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Tema lab 8

1. $h(t) = e^{-4|t|}$

$$H(k\omega_0) = \int_{-\infty}^{\infty} h(\tau)\cdot e^{-jk\omega_0\tau}\, d\tau = \int_{-\infty}^{\infty} e^{-4|t|}\cdot e^{-jk\omega_0\tau}\, d\tau =$$

$$= \int_{-\infty}^{0} e^{+4\tau}\cdot e^{-jk\omega_0\tau} + \int_{0}^{\infty} e^{-4\tau}\cdot e^{-jk\omega_0\tau}\, d\tau =$$

$$= \frac{1}{4-jk\omega_0}\cdot e^{(4-jk\omega_0)\tau}\Big|_{-\infty}^{0} + \frac{1}{-4-jk\omega_0}\cdot e^{(4-jk\omega_0)\tau}\Big|_{0}^{\infty} =$$

$$= \frac{1}{4-jk\omega_0} + \frac{1}{4+jk\omega_0} = \frac{4+jk\omega_0-4+jk\omega_0}{16+(k\omega_0)^2} = \frac{2jk\omega_0}{16+(k\omega_0)^2}$$

Fourier series repr. of the output $y(t)$ for these input signals

a) $x(t) = \cos(2\pi t)$

$$\Rightarrow \omega_0 = 2\pi;\quad x(t) = \frac{1}{2}\left(e^{2\pi t j} + e^{-2\pi t j}\right) = \sum_{k=-1}^{1} a_k\cdot e^{jk2\pi t}$$

where $a_0 = 0$ and $a_{-1} = a_1 = \frac{1}{2}$

$$y(t) = \sum_{k=1}^{1} b_k\cdot e^{jk2\pi t},\ \text{unde } b_0 = a_0\cdot H(0) = 0$$

$$b_1 = a_1\cdot H(\omega_0) = \frac{1}{2}H(2\pi) = \frac{1}{2}\cdot\frac{4\pi j}{16+4\pi^2} = \frac{\pi j}{8\left(1+\frac{1}{2}\pi^2\right)}$$

$$b_{-1} = a_{-1}\cdot H(-\omega_0) = \frac{1}{2}\cdot H(-2\pi) = \frac{1}{2}\cdot\frac{4\pi j}{16+4\pi^2} = \frac{\pi j}{8\left(1-\frac{1}{2}\pi^2\right)}$$

b) $x(t) = \sum_{m=-\infty}^{0} \delta(t-m) \Rightarrow T_0 = 1,\ \omega_0 = 2\pi$

$$a_k = \frac{1}{T_0}\int_{T_0} x(t)\cdot e^{-jk\omega_0 t}\, dt = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \delta(t)\cdot e^{-jk\omega_0 t}\, dt = \frac{1}{T_0}\cdot e^{-j\omega_0 k t}\Big|_{t=0} = \frac{1}{T_0} = 1$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k\cdot e^{jk\omega_0 t} = \frac{1}{T_0}\sum_{k=-\infty}^{\infty} e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} e^{jk2\pi t}$$

$$b_k = a_k\cdot H(2\pi k) = \frac{1}{T_0}H(2\pi k) = \frac{1}{T_0}\cdot\frac{4jk\pi_0}{16+4\pi^2k^2} = \frac{4\pi k j}{4+\pi^2k^2}$$

$$\Rightarrow y(t) = \sum_{k=-\infty}^{\infty} \frac{\pi k j}{4+(\pi k)^2}\cdot e^{jk2\pi t}$$

c) $x(t) = \sum_{n=-\infty}^{\infty} (-1)^n \delta(t-n) \Rightarrow T_0 = 2, \ w_0 = \pi$

$$a_k = \frac{1}{T_0}\int_{T_0} x(t)\cdot e^{-jkw_0t}\,dt = \frac{1}{T_0}\left(\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \delta(t)\cdot e^{-jkw_0t}\,dt + \int_{\frac{T_0}{2}}^{\frac{3T_0}{4}} -\delta\left(t-\frac{T_0}{2}\right)\cdot e^{-jkw_0t}\,dt\right)$$

$$= \frac{1}{T_0}\left(e^{-jkw_0t}\Big|_{t=0} - e^{-jkw_0t}\Big|_{t=\frac{T_0}{2}}\right) = \frac{1}{T_0}\left(1-e^{-jkw_0\cdot\frac{T_0}{2}}\right) = \frac{1}{T_0}\left(1-e^{-jk\pi}\right)$$

$$= \frac{1}{T_0}\left(1-(-1)^k\right) = \frac{1}{2}\left(1-(-1)^k\right)$$

$$x(t) = \sum_{n=-\infty}^{\infty} a_k\cdot e^{-jkw_0t} = \frac{1}{2}\sum_{n=-\infty}^{\infty}\left(1-(-1)^k\right)\cdot e^{jk\pi t}$$

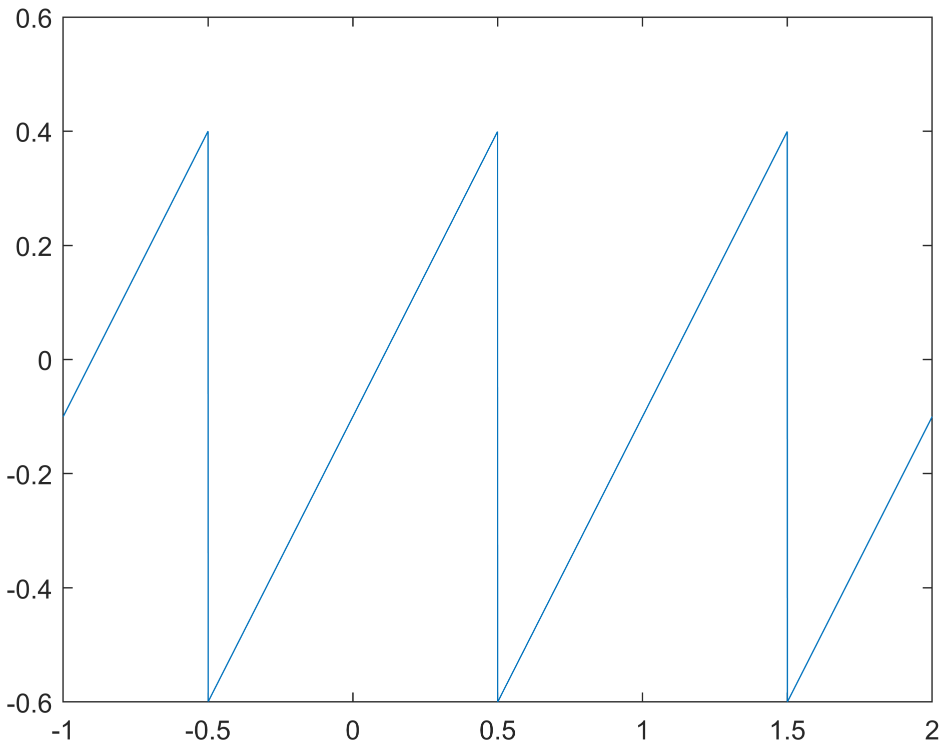
$$b_k = a_k\cdot H(kw_0) = \frac{1-(-1)^k}{T_0}\cdot H(k\pi) = \frac{1-(-1)^k}{T_0}\cdot\frac{2k\pi j}{16+(k\pi)^2}$$

$$\Rightarrow b_k = \begin{cases} \frac{2k\pi j}{16+(k\pi)^2}, & \text{for } k\text{-odd} \\ 0, & \text{for } k\text{-even} \end{cases}$$

$$y(t) = \sum_{n=-\infty}^{\infty} b_k\cdot e^{jk\pi t}$$

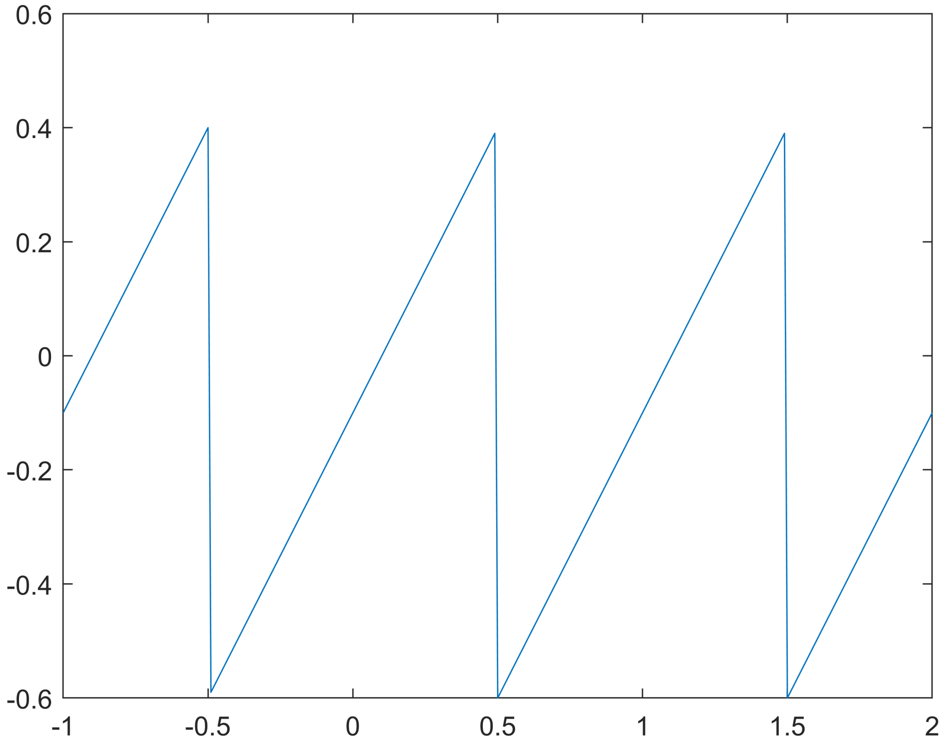
```
clear variables

T0 = 1; w0 = 2*pi/T0;
Ts = 0.0005; t = -1:Ts:2;
x = t - 0.1 - round(t);
figure;plot(t,x);
```



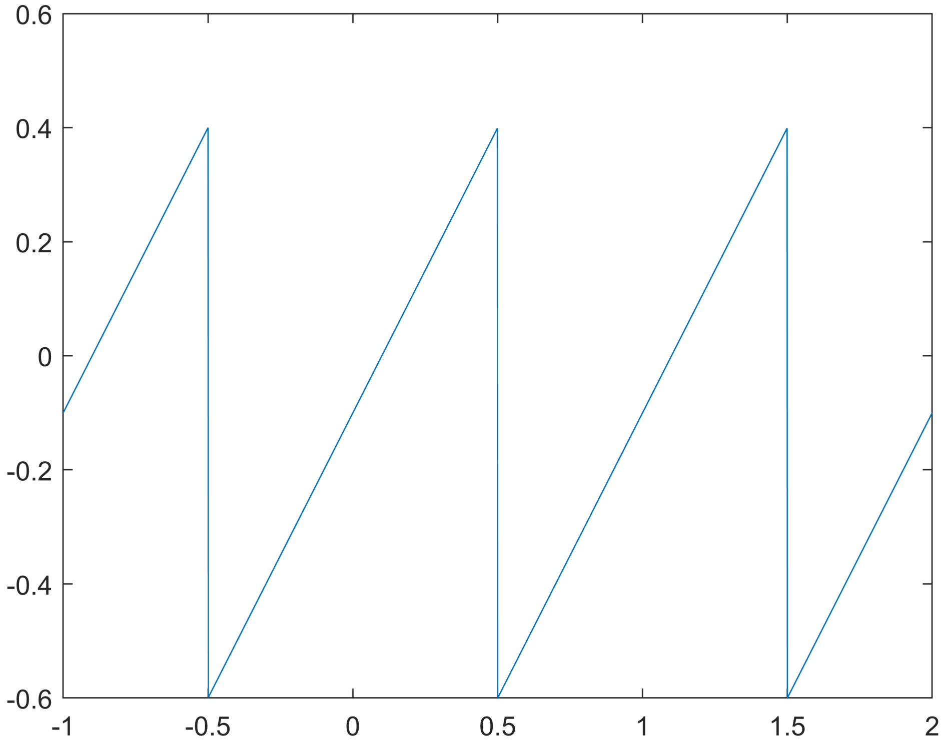

```
N = 8;
ak = fsAnalysis(x, t, T0, Ts, N);

%Pentru Ts = 0.01:
T0 = 1; w0 = 2*pi/T0;
Ts1 = 0.01; t = -1:Ts1:2;
x1 = t - 0.1 - round(t);
figure; plot(t,x1);
```




```
N = 3;
ak = fsAnalysis(x1, t, T0, Ts1, N);

%Pentru Ts = 0.001:
T0 = 1; w0 = 2*pi/T0;
Ts2 = 0.001; t = -1:Ts2:2;
x2 = t - 0.1 - round(t);
figure; plot(t,x2);
```




```
N = 10;
ak = fsAnalysis(x2, t, T0, Ts2, N);
a0 = ak(N+1);
ksi0 = 0;
AK = abs(ak(N+2:end));
ksi = angle(ak(N+2:end));

x_est = a0*ones(size(t));
for k = 1:N
x_est = x_est + 2*AK(k)*cos(w0*k*t+ksi(k));
end

figure; plot(t,x,'b',t,x_est,'r');
```

```
Error using plot
Vectors must be the same length.
```

```
xlabel('t');legend('x(t)','x_{est}(t)');

n0 = 0:N;
figure, subplot(3,1,1);stem(n0,[a0, AK],'.');
legend('Amplitude spectrum')
subplot(3,1,2),stem(n0,[ksi0, ksi],'.g')
legend('Phase spectrum')
subplot(3,1,3),stem(n0,[a0^2, (AK.^2)/2],'r')
label('n'), legend('Power spectrum');
```

```
function ak = fsAnalysis(x, t, T0, Ts, N)
% function for estimating the first N coefficients of the Fourier series for signal x
% T0 - fundamental period of x; Ts - sampling period

% exraction of one period from x
    t = t(1:floor(T0/Ts));
    x = x(1:length(t));

% estimation of the coeffiecients using the trapezoidal method
    w0 = 2*pi/T0;
    ak = [];
    for k = -N:N
        ak = [ak, (1/T0)*trapz(t, x.*exp(-j*k*w0*t))];
    end
end
```