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Real-Time Dynamic Pricing for Revenue Management with Reusable Resources, Advance Reservation, and Deterministic Service Time Requirements

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Abstract. We consider a dynamic pricing problem in a system with reusable resources. Customers arrive randomly over time according to a specified *nonstationary* rate, and each customer requests a service that uses a combination of different types of resources for a *deterministic* duration of time. The resources are *reusable* in the sense that they can be immediately used to serve a new customer on the completion of the previous service. Our objective is to construct a dynamic pricing control that maximizes expected total revenues. This is a fundamental problem faced by firms in many industries. We develop real-time heuristic controls based on the solution of the deterministic relaxation of the original stochastic problem and show that they are near optimal in the regime of large demand and large resource capacity. We further show that our results can be extended to a more general setting with heterogeneous service time and advance reservation.

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Keywords: dynamic pricing • reusable resources • asymptotic analysis

1. Introduction

Consider a firm managing a finite number of different types of resources to serve time-varying (nonstationary), price-dependent demand over a finite selling horizon. Each arriving customer requests a service that uses a combination of resources for a consecutive and *deterministic* duration of time (that is, deterministic *service time* requirement). If all of the required resources are available at the time of arrival, the new customer is immediately admitted into the system at the current price, and the service is immediately started without delay; otherwise, the customer will leave the system without waiting. After a service is completed, the corresponding resources are released and can be directly used to satisfy a new demand (that is, resources are *reusable*). The firm's objective is to maximize her expected total revenues throughout the selling horizon by setting prices dynamically.

The setting described above can be used to model many real-life business and operations problems. For example, the model has application in the emerging cloud computing business. Cloud computing service providers usually have a fixed amount of computational resources and lease their available resources to customers who arrive randomly over time with specific requests on usage time and capacity requirements. Because demand tends to be nonstationary (Borgs et al. 2014), one of the key operational challenges is how to dynamically match time-varying demand with the available pool of reusable resources. This has motivated many researchers to investigate dynamic pricing in various settings (Arshad et al. 2015). Aside from cloud computing service providers, our model can also be used to describe the operational process of firms managing *physical* resources. These include classic revenue management (RM) applications, such as car rental and hotel reservation, and also, newer applications, such as those in the context of many emerging on-demand service firms. On-demand service firms control a finite number of resources and offer them to customers who either book services in advance or on the spot through either the web or smartphone interface. Demands are usually characterized by both a specified service type and an intended usage time (that is, the starting and ending times of the service), and existing digital user interfaces enable firms to effortlessly adjust prices to manage demand. Some on-demand service firms have already used dynamic pricing on a daily basis: Project44 provides dynamic pricing solutions to a third-party freight transport company, Tock provides ticketing systems to high-end restaurants, and MyTime offers online scheduling solutions with dynamic pricing capability to local service businesses.

Our model captures the key operational tradeoffs faced by all of the aforementioned firms. On the demand side, arrival rates are typically nonstationary. On the supply side, the utilization of limited resources needs to be carefully controlled. The main technical challenge is how to properly balance resource utilization across different *service cycles*. Because the number of available resources is finite, resource utilization in one cycle directly affects resource utilization in the subsequent cycle. This dependency introduces a subtlety that is not present in the canonical RM model (see Section 2).

1.1 Main Contribution

In this paper, we propose two provably near-optimal and easily implementable heuristic controls. Our first heuristic control applies a precomputed price for each service type regardless of the actual demand realizations (that is, except when there is not enough resource available to serve the customer); our second heuristic control builds on the first heuristic control and dynamically adjusts the price in *real time* (using a simple price update rule without having to run an explicit reoptimization) based on the observed demand realizations. We show that the average losses of these heuristic controls are $\tilde{O}(\theta^{-\frac{1}{2}})$ and $\tilde{O}(\theta^{-\frac{2}{3}})$, respectively, where θ is the *size* of the problem. To the best of our knowledge, our adaptive control is the first control that guarantees an asymptotic performance of order $o(\theta^{-\frac{1}{2}})$ in the setting of reusable revenue management (RRM), especially in the presence of nonstationary arrivals and deterministic service time assumptions.

1.2. Organization of the Paper

The related literature is reviewed in Section 2. In Section 3, we formulate the basic model with homogeneous service time and no advance reservation. We introduce our heuristic controls in Sections 4 and 5. In Section 6, we discuss an extension to the setting with heterogeneous service time and advance reservation. Finally, in Section 7, we conclude the paper. The proof of all results can be found in the e-companion.

2. Literature Review

Our work in this paper is broadly related to three streams of literature: the canonical pricing and revenue management (PRM), the RRM, and the queueing and service operations. Because of space limitation, we will only briefly discuss each of them below.

Within PRM literature, our work is related to papers that develop provably near-optimal heuristic controls (Gallego and Van Ryzin 1997, Reiman and Wang 2008, Atar and Reiman 2012, Jasin and Kumar 2012, Golrezaei et al. 2014, Jasin 2014, Gallego et al. 2015, Chen et al. 2015, Lei et al. 2018). Among the existing PRM literature, our work is closest to Jasin (2014) and Chen et al. (2015) in that they also develop provably near-optimal real-time dynamic pricing controls for multiproduct and multiresource settings that do not require any reoptimization at all. Our work differs from their works in that they assume the canonical price-based RM model of Gallego and Van Ryzin (1997), whereas we consider the setting of RRM. As noted before, one of the key challenges in the RRM setting is that we need to properly balance resource utilization across different service cycles; in contrast, in the canonical PRM setting, a key assumption is that all resources are consumed at some time after the end of the selling horizon (that is, there is only one service cycle) instead of in a continuous manner, where a service can start and end at some times *within* the selling horizon. To further highlight this, in Section EC.5 in the e-companion, we show numerically that a heuristic control that performs very well in the canonical PRM setting may not perform as well in the RRM setting.

There has recently been a growing body of literature that studies the optimal control problem in the context of RRM. Existing works on RRM that study a single-resource setting include those by Levi and Radovanovic (2010), Xu and Li (2013), Borgs et al. (2014), Chen and Shi (2017), and Chen et al. (2017). The three papers by Levi and Radovanovic (2010), Chen and Shi (2017), and Chen et al. (2017) study admission control in a stationary infinite horizon setting with a general service time, and the authors develop provably near-optimal nonadaptive heuristic controls with constant factor performance guarantees. Xu and Li (2013) study some properties of the optimal pricing control under the assumption of exponential service time, and Borgs et al. (2014) consider a deterministic demand model with a preannounced price trajectory where customers can strategically choose their purchase timing. To the best of our knowledge, the only existing works on RRM that consider multiple types of reusable resources are the recent works of Owen and Simchi-Levi (2017) and Rusmevichientong et al. (2020). Both papers consider the joint assortment and pricing (with discrete price points) problem under a general choice model and provide constant factor performance guarantees for their proposed controls: Owen and Simchi-Levi (2017) analyze a randomized static control under the exponential service time, and Rusmevichientong et al. (2020) develop approximate dynamic programming-based heuristic controls. Compared with these papers, we propose heuristic controls that are applicable to the case of deterministic service time with advance reservation, and we characterize asymptotic performances of our heuristic controls.

In the queueing control literature, the continuous time version of our model is often called the *loss system*. Pricing decision in the loss system has been studied extensively under various settings (Lanning et al. 1999, Courcoubetis et al. 2001). These works use a deterministic approximation of the original stochastic problem to construct different heuristic controls, and most of them assume stationary arrival and exponential service time. An exception to this is the work by Hampshire et al. (2009), which develops a dynamic pricing control for nonhomogeneous Poisson arrival process under quality-of-service constraints. They show numerically that their control performs better than static or myopic pricing control. Aside from the literature on loss model, dynamic pricing has also been studied in the setting of delay model under stationary arrival assumption (Chen and Frank 2001, Yoon and Lewis 2004). Several papers study asymptotically optimal dynamic pricing controls (Maglaras and Zeevi 2005, Ata and Olsen 2009, Kim and Randhawa 2017). Closest to our work is the work by Kim and Randhawa (2017), which proposes a heuristic control that dynamically adjusts the baseline control given by a fluid approximation and shows that the average loss of their heuristic is $\tilde{O}(\theta^{-\frac{2}{3}})$. Aside from not permitting customers to wait, our model is different from the above cited works adopting asymptotic analysis in terms of the deterministic service time requirement and the nonstationary demand assumption. We choose to explicitly address these modeling differences, because it is more appropriate in most of the applications that we consider in Section 1. Thus, our work complements existing works in the queueing literature.

3. Basic Model

We consider a monopolistic firm that manages I types of resource and provides J types of service to customers. The selling horizon is divided into T periods, and at most, one new request arrives in each period. A request for service type j requires $a_{ij} \in \{0,1\}$ unit of resource type i for n consecutive periods. Without loss of generality, we assume that $\sum_i a_{ij} > 0$ for all j and $T/n \in \mathscr{Z}^+$. A service started in period t will be completed at the end of period $t+n-1$, and after completion, the corresponding resources are immediately available for a new service in period $t+n$. For convenience, we call any consecutive n periods a service cycle. We will also call matrix $A = [a_{ij}]$ the *resource consumption matrix*, and its jth column A^j is the *resource consumption vector* for product j. Initial capacity of resources is given by the vector $\boldsymbol{C} = (C_i) \geq \boldsymbol{e}$, where $\boldsymbol{e}$ is a vector of ones with a proper dimension. (Unless otherwise noted, all vectors are to be understood as column vectors.)

At the beginning period t, the firm posts a price vector $\boldsymbol{p}^t = (p_j^t)$, which in turn, induces an arrival of customer request. Demand rate in period t is given by $\boldsymbol{\lambda}^t(p^t) = (\lambda_j^t(p^t))$, and its corresponding revenue rate is given by $r^t(p^t) = (\boldsymbol{p}^t)^\top \boldsymbol{\lambda}^t(\boldsymbol{p}^t) = \sum_j p_j^t \cdot \lambda_j(p^t)$. Let $\boldsymbol{D}^t(\boldsymbol{p}^t) = (D_j^t(\boldsymbol{p}^t))$ denote the realized demand vector in period t under $\boldsymbol{p}^t$. By definition, $\mathbf{E}[\boldsymbol{D}^t(\boldsymbol{p}^t)] = \boldsymbol{\lambda}^t(\boldsymbol{p}^t)$ and $\mathbf{E}[(\boldsymbol{p}^t)^\top \boldsymbol{D}^t(\boldsymbol{p}^t)] = r^t(\boldsymbol{p}^t)$. Thus, $\lambda_j^t(\boldsymbol{p}^t)$ can be interpreted as the arrival probability of a new request for service j in period t under price $\boldsymbol{p}^t$. It is typically assumed in the literature that demand rate is invertible in price (see Assumption 1 below). Thus, by abuse of notation, we will also use $\boldsymbol{D}^t(\boldsymbol{\lambda}^t) = \boldsymbol{D}^t(\boldsymbol{p}^t(\boldsymbol{\lambda}^t))$ and $r^t(\boldsymbol{p}^t) = (\boldsymbol{p}^t)^\top \boldsymbol{\lambda}^t(\boldsymbol{p}^t) = (\boldsymbol{\lambda}^t)^\top \boldsymbol{p}^t(\boldsymbol{\lambda}^t) = r^t(\boldsymbol{\lambda}^t)$ to denote the direct dependency of realized demand and revenue rate on demand rate instead of on price (we use $\boldsymbol{p}^t(\cdot)$ to denote the inverse of $\boldsymbol{\lambda}^t(\cdot)$).

Let Ω_p and $\Omega_\lambda = [0, \lambda_U]^J$ (for some $\lambda_U \in (0,1]$) denote the convex and compact feasible set of price and demand rate, respectively. Let $\mathbf{int}(S)$ denote the strict interior of the set S. Below, we state some standard regularity conditions (Gallego and Van Ryzin 1997) on $\lambda^t(\cdot)$ and $r^t(\cdot)$.

Assumption 1. *$\boldsymbol{\lambda}^t(p^t): \Omega_p \to \Omega_\lambda$ is bounded, twice differentiable, and invertible.*

Assumption 2. *For each j, there exists a "turnoff" price $\bar{p}_j < \infty$ such that, for any feasible sequence $\{p^{t,(v)}\}$, $p_j^{t,(v)} \to \bar{p}_j$ implies $\lambda_j^t(\boldsymbol{p}^{t,(v)}) \to 0$.*

Assumption 3. *For any feasible sequences $\{\boldsymbol{\lambda}^{t,(v)}\}$, $\lambda_j^{t,(v)} \to 0$ implies that $\lambda_j^{t,(v)} \cdot p_j^t(\boldsymbol{\lambda}^{t,(v)}) \to 0$.*

Assumption 4. *$r^t(\boldsymbol{\lambda}^t)$ is bounded, is strictly concave, and has an interior maximizer $\boldsymbol{\lambda}^{t,u} \in \mathbf{int}(\Omega_\lambda)$.*

The turnoff price $\bar{p}_j$ can be used to effectively turn off demand whenever needed. Although the theoretical turnoff price can be infinite (for example, when $\lambda^t(p^t) = a \cdot e^{-p^t}$), because real-world price is never infinite, we can assume without loss of generality that $\bar{p}_j < \infty$.

3.1. Problem Formulations

Let Π denote the set of all nonanticipating controls, and let $\boldsymbol{p}^{t,\pi}$ denote the price to be applied during period t under control $\pi \in \Pi$. For any integer x, let $(x)^+ = \max\{1, x\}$. The optimal stochastic control formulation of our dynamic pricing problem is given below:

$$\textbf{OPT}: \quad J^* = \left\{ \max_{\pi \in \Pi} \mathbf{E}\left[\sum_{t=1}^{T} R^t(\boldsymbol{p}^{t,\pi})\right] : \sum_{s=(t-n+1)^+}^{t} A\boldsymbol{D}^s(\boldsymbol{p}^{s,\pi}) \leq \boldsymbol{C} \text{ for all } t \leq T \right\},$$

where the constraints must hold almost surely or with probability one. To understand the intuition behind the above constraints, note that the amount of available resources at the beginning of period t is given by $\boldsymbol{C} - \sum_{s=(t-n+1)^+}^{t-1} A\boldsymbol{D}^s(\boldsymbol{p}^{s,\pi})$. Here, we only need to consider total demands that arrive in the previous $n-1$ periods, because any resource being requested in period $s < (t-n+1)^+$ must already complete its assigned service and is either at an idle state at the beginning of period t or currently being used to satisfy a new request arriving in period $s \in [(t-n+1)^+, t-1]$ (by abuse of notation, we use $[t_1, t_2]$ to denote the set $\{t_1, t_1+1, \ldots, t_2\}$). This explains our constraints in **OPT**.

The exact stochastic formulation **OPT** is in general difficult to solve because of the curse of dimensionality of dynamic programming. The following is the deterministic analogue of **OPT**:

$$\textbf{DET}: \quad J^D = \left\{ \max_{p^t \in \Omega_p} \sum_{t=1}^{T} r^t(\boldsymbol{p}^t) : \sum_{s=(t-n+1)^+}^{t} A\boldsymbol{\lambda}^s(\boldsymbol{p}^s) \right. \\ \left. \leq \boldsymbol{C} \text{ for all } t \leq T \right\}.$$

Because demand rate is invertible in price (by Assumption 1), we can rewrite **DET** using demand rates instead of prices as the immediate decision variables as follows: $J^D = \{\max_{\lambda^t \in \Omega_\lambda} \sum_{t=1}^{T} r^t(\boldsymbol{\lambda}^t) : \sum_{s=(t-n+1)^+}^{t} A\boldsymbol{\lambda}^s \leq \boldsymbol{C}$ for all $t \leq T\}$. Let $\boldsymbol{p}^D := \{\boldsymbol{p}^{t,D}\}_{t=1}^T$ denote the optimal solution of **DET**, and $\boldsymbol{\lambda}^D := \{\boldsymbol{\lambda}^{t,D}\}_{t=1}^T$ denote the corresponding optimal demand rates (that is, $\boldsymbol{\lambda}^{t,D} = \boldsymbol{\lambda}^t(\boldsymbol{p}^{t,D})$). We assume without loss of generality that either $\lambda_j^{t,D} = 0$ or $\lambda_j^{t,D} \in (0, \lambda_U)$ for all j and t. This is a direct consequence of Assumption 4. The argument is straightforward. If $\lambda_j^{t,D} = \lambda_U$, by Assumption 4 and the unimodality of $r^t(\lambda)$, we can find $\tilde{\lambda}^{t,D}$ such that $r^t(\tilde{\lambda}^{t,D}) = r^t(\lambda^{t,D})$ and $\tilde{\lambda}_j^{t,D} \leq \lambda_j^{t,u} < \lambda_U$. Define $\varphi_L = (1/2) \cdot \min_{j,t:\lambda_j^{t,D}>0} \lambda_j^{t,D}$ and $\varphi_U = (1/2) \cdot (\lambda_U - \max_{j,t} \lambda_j^{t,D})$. Given the argument above, φ_L and φ_U are both strictly positive constant. We state some boundedness assumptions, which are easily verifiable for many commonly used demand functions.

Assumption 5. *There exists positive constant Ψ such that $\left|\frac{\partial r^t(\lambda)}{\partial \lambda_j}\right|$, $\left|\frac{\partial p_j^t(\lambda)}{\partial \lambda_j}\right|$, and the absolute eigenvalues of the Hessian matrix of $r^t(\boldsymbol{\lambda})$ are bounded from above by Ψ on $\Omega_\lambda \cap [\boldsymbol{\lambda}^{t,D} - \varphi_L \mathbf{e}, \boldsymbol{\lambda}^{t,D} + \varphi_U \mathbf{e}]$.*

Let R^π denote the total revenues collected under control π throughout the selling horizon. We define the *average loss* or simply, *loss* of a feasible control π as follows: $\mathcal{L}(\pi) = \frac{J^D - \mathbf{E}[R^\pi]}{T}$. The following lemma is the analogue of the standard result in the RM literature (Jasin 2014).

Lemma 1. Under Assumptions A1–A4, $J^* \leq J^D$.

3.2. Asymptotic Setting

In this paper, we are interested in the performance of heuristic controls in a system with large demand and large resource capacity. To do this, we consider a sequence of systems indexed by a scaling parameter θ, where the parameters in the θth system are given by

$$T^{(\theta)} = \theta \cdot T, \quad \boldsymbol{C}^{(\theta)} = \theta \cdot \boldsymbol{C}, \quad n^{(\theta)} = \theta \cdot n, \quad \text{and} \\ \boldsymbol{\lambda}^{t,(\theta)}(\cdot) = \boldsymbol{\lambda}^{\lceil t/\theta \rceil}(\cdot). \tag{1}$$

(Throughout the remainder of the paper, we will consistently use the superscript (θ) as a reference to the θth system.) Because we assume at most one arrival per period, the scaling in the length of selling horizon and in the service time reflects a change in timescale rather than in the actual length of selling horizon and service time. In other words, a scaled system has the same absolute length of selling horizon and service time compared with the unscaled system, but the length of one period is smaller than that in the unscaled system. Consequently, the scaling in the service time should be interpreted as increasing the demand arrival intensity as well as the frequency at which firm makes the pricing decisions. We define the average loss of a control π in the θth system as $\mathcal{L}(\pi, \theta) := (J^{D,(\theta)} - \mathbf{E}[R^{\pi,(\theta)}])/T^{(\theta)}$. We state a lemma.

Lemma 2. *For any $\theta \geq 1$, $\boldsymbol{\lambda}^{t,(\theta),D} = \boldsymbol{\lambda}^{\lceil t/\theta \rceil, D}$.*

One implication of Lemma 2 is that, in the fluid model, the total demand within any service cycle is on the order of $\Theta(\theta)$. Moreover, it is easy to see that $J^{D,(\theta)} = \theta \cdot J^D$.

4. Deterministic Price Control

In this section, we discuss a simple heuristic control called *deterministic price control* (DPC). Let C^t denote the vector of available resources at the beginning of period t: that is, after the service that terminates at the end of period $t-1$ has released the occupied resources and before the firm sets a new price $\boldsymbol{p}^t$. Let $|S|$ denote the cardinality of set S. For each service type j, define

$$\underline{n_j} := \min_{t \in [1, T-n+1]: \sum_{s=t}^{t+n-1} \lambda_j^{s,D} > 0} \\ \cdot \left| \left\{ \lambda_j^{s,D} : \lambda_j^{s,D} > 0, s \in [t, t+n-1] \right\} \right|. \tag{2}$$

By definition, $\underline{n_j}$ is the minimum number of nonzero $\lambda_j^{t,D}$ values within any service cycle during which there is a positive demand for service type j (under λ^D). By definition, $\underline{n_j} > 0$. Unlike in the canonical RM model, where the deterministic optimal solution is typically strictly positive (for example, assumption A5 in Jasin 2014), in RRM, it is not uncommon that $\lambda_j^{t,D} = 0$ for some j and t, even for some commonly used demand functions. See Section EC.6 in the e-companion for an example of this. Let $\underline{n} = \min_j \underline{n}_j$. Without loss of generality, we will assume that the number of periods within any service cycle where $\lambda_j^{t,D} > 0$ (for some j) must be at least $\underline{n}$: that is,

$$\min_{t\in[1,T-n+1]}\left|\left\{\lambda_j^{s,D} : \lambda_j^{s,D} > 0, s \in [t, t+n-1], j\in[1,J]\right\}\right| \geq \underline{n}.$$

To see why the above assumption is without loss of generality, note that, if the above inequality does not hold, by the definition of $\underline{n}$, there must exist a service cycle during which, in the deterministic system, there is no demand for any type of service at any time period within the cycle at all. If this is indeed the case, in the performance analysis of DPC, one can simply treat the periods before and after such a service cycle as two independent selling horizons, because any service that starts before such service cycle must have been terminated by the end of this service cycle.

Let $\textsc{Proj}_S(x) = \arg\min_{y\in S} \|y - x\|_2$ denote the projection function of a vector x onto the set S under the standard Euclidean norm. The formal definition of DPC is given below.

Deterministic Price Control with Parameter ϵ (DPC(ϵ)).

Step 1. Solve **DET**, and get $\boldsymbol{\lambda}^D$. Set $\boldsymbol{C}^1 = \boldsymbol{C}$.

Step 2. For $t = 1$ to T, at the beginning of each t, do

a. Compute $\hat{\boldsymbol{p}}^t \in \Omega_p$ such that

$$\lambda_j^t(\hat{p}^t) = \textsc{Proj}_{\Omega_\lambda}\left[\lambda_j^{t,D} - \frac{\epsilon}{\underline{n}} \cdot \mathbf{1}\left\{\lambda_j^{t,D} > 0\right\}\right].$$

b. For each j, if $\boldsymbol{C}^t \geq \boldsymbol{A}^j$, then set $p_j^t = \hat{p}_j^t$; otherwise, set $p_j^t = \bar{p}_j$.

Observe that DPC is parameterized by $\epsilon > 0$. To understand the impact of ϵ, consider what happens in the deterministic system where the realized demand in period t is exactly $\boldsymbol{\lambda}^t(\hat{p}^t)$. Suppose that $\lambda_j^{t,D} > 0$ for all j and t (that is, $\underline{n} = n$), and ϵ is chosen such that $\lambda_j^{t,D} - \epsilon/\underline{n} \in (0, \lambda_U)$ for all j and t. In such case, there are enough resources available for all types of service at any time period. Consequently, we have $\lambda_j^t = \lambda_j^{t,D} - \epsilon/n$ for all j and t, and, therefore, the total resource consumption during any service cycle is at most $C - \epsilon \cdot \mathbf{e}$. In the more general case where $\lambda_j^{t,D} = 0$ for some j and t, we only perturb $\lambda_j^{t,D}$ if its value is not 0. By definition of $\underline{n}$, in the deterministic system, there are enough resources available for service type j at all time periods t with $\lambda_j^{t,D} > 0$, and therefore, the total resource consumption during any service cycle is still at most $\boldsymbol{C} - \underline{n} \cdot (\epsilon/\underline{n}) \cdot \mathbf{e} = \boldsymbol{C} - \epsilon \cdot \mathbf{e}$.

The importance of the above observations is as follows. Assuming that ϵ is properly chosen, it can be shown that, in the actual stochastic system with random demand, there are enough resources available for service type j at all time periods t with $\lambda_j^{t,D} > 0$ with a very high probability. In other words, one can interpret ϵ as a "resource buffer" (or simply *buffer*) that we intentionally reserve for the purpose of hedging against demand uncertainty so that, in the implementation of DPC, we almost never have to apply the turnoff price to any service type at any time period when the corresponding deterministic demand rate is not 0 (that is, $\lambda_j^{t,D} > 0$). We state our result for DPC.

Theorem 1. *Suppose that* $\epsilon \in (1, \min\{\min\{\varphi_L, \varphi_U\} \cdot \underline{n}, 6 \cdot \min\{\max_i C_i, n\} + 1\}]$. *There exists a constant* $M_1 > 0$ *such that, for all* T, $\boldsymbol{C}$, *and* n,

$$\mathscr{L}(\mathrm{DPC}(\epsilon)) \leq M_1 \cdot \left[\frac{\epsilon}{\underline{n}} + \frac{T}{n}\exp\left\{-\frac{(\epsilon-1)^2}{36\cdot\min\{\max_i C_i, n\}}\right\}\right]. \quad (3)$$

In particular, using $\epsilon = 1 + 6\sqrt{b \cdot \underline{n}^{(\theta)} \cdot \log \underline{n}^{(\theta)}}$ *for some constant* $b > \min\{\max_i C_i, n\}$ *yields*

$$\mathscr{L}(DPC(\epsilon)) = O\left(\theta^{-\frac{1}{2}} \cdot \log^{\frac{1}{2}}\theta\right). \quad (4)$$

The first bound in Theorem 1 is very general; it highlights the impact of parameters T, n, C, and ϵ on the performance of DPC. As for the second bound, it follows from a simple algebraic substitution of the parameters defined in (1) into (3). (By Lemma 2, $\underline{n}^{(\theta)} = \theta \cdot \underline{n}$.)

5. Deterministic Price Control with Periodic Batch Adjustments

We now discuss an adaptive version of DPC, which we call deterministic price control with periodic batch adjustment (DPC-B). Similar to the original DPC, DPC-B also reserves some capacity (or buffer) for the purpose of hedging against demand uncertainty; however, unlike in the case of DPC, where the buffer needs to be sufficiently large to deal with demand uncertainty during a full service cycle, the buffer in DPC-B only needs to deal with demand uncertainty during much smaller cycle periods. To make sure that DPC-B guarantees a better performance than DPC while using a smaller buffer than DPC (in terms of order of magnitude), we need to adaptively adjust the prices by taking into account demand realizations in the previous periods. Let m be a positive integer. For each service type j, we define a partition of the selling horizon $\mathscr{P}_j := \{\mathscr{T}_j^b\}_{b=1}^{B_j}$, where

$$\left|\left\{\lambda_j^{t,D} : \lambda_j^{t,D} > 0, t \in \mathscr{T}_j^b\right\}\right| = m, \quad \mathscr{T}_j^{b_1} \cap \mathscr{T}_j^{b_2} = \emptyset \quad \text{for } b_1 \neq b_2, \quad \text{and } \cup_b \mathscr{T}_j^b = [1, T].$$

Without loss of generality, we assume that the above partition is done in such a way that $\max\{t : t \in \mathcal{T}_j^b\} + 1 = \min\{t : t \in \mathcal{T}_j^{b+1}\}$: that is, the partition $\mathcal{P}_j$ divides the whole selling horizon into a sequence of *adjacent* nonoverlapping batches of periods, where each batch contains exactly m periods in which $\lambda_j^{t,D} > 0$. In the special case where $\lambda_j^{t,D} > 0$ for all j and t, $\mathcal{T}_j^b \equiv [(b-1)m+1, bm]$ for all b and j: that is, all batches contain exactly m periods. (We implicitly assume that the last batch $\mathcal{T}_j^{B_j}$ also contains exactly m periods in which $\lambda_j^{t,D} > 0$. This may not always hold, and we assume this for expositional clarity only; see Remark 1 at the end of this section for additional discussions.) We also define $\mathcal{T}_j^0 = \emptyset$ and $\beta_j(t)$ to be the index of the batch that period t belongs to: that is, $t \in \mathcal{T}_j^{\beta_j(t)}$.

For any feasible control π, let $\boldsymbol{\Delta}^t(\boldsymbol{p}^{t,\pi}) = \boldsymbol{D}^t(\boldsymbol{p}^{t,\pi}) - \boldsymbol{\lambda}^t(\boldsymbol{p}^{t,\pi})$ (that is, $\boldsymbol{\Delta}^t(\boldsymbol{p}^{t,\pi})$ is the error from expected demand at period t under price $\boldsymbol{p}^{t,\pi}$). For brevity, we will often suppress the notational dependency on price and simply write $\boldsymbol{\Delta}^t = \boldsymbol{\Delta}^t(\boldsymbol{p}^{t,\pi})$. The complete definition of DPC-B is given below.

DPC-B with Parameters m and ϵ (DPC-B(m,ϵ)).

Step 1. Solve **DET**, and get $\boldsymbol{\lambda}^D$. Set $\boldsymbol{C}^1 = \boldsymbol{C}$.

Step 2. For $t = 1$ to T, at the beginning of each t, do

a. Compute $\hat{\boldsymbol{p}}^t \in \Omega_p$ such that, for all j

$$\lambda_j^t(\hat{p}^t) = \textsc{Proj}_{[0,\lambda_U]}\left[\lambda_j^{t,D} - \left(\frac{\epsilon}{\underline{n}} + \frac{1}{m}\sum_{s\in\mathcal{T}_j^{\beta_j(t)-1}} \Delta_j^s\right) \cdot \mathbf{1}\left\{\lambda_j^{t,D} > 0\right\}\right].$$

b. For each j, if $C^t \geq A^j$, then set $p_j^t = \hat{p}_j^t$; otherwise, set $p_j^t = \bar{p}_j$.

The price vector $\hat{\boldsymbol{p}}^t$ in Step 2(a) is well defined by the invertibility of $\boldsymbol{\lambda}^t$. As for the price update formula for period $t \in \mathcal{T}_j^{b+1}$, note that, in addition to having the perturbation term $\epsilon/\underline{n}$ as in DPC, we also have a new perturbation term $(\sum_{s\in\mathcal{T}_j^b} \Delta_j^s)/m$ with value that depends on the realized demands in the previous batch b. The purpose of this additional term is to adjust the total induced demands in the current batch $b+1$ so that the impact of cumulative demand randomness in batch b is completely absorbed (or corrected) in batch $b+1$ and does not directly affect future batches (that is, batch $b' \geq b+2$). The benefit of this error correction scheme is that the buffer ϵ only needs to hedge against the cumulative demand randomness during $m \leq \underline{n}$ periods instead of the whole $\underline{n}$ periods. Unlike the original DPC, DPC-B is parameterized by two parameters m and ϵ. The value of these parameters must be carefully chosen. If m is too small, the price adjustment scheme under DPC-B may not have sufficient power to fully correct the cumulative errors in the previous batch by adjusting the total induced demands in the current batch (for example, cumulative errors in the previous batch may have the same order of magnitude as total potential demands in the current batch); if, however, m is too large, we already incur a lot of loss in the previous batch that is not recoverable by simply adjusting the new demands in the current batch (for example, if $m = \underline{n}$, the performance of DPC-B is comparable with that of DPC, and our price adjustment scheme has a minimal benefit). The following theorem tells us the performance of DPC-B. We omit its proof, because it is a special case of Theorem 3 in Section 6.

Theorem 2. *Suppose that*

$$\epsilon \in \left(1, \min\left\{\underline{n}\cdot\min\left\{1, \frac{1+4m\cdot\min\{\varphi_L,\varphi_U\}}{4m+\underline{n}}\right\}, 1 + 16\min\left\{\max_i C_i, m\right\}\right\}\right]$$

and $m \leq \underline{n}$. There exists a constant $M_2 > 0$ such that, for all T, $\boldsymbol{C}$, and n,

$$\mathcal{L}(DPC - B(m,\epsilon)) \leq M_2 \cdot \left[\frac{\epsilon}{\underline{n}} + \frac{1}{m} + \frac{T}{m}\cdot\exp\left\{-\frac{(\epsilon-1)^2}{256\min\{\max_i C_i, m\}}\right\}\right]. \tag{5}$$

In particular, using $\epsilon = 1 + 16\sqrt{b\cdot(\underline{n}^{(\theta)})^c\cdot\log\underline{n}^{(\theta)}}$ and $m = \lceil(\underline{n}^{(\theta)})^c\rceil$ for some constant $b > \min\{\max_i C_i, \lceil\underline{n}^c\rceil\}$ and $c \in [0,1]$ yields

$$\mathcal{L}(DPC - B(m,\epsilon)) = O\left(\theta^{\frac{c}{2}-1}\cdot\log^{\frac{1}{2}}\theta + \theta^{-c}\right). \tag{6}$$

The performance of DPC-B is largely affected by the choice of c. Ignoring the logarithmic term in (6), the optimal bound is achieved when $c = 2/3$, which yields an average loss of order $\theta^{-2/3}\cdot\log^{1/2}\theta$. This is a significant improvement over the performance of DPC. It is curious to ask whether the bound in Theorem 2 can be further improved, perhaps by a different heuristic control. (In the canonical PRM setting, a heuristic control proposed in Jasin (2014) guarantees an average loss of order $\theta^{-1}\cdot\log\theta$. Unfortunately, a direct adoption of this heuristic control to our setting does not perform as well as DPC-B; see Section EC.5 in the e-companion.) We leave this for future research pursuit.

Remark 1 (When $\mathcal{T}_j^{B_j}$ Contains Less Than m Periods with Positive $\lambda_j^{t,D}$)**.** Throughout, we have assumed that all batches contain m periods with positive $\lambda_j^{t,D}$. When this assumption does not hold, we only need to make a minor change in the definition of the last two batches: that is, we simply combine the sets $\mathcal{T}_j^{B_j-1}$ and $\mathcal{T}_j^{B_j}$ together, and we end up with a total of $B_j - 1$ instead of B_j batches. After the join operation, the last batch may now have $m' \in (m, 2m)$ periods with positive $\lambda_j^{t,D}$. For all t in the last batch, we simply modify the definition of $\hat{p}^t$ in Step 2(a) as follows:

$$\lambda_j^t(\hat{p}^t) = \textsc{Proj}_{[0,\lambda_U]}\left[\lambda_j^{t,D} - \left(\frac{\epsilon}{\underline{n}} + \frac{1}{m'}\sum_{s\in\mathcal{T}_j^{B_j-2}} \Delta_j^s\right)\cdot \mathbf{1}\left\{\lambda_j^{t,D} > 0\right\}\right].$$

It is not difficult to check that the bounds in Theorem 2 still hold under this minor alteration.

6. The Setting with Heterogeneous Service Time and Advance Reservation

We discuss a generalization of the basic model in Section 3 that allows different service time requirements and advance reservation (that is, a customer may request a service to be started at a future period). This extension is important for modeling the wide variety of applications discussed in Section 1. The firm manages I types of resource and provides $K \geq 1$ types of services to customers. A request for service type k requires $a_{ik} \in \{0,1\}$ unit of resource type i for a duration of n_k periods that starts ℓ_k periods later (that is, a service type k requested at the beginning of period t will start at period $t + \ell_k$ and end at period $t + \ell_k + n_k - 1$). For ease of exposition, we will assume that $T/n_k \in \mathcal{Z}^+$ for all k. Moreover, without loss of generality, we also assume that the service types are indexed in such a way that (i) $n_1 + \ell_1 \leq n_2 + \ell_2 \leq \cdots \leq n_K + \ell_K$ and (ii) if $n_k + \ell_k = n_{k'} + \ell_{k'}$ and $k < k'$, then $\ell_k \leq \ell_{k'}$. The initial capacity of resources is C, and the resource consumption matrix is A. The price vector is denoted by $\boldsymbol{p}^t = (p_k^t)$, and its corresponding demand and rate vectors are $\boldsymbol{D}^t(\boldsymbol{p}^t) = (D_k^t(\boldsymbol{p}^t))$ and $\boldsymbol{\lambda}^t(\boldsymbol{p}^t) := (\lambda_k^t(\boldsymbol{p}^t))$, respectively. We assume the same feasible sets and regularity conditions as in Section 3. The stochastic control formulation of our problem is given by

$$\textbf{OPT}-\textbf{H}:\quad J_H^* = \left\{\max_{\pi\in\Pi} \mathbf{E}\left[\sum_{t=1}^T R^t(\boldsymbol{p}^{t,\pi})\right] : \sum_{k=1}^K \sum_{s=(t-n_k-\ell_k+1)^+}^{(t-\ell_k)^+} a_{ik} \cdot D_k^s(\boldsymbol{p}^{s,\pi}) \leq C_i \text{ for all } t \leq T\right\},$$

where the constraints must hold almost surely and Π is the set of all nonanticipating controls. A deterministic relaxation of **OPT-H** is given by

$$\textbf{DET}-\textbf{H}:\quad J_H^D = \left\{\max_{\lambda^t\in\Omega_\Lambda} \sum_{t=1}^T r^t(\boldsymbol{\lambda}^t) : \sum_{k=1}^K \sum_{s=(t-n_k-\ell_k+1)^+}^{(t-\ell_k)^+} a_{ik} \cdot \lambda_k^s \leq C_i \text{ for all } t \leq T\right\}.$$

As in Lemma 1, it is not difficult to show that J_H^D is an upper bound of J_H^*. We use $\boldsymbol{\lambda}^D := \{\boldsymbol{\lambda}^{t,D}\}_{t=1}^T$ (and $\boldsymbol{p}^D := \{\boldsymbol{p}^{t,D}\}_{t=1}^T$) to denote the optimal solution of **DET-H**. We assume that Assumptions 1–5 still hold, and we use a similar asymptotic regime as in Section 3, with $T^{(\theta)} = \theta \cdot T$, $\boldsymbol{C}^{(\theta)} = \theta \cdot \boldsymbol{C}$, $n_k^{(\theta)} = \theta \cdot n_k$, $\ell_k^{(\theta)} = \theta \cdot \ell_k$, and $\boldsymbol{\lambda}^{t,(\theta)}(\cdot) = \boldsymbol{\lambda}^{\lceil t/\theta \rceil}(\cdot)$. It can be shown that the result of Lemma 2 still holds.

6.1. A Generalized DPC-B

Let $\boldsymbol{m} = (m_1, \ldots, m_K)$ be a sequence of positive integers. For service type k, define a partition of the selling horizon $\mathcal{P}_k := \{\mathcal{T}_k^b\}_{b=1}^{B_j}$, where

$$\left|\left\{\lambda_k^{t,D} : \lambda_k^{t,D} > 0, t \in \mathcal{T}_k^b\right\}\right| = m_k, \quad \mathcal{T}_k^{b_1} \cap \mathcal{T}_k^{b_2} = \emptyset \quad \text{for } b_1 \neq b_2, \text{ and } \cup_{\mathrm{b}} \mathcal{T}_{\mathrm{k}}^{\mathrm{b}} = [1, \mathrm{T}].$$

As in Section 5, we will assume that the above partitioning is possible, and it is done in such a way that $\max\{t : t \in \mathcal{T}_k^b\} + 1 = \min\{t : t \in \mathcal{T}_k^{b+1}\}$ for all k. (If the last batch $\mathcal{T}_k^{B_k}$ does not contain exactly m_k periods in which $\lambda_k^{t,D} > 0$, we can slightly modify our batch definition as in Remark 1 at the end of Section 5.) Let $\beta_k(t)$ be the batch to which period t belongs under partition $\mathcal{P}_k$ (that is, $t \in \mathcal{T}_k^{\beta_k(t)}$), and let $\boldsymbol{\Delta}^t := (\Delta_k^t)_{k=1}^K = (D_k^t(\boldsymbol{p}^t) - \lambda_k^t(\boldsymbol{p}^t))_{k=1}^K$ denote the vector of errors from expected demands in period t under price $\boldsymbol{p}^t$. We use $\boldsymbol{\epsilon} = (\epsilon_1, \ldots, \epsilon_K)$ to denote the buffer for each service type. Given the possibility of advance reservation, we need to carefully track the amount of available resources at future periods in order to avoid overbooking. To do this, for all $1 \leq t \leq s \leq \min\{t + \max_k \ell_k, T\}$, we define $\tilde{C}_i(t,s)$ to be the number of resource type i that is available for service at period s at the beginning of period t before a new request in period t arrives. We will provide an update rule for $\tilde{\boldsymbol{C}}(t,s) = (\tilde{C}_i(t,s))$ in the description of our generalized DPC-B below. Let $\underline{n}_k$ denote the minimum number of nonzero $\lambda_k^{t,D}$ within any service cycle during which there is a positive demand for service type k in the optimal deterministic solution: that is,

$$\underline{n}_k := \min_{t\in[1,T-\ell_k-n_k+1]:\sum_{s=t+\ell_k}^{t+\ell_k+n_k-1}\lambda_k^{s,D}>0} \cdot \left|\left\{\lambda_k^{s,D} : \lambda_k^{s,D} > 0, s \in [t+\ell_k, t+\ell_k+n_k-1]\right\}\right|.$$

The complete definition of the generalized DPC-B is given below.

DPC-B with Parameters m and ϵ (DPC-B(m, ϵ)).

Step 1. Solve **DET-H**, and get $\boldsymbol{\lambda}^D$. Set $\boldsymbol{C}^1 = \tilde{\boldsymbol{C}}(1,t) = \boldsymbol{C}$ for all feasible t.

Step 2. For $t = 1$ to T, at the beginning of each t, do

a. For each i and $s \in [t, \min\{t + \max_k \ell_k, T\}]$, compute $\tilde{C}_i(t,s)$ according to

$$\tilde{C}_i(t,s) \leftarrow C_i^t + \sum_k \sum_{v=(t-\ell_k-n_k+1)^+}^{(\min\{t-\ell_k,s-\ell_k-n_k\})^+} a_{ik}\cdot D_k^v - \sum_k \sum_{v=(\max\{t-\ell_k+1,s-\ell_k-n_k+1\})^+}^{(\min\{t-1,s-\ell_k\})^+} a_{ik}\cdot D_k^v.$$

b. Compute $\boldsymbol{p}^t = \hat{\boldsymbol{p}}^t$ such that, for all k

$$\lambda_k^t(\hat{p}^t) = \textsc{Proj}_{[0,\lambda_U]}\left[\lambda_k^{t,D} - \left(\frac{\epsilon_k}{\underline{n}_k} + \frac{1}{m_k}\sum_{s\in\mathcal{T}_k^{\beta_k(t)-1}}\Delta_k^s\right) \cdot \mathbf{1}\{\lambda_k^{t,D} > 0\}\right].$$

c. For each k, if $\tilde{\boldsymbol{C}}(t, t+\ell_k) \geq \boldsymbol{A}^k$, then set $p_k^t = \hat{p}_k^t$; otherwise, set $p_k^t = \bar{p}_k$.

We now explain the logic behind the updating rule for $\tilde{C}_i(t,s)$. Given all admitted service requests at the beginning of period t, the number of resource type i that is available for service at period s and can be booked at period t can be written as the number of idle resource type i at the beginning of period t *plus* the number of resource type i that is currently being used and will be released at the end of periods t to $s-1$ *minus* the number of resource type i that will be used at period s under admitted services with service period that will commence at period $t' \in [t,s]$. Fix a service type k. A request for service type k that is already being processed and will be completed by the end of period $s-1$ should have arrived in period v, satisfying $v+\ell_k \leq t$ and $t+1 \leq v+\ell_k+n_k \leq s$. A request for service type k with service periods that will commence at a future period and go through at least period s should have arrived in period v, satisfying $v \leq t-1$, $t+1 \leq v+\ell_k \leq s$, and $v+\ell_k+n_k \geq s+1$. This explains the updating rule in Step 2(a). We state our result.

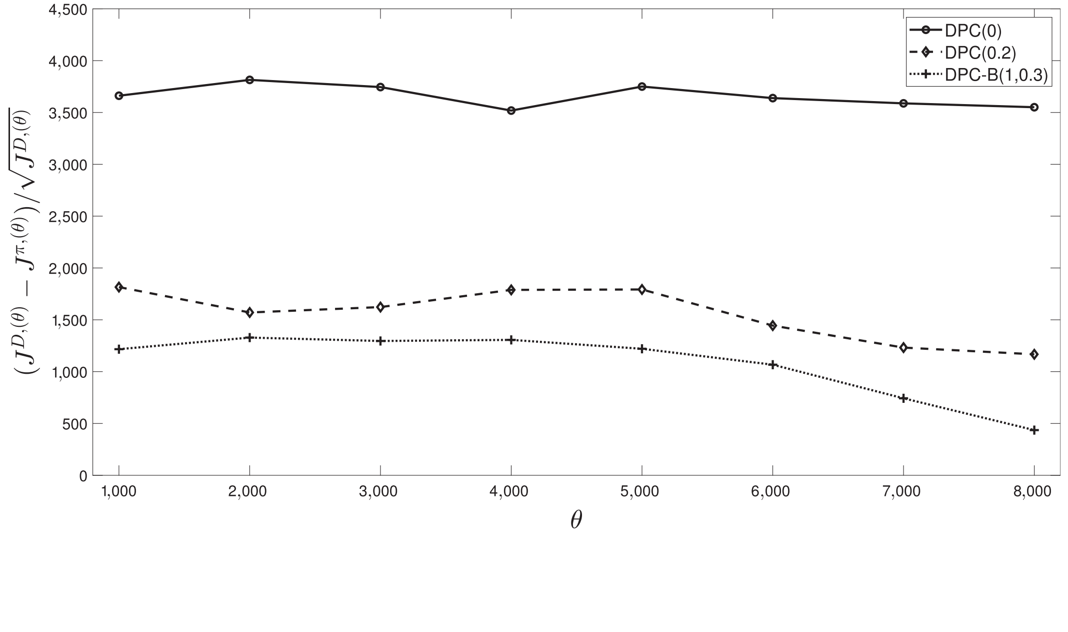
Theorem 3. *Suppose that*

$$\epsilon_k \in \left(1, \min\left\{\underline{n}_k \cdot \min\left\{1, \frac{1+4m_k\cdot\min\{\varphi_L,\varphi_U\}}{4m_k+\underline{n}_k}\right\}, 1 + 16\min\left\{\max_i C_i, m_k\right\}\right\}\right],$$

and $m_k \leq \underline{n}_k$. There exists a constant $M_3 > 0$ such that, for all T, $\boldsymbol{C}$, n_k,

$$\mathcal{L}(DPC-B(\boldsymbol{m},\boldsymbol{\epsilon})) \leq M_3 \cdot \sum_{k=1}^K \left[\frac{\epsilon_k}{\underline{n}_k} + \frac{1}{m_k} + \frac{T}{m_k} \cdot \exp\left\{-\frac{(\min_k \epsilon_k - 1)^2}{256K^2\cdot\min\{\max_i C_i, m_k\}}\right\}\right]. \tag{7}$$

Figure 1. Scaled Loss Under Varying θ



In particular, using $\epsilon_k = 1 + 16\sqrt{b_k \cdot (\underline{n}_k^{(\theta)})^c \cdot \log \underline{n}_k^{(\theta)}}$ *and* $m_k = \lceil (\underline{n}_k^{(\theta)})^c \rceil$ *for some constant* $b_k > K \cdot \min\{\max_i C_i, \lceil \underline{n}_k^c \rceil\}$ *and* $c \in [0,1]$ *yields*

$$\mathscr{L}(DPC - B(m,\epsilon)) = O\left(\theta^{\frac{c}{2}-1} \cdot \log^{\frac{1}{2}} \theta + \theta^{-c}\right). \quad (8)$$

Similar to Theorem 3, setting $c = 2/3$ yields an average loss of order $\theta^{-2/3} \cdot \log^{1/2} \theta$.

7. Numerical Experiments

We conduct numerical experiments to test the performance of the two proposed heuristic controls under different values of control parameters in the setting with nonstationary arrival, heterogeneous service time, and advance reservation. We focus on the asymptotic performances of all heuristic controls by varying the system scale θ from 1,000 to 8,000 (in other words, the total number of arrivals within a service cycle ranges from 10^3 to 10^4).

We consider a setting with four service types and two resource types. The (unscaled) value of the service times, advance reservation lead times, resource capacity levels, and the resource consumption matrix are given as follows:

$$\boldsymbol{n} = [3,5,6,8], \quad \boldsymbol{\ell} = [1,0,3,2], \quad \boldsymbol{C} = [2,1.5], \quad \text{and} \quad A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

The length of the selling horizon in the unscaled system is assumed to be $T = 40$. The demand rate is nonstationary over time, independent across products, and exponentially decreasing over its own price. Specifically, we use $\lambda_k^t(\boldsymbol{p}^t) = \exp(\alpha^t \cdot a_k^t - b_k^t \cdot p_k^t)$, where $\{\alpha^t\}$ is an increasing sequence of random numbers within $[0.9, 1.1]$, and a_k^t and b_k^t are sampled uniformly at random from $[-0.8, -1.3]$ and $[0.005, 0.015]$, respectively. The maximum possible price is \$10,000, and the minimum possible price is zero. The resulting optimal deterministic demand $\boldsymbol{\lambda}^D$ is clearly nonstationary, and the corresponding optimal value is $J^D = 2{,}146.05$.

We test both DPC and DPC-B under different values of control parameters. We set $\epsilon_k^{(\theta)} = \epsilon^0 \cdot \sqrt{\underline{n}^{(\theta)} \cdot \log \underline{n}^{(\theta)}}$ in DPC($\epsilon^{(\theta)}$), and $\epsilon_k^{(\theta)} = \epsilon^0 \cdot \sqrt{(\underline{n}^{(\theta)})^{2/3} \cdot \log \underline{n}^{(\theta)}}$ and $m_k^{(\theta)} = m^0 \cdot \lceil (\underline{n}^{(\theta)})^{2/3} \rceil$ in DPC-B($\epsilon^{(\theta)}, m^{(\theta)}$). By abuse of notation, we will simply use DPC(ϵ^0) and DPC-B(m^0, ϵ^0) to denote the corresponding controls (for example, DPC(1) means that we set $\epsilon_k^{(\theta)} = 1 \cdot \sqrt{\underline{n}^{(\theta)} \cdot \log \underline{n}^{(\theta)}}$ for all k). For each θ, we simulate the performances of DPC(0), DPC(0.2), and DPC-B(1,0.3). (We set $m^0 = 1$ by default and optimize for ϵ^0 for both DPC and DPC-B. Note that the exact value of the optimal ϵ^0 may change slightly as θ changes, but we simply use a fixed ϵ^0 for all θ for consistency.) In particular, we implement DPC(0) to understand the value of injecting a buffer. We report in Figure 1 the value of $(J^{D,(\theta)} - J^{\pi,(\theta)})/\sqrt{J^{D,(\theta)}}$ for all three controls; the detailed numerical results are summarized in Table EC.1 in Section EC.5.1 in the e-companion. In particular, we divide the loss by $\sqrt{J^{D,(\theta)}}$ to give a clearer understanding of the asymptotic performance (note that, by Lemma 2, $\sqrt{J^{D,(\theta)}} = \Theta(\sqrt{\theta})$). The results confirm the asymptotic optimality of the proposed heuristic controls.

Lastly, in order to test the impact of different values of control parameters, we further change the value of ϵ^0 for DPC and ϵ^0, and m^0 for DPC-B. The numerical results are reported in Table EC.2 in Section EC.5.1 in the e-companion. We observe that the performances of both DPC and DPC-B are sufficiently robust with respect to the exact value of control parameters as long as the variation is not too large. In general, it is better to have a larger buffer when the batch size is larger. Moreover, as long as the value of control parameters is too extreme (for example, batch size is too large, whereas the buffer size is too small), DPC-B consistently outperforms DPC.

8. Closing Remarks

In this paper, we address a general dynamic pricing problem for RM with reusable resources, advance reservation, and deterministic service time requirements. This is a fundamental problem faced by firms in many industries. To the best of our knowledge, our adaptive control is the first in the literature that guarantees an asymptotic performance of order $o(\theta^{-\frac{1}{2}})$ in such setting. As mentioned in Section 5, there is a big performance gap between the best heuristic controls in the canonical RM and RRM settings. It is curious to ask whether our adaptive control can be further improved. It would also be interesting to see how our adaptive control can be generalized to the setting with random service time under a general service time distribution.

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