PROCESS MODELING

COURSE for 2nd year,

Automation and Applied Informatics,

TUCN

Analogies between different energy domains

Analogical models, also called "analog" or "analogue" models, are associating :

the **analogue systems** (that share same properties) with the **target system**

The analog systems are made out of components which may differ in substance or structure but **share the properties of dynamic behaviour.**

Since the schematic (networked) representation is widely used in engineering, it is beneficial to seek those analogies that **preserve the network topology**.

The two directions to be followed are:

- **Equivalences at the level of the components** (energy handling and constitutive relations)
- Similarities at the level of the **laws** associated with the **interconnectivity**

Mechanics – Inter-connectivity rules

Sum of all forces (including inertial) applied to a point is zero – ***junction/node law*** (Newton's law)

$$\sum_{i;\, in\ a\ junction} \vec{F}_i = 0$$

Sum of all (relative) velocities around a closed loop defined on a system is zero – ***closed loop law*** (continuity of space law)

$$\sum_{i;around\ a\ loop} \vec{v}_i = 0$$

Electricity – Connectivity laws = Kirchoff's laws

Sum of all currents entering in a junction equals the sum of all currents exiting the junction – ***junction/node law*** (Kirchoff's current law)

$$\sum_{i;in\ the\ junction} I_i = \sum_{j;out\ of\ the\ junction} I_j$$

Sum of all voltages around a closed loop is zero – ***closed loop law*** (Kirchoffs's voltage law)

$$\sum_{i;around\ the\ loop} V_i =$$

Mechanical-Electrical Analogy

Considering the interconnection laws:

The forces **at** a (massless) node must sum to zero, just as the currents **at** a (capacitance-less) node must sum to zero.

The force **through** a set of series elements (except masses) must be the same, just as the current **through** a set of series elements must be the same.

Mechanical-Electrical Analogy

$F = I$ Force is analogous to current

$v = V$ velocity is analogous to voltage

$\frac{1}{b} = R$ Inverse of the friction coefficient is analogous to resistance

$\frac{1}{k} = L$ Inverse of the spring constant is analogous to inductance

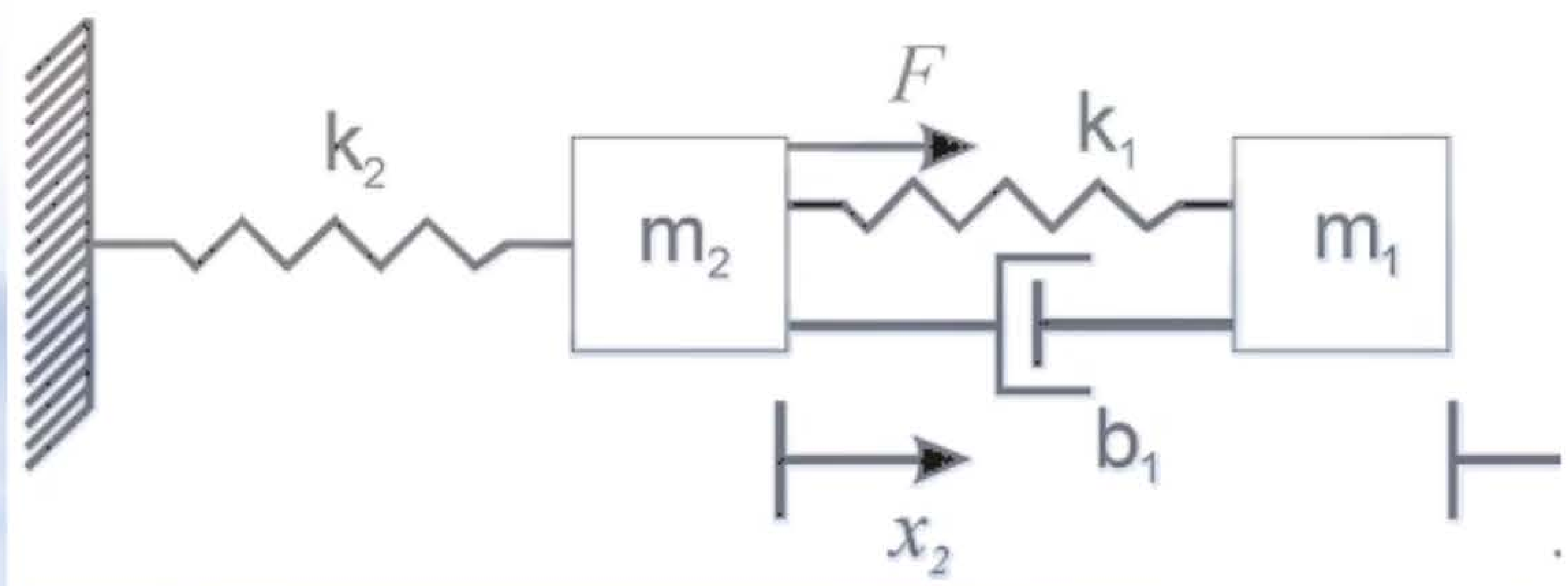
$M = C$ Mass is analogous to capacitance

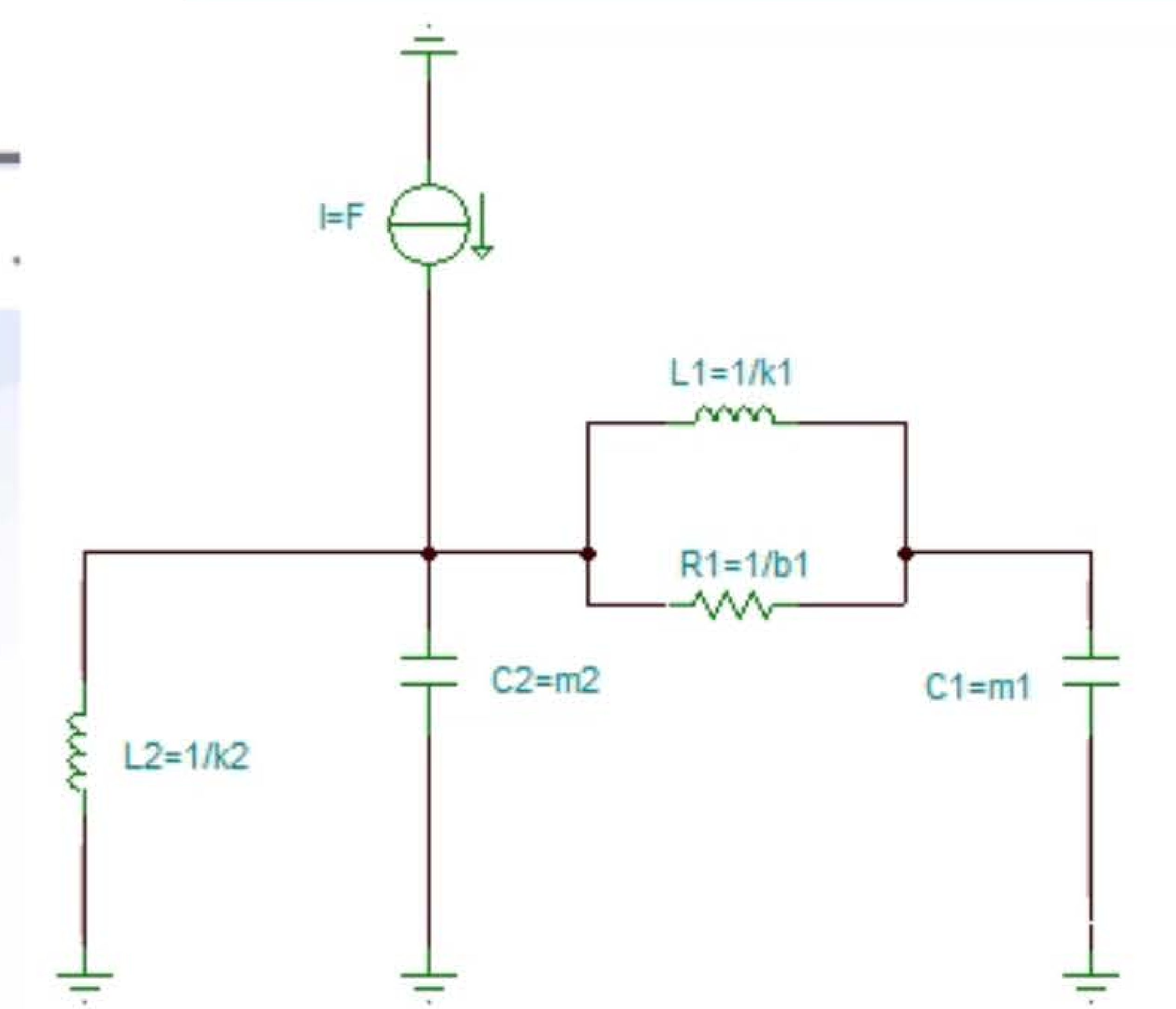
Mechanical-Electrical Analogy

Table 1: Mechanical-Electrical analogy

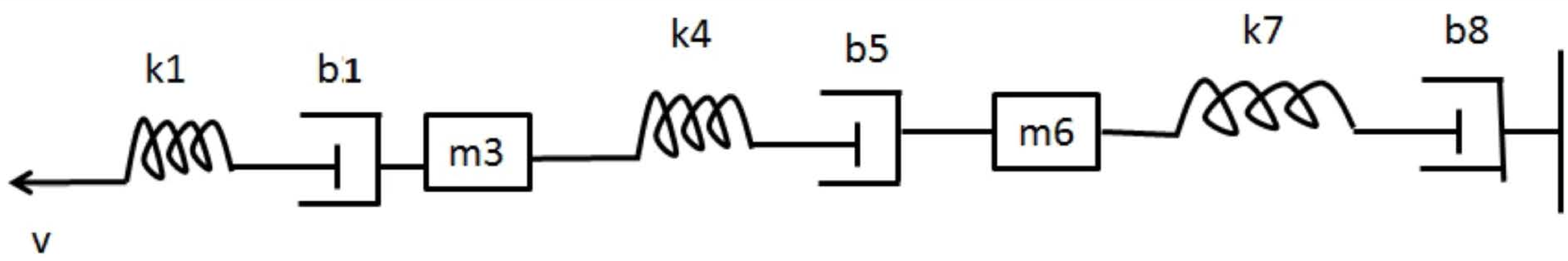
	Mechanical	Electrical
flow	force	current
effort	velocity	voltage
flow store	mass	electrical charge
effort store	displacement of a spring	magnetic flux linkage λ
Elements		
	mass	capacitor
	spring	inductor
	dissipator	resistor
Energy handling		
flow store	stores kinetic energy	stores electric energy
effort store	stores potential energy	stores magnetic energy
dissipator	dissipates kinetic energy	dissipates electric energy
Junction/node law $\sum(flow) = 0$	Newton's law $\sum F = 0$	Kirchoff's Current Law $\sum I = 0$
Closed loop law $\sum(Potential) = 0$	Continuity of space law $\sum v = 0$	Kirchoff's Voltage Law $\sum V = 0$

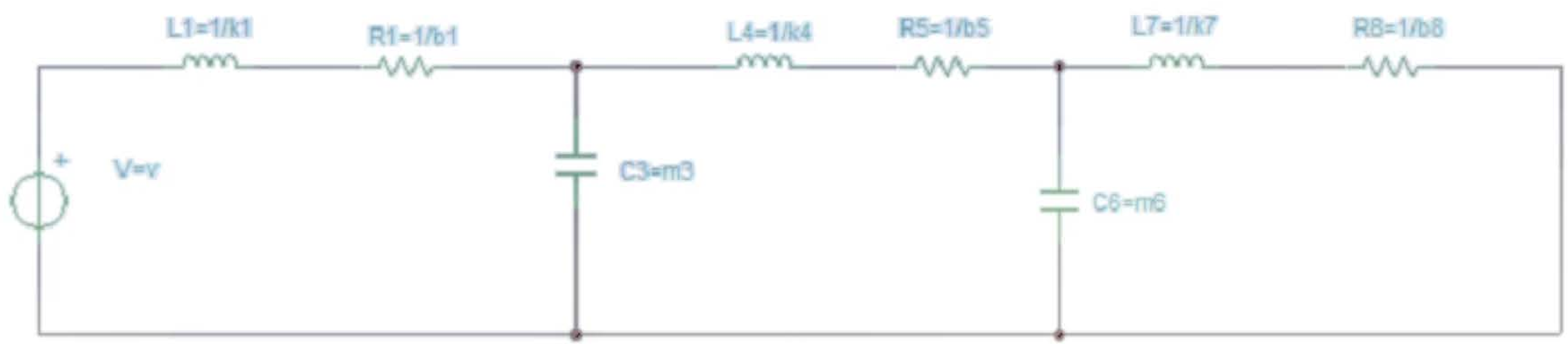
Mechanical-Electrical Analogy - Example 1





Mechanics-Electricity Analogy - Example 2





Mechanical-Electrical Analogy

Remarks on the "reversibility" of the mapping:

1. Translational masses (like thermal capacitors, and most fluid capacitors) have this characteristic mapping: one terminal of the associated electric capacitor must be "grounded" (the across variable is measured with one end at zero).
2. One can construct an electrical analogy for any translational mechanical system (by grounding one end of the associated component), but one can't construct always a thermal or a mechanical analogy for a general electrical network

Hydraulic-Electrical Analogy

A hydraulic system might be seen as an interconnected set of discrete components that transport liquid – a networked structure found also in electrical circuits, seen as networks travelled by movable charge carriers.

Since pressure might be induced by gravity (water reservoirs), the potential energy of the water head is the pressure source. This suggest the idea of associating electric potential with gravitational potential.

The similarities between equations governing the flow of fluid and the flow of charge can be used for building an analogy.

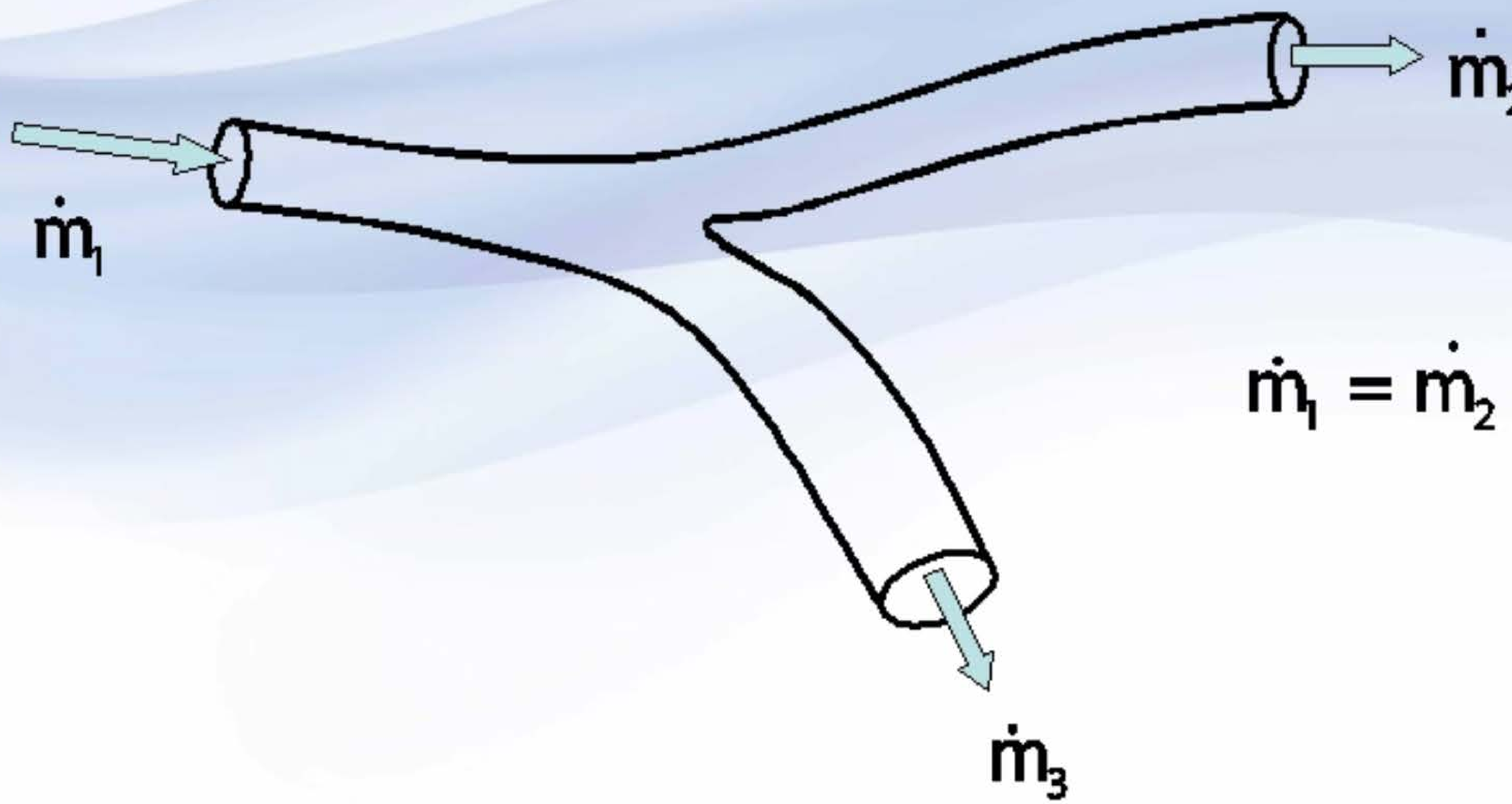
An ideal voltage source or ideal current source can be associated with a pump that (using feedback control) provides a constant pressure or a constant flowrate.

Fluid flow – Interconnectivity rules

Mass cannot be created or destroyed

The sum of all mass flow rates in a junction is zero – ***junction/node law*** (continuity law)

$$\sum_{i;in\ the\ junction} \frac{dm_i}{dt} = \sum_{j;out\ of\ the\ junction} \frac{dm_j}{dt}$$



$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3$$

Fluid flow – Interconnectivity rules

For **incompressible** fluids (liquids) the density remains essentially constant.

The continuity equation can therefore be written in terms of the **volumetric** flowrates:

The sum of all **volume flow rates** in a junction is zero – ***junction/node law*** (continuity law)

$$\sum_{i;in\ the\ junction} \frac{dV_i}{dt} = \sum_{j;out\ of\ the\ junction} \frac{dV_j}{dt}$$

Sum of all pressures measured around a closed loop is zero – ***closed loop law***

$$\sum_{i;around\ a\ loop} p_i = 0$$

Fluid flow – Constitutive relations

$V_c = C_f * P_c$ - volume (stored flow) as a function of pressure (effort)

$\Gamma_L = L_f * Q_L$ - fluid momentum (stored effort) as a function of flow rate (flow)

$P_R = R_f * Q_R$ - pressure (effort) as a function of flow rate (flow)

$C_f = \frac{A}{\varrho g}$ Fluid capacitance

$L_f = \frac{\rho l}{A}$ Fluid inertia

$R_f = \frac{128 \mu l}{\pi d^4}$ Hydraulic resistance

Fluid flow-Electrical Analogy

Q $= I$ Flow rate is analogous to current

$p = V$ Pressure is analogous to voltage

$R_f = R$ Hydraulic resistance is analogous to resistance

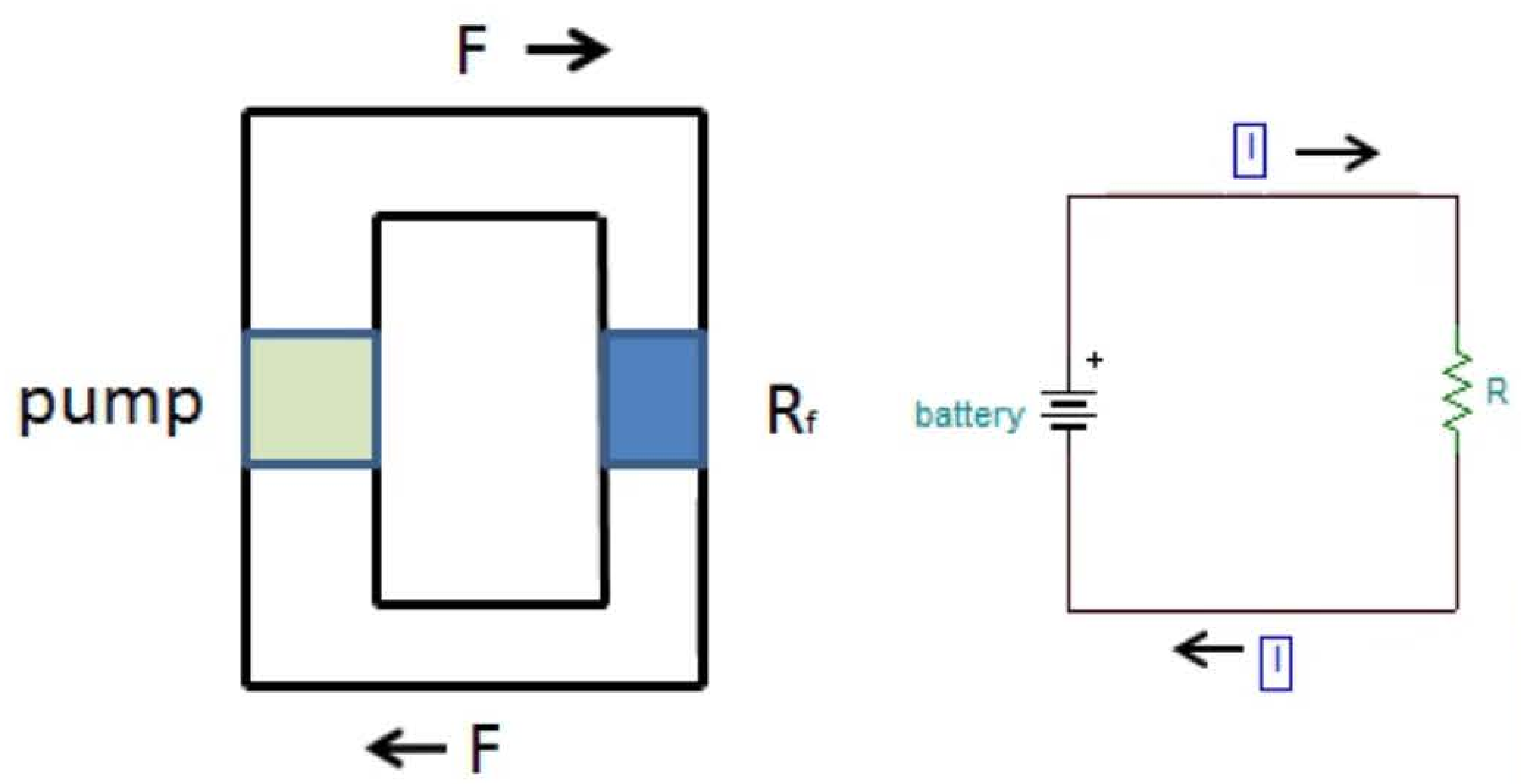
$L_f = L$ Fluid inertia is analogous to inductance

$C_f = C$ Fluid capacitance is analogous to capacitance

Fluid flow-electrical analogy

	Fluid	Electrical
flow	flow rate	current
effort	pressure	voltage
flow store	reservoir/pressurized tank	electrical charge
effort store	fluid flow in a pipe	magnetic flux linkage λ
Elements		
	fluid capacitance	capacitor
	fluid inertia	inductor
	hydraulic resistance	resistor
Energy handling		
flow store	stores potential energy	stores electric energy
effort store	stores kinetic energy	stores magnetic energy
dissipator	dissipates potential, kinetic energy	dissipates electric energy
Junction/node law $\sum(flow) = 0$	Continuity of mass	KCL $\sum I = 0$
Closed loop law $\sum(Potential) = 0$	Bernoulli's Eq.	KVL $\sum V = 0$

Illustration of the junction/node law



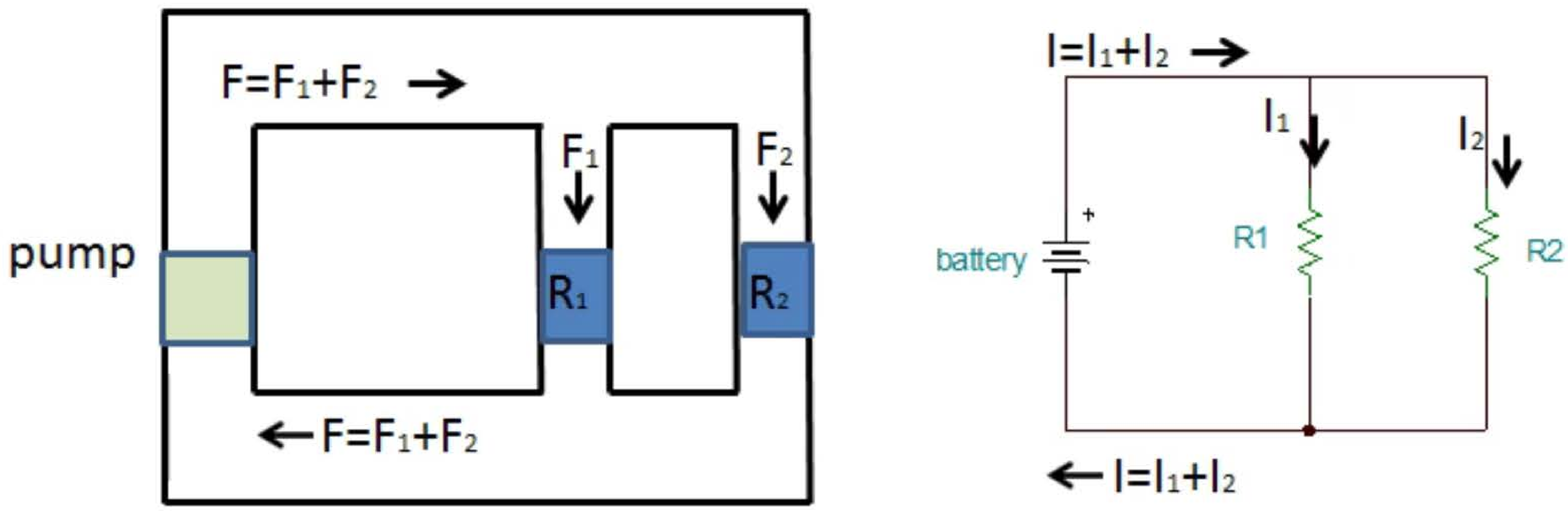
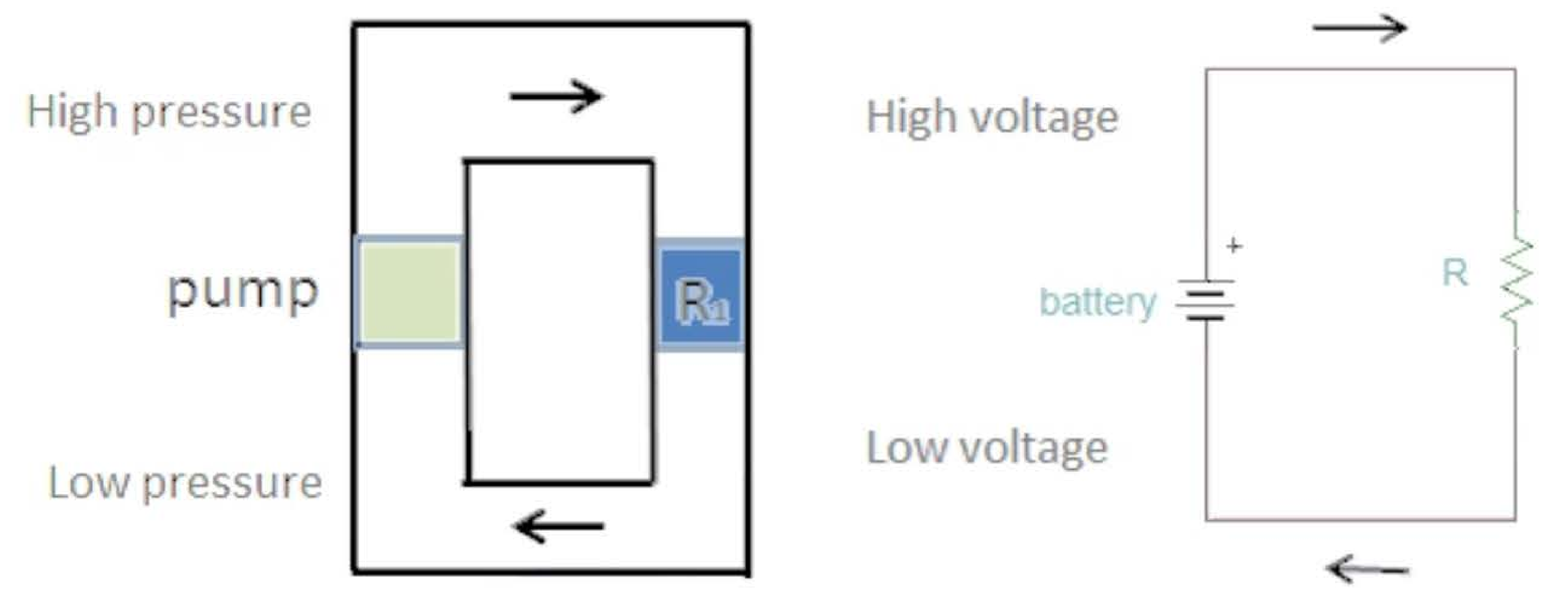
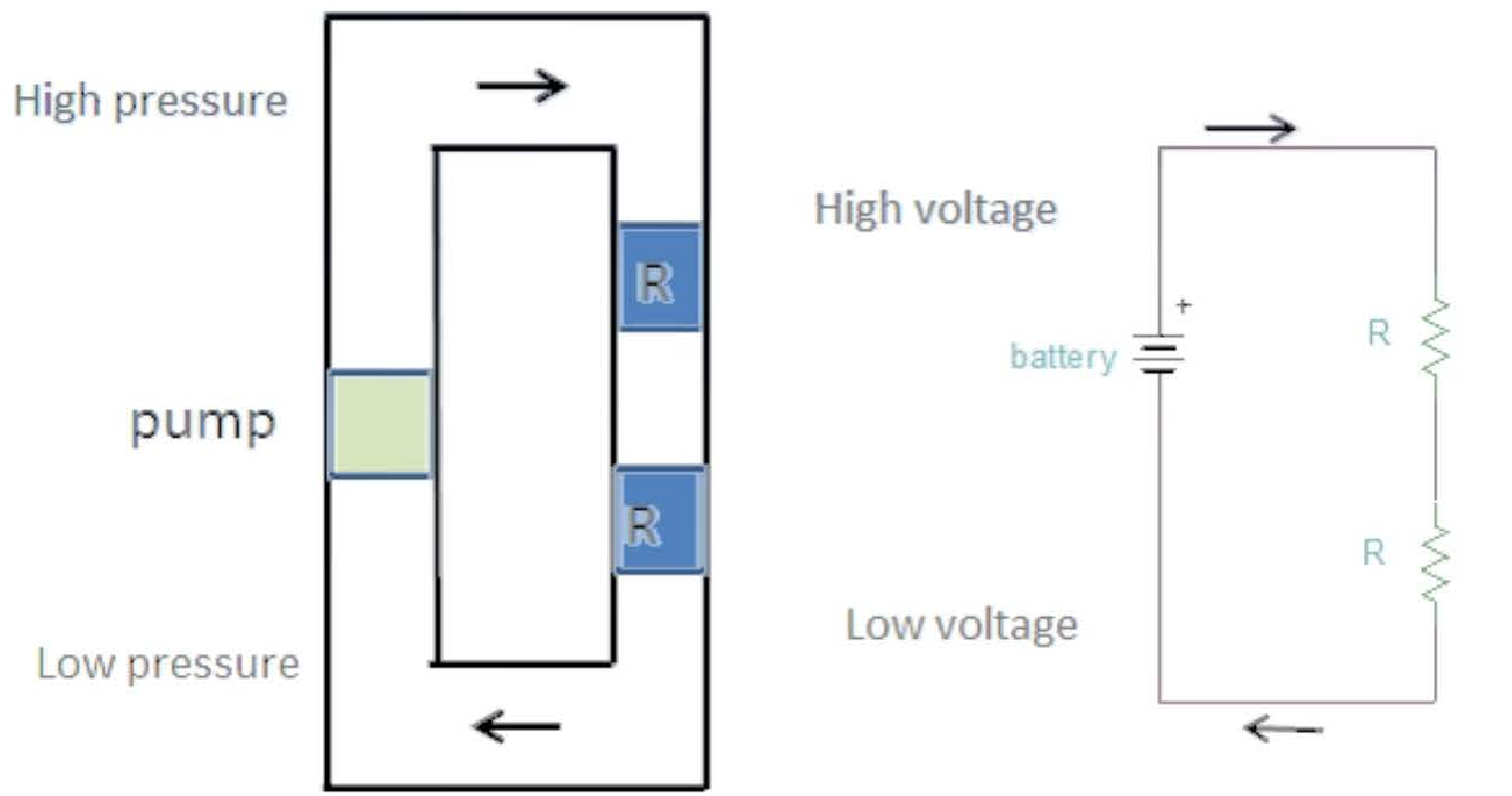
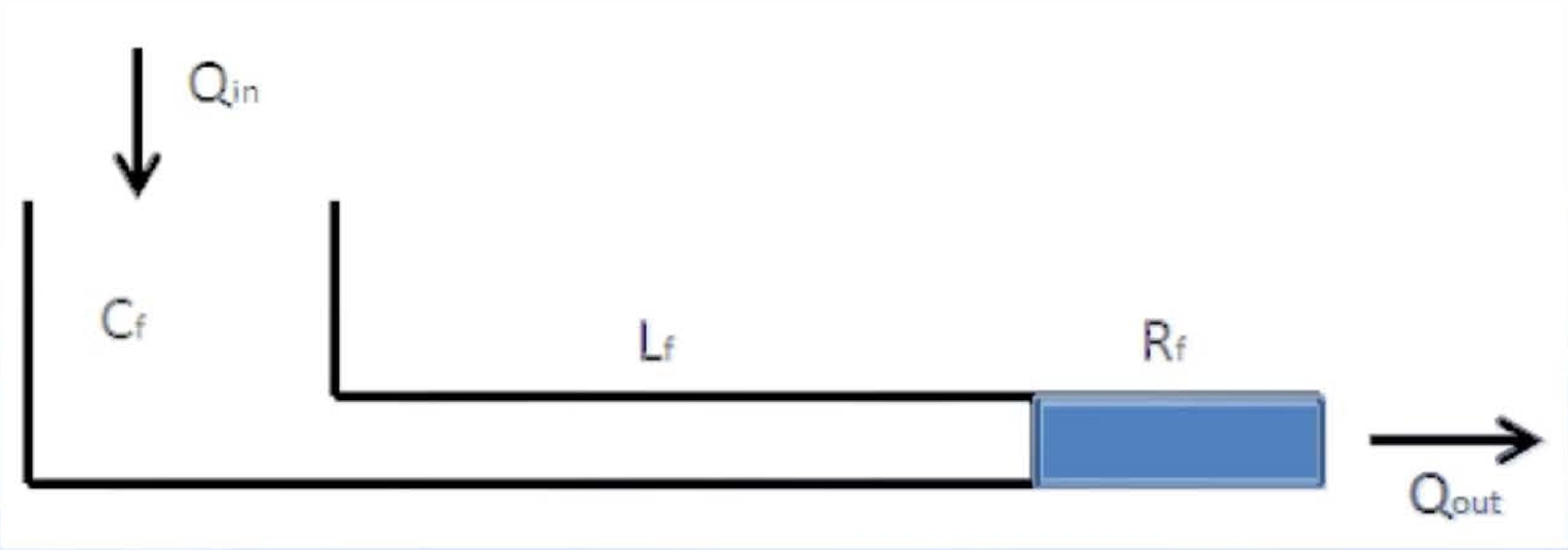


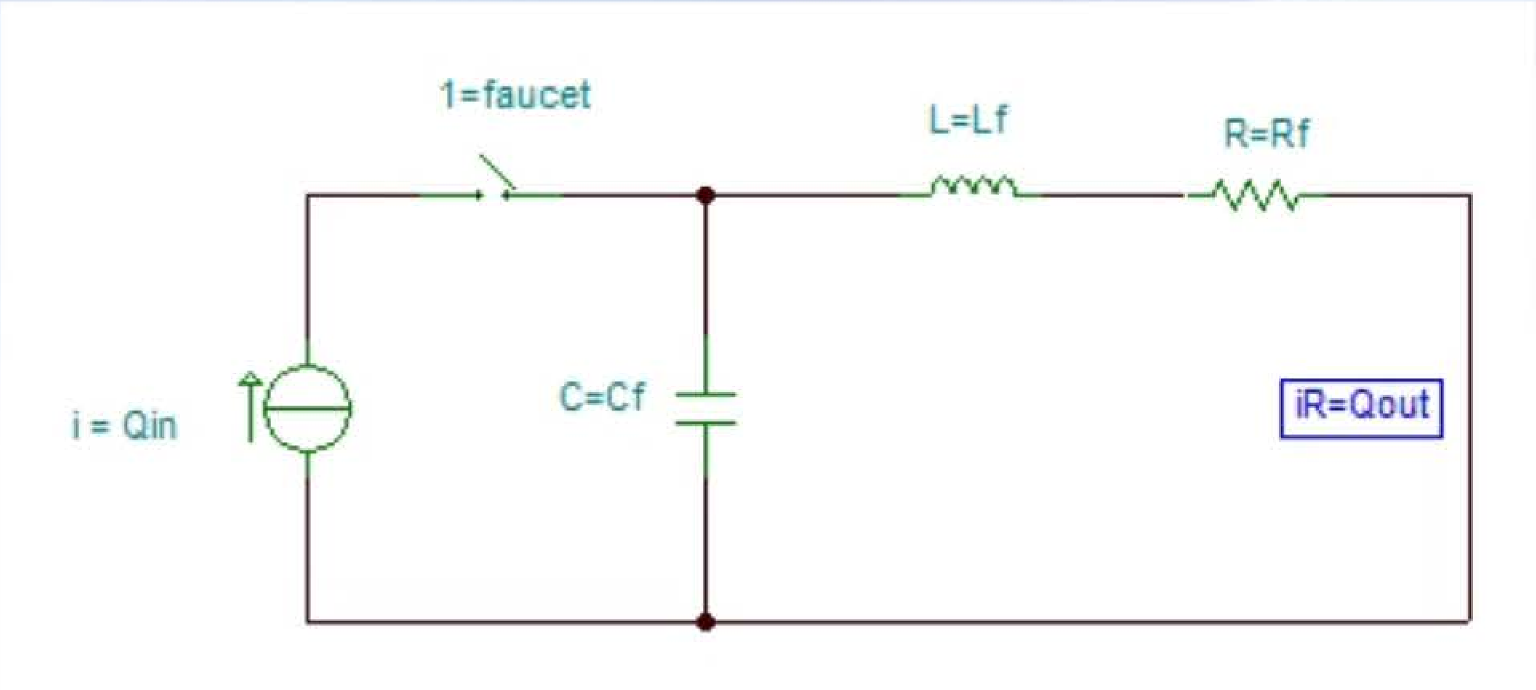
Illustration of the closed loop law





Fluid flow-Electricity Analogy - Example 1

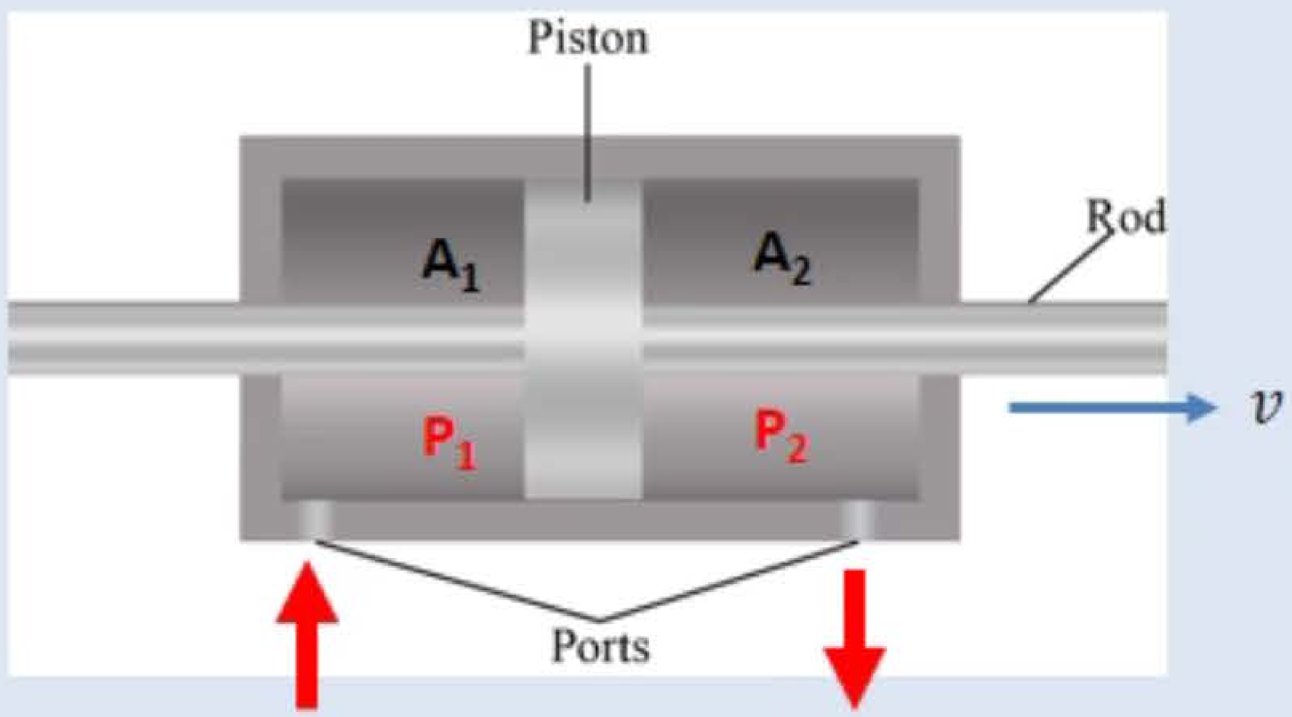


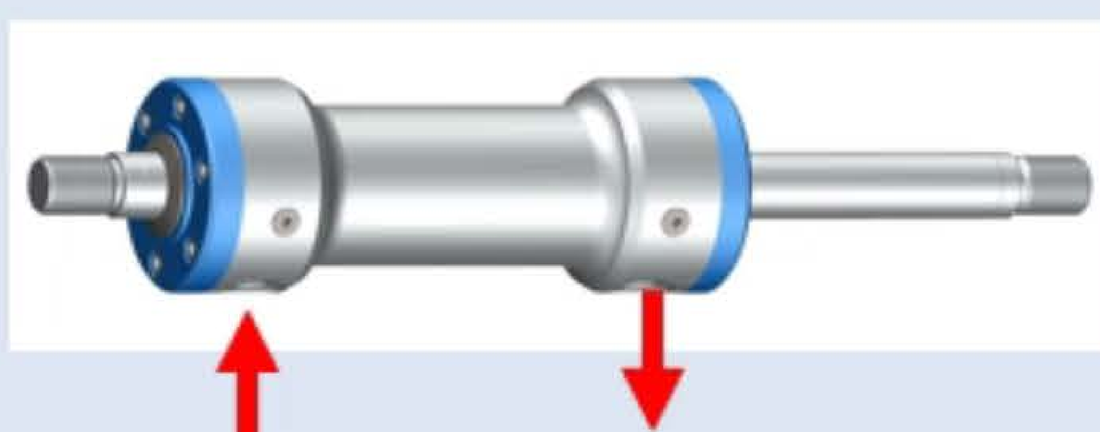


Fluid flow-Electricity Analogy - Example 2

Hydraulic actuators:

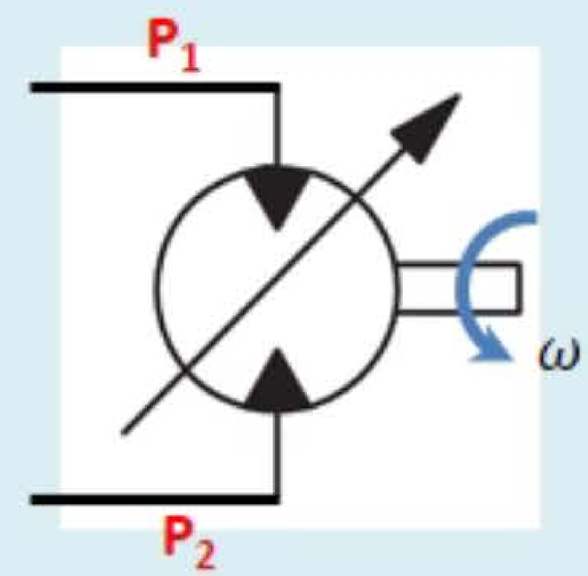
Double-rod cylinder (A_1=A_2) with a motor driven hydraulic pump





Linear actuator:

$$P_1A_1 - P_2A_2 = F_{\text{losses}} + F_{\text{load}} + M\frac{dv}{dt}$$



Pump and motor:

$$D_m(P_1 - P_2) = T_{\text{losses}} + T_{\text{load}} + J\frac{d\omega}{dt}$$

Linear actuator: $P_1A_1 - P_2A_2 = B_v v + M\frac{dv}{dt}$

$$Q = vA_1$$

$$P_1\gamma - P_2 = \boxed{\frac{B_v}{A_1A_2}Q} + \boxed{\frac{M}{A_1A_2}\frac{dQ}{dt}}$$

$$\gamma = \frac{A_1}{A_2}$$

Pump+motor:

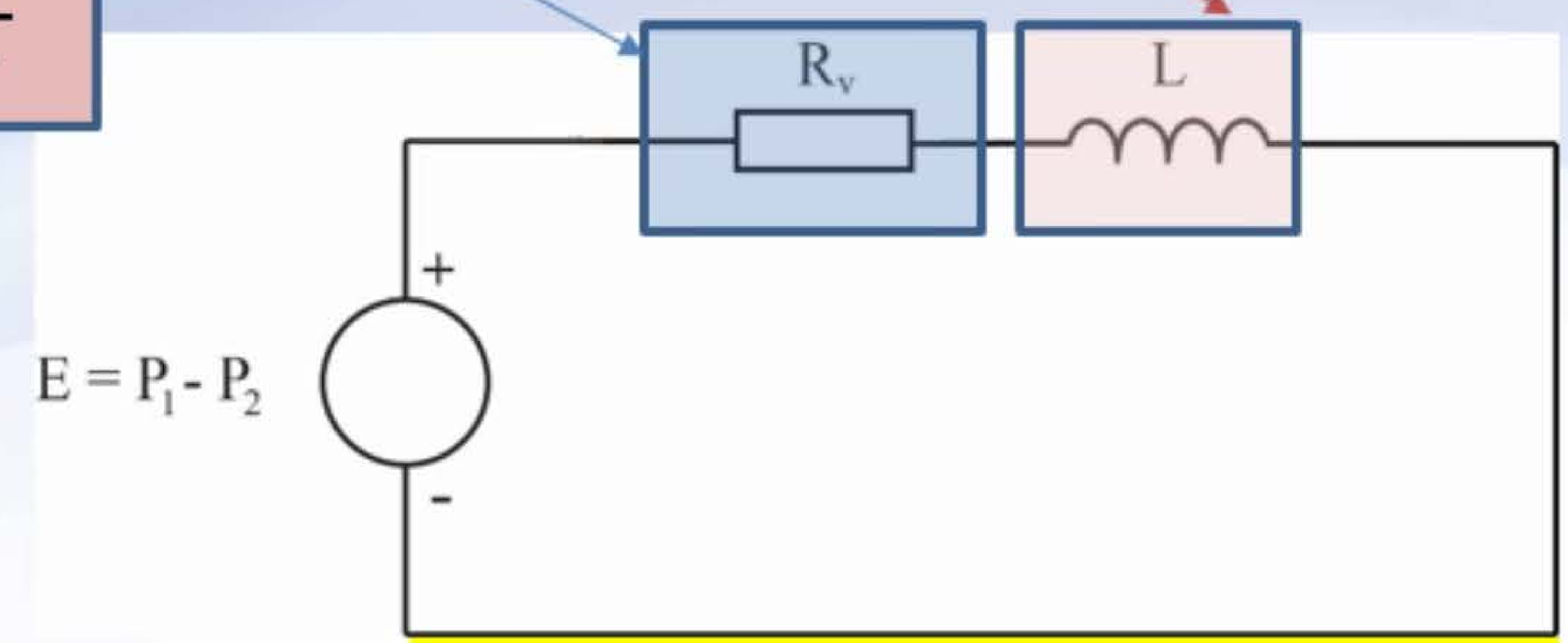
$$D_m(P_1 - P_2) = B_v\omega + J\frac{d\omega}{dt}$$

$$Q = D_m\omega$$

$$P_1 - P_2 = \boxed{\frac{B_v}{D_m^2}Q} + \boxed{\frac{J}{D_m^2}\frac{dQ}{dt}}$$

$$\boxed{Ri} \qquad \boxed{L\frac{di}{dt}}$$

The equivalence of the *effort type* variables (voltage drop = pressure drop), and of the *flow type* variables (current = flowrate), combined with the *constitutive equations* for resistor and inductor are pointing out toward the corresponding expressions for the ***R**$_v$* and ***L*** values



Linear actuator:

viscous resistance $R_v = \frac{B_v}{A_1A_2}$,

inductance, $L = \frac{M}{A_1A_2}$

Pump and motor:

viscous resistance, $R_v = \frac{B_v}{D_m^2}$, inductance, $L = \frac{J}{D_m^2}$

The form of the two equations leads to the above analogue electric circuit