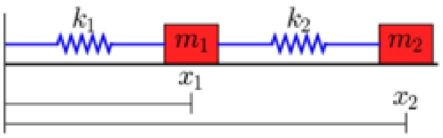
Examen PM

1) Considering the variables x1, x2 = position (left edge) of blocks, and the parameters m1, m2 = mass of blocks, w1, w2 = width of blocks, k1, k2 = spring constants, R1, R2 = rest length of springs for the system in the figure, the mathematical model can be written as:



$$(1)\quad \begin{cases} x_1' = -m_1k_1(x_1 - R_1) + m_2k_2(x_2 - x_1 - w_1 - R_2) \\ x_2' = -m_2k_2(x_2 - x_1 - w_1 - R_2) \end{cases}$$

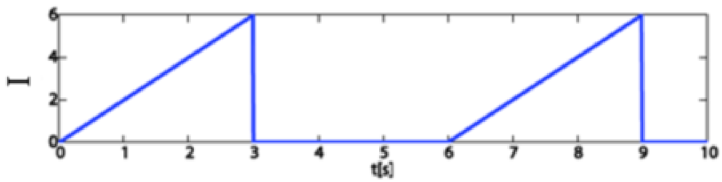
$$(2)\quad \begin{cases} x_1'' = -m_1k_1(x_1 - R_1) + m_2k_2(x_2 - x_1 - w_1 - R_2) \\ x_2'' = -m_2k_2(x_2 - x_1 - w_1 - R_2) \end{cases}$$

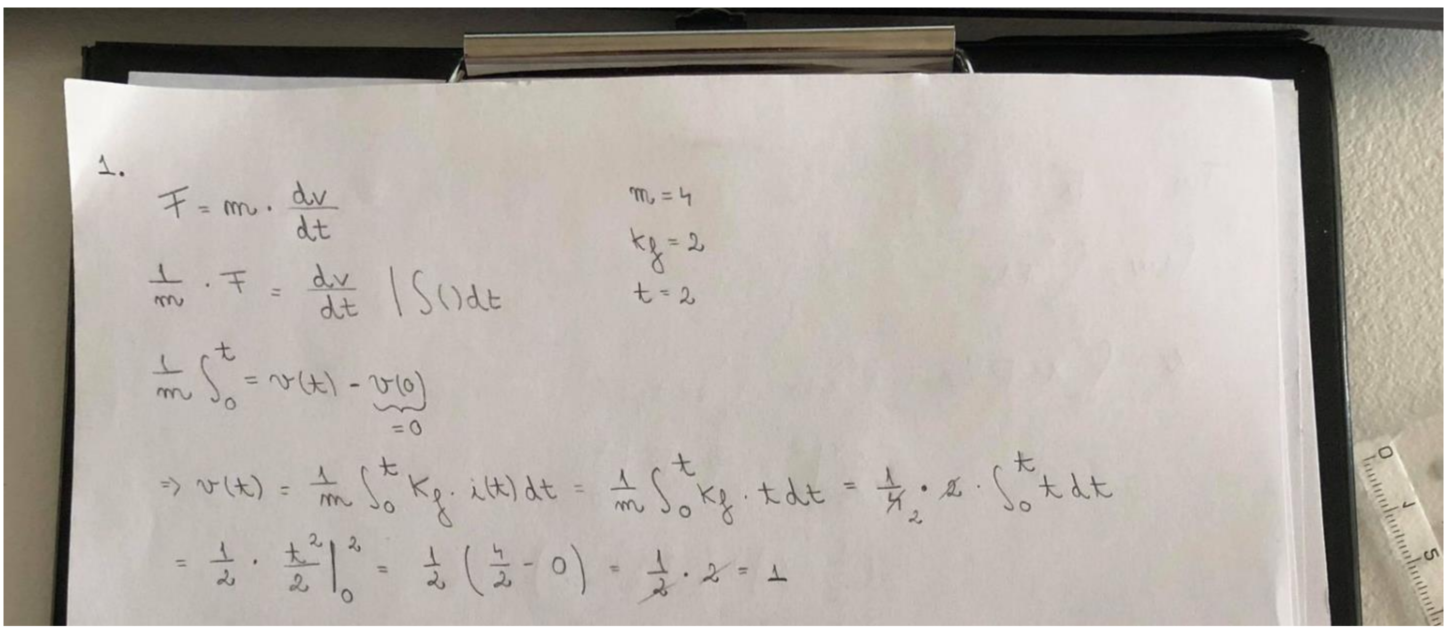
$$(3)\quad \begin{cases} m_1x_1' = -k_1(x_1 - R_1) + k_2(x_2 - x_1 - w_1 - R_2) \\ m_2x_2' = -k_2(x_2 - x_1 - w_1 - R_2) \end{cases}$$

$$(4)\quad \begin{cases} m_1x_1'' = -k_1(x_1 - R_1) + k_2(x_2 - x_1 - w_1 - R_2) \\ m_2x_2'' = -k_2(x_2 - x_1 - w_1 - R_2) \end{cases}$$

Raspuns: ecuatia 4

1) If the current profile presented in the figure is applied to a voice coil actuator, characterized by the equation F(t) = K i(t), having the mass m = 4 and Kf = 2, with the initial conditions i(0) = 0, v(0) = 0, x(0) = 0, then the value of the velocity at time t = 2s is:





1.

$F = m \cdot \frac{dv}{dt}$ $\qquad m = 4$

$\frac{1}{m} \cdot F = \frac{dv}{dt} \; \Big| \int ( ) dt$ $\qquad k_f = 2$

$t = 2$

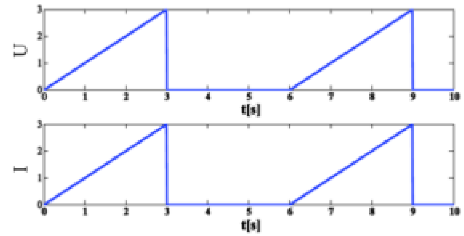
$\frac{1}{m} \int_0^t = v(t) - \underbrace{v(0)}_{=0}$

$\Rightarrow v(t) = \frac{1}{m} \int_0^t K_f \cdot i(t)\, dt = \frac{1}{m} \int_0^t K_f \cdot t\, dt = \frac{1}{\cancel{4}_2} \cdot \cancel{2} \cdot \int_0^t t\, dt$

$= \frac{1}{2} \cdot \frac{t^2}{2} \Big|_0^2 = \frac{1}{2} \left(\frac{4}{2} - 0 \right) = \frac{1}{\cancel{2}} \cdot \cancel{2} = 1$

Raspuns: 1

2) A semiconductor device mounted on a heatsink is characterized by the thermal resistances (junction-case (J-C), case-heatsink (C-HS), heatsink-ambient (HS-A)) whose values are mentioned in the figure. The figure presents the current (I) and voltage (U) profiles applied to the device. If Tambient = 24 C, the avera ge junction temperature Tj, estimated using the dissipated averaged power, is:



$R_{thJ\text{-}C} = 12$ C/W, $R_{thC\text{-}HS} = 6$ C/W, $R_{thHS\text{-}A} = 3$ C/W

2.

$R_\Sigma = 12 + 6 + 3 = 21\ °C/W$

$T_A = 24°C$

$T_j = T_A + P_{medie} \cdot R_\Sigma$

$P_{medie}\ (\text{grafic}) = \frac{1}{T}\int_0^T f(t)\,dt$

$T = 6$

$f(t) = U \cdot I = t \cdot t = t^2$

$\Rightarrow P_{medie} = \frac{1}{6}\int_0^3 t^2\,dt = \frac{1}{6}\cdot\frac{t^3}{3}\Big|_0^3 = \frac{1}{6}\left(\frac{27}{3} - 0\right) = \frac{1}{6}\cdot 9$

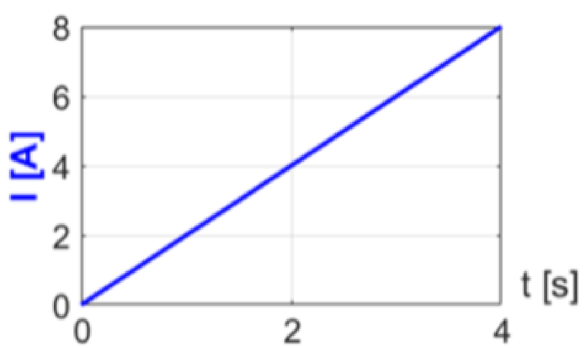
$= 1{,}5$

$\Rightarrow T_j = 24 + 1{,}5 \cdot 21 = 24 + 31{,}5 = 55{,}5$

Raspuns: 55.5 C

1) Consider an inductor described by the linear equation in the figure, where L is constant, expressed in H and i is expresses in A. For L=8mH and the current signal presented in the figure, the energy accumulated in inductor in the first 2 seconds is:

$$\lambda = L\,i$$



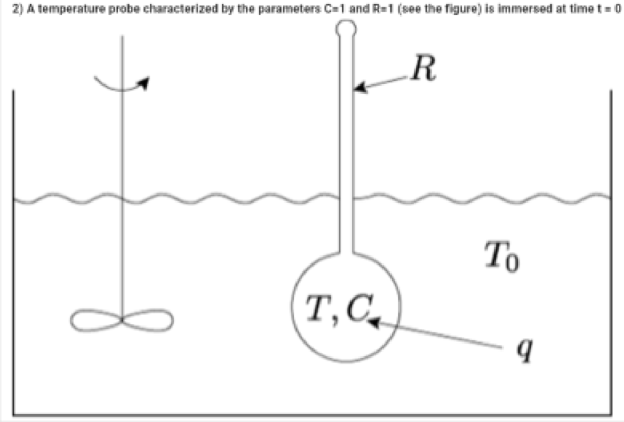
3. $\lambda = L \cdot i$

$T = \frac{1}{2L} \cdot \lambda^2 = \frac{1}{2L} \cdot L^2 i^2 = \frac{1}{2} L i^2 = \frac{1}{2} \cdot 0{,}008 \cdot 16 = 0{,}064$

$L = 8\,mH = 0{,}008\,H$

Raspuns: 0.064

2) A temperature probe characterized by the parameters C=1 and R=1 (see the figure) is immersed at time t = 0 in a liquid having the temperature T0 = 10 C. The temperature of the probe at time t = 0 is 20C. Which of the following statements are true?



Raspuns: After 3 seconds T=12 C

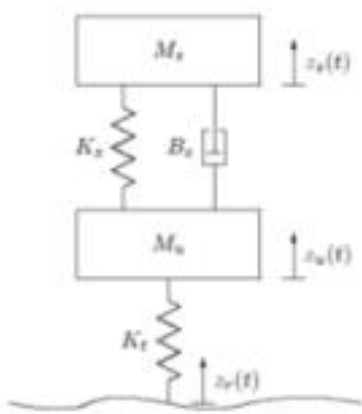
2) The mathematical model of the system in the figure is described by:

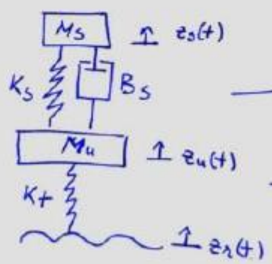
$$(1)\ \dot{\mathbf{x}}(t) = \begin{pmatrix} 0 & K_t & 0 & 0 \\ \frac{1}{M_u} & \frac{B_s}{M_u} & -\frac{1}{M_u} & -\frac{B_s}{M_u} \\ 0 & -K_s & 0 & K_s \\ 0 & -\frac{B_s}{M_s} & \frac{1}{M_s} & \frac{B_s}{M_s} \end{pmatrix} \mathbf{x}(t) + \begin{pmatrix} K_t \\ 0 \\ 0 \\ 0 \end{pmatrix} z_r(t)$$

$$(2)\ \dot{\mathbf{x}}(t) = \begin{pmatrix} 0 & -K_t & 0 & 0 \\ \frac{1}{M_u} & -\frac{B_s}{M_u} & -\frac{1}{M_u} & \frac{B_s}{M_u} \\ 0 & K_s & 0 & -K_s \\ 0 & \frac{B_s}{M_s} & \frac{1}{M_s} & -\frac{B_s}{M_s} \end{pmatrix} \mathbf{x}(t) + \begin{pmatrix} K_t \\ 0 \\ 0 \\ 0 \end{pmatrix} z_r(t)$$

$$(3)\ \dot{\mathbf{x}}(t) = \begin{pmatrix} 0 & K_t & 0 & 0 \\ -\frac{1}{M_u} & \frac{B_s}{M_u} & \frac{1}{M_u} & -\frac{B_s}{M_u} \\ 0 & -K_s & 0 & K_s \\ 0 & -\frac{B_s}{M_s} & -\frac{1}{M_s} & \frac{B_s}{M_s} \end{pmatrix} \mathbf{x}(t) + \begin{pmatrix} K_t \\ 0 \\ 0 \\ 0 \end{pmatrix} z_r(t)$$

$$(4)\ \dot{\mathbf{x}}(t) = \begin{pmatrix} 0 & -K_t & 0 & 0 \\ \frac{1}{M_u} & -\frac{B_s}{M_u} & -\frac{1}{M_u} & \frac{B_s}{M_u} \\ 0 & K_s & 0 & -K_s \\ 0 & \frac{B_s}{M_s} & \frac{1}{M_s} & -\frac{B_s}{M_s} \end{pmatrix} \mathbf{x}(t) + \begin{pmatrix} \frac{K_t}{M_u} \\ 0 \\ 0 \\ 0 \end{pmatrix} z_r(t)$$





equivalent forces:

$\longrightarrow F_s = F_{K_s} + F_{b_s} - M_s g$

$\longrightarrow F_u = F_{K_t} - F_{K_s} - F_{b_s} - M_u g$

string → elastic potential energy → Force

• for string with K_t

$$F_{K_t} = K_t \cdot \Delta l = K_t \cdot (\overset{\text{initial}}{z_r(t)} - \overset{\text{final}}{z_u(t)})$$

$$= K_t \cdot z_r(t) - K_t \cdot z_u(t)$$

$$\Leftrightarrow \dot{F}_{K_t} = K_t \cdot \dot{z}_r(t) - K_t \cdot \dot{z}_u(t)$$

$$\underline{= -K_t \cdot v_u + K_t \cdot \dot{z}_r(t)} \quad (1)$$

for string with K_s

$$F_{K_s} = K_s \cdot \Delta l = K_s \cdot (\overset{\text{initial}}{z_u(t)} - \overset{\text{final}}{z_s(t)})$$

$$\Leftrightarrow \dot{F}_{K_s} = K_s \cdot \dot{z}_u(t) - K_s \cdot \dot{z}_s(t)$$

$$\underline{= K_s \cdot v_u - K_s \cdot v_s} \quad (2)$$

for the mass M_u → mass → kinetic energy → velocity

$$M_u \cdot a_u = F_{K_t} - F_{K_s} - F_{b_s}$$

$$= F_{K_t} - F_{K_s} - B_s(\overset{\text{initial}}{v_u} - \overset{\text{final}}{v_s})$$

$$\Leftrightarrow \underline{\dot{v}_u = \frac{1}{M_u}(F_{K_t} - F_{K_s} - B_s v_u + B_s v_s)} \quad (3)$$

• for the mass M_s

$$M_s \cdot a_s = F_{K_s} + F_{b_s}$$

$$= F_{K_s} + B_s \cdot (v_u - v_s)$$

$$\underline{= F_{K_s} + B_s v_u - B_s v_s} \quad (4)$$

$$\Leftrightarrow \dot{v}_s = \frac{1}{M_s}(F_{K_s} + B_s v_u - B_s v_s)$$

(1), (2), (3), (4) and considering
$$X(t) = \begin{pmatrix} F_{K_t} \\ v_u \\ F_{K_s} \\ v_s \end{pmatrix} \quad \Leftrightarrow \quad \dot{X}(t) = \begin{pmatrix} 0 & -K_t & 0 & 0 \\ \frac{1}{M_u} & -\frac{B_s}{M_u} & -\frac{1}{M_u} & \frac{B_s}{M_u} \\ 0 & K_s & 0 & -K_s \\ 0 & \frac{B_s}{M_s} & \frac{1}{M_s} & -\frac{B_s}{M_s} \end{pmatrix} \cdot X(t) + K_t \cdot \dot{z}_r(t)$$

, where $K_t \cdot \dot{z}_r(t) \Leftrightarrow \begin{pmatrix} K_t \\ 0 \\ 0 \\ 0 \end{pmatrix} \cdot \dot{z}_r(t)$ ✓

Raspuns: eq (2)

3) Consider a translational spring at rest if the compression force F is related to the displacement x through the equation F(x)=x+sqrt(x). Compute the potential energy stored in the spring compressed at x = 1.

File

$F(x) = x + \sqrt{x}$, $x = 1$

$$\int dW = \int_0^x F\,dx$$

$$W = \int_0^x x + \sqrt{x}\,dx = \int_0^x x\,dx + \int_0^x \sqrt{x}\,dx$$

$$= \frac{x^2}{2}\Big|_0^1 + \frac{2}{3}x\sqrt{x}\Big|_0^1$$

$$= \left(\frac{1}{2} - 0\right) + \left(\frac{2}{3}\cdot 1 - 0\right)$$

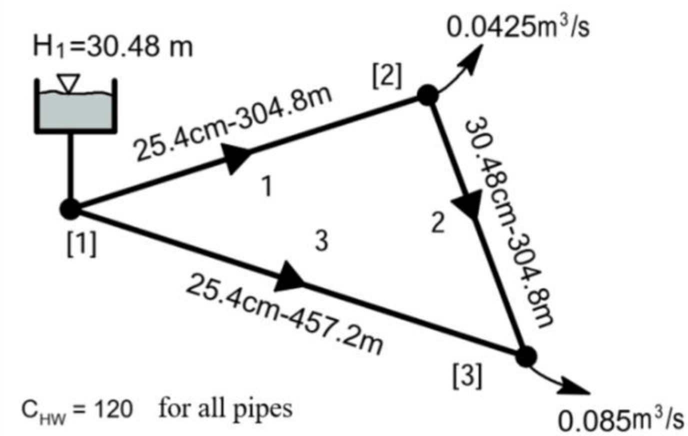
$$= {}^{3/}\frac{1}{2} + {}^{2/}\frac{2}{3}$$

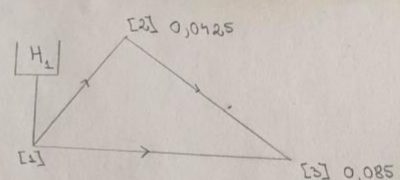
$$= \frac{7}{6}$$

$$= 1{,}16\ J$$

$\Rightarrow$ The potential energy of the spring : 1,16 J

4) Considering the hydraulic network from the figure, and assuming a linear relation between the flow rate and the pressure drop across each pipe, draw the analogue electrical circuit and write uations of the mathematical model of this circuit.





$$Q_{ij} = \frac{(H_i - H_j)^a}{f_{ij}}$$

$$Q_{12} = Q_{23} + n_2$$

$$Q_{13} = -Q_{23} + n_3$$

$$\frac{(H_1-H_2)^a}{(f_{12})^a} = \frac{(H_2-H_3)^a}{(f_{23})^a} + 0{,}0425$$

$$\frac{(H_1-H_3)^a}{(f_{13})^a} = -\frac{(H_2-H_3)^a}{(f_{23})^a} + 0{,}085$$

$$Q_{12} = \frac{(H_1-H_2)^a}{(f_{12})^a}$$

$$Q_{13} = \frac{(H_1-H_3)^a}{(f_{13})^a}$$

$$Q_{23} = \frac{(H_2-H_3)^a}{(f_{23})^a}$$