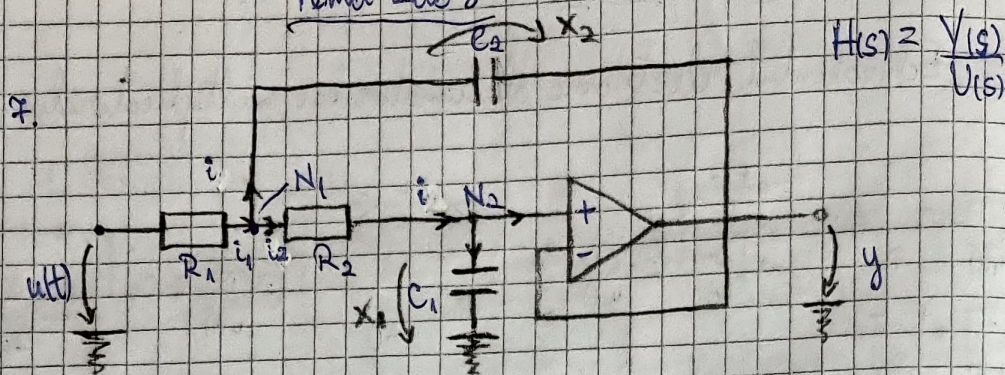


Tema Lab 8



$$a) \quad i_2 = C_2 \frac{dx_2}{dt} \quad i_3 = C_1 \frac{dx_1}{dt}$$

$$V_{N1} = y = x_2$$

$$V_{N2} = 0 = x_1$$

$$i_1 = i_2 + i_3$$

$$V - V_{N1} = V_{R1} = R_1 \cdot i_1$$

$$V - (x_1 + x_2) = R_1 \cdot i_1 \Rightarrow i_1 = \frac{V - x_1 - x_2}{R_1}$$

$$V_{N1} - V_{N2} = (x_1 + x_2) - x_1 = x_2 = V_{R2}$$

$$V_{R2} = R_2 \cdot i_3 = R_2 \cdot C_1 \cdot \frac{dx_1}{dt} \Rightarrow x_2 = R_2 C_1 \cdot \frac{dx_1}{dt} \Rightarrow \dot{x}_1 = \frac{x_2}{R_2 C_1}$$

$$\Rightarrow \frac{V - x_1 - x_2}{R_1} = C_2 \dot{x}_2 + C_1 \cdot \frac{1}{R_2 C_1} x_2 \quad | : C_2$$

$$\dot{x}_2 = \frac{V - x_1 - x_2}{R_1 C_2} - \frac{1}{R_2 C_2} x_2 \Rightarrow \dot{x}_2 = -\frac{x_1}{R_1 C_2} - \left(\frac{1}{R_1 C_2} + \frac{1}{R_2 C_2} \right) x_2 + \frac{V}{R_1 C_2}$$

$$A = \begin{pmatrix} 0 & \frac{1}{R_2 C_1} \\ -\frac{1}{R_1 C_2} & -\left(\frac{1}{R_1 C_2} + \frac{1}{R_2 C_2} \right) \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ \frac{1}{R_1 C_2} \end{pmatrix}$$

$$C = (1 \ 0), \quad D = 0$$

$$H(s) = C \cdot (s \cdot I_2 - A)^{-1} \cdot B + D$$

$$T = s \cdot I_2 - A = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - A = \begin{pmatrix} s & -\frac{1}{R_2 C_1} \\ \frac{1}{R_1 C_2} & s - \left(\frac{1}{R_1 C_2} + \frac{1}{R_2 C_2} \right) \end{pmatrix}$$

$$T^{-1} = \frac{1}{\det T} \begin{pmatrix} s - \frac{R_1 + R_2}{R_1 R_2 C_2} & \frac{1}{R_2 C_1} \\ -\frac{1}{R_1 C_2} & s \end{pmatrix} \Rightarrow H(s) = \frac{1}{C_1 C_2 R_1 R_2 s^2 + C_1 R_2 s + C R_1 s}$$

$$H(s) = \frac{1}{C_1 C_2 R_1 R_2} \cdot \frac{1}{s^2 + s \cdot \frac{R_1 + R_2}{C_2 R_1 R_2} + \frac{1}{C_1 C_2 R_1 R_2}}$$

$$\omega_n = \frac{\sqrt{C_1 C_2 R_1 R_2}}{C_1 C_2 R_1 R_2} = \frac{1}{\sqrt{C_1 C_2 R_1 R_2}}$$

$$2 \zeta \omega_n = \frac{R_1 + R_2}{C_2 R_1 R_2} \Leftrightarrow \zeta = \frac{1}{2} \cdot \frac{R_1 + R_2}{C_1 R_1 R_2} \sqrt{C_1 C_2 R_1 R_2}$$

$$= \frac{1}{2} \frac{(R_1 + R_2) C_2}{\sqrt{C_1 C_2 R_1 R_2}}$$

$\rightarrow \zeta = 1$ factor de proportionalitate