

## Tema lab 6

1. Compute the autocorrelation function for the signal  $x(t) = A \sin(\omega_0 t + \varphi)$

$$\begin{aligned}
 \varphi_x(\tau) &= \frac{1}{T_0} \int_0^{T_0} x(t) \cdot x(t-\tau) dt = \frac{1}{T_0} \int_0^{T_0} A \sin(\omega_0 t + \varphi) \cdot A \sin(\omega_0(t-\tau) + \varphi) dt \\
 &= \frac{A^2}{T_0} \int_0^{T_0} \sin(\omega_0 t + \varphi) \cdot \sin(\omega_0 t + \varphi - \omega_0 \tau) dt = \\
 &= \frac{A^2}{T_0} \int_0^{T_0} \frac{\cos(-\omega_0 \tau) - \cos(2\omega_0 t + 2\varphi - \omega_0 \tau)}{2} dt = \\
 &= \frac{A^2}{2T_0} \cdot \cos(\omega_0 \tau) \cdot T_0 - \frac{A^2}{2T_0} \int_0^{T_0} \cos(2\omega_0 t + 2\varphi - \omega_0 \tau) dt = \\
 &= \frac{A^2}{2} \cos(\omega_0 \tau) - \frac{A^2}{2T_0} \cdot \frac{\sin(2\omega_0 t + 2\varphi - \omega_0 \tau)}{2\omega_0} \Big|_0^{T_0} = \\
 &= \frac{A^2}{2} \cos(\omega_0 \tau) - \frac{A^2}{4T_0 \omega_0} (\sin(2\omega_0 T_0 + 2\varphi - \omega_0 \tau) - \sin(2\varphi - \omega_0 \tau))
 \end{aligned}$$

We know that  $T_0 = \frac{2\pi}{\omega_0} \Rightarrow \varphi_x(\tau) = \frac{A^2}{2} \cos(\omega_0 \tau) - \frac{A^2}{8\pi} (\sin(4\pi + 2\varphi - \omega_0 \tau) - \sin(2\varphi - \omega_0 \tau))$

$$\Rightarrow \varphi_x(\tau) = \frac{A^2}{2} \cos(\omega_0 \tau) - \frac{A^2}{8\pi} (\underbrace{\sin(4\pi)}_0 \cos(2\varphi - \omega_0 \tau) + \underbrace{\cos(4\pi)}_1 \sin(2\varphi - \omega_0 \tau) - \sin(2\varphi - \omega_0 \tau))$$

$$\Rightarrow \varphi_x(\tau) = \frac{A^2}{2} \cos(\omega_0 \tau) - \frac{A^2}{8\pi} \cdot 0 \Rightarrow \varphi_x(\tau) = \frac{A^2}{2} \cdot \cos(\omega_0 \tau)$$

```
% function:
function phi = autocorrelation(x,N,M)
if(length(x) < M+N)
phi = [];
return
end
phi = zeros(1,N);
for n = 1:N
for k = n+1:n+M
phi(n) = phi(n) + x(k) * x(k-n);
end
end
phi = phi / M;
```

### % Exercițiul 2:

```
clear , close all
```

```
N = 128; M = 1000;
```

```
A = 1; n = 0:N+M-1;
```

```
x = sin(2*pi*n/64) + A*randn(1,length(n));
```

```
phi = autocorrelation(x,N,M);
```

```
subplot(212); plot(x(1:N)); grid; title('Sine with white noise');
```

```
subplot(222); plot(phi); grid; title('Autocorrelation function');
```

