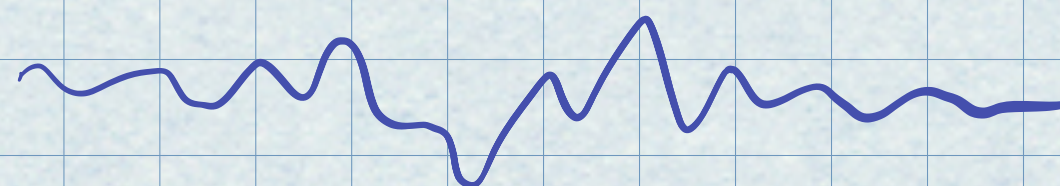
IS JOHN A GOOD SINGER?



or... CAN WE GET GWs IN THE "JOHN" THEORY?

PLAN

THEORY

$\downarrow$

EQUATIONS OF MOTION

$\nearrow$

BACKGROUND SOLUTIONS

$\downarrow$

STABILITY

$\nearrow$

GWs !

JOHN

COSM. CONST.

Ricci

$$S = \int d^4x \sqrt{-g}\left[\zeta R - \eta(\partial\phi)^2 + \beta G^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - 2\Lambda\right]$$

JOHN

KINETIC

JOHN

$$S = \int d^4x \sqrt{-g} \left[\zeta R - \eta (\partial\phi)^2 + \beta G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 2\Lambda \right]$$

SHIFT - SYMMETRY	$\phi \to \phi + c$	$(\partial\phi)^2 \to (\partial(\phi+c))^2 = (\partial\phi)^2$ $\partial_\mu\phi\partial_\nu\phi \to \partial_\mu(\phi+c)\partial_\nu(\phi+c) = \partial_\mu\phi\partial_\nu\phi$	✓

JOHN

$$S = \int d^4x \sqrt{-g}\left[\zeta R - \eta(\partial\phi)^2 + \beta G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - 2\Lambda\right]$$

SHIFT - SYMMETRY	$\phi \to \phi + c$	$(\partial\phi)^2 \to (\partial(\phi+c))^2 = (\partial\phi)^2$ $\partial_\mu\phi\partial_\nu\phi \to \partial_\mu(\phi+c)\partial_\nu(\phi+c) = \partial_\mu\phi\partial_\nu\phi$	✓
REFLECTION - SYMMETRY	$\phi \to -\phi$	$(\partial\phi)^2 \to (\partial(-\phi))(\partial(-\phi)) = (\partial\phi)^2$ $\partial_\mu\phi\partial_\nu\phi \to \partial_\mu(-\phi)\partial_\nu(-\phi) = \partial_\mu\phi\partial_\nu\phi$	✓

EQUATIONS OF MOTION

$$\frac{\delta S}{\delta g^{\mu\nu}} = 0 \quad : \quad \zeta G_{\mu\nu} - \eta\left(\partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2\right) + g_{\mu\nu}\Lambda$$

$$+ \frac{\beta}{2}\left((\partial\phi)^2 G_{\mu\nu} + 2P_{\mu\alpha\nu\beta}\nabla^\alpha\phi\nabla^\beta\phi + g_{\mu\nu}\,\delta^{\alpha\rho\sigma}_{\nu\gamma\delta}\nabla^\gamma\nabla_\rho\phi\,\nabla^\delta\nabla_\sigma\phi\right) = 0$$

$$\frac{\delta S}{\delta \phi} = 0 \quad : \quad \nabla_\mu J^\mu = 0 \quad , \quad J^\mu = \left(\eta g^{\mu\nu} - \beta G^{\mu\nu}\right)\partial_\nu\phi$$

BACKGROUND SOLUTION

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -f(r)\,dt^2 + f(r)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2$$

$$f(r) = \left(1 - \frac{2M}{r} - \frac{\Lambda_{eff}}{3} r^2\right)$$

$$\phi = qt + \psi(r)$$

$$\psi(r) = q \int dr \frac{\sqrt{1-f}}{f}$$

e.g. $$\varsigma G_{\mu\nu} - \eta\left(\partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2\right) + g_{\mu\nu}\Lambda$$

$$+ \frac{\beta}{2}\left((\partial\phi)^2 G_{\mu\nu} + 2P_{\mu\alpha\nu\beta}\nabla^\alpha\phi\nabla^\beta\phi + g_{\mu\nu}\,\delta^{\alpha\rho\sigma}_{\nu\gamma\delta}\,\nabla^\gamma\nabla_\rho\phi\,\nabla^\delta\nabla_\sigma\phi\right) = 0$$

e.g.

$$\varsigma G_{\mu\nu} - \eta\left(\partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2\right) + g_{\mu\nu}\Lambda$$

$$+ \frac{\beta}{2}\left((\partial\phi)^2 G_{\mu\nu} + 2P_{\mu\alpha\nu\beta}\nabla^\alpha\phi\nabla^\beta\phi + g_{\mu\nu}\delta^{\alpha\rho\sigma}_{\nu\gamma\delta}\nabla^\gamma\nabla_\rho\phi\nabla^\delta\nabla_\sigma\phi\right) = 0$$

$$\frac{\beta}{2}(\partial\phi)^2 G_{\mu\nu} = \frac{\beta}{2}(\partial\phi)^2\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)$$

e.g.

$$\varsigma G_{\mu\nu} - \eta\left(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2\right) + g_{\mu\nu}\Lambda$$

$$+ \frac{\beta}{2}\left((\partial\phi)^2 G_{\mu\nu} + 2P_{\mu\alpha\nu\beta}\nabla^\alpha\phi\nabla^\beta\phi + g_{\mu\nu}\delta^{\alpha\rho\sigma}_{\nu\gamma\delta}\nabla^\gamma\nabla_\rho\phi\nabla^\delta\nabla_\sigma\phi\right) = 0$$

$$\frac{\beta}{2}(\partial\phi)^2 G_{\mu\nu} = \frac{\beta}{2}(\partial\phi)^2\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)$$

$$-\frac{1}{4}Rg_{\mu\nu}(\partial\phi)^2$$

2 1

1

$$(\partial\phi)^2 = g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$$

$$= g^{tt}(\partial_t\phi)^2 + g^{rr}(\partial_r\phi)^2 + g^{\theta\theta}(\partial_\theta\phi)^2 + g^{\phi\phi}(\partial_\phi\phi)^2$$

1

$$(\partial\phi)^2 = g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$$

INVERSE $g^{\mu\nu} = (g_{\mu\nu})^{-1} = \begin{pmatrix} -\frac{1}{f} & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & r^{-2} & 0 \\ 0 & 0 & 0 & r^{-2}\sin^{-2}\theta \end{pmatrix}$

$$= g^{tt}(\partial_t\phi)^2 + g^{rr}(\partial_r\phi)^2 + g^{\theta\theta}(\partial_\theta\phi)^2 + g^{\phi\phi}(\partial_\phi\phi)^2$$

$$\phi = q\left[t + \int dr \frac{\sqrt{1-f}}{f}\right] \longrightarrow$$

$$\partial_t\phi = q$$

$$\partial_r\phi = q\frac{\sqrt{1-f}}{f}$$

$$\partial_\theta\phi = \partial_\phi\phi = 0$$

①

$$(\partial\phi)^2 = g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi$$

INVERSE $g^{\mu\nu} = (g_{\mu\nu})^{-1} = \begin{pmatrix} -\frac{1}{f} & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & r^{-2} & 0 \\ 0 & 0 & 0 & r^{-2}\sin^{-2}\theta \end{pmatrix}$

$$= g^{tt}(\partial_t\phi)^2 + g^{rr}(\partial_r\phi)^2 + g^{\theta\theta}(\partial_\theta\phi)^2 + g^{\phi\phi}(\partial_\phi\phi)^2$$

$$\phi = q\left[t + \int dr\,\frac{\sqrt{1-f}}{f}\right] \longrightarrow \begin{aligned} \partial_t\phi &= q \\ \partial_r\phi &= q\,\frac{\sqrt{1-f}}{f} \\ \partial_\theta\phi &= \partial_\phi\phi = 0 \end{aligned}$$

$$= -\frac{1}{f}q^2 + f\,q^2\,\frac{1-f}{f^2}$$

$$= q^2\left[-\cancel{\frac{1}{f}} + \cancel{\frac{1}{f}} - 1\right]$$

$$= -q^2$$

②

$$R = g^{\mu\nu} R_{\mu\nu}$$

$$= g^{\mu\nu} R^{\lambda}{}_{\mu\lambda\nu}$$

$$= g^{\mu\nu}\left(\partial_\lambda \Gamma^{\lambda}_{\mu\nu} - \partial_\nu \Gamma^{\rho}_{\rho\mu} + \Gamma^{\lambda}_{\lambda\alpha}\Gamma^{\alpha}_{\mu\nu} - \Gamma^{\lambda}_{\nu\alpha}\Gamma^{\alpha}_{\lambda\mu}\right)$$

CHRISTOFFEL:

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\sigma}\left(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}\right)$$

②

$$R = g^{\mu\nu} R_{\mu\nu}$$

CHRISTOFFEL :

$$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\sigma} (\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu})$$

$$= g^{\mu\nu} R^{\lambda}{}_{\mu\lambda\nu}$$

$$= g^{\mu\nu} \left(\partial_\lambda \Gamma^{\lambda}_{\mu\nu} - \partial_\nu \Gamma^{\rho}_{\rho\mu} + \Gamma^{\lambda}_{\lambda\alpha} \Gamma^{\alpha}_{\mu\nu} - \Gamma^{\lambda}_{\nu\alpha} \Gamma^{\alpha}_{\lambda\mu} \right)$$

$$= g^{tt} \left(\partial_\lambda \Gamma^{\lambda}_{tt} - \partial_t \Gamma^{\rho}_{\rho t} + \Gamma^{\lambda}_{\lambda\alpha} \Gamma^{\alpha}_{tt} - \Gamma^{\lambda}_{t\alpha} \Gamma^{\alpha}_{\lambda t} \right)$$

$$+ g^{rr} \left(\partial_\lambda \Gamma^{\lambda}_{rr} - \partial_r \Gamma^{\rho}_{\rho r} + \Gamma^{\lambda}_{\lambda\alpha} \Gamma^{\alpha}_{rr} - \Gamma^{\lambda}_{r\alpha} \Gamma^{\alpha}_{\lambda r} \right)$$

$$+ g^{\theta\theta} \left(\partial_\lambda \Gamma^{\lambda}_{\theta\theta} - \partial_\theta \Gamma^{\rho}_{\rho\theta} + \Gamma^{\lambda}_{\lambda\alpha} \Gamma^{\alpha}_{\theta\theta} - \Gamma^{\lambda}_{\theta\alpha} \Gamma^{\alpha}_{\lambda\theta} \right)$$

$$+ g^{\phi\phi} \left(\partial_\lambda \Gamma^{\lambda}_{\phi\phi} - \partial_\phi \Gamma^{\rho}_{\rho\phi} + \Gamma^{\lambda}_{\lambda\alpha} \Gamma^{\alpha}_{\phi\phi} - \Gamma^{\lambda}_{\phi\alpha} \Gamma^{\alpha}_{\lambda\phi} \right)$$

②

$$
\begin{aligned}
R = g^{tt}\Big(& \partial_t \Gamma^t_{tt} + \partial_r \Gamma^r_{tt} + \partial_\theta \Gamma^\theta_{tt} + \partial_\phi \Gamma^\phi_{tt} - \partial_t \Gamma^t_{tt} - \partial_t \Gamma^r_{rt} - \partial_t \Gamma^\theta_{\theta t} - \partial_t \Gamma^\phi_{\phi t} \\
& + \Gamma^t_{tt}\Gamma^t_{tt} + \Gamma^r_{rt}\Gamma^t_{tt} + \Gamma^\theta_{\theta t}\Gamma^t_{tt} + \Gamma^\phi_{\phi t}\Gamma^t_{tt} \\
& + \Gamma^t_{tr}\Gamma^r_{tt} + \Gamma^r_{rr}\Gamma^r_{tt} + \Gamma^\theta_{\theta r}\Gamma^r_{tt} + \Gamma^\phi_{\phi r}\Gamma^r_{tt} \\
& + \Gamma^t_{t\theta}\Gamma^\theta_{tt} + \Gamma^r_{r\phi}\Gamma^\phi_{tt} + \Gamma^\theta_{\theta\phi}\Gamma^\phi_{tt} + \Gamma^\phi_{\phi\phi}\Gamma^\phi_{tt} \\
& + \Gamma^t_{t\phi}\Gamma^\phi_{tt} + \Gamma^r_{r\phi}\Gamma^\phi_{tt} + \Gamma^\theta_{\theta\phi}\Gamma^\phi_{tt} + \Gamma^\phi_{\phi\phi}\Gamma^\phi_{tt} \\
& - \Gamma^t_{tt}\Gamma^t_{tt} - \Gamma^r_{tt}\Gamma^t_{rt} - \Gamma^\theta_{tt}\Gamma^t_{\theta t} - \Gamma^\phi_{tt}\Gamma^t_{\phi t} \\
& - \Gamma^t_{tr}\Gamma^r_{tt} - \Gamma^r_{tr}\Gamma^r_{rt} - \Gamma^\theta_{tr}\Gamma^r_{\theta t} - \Gamma^\phi_{tr}\Gamma^r_{\phi t} \\
& - \Gamma^t_{t\theta}\Gamma^\theta_{tt} - \Gamma^r_{t\theta}\Gamma^\theta_{rt} - \Gamma^\theta_{t\theta}\Gamma^\theta_{\theta t} - \Gamma^\phi_{t\theta}\Gamma^\theta_{\phi t} \\
& - \Gamma^t_{t\phi}\Gamma^\phi_{tt} - \Gamma^r_{t\phi}\Gamma^\phi_{rt} - \Gamma^\theta_{t\phi}\Gamma^\phi_{\theta t} - \Gamma^\phi_{t\phi}\Gamma^\phi_{\phi t} \Big) \\
& + g^{rr}(\cdots) + g^{\theta\theta}(\cdots) + g^{\phi\phi}(\cdots)
\end{aligned}
$$

② $\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\sigma} (\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu})$

$g_{rr} = \frac{1}{f(r)}$, $g_{rr} = f(r)$

e.g.

$$
\begin{aligned}
\Gamma^{r}_{rt} &= \frac{1}{2} g^{r\sigma} (\partial_r g_{\sigma t} + \partial_t g_{\sigma r} - \partial_\sigma \cancel{g_{rt}}) \\
&= \frac{1}{2} g^{rr} (\partial_r \cancel{g_{rt}} + \partial_t g_{rr}) \\
&= \frac{1}{2} g^{rr} \cancel{\partial_t g_{rr}} \\
&= 0
\end{aligned}
$$

② $$\Gamma^{\alpha}_{\mu\nu} = \frac{1}{2} g^{\alpha\sigma}\left(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}\right)$$

$$g_{rr} = \frac{1}{f(r)}, \quad g^{rr} = f(r)$$

e.g.

$$\begin{aligned}
\Gamma^{r}_{rt} &= \frac{1}{2} g^{r\sigma}\left(\partial_r g_{\sigma t} + \partial_t g_{\sigma r} - \partial_\sigma \cancel{g_{rt}}\right) \\
&= \frac{1}{2} g^{rr}\left(\partial_r \cancel{g_{rt}} + \partial_t g_{rr}\right) \\
&= \frac{1}{2} g^{rr} \cancel{\partial_t g_{rr}} \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\Gamma^{r}_{rr} &= \frac{1}{2} g^{r\sigma}\left(\partial_r g_{\sigma r} + \partial_r g_{r\sigma} - \partial_\sigma g_{rr}\right) \\
&= \frac{1}{2} g^{rr}\left(\partial_r g_{rr} + \cancel{\partial_r g_{rr}} - \cancel{\partial_r g_{rr}}\right) \\
&= \frac{1}{2} f\left(-\frac{1}{f^2} f'\right) \\
&= -\frac{1}{2}\frac{f'}{f}
\end{aligned}$$

```
In[247]:= ChristoffelCDPDSdS[b, -c, -d] // ToBasis[SdS]
          % // ComponentArray
          % // ToValues // ToValues
```

Out[247]= $\Gamma[\nabla,\mathcal{D}]^{b}{}_{cd}$

Out[248]= $\{\{\{\Gamma[\nabla,\mathcal{D}]^{t}{}_{tt}, \Gamma[\nabla,\mathcal{D}]^{t}{}_{tr}, \Gamma[\nabla,\mathcal{D}]^{t}{}_{t\theta}, \Gamma[\nabla,\mathcal{D}]^{t}{}_{t\phi}\}, \{\Gamma[\nabla,\mathcal{D}]^{t}{}_{rt}, \Gamma[\nabla,\mathcal{D}]^{t}{}_{rr}, \Gamma[\nabla,\mathcal{D}]^{t}{}_{r\theta}, \Gamma[\nabla,\mathcal{D}]^{t}{}_{r\phi}\},$
$\{\Gamma[\nabla,\mathcal{D}]^{t}{}_{\theta t}, \Gamma[\nabla,\mathcal{D}]^{t}{}_{\theta r}, \Gamma[\nabla,\mathcal{D}]^{t}{}_{\theta\theta}, \Gamma[\nabla,\mathcal{D}]^{t}{}_{\theta\phi}\}, \{\Gamma[\nabla,\mathcal{D}]^{t}{}_{\phi t}, \Gamma[\nabla,\mathcal{D}]^{t}{}_{\phi r}, \Gamma[\nabla,\mathcal{D}]^{t}{}_{\phi\theta}, \Gamma[\nabla,\mathcal{D}]^{t}{}_{\phi\phi}\}\},$
$\{\{\Gamma[\nabla,\mathcal{D}]^{r}{}_{tt}, \Gamma[\nabla,\mathcal{D}]^{r}{}_{tr}, \Gamma[\nabla,\mathcal{D}]^{r}{}_{t\theta}, \Gamma[\nabla,\mathcal{D}]^{r}{}_{t\phi}\}, \{\Gamma[\nabla,\mathcal{D}]^{r}{}_{rt}, \Gamma[\nabla,\mathcal{D}]^{r}{}_{rr}, \Gamma[\nabla,\mathcal{D}]^{r}{}_{r\theta}, \Gamma[\nabla,\mathcal{D}]^{r}{}_{r\phi}\},$
$\{\Gamma[\nabla,\mathcal{D}]^{r}{}_{\theta t}, \Gamma[\nabla,\mathcal{D}]^{r}{}_{\theta r}, \Gamma[\nabla,\mathcal{D}]^{r}{}_{\theta\theta}, \Gamma[\nabla,\mathcal{D}]^{r}{}_{\theta\phi}\}, \{\Gamma[\nabla,\mathcal{D}]^{r}{}_{\phi t}, \Gamma[\nabla,\mathcal{D}]^{r}{}_{\phi r}, \Gamma[\nabla,\mathcal{D}]^{r}{}_{\phi\theta}, \Gamma[\nabla,\mathcal{D}]^{r}{}_{\phi\phi}\}\},$
$\{\{\Gamma[\nabla,\mathcal{D}]^{\theta}{}_{tt}, \Gamma[\nabla,\mathcal{D}]^{\theta}{}_{tr}, \Gamma[\nabla,\mathcal{D}]^{\theta}{}_{t\theta}, \Gamma[\nabla,\mathcal{D}]^{\theta}{}_{t\phi}\}, \{\Gamma[\nabla,\mathcal{D}]^{\theta}{}_{rt}, \Gamma[\nabla,\mathcal{D}]^{\theta}{}_{rr}, \Gamma[\nabla,\mathcal{D}]^{\theta}{}_{r\theta}, \Gamma[\nabla,\mathcal{D}]^{\theta}{}_{r\phi}\},$
$\{\Gamma[\nabla,\mathcal{D}]^{\theta}{}_{\theta t}, \Gamma[\nabla,\mathcal{D}]^{\theta}{}_{\theta r}, \Gamma[\nabla,\mathcal{D}]^{\theta}{}_{\theta\theta}, \Gamma[\nabla,\mathcal{D}]^{\theta}{}_{\theta\phi}\}, \{\Gamma[\nabla,\mathcal{D}]^{\theta}{}_{\phi t}, \Gamma[\nabla,\mathcal{D}]^{\theta}{}_{\phi r}, \Gamma[\nabla,\mathcal{D}]^{\theta}{}_{\phi\theta}, \Gamma[\nabla,\mathcal{D}]^{\theta}{}_{\phi\phi}\}\},$
$\{\{\Gamma[\nabla,\mathcal{D}]^{\phi}{}_{tt}, \Gamma[\nabla,\mathcal{D}]^{\phi}{}_{tr}, \Gamma[\nabla,\mathcal{D}]^{\phi}{}_{t\theta}, \Gamma[\nabla,\mathcal{D}]^{\phi}{}_{t\phi}\}, \{\Gamma[\nabla,\mathcal{D}]^{\phi}{}_{rt}, \Gamma[\nabla,\mathcal{D}]^{\phi}{}_{rr}, \Gamma[\nabla,\mathcal{D}]^{\phi}{}_{r\theta}, \Gamma[\nabla,\mathcal{D}]^{\phi}{}_{r\phi}\},$
$\{\Gamma[\nabla,\mathcal{D}]^{\phi}{}_{\theta t}, \Gamma[\nabla,\mathcal{D}]^{\phi}{}_{\theta r}, \Gamma[\nabla,\mathcal{D}]^{\phi}{}_{\theta\theta}, \Gamma[\nabla,\mathcal{D}]^{\phi}{}_{\theta\phi}\}, \{\Gamma[\nabla,\mathcal{D}]^{\phi}{}_{\phi t}, \Gamma[\nabla,\mathcal{D}]^{\phi}{}_{\phi r}, \Gamma[\nabla,\mathcal{D}]^{\phi}{}_{\phi\theta}, \Gamma[\nabla,\mathcal{D}]^{\phi}{}_{\phi\phi}\}\}\}$

Out[249]= $\left\{\left\{\left\{0, \frac{B'[r]}{2\,B[r]}, 0, 0\right\}, \left\{\frac{B'[r]}{2\,B[r]}, 0, 0, 0\right\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\right\},\right.$
$\left\{\left\{\frac{1}{2} B[r]\, B'[r], 0, 0, 0\right\}, \left\{0, -\frac{B'[r]}{2\,B[r]}, 0, 0\right\}, \{0, 0, -B[r]\, r, 0\}, \left\{0, 0, 0, -B[r]\, r\, \mathrm{Sin}[\theta]^2\right\}\right\},$
$\left\{\{0, 0, 0, 0\}, \left\{0, 0, \frac{1}{r}, 0\right\}, \left\{0, \frac{1}{r}, 0, 0\right\}, \{0, 0, 0, -\mathrm{Cos}[\theta]\, \mathrm{Sin}[\theta]\}\right\},$
$\left.\left\{\{0, 0, 0, 0\}, \left\{0, 0, 0, \frac{1}{r}\right\}, \{0, 0, 0, \mathrm{Cot}[\theta]\}, \left\{0, \frac{1}{r}, \mathrm{Cot}[\theta], 0\right\}\right\}\right\}$

2

$$
\begin{aligned}
R = g^{tt}\Big(& \partial_t \Gamma^t_{tt} + \partial_r \Gamma^r_{tt} + \partial_\theta \Gamma^\theta_{tt} + \partial_\phi \Gamma^\phi_{tt} - \partial_t \Gamma^t_{tt} - \partial_t \Gamma^r_{rt} - \partial_t \Gamma^\theta_{\theta t} - \partial_t \Gamma^\phi_{\phi t} \\
& + \Gamma^t_{tt}\Gamma^t_{tt} + \Gamma^r_{rt}\Gamma^t_{tt} + \Gamma^\theta_{\theta t}\Gamma^t_{tt} + \Gamma^\phi_{\phi t}\Gamma^t_{tt} \\
& + \Gamma^t_{tr}\Gamma^r_{tt} + \Gamma^r_{rr}\Gamma^r_{tt} + \Gamma^\theta_{\theta r}\Gamma^r_{tt} + \Gamma^\phi_{\phi r}\Gamma^r_{tt} \\
& + \Gamma^t_{t\theta}\Gamma^\theta_{tt} + \Gamma^r_{r\phi}\Gamma^\phi_{tt} + \Gamma^\theta_{\theta\phi}\Gamma^\phi_{tt} + \Gamma^\phi_{\phi\phi}\Gamma^\phi_{tt} \\
& + \Gamma^t_{t\phi}\Gamma^\phi_{tt} + \Gamma^r_{r\phi}\Gamma^\phi_{tt} + \Gamma^\theta_{\theta\phi}\Gamma^\phi_{tt} + \Gamma^\phi_{\phi\phi}\Gamma^\phi_{tt} \\
& - \Gamma^t_{tt}\Gamma^t_{tt} - \Gamma^r_{tt}\Gamma^t_{rt} - \Gamma^\theta_{tt}\Gamma^t_{\theta t} - \Gamma^\phi_{tt}\Gamma^t_{\phi t} \\
& - \Gamma^t_{tr}\Gamma^r_{tt} - \Gamma^r_{tr}\Gamma^r_{rt} - \Gamma^\theta_{tr}\Gamma^r_{\theta t} - \Gamma^\phi_{tr}\Gamma^r_{\phi t} \\
& - \Gamma^t_{t\theta}\Gamma^\theta_{tt} - \Gamma^r_{t\theta}\Gamma^\theta_{rt} - \Gamma^\theta_{t\theta}\Gamma^\theta_{\theta t} - \Gamma^\phi_{t\theta}\Gamma^\theta_{\phi t} \\
& - \Gamma^t_{t\phi}\Gamma^\phi_{tt} - \Gamma^r_{t\phi}\Gamma^\phi_{rt} - \Gamma^\theta_{t\phi}\Gamma^\phi_{\theta t} - \Gamma^\phi_{t\phi}\Gamma^\phi_{\phi t} \Big) \\
& + g^{rr}(\cdots) + g^{\theta\theta}(\cdots) + g^{\phi\phi}(\cdots)
\end{aligned}
$$

②

$$R = g^{tt}\Big(\cancel{\partial_t\Gamma^t_{tt}} + \partial_r\Gamma^r_{tt} + \cancel{\partial_\theta\Gamma^\theta_{tt}} + \cancel{\partial_\phi\Gamma^\phi_{tt}} - \cancel{\partial_t\Gamma^t_{tt}} - \cancel{\partial_t\Gamma^r_{rt}} - \cancel{\partial_t\Gamma^\theta_{\theta t}} - \cancel{\partial_t\Gamma^\phi_{\phi t}}$$

$$+ \cancel{\Gamma^t_{tt}\Gamma^t_{tt}} + \cancel{\Gamma^r_{rt}\Gamma^t_{tt}} + \cancel{\Gamma^\theta_{\theta t}\Gamma^t_{tt}} + \cancel{\Gamma^\phi_{\phi t}\Gamma^t_{tt}}$$

$$+ \Gamma^t_{tr}\Gamma^r_{tt} + \Gamma^r_{rr}\Gamma^r_{tt} + \Gamma^\theta_{\theta r}\Gamma^r_{tt} + \Gamma^\phi_{\phi r}\Gamma^r_{tt}$$

$$+ \cancel{\Gamma^t_{t\theta}\Gamma^\theta_{tt}} + \cancel{\Gamma^r_{r\phi}\Gamma^\phi_{tt}} + \cancel{\Gamma^\theta_{\theta\phi}\Gamma^\phi_{tt}} + \cancel{\Gamma^\phi_{\phi\phi}\Gamma^\phi_{tt}}$$

$$+ \cancel{\Gamma^t_{t\phi}\Gamma^\phi_{tt}} + \cancel{\Gamma^r_{r\phi}\Gamma^\phi_{tt}} + \cancel{\Gamma^\theta_{\theta\phi}\Gamma^\phi_{tt}} + \cancel{\Gamma^\phi_{\phi\phi}\Gamma^\phi_{tt}}$$

$$- \cancel{\Gamma^t_{tt}\Gamma^t_{tt}} - \Gamma^r_{tt}\Gamma^t_{rt} - \cancel{\Gamma^\theta_{tt}\Gamma^t_{\theta t}} - \cancel{\Gamma^\phi_{tt}\Gamma^t_{\phi t}}$$

$$- \Gamma^t_{tr}\Gamma^r_{tt} - \cancel{\Gamma^r_{tr}\Gamma^r_{rt}} - \cancel{\Gamma^\theta_{tr}\Gamma^r_{\theta t}} - \cancel{\Gamma^\phi_{tr}\Gamma^r_{\phi t}}$$

$$- \cancel{\Gamma^t_{t\theta}\Gamma^\theta_{tt}} - \cancel{\Gamma^r_{t\theta}\Gamma^\theta_{rt}} - \cancel{\Gamma^\theta_{t\theta}\Gamma^\theta_{\theta t}} - \cancel{\Gamma^\phi_{t\theta}\Gamma^\theta_{\phi t}}$$

$$- \cancel{\Gamma^t_{t\phi}\Gamma^\phi_{tt}} - \cancel{\Gamma^r_{t\phi}\Gamma^\phi_{rt}} - \cancel{\Gamma^\theta_{t\phi}\Gamma^\phi_{\theta t}} - \cancel{\Gamma^\phi_{t\phi}\Gamma^\phi_{\phi t}}\Big)$$

$$+ g^{rr}(\cdots) + g^{\theta\theta}(\cdots) + g^{\phi\phi}(\cdots)$$

②

$$R = g^{tt}\Big(\cancel{\partial_t \Gamma^t_{tt}} + \partial_r \Gamma^r_{tt} + \cancel{\partial_\theta \Gamma^\theta_{tt}} + \cancel{\partial_\phi \Gamma^\phi_{tt}} - \cancel{\partial_t \Gamma^t_{tt}} - \cancel{\partial_t \Gamma^r_{rt}} - \cancel{\partial_t \Gamma^\theta_{\theta t}} - \cancel{\partial_t \Gamma^\phi_{\phi t}}$$

$$g^{tt} \to -\frac{1}{f}$$

$$\partial_r\left(\frac{1}{2} f f'\right) = \frac{1}{2}\left(f'^2 + f f''\right)$$

$$+ \cancel{\Gamma^t_{tt}\Gamma^t_{tt}} + \cancel{\Gamma^r_{rt}\Gamma^t_{tt}} + \cancel{\Gamma^\theta_{\theta t}\Gamma^t_{tt}} + \cancel{\Gamma^\phi_{\phi t}\Gamma^t_{tt}}$$

$$+ \Gamma^t_{tr}\Gamma^r_{tt} + \Gamma^r_{rr}\Gamma^r_{tt} + \Gamma^\theta_{\theta r}\Gamma^r_{tt} + \Gamma^\phi_{\phi r}\Gamma^r_{tt}$$

$$\left(-\frac{f'}{2f}\right)\left(\frac{1}{2}ff'\right) \qquad \left(\frac{1}{r}\right)\left(\frac{1}{2}ff'\right) \qquad \left(\frac{1}{r}\right)\left(\frac{1}{2}ff'\right)$$

$$+ \cancel{\Gamma^t_{t\theta}\Gamma^\theta_{tt}} + \cancel{\Gamma^r_{r\phi}\Gamma^\phi_{tt}} + \cancel{\Gamma^\theta_{\theta\phi}\Gamma^\phi_{tt}} + \cancel{\Gamma^\phi_{\phi\phi}\Gamma^\phi_{tt}}$$

$$+ \cancel{\Gamma^t_{t\phi}\Gamma^\phi_{tt}} + \cancel{\Gamma^r_{r\phi}\Gamma^\phi_{tt}} + \cancel{\Gamma^\theta_{\theta\phi}\Gamma^\phi_{tt}} + \cancel{\Gamma^\phi_{\phi\phi}\Gamma^\phi_{tt}}$$

$$- \cancel{\Gamma^t_{tt}\Gamma^t_{tt}} - \Gamma^r_{tt}\Gamma^t_{rt} - \cancel{\Gamma^\theta_{tt}\Gamma^t_{\theta t}} - \cancel{\Gamma^\phi_{tt}\Gamma^t_{\phi t}}$$

$$-\left(\frac{1}{2}ff'\right)\left(\frac{f'}{2f}\right)$$

$$- \Gamma^t_{tr}\Gamma^r_{tt} - \cancel{\Gamma^r_{tr}\Gamma^r_{rt}} - \cancel{\Gamma^\theta_{tr}\Gamma^r_{\theta t}} - \cancel{\Gamma^\phi_{tr}\Gamma^r_{\phi t}}$$

$$- \cancel{\Gamma^t_{t\theta}\Gamma^\theta_{tt}} - \cancel{\Gamma^r_{t\theta}\Gamma^\theta_{rt}} - \cancel{\Gamma^\theta_{t\theta}\Gamma^\theta_{\theta t}} - \cancel{\Gamma^\phi_{t\theta}\Gamma^\theta_{\phi t}}$$

$$- \cancel{\Gamma^t_{t\phi}\Gamma^\phi_{tt}} - \cancel{\Gamma^r_{t\phi}\Gamma^\phi_{rt}} - \cancel{\Gamma^\theta_{t\phi}\Gamma^\phi_{\theta t}} - \cancel{\Gamma^\phi_{t\phi}\Gamma^\phi_{\phi t}}\Big)$$

$$+ g^{rr}(\cdots) + g^{\theta\theta}(\cdots) + g^{\phi\phi}(\cdots)$$

②

$$R = -\frac{1}{2} f'' - \frac{1}{r} f'$$

$$+ g^{rr} (\cdots) + g^{\theta\theta} (\cdots) + g^{\phi\phi} (\cdots)$$

$$f = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2$$

$$f' = \frac{2M}{r^2} - \frac{2\Lambda}{3} r$$

$$f'' = -\frac{4M}{r^3} - \frac{2\Lambda}{3}$$

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$$f'' = -\frac{4M}{r^3} - \frac{2\Lambda}{3}$$

$$= -\frac{1}{2}\left(\cancel{-\frac{4M}{r^3}} - \frac{2\Lambda}{3}\right) - \frac{1}{r}\left(\cancel{\frac{2M}{r^2}} - \frac{2\Lambda}{3}r\right)$$

$$+ g^{rr}(\cdots) + g^{\theta\theta}(\cdots) + g^{\phi\phi}(\cdots)$$

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$$+ g^{rr}(\cdots) + g^{\theta\theta}(\cdots) + g^{\phi\phi}(\cdots)$$

$$= \Lambda_{eff}$$

$$+ g^{rr}(\cdots) + g^{\theta\theta}(\cdots) + g^{\phi\phi}(\cdots)$$

②

$$R = -\frac{1}{2} f'' - \frac{1}{r} f'$$

$$+ g^{rr} (\cdots) + g^{\theta\theta} (\cdots) + g^{\phi\phi} (\cdots)$$

$$= -\frac{1}{2}\left(-\cancel{\frac{4M}{r^3}} - \frac{2\Lambda}{3}\right) - \frac{1}{r}\left(\cancel{\frac{2M}{r^2}} - \frac{2\Lambda}{3} r\right)$$

$$+ g^{rr} (\cdots) + g^{\theta\theta} (\cdots) + g^{\phi\phi} (\cdots)$$

$$= \Lambda_{eff}$$

$$+ g^{rr} (\cdots) + g^{\theta\theta} (\cdots) + g^{\phi\phi} (\cdots)$$

$$= 4\Lambda_{eff}$$

$$f = 1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2$$

$$f' = \frac{2M}{r^2} - \frac{2\Lambda}{3} r$$

$$f'' = -\frac{4M}{r^3} - \frac{2\Lambda}{3}$$

$$R = 4\Lambda_{eff} \quad , \quad (\partial\phi)^2 = -q^2$$

$$\Rightarrow \quad -\frac{1}{4} R g_{\mu\nu} (\partial\phi)^2 = q^2 \Lambda_{eff}\, g_{\mu\nu} = q^2 \Lambda_{eff} \begin{pmatrix} -f & 0 & 0 & 0 \\ 0 & 1/f & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$

$$R = 4\Lambda_{eff} \quad , \quad (\partial\phi)^2 = -q^2$$

$$\Rightarrow \quad -\frac{1}{4} R g_{\mu\nu} (\partial\phi)^2 = q^2 \Lambda_{eff}\, g_{\mu\nu} = q^2 \Lambda_{eff} \begin{pmatrix} -f & 0 & 0 & 0 \\ 0 & 1/f & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{pmatrix}$$

$$\zeta G_{\mu\nu} - \eta \left(\partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} (\partial\phi)^2 \right) + g_{\mu\nu} \Lambda$$

$$+ \frac{\beta}{2} \left((\partial\phi)^2 \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) + 2 P_{\mu\alpha\nu\beta} \nabla^\alpha \phi \nabla^\beta \phi + g_{\mu\nu} \delta^{\alpha\rho\sigma}_{\nu\gamma\delta} \nabla^\gamma \nabla_\rho \phi \nabla^\delta \nabla_\sigma \phi \right) = 0$$

SOLUTION

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -f(r)\,dt^2 + f(r)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2$$

$$\phi = qt + \psi(r)$$

SOLVES EOMs

$$\zeta G_{\mu\nu} - \eta\left(\partial_\mu\phi\,\partial_\nu\phi - \tfrac{1}{2} g_{\mu\nu} (\partial\phi)^2\right) + g_{\mu\nu}\Lambda$$

$$+ \frac{\beta}{2}\left((\partial\phi)^2 \left(R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R\right) + 2 P_{\mu\alpha\nu\beta} \nabla^\alpha\phi\,\nabla^\beta\phi + g_{\mu\alpha}\, \delta^{\alpha\rho\sigma}_{\nu\gamma\delta}\, \nabla^\gamma\nabla_\rho\phi\,\nabla^\delta\nabla_\sigma\phi \right) = 0$$

$$\nabla_\mu J^\mu = 0$$

IFF

$$\Lambda_{eff} = -\frac{\eta}{\beta} \quad , \quad q^2 = \frac{\eta + \beta\Lambda}{\eta\beta}\,\zeta$$

PLAN

THEORY

EQUATIONS OF MOTION

BACKGROUND SOLUTIONS

STABILITY

GWs !

STABILITY

DO SMALL PERTURBATIONS GROW OR DECAY ?

STABILITY

DO SMALL PERTURBATIONS GROW OR DECAY ?

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \varepsilon h_{\mu\nu} \qquad \phi = \bar{\phi} + \varepsilon \delta\phi$$

ODD

EVEN

(SPHERICAL SYMMETRY)

Q

$Z \,,\, \Phi$

Regge-Wheeler

Zerilli

Scalar

Quadratic Action

$$
\begin{aligned}
\delta S^{(2)} = \int d^4x\, \frac{1}{8}\sqrt{-g}\,\Big(& -2\Lambda\, h^{1b}{}_{b}\, h^{1c}{}_{c} - 4\Lambda_{eff}\beta\, h^{1}{}_{ck}\, h^{1ck}\, \nabla_b\phi\,\nabla^b\phi + 2\eta\, h^{1}{}_{ck}\, h^{1ck}\,\nabla_b\phi\,\nabla^b\phi + 2\Lambda_{eff}\beta\, h^{1c}{}_{c}\, h^{1k}{}_{k}\,\nabla_b\phi\,\nabla^b\phi - \eta\, h^{1c}{}_{c}\, h^{1k}{}_{k}\,\nabla_b\phi\,\nabla^b\phi + \\
& 8\Lambda_{eff}\beta\, h^{1c}{}_{c}\,\nabla_b\delta\phi^1\,\nabla^b\phi - 8\eta\, h^{1c}{}_{c}\,\nabla_b\delta\phi^1\,\nabla^b\phi - 8\eta\,\nabla_b\delta\phi^1\,\nabla^b\delta\phi^1 + 2\xi\,\nabla_c h^{1k}{}_{k}\,\nabla^c h^{1b}{}_{b} + 8\beta\, G^{kd}\, h^{1}{}_{bk}\, h^{1}{}_{cd}\,\nabla^b\phi\,\nabla^c\phi + \\
& 32\Lambda_{eff}\beta\, h^{1}{}_{b}{}^{k}\, h^{1}{}_{ck}\,\nabla^b\phi\,\nabla^c\phi - 8\eta\, h^{1}{}_{b}{}^{k}\, h^{1}{}_{ck}\,\nabla^b\phi\,\nabla^c\phi - 8\beta\, G_b{}^k\, h^{1}{}_{ck}\, h^{1d}{}_{d}\,\nabla^b\phi\,\nabla^c\phi + 16\beta\, G_b{}^k\, h^{1}{}_{c}{}^{d}\, h^{1}{}_{kd}\,\nabla^b\phi\,\nabla^c\phi - \\
& 2\beta\, G_{bc}\, h^{1}{}_{kd}\, h^{1kd}\,\nabla^b\phi\,\nabla^c\phi + \beta\, G_{bc}\, h^{1d}{}_{d}\, h^{1k}{}_{k}\,\nabla^b\phi\,\nabla^c\phi + 2\beta\,\nabla_b h^{1k}{}_{k}\,\nabla^b\phi\,\nabla_c h^{1d}{}_{d}\,\nabla^c\phi - 2\beta\,\nabla_b h^{1kd}\,\nabla^b\phi\,\nabla_c h^{1}{}_{kd}\,\nabla^c\phi - \\
& 16\beta\, G_c{}^k\, h^{1}{}_{bk}\,\nabla^b\phi\,\nabla^c\delta\phi^1 - 16\beta\, G_b{}^k\, h^{1}{}_{ck}\,\nabla^b\phi\,\nabla^c\delta\phi^1 + 8\beta\, G_{bc}\, h^{1k}{}_{k}\,\nabla^b\phi\,\nabla^c\delta\phi^1 - 4\beta\,\nabla_b\nabla_c h^{1k}{}_{k}\,\nabla^b\phi\,\nabla^c\delta\phi^1 + \\
& 8\beta\, G_{bc}\,\nabla^b\delta\phi^1\,\nabla^c\delta\phi^1 - 4\beta\,\nabla^b\phi\,\nabla_c\nabla_b h^{1k}{}_{k}\,\nabla^c\delta\phi^1 + 4\, h^{1}{}_{bc}\,\Big(\Lambda\, h^{1bc} + \nabla^b\phi\,\big((-3\Lambda_{eff}\beta + \eta)\, h^{1k}{}_{k}\,\nabla^c\phi + 4\times(-2\Lambda_{eff}\beta + \eta)\,\nabla^c\delta\phi^1\big)\Big) + \\
& 8\beta\,\nabla^b\phi\,\nabla_c h^{1}{}_{kd}\,\nabla^c\phi\,\nabla^d h^{1}{}_{b}{}^{k} - 4\beta\,\nabla^b\phi\,\nabla^c\phi\,\nabla_d h^{1}{}_{ck}\,\nabla^d h^{1}{}_{b}{}^{k} + \beta\,\nabla_b\phi\,\nabla^b\phi\,\nabla_d h^{1}{}_{ck}\,\nabla^d h^{1ck} - 4\beta\,\nabla^b\phi\,\nabla_c h^{1d}{}_{d}\,\nabla^c\phi\,\nabla_k h^{1}{}_{b}{}^{k} + \\
& 4\beta\,\nabla^b\phi\,\nabla^c\phi\,\nabla^d h^{1}{}_{b}{}^{k}\,\nabla_k h^{1}{}_{cd} - 2\beta\,\nabla_b\phi\,\nabla^b\phi\,\nabla^d h^{1ck}\,\nabla_k h^{1}{}_{cd} - 4\xi\,\nabla^c h^{1b}{}_{b}\,\nabla_k h^{1}{}_{c}{}^{k} - 4\beta\,\nabla^b\phi\,\nabla_c h^{1}{}_{b}{}^{k}\,\nabla^c\phi\,\nabla_k h^{1d}{}_{d} + 8\beta\,\nabla^b\phi\,\nabla^c\delta\phi^1\,\nabla_k\nabla_b h^{1}{}_{c}{}^{k} + \\
& 8\beta\,\nabla^b\phi\,\nabla^c\delta\phi^1\,\nabla_k\nabla_c h^{1}{}_{b}{}^{k} - 8\beta\,\nabla_b\delta\phi^1\,\nabla^b\phi\,\nabla_k\nabla_c h^{1ck} - 8\beta\,\nabla^b\phi\,\nabla^c\delta\phi^1\,\nabla_k\nabla^k h^{1}{}_{bc} + 8\beta\,\nabla_b\delta\phi^1\,\nabla^b\phi\,\nabla_k\nabla^k h^{1c}{}_{c} - 4\beta\,\nabla^b\phi\,\nabla^c\phi\,\nabla_d h^{1}{}_{k}{}^{d}\,\nabla^k h^{1}{}_{bc} + \\
& 4\beta\,\nabla^b\phi\,\nabla^c\phi\,\nabla_k h^{1d}{}_{d}\,\nabla^k h^{1}{}_{bc} + 4\xi\,\nabla_c h^{1}{}_{bk}\,\nabla^k h^{1bc} - 2\xi\,\nabla_k h^{1}{}_{bc}\,\nabla^k h^{1bc} + 2\beta\,\nabla_b\phi\,\nabla^b\phi\,\nabla_d h^{1}{}_{k}{}^{d}\,\nabla^k h^{1c}{}_{c} - \beta\,\nabla_b\phi\,\nabla^b\phi\,\nabla_k h^{1d}{}_{d}\,\nabla^k h^{1c}{}_{c}\Big)
\end{aligned}
$$

QUADRATIC ACTION - ODD

$\delta\phi = 0 \quad , \quad h = h^{\mu}{}_{\mu} = 0 \; , \; \ldots$

$$\delta S^{(2)}_{ODD} = \int d^4x \, \frac{1}{8} \sqrt{-g} \Big(4\Lambda\, h^1{}_{bc}\, h^{1bc} - 4\Lambda_{eff}\,\beta\, h^1{}_{ck}\, h^{1ck}\, \nabla_b\phi\, \nabla^b\phi + 2\eta\, h^1{}_{ck}\, h^{1ck}\, \nabla_b\phi\, \nabla^b\phi + 8\beta\, G^{kd}\, h^1{}_{bk}\, h^1{}_{cd}\, \nabla^b\phi\, \nabla^c\phi +$$

$$32\Lambda_{eff}\,\beta\, h^1{}_b{}^k\, h^1{}_{ck}\, \nabla^b\phi\, \nabla^c\phi - 8\eta\, h^1{}_b{}^k\, h^1{}_{ck}\, \nabla^b\phi\, \nabla^c\phi + 16\beta\, G_b{}^k\, h^1{}_c{}^d\, h^1{}_{kd}\, \nabla^b\phi\, \nabla^c\phi -$$

$$2\beta\, G_{bc}\, h^1{}_{kd}\, h^{1kd}\, \nabla^b\phi\, \nabla^c\phi - 2\beta\, \nabla_b h^{1kd}\, \nabla^b\phi\, \nabla_c h^1{}_{kd}\, \nabla^c\phi + 8\beta\, \nabla^b\phi\, \nabla_c h^1{}_{kd}\, \nabla^c\phi\, \nabla^d h^1{}_b{}^k -$$

$$4\beta\, \nabla^b\phi\, \nabla^c\phi\, \nabla_d h^1{}_{ck}\, \nabla^d h^1{}_b{}^k + \beta\, \nabla_b\phi\, \nabla^b\phi\, \nabla_d h^1{}_{ck}\, \nabla^d h^{1ck} + 4\beta\, \nabla^b\phi\, \nabla^c\phi\, \nabla^d h^1{}_b{}^k\, \nabla_k h^1{}_{cd} -$$

$$2\beta\, \nabla_b\phi\, \nabla^b\phi\, \nabla^d h^{1ck}\, \nabla_k h^1{}_{cd} - 4\beta\, \nabla^b\phi\, \nabla^c\phi\, \nabla_d h^1{}_k{}^d\, \nabla^k h^1{}_{bc} + 4\xi\, \nabla_c h^1{}_{bk}\, \nabla^k h^{1bc} - 2\xi\, \nabla_k h^1{}_{bc}\, \nabla^k h^{1bc} \Big)$$

QUADRATIC ACTION - ODD COMPONENTS

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & h_0 \\ 0 & 0 & 0 & h_1 \\ 0 & 0 & 0 & 0 \\ h_0 & h_1 & 0 & 0 \end{pmatrix} \sin\theta \, P'_{\ell m} \quad , \qquad h_0(r,t) \, , \, h_1(r,t)$$

$$\downarrow$$

$$\delta S^{(2)}_{ODD} = \frac{2\pi}{2\ell+1} \int dt\, dr \left[a_1 h_0^2 + a_2 h_1^2 + a_3 \left(\dot{h}_1^2 - 2h_0' \dot{h}_1 + h_0'^2 + \frac{4 h_0 \dot{h}_1}{r} \right) + a_4 h_0 h_1 \right]$$

QUADRATIC ACTION - ODD COMPONENTS

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & h_0 \\ 0 & 0 & 0 & h_1 \\ 0 & 0 & 0 & 0 \\ h_0 & h_1 & 0 & 0 \end{pmatrix} \sin\theta \, P'_{\ell m} \quad , \qquad h_0(r,t) \, , \; h_1(r,t)$$

$$\downarrow$$

$$\delta S^{(2)}_{\mathrm{ODD}} = \frac{2\pi}{2\ell+1} \int dt\, dr \left[a_1 h_0^2 + a_2 h_1^2 + a_3 \left(\dot{h}_1^2 - 2h_0' \dot{h}_1 + h_0'^2 + \frac{4 h_0 \dot{h}_1}{r} \right) + a_4 h_0 h_1 \right]$$

$$\downarrow$$

$$\delta S^{(2)}_{\mathrm{ODD}} = \frac{2\pi \ell(\ell+1)}{(2\ell+1)\, 2(\ell-1)(\ell-2)} \int dt\, dr \left[b_1 \dot{Q}^2 - b_2 Q'^2 + b_3 \dot{Q} Q' - \ell(\ell+1) b_4 Q^2 - V Q^2 \right]$$

STABILITY EXAMPLE

$$S = \int d^4x \, \frac{1}{2}\left[a\dot{\psi}^2 - b\psi'^2 - c\psi^2 \right]$$

Stability Example

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$$\downarrow$$

$$a\ddot{\psi} - b\psi'' + c\psi = 0$$

STABILITY EXAMPLE

$$S = \int d^4x \, \frac{1}{2}\left[a\dot{\psi}^2 - b\,\psi'^2 - c\,\psi^2 \right]$$

$$\downarrow$$

$$a\ddot{\psi} - b\psi'' + c\psi = 0$$

$$\downarrow$$

$$\psi = Ae^{-i(kr-\omega t)} + A^* e^{i(kr-\omega t)}, \quad \omega = \sqrt{\frac{b}{a}k^2 + \frac{c}{a}}$$

$$\psi = A e^{-i(kr - wt)} + A^* e^{i(kr - wt)}, \quad \omega = \sqrt{\frac{b}{a}k^2 + \frac{c}{a}}$$

STABILITY CRITERIA : $a>0$, $b>0$, $c>0$

$$\psi = A e^{-i(kr - \omega t)} + A^* e^{i(kr - \omega t)} , \quad \omega = \sqrt{\frac{b}{a} k^2 + \frac{c}{a}}$$

STABILITY CRITERIA : $a > 0$, $b > 0$, $c > 0$

$a > 0$, $b > 0$, $c < 0$

TACHYONIC INSTABILITY

ω purely imaginary

$$\psi \sim e^{|\omega| t}$$

$$\psi = A e^{-i(kr - \omega t)} + A^* e^{i(kr - \omega t)}, \quad \omega = \sqrt{\frac{b}{a}k^2 + \frac{c}{a}}$$

STABILITY CRITERIA : $a > 0$, $b > 0$, $c > 0$

$a > 0$, $b > 0$, $c < 0$	$a > 0$, $b < 0$, $c > 0$ or $a < 0$, $b > 0$, $c > 0$
TACHYONIC INSTABILITY	**GRADIENT INSTABILITY**
ω purely imaginary	ω purely imaginary
$\psi \sim e^{\lvert\omega\rvert t}$	$\psi \sim e^{\lvert\omega\rvert t}$

$$\psi = A e^{-i(kr - wt)} + A^{*} e^{i(kr - wt)}, \quad \omega = \sqrt{\frac{b}{a}k^2 + \frac{c}{a}}$$

STABILITY CRITERIA : $a>0$, $b>0$, $c>0$

$a>0$, $b>0$, $c<0$	$a>0$, $b<0$, $c>0$ or $a<0$, $b>0$, $c>0$	$a<0$, $b<0$, $c>0$
TACHYONIC INSTABILITY	GRADIENT INSTABILITY	GHOST INSTABILITY
ω purely imaginary	ω purely imaginary	Hamiltonian unbounded from below
$\psi \sim e^{\lvert\omega\rvert t}$	$\psi \sim e^{\lvert\omega\rvert t}$	Arbitrary negative energies

$$S = \int d^4x \, \frac{1}{2} \left[a \dot{\psi}^2 - b \psi'^2 - c \psi^2 \right]$$

$$\boxed{a > 0, \quad b > 0, \quad c > 0}$$ STABILITY

EX: KLEIN-GORDON

$$S = \int d^4x \, \frac{1}{2} \left(-\partial_\mu \phi \partial^\mu \phi - m^2 \phi \right)$$

$$= \int d^4x \, \frac{1}{2} \left(\dot{\phi}^2 - \phi'^2 - m^2 \phi \right)$$

✓ STABLE

STABILITY OF ODD JOHN

$$\delta S^{(2)}_{\text{ODD}} = c \int dt\, dr \left[b_1 \dot{Q}^2 - b_2 Q'^2 + b_3 \dot{Q} Q' - \ell(\ell+1) b_4 Q^2 - V Q^2 \right]$$

[1510.07400] : $b_1 > 0$, $b_2 > 0$, $b_4 > 0$

However : $b_1 b_2 \simeq -4 (\cdots)^2 < 0$ @ horizon $f=0$

$\Downarrow$

either $b_1 < 0$ or $b_2 < 0$ $\Rightarrow$ UNSTABLE

STABILITY OF ODD JOHN

BUT!

[1803.11444] CLAIMS THAT CONCLUSIONS IN [1510.07400] ARE WRONG

- [1510.07400]'s ARGUMENT IS THAT HAMILTONIAN IS UNBOUNDED FROM BELOW
- THIS IS A COORDINATE-DEPENDENT STATEMENT
- HENCE DOES NOT NECESSARILY IMPLY INSTABILITIES
- [1803.11444] PROVIDES STABILITY CRITERIA IN TERMS OF "ALIGNMENT" OF CAUSAL CONES FOR ALL DOFs AND FIND A STABLE RANGE OF PARAMETERS