Stealth Black Holes in Cubic HOST Theories

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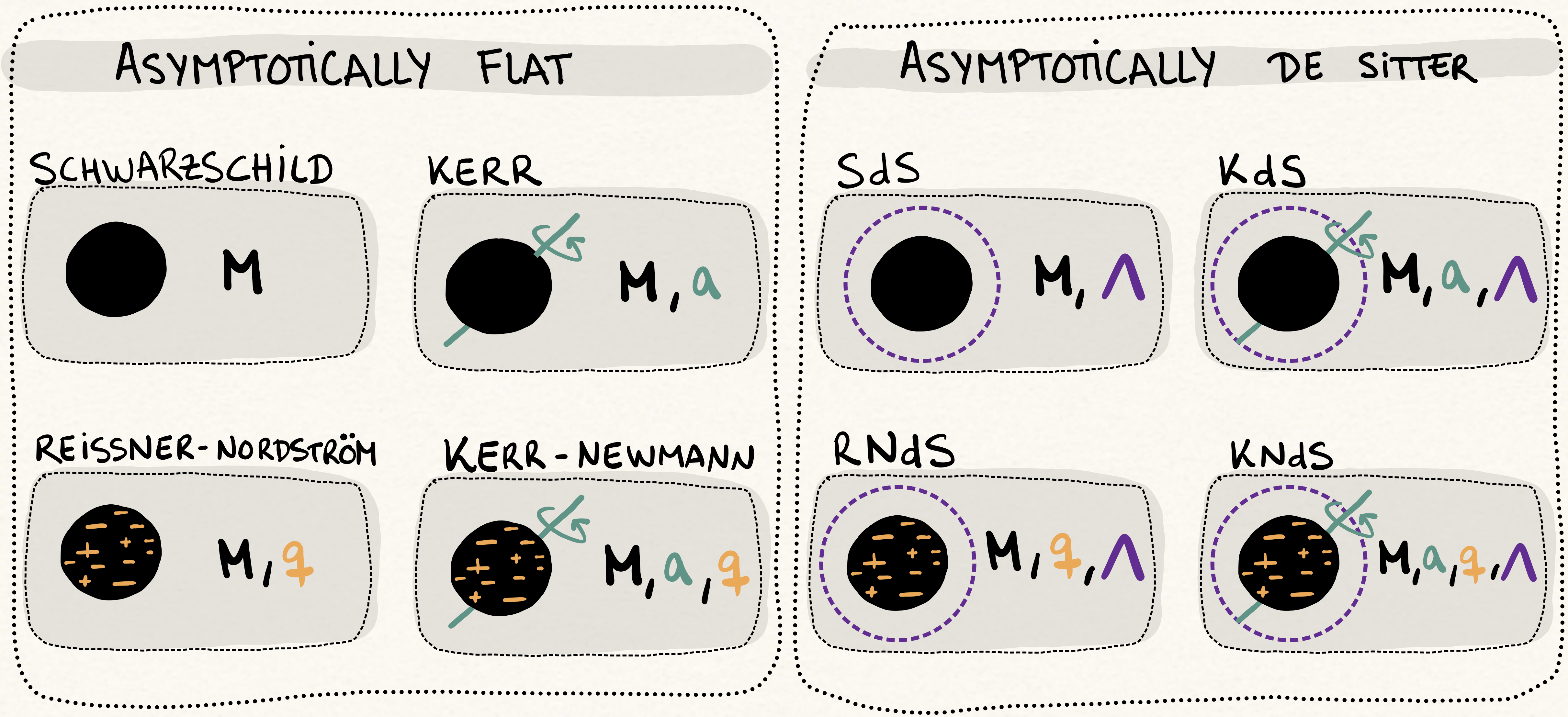
Stealth Black Holes: Metric Solutions of General Relativity



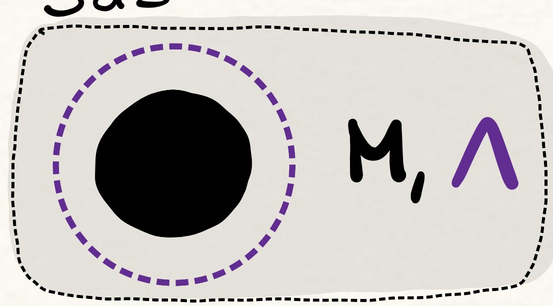
Stealth Black Holes: Metric Solutions of General Relativity

$$ds^2 = \overbrace{-f\,dt^2 + \frac{1}{f}\,dr^2 + r^2 d\Omega_2^2}^{(t,r,\theta,\varphi)} = \overbrace{-d\tau^2 + (1-f)\,d\rho^2 + r^2 d\Omega_2^2}^{(\tau,\rho,\theta,\varphi)}$$

SCHWARZSCHILD

$$f_{Schw} = 1 - \frac{2M}{r}$$

SdS



$$f_{SdS} = 1 - \frac{2M}{r} - \frac{1}{3}\Lambda r^2$$

Stealth Black Holes: Metric Solutions of General Relativity

Uniqueness Theorems

This family of black holes are the unique solutions for asymptotically flat (de Sitter), stationary, axisymmetric spacetimes in (electro-) vacuum.

Black holes are fully characterised by M, a, q.

These black holes are bald

STEALTH BLACK HOLES: METRIC SOLUTIONS OF GENERAL RELATIVITY

IF WE ADD MORE FUNDAMENTAL FIELDS, CAN WE HAVE BLACK HOLES WITH MORE PARAMETERS?

NO-HAIR THEOREMS

SATISFIED

BALD BLACK HOLES

EXAMPLE: EINSTEIN-MAXWELL-KLEIN-GORDON

$$S = \int d^4x \sqrt{-g} \left(R - 2\Lambda - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \pm \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi \right)$$

$M \quad a \quad \Lambda \quad q \quad \times$

STEALTH BLACK HOLES: METRIC SOLUTIONS OF GENERAL RELATIVITY

IF WE ADD MORE FUNDAMENTAL FIELDS, CAN WE HAVE BLACK HOLES WITH MORE PARAMETERS?

NO-HAIR THEOREMS

BALD BLACK HOLES

SATISFIED

VIOLATED

HAIRY BLACK HOLES

EXAMPLE: SCALAR GAUSS-BONNET

$$S = \int d^4x \sqrt{-g} \left(R - \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi + \alpha \phi \mathcal{G} \right)$$

$$\mathcal{G} \equiv R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\sigma\rho} R^{\mu\nu\sigma\rho}$$

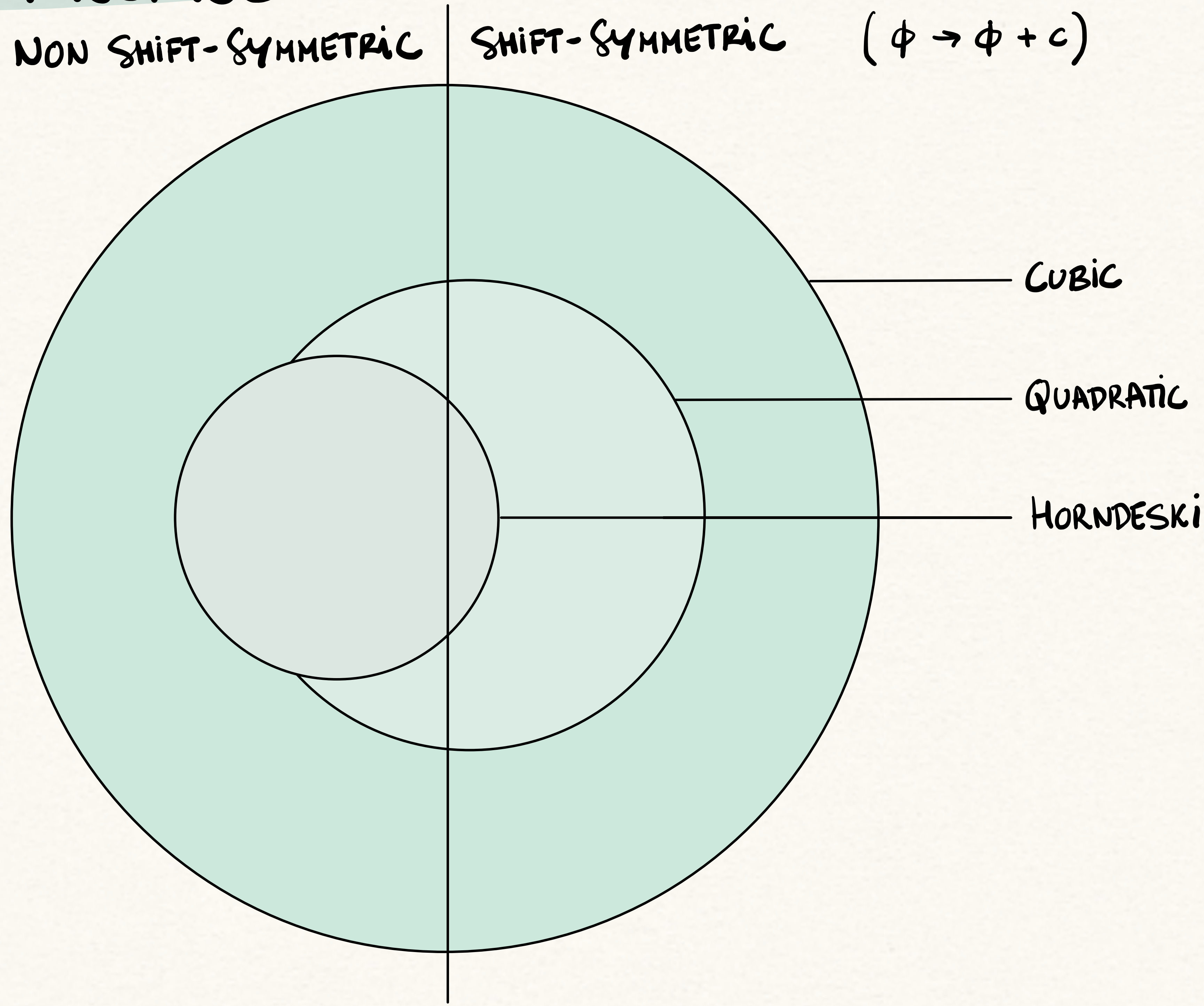
Cubic Host Theories

(LOVELOCK'S THEOREM)

$$\left.\begin{array}{c} 4D \\ g_{\mu\nu} \\ \text{LOCAL} \\ 2^{nd}\text{ order EOM} \end{array}\right\} \text{GR} \qquad \rightarrow \qquad S = \int d^4x \sqrt{-g}\, R[g_{\mu\nu}]$$

Cubic HOST Theories

Higher
Order
Scalar
Tensor



Cubic HOST Theories

$F(\phi, X)$ $\quad X \equiv -\frac{1}{2}\phi_\mu \phi^\mu$ $\quad \phi_\mu \equiv \nabla_\mu \phi$

$$S = \int d^4x \sqrt{-g}\left[F_0 + F_1 \Box\phi + F_2 R + \sum_{I=1}^{5} A_I L_I^{(2)} + F_3 G_{\mu\nu}\phi^{\mu\nu} + \sum_{J=1}^{10} B_J L_J^{(3)} + L_m \right]$$

Quadratic

$$L_1^{(2)} = \phi_{\mu\nu}\phi^{\mu\nu}$$

$$L_2^{(2)} = (\Box\phi)^2$$

$$L_3^{(2)} = (\Box\phi)\phi^\mu \phi_{\mu\nu}\phi^\nu$$

$$L_4^{(2)} = \phi^\mu \phi_{\mu\rho}\phi^{\rho\nu}\phi_\nu$$

$$L_5^{(2)} = (\phi^\mu \phi_{\mu\nu}\phi^\nu)^2$$

Cubic

$$L_1^{(3)} = (\Box\phi)^3$$

$$L_2^{(3)} = \Box\phi\, \phi_{\mu\nu}\phi^{\mu\nu}$$

$$L_3^{(3)} = \phi_{\mu\nu}\phi^{\nu\rho}\phi_\rho^\mu$$

$$L_4^{(3)} = (\Box\phi)^2 \phi_\mu \phi^{\mu\nu}\phi_\nu$$

$$L_5^{(3)} = \Box\phi\, \phi_\mu \phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho$$

$$L_6^{(3)} = \phi_{\mu\nu}\phi^{\mu\nu}\phi_\rho \phi^{\rho\sigma}\phi_\sigma$$

$$L_7^{(3)} = \phi_\mu \phi^{\mu\nu}\phi_{\nu\rho}\phi^{\rho\sigma}\phi_\sigma$$

$$L_8^{(3)} = \phi_\mu \phi^{\mu\nu}\phi_{\nu\rho}\phi^\rho \phi_\sigma \phi^{\sigma\lambda}\phi_\lambda$$

$$L_9^{(3)} = \Box\phi\,(\phi^\mu \phi_{\mu\nu}\phi^\nu)^2$$

$$L_{10}^{(3)} = (\phi^\mu \phi_{\mu\nu}\phi^\nu)^3$$

Cubic Host Theories

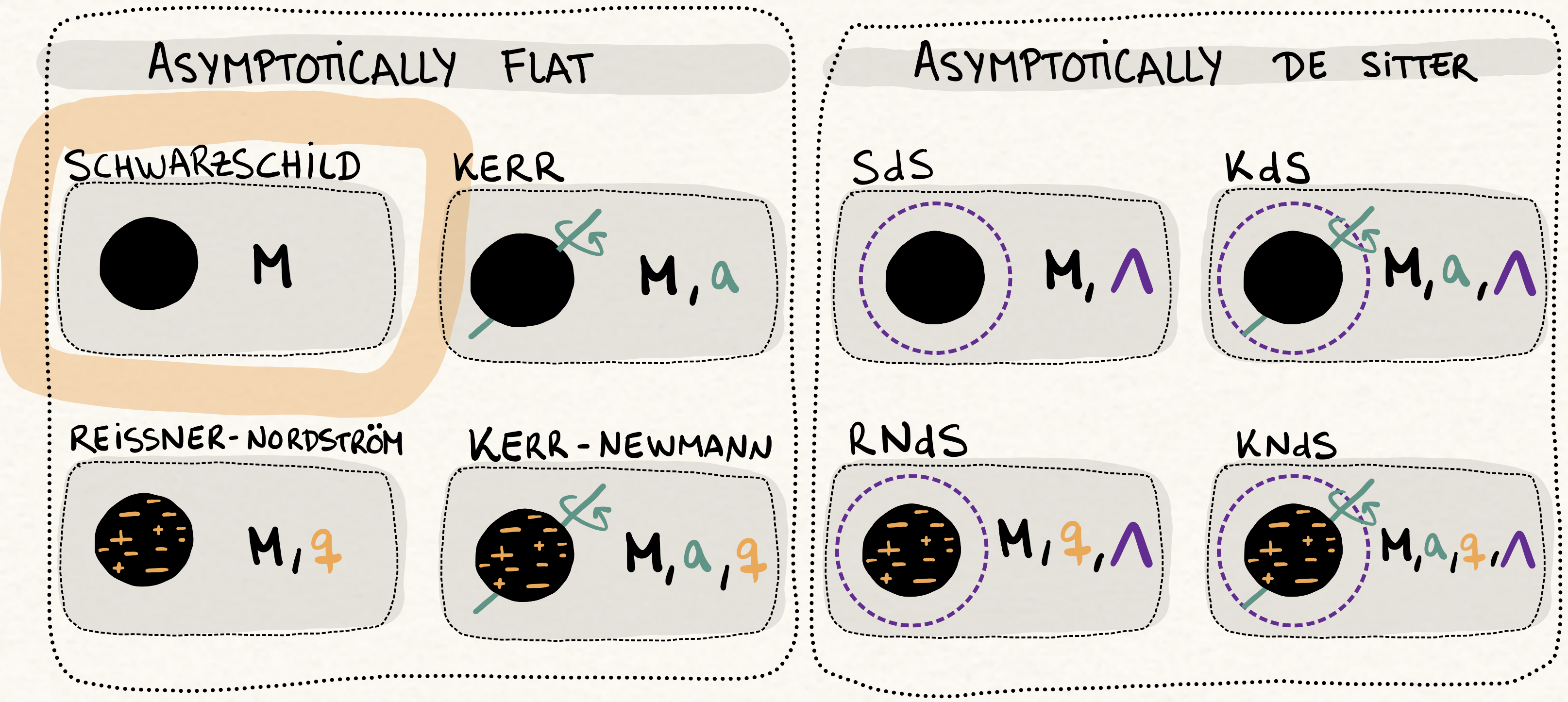
Equations of Motion

$$
\begin{aligned}
\mathcal{E}_{\mu\nu} =& (F_2 - X_0 F_{3\phi}) G_{\mu\nu} \\
& - \frac{1}{2}\Big\{ F_0 + 2X_0(F_{1\phi} + 2F_{2\phi\phi}) + (A_1 + A_2 + 2X_0 B_{2\phi})\phi^\beta_\alpha \phi^\alpha_\beta - 2[F_{2\phi} + X_0(F_{3\phi\phi} - 2A_{2\phi})]\Box\phi \\
& + (F_{3\phi} - A_2)\Big[(\Box\phi)^2 - \phi^\beta_\alpha\phi^\alpha_\beta - 2\phi^\alpha\phi^\beta R_{\alpha\beta}\Big] - 2B_1\Box\phi\Big[(\Box\phi)^2 - 3\phi^\beta_\alpha\phi^\alpha_\beta - 3\phi^\alpha\phi^\beta R_{\alpha\beta}\Big] \\
& + 6X_0 B_{1\phi}(\Box\phi)^2 + (2B_2 + B_3)\phi^\beta_\alpha\phi^\gamma_\beta\phi^\alpha_\gamma + 2B_2\phi^\lambda\phi^\sigma\phi^{\alpha\beta}R_{\lambda\alpha\sigma\beta}\Big\} g_{\mu\nu} \\
& - \frac{1}{2}\Big\{ F_{0X} + 2(F_{1\phi} + F_{2\phi\phi}) + (F_{2X} - F_{3\phi})R + \Big(F_{1X} - F_{3\phi\phi} - \frac{1}{2}RF_{3X} + 4A_{2\phi} - 2X_0 A_{3\phi}\Big)\Box\phi \\
& + [A_{1X} + 2(B_{2\phi} - X_0 B_{6\phi})]\phi^\beta_\alpha\phi^\alpha_\beta + [A_{2X} + 2(3B_{1\phi} - X_0 B_{4\phi})](\Box\phi)^2 + (B_{1X} + B_4)(\Box\phi)^3 \\
& + F_{3X}\phi^{\alpha\beta}R_{\alpha\beta} + A_3\Big[(\Box\phi)^2 - \phi^\beta_\alpha\phi^\alpha_\beta - \phi^\alpha\phi^\beta R_{\alpha\beta}\Big] + (B_{3X} - 2B_6)\phi^\beta_\alpha\phi^\gamma_\beta\phi^\alpha_\gamma \\
& + (B_{2X} - 2B_4 + B_6)\Box\phi\phi^\beta_\alpha\phi^\alpha_\beta - 2B_4\Box\phi\phi^\alpha\phi^\beta R_{\alpha\beta} - 2B_6\phi^\lambda\phi^\sigma\phi^{\alpha\beta}R_{\lambda\alpha\sigma\beta}\Big\}\phi_\mu\phi_\nu \\
& - \Big\{ F_{2\phi} + X_0(F_{3\phi\phi} + 2A_{1\phi}) - (F_{3\phi} + A_1 - 2B_{2\phi}X_0)\Box\phi - B_2\Big[(\Box\phi)^2 - \phi^\beta_\alpha\phi^\alpha_\beta - R_{\alpha\beta}\phi^\alpha\phi^\beta\Big]\Big\}\phi_{\mu\nu} \\
& - \frac{1}{2}\big[2(F_{3\phi} + A_1 + 3X_0 B_{3\phi}) + (2B_2 - 3B_3)\Box\phi\big]\phi_{\mu\lambda}\phi^\lambda_\nu - (2B_2 + 3B_3)\Box\phi_\lambda\phi_{(\mu}\phi^\lambda_{\nu)} \\
& - (F_{3\phi} + A_1 + B_2\Box\phi)\phi^\lambda\phi^\sigma R_{\mu\lambda\nu\sigma} + 2B_2\phi^\lambda R_{\lambda\sigma}\phi_{(\mu}\phi^\sigma_{\nu)} - 2(F_{3\phi} - A_2 - 3B_1\Box\phi)\phi^\lambda\phi_{(\mu}R_{\nu)\lambda} \\
& - 2[A_1 + A_2 + (3B_1 + B_2)\Box\phi]\phi_{(\mu}\Box\phi_{\nu)} - 3B_3\phi_{\lambda\sigma}\phi^\lambda_\mu\phi^\sigma_\nu - (2B_2 + 3B_3)\phi^{\lambda\sigma}\phi_{(\mu}\phi_{\nu)\lambda\sigma} \\
& + 3B_3\phi^\lambda\phi^\sigma\phi^\rho_{(\mu}R_{\nu)\lambda\sigma\rho} + 2B_2\phi^\lambda\phi^{\sigma\rho}\phi_{(\mu}R_{\nu)\sigma\lambda\rho} - T_{\mu\nu}\,, \qquad (2.8)
\end{aligned}
$$

Assuming X_0 = const

Stealth Black Holes in Cubic Host

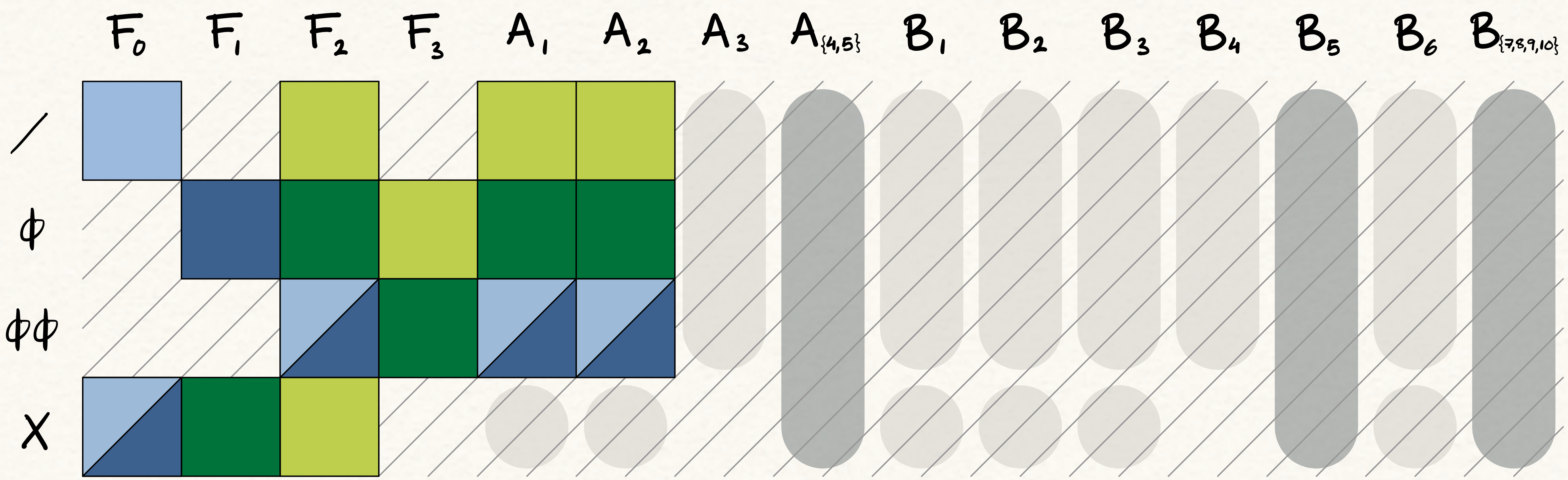
? What conditions do host theories need to satisfy in order to admit stealth black holes?



1. All GR metrics in the presence of matter
2. All GR metrics in vacuum
3. Schwarzschild-de Sitter
4. Schwarzschild

Stealth Black Holes in Cubic Host

① Existence conditions for all GR metrics with matter



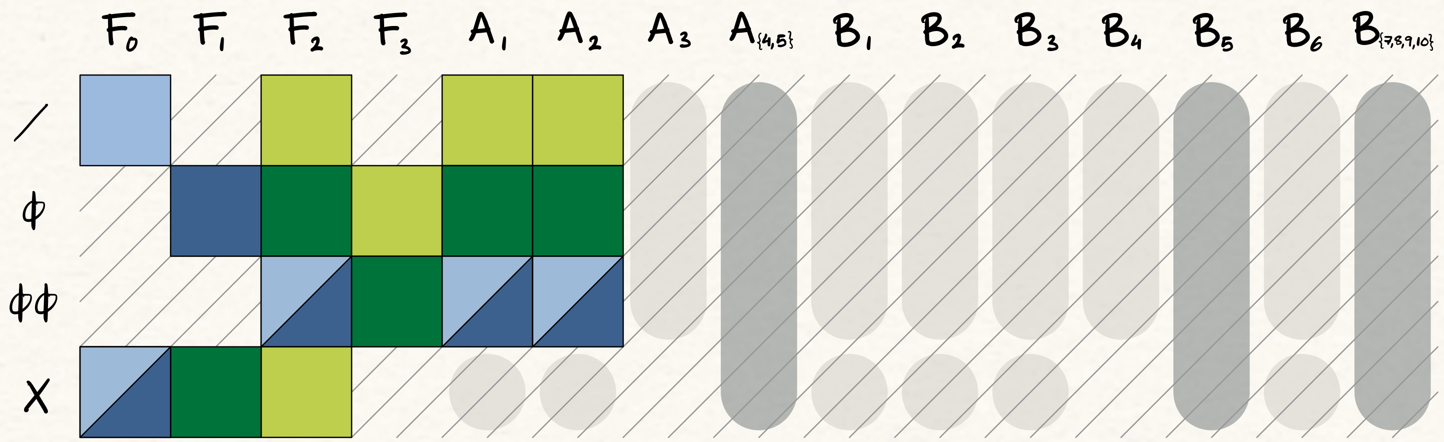
$$F_0 + 2\Lambda M_{\rm Pl}^2 = -2X_0(F_{1\phi} + 2F_{2\phi\phi}) = X_0(F_{0X} - 2F_{2\phi\phi}) \,,$$

$$X_0 F_{1X} = -3F_{2\phi} = -3X_0 F_{3\phi\phi} \,, \quad X_0^{-1}(F_2 - M_{\rm Pl}^2) = F_{2X} = F_{3\phi} = -A_1 = A_2 \,,$$

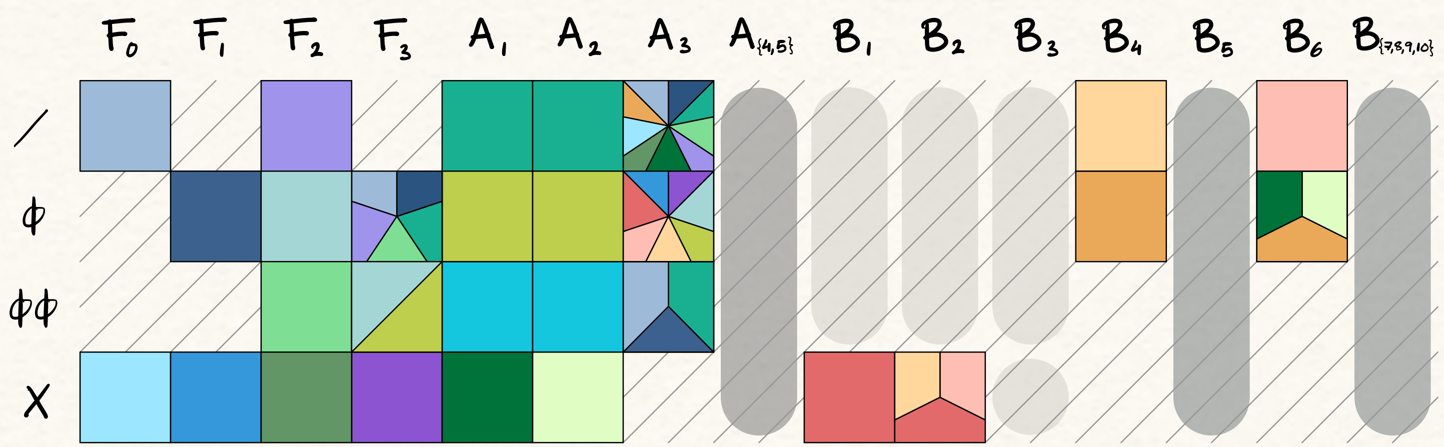
$$F_{3X} = A_{1X} = A_{2X} = A_3 = B_1 = B_{1X} = B_2 = B_{2X} = B_3 = B_{3X} = B_4 = B_6 = 0 \,,$$

Stealth Black Holes in Cubic Host

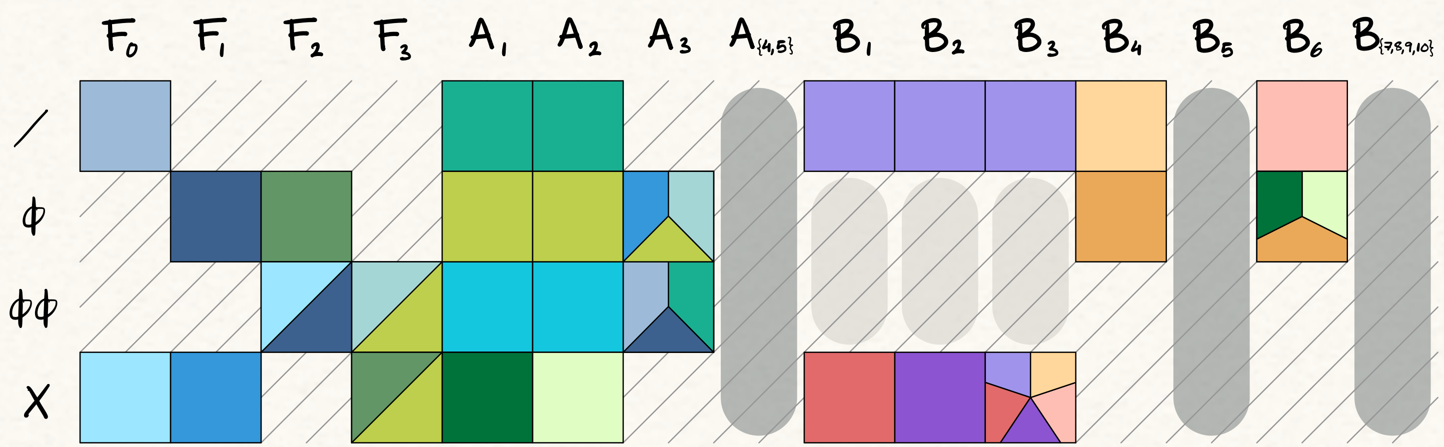
Existence conditions



① All GR metrics with matter



③ Schwarzschild-de Sitter



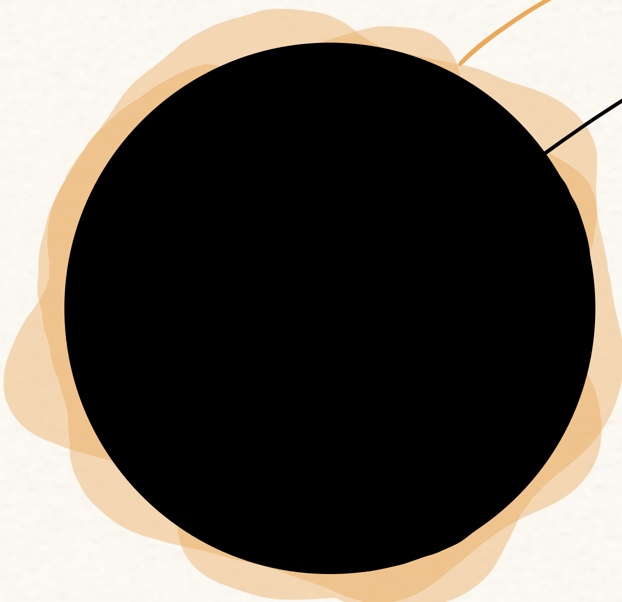
④ Schwarzschild

Stealth Black Holes in Cubic Host

HOST theory	Stealth metric	Existence conditions	GR-deviations in odd modes on S(dS)	Stability conditions
Cubic	General GR with matter	this work (2.14)	$\times$ (this work) (3.13)	$\checkmark$
	General GR vacuum	this work (2.15)	$\checkmark_1$ (this work) (3.14)	(4.6)
	SdS	this work (2.16)	$\checkmark_2$ (this work) (3.15)	(4.8)
	Schwarzschild	this work (2.17)	$\checkmark_3$ (this work) (3.17)	(4.10)
Shift-sym cubic	General GR with matter	this work (A.1)	$\times$ [35, 36] (3.13)	$\checkmark$
	General GR vacuum	this work (A.4)	$\times$ [35, 36] (C.1)	(4.7)
	SdS	[27] (A.7)	$\checkmark_1$ [35, 36] (C.2)	(4.9)
	Schwarzschild	[27] (A.10)	$\checkmark_2$ [35, 36] (C.3)	(4.11)
Quadratic	General GR with matter	[28] (A.2)	$\times$ (this work) (3.13)	$\checkmark$
	General GR vacuum	[28] (A.5)	$\checkmark_1$ (this work) (C.1)	(4.7)
	SdS	this work (A.8)	$\checkmark_2$ (this work) (C.2)	(4.9)
	Schwarzschild	this work (A.11)	$\checkmark_2$ (this work) (C.2)	(4.9)
Shift-sym quadratic	General GR with matter	[28] (A.3)	$\times$ [26, 37] (3.13)	$\checkmark$
	General GR vacuum	[28] (A.6)	$\times$ [26, 37] (C.1)	(4.7)
	SdS	[25] (A.9)	$\checkmark_1$ [26, 37] (C.2)	(4.9)
	Schwarzschild	[25] (A.12)	$\checkmark_1$ [26, 37] (C.2)	(4.9)

Stealth Black Holes in Cubic Host

Black Hole Perturbation Theory



$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$$

Background

S(dS)

Perturbations

ODD → Regge-Wheeler

EVEN → Zerilli

$$h^{odd}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & h_0 \\ 0 & 0 & 0 & h_1 \\ 0 & 0 & 0 & 0 \\ h_0 & h_1 & 0 & 0 \end{pmatrix} r^2 \sin\theta\, \partial_\theta P_\ell(\cos\theta)$$

Stealth Black Holes in Cubic Host

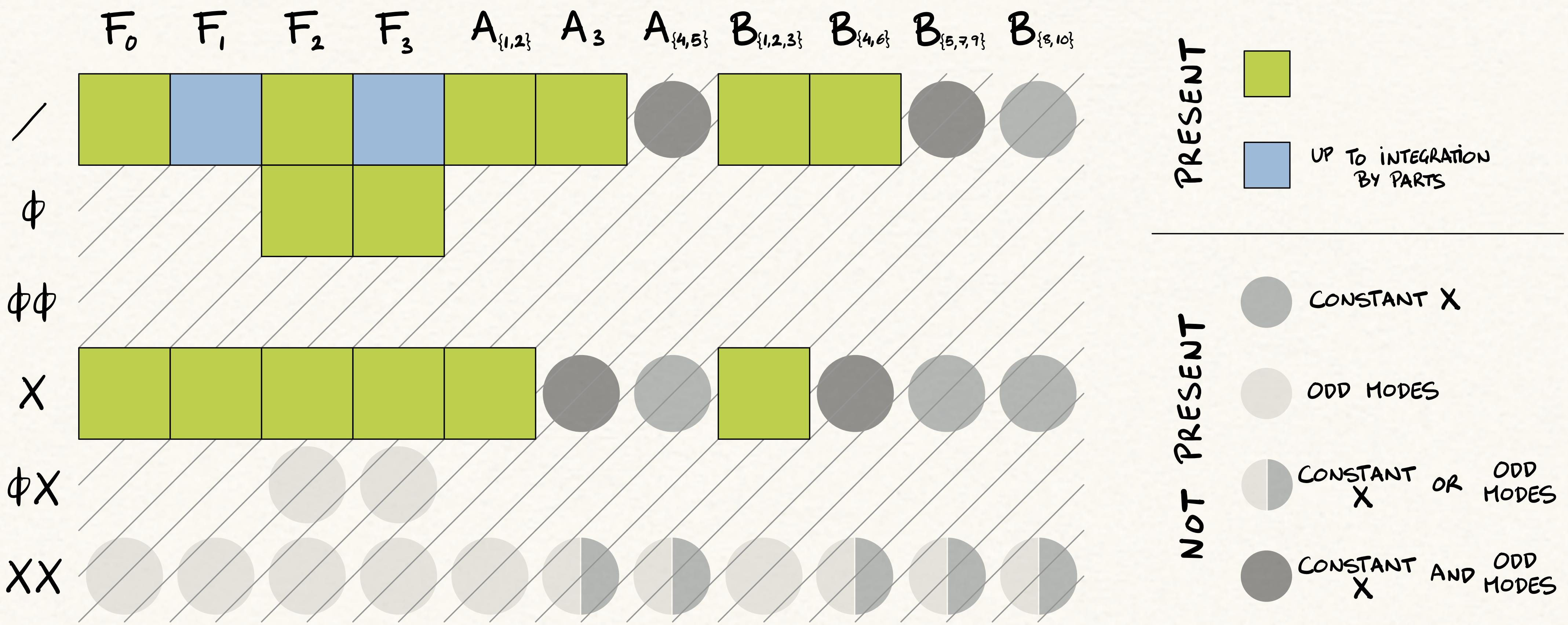
Covariant Quadratic Lagrangian

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \Rightarrow S$$

114 contributions
23 survive

$$S^{(2)} = \frac{1}{4}\int d^4x \sqrt{-g} \sum_a \left[\sum_{K=0}^{3} \delta\mathcal{L}_{F_{Ka}} F_{Ka} + \sum_{I=1}^{5} \delta\mathcal{L}_{A_{Ka}} A_{Ka} + \sum_{J=1}^{10} \delta\mathcal{L}_{B_{Ka}} B_{Ka} \right]$$

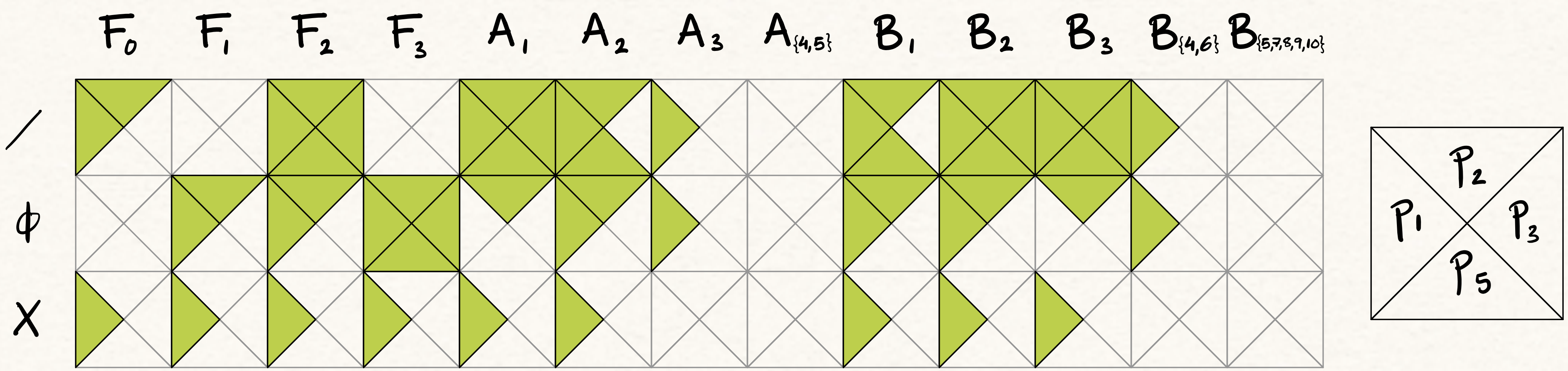
$a = \{\emptyset, \phi, X, \phi\phi, XX, \phi X\}$



Stealth Black Holes in Cubic Host

Quadratic Lagrangian in Components

$$\frac{2\ell+1}{2\pi\ell(\ell+1)}\mathcal{L}_2 = p_1 h_0^2 + p_2 h_1^2 + p_3\left[(\dot{h}_1 - \partial_r h_0)^2 + 2p_4 h_1 \partial_r h_0\right] + p_5 h_0 h_1$$



$P_4 = 0$

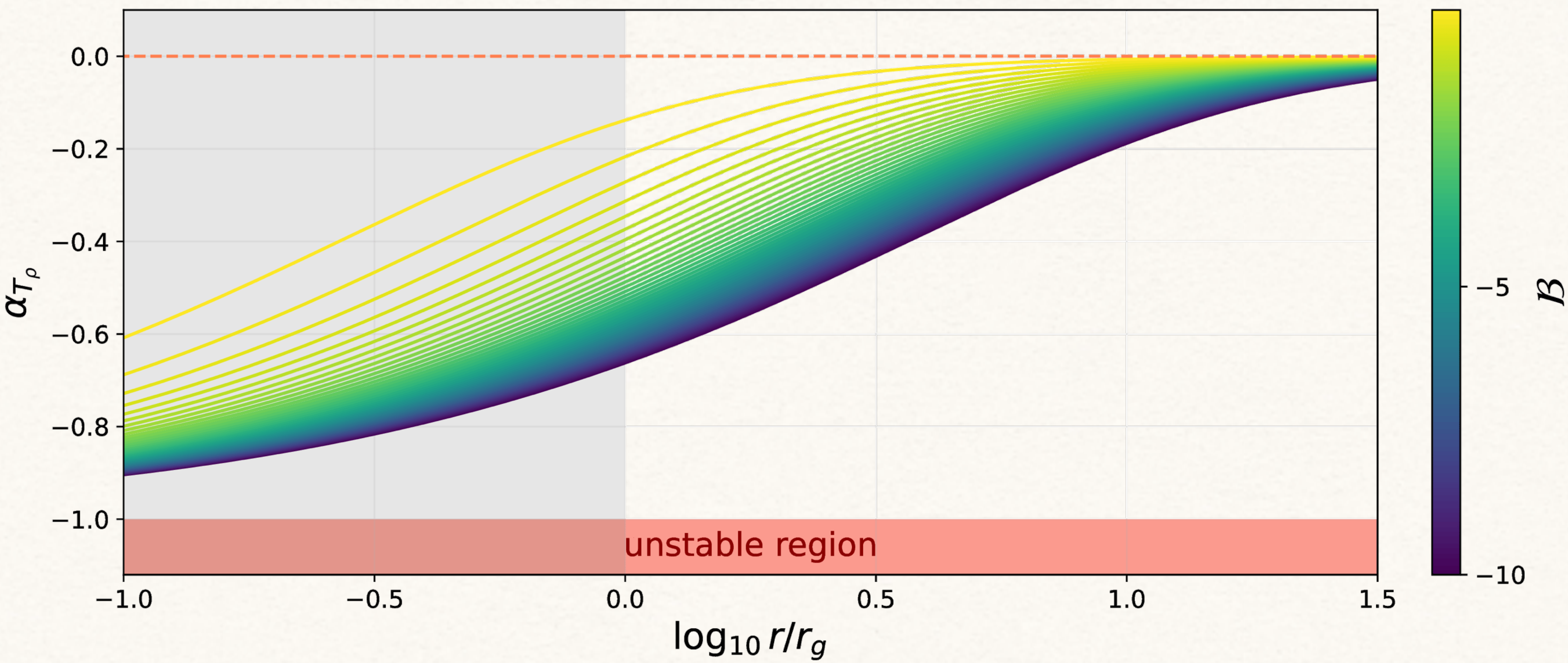
Stealth Black Holes in Cubic Host

Deviations from GR

Model	**Beyond-GR parameter(s)**	**Symbol**
$^{(\mathrm{SS})}$**Cubic**$_{\text{GR-mat}}$, $^{(\mathrm{SS})}$**Quadratic**$_{\text{GR-mat}}$, $^{\mathrm{SS}}$**Cubic**$_{\text{GR-vac}}$, $^{\mathrm{SS}}$**Quadratic**$_{\text{GR-vac}}$	None	$\times$
Cubic$_{\text{GR-vac}}$	$F_2 - X_0 F_{3\phi}$ at $(\phi, X) = (\phi_0, X_0)$	$\checkmark_1$
Cubic$_{\text{SdS}}$	$F_2 - X_0 F_{3\phi}$, $A_1 + F_{3\phi}$ at $(\phi, X) = (\phi_0, X_0)$	$\checkmark_2$
Cubic$_{\text{Schw}}$	$F_2 - X_0 F_{3\phi}$, $A_1 + F_{3\phi}$, B_1 at $(\phi, X) = (\phi_0, X_0)$	$\checkmark_3$
Quadratic$_{\text{GR-vac}}$	F_2 at $(\phi, X) = (\phi_0, X_0)$	$\checkmark_1$
Quadratic$_{\text{SdS}}$, **Quadratic**$_{\text{Schw}}$	F_2, A_1 at $(\phi, X) = (\phi_0, X_0)$	$\checkmark_2$
$^{\mathrm{SS}}$**Cubic**$_{\text{SdS}}$, $^{\mathrm{SS}}$**Quadratic**$_{\text{SdS}}$, $^{\mathrm{SS}}$**Quadratic**$_{\text{Schw}}$	A_1 at $X = X_0$	$\checkmark_1$
$^{\mathrm{SS}}$**Cubic**$_{\text{Schw}}$	A_1, B_1 at $X = X_0$	$\checkmark_2$

Stealth Black Holes in Cubic Host

Non-trivial speed of gravitational waves



SUMMARY [2503.05651]

- PROVIDED THE MOST GENERAL STUDY OF HOW REQUIRING GR SOLUTIONS RESTRICTS SCALAR-TENSOR THEORIES

- INVESTIGATED BEYOND-GR SIGNATURES IN THE ODD SECTOR

  - MOST MODELS SHOW
    - NO DEVIATIONS
    - 1 SIMPLE DEVIATION (QNMs ARE RESCALED)
  - 2 MODELS SHOW AN INTERESTING DEVIATION WITH A NON-TRIVIAL BUT HEALTY SPEED OF GWs

THANK YOU!

GitHub

ringdown-calculations Public

A collection of notebooks for black hole perturbation theory calculations in GR and modified gravity.

Mathematica ☆ 10 4