Ringdown Tests of Gravity

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[2301.10272, 2408.01720] SS + Johannes Noller

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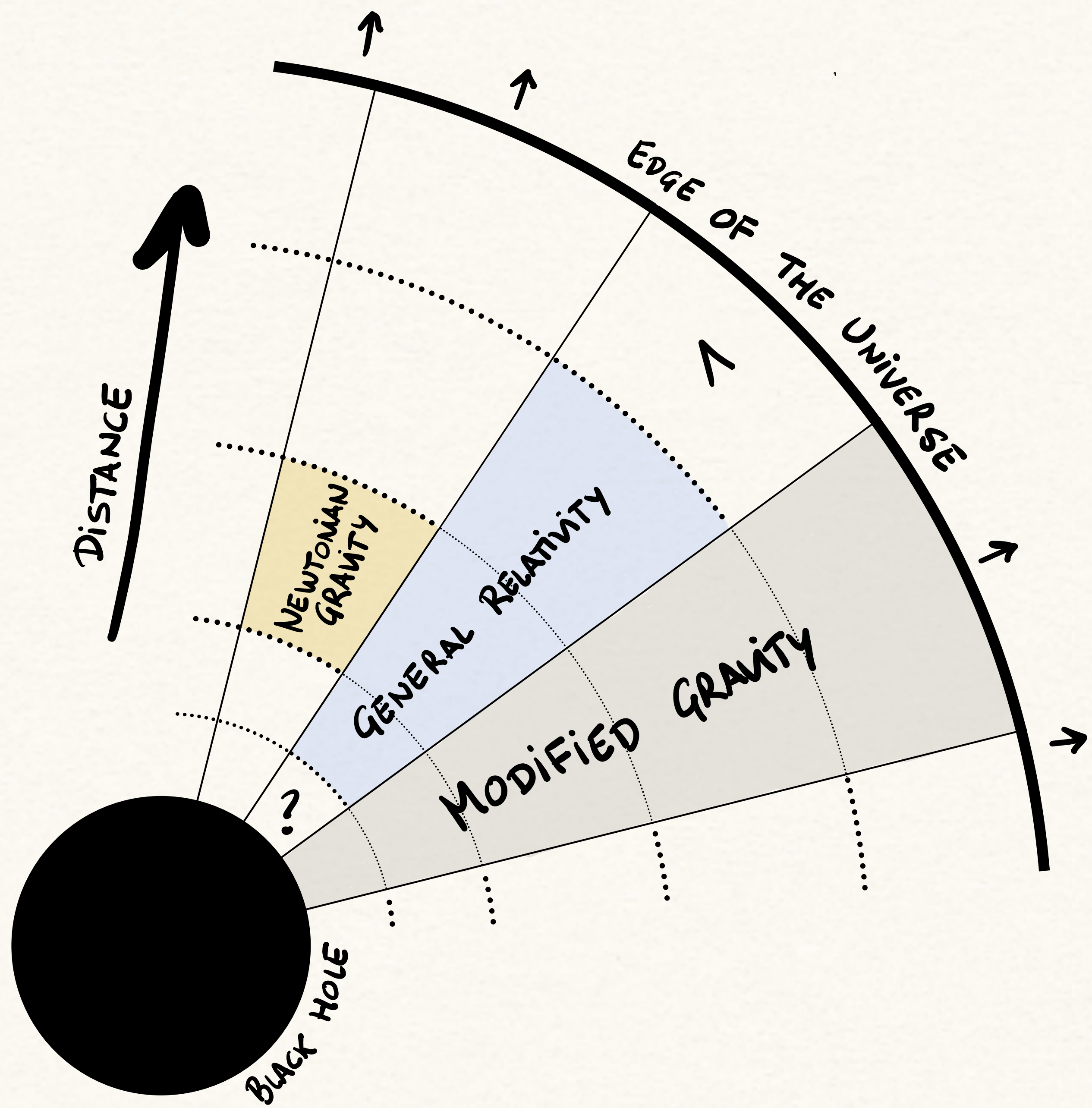
TESTING GRAVITY



TESTING GRAVITY

(LOVELOCK'S THEOREM)

$$\left.\begin{array}{c} 4D \\ g_{\mu\nu} \\ \text{LOCAL} \\ 2^{nd} \text{ order EOM} \end{array}\right\} \quad \text{GR} \quad \rightarrow \quad S = \int d^4x \sqrt{-g}\, R[g_{\mu\nu}]$$

$$\left.\begin{array}{c} 4D \\ g_{\mu\nu} + \phi \\ \text{LOCAL} \\ 2^{nd} \text{ order EOM} \end{array}\right\} \quad \text{HORNDESKI} \quad \rightarrow \quad S = \int d^4x \sqrt{-g}\, H[g_{\mu\nu}, \phi]$$

TESTING GRAVITY

$$S = \int d^4x \sqrt{-g} \left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 \right)$$

HORNDESKI GRAVITY

$$\mathcal{L}_2 = G_2(\phi, X)$$

$$\mathcal{L}_3 = G_3(\phi, X) \Box \phi$$

$$\mathcal{L}_4 = G_4(\phi, X) R + G_{4X}(\phi, X) \left[(\Box\phi)^2 - (\phi_{\mu\nu})^2 \right]$$

$$\mathcal{L}_5 = G_5(\phi, X)\, G_{\mu\nu} \phi^{\mu\nu} - \frac{1}{6} G_{5X}(\phi, X) \left[(\Box\phi)^3 - 3 (\phi_{\mu\nu})^2 \Box\phi + 2 (\phi_{\mu\nu})^3 \right]$$

WHERE $X = -\frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi$, $\phi_\mu := \nabla_\mu \phi$, $\phi_{\mu\nu} := \nabla_\nu \nabla_\mu \phi$, ...

$G_{4X} := \partial_X G_4$ $\quad$ $(\phi_{\mu\nu})^2 := \phi_{\mu\nu}\phi^{\mu\nu}$

$(\phi_{\mu\nu})^3 := \phi_{\mu\nu}\phi^{\mu\sigma}\phi_\sigma{}^\nu$

TESTING GRAVITY

THEORY $\longrightarrow$ OBSERVABLE

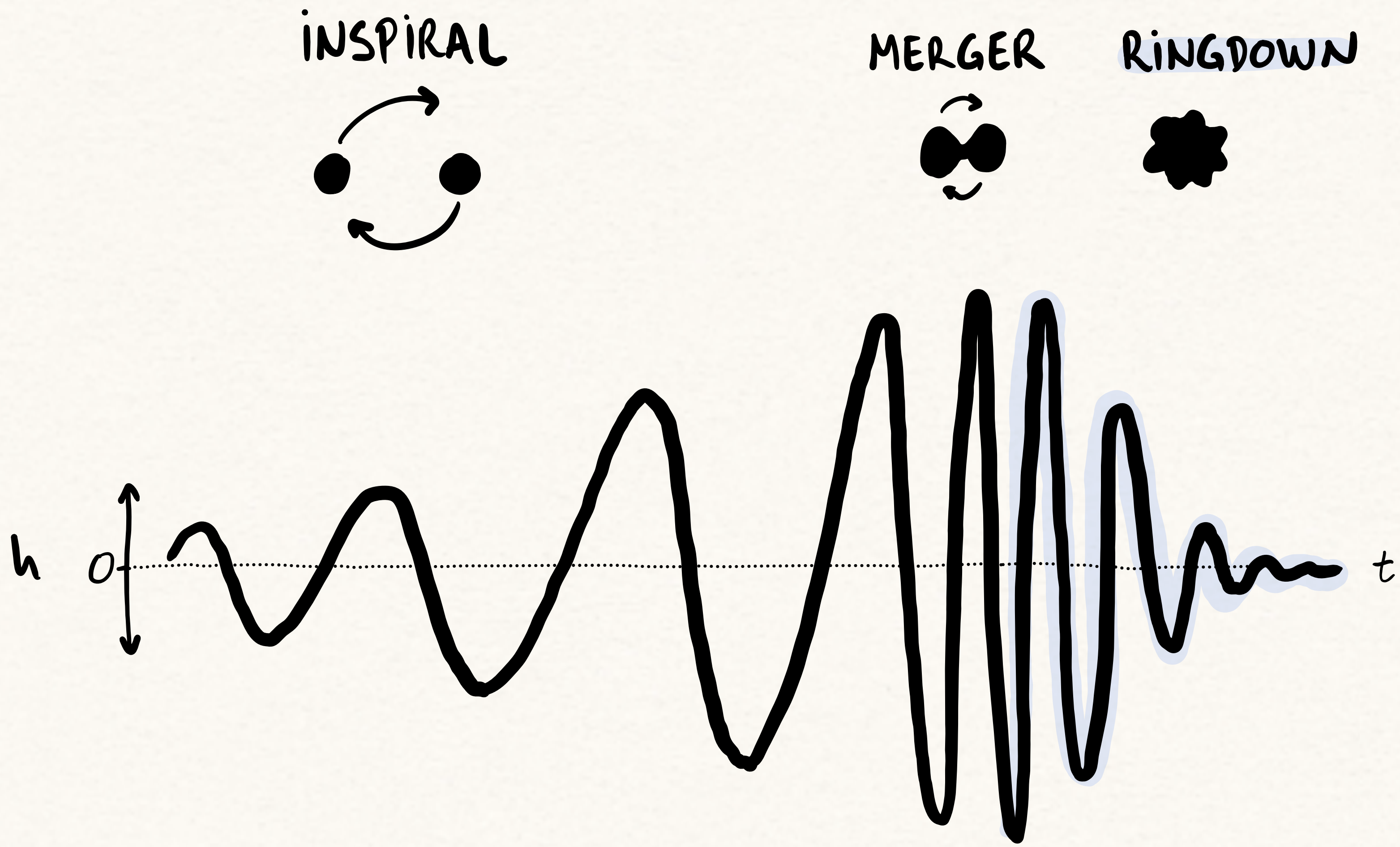
$$S = \int d^4x \sqrt{-g}\, R \quad \longrightarrow \quad \alpha = 0$$

$$S = \int d^4x \sqrt{-g}\, H \quad \longrightarrow \quad \alpha \neq 0$$

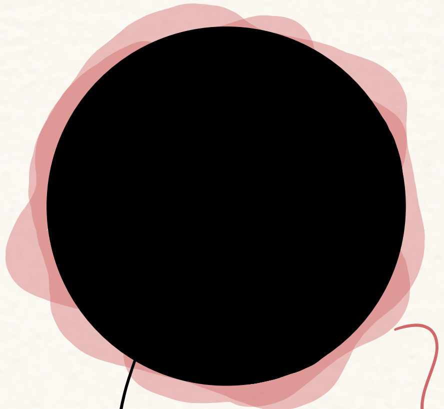
"SMOKING GUN SIGNAL"

Gravitational Waves : BH Perturbation Theory

Astrophysical sources : Mergers (black holes / neutron stars)



GRAVITATIONAL WAVES : BH PERTURBATION THEORY



$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

$$S = \int d^4x \sqrt{-g}\, R$$

$$\frac{d^2 h}{dr_*^2} + \left[\omega^2 - V\right] h = 0$$

$$\omega\,(M, a)$$

QUASINORMAL MODES

(NO HAIR THEOREM)

BACKGROUND

- SCHWARZSCHILD
- SCHWARZSCHILD - DE SITTER
- REISSNER-NORDSTRÖM
- ...

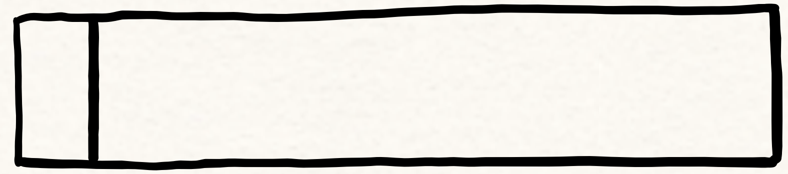
PERTURBATIONS

- ODD → REGGE-WHEELER
- EVEN → ZERILLI

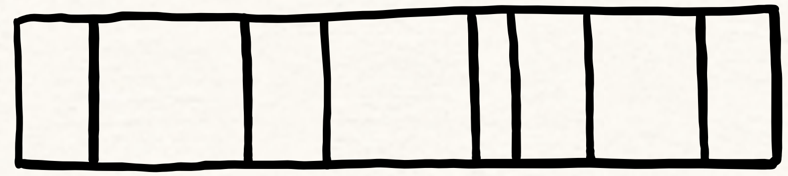
BLACK HOLE SPECTROSCOPY

$\omega(M,a)$

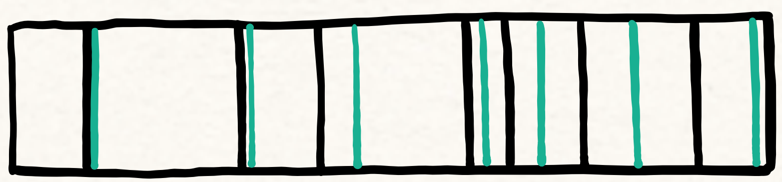
- 1st QNM sets (M,a)



- All other QNMs are fixed in GR



MEASURING QNMs PROVIDES CLEAN TESTS OF BACKGROUND GEOMETRY AND UNDERLYING THEORY



GR $\omega(M,a)$

MG $\omega(M,a,\alpha)$

RINGDOWN OF HAIRY BHs

IN SHIFT-SYMMETRIC SCALAR-TENSOR THEORIES, BLACK HOLES CANNOT SUPPORT SCALAR HAIR IF:

1. SPACETIME IS SPHERICALLY SYMMETRIC, STATIC AND ASSYMPTOTICALLY FLAT
2. SCALAR FIELD IS STATIC $\phi(r)$ AND HAS A VANISHING DERIVATIVE ϕ' AT INFINITY
3. NORM OF THE CURRENT ASSOCIATED W/ SHIFT SYMMETRY IS FINITE DOWN TO THE HORIZON
4. ACTION CONTAINS A CANONICAL KINETIC TERM $X \subset G_2$
5. ALL $G_i(X)$ FUNCTIONS ARE ANALYTICAL AT $X = 0$

[1312.5742] HU, RAVERI, FRUSCINANTE, SILVESTRI

Ringdown of Hairy BHs

Static $\phi(r)$

- E.G. Scalar Gauss-Bonet

$$G_2 = \eta X, \quad G_4 = \zeta, \quad G_5 = \alpha \ln|X|$$

[1312.3622, 1408.1698] Sotiriou, Zhou

- More general

$$G_2 \supseteq \sqrt{X}, \quad G_3 \supseteq \ln|X|,$$
$$G_4 \supseteq \sqrt{X}, \quad G_5 \supseteq \ln|X|$$

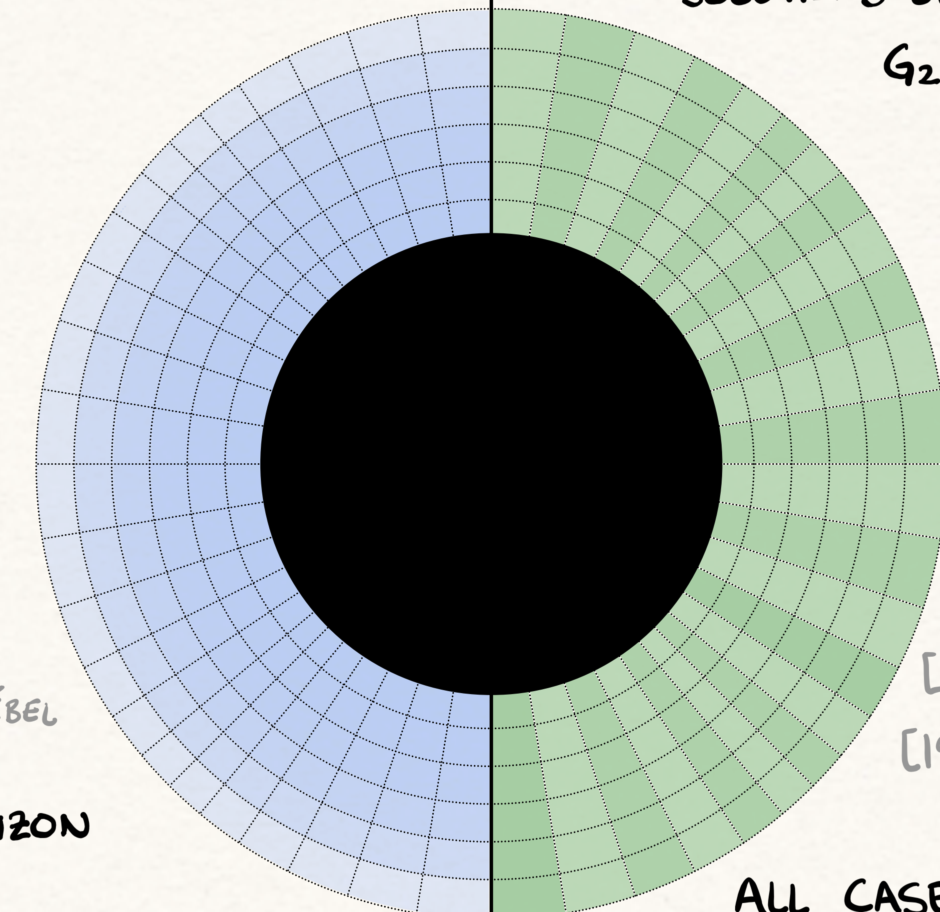
[1702.01938] Babichev, Charmousis, Lehébel

→ Finite-norm current at horizon

→ Lack Lorentz invariant ($X=0$) solution in Minkowski spacetime

[1903.02055] Saravani, Sotiriou

Real problem or not?



Time-dependent $\phi(t,r)$

Solutions of shift + reflection symm Horndeski $G_2(X)$, $G_4(X)$

[1312.3204] Babichev, Charmousis

[1403.4364] Kobayashi, Tanahashi

However, prone to instabilities

[1510.07400] Ogawa, Kobayashi, Suyama

[1803.11444] Babichev, Charmousis, Esposito-Farèse, Lehébel

[1610.00432] Takahashi, Suyama

[1904.03554] Takahashi, Motohashi, Minamitsuji

[1907.00699] de Rham, Zhang

All cases studied assumed $X = \text{constant}$

Solution with $X \neq \text{constant}$

[2310.11919] Bakopoulos, Charmousis, Kanti, Lecoeur, Nakas

STATIC HAIR AND SPEED OF GRAVITY

[2301.10272] SS + JOHANNES NOLLER

SET-UP

THEORY : HORNDESKI WITH $G_{4\phi} = 0$

BACKGROUND : METRIC : SCHWARZSCHILD

SCALAR : STATIC HAIRY DEVIATION $\bar{\Phi} = \hat{\phi} + \varepsilon\, \delta\phi(r)$, $\delta\phi = \varphi_c \frac{2M}{r}$

MODIFIED REGGE - WHEELER EQUATION

GRAVITATIONAL WAVE SPEED EXCESS

$$\frac{d^2Q}{dr_*^2} + \left[\omega^2(1+\alpha_T) + V + \alpha_T\, \delta V\right]Q = 0$$

$$\alpha_T = -f\,(2M)^2\, G_T\, \delta\phi'^2$$

$$G_T = \frac{G_{4X} - G_{5\phi}}{G_4}$$

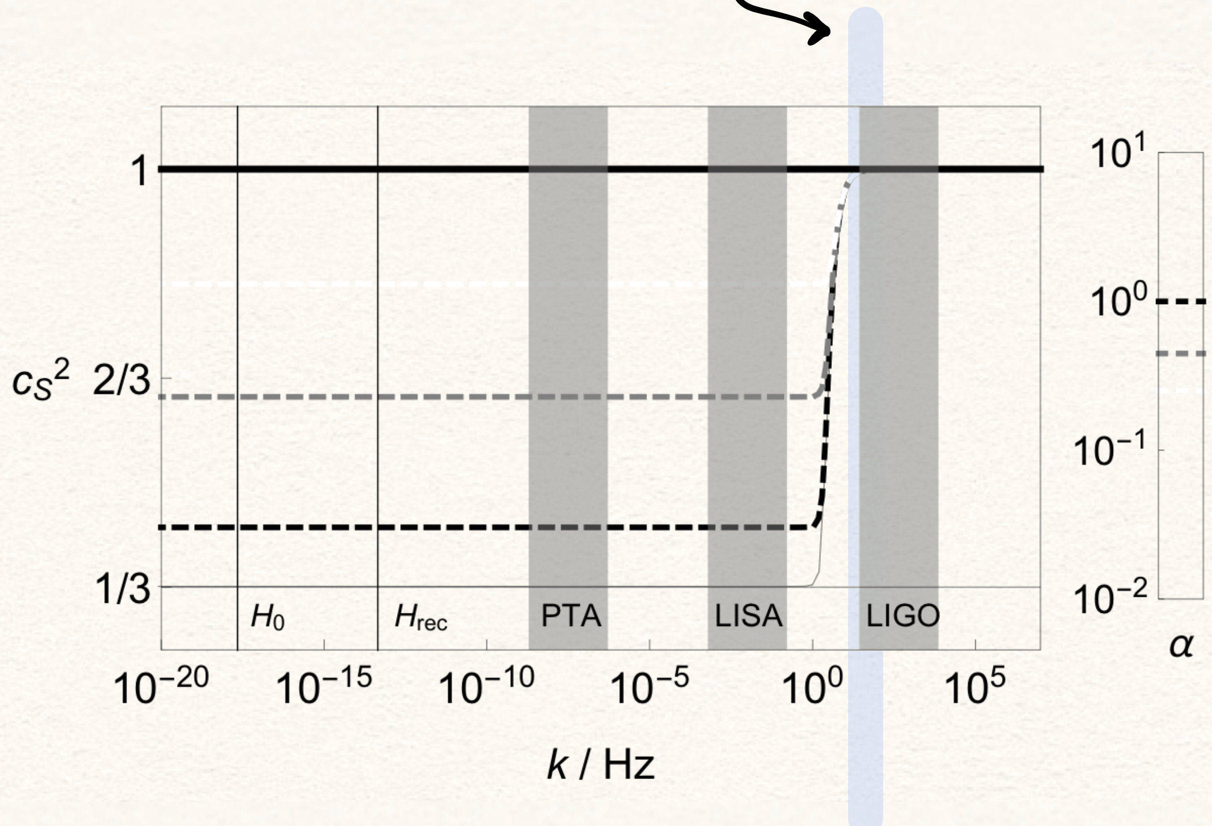
$$\delta V = -\frac{1}{f}\omega_0^2 + \frac{M(2r-5M)}{r^3(r-2M)} + \frac{(\ell+2)(\ell-1)}{r^2} - \frac{r-2M}{2r}\left(\left(\frac{\delta\phi''}{\delta\phi'}\right)^2 - \frac{\delta\phi'''}{\delta\phi'}\right) + \frac{r-5M}{r^2}\frac{\delta\phi''}{\delta\phi'}$$

STATIC HAIR AND SPEED OF GRAVITY

[2301.10272] SS + JOHANNES NOLLER

WHAT DO WE KNOW ABOUT α_T?

- LIGO : $\alpha_T \lesssim 10^{-15}$ (GW170817)
- DARK ENERGY EFTs : CUTOFF AT $\sim 10^2$ Hz [1806.09417] MELVILLE, DE RHAM



Static Hair and Speed of Gravity

[2301.10272] SS + Johannes Noller

Obtain QNM corrections with Parametrised Ringdown Formalism

[1901.01265] Cardoso, Kimura, Maselli, Berti, Macedo, McManus

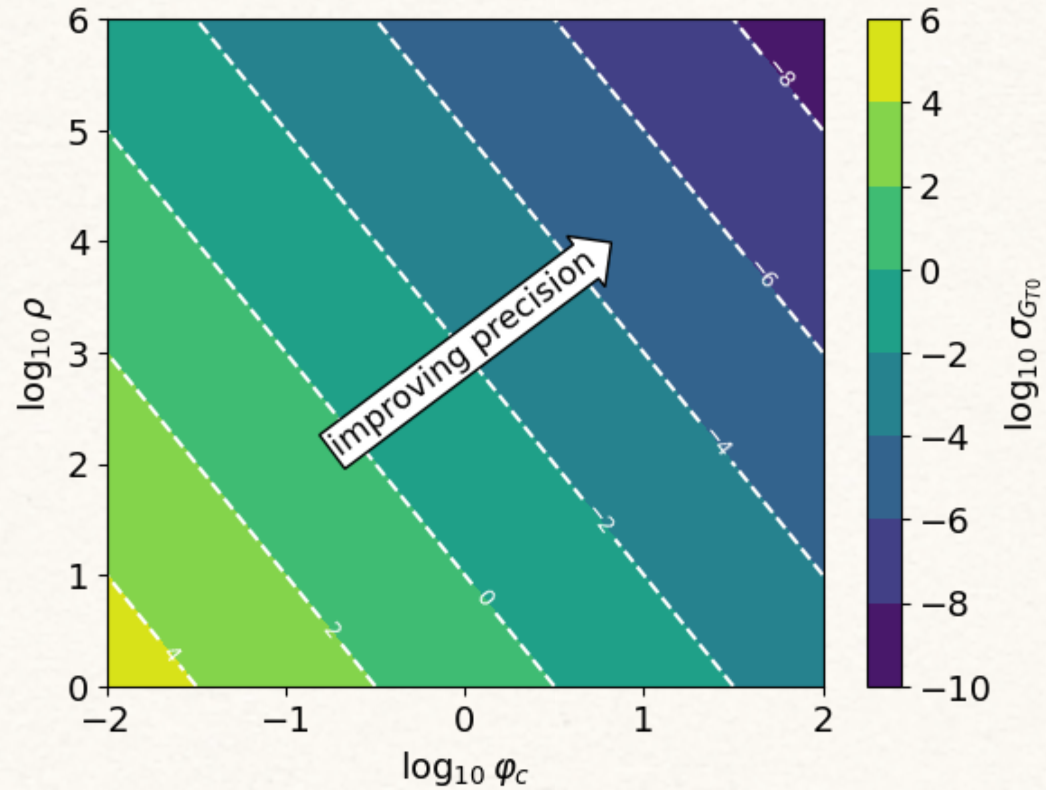
$$\underbrace{\omega}_{} = \underbrace{\omega_0}_{\text{GR}} + \delta\omega(\alpha_T)$$

$$\delta\omega = \sum_{i=0}^{\infty} A_{Ti}\left[(2M\omega_0)^2 e_{4+i} - (\ell(\ell+1)-9)\, e_{6+i} + (\ell(\ell+1)-20)\, e_{7+i} + \frac{45}{4} e_{8+i}\right]$$

$$A_{Ti} = G_{Ti}\,\varphi_c^2 \quad , \quad G_T = \frac{1}{(2M)^2}\frac{G_{4X} - G_{5\phi}}{G_4} = \sum_i G_{Ti}\left(\frac{2M}{r}\right)^i$$

Fisher Forecast Analysis

Detector(s)	Ringdown SNR (ρ)	Error on α_T
LVK	10 [136–138]	1
ET / CE	10^2 [138–141]	10^{-1}
pre-DECIGO	10^2 [142]	10^{-1}
DECIGO / AEDGE	10^3 [143, 144]*	10^{-2}
LISA	10^5 [137, 145]	10^{-4}
TianQin	10^5 [145]	10^{-4}
AMIGO	10^5 [130]	10^{-4}



Time-Dependent Hair

[2408.01720] SS + Johannes Noller

$$\phi = qt + \psi(r)$$

$\mathscr{L}$	Background solution $g_{\mu\nu}$	X	Stability odd	even
(Shift + refl)-sym Horndeski [29, 30]	S(dS)*	$\frac{q^2}{2} = \text{const}$	✓	✗
Cubic Galileon [31, 32]	S(dS)* (non-exact)	non-const (non-exact)	?	?
Shift-sym breaking Horndeski [65]	∄ (for large subclasses)	∄ (for large subclasses)	-	-
$G_2 = \eta X, G_4 = \zeta + \beta\sqrt{X}$ [33]	S(dS)* + RN(dS)* (non-exact)	non-const	?	?
Shift-sym beyond Horndeski [34]	SdS*	$\frac{q^2}{2} = \text{const}$	✓	✗
Shift-sym breaking quadratic DHOST [35]	S(dS)*	$\frac{q^2}{2} = \text{const}$	✓	✗
Shift-sym quadratic DHOST [36, 66]	S(dS)* + K*	const	✓	✗
Quadratic DHOST [37]	S(dS)* + (K)RN(dS)*	const	✓	✗
$G_2 = -2\Lambda + 2\eta\sqrt{X}, G_4 = 1 + \lambda\sqrt{X}$ [1]	S(dS)	$\frac{1}{2}\frac{q^2\lambda}{\lambda+\eta r^2}$	✓(this work)	?

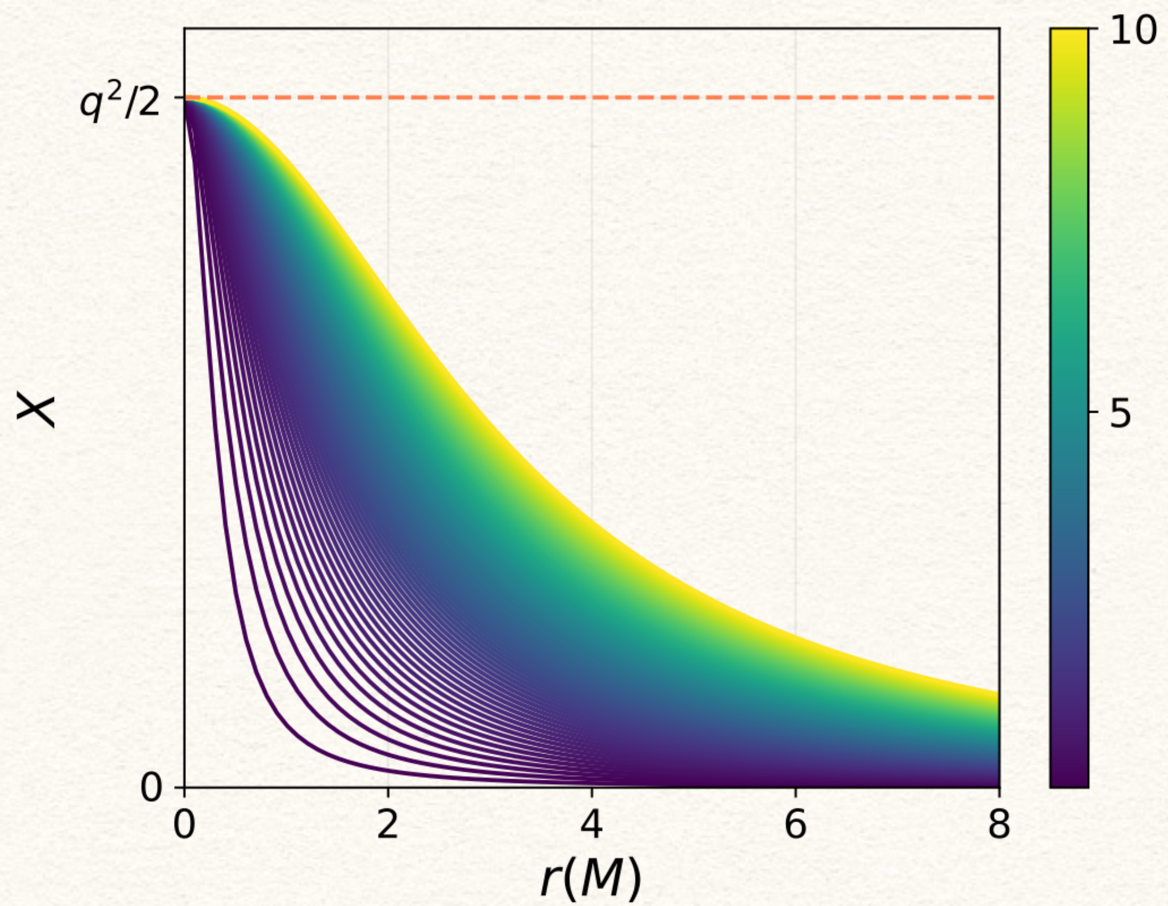
Time-Dependent Hair

Set-up

Theory : $$\mathcal{L} = -2\Lambda + 2\eta\sqrt{X} + (1 + \lambda\sqrt{X})R + \frac{\lambda}{2\sqrt{X}}\left(\Box\phi^2 + \phi_{\mu\nu}^2\right)$$

Background : Metric : Schwarzschild – de Sitter

Scalar : $\bar{\Phi} = qt + \psi(r)$, $\quad \psi'(r)^2 = \frac{q^2}{B}\left(1 - \frac{\lambda B}{\lambda + \eta r^2}\right)$, $\quad X = \frac{1}{2}\frac{q^2\lambda}{\lambda + \eta r^2}$



$\beta^2 = \frac{\lambda}{2\eta}$

Time-Dependent Hair

[2408.01720] SS + Johannes Noller

Quadratic action

$$S^{(2)} = \frac{1}{2}\int d^4x\sqrt{-g}\left[\mathscr{L}^{(2)}_{GR} + \mathscr{L}^{(2)}_{\eta} + \mathscr{L}^{(2)}_{\lambda}\right],$$

$$\begin{aligned}
\mathscr{L}^{(2)}_{GR} &= \frac{1}{2}\left(\nabla^\sigma h^{\mu\nu}(2\nabla_\nu h_{\mu\sigma} - \nabla_\sigma h_{\mu\nu}) + 2\Lambda h_{\mu\nu}h^{\mu\nu}\right),\\
\mathscr{L}^{(2)}_{\eta} &= \frac{-\eta}{\sqrt{X}}\left(X h_{\mu\nu}h^{\mu\nu} + \phi^\mu\phi^\nu h^\sigma_\mu h_{\nu\sigma}\right),\\
\mathscr{L}^{(2)}_{\lambda} &= \frac{-\lambda}{2\sqrt{X}}\Bigg[\nabla_\sigma h_{\mu\nu}\left(X\left(\nabla^\sigma h^{\mu\nu} - 2\nabla^\nu h^{\mu\sigma}\right) + \phi^\sigma\phi^\rho\left(\frac{1}{2}\nabla_\rho h^{\mu\nu} - 2\nabla^\nu h^\mu_\rho\right) + \phi^\mu\phi^\nu\nabla_\rho h^{\sigma\rho} + 2\phi^\mu\phi^\rho\nabla^{\{\nu}h^{\sigma\}}_\rho\right)\\
&\qquad + h_{\mu\nu}\Bigg(\frac{1}{2}h^{\mu\nu}((\Box\phi)^2 - \phi_{\rho\gamma}\phi^{\rho\gamma}) + 4h^\nu_\sigma(\Lambda\phi^\mu\phi^\sigma - \phi^{\sigma\mu}\Box\phi + \phi^\mu_\rho\phi^{\rho\sigma}) + 4h_{\sigma\rho}\phi^{\mu[\sigma}\phi^{\nu]\rho}\\
&\qquad\quad + \frac{1}{2X}\left(h^\nu_\sigma\phi^\mu\phi^\sigma(\phi_{\rho\gamma}\phi^{\rho\gamma} - (\Box\phi)^2) + 2\phi^{\sigma\rho}(\phi^\mu\phi^\nu\phi^\lambda_{\{\sigma}h_{\rho\}\lambda} - h_{\sigma\rho}\phi^\mu\phi^\nu\Box\phi)\right)\\
&\qquad\quad + 4\phi^\rho\phi^{\mu\sigma}(2\nabla_{[\sigma}h^\nu_{\rho]} + \nabla^\nu h_{\rho\sigma}) + 2\phi^\mu\phi^{\sigma\rho}(2\nabla_\sigma h^\nu_\rho - \nabla^\nu h_{\sigma\rho}) + 2\phi^\mu\phi^{\nu\sigma}\nabla_\rho h^\rho_\sigma\\
&\qquad\quad + 2\phi^\sigma\phi^\mu_\sigma\nabla_\rho h^{\nu\rho} + 2\phi^\sigma\phi_{\sigma\rho}\nabla^{[\nu}h^{\rho]\mu} + 2\Box\phi(\phi^\sigma\nabla_\sigma h^{\mu\nu} - 2\phi^\mu\nabla_\sigma h^{\nu\sigma} - 2\phi^\sigma\nabla^\nu h^\mu_\sigma)\\
&\qquad\quad + \frac{1}{2X}\phi^\mu\phi^\nu\phi^\sigma\left(\phi^\rho_\sigma\nabla_\gamma h^\gamma_\rho + \phi^{\rho\gamma}(2\nabla_{[\rho}h_{\gamma\}\sigma} - \nabla_\sigma h_{\rho\gamma})\right)\Bigg)\Bigg],
\end{aligned}$$

TIME-DEPENDENT HAIR

[2408.01720] SS + JOHANNES NOLLER

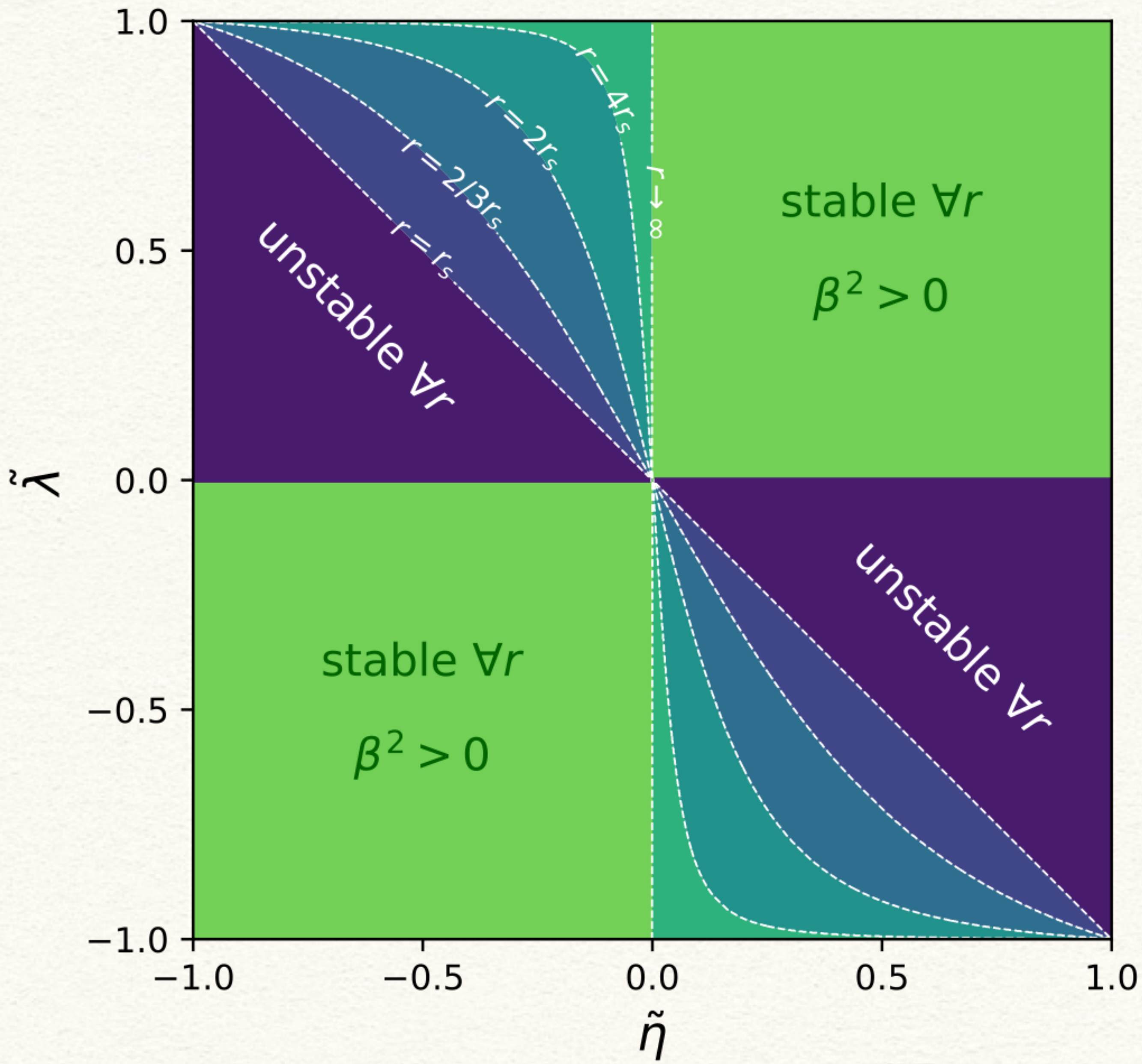
STABILITY CONDITIONS

$$S^{(2)} = \frac{\ell(\ell+1)}{4(\ell-1)(\ell+2)} \int dt\, dr \left[\tilde{b}_1 (\partial_t Q)^2 - b_2 Q'^2 - \left(\ell(\ell+1) b_4 + V_{eff} \right) Q^2 \right]$$

$\tilde{b}_1 > 0$

$b_2 \geq 0$

$b_4 \geq 0$



TIME-DEPENDENT HAIR

[2408.01720] SS + JOHANNES NOLLER

MODIFIED REGGE-WHEELER EQUATION

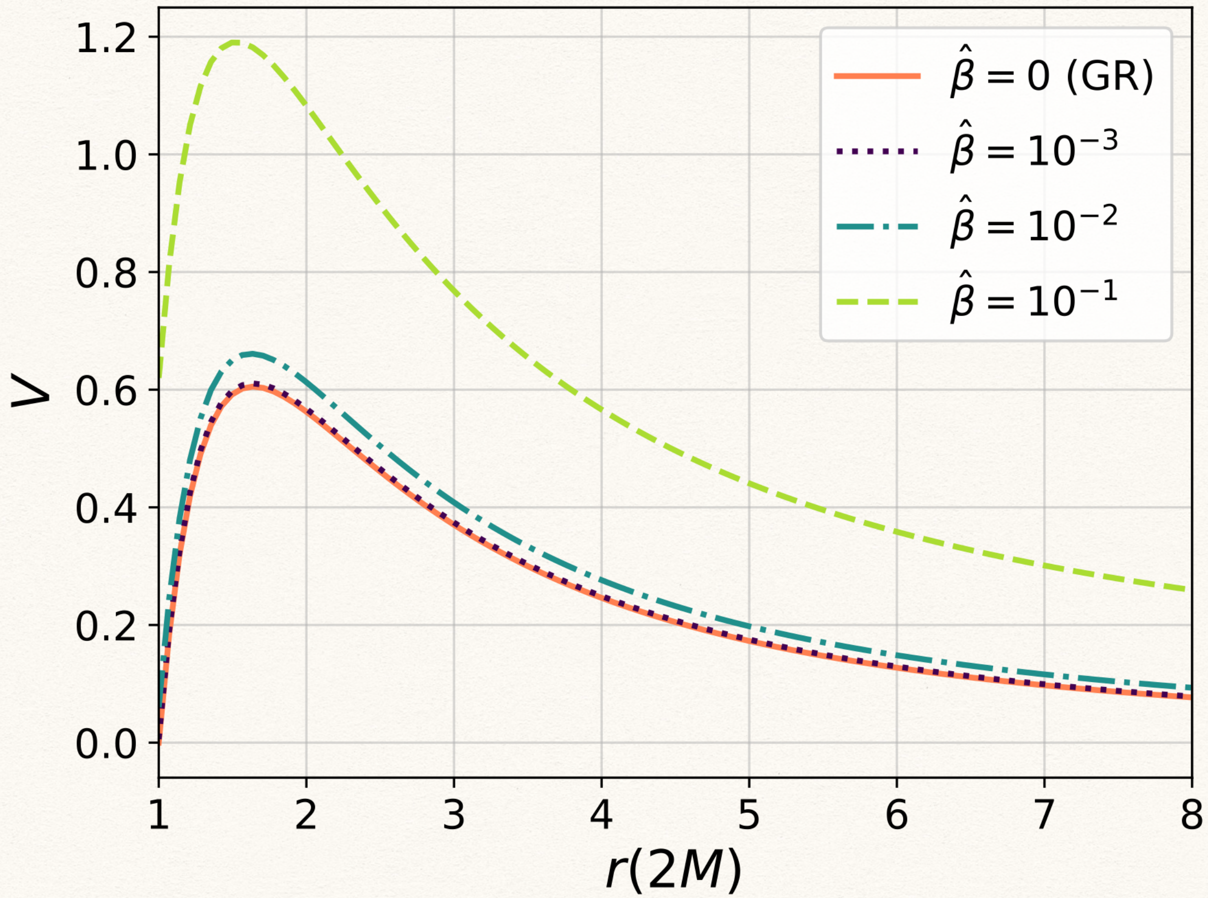
$$\frac{d^2 Q}{d r_*^2} + \left[\omega^2 - V \right] Q = 0$$

MODIFIED REGGE-WHEELER POTENTIAL

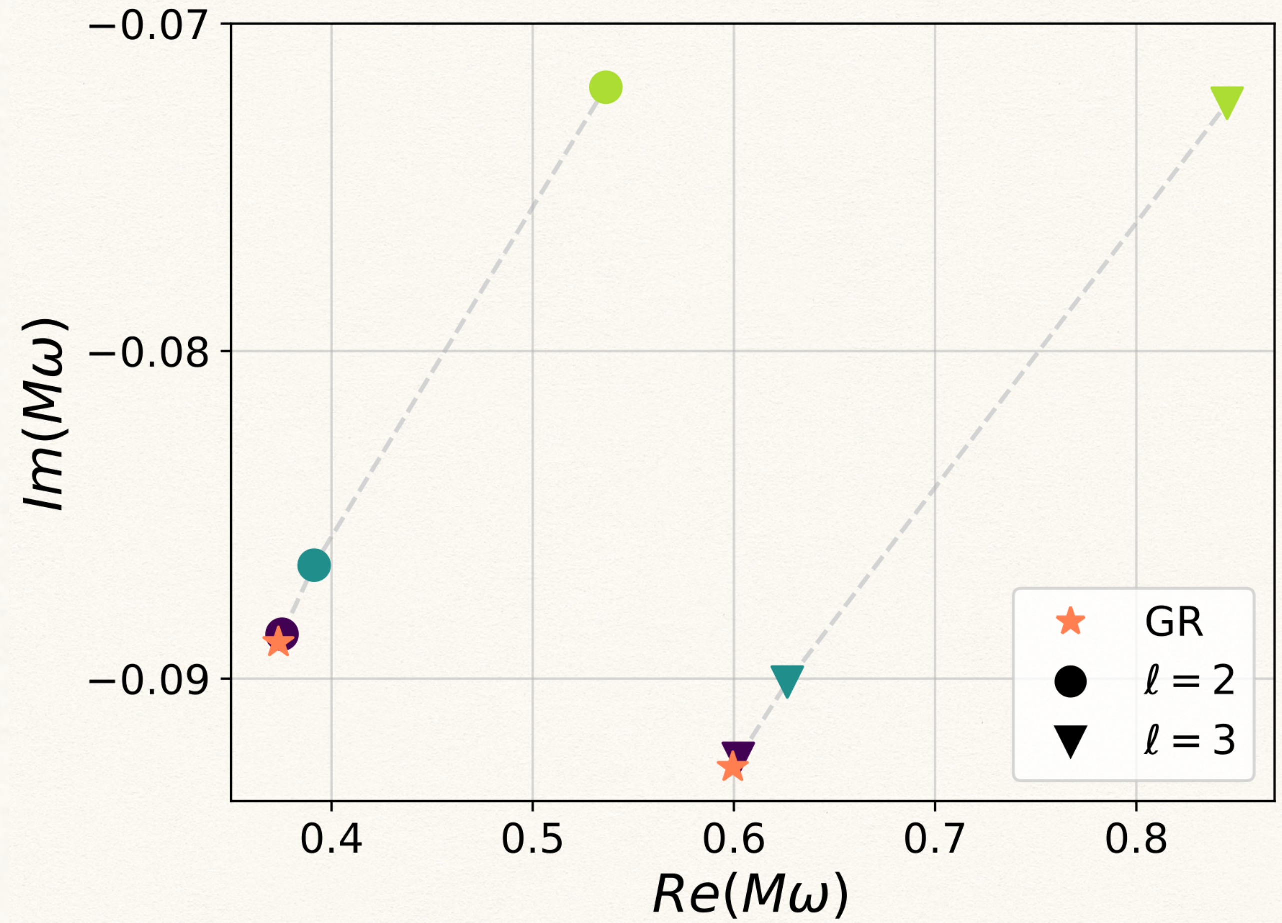
$$V = V_{RW} \left(1 + \frac{q\beta\sqrt{2\beta^2 + r^2}}{B} \right) + \frac{q\beta(B + q\beta\sqrt{2\beta^2 + r^2})}{4r^2(2\beta^2 + r^2)(2q\beta^3 + \sqrt{2\beta^2 + r^2})^3} \times$$
$$\times \Big[q^2\beta^6 \left(192\beta^4 + 104\beta^2 r^2 + 3r^4\right) + 2(2\beta^2 + r^2)\left(24\beta^4 + 17\beta^2 r^2 + 2r^4\right) + 6q\beta^3\sqrt{2\beta^2 + r^2}\left(384\beta^4 + 416\beta^2 r^2 + 117 r^4\right)$$
$$- \beta^2 B \left(2\left(48 q^2 \beta^6 (2\beta^2 + r^2) + 48\beta^4 + 56\beta^2 r^2 + 19 r^4 \right) \right) + \frac{q\beta^3}{\sqrt{2\beta^2 + r^2}} \left(384\beta^4 + 416\beta^2 r^2 + 117 r^4\right) \Big]. \tag{48}$$

Time-Dependent Hair

Modified Regge-Wheeler Potential



QNM Deviations



TIME-DEPENDENT HAIR

[2408.01720] SS + JOHANNES NOLLER

QNMS CALCULATED WITH WKB APPROXIMATION

[1904.1033] KONOPLYA, ZHIDENKO, ZINHAILO

$$\omega^2 = V_0 - i\left(n+\frac{1}{2}\right)\sqrt{2V_0^{(2)}} - i\sqrt{2V_0^{(2)}}\sum_{i=2}^{N}\Lambda_j.$$

	$\mathrm{Re}(M\omega)$	$\mathrm{Im}(M\omega)$
$\hat{\beta}=0$ (GR)	0.3736	−0.0889
$\hat{\beta}=10^{-3}$	0.3754	−0.0886
$\hat{\beta}=10^{-2}$	0.3914	−0.0865
$\hat{\beta}=10^{-1}$	0.5364	−0.0719

LIGHT RING EXPANSION

[1810.07706] FRANCIOLINI, HUI, PENCO, SANTONI, TRINCHERINI

POTENTIAL MAXIMUM

$$\frac{M_{\rm Pl}^2}{M} r_*^{\rm max} = 3.2808 - 3.0306\cdot\hat{\beta} + 0.8316\cdot\hat{\beta}^2 - \left(0.22819 + 1.8502\frac{M_{\rm Pl}^8}{M^4 q^2}\right)\cdot\hat{\beta}^3 + \mathcal{O}(\hat{\beta}^4)$$

SEMI-ANALYTIC QNMS

$$\frac{M\omega}{M_{\rm Pl}^2} = \frac{M\omega_0}{M_{\rm Pl}^2} + (1.80+0.25i)\hat{\beta} - (2.24+1.57i)\hat{\beta}^2 + \mathcal{O}(\hat{\beta}^3),$$

FISHER FORECAST ANALYSIS

Detector(s)	Ringdown SNR (ρ)	Error on $\hat{\beta}$
LVK	10 [132–134]	10^{-2}
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DECIGO / AEDGE	10^3 [139, 140]*	10^{-4}
LISA	10^5 [133, 141]	10^{-6}
TianQin	10^5 [141]	10^{-6}
AMIGO	10^5 [142]	10^{-6}

Summary

- QNMs can be used to test new gravitational physics

Static

- Set up: Horndeski w/ $G_{4\phi} = 0$
  Parametrised static hairy deviations
- QNMs are α_T-dependent iff $\exists$ scalar hair
- Calculated QNM corrections $\delta\omega(\alpha_T)$ w/ parametrised ringdown formalism
- LISA can constrain $|\alpha_T| \leq 10^{-4}$ w/ ringdown of one SMBH merger

Time-dependent

- Set up: Specific theory w/ $G_2, G_4 \supseteq \sqrt{X}$
  Linear t-dep scalar w/ $X \neq$ const
- Identified stable parameter space
- Calculated QNM corrections $\delta\omega(\hat{\beta})$ w/ WKB approximation
- LISA can constrain $\hat{\beta} \leq 10^{-6}$ w/ ringdown of one SMBH merger

Thank you!

GitHub

ringdown-calculations Public
A collection of notebooks for black hole perturbation theory calculations in GR and modified gravity.
Mathematica 7 4