University of Michigan, Department of Economics
Econ 502: Applied Macroeconomics, Winter 2023
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Midterm examination
February 23, 2023, 8:30am-9:50am

The exam consists of four problems. The total number of points attainable on the exam is 100. Students have 80 minutes to complete the test. During the test, students are not allowed to talk to each other or use any written sources of information. No telephones, smart watches, or other mobile communication devices are allowed, and they need to be put away. Violation of these rules will lead to immediate failing of the test.

<u>Problem 1.</u> (15 points) Global real interest rates decreased substantially during recent decades. What are possible explanations for this phenomenon? Give three possible reasons and explain.

Answer: Interest rates in the long run are determined in the global market for loanable funds by the balance of investment and saving. Therefore, reduction in the interest rates implies that supply of saving has increased and/or desire to invest decreased. There are a number of reasons why that could have happened. Saving could have increased because of population aging, so that people started to save more for a longer expected retirement. Alternatively, global saving could have increased because Chinese economy now takes up a bigger part of the world economy, while China historically has had a higher saving rate than other countries. Finally, in the recent decade, uncertainty caused by economic cataclysms such as the World Financial Crisis of 2008 led to an increase in precautionary saving. On the investment side, desired investment could have decreased for a number of reasons as well. First, increased risk not only increases saving, but also discourages investment. Second, structure of the world economy has shifted towards less capital-intensive industries, such as IT, which requires less investment than traditional industries. Third, increased degree of monopolization in developed countries led to a decrease in desire of firms to build facilities and produce.

<u>Problem 2.</u> (15 points) The US has been running trade deficits over the last few decades. Provide two fundamental reasons/motives why a country can run persistent deficits. How applicable are they for the US situation?

Answer: First of all, by equation $NX + NFP = S – I$, we see that the trade balance is primarily determined by the balance of saving and investment (NFP is a small number for most countries). A trade deficit is almost the same thing as the current account deficit, which means that the country borrows more from the rest of the world than it lends to it. In our lectures, we studied two motives for such behavior. First is consumption smoothing: if a country's citizens expects to have a higher income in the future than today, they borrow today to consume more. The second is return to investment: if a country's MPK is higher than the world interest rate r^*, then it makes sense to borrow and invest within the country. If either of these conditions are true for a *long time, a county can run a trade deficit for a long time. And a country can run a perpetual trade deficit if it accumulated positive net foreign assets, which yield yearly dividends, which can be spent on imports.*

The US situation is applicable to the first two explanations. Although US does not have to smooth its consumption, other countries want to save for the rainy day, and treat the US as a "safe haven", so they lend money to the US by buying its assets. The US then uses the lent money to buy imports. Likewise, US has attractive business climate with high MPK (especially the tech boom in the late 1990s), which draws in a lot of foreign investment.

<u>Problem 3. The Solow model.</u> (40 points)
Consider the Solow model with population growth and technological progress as derived in class. The production function is Cobb-Douglas:

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$$

Where capital K_t evolves according to the capital accumulation equation, with depreciation rate δ:

$$\dot{K}_t = I_t - \delta K_t$$

Households save a constant fraction of their income, s. The population grows at rate n and technology advances at rate g.

a) (7 points) Derive the capital accumulation equation in terms of capital per effective worker.

Starting with the given capital accumulation equation and dividing both sides by $A_t L_t$:

$$\frac{\dot{K}_t}{A_t L_t} = \frac{I_t}{A_t L_t} - \delta \frac{K_t}{A_t L_t}$$

Note that

$$\dot{k}_t = \frac{\dot{K}_t}{A_t L_t} - g\frac{K_t}{A_t L_t} - n\frac{K_t}{A_t L_t} \qquad \Longrightarrow \qquad \frac{\dot{K}_t}{A_t L_t} = \dot{k}_t + g k_t + n k_t$$

Plugging this result in and solving for $\dot{k}_t$ yields the capital accumulation equation in terms of capital per effective worker:

$$\dot{k}_t = i_t - (n + g + \delta)k_t$$

b) (8 points) Solve for the steady-state level of capital per effective worker in terms of exogenous parameters only.

In the steady-state, capital per effective worker is constant. Moreover, investment comes from saving a constant fraction of income. Plugging these results into the capital accumulation equation:

$$0 = sf(k) - (n + g + \delta)k$$

where the time-subscripts are dropped since capital per effective worker is constant. We can solve for output per effective worker from the production function:

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha} \qquad \Longrightarrow \qquad \frac{Y_t}{A_t L_t} = \frac{K_t^{\alpha}(A_t L_t)^{1-\alpha}}{A_t L_t}$$

$$\Longrightarrow \qquad y_t = f(k_t) = k_t^{\alpha}$$

Plugging this result in for $f(k)$ and solving for k yields the steady-state level of capital per effective worker:

$$k = \left(\frac{s}{n + g + \delta}\right)^{\frac{1}{1-\alpha}}$$

c) (7 points) Suppose the population growth rate, n, increases. Does the golden-rule savings rate that maximizes consumption in the steady-state change? Explain why or why not.

It does not. First of all, the production function is Cobb-Douglas, so the golden-rule savings rate is always the exponent on capital, α (see Problem Set 2, question 2c). Second, as described in lecture, consumption in the steady-state is maximized when the marginal productivity of capital per effective worker equals the rate of biological+technological+physical depreciation:

$$f'(k) = n + g + \delta$$

$$\Longrightarrow \qquad \alpha k^{\alpha-1} = n + g + \delta$$

$$\Longrightarrow \qquad k = \left(\frac{\alpha}{n + g + \delta}\right)^{\frac{1}{1-\alpha}}$$

This is the steady-state level of capital per effective worker that maximizes consumption in equilibrium. To solve for the savings rate that brings about this level, we plug into what we derived in part b):

$$\left(\frac{\alpha}{n + g + \delta}\right)^{\frac{1}{1-\alpha}} = \left(\frac{s}{n + g + \delta}\right)^{\frac{1}{1-\alpha}}$$

From here, we can see $s = \alpha$ is the savings rate that maximizes consumption in the steady-state. Given that this golden-rule savings rate does not depend on the population growth rate, n, the savings rate tha maximizes consumption in the steady-state is unchanged with an increase in the population growth rate.

d) We will now use this Solow model to analyze a sudden increase in labor force on wage growth. Assume we start in the steady-state. Then, at time T, there is a sudden inflow of immigrants so that the population doubles. Other than at time T, the population continues to grow at rate n. Answer the following questions:

   i) (6 points) Derive the expression for the real wage as a function of capital per effective worker, productivity A_t, and parameters. What is happening to the wage before the shock at time T?

Recall the firm's profit-maximization problem:

$$\max_{K_t, L_t}\{PF(A_t, K_t, L_t) - w_t L_t - R_t K_t\}$$

Plugging in the Cobb-Douglas production function and taking the first-order condition with respect to labor:

$$P(1-\alpha)A_t K_t^{\alpha}(A_t L_t)^{-\alpha} - w_t = 0$$

Solving for the real wage:

$$\frac{w_t}{P} = (1-\alpha)A_t k_t$$

Before the shock at time T, capital per effective worker k_t is constant, therefore wages grow at the rate of technological progress for A_t, which is g. This can also be shown with a growth decomposition of wages. Starting by taking the log of both sides:

$$ln\left(\frac{w_t}{P}\right) = ln(1-\alpha) + ln(A_t) + ln\,(k_t)$$

Deriving both sides with respect to t and plugging in given growth rates yields:

Growth Rate of Real Wages = 0 + g + 0 = g

   ii) (6 points) Does the wage rise, fall, or stay unchanged at time T? Explain intuition. Is the growth rate of the wage affected in the long run?

The wage falls. At time T, L_t suddenly increases, driving capital per effective worker k_t down and decreasing the wage (see the formula derived in part d)i) as the decline in capital per effective worker decreases the marginal product of labor.

In the long run, the growth rate of the wage is unaffected.

iii) (6 points) Justify your answer for the long-run growth rate of wages. How does capital adjust to increase/decrease/maintain the long-run growth rate of wages? Why?

The capital stock increases to return capital per effective worker, k_t, to its steady-state value. This occurs because the influx of labor increases the marginal productivity of capital. To see this, take the firm's first-order condition and solve for the real rental rate to yield the marginal productivity of capital:

$$P\alpha K_t^{\alpha-1}(A_t L_t)^{1-\alpha} - R_t = 0 \qquad \Longrightarrow \qquad \frac{R_t}{P} = \frac{\alpha}{k_t^{1-\alpha}}$$

With the influx labor at time T, capital per effective worker drops and the marginal productivity of capital rises. Firms will acquire more capital until its marginal productivity returns to the real rental rate.

Problem 4. The OLG model and the pension system. (30 points)
As the share of population that is elderly increases, many countries are facing pressure to adjust their pension systems to maintain fiscal sustainability. Consider the two-period OLG model with the PAYG pension system as derived in class: labor is inelastically supplied so L=1, the depreciation rate is 100%, workers have logarithmic preferences $U(C) = \ln(C)$, and the discount rate is ρ.

Suppose now that the government decides to reduce benefits by share $\varepsilon > 0$. That is, from now on, workers are taxed at rate τ, but the retirees earn $(1-\varepsilon)\tau W_{t+1}$ as their pension benefits in retirement. The government puts any surpluses from the pension system into national savings.

a) (7 points) Write down the budget constraint for the generation born at time t. That is, define consumption in period 2 by the generation born at time t as $C_{t,2}$, and express it in terms of wages, the interest rate, parameters, and first period consumption $C_{t,1}$.

$$C_{t,2} = \tau(1-\varepsilon)W_{t+1} + (1 + r_{t+1})((1-\tau)W_t - C_{t,1})$$

Intuition: the income available for consumption for the generation born at time t in the second comes from taxing workers born at time $t+1$ during their working years. These benefits are reduced by share ε. In addition, income is earned from the principal and interest of savings for the generation born at time t in their first period, which is after-tax income less consumption.

b) (16 points) Solve for optimal consumption in period 1 for the generation born at time t. That is, solve for $C_{t,1}$ as an expression of wages, the interest rate, and parameters.

Plugging in the given functional form for utility, generation t's maximization problem is:

$$\max_{C_{t,1},C_{t,2}} \left\{\ln(C_{1,t}) + \frac{1}{1+\rho}\ln(C_{2,t})\right\}$$
$$s.t.\ C_{t,2} = \tau(1-\varepsilon)W_{t+1} + (1+r_{t+1})\left((1-\tau)W_t - C_{t,1}\right)$$

Plugging in the budget constraint for $C_{2,t}$, taking the first-order condition with respect to $C_{1,t}$, and rearranging yields the Euler equation:

$$\frac{1}{C_{t,1}} - \frac{1+r_{t+1}}{(1+\rho)C_{t,2}} = 0 \qquad \Rightarrow \qquad \frac{C_{t,2}}{C_{t,1}} = \frac{1+r_{t+1}}{1+\rho}$$

Plugging the budget constraint into the Euler equation and solving for $C_{t,1}$ yields consumption for the generation born at time t in period 1:

$$\frac{\tau(1-\varepsilon)W_{t+1} + (1+r_{t+1})\left((1-\tau)W_t - C_{t,1}\right)}{C_{t,1}} = \frac{1+r_{t+1}}{1+\rho}$$

Cross-multiplying to get rid of fractions:

$$\left(\tau(1-\varepsilon)W_{t+1} + (1+r_{t+1})\left((1-\tau)W_t - C_{t,1}\right)\right)(1+\rho) = (1+r_{t+1})C_{t,1}$$

Bringing $C_{t,1}$ to one side:

$$-(1+r_{t+1})(1+\rho)C_{t,1} - (1+r_{t+1})C_{t,1} = -(\tau(1-\varepsilon)W_{t+1} + (1+r_{t+1})(1-\tau)W_t)(1+\rho)$$

Simplify:

$$(1+r_{t+1})(2+\rho)C_{t,1} = (\tau(1-\varepsilon)W_{t+1} + (1+r_{t+1})(1-\tau)W_t)(1+\rho)$$

Solve for $C_{t,1}$:

$$C_{t,1} = \frac{(\tau(1-\varepsilon)W_{t+1} + (1+r_{t+1})(1-\tau)W_t)(1+\rho)}{(1+r_{t+1})(2+\rho)}$$

c) (7 points) Assume that wages and the interest rate do not change when the government implements the reduction in benefits. Does saving increase in period 1? Why?

Savings increases in period 1. From the consumption function in part b), the reduction in benefits, ε, enters negatively. Receiving fewer benefits in retirement induces households to save more.