Hello everyone. I wanted to thank the organisers for allowing me to talk today. I'm Sergi, a PhD student at the ICG in Portsmouth, and in this talk I'll try to show you a cartoon picture of one way in which we can use gravitational waves to search for deviations from GR and therefore prove the fundamental nature of gravity in the strong regime.

I will mostly talk about some work I did in collaboration with Johannes Noller in this paper, but if I have time I'll also talk about some things I've been recently interested in.

Ok , so why do we want to test GR? I've listed here some of the commonly known issues in GR like the nature of dark energy, the existence of singularities, etcetera. But besides all of these, because GR is the most successful gravitational theory to date, we, as good scientists, should be trying our best to falsify it, even if we didn't have any of the intrinsic motivations above, hence why I also included "why not?".

To test GR, we first need to understand GR. Lovelock's theorem tells us that if we want to have a theory in 4 dimensions of a massless spin-2 field, the metric, which is local and gives second order equations of motion, GR is the most general thing you can write down, where here you can also add a cosmological constant. If we want to go beyond GR we have to modify at least one of these assumptions. If we choose to alter the field content of the theory, then the simplest thing one can add is a scalar field. Then, the most general theory we can write down that still satisfies the other assumptions is Horndeski gravity, which I've written here as this H. Just for reference, this is how it looks like in its full glory. In particular, note that these G functions are unspecified functions of the scalar field and its kinetic term X, so this really represents a big family of scalar tensor theories, which makes Horndeski a powerful, well-defined and well-motivated framework to test gravity.

Horndeski gravity is 50 years old this year, and in an anniversary review a couple of weeks ago Horndeski himself said that he and Lovelock wondered who would be crazy enough to work with such equations, and well now people even go beyond Horndeski.

Ok, so the goal is to connect the theory to an observable, ideally encoded in a parameter which vanishes for GR but becomes non-zero once we turn on these extra interactions. This is what is sometimes called a smoking-gun signal, because measuring it to be non-zero directly tells us we've hit something beyond GR.

Right, so with this motivation we can now move to gravitational waves. We know that in GR small perturbations on the spacetime propagate as waves travelling at the speed of light. Here we focus on gravitational waves coming from astrophysical sources, of the type that LVK measures. The signal for such events look schematically like this. In particular, we'll focus on the ringdown, which is this very last part after the objects have merged and the remaining object is ringing down to a settled state.

The tool we have to study this is black hole perturbation theory. Here, the idea is that we can approximate the spacetime as a black hole background, so here you can choose your favourite black hole solution and we add some small perturbations around it. Due to the spherical symmetry of the problem, perturbations separate into two sectors, the odd and the even sectors, both of which evolve independently of each other at linear order. Putting this choice for the spacetime in the action to quadratic order, we can study the linear evolution of perturbations. After several technical procedures we can arrive to this Schrodinger-like equations, and we have one for each sector. For the odd sector this is the Regge-Wheeler equation and for the even sector it's the Zerilli equation. After putting the relevant boundary conditions, the solutions to these differential equations give you the values for this omega, which are the quasinormal modes. These are complex numbers where the real part tells you about the oscillations in frequency and the imaginary part about the damping time for each mode, so how quickly it decays. In GR, as a consequence of the no hair theorem, these QNMs are fully given by the asymptotic black hole parameters, the mass and the angular momentum.

This neat result allows us to do something called black hole spectroscopy. This is a very oversimplified picture but here’s the idea, We measure the dominant QNM and we use it as a measurement of the mass and angular momentum, because it has the real and imaginary components. Then, in GR all the spectrum is fixed, so measuring the next mode would allow us to test if the prediction is right or not. The key take-home message is that measuring QNMs provides very clean tests of the background geometry and the underlying theory. For instance, in theories beyond GR QNMs could also depend on extra parameters and the spectrum could be shifted.

As a specific example, in Horndeski the Regge-Wheeler equations looks like this, where this alpha_T refers to the difference between the speed of gravitational waves compared to the speed of light, so this is our smoking-gun signal.

But what do we know about this aloha_T so far? WE know that it is constrained by the LVK detection of a neutron star binary to ten to the minus fifteen, so very small. However, we also have reasons to believe that things might look different at the LISA band. This is because these scalar-tensor theories, when meant as EFT descriptions of dark energy, come with a naïve cutoff scale that sits exactly on the LVK band. This means that the largest energy scale at which this theory is valid is exactly on the frequencies measured by LVK. So at that point one can no longer trust the theory, and this can in principle happen at several orders of magnitude lower. In the LISA band, we don’t really have any information yet on alpha_T so it is interesting to see how much we will be able to constrain it there. We’ve found that with just one loud supermassive BH event we will be able to constrain it to ten to minus 4, which will improve with the number of such detections which are expected to be between 10 to 100 yearly.

Let me give you a bit more background of the result. All the calculations are openly available and reproducible in a Mathematica notebook in this repository. So for our background spacetime we have Schwarzschild and for the background scalar we have a constant and then some small radial hair, which we parametrise like this. Then alpha_T appears in the Regge-Wheeler equation like this. And it’s interesting to point out this expression where we see that without this radial hair alpha_T would be zero, so we need hairy black holes to have a non-zero alpha_T> In this plot you can see the strength of the constraints on this G_T, which is given by this combination, for different values of the SNR and the dimensionless scalar amplitude.

In the last 5 minutes I wanted to talk about some work that we’ve been doing related to the time dependent solutions that Vicharit introduced earlier. As a quick recap, the characterising feature of these solutions is that the scalar field has a linearly time dependence. They have been found to be exact solutions of the subset of Horndeski which satisfies shift and reflection symmetry. However, in a number of papers they have been shown to generically be prone to instabilities. This is why going the EFT route as Vicharit explained is a good thing to do. However, one assumption that all the cases studied share is that the background kinetic term X is always a constant. Recently, a solution of this form was found for which X is not a constant. So we ask? Can this help in resolving the instabilities? The case that was found involves these terms in the lagrangian, where we have the square root of X.

And so, this is the set up that we wanted to test. This book by Yoko Ogawa stars with a tribute to the square root and among other things it says that the square root is a generous symbol, it gives shelter to all the numbers. So lets see if it also gives shelter to the stability of these solutions. The preliminary results I can show you know, which I stress are preliminary, is that odd parity modes are stable as shown in this plot. This is all a stable region. And I also show here the plot for X, which is no longer constant and has a radial profile that depends on the parameters of the theory.

So with this I'd like to finish and I'll leave the summary slide here.