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When is the economy monocentric?: von Thünen and Chamberlin unified

Masahisa Fujita[a,*], Paul Krugman[b]

[a]*Department of Economics, University of Pennsylvania, 3718 Locust Walk, Philadelphia, PA 19104-6209, USA*
[b]*Department of Economics, Stanford University, Stanford, CA 94305, USA*

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Abstract

Since *The Isolated State* by von Thünen, countless versions of the von Thünen model have appeared. It seems, however, that a fundamental question remains unanswered: Why should all manufacturing goods be produced in a single town? In this paper we develop a monopolistic competition model of an 'isolated state', and investigate the answer to this question. Since the answer to this question suggests when there will be more than one town in the isolated state, it is hoped the present model will lead to the development of a general equilibrium model of urban systems.

Keywords: Urban system; Monopolistic competition; von Thünen

JEL classification: R12; F12; O14

1. Introduction

Imagine a very large town, at the center of a fertile plain which is crossed by no navigable river or canal. Throughout the plain the soil is capable of cultivation and of the same fertility. Far from the town, the plain turns into an uncultivated wilderness which cuts off all communication between this state and the outside world. There are no other towns on the plain. The central town must therefore supply the rural areas with all manufactured products, and in return it will obtain all its provisions from the surrounding countryside.[1]

* Corresponding author. Present address: Kyoto Institute of Economic Research, Kyoto University, Yoshida-honmachi, Sakyo-ku, Kyoto 606, Japan.

[1] This quotation is from the English translation of von Thünen (1826) by Wartenberg (1966, p. 7).

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Thus starts von Thünen's classic, *The Isolated State*, which laid the foundation of land use theory in 1826. Countless versions of the von Thünen model have appeared since then.[2] In our opinion, however, it seems that a fundamental question remains unanswered. That is, *Why should all manufactured products be produced in a single town of the isolated state?* To our knowledge it seems that there is no work that provides a systematic answer to this question in the context of a general equilibrium model. In this paper we develop a monopolistic competition model of an 'isolated state', and examine conditions under which all manufactured goods will be produced in a single town. The answer to this question should in turn provide conditions under which there will be more than one town in the isolated state. In this way it is hoped that the present model will eventually lead to the development of a general equilibrium model of urban systems.

In the context of our monopolistic competition model below, the basic mechanism for the concentration of manufacturing firms in a city (or town) of the isolated state is as follows. Suppose that we have a homogeneous population of workers (=consumers). Each worker consumes a homogeneous agricultural good (A-good) together with a large variety of differentiated manufactured goods (M-goods). Owing to scale economies in product specialization, each variety of M-good is to be produced by a single firm (by using labor as its sole input) which chooses its f.o.b. price monopolistically (in the sense of Chamberlin). In this context, if a large variety of M-goods is produced in a city, this variety of goods can be purchased at lower prices there in comparison with more distant places. Thus, given a nominal wage, because of tastes for variety, the real income of workers rises in the city. This, in turn, induces more workers to migrate to the city. Then this increase in the number of workers (=consumers) creates a greater demand for M-goods in the city, supporting a greater number of specialized manufacturing firms. (Note that due to scale economies at the individual firm level, a large number of firms can be supported only when the total demand for M-goods is sufficiently large.) This implies the availability of an even greater variety of M-goods from the city. Fig. 1 depicts this circular causality in the spatial agglomeration of firms and workers through a *forward linkage* (where the availability of a greater variety of consumption goods increases the real income of workers there) and a *backward linkage* (where a greater number of consumers supports a greater number of specialized firms). In other words, through these forward and backward linkages, scale economies at the individual firm level are transformed into increasing returns at the city level.

A bigger city, however, requires a greater agricultural hinterland and

[2] In particular, for recent general equilibrium versions, see, for example, Samuelson (1983) and Nerlove and Sadka (1991).

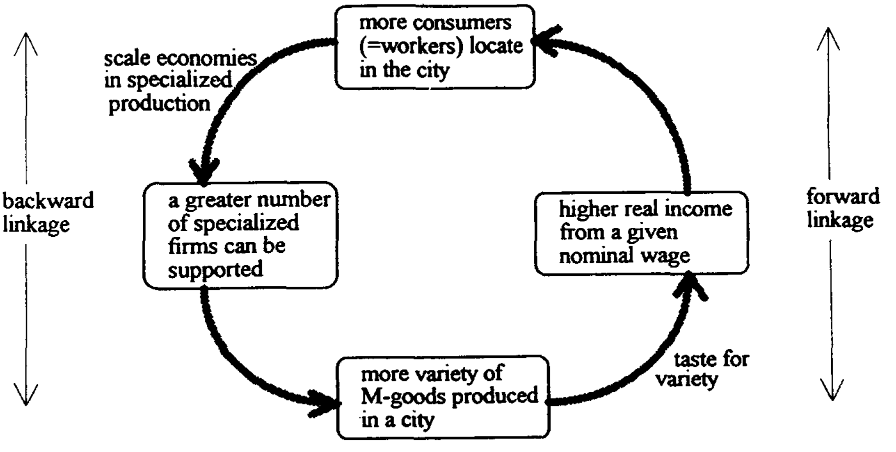

Fig. 1. Circular causality in spatial agglomeration of firms and workers.

hence a longer distance for transportation of the A-good to the city, causing diseconomies of spatial agglomeration.[3] If the spatial agglomeration force generated through the scale economies of M-good production and consumption is sufficiently strong so that it overwhelms this negative effect of the expansion of the agricultural hinterland, then the isolated state will have one city only.

It is of course not always necessary for all M-firms to agglomerate in a single city. For example, if M-goods are close substitutes for each other or if transport costs for M-goods are high, then some firms may find that by moving away from the city and primarily serving farmers in a fringe agricultural area, they can earn a greater profit. In this case, then, the isolated state will have more than one city. We investigate below the exact conditions under which the agglomeration of all M-firms in a single city is indeed in equilibrium.[4]

The plan of the paper is as follows. In Section 2 we present a formal model of a spatial economy which represents the basic idea described above in concrete terms. Then, first in Section 3, assuming a priori that all M-firms locate in a city, the equilibrium spatial configuration of the economy is determined uniquely. To claim, however, that this monocentric spatial configuration is really in equilibrium, we must make sure that no firm would

[3] The consumption of land by workers and M-firms (which is neglected in this paper for simplicity) will augment the agglomeration diseconomies of the city.

[4] In this paper we limit our study to the case of single-city economy. Fujita and Krugman (1993) present preliminary analyses of equilibrium spatial systems containing multiple cities.

desire to deviate from the city. Thus, in Section 4, using the concept of the market potential function, we examine the *location equilibrium condition* under which the city is indeed the optimal location for all M-firms. Next, in Section 5 we conduct the comparative statics of the monocentric equilibrium. Here we obtain a number of interesting results which could happen only in the context of a general spatial equilibrium. It is shown, in particular, that many interesting conclusions depend crucially on the satisfaction of the location equilibrium condition. Finally, in Section 6 we discuss possible future research directions.

2. A formal model of a spatial economy

Consider a *long-narrow country*, in which area is represented by one-dimensional unbounded location space, $X \equiv \mathbb{R}$. The quality of land is homogeneous and the density of land is equal to 1 everywhere. The country has a continuum of homogeneous workers with a given size, N. Each worker is endowed with a unit of labor, and is free to choose any location and job (i.e. manufacturing work or agricultural work) in the country. The consumers of the country consist of the workers and landlords. All landlords are attached to their land (like weeds), and consume the entire revenue from land (i.e. land rent) at their location.[5]

Each consumer consumes a homogeneous agricultural good (A-good) together with a continuum of differentiated manufacturing goods (M-goods) of size n. Here, n is to be determined endogenously. All consumers have the same utility function given by

$$u = \alpha_A \log z + \alpha_M \log\left\{\int_0^n q(\omega)^\rho \, \mathrm{d}\omega\right\}^{1/\rho}, \tag{2.1}$$

where z represents the amount of A-good consumed, $q(\omega)$ is the consumption (density) of each variety $\omega \in [0, n]$ of M-good; and α_A, α_M and ρ are positive constants such that $\alpha_A + \alpha_M = 1$ and $0 < \rho < 1$. Note that a *smaller ρ means that consumers have a stronger preference for variety* in M-goods.

[5] The model presented in Section 2 is a variation of Krugman (1993). Specifically, in Krugman (1993) there are fixed number of two distinct types of workers, i.e. agricultural workers and manufacturing workers, and the agricultural workers are distributed uniformly over a given fixed interval of space, while manufacturing workers can move freely. Furthermore, in agricultural production the use of land is not considered explicitly, and thus no land market appears in the model. In contrast, in this paper all workers are homogeneous, can move freely, and can work either in agriculture or in manufacturing. Furthermore, in the spirit of the von Thünen model, the use of land in agricultural production is explicitly considered, and hence the land market is introduced.

Suppose that a consumer has an income, Y, and faces a set of prices, p_A (for the A-good) and $p_M(\omega)$ (for each variety ω of M-good). Then, by choosing the consumption bundle that maximizes (2.1) subject to the budget constraint $p_A z + \int_0^n p_M(\omega)q(\omega)\,d\omega = Y$, demand functions of the consumer can be obtained as

$$z = (\alpha_A Y)/p_A \,, \tag{2.2}$$

$$q(\omega) = (\alpha_M Y/p_M(\omega))\left(p_M(\omega)^{-\gamma} \Big/ \int_0^n p_M(\omega)^{-\gamma}\,d\omega\right), \tag{2.3}$$

for each $\omega \in [0, n]$, where $\gamma \equiv \rho/(1-\rho)$. Note from (2.3) that the demand for any variety in M-good has the same price elasticity, E, given by

$$E = 1/(1-\rho) = 1 + \gamma \,. \tag{2.4}$$

Thus, E increases as ρ (or γ) increases. Substitution of (2.2) and (2.3) into (2.1) yields the following indirect utility function:

$$u = \log\{\alpha_A^{\alpha_A}\alpha_M^{\alpha_M} Y p_A^{\alpha_A}\} + \frac{\alpha_M}{\gamma}\log\left\{\int_0^n (\omega)^{-\gamma}\,d\omega\right\}. \tag{2.5}$$

Next, the A-good is assumed to be produced under constant returns, where each unit of A-good consumes a unit of land and a_A units of labor. (In this paper, land is used for A-good production only.) Each M-good is produced with labor only. All types of M-good have the same production technology under increasing returns such that the total labor input, L, for production of quantity Q of any product is given by

$$L = f + a_M Q \,, \tag{2.6}$$

where f is the fixed labor requirement and a_M the marginal labor input. We assume, for simplicity, that the transport cost of each good takes Samuelson's 'iceberg' form: if a unit of good i (i = A or M) is shipped over a distance d, only $e^{-t_i d}$ units actually arrive, where t_i is a positive constant.[6]

Owing to scale economies in production, each variety of M-good is assumed to be produced by a single, specialized firm. If a firm locates at $x \in X$ and produces an M-product, it chooses an f.o.b. price $p_M(x)$ so as to maximize its profit at the Chamberlinian equilibrium. By the assumption of iceberg transport technology, the effective (delivered) price, $p_M(y|x)$, for

[6] The 'iceberg' transport technology was formally introduced by Samuelson (1952). It is interesting to note, however, that von Thünen supposed the cost of grain transportation to consist largely of the grain consumed on the way by the horses pulling the wagon (von Thünen, 1826, ch. 4). Hence, the von Thünen model may be considered as the predecessor of the 'iceberg' transport technology.

consumers at location $y \in X$ of any M-product produced at location x is given by

$$p_M(y|x) = p_M(x)\,e^{t_M|y-x|}\,. \tag{2.7}$$

Given the delivered price function above, it can be readily verified that the price elasticity of the total demand for any M-product is independent of the spatial distribution of the demand, and equals the price elasticity, E, of each consumer's demand given by (2.4). Thus, given the equilibrium wage rate, $W(x)$, at x, by the equality of the marginal revenue and marginal cost, $p_M(x)(1 - E^{-1}) = a_M W(x)$, the optimal f.o.b. price for the firm (at location x) can be obtained as

$$p_M(x) = a_M W(x)/\rho\,, \tag{2.8}$$

which represents the familiar result that each monopolistic firm will charge its f.o.b. price at a markup over the marginal cost $a_M W(x)$. Thus, if Q is the output of the firm, its profit equals

$$\pi(x) = p_M(x)Q - W(x)(f + a_M Q) = a_M \gamma^{-1} W(x)(Q - \gamma f/a_M)\,. \tag{2.9}$$

Therefore, given any equilibrium configuration of the economy, if an M-firm operates at x, then by the zero-profit condition, its (equilibrium) output must be equal to

$$Q^* = \gamma f/a_M\,, \tag{2.10}$$

which is a constant independent of location.

Consequently, the remaining unknowns of the model are: (i) the price, $p_A(y)$, of the A-good at each y; (ii) the wage rate, $W(y)$, at each y; (iii) the land rent, $R(y)$, at each y; (iv) the equilibrium utility level, u, of workers; (v) the spatial distribution of workers; (vi) the spatial distribution of M-good production; and (vii) the trade pattern of each good. A spatial configuration is in equilibrium if all workers achieve the same highest utility, each active firm earns zero profit, and the equality of the demand and supply of each good is attained.

3. The monocentric equilibrium

Depending on parameters, there may exist many different patterns of equilibrium spatial configurations. In this paper, however, we focus on a specific spatial configuration depicted in Fig. 2. In this figure, the production of all M-goods is assumed to take place in the city located at the center, $y = 0$, and the agricultural area extends from $-l$ to l. This is the case of the *Isolated State* by von Thünen (1826). First, in this section we determine all unknowns *assuming* that all M-firms locate in the city. Then, in the next

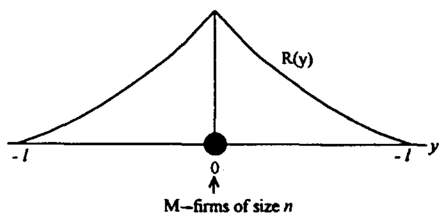

Fig. 2. The monocentric spatial configuration.

section, we examine the location equilibrium conditions under which indeed no M-firm would desire to deviate from the city.

Let the price curve, $p_A(y)$, of the A-good be normalized such that $p_A(0) = 1$. Then, since all excess A-goods are to be transported to the city, at each location y in the agricultural hinterland, it must hold that

$$p_A(y) = \mathrm{e}^{-t_A|y|}\,. \tag{3.1}$$

Next, let $p_M = p_M(0)$ be the f.o.b. price of each M-good produced in the city. Then, by (2.7) the (delivered) price of each M-good at each location is given by

$$p_M(y) \equiv p_M(y|0) = p_M\,\mathrm{e}^{t_M|y|}\,. \tag{3.2}$$

Furthermore, let n be the size of the M-industry established in the city; by definition, n is then the number (more precisely, the mass) of firms producing M-goods in the city.

Now, let $W(y)$ be the equilibrium wage rate at each $y \in [-l, l]$. Then, in equilibrium, since all workers must achieve the same utility level, say u, by the indirect utility function (2.5) we have that

$$W(y) = \mathrm{e}^{u}\alpha_A^{-\alpha_A}\alpha_M^{-\alpha_M}n^{-\alpha_M/\gamma}p_M^{\alpha_M}\,\mathrm{e}^{(\alpha_M t_M - \alpha_A t_A)|y|}\,. \tag{3.3}$$

By the zero-profit condition in A-good production, the land rent at each location can be obtained as

$$R(y) = p_A(y) - a_A W(y) \equiv \mathrm{e}^{-t_A|y|} - a_A W(y) \quad \text{for } y \in [-l, l]\,. \tag{3.4}$$

Since $R(l) = 0$ at the fringe location l, it holds by (3.4) that $\mathrm{e}^{-t_A l} = a_A W(l)$, which together with (3.3) yields the following wage curve:

$$\begin{aligned} W(y) &= \{a_A^{-1}\,\mathrm{e}^{-t_A l}\}\,\mathrm{e}^{(\alpha_A t_A - \alpha_M t_M)(l - |y|)} \\ &= a_A^{-1}\,\mathrm{e}^{-\alpha_M(t_A + t_M)l}\,\mathrm{e}^{(\alpha_M t_M - \alpha_A t_A)|y|}\,. \end{aligned} \tag{3.5}$$

Notice that $W(l) = a_A^{-1}\,\mathrm{e}^{-t_A l}$, which determines the wage rate at the fringe location (in terms of A-good units at the city) solely as a function of distance

l. In turn, $W(y)$ gives the equilibrium wage rate at each y, which compensates for the price differences (of A-good and M-goods) between location y and the fringe location. Substitution of (3.5) into (2.8) yields the following f.o.b. price of M-goods at the city:

$$p_M \equiv p_M(0) = a_M(a_A \rho)^{-1} e^{-\alpha_M(t_A+t_M)l} . \tag{3.6}$$

Next, to determine the number of firms, n, in the city, if we let N_A be the size of agricultural workers and N_M be that of manufacturing workers (in the city), then

$$N_A = 2a_A l , \tag{3.7}$$

$$N_M = n(f + a_M Q^*) = nf(1+\gamma) , \tag{3.8}$$

where the last equality follows from (2.10). Hence, by the full-employment condition, $N_A + N_M = N$, we have

$$n = \frac{N - 2a_A l}{f(1+\gamma)} . \tag{3.9}$$

Now, if we know l (the fringe distance of the agricultural area), all unknowns will be determined uniquely by (3.4)–(3.9). The equilibrium value of l can be determined by the equality of demand and supply of the A-good as follows. By (2.2) the excess supply of the A-good per unit distance at each $y \neq 0$ equals $1 - (\alpha_A Y(y)/p_A(y)) = 1 - \alpha_A = \alpha_M$ (where $Y(y) \equiv a_A W(y) + R(y) = p_A(y)$ by (3.4)). Thus, considering the consumption of the A-good in transportation, the total net supply of the A-good to the city equals

$$S_A(0) \equiv \int_{-l}^{l} \alpha_M e^{-t_A|y|} dy = 2\alpha_M t_A^{-1}(1 - e^{-t_A l}) , \tag{3.10}$$

while the total demand of the A-good at the city equals $\alpha_A Y(0)/p_A(0)$ (where $Y(0) = W(0)N_M$ and $p_A(0) = 1) = \alpha_A W(0) N_M$. Hence, the equality of the supply and demand requires that

$$\frac{S_A(0)}{\alpha_A W(0)} = N_M , \tag{3.11}$$

or, since $N_M = N - N_A = N - 2a_A l$, if we define

$$N_C(l) \equiv \frac{S_A(0)}{\alpha_A W(0)} = \frac{2\alpha_M t_A^{-1}(1 - e^{-t_A l})}{\alpha_A a_A^{-1} e^{-\alpha_M(t_A+t_M)l}} , \tag{3.12}$$

$$N_M(l) \equiv N - 2a_A l , \tag{3.13}$$

then (3.11) can be restated as

$$N_C(l) = N_M(l)\,. \tag{3.11a}$$

Here $N_C(l)$ represents the size of urban consumers, which is just sufficient to purchase all A-good units supplied from the hinterland when the wage rate at the city equals $W(0) \equiv a_A^{-1}\,e^{-\alpha_M(t_A+t_M)l}$, while $N_M(l)$ represents the size of urban workers when the agricultural fringe distance is l. We call $N_C(l)$ the A-good exhausting size of urban consumer, or *AE urban-consumer size*, and call $N_M(l)$ the *urban-labor force*.

Notice that the function $N_C(l)$ is increasing in l, $N_C(0) = 0$ and $N_C(\infty) = \infty$. Therefore, as demonstrated in Fig. 3, the equality of $N_C(l)$ and $N_M(l)$ determines the equilibrium fringe distance l^* uniquely. This in turn determines the equilibrium urban population N_M^* and rural population N_A^* uniquely, as indicated in Fig. 3.

Now, substituting l^* into (3.4)–(3.9), all other unknowns can be determined uniquely. In particular, we have that

$$n^* = \frac{N - 2a_A l^*}{f(1+\gamma)}\,, \tag{3.14}$$

$$u^* = -\alpha_A\alpha_M(t_A + t_M)l^* + \frac{\alpha_M}{\gamma}\log\frac{N - 2a_A l^*}{f(1+\gamma)} + \log \alpha_A^{\alpha_A}\alpha_M^{\alpha_M}a_A^{-\alpha_A}a_M^{-\alpha_M}\rho^{\alpha_M}\,, \tag{3.15}$$

$$W^*(y) = a_A^{-1}\,e^{-\alpha_M(t_A+t_M)l^*}\,e^{(\alpha_A t_M - \alpha_A t_M)|y|}\,. \tag{3.16}$$

Finally, notice in Fig. 3 that if $t_A = t_M = 0$, i.e. if there were no transport costs, then $N_C(l) = (2a_A\alpha_M/\alpha_A)l$, and hence $N_A^* = \alpha_A N$ and $N_M^* = \alpha_M N$, as expected. It must be emphasized that in general, point $\mathscr{E}$ in Fig. 3 is not always in the upper left of point $\mathscr{E}'$. It depends on the relative values of t_A

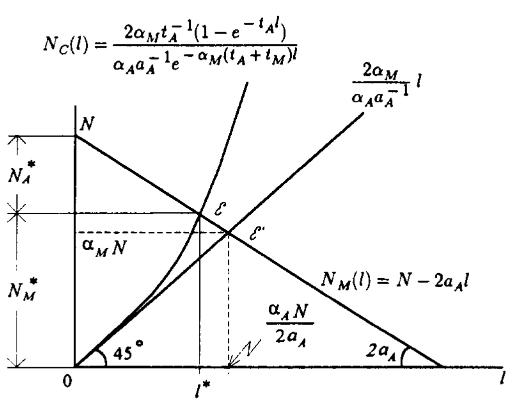

Fig. 3. Determination of the fringe distance l^* for a monocentric equilibrium.

and t_M. We will show below, however, that if *all M-firms are in location equilibrium*, then point $\mathscr{E}$ is indeed always in the upper left of $\mathscr{E}'$.

4. The potential function and location-equilibrium of M-firms

For the spatial configuration determined above to be really in equilibrium, we must make sure (i) no existing M-firms could increase their profit by moving away from the city, and (ii) no new M-firm would enter the market. In the present context, however, (i) and (ii) are the same. That is, *since each firm is of zero measure, a change in the location of one existing firm or the entry of a new firm at any location does not alter the present spatial configuration (including all associated price variables) of the economy.* Hence, for both (i) and (ii) it is sufficient to check that *given the present spatial configuration*, no (new or existing) M-firm can earn a positive profit at any location.

Suppose an M-firm locates at $x \in \mathbb{R}$. Then, given the market wage rate $W^*(x)$ at x, the firm will set its f.o.b. price at $a_M W^*(x)\rho^{-1}$ [recall (2.8)]. Hence, at each location $y \in \mathbb{R}$, the delivered price, $p_M(y|x)$, of the M-good produced by the firm (located at x) is given by

$$p_M(y|x) = a_M W^*(x)\rho^{-1}\, \mathrm{e}^{t_M|y-x|}\,. \tag{4.1}$$

Using (4.1), as a function of the market wage rate $W^*(x)$ there, the total demand for the firm located at x can be obtained as follows (see Appendix A for the derivation):

$$D(x, W^*(x)) = \frac{\alpha_A \gamma f}{2a_M}\left(\frac{W^*(0)}{W^*(x)}\right)^{1+\gamma} \varphi(x) \quad \text{for } x \geq 0\,, \tag{4.2}$$

where

$$\begin{aligned}\varphi(x) &\equiv \mathrm{e}^{-\gamma t_M x}\left\{\frac{2\alpha_M}{\alpha_A} + \frac{t_A}{1-\mathrm{e}^{-t_A l^*}}\, \mathrm{e}^{\gamma t_M x}\int_{-l^*}^{l^*} \mathrm{e}^{-t_A|y|}\, \mathrm{e}^{\gamma t_M(|y|-|y-x|)}\,\mathrm{d}y\right\} \\ &= \mathrm{e}^{-\gamma t_M x}\left\{\frac{1+\alpha_M}{\alpha_A} + \frac{t_A}{2\gamma t_M - t_A}\,\frac{\mathrm{e}^{(2\gamma t_M - t_A)x} - 1}{1-\mathrm{e}^{-t_A l^*}} + \frac{1-\mathrm{e}^{-t_A(l^*-x)}}{1-\mathrm{e}^{-t_A l^*}}\,\mathrm{e}^{(2\gamma t_M - t_A)x}\right\}.\end{aligned} \tag{4.3}$$

The resulting firm's profit is

$$\begin{aligned}\pi(x, W^*(x)) &= a_M W^*(x)\rho^{-1} D(x, W^*(x)) - W^*(x)[f + a_M D(x, W^*(x))] \\ &= a_M \gamma_M^{-1} W^*(x)\{D(x, W^*(x)) - \gamma f/\alpha_M\}\,,\end{aligned} \tag{4.4}$$

which implies that $\pi(x, W^*(x)) \gtreqless 0$ as $D(x, W^*(x)) \gtreqless \gamma f/a_M$. For convenience, let us define

$$\Omega(x) \equiv \frac{D(x, W^*(x))}{\gamma f / a_{\mathrm{M}}}\,, \tag{4.5}$$

where $\gamma f / a_{\mathrm{M}}$ represents the equilibrium output level (of each existing M-firm) determined by the zero-profit condition [recall (2.10)]. By definition, then, it is obvious that

$$\pi(x, W^*(x)) \gtreqqless 0 \Leftrightarrow \Omega(x) \gtreqqless 1\,. \tag{4.6}$$

Therefore, we can conclude that for the symmetric monocentric configuration defined in Section 3 to be in equilibrium, it is necessary and sufficient that

$$\Omega(x) \leqslant 1 \quad \text{for } x \geqslant 0\,, \tag{4.7}$$

where $\Omega(0) = 1$ by definition.

Following Krugman (1993), we call $\Omega(x)$ the (*market*) *potential function* of the M-industry, which represents the relative profitability of each location for M-firms. By (4.2) and (4.5), for $x \geqslant 0$ we have

$$\begin{aligned} \Omega(x) &= \frac{\alpha_{\mathrm{A}}}{2}\,\varphi(x)\,\mathrm{e}^{(1+\gamma)(\alpha_{\mathrm{A}} t_{\mathrm{A}} - \alpha_{\mathrm{M}} t_{\mathrm{M}})x} \\ &= \frac{\alpha_{\mathrm{A}}}{2}\,\mathrm{e}^{-\tau x}\left\{\frac{1+\alpha_{\mathrm{M}}}{\alpha_{\mathrm{A}}} + \frac{t_{\mathrm{A}}}{2\gamma t_{\mathrm{M}} - t_{\mathrm{A}}}\,\frac{\mathrm{e}^{(2\gamma t_{\mathrm{N}} - t_{\mathrm{A}})x} - 1}{1 - \mathrm{e}^{-t_{\mathrm{A}} l^*}}\right. \\ &\quad \left. + \frac{1 - \mathrm{e}^{-t_{\mathrm{A}}(l^* - x)}}{1 - \mathrm{e}^{-t_{\mathrm{A}} l^*}}\,\mathrm{e}^{(2\gamma t_{\mathrm{M}} - t_{\mathrm{A}})x}\right\}, \end{aligned} \tag{4.8}$$

where

$$\tau \equiv (1+\gamma)(\alpha_{\mathrm{M}} t_{\mathrm{M}} - \alpha_{\mathrm{A}} t_{\mathrm{A}}) + \gamma t_{\mathrm{M}}\,. \tag{4.9}$$

Examining condition (4.7), we can investigate under what conditions the monocentric configuration is in equilibrium. Table 1 summarizes the results (see Appendix B for the derivation).

Since $\Omega(0) = 1$, the monocentric configuration can be in equilibrium only if $\Omega(x)$ is not increasing at $x = 0$. By (4.2) and (4.8),

Table 1
Possibility of a monocentric equilibrium

<table>
<tr><td colspan="2">$\alpha_{\mathrm{A}} t_{\mathrm{A}} \leqslant (1+\rho)\alpha_{\mathrm{M}} t_{\mathrm{M}}$</td><td>$\alpha_{\mathrm{A}} t_{\mathrm{A}} > (1+\rho)\alpha_{\mathrm{M}} t_{\mathrm{M}}$</td></tr>
<tr><td>$\alpha_{\mathrm{M}} \geqslant \rho$</td><td>$\alpha_{\mathrm{M}} < \rho$</td><td rowspan="2">Never</td></tr>
<tr><td>Always</td><td>For small N</td></tr>
</table>

$$\Omega'(0) = (1+\gamma)\left\{-\frac{W^{*\prime}(0)}{W^*(0)} + \frac{\varphi'(0)}{\varphi(0)}(1+\gamma)^{-1}\right\}$$
$$= (1+\gamma)\left\{\underbrace{(\alpha_A t_A - \alpha_M t_M)}_{\substack{\text{wage-pull}\\ \text{towards}\\ \text{the fringe}}} - \underbrace{\rho\alpha_M t_M}_{\substack{\text{demand-pull}\\ \text{of city workers}\\ \text{towards the center}}}\right\}, \tag{4.10}$$

where $\Omega'(0) \equiv \mathrm{d}\Omega(x)/\mathrm{d}x$ at $x = 0$, for example. Hence, if $\alpha_A t_A - \alpha_M t_M > \rho\alpha_M t_M$, for example, then when a firm moves a short distance away from the city, the wage rate is decreasing sufficiently fast while the demand for its product is not decreasing much. In this case, the firm finds it more profitable to move away from the city; hence, as noted in the top row in Table 1, the monocentric configuration can never be in equilibrium. This can happen, for example, when the transport cost for the A-good (weighted by expenditure share α_A) is very high in comparison with that of M-goods.

Next, given that $\alpha_A t_A \leqslant (1+\rho)\alpha_M t_M$, the criterion on the second row in Table 1 is based on whether a firm can become more profitable by moving far away from the city (and by focusing on the local demand by agricultural workers in a periphery).[7] If M-goods are highly differentiated from each other so that $\alpha_M > \rho$, then the price elasticity of each M-good is very low. In this case, by moving into a periphery a firm does not find much increase in the local demand there while the demand from the rest of the economy decreases. Rather, each M-firm finds it most profitable to locate at the center of the entire demand of the economy, i.e. at the city. Thus, as noted in the table, when $\alpha_M > \rho$, the monocentric pattern is always an equilibrium configuration.[8] Conversely, if M-goods are highly substitutable for each other so that $\alpha_M < \rho$, then given that all other firms stay in the city, the firm that moves into a periphery location can capture a large share of local demand there. A large population N implies a large periphery at each side of the city. Thus, when N is large, a large local demand at the periphery makes a firm locating there more profitable than in the city. Therefore, as noted in the table, when $\alpha_M < \rho$, the monocentric configuration can be in equilibrium only if N is relatively small.

Fig. 4 demonstrates the last conclusion numerically. In this numerical example, except for population N, all parameters are fixed such that $\alpha_A = \alpha_M = 0.5$, $a_A = 0.5$, $t_A = 0.9$, $t_M = 1$, $\rho = 0.8$ (i.e. $\gamma = 4$), which implies

[7] Notice that this criterion (of the second row in Table 1) is identical to that of Krugman (1993) for a monocentric equilibrium. For example, the condition $\alpha_M < \rho$ is equivalent to that of eq. (25) in Krugman (1993).

[8] The same result that the central agglomeration is an equilibrium when products are sufficiently differentiated was also obtained in spatial competition models with differentiated products (see, for example, de Palma et al., 1985; and Ben-Akiva et al., 1989). This suggests the robustness of the result against alternative model specifications.

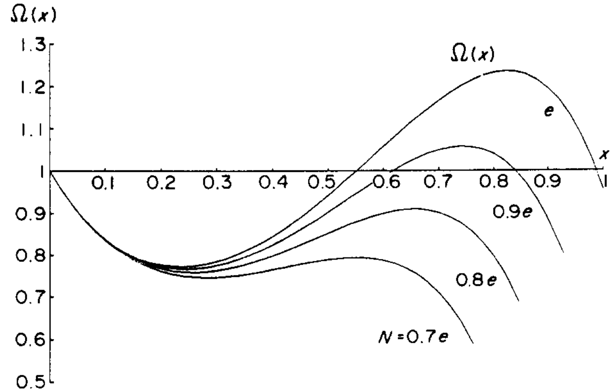

Fig. 4. Potential curves for the monocentric configuration under various N [$\alpha_A = \alpha_M = 0.5$, $t_A = 0.9$, $t_M = 1.0$, $\rho = 0.8$, $a_A = 0.5$, $a_M = f = 1$].

that $\alpha_A t_A < (1+\rho)\alpha_M t_M$ and $\alpha_M < \rho$.[9] Thus, we have the situation in the middle part of Table 1. We can see from Fig. 4 that as the population N continues to increase from $0.7e$, the potential curve $\Omega(x)$ eventually starts exceeding 1 at the periphery. Thus, the monocentric configuration can be in equilibrium only when N is relatively small. When N is sufficiently large, an equilibrium spatial configuration will contain more than one city.

5. Comparative statics

Using (3.7)–(3.16), we can examine the effect of a marginal increase in each parameter on major variables of the monocentric equilibrium. Table 2 summarizes the results. Many of the results in this table can be understood intuitively by using Fig. 3. In particular, many results depend crucially on the fact that in Fig. 3, the AE urban-consumer size, $N_C(l)$, is strictly convex in l. (This implies that in Fig. 3, point $\mathscr{E}$ is in the upper left of point $\mathscr{E}'$.)

It must be emphasized that in general the function $N_C(l)$ is not always convex in l. However, as noted before, if all M-firms are in location equilibrium, then it is indeed strictly convex. To see this, recall from Table 1 that in the context of the monocentric spatial configuration, M-firms are in location equilibrium only when $\alpha_A t_A \leqslant (1+\rho)\alpha_M t_M$. Since $\rho < 1$, this implies that

$$\alpha_A t_A < 2\alpha_M t_M \,. \tag{5.1}$$

[9] Note that both (3.11a) and (4.8) do not involve the parameters a_M and f, and hence the potential function $\Omega(x)$ is independent of these parameters.

Table 2
Effect of a marginal increase in each parameter on the monocentric equilibrium

	l^*	$N_M^* \equiv N - N_A^*$	n^*	u^*
$\alpha_A \equiv 1 - \alpha_M$	+	−	−	I
a_A	−	−	−	+ if $\alpha_M < \rho$ and both $\alpha_A t_A / \alpha_M t_M$ and N large − otherwise
a_M	0	0	0	−
t_A	$\pm^A$	$\mp^B$	$\mp^B$	+ if $\partial l^* / \partial t_A < 0$ and ρ small − otherwise
t_M	−	+	+	+ for small ρ − for large ρ
ρ	0	0	−	I
f	0	0	−	−
N	+	+	+	− if $\alpha_M < \rho$ and N large + otherwise

Notes: A: + if $\alpha_A > \alpha_M$ and N small, − otherwise.
B: − if $\alpha_A > \alpha_M$ and N small, + otherwise.
I: irrelevant.

Using (5.1) we can readily confirm mathematically that the function $N_C(l)$ is strictly convex in l. To see the economic logic behind this result, notice that condition (5.1) means that the transport cost for the A-good (weighted by expenditure share α_A) is not too high in comparison with that of M-goods. To illuminate the basic logic, let us take an extreme example where $t_A = 0 < t_M$; thus condition (5.1) is always satisfied. In this case, by (3.5) and (3.12),

$$N_C(l) = \frac{S_A(0)}{\alpha_A W(0)} = \frac{2\alpha_M l}{\alpha_A a_A^{-1} e^{-\alpha_M t_M l}}, \tag{5.2}$$

where

$$S_A(0) = 2\alpha_M l \quad \text{and} \quad W(0) = a_A^{-1} e^{-\alpha_M t_M l}. \tag{5.3}$$

That is, if $t_A = 0$, then the supply of the A-good to the city increases proportionally in l. On the other hand, because of the transport cost in M-goods, the equilibrium wage rate at the city, $W(0)$, must decrease in l.[10]

[10] Notice from (3.5) that when $t_A = 0$, the fringe wage $W(l)$ equals a_A^{-1}, a constant. While, since all M-goods are supplied from the city, their prices increase as l increases. Therefore, the compensating wage $W(0)$ in the city, which equates the workers' utility in the city with that at the fringe l, must decrease as l increases, as indicated by (5.3).

Therefore, the AE urban-consumer size must increase more than proportionally in l, i.e. the function $N_C(l)$ is strictly convex. The argument above suggest that *the strict convexity of the curve $N_C(l)$ in Fig. 3 does not depend on the particular forms of transport cost function*; rather, it depends on the fact that if the monocentric spatial configuration is in equilibrium, then the transport cost for the A-good (weighted by its expenditure share) should not be too high in comparison with that of the M-goods. Therefore, most results in Table 2 might be rather robust. In the rest of this section we explain the results in Table 2 by taking several examples. Since the effect of a parameter change on u^* is rather complex, we discuss it in the last part.

First, let us consider the impact of a marginal *decrease* in the labor–land ratio, a_A. That is, suppose that a labor-saving technology has been introduced in the agricultural sector. Then, reversing each sign in the third row in Table 2, we can see that as a_A decreases, the agricultural frontier expands ($l^*\uparrow$), the urban population increases ($N_M^*\uparrow$), and the variety of M-goods produced in the city also increases ($n^*\uparrow$). This can be readily seen from Fig. 3. In the figure, if a_A decreases, then not surprisingly the urban-worker supply curve $N_M(l)$ shifts upward, while the $N_C(l)$ curve shifts downward (since the urban wage rate, $W(0) \equiv a_A^{-1} e^{-\alpha_M(t_A+t_M)l}$, increases); consequently, the fringe distance l^* increases. Using the fact that the $N_C(l)$ curve is strictly convex, it is also not difficult to verify that as a_A decreases, urban population N_M^* increases,[11] which in turn increases the number of firms n^* in the city. *Hence, the development of a labor-saving technology in the agriculture causes both an expansion of the agricultural frontier and further industrialization and urbanization, as is often observed in developing countries.*

Next, let us examine the impact of transport cost changes. First, suppose that the M-good transport cost rate, t_M, increases marginally (while t_A remains the same). Then, in Eq. (3.12), not surprisingly, $S_A(0)$ is unaffected, while $W(0)$ diminishes. Hence, in Fig. 3, the AE urban-consumer size curve $N_C(l)$ shifts upward, which in turn reduces both l^* and N_A^* while increasing N_M^* (and hence also n^*), as indicated in the sixth row in Table 2. That is, as t_M increases, the wage rate at the city becomes relatively lower in comparison with that at the urban fringe. This encourages the M-industry in the city to expand, causing a population migration from the hinterland to the city. Next, suppose that the A-good transport cost rate, t_A, increases marginally. In this case, since both $S_A(0)$ and $W(0)$ decrease in Eq. (3.12), the net effect on $N_C(l)$ is not clear. A direct calculation, however, reveals that excluding a special case (i.e. $\alpha_A > \alpha_M$ *and* N is very small), $N_C(l)$

[11] This can be seen more easily if we replace $N_M(l)$ by $N_M(l)/(2a_A)$ and $N_C(l)$ by $N_C(l)/(2a_A)$ in Fig. 3.

increases.[12] Therefore, in Fig. 3, the $N_C(l)$ curve shifts upward as before, and hence again both l^* and N_A^* decrease while both N_M^* and n^* increase. Now, reversing all arguments above, we can conclude as follows. *A reduction in the transport costs of the A-good and/or M-goods is most likely to cause both an expansion of the agricultural frontier and a population migration from the city to the hinterland.* This may partly explain the strong westward migration of population in the United States over the nineteenth century and early twentieth century, during which transport costs decreased dramatically due to a successive expansion of railways and waterways.

For another example, the bottom row of Table 2 indicates that as N (the population size of workers in the economy) increases, not surprisingly the agricultural frontier expands further ($l^*\uparrow$), and both the population and the number of M-firms increase in the city ($N_M^*\uparrow$ and $n^*\uparrow$); then since $N_A^* \equiv 2a_A l^*$, the rural population also increases. In addition, since the $N_C(l)$ curve is strictly convex in Fig. 3, we can readily see that as N increases, the urban population share also increases ($N_M^*/N\uparrow$). Therefore, *an increase in the population has an effect similar to an increase in A-sector labor productivity: both cause an expansion of the agricultural frontier and further industrialization and urbanization.*

Turning to the impact of a parameter change on the equilibrium utility level u^*, we can see from Table 2 that it is generally complex. To see the reason for this complexity, by setting $y = 0$ in (3.3) and using (3.5) and (3.6), we may rewrite (3.15) as follows:

$$u^* = \alpha_A \log W(0) + \frac{\alpha_M}{\gamma} \log n^* + \log \alpha_A^{\alpha_A} \alpha_M^{\alpha_M} a_M^{-\alpha_M} \rho^{\alpha_M}\,, \tag{5.4}$$

where

$$W(0) = a_A^{-1}\, e^{-\alpha_M(t_A + t_M)l^*} \tag{5.5}$$

and

$$n^* = \frac{N_M^*}{f(1+\gamma)}\,. \tag{5.6}$$

In (5.4), if a parameter change affects $W(0)$ and n^* oppositely, then the net effect on u^* is generally ambiguous.

For example, let us consider the effect of an increase in population N on

[12] By totally differentiating both sides of (3.11a) by l and t_A, we can see that $dl/dt_A \propto -f(v)$, where $v \equiv t_A l$ and $f(v) \equiv v\,e^{-v} + \alpha_M v(1 - e^{-v}) + e^{-v} - 1$. Since $f(0) = 0$ and $f'(v) = \alpha_M e^{-v}\{e^v - 1 - (\alpha_A/\alpha_M)v\}$, $f(v) > 0$ for all $v > 0$ if $\alpha_A \leq \alpha_M$ While, if $\alpha_A > \alpha_M$, then $f(v) < 0$ only when v is sufficiently small (i.e. l is small, which means N is small). Therefore, l^* can increase (and hence N_A^* increases) in t_A only when $\alpha_A > \alpha_M$ *and* N is small.

u^*. When N increases, as shown before, the fringe distance l^* increases. This expansion in the agricultural frontier diminishes the urban wage rate, which contributes to lowering u^*, as indicated by (5.4) and (5.5). We may call this negative effect of a population increase (through the expansion of the agricultural frontier), the *scale diseconomies of population N in the A-supply*. On the other hand, as noted before, an increase in N causes the urban population N_M^* to increase, which in turn increases the product variety n^* of M-goods available for the economy. Then, as indicated in (5.4), u^* also increases. We may call this positive effect of a population increase (through the increase in product variety of M-goods) the *scale economies of population N in the M-good consumption*. The net effect on u^* depends on the relative sizes of the two opposing effects.

To examine the net effect of population increase on u^*, let $l^*(N)$ and $u^*(N)$ represent respectively the equilibrium values of l^* and u^* at each N. Then, by (5.4)–(5.6), we can show that

$$\frac{\mathrm{d}u^*(N)}{\mathrm{d}N} = A\left\{\frac{\alpha_M - \rho}{1-\rho} + \frac{t_A}{t_A + t_M}\frac{\mathrm{e}^{-t_A l^*(N)}}{1 - \mathrm{e}^{-t_A l^*(N)}}\right\}, \tag{5.7}$$

where A is a positive constant. Hence, if $\alpha_M \geqslant \rho$, then $\mathrm{d}u^*(N)/\mathrm{d}N > 0$ for all N. That is, suppose M-goods are highly differentiated so that $\alpha_M \geqslant \rho$ (which is the case where the monocentric configuration is in equilibrium for any N). Then, the scale economies of population N in the *M-goods* consumption are sufficiently strong so that they overwhelm the scale diseconomies of population in the A-good supply. Thus, u^* increases as N increases. Note also by (3.4) and (3.5) that since l^* increases always as N increases, the land rent $R(y)$ also increases everywhere if N increases. Therefore, if $\alpha_M \geqslant \rho$, then as the population size N of workers increases, all workers and landlords in the economy improve their welfare.[13]

Suppose conversely that M-goods are not much differentiated from one another so that $\alpha_M < \rho$. Then, as the population N continues to increase, the scale diseconomies of population N in the A-supply eventually start to overwhelm the scale economies of N in the M-good consumption. Setting the RHS of (5.7) equal to zero, this *critical population level*, $\hat{N}$, can be determined uniquely by solving the next equation for N:

$$\frac{\rho - \alpha_M}{1-\rho} = \frac{t_A}{t_A + t_M}\frac{\mathrm{e}^{-t_A l^*(N)}}{1 - \mathrm{e}^{-t_A l^*(N)}}. \tag{5.8}$$

Focusing on the parameter ρ, let $\hat{N}(\rho)$ be the solution to the above

[13] Substituting $R(y)$ for $W(y)$ in (3.3), we can readily see that at each y, landlords' utility $u(y)$ also increases as N increases.

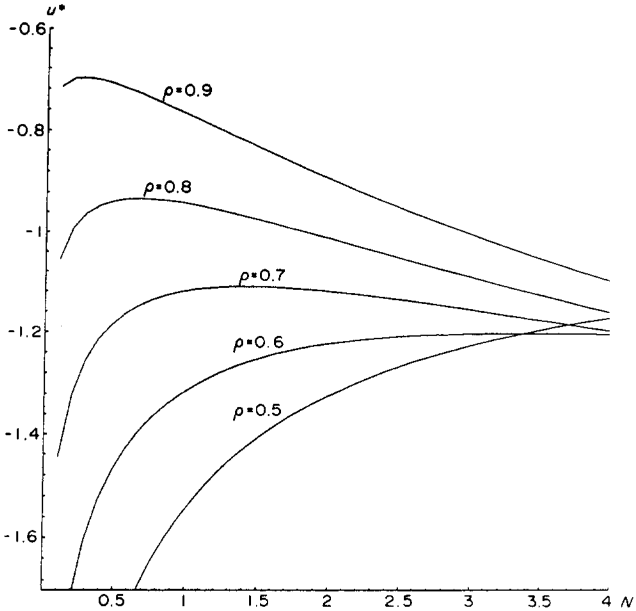

Fig. 5. The effect of N and ρ on u^* in monocentric equilibria [$\alpha_A = \alpha_M = 0.5$, $t_A = 0.9$, $t_M = 1$, $a_A = 0.5$, $a_M = f = 1$].

equation. Then, since l^* is increasing in N and independent of ρ (see Table 2), we can readily see that

$$\frac{d\hat{N}(\rho)}{d\rho} < 0 . \tag{5.9}$$

That is, the critical population level is greater when M-goods are more differentiated (i.e. ρ is smaller).

Fig. 5 demonstrates the effect of N and ρ on the equilibrium utility level u^*. (Other parameters are fixed at $\alpha_A = \alpha_M = 0.5$, $a_A = 0.5$, $a_M = 1$, $t_A = 0.9$, $t_M = 1$, and $f = 1$.) We can confirm by the figure that if $\rho > \alpha_M = 0.5$, the critical population indeed decreases in ρ, and that if $\rho \leq \alpha_M = 0.5$, then $u^*(N)$ increases for all N.

6. Conclusion

In this paper we presented a general spatial equilibrium model of an isolated state based on the monopolistic competition behavior of manufacturing firms producing differentiated consumption goods. We have examined conditions under which all manufacturing firms will agglomerate in a single city. Through the comparative statics of this monocentric equilibrium, we have examined the impact of technological changes on the equilibrium spatial configuration, in particular on the urbanization rate of the economy.

Our analysis of the model in this paper, however, is rather preliminary, and a great deal of work has been left for the future. First, although the location equilibrium conditions (obtained in Section 4) suggest when the economy will have more than one city, the study of equilibrium spatial configurations containing multiple cities has been left as an important future task.[14] Second, another important direction in extending our model is to introduce multiple groups of manufactured goods, with each group having different characteristics in terms of transport costs, degree of product differentiation, and production technology. Such an extended model may be able to generate a variety of hierarchical urban systems endogenously.[15]

Third, in our model above, the basic force of spatial agglomeration was generated through product variety in consumption goods. A similar model of spatial agglomeration can be developed by introducing product variety in intermediate goods. For example, suppose that a homogeneous M-good is produced (by M-firms) using labor and a variety of differentiated (producer) services. Owing to scale economies in product specialization, each service is assumed to be supplied monopolistically by a single S-firm. In this context, we can re-read Fig. 1 as follows. If a large variety of services is produced in a city, then this variety of services can be purchased at lower prices there (assuming the f.o.b. pricing by S-firms). Thus, because of complementarity among services, the productivity of M-firms becomes higher in the city. This, in turn, will induce more M-firms to agglomerate there. Then, this increase in the number of M-firms will create a greater demand for services there, which in turn can support a greater number of specialized S-firms. In turn, this implies a greater service variety in the city. Thus, as before, this process creates a circular causality in the spatial agglomeration of M-firms and S-firms (as well as their workers). Such an agglomeration force due to product variety in producer services can partly explain, for example, a concentration of high-technology firms (e.g. Silicon Valley) or business firms (e.g. New York).[16]

Fourth, another interesting direction of model extension is to consider

[14] As noted before, Fujita and Krugman (1993) present a preliminary study in this direction. For additional results on this subject (including a study of the bifurcation process from the monocentric economy to multicentric economies), see Fujita and Mori (1994).

[15] For recent results on this topic, see Fujita et al. (1994).

[16] For spatial agglomeration models based on product variety in intermediate goods, see, for example, Helpman and Krugman (1985, ch. 11), Fujita (1990), Krugman and Venables (1993), and Venables (1993). Except Fujita (1988), all these models adopt a discrete spatial economy (i.e. two-region economy). Fujita (1990) is concerned with the spatial structure of a metropolitan area, rather than a national economy.

multi-locational firms. That is, suppose that each firm consists of multiple units (e.g. HQs, R&D units, and manufacturing plants) which can be located separately. Then, since different units will follow different agglomeration forces (e.g. availability of business services and convenience of face-to-face communication for HQs, availability of engineers for R&D units, and accessibility to consumers for manufacturing plants), we will be able to develop a richer class of spatial models.[17] Eventually, by introducing into such basic models various realistic features (such as political restrictions, natural geographic features, and historical and cultural elements), we will be able to develop a realistic regional model which would be useful in studying the future economic geography of a nation or group of nations.

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Appendix A: Derivation of (4.2)

Because of the iceberg transport assumption, in order to deliver one unit of an M-good from the firm at x to consumers at y, $e^{t_M|y-x|}$ units of the good should be dispatched by the firm. Hence, setting $Y=(N-2a_A l)W^*(0)$ in Eq. (2.3), the total demand for the product (of the firm at x) by the consumers at the city is given by

$$\begin{aligned} Z(0) &\equiv \frac{\alpha_M(N-2a_A l)W^*(0)}{p_M(0|x)} \frac{p_M(0|x)^{-\gamma}}{np_M(0|0)^{-\gamma}} e^{t_M|x|} \\ &= \frac{\alpha_M \gamma f}{a_M}\left(\frac{W^*(0)}{W^*(x)}\right)^{1+\gamma} e^{-\gamma t_M|x|}, \qquad \text{(A.1)} \end{aligned}$$

using (3.9), (3.11) and (4.1). Next, at each $y \neq 0$, the total income (for A-workers and landlords) per unit of distance at y equals $p_A(y)=e^{-t_A|y|}$. Hence, setting $Y=e^{-t_A|y|}$ in (2.3) for $y \neq 0$, the total demand (including transport consumption) for the product (of the firm at x) by all A-workers and landlords can be obtained as

[17] For spatial economic models with multiunit firms, see, for example, Helpman and Krugman (1985, ch. 13), and Ota and Fujita (1993).

$$\int_{-l^*}^{l^*} \frac{\alpha_M \, e^{-t_A|y|}}{p_M(y|x)} \frac{p_M(y|x)^{-\gamma}}{np_M(y|0)^{-\gamma}} e^{t_M|y-x|} \, dy$$

$$= \frac{\alpha_A \gamma f}{2a_M} \left(\frac{W^*(0)}{W^*(x)}\right)^{1+\gamma} \frac{t_A}{1-e^{-t_A l^*}} \int_{-l^*}^{l^*} e^{-t_A|y|} \, e^{\gamma t_M(|y|-|y-x|)} \, dy \qquad \text{(A.2)}$$

using (3.9), (3.11) and (4.1). Hence, the total demand $D(x, W^*(x))$ for the product of the firm at x is given by

$$D(x, W^*(x)) = \frac{\alpha_A \gamma f}{2a_M} \left(\frac{W^*(0)}{W^*(x)}\right)^{1+\gamma} e^{-\gamma t_M|x|}$$
$$\times \left\{\frac{2\alpha_M}{\alpha_A} + \frac{t_A}{1-e^{-t_A l^*}} e^{\gamma t_M|x|} \int_{-l^*}^{l^*} e^{-t_A|y|} \, e^{\gamma t_M(|y|-|y-x|)} \, dy\right\}, \qquad \text{(A.3)}$$

which leads to (4.2).

Appendix B: Derivation of the results in Table 1

Differentiating (4.8) in $x > 0$,

$$\Omega'(x) = -\frac{\alpha_A}{2} \tau \, e^{\tau x} \frac{t_A}{1-e^{-t_A l^*}} f(x), \qquad \text{(B.1)}$$

where $\tau \equiv (1+\gamma)(\alpha_M t_M - \alpha_A t_A) + \gamma t_M$ and

$$f(x) \equiv \frac{1+\alpha_M}{\alpha_A} \frac{1-e^{-t_A l^*}}{t_A} + \int_0^x e^{(2\gamma t_M - t_A)y} \, dy$$
$$+ \frac{\tau - 2\gamma t_M}{\tau} e^{2\gamma t_M x} \int_x^{l^*} e^{-t_A y} \, dy. \qquad \text{(B.2)}$$

Here, notice that

(a) if $2\gamma t_M \neq t_A$,

$$f(x) = h(x) + k(x), \qquad \text{(B.3)}$$

where

$$h(x) = \frac{2-\alpha_A}{\alpha_A} \frac{1-e^{-t_A l^*}}{t_A} - \frac{1}{2\gamma t_M - t_A} + A \, e^{2\gamma t_M x}, \qquad \text{(B.4)}$$

$$k(x) = B \, e^{(2\gamma t_M - t_A)x} \qquad \text{(B.5)}$$

and

$$A \equiv \frac{2\gamma t_M - \tau}{\tau t_A} e^{-t_A l^*}, \qquad B = \frac{2\gamma t_M(\tau - 2\gamma t_M + t_A)}{(2\gamma t_M - t_A)\tau t_A}; \tag{B.6}$$

(b) while if $2\gamma t_M = t_A$, then

$$f(x) = \frac{2-\alpha_A}{\alpha_A} \frac{1 - e^{-t_A l^*}}{t_A} + x + \frac{\tau - 2\gamma t_M}{\tau} \frac{1 - e^{-t_A(l^* - x)}}{t_A}. \tag{B.7}$$

We examine the five different cases below in turn:

(I) $\tau < \alpha_A \gamma t_M$, i.e. $\alpha_A t_A > (1+\rho)\alpha_M t_M$. In this case, since $\Omega(0) = 1$ and $\Omega'(0) = -(\tau - \alpha_A \gamma t_M) > 0$, we have that $\Omega(x) > 1$ for small x. Hence, as indicated in Table 1, the monocentric configuration is never in equilibrium.

(II-1) $\tau \geq \alpha_A \gamma t_M$ (i.e. $\alpha_A t_A \leq (1+\rho)\alpha_M t_M$), $\tau \geq 2\gamma t_M - t_A$ (i.e. $\alpha_M \geq \rho$), and $\tau \geq 2\gamma t_M$. In this case, $\tau > 0$ and $f(x) > 0$ for all $x \leq l^*$ (by (B.2)), which implies that $\Omega'(x) < 0$ for all $x \leq l^*$. Hence, $\Omega(x) \leq 1$ for all $x \leq l^*$, and the monocentric configuration is always in equilibrium.

(II-2-i) $\tau \geq \alpha_A \gamma t_M$, $\tau \geq \gamma t_M - t_A$, $\tau < 2\gamma t_M$, and $2\gamma t_M > t_A$. In this case, $A > 0$ and $B \geq 0$. Hence, by (B.3), $f(x)$ is increasing in x. Furthermore,

$$f(0) = \frac{2(\tau - \alpha_A \gamma t_M)}{\alpha_A \tau} \frac{1 - e^{-t_A l^*}}{t_A} \geq 0. \tag{B.8}$$

Thus, $f(x) \geq 0$ for all $x \leq l^*$, and since $\Omega(0) = 1$. This implies that $\Omega(x) \leq 1$ for all $x \leq l^*$; thus, the monocentric configuration is always in equilibrium.

(II-2-ii) $\tau \geq \alpha_A \gamma t_M$, $\tau \geq 2\gamma t_M - t_A$, $\tau < 2\gamma t_M$, and $2\gamma t_M < t_A$. In this case, $A > 0$, $h(x)$ is increasing in x. Since $B < 0$, $k(x) < 0$; but, since $2\gamma t_M < t_A$, $|k(x)|$ is decreasing in x. Then, since (B.8) holds, $f(x) \geq 0$ for all $x \leq l^*$. Hence, as in (II-2-i), the monocentric configuration is always in equilibrium.

(II-2-iii) $\tau \geq \alpha_A \gamma t_M$, $\tau \geq 2\gamma t_M - t_A$, $\tau < 2\gamma t_M$, and $2\gamma t_M > t_A$. Since (B.8) holds, and since the absolute value of the last term in the RHS of (B.7) is decreasing in x, we can readily conclude that $\Omega(x) \leq 1$ for all $x \leq l^*$, and hence the monocentric configuration is always in equilibrium.

(III) $\tau \geq \alpha_A \gamma t_M$, and $\tau < 2\gamma t_M - t_A$ (and hence $2\gamma t_M > t_A$). In (4.8), let us denote l^* simply as l; and, in order to make explicit that $\Omega(x)$ is also a function of l, let us rewrite $\Omega(x)$ as $\Omega(x; l)$. Then, it can be readily verified that for $x > 0$,

$$\frac{\partial \Omega(x:l)}{\partial l} > 0 \quad \text{for all } x < l, \tag{B.9}$$

and that for each $x \geq 0$,

$$\lim_{l\to\infty} \Omega(x; l) = \frac{\alpha_A}{2}\left\{\left(\frac{2-\alpha_A}{\alpha_A} + \frac{t_A}{2\gamma t_M - t_A}\right) e^{-\tau x} + \frac{2\gamma t_M}{2\gamma t_M - t_A} e^{(2\gamma t_M - t_A - \tau)x}\right\}. \tag{B.10}$$

Therefore, there exists $\hat{x} > 0$ such that

$$\Omega(x; l = \infty) \gtreqless 1 \text{ as } x \gtreqless \hat{x} . \tag{B.11}$$

Together (B.9) and (B.11) imply that

$$\Omega(x; l) < 1 \quad \text{for } x < l < \hat{x} , \tag{B.12}$$

and that

$$\{\Omega(x; l') = 1 \text{ for } x < l'\} \;\not\Rightarrow\; \{\Omega(x; l) > 1 \text{ for all } l > l'\} . \tag{B.13}$$

Since population N and equilibrium $l(\equiv l^*)$ are in one-to-one correspondence and l is increasing in N, we can conclude by (B.13) that the monocentric configuration is in equilibrium only when N is sufficiently small.

Table 1 summarizes the results above.

References

Ben-Akiva, M., A. de Palma and J.-F. Thisse, 1989, Spatial competition with differentiated products, Regional Science and Urban Economics 19, 5–19.

de Palma, A., V. Ginsburgh, Y. Papageorgiou and J.-F. Thisse, 1985, The principle of minimum differentiation holds under sufficient heterogeneity, Econometrica 53, 767–781.

Fujita, M., 1988, A monopolistic competition model of spatial agglomeration: Differentiated product approach, Regional Science and Urban Economics 18, 87–124.

Fujita, M., 1990, Spatial interactions and agglomeration in urban economics, in: M. Chatterji and R.E. Kunne, eds., New frontiers in regional science (Macmillan, London) 185–221.

Fujita, M. and P. Krugman, 1993, A monopolistic competition model of urban systems and trade, Working Papers in Regional Science No. 163, Department of Regional Science, University of Pennsylvania, Philadelphia, PA.

Fujita, M. and T. Mori, 1994, Structural stability and evolution of urban systems, mimeograph, Department of Regional Science, University of Pennsylvania, Philadelphia, PA.

Fujita, M., P. Krugman and T. Mori, 1994, On the evolution of hierarchical urban systems, mineograph, Department of Regional Science, University of Pennsylvania, Philadelphia, PA.

Helpman, E. and P. Krugman, 1985, Market structure and foreign trade (MIT Press, Cambridge, MA).

Krugman, P., 1993, First nature, second nature, and metropolitan location, Journal of Regional Science 34, 129–144.

Krugman, P. and A.J. Venables, 1993, Integration, specialization, and adjustment, mimeograph, Department of Economics, Massachusetts Institute of Technology.

Nerlove, M.L. and E. Sadka, 1991, von Thünen's model of the dual economy, Journal of Economics 54, 97–123.

Ota, M. and M. Fugita, 1993, Communication technologies and spatial organization of multi-unit firms in metropolitan areas, Regional Science and Urban Economics 23, 695–729.

Samuelson, P.A., 1952, The transfer problem and transport costs: The terms of trade when impediments are absent, Economic Journal LXII, 278–304.

Samuelson, P.A., 1983, Thünen at two hundred, Journal of Economic Literature 21, 1468–1488.

Venables, A.J., 1993, Equilibrium location of vertically linked industries, mimeograph, London School of Economics.

von Thünen, 1826, Der Isolierte Staat in Beziehung auf landwrirtschaft und Nationalekonomie (Hamburg).

Wartenberg, C.M., 1966, von Thünen's isolated State (Pergamon, London).