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Tema lab 8

1. $h(t) = e^{-4|t|}$

$$H(k\omega_0) = \int_{-\infty}^{\infty} h(\tau)\cdot e^{-jk\omega_0\tau}\,d\tau = \int_{-\infty}^{\infty} e^{-4|t|}\cdot e^{-jk\omega_0\tau}\,d\tau =$$

$$= \int_{-\infty}^{0} e^{+4\tau}\cdot e^{-jk\omega_0\tau} + \int_{0}^{\infty} e^{-4\tau}\cdot e^{-jk\omega_0\tau}\,d\tau =$$

$$= \frac{1}{4-jk\omega_0}\cdot e^{(4-jk\omega_0)\tau}\Big|_{-\infty}^{0} + \frac{1}{-4-jk\omega_0}\cdot e^{(4-jk\omega_0)\tau}\Big|_{0}^{\infty} =$$

$$= \frac{1}{4-jk\omega_0} + \frac{1}{4+jk\omega_0} = \frac{4+jk\omega_0-4+jk\omega_0}{16+(k\omega_0)^2} = \frac{2jk\omega_0}{16+(k\omega_0)^2}$$

Fourier series repr. of the output $y(t)$ for these input signals

a) $x(t) = \cos(2\pi t)$

$$\Rightarrow \omega_0 = 2\pi \;;\quad x(t) = \frac{1}{2}\left(e^{2\pi t j} + e^{-2\pi t j}\right) = \sum_{k=-1}^{1} a_k\cdot e^{jk2\pi t}$$

where $a_0 = 0$ and $a_{-1} = a_1 = \frac{1}{2}$

$$y(t) = \sum_{k=1}^{1} b_k\cdot e^{jk2\pi t}\;, \text{ unde } b_0 = a_0\cdot H(0) = 0$$

$$b_1 = a_1\cdot H(\omega_0) = \frac{1}{2}H(2\pi) = \frac{1}{2}\cdot\frac{4\pi j}{16+4\pi^2} = \frac{\pi j}{8(1+\frac{1}{2}\pi^2)}$$

$$b_{-1} = a_{-1}\cdot H(-\omega_0) = \frac{1}{2}\cdot H(-2\pi) = \frac{1}{2}\cdot\frac{4\pi j}{16+4\pi^2} = \frac{\pi j}{8(1-\frac{1}{2}\pi^2)}$$

b) $x(t) = \sum_{n=-\infty}^{0} \delta(t-n) \Rightarrow T_0 = 1,\ \omega_0 = 2\pi$

$$a_k = \frac{1}{T_0}\int_{T_0} x(t)\cdot e^{-jk\omega_0 t}\,dt = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \delta(t)\cdot e^{-jk\omega_0 t}\,dt = \frac{1}{T_0}\cdot e^{-j\omega_0 k t}\Big|_{t=0} = \frac{1}{T_0} = 1$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k\cdot e^{jk\omega_0 t} = \frac{1}{T_0}\sum_{k=-\infty}^{\infty} e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} e^{jk2\pi t}$$

$$b_k = a_k\cdot H(2\pi k) = \frac{1}{T_0}H(2\pi k) = \frac{1}{T_0}\cdot\frac{4jk\pi_0}{16+4\pi^2k^2} = \frac{4\pi k j}{4+\pi^2k^2}$$

$$\Rightarrow y(t) = \sum_{k=-\infty}^{\infty} \frac{\pi k j}{4+(\pi k)^2}\cdot e^{jk2\pi t}$$

c) $x(t) = \sum_{n=-\infty}^{\infty} (-1)^n \delta(t-n) \quad \Rightarrow T_0 = 2, \; w_0 = \pi$

$$a_k = \frac{1}{T_0} \int_{T_0} x(t) \cdot e^{-jkw_0 t} dt = \frac{1}{T_0} \left(\int_{-\frac{T_0}{2}}^{\frac{T_0}{2}} \delta(t) \cdot e^{-jkw_0 t} dt + \int_{\frac{T_0}{2}}^{\frac{3T_0}{4}} -\delta\left(t - \frac{T_0}{2}\right) \cdot e^{-jkw_0 t} dt \right)$$

$$= \frac{1}{T_0} \left(e^{-jkw_0 t} \Big|_{t=0} - e^{-jkw_0 t} \Big|_{t=\frac{T_0}{2}} \right) = \frac{1}{T_0} \left(1 - e^{-jkw_0 \cdot \frac{T_0}{2}}\right) = \frac{1}{T_0} \left(1 - e^{-jk\pi}\right)$$

$$= \frac{1}{T_0} \left(1 - (-1)^k\right) = \frac{1}{2} \left(1 - (-1)^k\right)$$

$$x(t) = \sum_{n=-\infty}^{\infty} a_k \cdot e^{-jkw_0 t} = \frac{1}{2} \sum_{n=-\infty}^{\infty} \left(1 - (-1)^k\right) \cdot e^{jk\pi t}$$

$$b_k = a_k \cdot H(kw_0) = \frac{1 - (-1)^k}{T_0} \cdot H(k\pi) = \frac{1 - (-1)^k}{T_0} \cdot \frac{2k\pi j}{16 + (k\pi)^2}$$

$$\Rightarrow b_k = \begin{cases} \frac{2k\pi j}{16 + (k\pi)^2} & , \text{ for } k\text{-odd} \\ 0 & , \text{ for } k\text{-even} \end{cases}$$

$$y(t) = \sum_{n=-\infty}^{\infty} b_k \cdot e^{jk\pi t}$$