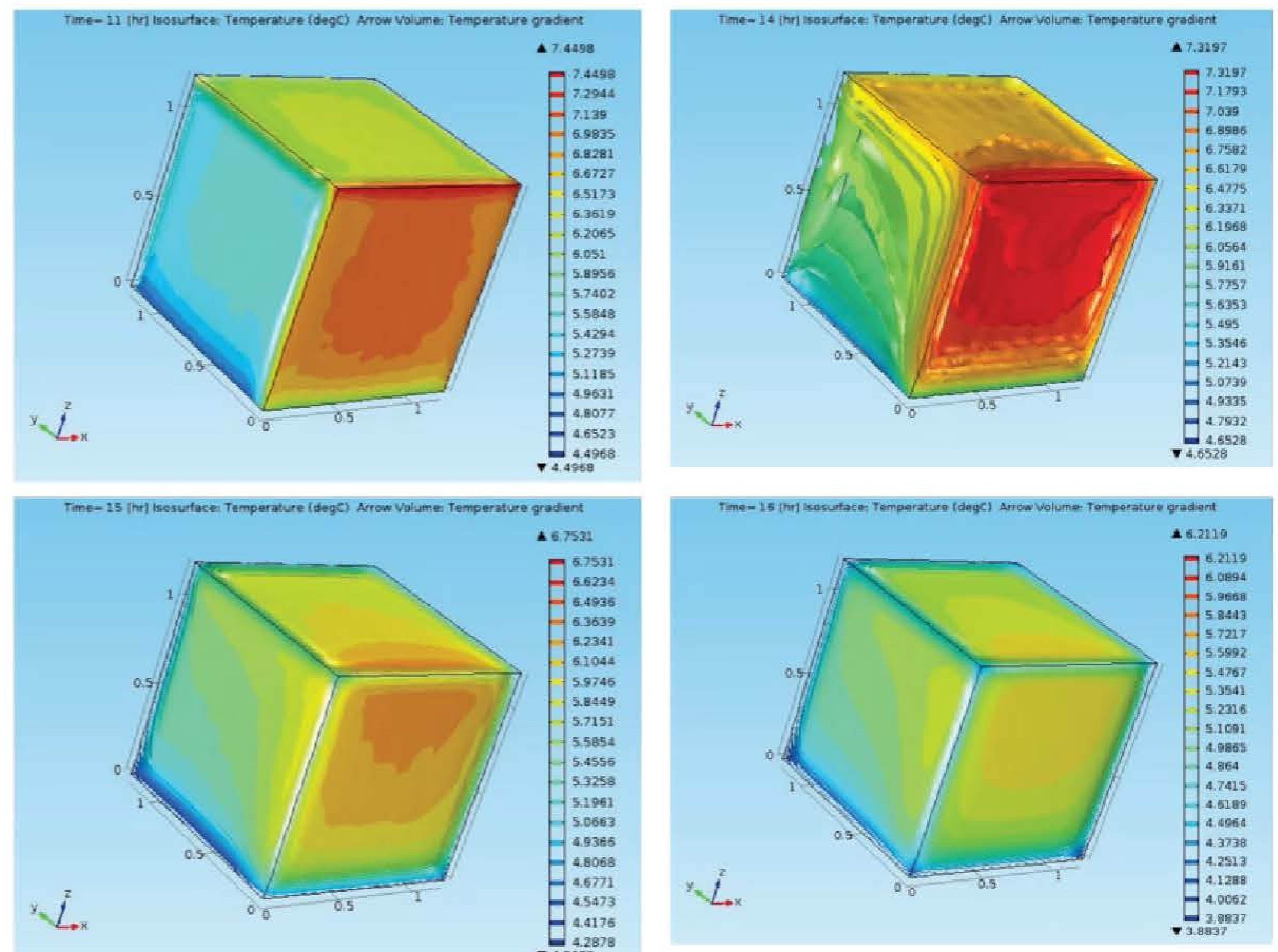
Modelling heat transfer processes

Basic principles and approaches

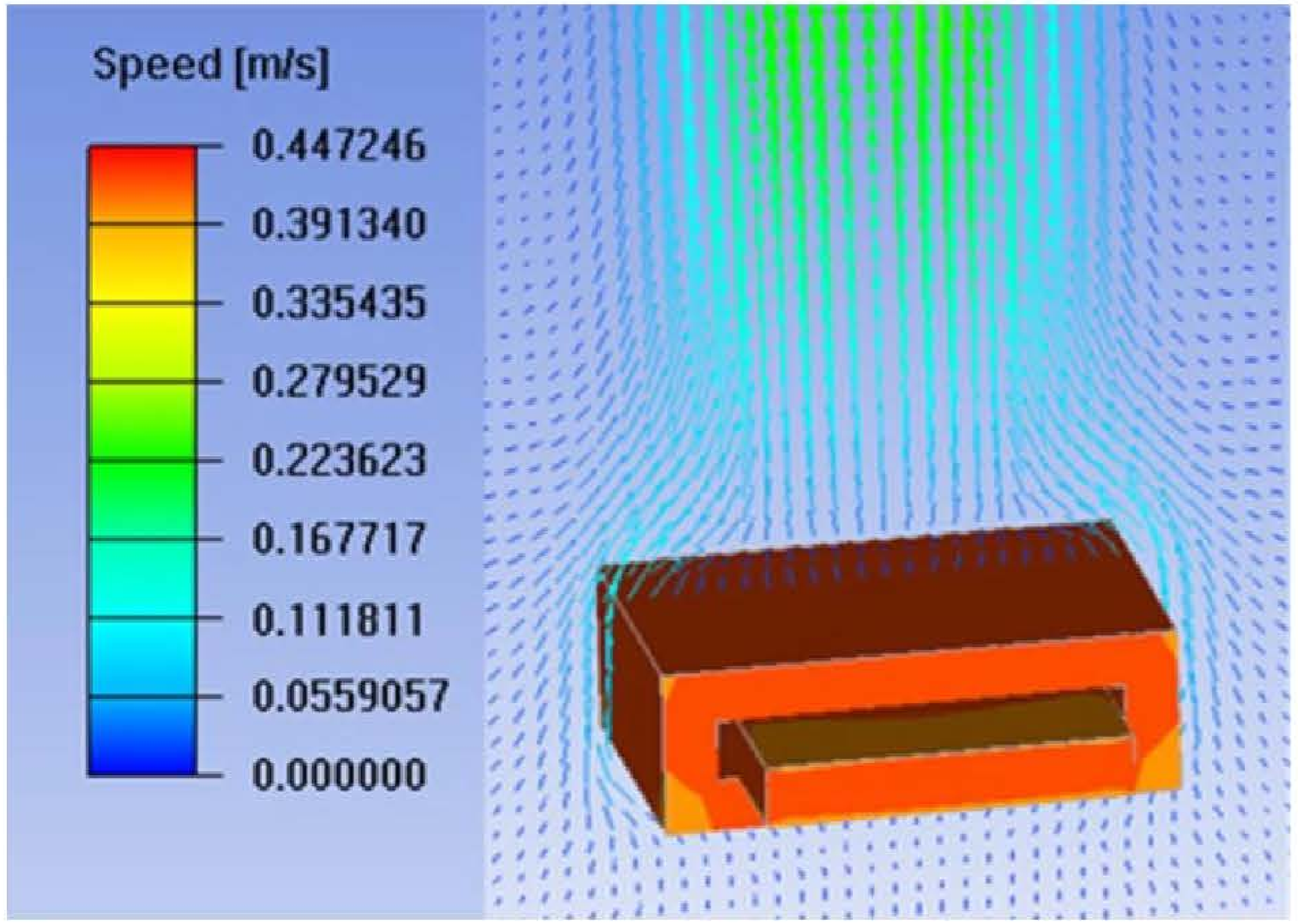
Modeling of thermal systems:

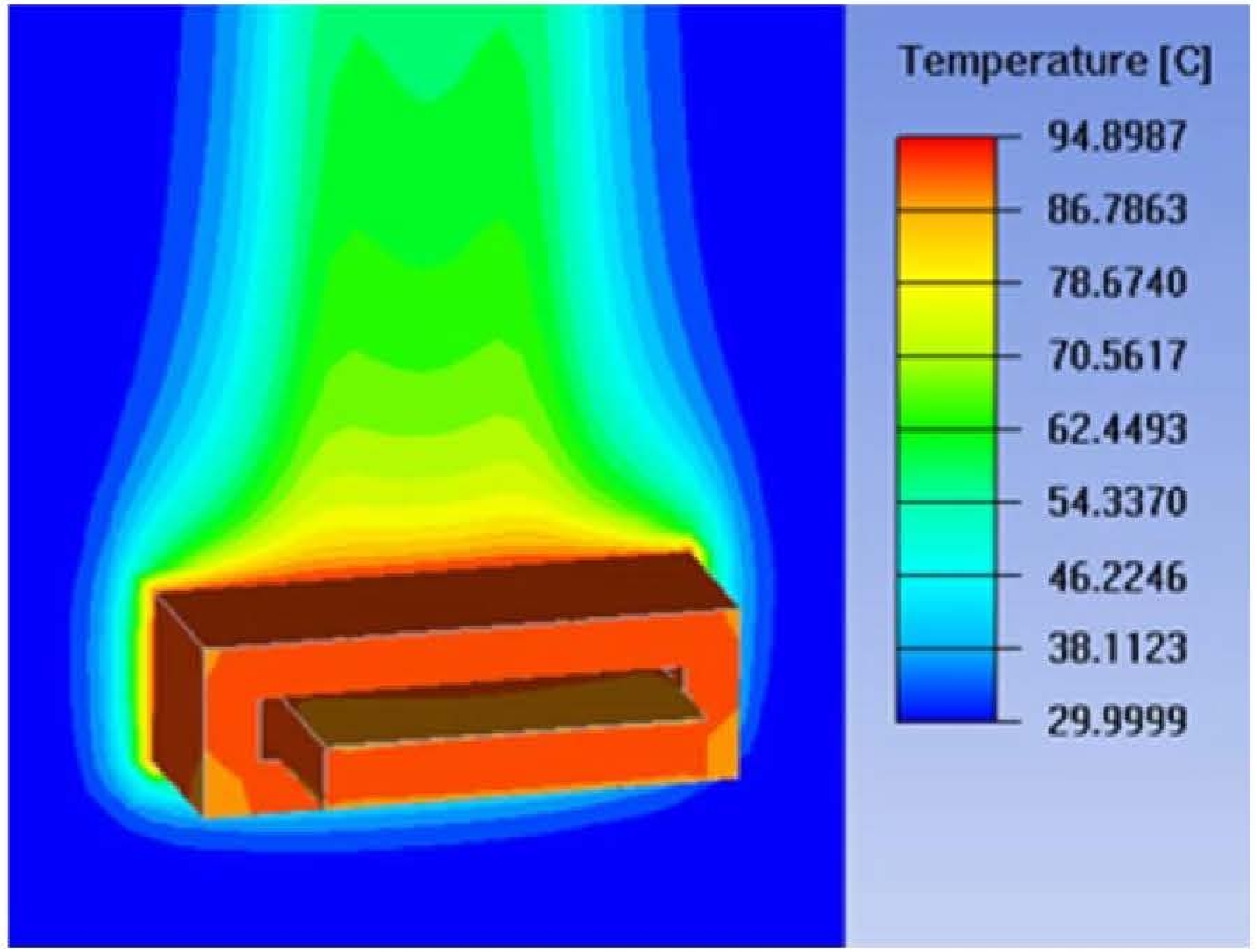
1. Reality: a rigorous description of thermal phenomena such as conduction or convection involves the use of partial derivative equations
2. Different numerical methods for solving Partial Differential Equations like *Finite Element Methods:* Finite Volume methods (FVM) or Finite Difference methods (FDM)



CFD simulation example

Temperature distribution obtained around a magnetic component (planar transformer) when a power P=6W is dissipated and the ambient air temperature is T_a=30C. The air movement is simulated the case of natural convection, when hot air moves upward, and cold air moved downward.



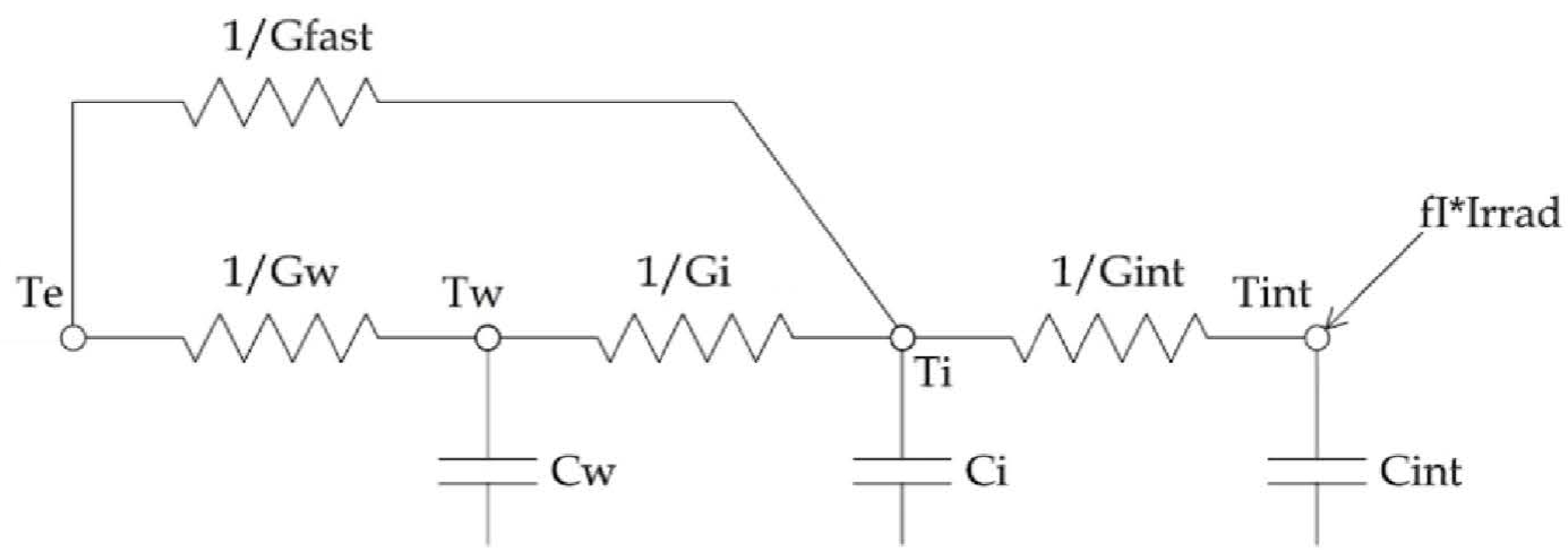


Basic principles and approaches

3. Simplifying hypotheses for building lumped parameter models:

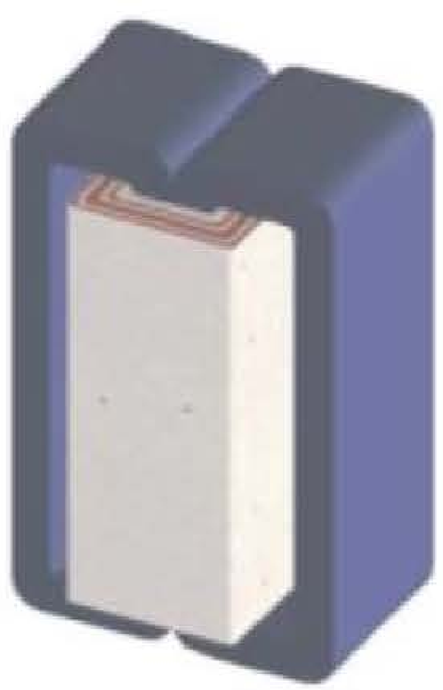
- uniform spatial distribution of temperature (constant temperature) allows usage of **lumped parameter models**
- hypothesis satisfied for bodies of small dimensions relative to the volume of fluid, and assuming a perfect mixing of the fluids

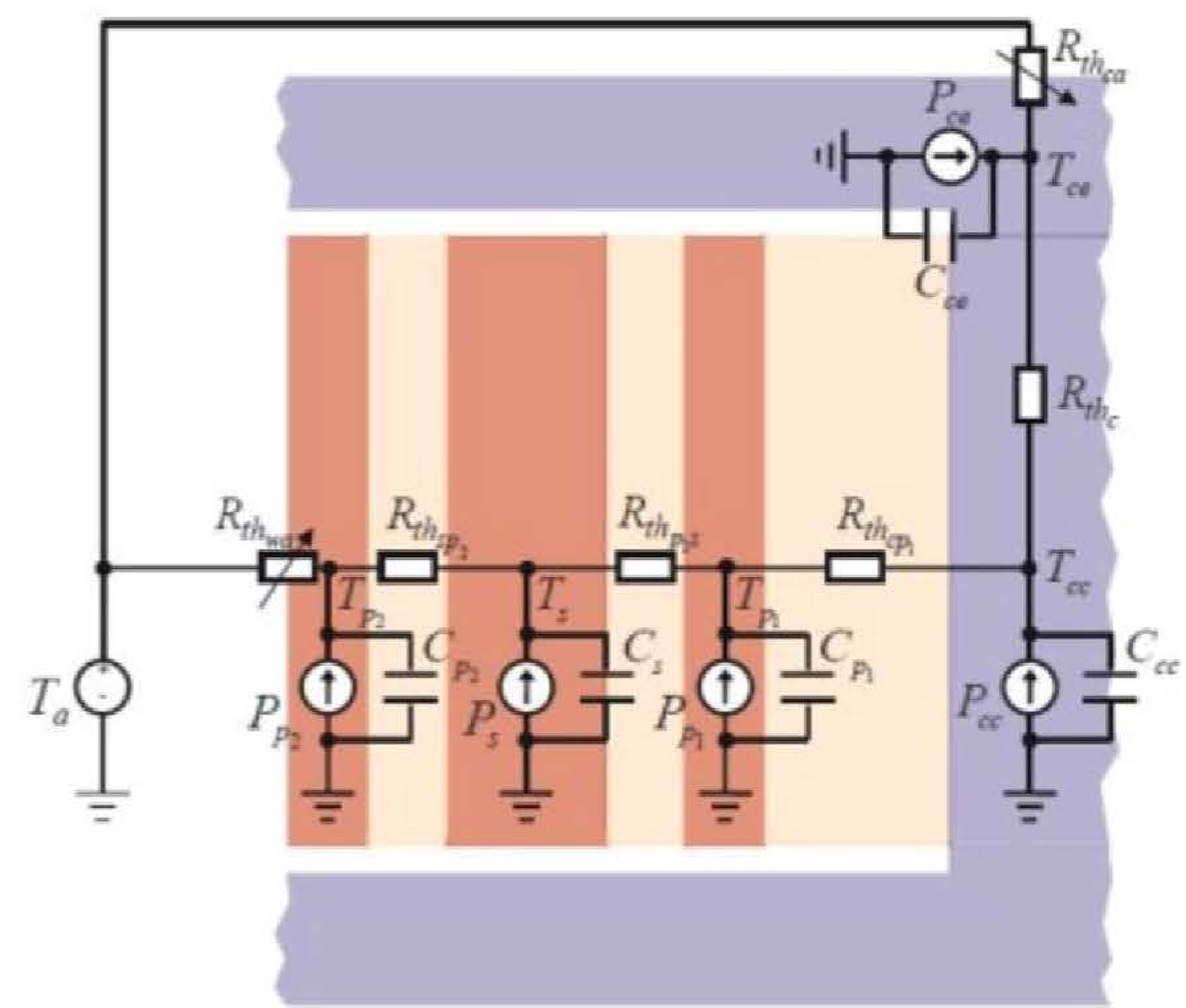
4. The analogies with the electric domain allow the development of the schematics (networks) that can be easily simulated . Partitioning of the geometry into a collection of individual regions (2D or 3D) might lead to the development of equivalent electric circuits based on repetitive use of cells (subnetworks) with same topology.



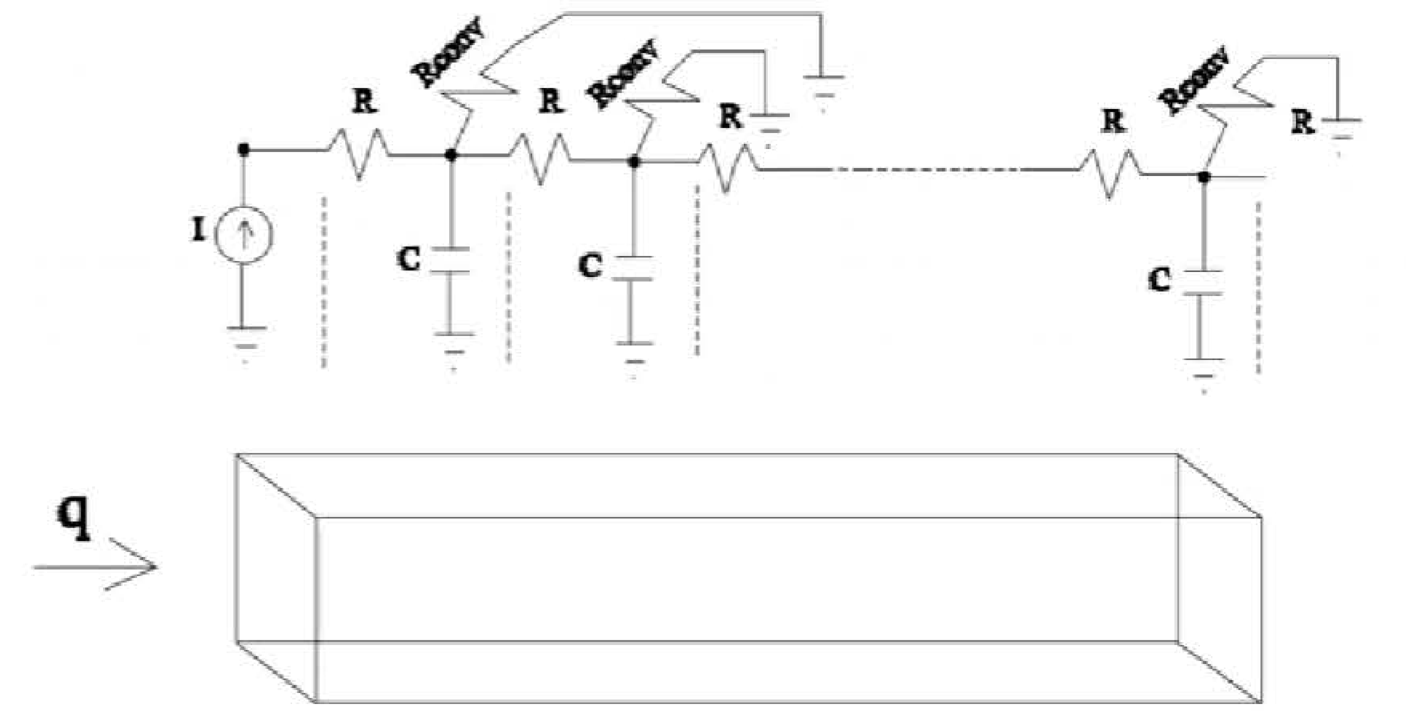
Ex. Transformer

- An equivalent thermal network
- The thermal resistances are considered for both convective and radiative transfers: this might lead to nonlinear constitutive laws (resistances depending on temperature)





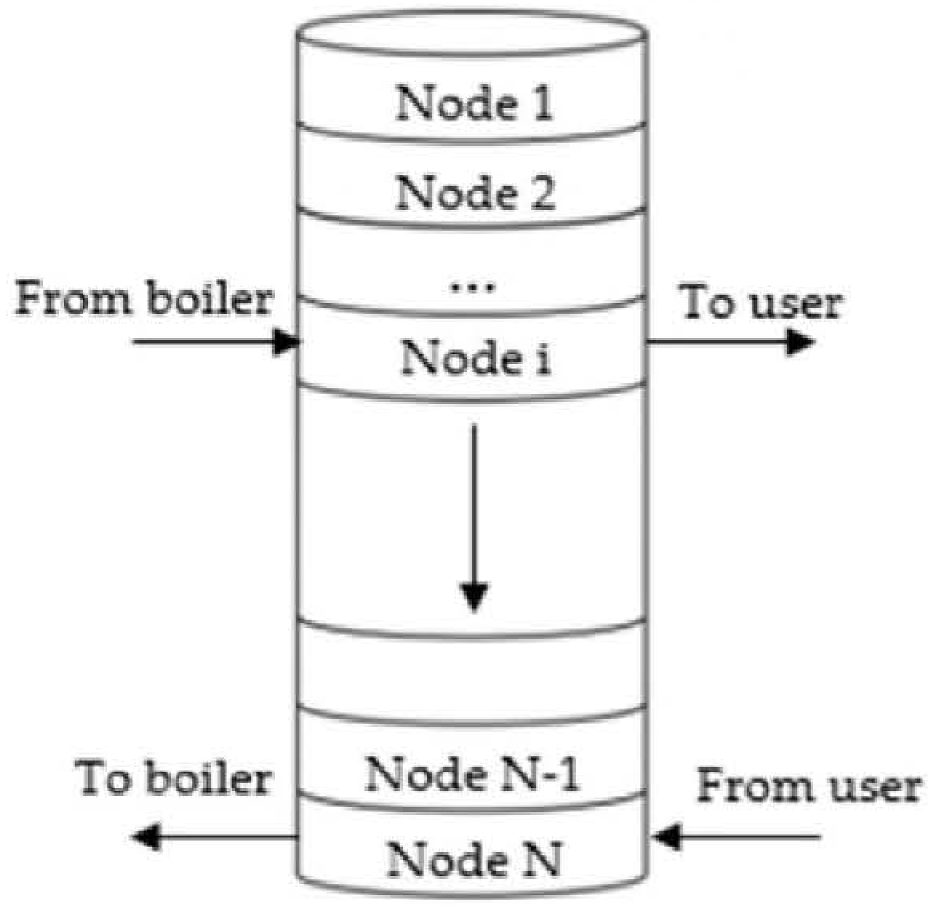
Ex. One-dimensional equivalent electrical model of heat conduction

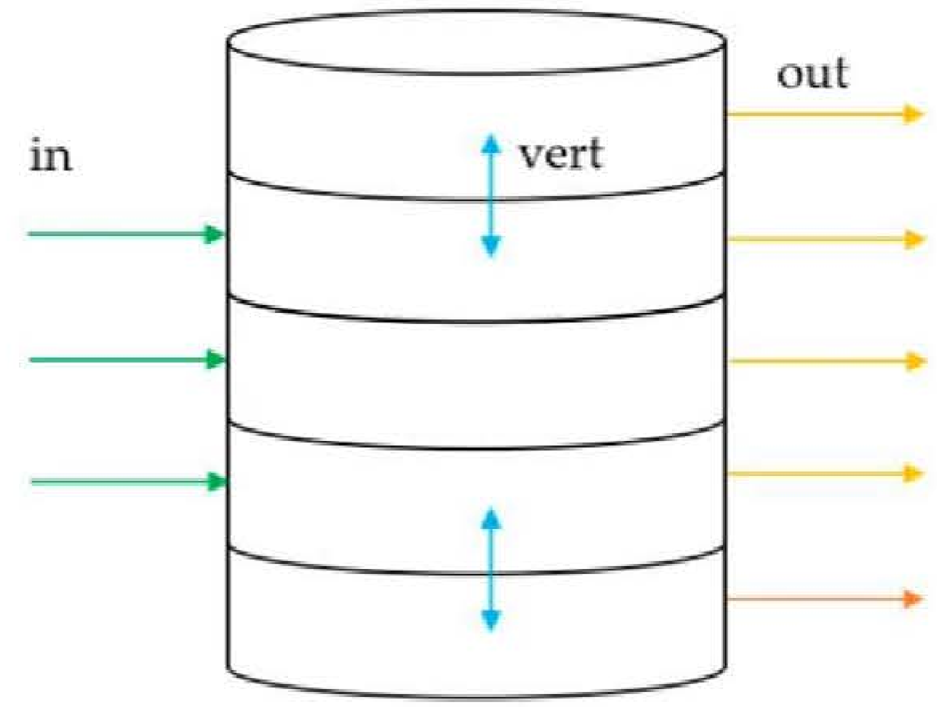


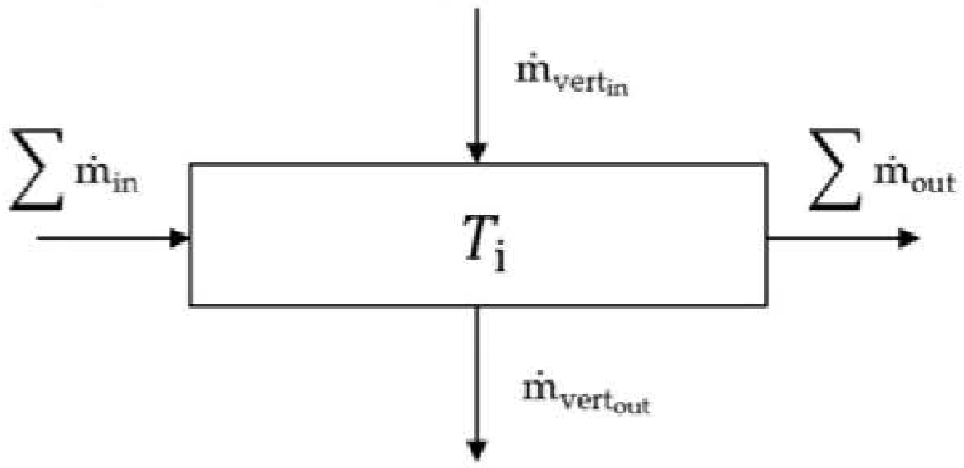
- Based on electro-thermal analogy, the heat conduction process can be modelled by dividing the length into smaller sections
- Consider the corresponding R and C value for each section
- A current source simulating a heat source (q) placed at one end of the body has to be considered
- Convection resistances R_{conv} can be connected (at each RC node) for augmenting the model for modelling also conductive heat exchanges

Ex. Liquid storage tank schematic representation for the multi-node modeling approach

- Schematic representation of nodal approach for stratified storage tank, showing incoming, outgoing, and vertical mass flows
- 2D or 3D decomposition (partitioning) of the space analyzed







Modeling of thermal conduction

Thermal capacity is a *measure characteristic for a* given ***system***

Specific heat is *a characteristic of the* considered body ***material***

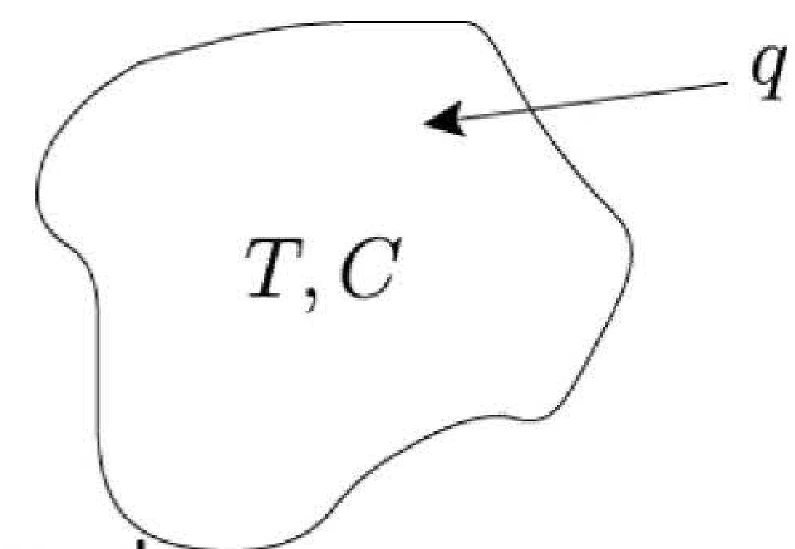
The heat capacity of an object, denoted by C, is the limit $C = \lim_{\Delta T \to 0} \frac{\Delta Q}{\Delta T}$

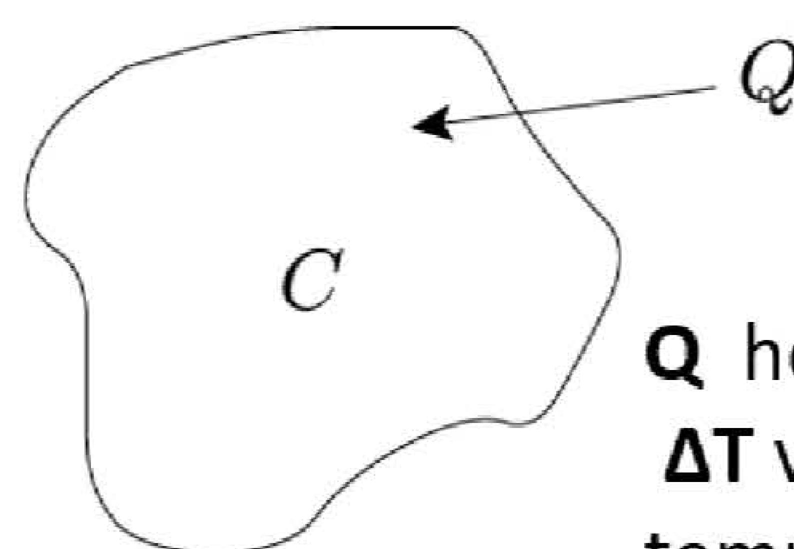
where ΔQ is the amount of heat that must be added to the object (of mass M) in order to raise its temperature by ΔT.

$$C = C_s\, M$$

$$Q = C\, \Delta T$$

$$P = q = \frac{dQ}{dt} = C\,\frac{dT}{dt}$$





$$c = \frac{C}{M} = \frac{1}{M} \cdot \frac{dQ}{dT}$$

The specific heat capacity of a substance, denoted by $\mathbf{C_s}$, is the heat capacity **C** of a sample of the substance, divided by the mass **M** of the sample

Q heat exchanged,
ΔT variation of the body temperature,
$\mathbf{C_s}$ specific heat
P=q thermal power

Modeling of thermal conduction

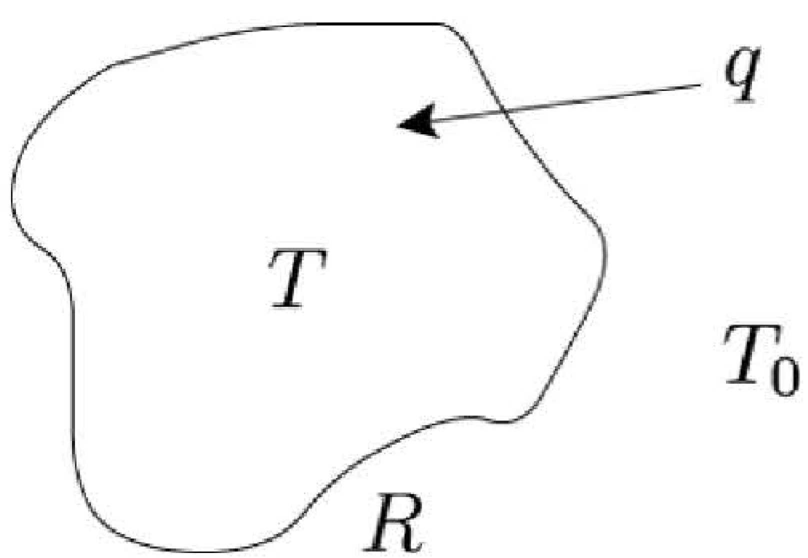
Thermal capacity is a *measure characteristic for a* given ***system***

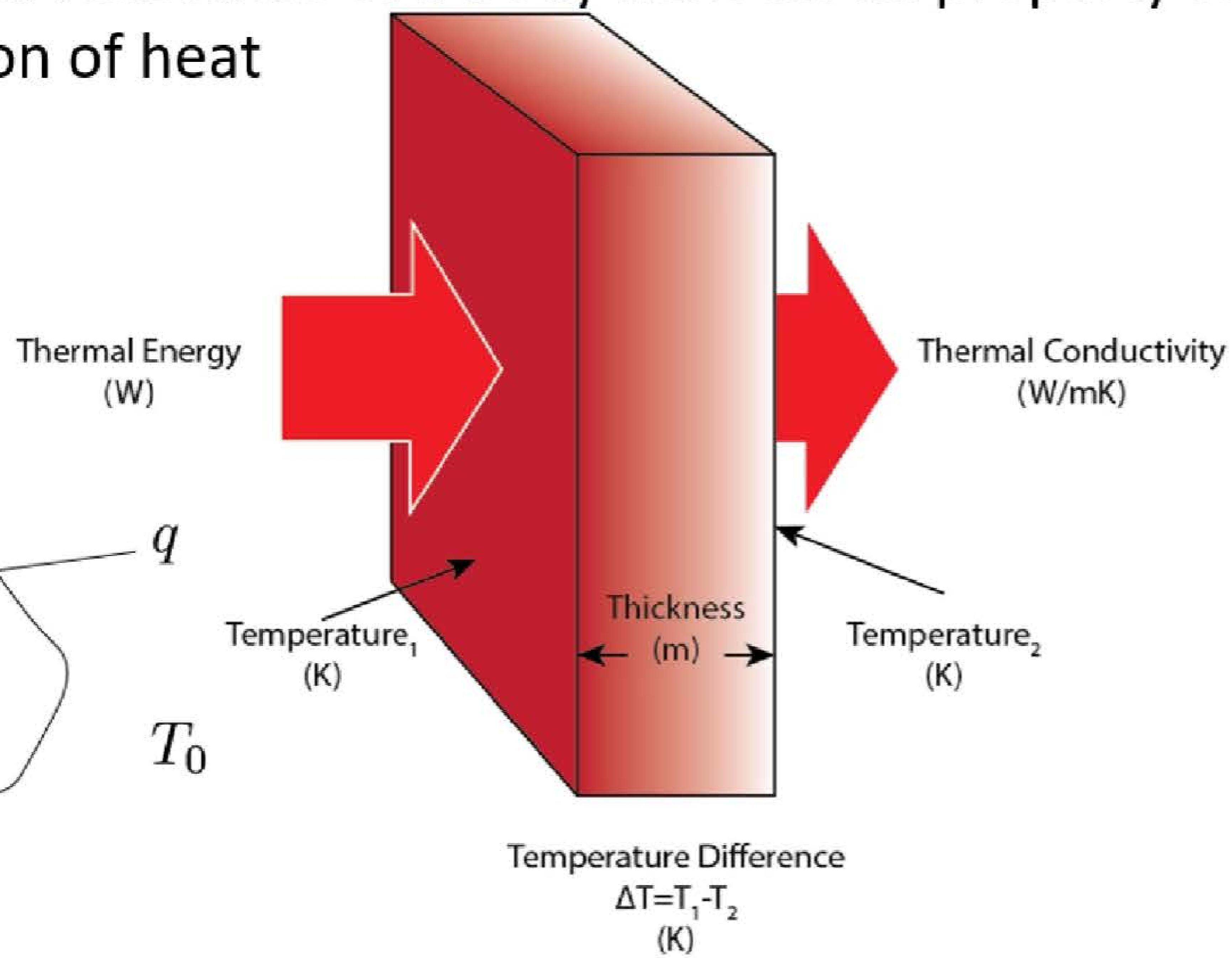
Specific heat is *a characteristic of the* considered body ***material***

The thermal resistance of a body denotes its property to prevent the transmission of heat

$$P = q = \frac{dQ}{dt}$$

$$q = \frac{1}{R}(T_0 - T)$$





Q heat exchanged,
ΔT variation of the body temperature,
C$_s$ specific heat
P=q thermal power
R thermal resistance

Thermal resistance is the temperature difference, at steady state, between **two defined surfaces** of a material or construction that induces a unit **heat flow** rate through a unit area

3D Modeling of thermal conduction

If ΔT is the Laplacian of the temperature $\Delta T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$

The heat conduction equation for an homogeneous isotropic body:

$$\rho C_p \frac{\partial T}{\partial t} = q_g + \lambda \Delta T$$

Can be derived based on Fourier law: $\varphi = -\lambda \nabla T$

$\boldsymbol{\lambda}$: Thermal conductivity of the material[W. m^{-1}. K^{-1}] (also denoted with ***k***)
$\boldsymbol{\varphi}$: Heat flow density [W. m^{-2}]
$\boldsymbol{T}$: Temperature [K]

Convection

- Convection

  $$q = k\,A\,\Delta T$$

  where:

  q = heat transferred per unit time (J/s)

  k (or h) = convective heat transfer coefficient of the process (W m^{-2} $°C^{-1}$)

  A = heat transfer area of the surface (m^2)

  ΔT = temperature difference between the surface and the bulk fluid (°C)

Radiation

- A net transfer of radiant heat requires a difference in the surface temperature of any two bodies between which the exchange is taking place
- "Gray Body" (vs. "Black Body") Radiation

$$q_r = A_1 \varepsilon_1 \sigma (T_1^4 - T_2^4)$$

where:

q = heat transfer per unit time (W)

ε = emissivity of the object (one for a black body)

σ = Stefan-Boltzmann constant =

$5.6703 * 10^{-8} (Wm^{-2}\ K^{-4})$

A = area of the object (m^2)

T_1= hot body absolute temperature (K)

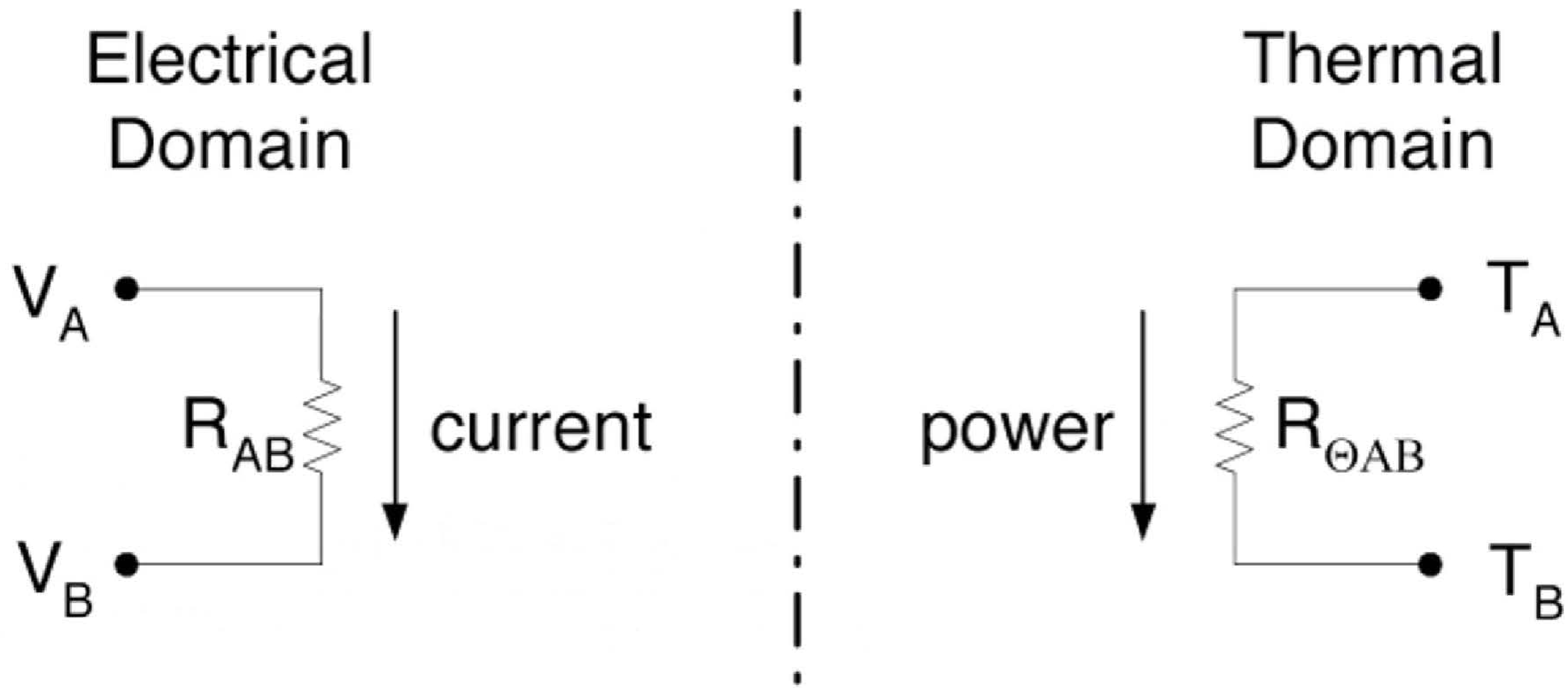
T_2 = cold surroundings absolute temperature (K)

If neither of two bodies is a perfect radiator and if the two bodies have a given geometric relationship to each other, the net heat transfer by radiation between them is given by the equation $q_r = A_1 \mathcal{F}_{1-2} \sigma (T_1^4 - T_2^4)$

$\mathcal{F}_{1-2}$ is a dimensionless modulus that modifies the equation for perfect radiators to account for the emittances and *relative geometries* of the actual bodies

Basic Principles

Fundamental Relations in the Electrical and Thermal Domains



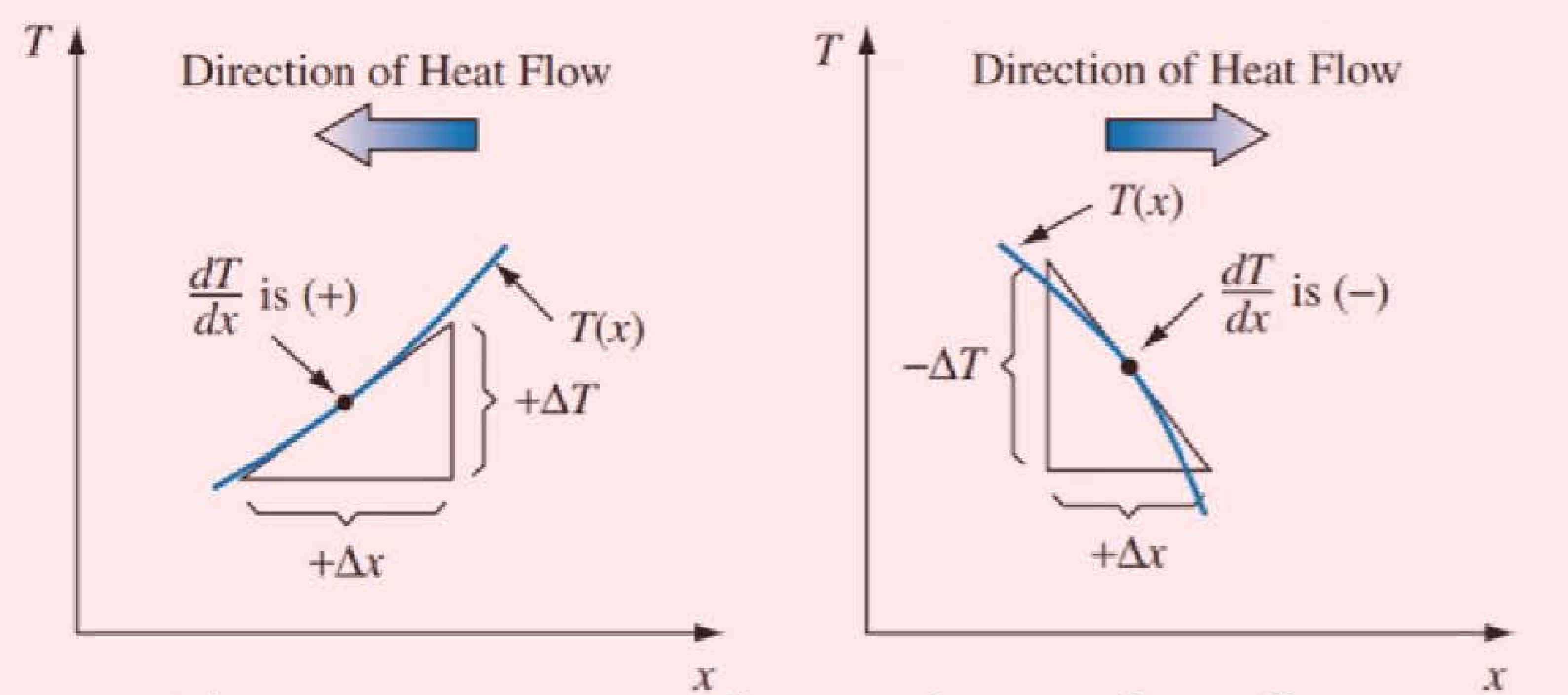
Basic Principles

Basic Relations in the Electrical and Thermal Domains

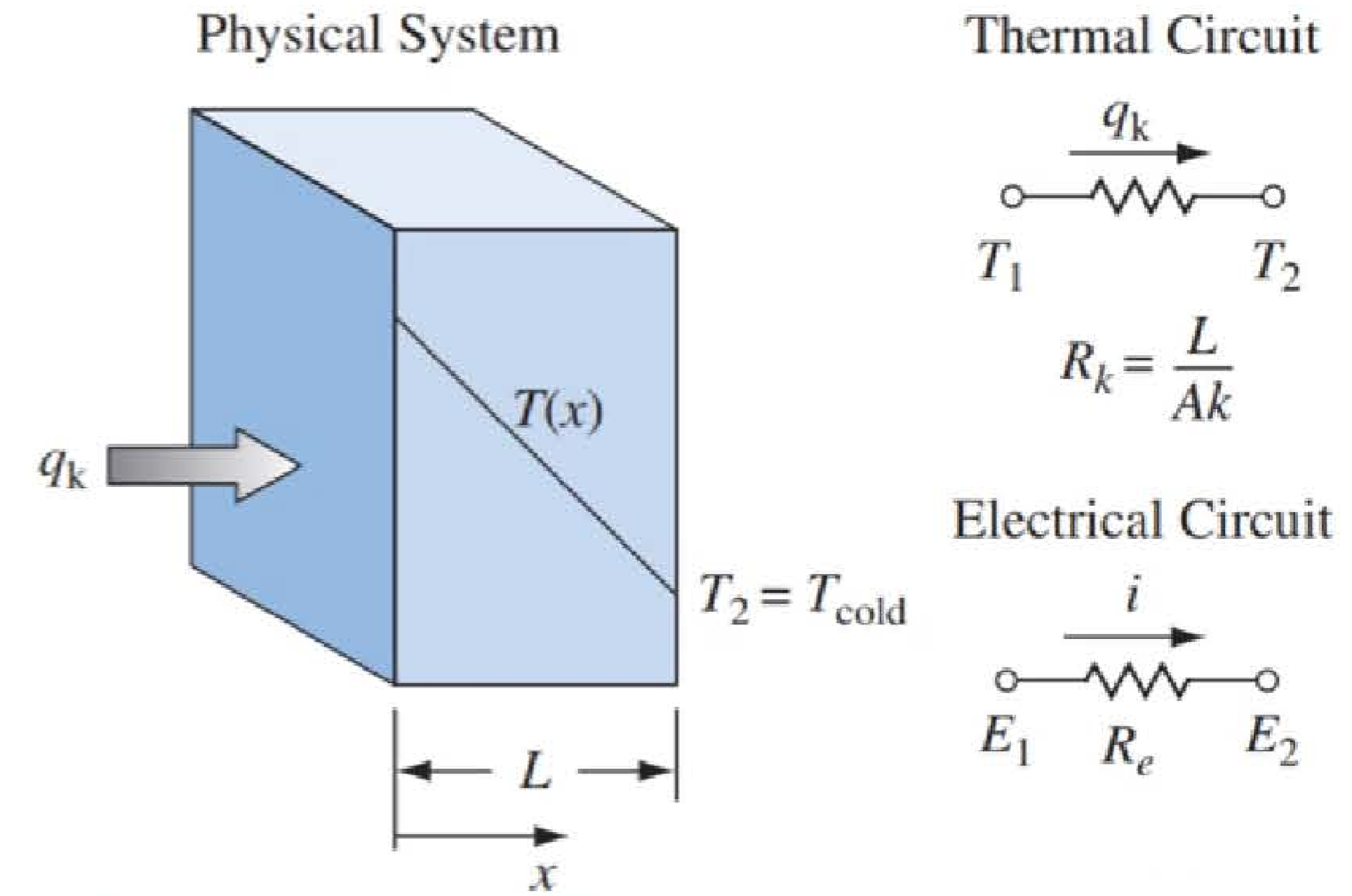
	Electrical Domain			Thermal Domain		
Through Variable *(FLOW)*	Current	I	Amperes or Coulombs/s	Power or heat flux	P_D	Watts or Joules/s
Across Variable *(EFFORT)*	Voltage	V	Volts	Temperature	T	°C or K
Resistance	Electrical resistance	R	Ohms	Thermal resistance	$R_{\Theta AB}$	°C/W or K/W
Capacitance	Electrical capacitance	C	Farads or Coulombs/V	Thermal Capacitance	C_{Θ}	Joules/°C
„Ohm's Law"	$\Delta V_{AB}=V_A-V_B=I\cdot R_{AB}$			$\Delta T_{AB}=T_A-T_B=P_D\cdot R_{\theta AB}$ (derived from Fourier's Law)		

1D Modeling of thermal conduction – Electrical analogy

- The thermal conductivity **k** is a physical property of the medium (is a material property that indicates the amount of heat that will flow per unit time across a unit area when the temperature gradient is unity)
- The ratio ***k/L*** (the thermal conductance per unit area), is called the *unit thermal conductance* for conduction *heat flow*, while the reciprocal, is called the *unit thermal resistance*



The sign convention for conduction heat flow



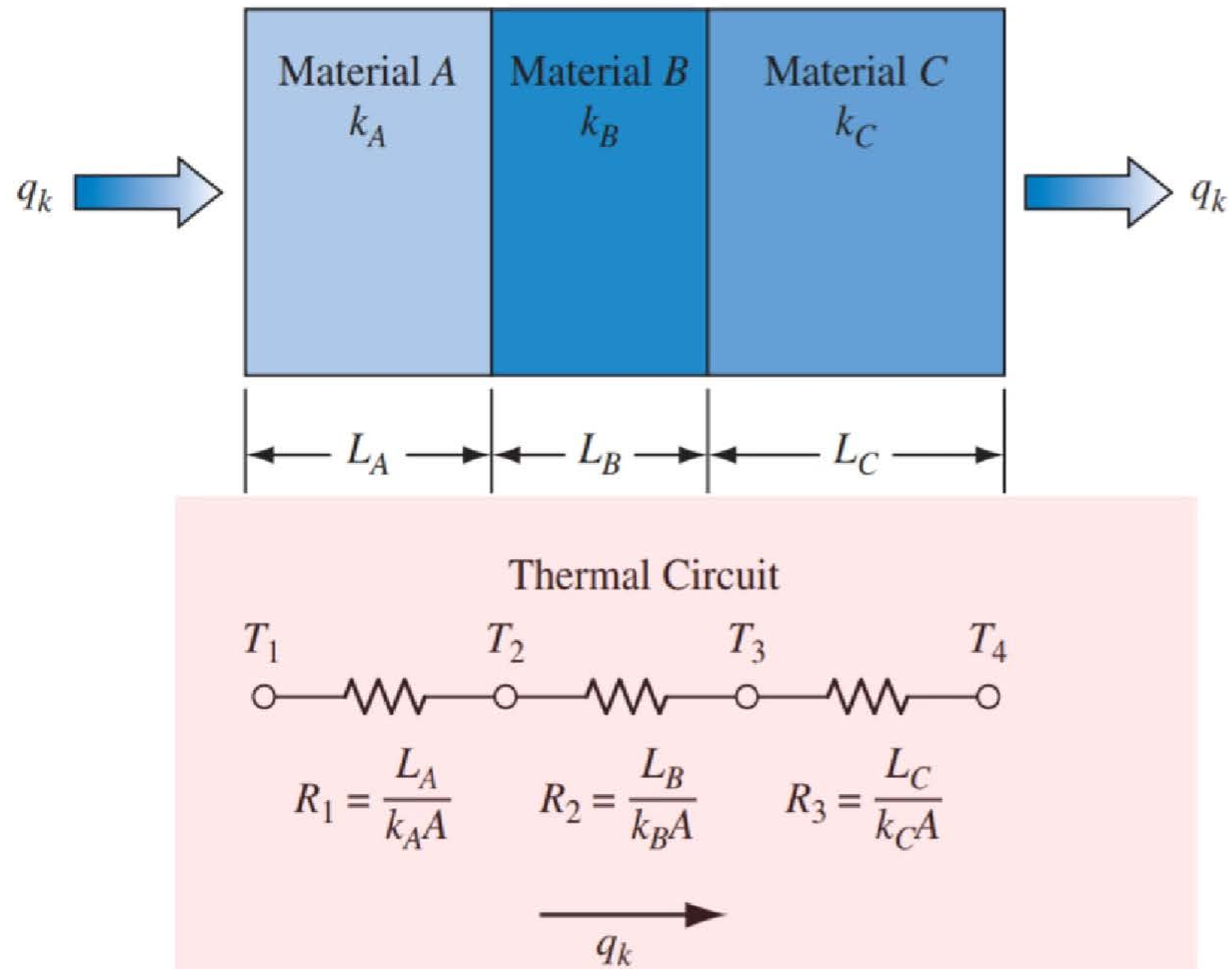
- conduction through a homogeneous medium

$$q_k = -kA\frac{dT}{dx}$$

<u>Example:</u> Typical thermal conductivity values for glass wools are between 0.023 and 0.040W/m·K
For a concrete block a typical value is 2.26W/m·K

1D Modeling of thermal conduction – Electrical analogy

Example1: Three-layer system in series



$$q_k = \left(\frac{kA}{L}\right)_A (T_1 - T_2) = \left(\frac{kA}{L}\right)_B (T_2 - T_3) = \left(\frac{kA}{L}\right)_C (T_3 - T_4)$$

$$q_k = \frac{T_1 - T_4}{(L/kA)_A + (L/kA)_B + (L/kA)_C}$$

N layers in series:

$$q_k = \frac{T_1 - T_{N+1}}{\sum_{n=1}^{n=N} (L/kA)_n}$$

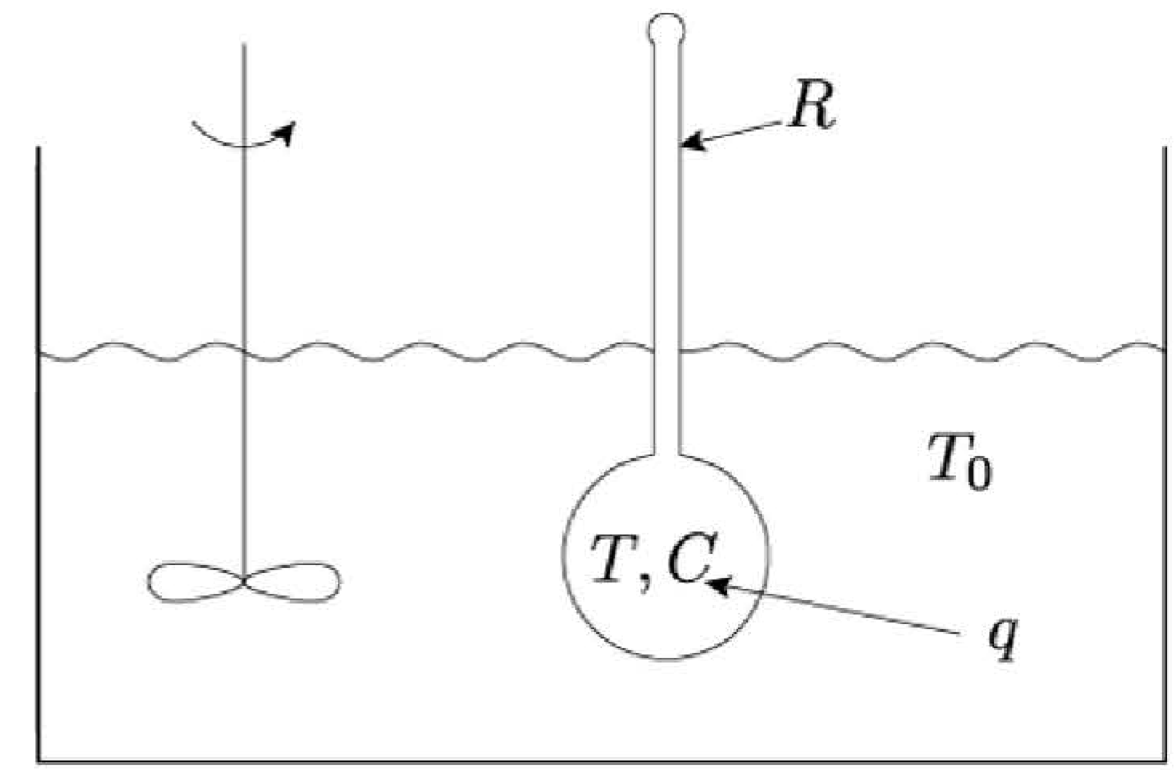
$$q_k = \frac{T_1 - T_{N+1}}{\sum_{n=1}^{n=N} R_{k,\,n}} = \frac{\Delta T}{\sum_{n=1}^{n=N} R_{k,\,n}}$$

Creating a mathematical model

- Writing the basic equations in the Electrical and Thermal Domains
- Identification of heat flow pathways (conductive transfer)
- Identification of heat gains or losses due to circulation of fluids or mass transport and of radiative transfers
- Heat balance equations
- Non-stationary state energy balance models for temperature given usually by the ordinary differential equations, where the independent variable is time **t** and the dependent variable is the temperature **T(t)**

Ex. 1 Thermometric probe

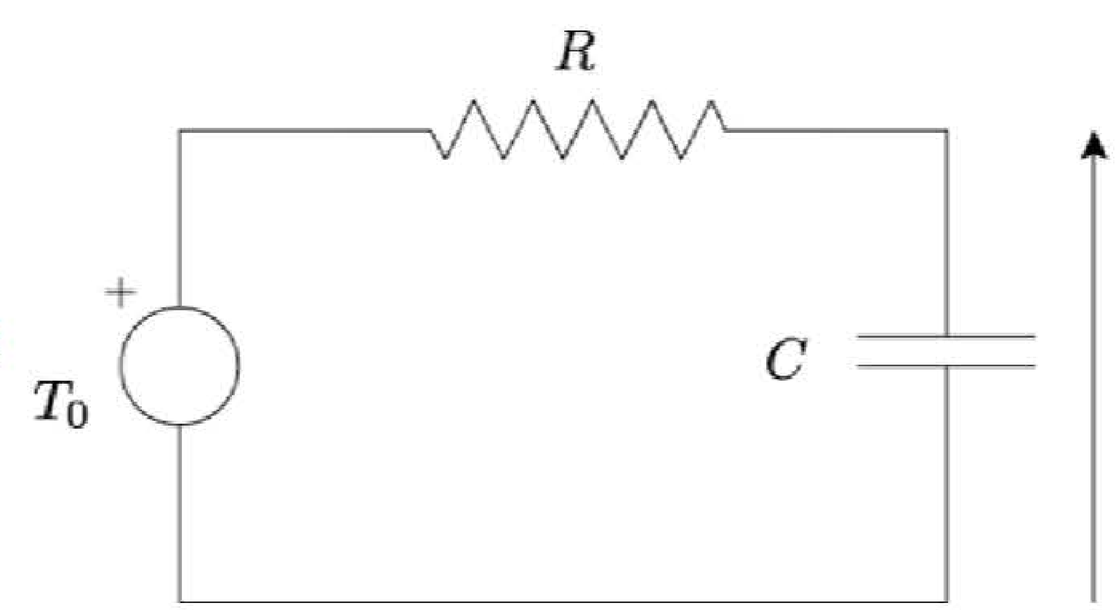
$$\begin{cases} q = \dfrac{1}{R}(T_0 - T) \\ q = C\dfrac{dT}{dt} \end{cases}$$

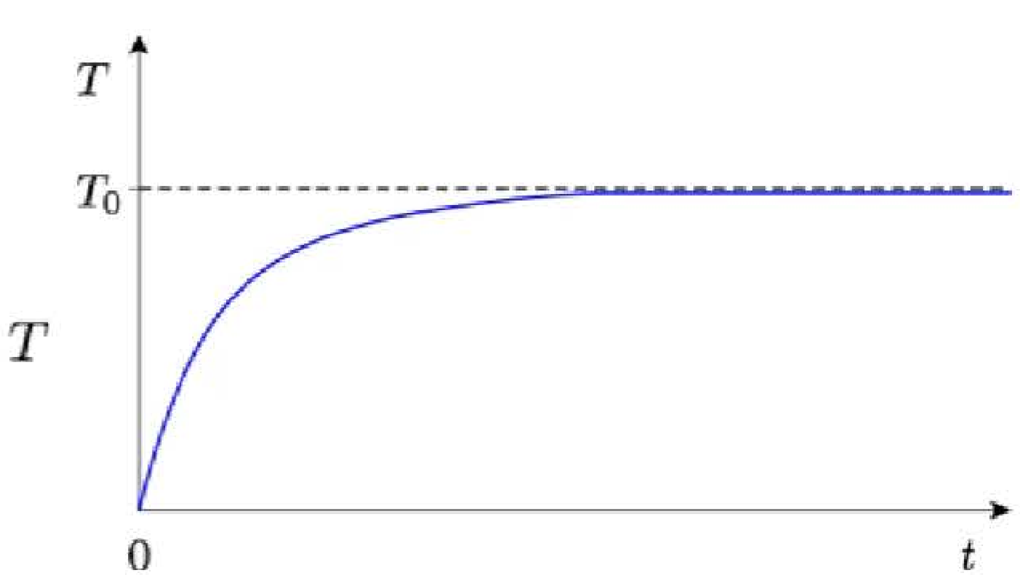


$$RC\frac{dT}{dt} = (T_0 - T)$$

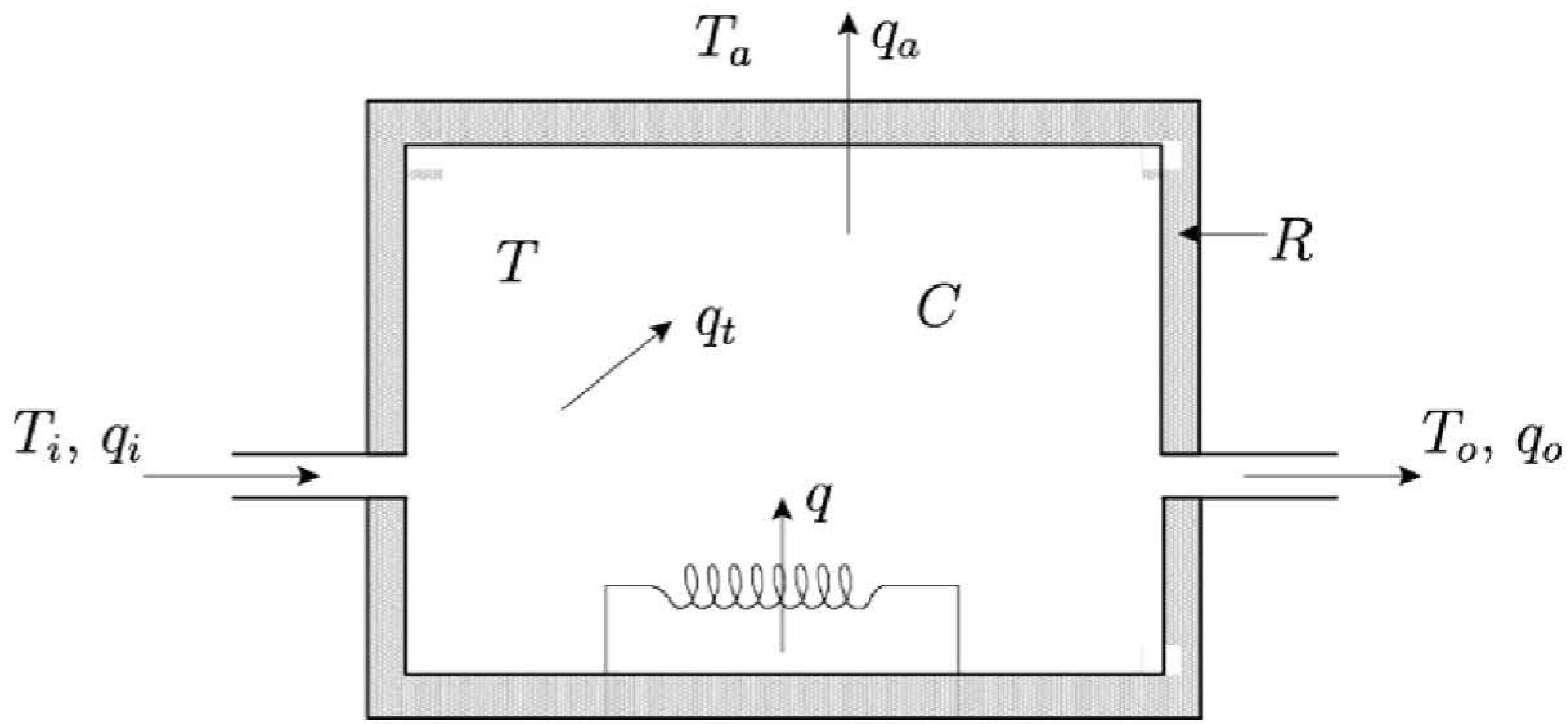
$$\dot{T} = -\frac{1}{RC}T + \frac{1}{RC}T_0$$

Equivalent electric circuit





Ex2. Electric Boiler



q_t heat absorbed by water,
R thermal resistance of the boilers walls,
n water flux through the boiler (in Kg/s)

$Q_{in} = Q_{acc} + Q_{out}$

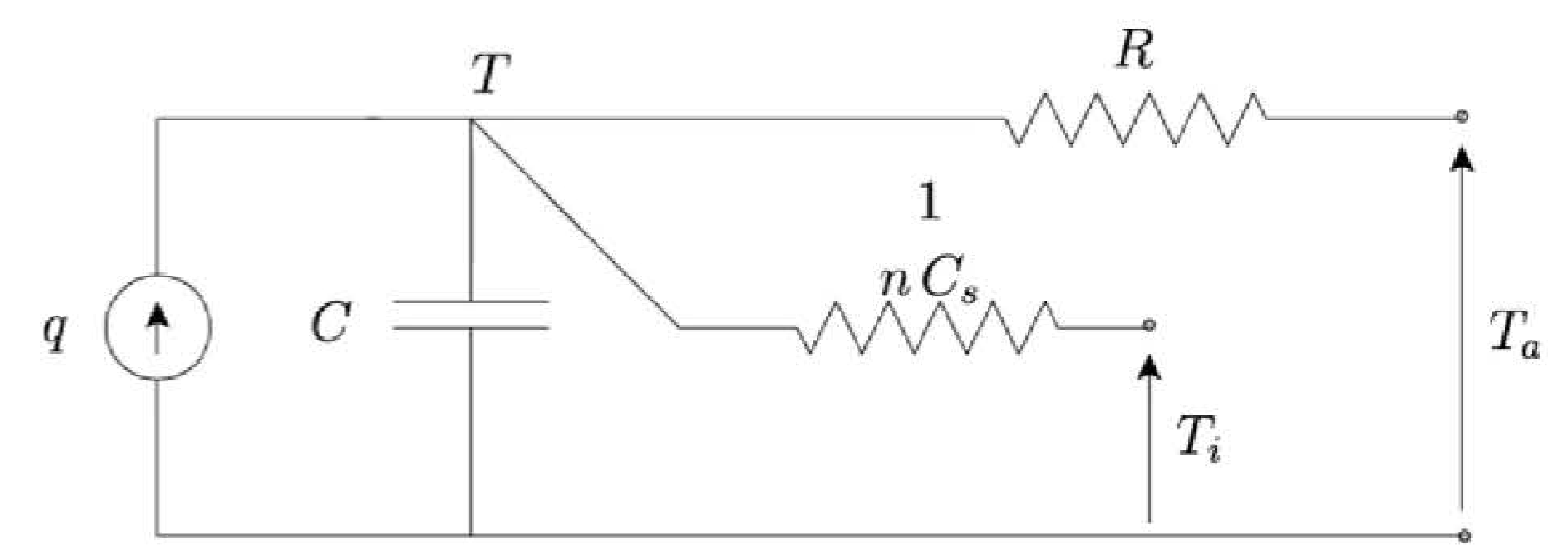
$$q_i + q = q_t + q_a + q_0$$

$$q_t = C\frac{dT}{dt}, \quad q_0 = nC_sT, \ \ q_i = nC_sT_i,$$

$$q_a = \frac{T - T_a}{R}$$

$$C\dot{T} + nC_s(T - T_i) + \frac{T - T_a}{R} = q$$

$$\dot{T} = -\frac{1}{C}\left(nC_s + \frac{1}{R}\right)T + \frac{nC_s}{C}T_i + \frac{1}{RC}T_a + \frac{1}{C}q$$



Equivalent electric circuit

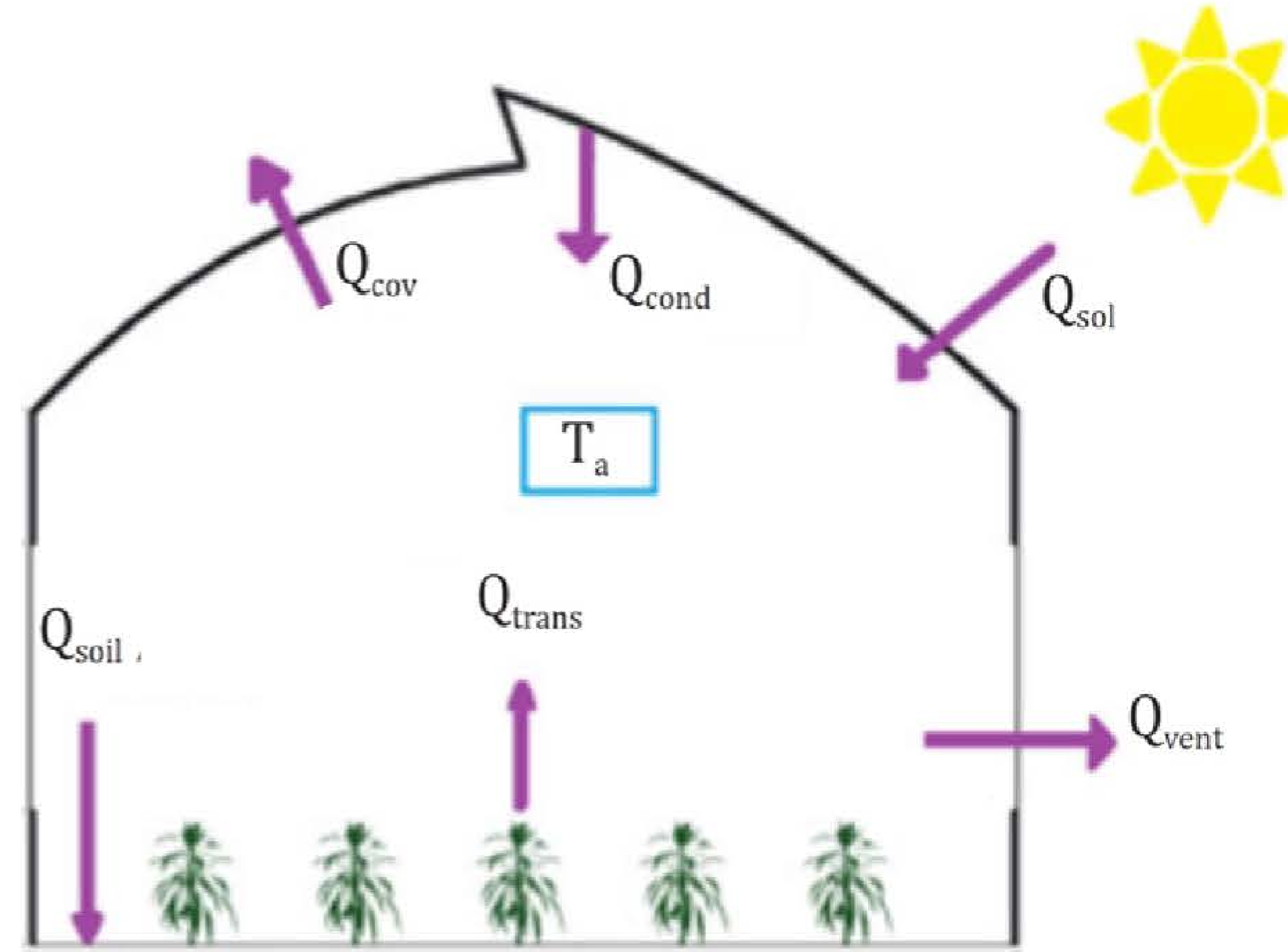
Ex. 3 Greenhouse

Identifying the **Energy flows**

A. Heat **gains**
   due to solar radiation Q_{sol} and
   due to water vapor
   condensation on the roof Q_{cond}

B. Heat **losses**
   through the cover Q_{cov},
   due to ventilation Q_{vent},
   through crop transpiration Q_{trans}
   and through the soil Q_{soil}

all expressed in $W \cdot m^{-2}$



$$V C_p \rho \frac{dT_a}{dt} = (Q_{sol} - Q_{cov} - Q_{trans} - Q_{vent} - Q_{soil} + Q_{cond})$$

V Greenhouse volume, **ρ** Air density, $\mathbf{C_p}$ Specific heat of the air

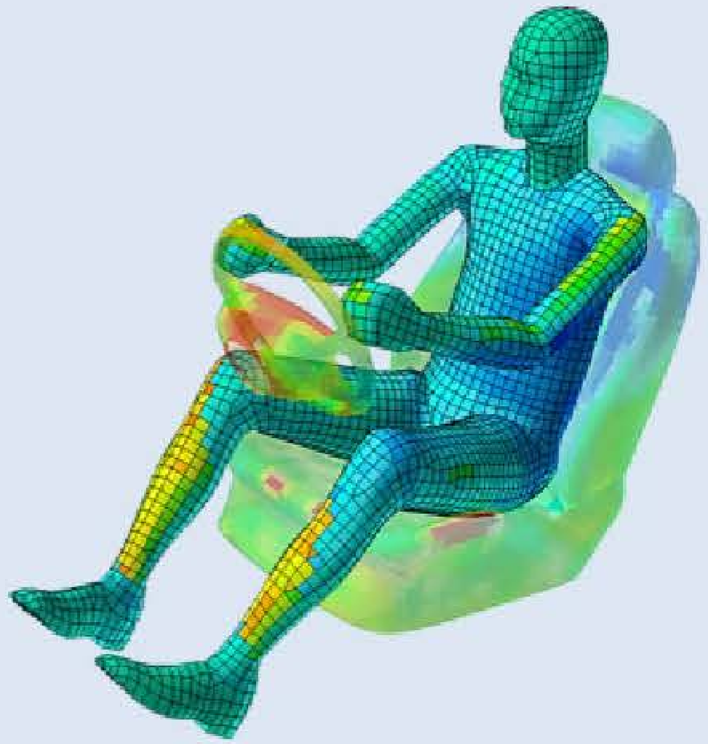
Ex. 4 Effects of the thermal environment on the human body

The most important **thermophysiological processes**:

-constriction or dilation of the peripheral blood vessels,

-sweating,

-production of heat by shivering.



Identifying the **Energy flows**

Metabolic heat production (**H**)

Convective heat flux (between the surface of the body and the ambient air) (**C**)

Radiative heat flux (heat exchanged by radiation between the body and the environment) (**R**)

Water vapor diffusion (heat associated to the water vapor diffusing from the subcutaneous tissues through the skin into the ambient air)

Respiratory heat fluxes (heat transferred from the surface of the respiratory tract into the respired air)

Humidification of respired air (The humidification necessary to reach the saturation of the respired air with water vapor is associated with a corresponding heat loss in the upper airways)

Sweat evaporation has an associated heat loss

Heat flux from food intake (If the temperature of food or drink is different from the core temperature this means a heat loss or gain of the body).

Heat storage (**S**)

Writing the heat balance equation

$$H + C + R + E_D + E_{Res} + E_{Rel} + E_{sw} + F = S$$

Nodal approach to Thermal Analysis

Multi-node modelling approach

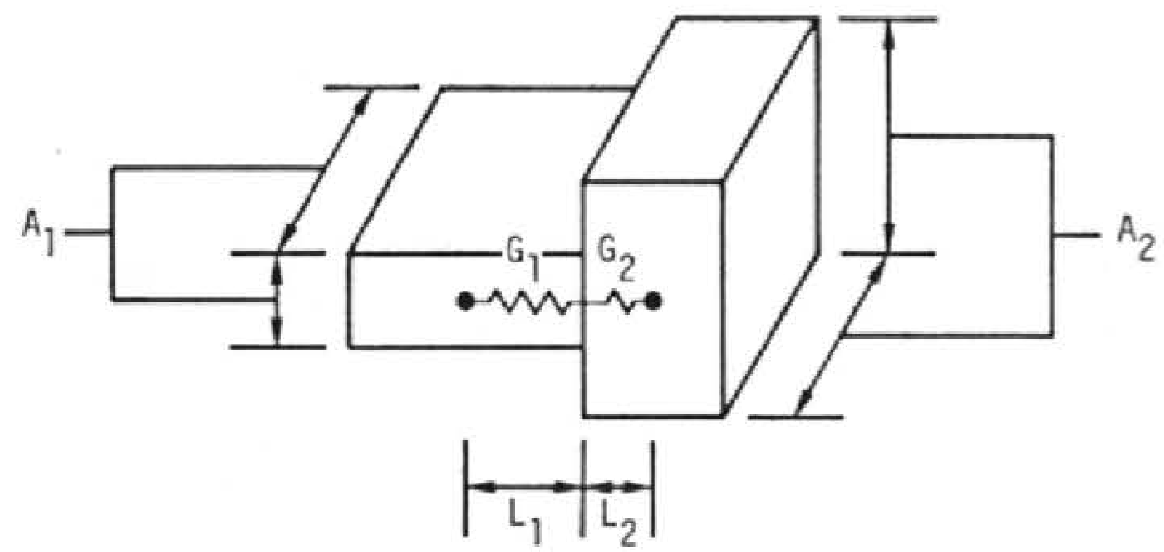
- The thermal system is partitioned into a number of finite subvolumes called **nodes**.
- Each node will have an **associated thermal network**
- The components of the network are thermal resistances and capacitances
- Each node has a **temperature** (potential) and an associated capacitance (thermal mass). For the analysis of stationary regimes, capacitances are often neglected.
- The interaction between neighboring nodes (heat flows) will be reflected by **interconnections** between their associated networks (usually through thermal resistances)

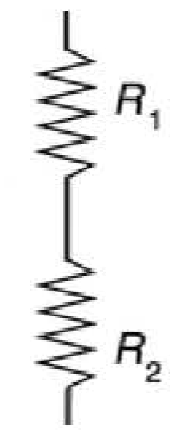
REMARKS:

- The temperature, **T**, assigned to a node represents the average mass temperature of the subvolume
- Its value is associated to the central point of the of the subvolume.
- The temperature at an arbitrary position inside the modelled body can be approximated by interpolation between adjacent nodal points (whose the temperatures are known), when an homogeneous isotropic body is considered
- The extension of the method for modelling other types of exchanges than the conductive one is possible, but for the case of radiative and convective transfers the associated thermal resistances will be temperature dependent
- The value of the created model depends on the quality of the partitioning

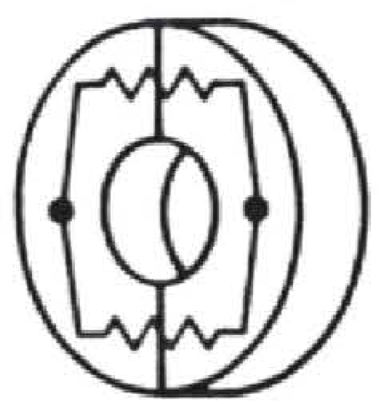
Examples of Network topologies

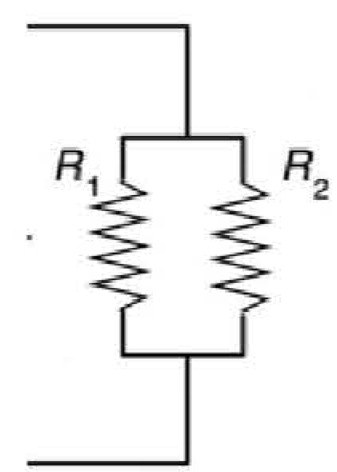
- Cascaded conducting blocks:





- Parallel conducting blocks:



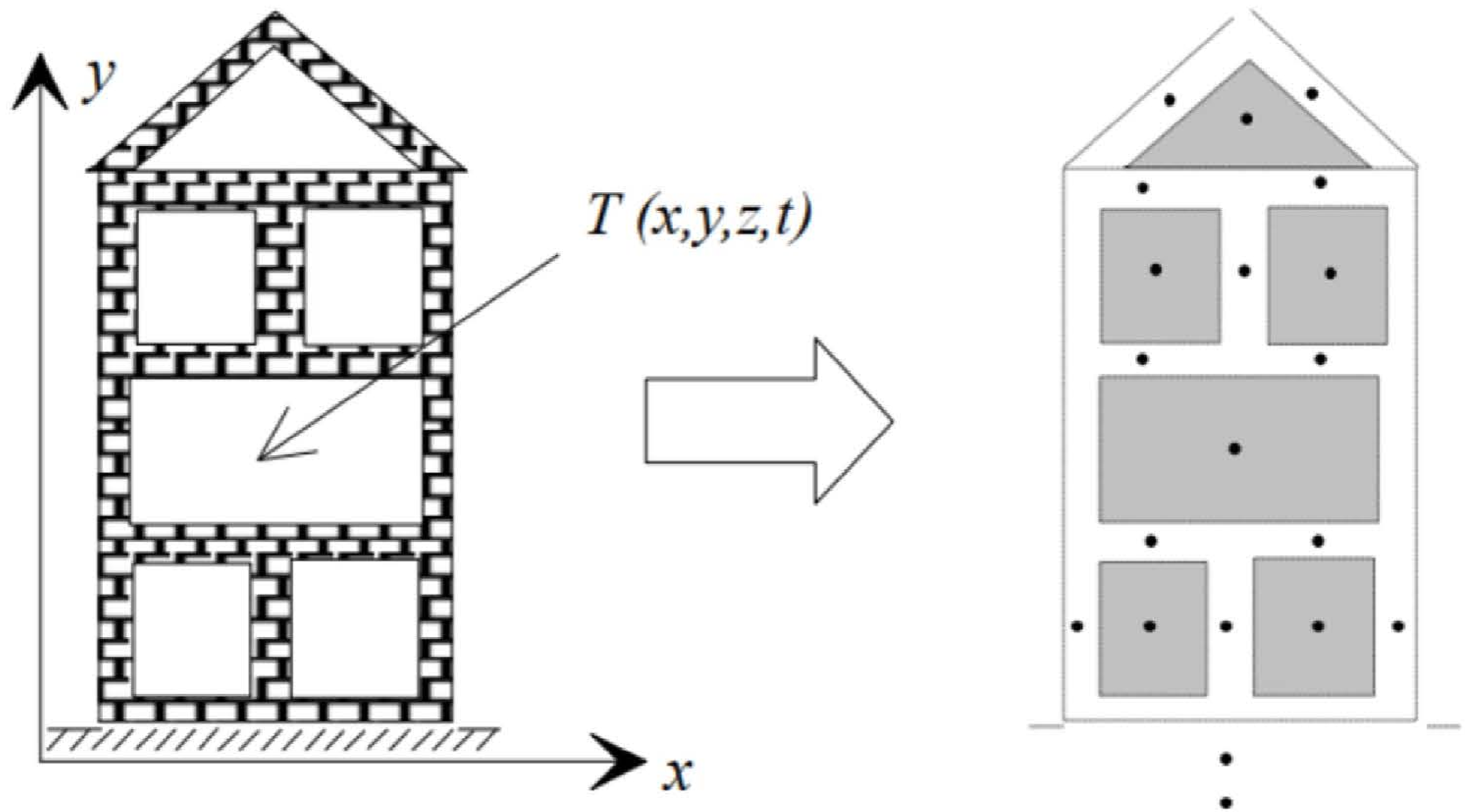


Principles for partitioning

- A first principle is based on the uniformity of the blocks considered:
  - Similar, simple, geometries are preferred
  - Homogenous subvolumes are preferred
  - Uniformity of the thermal transfer or of the heat generation
- A second principle is to consider the partitioning based on the functional blocks presents in the considered volume
- The different types of heat transfer must be considered for each block (Conduction, Convection, Radiation)
- A distinction should be made between blocks that are presenting heat transfer through all their volume (ex. Exchange through conduction) and blocks that are transferring heat just through a surface (ex. Exchange through forced or natural convection or radiation)
- *In order to validate the models*, it is obviously preferable to consider subvolumes centered on the points where the temperature probes are placed

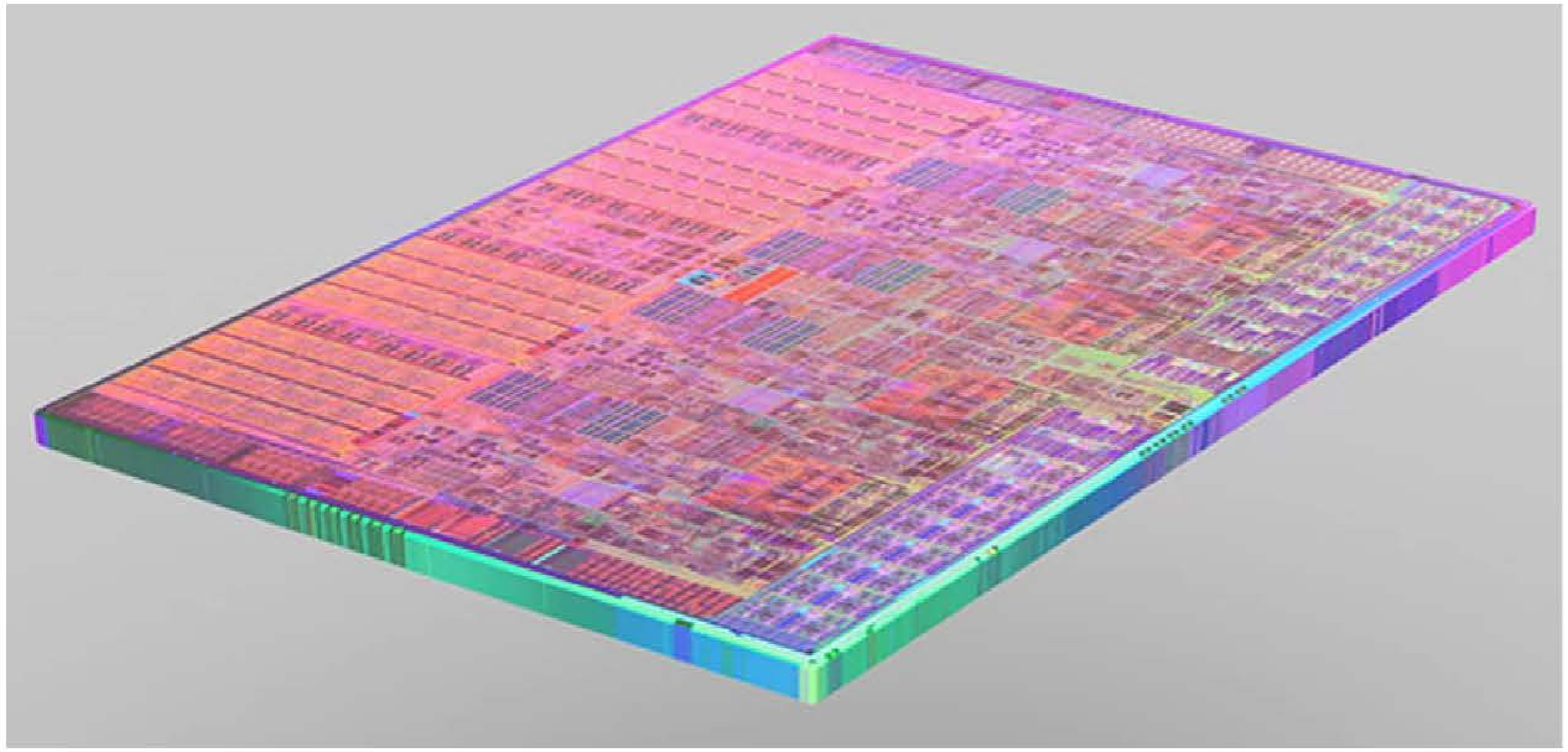
Examples of partitioning

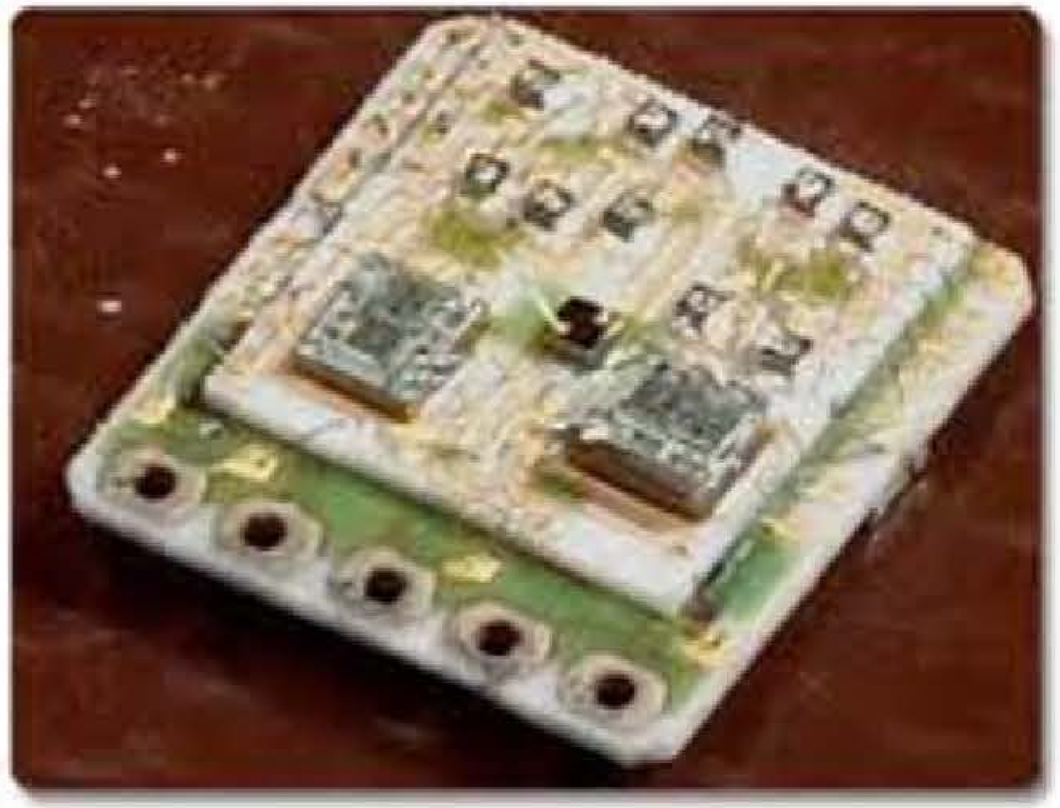
- Thermal partitioning of a building

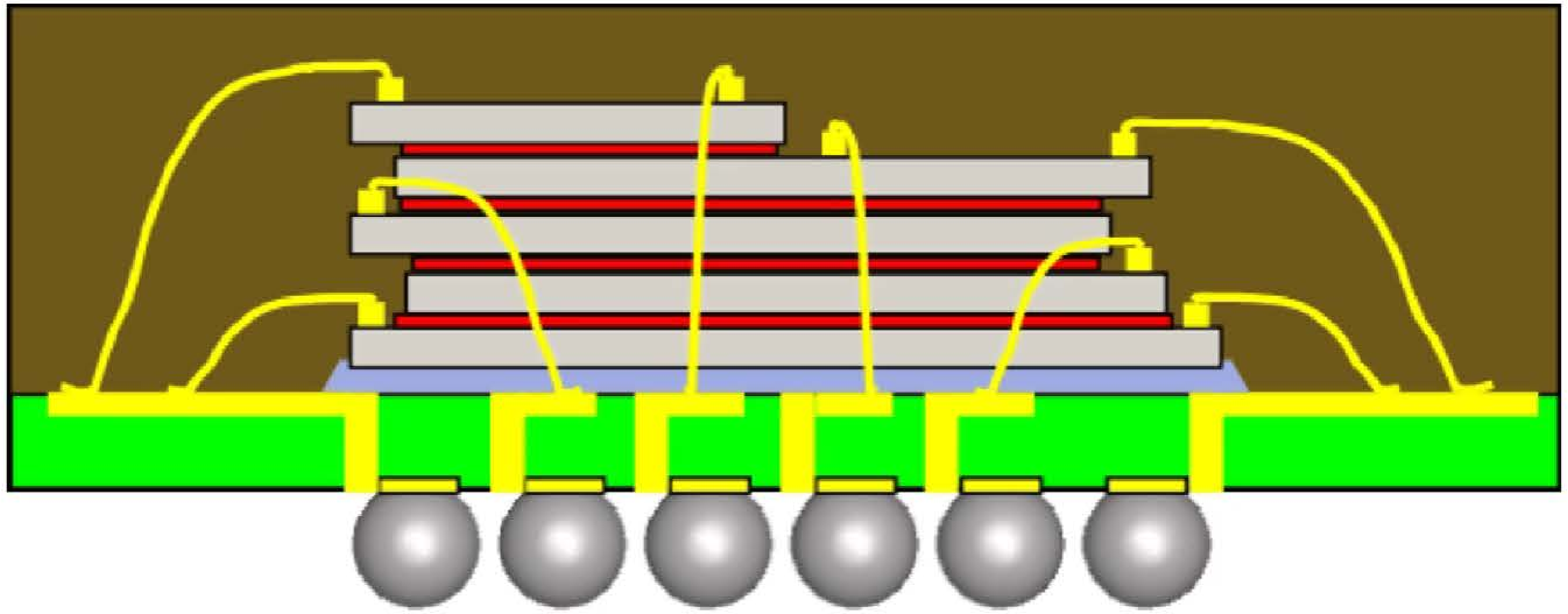


Thermal analysis of semiconductor systems using analogies with the electric domain

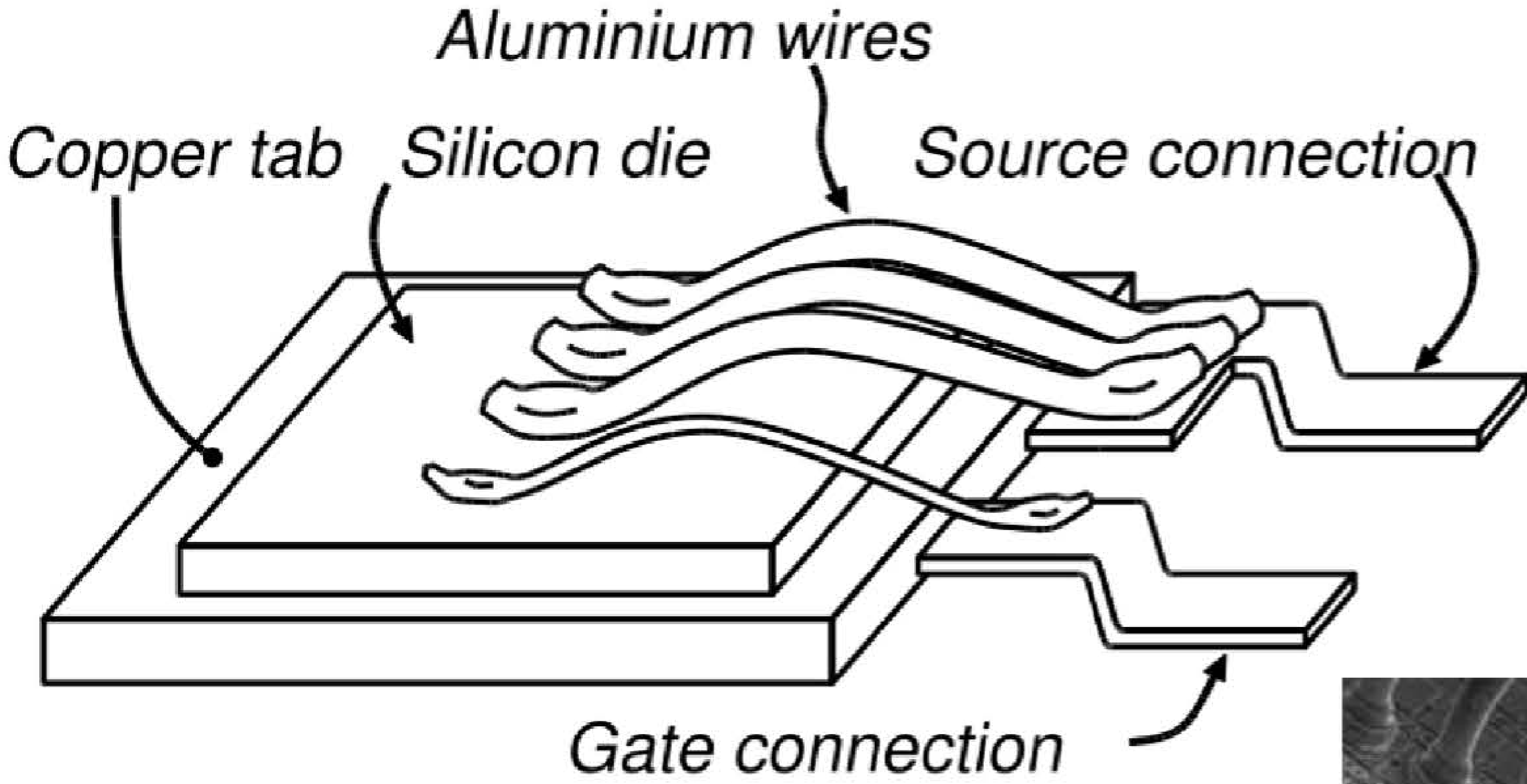
Evolution toward SIP

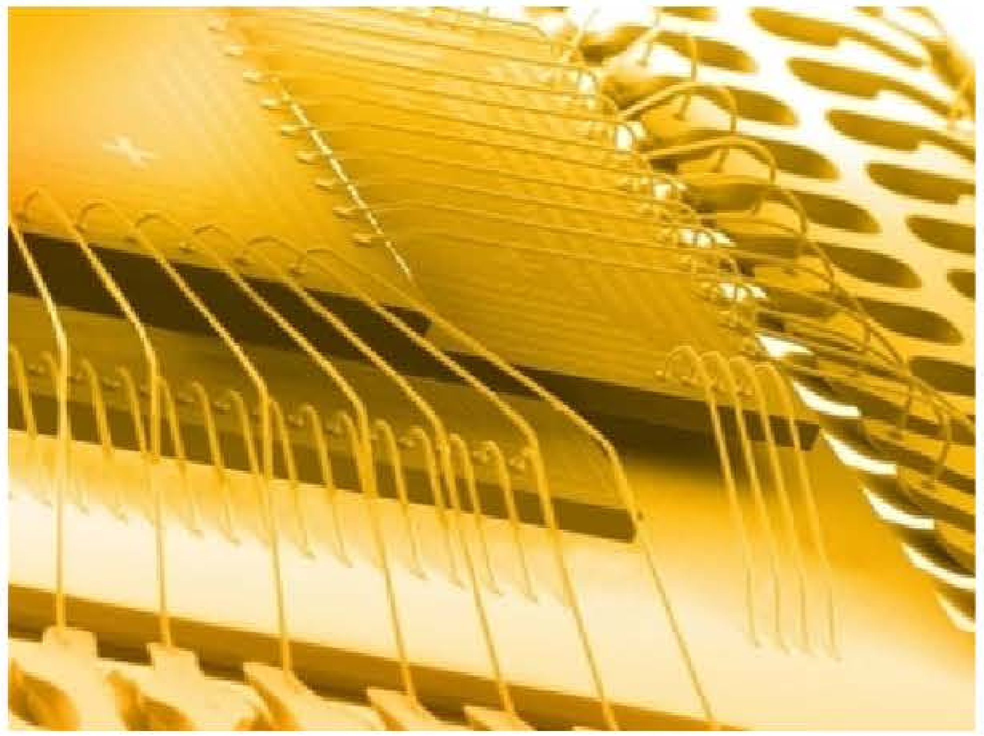


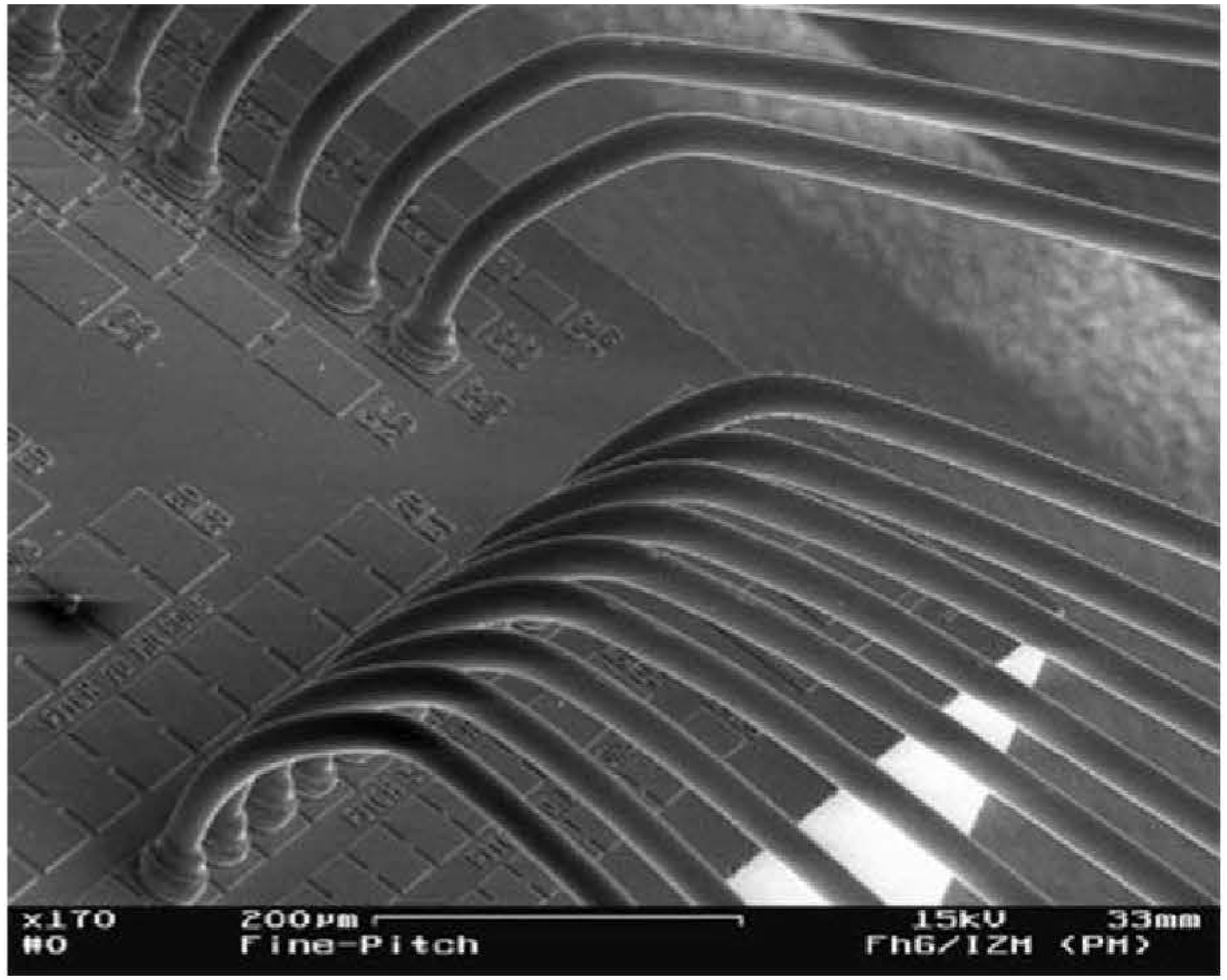


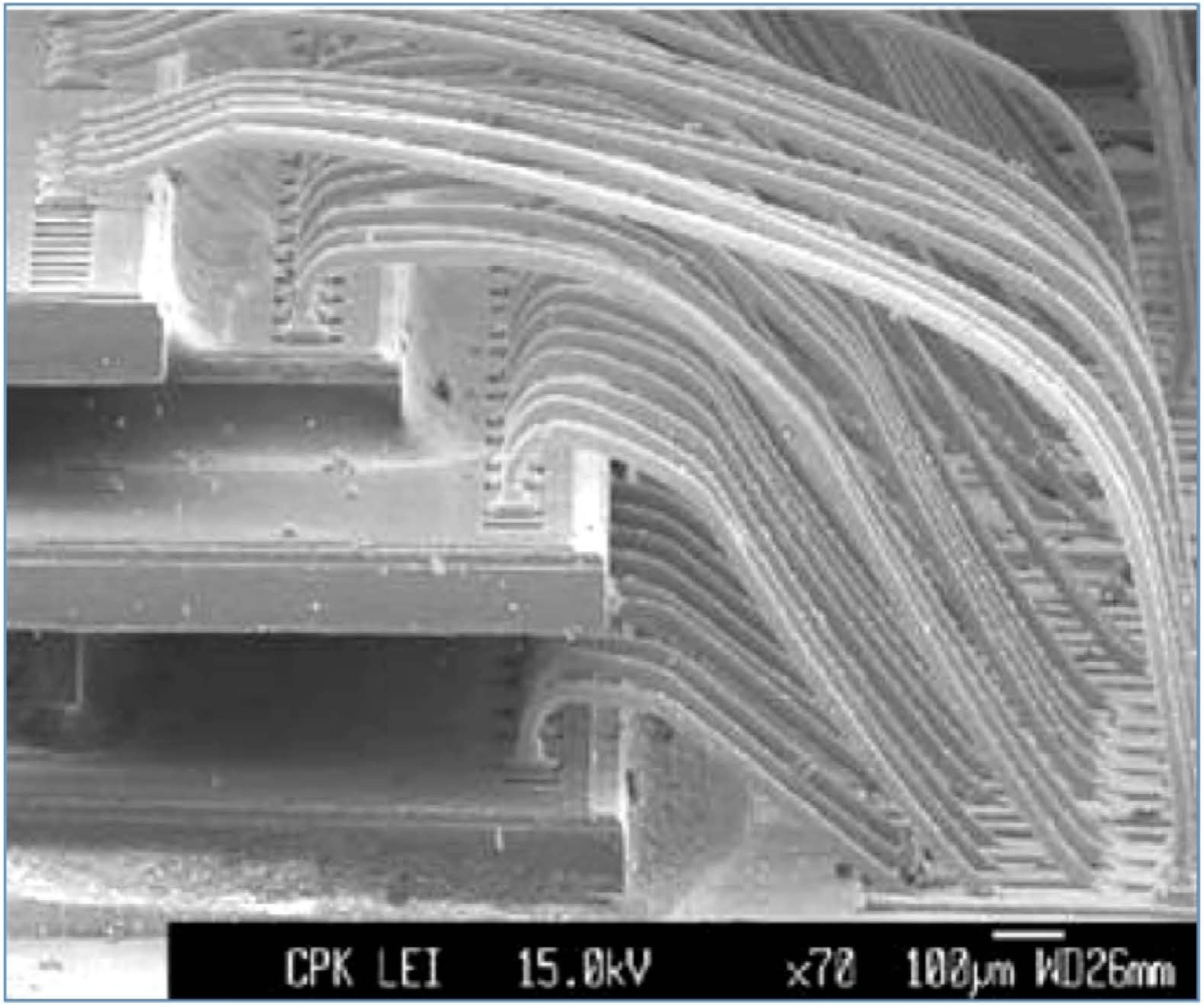


Heat transfer pathways








CPK LEI 15.0kV x70 100µm WD26mm

Definitions

- H Heat flux, rate of heat flow across a unit area ($J \cdot m^{-2} \cdot s^{-1}$)
- $R_{\Theta AB}$ Thermal resistance between reference points "A" and "B", or R_{THAB}
- $R_{\Theta JMA}$ Junction to moving air ambient thermal resistance
- $R_{\Theta JC}$ Junction to case thermal resistance of a packaged component from the surface of its silicon to its thermal tab, or R_{THJC}
- $R_{\Theta JA}$ Junction to ambient thermal resistance, or R_{THJA}
- $C_{\Theta AB}$ Thermal capacitance between reference points "A" and "B", or C_{THAB}
- °C or K Degrees Celsius or degrees Kelvin
- $Z_{\Theta AB}$ Transient thermal impedance between reference points "A" and "B", or Z_{THAB}

Definitions

- $R_{\Theta AB}$ Thermal resistance between reference points "A" and "B", or R_{THAB}
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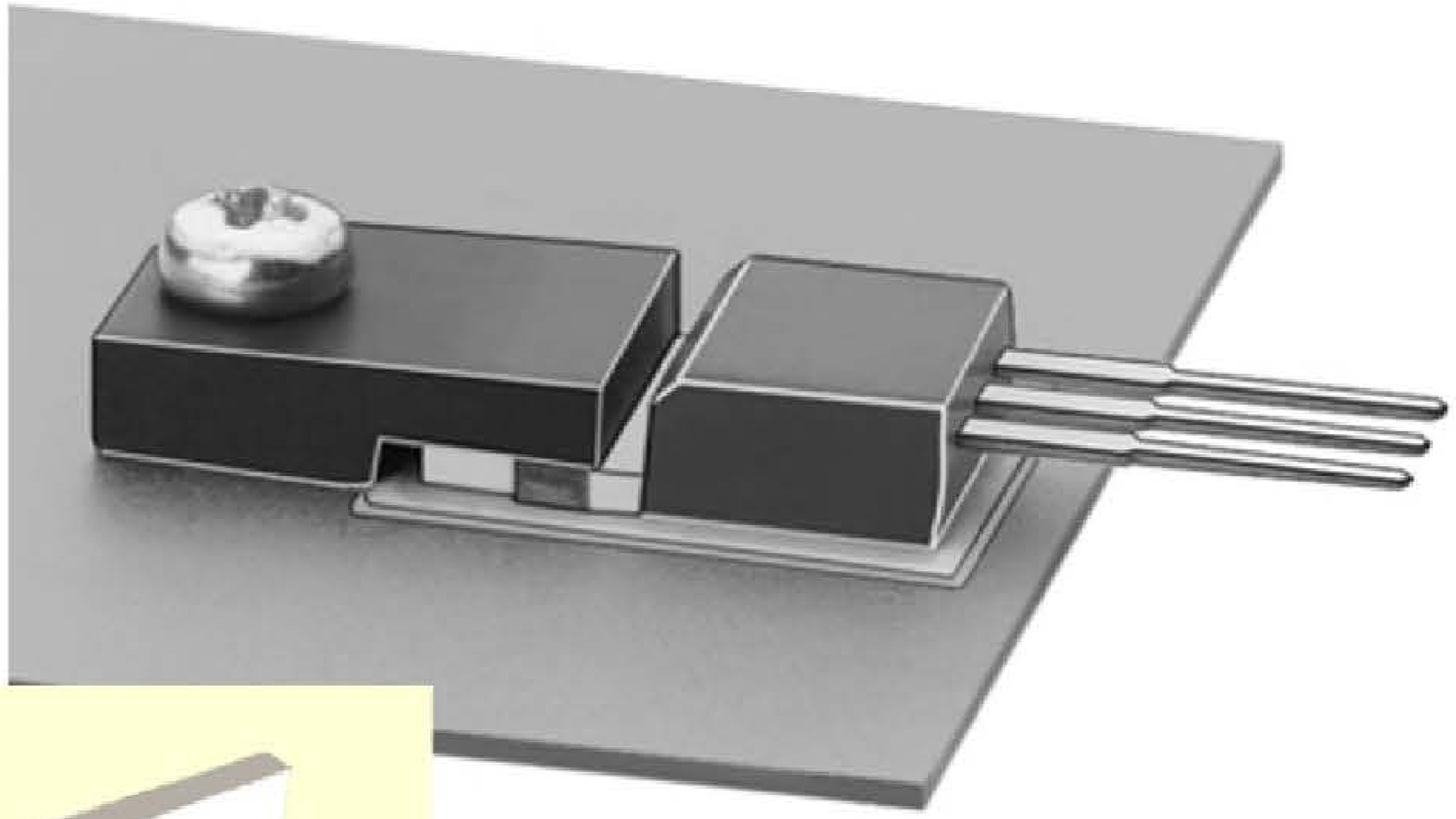

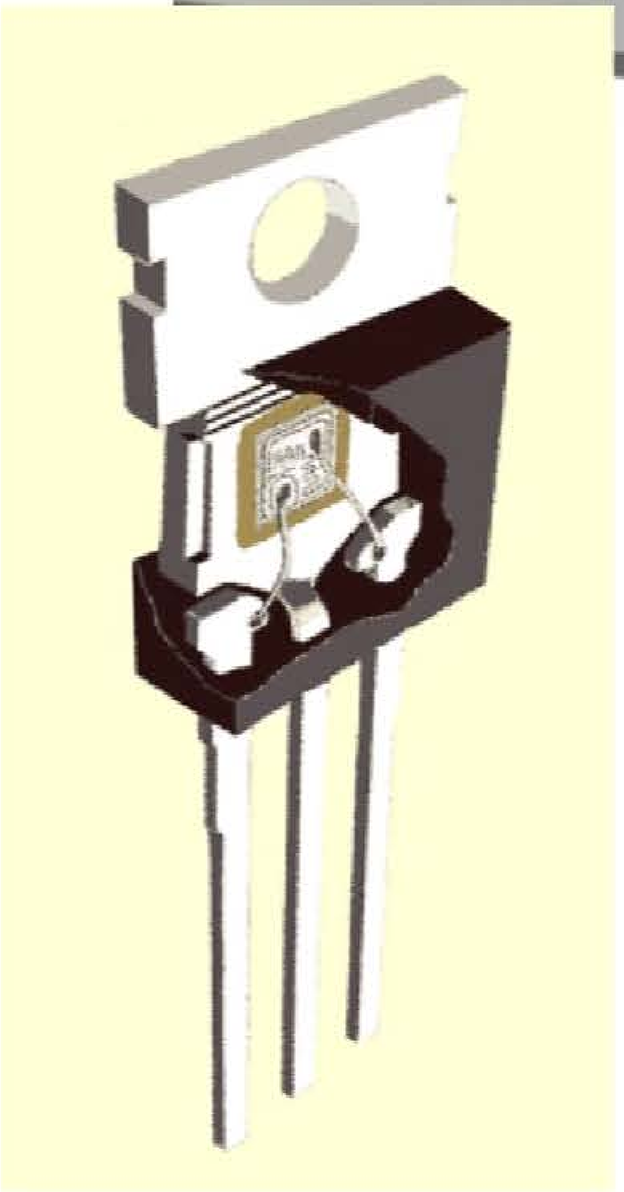

Basic Principles

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$$\Delta T_{JA} = (T_J - T_A) = P_D \, R_{\Theta JA}$$

$$T_J = T_A + (P_D \, R_{\Theta JA})$$

$$R_{\Theta JA} = 30^{o} C/W$$

$$P_D = 2.0W$$

$$T_A = 75^{o} C$$

$$T_J = 75^{o} C + (2.0W * 30^{o} C/W)$$

$$T_J = 75^{o} C + 60^{o} C$$

$$T_J = 135^{o} C$$

Transient Thermal Response

- Thermal time constant:

$$\tau_{\Theta} = R_{\Theta} C_{\Theta}$$

- Thermal capacitance:

$$C_{\Theta} = q\ t/\Delta T$$

where:

q = heat transfer per second (J/s)

t = time (s)

ΔT = the temperature increase (°C)

Transient Thermal Response

- Thermal capacitance as a function of mechanical properties:

  $$C_{\Theta} = cdV$$

  where:

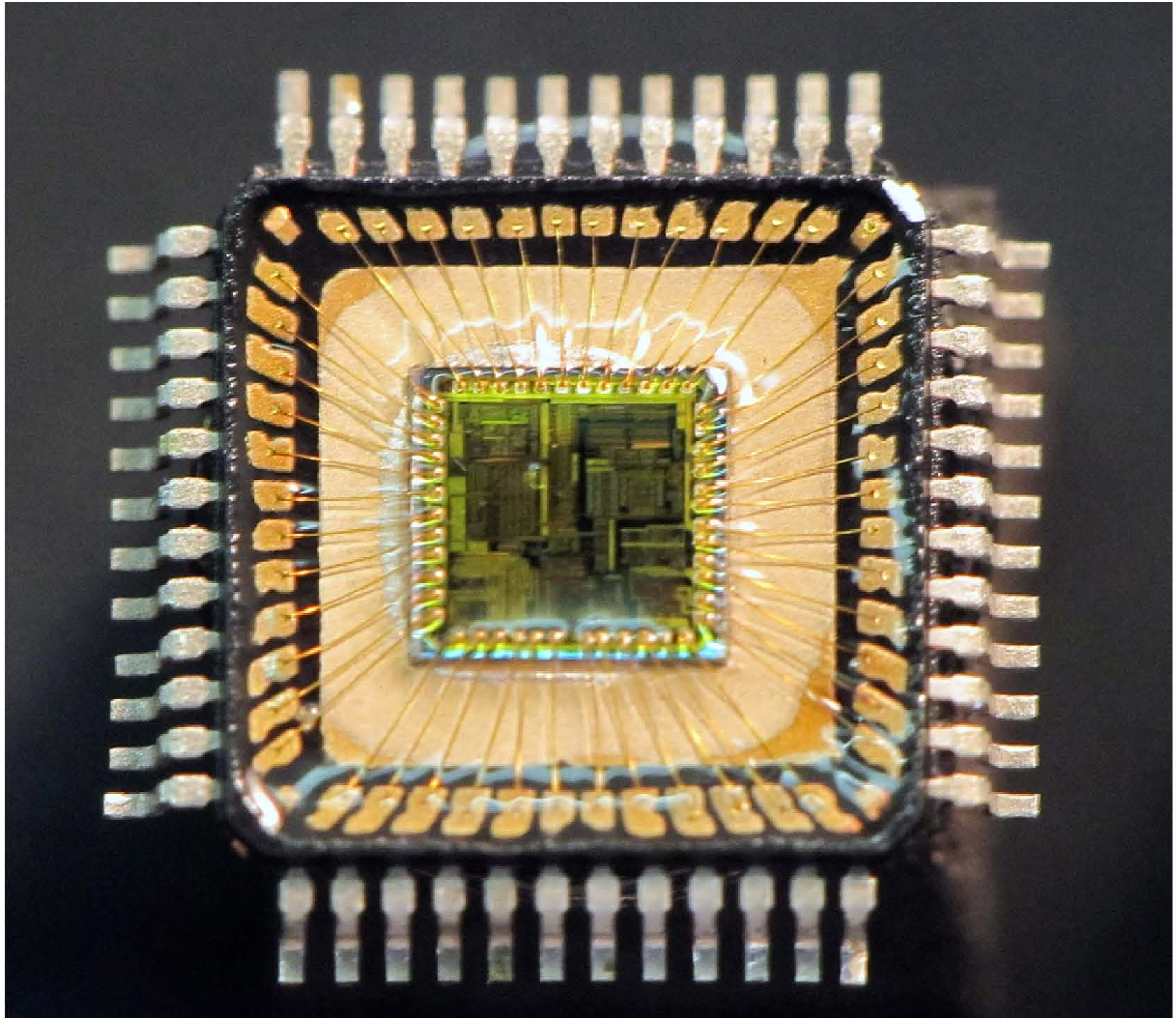
  c = specific heat ($J\ kg^{-1}\ K^{-1}$)

  d = density (kg/m^3)

  V = volume (m^3)

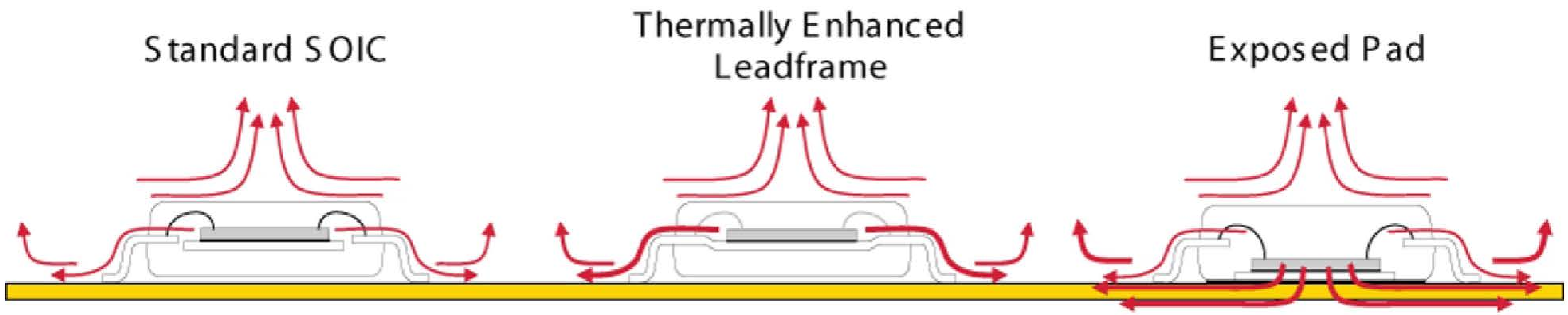
- The temperature of a thermal RC network responds to a step input of power according to:

  $$\Delta T_{AB} = R_{\Theta AB}\ P_D\ (1 - e^{(-t/\tau)})$$

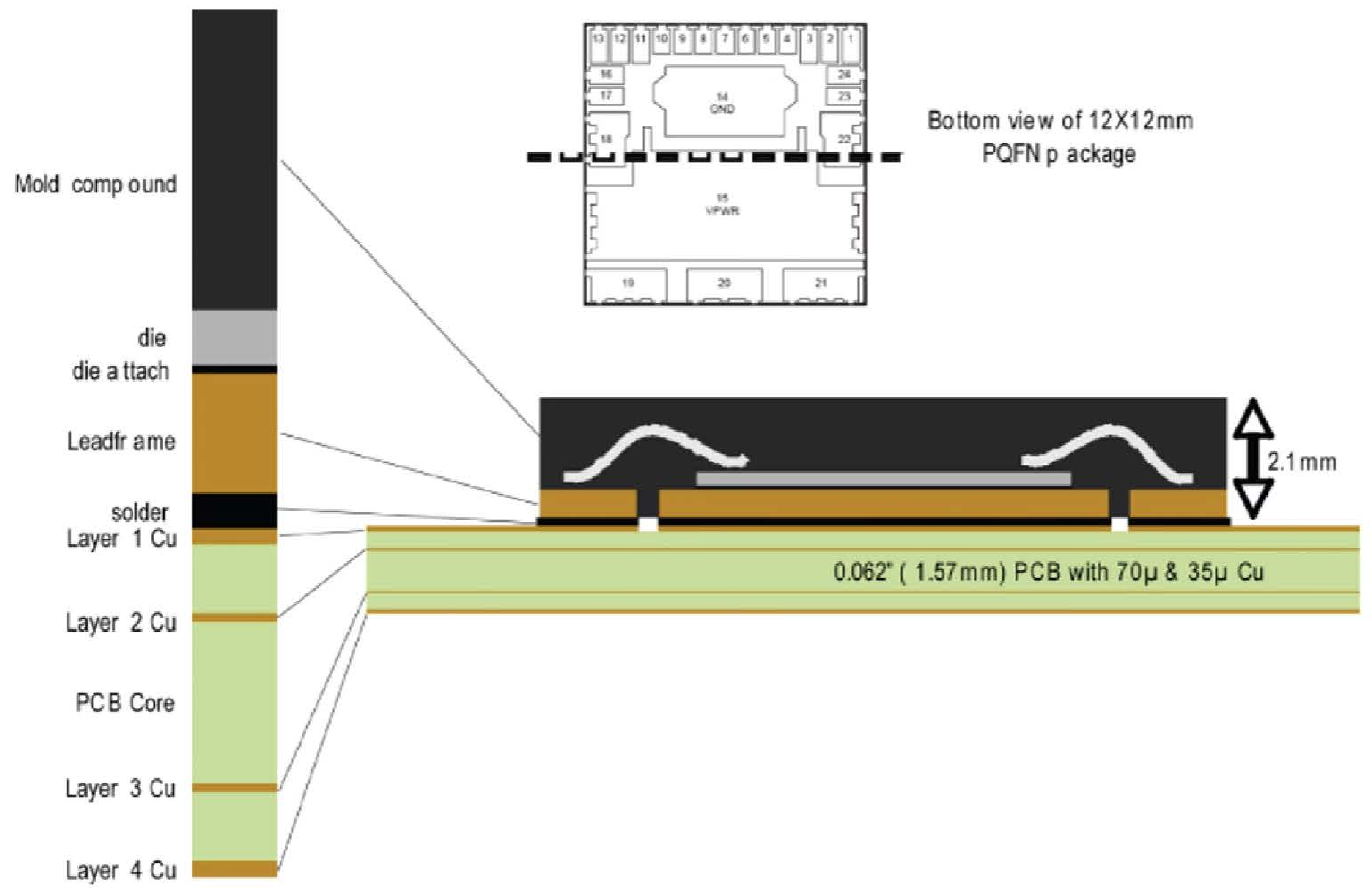


Thermal Resistance Ratings

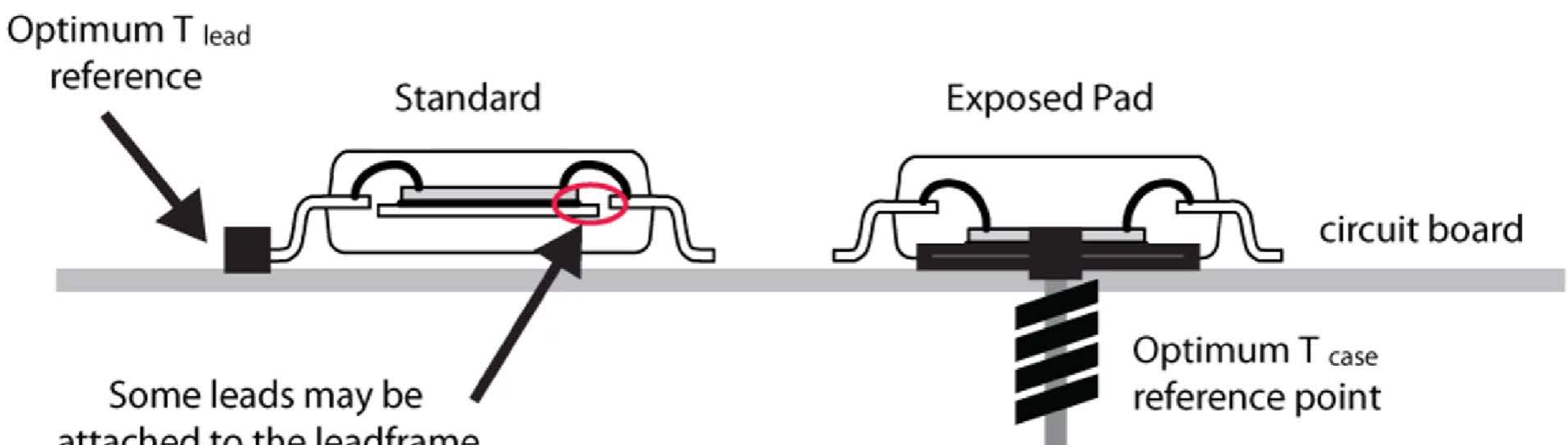
Cross Sections of standard and exposed pad SOICs



Cross section of a PCB and a PQFN package

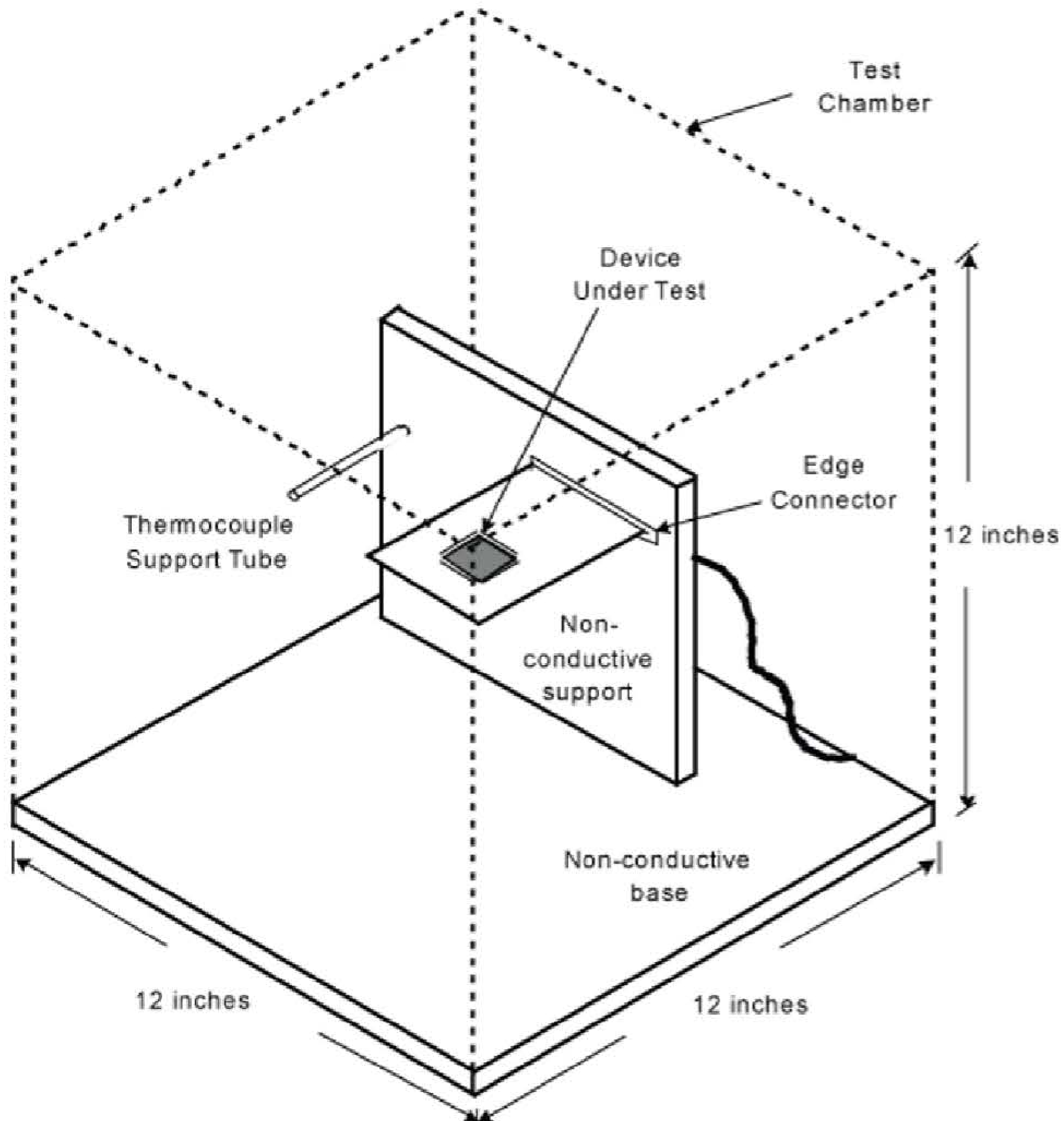


Cross Sections of standard and exposed pad SOICs

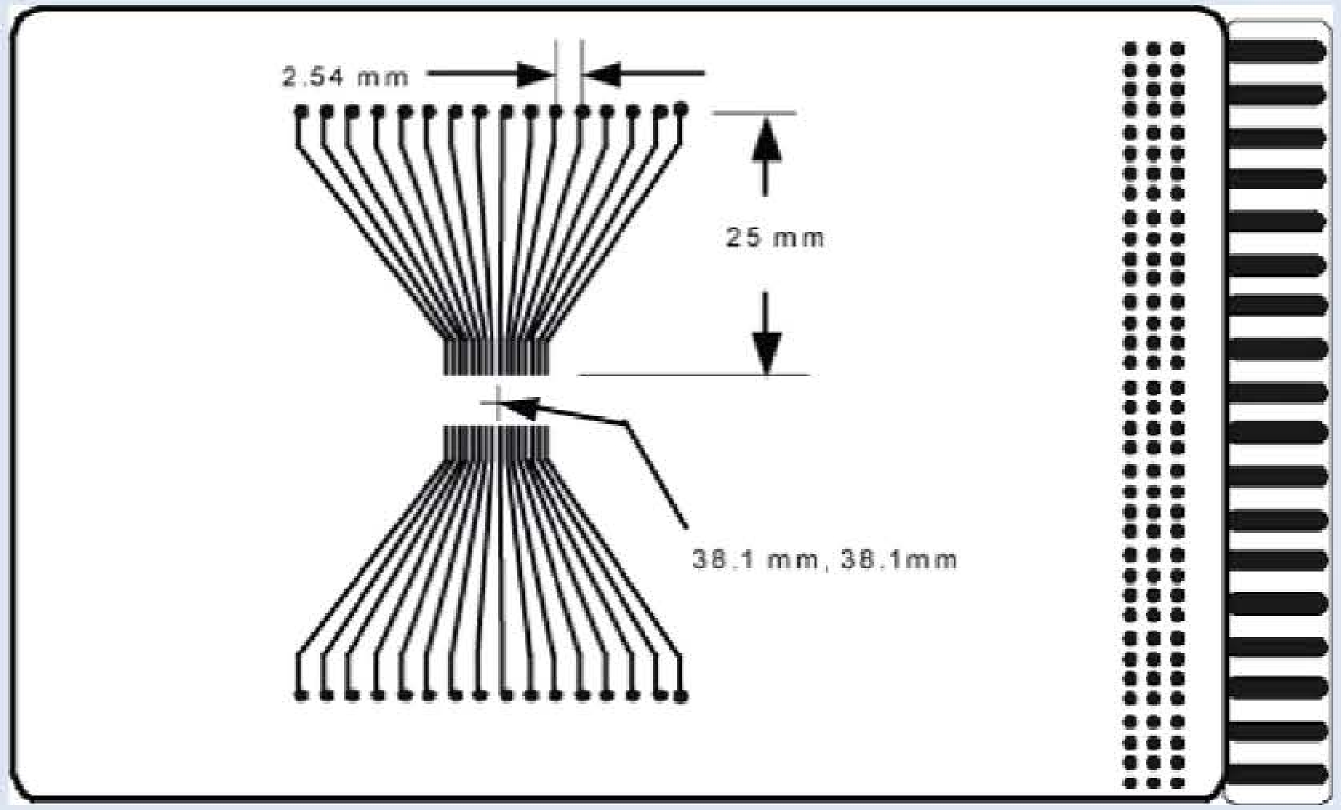


JEDEC Test Methods and Ratings

- Test chamber as recommended in JESD51-2



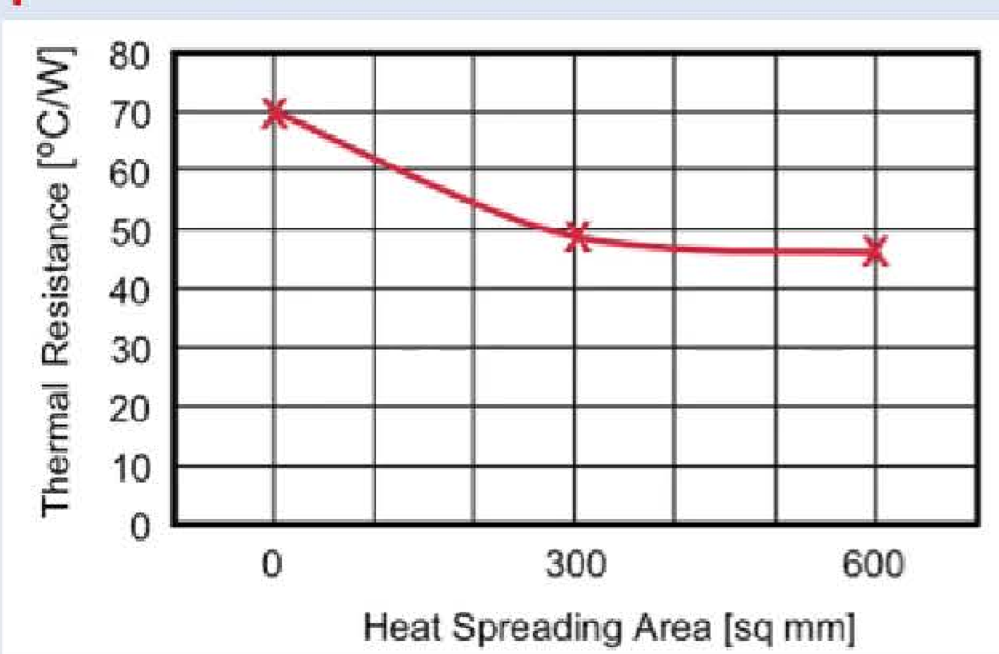
JEDEC specified PCB for J-A thermal characterization

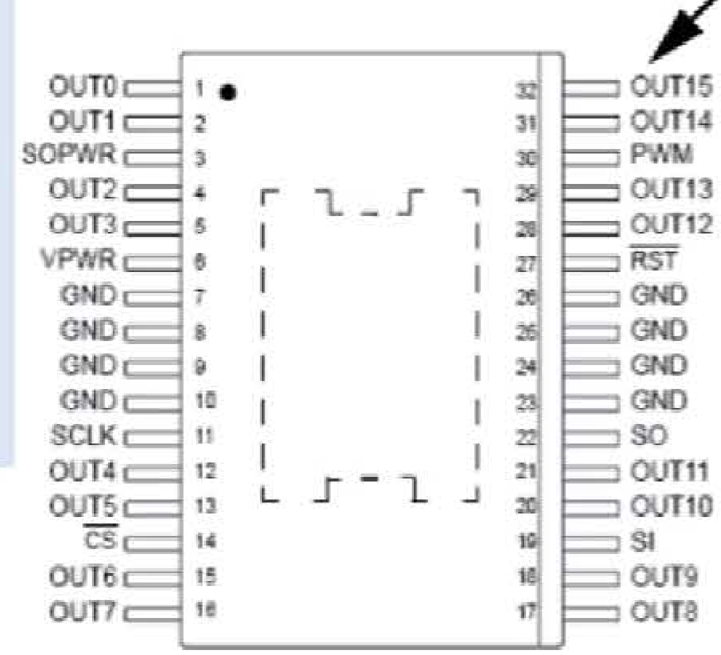


Thermally Enhanced Circuit Boards

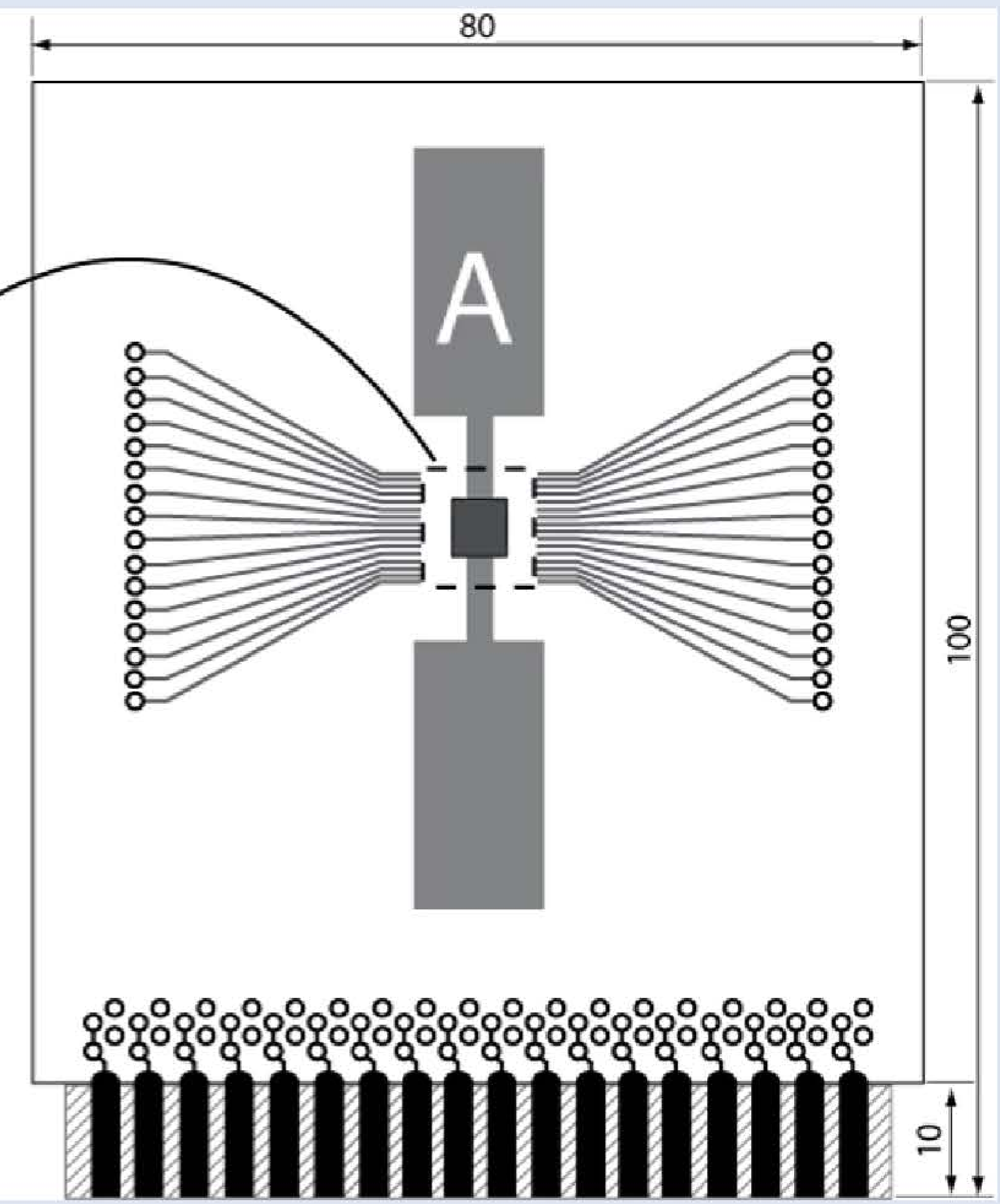
Thermal resistance test board with dedicated thermal pad

Junction to ambient thermal resistance decreases with dedicated thermal pad area



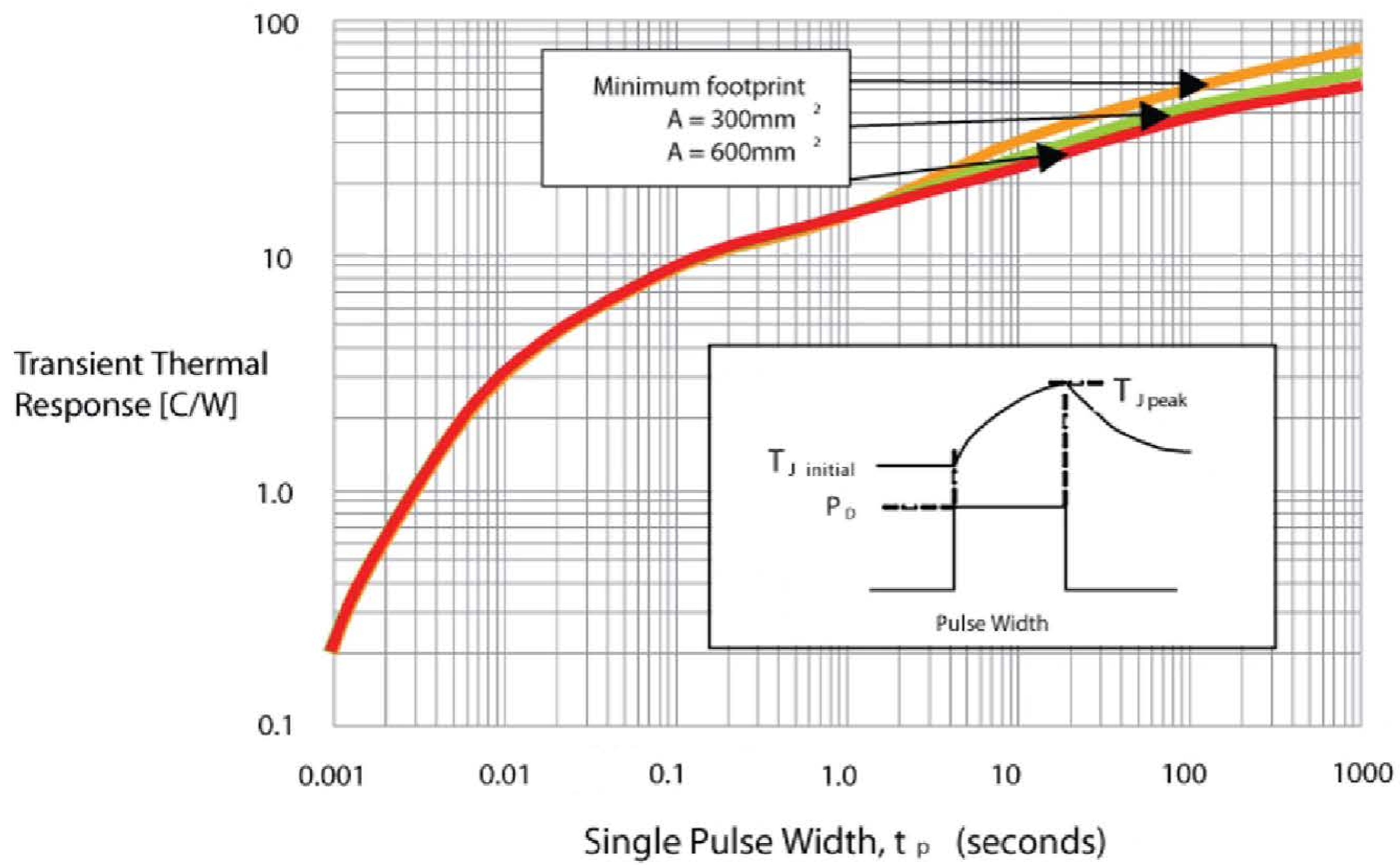


33996DWB Terminal Connections
32-Terminal SOICW EP
0.65 mm Pitch
11.0 mm x 7.5 mm Body
4.6 x 5.7 mm exposed pad



Transient Thermal Response Ratings

Transient thermal response curve



- Replacing $R_{\Theta JA}$ with $Z_{\Theta JA}$ in the basic thermal equation we get:

$$T_{Jpk} = T_A + (P_D \, Z_{\Theta JA})$$

- Assuming that a system experiences a power transient with the following characteristics and ambient temperature:

  Power pulse width, tp = 1ms

  $P_D = 50W$

  $T_A = 75°C$

- From Figure we can estimate $Z_{\Theta JA}$ for a 1ms pulse width:

  $Z_{\Theta JA}$ @ 1ms = 0.2°C/W

then,

$\Delta T_{JApk} = (T_{Jpk} - T_A) = PD \, Z_{\Theta JA}$

$T_{Jpk} = T_A + (P_D \, Z_{\Theta JA})$

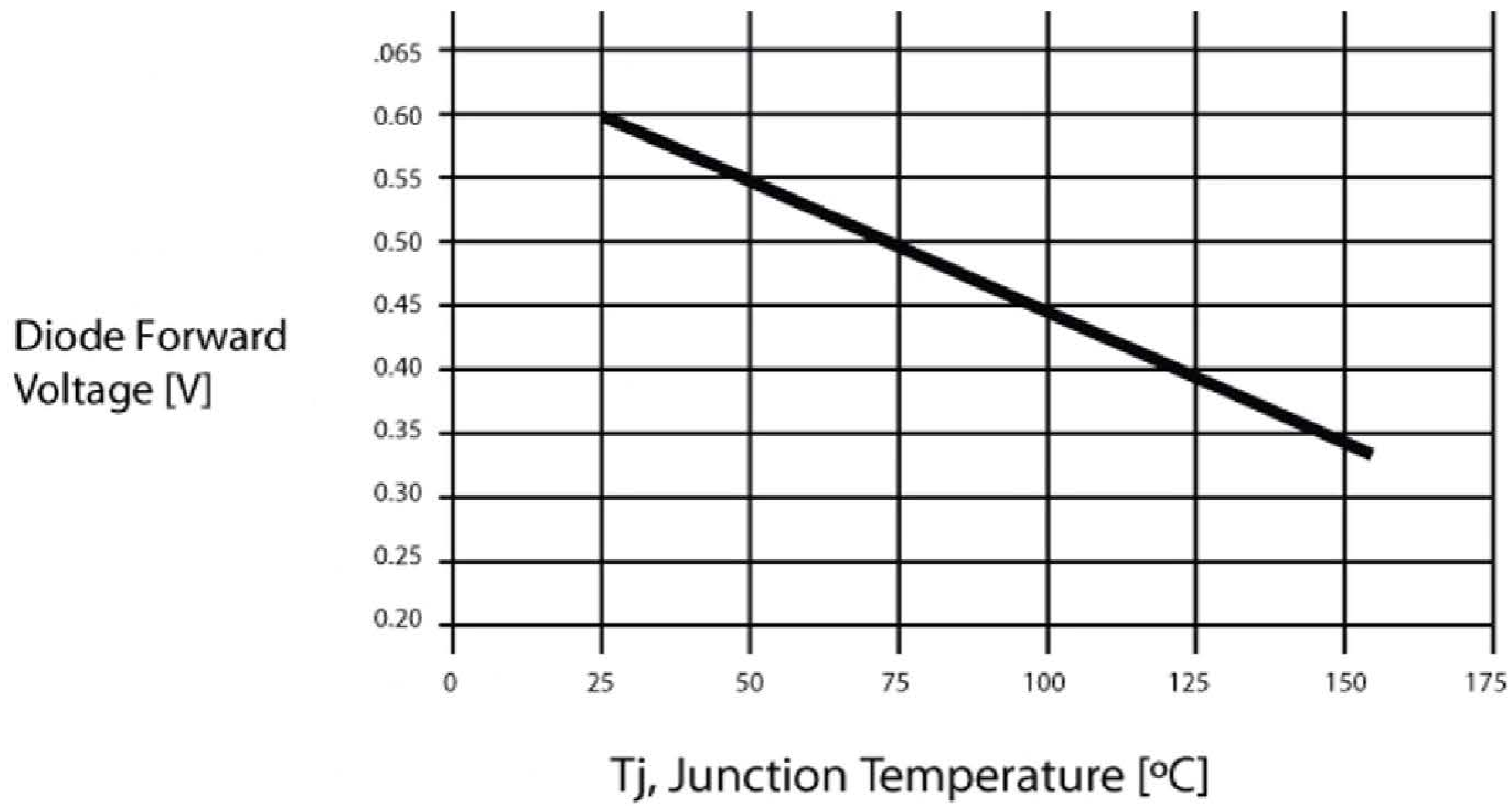
$T_{Jpk} = 75°C + (50W * 0.2°C/W)$

$T_{Jpk} = 75°C + 10°C$

$T_{Jpk} = 85°C$

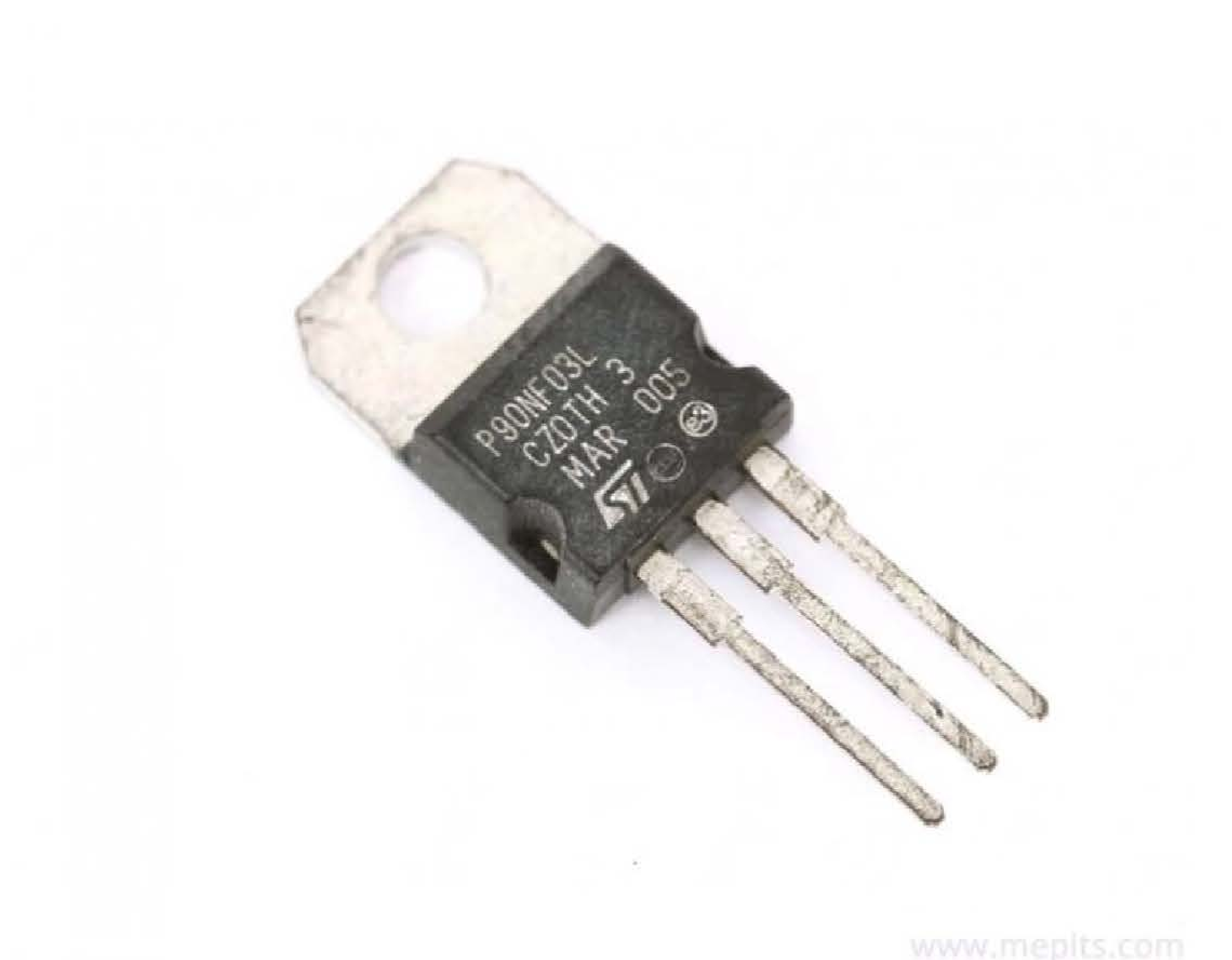
Ramifications of High Operating Temperature

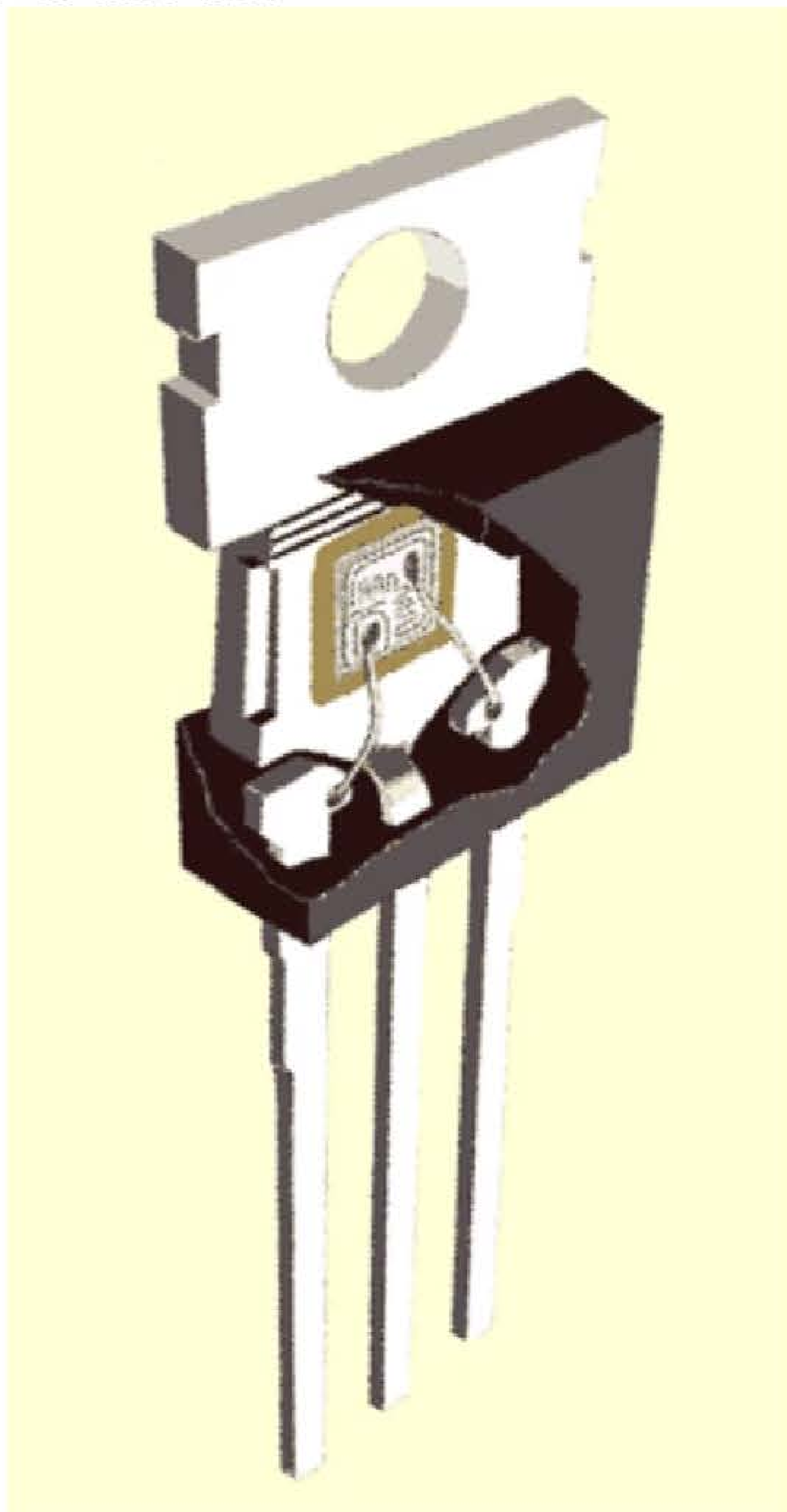
Diode forward voltage vs. operating temperature



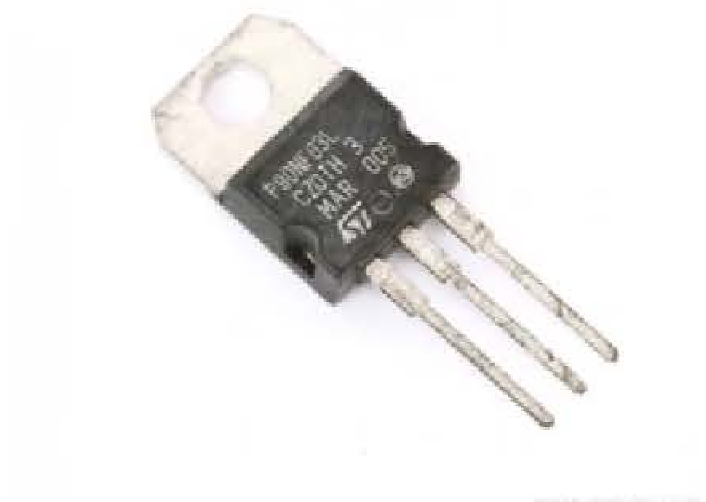
Thermal Circuits

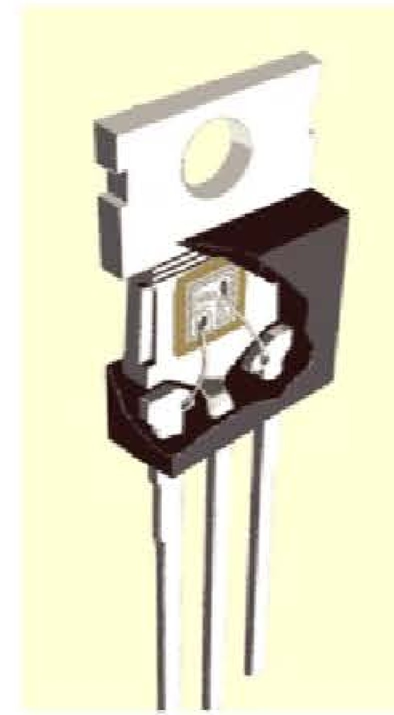
Electrical and Thermal Domain Circuits

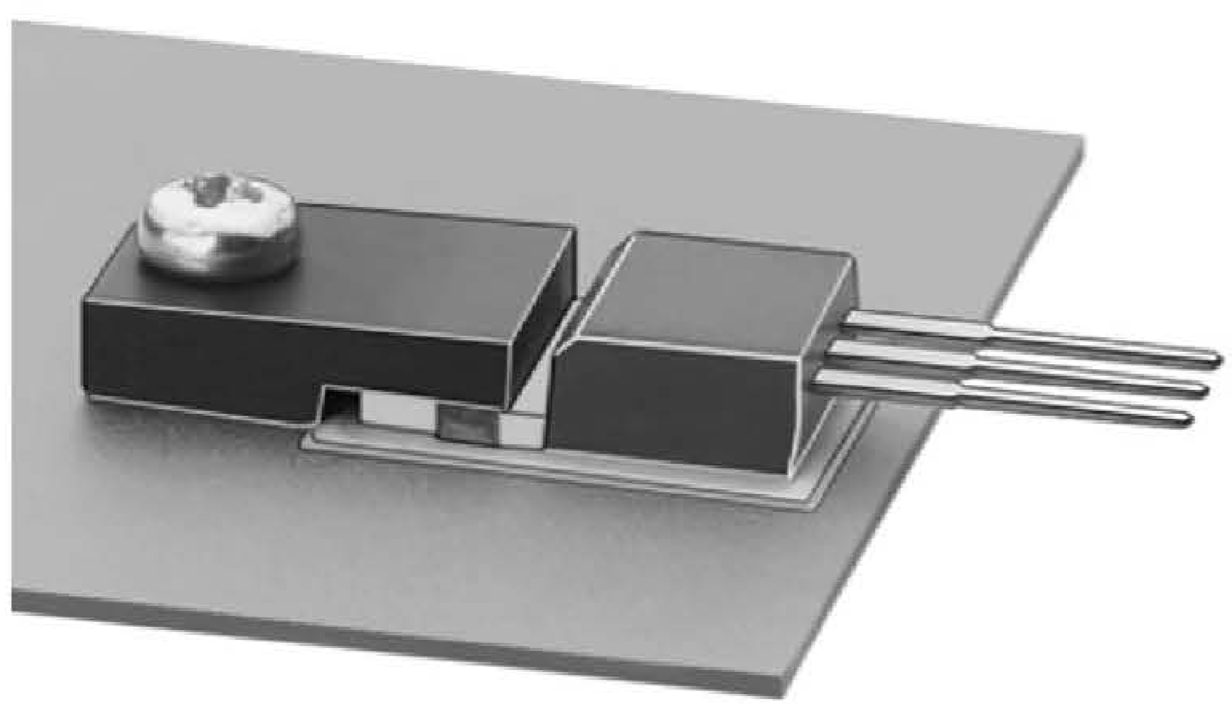


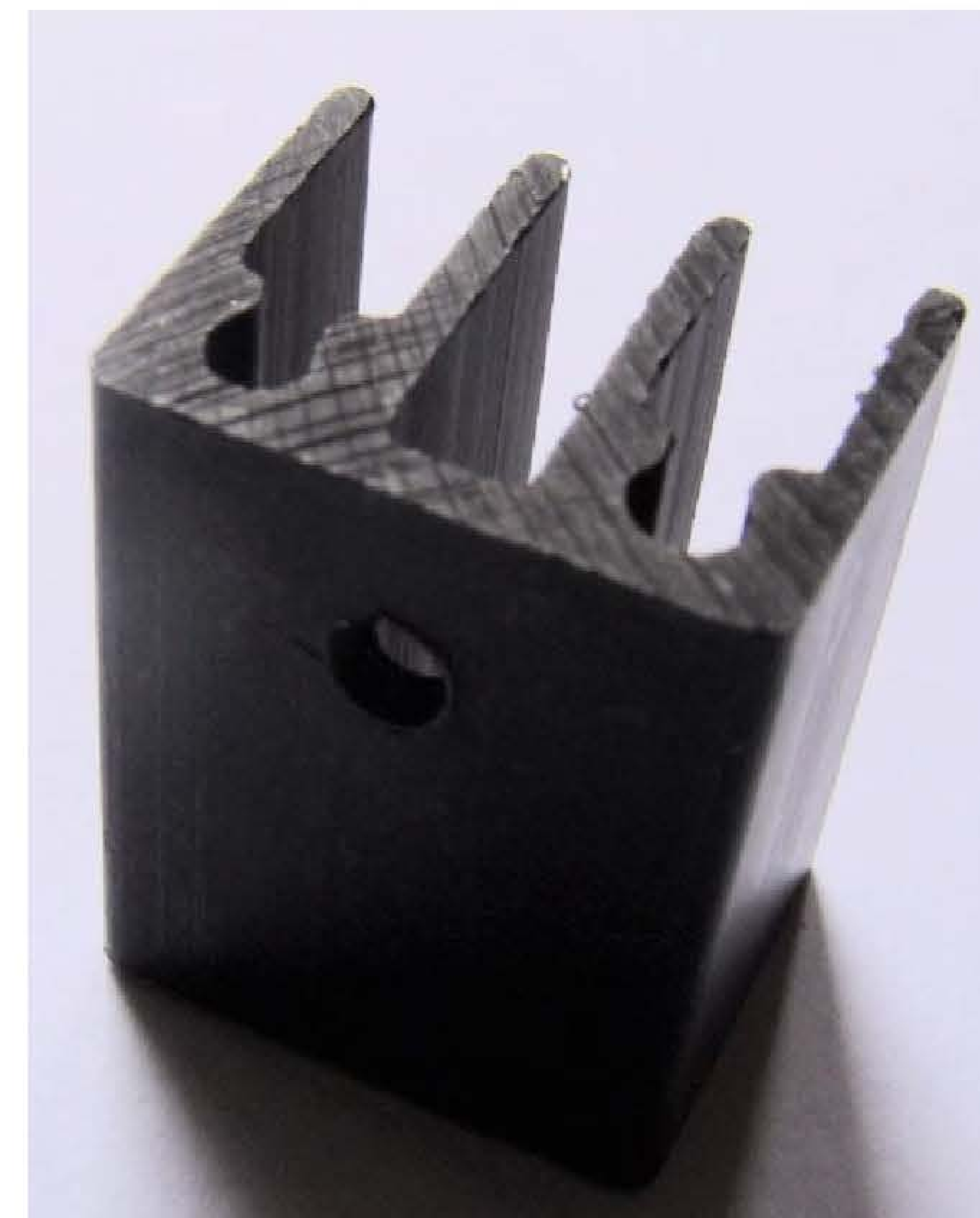


Thermal Circuits



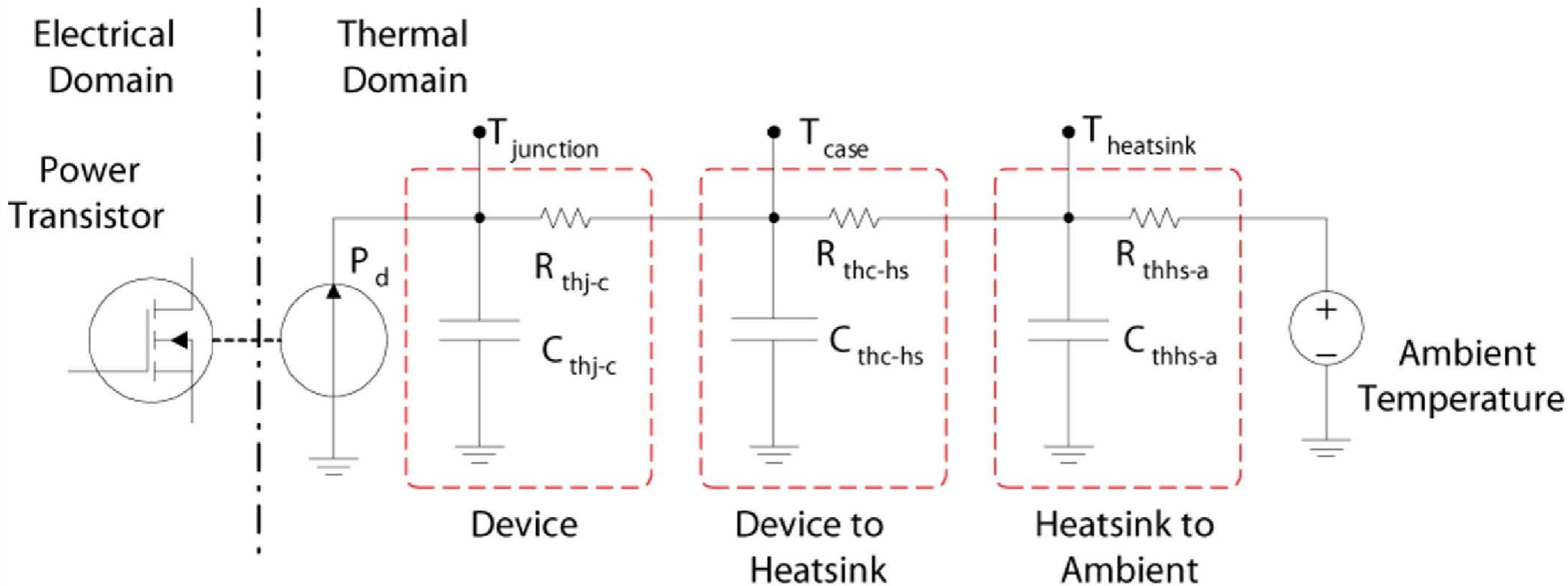






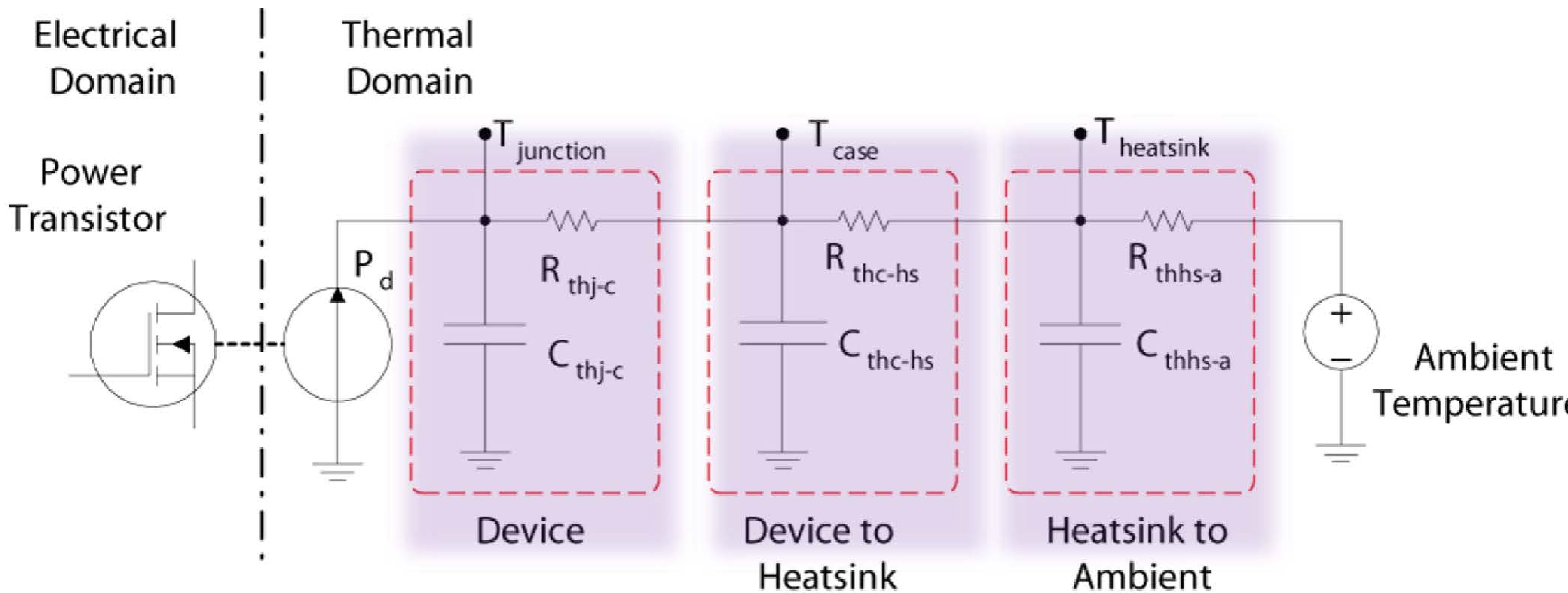
Thermal Circuits

Electrical and Thermal Domain Circuits



REMARKS

- The equivalent electrical circuit serves as an analogue model for the thermal system
- Repetitive structures are encountered **subcircuits** called **subnetworks** or **cells**

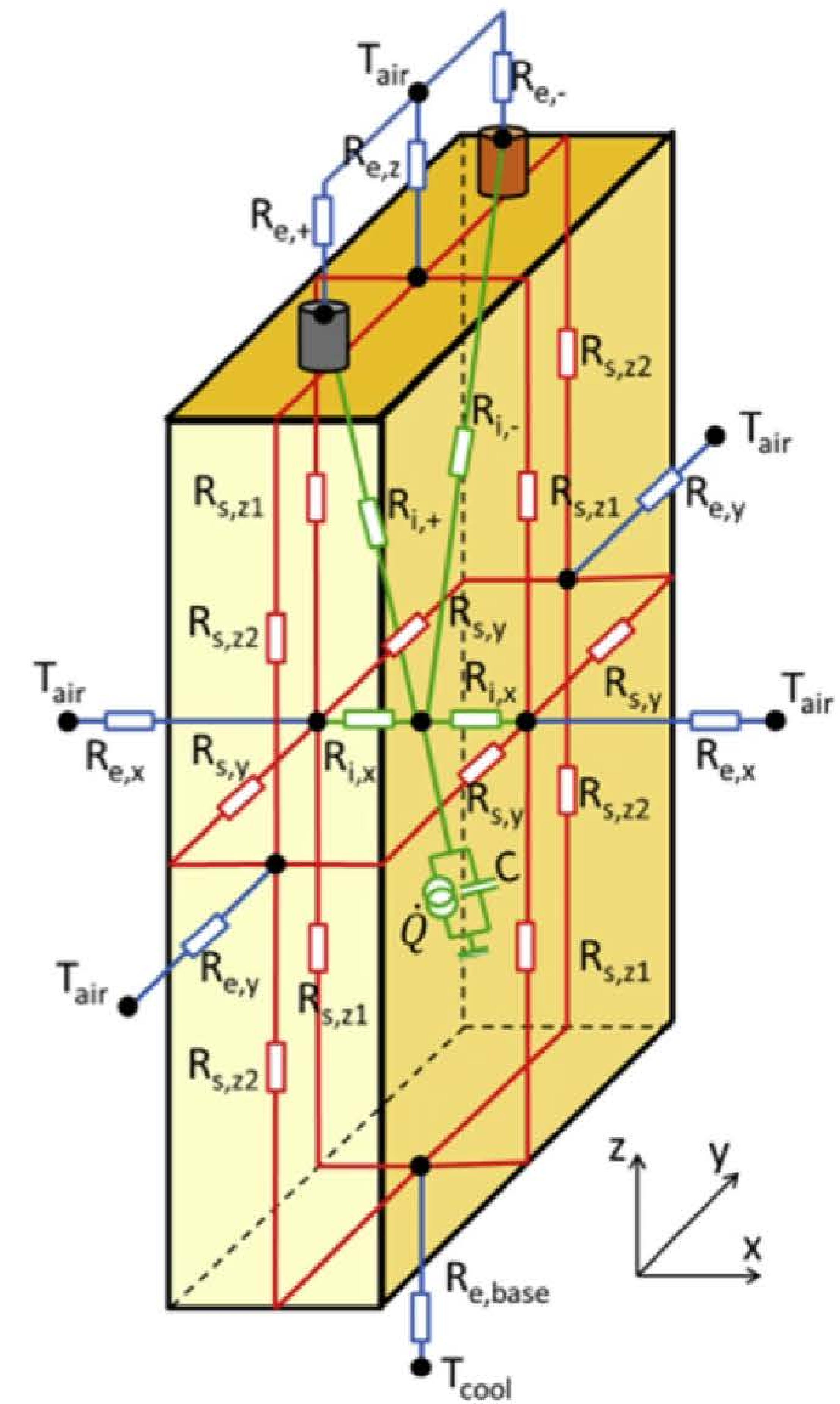


REMARKS

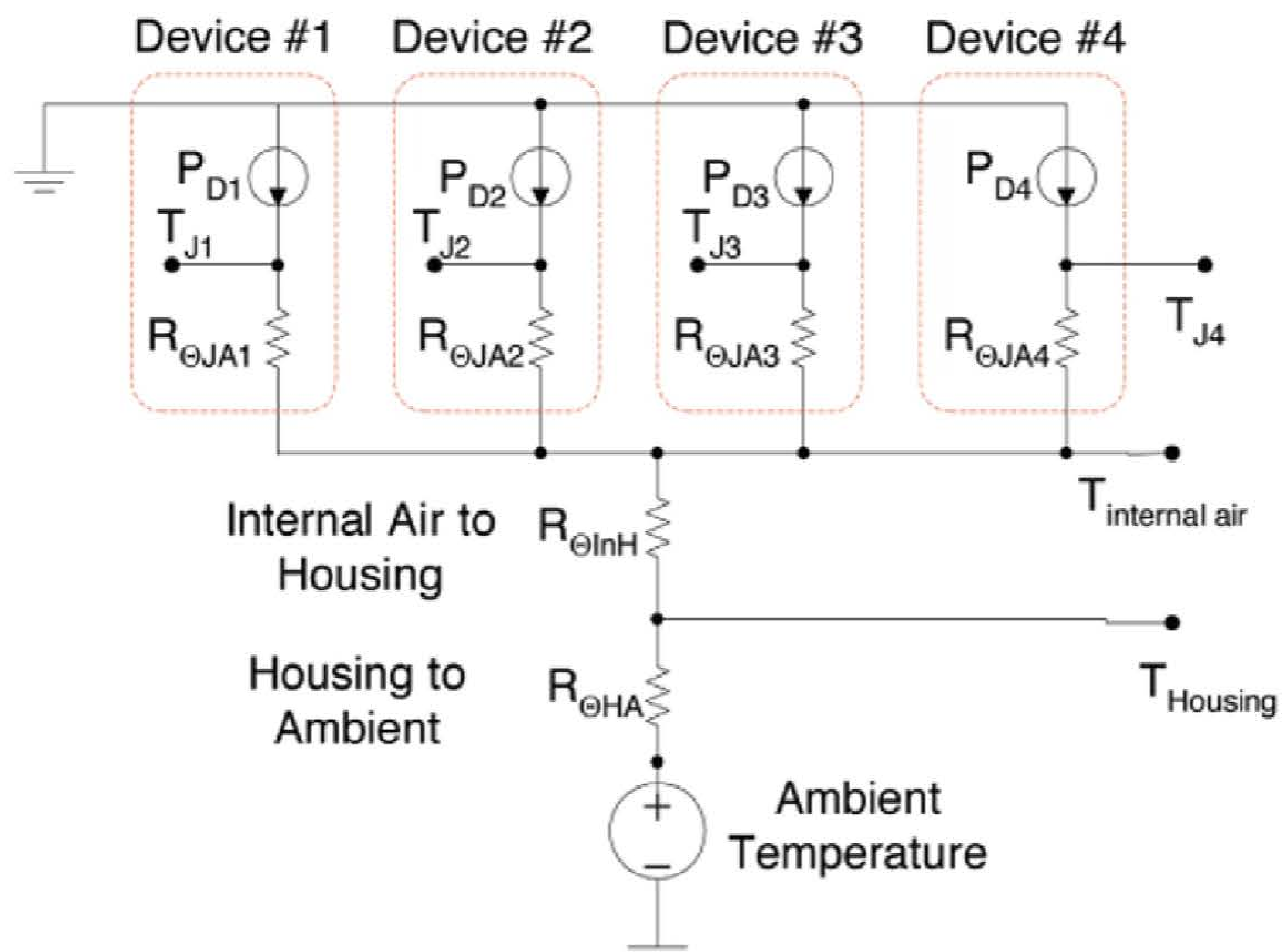
- These are suggesting to follow the nodal approach: partitioning the thermal system and creating an equivalent circuit made out of “standardised” cells:
  – The interaction between neighbouring elements (heat flows) will be reflected by interconnection between neighbouring cells
  - The heat storage associated with one element-> thermal capacitance
  - The heat transfer through the interfaces of an element -> thermal resistances

<u>Example</u>: An electric car battery is a system made of multiple cells tightly packaged together that needs to be cooled.

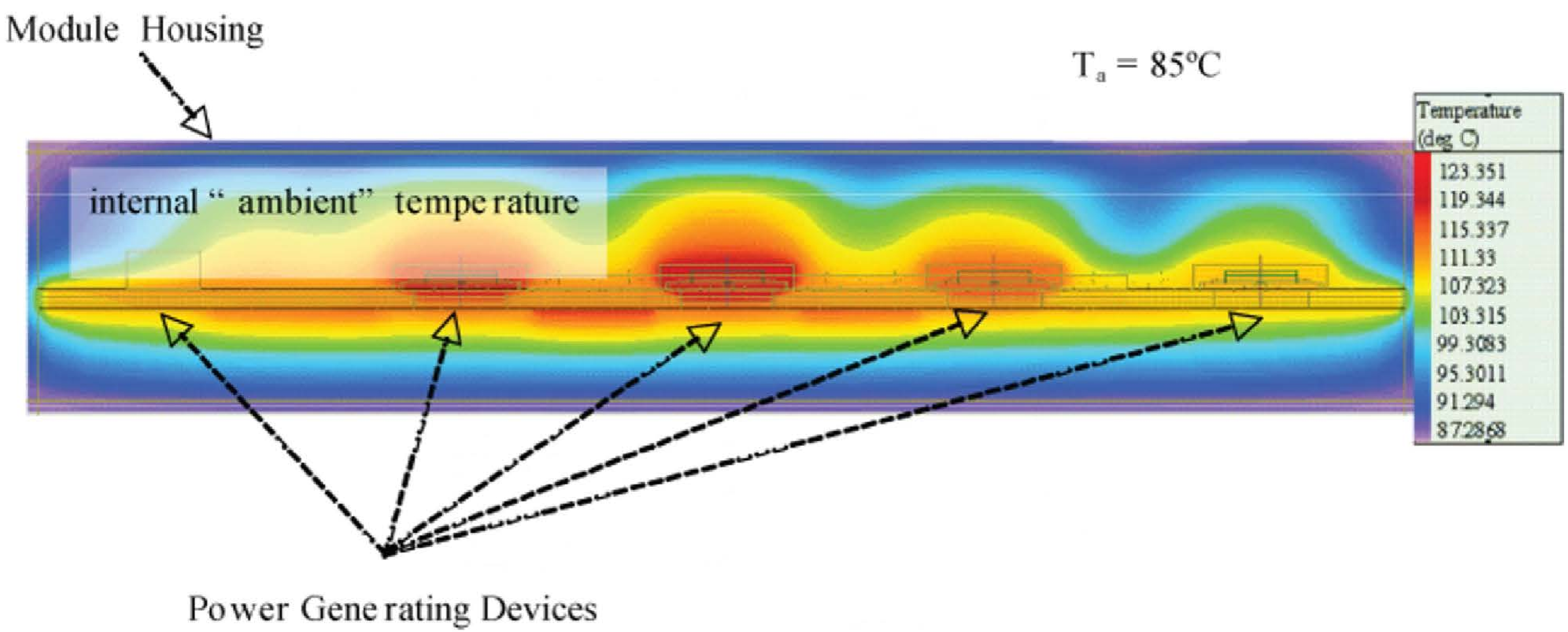
A complex thermal network model for a cell (the cells are cooled from the bottom)



Popular thermal model for a module with multiple power devices



Thermal gradients within a module



Matrix form and coupling coefficients

- The junction temperature of each device can be estimated by solving simultaneous equations:

$$\begin{vmatrix} T_{J1} \\ T_{J2} \\ T_{J3} \\ T_{J4} \end{vmatrix} = \begin{vmatrix} c_{11} & c_{21} & c_{31} & c_{41} \\ c_{12} & c_{22} & c_{32} & c_{42} \\ c_{13} & c_{23} & c_{33} & c_{43} \\ c_{14} & c_{24} & c_{34} & c_{44} \end{vmatrix} \times \begin{vmatrix} P_1 R_{\Theta JA1} \\ P_2 R_{\Theta JA2} \\ P_3 R_{\Theta JA3} \\ P_4 R_{\Theta A4} \end{vmatrix} + \begin{vmatrix} T_A \\ T_A \\ T_A \\ T_A \end{vmatrix}$$

where $\boldsymbol{c}_{mn}$ is the coupling coefficient between Device m and Device n, $\boldsymbol{R}_{\Theta JCm}$ is the junction to ambient thermal resistance of Device m, and $\mathbf{P}_m$ is the power dissipated by Device m.

Q1: **How can be such an approach extended to more complex architectures?**

A1: Main functional blocks of a complex IC can be each modelled as a heat source. The equivalent electrical circuit model of a block must consider all important heat flows. The overall equivalent model schematic will be *an interconnection of the cells associated with each block*. This schematic *can be easily simulated in a SPICE* framework. -> Nodal analysis approach

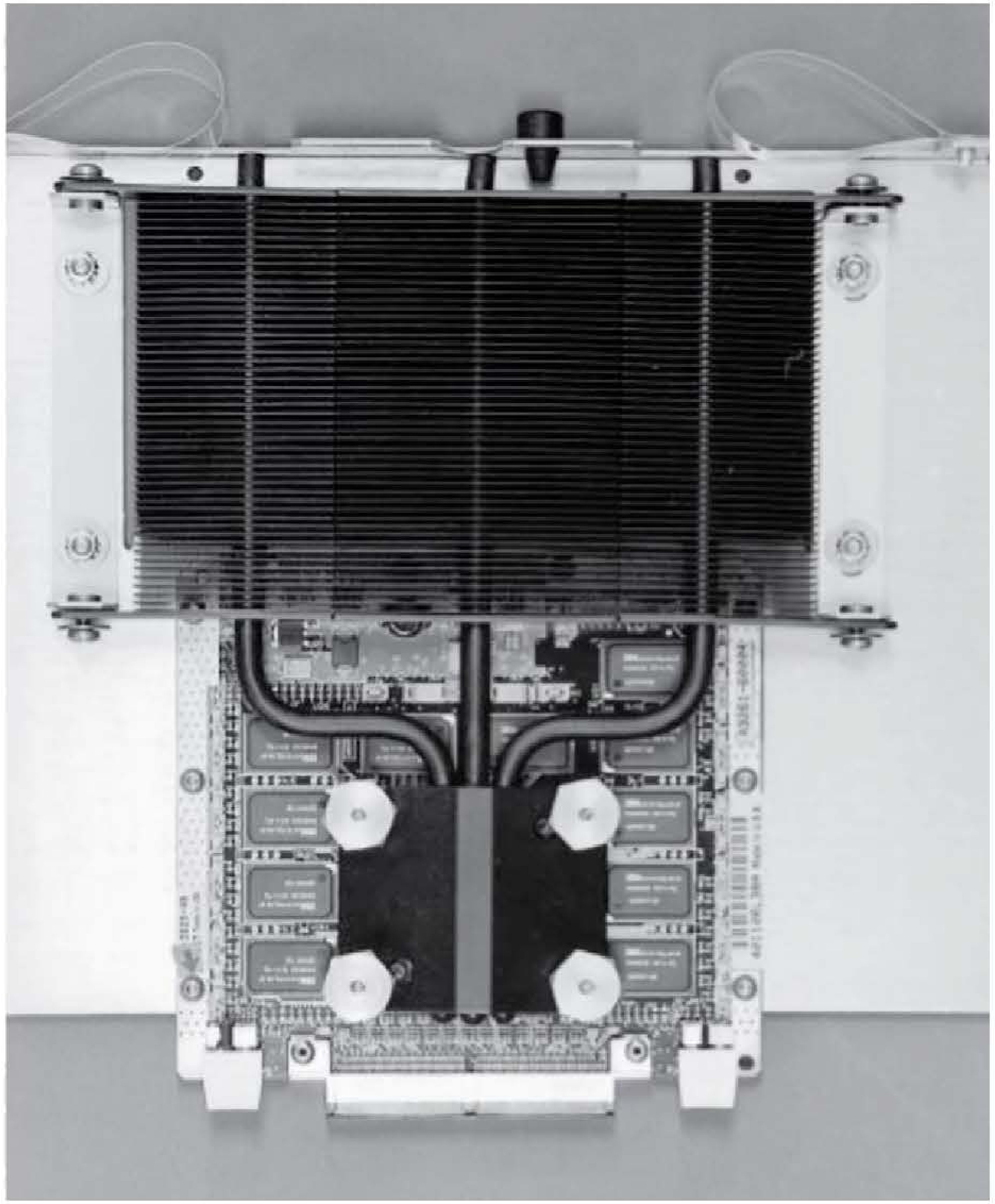
Q2: Sin**ce an accurate modelling of the thermal gradients can be achieved only starting with distributed models (based on partial derivatives) how good is the approximation based on concentrated parameters models?**

A2: The following example indicates good matching. The identification of accurate values for the equivalent R and C parameters is vital for precision.

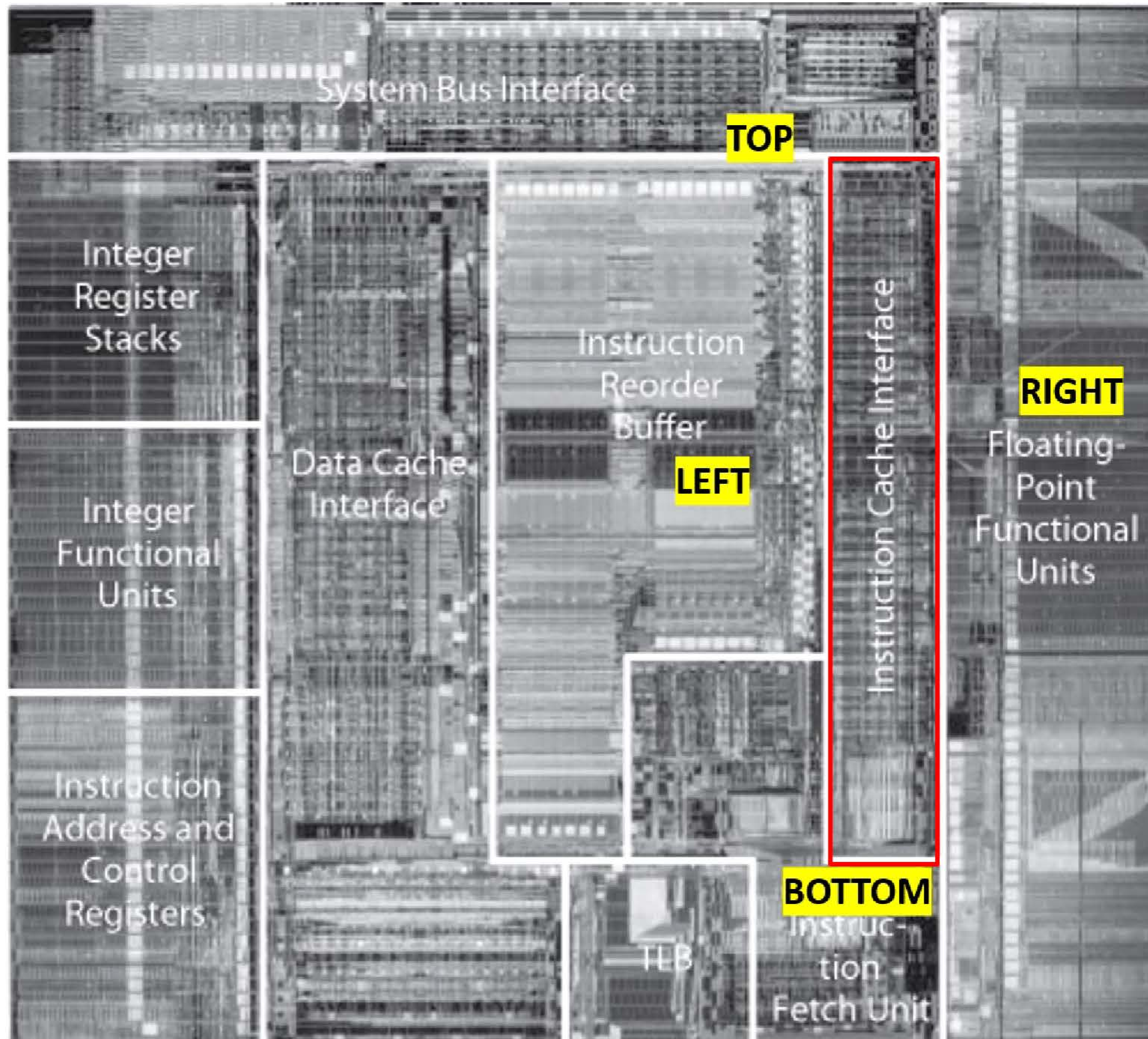
EXAMPLE

Comparison of Finite-Difference and SPICE Tools for Thermal Modeling of the Effects of Nonuniform Power Generation in **High-Power CPUs**

Fan/heat-sink forced air cooling solution

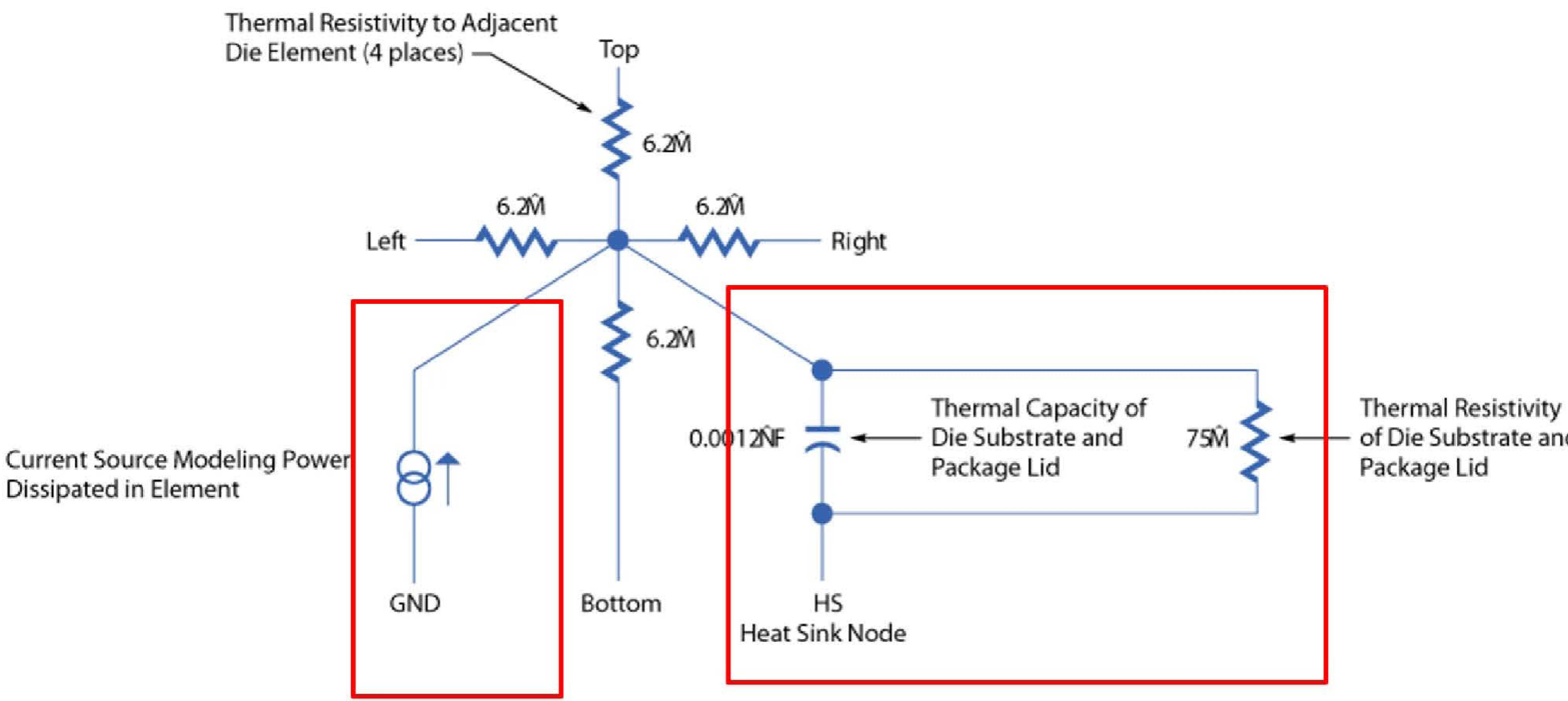


Heat pipe attached to the printed circuit board

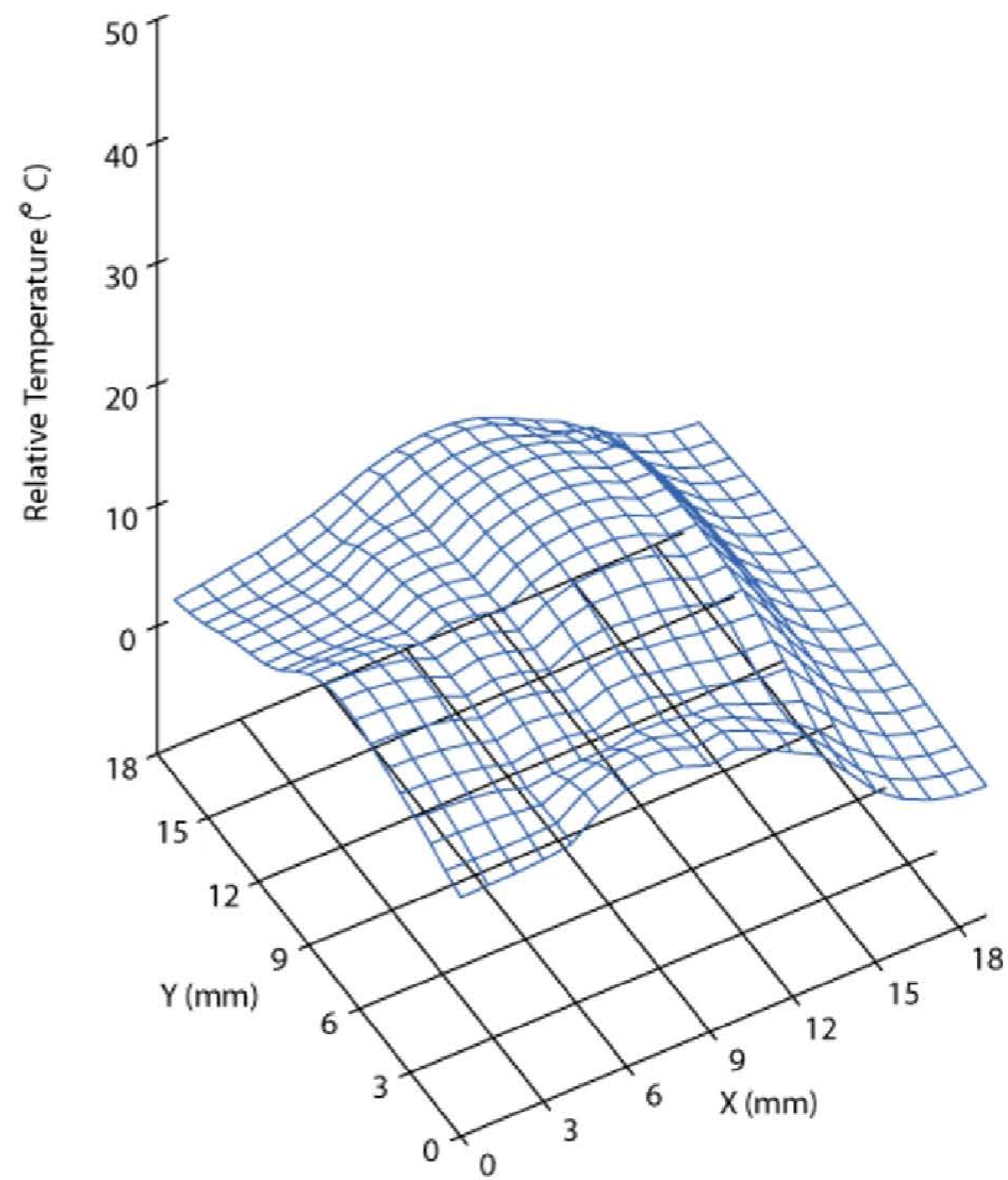


HP PA 8000 processor

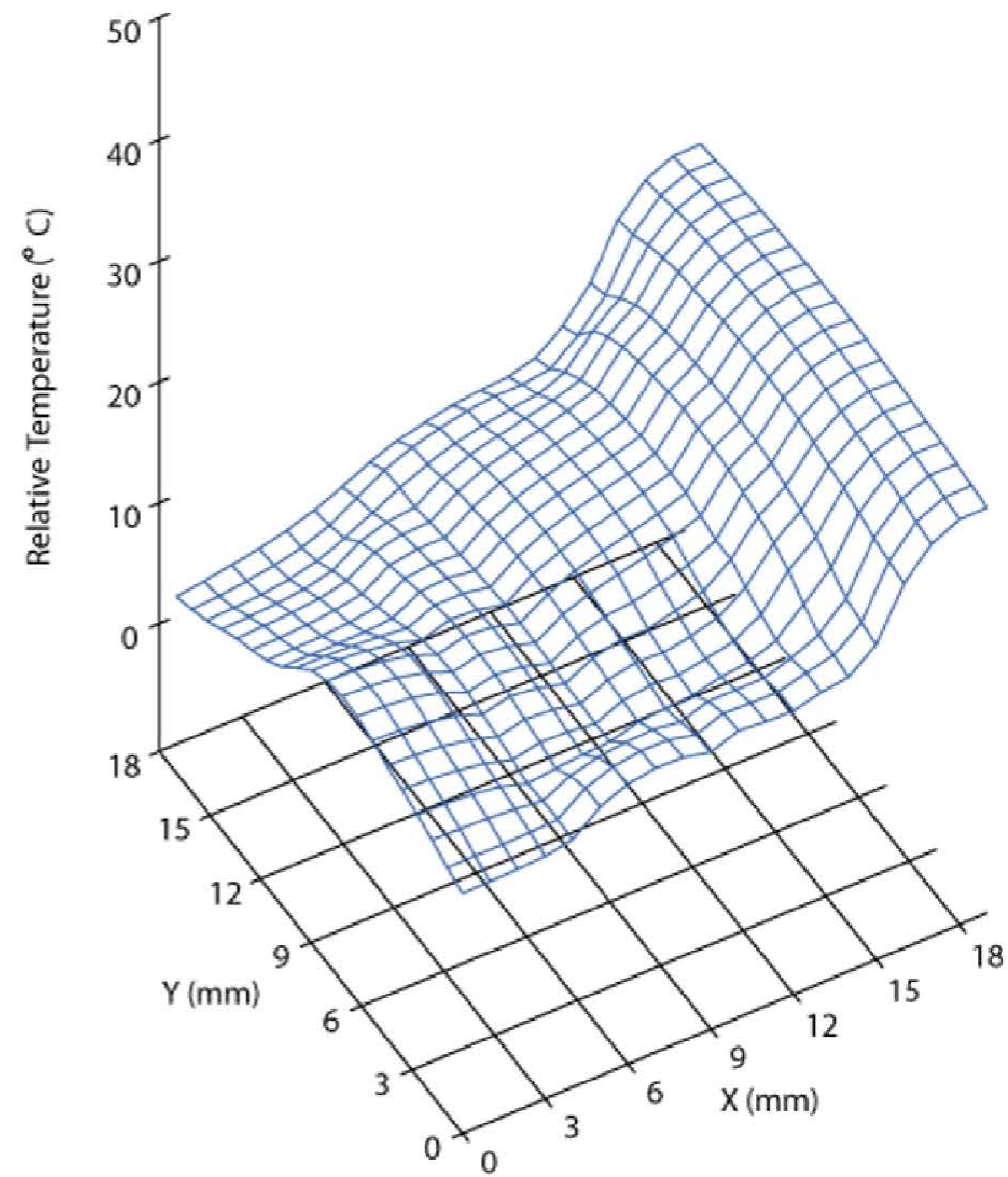
SPICE cell → **SPICE (Simulation Program with Integrated Circuit Emphasis)**[I]



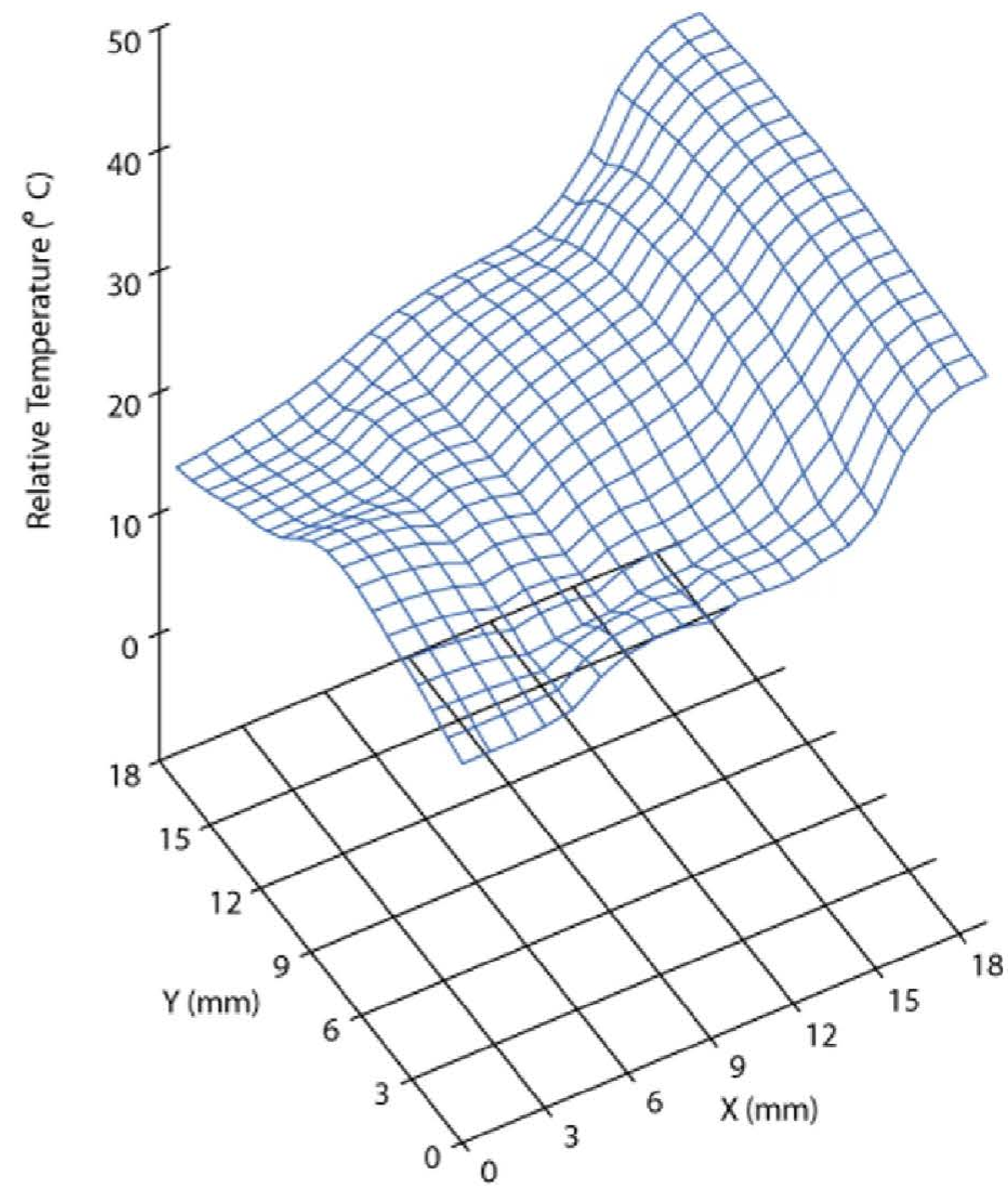
Temperature gradient with floating-point unit power off (from SPICE simulation)



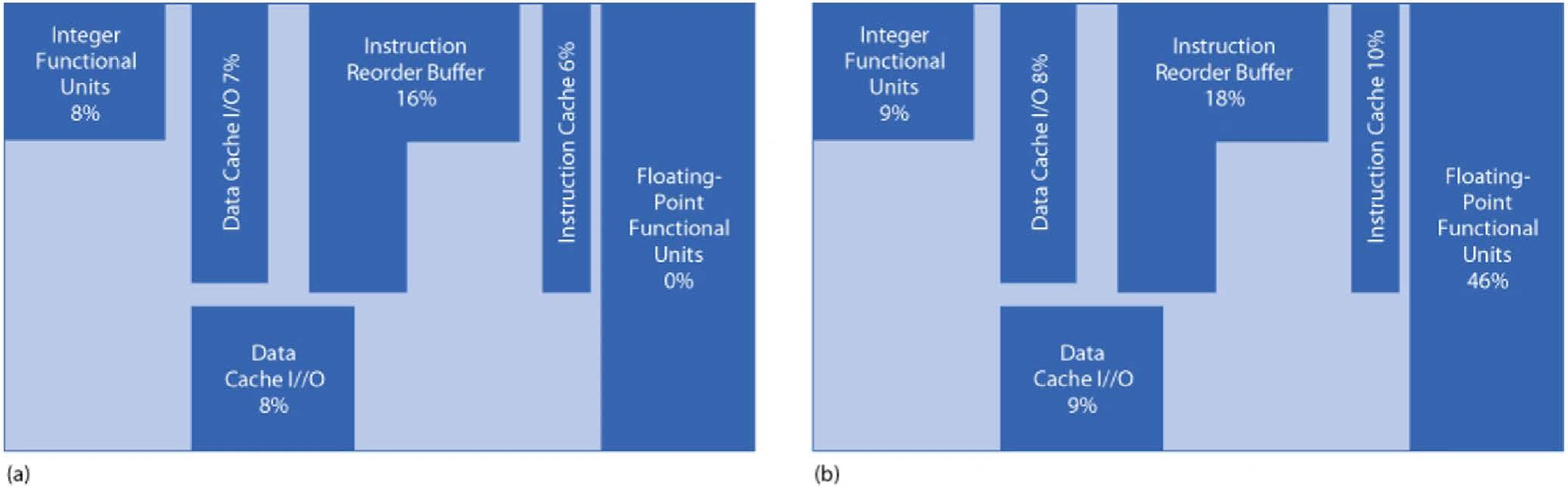
Temperature gradient profile established after floating point unit power-on (from SPICE simulation)



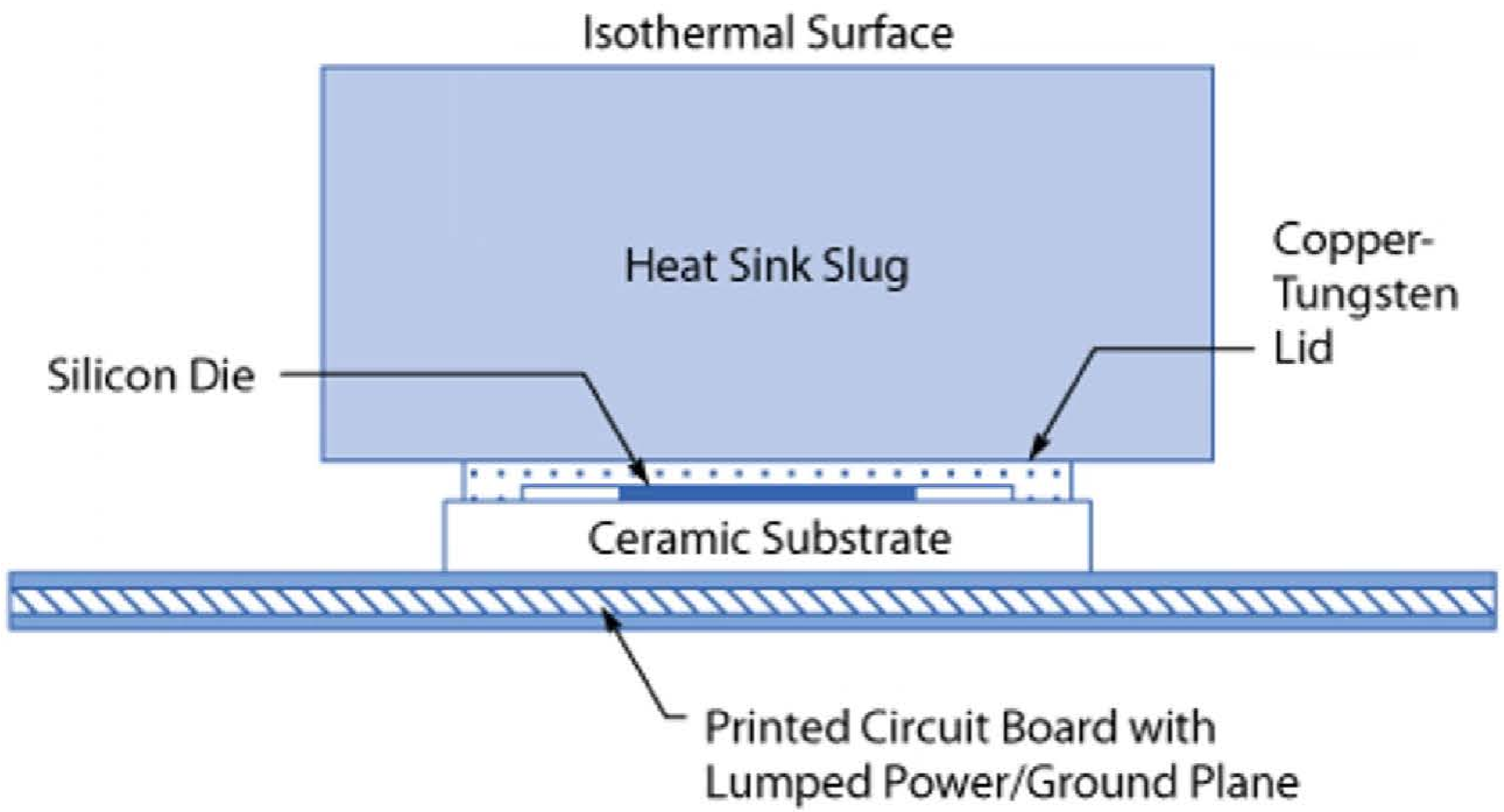
Temperature gradient final state after heat sink heating (from SPICE simulation).



Power distribution in the (a) low-power and (b) high-power states

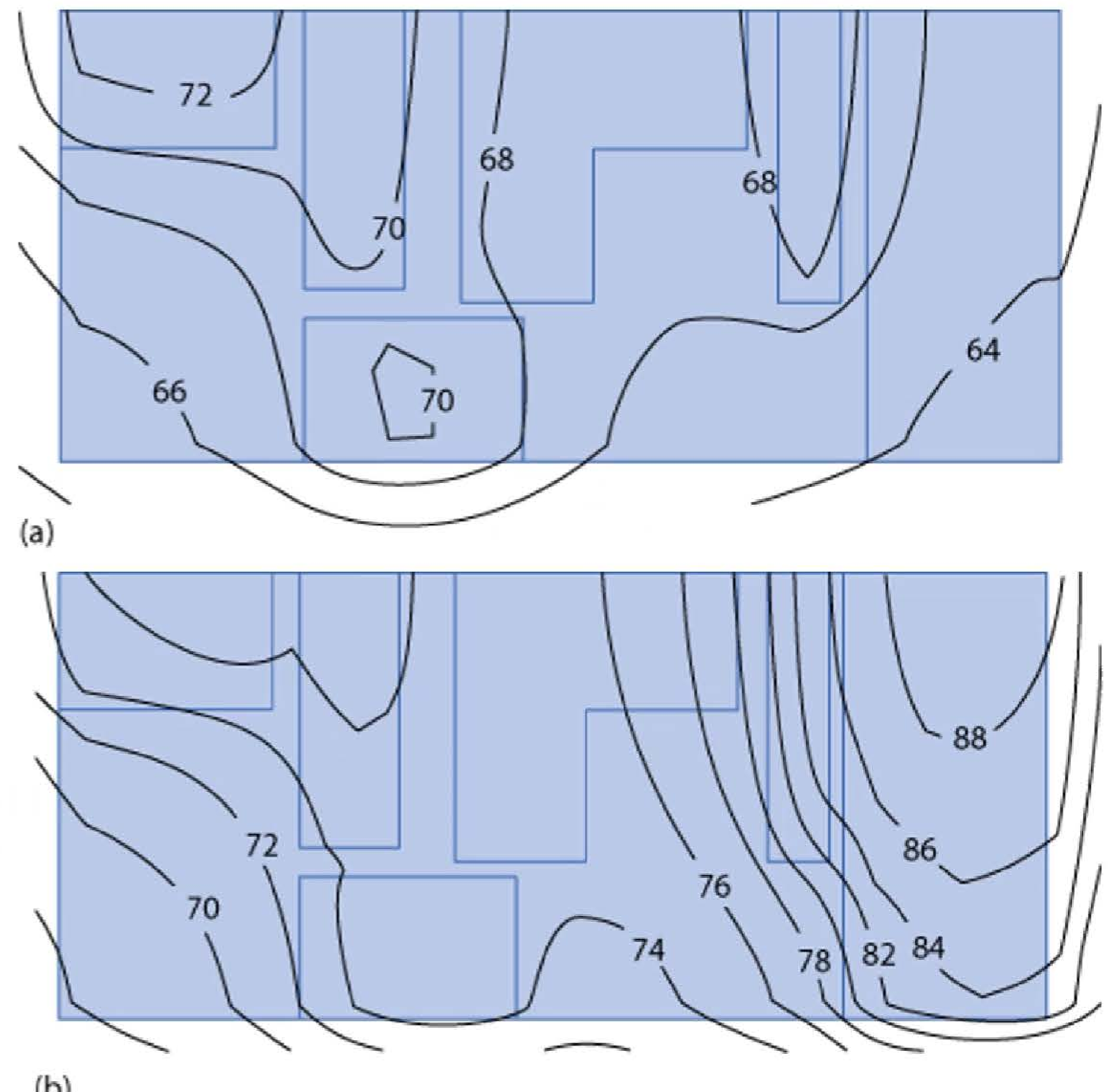


Model construction for finite-difference simulation



Temperature (°C) gradients during

(a) floating-point unit low-power mode and (b) full-power operation (from finite-difference simulation).



Comparison of SPICE and Finite-Difference Predictions

Simulation Tool	Gradient across Die with Floating-Point Unit On	Gradient across Die with Floating-Point Unit Off	Difference in Maximum Chip Temperature between On and Off States
Finite-Difference	20°C	10°C	16°C
SPICE	21°C	9°C	18°C

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1. F. Kreith, R.M. Manglik, M.S. Bohn. *Principles of Heat transfer*, Seventh Edition, 2011, Cengage Learning. ISBN: 978-0-495-66770-4
2. J.L. Deeney, C.M. Ramsey. “Comparison of Finite-Difference and SPICE Tools for Thermal Modeling of the Effects of Nonuniform Power Generation in High-Power CPUs”. The Hewlett-Packard Journal, vol. 50, no. 1, article 6, 1998.

   https://www.hpl.hp.com/hpjournal/98nov/nov98a6.pdf