Ringdown Tests of Gravity

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[2301.10272, 2408.01720] SS + Johannes Noller

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PLAN

1. MOTIVATION AND BACKGROUND
   - TESTING GRAVITY
   - GRAVITATIONAL WAVES : BLACK HOLE PERTURBATION THEORY
2. RINGDOWN OF HAIRY BLACK HOLES
   - STATIC HAIR AND SPEED OF GRAVITY
   - TIME-DEPENDENT HAIR
3. BONUS : YITP PROJECT
4. SUMMARY

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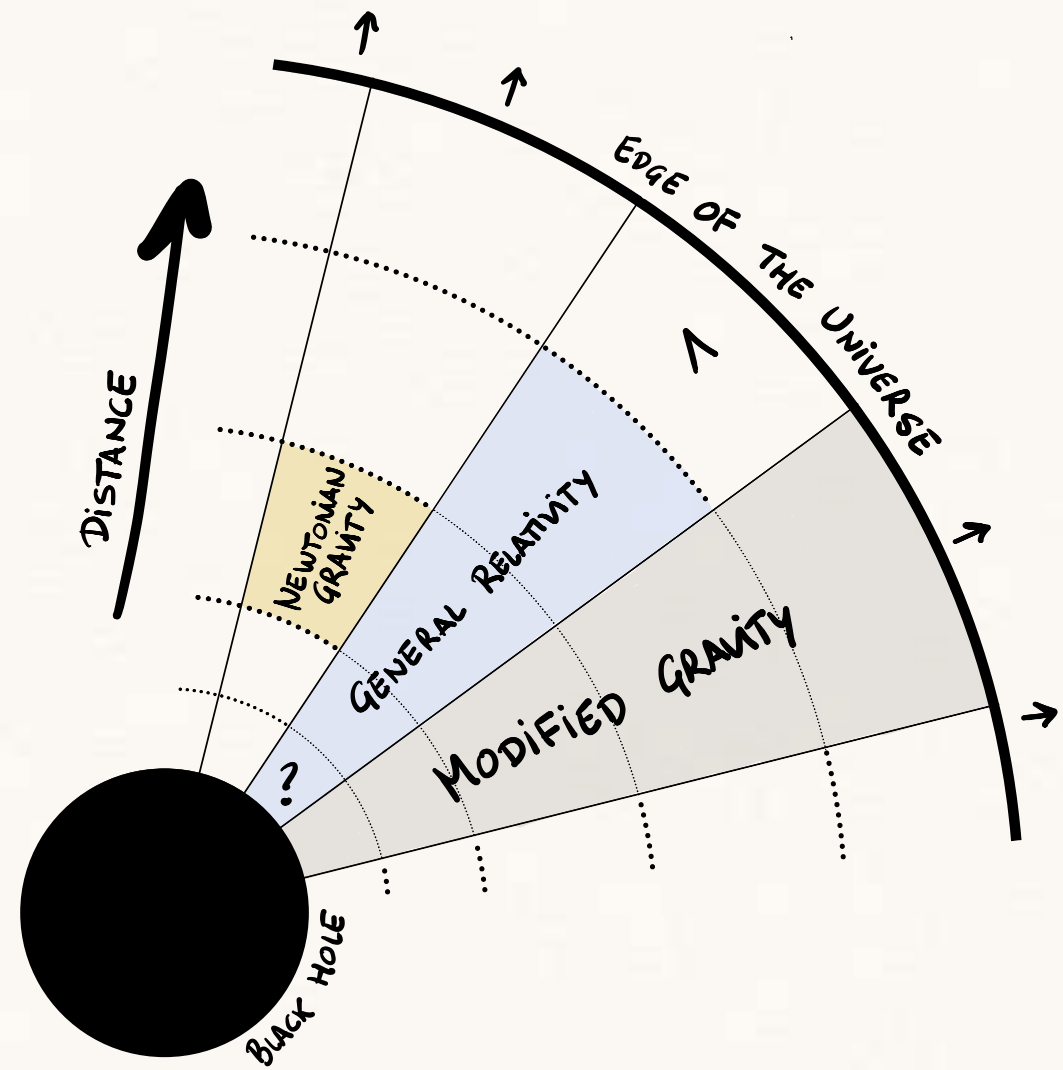
Motivation and Background

TESTING GRAVITY

- SO WHY SHOULD WE TEST GR?
  - DARK ENERGY
  - SINGULARITIES
  - NOT QUANTIZABLE
  - ...
  - WHY NOT?



TESTING GRAVITY

(LOVELOCK'S THEOREM)

$$\left.\begin{array}{c} 4\text{D} \\ g_{\mu\nu} \\ \text{LOCAL} \\ 2^{\text{nd}}\text{order EOM} \end{array}\right\} \quad \text{GR} \quad \rightarrow \quad S = \int d^4x \sqrt{-g}\, R[g_{\mu\nu}]$$

$$\left.\begin{array}{c} 4\text{D} \\ g_{\mu\nu} + \phi \\ \text{LOCAL} \\ 2^{\text{nd}}\text{order EOM} \end{array}\right\} \quad \text{HORNDESKI} \quad \rightarrow \quad S = \int d^4x \sqrt{-g}\, H[g_{\mu\nu}, \phi]$$

TESTING GRAVITY

$$S = \int d^4x \sqrt{-g} \left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 \right)$$

HORNDESKI GRAVITY

$$\mathcal{L}_2 = G_2(\phi, X)$$

$$\mathcal{L}_3 = G_3(\phi, X) \Box\phi$$

$$\mathcal{L}_4 = G_4(\phi, X) R + G_{4X}(\phi, X) \left[(\Box\phi)^2 - (\phi_{\mu\nu})^2 \right]$$

$$\mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu} \phi^{\mu\nu} - \frac{1}{6} G_{5X}(\phi, X) \left[(\Box\phi)^3 - 3(\phi_{\mu\nu})^2 \Box\phi + 2(\phi_{\mu\nu})^3 \right]$$

WHERE $X = -\frac{1}{2} \nabla_\mu\phi \nabla^\mu\phi$, $\phi_\mu := \nabla_\mu\phi$, $\phi_{\mu\nu} := \nabla_\nu \nabla_\mu \phi$, ...

$G_{4X} := \partial_X G_4$ $\quad$ $(\phi_{\mu\nu})^2 := \phi_{\mu\nu}\phi^{\mu\nu}$

$(\phi_{\mu\nu})^3 := \phi_{\mu\nu}\phi^{\mu\sigma}\phi_\sigma{}^\nu$

TESTING GRAVITY

THEORY → OBSERVABLE

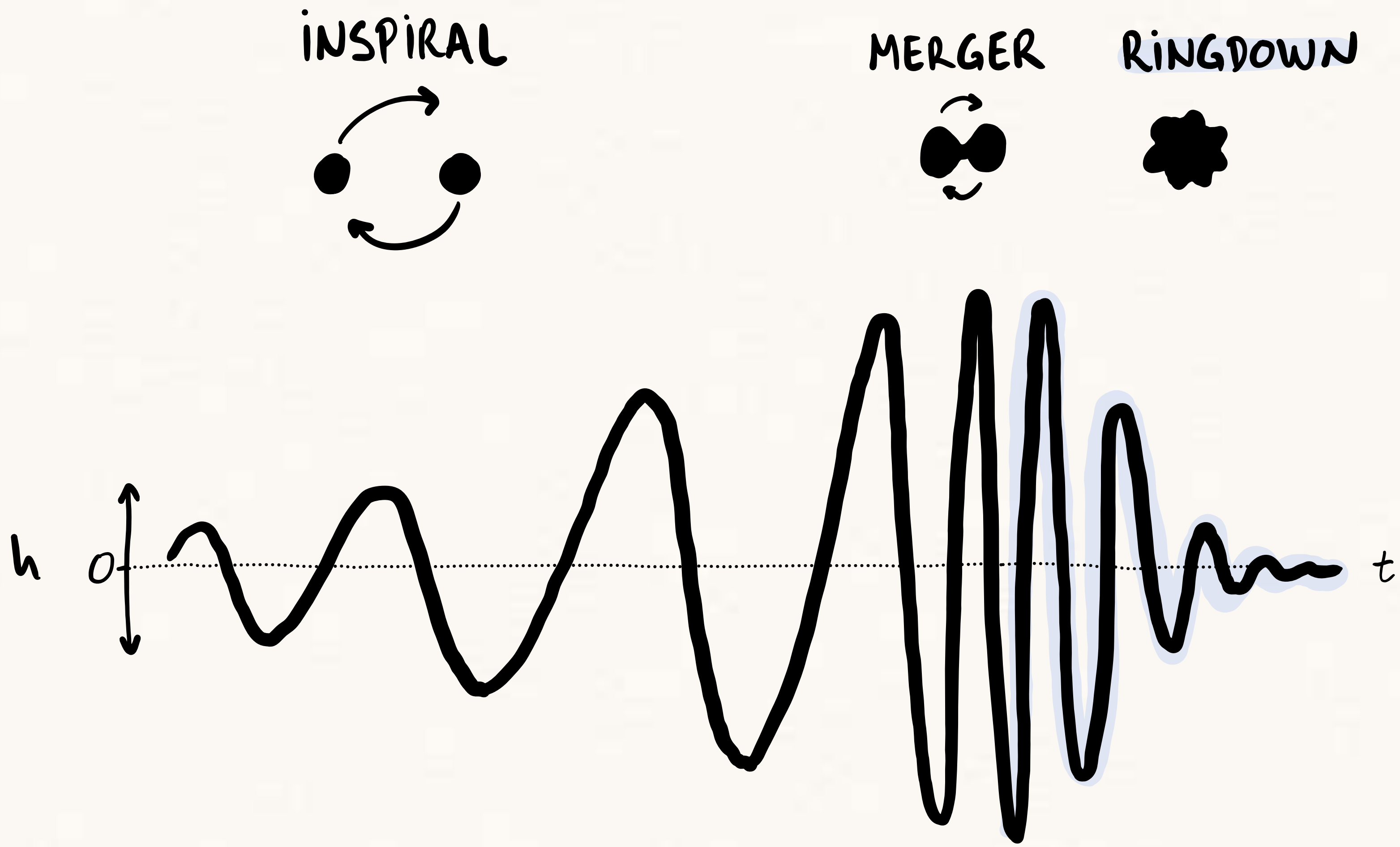
$S = \int d^4x \sqrt{-g}\, R \;\rightarrow\; \alpha = 0$

$S = \int d^4x \sqrt{-g}\, H \;\rightarrow\; \alpha \neq 0$

"SMOKING GUN SIGNAL"

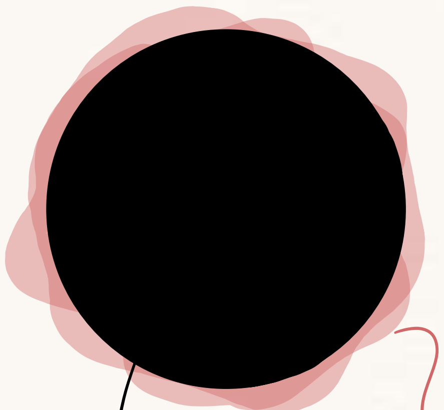
Gravitational Waves : BH Perturbation Theory

Astrophysical Sources : Mergers (black holes / neutron stars)



GRAVITATIONAL WAVES : BH PERTURBATION THEORY

RINGDOWN : BLACK HOLE PERTURBATION THEORY



$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$S = \int d^4x \sqrt{-g}\, R$$

$$\frac{d^2 h}{dr_*^2} + \left[\omega^2 - V\right] h = 0$$

$$\omega(M, a)$$

QUASINORMAL MODES

(NO HAIR THEOREM)

BACKGROUND

- SCHWARZSCHILD
- SCHWARZSCHILD - DE SITTER
- REISSNER-NORDSTRÖM
- ...

PERTURBATIONS

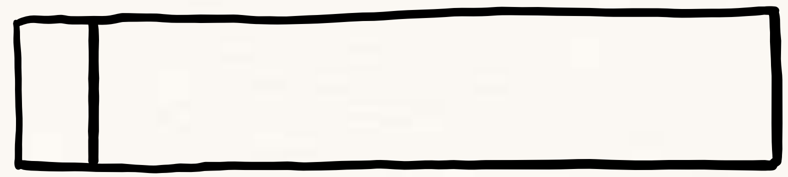
- ODD → REGGE-WHEELER
- EVEN → ZERILLI

GRAVITATIONAL WAVES : BH PERTURBATION THEORY

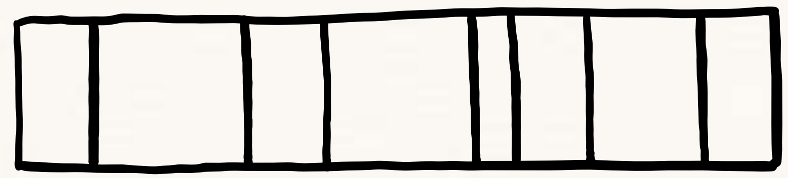
BLACK HOLE SPECTROSCOPY

$\omega(M, a)$

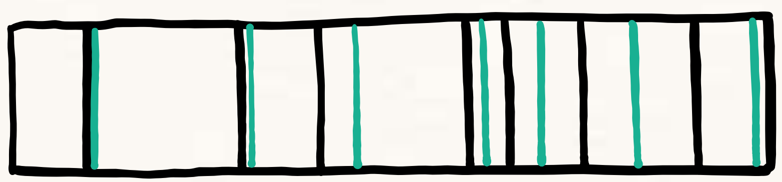
- 1st QNM sets (M, a)



- All other QNMs are fixed in GR



MEASURING QNMs PROVIDES CLEAN TESTS OF BACKGROUND GEOMETRY AND UNDERLYING THEORY



GR $\omega(M, a)$

MG $\omega(M, a, \alpha)$

GRAVITATIONAL WAVES : BH PERTURBATION THEORY

DERIVATION OF REGGE-WHEELER EQUATION IN GR

① SET-UP

THEORY : GR $\qquad S = \int d^4x \sqrt{-g}\, R$

BACKGROUND : SCHWARZSCHILD $\qquad ds^2 = \bar{g}_{\mu\nu} dx^\mu dx^\nu = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega_2^2 \quad , \quad f = 1 - \frac{2M}{r}$

② QUADRATIC ACTION

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

Odd perturbations traceless $h = 0$

$$\hookrightarrow S^{(2)} = \frac{1}{4}\int \sqrt{-g}\, d^4x \Big[-\nabla h_{\mu\nu} \nabla h^{\mu\nu} + \nabla_\sigma h \nabla^\sigma h - \nabla_\mu h_{\nu\sigma} \nabla^\nu h^{\mu\sigma} + 2\nabla_\mu h^{\mu\nu} \nabla_\sigma h_\nu{}^\sigma - 2\nabla^\mu h \nabla_\nu h_\mu{}^\nu$$
$$+ 2\left(h_\sigma{}^\nu h^{\sigma\mu} - h h^{\mu\nu}\right) R_{\mu\nu} - \left(h_{\mu\nu} h^{\mu\nu} - \frac{1}{2} h^2\right) R + 2 h^{\mu\nu} h^{\sigma\lambda} R_{\mu\sigma\nu\lambda} \Big]$$

Ricci-flat $R_{\mu\nu} = 0 = R$

③ REGGE WHEELER GAUGE

$$h_{\mu\nu} = h_{\mu\nu}^{odd} + h_{\mu\nu}^{even}$$

axisymmetric $\partial_\phi = 0$

$$\hookrightarrow h_{\mu\nu}^{odd} = \begin{pmatrix} 0 & 0 & -h_0 \frac{1}{\sin\theta} \partial_\phi Y_{\ell m} & h_0 \sin\theta\, \partial_\theta Y_{\ell m} \\ 0 & 0 & -h_1 \frac{1}{\sin\theta} \partial_\phi Y_{\ell m} & h_1 \sin\theta\, \partial_\theta Y_{\ell m} \\ -h_0 \frac{1}{\sin\theta} \partial_\phi Y_{\ell m} & -h_1 \frac{1}{\sin\theta} \partial_\phi Y_{\ell m} & 0 & 0 \\ h_0 \sin\theta\, \partial_\theta Y_{\ell m} & h_1 \sin\theta\, \partial_\theta Y_{\ell m} & 0 & 0 \end{pmatrix}$$

$$h_{\mu\nu}^{even} = \begin{pmatrix} f H_0 & H_1 & 0 & 0 \\ H_1 & \frac{1}{f} H_2 & 0 & 0 \\ 0 & 0 & r^2 K & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta\, K \end{pmatrix} Y_{\ell m}$$

GRAVITATIONAL WAVES : BH PERTURBATION THEORY

④ ODD QUADRATIC ACTION IN COMPONENT FORM (ANGLES INTEGRATED)

$$S^{(2)}_{odd} = \frac{\ell(\ell+1)\pi}{(1+2\ell)} \int dt\, dr \frac{1}{f r^2} \Big[(\ell^2+\ell-2)(h_0^2 - f^2 h_1^2) + f(h_0^2 + r^2(\dot{h}_0^2 + h_0'^2) - 2r\dot{h}_1(-2h_0 + r h_0')) + 2f^3 h_1(3h_1 - 2r h_1')) \Big]$$

⑤ VARIATIONAL PRINCIPLE

$$\frac{\delta S^{(2)}_{odd}}{\delta h_0} = 0 \longrightarrow h_0'' + i\omega h_1' + 2i\omega \frac{h_1}{r} - \frac{h_0}{r^2} f^{-1}\left(\ell(\ell+1) - \frac{4M}{r}\right) = 0$$

$$\frac{\delta S^{(2)}_{odd}}{\delta h_1} = 0 \longrightarrow f^{-1}\left(2i\omega \frac{h_0}{r} + \omega^2 h_1 - i\omega h_0'\right) - \frac{h_1}{r^2}(\ell+2)(\ell-1) = 0$$

→ r-DERIVATIVE OF 2ND EQ = 1ST EQ × $\frac{i\omega}{f}$ $\Rightarrow$ $h_0 = -\frac{f(h_1 f)'}{i\omega}$

⑥ DEFINE REGGE-WHEELER VARIABLE AND TORTOISE COORDS

$$Q_{odd} = \frac{f h_1}{r} \quad , \quad r_* = r + 2M \log\frac{r-2M}{2M} \quad , \quad f\partial_r = \partial_*$$

$$\Rightarrow \frac{d^2 Q_{odd}}{dr_*^2} + \left[\omega^2 - f V_{odd}\right] Q_{odd} = 0 \qquad \text{REGGE-WHEELER EQUATION}$$

$$\downarrow$$

$$V_{odd} = \frac{\ell(\ell+1)}{r^2} - \frac{6M}{r^3}$$

GRAVITATIONAL WAVES : BH PERTURBATION THEORY

$$\frac{d^2 Q_{o/e}}{dr_*^2} + \left[\omega^2 - f V_{o/e} \right] Q_{o/e} = 0$$

REGGE-WHEELER POTENTIAL (1957)

$$V_o = \frac{\ell(\ell+1)}{r^2} - \frac{6M}{r^3}$$

ZERILLI POTENTIAL (1970)

$$V_e = 2 \frac{L^2 r^2 \left((L+1) r + 3M \right) + 9M^2 (Lr + M)}{r^3 (Lr + 3M)^2} , \quad 2L = (\ell+2)(\ell-1)$$

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RINGDOWN OF HAIRY BLACK HOLES

RINGDOWN OF HAIRY BHs

IN SHIFT-SYMMETRIC SCALAR-TENSOR THEORIES, BLACK HOLES CANNOT SUPPORT SCALAR HAIR IF:

1. SPACETIME IS SPHERICALLY SYMMETRIC, STATIC AND ASSYMPTOTICALLY FLAT
2. SCALAR FIELD IS STATIC $\phi(r)$ AND HAS A VANISHING DERIVATIVE ϕ' AT INFINITY
3. NORM OF THE CURRENT ASSOCIATED W/ SHIFT SYMMETRY IS FINITE DOWN TO THE HORIZON
4. ACTION CONTAINS A CANONICAL KINETIC TERM $X \subset G_2$
5. ALL $G_i(X)$ FUNCTIONS ARE ANALYTICAL AT $X=0$

[1312.5742] HU, RAVERI, FRUSCINANTE, SILVESTRI

Ringdown of Hairy BHs

Static $\phi(r)$

- E.G. Scalar Gauss-Bonet

$G_2 = \eta X$, $G_4 = \zeta$, $G_5 = \alpha \ln|X|$

[1312.3622, 1408.1698] Sotiriou, Zhou

- More general

$G_2 \supseteq \sqrt{X}$, $G_3 \supseteq \ln|X|$,

$G_4 \supseteq \sqrt{X}$, $G_5 \supseteq \ln|X|$

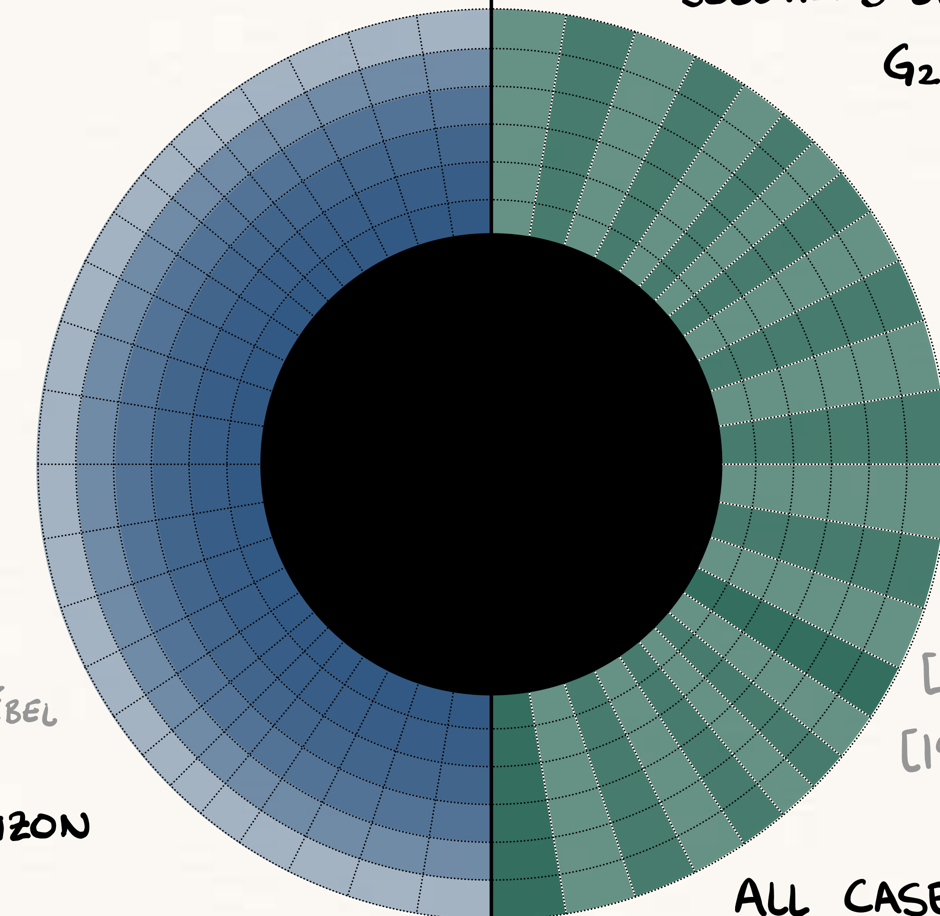
[1702.01938] Babichev, Charmousis, Lehébel

→ Finite-norm current at horizon

→ Lack Lorentz invariant ($X=0$) solution in Minkowski spacetime

[1903.02055] Saravani, Sotiriou

Real problem or not?



Time-dependent $\phi(t,r)$

Solutions of shift + reflection symm Horndeski

$G_2(X)$, $G_4(X)$

[1312.3204] Babichev, Charmousis

[1403.4364] Kobayashi, Tanahashi

However, prone to instabilities

[1510.07400] Ogawa, Kobayashi, Suyama

[1803.11444] Babichev, Charmousis, Esposito-Farèse, Lehébel

[1610.00432] Takahashi, Suyama

[1904.03554] Takahashi, Motohashi, Minamitsuji

[1907.00699] De Rham, Zhang

All cases studied assumed $X = \text{constant}$

Solution with $X \neq \text{constant}$

[2310.11919] Bakopoulos, Charmousis, Kanti, Lecoeur, Nakas

STATIC HAIR AND SPEED OF GRAVITY

[2301.10272] SS + JOHANNES NOLLER

SET-UP

THEORY : HORNDESKI WITH $G_{4\phi} = 0$

BACKGROUND : METRIC : SCHWARZSCHILD

SCALAR : STATIC HAIRY DEVIATION $\bar{\Phi} = \hat{\phi} + \varepsilon\, \delta\phi(r)$, $\delta\phi = \varphi_c \frac{2M}{r}$

QUADRATIC ACTION

$$S^{(2)} = \int dt\, dr \left[a_1 h_0^2 + a_2 h_1^2 + a_3 \left(\dot{h}_1^2 + h_0'^2 - 2 h_0' \dot{h}_1 + \frac{4}{r} \dot{h}_1 h_0 \right) \right]$$

MODIFIED REGGE-WHEELER EQUATION

GRAVITATIONAL WAVE SPEED EXCESS

$$\frac{d^2 Q}{dr_*^2} + \left[\omega^2 (1 + \alpha_T) + V + \alpha_T \delta V \right] Q = 0$$

$$\alpha_T = - f (2M)^2\, G_T\, \delta\phi'^2$$

$$G_T = \frac{G_{4X} - G_{5\phi}}{G_4}$$

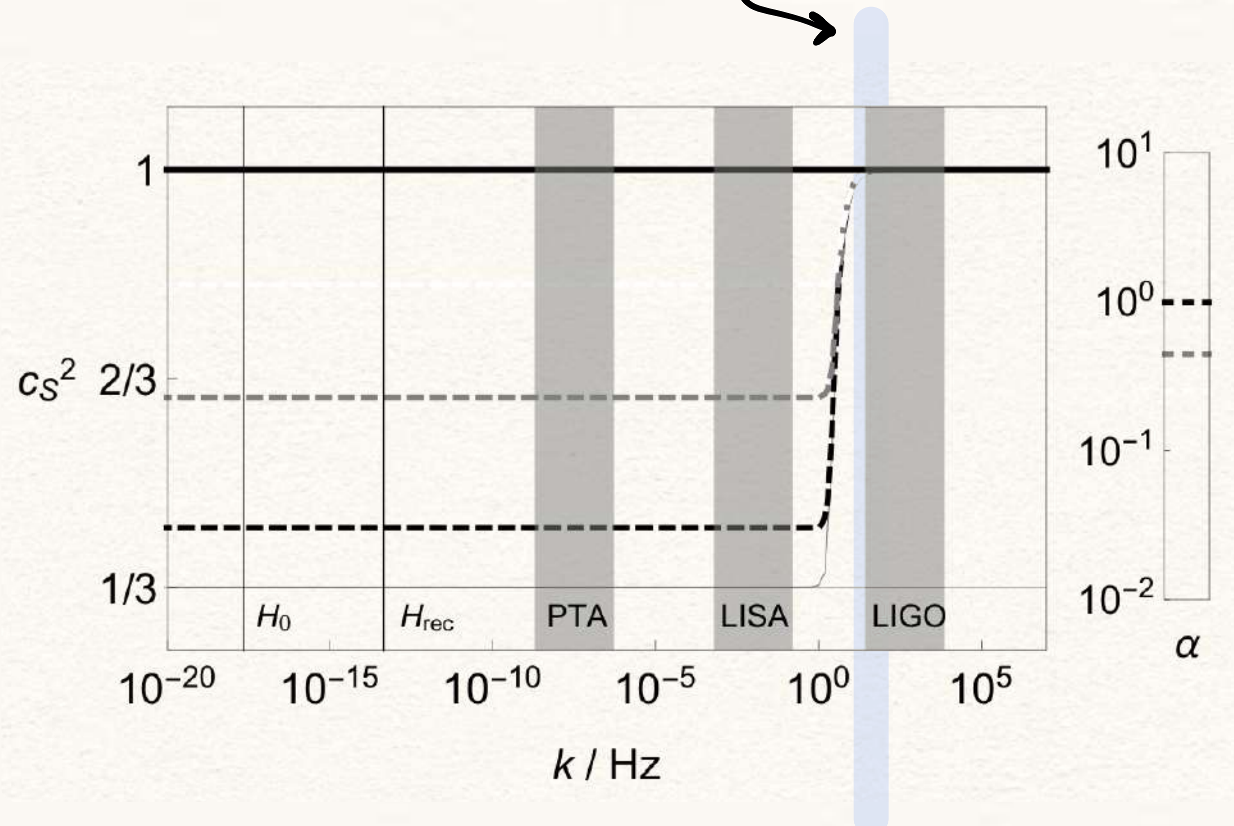
$$\delta V = -\frac{1}{f} \omega_0^2 + \frac{M(2r - 5M)}{r^3 (r - 2M)} + \frac{(\ell+2)(\ell-1)}{r^2} - \frac{r - 2M}{2r} \left(\left(\frac{\delta\phi''}{\delta\phi'} \right)^2 - \frac{\delta\phi'''}{\delta\phi'} \right) + \frac{r - 5M}{r^2} \frac{\delta\phi''}{\delta\phi'}$$

STATIC HAIR AND SPEED OF GRAVITY

[2301.10272] SS + JOHANNES NOLLER

WHAT DO WE KNOW ABOUT α_T ?

- LIGO : $\alpha_T \lesssim 10^{-15}$ (GW170817)
- DARK ENERGY EFTs : CUTOFF AT $\sim 10^{2}$ Hz [1806.09417] MELVILLE, DE RHAM



STATIC HAIR AND SPEED OF GRAVITY

[2301.10272] SS + JOHANNES NOLLER

OBTAIN QNM CORRECTIONS WITH PARAMETRISED RINGDOWN FORMALISM

[1901.01265] CARDOSO, KIMURA, MASELLI, BERTI, MACEDO, MCMANUS

$$\omega = \omega_0 + \delta\omega(\alpha_T)$$

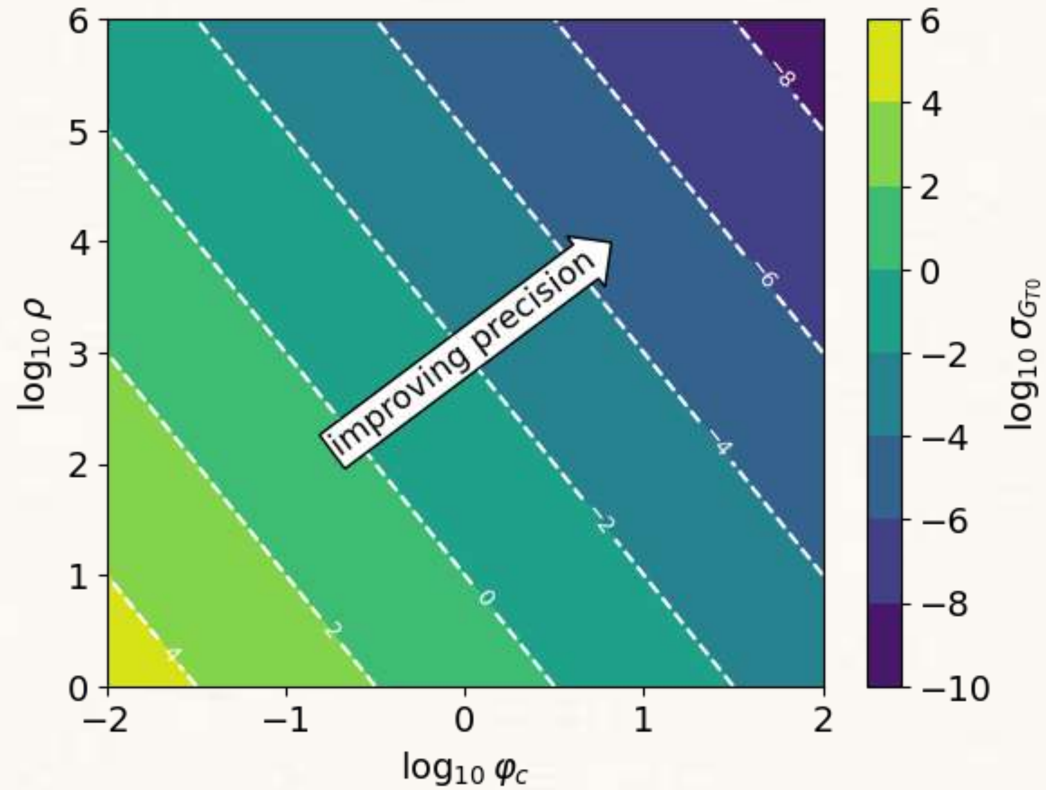
GR

$$\delta\omega = \sum_{i=0}^{\infty} A_{Ti}\left[(2M\omega_0)^2 e_{4+i} - (\ell(\ell+1)-9)\, e_{6+i} + (\ell(\ell+1)-20)\, e_{7+i} + \frac{45}{4} e_{8+i}\right]$$

$$A_{Ti} = G_{Ti}\,\varphi_c^2 \quad , \quad G_T = \frac{1}{(2M)^2}\frac{G_{4X} - G_{5\phi}}{G_4} = \sum_i G_{Ti}\left(\frac{2M}{r}\right)^i$$

FISHER FORECAST ANALYSIS

Detector(s)	Ringdown SNR (ρ)	Error on α_T
LVK	10 [136–138]	1
ET / CE	10^2 [138–141]	10^{-1}
pre-DECIGO	10^2 [142]	10^{-1}
DECIGO / AEDGE	10^3 [143, 144]*	10^{-2}
LISA	10^5 [137, 145]	10^{-4}
TianQin	10^5 [145]	10^{-4}
AMIGO	10^5 [130]	10^{-4}



TIME-DEPENDENT HAIR

[2408.01720] SS + JOHANNES NOLLER

$$\phi = qt + \psi(r)$$

$\mathscr{L}$	Background solution		Stability	
	$g_{\mu\nu}$	X	odd	even
(Shift + refl)-sym Horndeski [29, 30]	S(dS)*	$\frac{q^2}{2} = \text{const}$	✓	✗
Cubic Galileon [31, 32]	S(dS)* (non-exact)	non-const (non-exact)	?	?
Shift-sym breaking Horndeski [65]	∄ (for large subclasses)	∄ (for large subclasses)	-	-
$G_2 = \eta X, G_4 = \zeta + \beta\sqrt{X}$ [33]	S(dS)* + RN(dS)* (non-exact)	non-const	?	?
Shift-sym beyond Horndeski [34]	SdS*	$\frac{q^2}{2} = \text{const}$	✓	✗
Shift-sym breaking quadratic DHOST [35]	S(dS)*	$\frac{q^2}{2} = \text{const}$	✓	✗
Shift-sym quadratic DHOST [36, 66]	S(dS)* + K*	const	✓	✗
Quadratic DHOST [37]	S(dS)* + (K)RN(dS)*	const	✓	✗
$G_2 = -2\Lambda + 2\eta\sqrt{X}, G_4 = 1 + \lambda\sqrt{X}$ [1]	S(dS)	$\frac{1}{2}\frac{q^2\lambda}{\lambda+\eta r^2}$	✓(this work)	?

TABLE I. Stealth black hole solutions with a linearly time-dependent scalar (2). S(dS) corresponds to Schwarzschild(-de Sitter), while (K)RN(dS) refers to (Kerr-)Reissner-Nordsröm(-de Sitter). The cosmological constant in dS can either come from a bare cosmological constant in the action as in GR, or from an effective combination of beyond-GR parameters (i.e. self-tuned). We denote cases which can fall in the latter category with $*$. The symbol ∄ is used to indicate the non-existence of stealth black hole solutions – in particular such solutions were shown to be absent in large classes of shift symmetry breaking Horndeski theories in [65]. Further details related to all the solutions shown in this table are discussed in Appendix A. Let us only point out here that even modes have been shown to suffer from instabilities in all known hairy solutions for which $X = \text{const}$ [60]. This table builds on a pre-existing one [36].

TIME-DEPENDENT HAIR

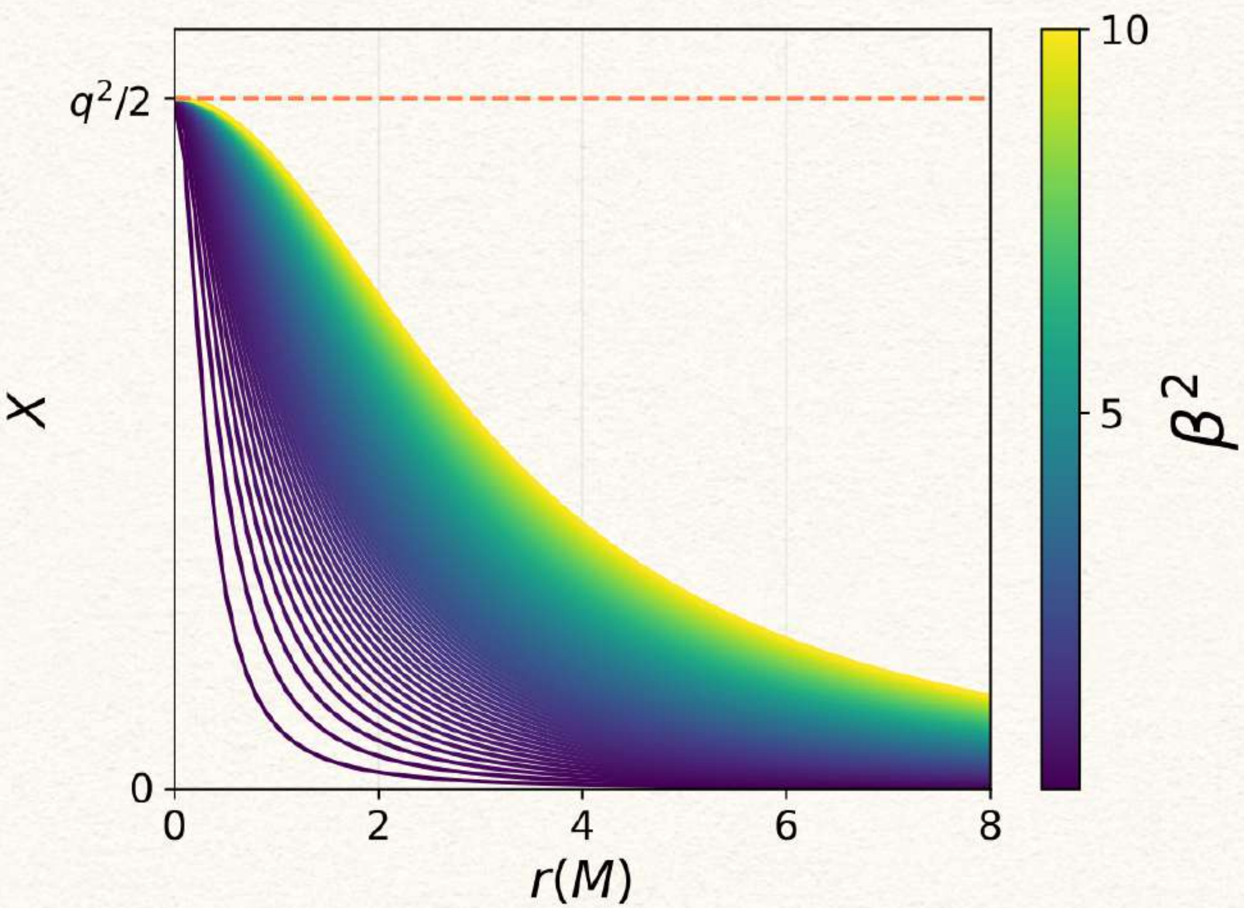
[2408.01720] SS + JOHANNES NOLLER

SET-UP

THEORY : $$\mathcal{L} = -2\Lambda + 2\eta\sqrt{X} + (1+\lambda\sqrt{X})R + \frac{\lambda}{2\sqrt{X}}\left(\Box\phi^2 + \phi_{\mu\nu}{}^2\right)$$

BACKGROUND : METRIC : SCHWARZSCHILD – DE SITTER

SCALAR : $\bar{\Phi} = qt + \psi(r)$, $\quad \psi'(r)^2 = \frac{q^2}{B}\left(1 - \frac{\lambda B}{\lambda + \eta r^2}\right)$, $\quad X = \frac{1}{2}\frac{q^2\lambda}{\lambda + \eta r^2}$



$\beta^2 = \frac{\lambda}{2\eta}$

TIME-DEPENDENT HAIR

[2408.01720] SS + JOHANNES NOLLER

QUADRATIC ACTION

$$S^{(2)} = \frac{1}{2}\int d^4x\sqrt{-g}\left[\mathscr{L}^{(2)}_{GR} + \mathscr{L}^{(2)}_{\eta} + \mathscr{L}^{(2)}_{\lambda}\right],$$

$$\mathscr{L}^{(2)}_{GR} = \frac{1}{2}\left(\nabla^{\sigma}h^{\mu\nu}(2\nabla_{\nu}h_{\mu\sigma} - \nabla_{\sigma}h_{\mu\nu}) + 2\Lambda h_{\mu\nu}h^{\mu\nu}\right),$$

$$\mathscr{L}^{(2)}_{\eta} = \frac{-\eta}{\sqrt{X}}\left(Xh_{\mu\nu}h^{\mu\nu} + \phi^{\mu}\phi^{\nu}h^{\sigma}_{\mu}h_{\nu\sigma}\right),$$

$$\begin{aligned}
\mathscr{L}^{(2)}_{\lambda} = \frac{-\lambda}{2\sqrt{X}}\Bigg[&\nabla_{\sigma}h_{\mu\nu}\left(X\left(\nabla^{\sigma}h^{\mu\nu} - 2\nabla^{\nu}h^{\mu\sigma}\right) + \phi^{\sigma}\phi^{\rho}\left(\frac{1}{2}\nabla_{\rho}h^{\mu\nu} - 2\nabla^{\nu}h^{\mu}_{\rho}\right) + \phi^{\mu}\phi^{\nu}\nabla_{\rho}h^{\sigma\rho} + 2\phi^{\mu}\phi^{\rho}\nabla^{\{\nu}h^{\sigma\}}_{\rho}\right)\\
&+ h_{\mu\nu}\Bigg(\frac{1}{2}h^{\mu\nu}((\Box\phi)^2 - \phi_{\rho\gamma}\phi^{\rho\gamma}) + 4h^{\nu}_{\sigma}(\Lambda\phi^{\mu}\phi^{\sigma} - \phi^{\sigma\mu}\Box\phi + \phi^{\mu}_{\rho}\phi^{\rho\sigma}) + 4h_{\sigma\rho}\phi^{\mu[\sigma}\phi^{\nu]\rho}\\
&\quad + \frac{1}{2X}\Big(h^{\nu}_{\sigma}\phi^{\mu}\phi^{\sigma}(\phi_{\rho\gamma}\phi^{\rho\gamma} - (\Box\phi)^2) + 2\phi^{\sigma\rho}(\phi^{\mu}\phi^{\nu}\phi^{\lambda}_{\{\sigma}h_{\rho\}\lambda} - h_{\sigma\rho}\phi^{\mu}\phi^{\nu}\Box\phi)\Big)\\
&\quad + 4\phi^{\rho}\phi^{\mu\sigma}(2\nabla_{[\sigma}h^{\nu}_{\rho]} + \nabla^{\nu}h_{\rho\sigma}) + 2\phi^{\mu}\phi^{\sigma\rho}(2\nabla_{\sigma}h^{\nu}_{\rho} - \nabla^{\nu}h_{\sigma\rho}) + 2\phi^{\mu}\phi^{\nu\sigma}\nabla_{\rho}h^{\rho}_{\sigma}\\
&\quad + 2\phi^{\sigma}\phi^{\mu}_{\sigma}\nabla_{\rho}h^{\nu\rho} + 2\phi^{\sigma}\phi_{\sigma\rho}\nabla^{[\nu}h^{\rho]\mu} + 2\Box\phi(\phi^{\sigma}\nabla_{\sigma}h^{\mu\nu} - 2\phi^{\mu}\nabla_{\sigma}h^{\nu\sigma} - 2\phi^{\sigma}\nabla^{\nu}h^{\mu}_{\sigma})\\
&\quad + \frac{1}{2X}\phi^{\mu}\phi^{\nu}\phi^{\sigma}\left(\phi^{\rho}_{\sigma}\nabla_{\gamma}h^{\gamma}_{\rho} + \phi^{\rho\gamma}(2\nabla_{[\rho}h_{\gamma\}\sigma} - \nabla_{\sigma}h_{\rho\gamma})\right)\Bigg)\Bigg],
\end{aligned}$$

$$\Big\downarrow$$

$$S^{(2)} = \int dt\,dr\left[a_1 h_0^2 + a_2 \dot{h}_1^2 + a_3\left(\dot{h}_1^2 + h_0'^2 - 2h_0'\dot{h}_1 + \frac{4}{r}\dot{h}_1 h_0\right) + a_4 h_0 \dot{h}_1\right]$$

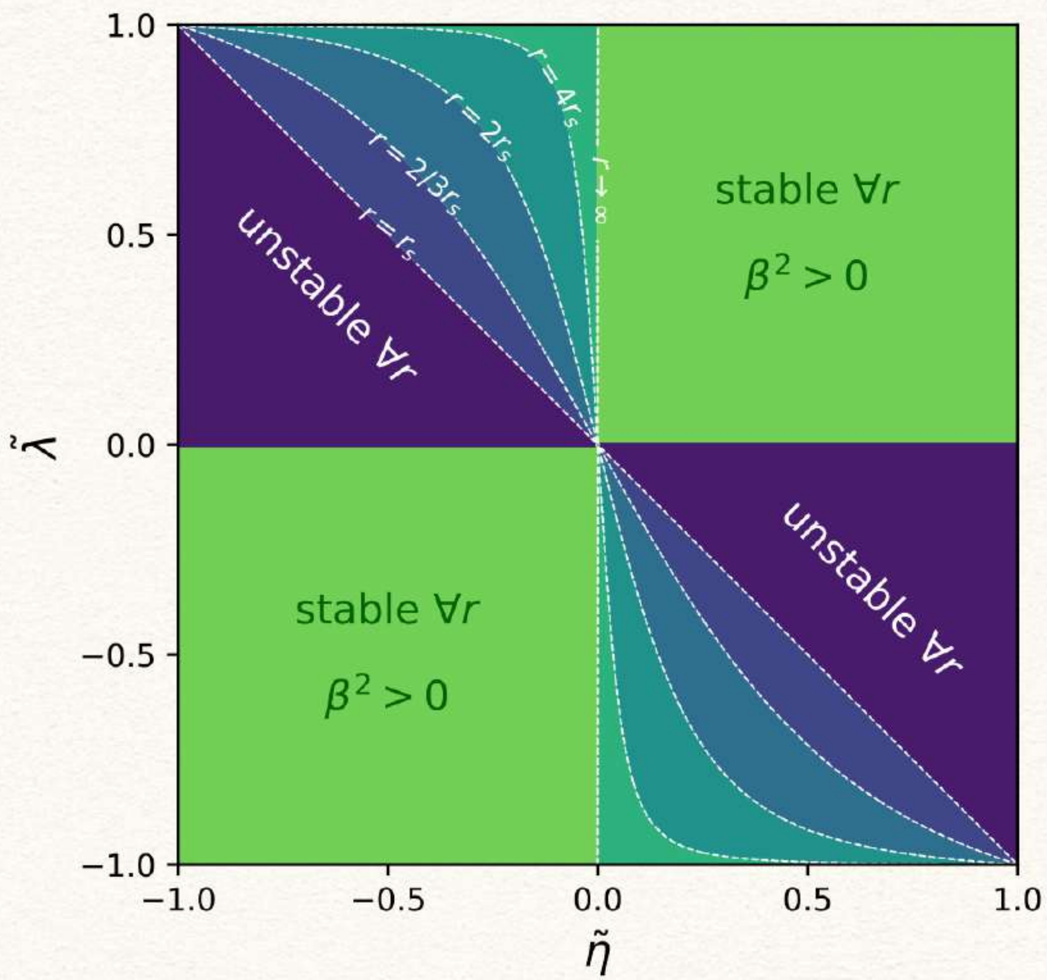
TIME-DEPENDENT HAIR

[2408.01720] SS + JOHANNES NOLLER

STABILITY CONDITIONS

$$S^{(2)} = \frac{\ell(\ell+1)}{4(\ell-1)(\ell+2)} \int dt\, dr \left[\tilde{b}_1 (\partial_t Q)^2 - b_2 Q'^2 - \left(\ell(\ell+1) b_4 + V_{\rm eff} \right) Q^2 \right]$$

$$\tilde{b}_1 > 0\,, \quad b_2 > 0\,, \quad b_4 > 0$$



TIME-DEPENDENT HAIR

[2408.01720] SS + JOHANNES NOLLER

MODIFIED REGGE-WHEELER EQUATION

$$\frac{d^2Q}{dr_*^2} + \left[\omega^2 - V\right] Q = 0$$

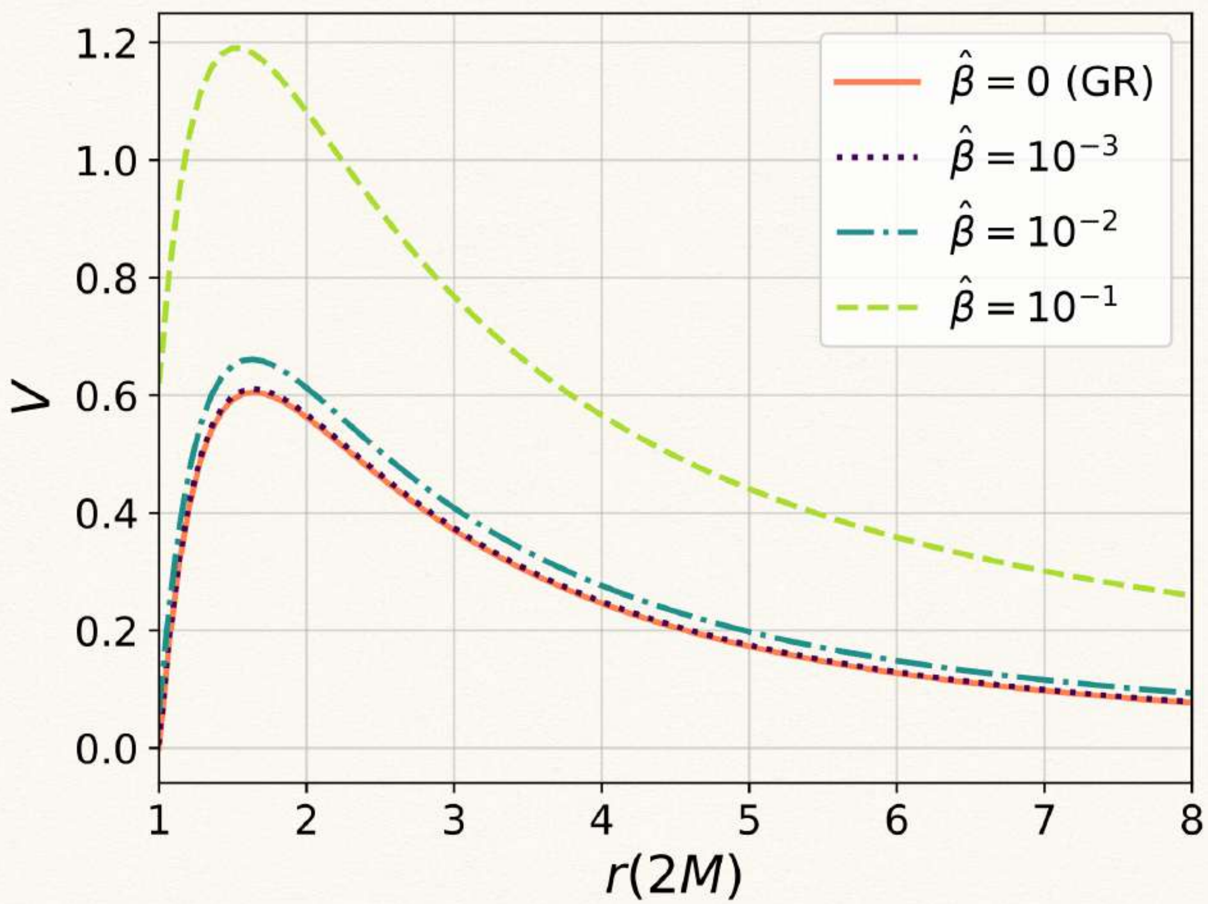
MODIFIED REGGE-WHEELER POTENTIAL

$$\begin{aligned} V = & V_{RW}\left(1+\frac{q\beta\sqrt{2\beta^2+r^2}}{B}\right)+\frac{q\beta(B+q\beta\sqrt{2\beta^2+r^2})}{4r^2(2\beta^2+r^2)(2q\beta^3+\sqrt{2\beta^2+r^2})^3}\times \\ & \times\left[q^2\beta^6\left(192\beta^4+104\beta^2r^2+3r^4\right)+2(2\beta^2+r^2)\left(24\beta^4+17\beta^2r^2+2r^4\right)+6q\beta^3\sqrt{2\beta^2+r^2}\left(384\beta^4+416\beta^2r^2+117r^4\right)\right. \\ & \left.-\beta^2B\left(2\left(48q^2\beta^6(2\beta^2+r^2)+48\beta^4+56\beta^2r^2+19r^4\right)\right)+\frac{q\beta^3}{\sqrt{2\beta^2+r^2}}\left(384\beta^4+416\beta^2r^2+117r^4\right)\right]. \end{aligned} \tag{48}$$

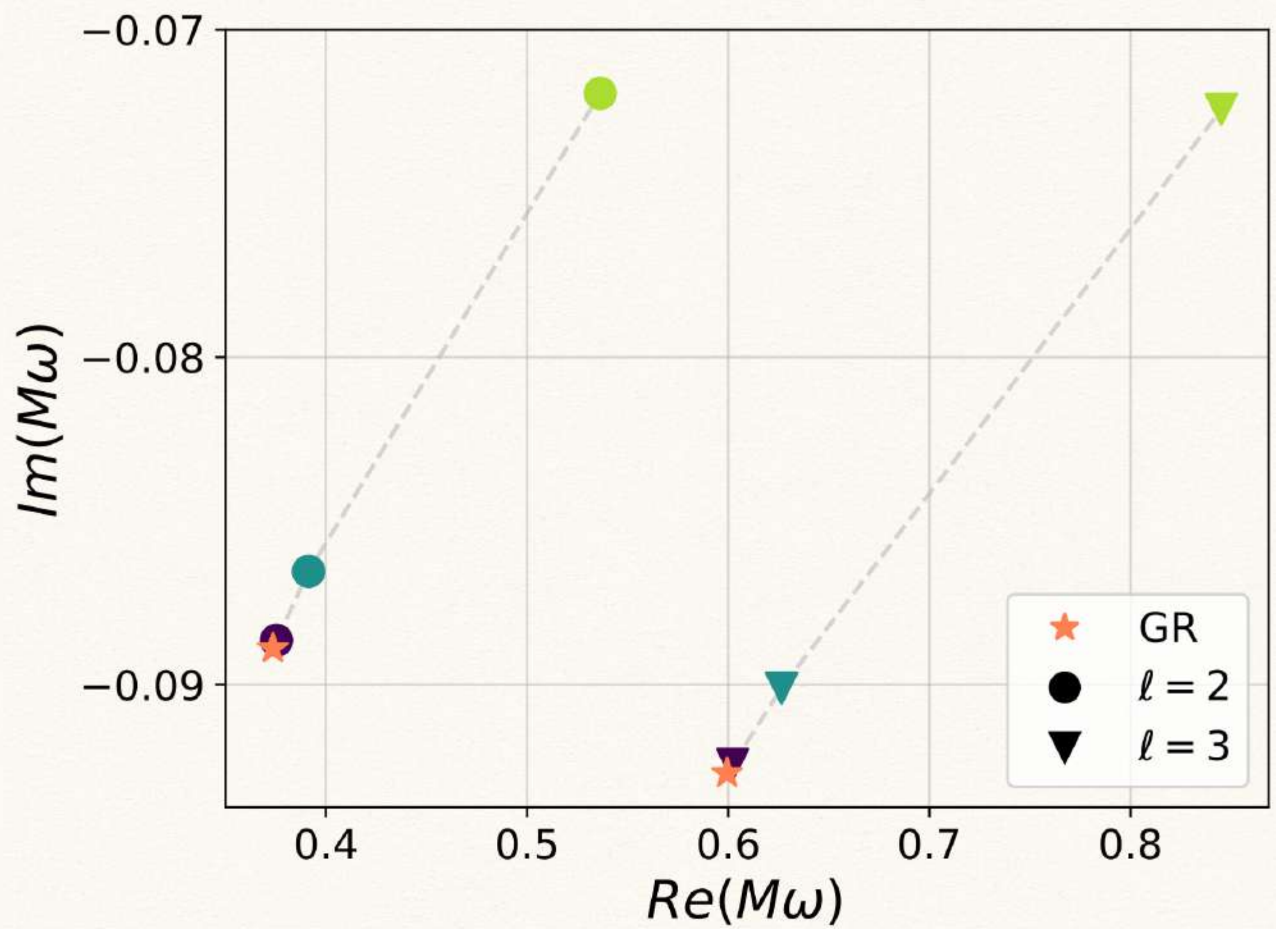
Time-Dependent Hair

[2408.01720] SS + Johannes Noller

Modified Regge-Wheeler Potential



QNM Deviations



TIME-DEPENDENT HAIR

[2408.01720] SS + JOHANNES NOLLER

QNMs CALCULATED WITH WKB APPROXIMATION

[1904.1033] KONOPLYA, ZHIDENKO, ZINHAILO

$$\omega^2 = V_0 - i\left(n+\frac{1}{2}\right)\sqrt{2V_0^{(2)}} - i\sqrt{2V_0^{(2)}}\sum_{i=2}^{N}\Lambda_j.$$

	$\mathrm{Re}(M\omega)$	$\mathrm{Im}(M\omega)$
$\hat{\beta}=0$ (GR)	0.3736	−0.0889
$\hat{\beta}=10^{-3}$	0.3754	−0.0886
$\hat{\beta}=10^{-2}$	0.3914	−0.0865
$\hat{\beta}=10^{-1}$	0.5364	−0.0719

LIGHT RING EXPANSION

[1810.07706] FRANCIOLINI, HUI, PENCO, SANTONI, TRINCHERINI

POTENTIAL MAXIMUM

$$\frac{M_{\rm Pl}^2}{M} r_*^{\rm max} = 3.2808 - 3.0306\cdot\hat{\beta} + 0.8316\cdot\hat{\beta}^2 - \left(0.22819 + 1.8502\frac{M_{\rm Pl}^8}{M^4 q^2}\right)\cdot\hat{\beta}^3 + \mathcal{O}(\hat{\beta}^4)$$

SEMI-ANALYTIC QNMs

$$\frac{M\omega}{M_{\rm Pl}^2} = \frac{M\omega_0}{M_{\rm Pl}^2} + (1.80+0.25i)\hat{\beta} - (2.24+1.57i)\hat{\beta}^2 + \mathcal{O}(\hat{\beta}^3),$$

Time-Dependent Hair

[2408.01720] SS + Johannes Noller

Fisher Forecast Analysis

Detector(s)	Ringdown SNR (ρ)	Error on $\hat{\beta}$
LVK	10 [132–134]	10^{-2}
ET / CE	10^2 [134–137]	10^{-3}
pre-DECIGO	10^2 [138]	10^{-3}
DECIGO / AEDGE	10^3 [139, 140]*	10^{-4}
LISA	10^5 [133, 141]	10^{-6}
TianQin	10^5 [141]	10^{-6}
AMIGO	10^5 [142]	10^{-6}

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Bonus: YITP Project

BONUS : PROJECT AT YITP

IN COLLABORATION WITH SHINJI MUKOHYAMA, KAZUFUMI TAKAHASHI,
HAJIME KOBAYASHI, VICHARIT YINGCHAROENRAT

STUDYING STEALTH BLACK HOLE SOLUTIONS IN CUBIC HOST THEORIES

↳ DERIVE CONDITIONS FOR THEIR EXISTENCE

↳ ODD PERTURBATION ANALYSIS

↳ QNMS AND OTHER RINGDOWN PHENOMENOLOGY

ALSO WITH THE AIM OF BETTER UNDERSTANDING RELATION BETWEEN

EFT OF BH PERTURBATIONS ⟷ CONCRETE THEORIES

$$
\begin{aligned}
S_{\text{grav}} = \int^4 x\sqrt{-g}\Big[&F_0(\phi,X) + F_1(\phi,X)\Box\phi + F_2(\phi,X)R + \sum_{I=1}^{5} A_I(\phi,X)L_I^{(2)} \\
&+ F_3(\phi,X)G_{\mu\nu}\phi^{\mu\nu} + \sum_{I=1}^{10} B_I(\phi,X)L_I^{(3)}\Big] + \int d^4x\sqrt{-g}L_m,
\end{aligned}
$$

$$
\begin{array}{lll}
L_1^{(2)} = \phi_{\mu\nu}\phi^{\mu\nu}, & L_1^{(3)} = (\Box\phi)^3, & L_2^{(3)} = (\Box\phi)\phi_{\mu\nu}\phi^{\mu\nu}, \\
L_2^{(2)} = (\Box\phi)^2, & L_3^{(3)} = \phi_{\mu\nu}\phi^{\nu\rho}\phi^{\mu}_{\rho}, & L_4^{(3)} = (\Box\phi)^2\phi_\mu\phi^{\mu\nu}\phi_\nu, \\
L_3^{(2)} = (\Box\phi)\phi^\mu\phi_{\mu\nu}\phi^\nu, & L_5^{(3)} = \Box\phi\phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho, & L_6^{(3)} = \phi_{\mu\nu}\phi^{\mu\nu}\phi_\rho\phi^{\rho\sigma}\phi_\sigma, \\
L_4^{(2)} = \phi^\mu\phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu, & L_7^{(3)} = \phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^{\rho\sigma}\phi_\sigma, & L_8^{(3)} = \phi_\mu\phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho\phi_\sigma\phi^{\sigma\lambda}\phi_\lambda, \\
L_5^{(2)} = (\phi^\mu\phi_{\mu\nu}\phi^\nu)^2, & L_9^{(3)} = \Box\phi(\phi_\mu\phi^{\mu\nu}\phi_\nu)^2, & L_{10}^{(3)} = (\phi_\mu\phi^{\mu\nu}\phi_\nu)^3.
\end{array}
$$

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Summary

SUMMARY

- QNMs CAN BE USED TO TEST NEW GRAVITATIONAL PHYSICS

STATIC

- SET UP: HORNDESKI W/ $G_{4\phi} = 0$
  PARAMETRISED STATIC HAIRY DEVIATIONS
- QNMs ARE α_T-DEPENDENT IFF $\exists$ SCALAR HAIR
- CALCULATED QNM CORRECTIONS $\delta\omega(\alpha_T)$
  W/ PARAMETRISED RINGDOWN FORMALISM
- LISA CAN CONSTRAIN $|\alpha_T| \leq 10^{-4}$
  W/ RINGDOWN OF ONE SMBH MERGER

TIME-DEPENDENT

- SET UP: SPECIFIC THEORY W/ $G_2, G_4 \supseteq \sqrt{X}$
  LINEAR T-DEP SCALAR W/ $X \neq$ CONST
- IDENTIFIED STABLE PARAMETER SPACE
- CALCULATED QNM CORRECTIONS $\delta\omega(\hat{\beta})$
  W/ WKB APPROXIMATION
- LISA CAN CONSTRAIN $\hat{\beta} \leq 10^{-6}$
  W/ RINGDOWN OF ONE SMBH MERGER

THANK YOU!

GitHub

ringdown-calculations Public

A collection of notebooks for black hole perturbation theory calculations in GR and modified gravity.

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