zero. Since we assume continuous time, the price level is, therefore, a predetermined variable (as in the standard Keynesian model).

An individual firm is assumed to set its price taking into account the expected average price and the 'state of the market' (given by 'excess demand' here) during the relevant future.

The above assumptions make the present model a close relative of the staggered-contracts model of Phelps (1978) and Taylor (1979, 1980). The main advantage of our formulation is its much greater analytical tractability, and also that it does not really depend on the existence of nominal contracts. Instead, the form in which the price-change 'signal' is emitted is more consistent with a situation where firms are subject to random shocks

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that either prevent them from making continuous price revisions, or, if the latter is not a constraint, that prevent them to observe and verify changes in the 'state of nature' that would otherwise lead to price changes.

We exploit the added simplicity of the price-setting technology by enriching the 'demand side' of the model. In the above-mentioned attempts, aggregate demand was assumed to be an increasing function of, essentially, real monetary balances. In particular, interest effects were not taken into account.<sup>1</sup>

We assume that the household sector is composed of a set of Sidrauski-Brock dynastic families whose (instantaneous) utilities depend on consumption and real money balances, and make their optimal plans under perfect foresight.<sup>2</sup> In this manner, we are able to place the 'staggered-prices' approach in a framework that makes it more directly comparable with Brock (1974), Fischer (1979) and Calvo (1979) — the latter being models of perfect price flexibility.

In section 2 the staggered-prices model is presented. We derive the interesting implication that the 'velocity' of inflation (i.e.,  $\dot{\Pi}$ , where  $\Pi$  is the rate of inflation) is a *decreasing* function of excess demand. Thus, showing that the model involves postulating what might be called a 'higher-order' inverse Phillips Curve.

Section 3 develops the demand side of the model. The main conceptual hurdle there is to decide what is the relevant price level for the representative family, given the fact that the price staggering hypothesis implies the existence of a non-degenerate distribution of prices. This is resolved by assuming that families after-tax price is the same in every firm (via a Price Regulating Mechanism, PRM). This assumption is made in order to isolate the analysis from microeconomic details that do not appear particularly relevant for an aggregate analysis.

Under these assumptions we show the existence and uniqueness of a converging path that satisfies all the axioms of the model.

In section 4 we show the usefulness of the model by demonstrating that a purely monetary policy is a welfare-superior instrument than a policy of expanding government expenditure, and by analyzing the implications of pegging the nominal interest rate. In this respect, we are able to show that the latter will lead to the existence of a continuum of equilibrium paths, implying, therefore that the indeterminacy problem encountered by Sargent and Wallace (1975) in connection with this policy is not just a mere artifact of their perfect-price-flexibility assumption.

<sup>1</sup>An exception is Rehm (1982) who simulated an extended version of Taylor's (1979) model to account for the effect of the real interest rate on demand. See also Calvo (1982) where such an effect is incorporated in an open-economy context.

<sup>2</sup>Perfect foresight is a viable assumption here because despite the uncertainty to which price-setters are subject, it disappears in the aggregate due to the further assumption that there is a 'large number' (more accurately, a continuum) of firms.

An important point to mention here is that all of our qualitative results are independent of the actual average length of price quotations.

## 2. A model of staggered prices

We assume that the production side of the economy is populated by a 'large' number (technically, a continuum) of identical firms. Without loss of generality, we assume that each of them is a point in the  $[0, 1]$  interval, thus making their 'total' equal to unity.

Each firm is capable of producing any amount of output up to a maximum of  $\bar{y}$  units at zero variable cost, and output is assumed to be non-storable. In this fashion, we isolate the analysis from decisions about capital and inventory accumulation, and labor employment, which although important in real life are not essential for our staggered-price story.<sup>3</sup>

We assume that each firm can set its price only in terms of domestic currency, and that it can change it only at the time when a price-change signal is received. The probability (density) of receiving such a signal  $h$  periods from today is assumed to be independent of the last time the firm got it (the signal), and to be given by

$$\delta e^{-\delta h}, \quad \delta > 0. \quad (1)$$

The above is an example of a situation where each firm is forced to announce prices in nominal terms and finds it prohibitively costly to be changing them at every point in time. In previous papers this was justified by introducing a non-zero cost to changing prices [see Barro (1972), Sheshinski and Weiss (1977)] but the emphasis there was in the determination of the optimal time for a price change. In contrast, here as in Phelps (1978) and Taylor (1979, 1980), we let such 'timing' be exogenous. In this connection notice that, by (1), the expected length of a price quotation is  $(1/\delta)$ .<sup>4</sup>

Let us now consider the problem of a firm that is able to change its price at time  $t$  under perfect foresight. It seems quite natural to expect that the firm's decision will be influenced by its forecast of the price set by other firms and aggregate demand. A simple form that would capture this is

$$V_t = \delta \int_t^{\infty} [P_s + \beta E_s] e^{-\delta(s-t)} ds, \quad \beta > 0, \quad (2)$$

<sup>3</sup>However, some extensions like allowing for labor to be a variable input could easily be accommodated within this framework.

<sup>4</sup>The exogeneity of  $\delta$  is, in principle, a not very satisfactory feature of the model. However, it is less restrictive than it may first appear, because all of our results are independent of the exact value of  $\delta$ . In other words, they hold true for any  $\delta > 0$ , as long as  $\delta$  is a constant.

where  $V_t$  is the (log of the) price quotation set at  $t$ ,  $P_s$  is the 'price level' at time  $s$ , or more specifically, the (log of the) geometric mean of outstanding price quotations at time  $s$ . Notice that the perfect-foresight assumption allows us to use actual future values in formula (2).

Formula (2) is in line with the corresponding assumption in Phelps (1978) and Taylor (1979, 1980). In words, it asserts that prices set at  $t$  will be an increasing function of the average price set by competitors ( $P$ ) and excess demand ( $E$ ). Moreover, we weigh  $[P_s + \beta E_s]$  by the probability (density) that the price quotation could be revised at time  $s$ , i.e., recalling (1),  $\delta e^{-\delta(s-t)}$ .

Furthermore, we approximate the price level at  $t$ , by the following expression:

$$P_t = \delta \int_{-\infty}^t V_s e^{-\delta(t-s)} ds. \quad (3)$$

Formula (3) will give us the exact value of  $P_t$  if we further assume that the price-change signal is 'stochastically independent across firms. To see this, notice that since there is a continuum of firms, we can appeal to the 'law of large numbers' to deduce that a number  $\delta$  of firms (also a continuum) will receive the price-change signal per unit of time. By the same principle, of the total number of firms that set their price at  $s < t$ , a share

$$e^{-\delta(t-s)} \quad (4)$$

will not have received the signal at time  $t$ .<sup>5</sup> Therefore,

$$\delta e^{-\delta(t-s)}$$

is the 'number' (i.e., the measure) of firms which set their prices at time  $s$ , and have not yet received a price-change signal at time  $t (> s)$ . If we now define  $P_t$  as the arithmetic average of the  $V$ 's outstanding at  $t$  — weighted by the 'number' of firms with the same  $V$  — (3) follows. See also the appendix for a proof in a discrete-time context.

It is important to realize that under our assumptions  $P_t$  becomes a predetermined variable at time  $t$  — its level being given by past price quotations. On the other hand,  $V_t$  is a function of the entire future, which can only be determined once the demand side of the model is incorporated. It should be noted, however, that, by (2), along a path where  $P$  and  $E$  are uniquely determined,  $V$  is necessarily a continuous function of time.

<sup>5</sup>To see this, note that  $e^{-\delta(t-s)} = \int_{t-s}^{\infty} \delta e^{-\delta s'} ds'$  which, by (1), is the probability that a price quotation at  $s$  will 'survive' for more than  $(t-s)$  periods. The continuum and independence assumptions allow us to conclude that (4) is also the share of those price quotations remaining at  $t$ .

At points in time where  $E_t$  is continuous (which is going to be the rule in our experiments) we can differentiate (2) and (3) with respect to time to get

$$\dot{V}_t = \delta[V_t - P_t - \beta E_t], \quad \dot{P}_t = \delta[V_t - P_t]. \quad (5a, b)$$

Consequently, if we identify the actual (=expected) rate of inflation,  $\Pi$ , with  $\dot{P}$ , i.e.,

$$\Pi_t = \dot{P}_t \quad (6)$$

we get, by (5) and (6),

$$\dot{\Pi}_t = \ddot{P}_t = \delta[\dot{V}_t - \dot{P}_t] = -bE_t, \quad \text{where} \quad (7)$$

$$b = \delta^2 \beta > 0. \quad (8)$$

It is interesting to contrast (7) with more naive models of the Phillips Curve that make  $\Pi$  and increasing function of excess demand,  $E$ ; here, instead, it is  $\dot{\Pi}$  the one that explicitly depends on excess demand, and it does so in a negative manner, i.e., it falls with an increase in excess demand. Interestingly enough, and in line with Taylor's papers, we will be able to show cases where *in equilibrium*  $\Pi$  is connected with  $E$  in the same fashion as is postulated by the more naive formulations.<sup>6</sup>

Given the production conditions postulated at the outset of this section, a profit maximizing firm will set its output equal to demand as long as the latter is less than or equal to  $\bar{y}$ ; otherwise output will just be  $\bar{y}$ . Hence, we have here a model where, as in the standard Keynesian framework, supply is demand-determined during periods of slack capacity.

<sup>6</sup>For the sake of comparison, notice that in the Phelps-Taylor models, contract length is finite, and non-stochastic. Thus denoting the latter by  $\theta$ , (2) becomes

$$V_t = (1/\theta) \int_t^{t+\theta} [P_s + \beta E_s] ds \quad (F.1)$$

while (3) takes the following form:

$$P_t = (1/\theta) \int_{t-\theta}^t V_s ds. \quad (F.2)$$

Differentiating the above expression with respect to  $t$ , leads to a difference-differential equations system with forward and backward lags. In particular,

$$\theta^2 \dot{\Pi}_t = -2[P_t + E_t] + P_{t+\theta} + E_{t+\theta} + P_{t-\theta} + E_{t-\theta} \quad (F.3)$$

and, thus,  $\dot{\Pi}_t$  is also a decreasing function of  $E_t$ , but the expression is now obviously much more complicated.

### 3. An aggregate general equilibrium model

The model discussed in the previous section is incomplete because it cannot be solved unless  $E$ , excess demand, is explicitly modelled. One possible way to close the model is to assume that  $E$  is functionally related to real monetary balances, like in Taylor (1979, 1980); another is to assume that the price-setting mechanism applies to 'home goods', and make  $E$  a function of the 'real exchange rate', like in Calvo (1982). However, our strategy here will be somewhat less direct, and therefore analytically more interesting. We will assume that the households sector is composed by Sidrauski-type families [see Brock (1974), Calvo (1979)] who maximize a discounted sum of (instantaneous) utilities that depend on consumption and real monetary balances with perfect foresight. In this fashion we will be able to *derive* an excess demand function from an optimization process that takes into account the path of all relevant variables over the entire future.

More specifically, we assume that utility of the representative individual or family (as perceived from the 'present',  $t=0$ ) is given by

$$\int_0^{\infty} [u(c_t) + v(m_t)] e^{-\rho t} dt, \quad (9)$$

where  $c$  and  $m$  denote consumption and real monetary balances, and, as usual

$$u(c) \text{ is increasing twice-continuously differentiable and strictly concave for } c > 0, \quad (10a)$$

$$v(m) \text{ is strictly concave and twice continuously differentiable for } m > 0. \quad (10b)$$

Furthermore, in order to insure interior solutions and existence of steady states, we also assume

$$\lim_{c \rightarrow 0} u'(c) = \infty, \quad (11a)$$

$$\lim_{m \rightarrow 0} v'(m) = \infty, \quad \lim_{m \rightarrow \infty} v'(m) \leq 0. \quad (11b)$$

Compared to similar models with flexible prices, we face the additional problem that, since at any point in time firms charge different prices, we have to specify how 'demand' is generated for each firm; and here, of course, we are faced with a full gamut of possibilities ranging from the one where demand is concentrated at the firm charging the lowest price, to the Lucas-type case where consumers concentrate their purchases in one firm, which

each of them chooses in a random and mutually independent manner. An important point to keep in mind, is that in selecting a given set-up one has to make sure that it is in principle compatible with the assumed price-setting behavior on the part of the firms (section 2).

We will assume that there exists a costless 'price-regulation mechanism' PRM which ensures that a consumer pays the same after tax price, whatever the firm at which he realizes his purchases. Furthermore, this uniform price is set equal to (the antilog of)  $P$  (recall section 2); in other words, the consumer price equals the geometric mean of all contract prices. In order to achieve this objective, consumers are subsidized (taxed) if they made their purchases at a firm whose  $V$  is larger (smaller) than  $P$ . In addition, and in order to make the price-setting formula (2) compatible with standard profit-maximizing behavior, the PRM includes a set of taxes that are charged to firms; the latter give disincentives to setting  $V$  larger than  $P$ , with penalties becoming stiffer the smaller is excess demand,  $E$ . Finally, any surplus or deficit associated with the operation of the PRM is covered by lump-sum subsidies or taxes to consumers.

The main advantage of the PRM is that it allows us to abstract from the details of the matching of customers to firms, and thus substitutes the associated diversity of individual situations and price levels by a scenario more appropriate for aggregative analysis where consumers are faced with a uniform price level (the antilog of  $P$ ). In this fashion we will be able to focus more sharply on the central issues.

Assuming that households have equal shares of all the firms, and that money is the only asset, the flow budget constraint for the representative consumer becomes

$$\dot{m}_t = y_t - c_t - \Pi_t m_t + \text{lump-sum subsidies at } t, \quad (12)$$

where  $y_t$  is income at time  $t$ . Thus, the problem faced by the representative family is to maximize (9), choosing the paths of  $c$  and  $m$ , subject to (12), and given  $m_0$  and the paths of  $\Pi$ ,  $y$ , lump-sum subsidies, and any 'quantity constraints' that may arise when aggregate consumption exceeds capacity output.

We will first study this economy under the additional assumption that output is demand determined *even when aggregate demand exceeds capacity*, i.e., when consumers are never subject to quantity constraints. In order to derive necessary and sufficient conditions for the family's optimum, we will apply the methods of Optimal Control [see Arrow and Kurz (1970)]. By (9) and (12), the (undiscounted) Hamiltonian becomes

$$H = u(c_t) + v(m_t) + \lambda_t [y_t - c_t - \Pi_t m_t + \text{lump-sum subsidies at } t], \quad (13)$$

where  $\lambda$  is the co-state variable. Therefore,

$$u'(c_t) = \lambda_t, \quad \dot{\lambda}_t = -v'(m_t) + \lambda_t(\rho + \Pi_t) \quad (14a, b)$$

are necessary conditions for an optimum.

For future reference, notice that if a path  $(c, m)$  satisfies all the constraints, (14), and, in addition  $(c, m)$  converges to a steady state, then by the Sufficiency Theorem of Optimal Control [see Arrow and Kurz (1970)], the  $(c, m)$  path is an optimal solution.<sup>7</sup>

Now, differentiating (14a) with respect to time and equating it to (14b) we get<sup>8</sup>

$$\dot{c} = (-u'(c)/u''(c))[v'(m)/u'(c) - \rho - \Pi]. \quad (15)$$

We are now going to specify the market-equilibrium conditions. In the first place, output market equilibrium requires<sup>9</sup>

$$y = c + g, \quad (16)$$

where  $g$  is government expenditure in goods and services. In order to sharply differentiate between  $c$  and  $g$ , we assume the latter is 'wasteful' consumption, it does not enter into utility functions. Secondly, equilibrium in the money market requires

$$\dot{m} = (\mu - \Pi)m, \quad (17)$$

where  $\mu$  is the rate of expansion of money supply.

Recalling section 2, excess demand,  $E$ , is naturally defined as

$$E = y - \bar{y}. \quad (18)$$

We are now ready to start examining 'the benchmark case' where

$$g = \bar{g}, \quad \mu = \bar{\mu}, \quad \text{constants.} \quad (19a, b)$$

Recalling (7), (15)–(17)

$$\dot{c} = (-u'(c)/u''(c))[v'(m)/u'(c) - \rho - \Pi], \quad (20a)$$

<sup>7</sup>In fact, if the path of subsidies,  $y$  and  $\Pi$  are continuous and  $c$  is constrained to be right-hand continuous, the  $(c, m)$  path described in the text is a unique optimum.

<sup>8</sup>From now on time subscripts will be deleted unless they are strictly necessary.

<sup>9</sup>In order to identify individual with aggregate market variables we assume, without loss of generality, that there is only one household or family.



$$\dot{m} = (\bar{\mu} - \Pi)m, \quad \dot{\Pi} = b(\bar{y} - c - \bar{g}). \quad (20b, c)$$

Consequently, a  $(c, m, \Pi)$  path that satisfies (20) and converges to a steady state satisfies all the behavioral, optimal and market conditions under the assumption that individuals, and firms are endowed with perfect foresight (PF).

We will now characterize the convergent PF path assuming (the proof of the following is in the appendix) uniqueness, and that *all variables converge in a monotonic fashion*.

Notice, first that if  $(\rho + \bar{\mu}) > 0$  — an assumption that will be maintained throughout the analysis unless otherwise asserted — there exists, by (10) and (11), a unique steady state where

$$\Pi = \bar{\mu}, \quad c + \bar{g} = \bar{y}, \quad \frac{v'(m)}{u'(\bar{y} - g)} = \rho + \bar{\mu}. \quad (21a, b, c)$$

Secondly, at any time  $t$  the only predetermined variable is  $m_t$ , for, all the others (i.e.,  $c$  and  $\Pi$ ) are allowed to 'jump', i.e., are allowed to instantaneously 'jump' to their equilibrium levels. Thus, if the solution is unique,  $\Pi$  and  $c$  ought to be uniquely related to  $m$ . We indicate this relationship by

$$\Pi = \Pi^*(m), \quad c = c^*(m). \quad (22a, b)$$

In particular, indicating steady-state levels of variables by an 'upper' bar, we get, recalling (21)

$$\Pi^*(\bar{m}) = \bar{\mu}, \quad c^*(\bar{m}) = \bar{y} - \bar{g}. \quad (23a, b)$$

Our objective is now to characterize the slope of the above functions.

Consider the case where  $m > \bar{m}$ . Since variables converge to their steady-state values in a monotonic fashion, it follows from (20b) and (23a) that

$$\Pi^*(m) > \bar{\mu} = \Pi^*(\bar{m}), \quad (24)$$

showing that  $\Pi^*(m)$  increases with  $m$ .

Now using (23a) in (20a and b), and employing the just proved condition that  $\Pi$  increases with  $m$  along and equilibrium path, we get the phase diagram depicted in fig. 1.

The arrowed curve shows the unique equilibrium relationship between  $c$  and  $m$  (assuming right-continuity of  $c$  with respect to time, see footnote 7). In the first place note that the region in fig. 1, where  $m < \bar{m}$  is one of excess capacity, and that if the economy follows the arrowed curve to the steady

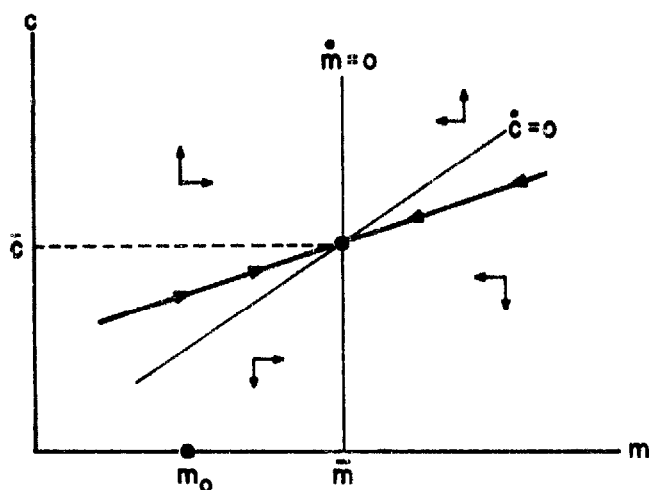


Fig. 1. Determination of equilibrium  $(c, m)$  pairs when output is demand-determined.

state it will never transverse the excess demand region (where  $m > \bar{m}$ ). It is then easy to convince oneself that the resulting path satisfies, in addition, the supply condition of section 2, namely that output is demand determined only when demand is less than or equal to capacity. Therefore, we have been able to characterize equilibrium when the economy starts at or below full equilibrium.

The region to the right of  $\bar{m}$  in fig. 1 contradicts the postulate of section 2, because output would have to exceed  $\bar{y}$ . However, by construction, we know that it would give us the demand that would be generated if output was perfectly elastic. Thus, it gives us a possible measure of 'notional demand'. If we adopt it, we would be saying that the relevant excess demand for firms is given by  $[c^*(m) + g - \bar{y}]$ , in which case the associated  $(\Pi, m)$  path would exactly coincide with that implied by the arrowed curve in fig. 1; the  $c$  path however would have to be flat and equal to  $\bar{y}$ .

In what follows we will concentrate our attention on the excess-supply region.

An interesting result that was advanced in section 2, is that along an equilibrium path there exists a positive association between the inflation rate and excess demand, the Phillips Curve; but it should also be clear that this Phillips-type relationship is, contrary to naive formulations, a function of policy and all the parameters of the system.

It is easy to see that there is an (implicit) nominal interest rate,  $i$ , and that in equilibrium<sup>10</sup>

$$i = v'(m)/u'(c). \quad (25)$$

<sup>10</sup>This can be shown by introducing an instantaneous-maturity bond yielding no liquidity or other services except for a nominal interest rate  $i$ .

Now, since by monotonicity and the fact that  $c$  is, in equilibrium, an increasing function of  $m$ , it follows that, at equilibrium

$$\dot{c} > 0 \quad \text{as } m < \bar{m} \quad (26)$$

which, by (20a) and (25) implies

$$i - \Pi > \rho \quad \text{as } m < \bar{m}. \quad (27)$$

In words, in periods of slack capacity the 'real' rate of interest (i.e.,  $i - \Pi$ ) will be larger than in full equilibrium. This is similar to what is obtained in a standard IS-LM framework when the system is perturbed from full equilibrium by a fall of  $m$ .

#### 4. Effect of policy

The model developed in the previous sections is a striking example where the 'money matters' hypothesis acquires full force. This can be seen in fig. 1; for, imagine that we start from an excess-supply situation, like when  $m = m_0$ ; then it is quite clear that a once-and-for-all unanticipated increase in money supply could immediately drive the system to full employment by shifting  $m$  to  $\bar{m}$ . As a matter of fact the first best could be attained by, in addition, announcing that the rate of expansion of money supply,  $\mu$ , will be such that

$$\mu = -\rho. \quad (28)$$

For, by (21c), this would imply that at steady state

$$v'(m) = 0 \quad (29)$$

yielding the optimal quantity of money.

Fiscal policy is also an effective but Pareto-inferior tool. Imagine, again, that we start at a point like  $m_0$  in fig. 1, and that government expenditure,  $g$ , is set so as to cover the full-employment gap; thus

$$g_t = \bar{y} - c_t \quad \text{for all } t. \quad (30)$$

Therefore, by (20c),

$$\dot{\Pi}_t = 0 \quad \text{for all } t \quad (31)$$

and, hence, for convergence of  $m$  [see (20b)]

$$\Pi_t = \bar{\mu} \quad \text{for all } t, \quad \text{implying} \quad (32)$$

$$m_t = m_0 \quad \text{for all } t. \quad (33)$$

Thus, by (20a), (32), and (33),

$$\dot{c} = (-u'(c)/u''(c))[v'(m_0)/u'(c) - \rho - \bar{\mu}] \quad (34)$$

which is a first-order differential equation in  $c$ . At steady state,

$$\partial \dot{c} / \partial c = v'(m_0)/u'(c) = \rho + \bar{\mu} > 0. \quad (35)$$

Hence, convergence of  $c$  requires setting  $c$  at its steady-state level where, by, (35),

$$v'(m_0) = (\rho + \bar{\mu})u'(c). \quad (36)$$

Since, before the above policy is implemented, the steady-state values of  $m$  and  $c$  ( $\bar{m}$  and  $\bar{c}$ , respectively) also satisfy

$$v'(\bar{m}) = (\rho + \bar{\mu})u'(\bar{c}) \quad (37)$$

and, recalling fig. 1,  $\bar{m} > m_0$ , the strict concavity of  $u(\cdot)$  and  $v(\cdot)$  imply that after the policy is implemented

$$c_t < \bar{c}. \quad (38)$$

Hence, the policy requires a once-and-for-all increase in  $g$ , which under our assumptions, implies that the level of utility will be permanently lower than the one attained with the monetary policy discussed at the outset of this section.

Examples of the applicability of the present framework could be easily multiplied. For the sake of brevity, however, the rest of this section will be devoted to the discussion of a fundamental indeterminacy that arises if the monetary authority attempts to peg the nominal interest rate at a predetermined level.

Sargent and Wallace (1975) studied this case in the context of an ad-hoc flexible prices model, and showed that a constant-interest-rate policy gives rise to indeterminacy of the price level. To see this in a simple way let us, momentarily, write the money market equilibrium as

$$M - P = L(i),$$

where the left-hand side stands for the supply of (the log of) real monetary balances ( $M$  being the log of money supply), and the right-hand side is its demand;  $\bar{i}$  stands for the policy-determined interest rate. Clearly, since  $P$  is free to jump and  $M$  is adjusted to keep  $i = \bar{i}$ , the value of  $P$  is undetermined.

The previous example may give the impression that such indeterminacy is linked to the price flexibility assumption (or, perhaps, also to the ad-hoc nature of the model). As we now show, however, although in our model  $P$  is a predetermined variable, setting  $i \equiv \bar{i}$  leads to a higher-order indeterminacy, namely, there exists a multiplicity of equilibrium rates of inflation consistent with the model.

The proof is straightforward. By (25)

$$i = v'(m)/u'(c) = \bar{i}. \quad (39)$$

Thus, system (20) becomes

$$\dot{c} = -u'(c)/u''(c)[\bar{i} - \rho - \Pi], \quad (40a)$$

$$\dot{\Pi} = b(\bar{y} - c - \bar{g}) \quad (40b)$$

and  $m$  is determined by (39).

The phase diagram for this system is depicted in fig. 2.

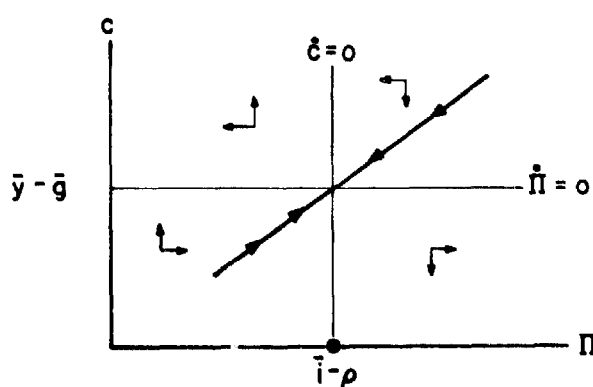


Fig. 2. Equilibrium  $(c, \Pi)$  pairs with  $i \equiv \bar{i}$ .

Clearly, any  $(c_0, \Pi_0)$  pair that lies on the arrowed curve is consistent with the assumption of equilibrium and convergence to a steady state; but since these variables are free to take on any value at  $t=0$ , it follows that the system is fundamentally undetermined.<sup>11</sup>

<sup>11</sup>The reader is invited to study the case where  $\mu_t = \alpha(i_t + \bar{i}) + \bar{i} - \rho$ ,  $\alpha > 0$ , i.e., where the rate of monetary expansion is an increasing function of the difference between the actual and target interest rate. Surprisingly enough, it is possible to extend (with no change) our recent results for an ad-hoc flexible price model [Calvo (1982b)], according to which non-uniqueness prevails if  $\alpha > 1$ ; but if  $\alpha < 1$  uniqueness always holds.

## Appendix

A.1. We will show that eq. (5b) can be seen as the limit of a discrete-time version of the model (when the number of price setters tends to infinity and the length of the period converges to zero).

Let

$$\rho \equiv \begin{array}{l} \text{probability that a price quotation} \\ \text{will expire in the next period} \end{array} \equiv \frac{\delta k}{1 + \delta k}, \quad (\text{A.1})$$

where  $k$  is the length of a period, and  $\delta$  is a positive number. In order to show that this probability model converges to the one given in the text when  $k \rightarrow 0$ , notice that the probability that a price quotation will expire within the next  $h$  units of time (where  $h$  is a multiple of  $k$ ) is

$$1 - (1 - \rho)^{h/k} = 1 - \left( \frac{1}{1 + \delta k} \right)^{h/k}. \quad (\text{A.2})$$

Letting  $k \rightarrow 0$ , the above expression is easily seen to converge to

$$1 - e^{-\delta h}, \quad (\text{A.3})$$

the p.d.f. of (A.3) is clearly (1) in the text.

Let

$\bar{N}$  = total (fixed) number of price setters,

$N(t, s)$  = number of price setters that set the price at time  $s$ , and have not been able to change it before time  $t$  ( $\geq s$ ).

If we assume, as in the text, that the price-change signals are mutually independent random variables across price setters, it follows from the 'law of large numbers' that as  $\bar{N} \rightarrow \infty$ , the number of firms that receive the price-change signal at any given time,  $t$ ,  $N(t, t)$ , tends to

$$N(t, t) = \rho \bar{N}. \quad (\text{A.4})$$

An implication of the above is that the number of price setters in any period tends to infinity together with  $\bar{N}$ ; thus, we can again apply the 'law of large numbers' to conclude that

$$N(t + h, s) \rightarrow (1 - \rho)N(t, s) \quad \text{as} \quad \bar{N} \rightarrow \infty. \quad (\text{A.5})$$

We define  $P$  and  $V$  as in the text. Thus

$$P_t = \frac{1}{\bar{N}} \sum_{j=0}^{\infty} V_{t-kj} N(t, t-kj). \quad (\text{A.6})$$

Thus, by (A.1), (A.4), (A.5) and (A.6), we have that

$$(P_{t+k} - P_t)/k \rightarrow \frac{\delta}{1 + \delta k} (V_{t+k} - P_t) \quad \text{as } \bar{N} \rightarrow \infty. \quad (\text{A.7})$$

Now, letting  $\bar{N} \rightarrow \infty$  and  $k \rightarrow 0$ , we recover (5b). Q.E.D.

A.2. We will show that in a neighborhood of the steady state of system (20), and given  $m_0$ , there exists a unique path that converges to the steady state. This requires showing that the matrix associated with the linear approximation at the steady state has one negative root and two roots with positive real parts,<sup>12</sup>

$$A \equiv \begin{bmatrix} \partial \dot{c} / \partial c, & \partial \dot{c} / \partial m, & \partial \dot{c} / \partial \Pi \\ \partial \dot{m} / \partial c, & \partial \dot{m} / \partial m, & \partial \dot{m} / \partial \Pi \\ \partial \dot{\Pi} / \partial c, & \partial \dot{\Pi} / \partial m, & \partial \dot{\Pi} / \partial \Pi \end{bmatrix} = \begin{bmatrix} \rho + \bar{\mu}, & -\frac{v''}{u''}, & \frac{u'}{u''} \\ 0, & 0, & -\bar{m} \\ -b, & 0, & 0 \end{bmatrix}. \quad (\text{A.8})$$

Let  $\theta_i$ ,  $i=1,2,3$ , be the characteristic roots of  $A$ . As is well known [see Gantmacher (1956)],

$$\theta_1 \theta_2 \theta_3 = \det A = -b(v''/u'')\bar{m} < 0, \quad (\text{A.9a})$$

$$\theta_1 + \theta_2 + \theta_3 = \text{Tr } A = \rho + \bar{\mu} > 0, \quad (\text{A.9b})$$

$$\theta_1 \theta_2 + \theta_1 \theta_3 + \theta_2 \theta_3 = b(u'/u'') < 0. \quad (\text{A.9c})$$

By (A.9a) and (A.9c), one root, say  $\theta_1$ , is real and negative, and  $\theta_2$  and  $\theta_3$  have non-negative real parts; but, by (A.9b), we can also rule out the case where  $\theta_i$ ,  $i=2,3$ , have zero real parts. Q.E.D.

Let  $h=(h_1, h_2, h_3)$  be a characteristic vector associated with the negative root,  $\theta_1$ . Then the convergent solution (for the linear system) takes the following form:

$$(c - \bar{y}, m - \bar{m}, \Pi - \bar{\mu}) = \eta h e^{\theta_1 t}, \quad (\text{A.10})$$

<sup>12</sup>For a related problem see Calvo (1979), Fischer (1979).

where  $\eta$  is chosen so that

$$\eta h_2 = m_0 - \bar{m}$$

implying that the linear system converges in a monotonic way. Given our regularity conditions the latter is also a property of the non-linear system [see Coddington and Levinson (1955)], for some neighborhood of the steady state.

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