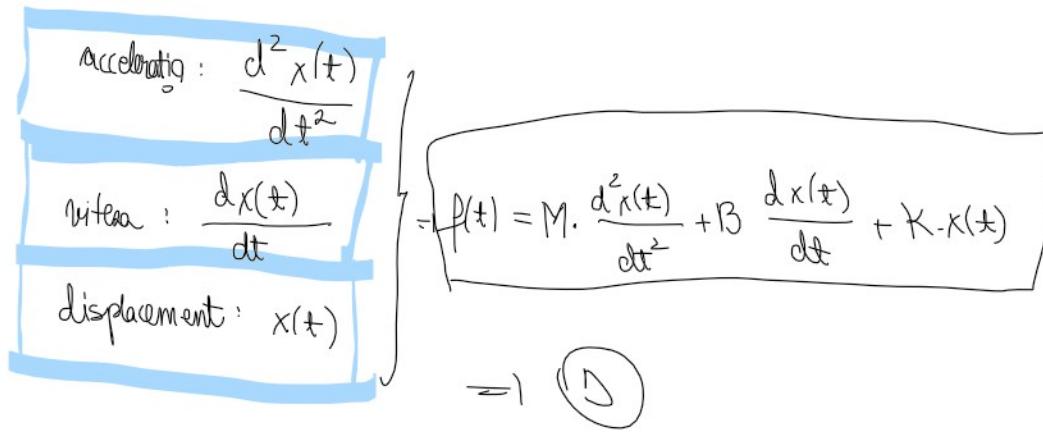


1. Modelul matematic al sistemului mecanic din Fig1 este dat de ecuațiile:

- A. $f(t) = M \frac{dx(t)}{dt} + B \frac{d^2x(t)}{dt^2} + Kx(t)$
 B. $f(t) = M \frac{d^2x(t)}{dt^2} - B \frac{dx(t)}{dt} + Kx(t)$
 C. $f(t) = M \frac{d^2x(t)}{dt^2} + B \frac{d^2x(t)}{dt^2} + Kx(t)$
 D. $f(t) = M \frac{d^2x(t)}{dt^2} + B \frac{dx(t)}{dt} + Kx(t)$

Mecanic \rightarrow Electric

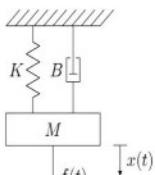
$$\begin{array}{ll} f(t) & u \\ B & R \\ K & L \\ M & C \end{array}$$



2. Considerand variabilele de stare x_1 =pozitia si x_2 =viteza masei M, pt sistemul din Fig1, ecuațiile de stare sunt:

2. Considering the state variables x_1 = position and x_2 =velocity of mass M, for the system in figure, the state equations are:

- A. $\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{B}{M} & -\frac{K}{M} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ M \end{pmatrix} u$
 B. $\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{B}{M} & -\frac{K}{M} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{M} \end{pmatrix} u$
 C. $\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{B}{M} & -\frac{B}{M} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{M} \end{pmatrix} u$
 D. $\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{B}{M} & \frac{B}{M} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$



$$\left\{ \begin{array}{l} x_1 = x(t) \quad \leftarrow \text{pozitie} \\ x_2 = \frac{dx(t)}{dt} \quad \leftarrow \text{viteza} \end{array} \right.$$

$$\left\{ \begin{array}{l} \dot{x}_3 = \ddot{x}_2 \\ \dot{x}_2 = \ddot{x}_1 \end{array} \right. \quad \leftarrow \text{acceleration} \quad \downarrow \quad \Rightarrow \ddot{x}_3 = \ddot{x}_1$$

$$f(t) = M \cdot \frac{d^2 x_1(t)}{dt^2} + B \cdot \frac{dx_1(t)}{dt} + k x_1(t) \quad \xrightarrow{\text{eigener } f(t)=u(t)}$$

$$u(t) = M \cdot \frac{d^2 x_1(t)}{dt^2} + B \cdot \frac{dx_1(t)}{dt} + k x_1(t)$$

$$\Rightarrow -M \cdot \frac{d^2 x_1(t)}{dt^2} = -u(t) + B \cdot \frac{dx_1(t)}{dt} + k x_1(t) \quad : \boxed{-M}$$

$$\dot{x}_2 = \frac{d^2 x_1(t)}{dt^2} = \frac{u(t)}{M} - \frac{B}{M} \frac{dx_1(t)}{dt} - \frac{k x_1(t)}{M}$$

$x_1 = x_2$

$\dot{x}_1 = \frac{dx_1(t)}{dt} = x_2$

$$\dot{x}_1 = x_2 \Rightarrow \dot{x}_2 = \ddot{x}_1$$

$$\dot{x}_2 = \frac{u(t)}{M} - \frac{B}{M} \cdot x_2 - \frac{k}{M} x_1$$

$$\left\{ \begin{array}{l} \dot{x}_2 = \frac{u(t)}{M} - \frac{B}{M} x_2 - \frac{k}{M} x_1 \\ \dot{x}_1 = x_2 \end{array} \right. \quad \leftarrow$$

$$\dot{x}_1 = x_2$$

$$\left(\begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \end{array} \right) = \left(\begin{array}{cc} 0 & 1 \\ -\frac{k}{M} & -\frac{B}{M} \end{array} \right) \left(\begin{array}{c} x_1 \\ x_2 \end{array} \right) + \left(\begin{array}{c} 0 \\ \frac{1}{M} \end{array} \right) u(t)$$

$0 \cdot x_1 + 1 \cdot x_2 + 0 \cdot u(t) + -\frac{k}{M} \cdot x_1 + -\frac{B}{M} \cdot x_2 + \frac{1}{M} \cdot u(t)$

3. Pentru un compresor care are diametrul conductei de admisie 50mm și cel al conductei de evacuare 20mm se cere viteza v_1 a aerului la admisie și viteza v_2 a aerului în conductă de evacuare știind următoarele: presiunea la intrare 1 atm, presiunea la ieșire 2 atm, temp. aerului la intrare 26,85°C, temp. aerului la ieșire 46,85°C, debitul la intrare 3,14 m³/min. Se consideră valoarea relativă pt. densitatea $\rho = f(T)$, unde $f = \text{presiunea și } T = \text{temp. în grade Kelvin, R constantă.}$

la intrare: $P = 1 \text{ atm}$

$t = 26,85^\circ\text{C}$

debit $Q = 3,14 \text{ m}^3/\text{min}$

la ieșire: $P = 2 \text{ atm}$

$t = 46,85^\circ\text{C}$

$m = \text{masă}$
 $P = \text{Pa}$

$\mu = \text{mild}$

$$P = \rho R T$$

$K \leftarrow \text{greșit}$
 $K \leftarrow \text{corect}$

Se dă: la intrare: $P = 1 \text{ atm}$
debit $\rightarrow Q_1 = 3,14 \text{ m}^3/\text{min}$
diametru = 50 mm
 $t = 26,85^\circ\text{C} \Rightarrow T = 300 \text{ K}$

la ieșire: $P = 2 \text{ atm}$

$t = 46,85^\circ\text{C} \Rightarrow T = 320 \text{ K}$
diametru = 20 mm

Se cere $v_1 \leftarrow$ viteza aer la admisie
 $v_2 \leftarrow$ viteza aer la evacuare

1) Pentru admisie

$$Q = A v$$

$$A = \pi \cdot r^2$$

$$\text{diam} = 2 \cdot r$$

$$\Rightarrow r = 25 \text{ mm}$$

$$A = 3,14 \cdot 25^2 =$$

$$Q_1 = A \cdot v_1$$

$$Q = \frac{3,14}{60} \text{ m}^3/\text{s} ; \quad A = 3,14 \cdot 625 \cdot 10^{-4}$$

$$v_1 = \frac{Q}{A} = \frac{3,14}{60} \frac{\text{m}^3}{\text{s}} \cdot \frac{1}{3,14 \cdot 625 \cdot 10^{-4} \text{ m}^2} = \frac{10}{60 \cdot 625} \frac{\text{m}}{\text{s}}$$

$$\Rightarrow v_1 = 2 \frac{\text{m}}{\text{s}}$$

2) Pentru evacuare

$$P_1 = 1 \text{ atm} = 10^5 \text{ Pa} \Rightarrow R = 8,31 \cdot \frac{J}{\text{mol} \cdot \text{K}}$$

$$P \cdot V = \gamma RT$$

$$\frac{P_1 \cdot V_1}{R \cdot T_1} = \gamma \Rightarrow \gamma = \frac{P_1 \cdot V_1}{R \cdot T_1} \Rightarrow \gamma = \frac{10^5 \cdot V_1}{R \cdot 300}$$

$$\gamma = \frac{P \cdot V}{RT}$$

$$P \cdot V = \gamma RT$$

$$\underline{P \cdot V = \frac{m}{M} RT} : V$$

$$\underline{\underline{P = \frac{m}{M} RT}}$$

$$\frac{P_1}{\frac{m}{M} V_1} = \frac{P_2}{\frac{m}{M} V_2} \Rightarrow \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} - \text{Volume}$$

$$V_2 = \frac{8}{15} V_1 - \text{volume}$$

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\frac{P_1 \cdot \cancel{m}}{\cancel{T_1} \cdot \cancel{V_1}} = \frac{P_2 \cdot \cancel{m}}{\cancel{T_2} \cdot \cancel{V_2}}$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

m - constantă
 γ - constantă

P_1 V_1 T_1 P_2 V_2 T_2

$P_1 A_1 V_1 = P_2 A_2 V_2$

$$\gamma = \frac{P}{RT} \Rightarrow \begin{cases} P_1 = \frac{\gamma \cdot R \cdot T_1}{V_1} \\ P_2 = \frac{\gamma \cdot R \cdot T_2}{V_2} \end{cases}$$

$$\gamma r_2 = \frac{P_1 A_1 V_1}{P_2 A_2}$$

$$A > \frac{\pi r_2^2}{4}$$

$$r_2 = \frac{d_2}{2} = \frac{20}{2} = 10 \text{ mm} = 0,1 \text{ m}$$

$$A_2 = 3,14 \cdot (0,1)^2 = 3,14 \cdot 0,0001$$

$$\frac{P_1}{RT_1} \cdot A_1 V_1 = \frac{P_2}{RT_2} \cdot A_2 V_2$$

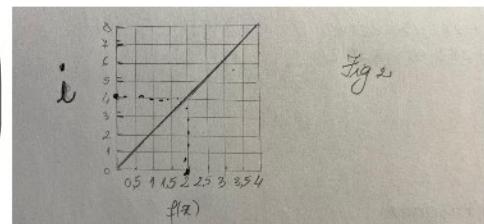
$$\begin{aligned} A_1 V_2 &= \frac{P_1}{RT_1} A_1 V_1 \cdot \frac{RT_2}{P_2 A_2} \\ &= \frac{1}{P} \cdot \frac{3,14 \cdot 0,000025}{300} \cdot 20 \cdot \frac{1}{2 \cdot 3,14 \cdot 0,0001} \\ &= \frac{32}{2 \cdot 30} \cdot \frac{0,000025}{0,0001} \cdot 20 \\ &= 3,33 \cdot V_1 \end{aligned}$$

4. Considerăm un inductor pentru care relația $\lambda = \varphi(i)$ poate fi descrisă prin $\lambda = Li$, unde L este exprimat în H și i în A . Pentru $L = 10 \text{ mH}$ și semnalele de curent descrise în Figura 2, energia acumulată în inductor în primele 2 secunde este

Liniar inductanțe

$$\lambda = L \cdot i$$

$$T = T^* = \left(\frac{1}{2L}\right) \cdot \lambda^2 = \frac{1}{2} \cdot Li^2$$

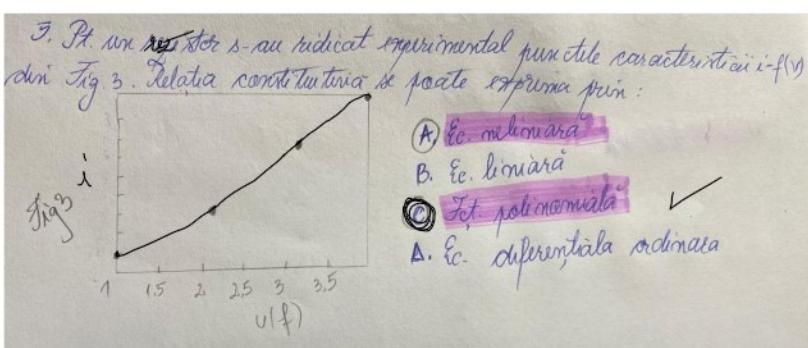


$$L = 10 \text{ mH} = 0,01 \text{ H}$$

$$\lambda = 10 \text{ mH} \cdot i$$

$$T = \frac{1}{2L} \cdot \lambda^2 = \frac{1}{2L} \cdot L \cdot i^2 = \frac{L \cdot i^2}{2} = \frac{0,01 \cdot i^2}{2} = 8 \cdot 10^{-2}$$

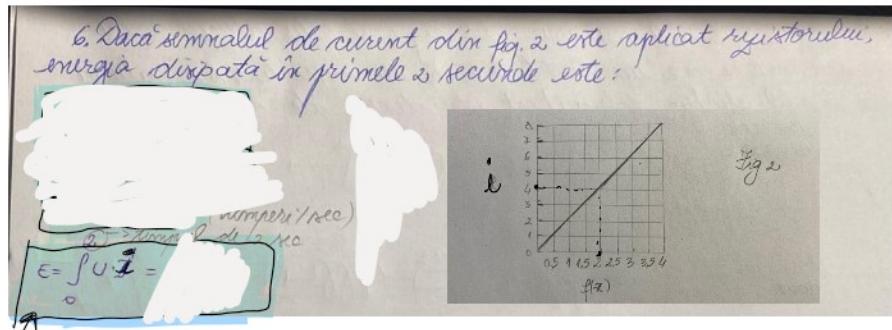
cheie



Dreptele
are curva

are curva

polinomiale \rightarrow nelineare

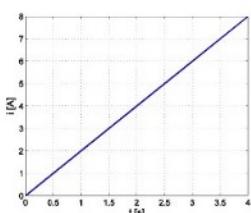
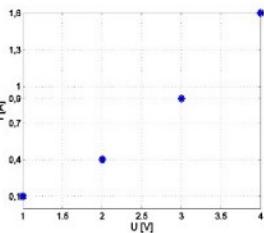


6. If the current signal shown in Figure is applied to the resistor, the energy dissipated in the first two seconds is:

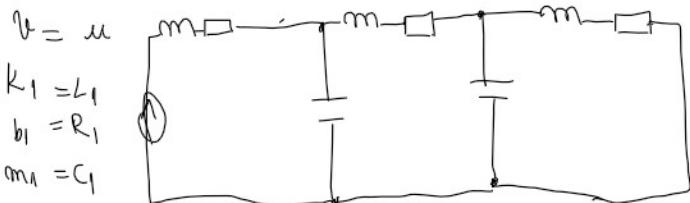
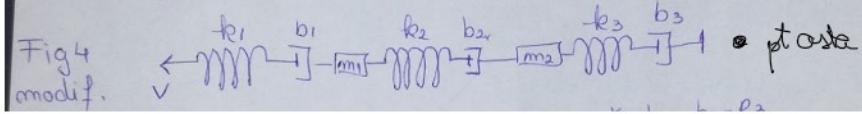
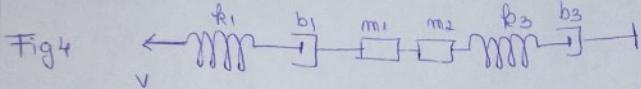
$$U = \sqrt{10} I^{1/2}, I = 2t$$

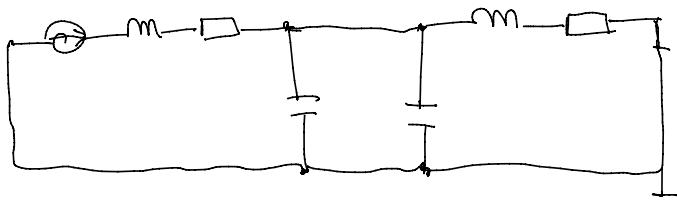
$$E = \int U \cdot I \, dt = \sqrt{10} \int_0^2 I^{3/2} \, dt = \sqrt{10} \int_0^2 (2t)^{3/2} \, dt \\ = \sqrt{10} \cdot 2^{3/2} \int_0^2 t^{3/2} \, dt$$

$$\text{Calea} \\ \text{către} \\ = \sqrt{10} \cdot 2^{\frac{3}{2}} \cdot \frac{t^{\frac{3}{2}+1}}{\frac{3}{2}+1} \Big|_0^2 \\ = \frac{\sqrt{10}}{5} \cdot 2^{\frac{3}{2}} \cdot 2 \cdot 2^{\frac{5}{2}} = \frac{\sqrt{10}}{5} 2^5$$

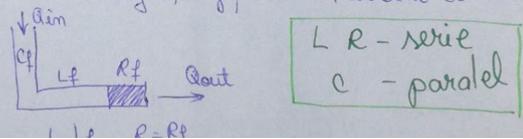


7. Dacă în sistemul mecanic din Figura 4 se intercalează între masele m_1 și m_2 un resort fix și un amortizor b_2 , atunci un model electric echivalent este descris de circuitul

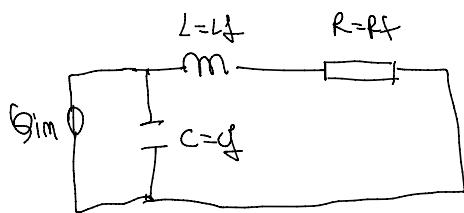




8. Pentru sistemul hidraulic de mai jos, alegeți modelul electric echivalent:



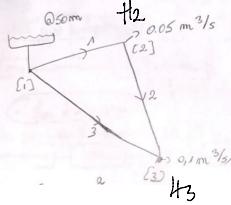
L - R - serie
 C - paralel



② For the hydraulic network in the figure above, the relation between Head loss and flow can be modeled as: $Q_{ij} = \frac{(H_i - H_j)^{\alpha}}{f^{\alpha}}$. The value of the resistance coefficients for pipes are: $f_{12} = f_{13} = 200$; $f_{23} = 100$.

a) find the mathematical model that allows the computation of the flow rate in each pipe, as a system of equations, when $\alpha = 0.5$.

b) Calculate the power dissipated on each pipe for the linear case (when $\alpha = 1$).



$$H_1 = 50$$

$$H_2 : Q_{12} - Q_{23} - n_2 = 0$$

$$H_3 : Q_{13} + Q_{23} - n_3 = 0$$

$$H_2 : \operatorname{sgn}(H_1 - H_2) \left(\frac{(H_1 - H_2)^{0.5}}{(200)^{0.5}} \right) - \operatorname{sgn}(H_2 - H_3) \left(\frac{(H_2 - H_3)^{0.5}}{100^{0.5}} \right) - n_2 = 0$$

$$H_3 : \operatorname{sgn}(H_1 - H_3) \left(\frac{|H_1 - H_3|^{0.5}}{200} \right) + \operatorname{sgn}(H_2 - H_3) \left(\frac{|H_2 - H_3|^{0.5}}{100} \right) - n_3 = 0$$

b) $I = \frac{U}{R}$ $P = U \cdot I$
 $P = \frac{U^2}{R}$
 $\alpha = 1 \Rightarrow$

$$P_{12} = \frac{(H_1 - H_2)^2}{(f_{12})^1}$$

$$H = 0.05 \cdot R$$

$$P_{13} = \frac{(H_1 - H_3)^2}{(f_{13})^1}$$

$$P_{23} = \frac{(H_2 - H_3)^2}{(f_{23})^1}$$

$$Q_{ij} = \frac{(H_i - H_j)^{\alpha}}{f^{\alpha}}$$

$$f_{12} = f_{13} = 200$$

$$f_{23} = 100$$

$$\left. \begin{aligned} Q_{12} &= \frac{(H_1 - H_2)^{0.5}}{(200)^{0.5}} \\ Q_{13} &= \frac{(H_1 - H_3)^{0.5}}{200^{0.5}} \\ Q_{23} &= \frac{(H_2 - H_3)^{0.5}}{100^{0.5}} \end{aligned} \right\} \text{flow rates}$$

$$H_2 : \operatorname{sgn}(H_1 - H_2) \left(\frac{(H_1 - H_2)^{0.5}}{(200)^{0.5}} \right) - \operatorname{sgn}(H_2 - H_3) \left(\frac{(H_2 - H_3)^{0.5}}{100^{0.5}} \right) - n_2 = 0$$

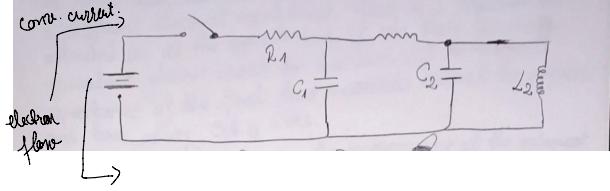
$$H_3 : \operatorname{sgn}(H_1 - H_3) \left(\frac{|H_1 - H_3|^{0.5}}{200} \right) + \operatorname{sgn}(H_2 - H_3) \left(\frac{|H_2 - H_3|^{0.5}}{100} \right) - n_3 = 0$$

force analogy hydro - el

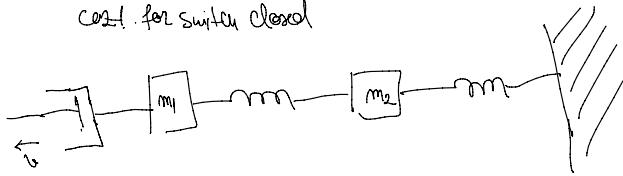
$$\Rightarrow U = H$$

$$R = f$$

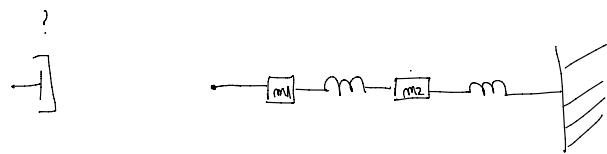
③ For the electrical circuit in the figure below, find and draw the mechanical analogies for the system.



case 1 for switch closed



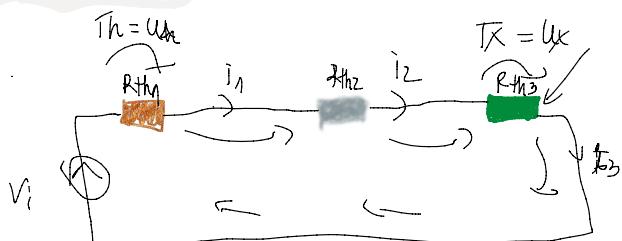
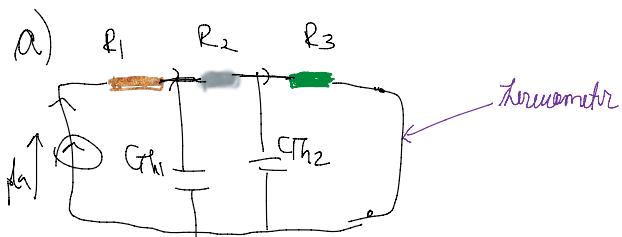
case 2 for switch open



④ Consider a heat source attached to a block composed of three materials with different characteristics (each with a different thermal resistance). There is a thermometer attached on at the opposite end of the block. Draw the equivalent electric circuit for two cases: ① the thermal capacitance of the first two materials cannot be neglected and have values C_{Th1} , C_{Th2} .

For case ①, determine the temperature of the innermost wall of the block if the next source has the temperature T_h .

$$U = i \cdot R$$



temperature - cadre de tensionne

$$U_{Th3} = V_i - U_{Th1} - U_{Th2}$$

	Electrical Domain		Thermal Domain			
	Through Variable (FLOW)	Current I	Amperes or Coulombs/s	Power or heat flux	P _D	Watts or Joules/s
Across Variable (EFFORT)	Voltage V	Volts	Temperature T	°C or K		
Resistance	Electrical resistance	R	Ohms	Thermal resistance	R _{θAB}	°C/W or K/W
Capacitance	Electrical capacitance	C	Farads or Coulombs/V	Thermal Capacitance	C _θ	Joules/°C
„Ohm's Law“	$\Delta V_{AB} = V_A - V_B = I \cdot R_{AB}$			$\Delta T_{AB} = T_A - T_B = P_D \cdot R_{\theta AB}$ (derived from Fourier's Law)		

⑥ A sensor provides N readings for the output variables (y), corresponding to N temperature values (x)

a) Assuming a linear transfer curve ($y = ax + b$) and, by employing the method of least squares, find an analytical formula for the values of the parameters a and b that minimize the error $J(a, b)$, of the assumption, where

$$J(a, b) = \sum_{n=1}^N (y_n - (ax_n + b))^2$$

b) Let $N = 5$ and the input/output values

$x = \{120, 150, 180, 200, 250\}$, $y = \{130, 170, 200, 250, 300\}$. What is the error associated to the parameters values?

$a = 10$ and $b = 1$?



d

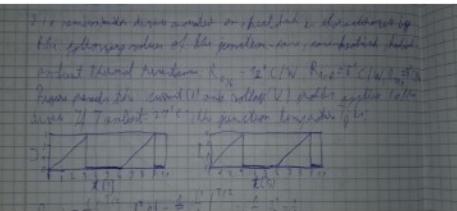
Me Witten per lab

9. A semiconductor device mounted a heatsink is characterized by the following ... of the ... heatsink, heatsink-ambient thermal resistances: $R_{SAC} = 12^\circ\text{C}/\text{W}$, $R_{SOC} = 6^\circ\text{C}/\text{W}$, $R_{RA} = 3^\circ\text{C}/\text{W}$

a) the current (i) and the voltage (u) profiles applied to the device

b) If $T_{\text{ambient}} = 27^\circ\text{C}$, the $a \dots$ is:

A. 135°C , B. 90.5°C , C. 58.5°C , D. 21°C



$$T_{\text{ambient}} = 27^\circ\text{C} \Rightarrow U = 27\text{V}$$

$$R_T = R_{Sc} + f_{Ck} + R_{FA} = 21 \Omega$$

$$T_A = 24^\circ\text{C}$$

$$T_{\text{Total}} = T_A + T_{\text{med}}$$

$$T_{\text{Total}} = T_A + P_{\text{med}} \cdot R_T$$

$$P_{\text{med}} = \frac{1}{T_A} \cdot \int_0^T f(t) dt$$

periodic

P_Wireless d. $T = 6$

$$\downarrow f(t) = V \cdot I = t \cdot t = t^2 \quad (\text{Ass. c' intro o g. 3 } u = i = t)$$

$$\Rightarrow P_{\text{med}} = \frac{1}{6} \cdot \int_0^6 t^2 dt = \frac{1}{6} \cdot \frac{t^3}{3} \Big|_0^6 = \frac{1}{6} \cdot \frac{24}{3}$$

$$= \text{P}_{\text{max}} = \frac{1}{6} \cdot \int_0^6 t^2 dt = \frac{1}{6} \cdot \frac{t^3}{3} \Big|_0^3 = \frac{1}{6} \cdot \frac{27}{3}$$

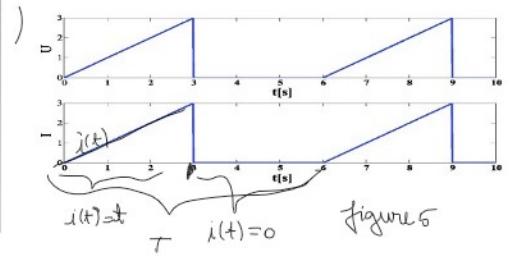
= 1,5

$$\Rightarrow T_{\text{total}} = T_A + 1,5 \cdot 21$$

$$T_{\text{total}} = 27 + 1,5 \cdot 21 = 58,5^\circ \text{C}$$

10. If the current profile of Figure 5 is applied to a rotating coil acuator, characterized by the equation $F(t) = K_f i(t)$ having the mass $m = 3$ and $K_f = 2$, with the initial conditions $i(0) = 0$, $x(0) = 0$, ~~$v(0) = 0$~~ , then the value at time $t = 3$ is:

A. 3/2, B. 9, C. 6, D. 3



$$F(t) = K_f \cdot i(t)$$

$$K_f = 2, m = 3, i(0) = 0, x(0) = 0, v(0) = 0$$

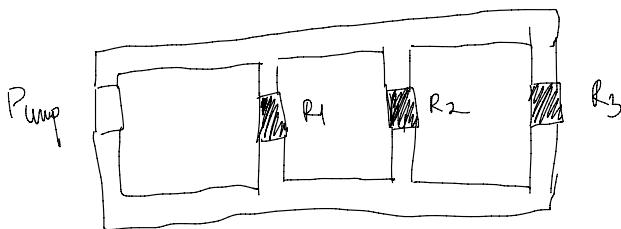
$t = 3$

$$i(t) = \begin{cases} \frac{t}{3}, & t \in [0, \frac{3}{2}) \\ 0, & t \in [\frac{3}{2}, 10] \end{cases}$$

$$v = \frac{1}{m} \cdot \int_0^T K_f \cdot i(t) dt$$

$$v = \frac{1}{3} \int_0^3 2 \cdot t dt = \frac{2 \cdot t^2}{3} \Big|_0^3 = \frac{2}{3} \cdot \frac{9}{2} = 3$$

11. Draw the equivalent hydraulic system for the circuit shown in the following figure.



12. Draw the equivalent hydraulic system for the circuit shown in the following figure.

