

Tema Lab 5

$$A = \begin{pmatrix} -2 & -3 \\ 0 & 1 \end{pmatrix}$$

1) eigenvalues $\rightarrow \det(\lambda I_2 - A) = 0$

$$\Rightarrow \begin{vmatrix} \lambda+2 & -3 \\ 0 & \lambda-1 \end{vmatrix} = 0 \quad \Rightarrow (\lambda+2)(\lambda-1) = 0 \quad \Rightarrow \begin{matrix} \lambda_1 = -2 \\ \lambda_2 = 1 \end{matrix}$$

2) $\Phi(s) = (s^* I_2 - A)^{-1}$

$$= \left[\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} -2 & -3 \\ 0 & 1 \end{pmatrix} \right]^{-1} = \begin{pmatrix} s+2 & 3 \\ 0 & s-1 \end{pmatrix}^{-1}$$

$$= \frac{1}{(s+2)(s-1)} \begin{pmatrix} s-1 & -3 \\ 0 & s+2 \end{pmatrix} = \begin{pmatrix} \frac{1}{s+2} & \frac{-3}{(s+2)(s-1)} \\ 0 & \frac{1}{s-1} \end{pmatrix}$$

3) $\Phi(t) = \mathcal{L}^{-1}[\Phi(s)]$

$$\boxed{\mathcal{L}^{-1}\left[\frac{1}{s+a}\right] = e^{-at}}$$

$$\frac{-3}{(s+2)(s-1)} = \frac{s+1}{s+2} + \frac{s+2}{s-1} = \frac{As-A+Bs+2B}{(s+2)(s-1)} = \frac{s(A+B)+2B-A}{(s+2)(s-1)}$$

$$\begin{cases} A+B=0 \\ 2B-A=-3 \end{cases} \quad (*)$$

$$3B = -3 \Rightarrow B = -1 \Rightarrow A = 1 \Rightarrow \Phi(t) = \begin{pmatrix} e^{-2t} & e^{-2t} - e^{-t} \\ 0 & e^t \end{pmatrix}$$

4) $x(t) = \Phi(t) \cdot x_0$

$$x(t) = \Phi(t) \cdot \begin{pmatrix} x_{01} \\ x_{02} \end{pmatrix} = \begin{pmatrix} e^{-2t} & e^{-2t} - e^{-t} \\ 0 & e^t \end{pmatrix} \begin{pmatrix} x_{01} \\ x_{02} \end{pmatrix}$$

$$\Rightarrow \begin{cases} x_1 = e^{-2t} \cdot x_{01} + (e^{-2t} - e^{-t}) \cdot x_{02} \\ x_2 = e^t \cdot x_{02} \end{cases}$$