JOURNAL OF URBAN ECONOMICS 10, 201–226 (1981)

Growth, Speculation and Sprawl in a Monocentric City

DAVID E. MILLS[1]

Department of Economics, Rouss 114, Charlottesville, Virginia 22901

Received March 3, 1980; revised July 28, 1980

An economic theory of sprawl in a growing, monocentric city is presented. Where decision-makers have perfect foresight, leapfrog development and discontinuous land-rent functions may occur and be efficient in both an *ex post* and *ex ante* sense. Where the extent of future growth is uncertain, decision-makers become speculators and the spatial pattern of development is more complicated. *Ex post* inefficiency generally occurs.

I. INTRODUCTION

The conventional monocentric-city model is a characterization of a static, long-run spatial equilibrium. It is usually marked by a continuous, downward-sloping land-rent function and a radial sequence of internally homogeneous von Thünen rings of development. While it makes many predictions that apply broadly to urban areas, this model cannot portray land-use changes brought on by economic growth.

This paper shows that growth has much to do with some of the discrepancies between this model and actual urban land-use configurations. In particular, it presents an economic theory of *sprawl*, a serviceable definition of which is "the lack of continuity in expansion."[2] Sprawl is widely discussed but poorly understood. Critics hold that it is aesthetically unattractive, that it impedes efficient local public service delivery by scattering users, and that it wastes and misallocates land [2, 3, 5, 11].

In the context of a formal monocentric-city model, there are three land–use patterns that qualify as examples of sprawl. *Leapfrog development* occurs when a von Thünen ring of undeveloped land separates rings of developed land. This form of sprawl involves radial discontinuity; the other two forms involve circumferential discontinuity. *Scattered development* occurs when there are annuli with both developed (homogeneously) and

[1]The author thanks Richard Arnott, David Crowe, R. Blaine Roberts, and a referee for helpful comments on an earlier draft. This paper reports work supported by Grant H-5036RG from the U.S. Department of Housing and Urban Development. The author is solely responsible for the accuracy of the statements and interpretations contained in this paper and such interpretations do not necessarily reflect the views of the Government.

[2]Clawson [5, p. 99].

0094-1190/81/050201-26$02.00/0

undeveloped land in them. *Mixed development* occurs when there are annuli with more than one developed use.

Theoretical explanations for all three are offered in the paper. The first pattern can be explained by mere intertemporal planning on the part of decision-makers who anticipate future growth with certainty. The essential idea here is similar to Ohls' and Pines' [10], that land inside the urban fringe is sometimes withheld from early development and preserved for more remunerative future options.[3] Explanations for the last two patterns require that decision-makers are uncertain about the extent of future growth and make speculative decisions.

Several dynamic, monocentric-city models have appeared in recent years. Anas [1] constructed one that portrayed a sequence of short-run spatial equilibria with myopic decision-makers. Fujita [6] and Brueckner and von Rabenau [4] constructed models where decision-makers have perfect foresight. The model presented here begins with perfect foresight but is extended to deal with uncertainty and speculation.

II. THE MODEL

Consider an open city in a national spatial economy where intercity trade and migration occur. It lies on a featureless plane and has a central point around which all activities position themselves. Intracity transportation costs are symmetric around this point and proximity to it is the only characteristic that meaningfully distinguishes land parcels. This means location can be represented by a single variable, x, the distance to the central point. We examine the use of land in this city in two time periods of fixed but unspecified lengths.

A single composite commodity, Q, is produced by a competitive industry in the city in both periods. Using subscripts to designate time periods, the endogenous quantities produced are Q_1 and Q_2. There is no storage between periods. Some Q is produced for export and the rest for local consumption.

[3] Several other explanations for sprawl have been offered. Harvey and Clark [7] argue that topography (steep slopes, low marshes, etc.) accounts for part of observed patterns, and that "dead land"—parcels withheld from development because of clouded titles, tax delinquency, and other institutional factors—accounts for much of the rest. Another explanation is that some landowners receive nonpecuniary benefits from maintaining their land in a use that does not maximize land rent. The family farm at the edge of the city is often cited in this connection. Northam [9] identifies considerable quantities of urban land in the U.S. which he calls "remnant parcels" because they are too small or too irregular in shape to be attractive for development. Markusen and Scheffman [8] offer the suggestion that where land-ownership concentration is high, land near the city-center may be withheld from development to increase demand for peripheral land. Under certain conditions, they show that aggregate land rents are maximized by following such a strategy. Finally, part of the observed pattern can be attributed to local government land-use controls. Open-space zoning and density ceilings for planned-unit and cluster development projects are prominent examples.

Regardless of whether it is bound for export or local use, every unit of Q must be transported to the central point at a cost of t per unit of Q, per unit of x. It is sold there at the (endogenous) price P_i, $i = 1, 2$. The Q production function has fixed factor-proportions. It requires exactly λ units of land, μ of labor and ν of capital to produce each unit of Q. Land rent and labor wages are endogenous to the model, but the cost of capital is exogenous; it is s per unit throughout the city in both periods.

Households who reside in the city are identical. Each provides a single unit of labor to the Q industry per period, receives (endogenous) wages of w per period, and has the same utility function. The number of households residing in the city in either period is endogenous. Because the city is open, migration will assure that every resident household attains the (exogenous) level of utility attained elsewhere in the national economy. Arguments of the utility function are Q, residential land, and another composite commodity, Z, that is imported from outside the city and sold at the central point at unitary price in both periods. Nonland components of housing services are absorbed in Q and Z. For analytical convenience, we suppress the substitutability of consumption goods and assume that to attain the exogenous level of utility, households must consume exactly q units of Q, z of Z, and 1 unit of residential land in each period.[4] To ensure that local consumption of Q does not exhaust production, we assume that

$$1/\mu > q.$$

Households value residential proximity to the central point because workers must commute to work by passing through it and shoppers must shop at it. The combined cost of these trips is T per household, per unit of x, per period.

The exogenous growth mechanism in the model is an increase in export demand for Q between the two periods.[5] The export demand function is $f(P_i, \Gamma_i)$ where Γ_i, $i = 1, 2$, is an exogenous demand-shift variable that encodes all the information required to predict the usual price-quantity demand relation. Further,

$$\partial f/\partial P_i < 0, \qquad \partial f/\partial \Gamma_i > 0, \qquad i = 1, 2; \qquad \text{and} \qquad \Gamma_2 > \Gamma_1.$$

No other exogenous variables change between periods.

[4] The suppression of substitution in consumption and the assumption that production uses fixed factor-proportions ensures that land-rent functions are linear. Relaxation of either assumption would make some of them convex downward and complicate the present analysis without adding anything essential.

[5] There are, of course, alternative growth mechanisms. In a closed city, growth could result from an exogenous population increase. It could also come from a change in production, housing, or transportation technology

The agents who make land-use decisions in the model are nonresident, profit-maximizing landowners. Their nonresidency means land rents do not reappear in the analysis as household income.[6] Landownership is sufficiently diffuse that the land market is competitive. Development strategies open to landowners include developing industrially or residentially in either period, and leaving land undeveloped in both periods. Conversion from one use to another in the second period is precluded by prohibitive conversion costs. Structures are durable and nonmalleable, and construction costs are amortized throughout their lives.

III. PERFECT-FORESIGHT PLANNING

Suppose landowners have perfect foresight and know at the outset what their land rent will be in both periods for all possible development strategies. This requires that Γ_2 be known with certainty in period 1, and means that all development decisions—whether executed in the first or second period—are made simultaneously in period 1. We call this perfect-foresight planning. It is to be distinguished from speculation, examined later, which occurs when landowners are uncertain about Γ_2 and future land rents.

In first-period equilibrium, households will reside in a von Thünen ring *outside* another where Q is produced if only

$$t/\lambda > T.$$

Similarly, second-period residential development will lie more distant from the central point than second-period industrial development. It is not certain, however, that all second-period development will lie beyond the first-period residential ring. Depending on the relative magnitudes of t/λ and T, it may be advantageous for landowners to withhold from development in period 1 a ring of land between the industrial and residential zones, and preserve it for second-period industrial use. This would provide an example of leapfrog development in period 1.

In equilibrium, land uses will conform to one of the following patterns:

(1) | Ind'y − 1 | Res − 1 | Ind'y − 2 | Res − 2 | → x

(2) | Ind'y − 1 | Ind'y − 2 | Res − 1 | Res − 2 | → x

There are four development zones in each. Every zone corresponds to a von

[6] We might instead have made households in the city landowners and allowed them to share equally in total land rent.

Thünen ring. The second pattern, which involves leapfrog development in period 1, is more likely to occur the greater is t/λ relative to T. In particular, it will *always* occur if

$$t/\lambda > (2 + r)T,$$

where r is the (universal) discount rate between periods. (This is proved in Appendix A.) Since $-t/\lambda$ and $-T$ are shown below to be the slope of industrial and residential bid-rent functions, this condition requires that industrial bid-rents decrease more than twice as rapidly with x as residential bid-rents. This is scarcely implausible. Since this paper is concerned with spatial discontinuities, we assume this to hold and confine ourselves to the second pattern exclusively.

A. First-Period Equations

Equilibrium in period 1 is characterized by eight endogenous variables. Three of them are Q_1, P_1 and w_1. Two others indicate land rents in the residential and industrial zones. The remaining three are spatial boundaries: x_a, the outer edge of the industrial zone; and x_b and x_c, the inner and outer boundary of the residential zone. From previous assumptions, these will satisfy

$$0 \leq x_a \leq x_b \leq x_c.$$

To identify the industrial land rent variable, note that firms in the competitive Q industry must earn zero profit. Thus land rent and transportation charges must exhaust revenues after payments are made to capital and labor. Since the latter costs are the same for every firm, on a per-unit-of-Q basis, land rent and transportation charges per-unit-of-Q must be the same. And since each unit of Q requires exactly λ units of land, this sum is the same for every firm on a per-unit-of-land basis as well. In period 1, we call this amount R_1^Q per unit of land. Because total transportations charges per-unit-of-land are tx/λ at x, the rent on industrial land is indicated by the linear function $R_1^Q - tx/\lambda$.

To identify the residential land rent variable, note that each household has the same income, consumes the same consumption bundle, and faces the same prices for Q and Z. Their expenditure on (one unit of) land and transportation charges must therefore be the same at every location. In period 1, we call this amount R_1^H. Because household transportation charges are Tx at x, the rent on residential land is indicated by another linear function, $R_1^H - Tx$.

First-period equilibrium is characterized by eight conditions. One of them equates the supply and demand for Q_1. This means local production of Q

must equal the sum of quantities demanded for export and local consumption. Where Q_1 units are produced, μQ_1 households are required to supply the corresponding amount of labor. This means local consumption of Q_1 must be $\mu q Q_1$. Q-market equilibrium requires that

$$Q_1(1 - \mu q) = f(P_1, \Gamma_1). \tag{1}$$

A second condition is that household budgets balance:

$$w_1 = R_1^H + P_1 q + z. \tag{2}$$

A third is that competitive Q-firms earn zero profit:

$$P_1 = \mu w_1 + \nu s + \lambda R_1^Q. \tag{3}$$

Each unit of Q requires λ units of land, so Q_1 is related to x_a by

$$Q_1 = \pi x_a^2/\lambda. \tag{4}$$

Similarly, each household resides on one unit of land and there are μQ_1 households in the city, so Q_1 is related to x_b and x_c by

$$Q_1 = \pi(x_c^2 - x_b^2)/\mu. \tag{5}$$

The last three conditions concern equilibrium in the land market. At x_a, industrial land rent must be zero, since otherwise land beyond it would be offered for first-period industrial development or land inside it withheld from development until later. This requires that

$$R_1^Q = tx_a/\lambda. \tag{6}$$

For similar reasons, residential land rent at x_c must be zero:

$$R_1^H = Tx_c. \tag{7}$$

The equilibrium condition concerning x_b is less easily stated. Owners of land near x_a must decide only whether to develop industrially in the first or second period. Those near x_c make a similar decision for residential development. In both cases the decision rule is simply to develop if the resulting land rent is positive but not otherwise. For owners near x_b, however, the decision is whether to develop residentially in the first period, or wait and develop industrially in the second. The present value of the first option is

$$R_1^H - Tx + \frac{1}{1+r}\left(R_2^H - Tx\right).$$

The present value of the second option is

$$\frac{1}{1+r}\left(R_2^Q - tx/\lambda\right).$$

x_b is the location where these strategies are equally profitable:

$$R_1^H - Tx_b = \frac{1}{1+r}\left[\left(R_2^Q - tx_b/\lambda\right) - \left(R_2^H - Tx_b\right)\right]. \tag{8}$$

Expressing it this way emphasizes that the gain received from early residential development (the left-hand side of (8)) equals the loss incurred by forgoing lucrative industrial development in the second period (the right-hand side).

Equations (1)–(8) are not a closed system since (8) includes the second-period, endogenous variables, R_2^H and R_2^Q. To solve for equilibrium, it is necessary to solve both periods' equations simultaneously, as in a two-period, dynamic program.

B. Second-Period Equations

Equilibrium in period 2 is characterized by six variables, five of which are Q_2, P_2, w_2, R_2^Q and R_2^H. The other one is x_d, the outer boundary of second-period, residential expansion. Of course

$$x_d \geq x_c.$$

The second-period conditions corresponding to (1)–(3) are

$$Q_2(1 - \mu q) = Q(P_2, \Gamma_2), \tag{9}$$

$$w_2 = R_2^H + P_2 q + z, \tag{10}$$

$$P_2 = \mu w_2 + \nu s + \lambda R_2^Q. \tag{11}$$

Since $\Gamma_2 > \Gamma_1$, Q-production will be greater in period 2 and the industrial zone will expand from x_a to x_b. Thus, Q_2 is related to x_b by

$$Q_2 = \pi x_b^2/\lambda. \tag{12}$$

The increase in production requires an expansion in the residential zone from x_c to x_d. In order that this expansion allow a total of Q_2 households to reside in the city:

$$Q_2 = \pi(x_d^2 - x_b^2)/\mu. \tag{13}$$

The final condition is that land rent at x_d is zero:

$$R_2^H = Tx_d. \tag{14}$$

Conditions (1)–(14) is a closed system of fourteen, independent equations in fourteen endogenous variables: x_a, x_b, x_c, x_d, R_1^H, R_1^Q, R_2^H, R_2^Q, Q_1, P_1, w_1, Q_2, P_2 and w_2.

C. The Equilibrium

Figure 1 illustrates an equilibrium configuration of land-rent functions. Land inside x_a is developed industrially in the first period; land between x_a and x_b is leapfrogged over in the first period and developed industrially in the second period; land between x_b and x_c is developed residentially in the first period; and land between x_c and x_d is developed residentially in the second period. The amounts of land in the residential and industrial zones in the first period are $\pi(x_c^2 - x_b^2)$ and πx_a^2, respectively. From (4) and (5), this means the ratio of residential to industrial land is λ/μ. The same ratio prevails in the second period, as dictated by model assumptions and as seen from (12) and (13).

The equilibrium land-rent function in the first period is

$$\begin{aligned} ER_1(x) &= R_1^Q - tx/\lambda, & x &\in [0, x_a] \\ &= R_1^H - Tx, & x &\in (x_b, x_c] \\ &= 0, & & \text{elsewhere}. \end{aligned}$$

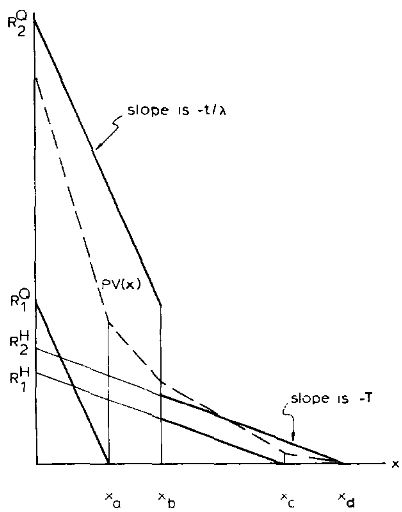

FIGURE 1

It is continuous at every x except x_b. Land just beyond x_b earns a positive rent while land just inside it earns none. The equilibrium land-rent in the second period is

$$\begin{aligned} ER_2(x) &= R_2^Q - tx/\lambda, && x \in [0, x_b] \\ &= R_2^H - Tx, && x \in (x_b, x_d] \\ &= 0, && \text{elsewhere.} \end{aligned}$$

Note first that

$$ER_2(x) \geq ER_1(x), \qquad \text{for all } x \geq 0,$$

and second that $ER_2(x)$ is also continuous at every x but x_b. In the second period, however, land just beyond x_b earns less rent than land just inside it.

The discontinuities at x_b are at variance with equilibrium properties of static monocentric-city models, but are easily explained. According to (8), they offset each other exactly once second-period rent is discounted. Because of this, the function indicating equilibrium present value of both period's land rent,

$$PV(x) = ER_1(x) + \frac{1}{1+r} ER_2(x),$$

is continuous at every x. This function is drawn in Fig. 1 using a value of 0.5 for r.

D. *Comparative Statics*

Consider now the effect on both periods' spatial equilibrium of a change in Γ_2, all other exogenous variables remaining the same. The following inequalities are proved in Appendix B:

$$\begin{aligned} dx_a/d\Gamma_2 &< 0, \\ dx_b/d\Gamma_2 &> 0, \\ dx_c/d\Gamma_2 &> 0, \\ dx_d/d\Gamma_2 &> 0, \end{aligned}$$

All boundaries move outward as Γ_2 increases except x_a, which moves inward.

Since x_b increases and x_a decreases with Γ_2, the second-period industrial zone is larger to accomodate increased production of Q. Since the residential-to-industrial land ratio is constant, the second-period residential zone must also be larger. Thus, the annulus of land added by the increase in x_d is greater than that lost by the increase in x_b.

In contrast to these results, both zones become smaller in the first period. The industrial zone contracts as x_a decreases, and preserving the size-of-zones ratio, less land is added to the residential zone by the increase in x_c than is lost by the increase in x_b. Since x_b increases and x_a decreases, the amount of land in the leapfrog zone increases with Γ_2. The first-period residential zone is not only smaller than before, but also more remote.

At first glance these results are anomalous. Why should a *ceteris paribus* increase in *future* export demand for the city's product cause a reduction in *current* production and city size? The answer lies in the incentives weighed by landowners in the leapfrog zone. By preserving more land for future industrial use, they inadvertently force early residential development farther away from the central point. This increases the transportation costs borne by the residential sector which has an effect best represented as an upward shift in the first-period supply curve for Q. Because there is no off-setting effect on the demand side of the Q-market, first period production and city-size fall.

IV. UNCERTAINTY AND SPECULATION

We now drop the perfect foresight assumption and suppose instead that landowners are uncertain in period 1 about second-period land rent. In particular, they share the probability distribution $g(\Gamma_2)$ over Γ_2 in the first period where the true value of Γ_2 is resolved in period 2. We retain the assumption that $\Gamma_2 > \Gamma_1$, so

$$\int_0^{\Gamma_1} g(\Gamma_2)\,d\Gamma_2 = 0.$$

We assume landowners are risk neutral.[7]

In this environment, landowners' first-period decisions are speculative. Decisions concerning second-period development are postponed until the second period rather than being incorporated in a complete strategy at the outset as in perfect-foresight planning. This means the monocentric-city model equilibrium is the result of a *sequential* decision process.

The main difference between this and the perfect-foresight environment is that landowners cannot know for certain how much land should be withheld from first-period development and preserved for future industrial expansion. (That some will be withheld is assured by the assumption $t/\lambda > (2 + r)T$.) The smaller the value of Γ_2 resolved, the less will be needed. A related difference is that depending upon the value of Γ_2 resolved, the outer boundary of industrial expansion may not be x_b. Because of this, we

[7] If not, they make first-period decisions on "certainty-equivalent" measures rather than "expected values."

introduce x_e as that boundary. While x_b is determined in period 1 on speculation, x_e is determined along with x_d in period 2 once Γ_2 is resolved. If Γ_2 is small, then $x_e < x_b$, indicating that more land than necessary was preserved. If Γ_2 is sufficiently large, then $x_e > x_c$, indicating that all preserved land is used and that a second industrial ring occurs beyond the first-period residential zone.

A final difference is that if Γ_2 is sufficiently small, land rents can be *negative* at some locations in the second period where residential development occured in the first period. It might seem that landowners would avoid this by withholding their developed properties from use in period 2 and earning 0 rent. But this will not occur for the following reason. Land rents are the difference between building rents and amortized construction costs (neither shown explicitly in the model). The latter must be borne in period 2 whether or not the building is leased because first-period development decisions are irrevocable. We assume building rents are always positive. This means landowners can always cover part of their unavoidable costs by leasing—even if all of them are not covered and land rents are negative—and will not refuse to do so.

There are five distinct cases concerning the equilibrium location of x_d and x_e vis-à-vis x_a, x_b and x_c. Each case corresponds to an endogenously determined range of Γ_2. Because second-period equilibrium conditions differ among the cases, it is better to begin with them. Afterward, we return to the first-period speculative equilibrium.

A. Second-Period Equilibrium

The second-period equilibrium is characterized by seven variables—six from perfect-foresight planning, and x_e—and seven independent equations. These equations reflect the actual values of Γ_2 and all variables determined in period 1. Three of them come from the perfect-foresight equilibrium: (9)–(11). The other four differ among the five cases described below. Some of them apply to more than one case, but none applies to all five. Figure 2 is helpful in elucidating these conditions. The values of x_a, x_b, x_c, R_1^H and R_1^Q in the figure are the same in every case since they are determined in the first period.

Case 1. We begin with the lower tail of $g(\Gamma_2)$. Suppose

$$\Gamma_2 = \Gamma_1 + \delta,$$

where δ is positive but very small, indicating miniscule growth in export demand for Q between periods. The amount of land needed for both industrial and residential expansion is much less than that preserved by speculators between x_a and x_b. Naturally, since the industrial rent function is steeper than the residential one, industrial expansion will occur adjacent

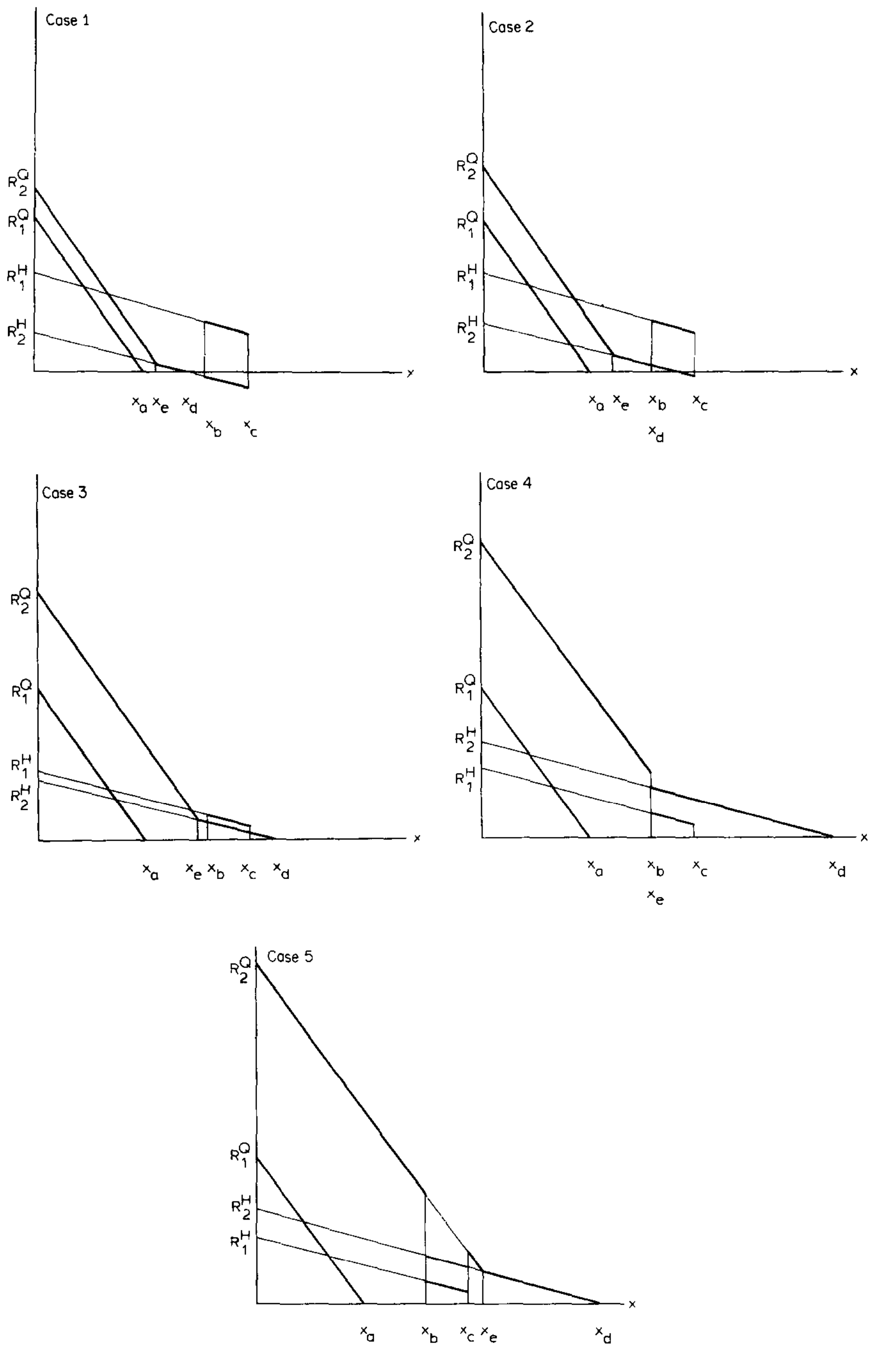
Case 1
R2Q
R1Q
R1H
R2H
x
xa xe xd
xb xc
Case 2
R2Q
R1Q
R1H
R2H
x
xa xe xb xc
xd
Case 3
R2Q
R1Q
R1H
R2H
x
xa xe xb xc xd
Case 4
R2Q
R1Q
R2H
R1H
x
xa xb xc xd
xe
Case 5
R2Q
R1Q
R2H
R1H
x
xa xb xc xe xd

FIGURE 2

to the first-period industrial zone. It will occupy an annulus of land between x_a and x_e. The expanded workforce will be accomodated by an annulus of residential development between x_e and x_d, where[8]

$$x_a < x_e < x_d < x_b < x_c.$$

This leaves three von Thünen rings of developed land—one industrial and two residential—and a ring of undeveloped land between residential zones.

The amount of land in the industrial zone must equal λQ_2, and that in the two residential zones μQ_2. This provides two of the remaining four equilibrium conditions:

$$Q_2 = \pi x_e^2/\lambda, \tag{15}$$

$$Q_2 = \pi\left(x_c^2 - x_b^2 + x_d^2 - x_e^2\right)/\mu. \tag{16}$$

The other two conditions concern equilibrium in the land market. In order that land beyond x_d not be offered for residential development, or land inside it withheld, residential land rent must be zero at x_d:

$$R_2^H = Tx_d. \tag{17}$$

In order that land beyond x_e not be developed industrially, or land inside it residentially, land rent must be the same for both uses at x_e:

$$R_2^Q - R_2^H = (t/\lambda - T)x_e. \tag{18}$$

An important feature of equilibrium in Case 1, and a direct implication of (17), is that second-period land rent is *negative* throughout the residential zone developed in the first period.

As Γ_2 increases, more land is required for both kinds of expansion, so x_e and x_d increase. But there is a limit to this kind of expansion. Once x_d reaches x_b, there is no longer any vacant land in the leapfrog zone to keep residential land rent zero at x_d. Thus, (17) ceases to hold and Case 1 no longer applies. Let the value of Γ_2 for which

$$R_2^H = Tx_b$$

be called Γ_2^1. This is the minimum value of Γ_2 for which

$$x_d = x_b. \tag{19}$$

[8] x_d, recall, is the outer boundary of second-period residential *expansion*. As this case shows, it is not necessarily the outer boundary of all residential development.

Case 1 and the set of conditions (15)–(18) apply only when

$$\Gamma_1 < \Gamma_2 < \Gamma_2^1.$$

Case 2. Now consider

$$\Gamma_2 = \Gamma_2^1.$$

All land in the leapfrog zone will be developed here. The equilibrium condition replacing (17) in this case is (19). The other equilibrium conditions are the same as before: (15), (16) and (18). This leaves two von Thünen rings of developed land and eliminates the undeveloped ring seen in Case 1. Taken together, (15), (16) and (19) imply that x_e can be solved for as a function of x_c:

$$x_e = x_c\sqrt{\lambda/(\mu+\lambda)}.$$

This case is distinguished from the previous one in two ways. First, as Γ_2 increases above Γ_2^1, x_d and x_e remain stationary. Nevertheless, P_2 increases and land rent rises throughout both zones. Second, land rent is negative in only part of the residential zone. As Γ_2 and consequently R_2^H increase, the point beyond which rent is negative (and within which it is positive) becomes more remote. Eventually it reaches x_c so that land rent is positive throughout the residential zone. Once Γ_2 exceeds this level, development beyond x_c becomes advantageous, and (19) ceases to hold. Let the value of Γ_2 for which

$$R_2^H = Tx_c$$

be called Γ_2^2. This is the maximum value of Γ_2 for which (19) holds. Thus, Case 2 and the set of conditions (15), (16), (18) and (19) apply only when

$$\Gamma_2^1 \leq \Gamma_2 \leq \Gamma_2^2.$$

Case 3. Suppose

$$\Gamma_2 = \Gamma_2^2 + \delta.$$

In this case land in an annulus with inside radius x_c is developed residentially. This is the first instance we have examined where the outer boundary of the city moves in the second period. It is also the first instance where land rent is greater everywhere in the city during the second period. We still have only two von Thünen rings of developed land, but because the residential zone expands in both directions, we now have

$$x_a < x_e < x_b < x_c < x_d.$$

Since the amount of land in the industrial zone must be λQ_2, (15) holds; since that in the residential zone must be μQ_2,

$$Q_2 = \pi(x_d^2 - x_e^2)/\mu. \tag{20}$$

The remaining conditions are (17) and (18), which ensure equilibrium in the land market.

As Γ_2 increases, both zones expand as x_e and x_d increase. But this expansion cannot continue indefinitely because x_e eventually reaches x_b. Once this happens, there is no longer any residential development in the leapfrog zone to keep both land rents equal at x_e. Condition (18) will cease to hold and Case 3 will no longer apply. Let the value of Γ_2 for which

$$R_2^Q - R_2^H = (t/\lambda - T)x_b$$

be called Γ_2^3. This is the minimum value of Γ_2 for which

$$x_e = x_b. \tag{21}$$

Case 3 and the set of conditions (15), (17), (18) and (20) therefore apply only when

$$\Gamma_2^2 < \Gamma_2 < \Gamma_2^3.$$

Case 4. Let

$$\Gamma_2 = \Gamma_2^3,$$

so that all land in the leapfrog zone is developed industrially. Condition (21) now becomes an equilibrium condition. All residential development occurs beyond x_c, and (17) must still hold. The last two conditions are (15) and (20). This leaves two von Thünen rings of developed land and resembles second-period equilibrium under perfect-foresight planning more than any other of the five cases. Taken together, (15), (20) and (21) imply that x_d can be solved for as a function of x_b:

$$x_d = x_b\sqrt{1 + \mu/\lambda}.$$

Just as with Case 2 above, x_d and x_e remain stationary as Γ_2 increases above Γ_2^3. P_2 rises, however, and so does industrial land rent. Residential land rent does not change here because (17) must hold. Eventually, industrial rent will rise to the point where landowners beyond x_c wish to develop industrially and (21) ceases to hold. Let the value of Γ_2 for which

$$R_2^Q - R_2^H = (t/\lambda - T)x_c$$

be called Γ_2^4. This is the maximum value of Γ_2 for which (21) holds, so Case 4 and the set of conditions (15), (17), (20) and (21) apply only when

$$\Gamma_2^3 \leq \Gamma_2 \leq \Gamma_2^4.$$

Case 5. This case applies when Γ_2 falls in the upper tail of $g(\Gamma_2)$:

$$\Gamma_2 > \Gamma_2^4.$$

Second-period export demand for Q is especially strong here, and landowners find it advantageous to develop industrially not only the entire leapfrog zone, but another ring between x_c and x_e. Residential expansion occurs even farther out between x_e and x_d. Thus, there are four von Thünen rings of developed land—two of each kind, alternating as x increases.

Because the amount of land in the two industrial zones must be λQ_2,

$$Q_2 = \pi\left(x_b^2 + x_e^2 - x_c^2\right)/\lambda \tag{22}$$

must hold. Condition (16) must hold so that μQ_2 units of land are in the residential zone. The other equilibrium conditions pertain to land market equilibrium; they are (17) and (18).

The second-period land-rent function, $ER_2(x)$, differs among the cases. In Case 1, it is

$$\begin{aligned} ER_2(x) &= R_2^Q - tx/\lambda, && x \in [0, x_e] \\ &= R_2^H - Tx, && x \in (x_e, x_d], x \in (x_b, x_c] \\ &= 0, && \text{elsewhere}. \end{aligned}$$

For *Case 2*, it is

$$\begin{aligned} ER_2(x) &= R_2^Q - tx/\lambda, && x \in [0, x_e] \\ &= R_2^H - Tx, && x \in (x_e, x_c] \\ &= 0, && \text{elsewhere}. \end{aligned}$$

TABLE 1
Equilibrium Conditions

	Γ_2 Range	First-period	Second-period
Case 1	$\Gamma_1 < \Gamma_2 < \Gamma_2^1$	(1, 2, 3, 4, 5, 6, 24, 25)	(9, 10, 11, 15, 16, 17, 18)
Case 2	$\Gamma_2^1 \leq \Gamma_2 \leq \Gamma_2^2$	(1, 2, 3, 4, 5, 6, 24, 25)	(9, 10, 11, 15, 16, 18, 19)
Case 3	$\Gamma_2^2 < \Gamma_2 < \Gamma_2^3$	(1, 2, 3, 4, 5, 6, 24, 25)	(9, 10, 11, 15, 17, 18, 20)
Case 4	$\Gamma_2^3 \leq \Gamma_2 \leq \Gamma_2^4$	(1, 2, 3, 4, 5, 6, 24, 25)	(9, 10, 11, 15, 17, 20, 21)
Case 5	$\Gamma_2 > \Gamma_2^4$	(1, 2, 3, 4, 5, 6, 24, 25)	(9, 10, 11, 16, 17, 18, 22)

For Cases 3 and 4,

$$\begin{aligned} ER_2(x) &= R_2^Q - tx/\lambda, && x \in [0, x_e] \\ &= R_2^H - Tx, && x \in (x_e, x_d] \\ &= 0, && \text{elsewhere.} \end{aligned}$$

And for Case 5,

$$\begin{aligned} ER_2(x) &= R_2^Q - tx/\lambda, && x \in [0, x_b], x \in (x_c, x_e] \\ &= R_2^H - Tx, && x \in (x_b, x_c], x \in (x_e, x_d] \\ &= 0, && \text{elsewhere.} \end{aligned}$$

Under perfect-foresight planning, $ER_2(x)$ was continuous at every x but x_b. With speculation under conditions of uncertainty, the same is true only for Cases 1 and 4. In Case 5 it is discontinuous at x_b and x_c, and in Cases 2 and 3 it is continuous everywhere.

For each of the cases, second-period equilibrium conditions, as well as the first-period ones taken up next, are summarized in Table 1.

B. First-Period Equilibrium

Under the stated conditions of uncertainty, the equilibrium in period 1 is characterized by the same eight variables as in perfect-foresight planning. Six of the eight equilibrium conditions are also borrowed from the perfect-foresight model: (1)–(6). Conditions (7) and (8), the conditions related to the location of x_c and x_b in that model, are replaced here with (24) and (25) which reflect landowners' uncertainty about Γ_2.

Under perfect foresight, (7) indicated that land rent must be zero at x_c to keep landowners beyond x_c from developing residentially in period 1. With Γ_2 and second-period strategies perfectly foreseen, there was no chance that residential development between x_b and x_c would later be regretted. That is not true here. As we have already seen, if Γ_2 is sufficiently small, some landowners who develop residentially in period 1 will actually earn negative rent in the second period. Hence, they will not develop at all in the first period unless the expected present value of doing so is greater than that of waiting until period 2 to decide.

In assessing the expected present value of these two strategies, landowners are assumed to form expectations consistently (rationally). That is, they form them on the basis of all information they possess in period 1. This information includes $g(\Gamma_2)$ and the equilibrium value of all first-period variables which, for simplicity, we designate

$$\phi_1 = \left(x_a, x_b, x_c, R_1^H, R_1^Q, Q_1, P_1, w_1\right).$$

The expected present value, then, of the strategy to develop residentially at x_c in period 1 is

$$R_1^H - Tx_c + \frac{1}{1+r}\left[\int_{\Gamma_1}^{\infty} g(\Gamma_2)\left(R_2^H(\Gamma_2, \phi_1) - Tx_c\right)d\Gamma_2\right], \tag{23}$$

where the notation $R_2^H(\Gamma_2, \phi_1)$ indicates the dependence of R_2^H on ϕ_1 and the value Γ_2 that occurs in period 2. Under the strategy to wait until the second period to develop at x_c, it is

$$\frac{1}{1+r}\left[\int_{\Gamma_2^2(\phi_1)}^{\Gamma_2^4(\phi_1)} g(\Gamma_2)\left(R_2^H(\Gamma_2, \phi_1) - Tx_c\right)d\Gamma_2 + \int_{\Gamma_2^4(\phi_1)}^{\infty} g(\Gamma_2)\left(R_2^Q(\Gamma_2, \phi_1) - tx_c/\lambda\right)d\Gamma_2\right],$$

since the best strategy will be not to develop if Case 1 or 2 obtains, to develop residentially if Case 3 or 4 obtains, and industrially if Case 5 obtains. Thus, x_c must in equilibrium satisfy:

$$R_1^H - Tx_c = \frac{1}{1+r}\left\{\int_{\Gamma_2^4(\phi_1)}^{\infty} g(\Gamma_2)\left[R_2^Q(\Gamma_2, \phi_1) - R_2^H(\Gamma_2, \phi_1) + (T - t/\lambda)x_c\right]d\Gamma_2 - \int_{\Gamma_1}^{\Gamma_2^2(\phi_1)} g(\Gamma_2)\left(R_2^H(\Gamma_2, \phi_1) - tx_c/\lambda\right)d\Gamma_2\right\}. \tag{24}$$

The certain gain from first-period residential development must equal the expected gain from waiting instead until the second period and developing industrially if Γ_2 is great enough, and leaving land undeveloped if Γ_2 is too small.

Condition (8) under perfect foresight held that x_b was at a distance where landowners are indifferent between developing residentially in period 1 and waiting to develop industrially in period 2. The analogous condition here is that the expected present value of first-period residential development is equal to the expected present value of waiting and developing the best use in the second period. The expected present value of the former strategy at x_b is (23), replacing x_c with x_b. Under the latter, it is

$$\frac{1}{1+r}\left[\int_{\Gamma_2^1(\phi_1)}^{\Gamma_2^3(\phi_1)} g(\Gamma_2)\left(R_2^H(\Gamma_2,\phi_1) - Tx_b\right)d\Gamma_2 + \int_{\Gamma_2^3(\phi_1)}^{\infty} g(\Gamma_2)\left(R_2^0(\Gamma_2,\phi_1) - tx_b/\lambda\right)d\Gamma_2\right],$$

since the best strategy will be not to develop if Case 1 obtains, to develop residentially if Case 2 or 3 obtains, and industrially if Case 4 or 5 obtains. This means x_b must in equilibrium satisfy

$$R_1^H - Tx_b = \frac{1}{1+r}\left\{\int_{\Gamma_2^3(\phi_1)}^{\infty} g(\Gamma_2)\left[R_2^Q(\Gamma_2,\phi_1) - R_2^H(\Gamma_2,\phi_1) + (T - t/\lambda)x_b\right]d\Gamma_2 - \int_{\Gamma_1}^{\Gamma_2^1(\phi_1)} g(\Gamma_2)\left(R_2^H(\Gamma_2,\phi_1) - Tx_b\right)d\Gamma_2\right\}. \tag{25}$$

The certain gain from first-period residential development must equal the expected gain from waiting instead until the second period and developing industrially if Γ_2 is sufficiently great and not developing at all if Γ_2 is too small.

The first-period land-rent function, $ER_1(x)$, is defined here just as it was with perfect-foresight planning. It is continuous at every x but x_b. There are several differences between land-market equilibrium with speculation and perfect-foresight planning that should be mentioned. These are demonstrated by comparing the $ER_1(x)$ and $ER_2(x)$ functions produced by the two models.

Under perfect-foresight planning,

$$ER_2(x) \geq ER_1(x) \geq 0, \qquad \text{for } x \geq 0.$$

But with speculation this may not hold. In particular, in Cases 1–3,

$$ER_2(x) < ER_1(x), \qquad \text{for } x \geq x_b.$$

In Case 3, all landowners who developed residentially in period 1 earn less rent in the second period than in the first. In Case 2 some of them earn negative rent in period 2, and in Case 1 all of them do. This happens because the growth in export demand for Q between periods is insufficient to fill the leapfrog zone with industrial expansion.

Another important difference concerns the discontinuities in $ER_1(x)$ and $ER_2(x)$. Under perfect foresight it was seen that discontinuities in these functions were exactly offsetting and that $PV(x)$ was continuous at every x. That may not be the case with speculation. $PV(x)$ under speculation is generally discontinuous at one or both of the points x_b and x_c. The reason for this is simply that landowners have imperfect foresight. Just as in other markets where speculative decisions are made, there will be *ex post* winners and losers. If Γ_2 is great (Case 5), landowners near x_b who developed residentially in period 1 regret that decision later. If Γ_2 is small (Case 1), those beyond x_c who refrained from first-period development in spite of certain, immediate profit are later relieved with that choice.[9]

V. HETEROGENOUS EXPECTATIONS

In this section we retreat from the assumption that landowner uncertainty over Γ_2 is characterized by a single probability distribution and introduce "subjective," heterogeneous expectations. Suppose every landowner i forms a first-period development strategy on the basis of a subjective probability distribution $g^i(\Gamma_2)$. These distributions differ among landowners because they differ in their understanding of the economic process that produces Γ_2. Nevertheless, all of them reflect the opinion that export demand for Q will increase in period 2:

$$\int_0^{\Gamma_1} g^i(\Gamma_2) = 0, \qquad \text{for all } i.$$

The main consequence of this change occurs in the first-period where there will no longer be well-defined points x_b and x_c marking the inner and outer boundary of the residential zone. In the previous environment, x_b and x_c were the result of speculative behavior based on uniform expectations across landowners. There was a consensus among them (represented by (24)

[9]Because first-period equilibrium is expectational and depends on $g(\Gamma_2)$, the comparative static results analogous to those presented above under perfect-foresight planning would indicate how x_a, x_b, and x_c vary with changes in $g(\Gamma_2)$. The expected changes in x_d and x_e would be even more complicated. No results of this kind have been successfully derived.

and (25)) as to where these boundaries should be. In the present environment there is no such consensus.

The result is that x_b and x_c are replaced in first-period equilibrium by "transition zones" that contain scattered residential development. One of them lies inside the residential zone, separating it from the totally vacant leapfrog zone, and the other lies outside the residential zone. In general, the density of residential development in an annulus of land in either transition zone is greater the closer it is to the residential zone. Indeed if expectations are sufficiently diverse, the transition zones will converge on each other and the homogeneous residential zone will disappear to be replaced by a single zone with scattered residential development of variable density. Since x_a is not the direct result of speculative decision-making, it remains a distinct boundary as before.

In the second period, uncertainty about Γ_2 is resolved and there is a consensus among landowners about the proper location of x_d and x_e. Even so, because of scattered development in period 1, there may remain zones of mixed and scattered development in the second period. Suppose, for example, that a high value of Γ_2 occurs. Then all the undeveloped land in the inner transition zone will be developed industrially, and a mixed industrial-residential zone appears. (If Γ_2 is sufficiently great, the undeveloped land in the outer transition zone will be developed industrially also.) If a low value of Γ_2 occurs, then the outer transition zone is unchanged in period 2 and scattered residential development remains.

One implication of both period equilibria here is that land rent cannot be expressed as a simple function of x as with $ER_1(x)$ and $ER_2(x)$ previously. It varies not only with radial but also with circumferential movement since some annuli of land contain mixed or scattered development. And if there are circumferential as well as radial discontinuities in both period land rent functions, we certainly cannot expect a continuous $PV(x)$ function. *Ex post* winners and losers abound throughout the transition zones.

With heterogenous landowner expectations, it might appear that land transactions between more and less "optimistic" (regarding the value of Γ_2 expected) landowners would reduce or eliminate the variance of opinion before development plans are formulated. This will occur to some extent, but there are a number of institutional barriers and market imperfections that impede arbitrage. One is that landowners have incomplete information concerning the reservation price of various land parcels. Another is imperfect access among landowners to capital markets. There are also transaction costs associated with the transfer of real property. Sales commissions, deed recording fees, survey and legal expenses are several examples. Others are transfer and capital-gains taxes levied at the time of sale. All such charges have the effect of diminishing or eliminating gains from trade and will greatly inhibit arbitrage when buyer and seller reservation prices are not widely divergent. For this reason, it is unlikely that preliminary land transitions will eliminate altogether the variance of opinion among decision-makers concerning Γ_2. To the extent such arbitrage does occur, land falls into the hands of more optimistic agents. This will increase the size of the leapfrog zone and push the first-period residential zone to a more remote location.

VI. CONCLUSION

Under perfect-foresight planning, landowners may preserve a ring of undeveloped land between the first-period industrial and residential zones, providing an example of leapfrog development. In the second period this land is filled-in with industrial development to accomodate increased demand for industrial production.

Under uncertainty of the first type, where landowners share common expectations about growth, the first-period, leapfrog development pattern recurs. In the second period however, once uncertainty has been resolved, things may be different than under perfect foresight. If actual growth is modest, only part of the leapfrog zone is developed industrially; the rest either is developed residentially or left undeveloped. If growth is large enough, the leapfrog zone is insufficient to contain all industrial expansion and a second, outlying industrial zone appears.

Under uncertainty of the second type, where landowner expectations are heterogeneous, the equilibrium development patterns are altogether different. Besides the leapfrog pattern, scattered development may occur in period 1 and possibly in period 2. Mixed development may also occur in period 2.

One criticism of sprawl cited at the outset was that it wastes and misallocates land. This view is too much the product of myopic thinking.[10] Urban land conversion is a dynamic process and as such should be evaluated by dynamic rather than static criteria. It does not follow just because a land-use configuration is inefficient at one moment in time, that it is inefficient in the larger scheme of things where it is evolving. Due to the cost, immobility, nonmalleability and durability of buildings, the efficient development process is not, by definition, one that links together a sequence of states each of which is efficient in its moment. Rather, it is one that makes optimal intertemporal tradeoffs between costs and benefits. There is, in principle, no reason to expect that efficient urban land conversion is marked by spatial continuity. Indeed in a *growing* city, efficiency will require that interior parcels are sometimes withheld from early development and preserved for alternative future uses.

[10] This judgement is shared by Harvey and Clark [7], who wrote that "[t]he application of static measures to dynamic areas can easily result in misidentification of an area as sprawl when it is in reality a viable, expanding, compacting portion of a city" (p. 6).

It is true, in an environment of uncertainty, that the land market will waste or misallocate land in an *ex post* sense. It is all but certain that some early decisions would be altered if landowners could foresee actual future conditions, rather than being uncertain. And these changes, it is needless to say, would diminish sprawl. But this is an empty concession. A fair criticism of the land conversion process and the development patterns produced must be based on *ex ante* considerations; to do otherwise is to confuse good *decisions* with good *outcomes*. Unless competitive landowners, in making their decisions, ignore part of the information at their disposal or disregard credible instruction related to the underlying source of uncertainty, and it is certainly not in their interest to do so, the collective result of their decentralized decision-making is *ex ante* efficient.

A second criticism of sprawl was that is it aesthetically unattractive. This of course raises issues well beyond the purview of this paper. Nevertheless, even here it must be asked whether a measure of early spatial irregularity should not be tolerated to achieve greater future regularity.

The final criticism mentioned was that sprawl contributes to public-sector inefficiency. There is much truth in this claim; the infrastructure costs associated with many local public services are surely greater when leapfrog, scattered, and mixed developments occur [12]. But the blame for those spillover effects from spatial development patterns is more accurately laid upon local governments who fail to implement marginal-cost pricing for their services. It is when the cost of these services to users is disengaged from the cost to government (as, for instance, when they are financed by a property tax) that market forces fail to produce efficient outcomes. This is not due to a failure of the land market to process information, but rather of the government to give accurate cost signals.

APPENDIX A

Assume $t/\lambda > (2 + r)T$, and suppose that pattern (1) applies. Let $x_a \cdots x_d$ have the same definitions as in III. Pattern (1) implies

$$0 < x_a = x_b < x_c < x_d.$$

The (unlabeled) boundary between second-period industrial and residential development lies in (x_c, x_d). At this point x,

$$R_2^Q - tx/\lambda = R_2^H - Tx.$$

Together with (14), which must hold in pattern (1) equilibrium, this implies that for any $x < x_c$:

$$R_2^Q - tx/\lambda > t(x_c - x)/\lambda + T(x_d - x_c). \tag{A1}$$

Now consider x_a. With a pattern (1) equilibrium, the present value of both periods' *industrial* land rent must equal the present value of both periods' *residential* land rent at x_a:

$$R_1^Q - tx_a/\lambda + \frac{\left(R_2^Q - tx_a/\lambda\right)}{1+r} = R_1^H - Tx_a + \frac{\left(R_2^H - Tx_a\right)}{1+r}.$$

Equivalently from (7) and (14), both of which must hold,

$$R_1^Q - tx_a/\lambda + \frac{\left(R_2^Q - tx_a/\lambda\right)}{1+r} = T(x_c - x_a) + \frac{T(x_d - x_a)}{1+r}.$$

Combined with (A1), evaluated at x_a, this implies

$$\begin{aligned} R_1^Q - tx_a/\lambda &+ \frac{t(x_c - x_a)/\lambda}{1+r} + \frac{T(x_d - x_c)}{1+r} \\ &< T(x_c - x_a) + \frac{T(x_d - x_a)}{1+r}. \end{aligned}$$

Rearranging terms, this becomes

$$\frac{\left(R_1^Q - tx_a/\lambda\right)(1+r)}{\left(x_c - x_a\right)} + t/\lambda < (2+r)T.$$

The first term on the lhs is nonnegative, so this contradicts the assumption that $t/\lambda > (2+r)T$. This assumption therefore implies that pattern (2) applies.

APPENDIX B

PROPOSITION 1. $dx_b/d\Gamma_2 > 0$.

Proof (by contradiction). Let Γ_2 increase and suppose $dx_b \leq 0$ in response. This implies that $dQ_2 \leq 0$, from (12), and that $dR_2^H \leq 0$, from (13) and (14). From (9) and from $d\Gamma_2 > 0$ and $dQ_2 \leq 0$, it follows that $dP_2 > 0$. Even though P_2 increases, the change in wages is less than proportional. Since $dR_2^H \leq 0$, (10) implies that $dw_2 < q dP_2$. From (11), we see that

$$dP_2 - \mu dw_2 = \lambda dR_2^Q,$$

from which it follows that

$$dP_2(1 - \mu q) < \lambda dR_2^Q.$$

This with the previous result that $dP_2 > 0$ implies that $dR_2^Q > 0$. We shift back into period 1 by rewriting (8) as

$$\frac{R_2^Q - R_2^H}{(1+r)} = R_1^H + x_b\left[\frac{t}{\lambda(1+r)} - \frac{T}{(1+r)} - T\right].$$

Having assumed that $dx_b \leq 0$ and shown it to follow that $dR_2^H \leq 0$ and $dR_2^Q > 0$, this equation implies that $dR_1^H > 0$, since the expression in brackets is positive. This implies, by (7), that $dx_c > 0$; by (5), that $dQ_1 > 0$; by (4), that $dx_a > 0$; and finally by (6), that $dR_1^Q > 0$. From (1), $dQ_1 > 0$ implies that $dP_1 < 0$ (since Γ_1 does not change). But since $dR_1^H > 0$, (2) implies that the change in wages is greater than proportional to the decrease in P_1: $dw_1 > q dP_1$. From (3)

$$dP_1 - \mu dw_1 = \lambda dR_1^Q,$$

so it follows that

$$dP_1(1 - \mu q) > \lambda dR_1^Q,$$

which together with $dP_1 < 0$ implies that $dR_1^Q < 0$. But this contradicts the previous result that $dR_1^Q > 0$ and establishes the result that $dx_b/d\Gamma_2$ must be positive.

PROPOSITION 2. $dx_d/d\Gamma_2 > 0$.

Proof. From (12) and Proposition 1, $dQ_2/dx_b > 0$. This with (13) establishes the result immediately.

PROPOSITION 3. $dx_c/d\Gamma_2 > 0$.

Proof (by contradiction). Let Γ_2 increase and suppose $dx_c \leq 0$ in response. This implies, by (7), that $dR_1^H \leq 0$; by (5) and Proposition 1, that $dQ_1 < 0$; by (4), that $dx_a < 0$; and by (6), that $dR_1^Q < 0$. From (1), $dQ_1 < 0$ implies that $dP_1 > 0$ (since Γ_1 is unchanged). The increase in P_1 is proportionately greater than the change in wages; since $dR_1^H < 0$, (2) implies that $dw_1 < q dP_1$. From (3) we see that

$$dP_1 - \mu dw_1 = \lambda dR_1^Q,$$

from which it follows that

$$dP_1(1 - \mu q) < \lambda dR_1^Q.$$

Since $dP_1 > 0$, this implies that $dR_1^Q > 0$ and establishes a contradiction. Therefore, $dx_c/d\Gamma_2 > 0$.

PROPOSITION 4. $dx_a/d\Gamma_2 < 0$.

Proof (by contradiction). Let Γ_2 increase and suppose $dx_a \geq 0$. This implies, by (4), that $dQ_1 \geq 0$; and by (6), that $dR_1^Q \geq 0$. From (1), $dQ_1 \geq 0$ implies that $dP_1 < 0$ (since Γ_1 does not change). From (7) and Proposition 3, $dR_1^H > 0$. This together with (2) implies that $dw_1 > q dP_1$. From (3)

$$dP_1 - \mu dw_1 = \lambda dR_1^Q,$$

so it follows that

$$dP_1(1 - \mu q) > \lambda dR_1^Q,$$

which together with $dP_1 < 0$ implies that $dR_1^Q < 0$, which establishes a contradiction. Therefore, $dx_a/d\Gamma_2 < 0$.

REFERENCES

1. A. Anas, Dynamics of urban residential growth, *J. Urban Econ.*, **5**, 66–87 (1978).
2. R. W. Archer, Land speculation and scattered development; Failures in the urban fringe land market, *Urban Studies*, **10**, 367–372 (1973).
3. R. Boyce, Myth vs. reality in urban planning, *Land Econ.*, **39**, 241–251 (1963).
4. J. K. Brueckner and B. von Rabenau, "Dynamics of Land-Use for a Closed City," Working Paper in Economics No. 139, University of California, Santa Barbara.
5. M. Clawson, Urban sprawl and speculation in urban land, *Land Econ.*, **38**, 99–111 (1962).
6. M. Fujita, Spatial patterns of urban growth, *J. Urban Econ.*, **3**, 209–241 (1976).
7. R. O. Harvey and W. A. V. Clark, The nature and economics of urban sprawl, *Land Econ.*, **41**, 1–9, (1965).
8. J. R. Markusen and D. T. Scheffman, "Ownership concentration and market power in urban land markets, *Rev. Econ. Studies*, **45**, 519–527 (1978).
9. R. M. Northam, Vacant urban land in the American city, *Land Econ.*, **47**, 345–355 (1971).
10. J. C. Ohls and D. Pines, Discontinuous urban development and economic efficiency, *Land Econ.*, **51**, 224–234 (1975).
11. Real Estate Research Corporation, "The Cost of Sprawl" (1974).