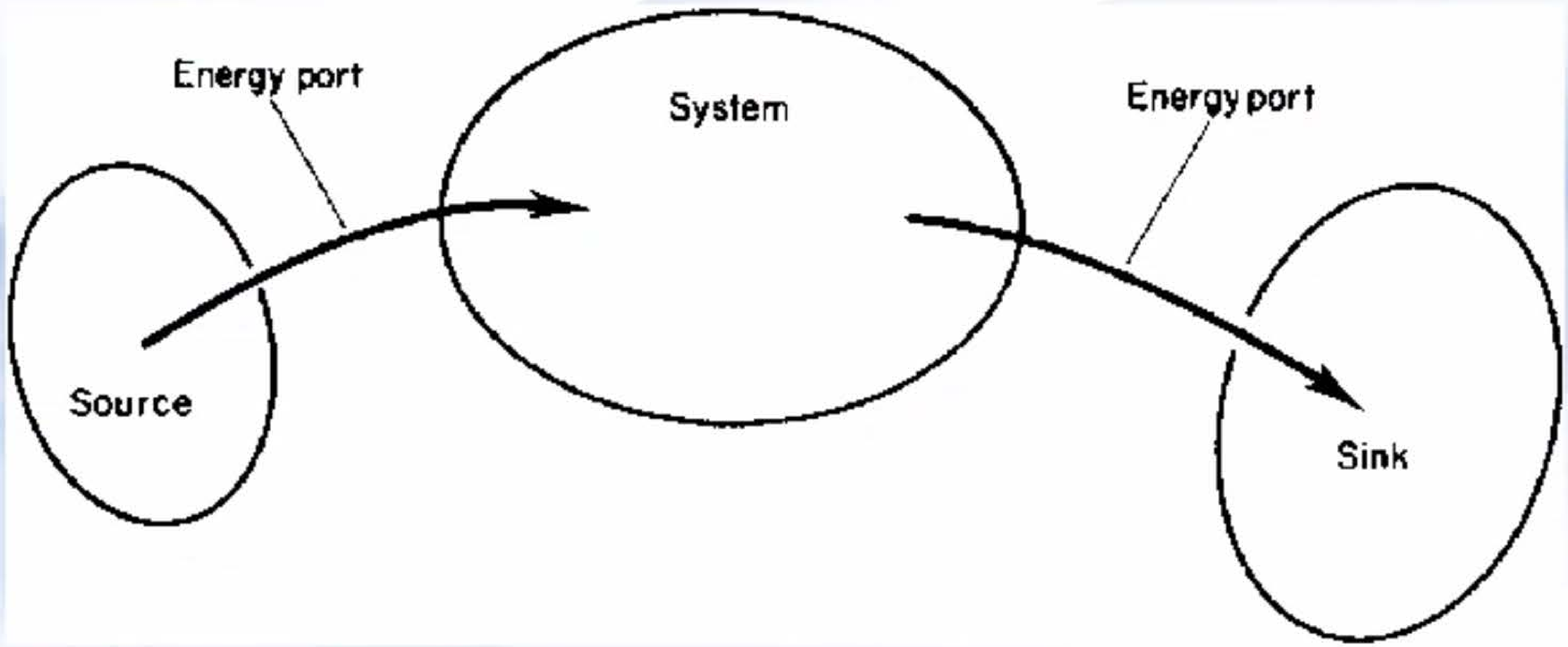
PROCESS MODELING

COURSE for 2nd year,

Automation and Applied Informatics,

TUCN

Generalized Variables and Basic Components



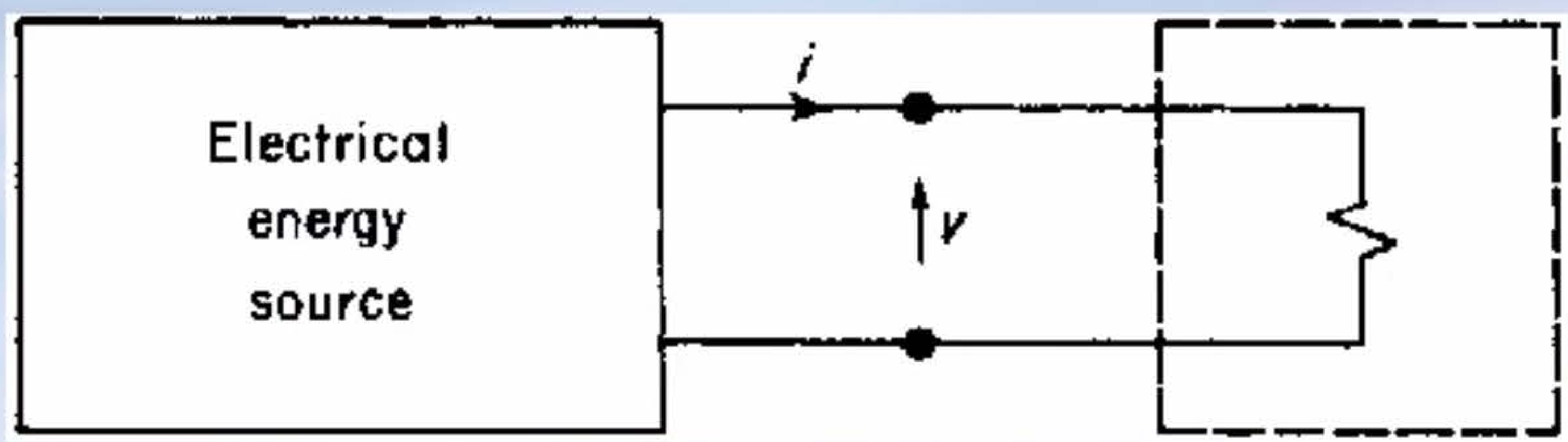
The idea of **systems as energy manipulators** which interact with inputs and outputs via energy ports is a conceptual model which encompasses a wide range of **physical systems**.

Places at which subsystems can be interconnected are places at which power can flow between the subsystems.

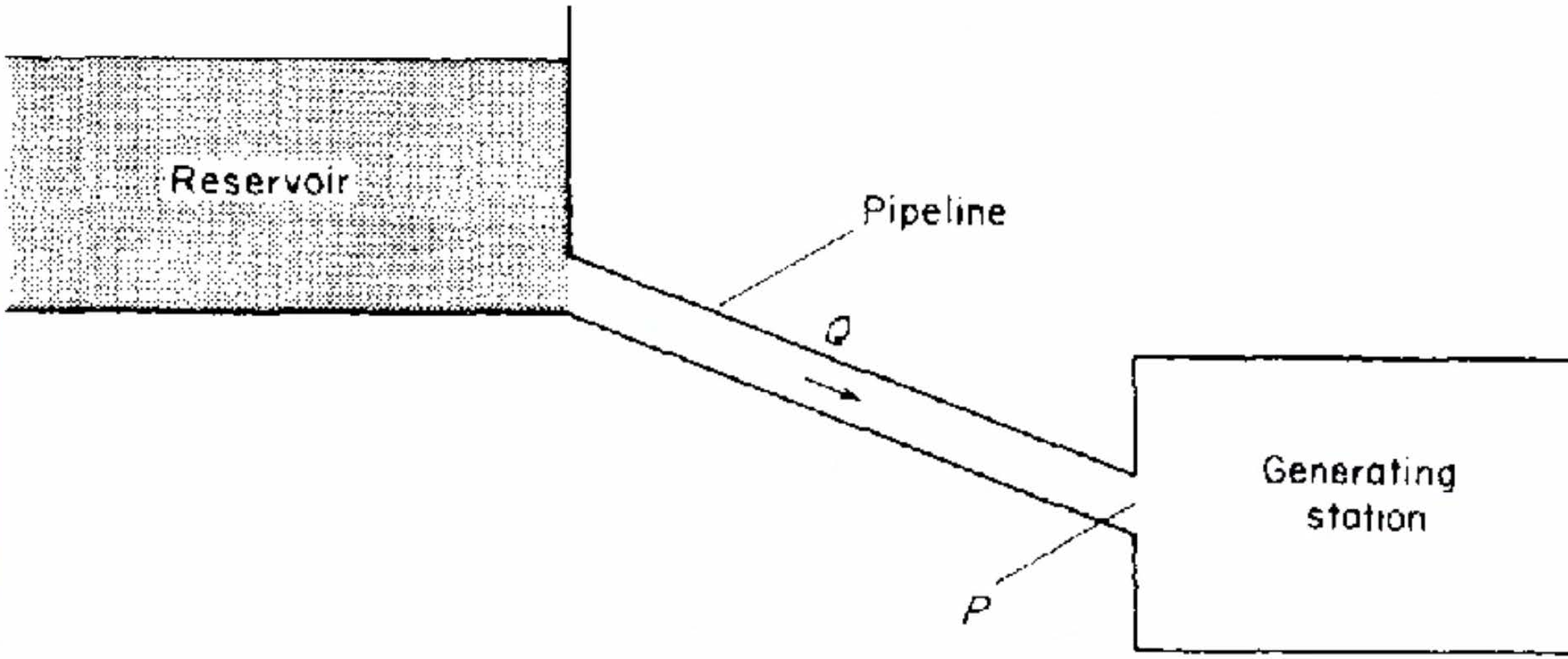
Such places are called ***ports***, and physical subsystems with one or more ports are called *multiports*.

A system with a single port is called a *1-port*, a system with two ports is called a *2-port*, and so on.

When two subsystems or components are joined together physically, *two complementary variables are simultaneously constrained to be equal for the two subsystems*.

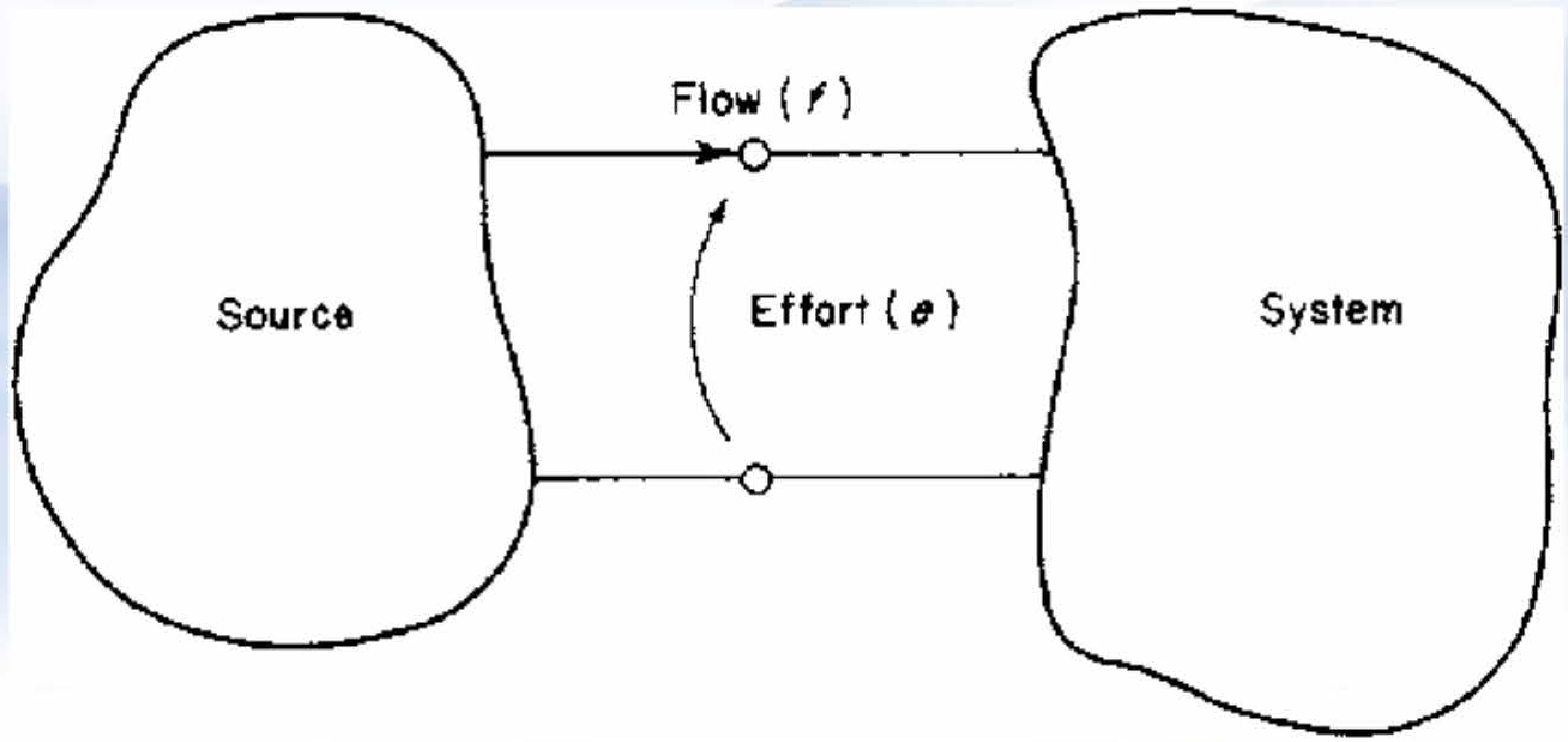


The power supply is an ***energy source***, the resistor is ***the system*** and the ***energy port*** connecting them ***is the pair of conducting wires***.
The transmission of power to the resistor is given as the product of the system variables voltage **v** and current **i**

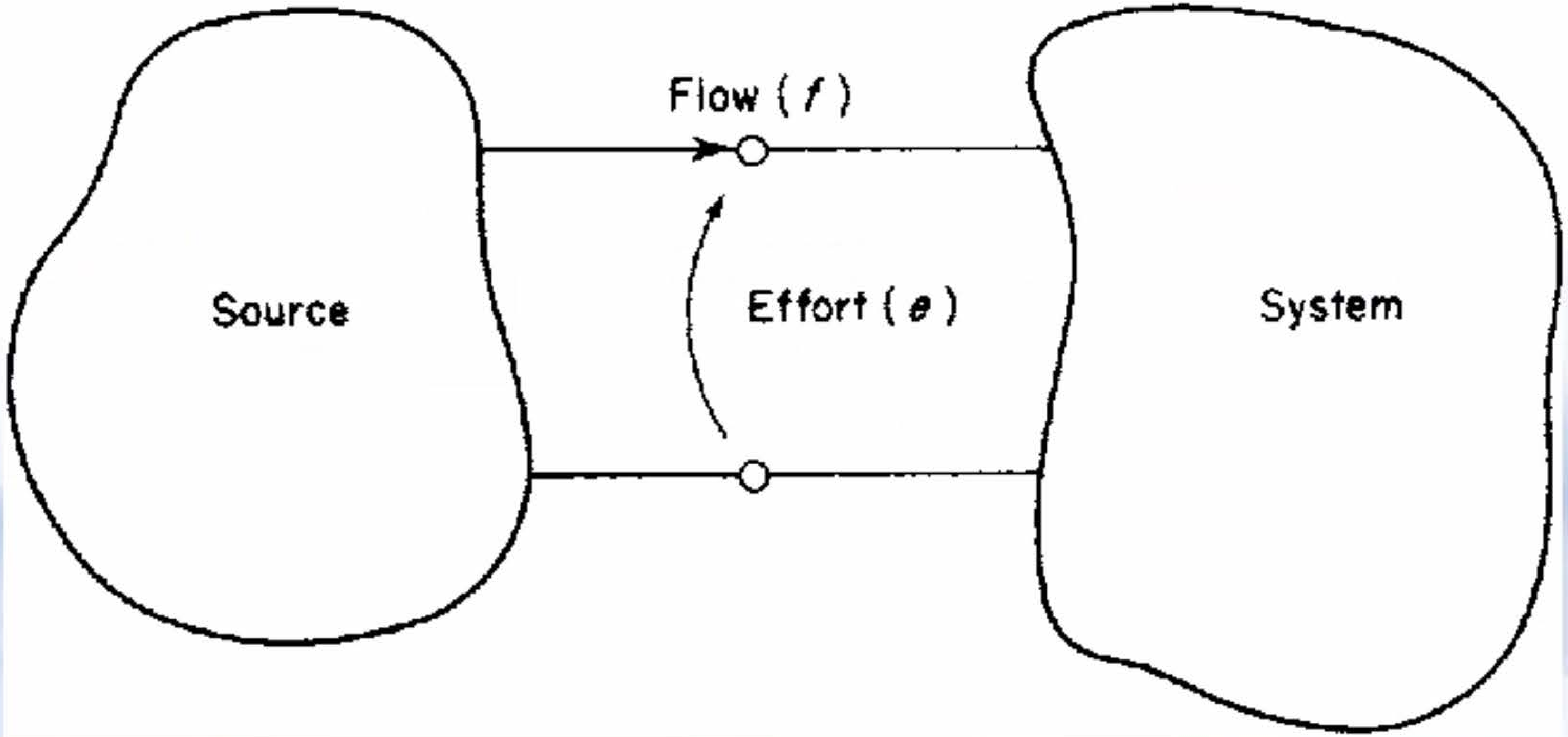


The reservoir is a source of hydro-energy connected by ***an energy port (the pipeline)*** to **a system (the hydro-energy generating station).** The system variables which give the power supplied to the generating station are the fluid volumetric flow rate **Q** and the pressure **P** measured at the intake with respect to some pressure datum

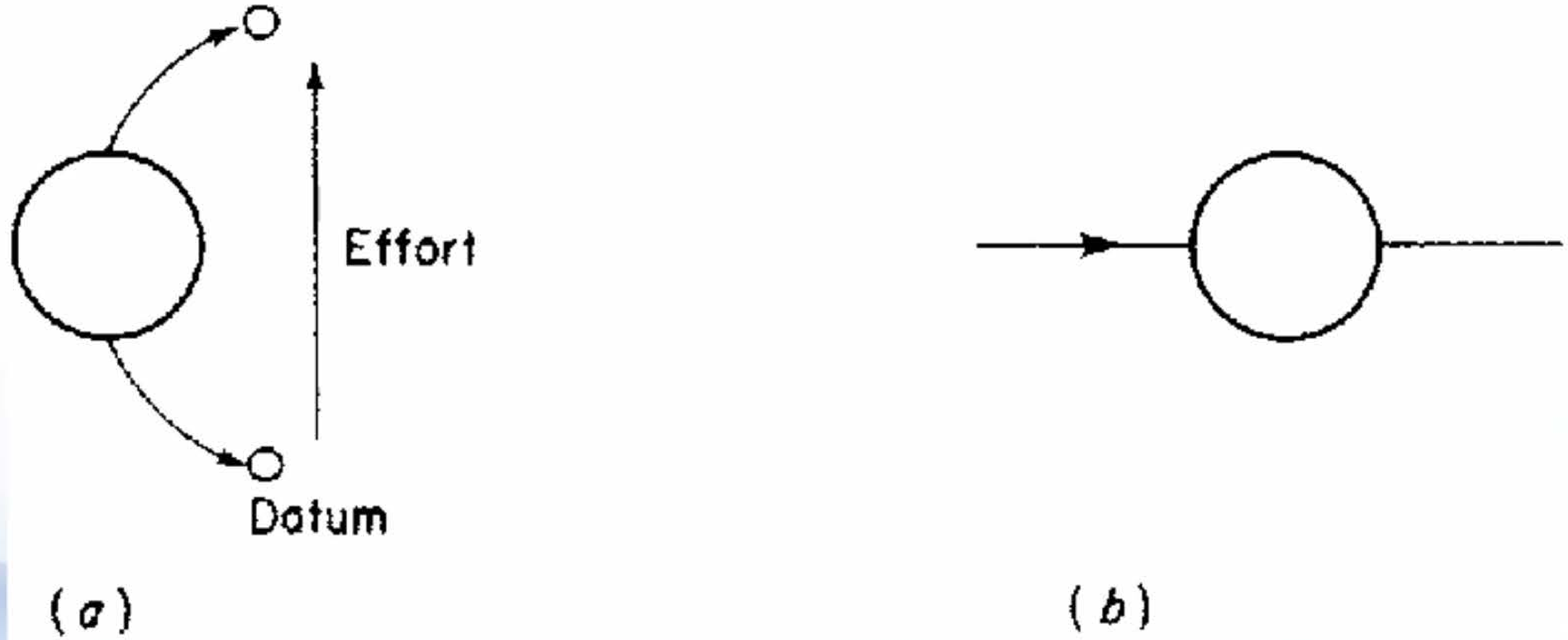
Further examples can be drawn to show that **the energy coupling of many systems** can be represented by ***a pair of system variables whose product is the instantaneous power*** being **transmitted through an energy port**



Since power interactions are always present when two multiports are connected, it is useful to **classify the various power variables in a universal scheme** and to describe *all types of multiports* in a *common language*.



The delivering of energy is associated with one intensive variable (e.g. current, fluid flow) giving the flux of energy flow, and an extensive variable (e.g. voltage, pressure) giving the pitch of energy flow. In a generalized sense the two energy variables can be thought of as an ***effort variable and a flow variable***. ***An abstract energy port*** *can be diagrammatically* represented by **a pair of terminals with a pair of generalized variables, effort (e) and flow (f)** which together represent the energy transfer mechanism.

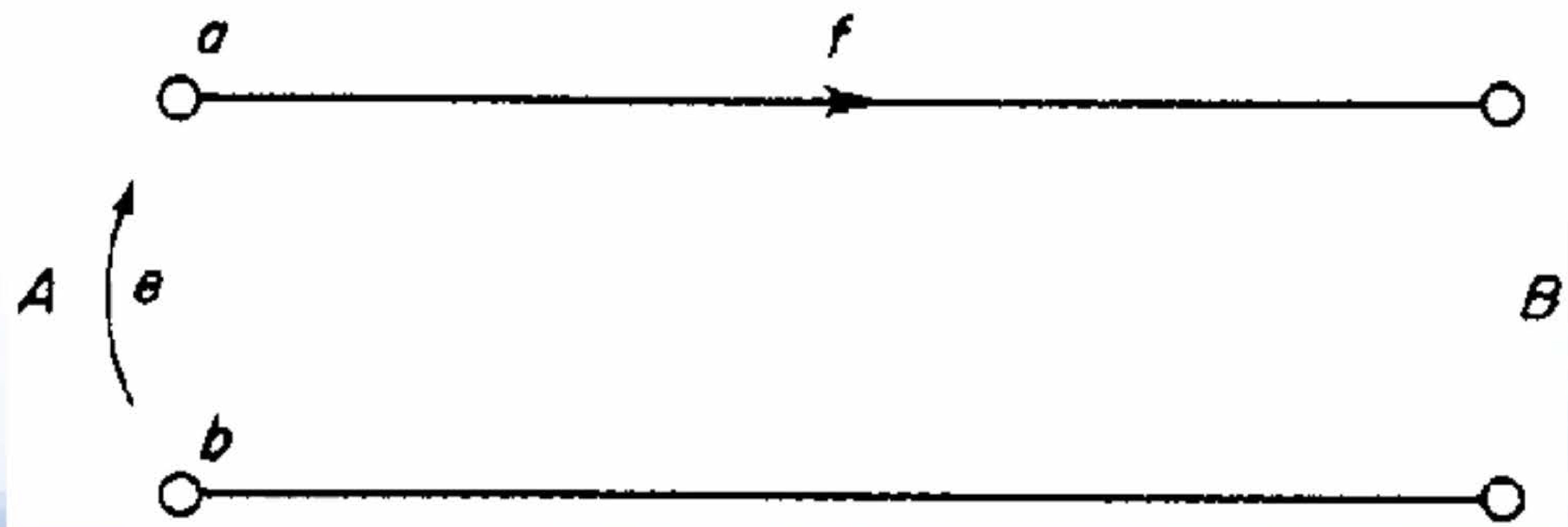


The classification of system variables according to the measurement scheme required to meter them

Two measuring devices are required, an across meter and a through meter:

-a two-terminal meter, connected ***across two points in space and thus measures a spatially extensive variable*** *(Fig. (a)).* Ex: voltmeters, pressure gauges, thermometers and velocity
transducers.

-a through meter, on the other hand, requires no separate datum point, it is **inserted *into a system and measures the rate of flow through itself of a physically intensive* variable** (Fig. (b)). Ex: flow meters, ammeters and force transducers.



In the generalized scheme of energy handling systems the product of the flow variable (f)
and the effort variable (e) is the instantaneous power associated with the energy port or
terminal pair which the pair (e,f) characterizes. The energy which is transferred over the terminal pair *ab in the interval of time 0 to t_1 is given by*

$$E_{ab}(t_1) = \int_0^{t_1} e\, f\, dt$$

<u>REMARK:</u> power and energy are both directed variables, whose sign depends upon the arbitrary sign conventions used for the effort and flow variables.

Stored energy

$$\text{Stored energy} \equiv \int_0^t e\,f\,dt$$

Two fundamental mechanisms exist for the storage of energy:

- in terms of stored effort

- in terms of stored flow.

Two new variables can be defined to account for energy stored in this manner.

For effort the stored effort can be defined as the effort accumulation $\boldsymbol{e_a}$ associated with a component:

$$e_a = \int_0^t e\,dt \qquad e = \frac{de_a}{dt}$$

$$\text{Stored energy} = \int_0^{e_a} f\,de_a$$

The stored flow can be defined as the flow accumulation f_a associated with a system component and given by:

$$f_a = \int_0^t f\,dt \qquad \text{or} \qquad f = \frac{df_a}{dt}$$

$$\text{Stored energy} = \int_0^{f_a} e\,df_a$$

$$\text{Stored energy } \equiv \int_0^t e\,f\,dt$$

$$f_a = \int_0^t f\,dt$$

$$f = \frac{df_a}{dt}$$

$$\text{Stored energy } = \int_0^{f_a} e\,df_a$$

$$e_a = \int_0^t e dt$$

$$e = \frac{de_a}{dt}$$

$$\text{Stored energy } = \int_0^{e_a} f\,de_a$$

- The physical variables at work in a system are the media by which energy is manipulated, and by suitable interpretation of physical variables, many systems can be reduced to a common, energy handling, basis.
- In order to extend the idea of energy handling system further it is necessary **to examine the components which make up physical systems, and classify them according to how they process energy.**

- In this manner, it should be possible **to build up a catalogue of system elements with distinct energy handling properties.**
- The basic components of Electrical , Mechanical, Hydraulic and Thermal systems will be examined in what follows for identifying their similar behavior in the frame of generalized variables
- The defining **constitutive relation (equation), i.e. the equation relating e and f** will be examined for each such component and **the expression of the stored or dissipated energy will be presented**

Electrical Systems

Voltage and Current

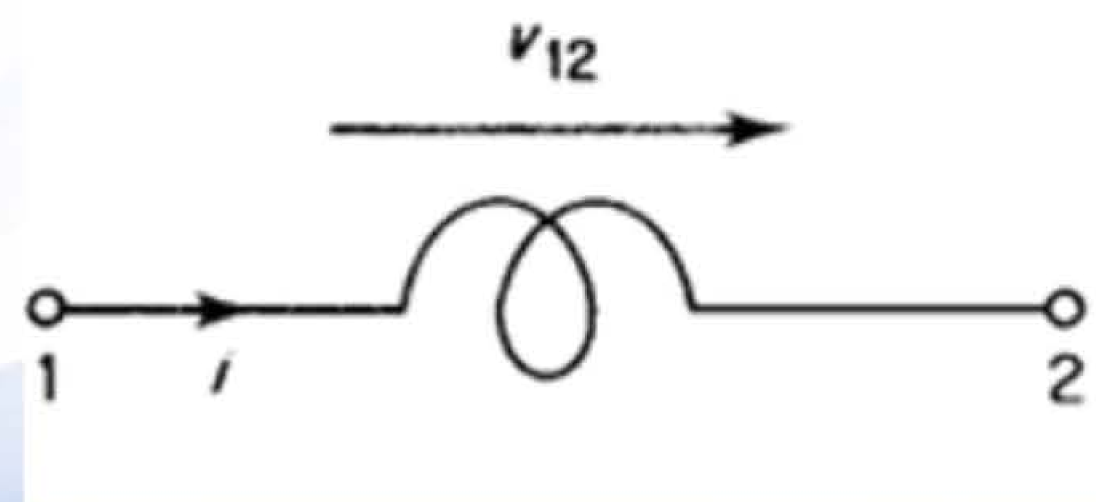
- Voltage is analogous to **effort**.
- Current is analogous to **flow**.

Electrical Systems

Inductance: Electrical Effort Storage

$$\lambda = \varphi(i)$$

"flux linkages" is the total magnetic flux linked by the electrical circuit.



$$v = d\lambda/dt$$

The energy T stored in the inductance $$T = \int_0^{\lambda} i\,d\lambda$$

The co-energy T^* $$T^* = \int_0^{i} \lambda\,di$$

Inductance – storage effort
Nonlinear case

$$\lambda = \varphi(i)$$

i = 1.5 sin (60 t);
v = 9 L_0 cos (60 t);

for $t = 1{:}100,$

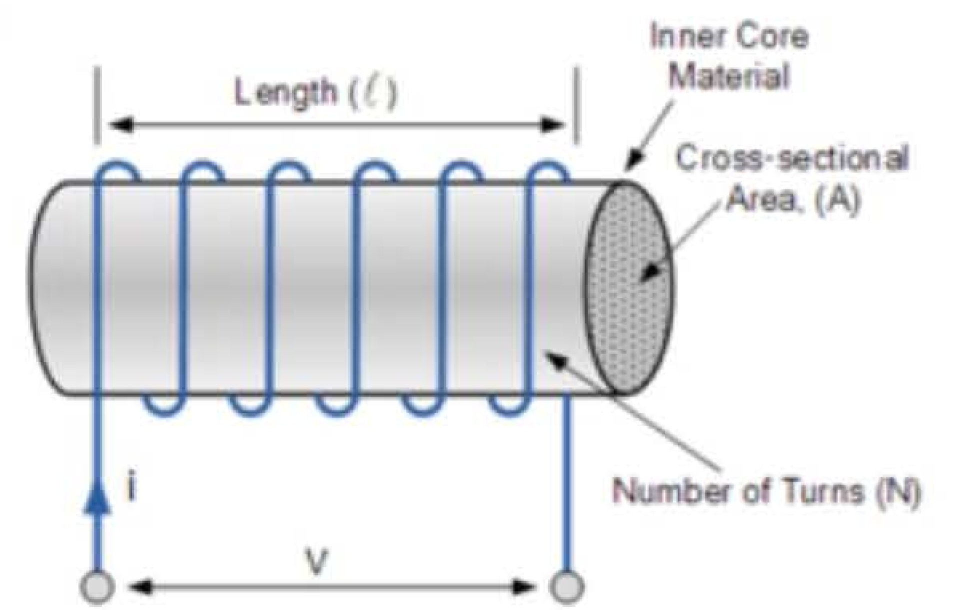
$\lambda(i) = L_0 * i(t);$

if $(abs(v(t)) > 120)$

$\lambda(i) = L_{sat} * i(t);$

$$L = \frac{N^2 \mu A}{l}$$

$$\mu = \mu_r \mu_0$$



Where,

L = Inductance of coil in Henrys
N = Number of turns in wire coil (straight wire = 1)
μ = Permeability of core material (absolute, not relative)
μ_r = Relative permeability, dimensionless (μ_0=1 for air)
$\mu_0 \approx 4\pi \times 10^{-7}$ H/m permeability of free space
A = Area of coil in square meters = πr^2
l = Average length of coil in meters

The inductance relation with the geometry of a solenoid

Inductance – storage effort
Nonlinear case

$$\lambda = \varphi(i)$$

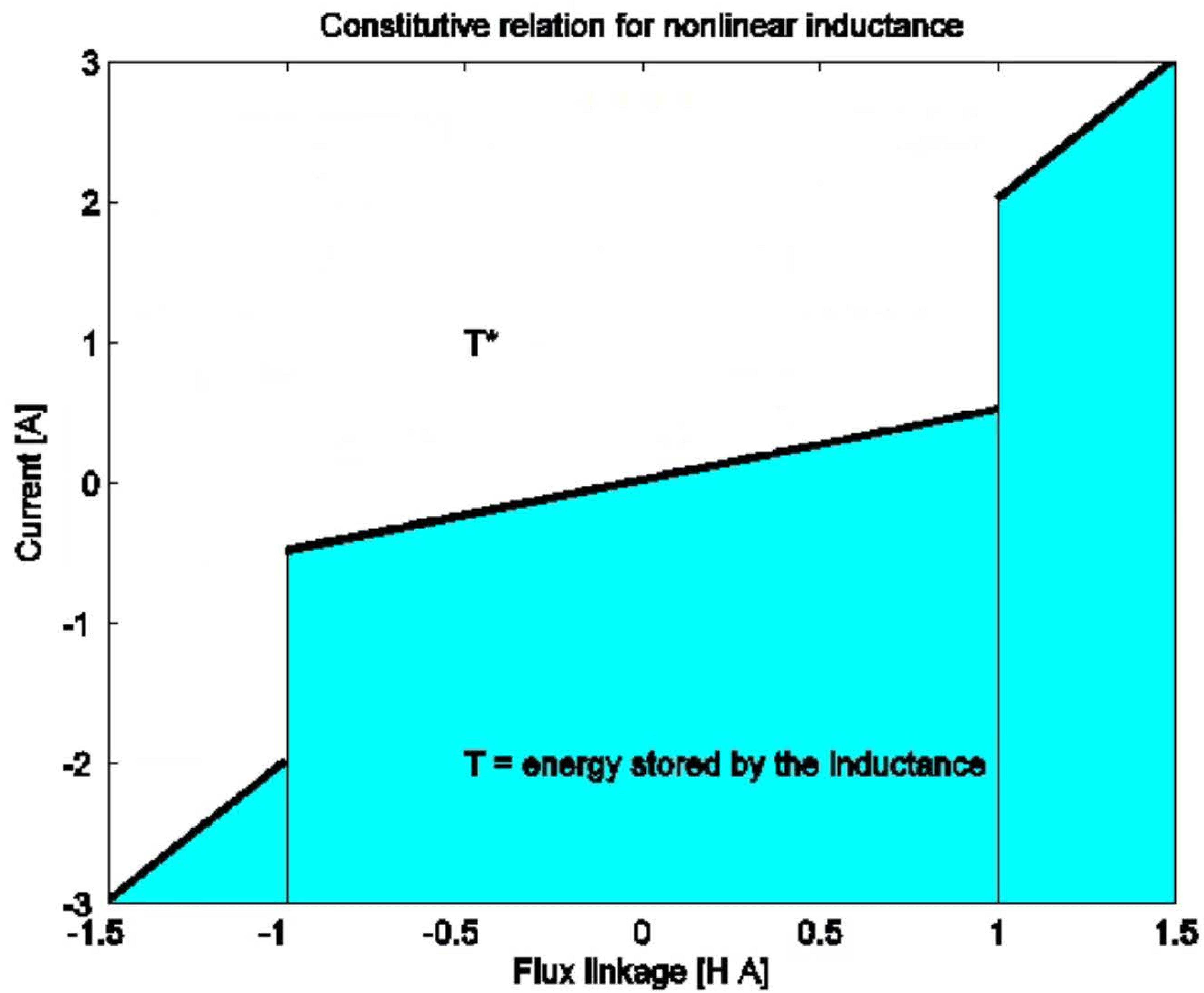
i = 1.5 sin (60 t);
v = 9 L_0 cos (60 t);

for $t = 1{:}100,$

$\lambda(i) = L_0 * i(t);$

if $(abs(v(t)) > 12$

$\lambda(i) = L_{sat} * i(t);$

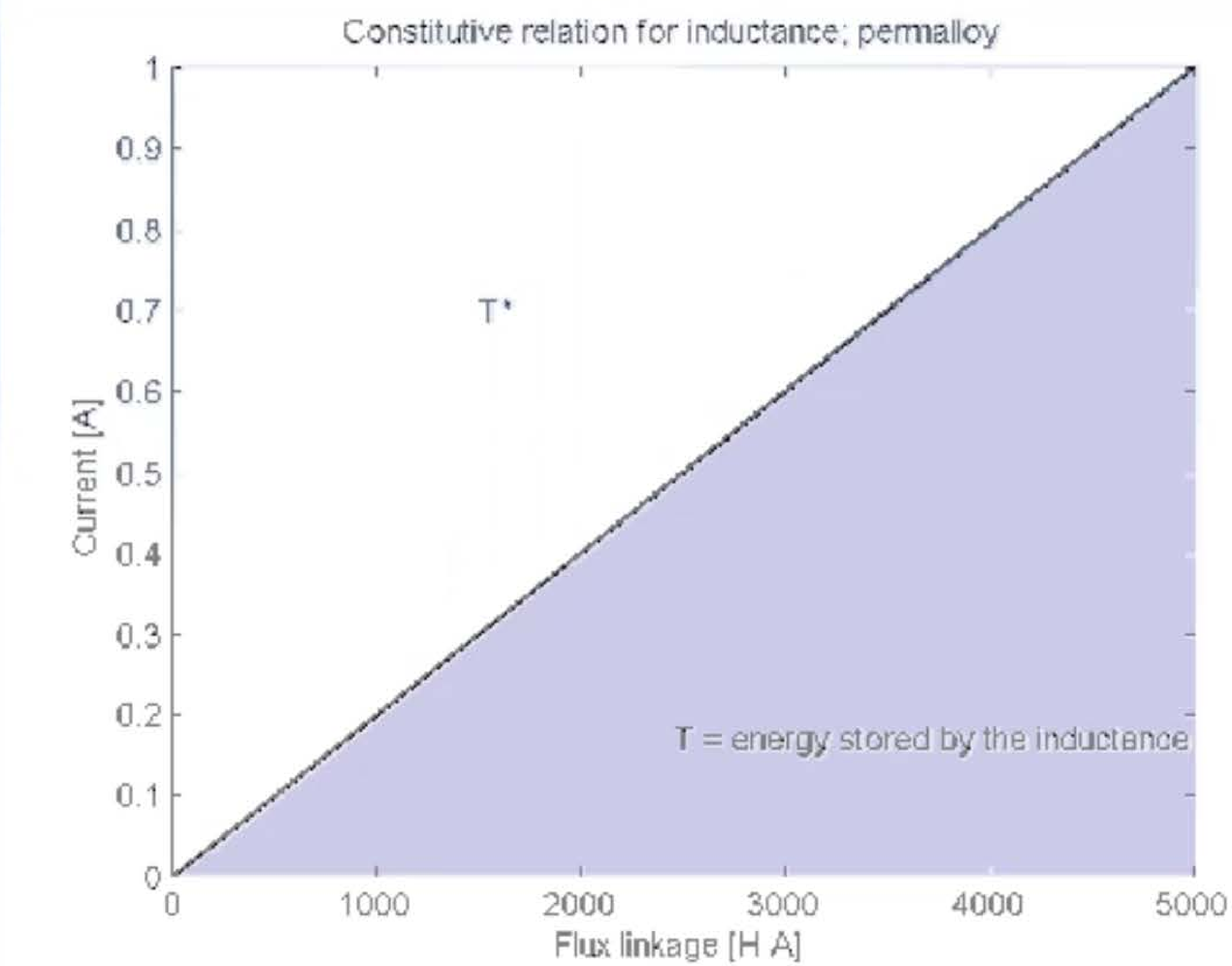


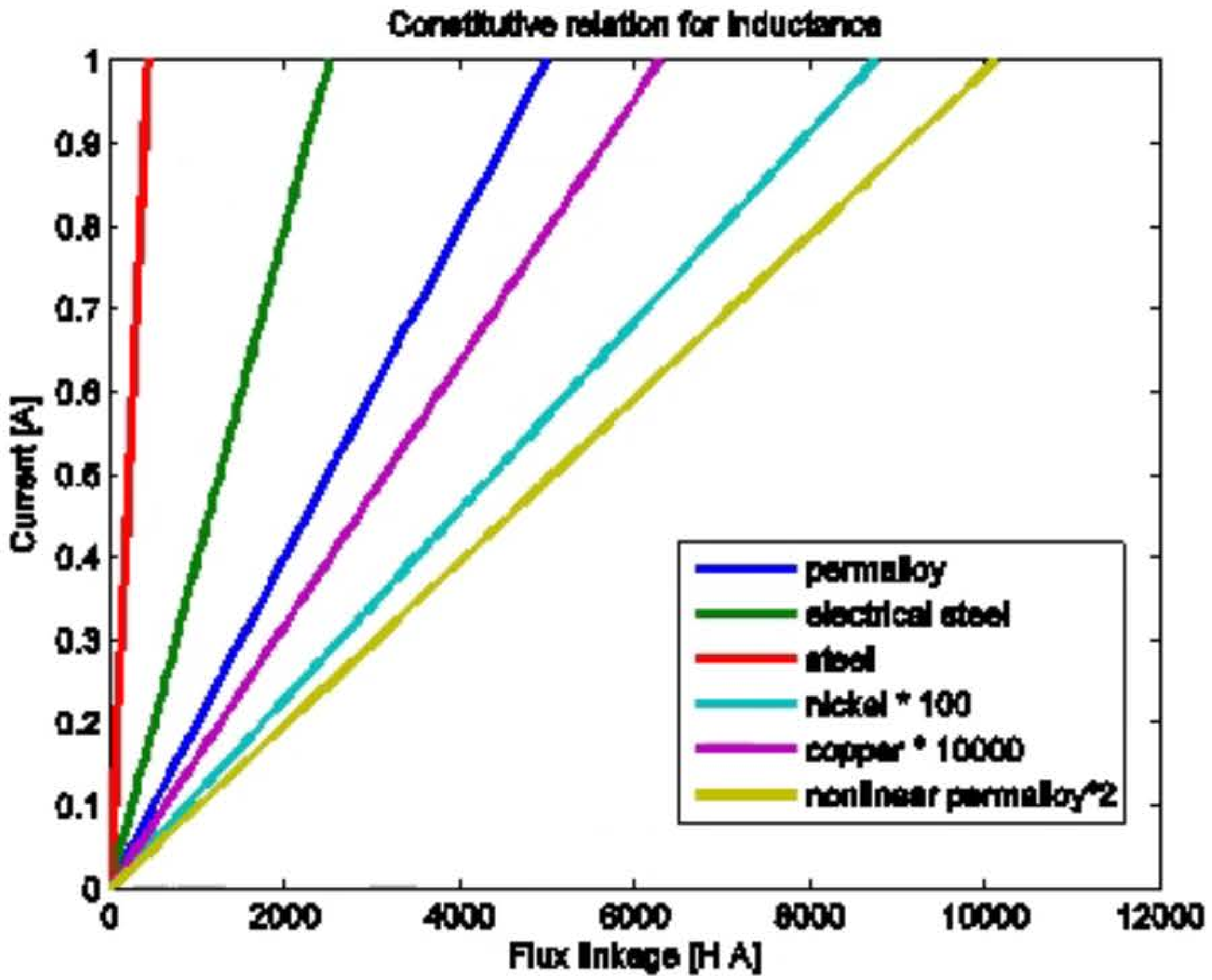
Linear Inductance

$$\lambda = Li$$

$$L = N^2 \mu a / l$$

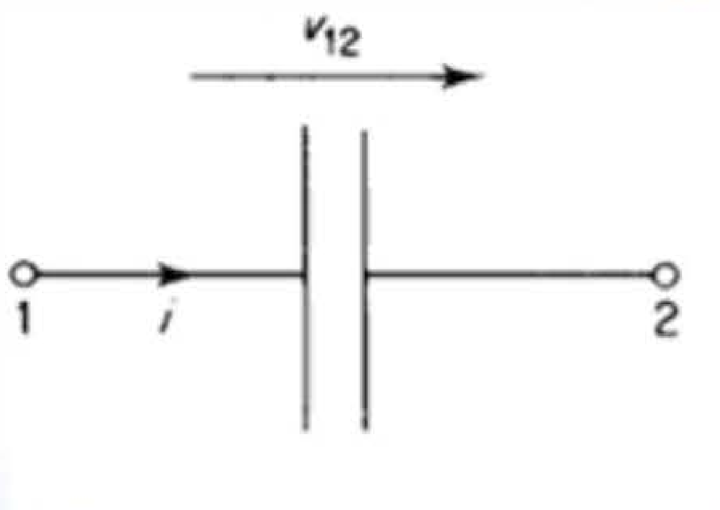
$$T = T^* = (1/2L)\lambda^2 = \frac{1}{2}Li^2$$





Electrical Systems

Capacitance: Electrical Flow Storage

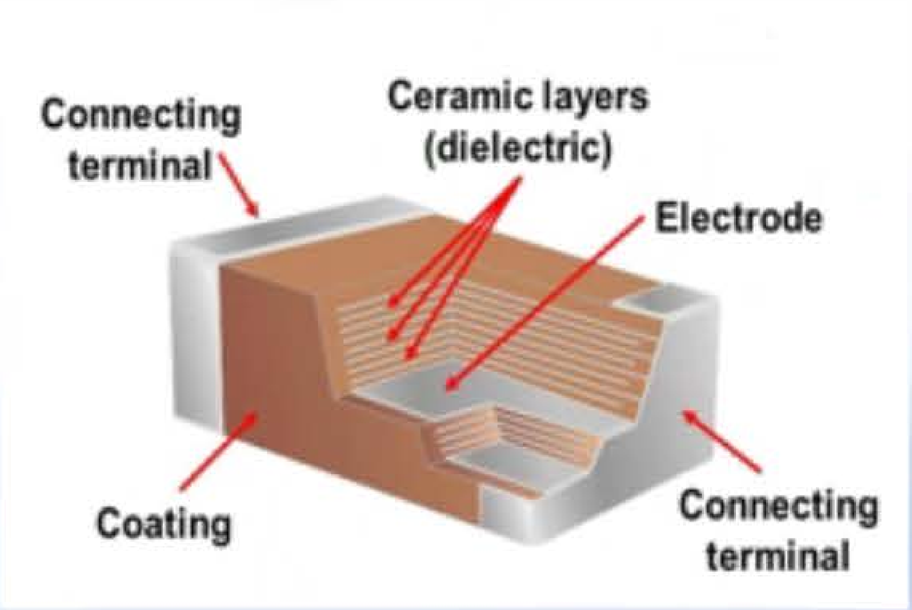


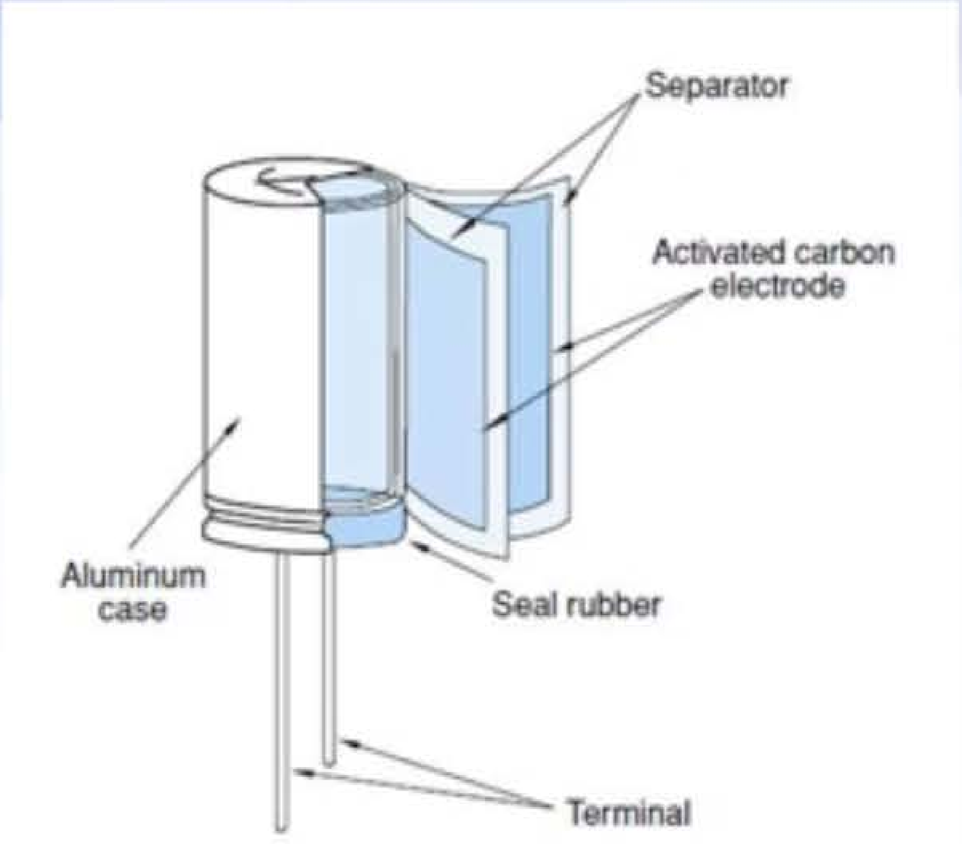
$$q = \varphi(v)$$

$$i = dq/dt$$

$$U = \int_0^q v\,dq$$

$$U^* = \int_0^v q\,dv$$



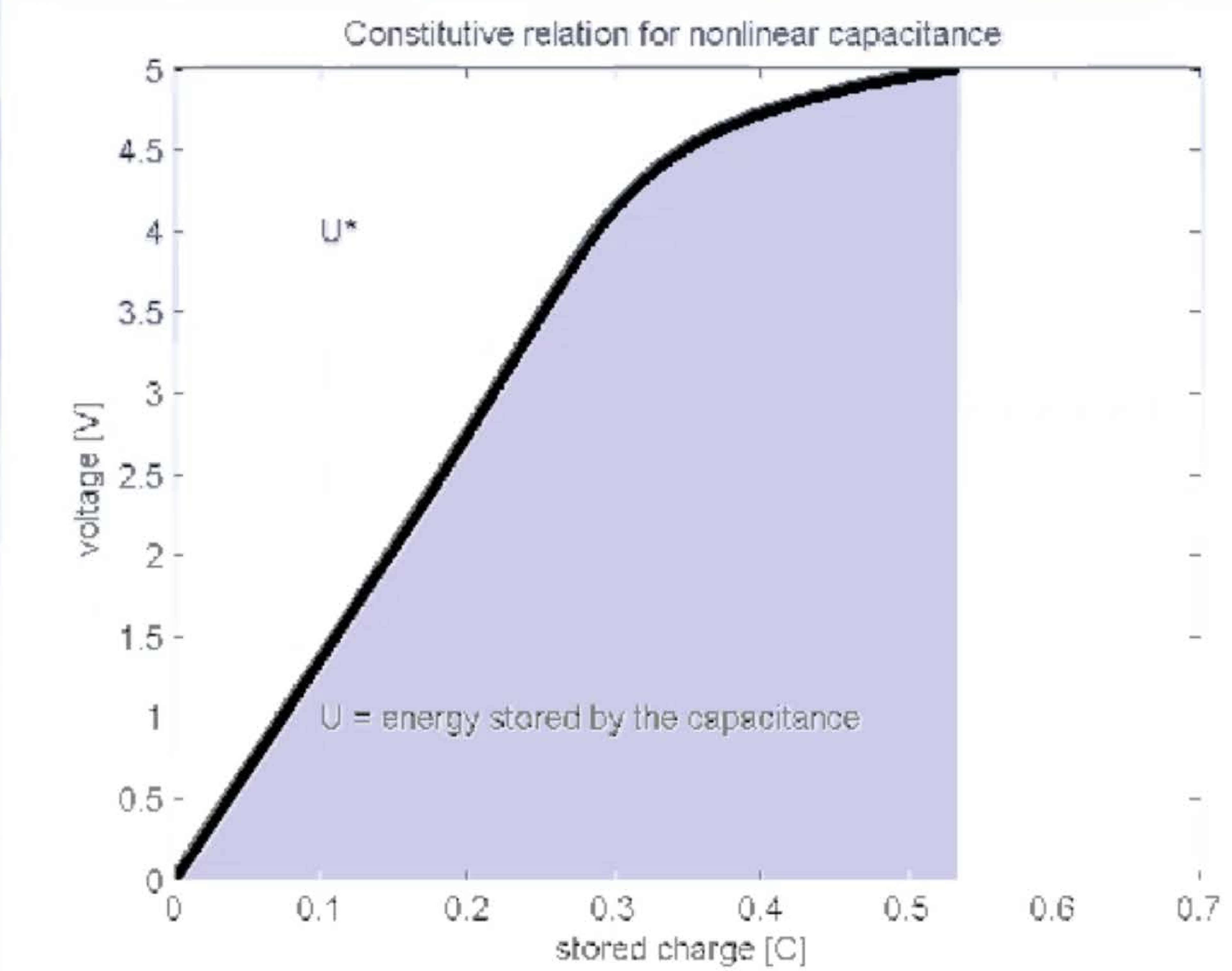


Capacitance: Electrical Flow Storage
Nonlinear case

Example of nonlinear constitutive relation $C(v) = 0.07099 + 0.00295v - 0.00105v^2 + 8.30549 \times 10^{-5}v^3 - 3.25626 \times 10^{-6}v^4 + 9.9665 \times 10^{-8}v^5$

v = 5 cos (60 t);

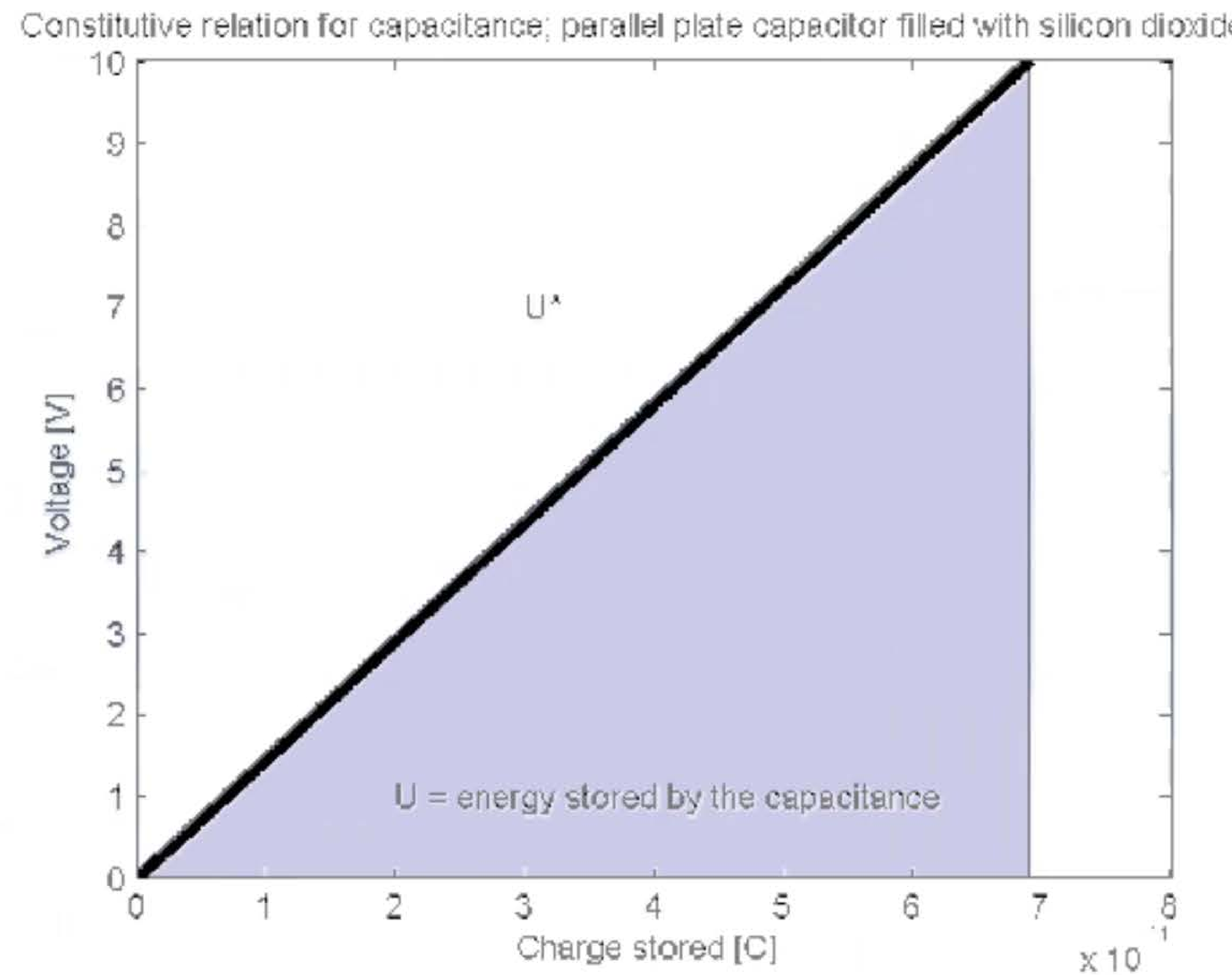
Data for capacitor Y5U



Linear Capacitance

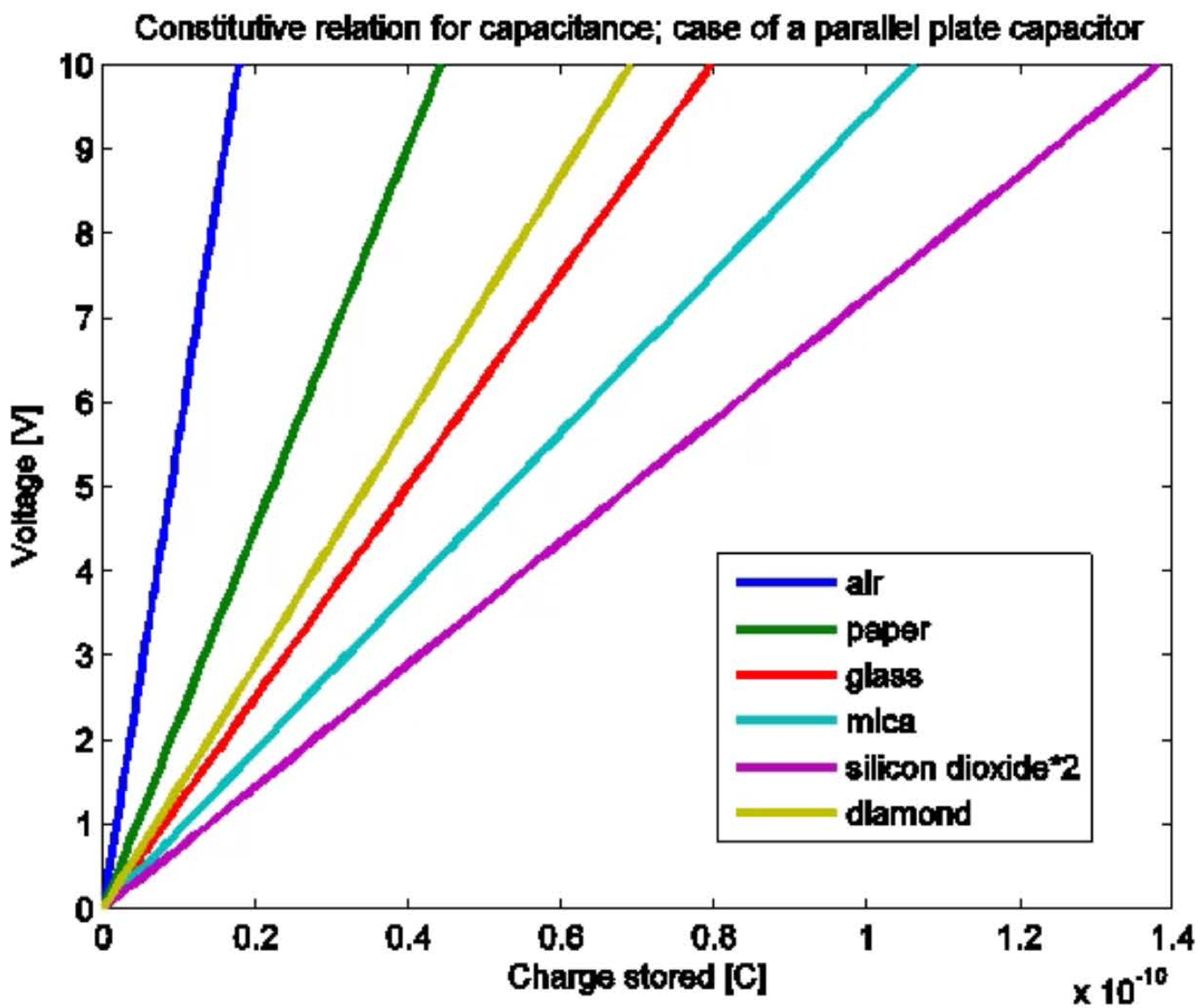
$$q = Cv$$

$$C = \varepsilon a / d$$



$$U = U^* = \frac{1}{2} C v^2 = \frac{1}{2C} q^2$$

Linear Capacitance

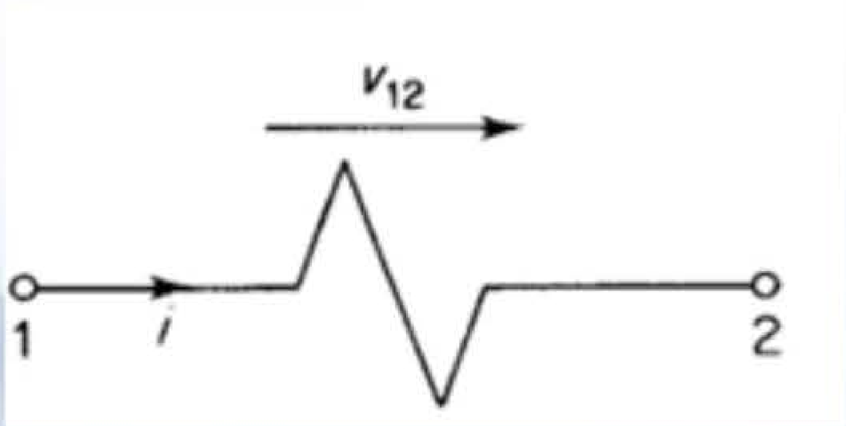


Electrical Systems

Resistance: Electrical Dissipation

The constitutive relation

$$v = \varphi(i)$$

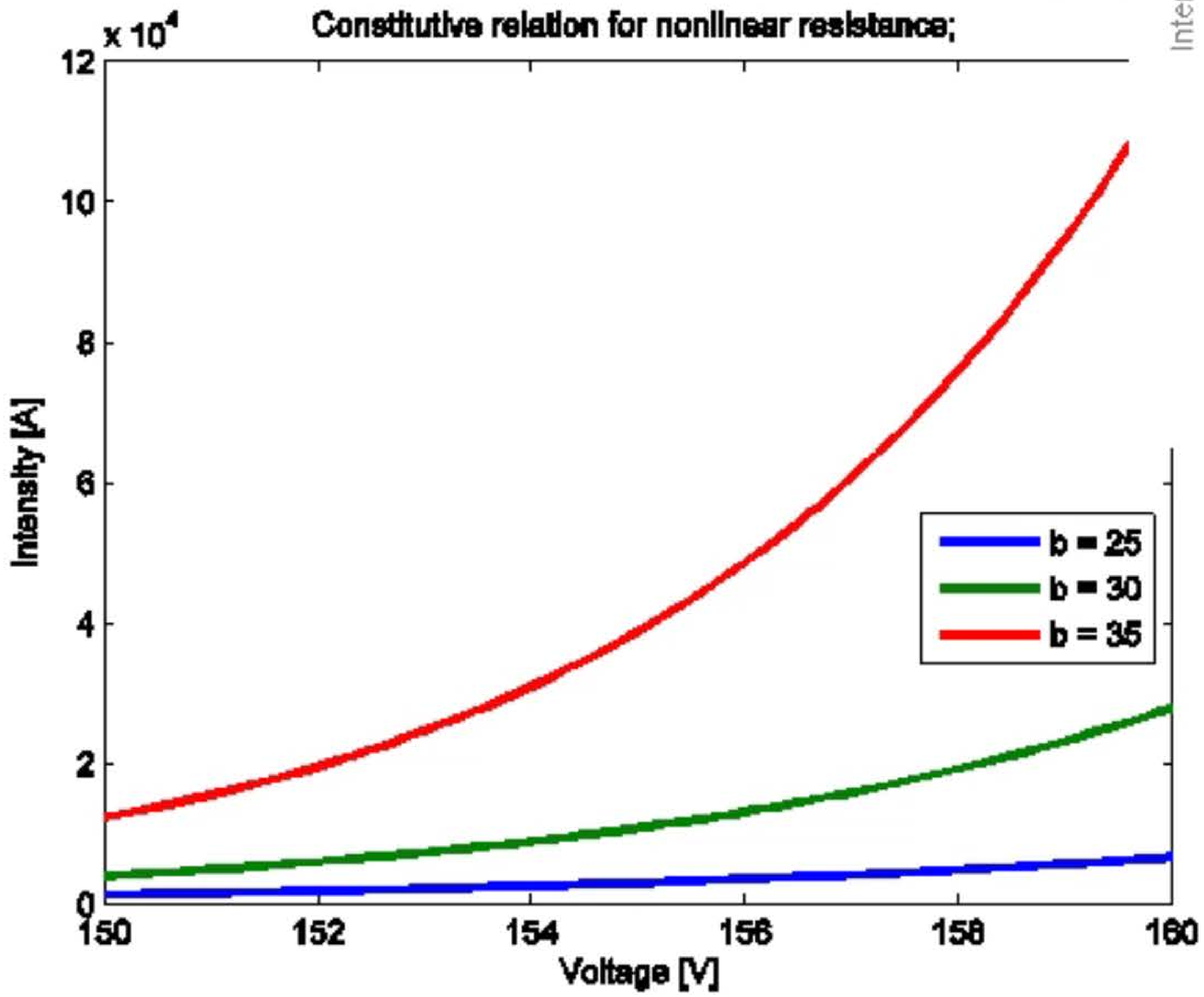


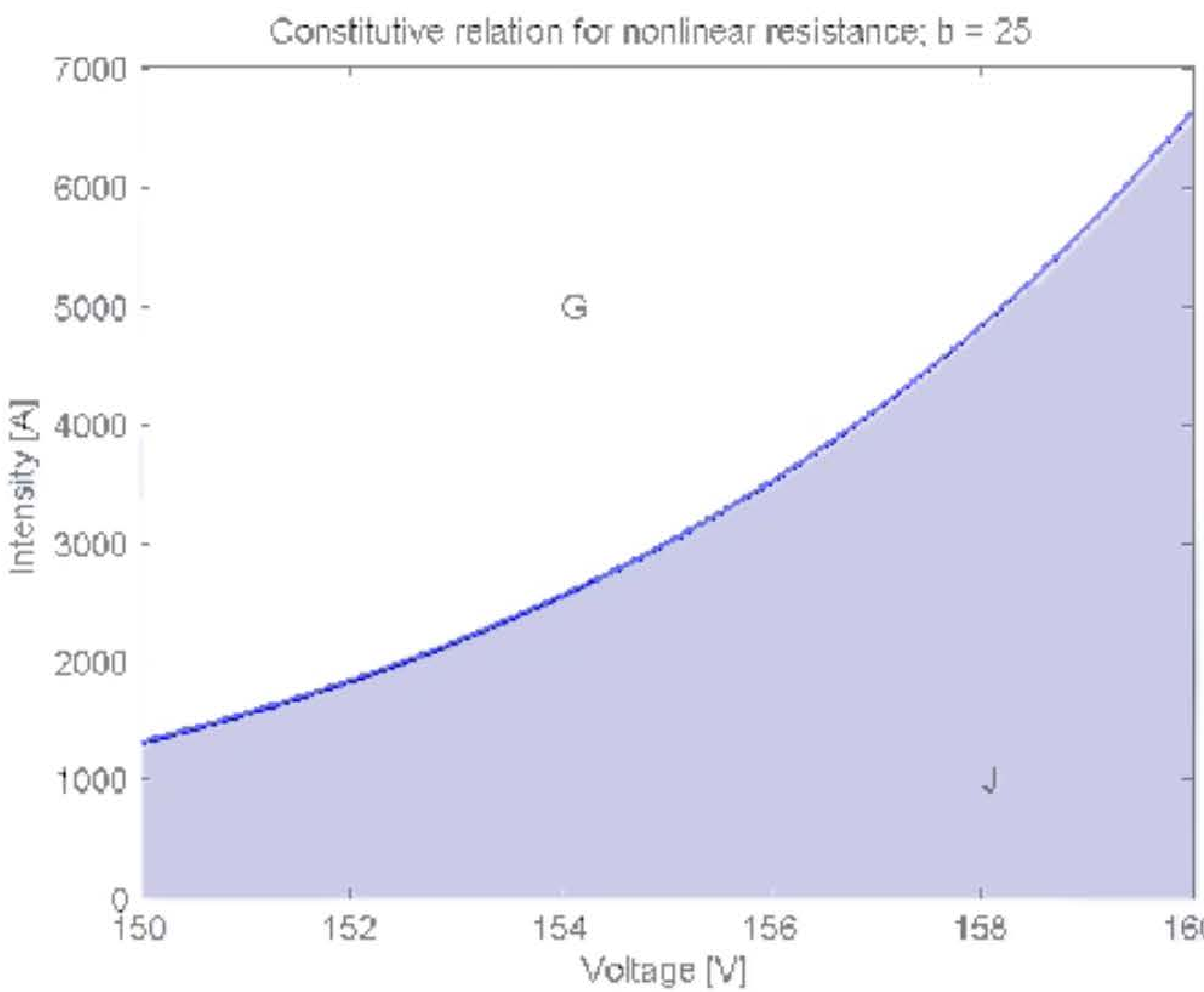
The instantaneous power absorbed by the systems

$$vi = \int_0^i v \; di + \int_0^v i \; dv = G + J.$$

Resistance: Electrical Dissipation
Nonlinear case

$$i = 5\left(\frac{v}{120}\right)^{b}$$

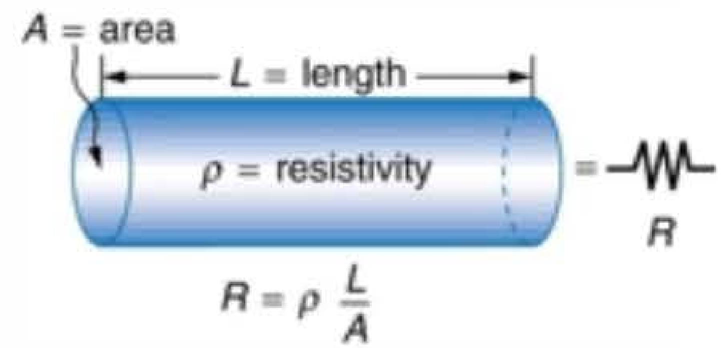


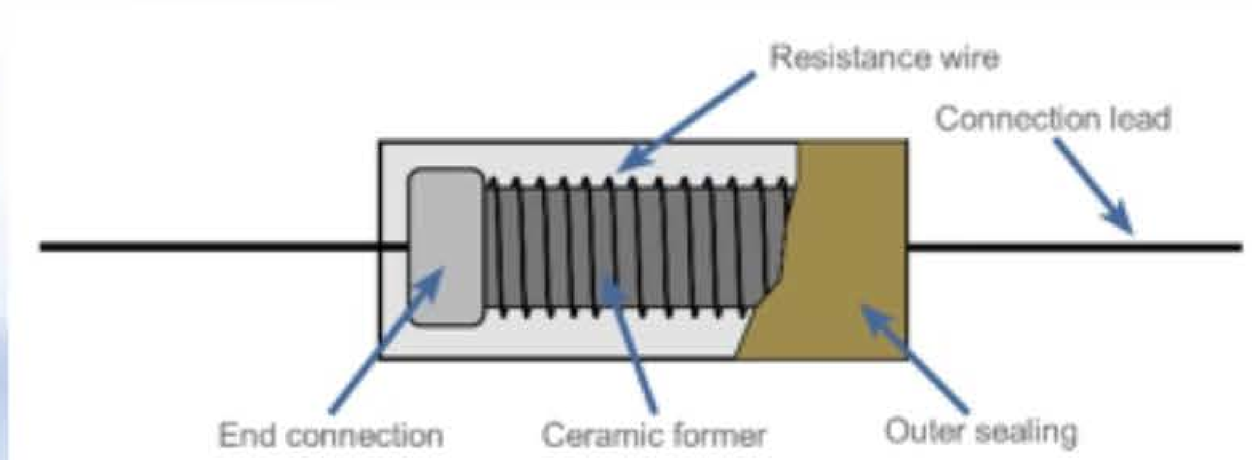


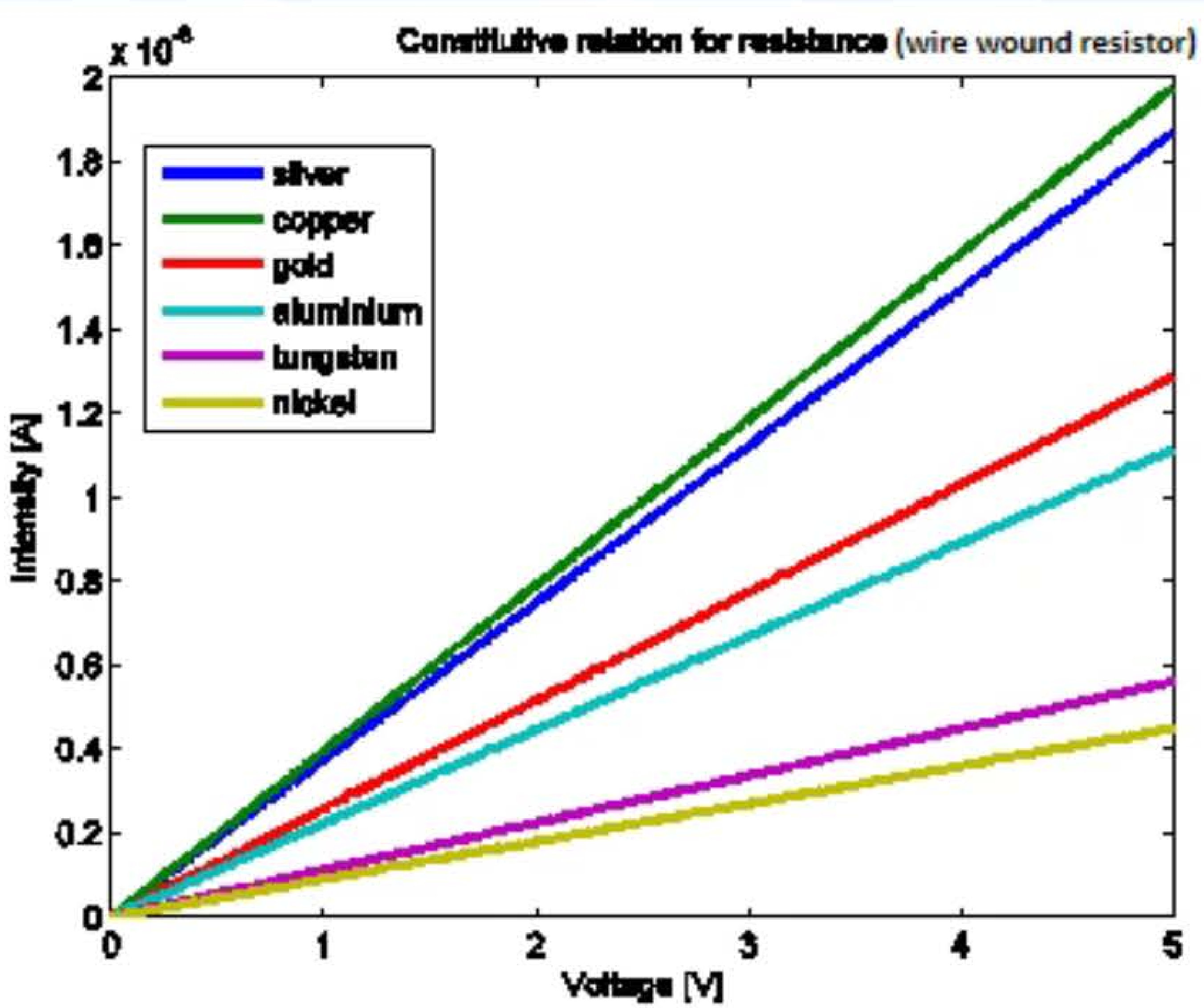
Linear Resistance

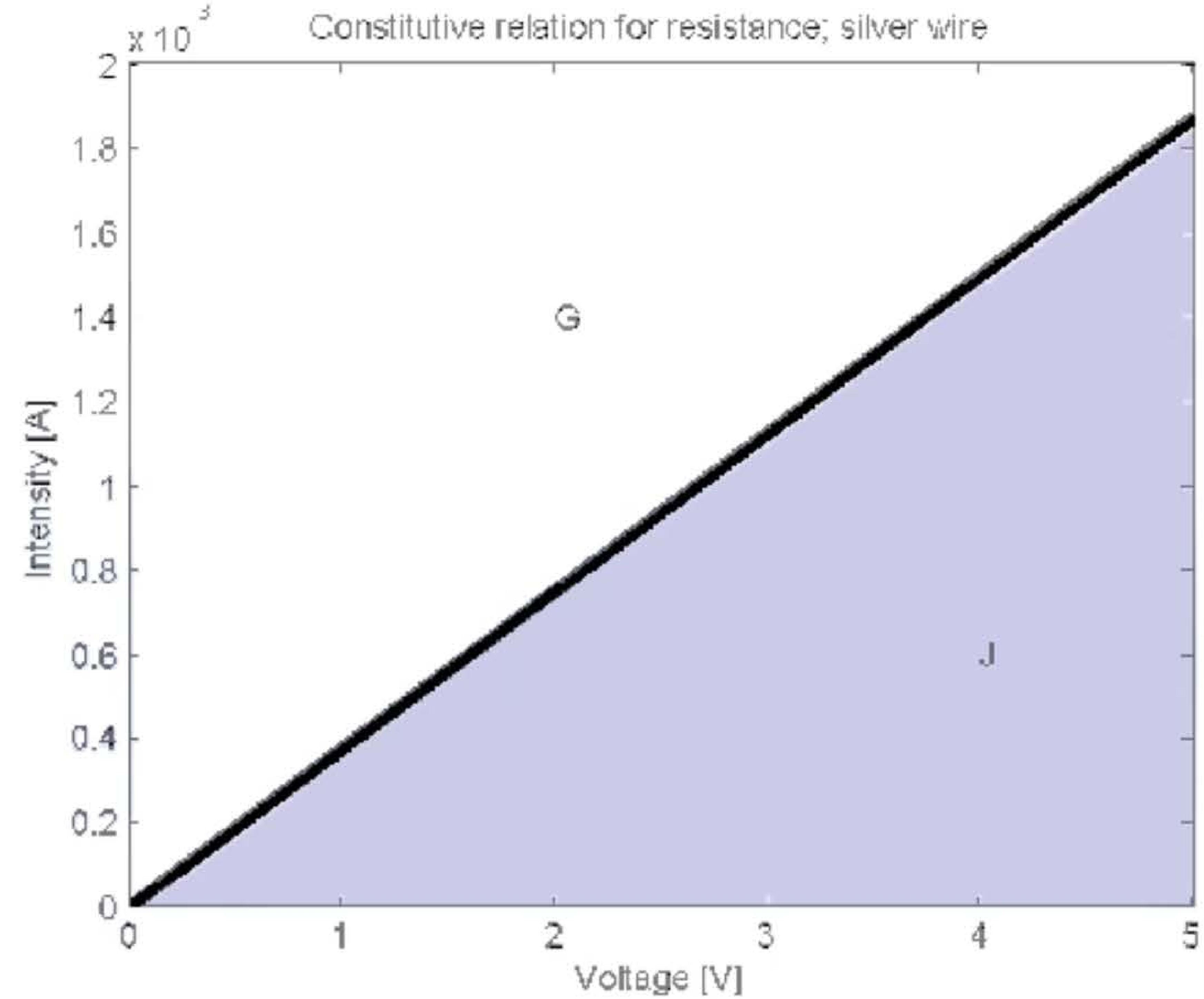
$$v = Ri$$

$$R = \rho l / a$$









$$G = J = \frac{1}{2} R i^2$$

$$J = vi / 2$$

Mechanical Systems

The dynamical behaviour of mechanical systems is specified by a set of:

- vector velocities,
- displacements,
- forces
- and moments.

An appropriate set of these variables is sufficient to specify the general motion of a mechanical assembly moving in three-dimensional space.

Mechanical Systems

Mechanical Elements (Translational)

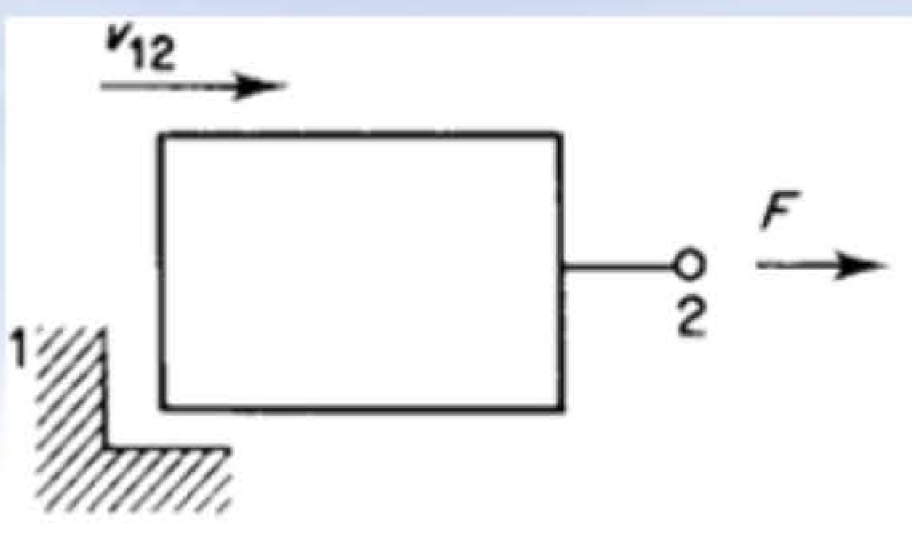
Two common choices of the effort and flow variables:

- **A.1** (Mobility Analogy)
  - Velocity is analogous to effort.
  - Force is analogous to flow.
- **A.2** (Classical Analogy)
  - Velocity is analogous to flow.
  - Force is analogous to effort.

Mechanical Systems

Translational Mass

A pure translational mass is a rigid mechanical object which is moving through a nondissipative environment. Then, according to Newton's second law, the momentum p of the mass is linearly related to the object's velocity v: $p = mv$



$$p = \int_{t_0}^{t} F\,dt + p(t_0)$$

The analogy A.1 (Mobility Analogy) indicates that ***the quantity momentum is formally analogous to flow accumulation and thus a pure translational mass can be classified as a flow store*** with the constitutive relation and symbol shown in figure.

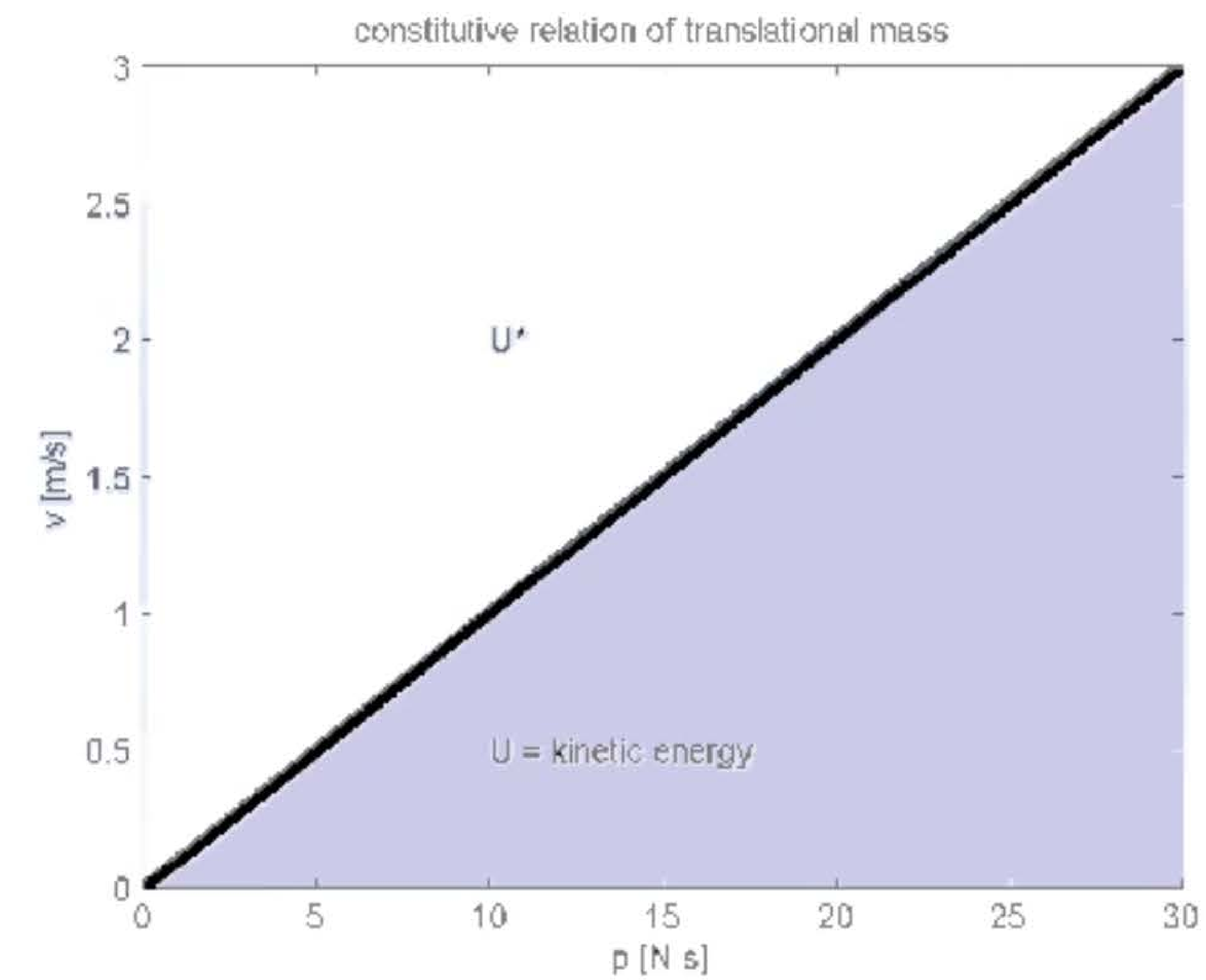
Translational mass

$$p = mv$$

$$p = \int_{t_0}^{t} F\,dt + p(t_0)$$

$$F = \frac{dp}{dt}$$

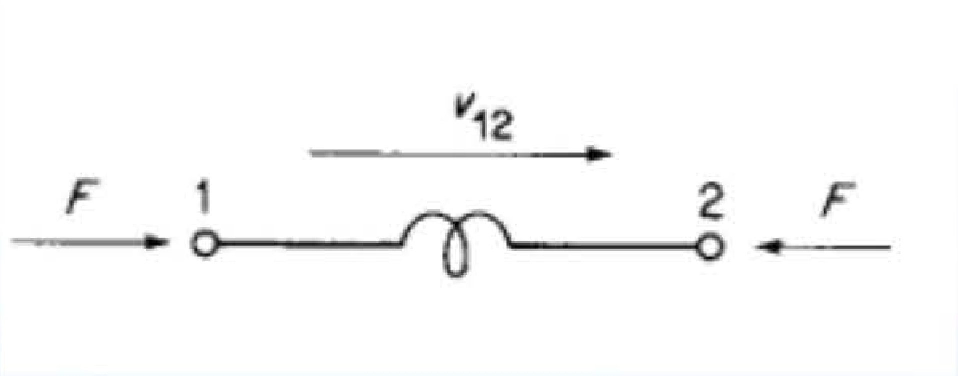
$$U = U^* = \frac{1}{2} m v_{12}^2$$



Mechanical Systems

Translational Spring

A mechanical object which when subject to a force either compresses or elongates without significant acceleration of its component parts, or loss of energy due to friction or unrecoverable deformation is a pure translational spring.



The constitutive relation

$$x_{12} = \phi(F)$$

The variable displacement is defined as:

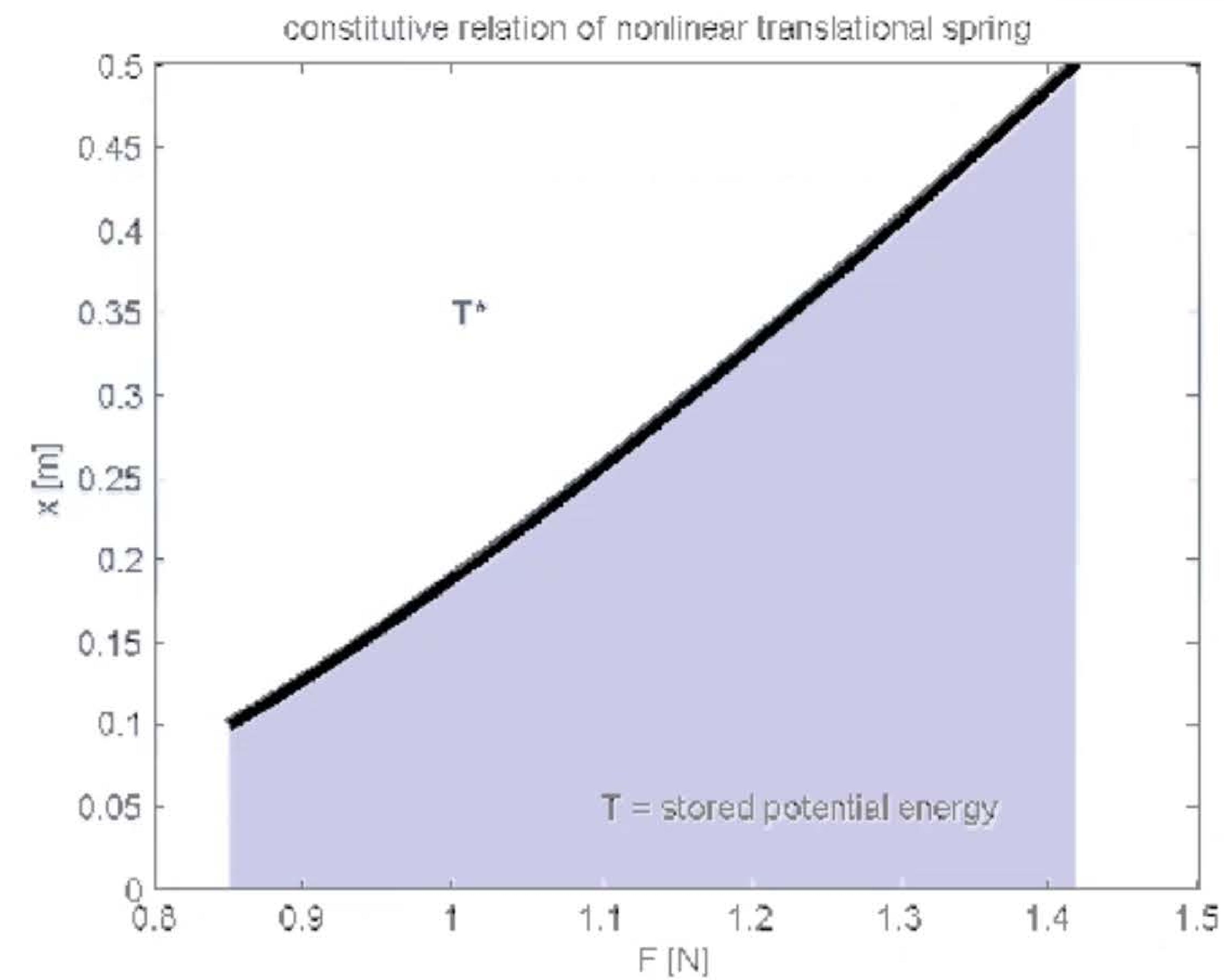
$$x = \int_{t_0}^{t} v\,dt + x(t_0)$$

$$T = \int_0^x F\,dx \qquad T^* = xF + T.$$

Translational Spring – Nonlinear case

$$x_{12} = \phi(F)$$

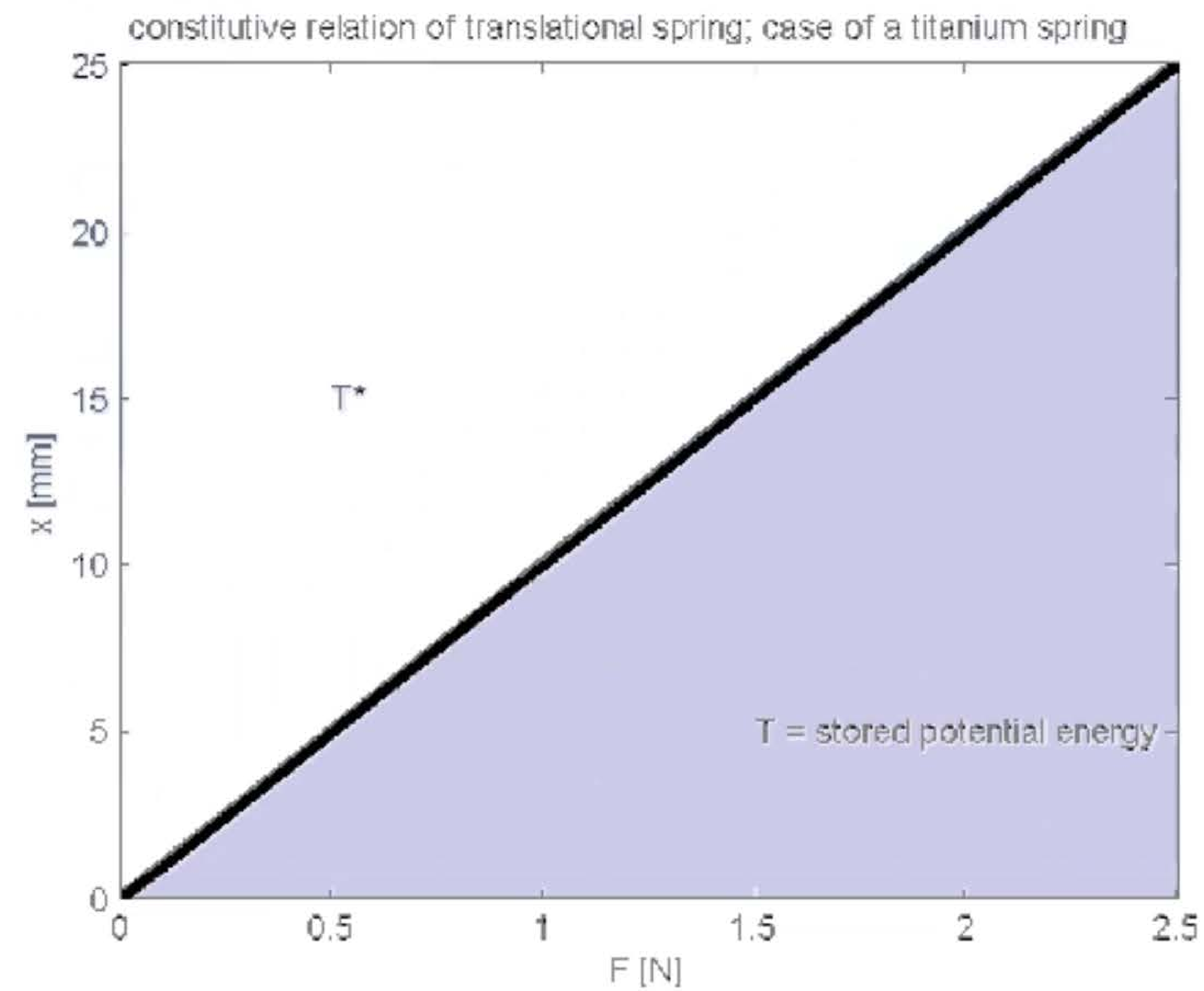
$$F = x + x^{1/8}$$



Translational Spring – Linear case

$$x_{12} = (1/k)F \qquad T = T^* = (1/2k)F^2$$

The spring is extending with a velocity of 5 mm/s for 5 s.



Mechanical Systems

Translational Dissipation

A mechanical object which requires a steady force to maintain a certain velocity displays *dissipative effects*.

Usually, the dissipation of power occurs because e*nergy is being transformed from kinetic energy to thermal energy by viscous friction*. Viscous forces have to be overcomed whenever neighbouring bodies have a *relative* velocity.

Any arrangement which involves the relative motion of *adjacent objects* will incur power dissipation.

Mechanical Systems

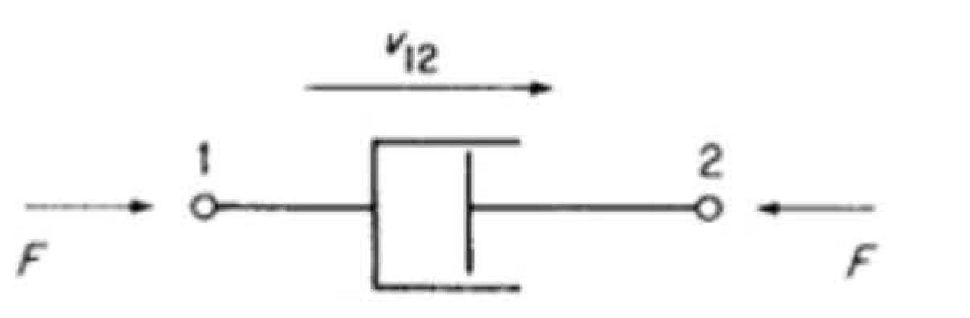
Translational Dissipation

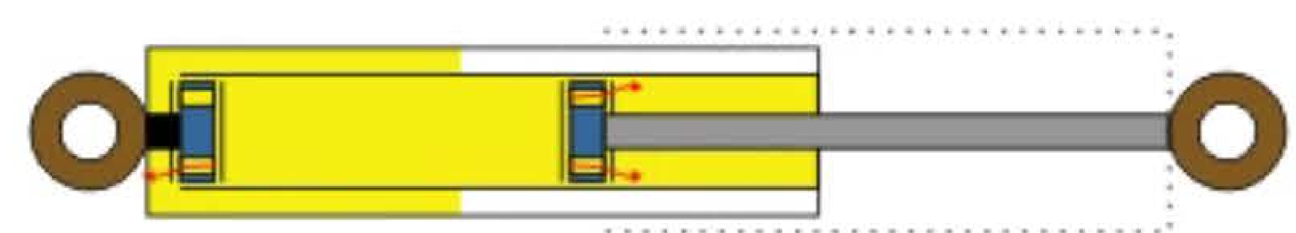
A pure dissipator is one in which the kinetic and potential energy storage phenomena are absent.

A light, rigid object moving through a viscous fluid or sliding along a rough surface will have a *constitutive relation* which statically relates the applied force and relative velocity of the object:

$$F = \phi\left(v_{12}\right)$$

This schematically evokes
the dashpot devices used to
damp the motions of motor car suspension



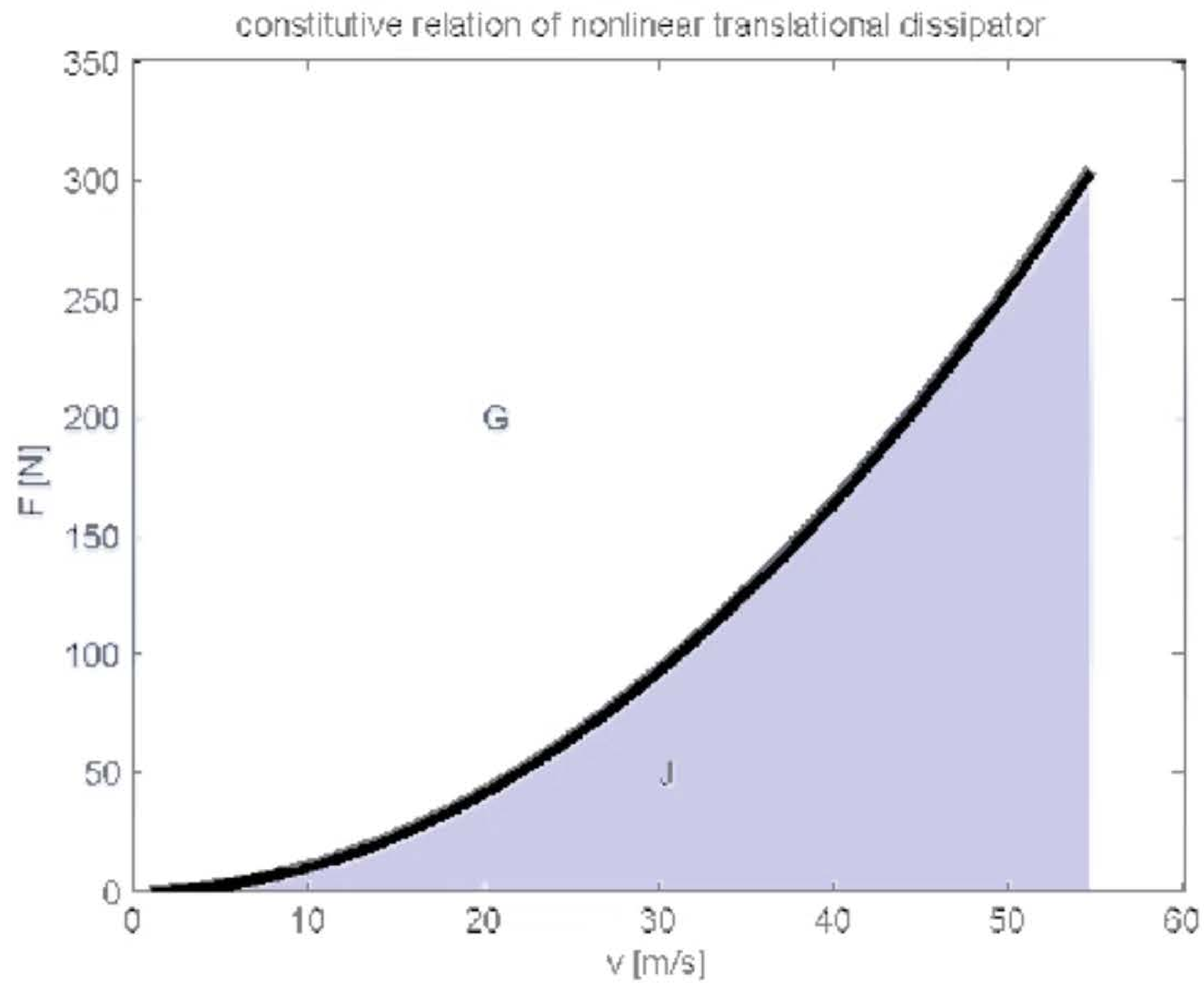


Translational Dissipation – nonlinear case

$$F = \phi(v_{12})$$

$$Ff = 0.1v + 0.1v^2 / v_0$$

$$v_0 = 1$$

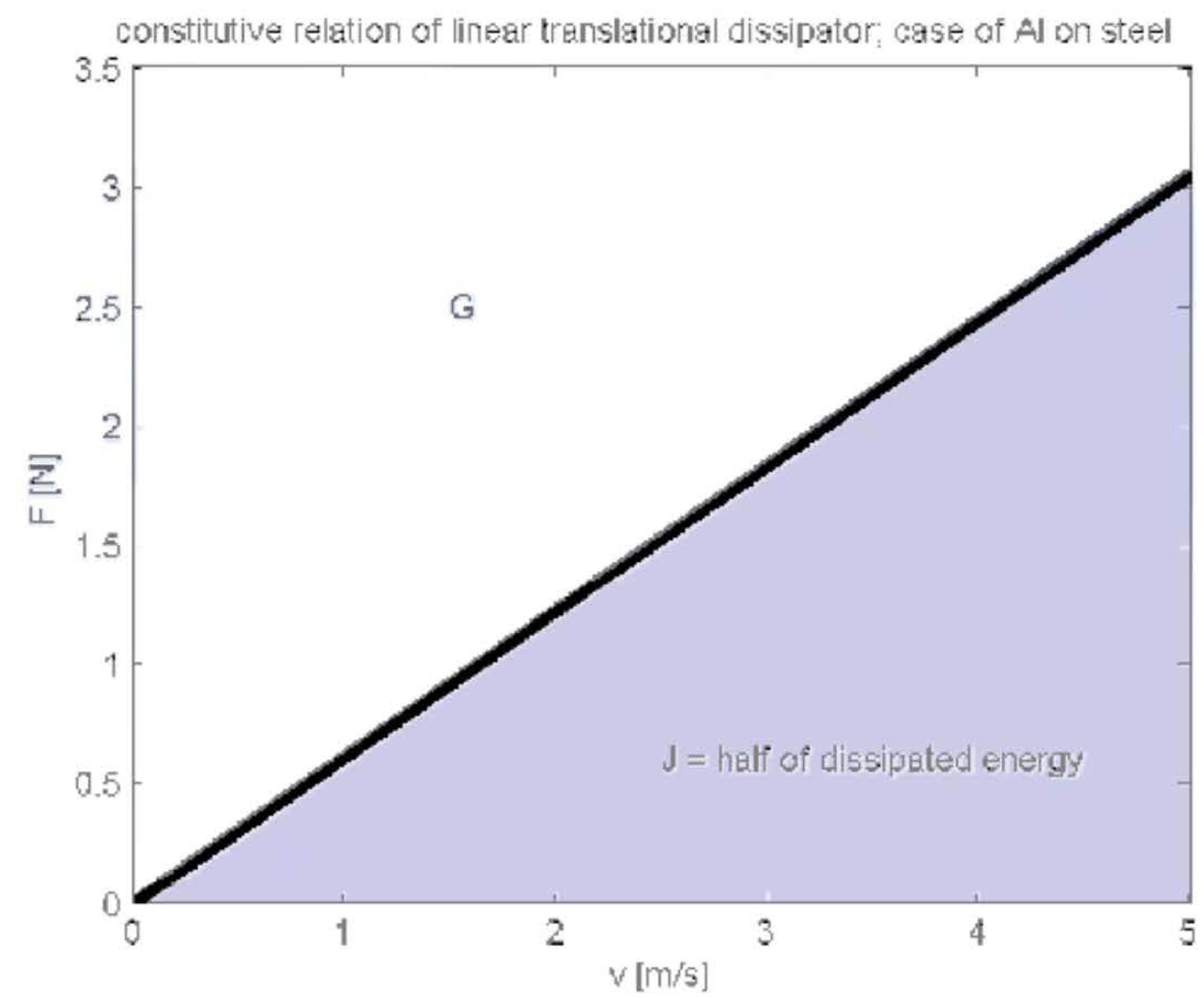


Translational Dissipation – linear case

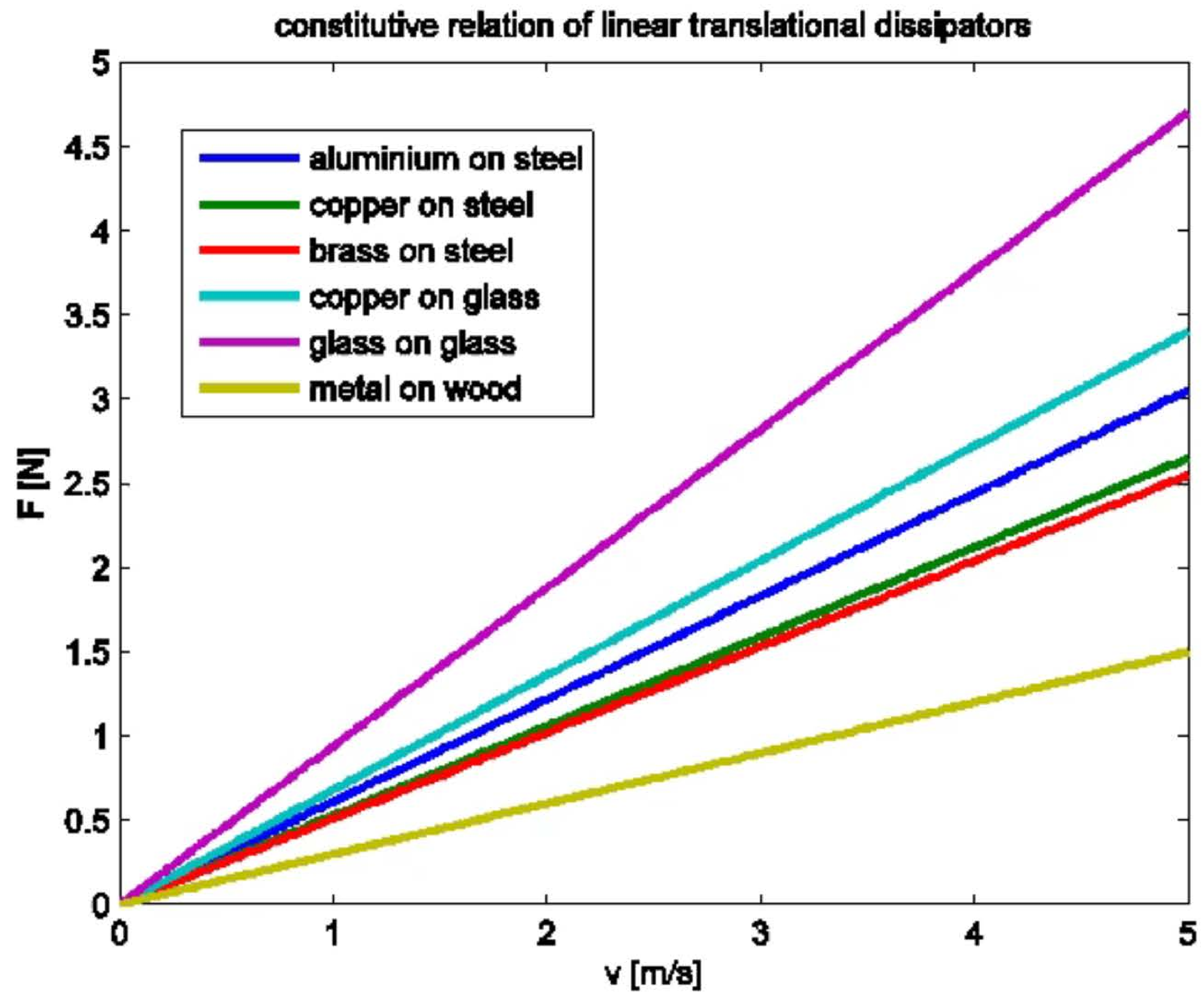
$$F = bv_{12}$$

$$J = G = \frac{1}{2}bv_{12}^{\ 2}$$

$$\text{Power} = bv_{12}^{\ 2} = F^2/b$$



Translational Dissipation – linear case



Electrical components – Constitutive relations

$\lambda_L = L\, I_L$ – flux linkage (stored effort)as a function of current (flow)

$q_C = C\, V_c$ – electric charge (stored flow)as a function of voltage (effort)

$V_R = R\, I_R$ – voltage (effort) as a function of current (flow)

Translational mechanical components – Constitutive relations

$p = mv$ – momentum (stored flow) as a function of velocity (effort)

$x = \frac{F}{k}$ – displacement (stored effort) as a function of force (flow)

$v = \frac{F_{friction}}{b}$ - velocity (effort) as a function of force (flow)