Testing the Speed of Gravity with Black Hole Ringdown

YITP Kyoto - February 2024

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[2301.10272] SS + Johannes Noller

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UCL

TESTING GRAVITY

- SO WHY SHOULD WE TEST GR ?
  - DARK ENERGY
  - SINGULARITIES
  - NOT QUANTIZABLE
  - ...
  - WHY NOT?

TESTING GRAVITY

(LOVELOCK'S THEOREM)

$$\left.\begin{array}{c} 4D \\ g_{\mu\nu} \\ \text{LOCAL} \\ 2^{nd}\text{ order EOM} \end{array}\right\} \quad \text{GR} \quad \rightarrow \quad S = \int d^4x \sqrt{-g}\, R[g_{\mu\nu}]$$

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$$\left.\begin{array}{c}\text{4D}\\ g_{\mu\nu} + \phi\\ \text{LOCAL}\\ 2^{\text{nd}}\text{ order EOM}\end{array}\right\}\ \text{HORNDESKI} \quad \rightarrow \quad S = \int d^4x\,\sqrt{-g}\;H[g_{\mu\nu},\phi]$$

TESTING GRAVITY

$$S = \int d^4x \sqrt{-g} \left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 \right)$$

HORNDESKI GRAVITY

$$\mathcal{L}_2 = G_2(\phi, X)$$

$$\mathcal{L}_3 = G_3(\phi, X) \Box\phi$$

$$\mathcal{L}_4 = G_4(\phi, X) R + G_{4X}(\phi, X) \left[(\Box\phi)^2 - (\phi_{\mu\nu})^2 \right]$$

$$\mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu} \phi^{\mu\nu} - \frac{1}{6} G_{5X}(\phi, X) \left[(\Box\phi)^3 - 3(\phi_{\mu\nu})^2 \Box\phi + 2(\phi_{\mu\nu})^3 \right]$$

WHERE $X = -\frac{1}{2} \nabla_\mu\phi \nabla^\mu\phi$, $\phi_\mu := \nabla_\mu\phi$, $\phi_{\mu\nu} := \nabla_\nu\nabla_\mu\phi$, ...

$G_{4X} := \partial_X G_4$ $\quad$ $(\phi_{\mu\nu})^2 := \phi_{\mu\nu}\phi^{\mu\nu}$

$(\phi_{\mu\nu})^3 := \phi_{\mu\nu}\phi^{\mu\sigma}\phi_\sigma{}^\nu$

TESTING GRAVITY

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When Prof. Lovelock and I saw how complex the Lagrangian which yields the most general second-order scalar-tensor field equations in a space of four-dimensions were, we felt that clearly puts the kibosh on scalar-tensor field theories. There were just too many of them, and they are way too complicated. We wondered who would be crazy enough to work with such equations. Then crazy showed up! And is still here today. It will be the task of my

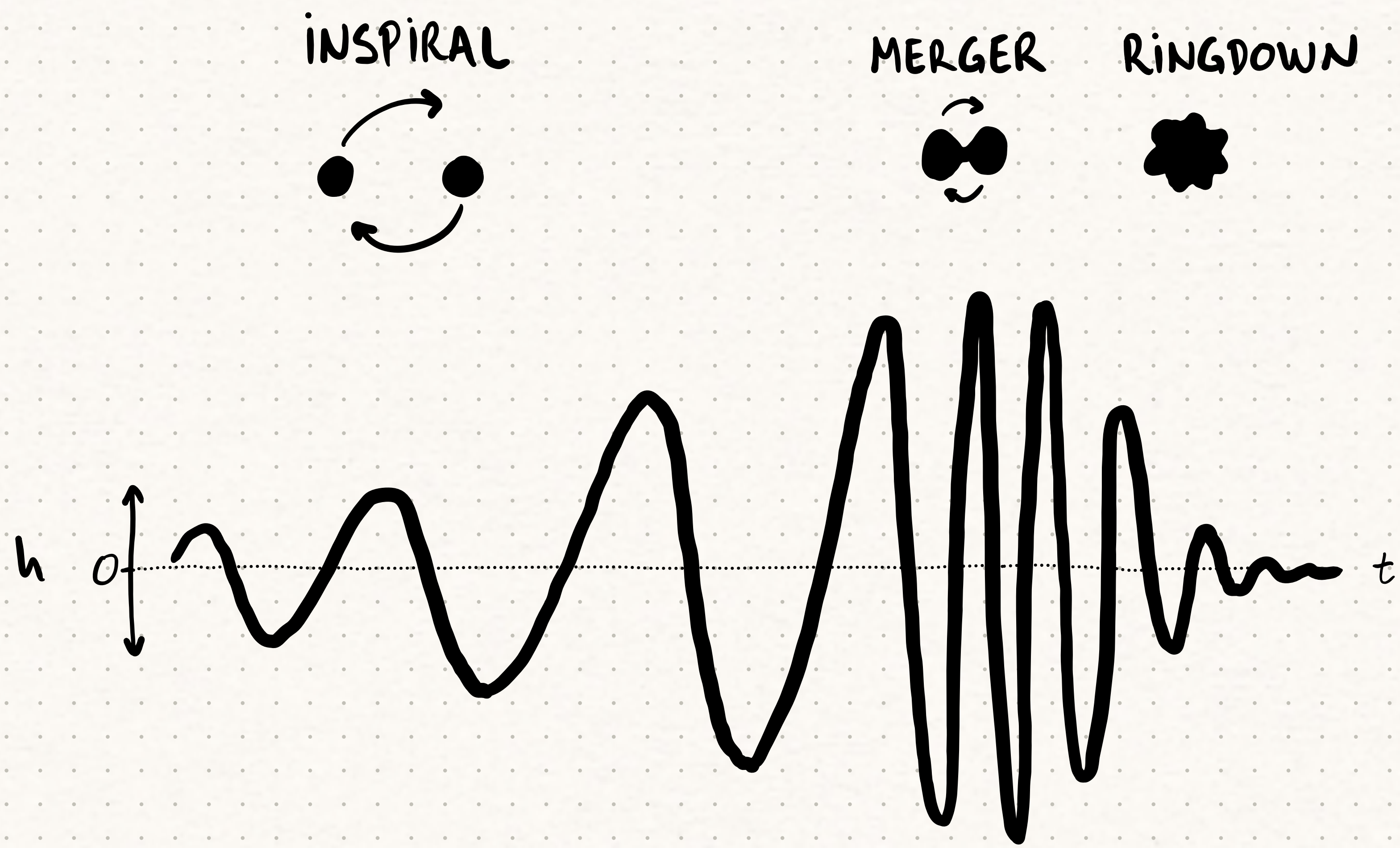
[2402.07538] HORNDESKI + SILVESTRI

TESTING GRAVITY

THEORY	$S = \int d^4x \sqrt{-g}\, R$	$S = \int d^4x \sqrt{-g}\, H$
$\downarrow$	$\downarrow$	$\downarrow$
OBSERVABLE	$\alpha = 0$	$\alpha \neq 0$ $\longrightarrow$ "SMOKING GUN SIGNAL"

GRAVITATIONAL WAVES

ASTROPHYSICAL SOURCES : MERGERS (BLACK HOLES / NEUTRON STARS)



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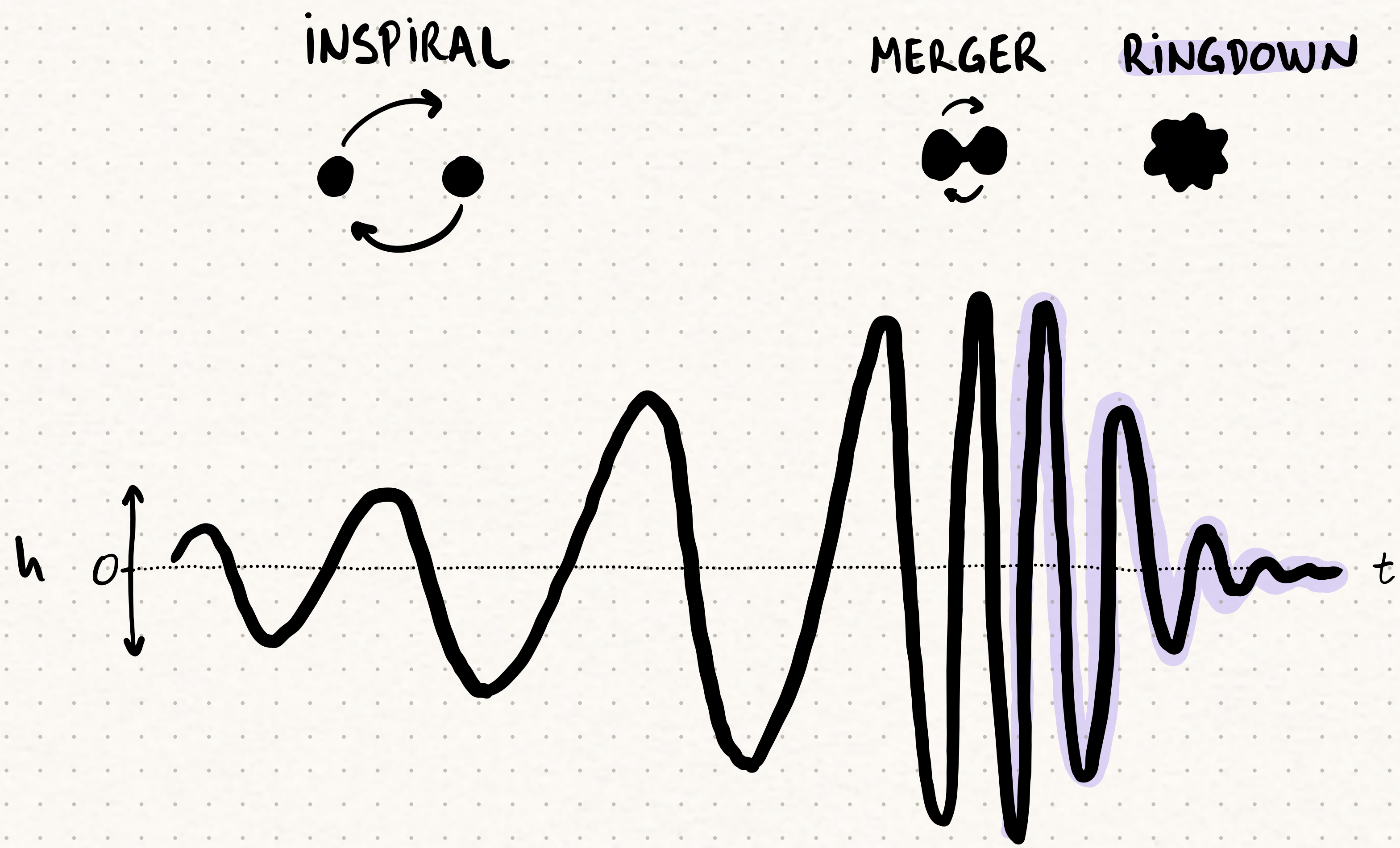
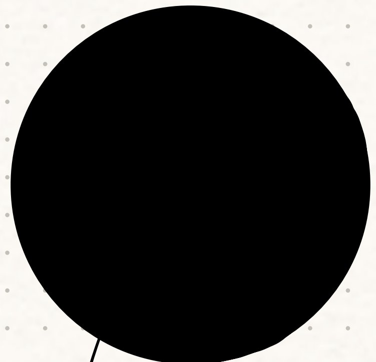


GRAVITATIONAL WAVES

RINGDOWN : BLACK HOLE PERTURBATION THEORY



$$g_{\mu\nu} = \bar{g}_{\mu\nu}$$

BACKGROUND

- SCHWARZSCHILD
- SCHWARZSCHILD - DE SITTER
- REISSNER-NORDSTRÖM
- ...

GRAVITATIONAL WAVES

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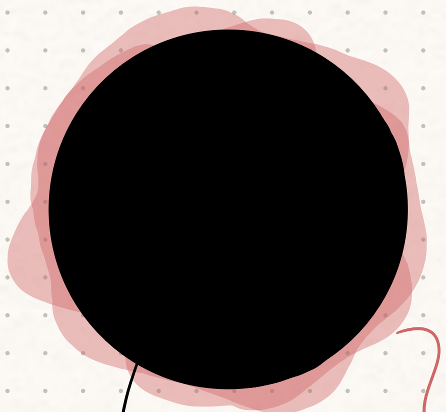

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

BACKGROUND

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- ...

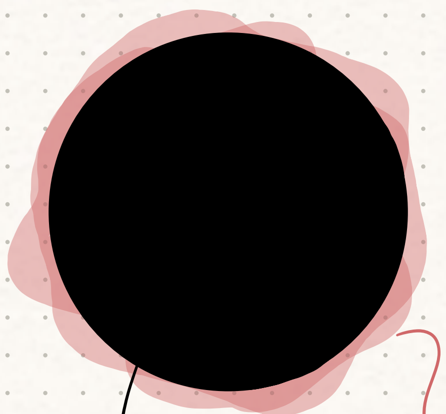
PERTURBATIONS

ODD

EVEN

GRAVITATIONAL WAVES

RINGDOWN : BLACK HOLE PERTURBATION THEORY



$$S = \int d^4x \sqrt{-g}\, R$$

$$\frac{d^2h}{dr_*^2} + \left[\omega^2 - V\right] h = 0$$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

BACKGROUND

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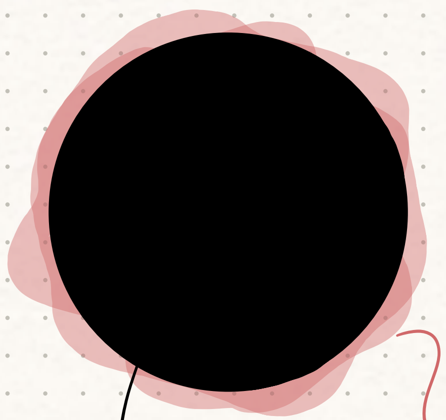
PERTURBATIONS

ODD → REGGE-WHEELER

EVEN → ZERILLI

GRAVITATIONAL WAVES

RINGDOWN : BLACK HOLE PERTURBATION THEORY



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$\omega(M, a)$

QUASINORMAL MODES

(NO HAIR THEOREM)

BACKGROUND

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PERTURBATIONS

ODD → REGGE-WHEELER

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GRAVITATIONAL WAVES

BLACK HOLE SPECTROSCOPY

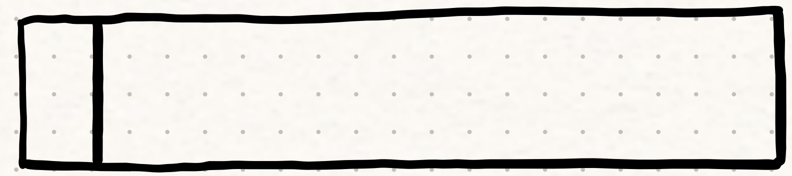
$\omega(M, a)$ • 1st QNM sets (M, a) 

GRAVITATIONAL WAVES

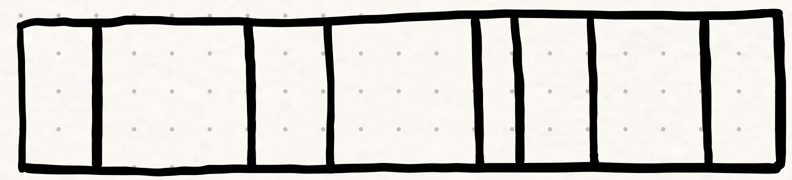
BLACK HOLE SPECTROSCOPY

$\omega(M,a)$

- 1st QNM sets (M, a)



- All other QNMs are fixed in GR



GRAVITATIONAL WAVES

BLACK HOLE SPECTROSCOPY

$\omega(M,a)$

- 1st QNM sets (M,a)

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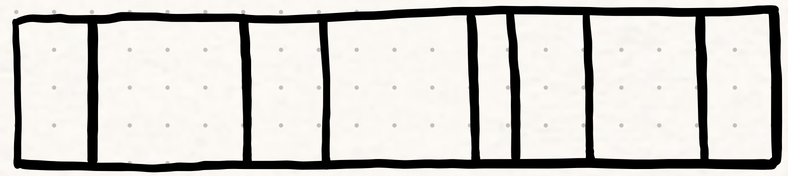
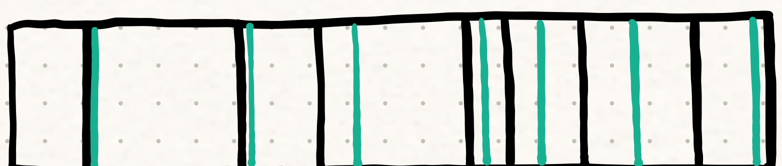


MEASURING QNMs PROVIDES CLEAN TESTS OF BACKGROUND GEOMETRY AND UNDERLYING THEORY



GR $\omega(M,a)$

MG $\omega(M,a,\alpha)$

SPEED OF GRAVITY

GR

$$S = \int d^4x \sqrt{-g}\, R(g_{\mu\nu})$$

$$\downarrow$$

$$\frac{d^2 h}{dr_*^2} + \left[w^2 + V\right] h = 0$$

$$\downarrow$$

$$w(M, a)$$

$$\alpha_T = 0$$

HORNDESKI

$$S = \int d^4x \sqrt{-g}\, H(g_{\mu\nu}, \phi)$$

$$\downarrow$$

$$\frac{d^2 h}{dr_*^2} + \left[w^2 (1 + \alpha_T) + V + \alpha_T \delta V\right] h = 0$$

$$\downarrow$$

$$w(M, a, \alpha_T)$$

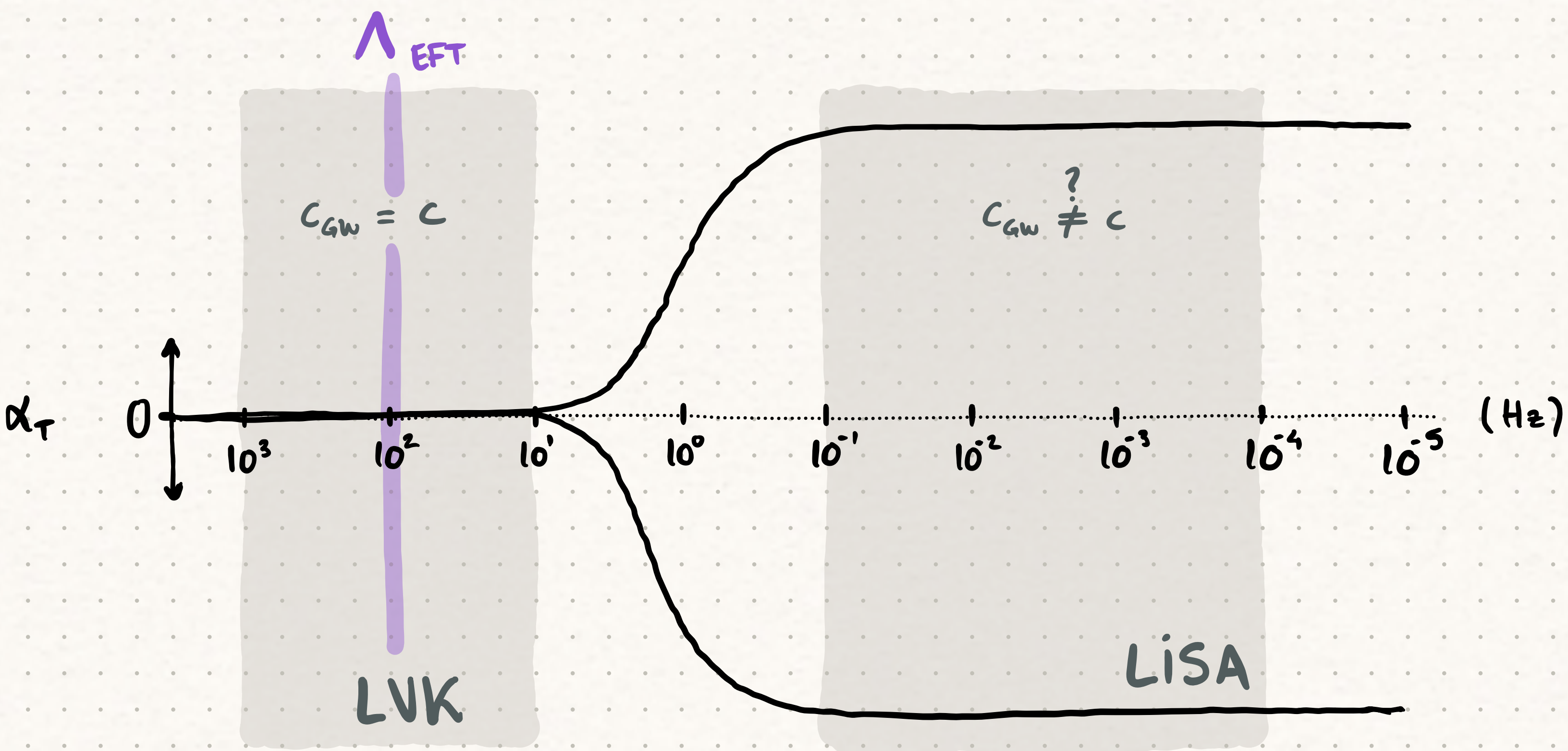
$$\alpha_T = \frac{c_{GW} - c}{c} \neq 0$$

GRAVITATIONAL WAVE SPEED EXCESS

SPEED OF GRAVITY

WHAT DO WE KNOW ABOUT α_T ?

- LIGO : $\alpha_T \lesssim 10^{-15}$ (GW170817)
- DARK ENERGY EFTs : CUTOFF AT $\sim 10^{2}$ Hz [1806.09417] MELVILLE + DE RHAM



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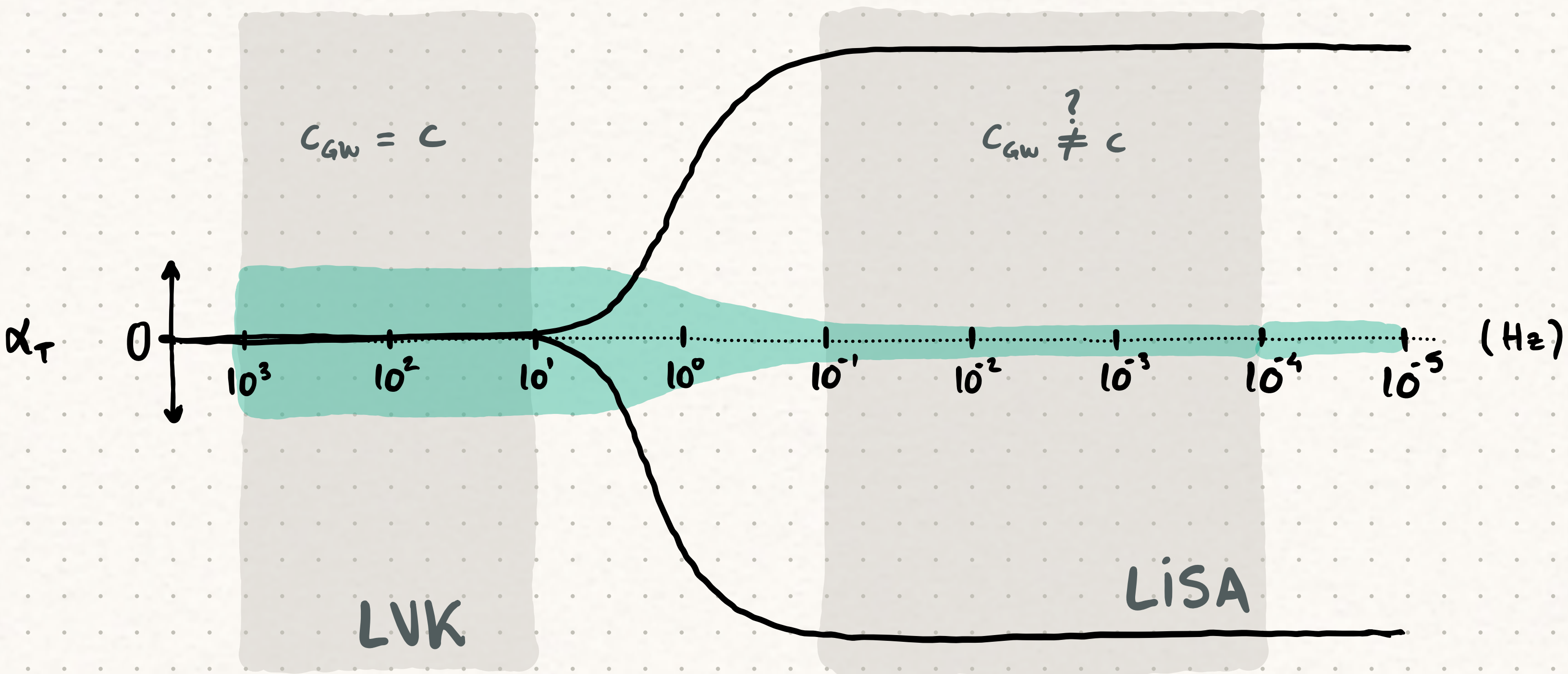

FISCHER FORECASTS :

FOR 1 LOUD MERGER :

- LISA : $\alpha_T \lesssim 10^{-4}$
- LIGO/ET $\alpha_T \lesssim 10^{-1}$

[2301.10272]

└ SS, JOHANNES NOLLER

sergisl / ringdown-calculations Public

SPEED OF GRAVITY

[2301.10272] SS + JOHANNES NOLLER

BACKGROUND:

$$\bar{g}_{\mu\nu} = -f\,dt^2 + f^{-1}dr^2 + r^2 d\Omega_2^2 , \qquad f = 1 - \frac{2M}{r}$$

$$\bar{\phi} = \hat{\phi} + \varepsilon\, \delta\phi(r) \quad , \qquad \delta\phi = \varphi_c \frac{2M}{r}$$

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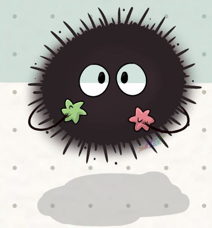
SPEED OF GRAVITY



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THEORY:

HORNDESKI WITH $G_{4\phi} = 0$

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MODIFIED REGGE - WHEELER EQUATION

$$\frac{d^2 h}{dr_*^2} + \left[w^2(1+\alpha_T) + V + \alpha_T \delta V\right]h = 0$$

$$\alpha_T = -f(2M)^2 G_T\, \delta\phi'^2 \qquad G_T = \frac{G_{4X} - G_{5\phi}}{G_4}$$

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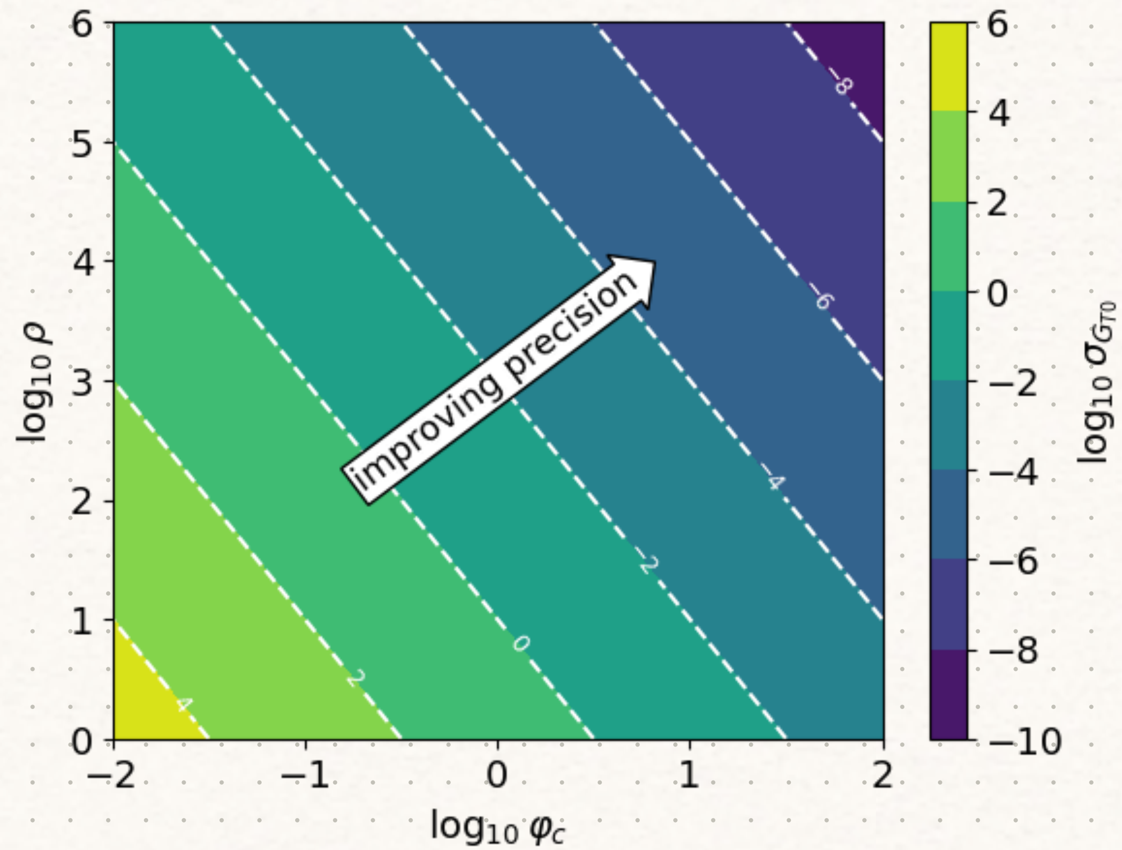

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Ongoing Work: Time-Dependent Solutions

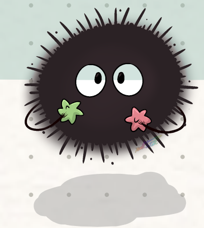
$$\bar{g}_{\mu\nu} = -f\,dt^2 + f^{-1}dr^2 + r^2 d\Omega_2^2\,, \qquad f = 1 - \frac{2M}{r} - \frac{1}{3}\Lambda r^2$$

$$\bar{\phi} = qt + \psi(r)$$

Solutions for shift + reflection symmetric Horndeski: $G_2(X)$, $G_4(X)$

[1312.3204] Babichev + Charmousis, [1403.4364] Kobayashi + Tanahashi

ONGOING WORK : TIME-DEPENDENT SOLUTIONS



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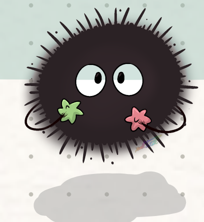
SOLUTIONS FOR SHIFT + REFLECTION SYMMETRIC HORNDESKI : $G_2(X)$, $G_4(X)$

└ [1312.3204] BABICHEV + CHARMOUSIS , [1403.4364] KOBAYASHI + TANAHASHI

HOWEVER, THEY ARE PRONE TO INSTABILITY ISSUES

└ [1510.07400] OGAWA + KOBAYASHI + SUYAMA , [1803.11444] BABICHEV + CHARMOUSIS + ESPOSITO - FARÈSE + LEHÉBEL
[1610.00432] TAKAHASHI + SUYAMA, [1904.03554] TAKAHASHI + MOTOHASHI + MINAMITSUJI, [1907.00699] DE RHAM + ZHANG

ONGOING WORK : TIME-DEPENDENT SOLUTIONS



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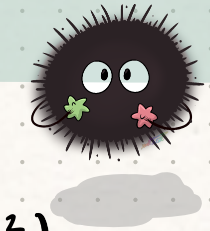
ALL CASES STUDIED ASSUMED $X := -\frac{1}{2}\partial_\mu\bar{\phi}\,\partial^\mu\bar{\phi}$ = constant

SOLUTION WITH $X \neq$ constant FOR $G_2 = -2\Lambda + 2\eta\sqrt{X}$, $G_4 = 1 + \lambda\sqrt{X}$

└ [2310.11919] BAKOPOULOS + CHARMOUSIS + KANTI + LECOEUR + NAKAS

CAN $X \neq$ constant HELP?

Ongoing work: Time-dependent solutions



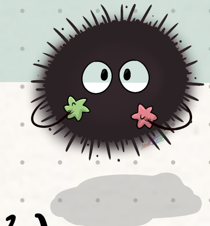
Background

SdS, $\bar{\phi} = qt + \psi(r)$

Theory

$$\mathcal{L} = -2\Lambda + 2\eta\sqrt{X} + (1 + \lambda\sqrt{X})R + \frac{\lambda}{2\sqrt{X}}\left(\Box\phi^2 + \phi_{\mu\nu}^2\right)$$

Ongoing Work: Time-Dependent Solutions



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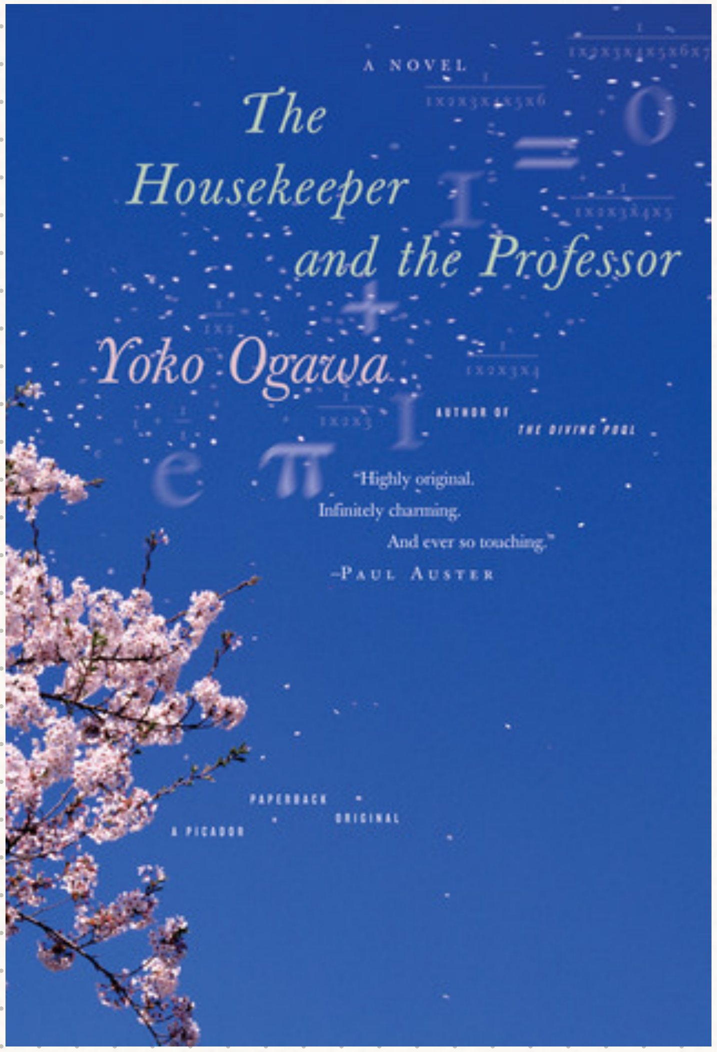

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We called him the Professor. And he called my son Root, because, he said, the flat top of his head reminded him of the square root sign.

"There's a fine brain in there," the Professor said, mussing my son's hair. Root, who wore a cap to avoid being teased by his friends, gave a wary shrug. "With this one little sign we can come to know an infinite range of numbers, even those we can't see." He traced the symbol in the thick layer of dust on his desk.

$\sqrt{}$

Of all the countless things my son and I learned from the Professor, the meaning of the square root was among the most important. No doubt he would have been bothered by my use of the word *countless*--too sloppy, for he believed that the very origins of the universe could be explained in the exact language of numbers--but I don't know how else to put it. He taught us about enormous prime numbers with more than a hundred thousand places, and the largest number of all, which was used in mathematical proofs and was in the *Guinness Book of Records*, and about the idea of something beyond infinity. As interesting as all this was, it could never match the experience of simply spending time with the Professor. I remember when he taught us about the spell cast by placing numbers under this square root sign. It was a rainy evening in early April. My son's schoolbag lay abandoned on

"I'm going to call you Root," he said. "The square root sign is a generous symbol, it gives shelter to all the numbers." And he quickly took off the note on his sleeve and made the addition: "The new housekeeper ... and her son, ten years old, $\sqrt{}$."

ONGOING WORK : TIME-DEPENDENT SOLUTIONS

BACKGROUND

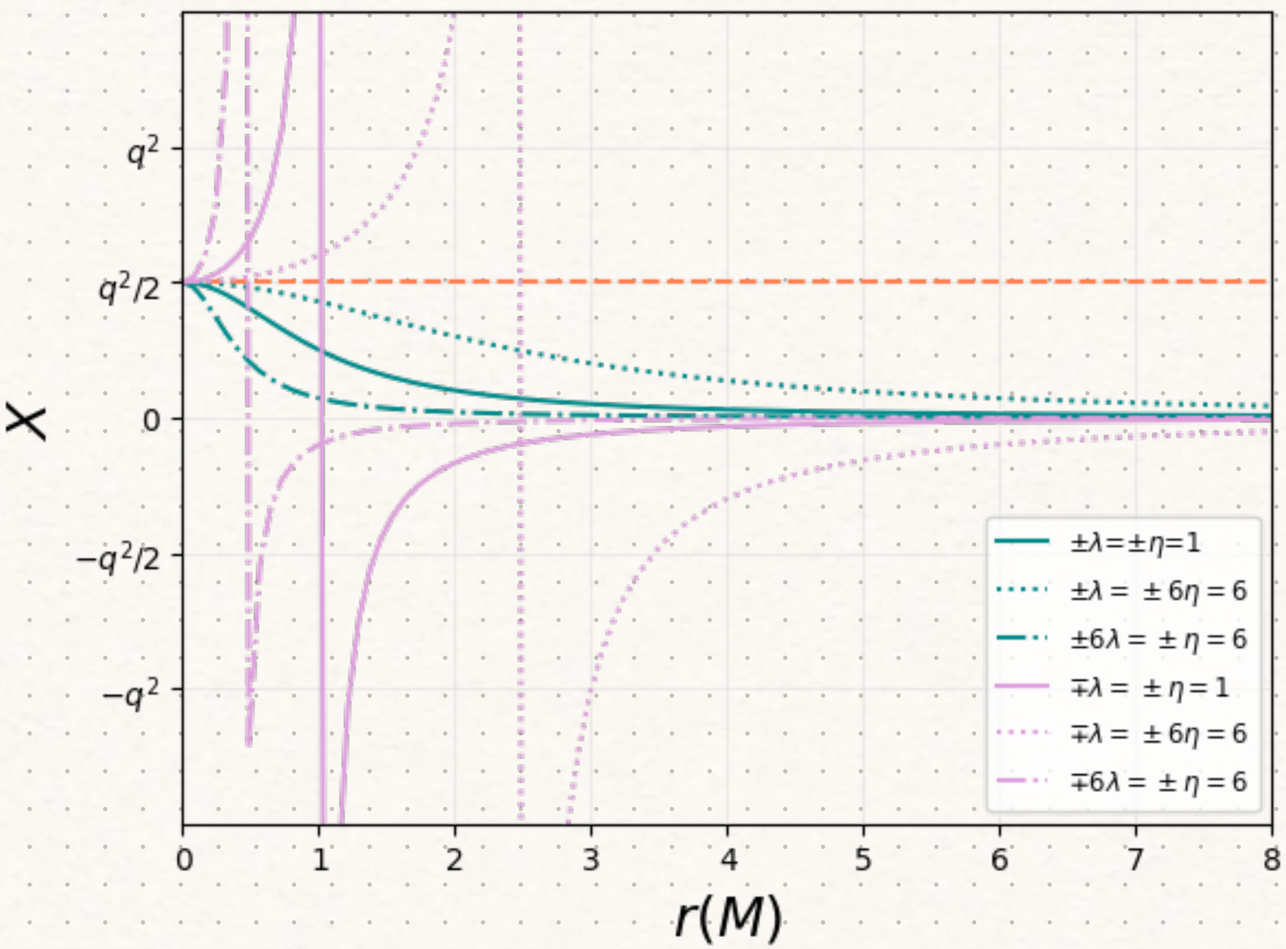
SdS , $\bar{\phi} = qt + \psi(r)$

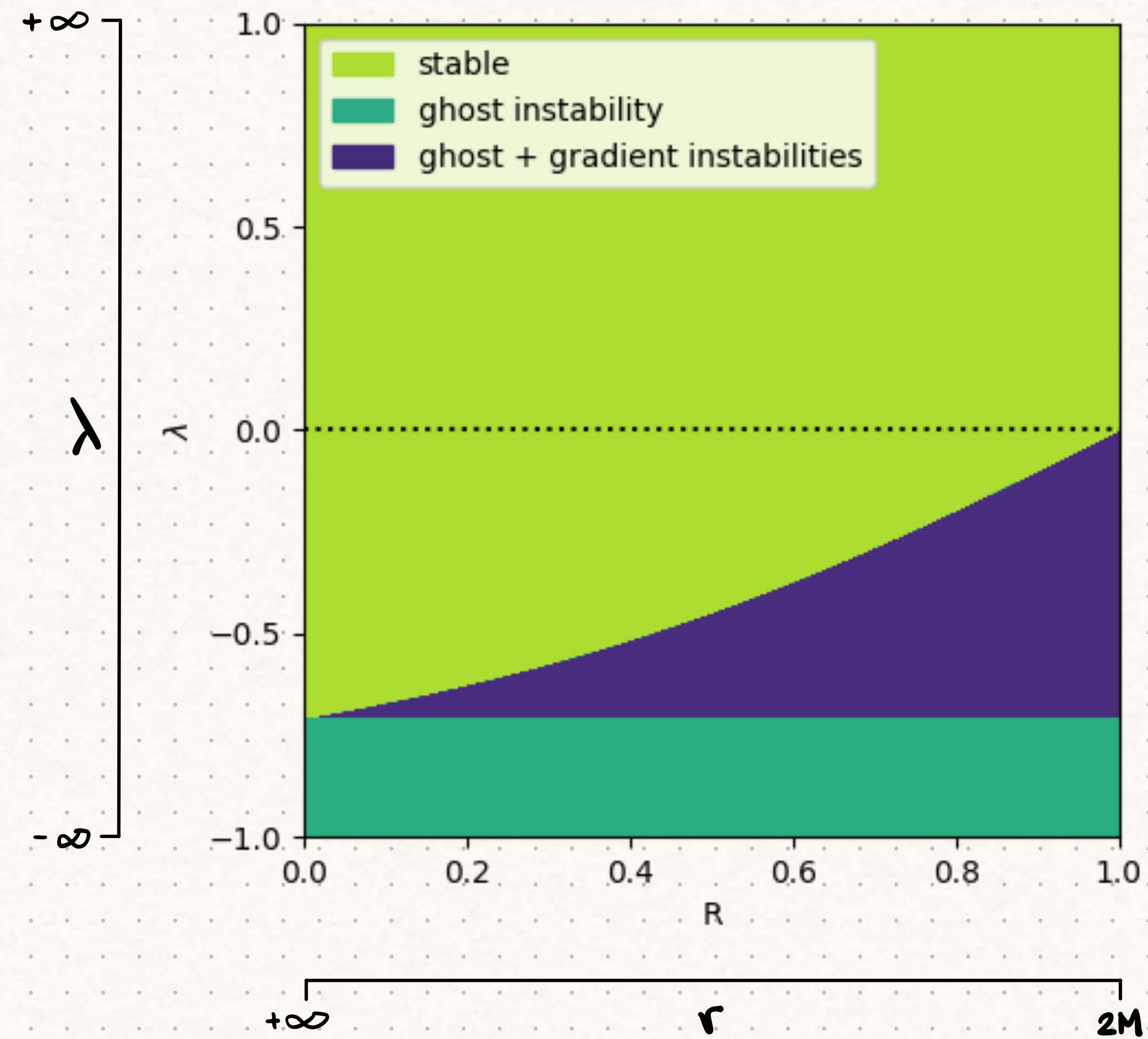
THEORY

$$\mathcal{L} = -2\Lambda + 2\eta\sqrt{X} + (1 + \lambda\sqrt{X})R + \frac{\lambda}{2\sqrt{X}}(\Box\phi^2 + \phi_{\mu\nu}{}^2)$$

PRELIMINARY RESULTS:

ODD PARITY MODES CAN BE STABLE





SUMMARY

- QNMs CAN BE USED TO TEST NEW GRAVITATIONAL PHYSICS
- SPEED OF GWs $\neq$ SPEED OF LIGHT IN SOME HORNDESKI THEORIES
- QNMs ARE α_T-DEPENDENT IF $\exists$ SCALAR HAIR
- LISA CAN CONSTRAIN $|\alpha_T| \lesssim 10^{-4}$ WITH RINGDOWN OF ONE SMBH MERGER

ONGOING WORK:

- PROVING STABILITY OF ODD PARITY MODES FOR TIME-DEPENDENT SOLUTIONS IN 'ROOT-X' THEORY
- QUASINORMAL MODES ?

THANKS!

@sergisirera

Backup Slides

BLACK HOLE PERTURBATIONS IN REGGE-WHEELER GAUGE

$$h_{\mu\nu} = h_{\mu\nu}^{odd} + h_{\mu\nu}^{even}$$

$$h_{\mu\nu}^{odd} = \begin{pmatrix} 0 & 0 & -h_0 \frac{1}{\sin\theta} \partial_\phi Y_{\ell m} & h_0 \sin\theta \partial_\theta Y_{\ell m} \\ 0 & 0 & -h_1 \frac{1}{\sin\theta} \partial_\phi Y_{\ell m} & h_1 \sin\theta \partial_\theta Y_{\ell m} \\ -h_0 \frac{1}{\sin\theta} \partial_\phi Y_{\ell m} & -h_1 \frac{1}{\sin\theta} \partial_\phi Y_{\ell m} & 0 & 0 \\ h_0 \sin\theta \partial_\theta Y_{\ell m} & h_1 \sin\theta \partial_\theta Y_{\ell m} & 0 & 0 \end{pmatrix}$$

$$h_{\mu\nu}^{even} = \begin{pmatrix} f H_0 & H_1 & 0 & 0 \\ H_1 & \frac{1}{f} H_2 & 0 & 0 \\ 0 & 0 & r^2 K & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta K \end{pmatrix} Y_{\ell m}$$

QUADRATIC ACTION IN GR

$$S^{(2)} = \frac{1}{4}\int\sqrt{-g}\,d^4x\Big[-h_{\mu\nu}h^{\mu\nu} + \nabla_\sigma h\nabla^\sigma h - \nabla_\mu h_{\nu\sigma}\nabla^\mu h^{\nu\sigma} + 2\nabla_\mu h^{\mu\nu}\nabla_\sigma h_\nu{}^\sigma - 2\nabla^\mu h\nabla_\nu h_\mu{}^\nu$$
$$+ 2\left(h_\sigma{}^\nu h^{\sigma\mu} - hh^{\mu\nu}\right)R_{\mu\nu} - \left(h_{\mu\nu}h^{\mu\nu} - \tfrac{1}{2}h^2\right)R + 2h^{\mu\nu}h^{\sigma\lambda}R_{\mu\sigma\nu\lambda}\Big]$$

QUADRATIC ACTION HORNDESKI ODD SECTOR IN COMPONENTS

$$S^{(2)} = \int dt\,dr\left[a_1h_0^2 + a_2h_1^2 + a_3\left(\dot{h}_1^2 + h_0'^2 - 2h_0'\dot{h}_1 + \frac{4}{r}\dot{h}_1h_0\right)\right]$$

$$a_1 = \frac{\ell(\ell+1)}{2r^2}\left[(r\mathcal{H})' + \frac{(\ell-1)(\ell+2)\mathcal{F}}{2B} + \frac{r^2}{B}\mathcal{E}_A\right]$$

$$a_2 = -\frac{\ell(\ell+1)}{2}B\left[\frac{(\ell-1)(\ell+2)\mathcal{G}}{2r^2} + \mathcal{E}_B\right]$$

$$a_3 = \frac{\ell(\ell+1)}{4}\mathcal{H}$$

$$\mathcal{F} = 2\left(G_4 + \tfrac{1}{2}B\phi'X'G_{5X} - XG_{5\phi}\right)$$

$$\mathcal{G} = 2\left[G_4 - 2XG_{4X} + X\left(\frac{B'}{2}\phi'G_{5X} + G_{5\phi}\right)\right]$$

$$\mathcal{H} = 2\left[G_4 - 2XG_{4X} + X\left(\frac{B}{r}\phi'G_{5X} + G_{5\phi}\right)\right]$$

$$S^{(2)} = \frac{\ell(\ell+1)}{4(\ell-1)(\ell+2)}\int dt\,dr_*\left[\frac{\mathcal{F}}{\mathcal{G}}\dot{Q}^2 - \left(\frac{dQ}{dr_*}\right)^2 - V(r)Q^2\right]$$

$$h_0 = -\frac{(r^2a_3q)'}{r^2a_1 - 2(ra_3)'}$$

$$h_1 = \frac{a_3}{a_2}\dot{q}$$

$$q = \frac{\sqrt{\mathcal{F}}}{r\mathcal{H}}Q$$

EXISTING & UPCOMING α_T CONSTRAINTS

$\lvert\alpha_T\rvert \lesssim 10^{0}$	$f \sim 10^{-18} - 10^{-14}$ Hz	CMB & LSS
$\lvert\alpha_T\rvert \lesssim 10^{-2}$	$f \sim 10^{-5}$ Hz	HULSE - TAYLOR BINARY [1507.05047] BELTRAN JIMENEZ ET. AL.
$\lvert\alpha_T\rvert \lesssim 10^{-12}$	$f \sim 10^{-1} - 10^{-4}$ Hz	ECLIPSING WHITE-DWARF BINARY [1908.00678] LITTENBERG ET. AL.
$\lvert\alpha_T\rvert \lesssim 10^{-4}$	$f \sim 10^{-1} - 10^{-4}$ Hz	REDSHIFT-INDUCED f-DEPENDENCE [2203.00566] BAKER ET. AL.
$\lvert\alpha_T\rvert \lesssim 10^{-17}$	$f \sim 10^{-1} - 10^{-4}$ Hz	f-DEPENDENT WAVEFORMS [2207.10096] HARRY AND NOLLER
$\lvert\alpha_T\rvert \lesssim 10^{-15}$	$f \sim 10^{3} - 10^{-4}$ Hz	MULTIBAND [2209.14398] BAKER ET. AL. [1602.06951] SESANA [2207.10096] HARRY AND NOLLER
$\lvert\alpha_T\rvert \lesssim 10^{-15}$	$f \sim 10^{2}$ Hz	GW170817