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Quasi-hyperbolic discounting, paternalism and optimal mixed taxation☆

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HIGHLIGHTS

- We characterize optimal tax policy under quasi-hyperbolic discounting.
- Private consumption plans may either be time consistent or not.
- Consumption and savings taxes are typically not sufficient for correction.
- Also labor income taxes may be used to correct private behavior.

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ABSTRACT

This paper develops a dynamic model with endogenous labor supply, savings and health capital, where the consumers differ in ability as well as suffer from a self-control problem generated by quasi-hyperbolic discounting. The purpose is to analyze how a paternalistic government, which implements a time-consistent mix of labor income taxation, capital income taxation and commodity taxation, ought to use this tax system for purposes of redistribution and correction when individual ability is private information. Among the results, we show how the (nonlinear) income taxes ought to be used as indirect instruments for influencing the commodity demand behavior at the individual level: the intuition is that linear commodity taxes are not flexible enough to achieve proper incentives for consumption of unhealthy goods.

1. Introduction

The classical approach for studying corrective taxation is based on the notion of market failure; for instance, an externality that provides a rationale for the government to intervene in a specific market. More recently, however, a literature dealing with optimal paternalism has evolved, in which the corrective role of government is based on the idea that individuals may not be fully rational. The underlying behavioral failure is a self-control problem caused by quasi-hyperbolic discounting, where the individual at any time t attaches a higher utility discount rate to tradeoffs between periods t and $t+1$ than to similar tradeoffs in the more distant future. In the present paper, we develop a dynamic model with endogenous labor supply, savings, and health capital, where the consumers are described as quasi-hyperbolic discounters. The purpose is to examine how a paternalistic government that does not share the consumer preference for immediate gratification ought to use a mixed tax system, i.e. an optimal combination of nonlinear income taxes and linear commodity taxes. As in many earlier studies on optimal taxation, we assume that individual ability (productivity) is private information. A mixed tax system constitutes a reasonably realistic description of the set of tax instruments that many countries have at their disposal. Furthermore, it means that the appearance of tax distortions at the second best optimum is a consequence of optimization subject to informational limitations.

Why is it interesting to consider quasi-hyperbolic discounting in the context of optimal mixed taxation? First, although a number of important aspects of hyperbolic discounting have been recognized and analyzed in earlier studies,[1] the literature dealing with corrective taxation in this particular context (see below) is quite

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1 These aspects include consumption and savings behavior in different contexts (e.g., Laibson, 1996, 1997) and retirement behavior (e.g., Diamond and Köszegi, 2003).

small and typically focuses on issues other than redistribution under asymmetric information. As a government that attempts to correct for a self-control problem also may want to redistribute in the most efficient way given the available information, we believe that redistributive and corrective aspects of taxation ought to be addressed simultaneously. Second, in our study, the consumers attach less weight to the future health consequences of their current consumption than the paternalistic government wants them to do. Since the consumers are heterogeneous, linear commodity taxes may not be flexible enough instruments to internalize the externalities that each consumer imposes on his/her future selves. As a consequence, there is an incentive for the government to supplement the commodity taxes by corrective income taxation in order to affect the commodity demand behavior at the individual level; a role of income taxation which has not been addressed in the earlier literature on self-control problems and optimal taxation.

There is a relatively small, yet growing, literature dealing with paternalistic motives for optimal taxation in the context of quasi-hyperbolic discounting. O'Donoghue and Rabin (2003, 2006) analyze optimal commodity taxation in economies where the current consumption of a particular commodity gives rise to disutility caused by a negative health effect in the next period, and where the consumers have a preference for immediate gratification due to quasi-hyperbolic discounting. Therefore, the self-control problem is dealt with by a modification of the commodity tax structure, and the results show that the optimal public policy may imply higher taxes on unhealthy commodities and lower taxes on other commodities than in the absence of the underlying self-control problem. In addition, it is generally desirable to implement sin-taxes paid at the same (positive) rate by all consumers, even if some of them do not suffer from the self-control problem. Aronsson and Thunström (2008) use a similar model with the modification that health is a capital concept, implying that the externality that the individual imposes on his/her future self is a stock-externality. Their contribution is to show that a subsidy directed at the stock of health capital is part of the policy package that implements the first best social optimum, defined as the allocation that would be implemented by a social planner with exponential discounting. Gruber and Köszegi (2001, 2004) consider cigarette taxation in an economy where smokers suffer from a self-control problem. They find, among other things, that cigarette taxes ought to be considerably higher in such an economy than under rational addiction, and that cigarette taxes are less regressive than found in earlier studies.

In a framework with nonlinear taxation and asymmetric information, Blomquist and Micheletto (2006) analyze paternalistic aspects of tax policy. Their study is based on a static model, in which the objective function of the government is assumed to differ from that of the consumers, i.e. a non-welfarist approach to optimal taxation, and the contribution is to examine how this change of assumption modifies the effective marginal tax rates and the structure of commodity taxation, respectively, by comparison with the standard two-type model for mixed taxation.[2] A similar framework is used by Pirttilä and Tenhunen (2008) to address paternalistic motives behind publicly provided private goods. Aronsson and Granlund (2014) also examine the welfare consequences of a publicly provided private good under optimal income taxation, although their study is based on a framework similar to ours where quasi-hyperbolic discounting generates the behavioral failure that the government attempts to correct for. Tenhunen and Tuomala (2007) use a two-period model to analyze how differences in the utility discount rate between the consumers and the government affect the optimal use of capital income taxation. However, none of these earlier studies have examined how a mixed tax system can be used to simultaneously achieve redistribution and correction in an environment with quasi-hyperbolic discounting.

Our study is based on an overlapping generations (OLG) model, in which the consumers differ in ability, and where each consumer lives for three periods (the minimum number of periods required to analyze the consequences of quasi-hyperbolic discounting). Following Stern (1982) and Stiglitz (1982), we simplify by considering a model with two ability-types. The instantaneous utility depends on the current consumption of two commodities, the use of leisure, and the stock of health capital,[3] respectively, where the latter accumulates via the consumption of one of the commodities, to be called 'unhealthy good'.

Earlier literature dealing with self-control problems often makes a distinction between naive and sophisticated consumers. At any time, t, a naive consumer erroneously expects to be time-consistent in the future, meaning that he/she may have an incentive to revise the optimal plan in each subsequent period, whereas a sophisticated consumer recognizes that the self-control problem also arises in future periods and implements a time-consistent plan. In this paper, we will not take a stand on whether the consumers are naive or sophisticated, since none of these behavioral assumptions can be ruled out based on empirical evidence.[4] Instead, tax policy responses to quasi-hyperbolic discounting will be examined both for naive and sophisticated consumers. This is also motivated by the fact that the tax policy implications of quasi-hyperbolic discounting to some extent depend on this distinction.

The government in our model uses a labor income tax, a capital income tax, and a commodity tax attached to the unhealthy good for purposes of redistribution and correction. We also recognize that once each consumer reveals his/her ability, which the consumer does at the end of the first period of life in our model (given the appropriate incentives), the government may implement income tax policies for the second and third periods of life for each consumer under perfect information about ability. Therefore, and in contrast to many earlier studies on optimal nonlinear taxation in dynamic economies, where the income tax policy for the whole individual life-cycle is decided upon before the individual reveals his/her ability,[5] the tax policy that the government in our study implements is time-consistent. A similar approach to optimal taxation – yet within the context of a two-period model and with no reference to paternalism – was taken by Brett and Weymark (2008).

Similar to our paper, Guo and Krause (2015) analyze a model of optimal nonlinear income taxation without commitment when individuals are facing a self-control problem caused by quasi-hyperbolic discounting. They examine whether quasi-hyperbolic discounting leads to higher or lower social welfare in a model where the government (or social planner) does not share the consumer preference for immediate gratification. Without any information asymmetries (i.e., if first best taxation can be used), their results show that quasi-hyperbolic discounting has no effect on the social welfare; with asymmetric information, on the other

2 See also Kanbur et al. (2006), who consider a non-welfarist approach to optimal nonlinear and mixed taxation in a static model. They also illustrate their analysis for a number of more specific cases (such as poverty reduction, merit goods and motives for sin-taxes).

3 By describing health as a capital concept, our model bears some resemblance to the classical health economics model developed by Grossman (1972).

4 Hey and Lotito (2009) present experimental results showing behavioral patterns consistent with both naivety and sophistication, even if naivety seems to be a more common type of behavior. See also, e.g., DellaVigna and Malmendier (2006) and Gine et al. (2010).

5 This approach is taken by, e.g., Brett (1997), Pirttilä and Tuomala (2001), Aronsson et al. (2009) and Aronsson and Johansson-Stenman (2010).

hand, quasi-hyperbolic discounting may either lead to higher or lower social welfare, depending on whether separating or pooling taxation is optimal. However, their study does not address the implications of quasi-hyperbolic discounting for optimal mixed taxation, which is the key issue in the present paper.[6]

Our study contributes to the literature in at least two ways. The first is that we integrate quasi-hyperbolic discounting into the theory of optimal mixed taxation. This means that the paper (i) extends the literature on optimal sin taxes, where earlier studies abstract from nonlinear income taxes, and (ii) extends the literature on non-welfarist motives for tax policy under asymmetric information by thoroughly examining the self-control problem in an intertemporal environment. The second contribution is to highlight a corrective role of income taxation in economies where agents have self-control problems: the lack of individual-specific commodity taxes necessitates using income taxation as an (imperfect) instrument for influencing the pattern of commodity demand. The intuition is that agents are heterogeneous, which means that linear commodity taxes are not flexible enough to adjust the consumer demand behavior in the way ideally preferred by the government. We believe that this aspect is particularly interesting, as several earlier studies on optimal taxation under quasi-hyperbolic discounting have focused on commodity taxes.

If there is no income effect on the labor supply, our results show that quasi-hyperbolic discounting will induce a sophisticated individual to (i) consume more of the unhealthy good when young and (ii) save more when young compared with a naive individual. Behavioral effect (i) implies that the paternalistic government has an incentive to implement a higher corrective tax on the unhealthy good if the consumers are sophisticated than if they are naive. We also show that the commodity tax on the unhealthy good is not a flexible enough instrument to adjust the consumption of the unhealthy good in the way ideally preferred by the government. More specifically, the actual commodity tax (faced by all consumers in a given time period) is too high for the old and most likely too low for the young and middle-aged, compared to a hypothetical system of individualized commodity taxes. This means that the income taxes are used as supplemental instruments for influencing the commodity demand, and we show how this motive for marginal labor and capital income taxation depends on (1) the discrepancy between the actual and ideal (individualized) commodity taxes and (2) how the labor supply and savings behavior indirectly influences the pattern of commodity demand. Behavioral effect (ii) above provides an incentive for the government to tax capital income of middle-aged consumers at a lower marginal rate when the consumers are naive than when they are sophisticated. Our results also show that if the government cannot commit to the tax policy from the outset (meaning that if must implement a time-consistent tax policy), then this produces an additional channel through which the self-selection constraint affects the optimal tax policy.

The outline of the study is as follows. In Section 2, we present the model and analyze the behavior of consumers and firms. Section 3 contains the optimal tax problem as well as the outcome in terms of optimal taxation. We summarize and discuss the results in Section 4.

6 Also Bassi (2010) considers optimal taxation in a dynamic economy where some agents are time inconsistent. Among the results, he shows that the optimal capital tax will include a paternalistic component which reduces the optimal tax compared to the case when the agents are not time inconsistent. Aronsson and Sjögren (2014) analyze optimal income taxation under quasi-hyperbolic discounting from the perspective of an open economy with international capital mobility (although without information asymmetries between the policy maker and the private sector).

2. The model

In this section, we describe the optimization problems facing the consumers and firms. The optimization problem facing the government and the outcome in terms of optimal taxation are addressed in Section 3.

2.1. Consumers

Consider an economy comprising two ability-types: a low-ability type (denoted by superindex 1) and a high-ability type (denoted by superindex 2), where the high-ability type is more productive and, therefore, earns a higher gross wage rate than the low-ability type. Each consumer lives for three periods: works in the first two and is retired in the third. As the number of consumers of each ability-type and generation is not important for the qualitative results to be derived below, it will be normalized to one for notational convenience. This means that one new consumer of each ability-type enters the economic system in each time-period and that the population is constant.

Individuals entering the economy in period t (who are young in period t, middle-aged in period $t+1$ and old in period $t+2$) will be referred to as generation t. The instantaneous utility functions facing an individual of ability-type i and generation t can be written as

$$\begin{aligned} u^i_{0,t} &= a(c^i_{0,t}, x^i_{0,t}, z^i_{0,t}) + \upsilon(h^i_{0,t}) \\ u^i_{1,t+1} &= a(c^i_{1,t+1}, x^i_{1,t+1}, z^i_{1,t+1}) + \upsilon(h^i_{1,t+1}) \\ u^i_{2,t+2} &= a(c^i_{2,t+2}, x^i_{2,t+2}, \bar{l}) + \upsilon(h^i_{2,t+2}) \end{aligned} \tag{1}$$

where c is the consumption of an ordinary (not unhealthy) good, x the consumption of an unhealthy good, z leisure and h the stock of health capital. Leisure is, in turn, defined as a time endowment, $\bar{l}$, less the time spent in market work, l. Subindices 0, 1 and 2 refer to the phase of the life-cycle, i.e. young, middle-aged and old, respectively. We assume that the functions $a(\cdot)$ and $\upsilon(\cdot)$ are increasing and strictly concave in their arguments, and all goods are normal. The accumulation of health capital is governed by

$$h^i_{1,t+1} = h^i_{0,t} + g(x^i_{0,t}), \quad h^i_{2,t+2} = h^i_{1,t+1} + g(x^i_{1,t+1}) \tag{2}$$

where $g(x)$ is a decreasing function of x. The initial endowment of health capital, $h^i_{0,t}$, is exogenously given.

The concept of present biased preferences is operationalized by using an approach developed by Phelps and Pollak (1968) and later used by, e.g., Laibson (1997) and O'Donoghue and Rabin (1999, 2003). The intertemporal objective of any generation, t, is given by

$$U^i_t = u^i_{0,t} + \beta \sum_{j=1}^{2} \Theta^j u^i_{j,t+j} \tag{3}$$

where $\Theta^j = 1/(1+\theta)^j$ is a conventional utility discount factor with utility discount rate θ, whereas β is the preference for immediate gratification, meaning that $\beta \in (0, 1)$.

Let $y^i_{0,t} = w^i_{0,t} l^i_{0,t}$ and $I^i_{1,t+1} = s^i_{0,t} r_{t+1}$ denote labor income when young and capital income when middle-aged, respectively, where w is the before-tax wage rate, s saving and r the interest rate. The labor income when middle-aged and capital income when old are defined analogously. We can then define $T^i_{0,t} = T_t(y^i_{0,t}, 0)$, $T^i_{1,t+1} = T_{t+1}(y^i_{1,t+1}, I^i_{1,t+1})$ and $T^i_{2,t+2} = T_{t+2}(0, I^i_{2,t+2})$ to represent the income tax payments made by ability-type i of generation t when young, middle-aged and old, respectively. The life-time budget constraint is given by the following equations:

$$0 = y^i_{0,t} - T^i_{0,t} - s^i_{0,t} - c^i_{0,t} - q_{x,t} x^i_{0,t} \tag{4}$$

$$0 = s^i_{0,t} + I^i_{1,t+1} + y^i_{1,t+1} - T^i_{1,t+1} - s^i_{1,t+1} - c^i_{1,t+1} - q_{x,t+1}x^i_{1,t+1} \tag{5}$$

$$0 = s^i_{1,t+1} + I^i_{2,t+2} - T^i_{2,t+2} - c^i_{2,t+2} - q_{x,t+2}x^i_{2,t+2}. \tag{6}$$

The variable q_x is the consumer price of the unhealthy good and it is given by $q_{x,t} = p_{x,t} + \tau_{x,t}$ where $p_{x,t}$ is the producer price, while $\tau_{x,t}$ is the commodity tax on the unhealthy good implemented in period t. The ordinary (not unhealthy) consumption good, c, is the numeraire good and its price is normalized to one.

As we mentioned in the introduction, although the consumers apply quasi-hyperbolic discounting, it is not clear whether they should be treated as being naive or sophisticated. We will, therefore, consider both these possibilities in the analysis below. Since the individual first order conditions, as well as the policy rules for optimal taxation, under naivety are interpretable as technical special cases of the corresponding first order conditions and policy rules that follow under sophistication, we first derive the results under the assumption that the consumers are sophisticated, and then explain how the results are modified if the consumers instead are naive. A sophisticated agent will choose a time-consistent consumption plan in the sense that he/she will recognize (when making the current decisions) that the self-control problem also appears in the future. To arrive at a time-consistent solution, the consumer's decision-problem will be solved sequentially: we begin by deriving the commodity demand system for the middle-aged and old selves as well as the labor supply and savings behavior of the middle-aged self, and then continue with the commodity demand, labor supply and saving by the young self.

Behavior of the Middle-Aged and Old

As in the bulk of earlier literature on optimal mixed taxation, we solve each consumer's decision-problem in two stages. In the first stage, we derive commodity demand functions conditional on the hours of work and savings; in the second, we derive the labor supply and the savings functions. The first stage problem means choosing $c^i_{1,t+1}, x^i_{1,t+1}, c^i_{2,t+2}$ and $x^i_{2,t+2}$ to maximize $u^i_{1,t+1} + \beta\Theta u^i_{2,t+2}$ subject to $h^i_{2,t+2} = h^i_{1,t+1} + g(x^i_{1,t+1})$ and subject to the following budget constraints

$$b^i_{1,t+1} = c^i_{1,t+1} + q_{x,t+1}x^i_{1,t+1}, \quad b^i_{2,t+2} = c^i_{2,t+2} + q_{x,t+2}x^i_{2,t+2} \tag{7}$$

where b is a fixed net income (adjusted for saving; see below). Note also that $h^i_{1,t+1}$ is treated as fixed here. Next, we substitute the equations in (7) into the utility function and maximize w.r.t. $x^i_{1,t+1}$ and $x^i_{2,t+2}$. Combining the resulting first order conditions with the budget constraints defined in (7) provides us with a system of equations which implicitly define the following conditional demand functions:

$$\eta^i_{1,t+1} = \eta_1(b^i_{1,t+1}, q_{x,t+1}, z^i_{1,t+1,t}, x^i_{0,t}) \text{ for } \eta = c, x \tag{8}$$

$$\eta^i_{2,t+2} = \eta_2(b^i_{2,t+2}, q_{x,t+2}, \bar{l}) \text{ for } \eta = c, x. \tag{9}$$

Note in particular from Eq. (8) that $c^i_{1,t+1}$ and $x^i_{1,t+1}$ become functions of $x^i_{0,t}$ via $h^i_{1,t+1}$. Let us then turn to the hours of work and saving. For given $s^i_{0,t}$ and $x^i_{0,t}$, we choose $l^i_{1,t+1}$ and $s^i_{1,t+1}$ to maximize $u^i_{1,t+1} + \beta\Theta u^i_{2,t+2}$ subject to $h^i_{2,t+2} = h^i_{1,t+1} + g(x^i_{1,t+1})$ and subject to Eqs. (8) and (9), as well as subject to the following budget constraints:

$$b^i_{1,t+1} = s^i_{0,t}(1 + r_{t+1}) + w^i_{1,t+1}l^i_{1,t+1} - T^i_{1,t+1} - s^i_{1,t+1} \tag{10}$$

$$b^i_{2,t+2} = s^i_{1,t+1}(1 + r_{t+2}) - T^i_{2,t+2}. \tag{11}$$

Substituting (10) and (11) into the objective function and maximizing the resulting expression w.r.t. work hours and saving provides us with two equations which implicitly define the following reaction functions:

$$l^i_{1,t+1} = l(s^i_{0,t}, x^i_{0,t}), \quad s^i_{1,t+1} = s(s^i_{0,t}, x^i_{0,t}). \tag{12}$$

It is straightforward to show that the consumer in this setting saves less than she would have done in a standard model without quasi-hyperbolic discounting (i.e. where $\beta = 1$). Note finally that Eqs. (8), (9), and (12) take the same form irrespective of whether the consumers are sophisticated or naive.

Behavior of the Young

By analogy to the analysis carried out for the middle-aged and old selves the consumption, labor supply, and savings behavior of the young self will also be analyzed by a two stage procedure. Once again, therefore, we begin by deriving the conditional demand functions and then turn to the labor supply and saving. For notational convenience, define the intertemporal objective that the young self **would like** the middle-aged self to maximize (instead of $u^i_{1,t+1} + \beta\Theta u^i_{2,t+2}$, which is the objective that the middle-aged self **actually** maximizes) as

$$V^i_{1,t} = u^i_{1,t+1} + \Theta u^i_{2,t+2}. \tag{13}$$

Note that Eq. (13) does not contain the preference for immediate gratification that the middle-aged self attaches to her own preferences. Therefore, the objective characterizing the young ability-type i in period t can be written as

$$V^i_{0,t,\beta} = u^i_{0,t} + \beta\Theta V^i_{1,t}. \tag{14}$$

To derive the conditional commodity demand functions for the young self, we maximize Eq. (14) with respect to $c^i_{0,t}$ and $x^i_{0,t}$ subject to Eqs. (2) and (7)–(12) as well as subject to the budget constraint

$$b^i_{0,t} = c^i_{0,t} + q_{x,t}x^i_{0,t}. \tag{15}$$

Define

$$MRS^{i,t+1}_{c_1,c_2} = \frac{\partial u^i_{1,t+1}/\partial c^i_{1,t+1}}{\beta\Theta(\partial u^i_{2,t+2}/\partial c^i_{2,t+2})}$$

to be the marginal rate of substitution between the consumption of the numeraire good in periods $t+1$ and $t+2$, and let $\tilde{x}^i_{1,t+1}$ denote the conditional compensated demand for commodity x by the middle-aged ability-type i of generation t. To simplify the presentation below, we will also use the following short notation for how the health capital stock of the old self responds to a utility compensated increase in the use of leisure and increased private disposable income, respectively, when middle-aged:

$$\frac{\partial \tilde{h}^i_{2,t+2}}{\partial z^i_{1,t+1}} = \frac{\partial h^i_{2,t+2}}{\partial x^i_{1,t+1}}\frac{\partial \tilde{x}^i_{1,t+1}}{\partial z^i_{1,t+1}}, \quad \frac{\partial h^i_{2,t+2}}{\partial b^i_{1,t+1}} = \frac{\partial h^i_{2,t+2}}{\partial x^i_{1,t+1}}\frac{\partial x^i_{1,t+1}}{\partial b^i_{1,t+1}}$$

where

$$\frac{\partial \tilde{x}^i_{1,t+1}}{\partial z^i_{1,t+1}} = \frac{\partial x^i_{1,t+1}}{\partial z^i_{1,t+1}} - MRS^{i,t+1}_{z_1,c_1}\frac{\partial x^i_{1,t+1}}{\partial b^i_{1,t+1}},$$

and where $MRS^{i,t+1}_{z_1,c_1} = (\partial u^i_{1,t+1}/\partial z^i_{1,t+1})/(\partial u^i_{1,t+1}/\partial c^i_{1,t+1})$. The first order condition for $x^i_{0,t}$ can then be written as

$$0 = \frac{\partial u^i_{0,t}}{\partial x^i_{0,t}} - \frac{\partial u^i_{0,t}}{\partial c_{0,t}}q_{x,t} + \beta\left(\Theta\frac{\partial u^i_{1,t+1}}{\partial h^i_{1,t+1}}\frac{\partial h^i_{1,t+1}}{\partial x^i_{0,t}} + \Theta^2\frac{\partial u^i_{2,t+2}}{\partial h^i_{2,t+2}}\frac{\partial h^i_{2,t+2}}{\partial x^i_{0,t}}\right) + \Omega^{i,x}_{0,t} \tag{16}$$

where[7]

$$\Omega_{0,t}^{i,x} = B\frac{\partial u_{2,t+2}^i}{\partial h_{2,t+2}^i}\frac{\partial h_{2,t+2}^i}{\partial x_{1,t+1}^i}\frac{\partial x_{1,t+1}^i}{\partial x_{0,t}^i} - B\frac{\partial u_{2,t+2}^i}{\partial h_{2,t+2}^i}\frac{\partial \tilde{h}_{2,t+2}^i}{\partial z_{1,t+1}^i}\frac{\partial l_{1,t+1}^i}{\partial x_{0,t}^i} + B\left(MRS_{c_1,c_2}^{i,t+1}\frac{\partial u_{2,t+2}^i}{\partial c_{2,t+2}^i} - \frac{\partial u_{2,t+2}^i}{\partial h_{2,t+2}^i}\frac{\partial h_{2,t+2}^i}{\partial b_{1,t+1}^i}\right)\frac{\partial s_{1,t+1}^i}{\partial x_{0,t}^i} \quad (17)$$

and where $B = (1-\beta)\Theta$. The terms in $\Omega_{0,t}^{i,x}$ arise because the young sophisticated consumer acts as a strategic leader vis-a-vis her future self. On the other hand, if the consumer is naive, $\Omega_{0,t}^{i,x}$ vanishes from the first order condition and Eq. (16) simplifies to

$$0 = \frac{\partial u_{0,t}^i}{\partial x_{0,t}^i} - \frac{\partial u_{0,t}^i}{\partial c_{0,t}^i}q_{x,t} + \beta\left(\Theta\frac{\partial u_{1,t+1}^i}{\partial h_{1,t+1}^i}\frac{\partial h_{1,t+1}^i}{\partial x_{0,t}^i} + \Theta^2\frac{\partial u_{2,t+2}^i}{\partial h_{2,t+2}^i}\frac{\partial h_{2,t+2}^i}{\partial x_{0,t}^i}\right).$$

To evaluate how sophistication (i.e. a nonzero $\Omega_{0,t}^{i,x}$) affects the choice of $x_{0,t}^i$, let us consider the special case when the instantaneous utility functions in (1) take the following form: $u = a(c-m(l)) + \phi(x) + \upsilon(h)$ where $m'(l), m''(l) > 0$. This functional form is chosen to ensure that labor supply is only a function of the marginal wage rate ω which means that the reaction function for $l_{1,t+1}^i$ in (12) is redundant, i.e. $\partial l_{1,t+1}^i/\partial x_{0,t}^i = 0$. Our choice of functional form also implies $\partial x_{1,t+1}^i/\partial x_{0,t}^i < 0$ and $\partial s_{1,t+1}^i/\partial x_{0,t}^i > 0$ so that $\Omega_{0,t}^{i,x} > 0$. This leads to the following result:

Proposition 1. *If there is no income effect on labor supply, then a young sophisticated agent will consume more of the unhealthy good than a young naive agent.*

Since this result depends on the properties of the personal reaction functions, we first need to understand why $\partial x_{1,t+1}^i/\partial x_{0,t}^i < 0$. This is a consequence of that the young sophisticated agent recognizes that her middle-aged self (due to quasi-hyperbolic discounting) will consume more of the unhealthy good than preferred by the young self. To induce her middle-aged self to consume less of this good, the agent may (on the margin) consume more of the unhealthy good when young because this has a negative effect on her health stock in the future (i.e. when middle-aged and old). With a lower health stock, the marginal disutility (in terms of reduced health) of consuming the unhealthy good when middle-aged will increase which, all else equal, works to reduce $x_{1,t+1}^i$. This also explains why $\partial s_{1,t+1}^i/\partial x_{0,t}^i > 0$: since an increase in $x_{0,t}^i$ reduces $x_{1,t+1}^i$, there is room for more saving in period $t+1$.

A key conclusion from this analysis is that a sophisticated consumer needs not necessarily have a smaller lifetime consumption $(x_{0,t}^i + x_{1,t+1}^i + x_{2,t+2}^i)$ of the unhealthy compared to a naive consumer. Rather, since the sophisticated consumer has an incentive to consume more of the unhealthy good when young, the negative health effect of this behavior is more long lasting than the negative health effect that arises because a naive agent consumes more of the unhealthy good when middle-aged (this only affects the health stock when old). These arguments suggest that the motivation for a paternalistic policy-maker to correct the consumption behavior of sophisticated agents may be as strong as the motive to correct the consumption behavior of naive agents.

Eqs. (15)–(17) imply the following conditional demand functions:

$$\eta_{0,t}^i = \eta_0(b_{0,t}^i, q_{x,t,}, z_{0,t}^i, q_{x,t+1,}) \text{ for } \eta = c, x. \quad (18)$$

With Eq. (18) at our disposal, let us then turn to the labor supply and saving decided upon by the young self. Therefore, we maximize Eq. (14) with respect to $l_{0,t}^i$ and $s_{0,t}^i$ subject to Eqs. (2), (7)–(12), (15) and (18), as well as subject to the budget constraint

$$b_{0,t}^i = w_{0,t}^i l_{0,t}^i - T_{0,t}^i - s_{0,t}^i. \quad (19)$$

By using Eq. (16), the first order conditions for work hours and saving, respectively, can be written as

$$0 = \frac{\partial u_{0,t}^i}{\partial c_{0,t}^i}\omega_{0,t}^i - \frac{\partial u_{0,t}^i}{\partial z_{0,t}^i} \quad (20)$$

$$0 = \beta\Theta\left(1+\rho_{1,t+1}^i\right)\frac{\partial u_{1,t+1}^i}{\partial c_{1,t+1}^i} - \frac{\partial u_{0,t}^i}{\partial c_{0,t}^i} + \Omega_{0,t}^{i,s} \quad (21)$$

where

$$\Omega_{0,t}^{i,s} = \beta(1-\beta)\Theta^2\frac{\partial u_{2,t+2}^i}{\partial h_{2,t+2}^i}\left(\frac{\partial h_{2,t+2}^i}{\partial b_{1,t+1}^i}(1+\rho_{1,t+1}^i) - \frac{\partial \tilde{h}_{2,t+2}^i}{\partial z_{1,t+1}^i}\frac{\partial l_{1,t+1}^i}{\partial s_{0,t}^i}\right) + \beta(1-\beta)\Theta^2\left(MRS_{c_2,c_3}^{i,t+1}\frac{\partial u_{2,t+2}^i}{\partial c_{2,t+2}^i} - \frac{\partial u_{2,t+2}^i}{\partial h_{2,t+2}^i}\frac{\partial h_{2,t+2}^i}{\partial b_{1,t+1}^i}\right)\frac{\partial s_{1,t+1}^i}{\partial s_{0,t}^i} \quad (22)$$

and where $\omega_{0,t}^i = w_{0,t}^i\left(1-\partial T_{0,t}^i/\partial y_{0,t}^i\right)$ denotes the marginal wage rate and $\rho_{1,t+1}^i = r_{t+1}\left(1-\partial T_{1,t+1}^i/\partial I_{1,t+1}^i\right)$ denotes the marginal interest rate. The labor supply condition summarized by Eq. (20) is standard whereas the first order condition for saving incorporates effects (via $\Omega_{0,t}^{i,s}$) from the personal reaction functions. By analogy to the consumption behavior examined above, if the consumer is naive $\Omega_{0,t}^{i,s}$ vanishes from the first order condition, i.e., setting $\Omega_{0,t}^{i,s} = 0$ in Eq. (21) gives the savings condition chosen by a young naive consumer.[8]

Since the sign of $\Omega_{0,t}^{i,s}$ is ambiguous it is not possible to infer whether a sophisticated agent will save more or less than a naive agent in general. However, in the special case when the instantaneous utility functions in (1) take the following form: $u = a(c-m(l))$ (i.e. the agents have no preferences for the unhealthy good which means that the health stock is exogenous and can be omitted from the utility function, and the labor supply is only a function of the marginal wage rate ω), then $\Omega_{0,t}^{i,s}$ reduces to

$$\Omega_{0,t}^{i,s} = \beta(1-\beta)\Theta^2 MRS_{c_2,c_3}^{i,t+1}\frac{\partial u_{2,t+2}^i}{\partial c_{2,t+2}^i}\frac{\partial s_{1,t+1}^i}{\partial s_{0,t}^i} > 0 \quad (23)$$

where $\partial s_{1,t+1}^i/\partial s_{0,t}^i > 0$. This leads to the following result:

Proposition 2. *If the agent has no preferences for the unhealthy good and if there is no income effect on labor supply, then a young sophisticated agent will save more than a young naive agent.*

[7] Note that $l_{1,t+1}^i$ affects $V_{0,t,\beta}^i$ directly as well as indirectly via $b_{1,t+1}^i$. This explains why the compensated derivative $\partial \tilde{x}_{1,t+1}^i/\partial z_{1,t+1}^i$ is part of the first order condition for $x_{0,t}^i$.

[8] The approach we follow when we characterize the sophisticated agents can be viewed as a discrete (and extended) analog to the approach used by Barro (1999). In his model, Barro incorporates a variable rate of time preference into the standard neoclassical growth model and the main analysis is conducted in a setting where the agent has no access to any commitment technology. Instead, the agent has to figure out how her choice of consumption at each point in time will influence the choices of consumption at later dates.

2.2. Production

Following the bulk of earlier literature on the self-selection approach to optimal taxation, we assume that the technology is linear in the sense that the producer and factor prices are fixed in each period (although not necessarily constant over time).

3. The optimal tax problem

Following earlier comparable literature, we assume that the government wants to correct for the self-control problem described in the previous section. To simplify the notation in later parts of the paper, let us reintroduce the sub-value function $V^i_{1,t} = u^i_{1,t+1} + \Theta u^i_{2,t+2}$ and then define the utility function that the government imposes on ability-type i of generation t as[9]

$$V^i_{0,t} = u^i_{0,t} + \Theta V^i_{1,t}. \tag{24}$$

Note that the discount factor β does not appear in the government's objective function, meaning that it differs from the corresponding objective function of the young consumer in Eq. (14), i.e. $V^i_{0,t,\beta} = u^i_{0,t} + \beta\Theta V^i_{1,t}$.

To simplify the analysis as much as possible, we also assume that the government uses a utilitarian social welfare function with the adjustment that the individual contribution to this social welfare function is given by Eq. (24) – instead of Eq. (14) – so that the social welfare function is written as follows[10]:

$$W = \sum_t \sum_i \Theta^t V^i_{0,t}. \tag{25}$$

The tax instruments facing the government are nonlinear taxes on labor income and capital income as well as a linear commodity tax on the unhealthy good. The resource constraint can be written as

$$\begin{aligned} 0 = &\sum_i [w^i_{0,t} l^i_{0,t} + w^i_{1,t} l^i_{1,t}] + K_t(1 + r_t) - K_{t+1} \\ &- \sum_i [c^i_{0,t} + c^i_{1,t} + c^i_{2,t} + p_{x,t}(x^i_{0,t} + x^i_{1,t} + x^i_{2,t})] \end{aligned} \tag{26}$$

for all t, where K is the capital stock.

We assume that the government can observe (labor and capital) income at the individual level, whereas ability is private information. We will concentrate on the 'normal case', where the government wants to redistribute from the high-ability to the low-ability type, in which we would like to prevent the high-ability type from pretending to be a low-ability type. This is accomplished by imposing a self-selection constraint such that the high-ability type does not prefer to become a mimicker (by reporting the same observable income as the low-ability type). However, once each consumer reveals her ability, which the consumer does at the end of the first period of life (given the appropriate self-selection constraint), the government will be able to implement income tax policies for the remaining life-cycle without consideration of mimicking. In other words, the income tax policies facing each individual during the second and third periods of life can be implemented with perfect information about individual ability.

To avoid a time-inconsistent solution, we solve the optimal tax problem sequentially. First, Eq. (25) is maximized w.r.t the variables characterizing the policies implemented for the middle-aged and old. This policy is implemented with perfect information about individual ability and the policy variables associated with this maximization problem are summarized by the following policy vector:

$$\begin{aligned} \mathbf{PV}_{t+1} = &(b^1_{1,t+1}, l^1_{1,t+1}, b^1_{2,t+2}, b^2_{1,t+1}, l^2_{1,t+1}, b^2_{2,t+2}, \\ &t_{x,t+1}, t_{x,t+2}, K_{t+2}), \end{aligned} \tag{27}$$

Since we solve the optimal tax problem via backward induction, the government's optimal policies for the middle-aged and old are conditioned on the policies implemented for the young generation. In other words, each policy variable on the right hand side of (27) can be written as a function such as[11]

$$PV_{j,t+1} = PV_{j,t+1}\left(b^1_{0,t}, l^1_{0,t}, b^2_{0,t}, l^2_{0,t}, t_{x,t}, K_{t+1}\right) \tag{28}$$

for all j, where $PV_{j,t+1}$ is the jth element of $\mathbf{PV}_{t+1}$. Eq. (28) will be used below when we derive the income tax policy implemented for the young as well as the commodity tax. The reason is that the policy reaction function provides a channel via which the government may relax the self-selection constraint.

Second, we maximize Eq. (25) w.r.t the variables characterizing the policies implemented for the young. Here, we introduce the self-selection constraint and to do this, let $\phi_{0,t} = w^1_{0,t}/w^2_{0,t}$ and $\phi_{1,t+1} = w^1_{1,t+1}/w^2_{1,t+1}$ denote the relative wage rate facing the young and middle-aged, respectively, of generation t. Recall from Section 2 that the intertemporal objective that the young ability-type i would like the middle-aged self to maximize is given by $V^i_{1,t} = u^i_{1,t+1} + \Theta u^i_{2,t+2}$, and note that we may define a corresponding objective that a young mimicker (denoted by superindex "m") would like the middle-aged self to maximize, i.e.

$$V^m_{1,t} = u^m_{1,t+1} + \Theta u^m_{2,t+2}. \tag{29}$$

In Eq. (29), we have used

$$\begin{aligned} u^m_{1,t+1} &= a(c^m_{1,t+1}, x^m_{1,t+1}, z^m_{1,t+1}) + \upsilon(h^m_{1,t+1}) \\ u^m_{2,t+2} &= a(c^m_{2,t+2}, x^m_{2,t+2}, \bar{l}) + \upsilon(h^m_{2,t+2}) \end{aligned}$$

where $z^m_{1,t+1} = \bar{l} - \phi_{1,t+1} l^1_{1,t+1}$. We can interpret $\phi_{1,t+1} l^1_{1,t+1}$ as the hours of work that the middle-aged mimicker needs to supply in order to reach the same labor income as the middle-aged low-ability type. Following the same line of argument as in Section 2.1, we can define the conditional demand functions for the mimicker

$$\eta^m_{1,t+1} = \eta^m_1(b^1_{1,t+1}, q_{x,t+1}, z^m_{1,t+1,t}, x^m_{0,t}) \text{ for } \eta = c, x \tag{30}$$

$$\eta^m_{0,t} = \eta^m_0(b^1_{0,t}, q_{x,t,}, z^m_{0,t}, q_{x,t+1}) \text{ for } \eta = c, x. \tag{31}$$

Note that the mimicker receives the same labor and capital income as the low-ability type; however, as the mimicker uses more leisure than the low-ability type, Eqs. (30) and (31) generally differ from the corresponding conditional demand functions derived for the low-ability type.[12] As a consequence, the mimicker and the low-ability type will typically have different health capital stocks when middle-aged and old, respectively. The self-selection constraint that may bind can then be written as

$$V^2_{0,t,\beta} = u^2_{0,t} + \beta\Theta V^2_{1,t} \geq V^m_{0,t,\beta} = u^m_{0,t} + \beta\Theta V^m_{1,t}, \tag{32}$$

where $u^m_{0,t} = a(c^m_{0,t}, x^m_{0,t}, z^m_{0,t}) + \upsilon(h^2_{0,t})$ and $z^m_{0,t} = \bar{l} - \phi_{0,t} l^1_{0,t}$.

It is important to bear in mind that the income tax policy implemented for each young consumer must be derived subject

[9] This means that one may view the government's objective function as coinciding with the young private agents´ long-run objective functions.

[10] This choice of social welfare function is not important for the qualitative results to be derived below.

[11] Note that the elements of PV_{t+1} are optimally chosen subject to the resource constraint defined by Eq. (26). Therefore, each element of PV_{t+1} is, in general, a function of all variables on the right hand side of Eq. (28).

[12] The special case where leisure is weakly separable from c and x in the utility function, in which the mimicker and the low-ability type would choose the same consumption bundle, will be discussed below.

to the self-selection constraint given by (32), whereas the income tax policy facing the middle-aged is implemented with perfect information about individual ability. This is also the reason why the policy reaction function presented in Eq. (28) is useful: the elements of $\mathbf{PV}_{t+1}$ affect $V^2_{1,t}$ and $V^m_{1,t}$ and, therefore, the self-selection constraint. As a consequence, $l^1_{0,t}, b^1_{0,t}, l^2_{0,t}, b^2_{0,t}$ and $t_{x,t}$ may have indirect effects on the self-selection constraint via the policy reaction function.[13]

The government's optimization problem is formally set-up and solved in the Appendix. The Lagrangian corresponding to this problem is presented in the Appendix together with the first order conditions. For the analysis to be carried out below, we define λ_t and γ_t to be the Lagrange multipliers associated with the self-selection constraint and the resource constraint in period t, respectively. We begin by characterizing the optimal commodity tax in Section 3.1. Section 3.2 analyzes the marginal labor income tax rates implemented for the middle-aged and the marginal capital income tax rates implemented for the old. In Section 3.3, we characterize the marginal labor and capital income tax structure designed to influence the labor supply and savings behavior of the young.

Finally, note that we focus on a separating equilibrium where we derive an income tax policy for each ability type.

3.1. Commodity taxation

To simplify the presentation of our results, let us begin by posing the following hypothetical question: if the government would be able to implement an individualized commodity tax on good x, what would this optimal commodity tax look like for the middle-aged and old agents of ability-type i and generation t? We show in the Appendix that the answer can be summarized by the following set of individualized commodity taxes

$$\tau^{i,x}_{1,t+1} = \frac{(\beta-1)\,\Theta}{\gamma_{t+1}}\frac{\partial u^i_{2,t+2}}{\partial h^i_{2,t+2}}\frac{\partial h^i_{2,t+2}}{\partial x^i_{1,t+1}} > 0 \tag{33}$$

$$\tau^{i,x}_{2,t+2} = 0. \tag{34}$$

We refer to Eqs. (33) and (34) as measuring *hypothetical individualized commodity taxes*, henceforth called HIC taxes, that the government attaches to the consumption of the unhealthy good by the middle-aged and old ability-type i, respectively.

The interpretation of the HIC taxes is straightforward. Since the middle-aged consumer attaches less weight to her future instantaneous utility (the instantaneous utility when old) than would be preferred by the government, the consumption of the unhealthy good will be too large from the point of view of the government. The HIC tax in Eq. (33) would correct for this behavioral failure and implies that the consumer behaves as if $\beta = 1$; in other words, if the government was able to implement this HIC tax, then the private first order condition for $x^i_{1,t+1}$ would take the same form as in the absence of quasi-hyperbolic discounting. For the old consumer, the consumption of the unhealthy good does not lead to any further deterioration of the health capital. Therefore, there is no paternalistic motive for the government to exercise control over $x^i_{2,t+2}$, which explains why $\tau^{i,x}_{2,t+2} = 0$.

We can analogously define the HIC tax for the young *sophisticated* ability-type i as

$$\tau^{i,x}_{0,t} = \frac{(\beta-1)\Theta}{\gamma_t}\left(\frac{\partial u^i_{1,t+1}}{\partial h^i_{1,t+1}}\frac{\partial h^i_{1,t+1}}{\partial x^i_{0,t}} + \Theta\frac{\partial u^i_{2,t+2}}{\partial h^i_{2,t+2}}\frac{\partial h^i_{2,t+2}}{\partial x^i_{0,t}}\right) + \frac{\Omega^{i,x}_{0,t}}{\gamma_t} \tag{35}$$

where $\Omega^{i,x}_{0,t}$ was defined in Eq. (17). If the young ability-type i is *naive*, the HIC tax simplifies to read

$$\tau^{i,x}_{0,t} = \frac{(\beta-1)\Theta}{\gamma_t}\left(\frac{\partial u^i_{1,t+1}}{\partial h^i_{1,t+1}}\frac{\partial h^i_{1,t+1}}{\partial x^i_{0,t}} + \Theta\frac{\partial u^i_{2,t+2}}{\partial h^i_{2,t+2}}\frac{\partial h^i_{2,t+2}}{\partial x^i_{0,t}}\right) > 0 \tag{36}$$

which is analogous to the corresponding HIC tax for the middle-aged ability-type i given by Eq. (33). The only difference is that the young consumer imposes an externality both on her middle-aged and old selves, whereas the middle-aged consumer only imposes an externality on her old self. In the case with naive consumers, therefore, the HIC tax solely reflects intertemporal externality correction, which is achieved by taxing good x.

Returning to the HIC tax for the young sophisticated consumer, Eq. (35) also contains the term $\Omega^{i,x}_{0,t}/\gamma_t$. If we make the same assumption as that underlying Proposition 1 above, which implies $\Omega^{i,x}_{0,t} > 0$, then the following result is immediately available:

Proposition 3. *If there is no income effect on labor supply, then the HIC tax for a young sophisticated agent will exceed the HIC tax for a young naive agent.*

To interpret Proposition 3, recall that if $\Omega^{i,x}_{0,t} > 0$ then a young sophisticated agent consumes more of the unhealthy good than a young naive agent of the same ability type. From the paternalistic government's perspective, therefore, the behavioral failure of a young sophisticated agent exceeds that of a young naive agent, and the HIC tax on the unhealthy good will thus be larger for a young sophisticated agent than for a young naive agent.

Let us now turn to the actual commodity tax implemented on good x at time t. To facilitate the presentation, let us introduce the following short notations:

$$\xi_{x,t} = \sum_i\left(\tau^{i,x}_{0,t}\frac{\partial\tilde{x}^i_{0,t}}{\partial q_{x,t}} + \tau^{i,x}_{1,t}\frac{\partial\tilde{x}^i_{1,t}}{\partial q_{x,t}}\right) \tag{37}$$

$$\frac{\partial\tilde{X}_t}{\partial q_{x,t}} = \sum_i\left(\frac{\partial\tilde{x}^i_{0,t}}{\partial q_{x,t}} + \frac{\partial\tilde{x}^i_{1,t}}{\partial q_{x,t}} + \frac{\partial\tilde{x}^i_{2,t}}{\partial q_{x,t}}\right) \tag{38}$$

$$\sigma^x_t = \left(\frac{\partial V^2_{1,t}}{\partial q_{x,t}} - \frac{\partial V^m_{1,t}}{\partial q_{x,t}}\right) + \sum_i\left(\frac{\partial V^2_{1,t}}{\partial b^i_{0,t}} - \frac{\partial V^m_{1,t}}{\partial b^i_{0,t}}\right)x^i_{0,t} \tag{39}$$

where σ^x_t reflects how $q_{x,t}$, $b^1_{0,t}$, and $b^2_{0,t}$ affect $V^2_{1,t}$ and $V^m_{1,t}$ via the policy reaction function defined by Eq. (28). These effects arise because the government's first order conditions for $\tau_{x,t}$, $b^1_{0,t}$, and $b^2_{0,t}$ (which are part of the set of conditions governing the commodity tax structure) directly affect the elements of $\mathbf{PV}_{t+1}$. Using these short notations, we can characterize the optimal commodity tax. In the Appendix we derive the following result:

Proposition 4. *If the consumers are sophisticated, the optimal commodity tax in period t can be written as*

$$\tau_{x,t} = \frac{1}{\left(\partial\tilde{X}_t/\partial q_{x,t}\right)}\left[\xi_{x,t} + \sum_i(\tau^{i,x}_{1,t+1} - \tau_{x,t+1})\frac{\partial\tilde{x}^i_{0,t}}{\partial q_{x,t}}\frac{\partial x^i_{1,t+1}}{\partial x^i_{0,t}}\right] + \frac{\lambda_t}{\gamma_t\left(\partial\tilde{X}_t/\partial q_{x,t}\right)}\left[\frac{\partial u^m_{0,t}}{\partial c^m_{0,t}}\left(x^1_{0,t} - x^m_{0,t}\right) - \sigma^x_t\right]. \tag{40}$$

[13] Since the tax structure facing the middle-aged generation is optimally chosen, all other indirect welfare effects of $l^1_{0,t}$, $b^1_{0,t}$, $l^2_{0,t}$ and $b^2_{0,t}$ via $\mathbf{PV}_{t+1}$ vanish by the Envelope Theorem.

If the consumers instead are naive, then the commodity tax formula reduces to

$$\tau_{x,t} = \frac{\xi_{x,t}}{\left(\partial \tilde{X}_t / \partial q_{x,t}\right)} + \frac{\lambda_t}{\gamma_t \left(\partial \tilde{X}_t / \partial q_{x,t}\right)} \times \left[\frac{\partial u^m_{0,t}}{\partial c^m_{0,t}} \left(x^1_{0,t} - x^m_{0,t}\right) - \sigma^x_t \right]. \tag{41}$$

To interpret Proposition 4, note first that (i) without quasi-hyperbolic discounting (in which case all terms in the first row of Eq. (40) vanish) and (ii) under full policy commitment (in which case $\sigma^x_t = 0$) both the tax formulas in Proposition 4 reduce to

$$\tau_{x,t} = \frac{\lambda_t}{\gamma_t \left(\partial \tilde{X}_t / \partial q_{x,t}\right)} \frac{\partial u^m_{0,t}}{\partial c^m_{0,t}} \left(x^1_{0,t} - x^m_{0,t}\right). \tag{42}$$

This result is well known from earlier research (see, e.g. Edwards et al., 1994) and implies that the government may relax the self-selection constraint by exploiting the difference in commodity demand between the low-ability type and the mimicker. It also means that if leisure is weakly separable from the other goods in the utility function (in which case $x^1_{0,t} = x^m_{0,t}$) then the optimal commodity tax will be zero. In the remaining discussion of commodity taxation in this subsection, we will therefore assume leisure separability, as it means that any non-zero commodity tax is entirely attributable to the features examined in this paper: no-commitment and quasi-hyperbolic discounting.[14]

For presentational convenience, consider first the effects of quasi-hyperbolic discounting under full policy commitment. If the consumers are naive, these assumptions imply that the commodity tax reduces to $\tau_{x,t} = \xi_{x,t} / \left(\partial \tilde{X}_t / \partial q_{x,t}\right) > 0$. From the definitions in (37) and (38) it follows that $\tau_{x,t}$ is a weighted average of the HIC taxes for the agent groups alive at time t. Since $\tau^{i,x}_{0,t}, \tau^{i,x}_{1,t} > 0$ while $\tau^{i,x}_{2,t} = 0$, the following result is immediately available:

Corollary 1. *Under leisure separability and full policy commitment, and if the consumers are naive, $\tau_{x,t}$ is set above the HIC tax for the old consumers but below at least one of the HIC taxes for the young and middle-aged consumers.*

This targeting problem (i.e., that the actual commodity tax satisfies $\tau_{x,t} > \tau^{i,x}_{2,t} = 0$ and most likely also satisfies $\tau_{x,t} < \tau^{i,x}_{0,t}, \tau^{i,x}_{1,t}$) reflects that the government has too few instruments available (only one commodity tax can be implemented at each point in time) compared with what is ideal from a first-best perspective (where each consumer type should face a unique commodity tax). As such, the commodity tax will not be a sufficient instrument to fully correct the decisions made by the consumers, and this implies that other policy instruments (such as the labor income tax and the capital income tax) will also be used to influence these consumption decisions. This will be discussed further below.

Let us now consider how the corrective part of the commodity tax will change if the consumers are sophisticated instead of naive. In this situation the commodity tax will be given by the expression in the first row on the right hand side (RHS) of Eq. (40). As long as the compensated demand functions are decreasing functions of the consumer price, the following result is readily available:

Corollary 2. *Suppose that the consumers are sophisticated, while the other assumptions underlying Corollary 1 still apply. If the HIC taxes for the middle-aged in period $t+1$ exceed the actual commodity tax implemented in period $t+1$, i.e. $\tau^{i,x}_{1,t+1} > \tau_{x,t+1}$ for $i = 1, 2$, then sophistication works to reduce the commodity tax in period t as long as $\partial x^i_{1,t+1} / \partial x^i_{0,t} < 0$.*

If $\tau^{i,x}_{1,t+1} > \tau_{x,t+1}$ for $i = 1, 2$, then the middle-aged agents in the next time period consume too much of the unhealthy good (from the paternalistic government's perspective). However, since the reaction function defined in Eq. (8) implies that $x^i_{1,t+1}$ is a function of $x^i_{0,t}$, and as long as $\partial x^i_{1,t+1} / \partial x^i_{0,t} < 0$, the government can indirectly induce an agent to reduce her consumption of $x^i_{1,t+1}$ by inducing her to consume more of the unhealthy good when young. This mechanism provides an incentive to set $\tau_{x,t}$ at a slightly lower level at time t and explains the result in Corollary 2.

Finally, let us also relax the assumption of policy commitment on which Corollaries 1 and 2 are based, in which case σ^x_t reappears. Then we see that without full commitment from the government there is a second channel via which the self-selection constraint affects the optimal commodity tax: the policy reaction function summarized by Eqs. (27) and (28). Since the elements of $\mathbf{PV}_{t+1}$ are functions of $q_{x,t}$ and $b^i_{0,t}$, and these elements directly affect $V^2_{1,t}$ and $V^m_{1,t}$, there is an induced effect on the self-selection constraint following a change in the commodity tax at time t.[15] This effect is summarized by the terms that appear in the definition of σ^x_t. If, for example, $\partial V^2_{1,t} / \partial q_{x,t} > \partial V^m_{1,t} / \partial q_{x,t}$, this effect means that a higher commodity tax on the unhealthy good contributes to relax the self-selection constraint via the policy reaction function, which provides an incentive for the government to implement a higher commodity tax at time t than it would otherwise have done.

3.2. Income taxation: incentives for the middle-aged

The marginal labor income tax rates for the middle-aged and the marginal capital income tax rates for the old (defined conditional on the income tax policy implemented for each consumer's young self as well as conditional on the commodity taxes) can be derived by maximizing Eq. (25) with respect to $b^i_{1,t+1}$, $l^i_{1,t+1}$, $b^i_{2,t+2}$ (for $i = 1, 2$) and K_{t+2} subject to the resource constraint, and then combining the resulting first order conditions with the private first order conditions for work hours and saving. As we showed in the previous part, linear commodity taxes do not in general constitute perfect instruments for adjusting the commodity demand in the way that the government would ideally prefer. Therefore, income taxation plays a supplemental role here: it is used, at least in part, to influence the demand for the unhealthy good. This is the issue to which we turn next.

Let $R_{t+2} = (1 + r_{t+2})$ denote the interest factor in period $t+2$. As before, we use $\tilde{x}$ to denote compensated conditional demand for commodity x. The marginal income tax structure is derived in the Appendix and presented in Proposition 5.

Proposition 5. *The optimal tax policy means that ability-type $i = 1, 2$ of generation t faces the following marginal labor income tax rate when middle-aged in period $t+1$:*

$$\frac{\partial T^i_{1,t+1}}{\partial y^i_{1,t+1}} = \frac{1}{w^i_{1,t+1}} \left(\tau_{x,t+1} - \tau^{i,x}_{1,t+1}\right) \frac{\partial \tilde{x}^i_{1,t+1}}{\partial z^i_{1,t+1}}, \tag{43}$$

[14] In terms of Eq. (1), this means that the function $a(\cdot)$ takes the form $a(c, x, z) = g(d(c, x), z)$.

[15] Note that this effect is a combined influence of $q_{x,t}$ and $b^i_{0,t}$, as the government's first order conditions for both these variables are used to derive the commodity tax.

and the following marginal capital income tax rate when old in period $t+2$:

$$\frac{\partial T^i_{2,t+2}}{\partial l^i_{2,t+2}} = \frac{\beta-1}{\beta}\frac{R_{t+2}}{r_{t+2}} - \frac{1}{r_{t+2}} MRS^{i,t}_{c_1,c_2}\tau_{x,t+2}\frac{\partial x^i_{2,t+2}}{\partial b^i_{2,t+2}} + \frac{R_{t+2}}{\beta r_{t+2}}\left(\tau_{x,t+1}-\tau^{i,x}_{1,t+1}\right)\frac{\partial x^i_{1,t+1}}{\partial b^i_{1,t+1}}. \quad (44)$$

Let us begin by emphasizing that the tax structure defined in Proposition 5 applies both to naive and to sophisticated agents. The reason is that the private first order conditions for labor supply, saving, and the consumption of good x look the same for a naive and for a sophisticated agent when they are middle-aged and old, respectively (recall from Section 2.1 that a sophisticated agent does not act strategically vis-a-vis her future self when she is middle-aged).

The expression for the marginal labor income tax rate implemented for the middle-aged ability-type i reflects the discrepancy between the actual commodity tax on good x and the corresponding HIC tax. Therefore, Proposition 5 suggests that the government uses the labor income tax because the actual commodity tax is not a perfect instrument for correction at the individual level. If $\tau_{x,t+1} < \tau^{i,x}_{1,t+1}$, the actual commodity tax underestimates the behavioral failure of the middle-aged ability type i. As a consequence, the government uses the labor income tax to further reduce the individual's consumption of the unhealthy good. This is accomplished by implementing a lower marginal labor income tax rate than it would otherwise have done if leisure is complementary with the unhealthy good ($\partial\tilde{x}^i_{1,t+1}/\partial z^i_{1,t+1} > 0$), and a higher marginal labor income tax rate if leisure is substitutable for the unhealthy good ($\partial\tilde{x}^i_{1,t+1}/\partial z^i_{1,t+1} < 0$).

The marginal capital income tax rate implemented for the old ability-type i reflects a combination of two motives for correction, both of which are due to the underlying self-control problem: (i) a desire to increase the saving and (ii) a desire to influence the consumption of the unhealthy good. The first motive is captured by the first term on the right hand side. Since $\beta < 1$, this effect contributes to decrease the marginal capital income tax rate.[16]

The second motive for using a corrective capital income tax is analogous to the motive for using labor income taxation described above, and arises because the government must implement the same set of commodity taxes for everybody. As all goods are normal by assumption, the second term in the first row contributes to decrease the marginal capital income tax rate (as long as $\tau_{x,t+2} > 0$) which, in turn, leads to an increase in $x^i_{2,t+2}$. The intuition is that the consumption pattern chosen when old does not affect the individual's health capital (in the model) – recall that the HIC tax is zero for the old consumer – which renders the commodity tax paid when old a useless instrument from the perspective of correction. Therefore, the lower marginal capital income tax serves to compensate for the distortion caused by the commodity tax paid when old. The second row is analogous to – and has the same interpretation as – the corresponding component in the expression for the marginal labor income tax rate.

3.3. Income taxation: incentives for the young

The socially optimal labor supply and disposable income combinations for the young generation in period t can be derived by maximizing the social welfare function in Eq. (25) with respect to $l^1_{0,t}$, $b^1_{0,t}$, $l^2_{0,t}$ and $b^2_{0,t}$ subject to the resource constraint and the self-selection constraint given by Eqs. (26) and (32), respectively, as well as the private reaction functions in (8), (9), (12), (18), (30) and (31), and the policy reaction function defined in Eq. (28).

3.3.1. Marginal labor income tax rates

Recall that the income tax policy implemented for each young consumer is derived subject to the self-selection constraint given by (32), whereas the income tax policy facing the middle-aged is implemented with perfect information about individual ability. This is the reason why the policy reaction function presented in Eq. (28) is useful: the elements of $\mathbf{PV}_{t+1}$ affect $V^2_{1,t}$ and $V^m_{1,t}$ and, therefore, the self-selection constraint. As a consequence, $l^1_{0,t}$, $b^1_{0,t}$, $l^2_{0,t}$ and $b^2_{0,t}$ may have indirect effects on the self-selection constraint via the policy reaction function. In the tax formulas presented below, the joint indirect effect that $l^i_{0,t}$ and $b^i_{0,t}$ has on $V^2_{1,t}$ and $V^m_{1,t}$, respectively, will be summarized by the following short notation:

$$\frac{\partial\check{V}^2_{1,t}}{\partial l^i_{0,t}} = \frac{\partial V^2_{1,t}}{\partial l^i_{0,t}} + MRS^{2,t}_{z_0,c_0}\frac{\partial V^2_{1,t}}{\partial b^i_{0,t}} \quad (45)$$

$$\frac{\partial\check{V}^m_{1,t}}{\partial l^i_{0,t}} = \frac{\partial V^m_{1,t}}{\partial l^i_{0,t}} + MRS^{m,t}_{z_0,c_0}\frac{\partial V^m_{1,t}}{\partial b^i_{0,t}} \quad (46)$$

where the *MRSs* refer to the marginal rate of substitution between leisure and private consumption of the numeraire good. By using these definitions, the marginal labor income tax rates can be presented as follows:

Proposition 6. *When consumers are sophisticated, the marginal labor income tax rates implemented for the young generation t can be written as*

$$\frac{\partial T^1_{0,t}}{\partial y^1_{0,t}} = \frac{\lambda^*_t}{w^1_{0,t}}\left(MRS^{1,t}_{z_0,c_0} - \phi_{0,t}\,MRS^{m,t}_{z_0,c_0}\right) - \frac{\lambda_t}{\gamma_t w^1_{0,t}}\left(\frac{\partial\check{V}^2_{1,t}}{\partial l^1_{0,t}} - \frac{\partial\check{V}^m_{1,t}}{\partial l^1_{0,t}}\right) + \frac{\gamma_{t+1}}{\gamma_t w^1_{0,t}}\Delta^1_{x,t}\frac{\partial\tilde{x}^1_{0,t}}{\partial z^1_{0,t}} + \frac{1}{w^1_{0,t}}\left(\tau_{x,t}-\tau^{1,x}_{0,t}\right)\frac{\partial\tilde{x}^1_{0,t}}{\partial z^1_{0,t}} \quad (47)$$

$$\frac{\partial T^2_{0,t}}{\partial y^2_{0,t}} = -\frac{\lambda_t}{\gamma_t w^2_{0,t}}\left(\frac{\partial\check{V}^2_{1,t}}{\partial l^2_{0,t}} - \frac{\partial\check{V}^m_{1,t}}{\partial l^2_{0,t}}\right) + \frac{\gamma_{t+1}}{\gamma_t w^2_{0,t}}\Delta^2_{x,t}\frac{\partial\tilde{x}^2_{0,t}}{\partial z^2_{0,t}} + \frac{1}{w^2_{0,t}}\left(\tau_{x,t}-\tau^{2,x}_{0,t}\right)\frac{\partial\tilde{x}^2_{0,t}}{\partial z^2_{0,t}} \quad (48)$$

where $\Delta^i_{x,t} = \left(\tau_{x,t+1}-\tau^{i,x}_{1,t+1}\right)\frac{\partial x^i_{1,t+1}}{\partial x^i_{0,t}}$ and $\lambda^*_t = \lambda_t(\partial u^m_{0,t}/\partial c^1_{0,t})/\gamma_t > 0$.

The tax policy in (47) and (48) also applies when the consumers are naive, with the exception that $\Delta^i_{x,t}$ vanishes from the tax formulas.

The proof of Proposition 6 is analogous to the proof of Proposition 5 (the first part) and is, therefore, omitted. The tax formulas in Proposition 6 identify three basic motives for influencing the hours of work: (i) to relax the self-selection constraint, (ii) to affect the incentives for strategic leadership facing the young consumer, and (iii) to affect the incentives for health capital accumulation facing the young consumer.

The first motive is captured by the first and second rows in Eq. (47), and the first row in Eq. (48). For the low-ability type, the first

[16] This effect has also been discussed in earlier research on quasi-hyperbolic discounting and capital income taxation (e.g. Laibson, 1997; Aronsson and Sjögren, 2014).

term on the right hand side contributes to increase the marginal labor income tax rate (since $MRS^{1,t}_{z_0,c_0} > \phi_{0,t}\, MRS^{m,t}_{z_0,c_0}$). This effect is well understood from earlier research (Stiglitz, 1982). However, in our framework, there is also a second channel via which the self-selection constraint affects the marginal labor income tax rates: the policy reaction function summarized by Eq. (28). This effect is summarized by the second term on the right hand side in the tax formula for the low-ability type and the analogous first row in the tax formula for the high-ability type. If $\partial\check{V}^2_{1,t}/\partial l^i_{0,t} - \partial\check{V}^m_{1,t}/\partial l^i_{0,t} > 0\,(<0)$, this effect means that increased work hours by the young ability-type i contributes to relax (tighten) the self-selection constraint which, in turn, provides an incentive for the government to implement a lower (higher) marginal labor income tax rate for the young ability-type i than it would otherwise have done.

The second motive for distorting the hours of work is captured by the first term in the first row in each tax formula. This component did not appear in the marginal labor income tax rate implemented for the middle-aged, as the middle-aged consumer has no incentive (in our model) to modify the behavior of her old self. A sophisticated young consumer, on the other hand, faces this particular incentive, as the young and middle-aged selves have different discount functions.

To exemplify, suppose that leisure is complementary with the unhealthy good in the sense that $\partial\tilde{x}^i_{0,t}/\partial z^i_{0,t} > 0$. In this case, if $\tau_{x,t+1} < \tau^{i,x}_{1,t+1}$ (as implied by Corollary 1) then the actual commodity tax is less distortive for the young consumer's middle-aged self than would ideally be preferred by the government. Therefore the government has an incentive decrease $x^i_{0,t}$ by implementing a lower marginal labor income tax rate for the young ability-type i than it would otherwise have done. As explained above, this policy incentive is absent under naivety.

The third motive for distorting the hours of work is summarized by the second term in the final row in each tax formula, which takes the same general form as in the expression for the marginal labor income tax rate implemented for the middle-aged consumer presented in Proposition 5. Therefore, the interpretation that we presented in the context of Proposition 5 applies here as well.

3.3.2. Marginal capital income tax rates

To simplify the presentation and interpretation of the results, let us begin by examining the marginal capital income tax rates that would be implemented in the hypothetical special case where (i) the self-selection constraint does not bind, and (ii) the government is able to implement HIC taxes to perfectly control the commodity demand behavior at the individual level. This results in a pure savings subsidy that only serves to affect the direct incentives for the young consumer to accumulate financial wealth; it neither reflects the desire to relax the self-selection constraint nor the incentive to supplement the (non-individualized) commodity taxes by corrective income taxation. In our model, this pure savings subsidy would take the following form for a sophisticated consumer of ability-type i ($i = 1, 2$):

$$\Gamma^i_{1,t+1} = \frac{(\beta-1)}{\beta}\frac{R_{t+1}}{r_{t+1}} + \frac{1}{\beta r_{t+1}}\frac{\Omega^{i,s}_{0,t}}{\alpha^i_{t+1}} \tag{49}$$

where $\alpha^i_{t+1} = \Theta(\partial u^i_{1,t+1}/\partial c^i_{1,t+1})$ and where $\Omega^{i,s}_{0,t}$ was defined in Eq. (22). If the consumers instead are naive, then Eq. (49) reduces to $\Gamma^i_{1,t+1} = (\beta-1)R_{t+1}/(\beta r_{t+1}) < 0$ which means that the government subsidizes saving because the young consumer discounts the future utility at a higher rate than would be preferred by the government, ceteris paribus. The second term on the RHS of Eq. (49) is due to sophistication and by making the same assumptions as those underlying Proposition 2, the following result is readily available:

Proposition 7. *Suppose that* (1) *the self-selection constraint does not bind, and* (2) *the government is able to implement HIC taxes to perfectly control the commodity demand behavior. If the individuals have no preferences for the unhealthy good and there is no income effect on the labor supply, then* $\Omega^{i,s}_{0,t} > 0$ *such that the saving subsidy is larger if the consumers are naive than if they are sophisticated.*

The intuition is that, given the assumptions underlying Proposition 7, a young sophisticated consumer saves more than a young naive consumer. As a consequence, the paternalistic government does not need to subsidize saving as much if the consumers are sophisticated as it must do if they are naive.

We are now in the position to analyze the marginal capital income tax rates implemented for the young generation t (and paid by each consumer's middle-aged self). By relaxing the simplifying assumptions on which Proposition 7 is based, our results are summarized as follows:

Proposition 8. *When consumers are sophisticated, the marginal capital income tax rates implemented for the middle-aged generation* t *can be written as*

$$\frac{\partial T^1_{1,t+1}}{\partial I^1_{1,t+1}} = \Gamma^1_{1,t+1} + \frac{\lambda_t}{\beta r_{t+1}\gamma_{t+1}}\left[\left\{\frac{\partial V^2_{1,t}}{\partial b^1_{0,t}} - \frac{\partial V^m_{1,t}}{\partial b^1_{0,t}}\right\} - \frac{\partial u^m_{0,t}}{\partial c^m_{0,t}}\right] + \Psi^1_t \tag{50}$$

$$\frac{\partial T^2_{1,t+1}}{\partial I^2_{1,t+1}} = \Gamma^2_{1,t+1} + \frac{\lambda_t}{\beta r_{t+1}\gamma_{t+1}}\left[\left\{\frac{\partial V^2_{1,t}}{\partial b^2_{0,t}} - \frac{\partial V^m_{1,t}}{\partial b^2_{0,t}}\right\} + \frac{\partial u^2_{0,t}}{\partial c^2_{0,t}}\right] + \Psi^2_t \tag{51}$$

where (for $i = 1, 2$*)*

$$\Psi^i_t = \left[(1+r_{t+1})\left(\tau_{x,t} - \tau^{i,x}_{0,t}\right) + \Delta^i_{x,t}\right]\frac{1}{\beta r_{t+1}}\frac{\partial x^i_{0,t}}{\partial b^i_{0,t}} - \left(t_{x,t+1} - \tau^{i,x}_{1,t+1}\right)\frac{1}{r_{t+1}} MRS^{i,t}_{c_0,c_1}\frac{\partial x^i_{1,t+1}}{\partial b^i_{1,t+1}}.$$

The tax policy in (50) *and* (51) *also applies when the consumers are naive, with the exception that* $\Delta^i_{x,t}$ *vanishes from the tax formulas.*

The proof is analogous to the proof of Proposition 5 (the second part) and is, therefore, omitted. In Proposition 8, the marginal capital income tax rate facing the young ability-type i (and paid by her middle-aged self) comprises three parts. The first, $\Gamma^i_{1,t+1}$, is the pure savings subsidy defined in Eq. (49) above, which disregards the desire to relax the self-selection constraint as well as assumes that the government is able to implement HIC taxes in order to adjust the commodity demand behavior at the individual level. Therefore, $\Gamma^i_{1,t+1}$ represents in a sense a first best corrective capital income subsidy (with analogies in earlier comparable literature on savings and quasi-hyperbolic discounting, e.g., Laibson, 1996).

The second part of each tax formula, i.e. the terms proportional to λ_t, reflects the effects caused by the self-selection constraint. The component in curly brackets is analogous to its counterpart in the expression for the marginal labor income tax rate presented in Proposition 6, and appears as a direct consequence of the policy reaction function in Eq. (28): as the elements of $\mathbf{PV}_{t+1}$ are functions of $b^i_{0,t}$, and as $\mathbf{PV}_{t+1}$ directly affects $V^2_{1,t}$ and $V^m_{1,t}$, there is an induced effect on the self-selection constraint following a change in $b^i_{0,t}$. If $\partial V^2_{1,t}/\partial b^i_{0,t} - \partial V^m_{1,t}/\partial b^i_{0,t} > 0$, this effect means that an increase in the disposable income (net of savings) for the young ability-type i, ceteris paribus, contributes to relax the self-selection constraint which, in turn, provides an incentive for the government to discourage savings by implementing a higher marginal capital income tax rate than it would otherwise have done.

The other effect associated with the self-selection constraint, represented by the final term within the square bracket, differs between ability-types. For the low-ability type, this component is in a sense an analogue to the result that the marginal capital income tax rate depends on the difference between $MRS^{1,t}_{c_0,c_1}$ and $MRS^{m,t}_{c_0,c_1}$ (e.g. Pirttilä and Tuomala, 2001; Aronsson et al., 2009), which the earlier literature derived under the assumption that the government commits to a policy decided upon before the individual abilities are revealed. Contrary to this earlier literature, however, we consider a time-consistent tax policy meaning, in particular, that the government's optimal choice of $b^1_{1,t+1}$ is not restricted by the self-selection constraint (as it is decided upon when all consumers of generation t have revealed their abilities). The government's choice of $b^1_{0,t}$, on the other hand, is restricted by the self-selection constraint; therefore, as an increase in $b^1_{0,t}$ leads to increased utility for the mimicker, ceteris paribus, it also contributes to tighten the self-selection constraint. This constitutes an incentive for the government to implement a smaller $b^1_{0,t}$ than it would otherwise have done, which explains why the final term in the square-bracket leads to a lower marginal capital income tax rate for the low-ability type. The argument for why the corresponding component is positive in the tax formula for the high-ability type is analogous: an increase in $b^2_{0,t}$ increases the utility of the high-ability type with the utility of the mimicker held constant.[17]

The final part of each tax formula, Ψ^i_t, appears because the government lacks the option of using individualized commodity taxes. As explained above, this means that the government uses income taxation as an indirect instrument for influencing the accumulation of health capital via the commodity demand functions. This mechanism is relevant also for capital income taxation, because $b^i_{0,t}$ and $b^i_{1,t+1}$ directly affect the choice of commodity x made by the young and middle-aged ability-type i, respectively. The intuition behind each component in the expression for Ψ^i_t is the same as that behind the corresponding effects included in the marginal labor income tax rate: the discrepancy between a commodity tax and the corresponding HIC tax shows whether this commodity tax is too high or two low for ability-type i – from the point of view of correcting for the self-control problem – and explains, therefore, also how the government uses the income tax to counteract the remaining distortion.

4. Summary and discussion

This paper develops an OLG model with endogenous labor supply, savings, and health capital, where the consumers suffer from a self-control problem generated by quasi-hyperbolic discounting. The health capital is assumed to deteriorate through the consumption of an unhealthy good. Consumers live for three periods, and differ in earnings ability (which is private information not known before-hand by the government). The purpose of the paper is to analyze how a paternalistic government, which attempts to correct for the self-control problem, ought to use a mix of nonlinear income taxes and a linear commodity tax on the unhealthy good.

Our study makes a distinction between naive and sophisticated consumers, and we show that the paternalistic government will under certain conditions implement a higher corrective tax on the unhealthy good if the consumers are sophisticated than if they are naive, which may seem surprising at first thought. The intuition is that sophisticated agents tend to consume more of the unhealthy good when young than naive agents. Furthermore, and irrespective of whether the consumers are naive or sophisticated, the commodity tax is not a flexible enough instrument to adjust the consumption of the unhealthy good in the way ideally preferred by the government. This is important not least because some earlier studies have analyzed commodity taxes on unhealthy goods (e.g., sin taxes) as means of correcting for self-control problems. More specifically, we find that the optimal linear commodity tax is too high for the old and most likely too low for the young and middle-aged compared to a system of hypothetical individualized commodity taxes (that would allow the government to exercise perfect control over the consumption of the unhealthy good by each ability-type and age-group). In turn, this means that the income taxes are used as supplemental instruments for influencing the consumption of the unhealthy good. The major contribution of the present paper is that it explains how the optimal marginal income tax rates are related to the discrepancy between the actual and ideal (individualized) commodity taxes.

In accordance with some recent (albeit contrary to most earlier) studies on optimal taxation in dynamic economies, we assume that the government decides on a time-consistent tax policy. As such, it recognizes that each individual reveals his/her true ability during the first period of life (if subject to a relevant incentive constraint) and, as a consequence, that the income tax policies for the second and third periods of the consumer's life can be implemented with perfect information about individual ability. By comparison with the time-inconsistent commitment solution, this gives rise to an additional channel through which the self-selection constraint affects the optimal tax policy.

Appendix

Taxation of the Middle-Aged and Old

For computational convenience, let us slightly rewrite the resource constraint by using $p_{x,t} = q_{x,t} - \tau_{x,t}$ and then substituting Eqs. (7) and (15) into Eq. (26). To avoid unnecessary notation, we shall here focus on variables relevant for the middle-aged in period $t+1$ and write the Lagrangian of the government's decision-problem as follows (by suppressing irrelevant terms):

$$\begin{aligned}
\mathcal{L}_{t+1} = \cdots &+ \sum_{i=1,2}\left[u^i_{1,t+1} + \Theta u^i_{2,t+2}\right] \\
&+ \breve{\gamma}_{t+1}\sum_{i=1,2}\left[+w^i_{1,t+1}l^i_{1,t+1} + \tau_{x,t+1}x^i_{1,t+1} - K_{t+1} - b^i_{1,t+1} +\right] \\
&+ \breve{\gamma}_{t+2}\sum_{i=1,2}\left[\ldots + \tau_{x,t+2}x^i_{2,t+2} + (1+r_{t+1})K_{t+1} - b^i_{2,t+2} + \cdots\right] \\
&+ \sum_{i=1,2}\breve{\mu}^i_{1,t+1}\left[x^i_{1,t+1} - x^i_{1,t+1}\left(b^i_{1,t+1}, q_{x,t+1}, l^i_{1,t+1}, x^i_{0,t}\right)\right] + \cdots \\
&+ \sum_{i=1,2}^{i}\breve{\mu}^i_{2,t+2}\left[x^i_{2,t+2} - x^i_{2,t+2}\left(b^i_{2,t+2}, q_{x,t+2}\right)\right] + \cdots . \qquad (A.1)
\end{aligned}$$

where the symbol "$\smile$" above the shadow prices indicates that they are expressed in present value terms as of time $t+1$. Note that we (instead of substituting the conditional commodity demand function into the objective function) have followed the equivalent approach of introducing the conditional commodity demand function as a separate restriction, which explains the fourth and fifth rows of Eq. (A.1). As a consequence, $x^i_{1,t+1}$ and $x^i_{2,t+2}$ will also be treated as (artificial) decision-variables for the

[17] This effect would vanish in the standard model with commitment. The reason is that the government's choices of $b^2_{0,t}$ and $b^2_{1,t+1}$ are, in this case, restricted by the self-selection constraint in the same way, meaning that the marginal rate of substitution between them will not depend directly on λ_t.

government in what follows. The associated Lagrange multipliers are $\breve{\mu}^i_{1,t+1}$ and $\breve{\mu}^i_{2,t+2}$.

Let us begin with the HIC taxes for the middle-aged and old. These can be derived by replacing $\tau_{x,t+1}$ with $\tau^{i,x}_{1,t+1}$, replacing $q_{x,t+1}$ with $q^{i,x}_{1,t+1}$ and by replacing $\tau_{x,t+2}$ with $\tau^{i,x}_{2,t+2}$ for $i = 1, 2$. The first order conditions for $\tau^{i,x}_{1,t+1}$, $x^i_{1,t+1}$, $b^i_{1,t+1}$, $l^i_{1,t+1}$ (for $i = 1, 2$) and K_{t+2} can then be written as

$$0 = \left(\breve{\gamma}_{t+1} - \frac{\partial u^i_{1,t+1}}{\partial c^i_{1,t+1}}\right) x^i_{1,t+1} + \breve{\mu}^i_{1,t+1} \frac{\partial x^i_{1,t+1}}{\partial q^{i,x}_{1,t+1}} \tag{A.2}$$

$$0 = (1-\beta)\,\Theta \frac{\partial u^i_{2,t+2}}{\partial h^i_{2,t+2}} \frac{\partial h^i_{2,t+1}}{\partial x^i_{1,t+1}} + \breve{\gamma}_{t+1}\tau^{i,x}_{1,t+1} + \breve{\mu}^i_{1,t+1} \tag{A.3}$$

$$0 = \frac{\partial u^i_{1,t+1}}{\partial c^i_{1,t+1}} - \breve{\gamma}_{t+1} - \breve{\mu}^i_{1,t+1} \frac{\partial x^i_{1,t+1}}{\partial b^i_{1,t+1}} \tag{A.4}$$

$$0 = \breve{\gamma}_{t+1} w^i_{1,t+1} - \frac{\partial u^i_{1,t+1}}{\partial z^i_{1,t+1}} + \breve{\mu}^i_{1,t+1} \frac{\partial x^i_{1,t+1}}{\partial z^i_{1,t+1}} \tag{A.5}$$

$$0 = \breve{\gamma}_{t+2}(1 + r_{t+2}) - \breve{\gamma}_{t+1}. \tag{A.6}$$

where we have used the private first order condition for $x^i_{1,t+1}$ to simplify (A.3).

To derive the HIC tax in Eq. (33), let us first solve for $\breve{\gamma}_{t+1}$ in (A.4) and use the resulting expression to replace $\breve{\gamma}_{t+1}$ in (A.2). This produces $\breve{\mu}^i_{1,t+1} \frac{\partial \tilde{x}^i_{1,t+1}}{\partial q^{i,x}_{1,t+1}} = 0$ and since $\frac{\partial \tilde{x}^i_{1,t+1}}{\partial q^{i,x}_{1,t+1}} \neq 0$, it follows that $\breve{\mu}^i_{1,t+1} = 0$. Using the latter result in (A.3) and then solving for $\tau^{i,x}_{1,t+1}$ produces the HIC tax in Eq. (33). Using an analogous approach produces the HIC tax in (34).

To derive the marginal labor income tax rate implemented for the middle-aged ability type i in the absence of HIC taxes, which means that we replace $\tau^{i,x}_{1,t+1}$ with $\tau_{x,t+1}$ in the government's first order conditions, use Eq. (A.3) to solve for $\breve{\mu}^{i,x}_{1,t+1}$ and then substitute the resulting expression into the first order conditions for $b^i_{1,t+1}$ and $l^i_{1,t+1}$ given by Eqs. (A.4) and (A.5). This gives

$$\frac{\partial u^i_{1,t+1}}{\partial c^i_{1,t+1}} = \breve{\gamma}_{t+1} - \breve{\gamma}_{t+1}\left(\tau_{x,t+1} - \tau^{i,x}_{1,t+1}\right) \frac{\partial x^i_{1,t+1}}{\partial b^i_{1,t+1}} \tag{A.7}$$

$$\frac{\partial u^i_{1,t+1}}{\partial z^i_{1,t+1}} = \breve{\gamma}_{t+1} w^i_{1,t+1} - \breve{\gamma}_{t+1}\left(\tau_{x,t+1} - \tau^{i,x}_{1,t+1}\right) \frac{\partial x^i_{1,t+1}}{\partial z^i_{1,t+1}} \tag{A.8}$$

in which we have used the HIC tax given by Eq. (33). Next, divide Eq. (A.8) by Eq. (A.7)

$$MRS^{i,t+1}_{z_1,c_1}\left[1 - \left(\tau_{x,t+1} - \tau^{i,x}_{1,t+1}\right)\frac{\partial x^i_{1,t+1}}{\partial b^i_{1,t+1}}\right]$$
$$= w^i_{1,t+1} - \left(\tau_{x,t+1} - \tau^{i,x}_{1,t+1}\right)\frac{\partial x^i_{1,t+1}}{\partial z^i_{1,t+1}}. \tag{A.9}$$

Finally, by using the Slutsky-type equation

$$\frac{\partial \tilde{x}^i_{1,t+1}}{\partial z^i_{1,t+1}} = \frac{\partial x^i_{1,t+1}}{\partial z^i_{1,t+1}} - MRS^{i,t+1}_{z_1,c_1}\frac{\partial x^i_{1,t+1}}{\partial b^i_{1,t+1}}$$

and by using the private first order condition for the hours of work, $w^i_{1,t+1}\partial T^i_{1,t+1}/\partial y^i_{1,t+1} = w^i_{1,t+1} - MRS^{i,t+1}_{z_1,c_1}$, we obtain the expression for the marginal labor income tax rate implemented for ability-type i in Proposition 5.

To derive the marginal capital income tax rate in Proposition 5, note that the government's first order conditions for $b^i_{2,t+2}$ and $x^i_{2,t+2}$ can be written as

$$0 = \Theta\frac{\partial u^i_{2,t+2}}{\partial c^i_{2,t+2}} - \breve{\gamma}_{t+2} - \breve{\mu}^i_{2,t+2}\frac{\partial x^i_{2,t+2}}{\partial b^i_{2,t+2}} \tag{A.10}$$

$$0 = \breve{\gamma}_{t+2}\tau_{x,t+2} + \breve{\mu}^i_{2,t+2}. \tag{A.11}$$

Substituting (from (A.11)) $\mu^i_{2,t+2} = -\gamma_{t+2}\tau_{x,t+2}$ into (A.10) and then combining the resulting expression with (A.7) while using that $\gamma_{t+1} = \gamma_{t+2}(1 + r_{t+2})$ produces

$$\beta MRS^{i,t+1}_{c_1,c_2} = \frac{(1 + r_{t+2})}{1 - \tau_{x,t+2}\partial x^i_{2,t+2}/\partial b^i_{2,t+2}}$$
$$\times \left[1 - \left(\tau_{x,t+1} - \tau^{i,x}_{1,t+1}\right)\frac{\partial x^i_{1,t+1}}{\partial b^i_{1,t+1}}\right]. \tag{A.12}$$

By using the private first order condition for saving, we can derive the expression for the marginal capital income tax rate implemented for ability-type i in Proposition 5.

Taxation of the Young

By focusing on variables relevant for the young generation in period t, and to save notational space, we write the Lagrangian as follows (by suppressing irrelevant terms):

$$\begin{aligned}
\mathcal{L}_t = {} & \cdots + \sum_{i=1,2}\left[u^i_{0,t} + \Theta u^i_{1,t+1} + \Theta^2 u^i_{2,t+2}\right] + \cdots \\
& + \lambda_t\left[\left(u^2_{0,t} + \beta\Theta u^2_{1,t+1} + \beta\Theta^2 u^2_{2,t+2}\right)\right. \\
& - \left.\left(u^m_{0,t} + \beta\Theta u^m_{1,t+1} + \beta\Theta^2 u^m_{2,t+2}\right)\right] \\
& + \gamma_t \sum_{i=1,2}\left[\cdots + w^i_{0,t} l^i_{0,t} - K_t - b^i_{0,t} + \tau_{x,t} x^i_{0,t} + \cdots\right] \\
& + \gamma_{t+1}\sum_{i=1,2}\left[(1 + r_{t+1}) K_t + \cdots\right] \\
& + \sum_{i=1,2}\mu^{i,x}_{0,t}\left[x^i_{0,t} - x^i_{0,t}\left(b^i_{0,t}, q_{x,t}, l^i_{0,t}\right)\right] \\
& + \sum_{i=1,2}\mu^{i,x}_{1,t+1}\left[x^i_{1,t+1} - x^i_{1,t+1}\left(b^i_{1,t+1}, q_{x,t+1}, l^i_{1,t+1}, x^i_{0,t}\right)\right] \\
& + \cdots.
\end{aligned} \tag{A.13}$$

Note that Eq. (A.13) incorporates the policy reaction function given by Eq. (28). The first order conditions can be written as (where we use the private first order condition for the unhealthy good to simplify the government's first order conditions for that good)

$$\begin{aligned}
\frac{\partial \mathcal{L}_t}{\partial x^1_{0,t}} &= 0 = \frac{\partial u^1_{0,t}}{\partial x^1_{0,t}} - \frac{\partial u^1_{0,t}}{\partial c^1_{0,t}} q_{x,t} + \Theta\frac{\partial u^1_{1,t+1}}{\partial h^1_{1,t+1}}\frac{\partial h^1_{1,t+1}}{\partial x^1_{0,t}} \\
&\quad + \Theta^2\frac{\partial u^1_{2,t+2}}{\partial h^1_{2,t+2}}\frac{\partial h^1_{2,t+2}}{\partial x^1_{0,t}} + \gamma_t\tau_{x,t} + \mu^{1,x}_{0,t} - \mu^{1,x}_{1,t+1}\frac{\partial x^1_{1,t+1}}{\partial x^1_{0,t}} \\
\frac{\partial \mathcal{L}_t}{\partial b^1_{0,t}} &= 0 = \frac{\partial u^1_{0,t}}{\partial c^1_{0,t}} - \lambda_t\frac{\partial u^m_{0,t}}{\partial c^1_{0,t}} - \gamma_t - \mu^{1,x}_{0,t}\frac{\partial x^1_{0,t}}{\partial b^1_{0,t}} \\
&\quad + \lambda_t\left(\frac{\partial V^2_{1,t}}{\partial b^1_{0,t}} - \frac{\partial V^m_{1,t}}{\partial b^1_{0,t}}\right) \\
\frac{\partial \mathcal{L}_t}{\partial l^1_{0,t}} &= 0 = -\frac{\partial u^1_{0,t}}{\partial z^1_{0,t}} + \lambda_t\phi_{0,t}\frac{\partial u^m_{0,t}}{\partial z^m_{0,t}} + \gamma_t w^1_{0,t} + \mu^{1,x}_{0,t}\frac{\partial x^1_{0,t}}{\partial z^1_{0,t}} \\
&\quad + \lambda_t\left(\frac{\partial V^2_{1,t}}{\partial l^1_{0,t}} - \frac{\partial V^m_{1,t}}{\partial l^1_{0,t}}\right) \\
\frac{\partial \mathcal{L}_t}{\partial K_{t+1}} &= 0 = \gamma_{t+1}(1 + r_{t+1}) - \gamma_t + \lambda_t\left(\frac{\partial V^2_{1,t}}{\partial K_{t+1}} - \frac{\partial V^m_{1,t}}{\partial K_{t+1}}\right)
\end{aligned}$$

$$\frac{\partial \mathcal{L}_t}{\partial x_{0,t}^2} = 0 = (1+\beta\lambda_t)\left(\Theta \frac{\partial u_{1,t+1}^i}{\partial h_{1,t+1}^i}\frac{\partial h_{1,t+1}^i}{\partial x_{0,t}^i} + \Theta^2 \frac{\partial u_{2,t+2}^i}{\partial h_{2,t+2}^i}\frac{\partial h_{2,t+2}^i}{\partial x_{0,t}^i}\right) + (1+\lambda_t)\left(\frac{\partial u_{0,t}^2}{\partial x_{0,t}^2} - \frac{\partial u_{0,t}^2}{\partial c_{0,t}^2}q_{x,t}\right) + \gamma_t \tau_{x,t} + \mu_{0,t}^{2,x} - \mu_{1,t+1}^{2,x}\frac{\partial x_{1,t+1}^2}{\partial x_{0,t}^2}$$

$$\frac{\partial \mathcal{L}_t}{\partial b_{0,t}^2} = 0 = (1+\lambda_t)\frac{\partial u_{0,t}^2}{\partial c_{0,t}^2} - \gamma_t - \mu_{0,t}^{2,x}\frac{\partial x_{0,t}^2}{\partial b_{0,t}^2} + \lambda_t\left(\frac{\partial V_{1,t}^2}{\partial b_{0,t}^2} - \frac{\partial V_{1,t}^m}{\partial b_{0,t}^2}\right)$$

$$\frac{\partial \mathcal{L}_t}{\partial l_{0,t}^2} = 0 = -(1+\lambda_t)\frac{\partial u_{0,t}^2}{\partial z_{0,t}^2} + \gamma_t w_{0,t}^2 + \mu_{0,t}^{2,x}\frac{\partial x_{0,t}^2}{\partial z_{0,t}^2} + \lambda_t\left(\frac{\partial V_{1,t}^2}{\partial l_{0,t}^2} - \frac{\partial V_{1,t}^m}{\partial l_{0,t}^2}\right)$$

$$\frac{\partial \mathcal{L}_t}{\partial t_{x,t}} = 0 = -x_{0,t}^1\frac{\partial u_{0,t}^1}{\partial c_{0,t}^1} - (1+\lambda_t)x_{0,t}^2\frac{\partial u_{0,t}^2}{\partial c_{0,t}^2} - \sum_{i=1,2} x_{1,t}^i\frac{\partial u_{1,t}^i}{\partial c_{1,t}^i} - \sum_{i=1,2} x_{2,t}^i\frac{\partial u_{2,t}^i}{\partial c_{2,t}^i} + \lambda_t\left(x_{0,t}^m\frac{\partial u_{0,t}^m}{\partial c_{0,t}^m} + \frac{\partial V_{1,t}^2}{\partial q_{x,t}} - \frac{\partial V_{1,t}^m}{\partial q_{x,t}}\right) - \sum_{i=1,2}\left[\mu_{0,t}^{i,x}\frac{\partial x_{0,t}^i}{\partial q_{x,t}} + \mu_{1,t}^{i,x}\frac{\partial x_{1,t}^i}{\partial q_{x,t}}\right] + \gamma_t\left[\sum_{i=1,2}x_{0,t}^i + \sum_{i=1,2}x_{1,t}^i + \sum_{i=1,2}x_{2,t}^i\right]$$

where all shadow prices are expressed in present value terms as of time t. The expressions for the HIC taxes and the marginal income tax rates for the young generation can be derived in the same general way as the corresponding expressions for the middle-aged.

Finally, to derive Eq. (40), use (A.3) together with the first order conditions w.r.t. $b_{0,t}^1$ and $b_{0,t}^2$ to solve for $\partial u_{0,t}^i/\partial c_{0,t}^i$, $\partial u_{1,t}^i/\partial c_{1,t}^i$ and $\partial u_{2,t}^i/\partial c_{2,t}^i$, respectively, and substitute the resulting expressions into $\frac{\partial \mathcal{L}_t}{\partial t_{x,t}} = 0$. Then, use the definitions of the HIC taxes and solve for $t_{x,t}$. This produces the tax formula in Proposition 4.

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