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Rural land development under hyperbolic discounting: a real option approach

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Abstract This article presents a simple model of land development under uncertainty and hyperbolic discounting. Land kept in rural use pays an uncertain rent, while net returns from land development are known and constant. The landowner is viewed here as a sequence of *selves* with time-inconsistent preferences. We solve the underlying timing game under both *naïve* and *sophisticated* beliefs about the landowner's time-inconsistency and show that (i) land development is accelerated due to his *present-bias* and (ii) a higher acceleration is associated with *sophistication*.

Keywords Land development · Stochastic hyperbolic discounting · Timing game · Real options

JEL Classification C61 · Q15 · R11

1 Introduction

A constant rate of time preference is a strong assumption that has been hotly debated since Strotz (1956) proposed, as a more "*reasonable*" alternative,[1] time-declining discount rates. Strotz's conjecture has since found empirical support, as several experiments have provided evidence of individuals' taste for immediate gratification (see Loewenstein and Prelec 1992, for a review). This has led to a series of papers

[1] Referring to exponential discounting, Strotz (1956) argues that there is "*no reason why an individual should have such a special discount function*" (p.172).

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using hyperbolic rather than exponential functions in model set-up (see Frederick et al. 2002, for a review).

Concerning farmers, empirical support for quasi-hyperbolic discounting has been found by Salois and Moss (2011) in a study based on US farmland values. Duflo et al. (2011) propose hyperbolic time preferences together with partial *naïveté* to explain why farmers in Western Kenya may not consider investing in fertilizers to boost agricultural productivity. Clot and Stanton (2014) find that Ugandan farmers exhibiting hyperbolic time preferences are significantly more likely to enroll into conservation programs paying a substantial transfer upfront. However, in spite of this evidence, efforts to introduce hyperbolic discounting in the theoretical frame investigating land development decisions have been quite limited and, to our knowledge, are represented solely by the model provided in Salois (2012). Salois introduces a quasi-hyperbolic landowner in the standard model of land development proposed by Irwin and Bockstael (2002) and shows that, by increasing the rate of land conversion, quasi-hyperbolic discounting may induce overdevelopment of rural land.

The analysis in Salois (2012) is performed in a deterministic frame and thus does not permit consideration of the relevant impact that uncertainty about payoffs associated with land uses may have on the decision to develop and on its timing. In fact, as widely recognised in the literature (see for instance Capozza and Helsley 1990; Capozza and Li 1994; Geltner et al. 1996), the option value associated with the decision to be taken may make postponing development the optimal choice in order to gather information about its future benefits.

The present study aims to fill this gap by investigating the land development problem under hyperbolic discounting in a real option frame. This enables examination of how two conflicting elements, namely hyperbolic discounting on the one hand, inducing a rush in land development and option value on the other hand inducing delay, affect the decision and timing of land development. Since, as noted by Strotz, declining discount rates may lead to time inconsistent planning, the impact of agents' self-awareness about their time inconsistency on the decision and timing of land development is also explicitly investigated. This is done by considering (i) a landowner who fully ignores his time inconsistency (*naïve*) and (ii) a landowner who perfectly foresees that his time preferences change over time (*sophisticated*) (see O'Donoghue and Rabin 1999a).[2]

Previous papers studying real options in the framework of time inconsistent preferences are those by Grenadier and Wang (2007) and Di Corato (2012). Grenadier and Wang (2007) investigate the impact of hyperbolic discounting on investment plans and show that the timing of investment differs from that set by an exponential discounter and depends on: (i) agents' self-awareness about their time inconsistency and (ii) the payoff form, i.e. a lump sum rather than a sequence of cash flows. Di Corato (2012) considers a forest-cutting problem in a dynamic game played by three non-overlapping, imperfectly altruistic generations, each of which is viewed as a hyperbolic discounting agent, and shows that, compared with the harvesting policy set by an exponential agent, the option value attached to the decision to harvest is lower and earlier harvest occurs.

[2] For more discussion on self-awareness and its empirical support see O'Donoghue and Rabin (1999b, 2001) and Frederick et al. (2002, Section 5.1, pp. 366–368).

That result holds under both *naïve* and *sophisticated* beliefs but under *sophistication* harvest occurs earlier than under *naïveté*.

The present study considers a standard land development problem where land kept in rural use pays a rent following geometric Brownian motion and net returns from land development are known and constant.[3] In line with the literature, the hyperbolic landowner is viewed as a sequence of infinite autonomous *selves* with time-inconsistent preferences (see Grenadier and Wang 2007; Harris and Laibson 2013). Each *self* is allowed to exercise the option to develop only during his life and cares about, but has no control over, the decisions taken by his future *selves*. The life of these *selves* is random and regulated by a Poisson death process. Each *self* determines his optimal threshold for exercise of the option to develop by solving the underlying non-cooperative intra-personal stopping game on the basis of his beliefs about the behaviour of future *selves*.

It is found that a hyperbolic landowner always develops land earlier than an exponential landowner. The reasoning is straightforward. Under hyperbolic discounting, the bias for present gratification lowers the value associated with holding the option to develop. Hence, each *self* finds convenient anticipating future *selves* by killing the option earlier. Interestingly, this study shows that a *sophisticated* landowner rushes land development more than a *naïve* landowner. In fact, due to his perfect foresight, a *sophisticated* landowner, takes into account the loss due to changes in the development timing strategy that are sub-optimal from his time perspective. Hence, compared with a *naïve* landowner totally ignoring this loss, the sophisticated landowner has a further argument for anticipating future *selves* by killing the option to develop.

These findings may have important implications for the design of policies governing land-use change, in particular when the timing of change is targeted. This may, for instance, be the case when considering issues such as land conservation, competition for land for cultivation of food or energy crops, farmland abandonment and urban development. As discounting matters for the timing of action, ignoring the presence of hyperbolic discounters when designing a policy addressing these issues may lead to unwanted outcomes as the incentives[4] given may not be strong enough to hit the target (see e.g. Shogren 2007; Hepburn et al. 2010).

The remainder of this paper is organised as follows. Section 2 presents the basic set-up. In Sect. 3, the value of land and the timing of land development under hyperbolic discounting are determined, under the assumption of both *naïve* and *sophisticated* beliefs. The results are then compared and discussed using the case of an exponential

[3] Grenadier and Wang (2007) study the optimal exercise of an American call option on investment under hyperbolic discounting. The present study, in contrast, is dealing with the optimal exercise of an American put-like option to "disinvest", by selling land to a developer. Note also that in our frame holding the option pays a periodic rent (associated with rural use) as a sort of dividend. On the optimal exercise of an American put-like option under time-inconsistent preferences see also Di Corato (2008) studying a forest-cutting problem and Tian (2016) studying optimal capital structure and investment decisions in the presence of a default option.

[4] Incentives may include compensations paid in order to induce land conservation (see e.g. Tegene et al. 1999; Schatzki 2003; Isik and Yang 2004), subsidies offered in order to favor the cultivation of energy crops (see e.g. Song et al. 2011; Di Corato et al. 2013) and growth controls set to limit urban sprawl (see e.g. Cunningham 2007).

landowner as the benchmark. Section 4 concludes. The Appendix contains the proofs omitted from the text.

2 The basic set-up

Consider a risk-neutral, infinite-lived landowner owning a parcel of land currently kept in rural use (agriculture, forestry, open space etc.). Under the current destination, the landowner earns a rent $\{R(t) : t \geq 0\}$. Assume that $R(t)$ is stochastic and evolves according to the following diffusion:

$$dR(t)/R(t) = \alpha dt + \sigma dQ(t), \text{ with } R(0) = R \tag{1}$$

where α and σ are the drift and volatility rates, respectively, and $\{Q(t)\}$ is a standard Wiener process with $E[dQ] = 0$ and $E\left[(dQ)^2\right] = dt$.

At any $t > 0$, the landowner can sell his land to a developer, converting it into an urban use. Here it is assumed that land development is irreversible[5] and the net pay-off,[6] accruing to the landowner selling his land is denoted $P > 0$.

2.1 The discount function

The time preferences of a hyperbolic landowner are characterised here by adopting the discount function proposed by Harris and Laibson (2013), where the hyperbolic discounting agent is viewed as an infinite sequence of risk-neutral *selves*, $i = 0, 1, 2, \ldots$. Each *self* i divides time into *present* and *future*. The *present* lasts a random time period, $s_{i+1} - s_i$, where s_i and s_{i+1} are the birth dates of *self* i and *self* $i + 1$. The transition is regulated by a Poisson death process with intensity $\lambda \in (0, \infty)$. The *future*, from the perspective of *self* i, starts at s_{i+1} and lasts forever. Each *self* i discounts 1) exponentially, at a constant rate ρ, both *present* and *future* payoffs and 2) additionally, by the factor $\beta \in (0, 1)$ all *future* payoffs. Summing up, his discount function is defined as follows:

Definition 1 *At time t for any $\beta \in (0, 1)$ and $\lambda \in (0, \infty)$, the self i's discount function is given by the following function:*

$$D_i(s, t) = \begin{cases} e^{-\rho(t-s)}, & \text{if } t \in [s_i, s_{i+1}), \\ \beta e^{-\rho(t-s)}, & \text{if } t \in [s_{i+1}, \infty), \end{cases}$$
$$\text{for } t > s \text{ and } s_i \leq s \leq s_{i+1}. \tag{2}$$

From Eq. (2), as $\beta \in (0, 1)$, a dollar paid to the generic *self* i in the *future* would be worth less than a dollar paid in the *present*. Note that in the limit cases $\beta \to 1$

[5] Note that, taking a real options perspective, the landowner can be viewed as holding an American put option, i.e. the option to develop, paying P if exercised. Otherwise, i.e. the land kept in rural use, the landowner receives $R(t)$ as a sort of dividend.

[6] As in Salois and Moss (2011) the pay-off P results from the sales price net of any conversion costs (administrative fees, permit expenses, institutional costs or necessary infrastructure expenditures, etc.).

and $\beta \to 0$, the agent would discount exponentially at rate ρ and $\rho + \lambda$, respectively, where λ is the "*speed*" at which time preferences change over time.[7] Note also that:

(i) The expected value of $D_i(s, t)$, i.e.

$$E[D_i(s,t)] = e^{-(\rho+\lambda)(t-s)} + \beta(1 - e^{-\lambda(t-s)})e^{-\rho(t-s)}$$
$$= \beta e^{-\rho(t-s)} + (1-\beta)e^{-(\rho+\lambda)(t-s)}, \quad (2.1)$$

is a convex combination of two exponential discount functions using ρ and $\rho + \lambda$ as discount rates.

(ii) The derivative of $E[D_i(s, t)]$ with respect to time is:

$$\partial E[D_i(s,t)]/\partial t = -[\rho\beta e^{-\rho(t-s)} + (\rho+\lambda)\,(1-\beta)e^{-(\rho+\lambda)(t-s)}\}] < 0. \quad (2.2)$$

This implies that the discount factor increases as t comes nearer or that, in other words, payoffs are discounted at the following time-declining rate:

$$-(\partial E[D_i(s,t)]/\partial t)/E[D_i(s,t)] = \rho+\lambda(1-\beta)e^{-(\rho+\lambda)(t-s)}/E[D_i(s,t)]. \quad (2.3)$$

3 Value of land and timing of land development

In order to provide a benchmark, in this section the standard land development problem faced by a landowner who discounts future payoffs exponentially is first solved. A hyperbolic landowner with the time preferences presented above is then considered. This landowner discounts future payoffs at time-declining rates and may formulate time-inconsistent plans, i.e. he may formulate plans which may later be disobeyed and revised. The analysis then proceeds by allowing for two extreme assumptions concerning the landowner's self-awareness about his time-inconsistency. First a *naïve* agent, i.e. an agent who ignores his time inconsistency and believes that his future *selves* will act according to his current time preferences, is considered. A *sophisticated* agent, i.e. an agent who perfectly foresees that his time preferences change over time, is then considered. Under *naïveté*, the timing strategy set by the current *self* will be formulated viewing future *selves* as "*committed*" to his plan of action. Under *sophistication*, in contrast, the current *self* sets the strategy, being aware that future *selves* are going to "*disobey*".[8] The resulting timing strategy then takes into account the sub-optimality, from the present *self*'s time perspective, associated with the timing strategies set by his future *selves*.

[7] The probability of having a new self born in the next time interval dt is equal to λdt. Hence, consistently, when $\beta \to 0$ the discount rate is adjusted in order to account for the "*sudden death*" of the current self.

[8] The present study focuses on "*consistent planning*", i.e. the agent do not choose plans that are going to be disobeyed. It does not consider the alternative possibility of selecting a "*strategy of precommitment*" which would require committing to a certain plan of action (see e.g Pollak 1968; Strotz 1956).

3.1 An exponential landowner

The landowner discounts exponentially future payoffs[9] at the constant discount rate $\rho > \alpha$.[10] Assume that it is currently worth keeping land in rural use, i.e. $R > R^*$ where R^* is the revenue threshold triggering land development.

The value of land, $V(R)$, is the solution of the following problem:

$$V(R) = \max_{R} \left\{ P, Rdt + e^{-\rho dt} E\left[V(R + dR)\right] \right\}. \tag{3}$$

Using standard arguments, in the continuation region $R > R^*$ the Bellman equation (3) becomes:

$$(1/2)\sigma^2 R^2 V''(R) + \alpha R V'(R) - \rho = -R. \tag{3.1}$$

Solving Eq. (3.1) yields the following proposition:

Proposition 1 *Under exponential discounting,*

(i) *land development occurs when revenues from rural land use reach the threshold*

$$R^* = [\gamma/(\gamma - 1)]P(\rho - \alpha); \tag{4}$$

(ii) *the value of land is equal to:*

$$V(R; R^*) = [P - R^*/(\rho - \alpha)](R/R^*)^{\gamma} + R/(\rho - \alpha), \tag{5}$$

where γ *is the negative root of the characteristic equation* $\Lambda(\gamma) = (1/2)\sigma^2\gamma(\gamma - 1) + \alpha\gamma - \rho = 0$.

Proof See Sect. A.1 in Appendix. □

As one can immediately see, the threshold in Eq. (4) sets a time-consistent timing rule for the development of land. This is of course in line with the assumed exponential discount function which in turn implies a constant rate of time preference.

Examining the threshold in Eq. (4), it can be seen that[11]: (i) the higher the expected growth rate, α, for returns from rural uses, the lower the critical threshold and the later, in expected terms, land is converted in urban use, i.e. $\partial R^*/\partial\alpha < 0$; (ii) as returns from rural uses become more volatile, the critical threshold is lowered and land development is postponed, i.e. $\partial R^*/\partial\sigma^2 < 0$; (iii) the higher the discount rate, the earlier land is converted, i.e. $\partial R^*/\partial\rho > 0$; and (iv) the higher the future net returns from urban use, the earlier land is converted, i.e. $\partial R^*/\partial P > 0$.

In Eq. (5), the first term represents the value of the option to develop land, i.e. the net returns from development, P, minus the present value of forgone returns from rural

[9] The analysis in this section is consistent with the limit case where $\beta \to 1$ in Eq. (2).

[10] This restriction is needed in order to ensure convergence. Note that if $\rho \leq \alpha$ land development would be never optimal.

[11] These results are pretty standard in the literature, see for instance Capozza and Helsley (1990) and Capozza and Li (1994).

land use, $R^*/(\rho-\alpha)$. Note that this payoff materialises only if R reaches R^*. Hence, consistently, the payoff is discounted using the stochastic discount factor $(R/R^*)^\gamma$ which accounts for the probability of hitting R^*.[12] The second term in Eq. (5) is the expected present value of returns from rural land use. This would be the value accruing to the landowner if the condition triggering land development were not achieved. In this respect, note that as $R^* \to 0$, the value of the option to develop vanishes as, by Eq. (1), $R(t) > 0$ and $\lim_{R^*\to 0} V(R; R^*) = R/(\rho-\alpha)$.

3.2 A hyperbolic naïve landowner

The present *self* i discounts by $e^{-\rho dt}$ any payoff accruing in the time interval (s, s_{i+1}) and by $\beta e^{-\rho dt}$ the pay-offs accruing in the interval $[s_{i+1}, \infty)$. Under *naïveté, self* i believes that the entire sequence of successive *selves*, i.e. *selves* in the interval $[s_{i+1}, \infty)$, will take decisions concerning the destination of land according to his own time preferences, i.e. $D_i(s, t)$. This implies that *self* i views these *selves* as standard exponential discounters using ρ as discount rate and having the following plan of action:

$$\begin{array}{ll} \text{keep land in rural use,} & \text{if } R > R^*, \\ \text{sell land,} & \text{if } R \le R^*. \end{array} \tag{S.1}$$

Self i solves the land development problem on the basis of his beliefs and sets R^n as the critical threshold. If $R(t)$ hits R^n before the next *self* is born, *self* i cashes the flow of rents accruing up to the land sale plus the sale net pay-off P. Otherwise, if *self* $i+1$ is born before $R(t)$ hits R^n, *self* i benefits from the flow of rents accruing in the period (s, s_{i+1}) plus the expected present value of the pay-offs cashed by future *selves*. Consistently with his beliefs, i.e. future *selves* developing at R^*, this continuation value is equal to:

$$V_c^n\left(R; R^*\right) = \beta \cdot V(R; R^*)$$

Taking the present self i's perspective, the value of land, $V^n(R)$, is the solution of the following problem:

$$\begin{aligned} V^n(R) = \max_R \Big\{ & P,\, R dt + e^{-\lambda dt} E\left[e^{-\rho dt} V^n(R+dR)\right] \\ & + \left(1-e^{-\lambda dt}\right) E\left[e^{-\rho dt} V_c^n(R+dR; R^*)\right]\Big\}. \end{aligned} \tag{6}$$

Assume that it is currently worth keeping land in rural use, i.e. $R > R^n$. Hence, in the continuation region $R > R^n$ the Bellman equation (6) becomes:

$$(1/2)\sigma^2 R^2 V^{n\,\prime\prime}(R) + \alpha R V^{n\,\prime}(R) - \rho V^n(R) = -\{R + \lambda[V_c^n(R; R^*) - V^n(R)]\}. \tag{6.1}$$

[12] Note that $(R/R^*)^\gamma = E[e^{-\rho T}]$ where T is the first hitting time for the stochastic process $\{R\}$ to reach the barrier R^* (see Dixit and Pindyck 1994, pp. 315–316).

Solving Eq. (6.1) yields the following proposition:

Proposition 2 *Under hyperbolic discounting and naïve beliefs,*

(i) *the land development threshold R^n is the solution of the following equation:*

$$\beta[(\theta-\gamma)/(\theta-1)][P-R^*/(\rho-\alpha)](R^n/R^*)^\gamma+\eta[R^n/(\rho-\alpha)]-[\theta/(\theta-1)]P=0; \tag{7}$$

(ii) *land is, in expected terms, developed earlier than under exponential discounting, i.e. $R^n > R^*$;*

(iii) *the value of land is equal to:*

$$\begin{aligned} V^n(R; R^n, R^*) = \{&P - \beta[P - R^*/(\rho - \alpha)](R^n/R^*)^\gamma \\ &-\eta[R^n/(\rho - \alpha)]\}(R/R^n)^\theta + \beta[P - R^*/(\rho - \alpha)](R/R^*)^\gamma \\ &+\eta[R/(\rho - \alpha)],\ \textit{for}\ R > R^n, \end{aligned} \tag{8}$$

where $\eta = (\rho + \lambda\beta - \alpha)/(\rho + \lambda - \alpha) \leq 1$ and θ is the negative root of the characteristic equation $\Omega(\theta) = (1/2)\sigma^2\theta(\theta - 1) + \alpha\theta - (\rho + \lambda) = 0$.

Proof See Sect. A.2 in Appendix. □

The critical threshold R^n set by a hyperbolic *naïve* landowner is higher than that set by an exponential landowner. This implies that, in expected terms, land is developed earlier. This makes sense considering that, due to his present-biased time preferences, the value of keeping open the option to develop has lower value for *self* i. Note in fact that the value associated with this option when held by his future *selves*, i.e. the flow of rents accruing up to the land sale plus the sale net pay-off P, is lowered by the terms $\eta < 1$ and $0 < \beta < 1$, respectively. Hence, the optimal timing strategy for *self* i is anticipating his future *selves* by setting a higher threshold for exercise of the option to develop. Note that $dR^n/d\beta < 0$ and $dR^n/d\lambda > 0$. This means that the higher the present bias, i.e. the lower β and/or the higher λ, the earlier land development occurs.

Finally, note that the timing strategy set by a hyperbolic *naïve* landowner is based on the belief that his future *selves* are committed to S.1, i.e. the timing strategy set by the current *self*. This belief is, of course, unfounded. In fact, as soon as the next *self* $i + 1$ is born, he will not adopt the threshold R^* as believed by *self* i, but will set his timing strategy on the basis of his own time preferences, i.e. using $D_{i+1}(s, t)$. Given that, by assumption, the sequence of *selves* is infinite and *naïveté* persistent, each newly born *self* faces the same decision problem solved above for *self* i. Hence, it can be concluded that the actual critical threshold for exercise of the option to develop land is unique and equal to $R^n > R^*$.

3.3 A hyperbolic sophisticated landowner

A *sophisticated self* i has perfect foresight and anticipates that his future *selves* may later revise his plan of action. Hence, his plan should be set taking into account that, from his time perspective, future *selves*' timing strategies may be sub-optimal. In the present frame, the landowner is viewed as a sequence of infinite *selves* each having their own time preferences, i.e. $D_i(s,t)$. However, as each *self* is followed by an infinite number of *selves*, the land development problem that each generic *self* must solve is the same. Hence, as they would all consider optimal developing land when the same critical time threshold is reached, their timing strategy can be determined by imposing stationarity to the solution of the underlying non-cooperative intra-personal timing game.

Consider the generic *self* i and denote (i) by $\widetilde{R}$ his conjecture about the critical development threshold set by his future *selves* and (ii) by $g(\widetilde{R})$ his own critical development threshold. Note that, as his timing strategy must be consistent with his beliefs, the threshold $g(\widetilde{R})$ is a function of the conjectured threshold $\widetilde{R}$.

Future *selves* have the following plan of action:

$$\begin{array}{ll} \text{keep land in rural use,} & \text{if } R > \widetilde{R}, \\ \text{sell land,} & \text{if } R \leq \widetilde{R}. \end{array} \tag{S.2}$$

Hence, as above, given S.2 and his time perspective, *self* i's continuation value is equal to:

$$V_c^s(R;\widetilde{R}) = \beta\{[P - \widetilde{R}/(\rho-\alpha)](R/\widetilde{R})^\gamma + R/(\rho-\alpha)\}.$$

Taking, as above, the present self i's perspective, the value of land, $V^s(R)$, is the solution of the following problem:

$$\begin{aligned} V^s(R) = \max_R\{P, Rdt + e^{-\lambda dt} E\left[e^{-\rho dt} V^s(R+dR)\right] \\ + \left(1-e^{-\lambda dt}\right) E[e^{-\rho dt} V_c^s(R+dR;\widetilde{R})]\}, \end{aligned} \tag{9}$$

In the continuation region $R > g(\widetilde{R})$ the Bellman equation (9) becomes:

$$(1/2)\sigma^2 R^2 V^{s\,\prime\prime}(R) + \alpha R V^{s\,\prime}(R) - \rho V^s(R) = -\{R + \lambda[V_c^s(R;\widetilde{R}) - V^s(R)]\}. \tag{9.1}$$

Solving Eq. (9.1) gives:

$$\begin{aligned} V^s(R; h(\widetilde{R}), \widetilde{R}) = \{P - \beta[P - \widetilde{R}/(\rho-\alpha)](g(\widetilde{R})/\widetilde{R})^\gamma \\ -\eta[g(\widetilde{R})/(\rho-\alpha)]\}(R/g(\widetilde{R}))^\theta + \beta[P - \widetilde{R}/(\rho-\alpha)](R/\widetilde{R})^\gamma \\ +\eta[R/(\rho-\alpha)], \text{ for } R > g(\widetilde{R}), \end{aligned} \tag{9.2}$$

where $g(\widetilde{R})$ is the solution of the following equation:

$$\beta[(\theta-\gamma)/(\theta-1)][P - \widetilde{R}/(\rho-\alpha)](g(\widetilde{R})/\widetilde{R})^\gamma + \eta[g(\widetilde{R})/(\rho-\alpha)] - [\theta/(\theta-1)]P = 0. \tag{9.3}$$

By imposing the stationarity condition $g(\widetilde{R}) = \widetilde{R} = R^s$, it can be shown that:

Proposition 3 *Under hyperbolic discounting and sophisticated beliefs,*

(i) *the land development threshold is*

$$R^s = \frac{\theta + \beta(\gamma - \theta)}{\eta(\theta - 1) + \beta(\gamma - \theta)} P(\rho - \alpha); \tag{10}$$

(ii) *land is, in expected terms, developed earlier than under naïve beliefs, i.e.* $R^s > R^n$, *and*

$$R^* < R^s < R^{**} = [\theta/(\theta - 1)]P(\rho + \lambda - \alpha); \tag{10.1}$$

(iii) *the value of land is equal to:*

$$\begin{aligned} V(R; R^s) = (1 - \beta)\{[P - R^s/(\rho + \lambda - \alpha)](R/R^s)^\theta + [R/(\rho + \lambda - \alpha)]\} \\ +\beta\{[P - R^s/(\rho - \alpha)](R/R^s)^\gamma + [R/(\rho - \alpha)]\}, \textit{ for } R > R^s \end{aligned} \tag{11}$$

Proof See Sects. A.2 and A.3 in Appendix. □

The critical threshold, R^s, is increasing in the degree of present bias, i.e. $dR^s/d\beta < 0$ and $dR^s/d\lambda > 0$. In the limit cases $\beta \to 1$ and $\beta \to 0$, the threshold converges toward $R^*$and R^{**}, i.e. the critical thresholds that would be set by an exponential discounter using ρ and $\rho + \lambda$ as discount rates, respectively. The threshold R^s is higher than R^*. As for the case of a *naïve* landowner, this is again due to the need for avoiding, by exercising the option to develop, the passage of *self* i's holdings, i.e. the flow of rents associated with rural use plus the option to develop, to future *selves*. In this respect, note that under *sophistication* the incentive for anticipating future *selves* is even higher and leads to the definition of a threshold R^s higher than that set by a *naïve* landowner. This makes the actual exercise by *self* i of the option to develop even more likely than under *naïveté*. The reasoning behind this result is straightforward. *Self* i in fact, being aware of his time inconsistency, fully internalises the expected loss due to the sub-optimal (from his time perspective) timing strategies set by his future *selves*. This loss makes the value associated with holding the option to develop even lower than for the case of a *naïve* landowner. Last, the threshold R^s is lower than R^{**}, i.e. the threshold corresponding to the case where the arrival of the next *self* would, as $\beta \to 0$, substantially absorb any value associated with *self* i's holdings.

Finally, the value function in Eq. (11) results from the weighted sum of the two value functions that an exponential discounter would have using $\rho+\lambda$ and ρ as discount rates, respectively, and fixing R^s as the threshold for exercise of the option to develop. This is of course consistent, as clearly illustrated by Eq. (2.1), with the expected discount rate associated with the time preferences of the *sophisticated* hyperbolic agent considered.

4 Conclusions

This paper examines the implications of hyperbolic time preferences for the timing of development of rural land. It extends the previous model by Salois (2012) in order to account for (i) uncertainty about payoffs associated with the asset land and (ii) self-awareness about the planning time inconsistency implied by the assumed time preferences. The results show that the impact of option value considerations leading, as uncertainty increases, to the postponement of land development is lower under hyperbolic time preferences. In fact, due to the present bias induced by the implicit time-declining discount rate, the value associated with holding open the option to develop is lower and, as a consequence, the hyperbolic landowner prefers earlier land development than in the case of an exponential landowner. It is found that a *sophisticated* hyperbolic landowner sets a higher threshold for exercise of the option to develop compared with a *naïve* landowner. In fact, a *sophisticated* landowner, being aware that the plan initially set may be later revised, sets his timing strategy by fully internalising the cost associated with the sub-optimality of future changes in the strategy. In contrast, this cost is totally missed by a *naïve* landowner, who believes that the development timing strategy initially set would not be violated. Hence, a further argument rushing land development emerges under *sophistication*.

Concerning the empirical testing of the model, two possible directions could be considered. If a sufficiently rich data set of parcel characteristics and land transactions is available, one may test the model's predictions by (i) checking for the presence of an option premium and/or looking at the extent to which land prices internalize an option value component or by (ii) examining whether the suggested exercise policy has actually been followed (see e.g. Quigg 1993; Plantinga et al. 2002; Schatzki 2003; Cunningham 2005, 2007; Bulan et al. 2009). Otherwise, to overcome potential data problems, one may test the model by using laboratory experiments to generate data about individual land development decisions (see e.g. Yavas and Sirmans 2005; Ihli et al. 2013; Sandri et al. 2010).

Last, two final remarks are in order. First, the discount function (2) may also be used to illustrate present-bias due to political turnover or imperfect intergenerational altruism.[13] This implies that our frame may easily apply to the analysis of land development decisions taken by agents such as, for instance, Governments or public agencies. Second, note that using the net payoff P as numeraire we could also, by invoking the homogeneity of the option value function, allow for stochastic net returns from land development following a geometric Brownian motion. The model would in fact be the same once $R(t)$ is normalised in terms of P.[14]

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[13] See Di Corato (2012).

[14] See for instance Dixit and Pindyck (1994, pp. 207–211) and Geltner et al. (1996, Section 1, p. 24).

Appendix

A.1 Exponential discounting

As standard, to guarantee optimality,[15] the solution of the differential Eq. (3.1) must meet the following *value-matching* and *smooth-pasting* conditions:

$$V(R^*) = P, \tag{A.1.1}$$
$$V'(R^*) = 0. \tag{A.1.2}$$

A candidate solution for Eq. (3.1) takes the form[16]:

$$V(R) = kR^{\gamma} + R/(\rho - \alpha), \tag{A.1.3}$$

where k is a constant to be determined and γ is the negative root of the characteristic equation $\Lambda(\gamma) = (1/2)\sigma^2\gamma(\gamma - 1) + \alpha\gamma - \rho = 0$.

Substituting Eq. (A.1.3) into Eq. (A.1.1) and Eq. (A.1.2) yields:

$$\begin{cases} kR^{*\gamma} + R^*/(\rho - \alpha) = P, \\ k\gamma R^{*\gamma-1} + 1/(\rho - \alpha) = 0. \end{cases}$$

Solving this system gives k and R^*. Plugging k into Eq. (A.1.3) yields Eq. (5).

A.2 Hyperbolic discounting

Equations (6.1) and (9.1) are technically similar. Proceed by solving first the underlying common problem and then characterising the solution according to the assumed beliefs concerning future *selves*' time preferences.

The equation to be solved is:

$$\begin{aligned}(1/2)\sigma^2 R^2 V^{q\,\prime\prime}(R) + \alpha R V^{q\,\prime}(R) - (\rho + \lambda)V^q(R) \\ = -\{R[1 + \lambda\beta/(\rho - \alpha)] + \lambda\beta\{[P - \overline{R}^q/(\rho - \alpha)](R/\overline{R}^q)^{\gamma}\}\end{aligned} \tag{A.2.1}$$

where

$$\begin{aligned} &q = n \text{ and } \overline{R}^n = R^*, \text{ if the agent is } \textit{naïve}, \\ &q = s \text{ and } \overline{R}^s = \widetilde{R}, \quad \text{if the agent is } \textit{sophisticated.} \end{aligned}$$

Suppose that the particular solution for Eq. (A.2.1) takes the form $V^q(R) = c_1 R^{\gamma} + c_2 R$. Substituting this candidate form and its first two derivatives, $\partial V^q(R)/\partial R =$

[15] See Dixit and Pindyck (1994, Chapter 4).

[16] The solution for the homogeneous part of Eq. (3.1) should have the form $V_h(R) = k_1 R^{\gamma_1} + k_2 R^{\gamma_2}$ where k_1 and k_2 are constants to be determined while $\gamma_1 > 0$ and $\gamma_2 < 0$ are the roots of the characteristic equation $\Lambda(\gamma) = 0$. However, as $R \to \infty$, the value of the option to develop goes to zero. Thus, as $\gamma_1 > 0$, then k_1 must be zero, otherwise $\lim_{R\to\infty} V_h(R) = \infty$.

$c_1\gamma R^{\gamma-1} + c_2$ and $\partial^2 V^q(R)/\partial R^2 = c_1\gamma(\gamma-1)R^{\gamma-2}$ into Eq. (A.2.1) yields:

$$\begin{aligned}&\{[(1/2)\sigma^2\gamma\,(\gamma-1)+\alpha\gamma-\rho]-\lambda\}c_1 R^\gamma + [\alpha-(\rho+\lambda)]\,c_2 R\\ &\quad = -\{R[1+\lambda\beta/(\rho-\alpha)] + \lambda\beta\{[P-\overline{R}^q/(\rho-\alpha)](R/\overline{R}^q)^\gamma\}\end{aligned} \tag{A.2.2}$$

The coefficients c_1 and c_2 can be determined by solving the following two equations:

$$\begin{aligned}\{[(1/2)\sigma^2\gamma\,(\gamma-1)+\alpha\gamma-\rho]-\lambda\}c_1 &= -\lambda\beta[P-\overline{R}^q/(\rho-\alpha)](1/\overline{R}^q)^\gamma\\ [\alpha-(\rho+\lambda)]\,c_2 &= -[1+\lambda\beta/(\rho-\alpha)]\end{aligned}$$

Solving both equations yields:

$$\begin{aligned}c_1 &= \beta[P-\overline{R}^q/(\rho-\alpha)](1/\overline{R}^q)^\gamma\\ c_2 &= [1+\lambda\beta/(\rho-\alpha)]/\,(\rho+\lambda-\alpha) = \eta/(\rho-\alpha)\end{aligned}$$

where $\eta = (\rho+\lambda\beta-\alpha)/(\rho+\lambda-\alpha) \le 1$.

The general solution then takes the form[17]:

$$V^q(R;\overline{R}^q) = kR^\theta + \beta[P-\overline{R}^q/(\rho-\alpha)](R/\overline{R}^q)^\gamma + \eta[R/(\rho-\alpha)] \tag{A.2.3}$$

where k is a constant to be determined and θ is the negative root of the characteristic equation $\Omega(\theta) = (1/2)\sigma^2\theta(\theta-1)+\alpha\theta-(\rho+\lambda) = 0$.

At the critical threshold value, $g(\overline{R}^q)$, optimality requires that the following *value-matching* and *smooth-pasting* conditions hold:

$$V^q(R^q;\overline{R}^q) = P, \tag{A.2.4}$$

$$\partial V^q(R;\overline{R}^q)/\partial R\Big|_{R=g(\overline{R}^q)} = 0 \tag{A.2.5}$$

Solving the system [A.2.4–A.2.5] yields:

$$k = \{P-\beta[P-\overline{R}^q/(\rho-\alpha)](g(\overline{R}^q)/\overline{R}^q)^\gamma + \eta[g(\overline{R}^q)/(\rho-\alpha)]\}(1/g(\overline{R}^q))^\theta \tag{A.2.6}$$

and

$$\begin{aligned}&\beta[(\theta-\gamma)/(\theta-1)][P-\overline{R}^q/(\rho-\alpha)](g(\overline{R}^q)/\overline{R}^q)^\gamma\\ &\quad +\eta[g(\overline{R}^q)/(\rho-\alpha)] - [\theta/(\theta-1)]P = 0\end{aligned} \tag{A.2.7}$$

[17] The solution for the homogeneous part of Eq. (A.2.1) should have the form $V_h^q(R) = k_1 R^{\theta_1} + k_2 R^{\theta_2}$ where k_1 and k_2 are constants to be determined while $\theta_1 > 0$ and $\theta_2 < 0$ are the roots of the characteristic equation $\Omega(\theta) = 0$. However, as $R \to \infty$, the value of the option to develop goes to zero. Thus, as $\theta_1 > 0$, then k_1 must be zero, otherwise $\lim_{R\to\infty} V_h^q(R) = \infty$.

A.2.1 Naïve beliefs

Substituting $q = n$, $g(\overline{R}^n) = R^n$ and $\overline{R}^n = R^*$ into Eqs. (A.2.6) and (A.2.7) yields:

$$k = \{P - \beta[P - R^*/(\rho - \alpha)](R^n/R^*)^\gamma - \eta[R^n/(\rho - \alpha)]\}(1/R^n)^\theta$$

and

$$\beta[(\theta - \gamma)/(\theta - 1)][P - R^*/(\rho - \alpha)](R^n/R^*)^\gamma$$
$$+\eta[R^n/(\rho - \alpha)] - [\theta/(\theta - 1)]P = 0$$

Plugging k into Eq. (A.2.3) gives Eq. (8).

A.2.2 Sophisticated beliefs

Substituting $q = s$ and $\overline{R}^n = \widetilde{R}$ into Eqs. (A.2.6) and (A.2.7) yields:

$$k = \{P - \beta[P - \widetilde{R}/(\rho - \alpha)](g(\widetilde{R})/\widetilde{R})^\gamma - \eta[g(\widetilde{R})/(\rho - \alpha)]\}(1/g(\widetilde{R}))^\theta,$$

and

$$\beta[(\theta - \gamma)/(\theta - 1)][P - \widetilde{R}/(\rho - \alpha)](g(\widetilde{R})/\widetilde{R})^\gamma$$
$$+\eta[g(\widetilde{R})/(\rho - \alpha)] - [\theta/(\theta - 1)]P = 0.$$

Then, imposing the stationarity condition $g(\widetilde{R}) = \widetilde{R} = R^s$ yields:

$$k = \{P - \beta[P - R^s/(\rho - \alpha)] - \eta[R^s/(\rho - \alpha)]\}(1/R^s)^\theta,$$

and

$$R^s = \frac{\theta + \beta(\gamma - \theta)}{\eta(\theta - 1) + \beta(\gamma - \theta)} P(\rho - \alpha).$$

Plugging k into Eq. (A.2.3) gives Eq. (11).

A.3 Timing thresholds: properties

Define the function $Z(x) = \beta[(\theta - \gamma)/(\theta - 1)][P - R^*/(\rho - \alpha)](x/R^*)^\gamma + \eta[x/(\rho - \alpha)] - [\theta/(\theta - 1)]P$. Note that $Z(x)$ is convex in x, $Z(R^n) = 0$ and $Z(R^*) < 0$. Optimality requires that $Z'(R^n) > 0$. Hence, it follows that $R^* < R^n$. Then, as $\theta > \gamma$, it can easily be shown that $[\gamma/(\gamma - 1)] < [\theta + \beta(\gamma - \theta)]/[\eta(\theta - 1) + \beta(\gamma - \theta)] < [\theta/(\theta - 1)]$. This in turn implies that $R^* < R^s < R^{**}$.

Finally, define the function $Z(x; \widetilde{x}) = \beta[(\theta - \gamma)/(\theta - 1)][P - \widetilde{x}/(\rho - \alpha)](x/\widetilde{x})^\gamma + \eta[x/(\rho - \alpha)] - [\theta/(\theta - 1)]P$. Denote by $g(\widetilde{x})$ the solution of the equation $Z(g(\widetilde{x}); \widetilde{x}) = 0$. Totally differentiating with respect to $\widetilde{x}$ yields:

$$\beta[(\theta-\gamma)/(\theta-1)]\{-1/(\rho-\alpha) \\ +[P-\widetilde{x}/(\rho-\alpha)][(1/g(\widetilde{x}))\partial g(\widetilde{x})/\partial\widetilde{x}-(\gamma/\widetilde{x})]\}(g(\widetilde{x})/\widetilde{x})^{\gamma} \\ +\eta(\partial g(\widetilde{x})/\partial\widetilde{x})/(\rho-\alpha)=0$$

Rearranging gives:

$$\partial g(\widetilde{x})/\partial\widetilde{x} \\ =\frac{\beta[(\theta-\gamma)/(\theta-1)](g(\widetilde{x})/\widetilde{x})^{\gamma}\widetilde{x}(\gamma-1)}{\beta[(\theta-\gamma)/(\theta-1)][P(\rho-\alpha)-\widetilde{x}](g(\widetilde{x})/\widetilde{x})^{\gamma-1}+\eta\widetilde{x}}\left(\frac{R^*}{\widetilde{x}}-1\right)<0, \text{ for } R^*<\widetilde{x}$$

Note that:

$$Z(R^n;R^*)=Z(R^s;R^s)=0$$

Hence, as $R^* < R^s$, it follows that $R^n < R^s$.

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