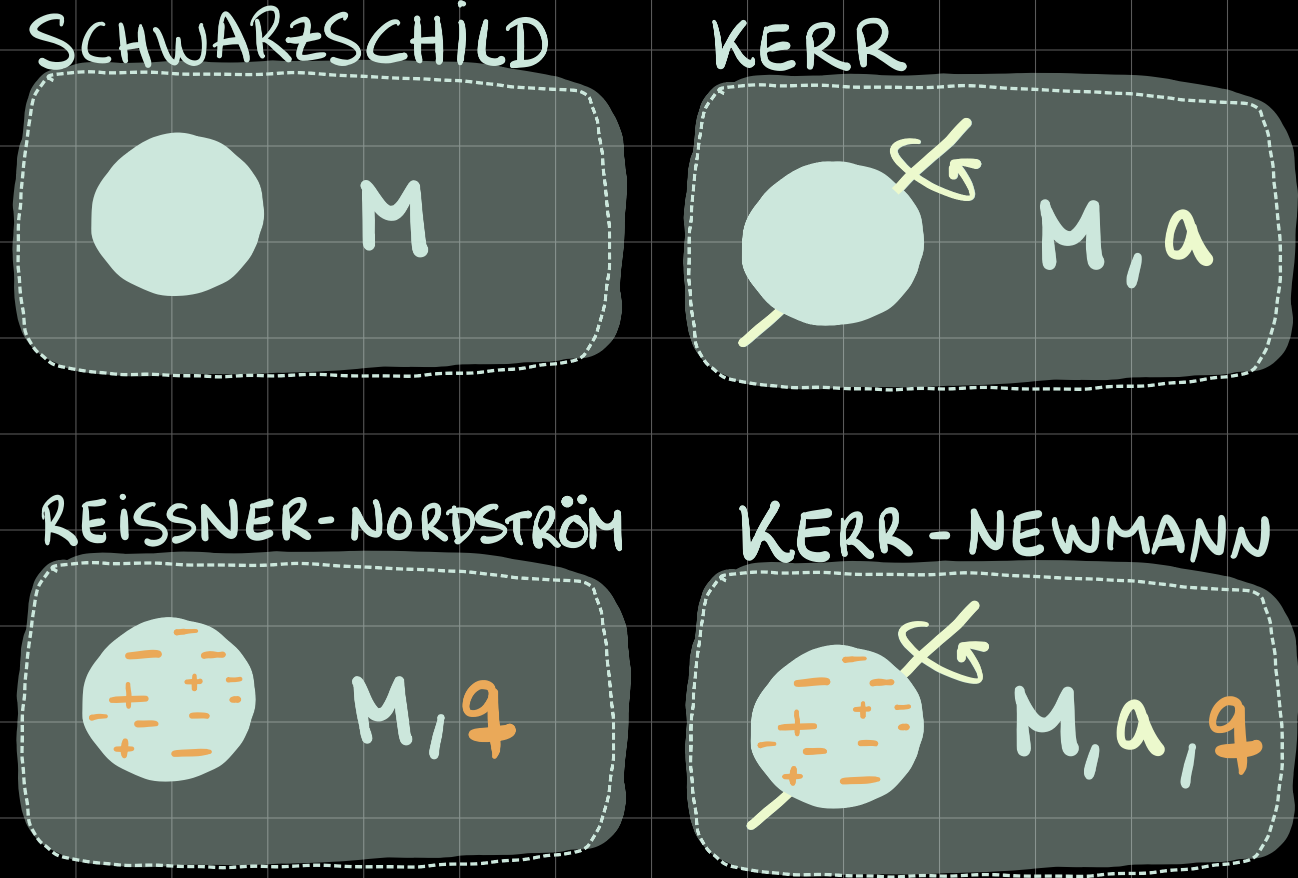
SOME ASPECTS OF BLACK HOLES

SERGI SIRERA LAHOZ

GW GROUP MEETING

22 / 06 / 2023

TYPES OF BLACK HOLES



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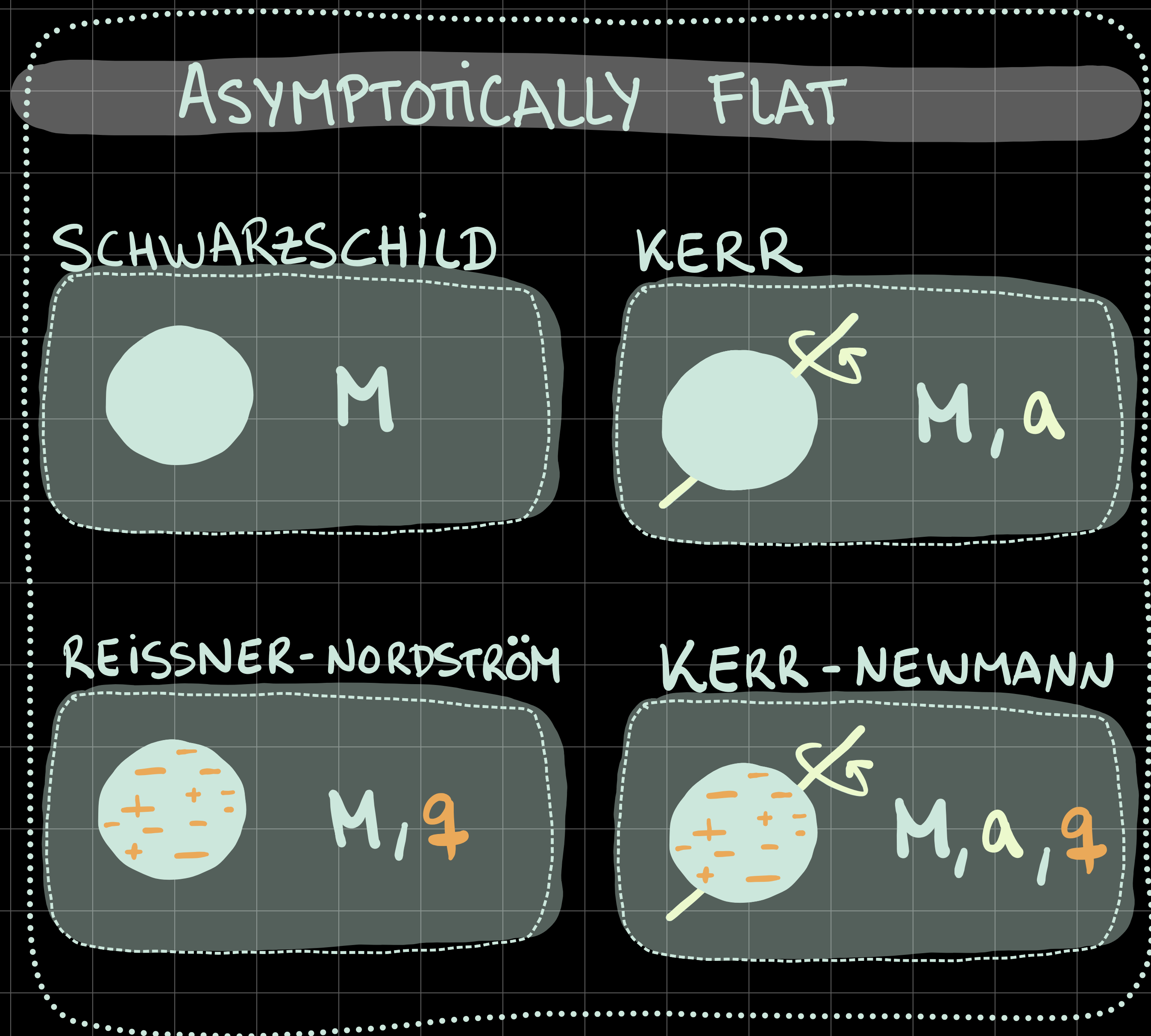
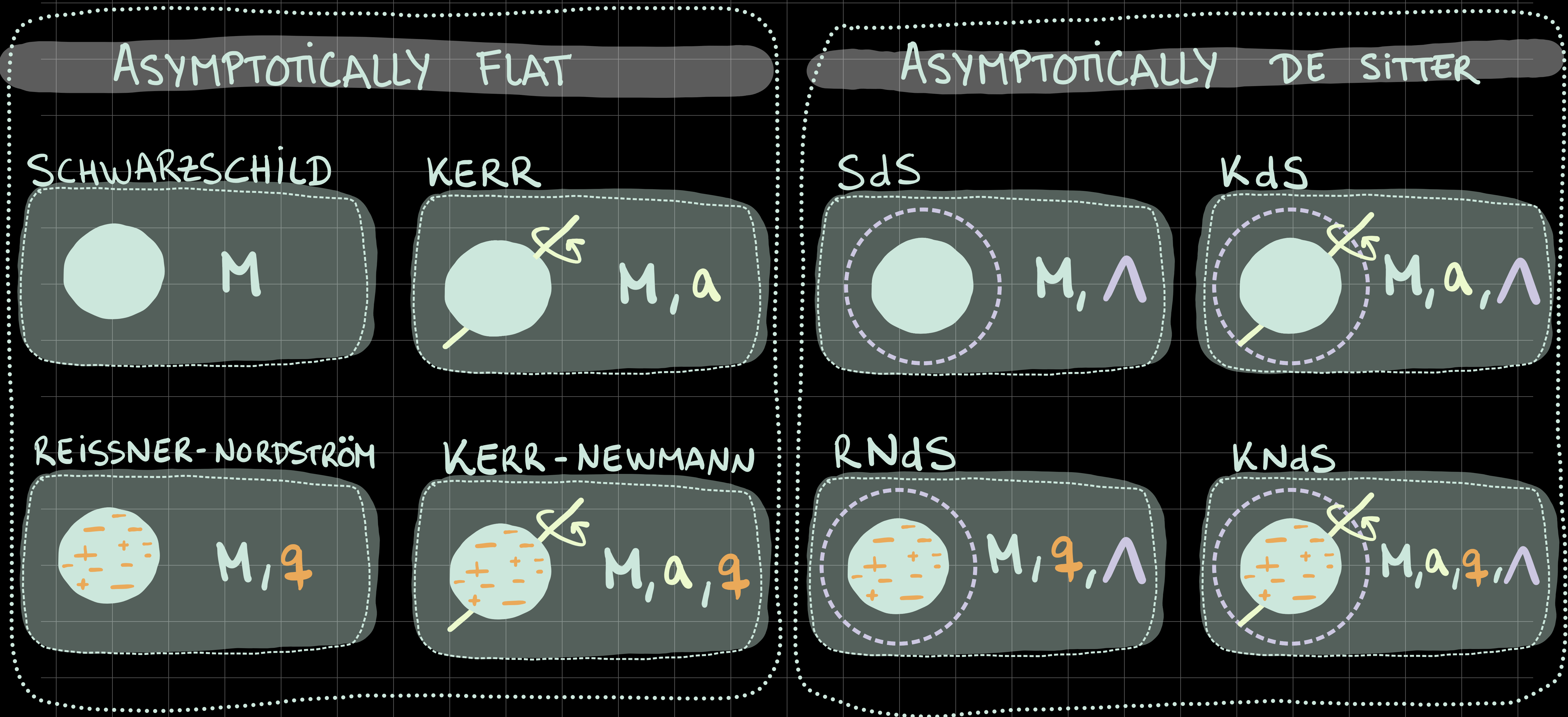
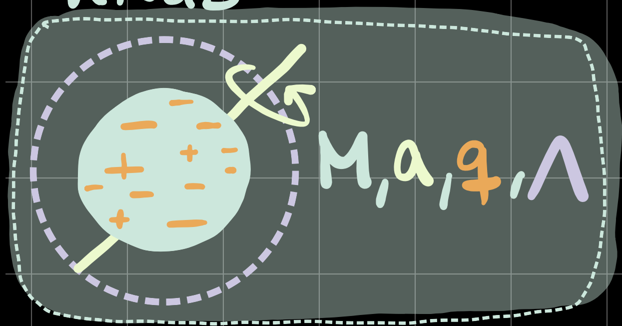


TYPES OF BLACK HOLES



KNdS



$$ds^2 = -\frac{\Delta_r}{I^2\rho^2}\left(dt - a\sin^2\theta\, d\phi\right)^2 + \frac{\Delta_\theta \sin^2\theta}{I^2\rho^2}\left(a\,dt - (r^2+a^2)\,d\phi\right)^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2$$

$$\Delta_r = \left(1 - \frac{\Lambda}{3}r^2\right)(r^2+a^2) - 2Mr + q^2$$

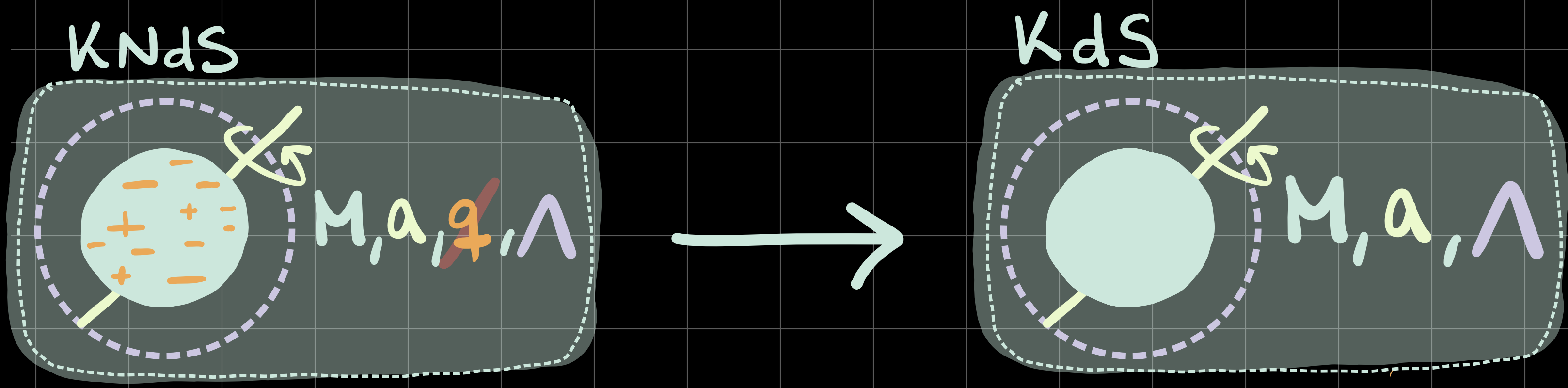
$$\Delta_\theta = 1 + \frac{\Lambda}{3}a^2\cos^2\theta$$

$$I = 1 + \frac{\Lambda}{3}a^2$$

$$\rho^2 = r^2 + a^2\cos^2\theta$$

GEOMETRIC UNITS

$G = c = 1$



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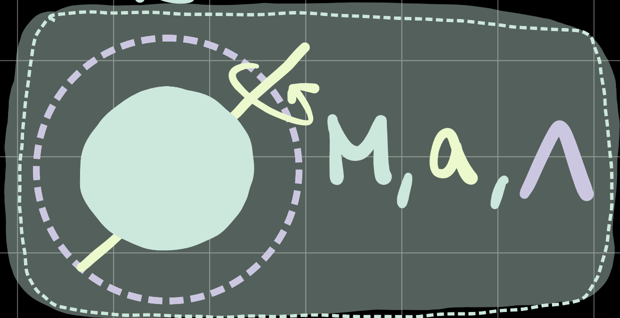

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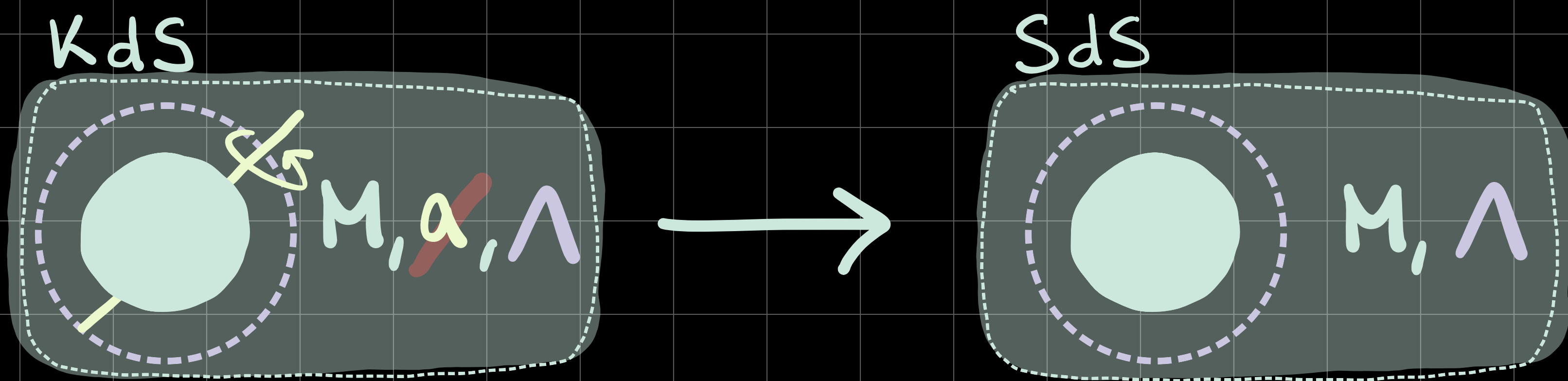

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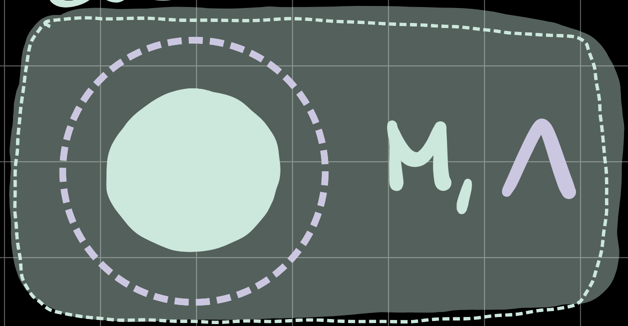
SdS



$$ds^2 = -\frac{\Delta_r}{\rho^2}dt^2 + \frac{\rho^2}{\Delta_r}dr^2 + \frac{\rho^2}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta \sin^2\theta}{\rho^2} r^4 d\phi^2$$

$$\Delta_r = \left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right) r^2$$

$$\Delta_\theta = 1$$

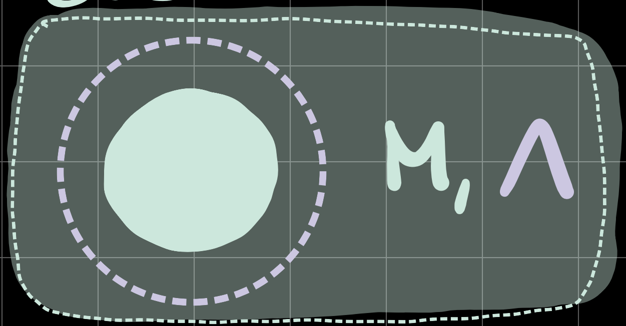
$$I = 1$$

$$\rho^2 = r^2$$

$$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

$$f = \left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right)$$

SdS



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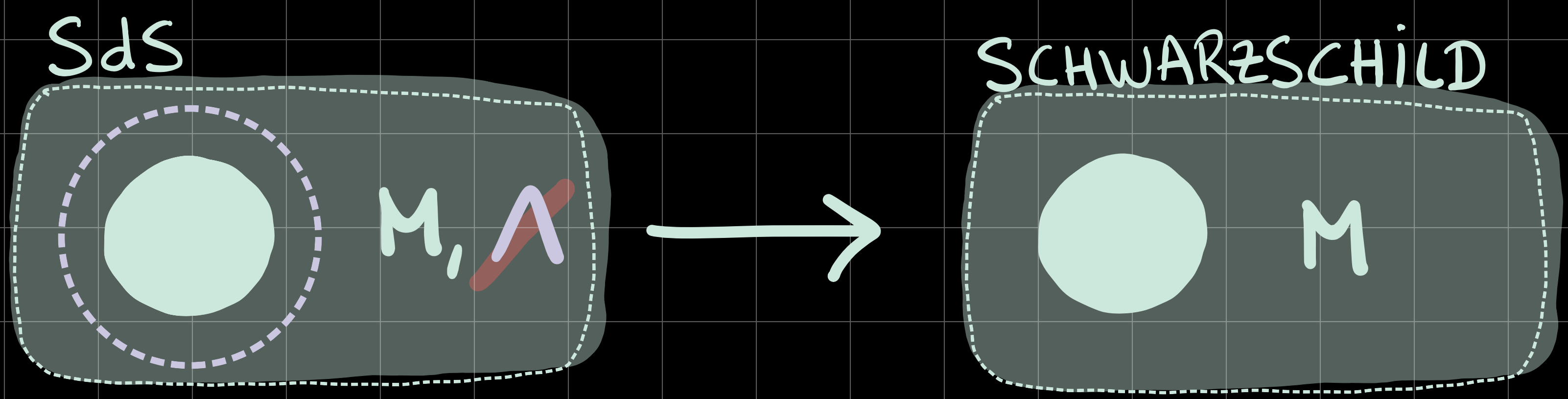

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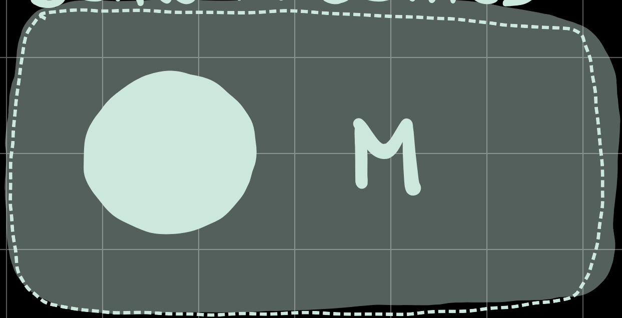
SCHWARZSCHILD



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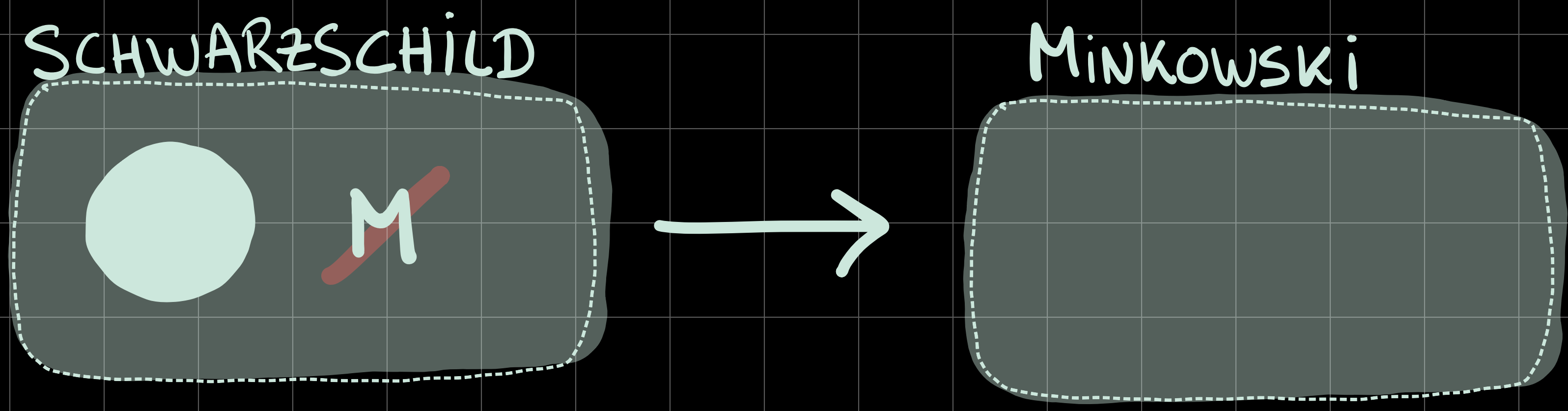

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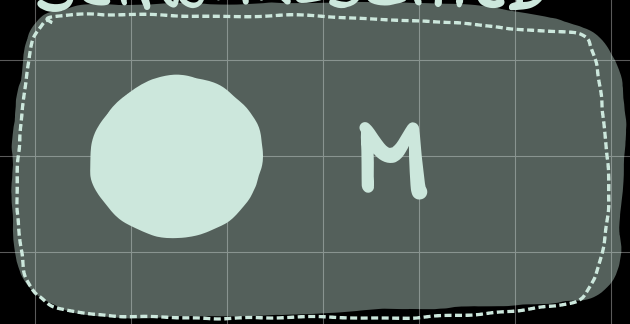
$$ds^2 = -dt^2 + dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

$$= -dt^2 + dx^2 + dy^2 + dz^2$$

SCHWARZSCHILD



(t, r, θ, ϕ)

$$ds^2 = -f\,dt^2 + f^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right), \qquad f = \left(1 - \frac{2M}{r}\right)$$

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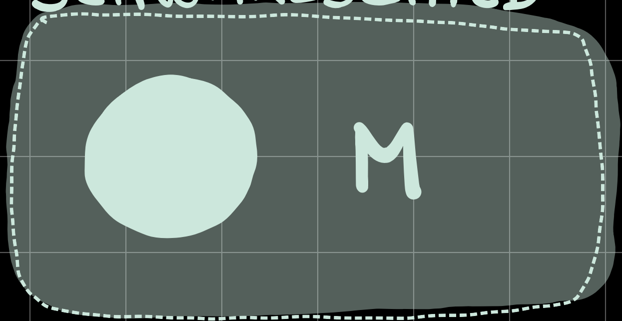

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$$dr_* = f^{-1} dr, \longrightarrow \begin{aligned} du &= dt - dr_* \\ dv &= dt + dr_* \end{aligned} \rightarrow dudv = dt^2 - dr_*^2$$

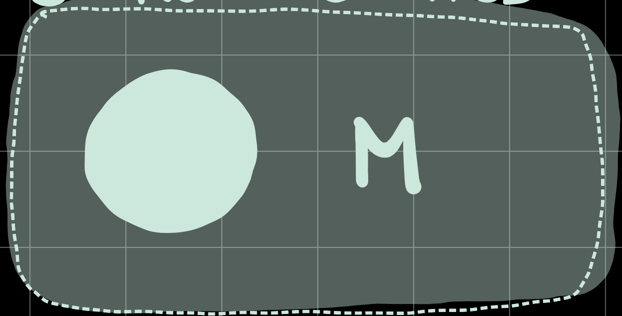
$$\downarrow$$

$$\begin{aligned} -f\,dudv &= -f\,dt^2 + f\,dr_*^2 \\ &= -f\,dt^2 + f^{-1} dr^2 \end{aligned}$$

(u, v, θ, ϕ)

$$ds^2 = -f\,dudv + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2 \right)$$

SCHWARZSCHILD



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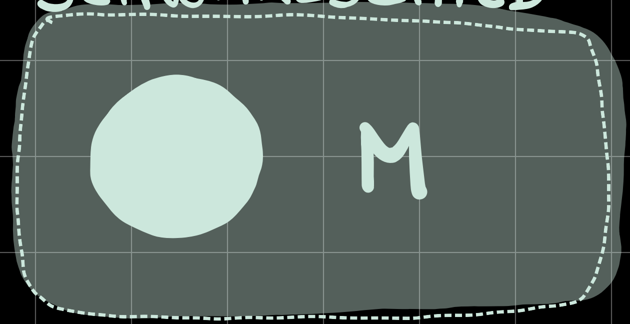


(u, v, θ, ϕ)

$$ds^2 = -f\,du\,dv + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)$$

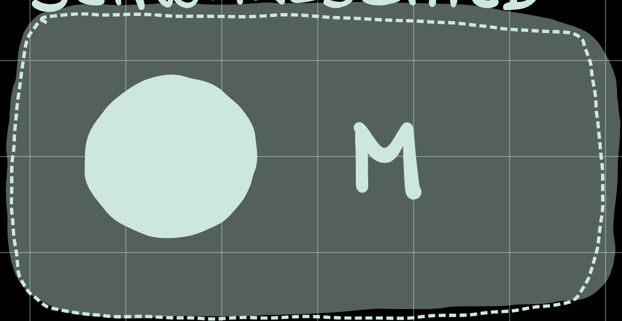
$$U = -\exp\left(-\frac{u}{4M}\right), \quad V = \exp\left(\frac{v}{4M}\right) \to UV = -\exp\left(\frac{r_*}{2M}\right) = -e^{\frac{r}{2M}}\left(\frac{r}{2M} - 1\right)$$

$$\searrow \frac{V}{U} = -e^{\frac{t}{2M}}$$

(U, V, θ, ϕ)

$$ds^2 = -\frac{32M^3}{r} e^{-\frac{r}{2M}}\,dU\,dV + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)$$

SCHWARZSCHILD



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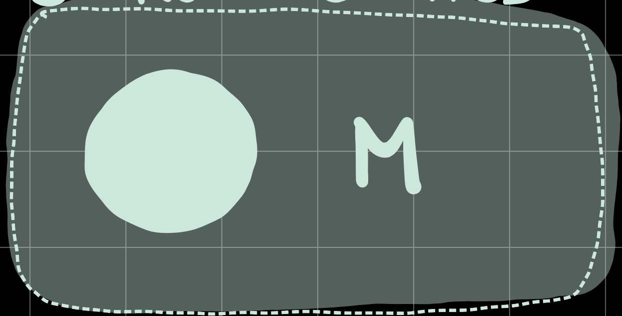

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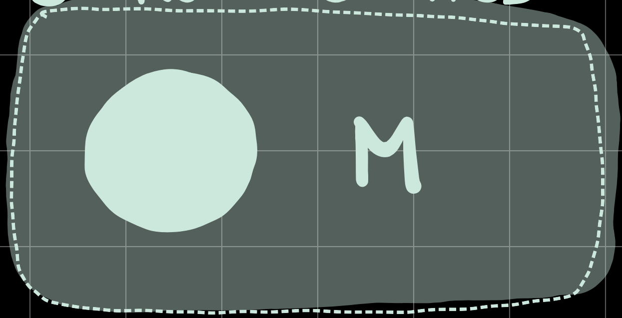
$$ds^2 = -\frac{32M^3}{r} e^{-\frac{r}{2M}} dU\,dV + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

$$U = \tan\tilde{U}, \quad V = \tan\tilde{V}$$

$(\tilde{U}, \tilde{V}, \theta, \phi)$

$$ds^2 = (2\cos\tilde{U}\cos\tilde{V})^{-2} \left[-4\,\frac{32M^3}{r} e^{-\frac{r}{2M}} d\tilde{U}\,d\tilde{V} + r^2 \cos^2\tilde{U} \cos^2\tilde{V}\, d\Omega^2 \right]$$

SCHWARZSCHILD



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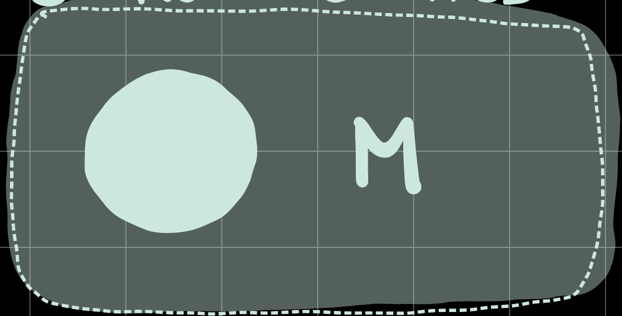

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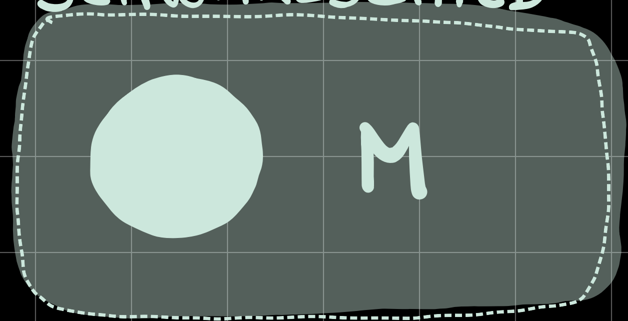
CAUSAL STRUCTURE PRESERVED BY WEYL TRANSFORMATION

$$ds^2 \rightarrow d\tilde{s}^2 = \Lambda^2(x)\,ds^2 \quad , \quad \Lambda = 2\cos\tilde{u}\cos\tilde{V}$$

$(\tilde{u}, \tilde{v}, \theta, \phi)$

$$ds^2 = -4\,\frac{32M^3}{r}\,e^{-\frac{r}{2M}}\,d\tilde{U}\,d\tilde{V} + r^2\cos^2\tilde{u}\,\cos^2\tilde{V}\,d\Omega^2$$

SCHWARZSCHILD



$(\tilde{u}, \tilde{v}, \theta, \phi)$

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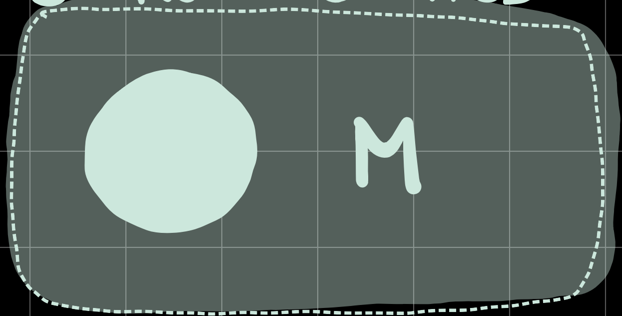
$$-\frac{\pi}{2} \leq \tilde{u}, \tilde{v} \leq \frac{\pi}{2} \longrightarrow \text{singularity } r=0 \text{ at } \tilde{u} + \tilde{v} = \pm\frac{\pi}{2}$$

CONVENIENT TO DEFINE

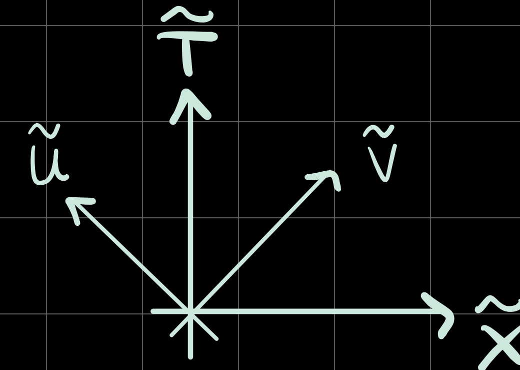
$$\tilde{u} = \tilde{T} - \tilde{X}$$
$$\tilde{v} = \tilde{T} + \tilde{X}$$

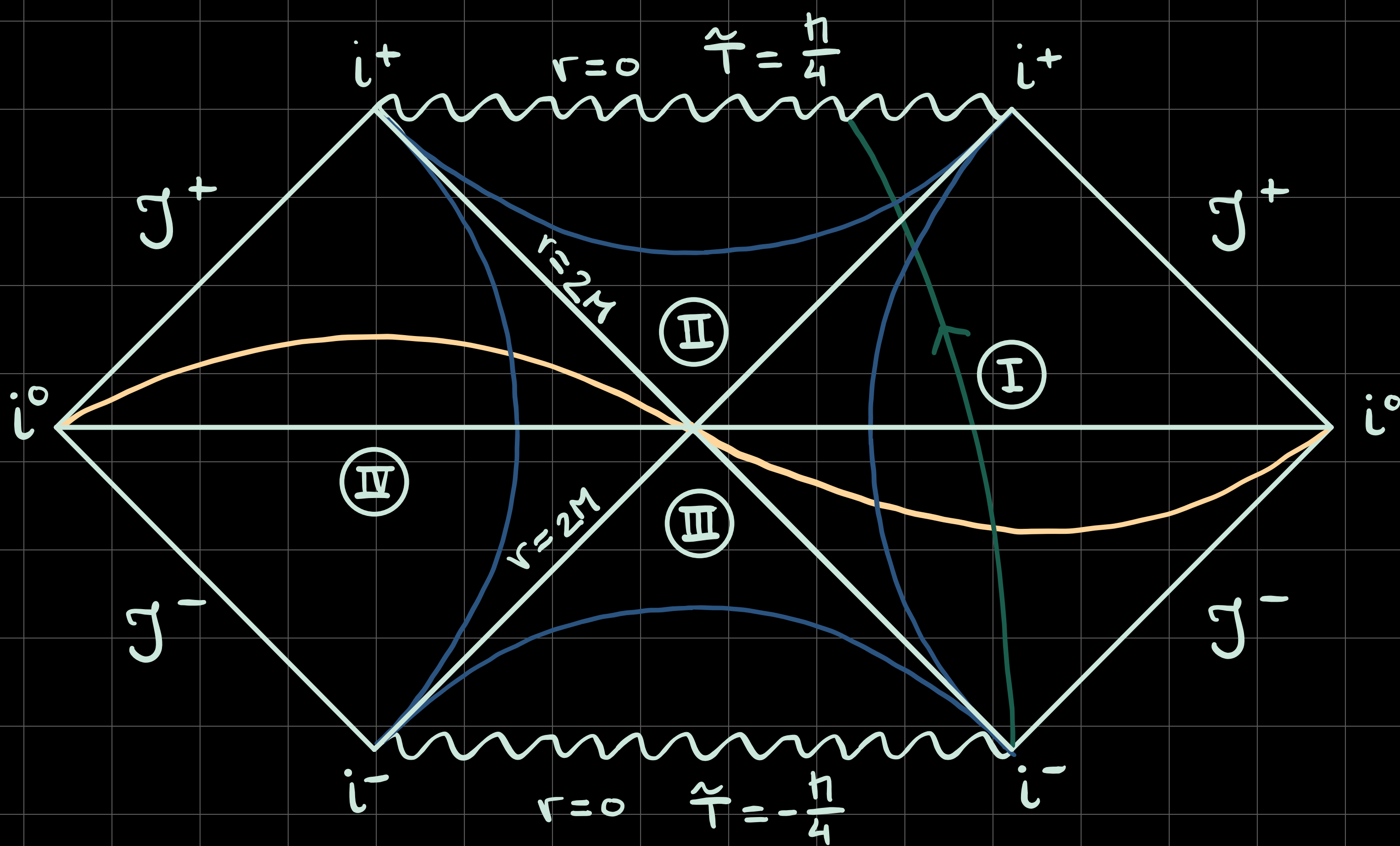
$\rightarrow$ singularity at $\tilde{T} = \pm\frac{\pi}{4}$

SCHWARZSCHILD



CARTER-PENROSE DIAGRAM





lines of constant t

lines of constant r

infalling particle trajectory

BIRKHOFF'S THEOREM

ANY SPHERICALLY SYMMETRIC SOLUTION OF THE VACUUM FIELD EQUATIONS ($G_{\mu\nu} = 0$) MUST BE STATIC AND ASYMPTOTICALLY FLAT.

$\Rightarrow$ SCHWARZSCHILD WITH CONSTANT M.

NO-HAIR THEOREMS:

SET OF PROOFS THAT, UNDER SOME CONDITIONS, KERR-NEWMANN (-DE SITTER) IS THE ONLY ASYMPTOTICALLY FLAT (DE SITTER) AND REGULAR SOLUTION OF THE FIELD EQUATIONS IN THE PRESENCE OF FUNDAMENTAL FIELDS.

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EINSTEIN-MAXWELL-KLEIN-GORDON

$$S = \int d^4x \left(R - 2\Lambda - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \pm \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi \right)$$

$M \quad a \quad \Lambda \quad q$

- NON-INTERACTING SCALARS
- FERMIONS
- ...

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- NON-INTERACTING SCALARS
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- ...

COROLLARY: EXTERIOR REGION OF STATIONARY BLACK HOLES HAS A FINITE NUMBER OF "HAIRS" (M, a, q, Λ).

KERR HYPOTHESIS:

ALL PHYSICAL BLACK HOLES IN THE UNIVERSE CAN BE DESCRIBED BY THE KERR GEOMETRY.

$$q = 0 \quad , \quad \Lambda \ll \frac{1}{9M^2}$$

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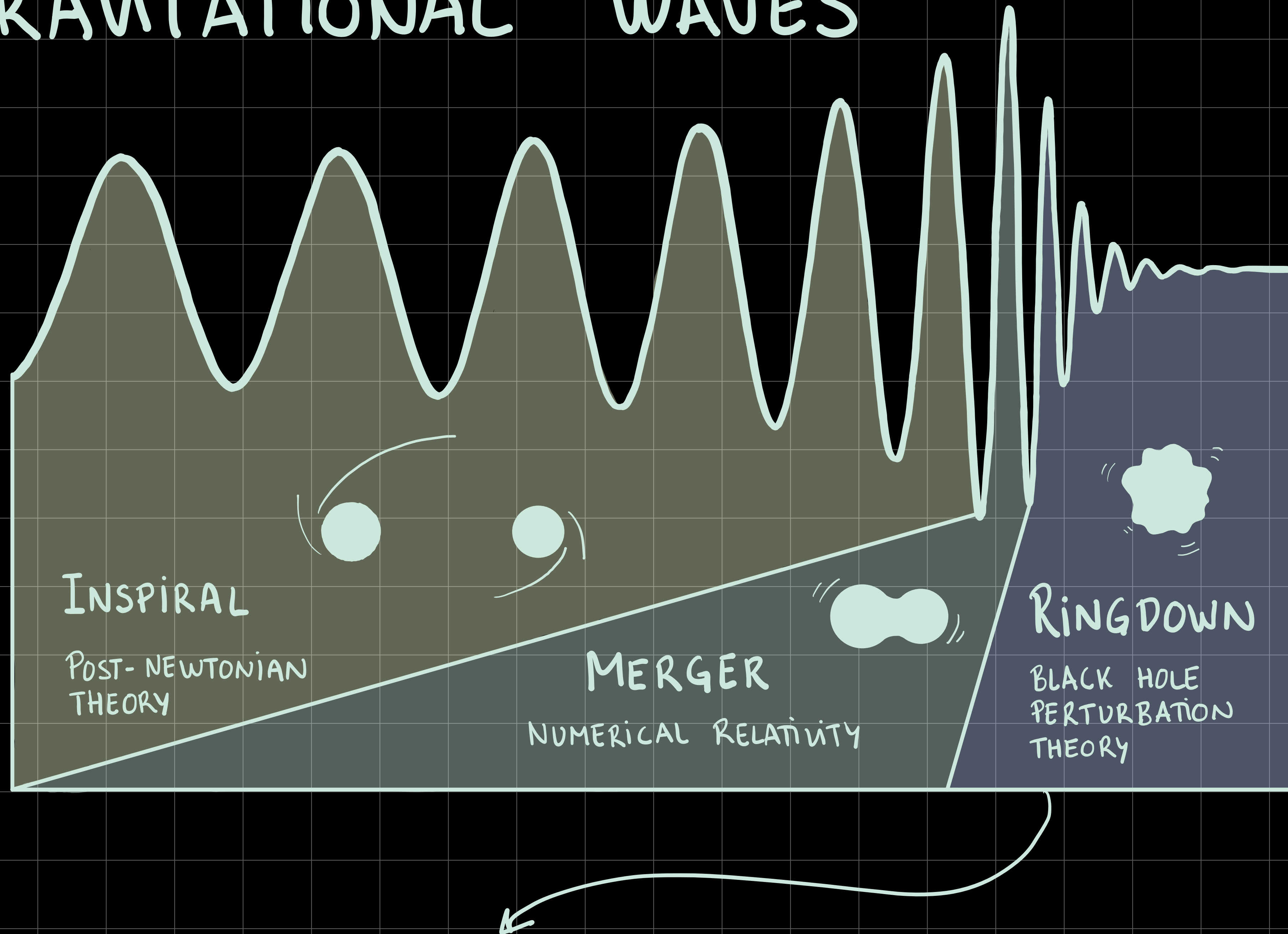
AN OBSERVATION OF A "NON-KERR" BH WOULD SIGNAL NEW PHYSICS.

TESTS OF GRAVITY IN THE STRONG-FIELD REGIME AIM AT VERIFYING THE KERR HYPOTHESIS.

GRAVITATIONAL WAVES



QUASINORMAL MODES: $\omega(M, a)$ (KERR HYPOTHESIS)

HAIRY BLACK HOLES

- PRIMARY HAIR : NEW GLOBAL CHARGE

$$\omega(M, a, \alpha) \quad , \quad \alpha \neq \alpha(M, a)$$

- SECONDARY HAIR: NEW COMBINATION OF GLOBAL CHARGES

$$\omega(M, a, \alpha) \quad , \quad \alpha = \alpha(M, a)$$

HOW TO INTRODUCE HAIR? $\longrightarrow$ NEW INTERACTIONS

$\searrow$ VIOLATING SYMMETRIES

EXAMPLE: SCALAR GAUSS-BONET GRAVITY

$$S = \int d^4x \sqrt{-g} \left(R - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi + \alpha \phi \mathcal{G} \right)$$

$$\mathcal{G} = R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\lambda\kappa} R^{\mu\nu\lambda\kappa}$$

SCALAR EQUATION OF MOTION:

$$\nabla_\mu \nabla^\mu \phi + \alpha \mathcal{G} = 0$$

SCALAR CHARGE: $P = \dfrac{2\alpha}{M}$ (SECONDARY)

QNMs: $\omega(M, \alpha, P(M))$