Modeling of energy conversion

ELECTROMAGNETIC ACTUATORS

- The electromagnetic actuators are used to convert electrical and mechanical energy into one another.
- These actuators produce force and torque by means of magnetic field.
- The fundamental principles that govern electromagnetic actuators are Faraday's laws of electromagnetic induction and Lorenz's law of electromagnetic force.
- The electric motor converts the electrical energy into mechanical energy. Electric motor is the most used electromagnetic actuator for widespread applications.
- The solenoid actuators are the simplest common electromagnetic actuators that convert the electrical energy into linear or rotational motion. Their applications include: conveyor diverters, relays, coin dispensers, electric lock mechanisms, etc.

Electromagnetism

- The electricity and the magnetism are interconnected – danish physicist H. C. Oersted proves at the beginning of the 19^{th} century that a current produces a magnetic field, showing that electric currents can influence a magnetized object
- Later on, Ampère introduces a formula connecting those phenomena known today as *Ampère's law.*
- Few years later, Faraday demonstrates the fact that a magnetic field might generate an electric field.
- **The magnetic flux through a surface (Fluxul magnetic)** ***φ*****, weber** (Wb)
- **Magnetic flux density (Densitatea fluxului magnetic) B** (Wb/m2), or **tesla** (T)
- **Magnetic field intensity (Intensitatea câmpului magnetic) H** (A/m)

Magnetic flux density B

The total magnetic flux through the surface is the surface integral of B.

When the flux lines are perpendicular to A, this becomes:

$$\phi = \int_A B \, \mathrm{d}A$$

If the value of B is constant, then: $\phi = B \cdot A$

Faraday's law

Faraday's law of induction:
When the flux through a wire loop changes(**B** changes, the wire loop geometry changes, the wire loop is moved) the wire loop acquires an EMF (electromotive force).

EMF is also given by the rate of change of the magnetic flux:

$$e = -\frac{d\phi}{dt}$$

For a winding with N turns:

$$e = N\frac{d\phi}{dt} \qquad \lambda = N\phi \qquad e = \frac{d\lambda}{dt}$$

Magnetic flux density and Magnetic field intensity

The relation between B and H:

$$\mathbf{B} = \mu \mathbf{H} = \mu_r \mu_0 \mathbf{H}$$

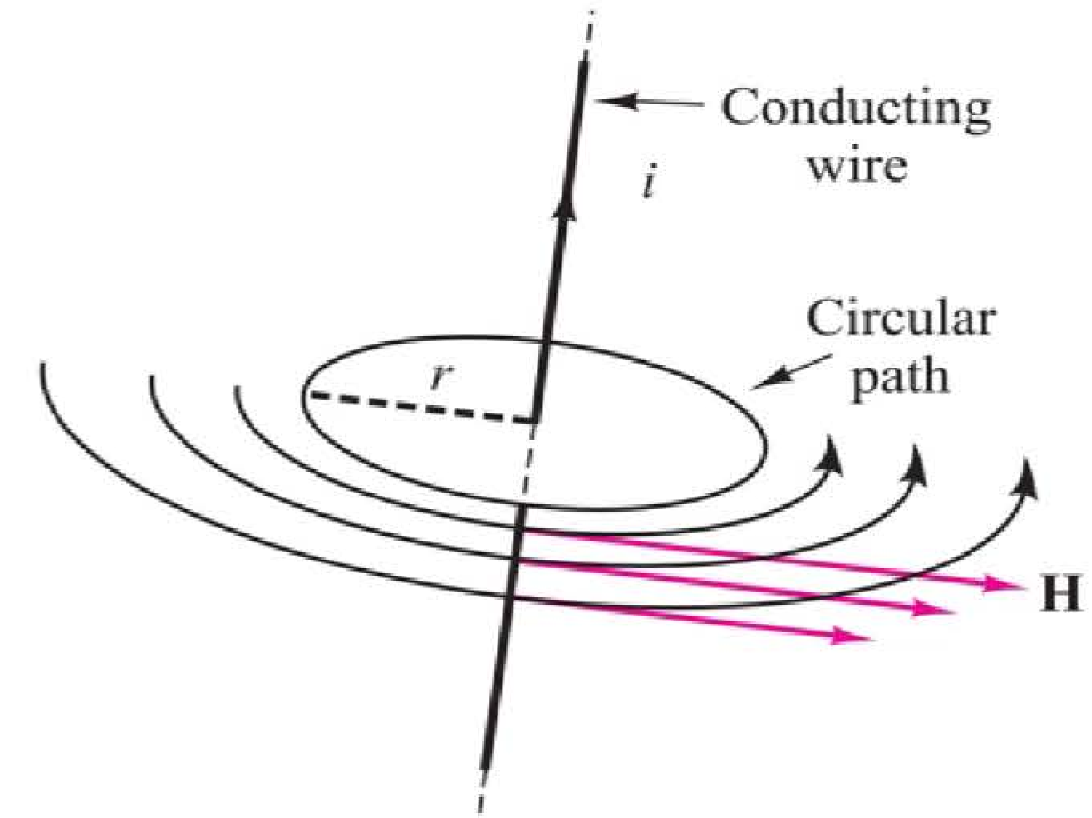
Ampère's circuital law

Ampère's circuital law relates the integrated magnetic field around a closed loop to the electric current passing through the loop:

$$\oint \mathbf{H} \cdot d\mathbf{l} = \sum i$$

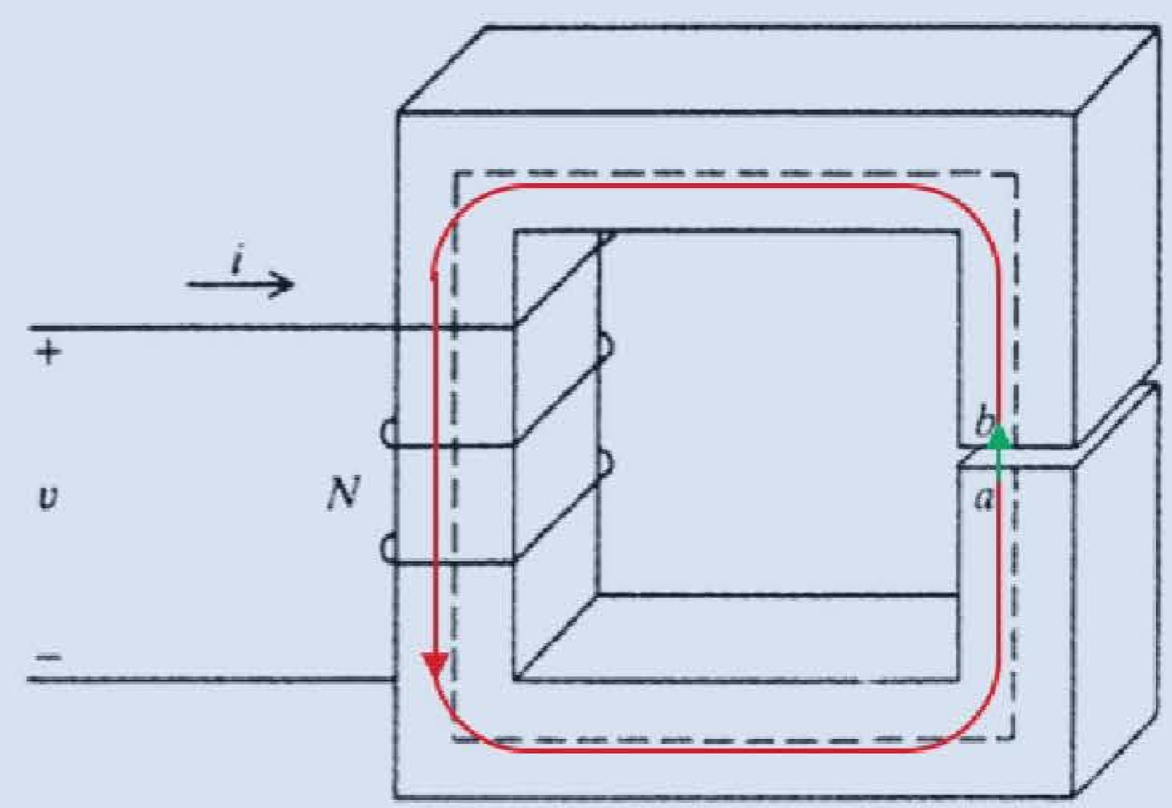
When dl and H have the same direction, the dot product simplifies to:

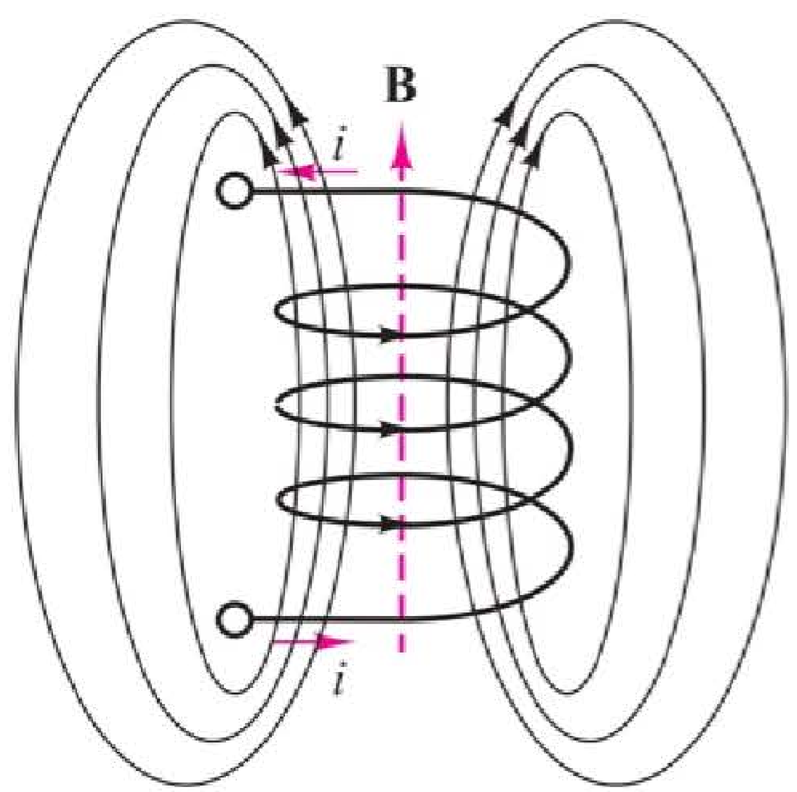
$$\int H\, dl = \sum i$$

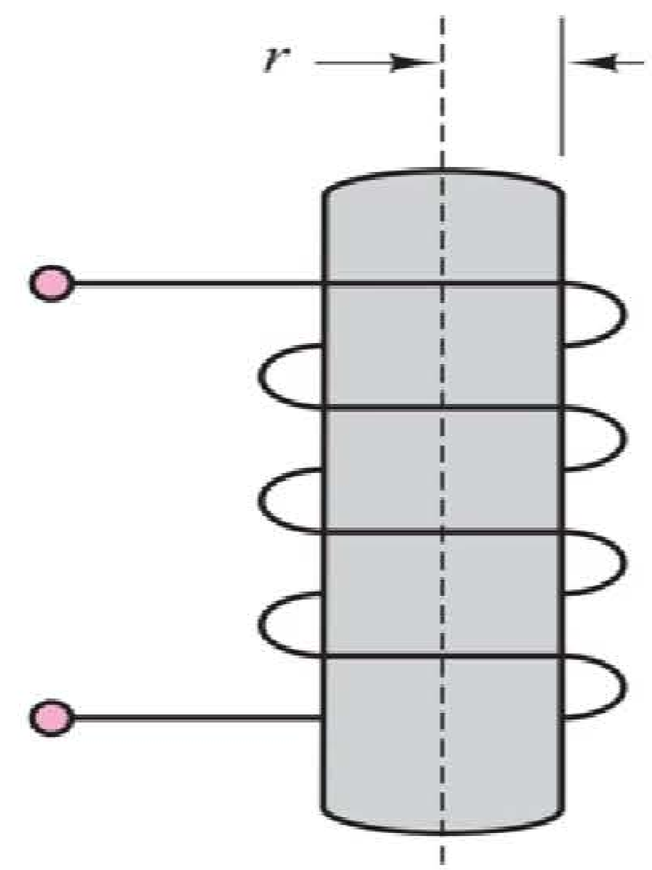


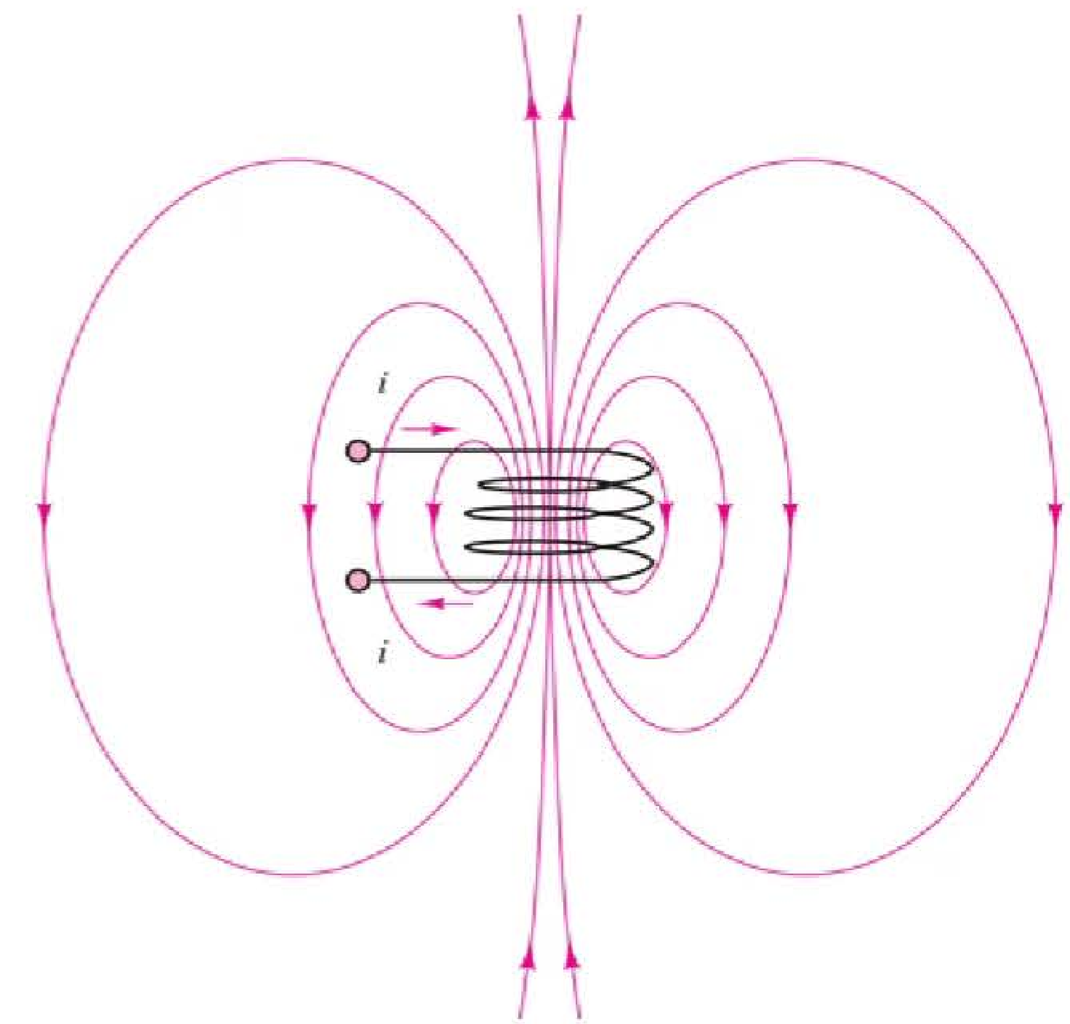
Example: One can apply Ampere's law to the closed path (depicted as a dashed line) and split the integral according to the different media travelled by the path: $\boldsymbol{H}_i$ (iron) denotes the field intensity in the ferromagnetic material and $\boldsymbol{H}_g$ (gap) denotes the field intensity in the air gap.

$$\boxed{\int_a^b H_i dL} + \boxed{\int_b^a H_g dL} = Ni$$









$$B = \frac{\mu N i}{l}$$

The presence of a high permeability material produces the concentration of the flux in the material with higher values of ***μ.***

Simplifying assumptions used in building models:

1. There is an average path for the magnetic flux (its length is denoted by ***l***):
2. An averaged flux density might be considered, and this is constant along the cross section of the magnetic structure

$$B = \frac{\phi}{A} \qquad H = \frac{B}{\mu} = \frac{\phi}{A\mu}$$

3. The term magnetomotive force (**tensiune magnetica**) was coined by Henry Augustus Rowland in 1880. Rowland intended this to indicate a direct analogy with electromotive force.

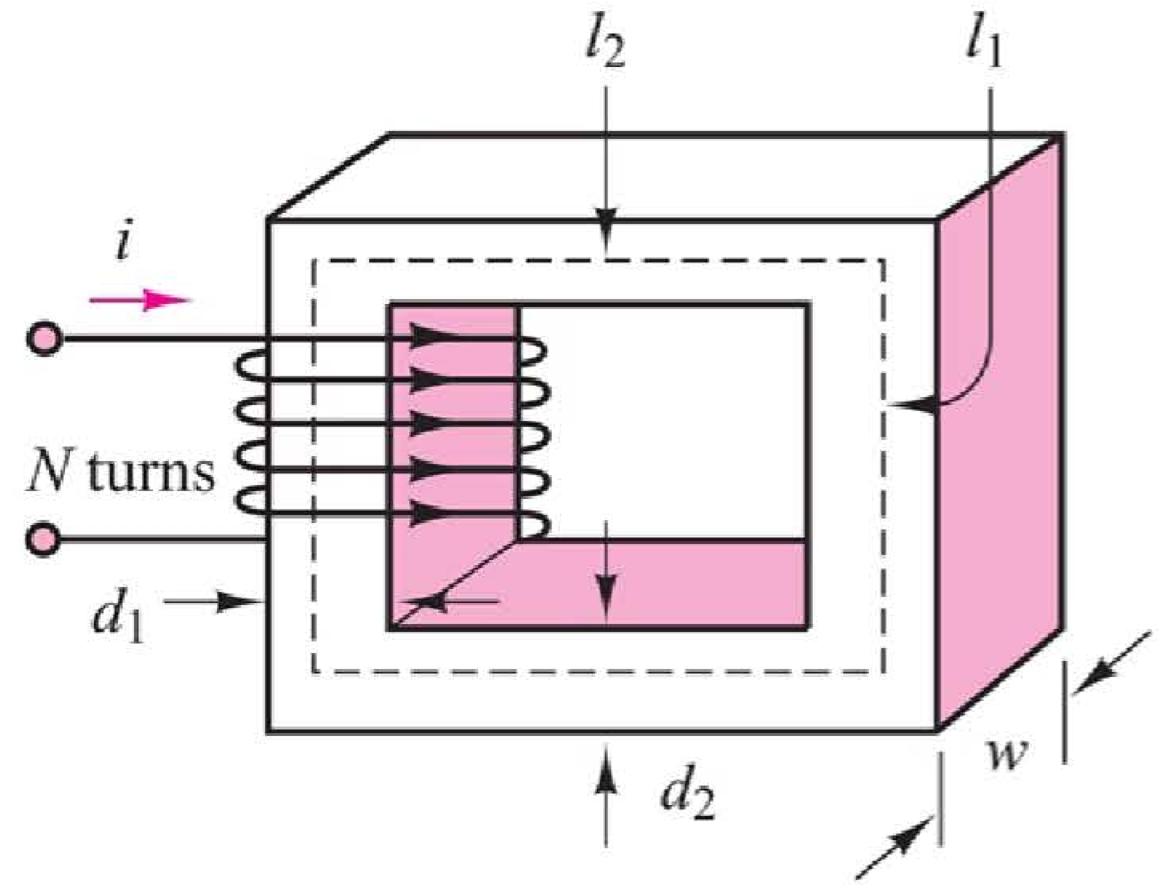
$$\mathcal{F} = N \cdot i = H \cdot l \qquad \mathcal{F} = \phi \frac{l}{\mu A}$$

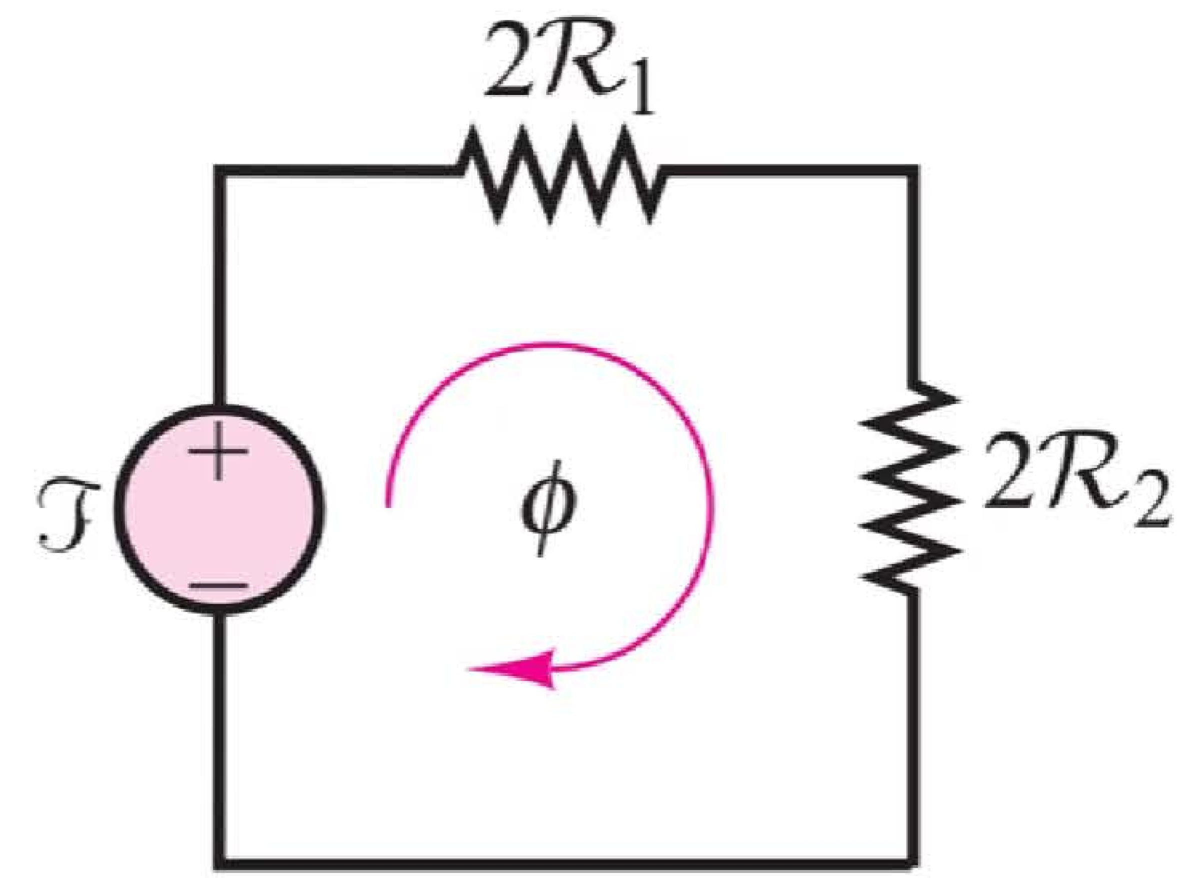
The magnetomotive force $\mathcal{F}$ can be seen as the analog of the electric voltage source in a circuit, and the flux φ as the equivalent of the electric current.

The expression $l/\mu A$ has the role of a *magnetic resistor for a path* of the magnetic circuit; it is called *reluctance* and has the symbol $\mathfrak{R}$.

$$L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{N}{i}\frac{Ni}{\mathfrak{R}} = \frac{N^2}{\mathfrak{R}}$$

Example: Ohm's law for magnetic circuits

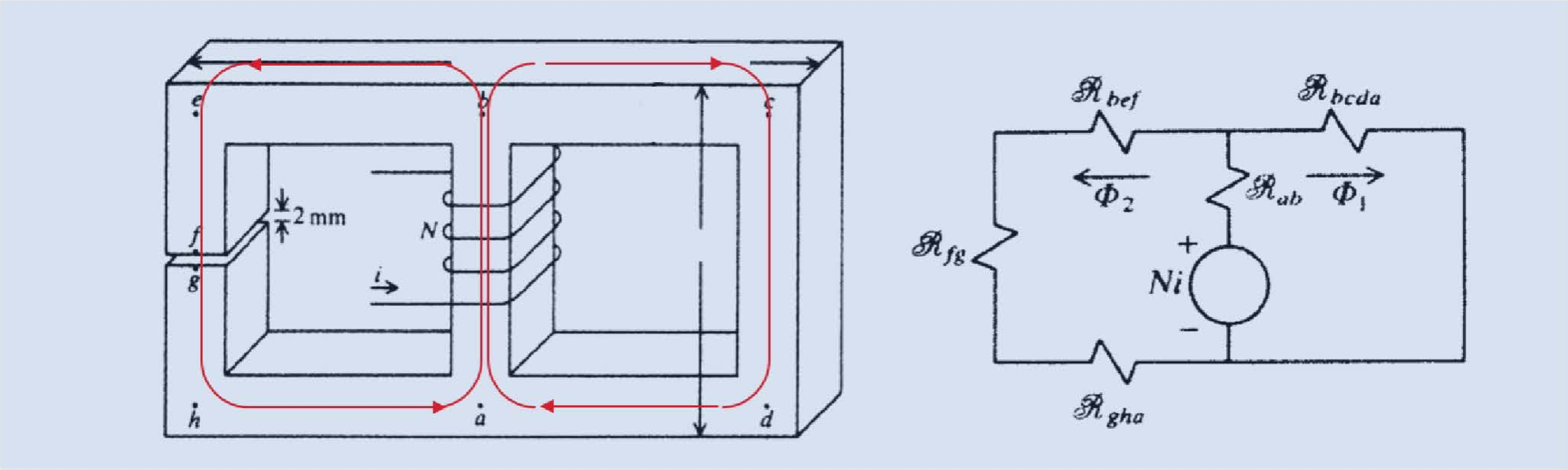




$$\mathcal{R}_{\text{series}} = 2\mathcal{R}_1 + 2\mathcal{R}_2 \qquad \mathcal{R}_1 = \frac{l_1}{\mu A_1} \qquad \mathcal{R}_2 = \frac{l_2}{\mu A_2}$$

$$A_1 = d_1 w \qquad A_1 = d_2 w$$

Example: Magnetic circuit representation



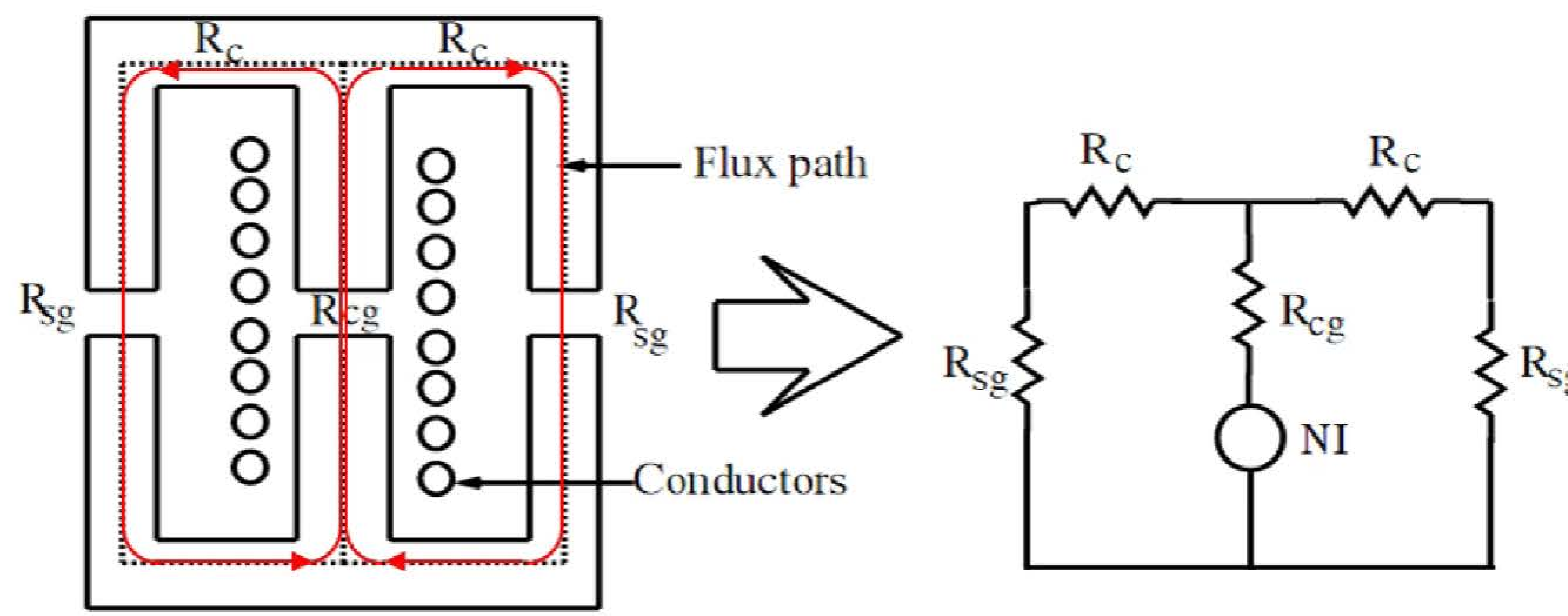
- Identification of the flux paths
- Identification of the reluctances
- Reduce the series-parallel circuit to an equivalent reluctance:

$$\Re_{eq} = \frac{(\Re_{bcda})(\Re_{bef} + \Re_{fg} + \Re_{gha})}{\Re_{bcda} + \Re_{bef} + \Re_{fg} + \Re_{gha}}$$

- The relation between total magnetic flux an total reluctance :

$$\Phi_1 + \Phi_2 = \frac{Ni}{\Re_{ab} + \Re_{eq}}$$

Example: Magnetic circuit representation

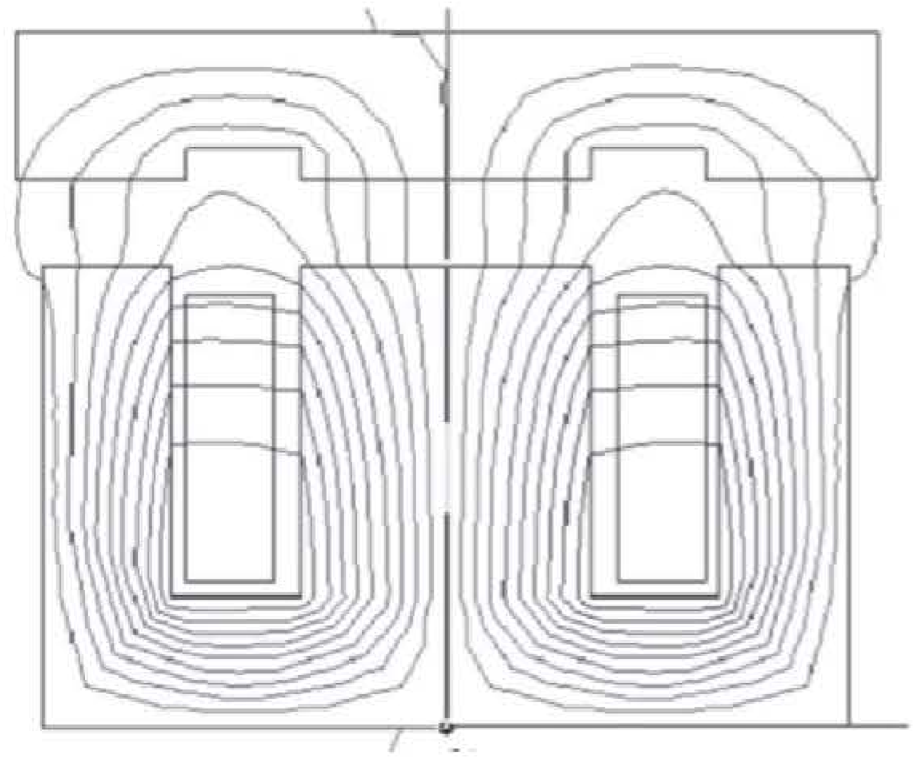


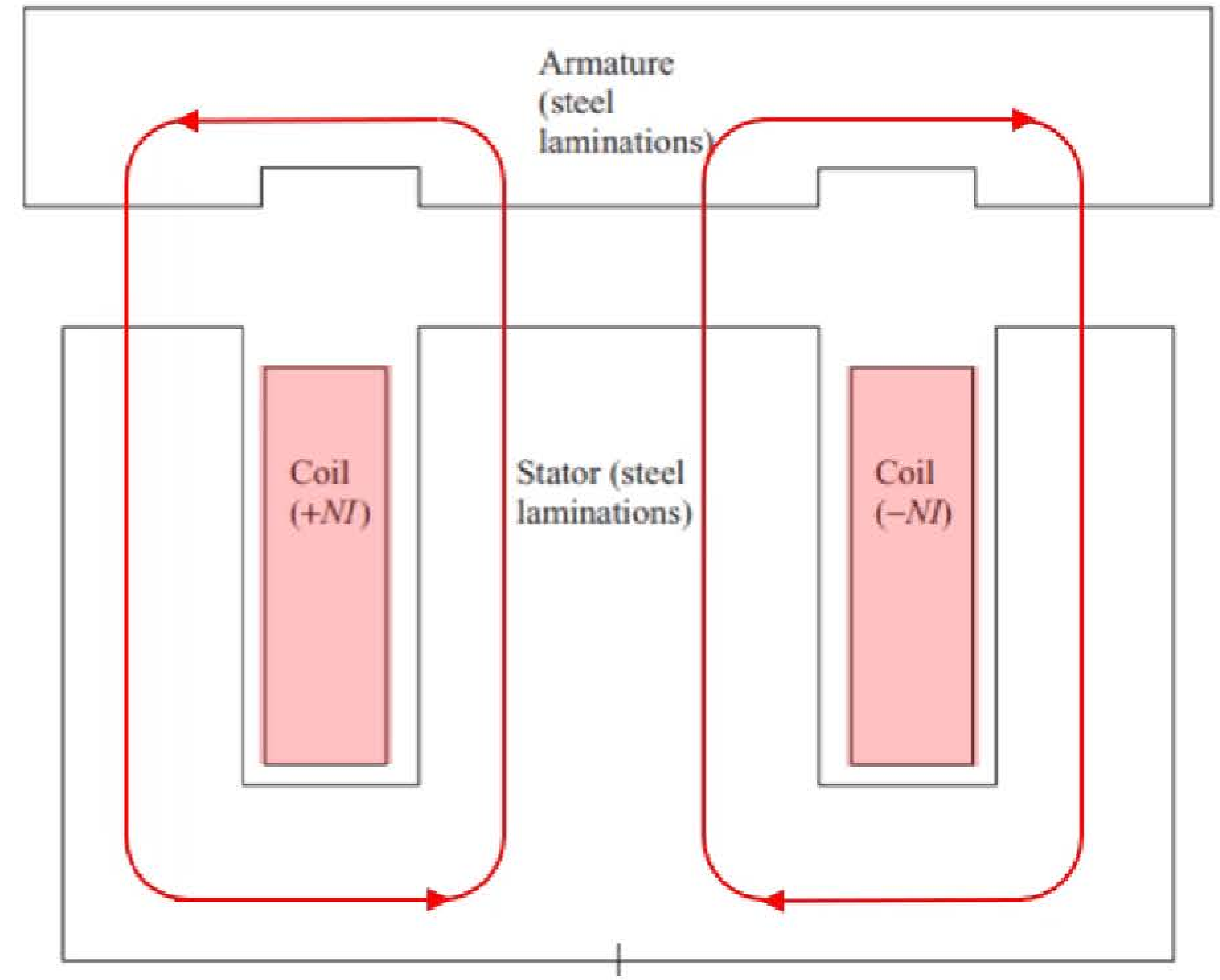
- Identification of the flux paths
- Identification of the reluctances:
  - Reluctance of the core $\mathfrak{R}_c$,
  - Reluctance of the centre leg gap $\mathfrak{R}_{cg}$
  - Reluctance of the side leg gap $\mathfrak{R}_{sg}$
- The total reluctance of the magnetic path :

  $\mathfrak{R}_t = \mathfrak{R}_{cg} + \mathfrak{R}_{sg}/2 + \mathfrak{R}_c/2$

Example: Clapper-type solenoid

- Identification of the flux paths



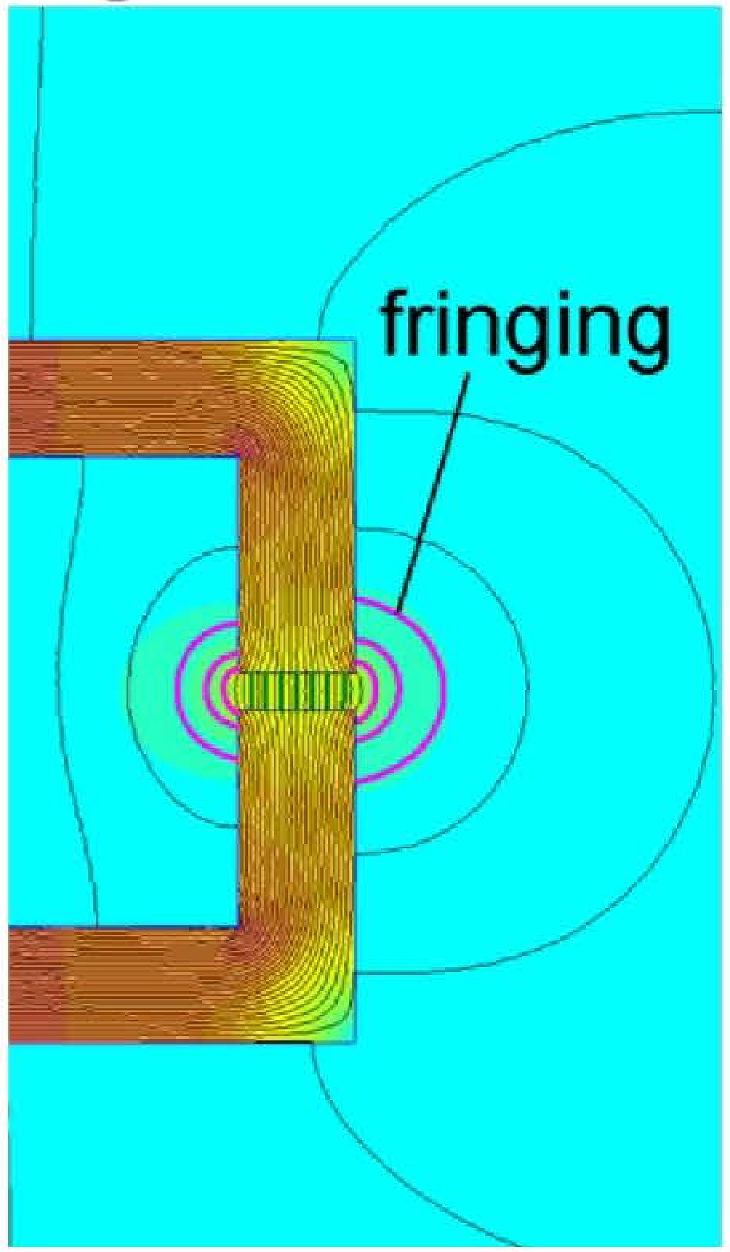


- Identification of the reluctances:
  - Reluctance of the stator $\Re_s$,
  - Reluctance of the left airgap $\Re_{gL}$
  - Reluctance of the centre leg gap $\Re_{gC}$
  - Reluctance of the right airgap $\Re_{gR}$
  - Reluctance of the armature, $\Re_a$
- The total reluctance :

$$\Re_t = \Re_s/2 + \Re_a/2 + \Re_{gL}/2 + \Re_{gC}$$

Airgaps in magnetic circuits

- The flux density in the air gap is given by: $B_g = \frac{\Phi_g}{A_g}$

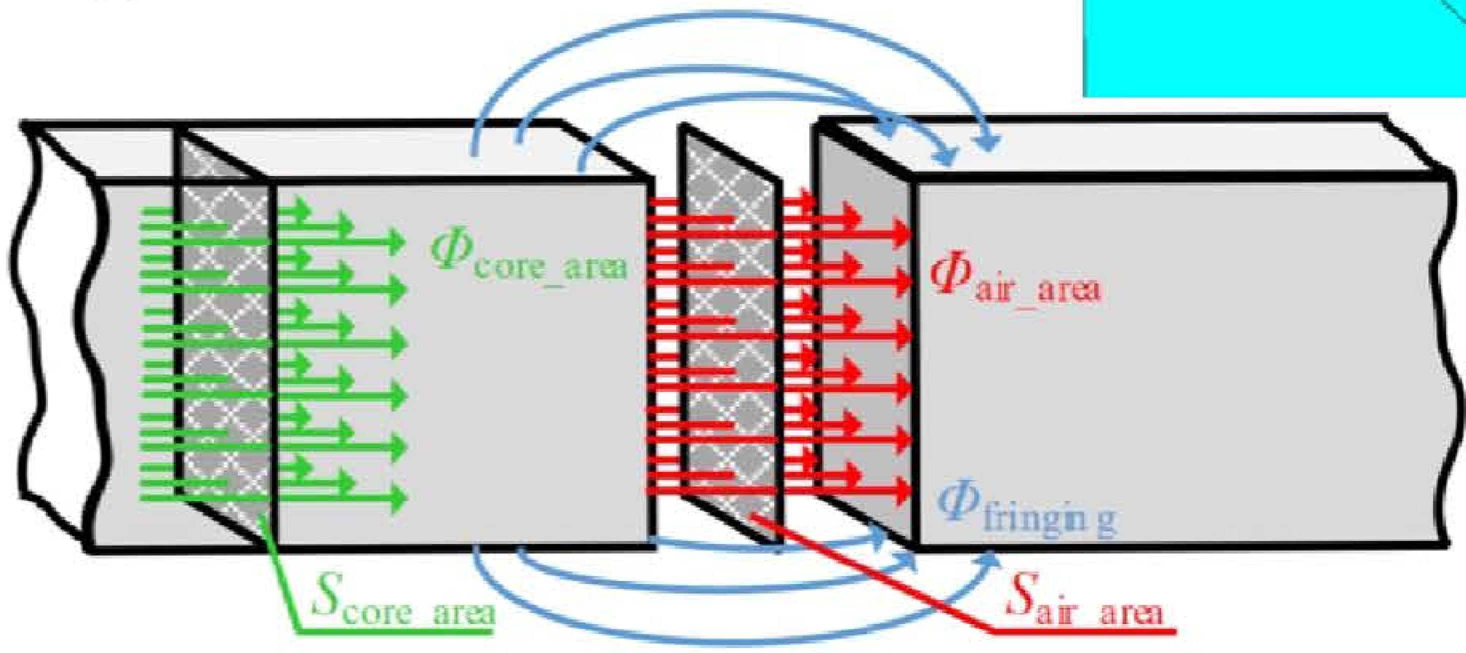


- The flux through the air gap is many times approximated by the flux through the core:

$$\Phi_g = \Phi_{core}$$

- In fact, due to the fringing effect (flux near the air-gap that bends out), the effective area > A_{core}, such that

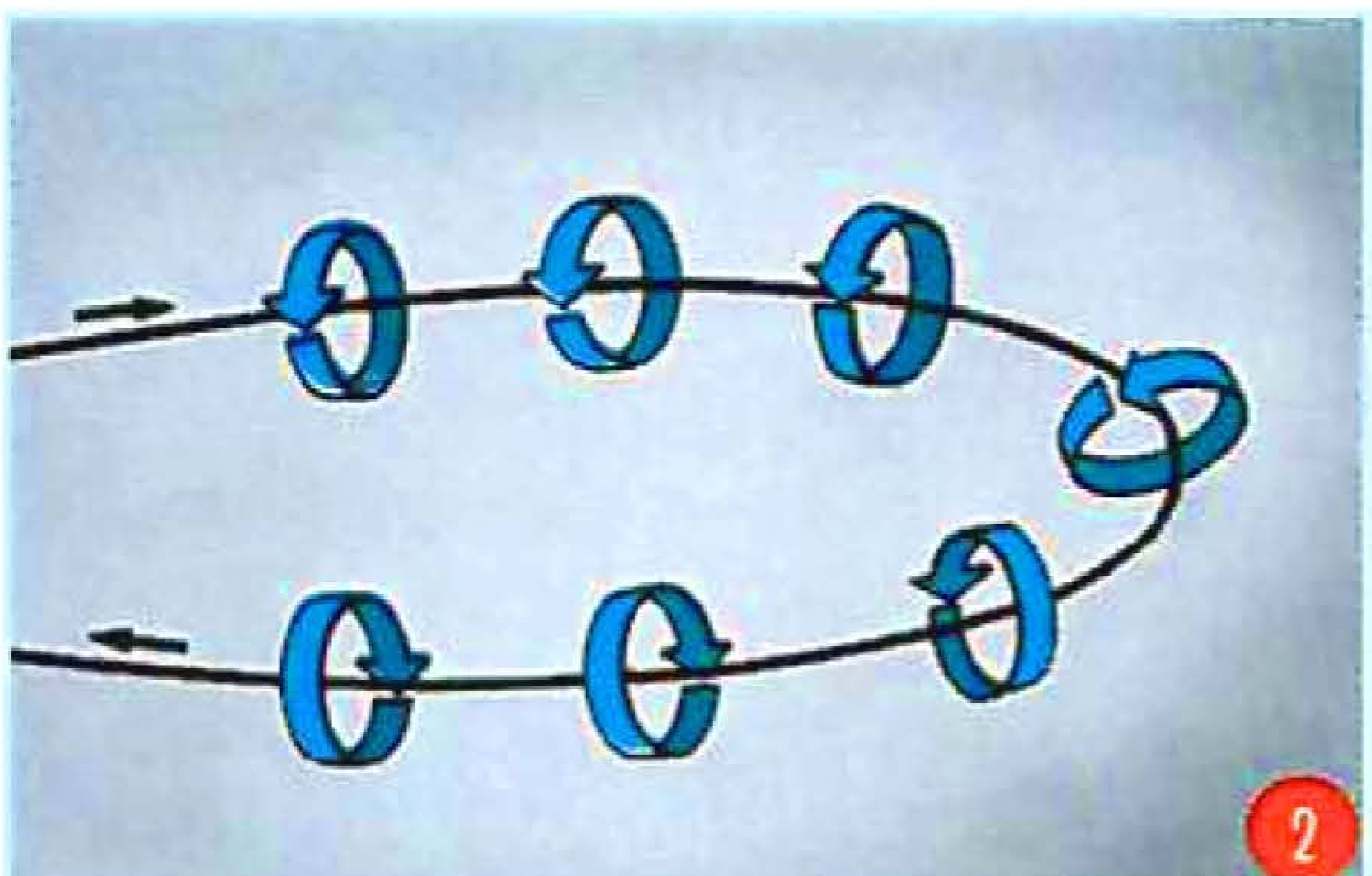
$$\mathcal{R} < \frac{\ell_g}{\mu_0 A_{core}}$$

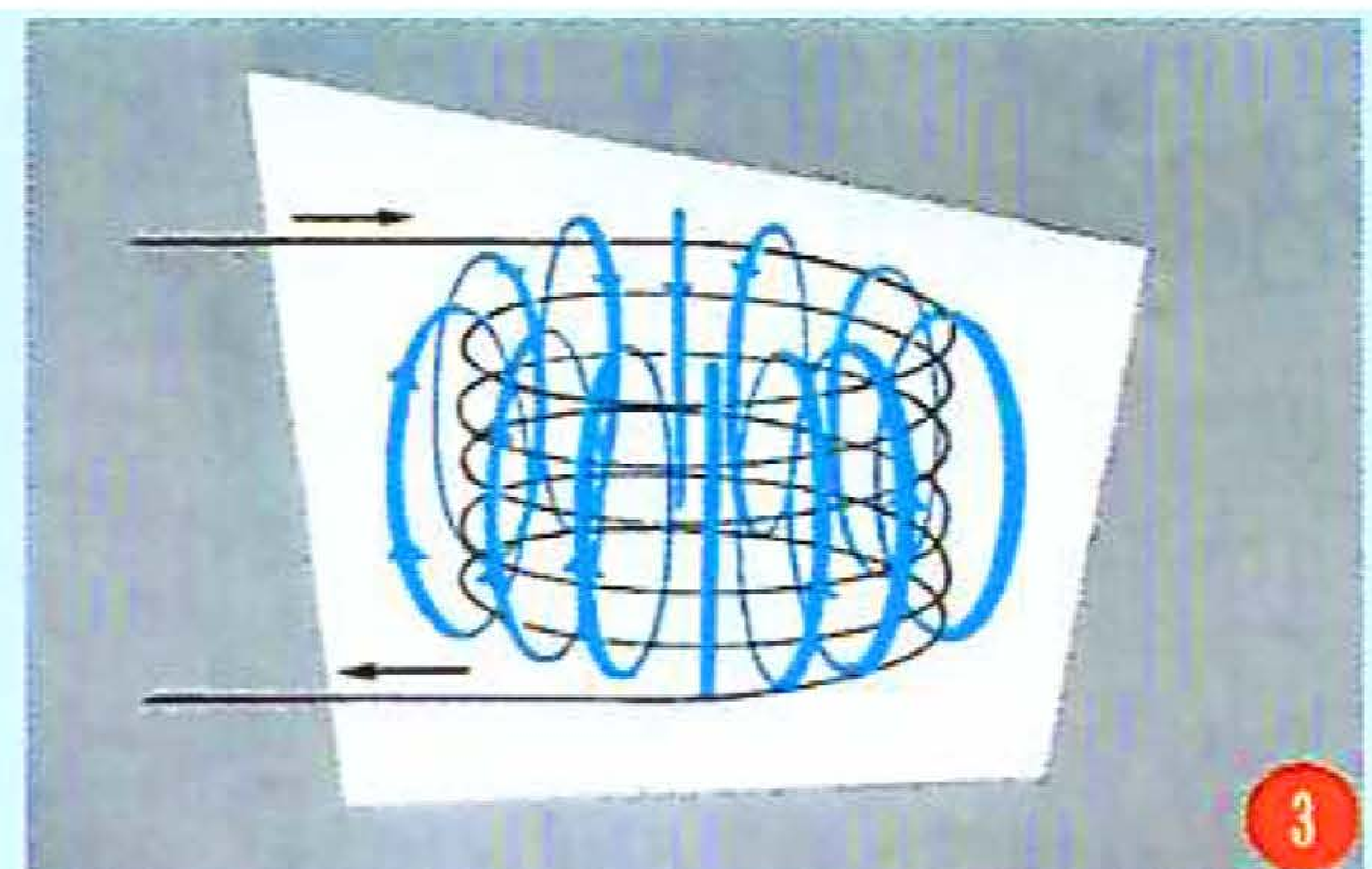


Remark: The term “air- gap” is referring a slot inside a magnetic core, however there are situations when this slot can be filled by a different material with permeability other than the air.

SOLENOIDS

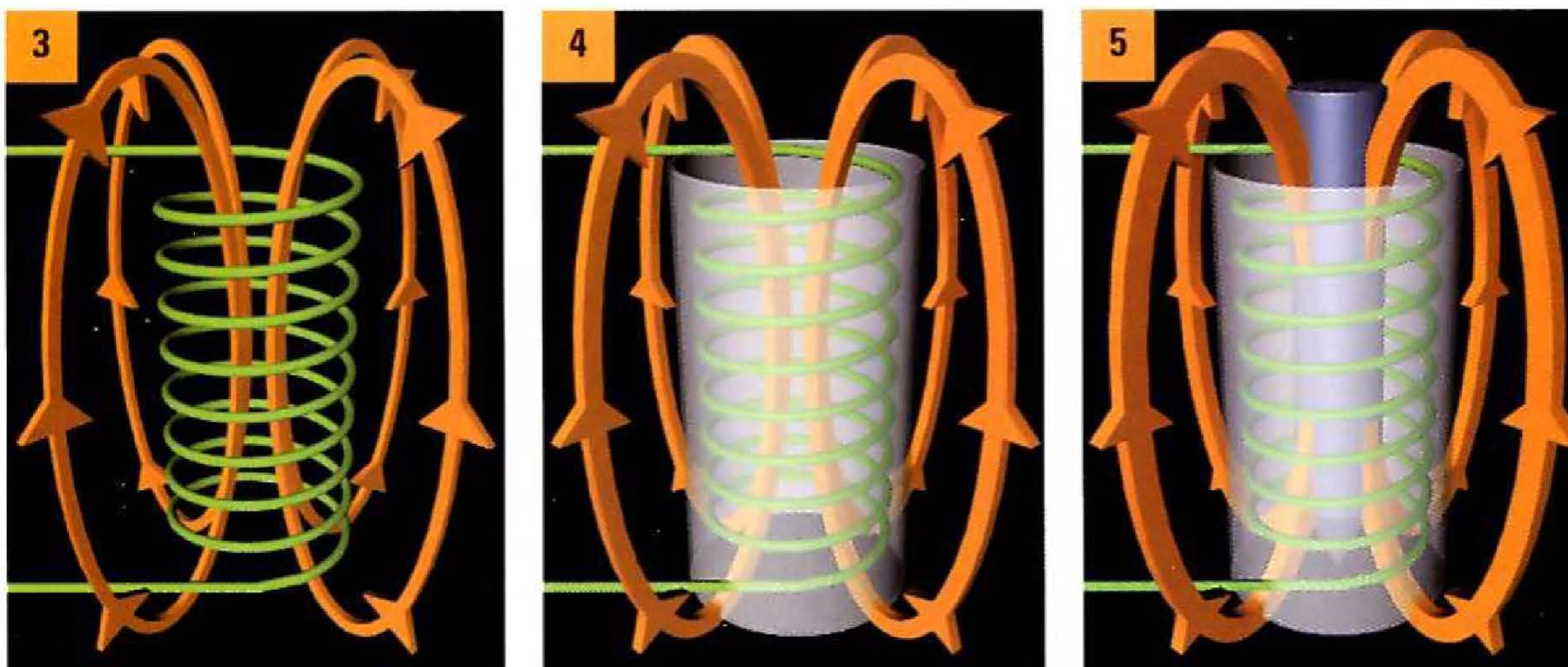
Solenoids use the physical effect that current flowing through an electric conductor generates a magnetic field.





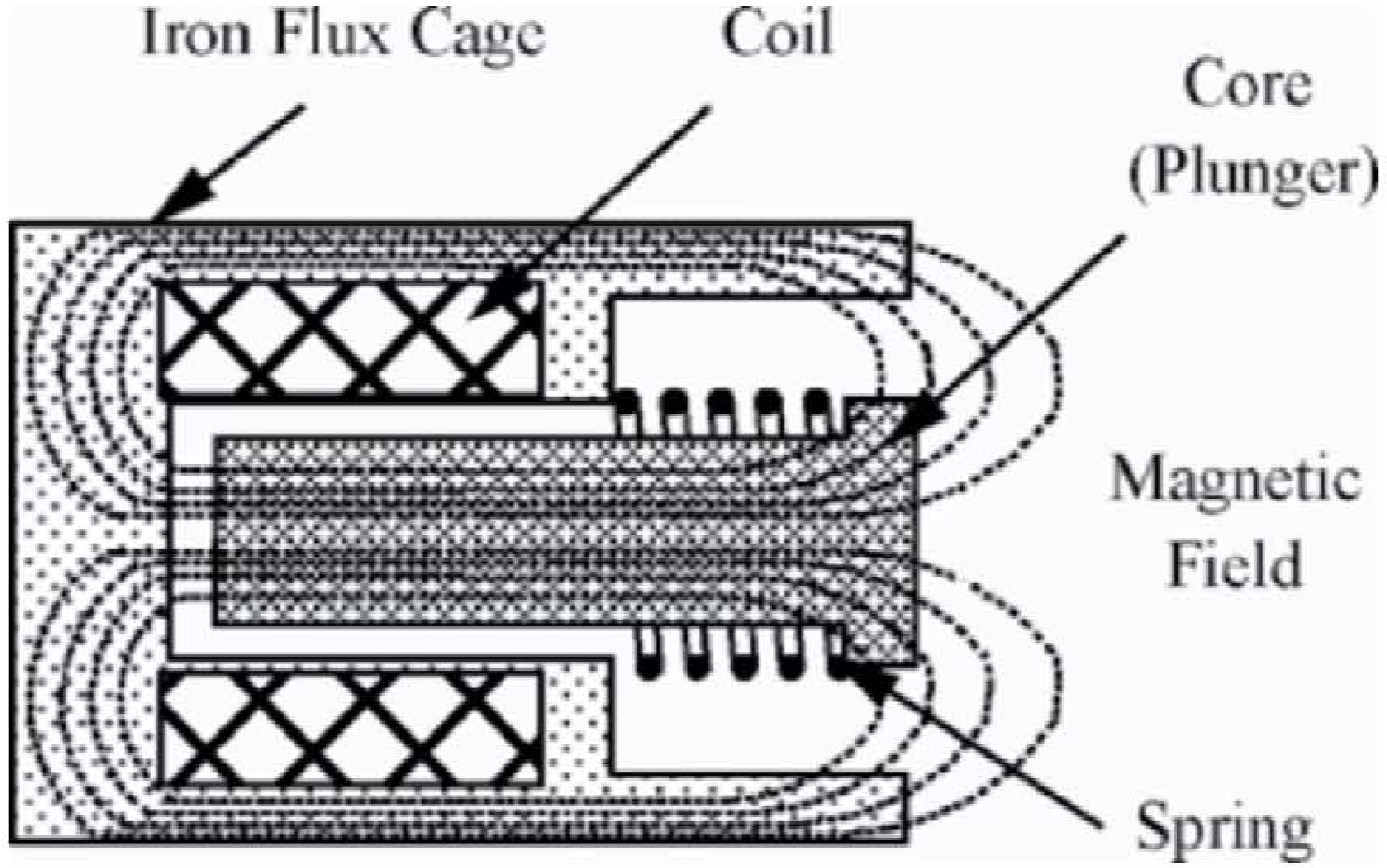
SOLENOIDS

The **magnetic field can be amplified** by arranging the wire in the form of a **coil with a large number of windings**.



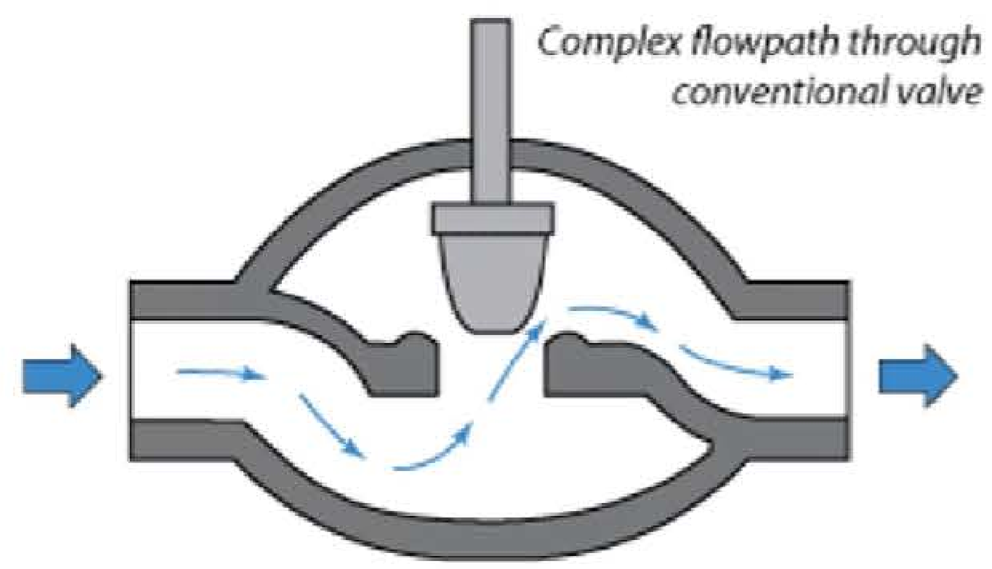
SOLENOIDS

- An iron flux cage around the solenoid coil has a substantial impact on the resultant magnetic forces.
- The magnetic field generated by the solenoid coil causes a force of attraction on the magnetic core located at the centre which is also frequently referred to as *plunger*



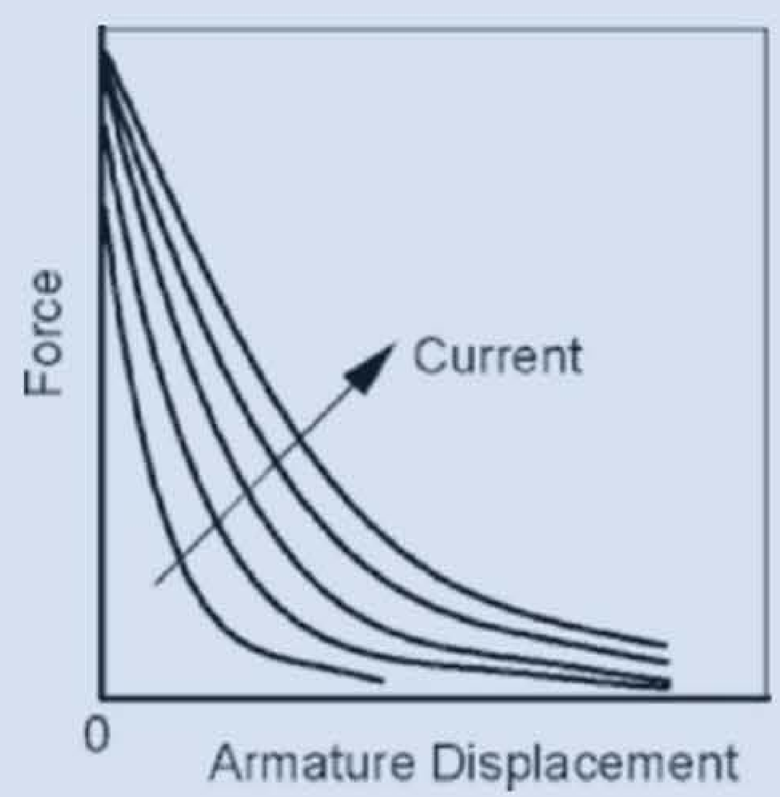
SOLENOIDS

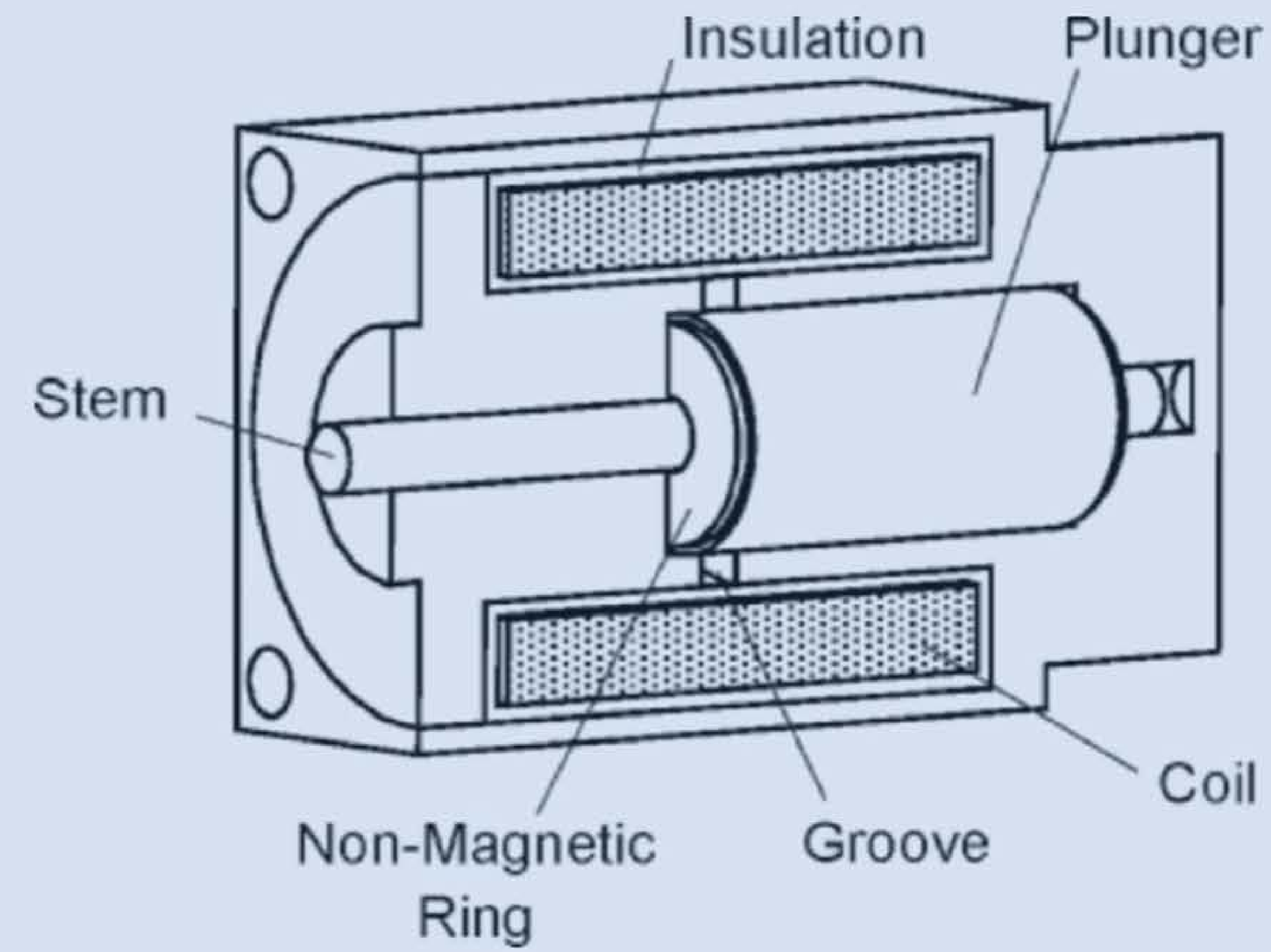
- Used in both **on-off control valves** and **proportional control valves**
- The force of the conventional DC solenoid is ***highly dependent on the position*** *of the armature* because *the air gap which can be seen as a resistance for the flux between flux cage and plunger changes with the position*
- Unlike voice-coil systems, the proportional solenoid has no permanent magnet → Its force is generated in only one direction, regardless of current direction


Complex flowpath through
conventional valve

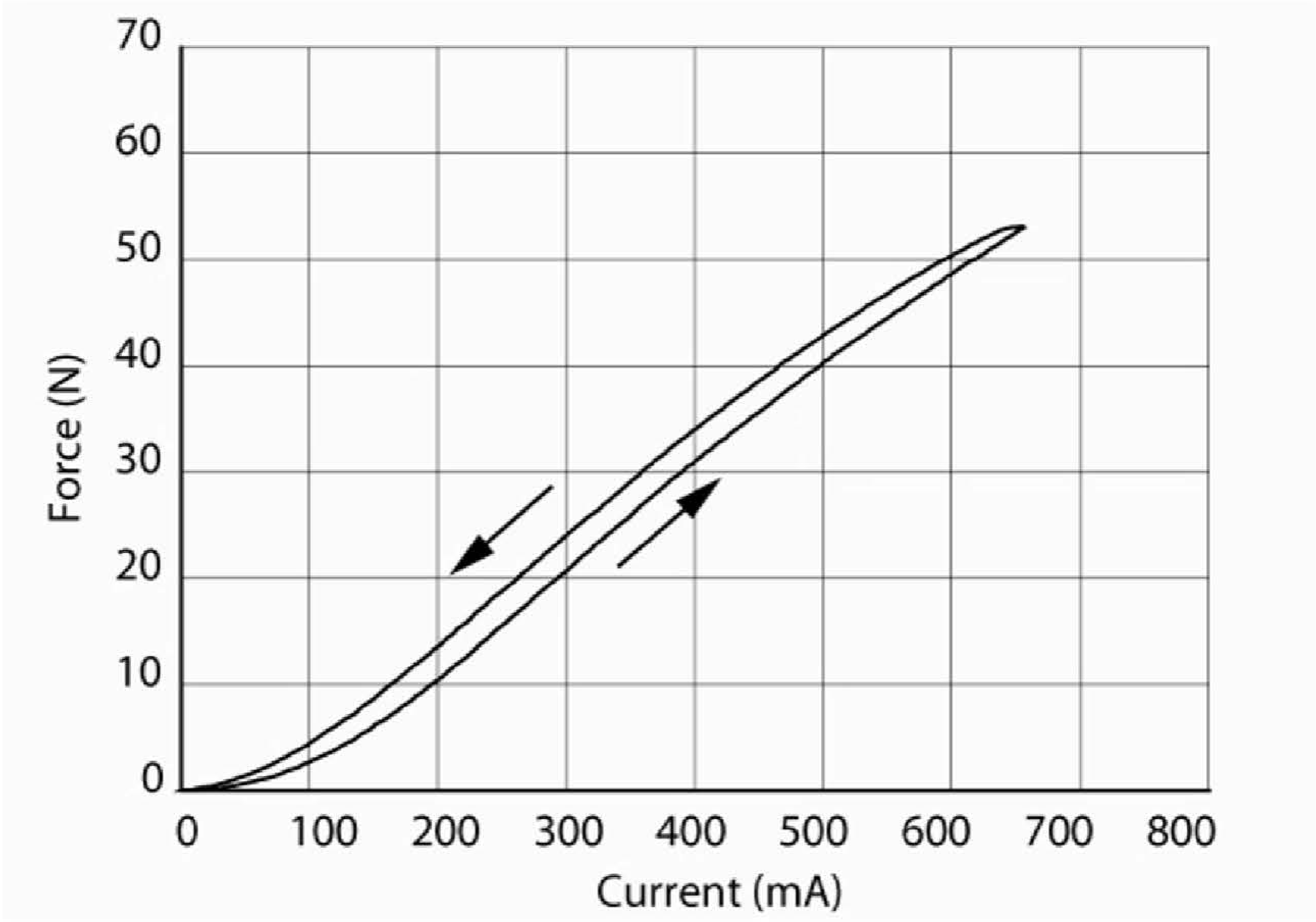
Force as a function of plunger displacement for several values of DC current

Cut-away view of a proportional solenoid





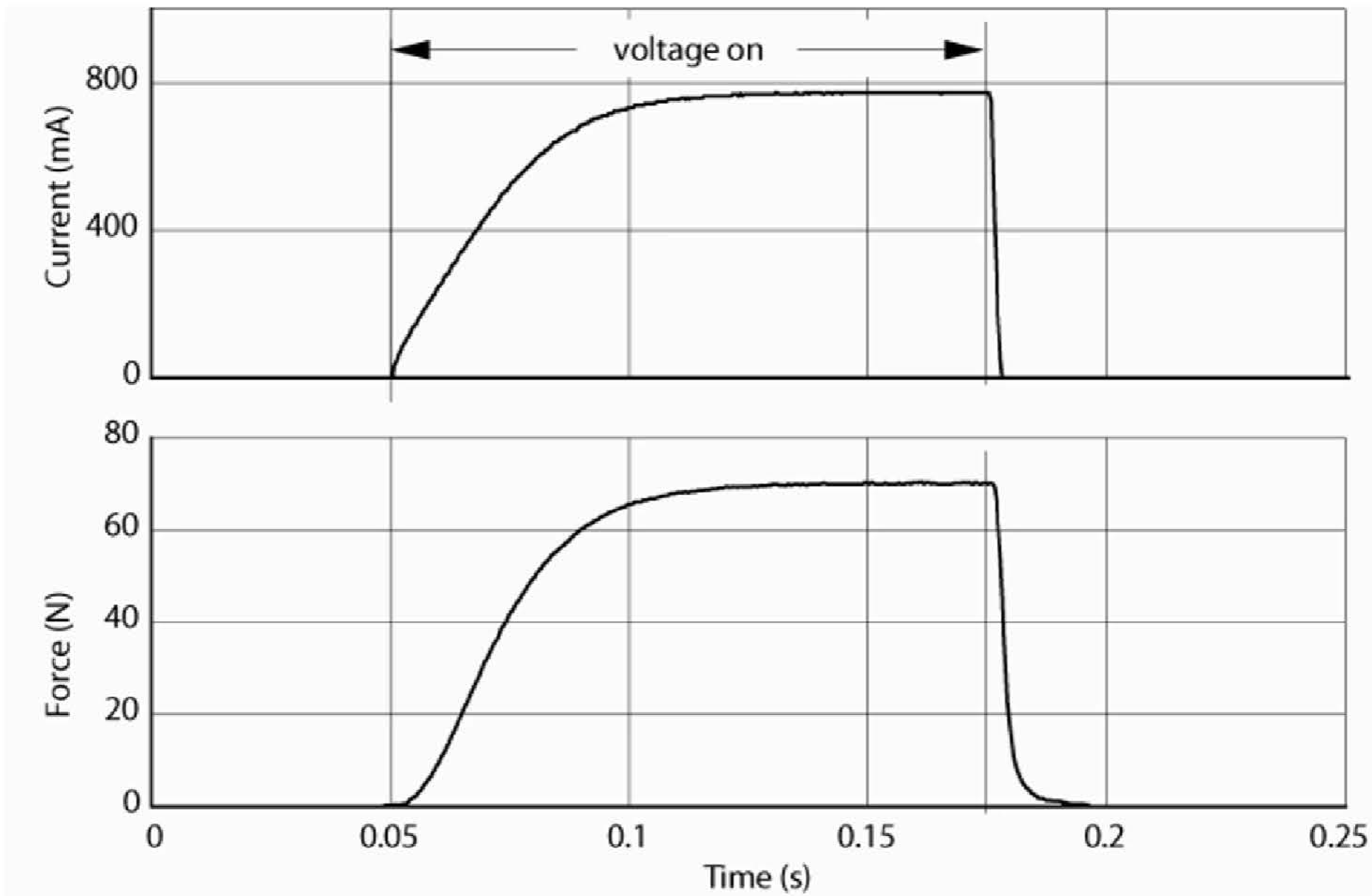
Solenoid force as a function of electric current (stabilized DC signal)



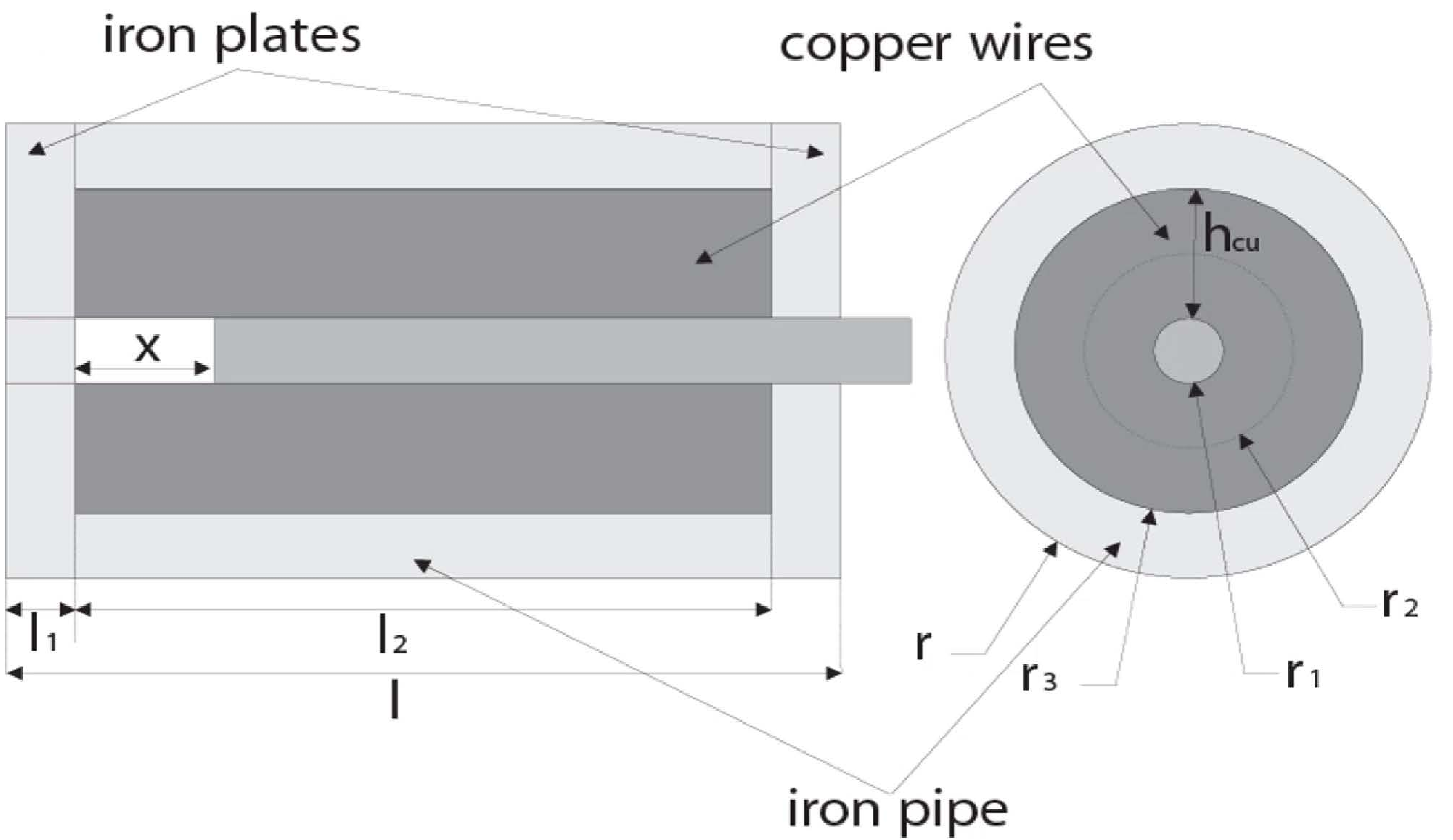
Comparison of DC and AC solenoids

DC solenoid	AC solenoid
Quieter	Direct operation with line voltage
Less wear of solenoid core	Tendency to hum
High solenoid holding force	Risk of burn-out of the solenoid coil if solenoid core is jammed
Same pick-up and holding power	
Simpler design (No shading ring required to avoid humming)	Faster switching speed than DC solenoids

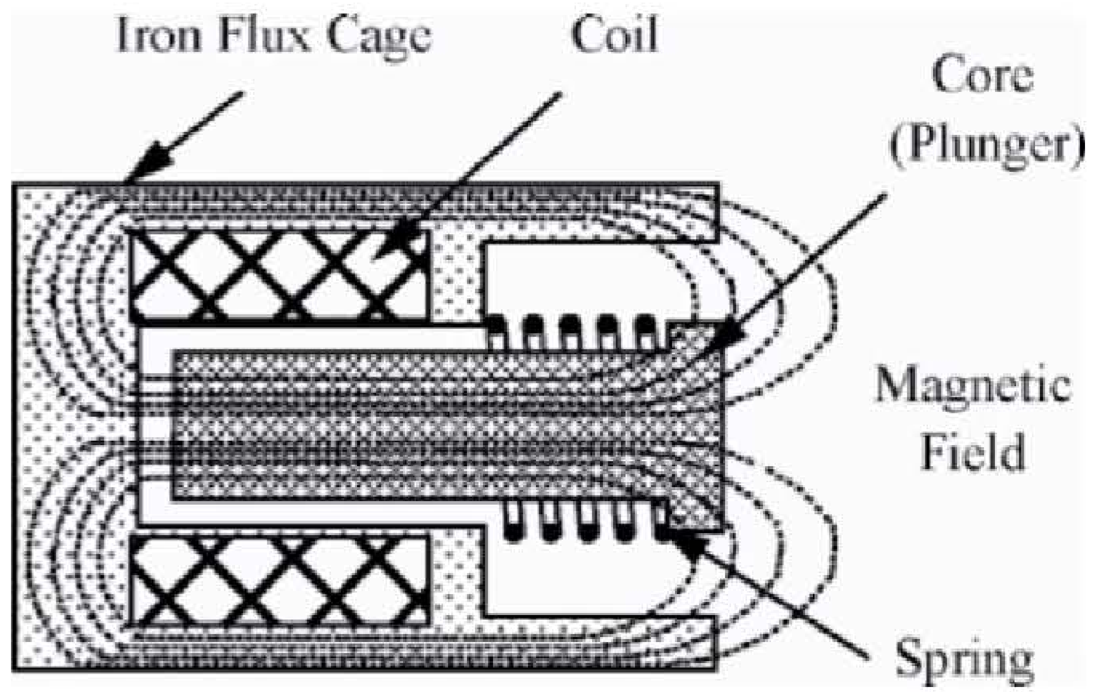
Step response of a proportional solenoid, armature fixed

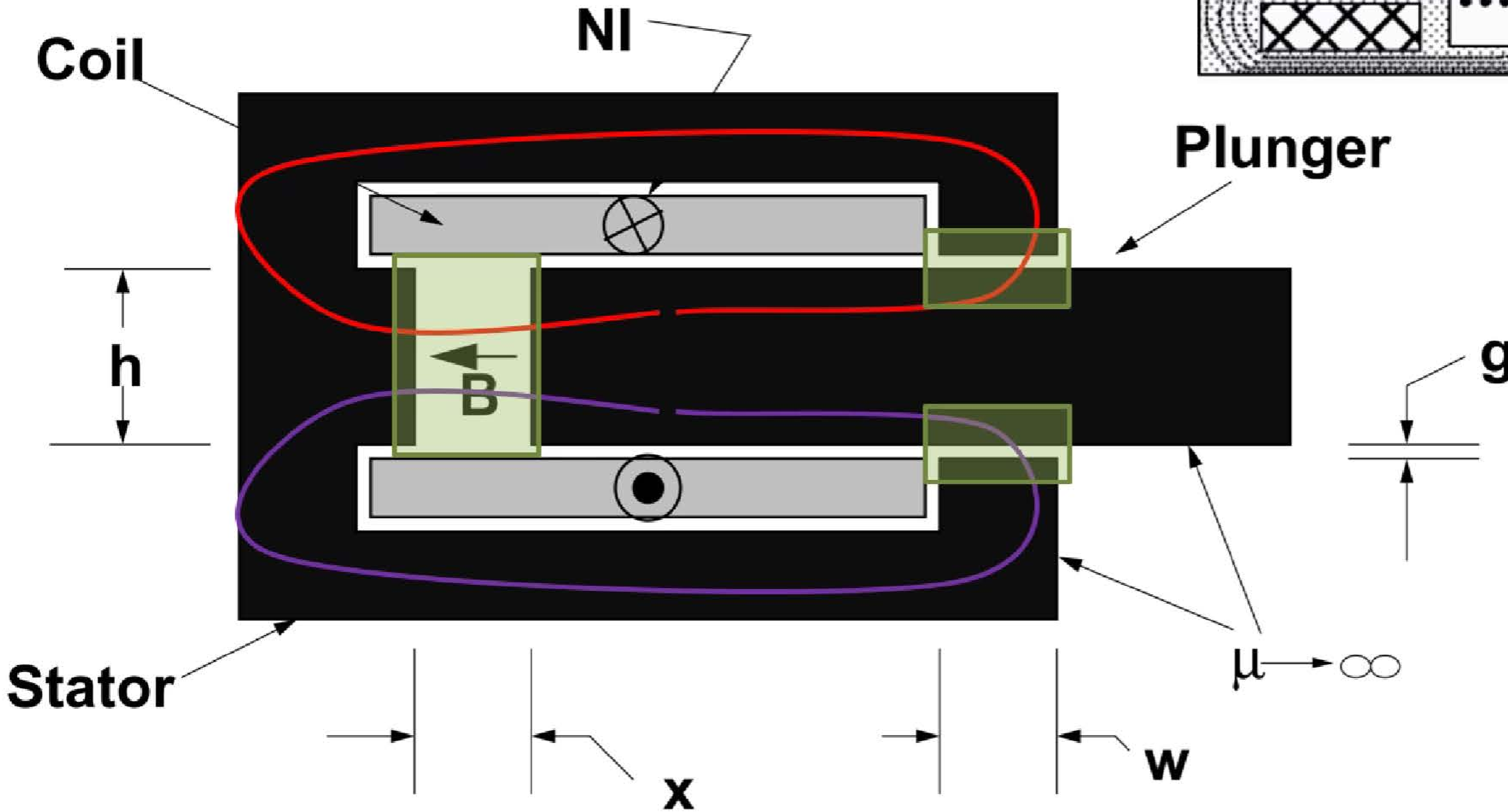


Solenoid – geometrical model

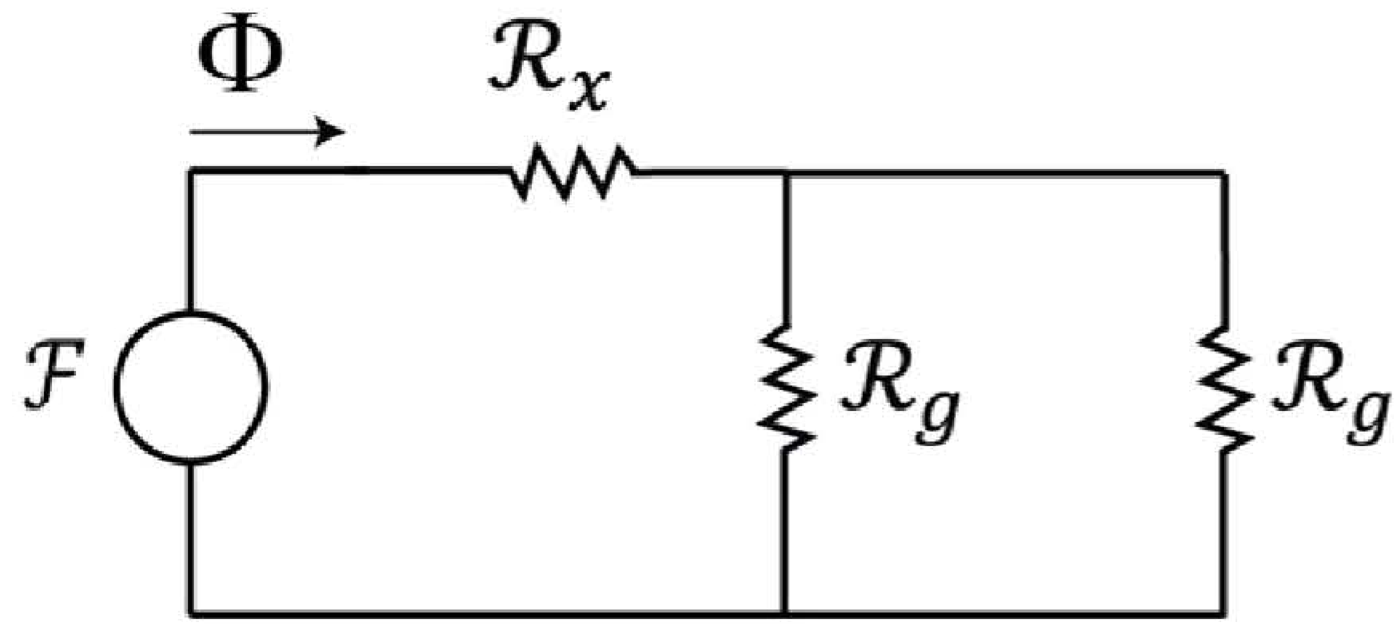


Solenoid model- Airgaps



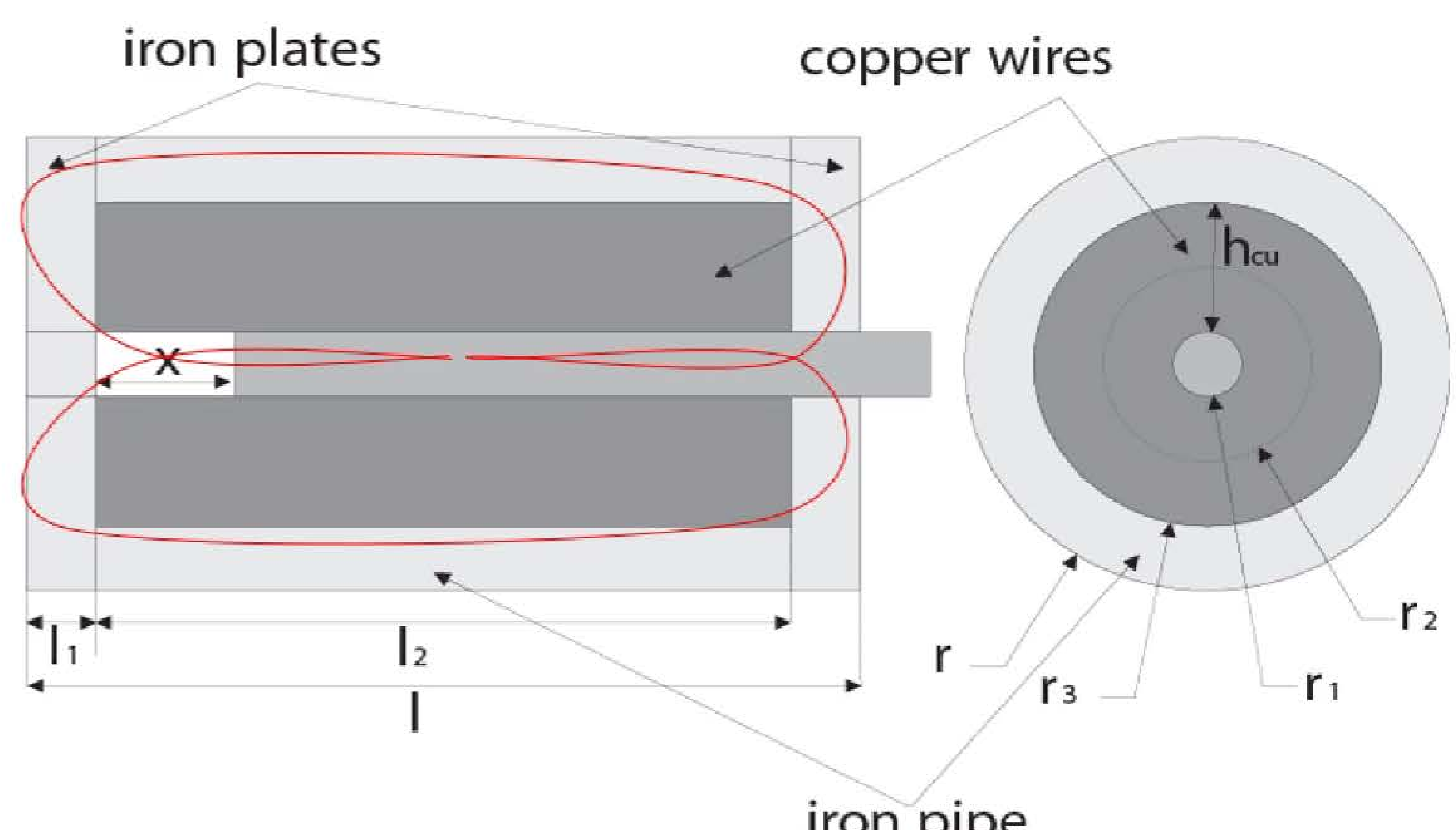


The magnetic circuit with the reluctances



$$\frac{x}{\mu_0 S} + \frac{l_2 - x}{\mu_r \mu_0 S} + \frac{l_{eq}}{\mu_r \mu_0 S}$$

The third term in introduced as an equivalent reluctance, in order to account for the reluctance of the plates and of the pipe



Solenoid Model

The magnetic flux flowing inside a solenoid can be written as:

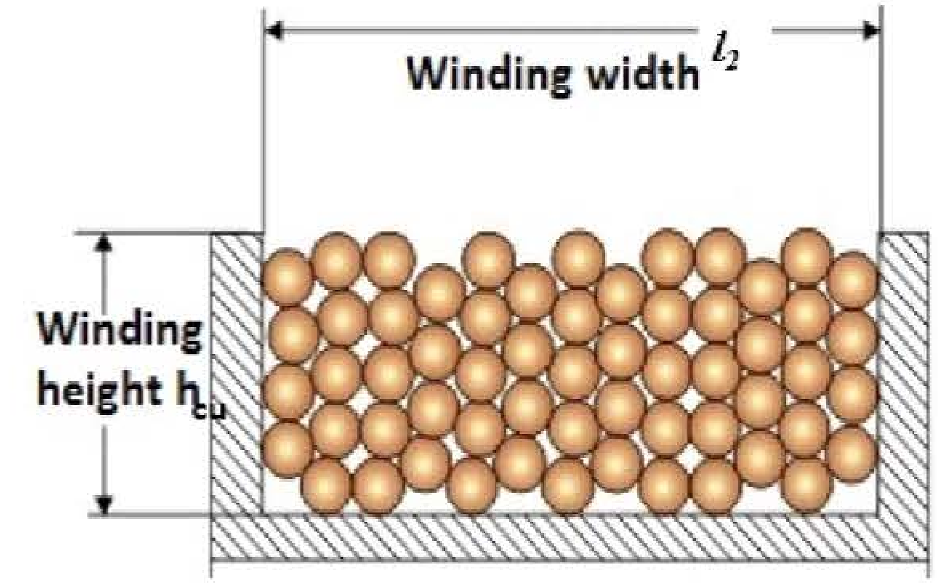
$$\varphi = \frac{F_{mm}}{\Re} = \frac{Ni}{\frac{x}{\mu_0 S} + \frac{l_2 - x + l_{eq}}{\mu_r \mu_0 S}} = \frac{Ni\mu_r \mu_0 S}{l_2 + l_{eq} + x(\mu_r - 1)}$$

where $\mathcal{R}$ is the reluctance expressed as a function of the magnetic properties of the iron μ_r, the length l_2, the cross-section of the plunger $S = \pi r_1^2$ and the length l_{eq}, which is *the plunger length with a reluctance equivalent to the reluctance of the plates and the pipe*.

F_{mm} is the magneto-motive force, equal to the number of turns *N* times the current $\boldsymbol{i}$.

Solenoid Model

The number of turns can be expressed as a function of the actuator dimensions as:



$$N = \frac{h_{cu} l_2 k_{ff}}{A_w}$$

where h_{cu} is the thickness of copper, l_2 the coil's length, k_{ff} the filling factor and A_w the cross-section of a single wire.

The solenoid force is produced for the change of reluctance due to the change of the air gap distance. Its expression can be derived from the energy stored in a solenoid $W_m = \int i d\lambda = \int N i d\phi$

It yields:

$$F = \frac{dW_m}{dx} = \frac{S N^2 i^2 \mu_r^2 \mu_0}{2\left(l_2 + l_{eq} + (\mu_r - 1)x\right)^2}$$

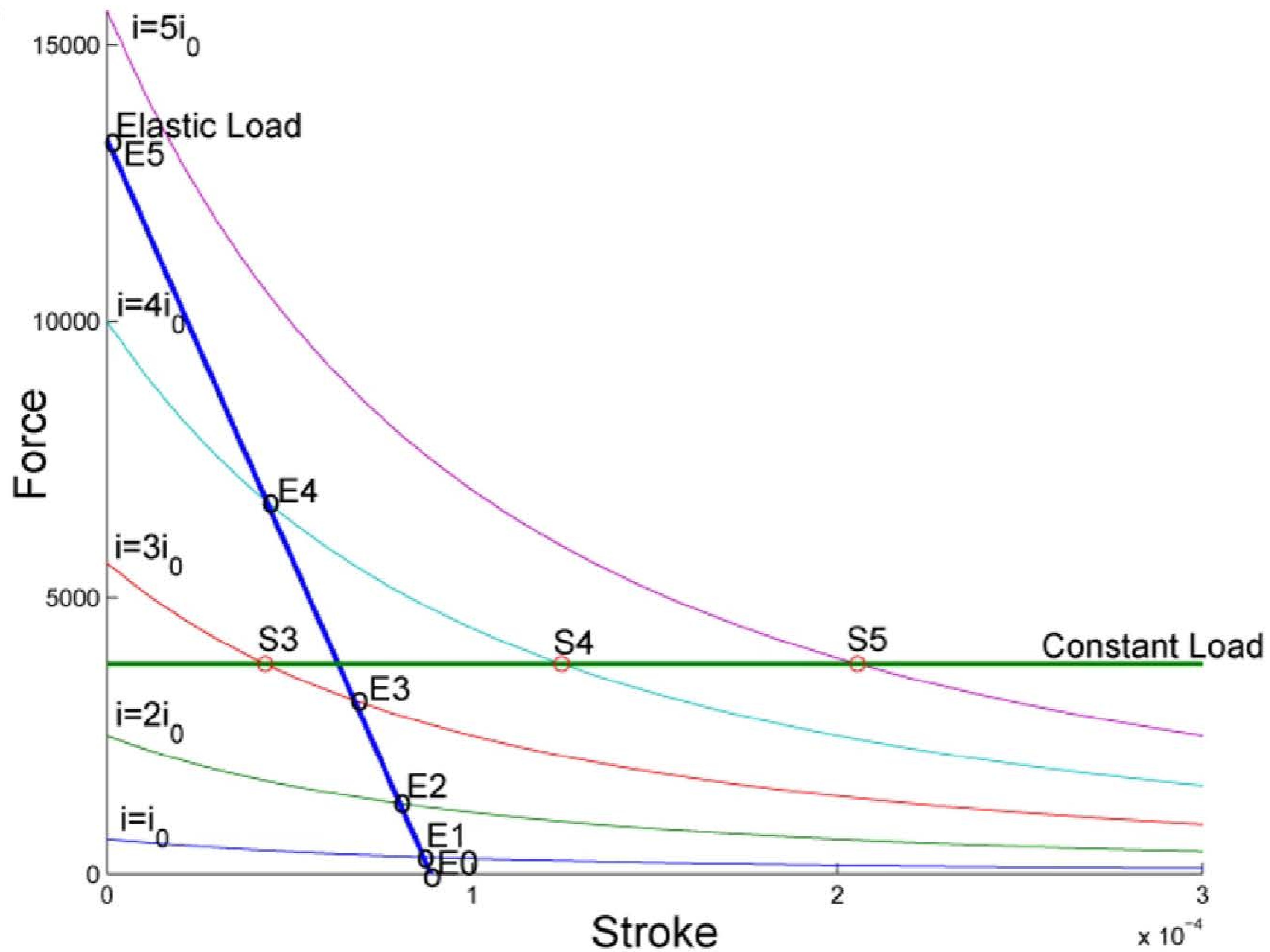
Solenoid Model

The energy can be obtained integrating the force between a given displacement *x* and 0 as

$$W = \int_0^x F dx = \frac{SN^2 i^2 \mu_r^2 \mu_0 x}{2\left(l_2 + l_{eq} + (\mu_r - 1)x\right)\left(l_2 + l_{eq}\right)}$$

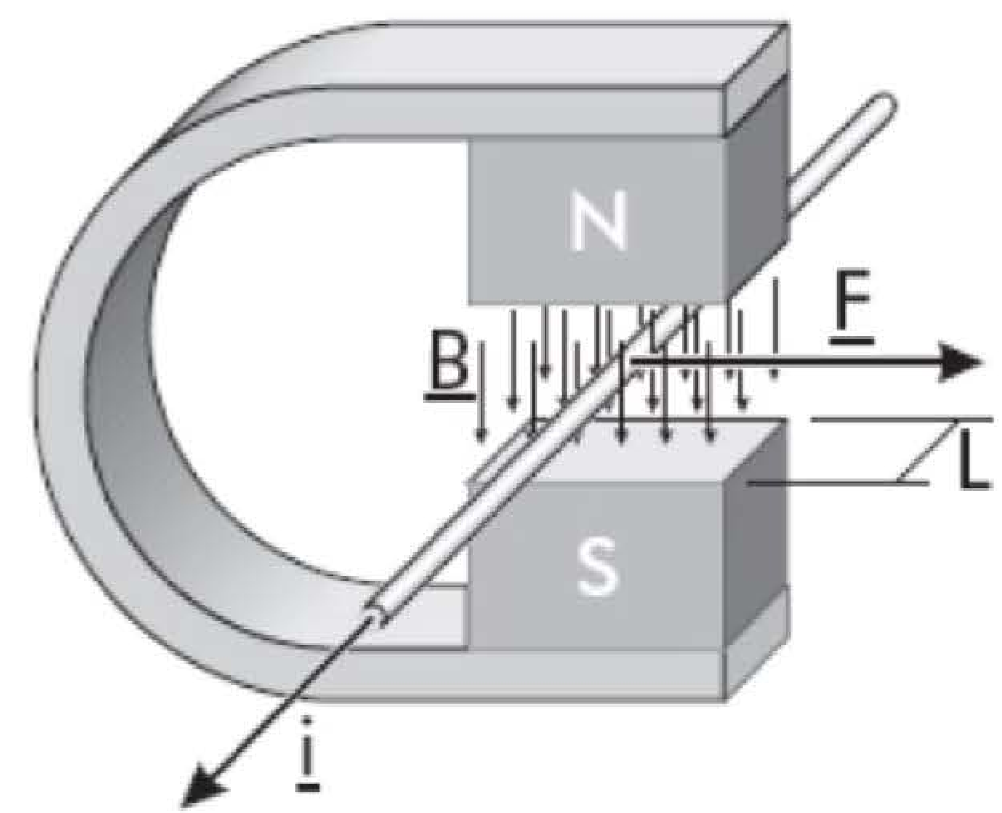
Solenoid Model

Force-Displacement curves for elastic loads and constant loads



Voice coil system

- the *Lorentz Force* principle: if a current-carrying conductor is placed in a magnetic field, **a force** will act upon it, which is **proportional to the current and the magnetic flux density.**



Lorentz Force

The Lorentz force is *the force experienced by a moving charge in an electromagnetic field*.

For a charge q moving with a velocity $\vec{v}$ in a space where an electromagnetic field exists, the force is given by:

$$\vec{F} = q\,(\vec{E} + \vec{v} \times \vec{B})$$

where $\vec{E}$ denotes the electric field, $\vec{B}$ denotes the magnetic field

For a wire where a uniform current $\boldsymbol{i}$ is flowing, under the assumption that there is no electric field, the force can be found as follows:

$$\begin{aligned} d\vec{F} &= dq\,\vec{v} \times \vec{B} \\ &= dq\,\frac{d\vec{l}}{dt} \times \vec{B} \\ &= \frac{dq}{dt} d\vec{l} \times \vec{B} \\ &= i\,d\vec{l} \times \vec{B} \end{aligned}$$

$\boldsymbol{dq}$ is the quantity of charge found in an infinitesimal section $\boldsymbol{dl}$ of the wire.

If $\boldsymbol{\alpha}$ denotes the angle of incidence between the magnetic field and a *straight wire* of length $\boldsymbol{l}$, and exogenous magnetic field can be considered uniform, then the magnitude of the force $\boldsymbol{F}$ is given by $|\vec{F}| = i\,l|\vec{B}|\sin\alpha$

For $\alpha = \pm\pi/2$, $|\vec{F}| = i\,l|\vec{B}|$

Principles of Operation

- Applying a current through the coil windings generates a force
- This force and the induced voltage can be calculated by

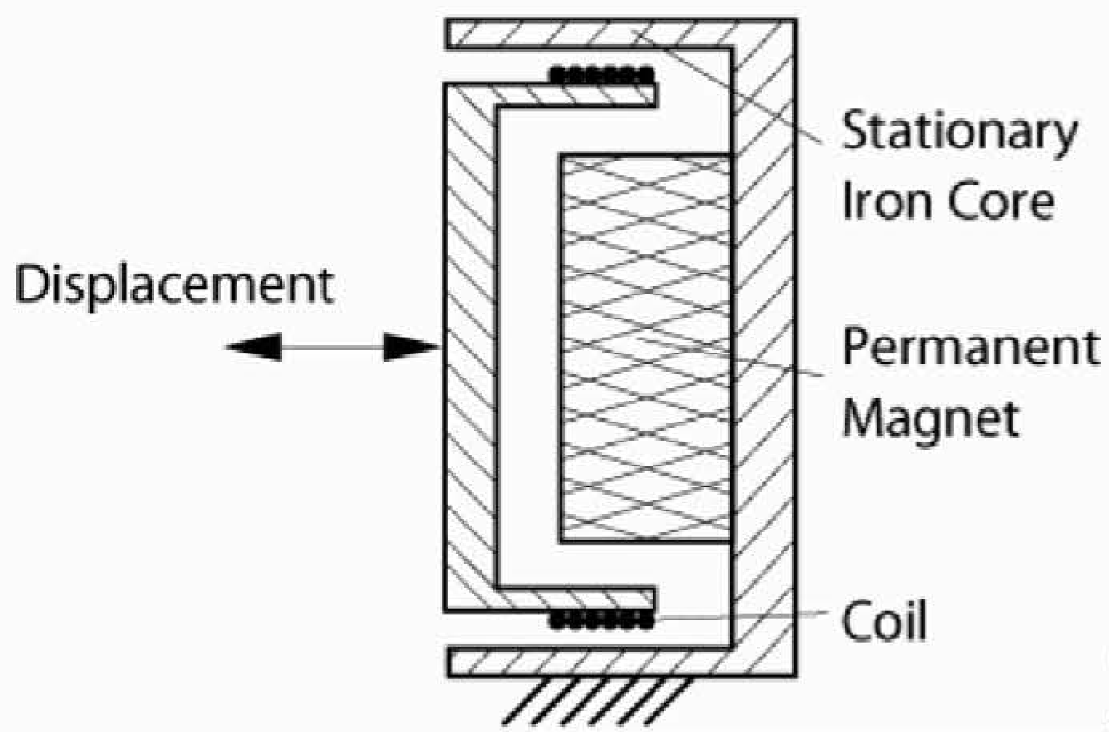
$$F = NBiL$$
$$U_i = NLvB$$

where:

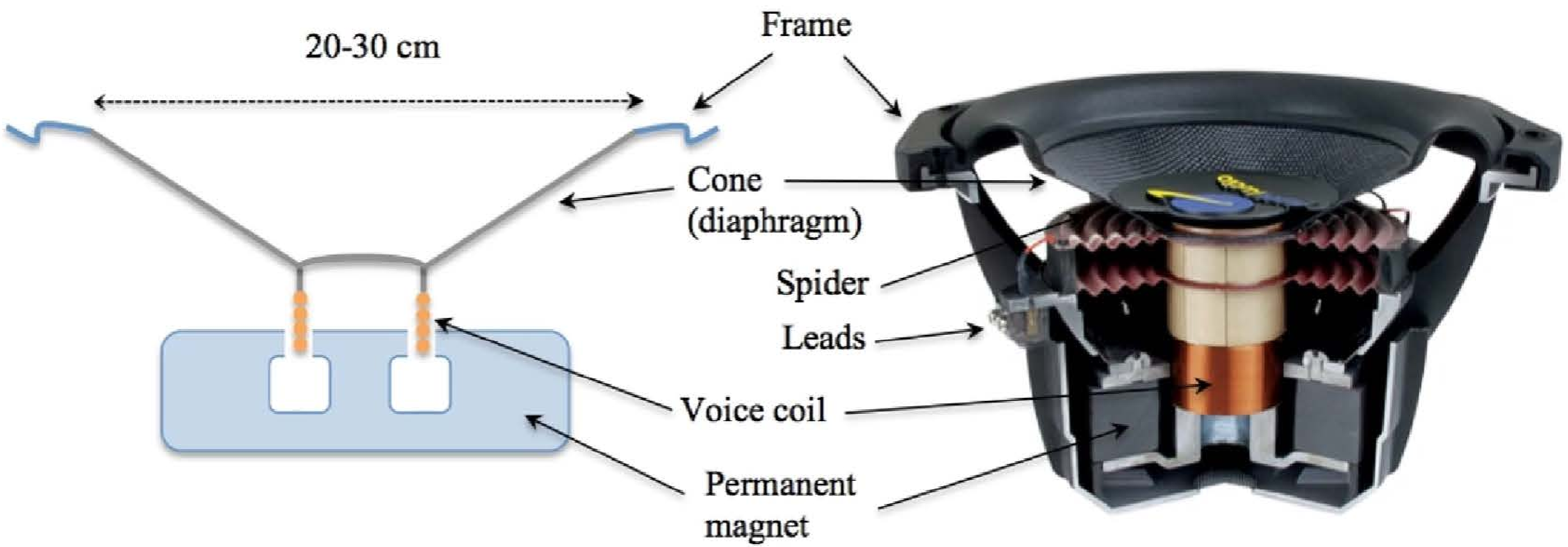
- N : Number of coil windings
- L : Length of **one** coil winding

Voice coil system

Schematic view of a voice coil system



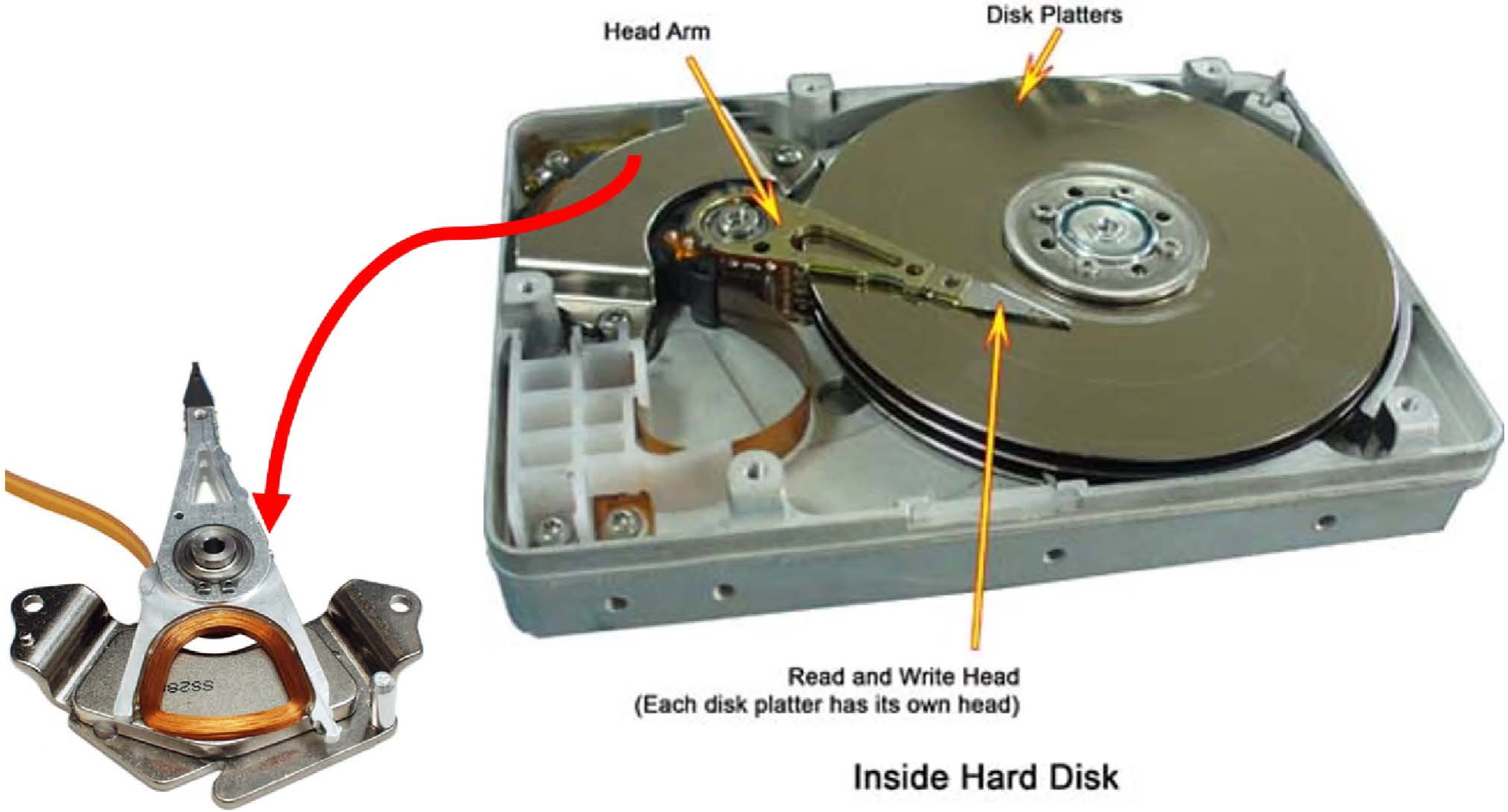
Sections through a loudspeaker



Voice coil system

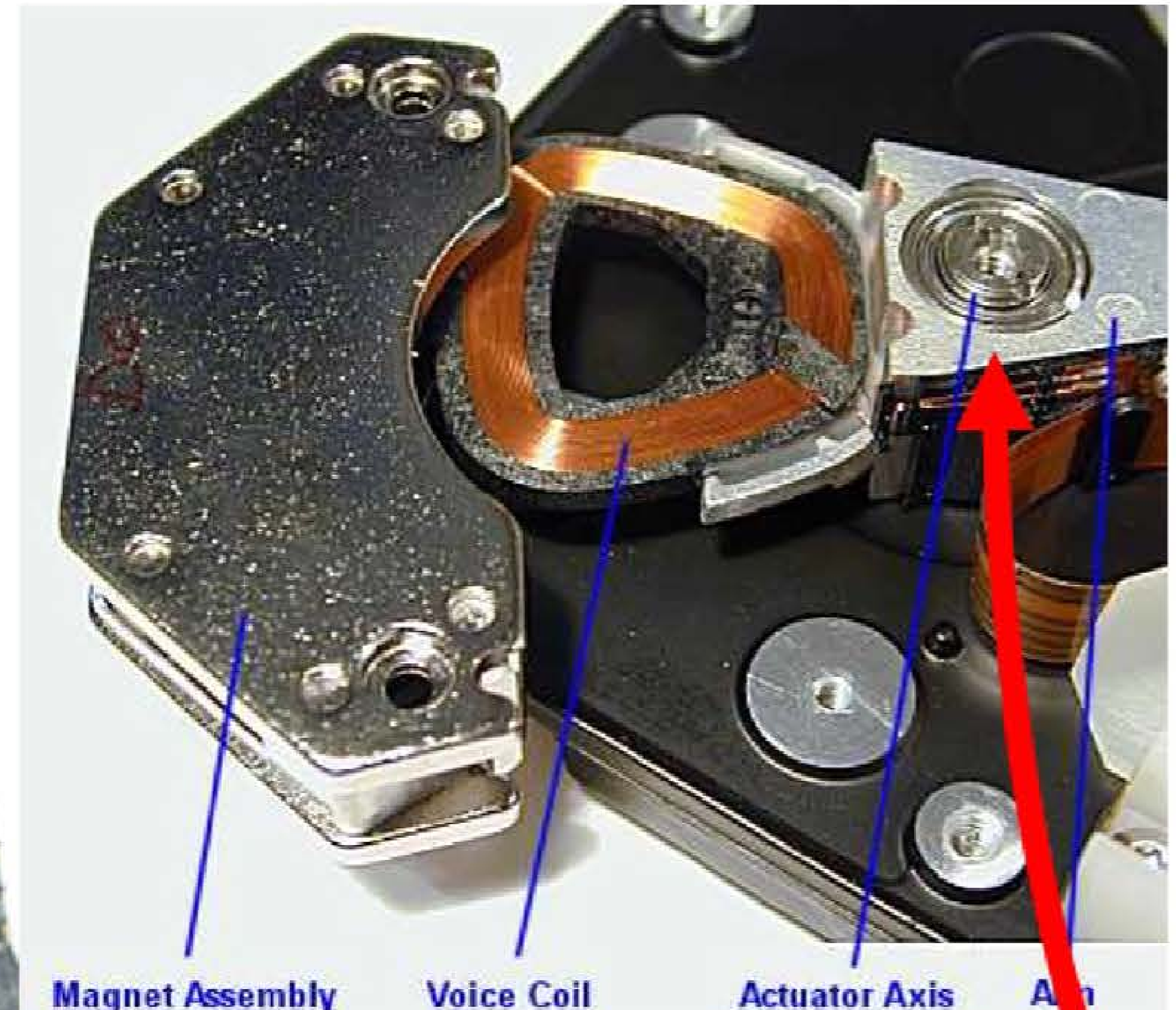
- To convert electrical energy into acoustical signals, many audio loudspeakers use a tubular coil of wire moving in a strong radially oriented magnetic field. (Permanent magnets lining the inside diameter of a ferromagnetic cylinder produce the field)
- When current flows through the coil, it generates an axial force on the coil and produces relative motion between the field assembly and the coil.
- Due to its origin this kind of electromechanical converter is often called *voice coil* or moving coil.

Rotational Voice Coil as hard-disk head actuator

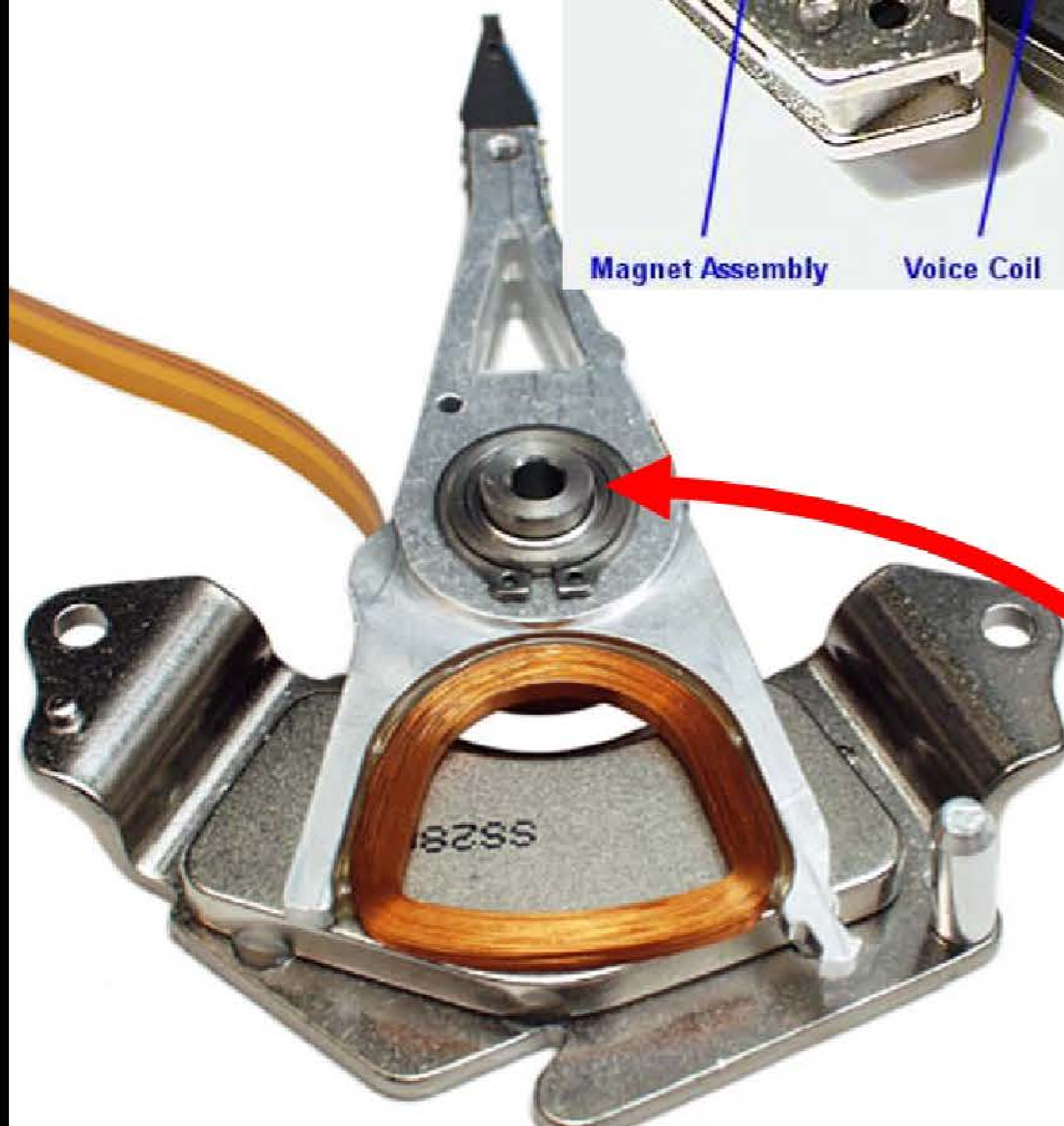


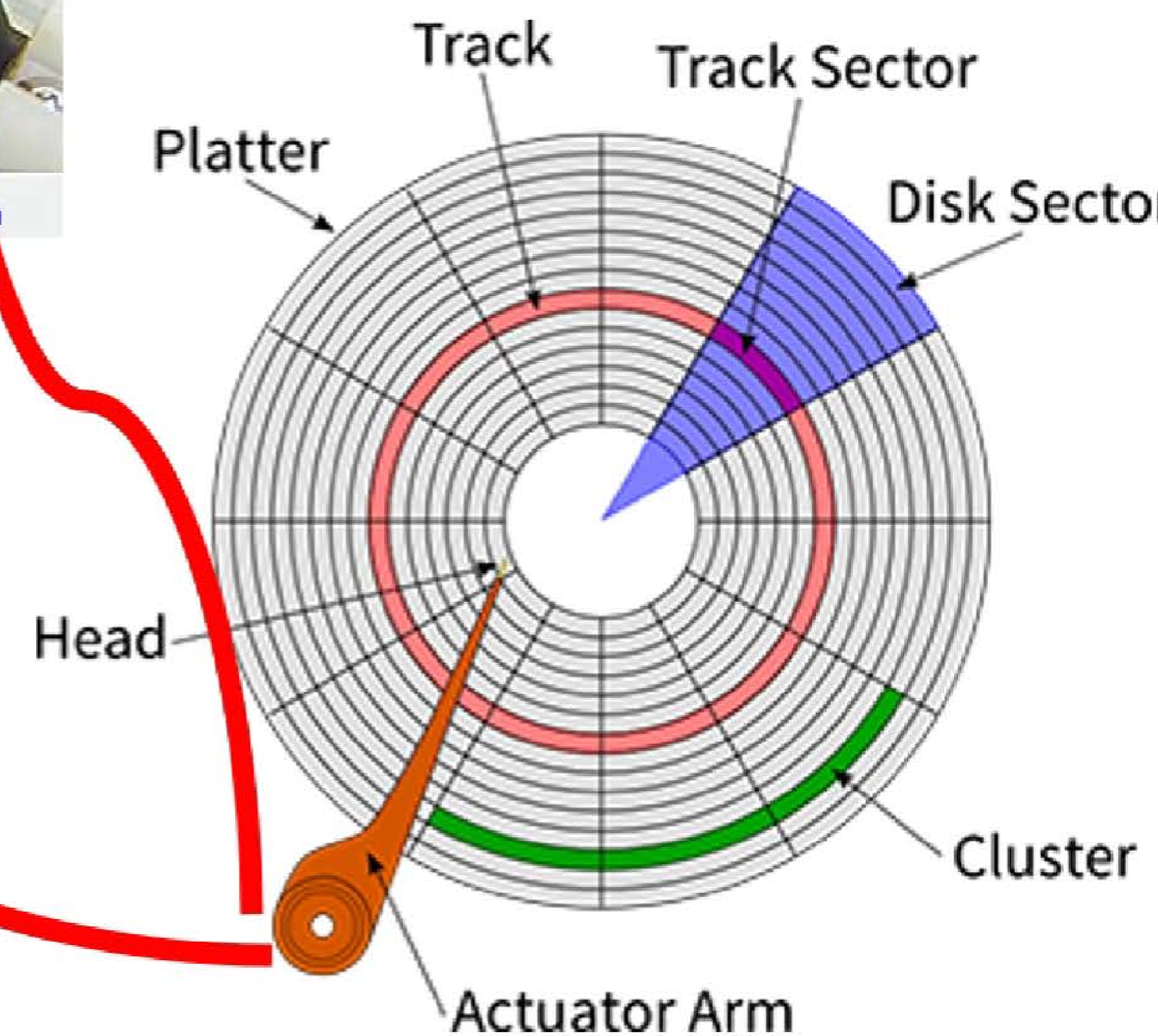
A conventional Hard Disk Drive (HDD) has two separate motors: a **Spindle Motor(SPM)** which rotates the disk at high velocities (typically 5400-1200 RPM and a **Positioner** or **Voice Coil Motor (VCM)**. This second motor moves the head assembly across the rotating disk to access individual packets of data written on the disk's magnetic media.

Rotational Voice Coil as hard-disk head actuator

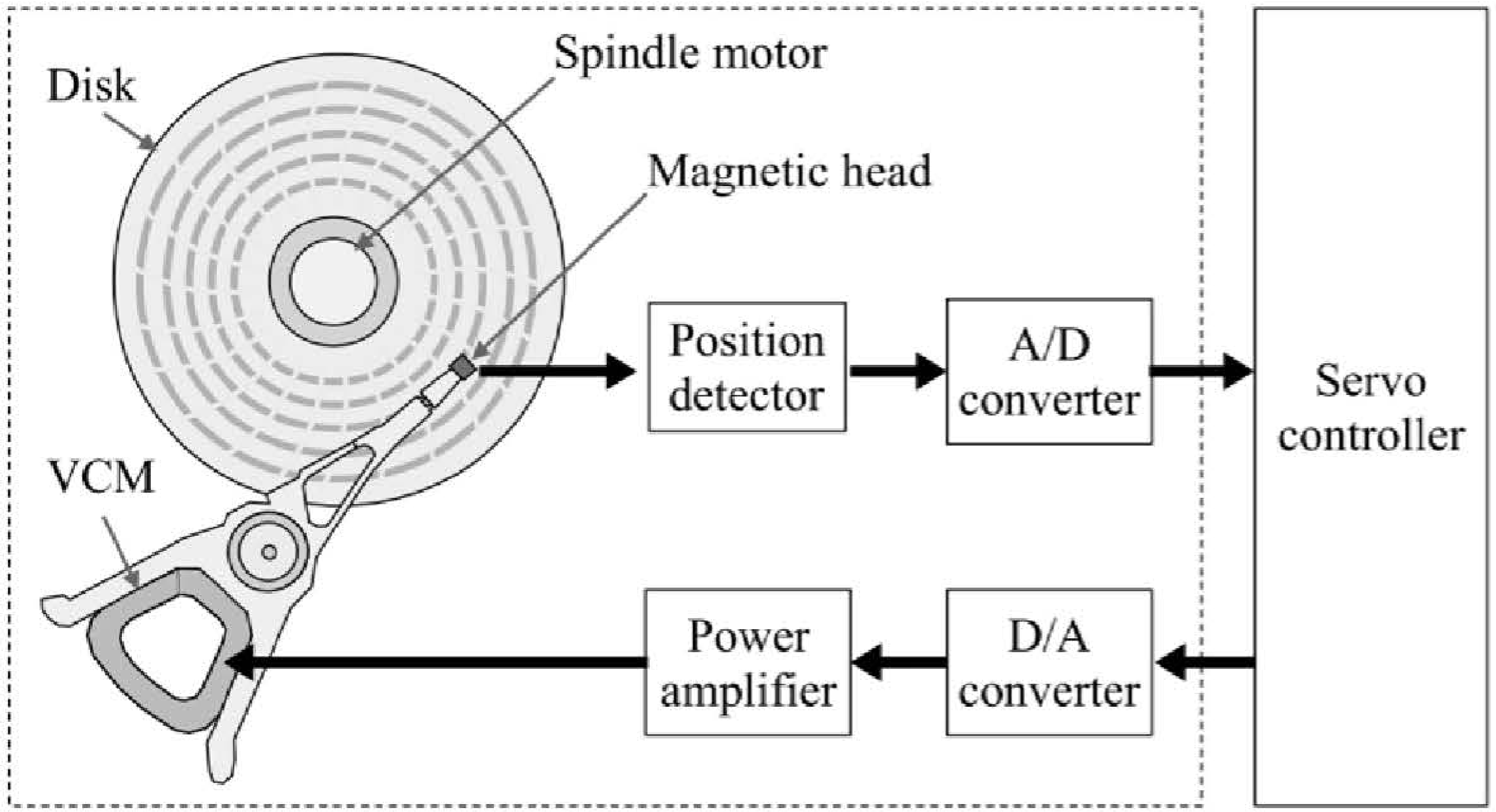


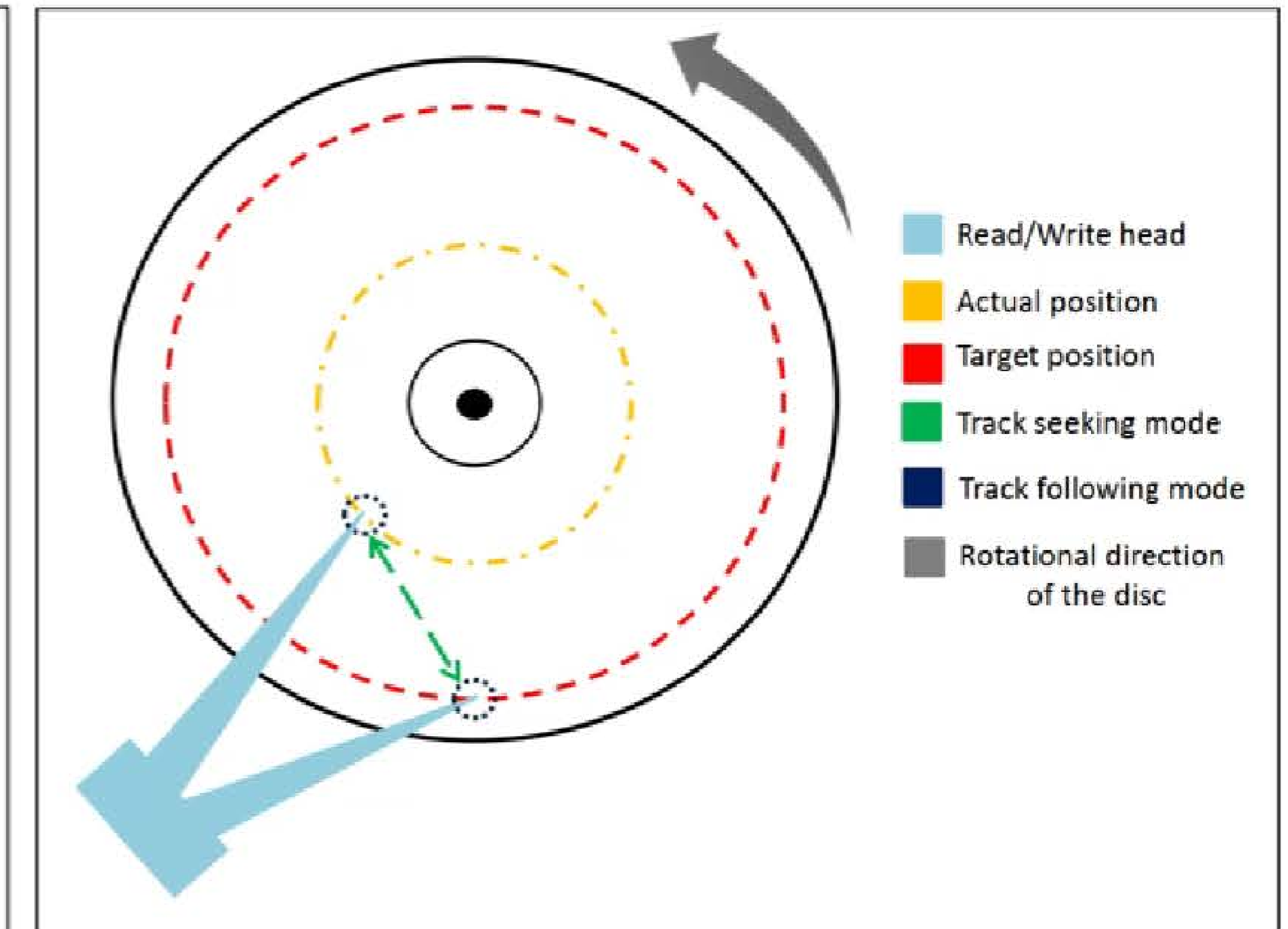
The magnets dimensions restrict how far the arm can rotate.





HDD head-positioning: control system

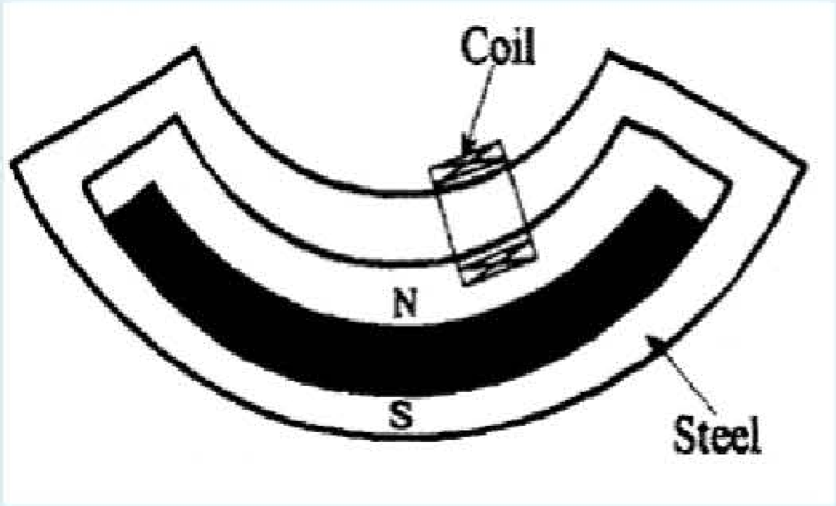


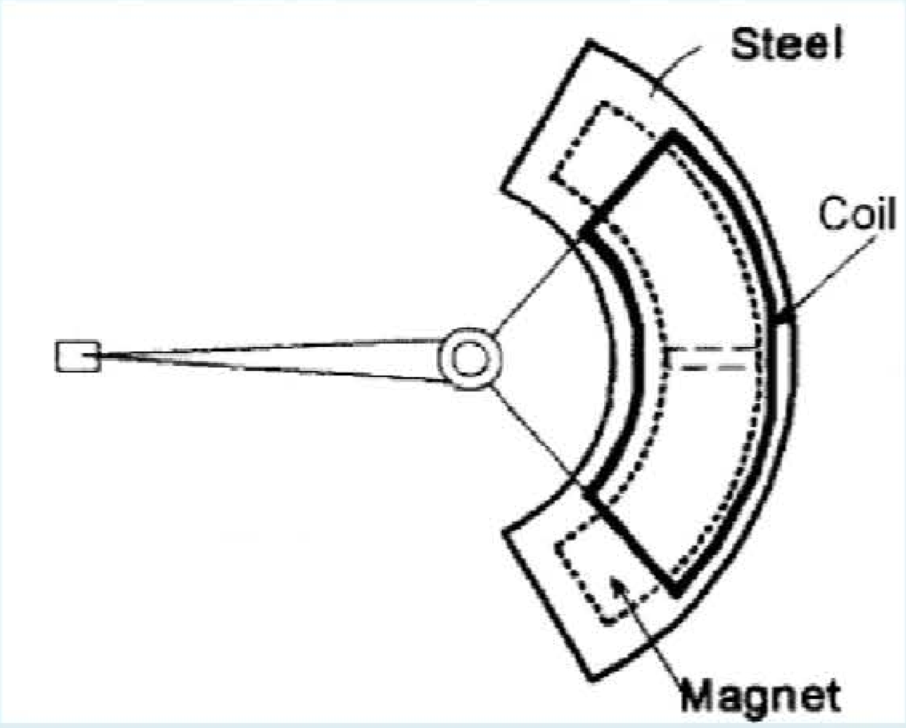


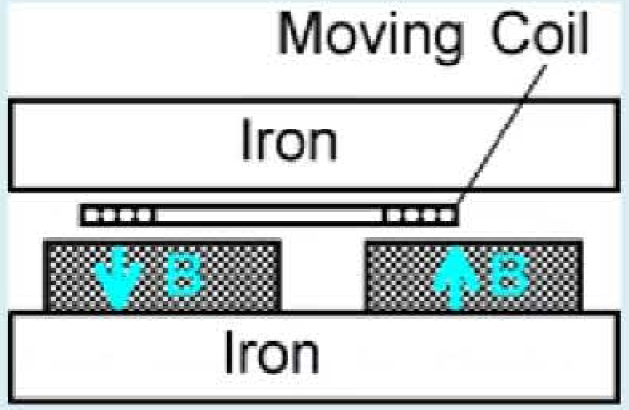
The VCM control system must address two different functions:
Seek mode, when the VCM moves from one concentric data track to another. The average time it takes to move from one track to another is one of the main operating parameters of the HDD, and is called the seek time.
Track Follow mode, when the VCM reads special magnetic markings on the disk surface (known as servo marks) in order to generate a position error signal used for positioning the read head to being directly over the data track as the disk spins underneath it.

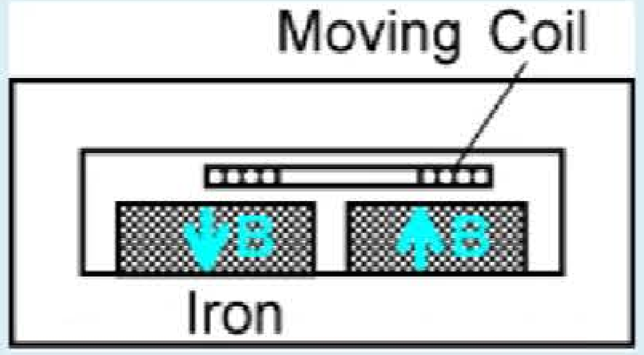
Magnetic Circuits

- Cross sections of rotary voice coil motors used in hard disk drives

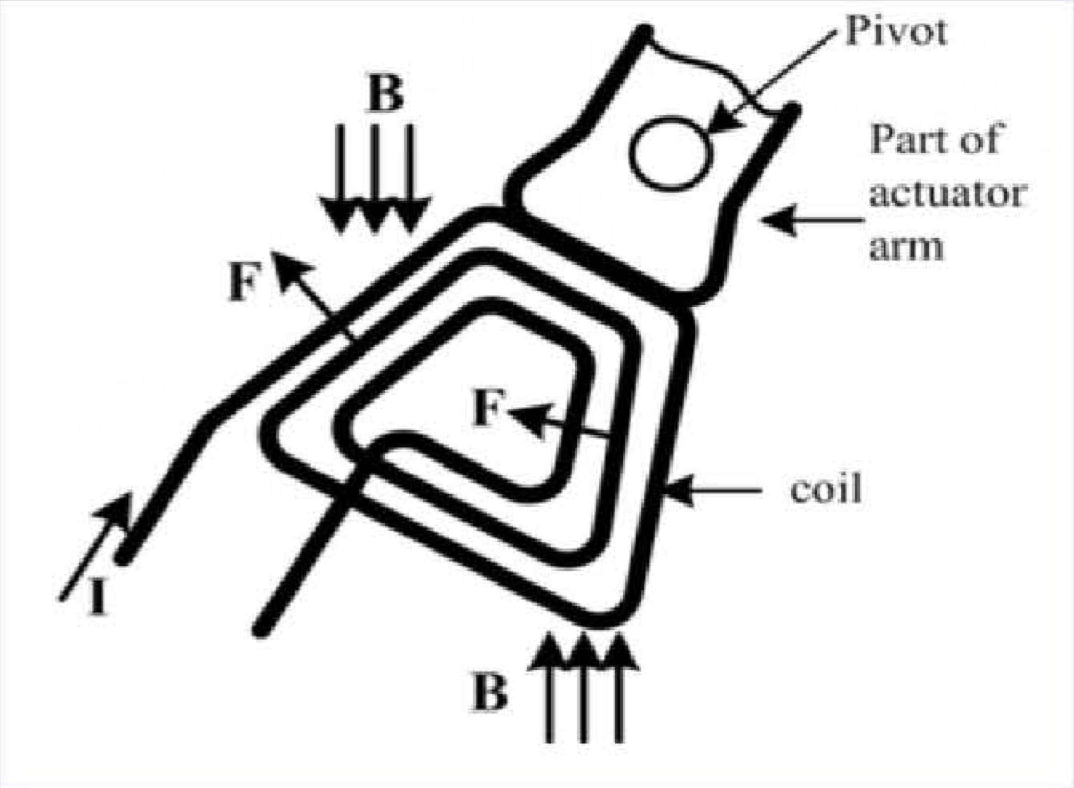




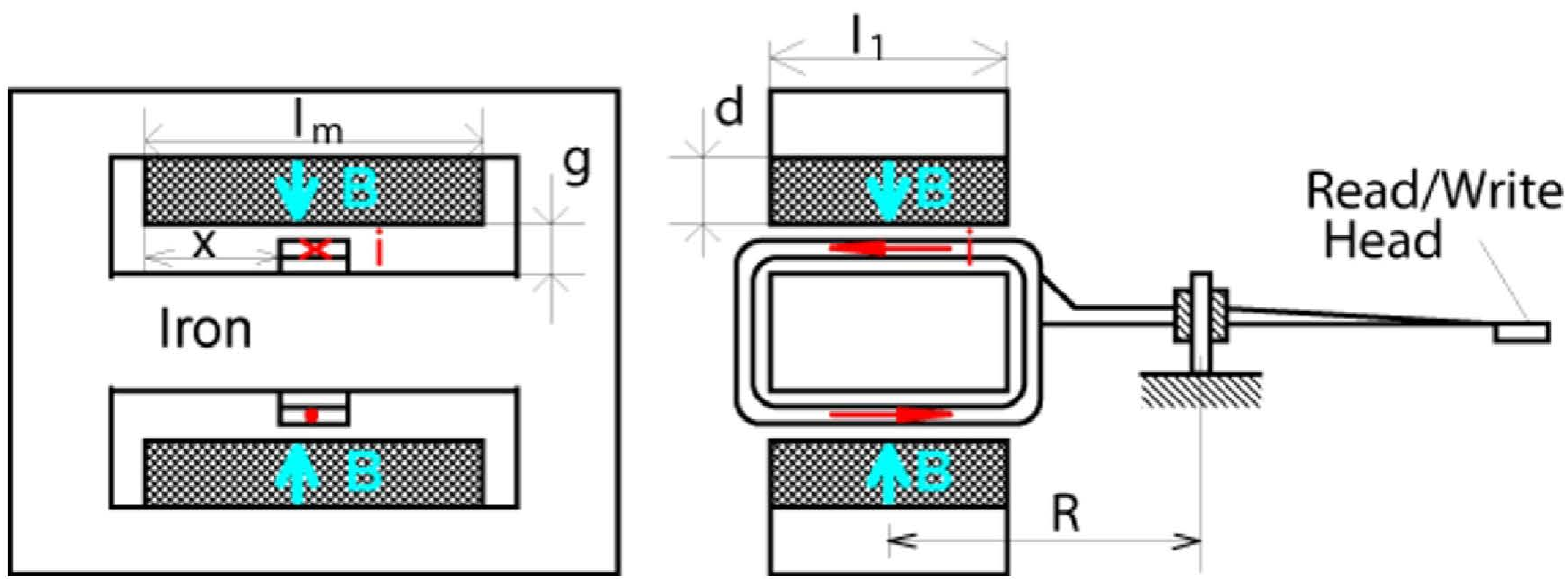




Generation of torque in rotary voice coil actuator



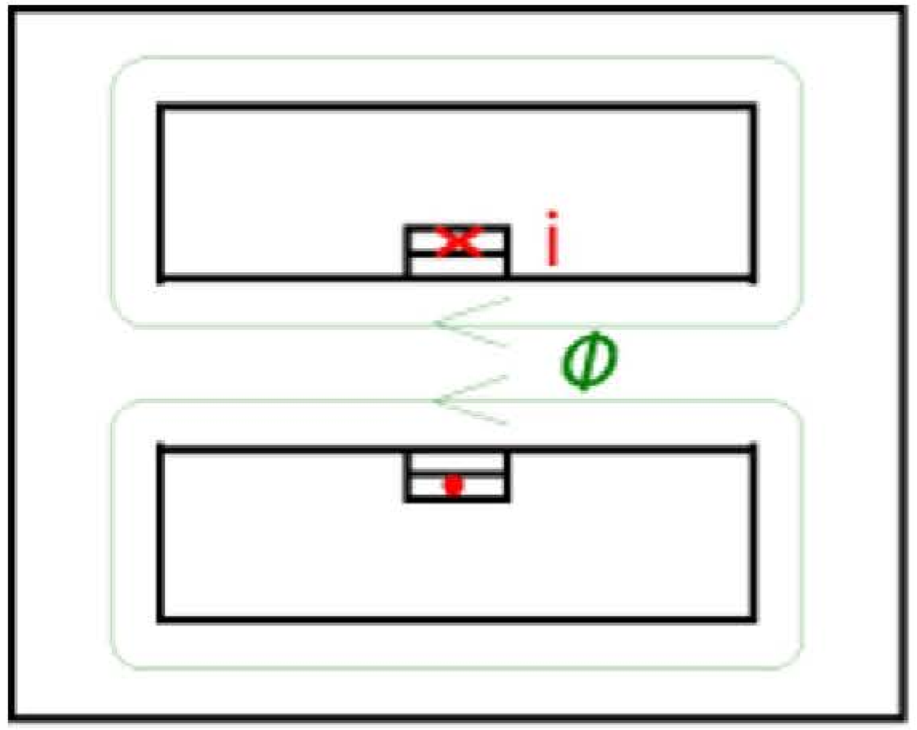
EMF in Voice Coil

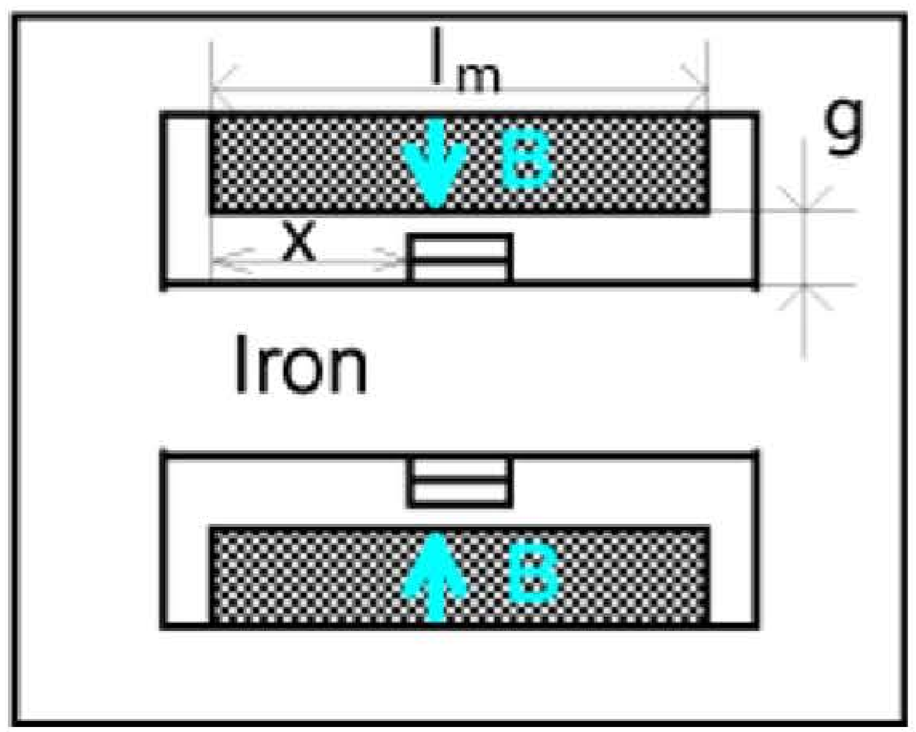


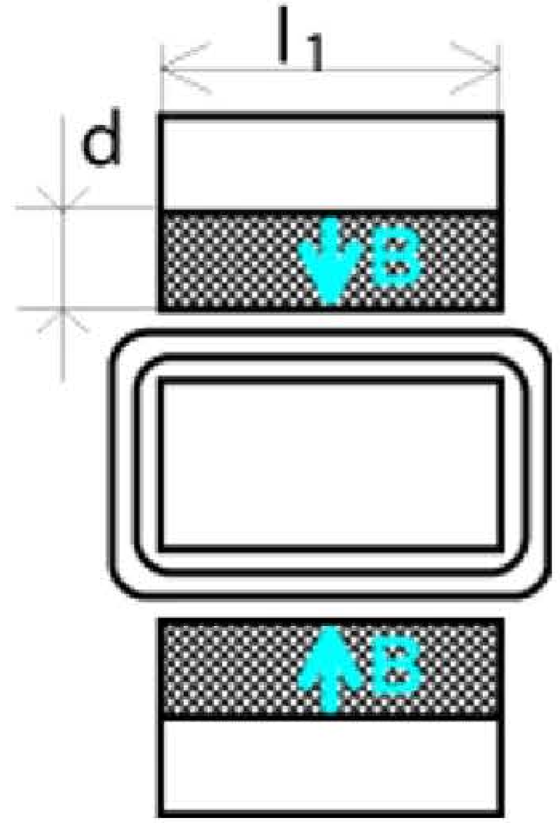
The *emf* in the voice coil can be calculated by the Faraday's law $e = d\lambda/dt$ where $\lambda = \lambda_1 + \lambda_m$, $\lambda_1 = L_1 i$, and λ_m are the flux linkages of the voice coil due to the current in the coil and the permanent magnet. Therefore:

$$e = L_1 \frac{di}{dt} + \frac{d\lambda_m}{dt}$$

Note that L_1 is independent of coil position.







EMF in Voice Coil

If the coil has N_1 turns, the self inductance is independent of the coil position and can be determined as:

$$L_1 = \frac{N_1^2}{R_{core}}$$

where R_{core} is the reluctance of the core.

The induced *emf* in the coil due to the permanent magnets can be calculated as:

$$e_m = \frac{d\lambda_m}{dt} = 2B_g N_1 l_1 v = K_g v$$

where $v = dx/dt$ is the speed and B_g the flux density in the air gap.

$$\lambda = N\phi$$

$$e = \frac{d\lambda}{dt} = N\frac{d\phi}{dt}$$

$$\phi = ABg = B_g l_1 x$$

Force and Torque

The total force acting on the coil can be calculated by

$$F = 2B_g N_1 l_1 i = K_f i$$

Assuming that the distance between the coil center and the shaft is *R*, the torque produced by the coil is:

$$T = K_f i R$$

Note that the force can also be calculated by:

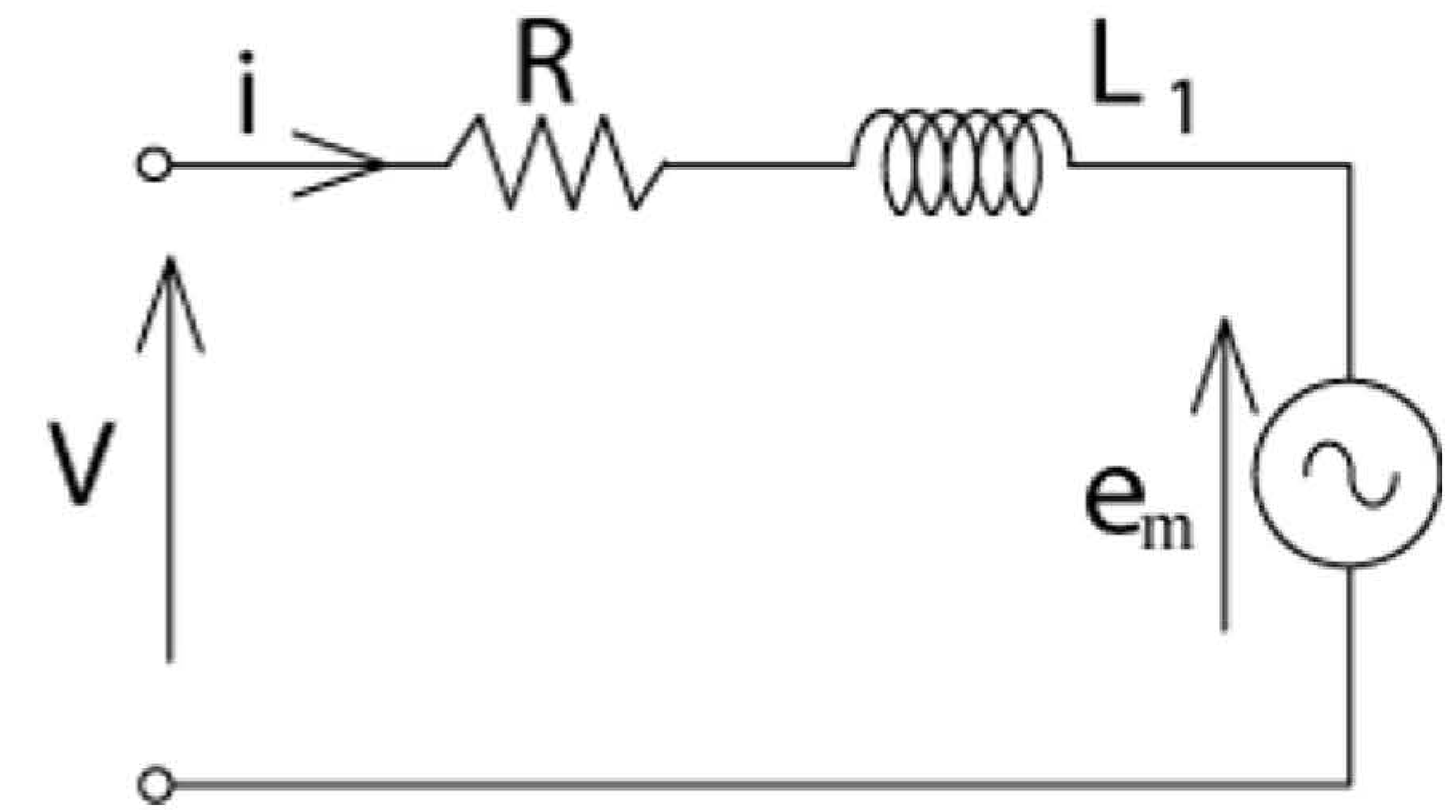
$$F = \frac{\partial W_f{}'}{\partial x} = \frac{1}{2} i^2 \frac{dL_1}{dx} + \frac{1}{2} I_m^2 \frac{dL_m}{dx} + i I_m \frac{dL_{1m}}{dx}$$

Since $dL_1/dx = 0$ and $dL_m/dx = 0$, this is the same with the previous results.

Mathematical Model

The electrical circuit equation for the voice coil is

$$V = Ri + \frac{d(L_1 i)}{dt} + \frac{d\lambda_m}{dt}$$



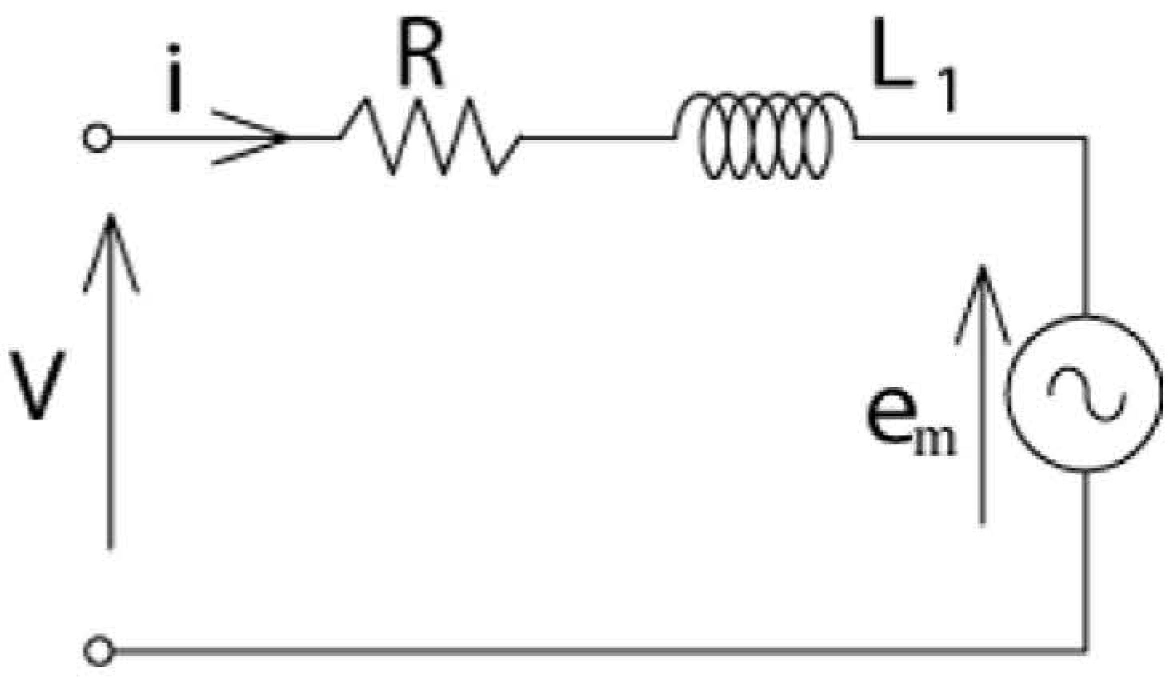
For the arc type rotatry voice coil motor, the self inductance is independent of the coil position, and the above equation becomes

$$V = Ri + L_1 \frac{di}{dt} + e_m$$

$$\lambda = N\phi \qquad e = \frac{d\lambda}{dt} = N\frac{d\phi}{dt}$$

where $e_m = \frac{d\lambda_m}{dt} = 2B_g N_1 l_1 v = K_g v$ is the induced *emf* in the coil due to the permanent magnet, $K_g = 2B_g N_1 l_1$ is the *emf constant*, and *v* = *dx/dt* is the speed.

The parameters of the mathematical model



<u>Remark</u>:

The resistance **R** is largely influenced

by **the temperature of the VCM windings**.

Expected variation of R is in the range of 30%, corresponding to a temperature variation around 60 °C for a copper coil (this has a temperature coefficient around 0.4%/ °C).

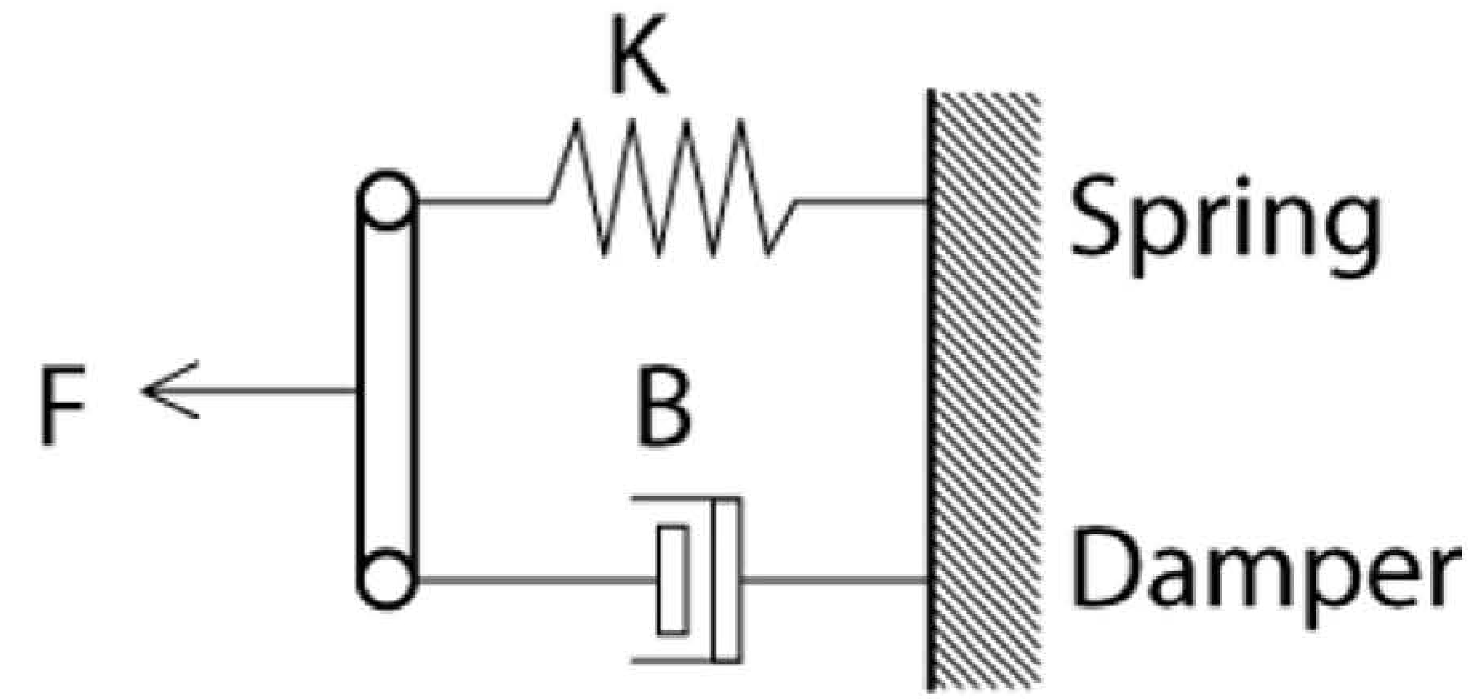
Algorithms for on line estimation of the actual **R** value are used, under the assumptions that all the other parameters of the VCM (inertia, torque constant, inductance L) are fixed or present smaller relative variations.

Mathematical Model

The mechanical system for a hard disk drive is illustrated on the right hand side.

By Newton's law, we have

$$F - F_{load} = m\frac{dv}{dt}$$



where

$$F = K_f i$$

is the electromagnetic force produced by the current in the coil, $K_f = 2B_g N_1 l_1$ the *electromagnetic force constant*, and

$$F_{load} = Kx + Bv$$

is the load force due to the spring and damper.

Mathematical Model

When the electrical and mechanical equations are expressed in the form of state equations, we obtain

$$\frac{di}{dt} = -\frac{R}{L_1}i - \frac{K_g}{L_1}v + \frac{1}{L_1}V$$

$$\frac{dv}{dt} = \frac{K_f}{m}i - \frac{K}{m}x - \frac{B}{m}v$$

$$\frac{dx}{dt} = v$$

These equations can be solved together with initial conditions

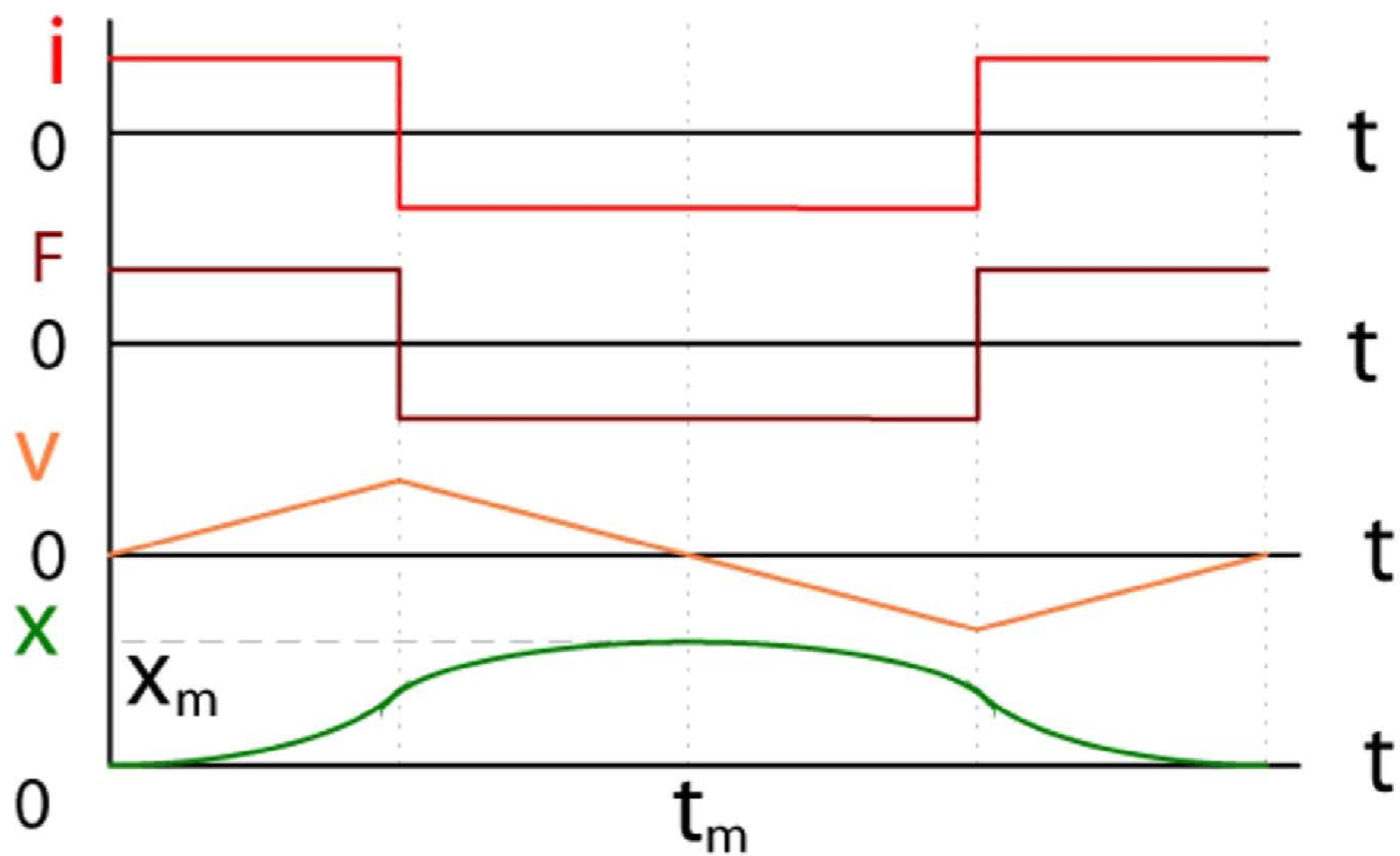
$$i(0) = i_0, \quad v(0) = v_0, \quad and \quad x(0) = x_0$$

The following waveforms illustrate the response of an arc type voice coil motor under a square wave current source excitation

$$F = K_f i$$

$$v = \frac{1}{m}\int_0^t F dt$$

$$x = \int_0^t v dt$$

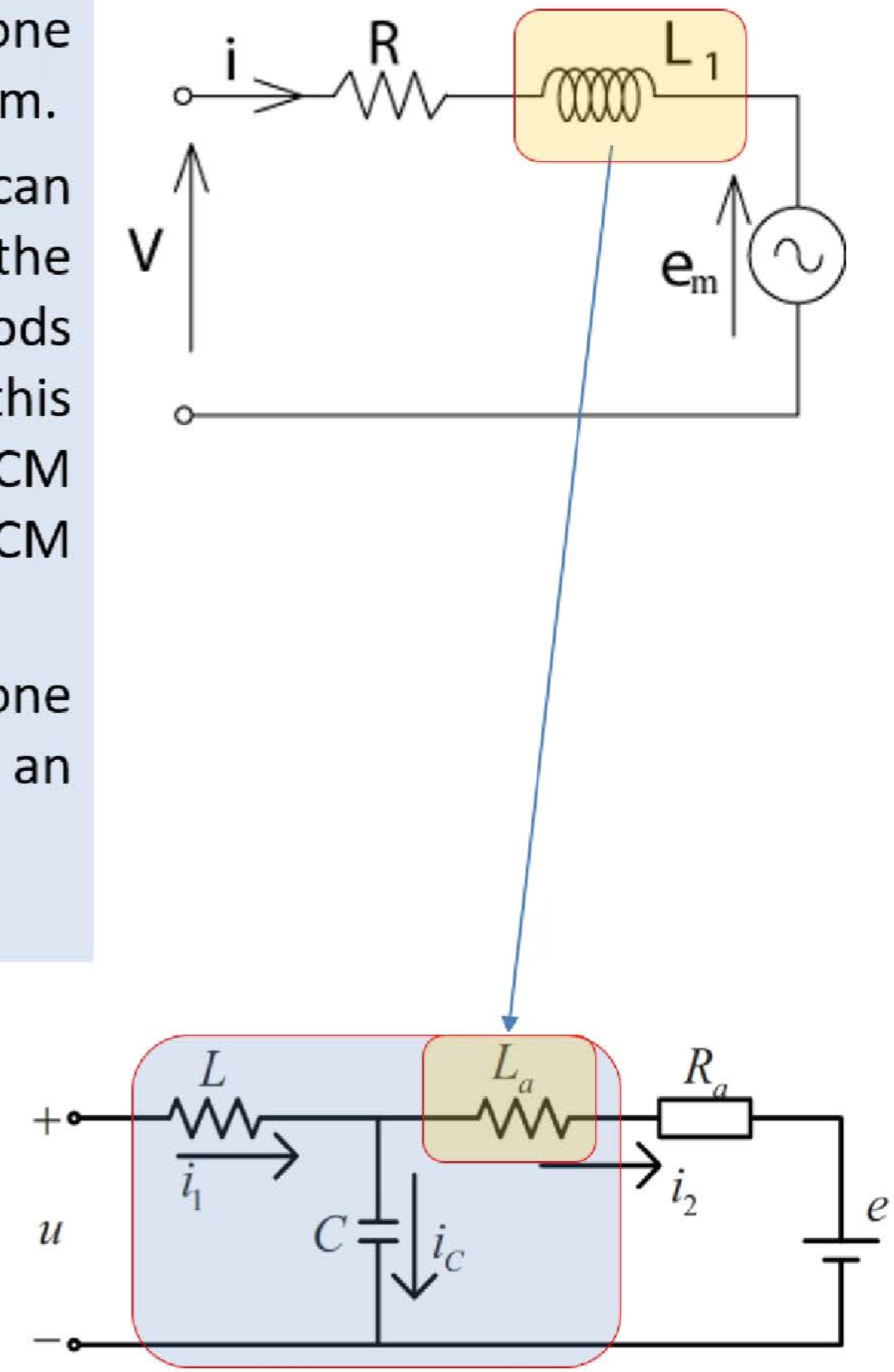


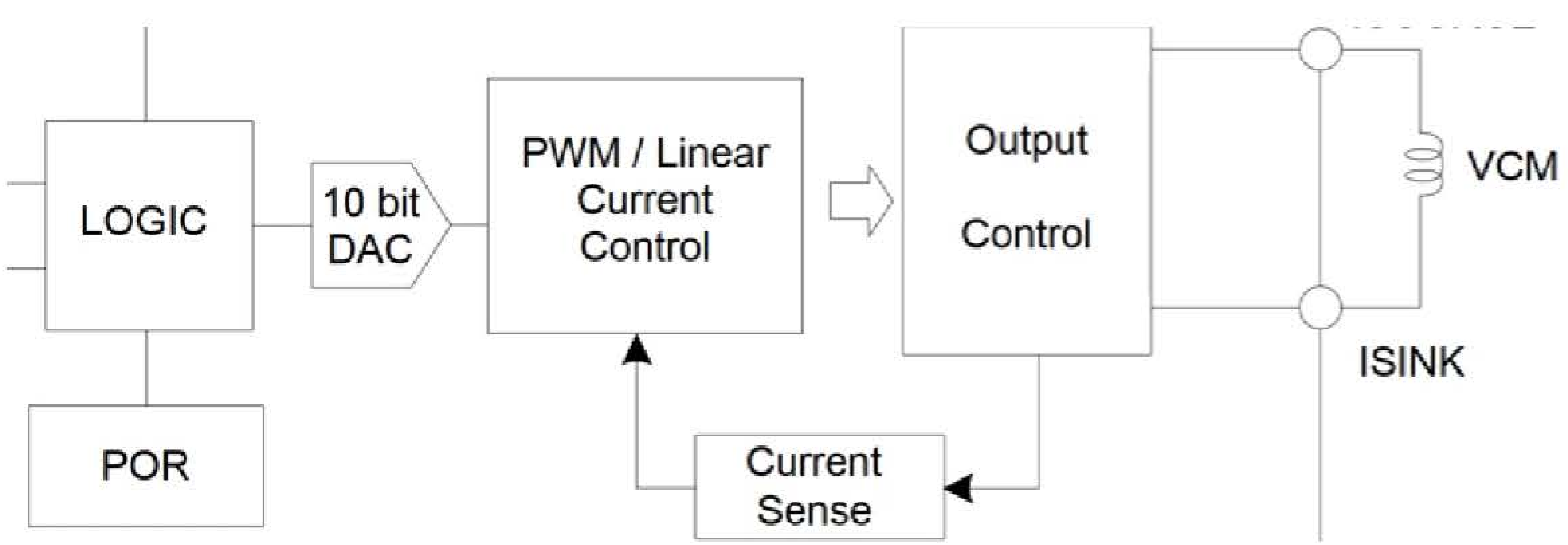
Remark:

Digital driving method with PWM chopping is one commonly used driving method for VCM system.

The low armature inductance of the VCM can lead to big current ripple (which will affect the positioning accuracy) when digital control methods with PWM chopper are used. To solve this problem, a LCL filter is applied before the VCM armature to reduce the current ripple in VCM and improve its positioning accuracy.

The armature inductance of the VCM can act as one inductance of the LCL filter, so only an additional inductor and a capacitor are added.

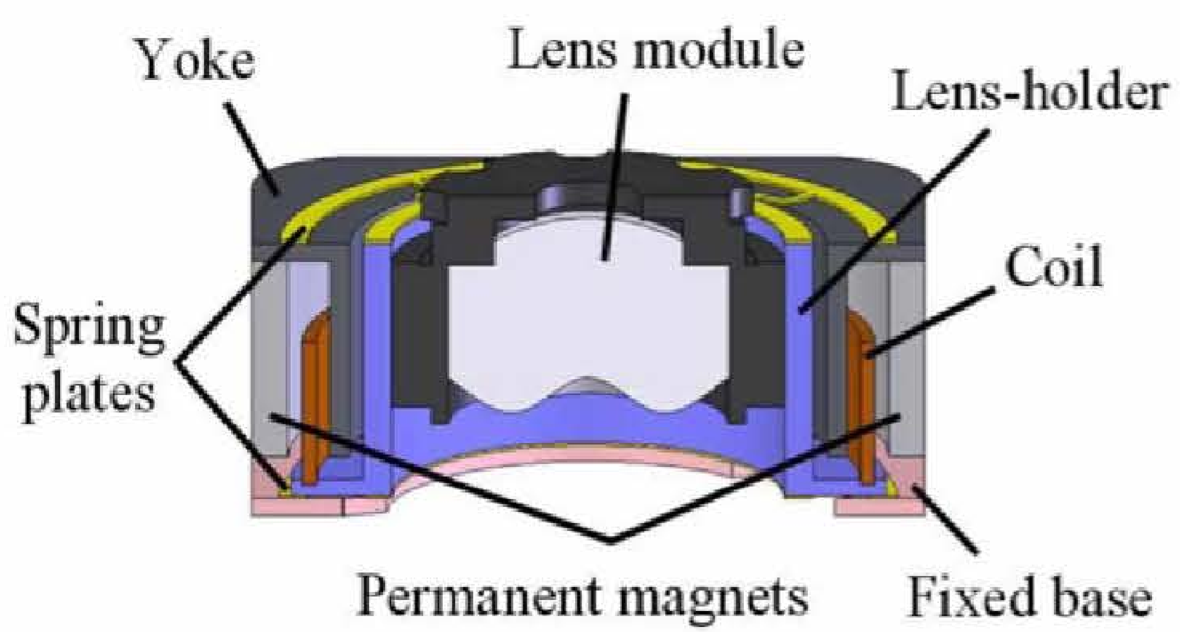




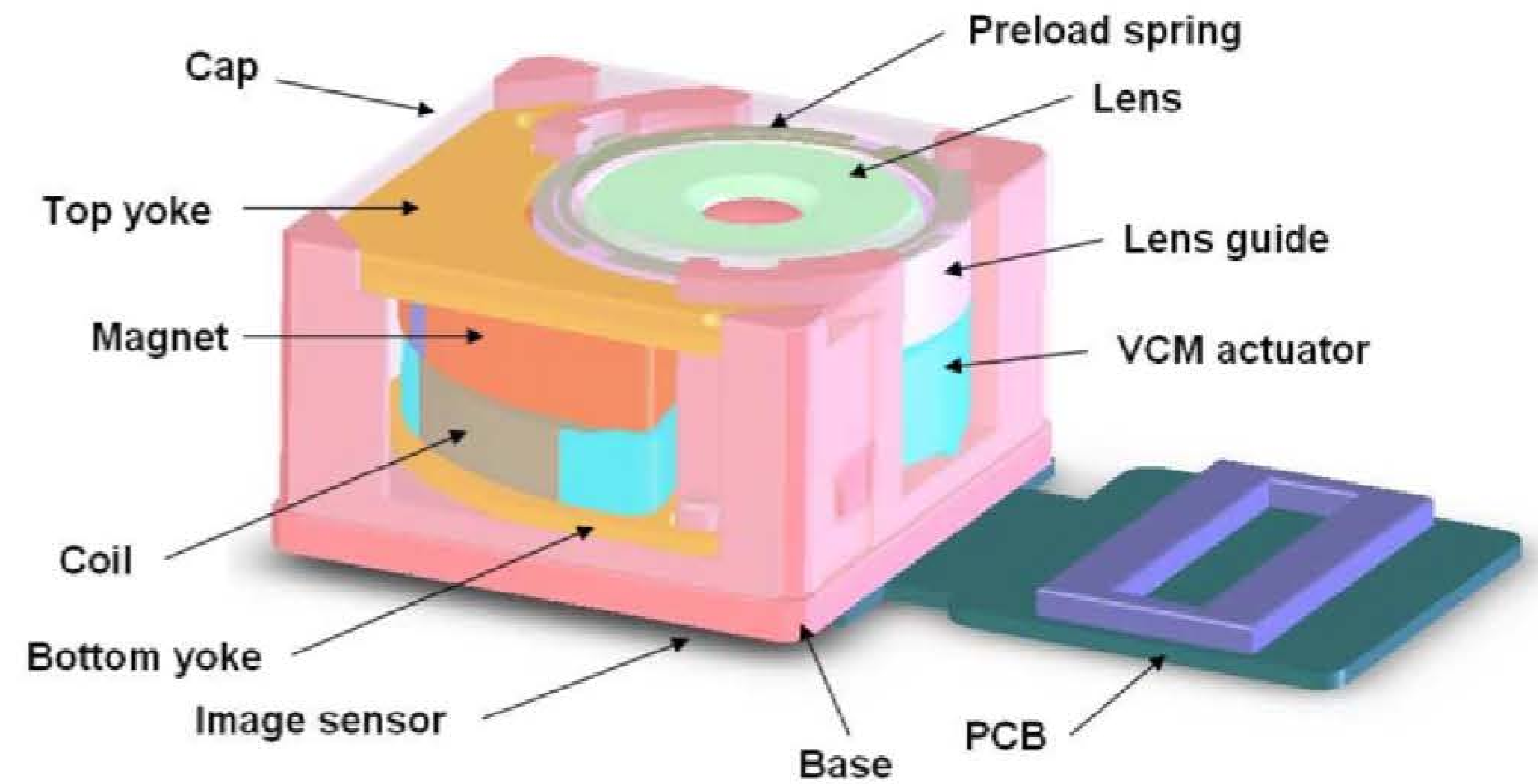
Other applications for VCM actuators

- Low power auto-focus actuator for camera in mobile phones:

linear setup

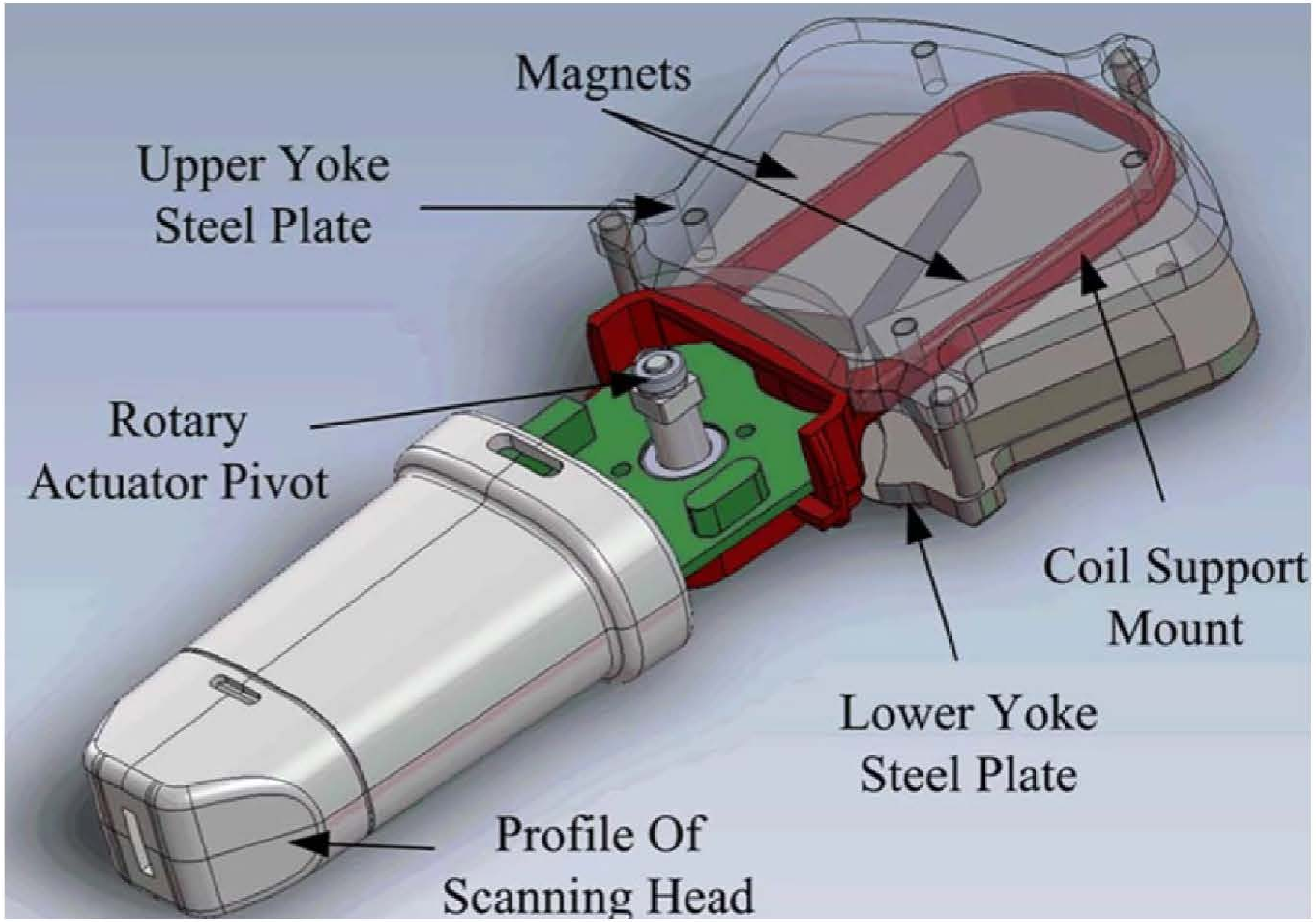


rotary setup



Other applications for VCM actuators

Cost effective single transducer element ultrasound scanner require an actuator in order to create a synthetic array by mechanically moving the transducer. The classic (multichannel phase array) transducer setup is significantly more expensive.



Linear(translational) vs. Rotational movement

$$\boxed{\text{Electrical Equations}} \Rightarrow \begin{cases} V_s(t) = i(t)(R + R_{\text{coil}}) + L\dfrac{di(t)}{dt} + \dfrac{\partial \Lambda(i,x)}{\partial x}\ v(t) \\ \text{(linear motion)} \\ V_s(t) = i(t)(R + R_{\text{coil}}) + L\dfrac{di(t)}{dt} + \dfrac{\partial \Lambda(i,\theta)}{\partial \theta}\ \omega(t) \\ \text{(rotational motion)} \end{cases}$$

Linear(translational) vs. Rotational movement

$$\boxed{\begin{array}{c}\text{Mechanical}\\ \text{Equations}\end{array}} \Rightarrow \begin{cases} m \dfrac{d^2x(t)}{dt^2} = F(i, x) & \text{(linear motion)} \\ \mathcal{J}_m \dfrac{d^2\theta(t)}{dt^2} = T(i, \theta) & \text{(rotational motion)} \end{cases}$$

General form of the mathematical model

Linear Motion

$$V_s(t) = i(t)(R + R_{\text{coil}}) + L\frac{di(t)}{dt} + \frac{\partial \Lambda(i,x)}{\partial x}\ v(t)$$

$$m\frac{d^2x(t)}{dt^2} = F(i,x)$$

Rotational Motion

$$V_s(t) = i(t)(R + R_{\text{coil}}) + L\frac{di(t)}{dt} + \frac{\partial \Lambda(i,\theta)}{\partial \theta}\ \omega(t)$$

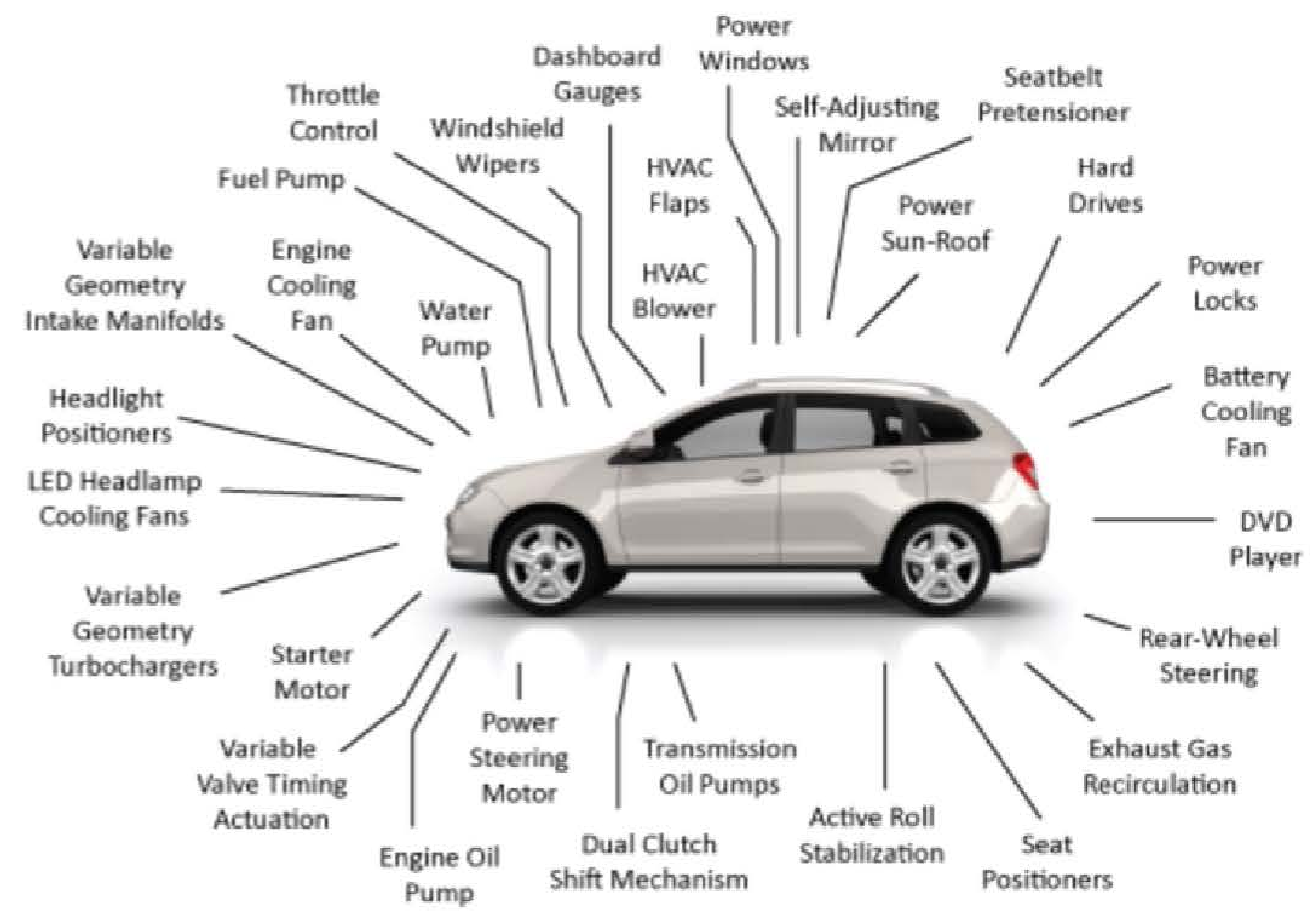
$$\mathcal{J}_m\frac{d^2\theta(t)}{dt^2} = T(i,\theta),$$

Modeling of energy conversion **DC motors** (with permanent magnets)

Electrical motor- the most common actuator in use today

Example : Motors found in a car
can be divided into three categories:

1. Performance related motors
2. Comfort related motors
3. Volume motors



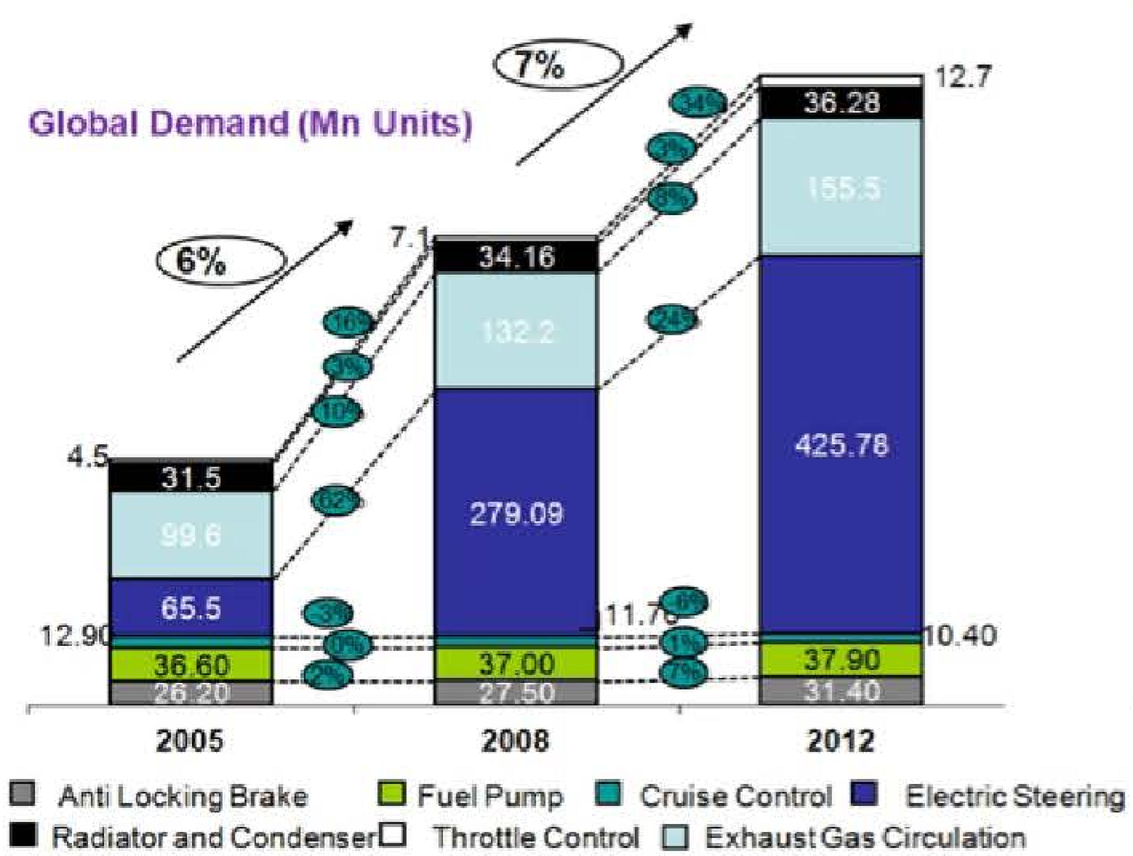
Automotive Applications

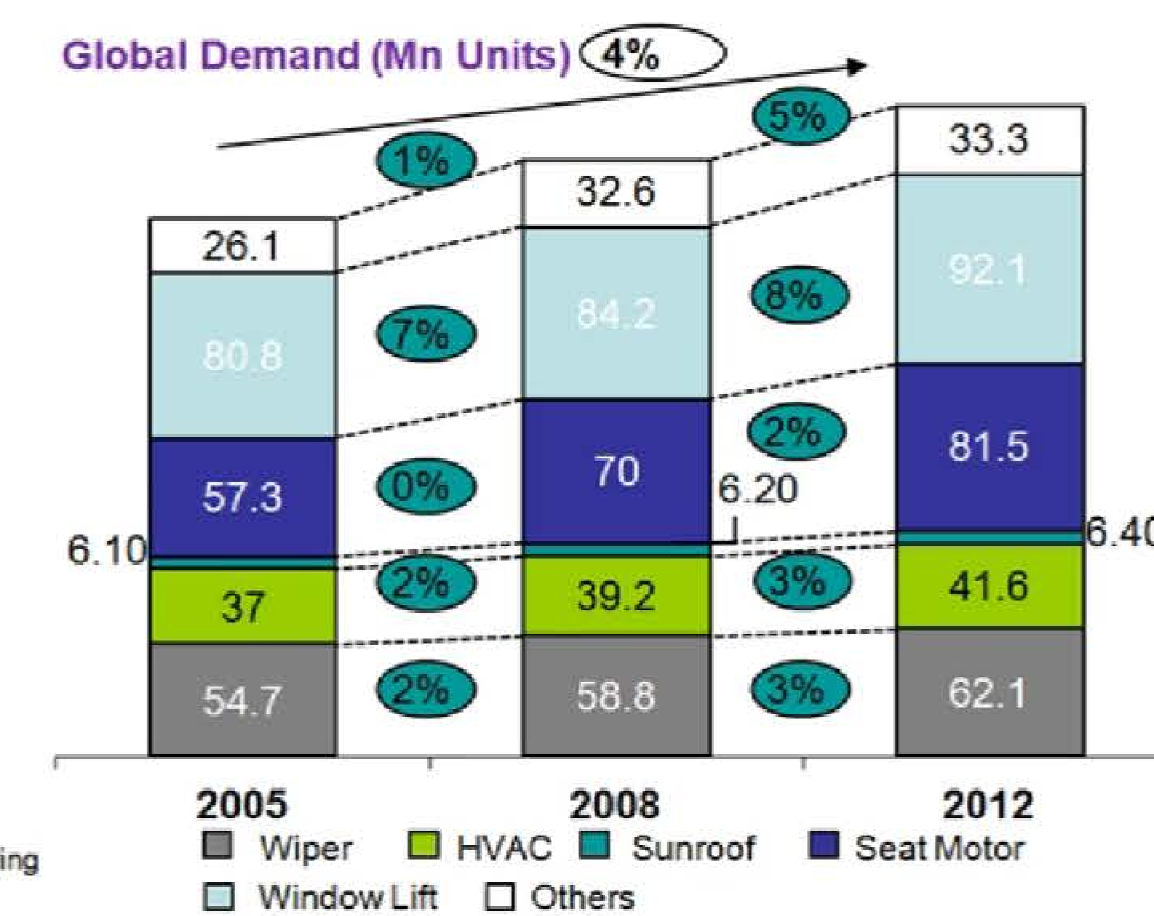
Performance Related Motors	Comfort Related Motors	Volume Motors
• Performance related motors include all motors associated with the driving operations of vehicles	• Comfort related motors are defined as motors used in comfort applications and are not essential for the operational of vehicles	• Volume motors are low power motors which are produced in large volumes and can be used for multiple applications
Examples	**Examples**	**Examples**
• Engine-cooling motors	• Window lift motors	• Door lock motors
• Anti-locking brake system (ABS) motors	• Seat motor	• Air-conditioning actuator motors
• Fuel-pump motors	• Sunroof motors	• Mirror motors
• Cruise control motors	• HVAC blower motors	• Washer motors
• Electric steering motors	• Wiper motors	
• Throttle control motors		
• Starter motor & alternators		

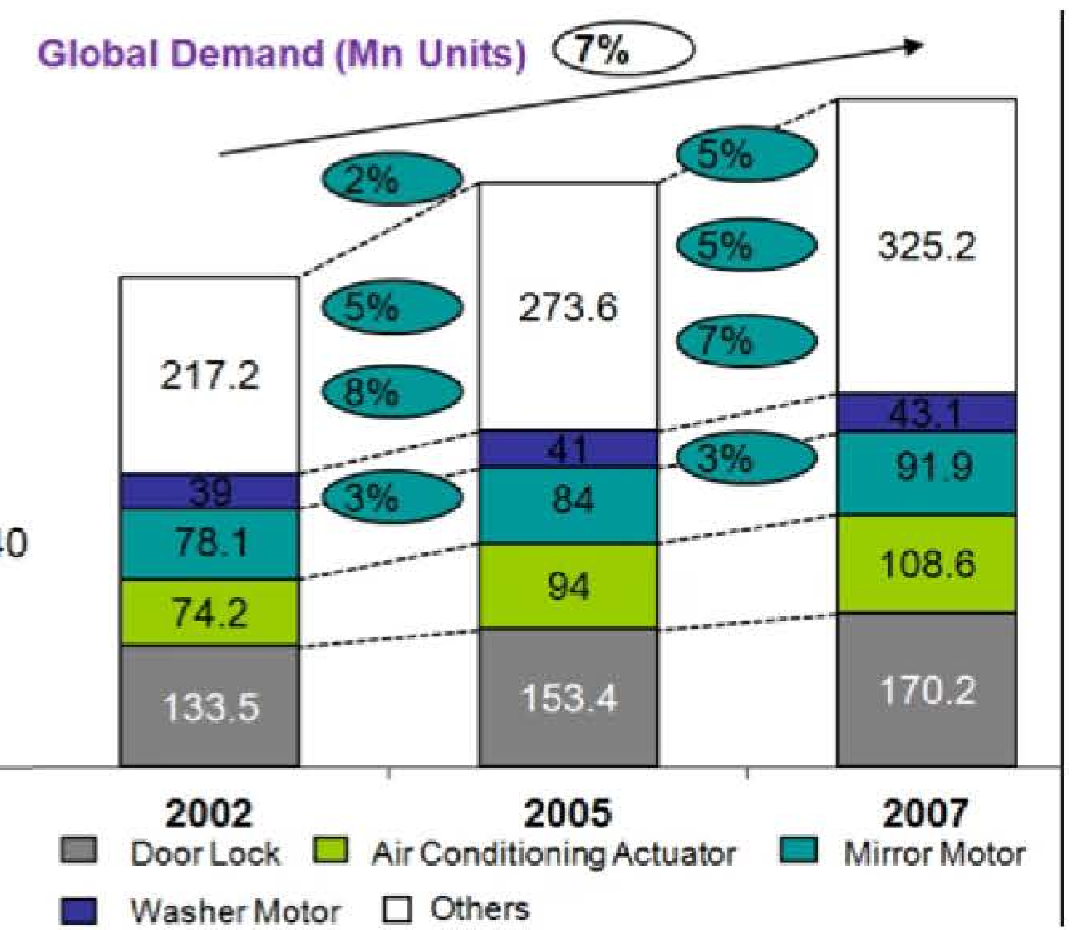
Electrical motor- the most common actuator in use today

Example : Motors, found in a car, can be divided into three categories:

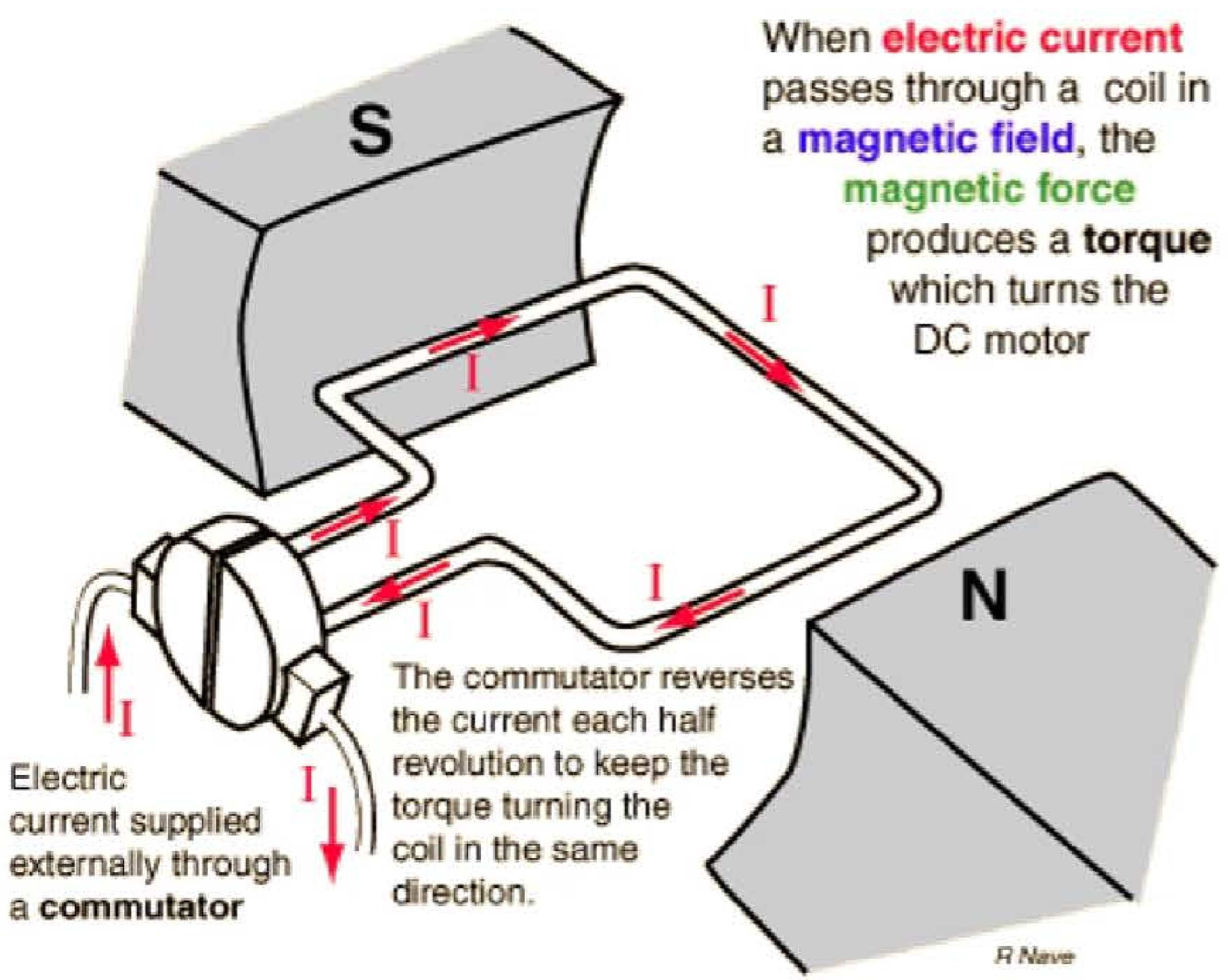
1. Performance related motors
2. Comfort related motors
3. Volume motors

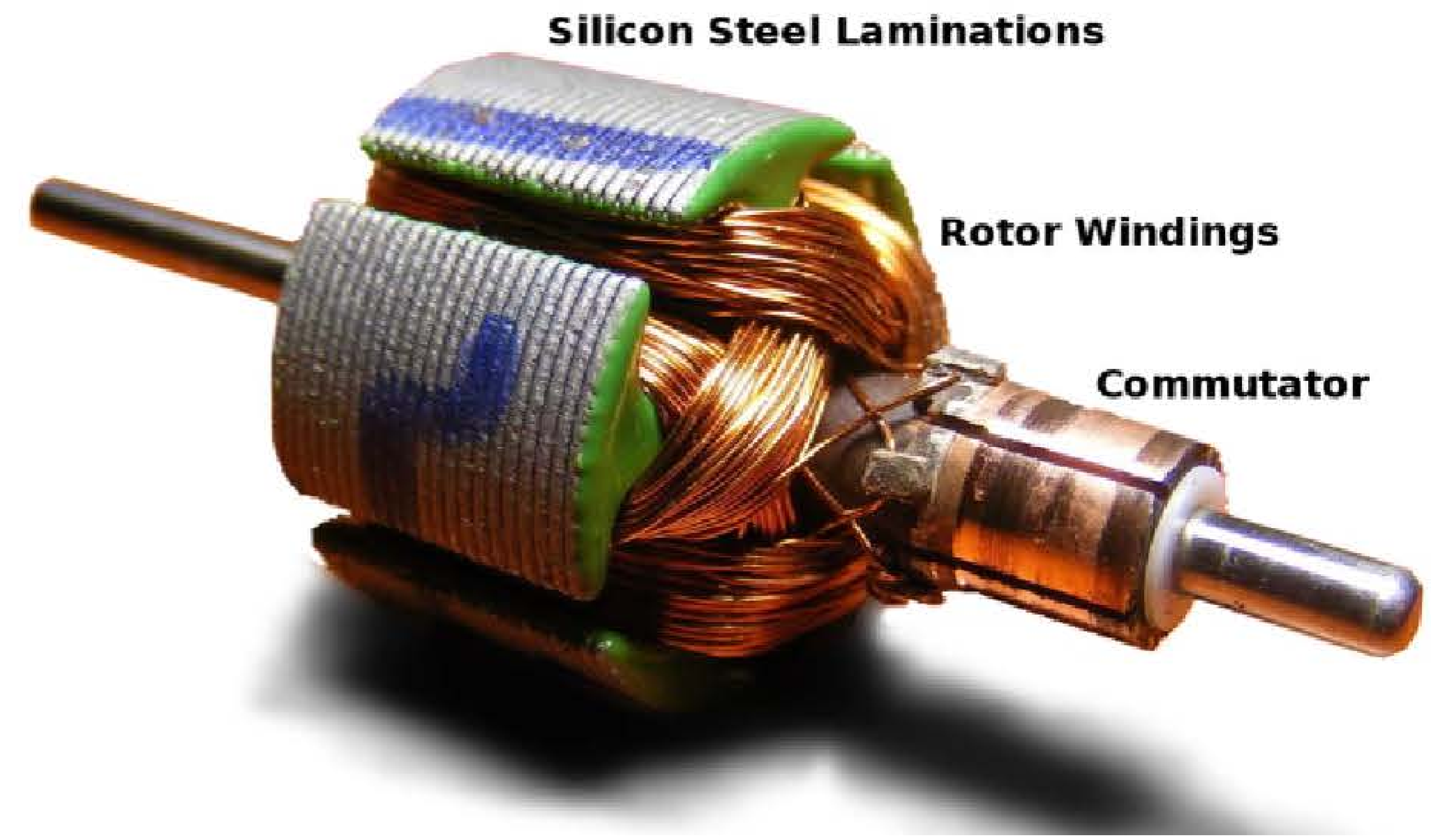


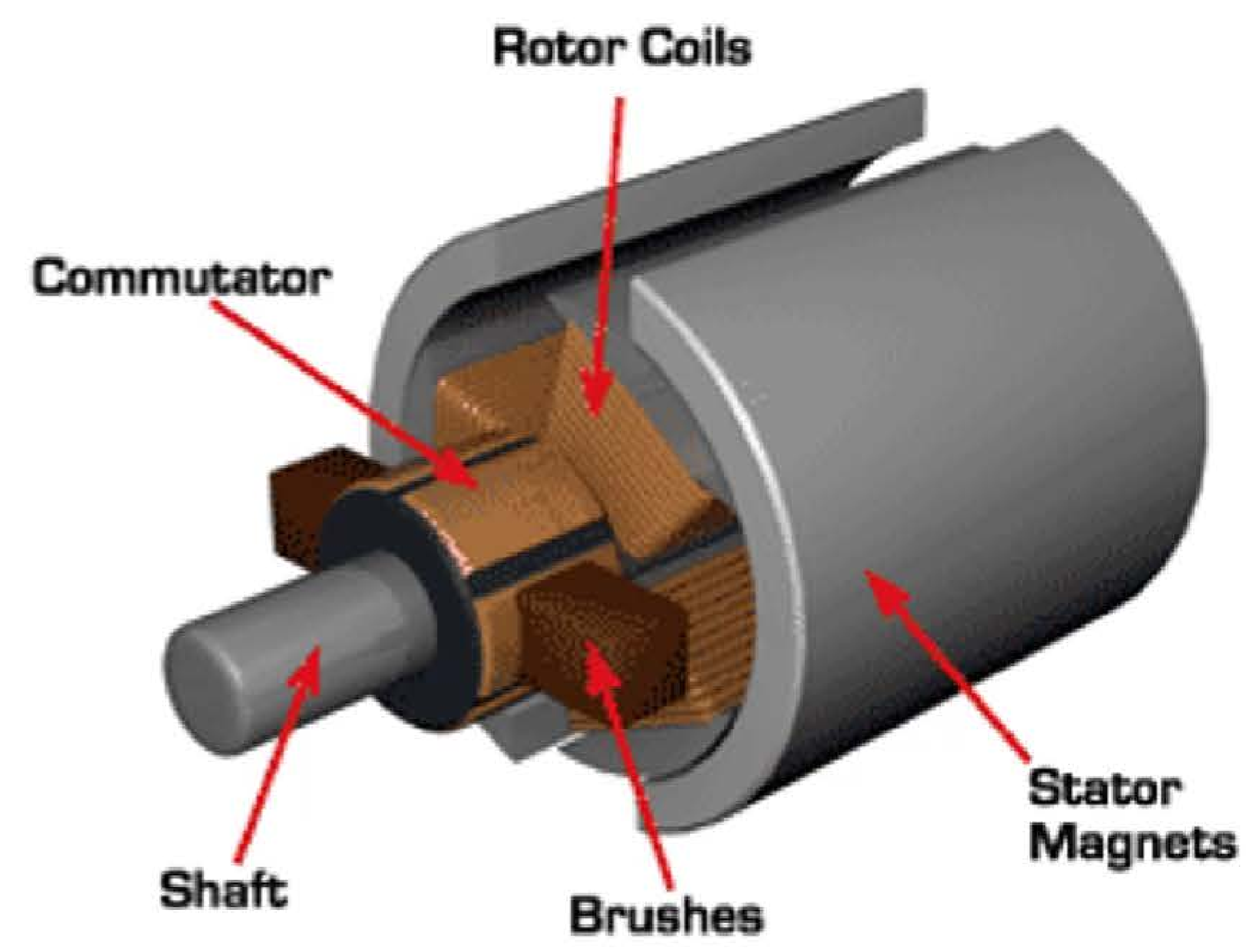




Summary of the physics of DC motors

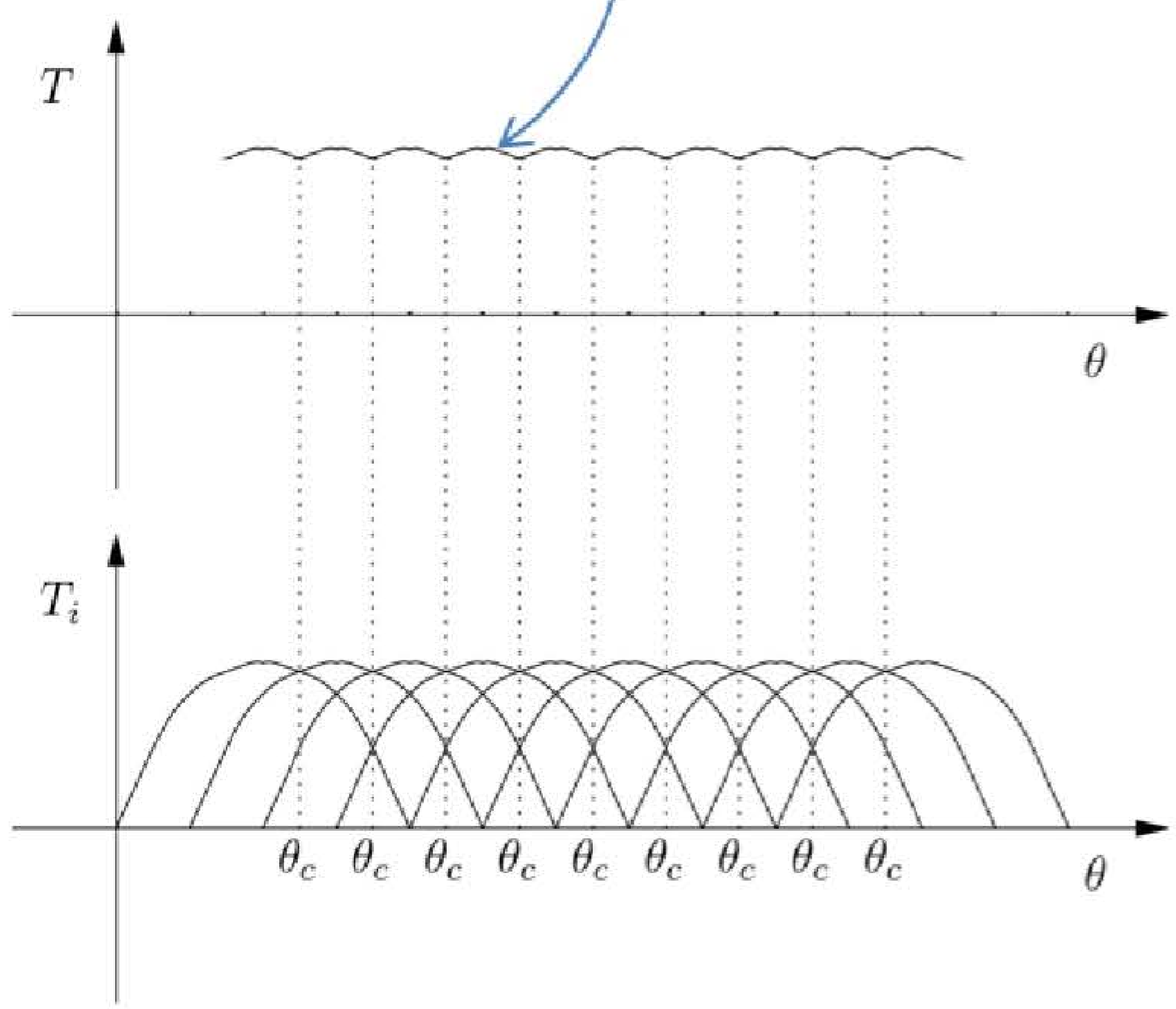




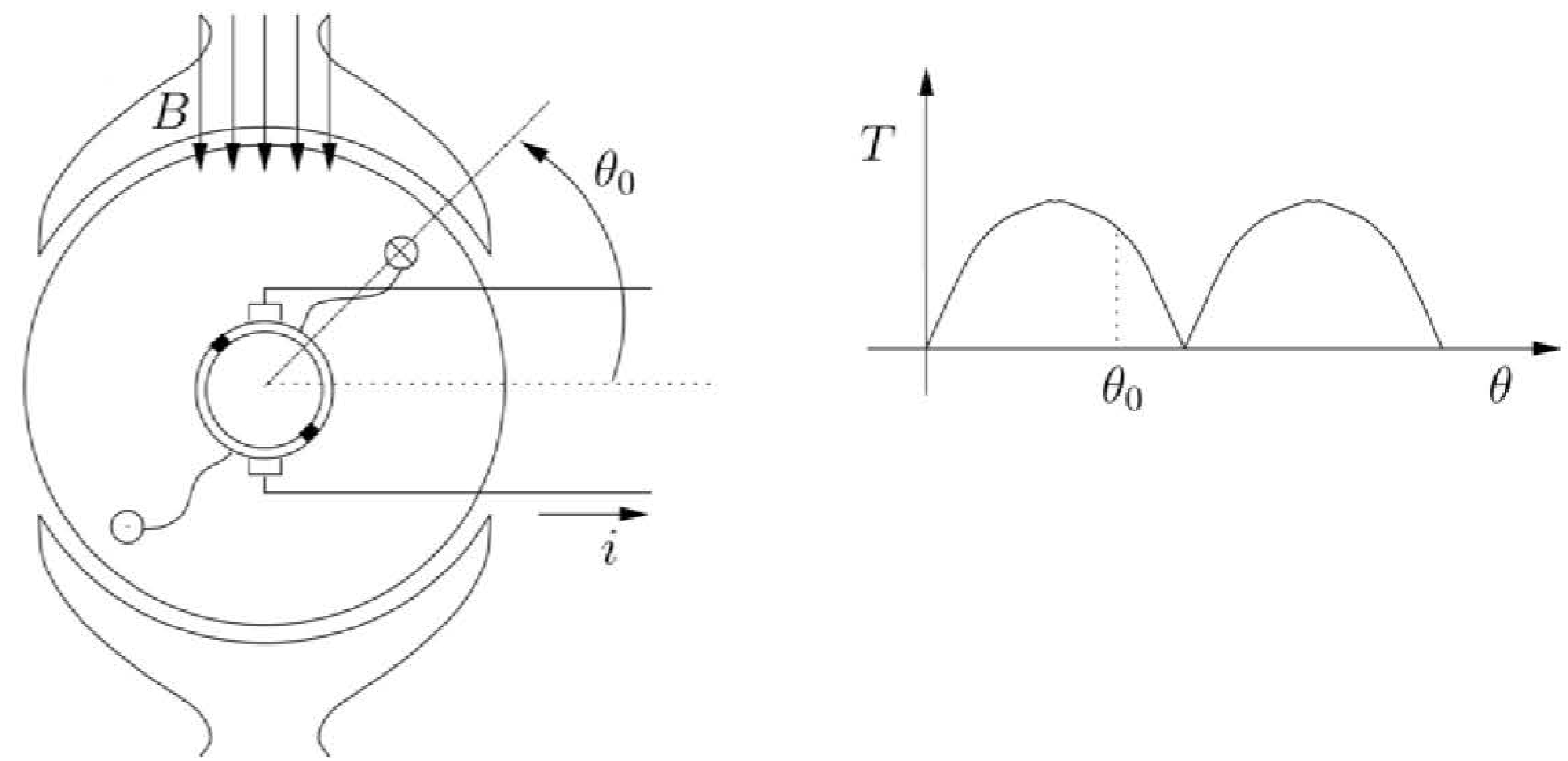


Torque of DC motors

The torque ripple may be decreased as much as needed by *increasing the number of segments*
The *residual ripple* may be assumed to be filtered by the mechanical system, so that the resulting torque may be approximated as being proportional to the current.



Torque exerted by the motor, when a multiple segments commutator is used



Torque exerted by the motor, when a two segments commutator is used

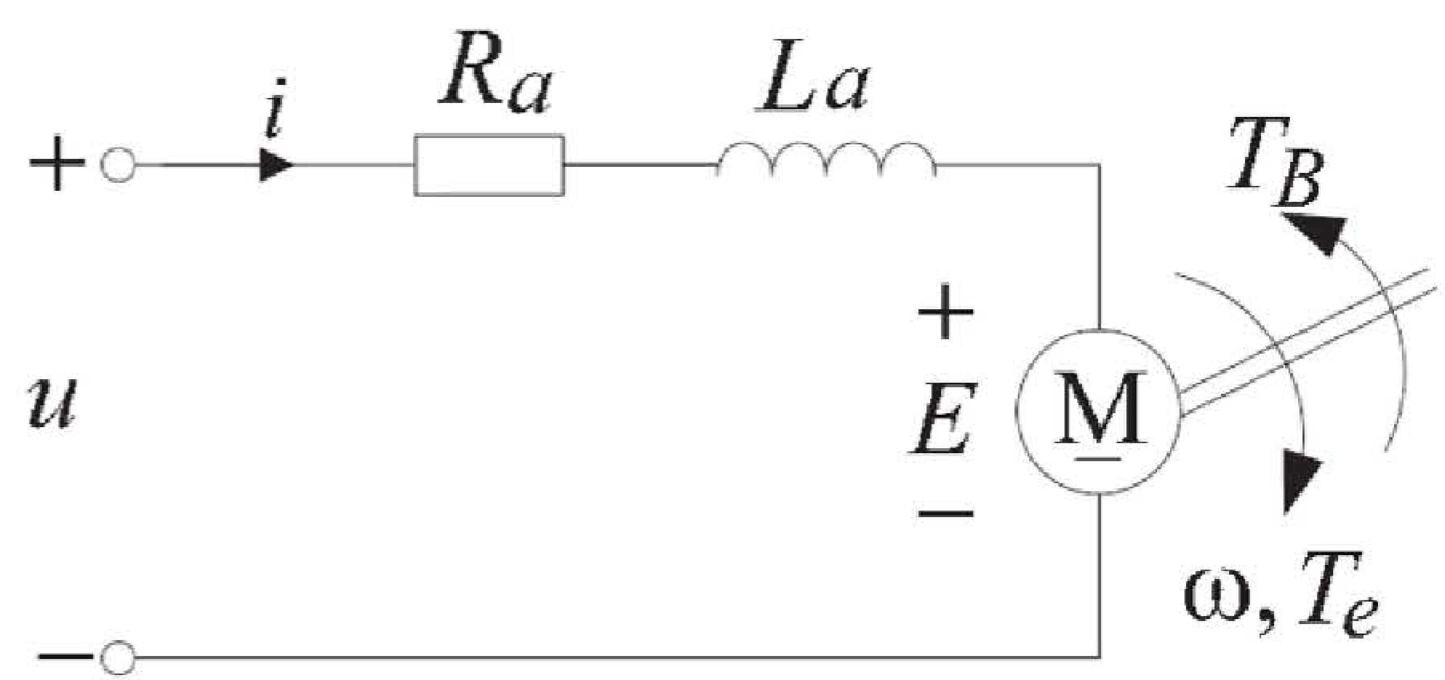
DC motor parameters

$E = k_e \omega$ The voltage E produced by the rotor is proportional to the angular velocity

$T_B = k_t i$ The useful torque T produced by the motor, to be applied on a load B, is proportional with the current i in the coil (ideal case)

The magnetic flux of the field generated by the own current is varying, and that leads to a generated voltage $V_L = L\frac{di}{dt}$. This voltage opposes the voltage V applied to the motor, and therefore it must be subtracted:

$$T_B = \frac{k_t}{R_a}\left(u - k_e\omega - \frac{L_a di}{dt}\right)$$



1 DC motor equivalent circuit

DC motor – zero load parameters

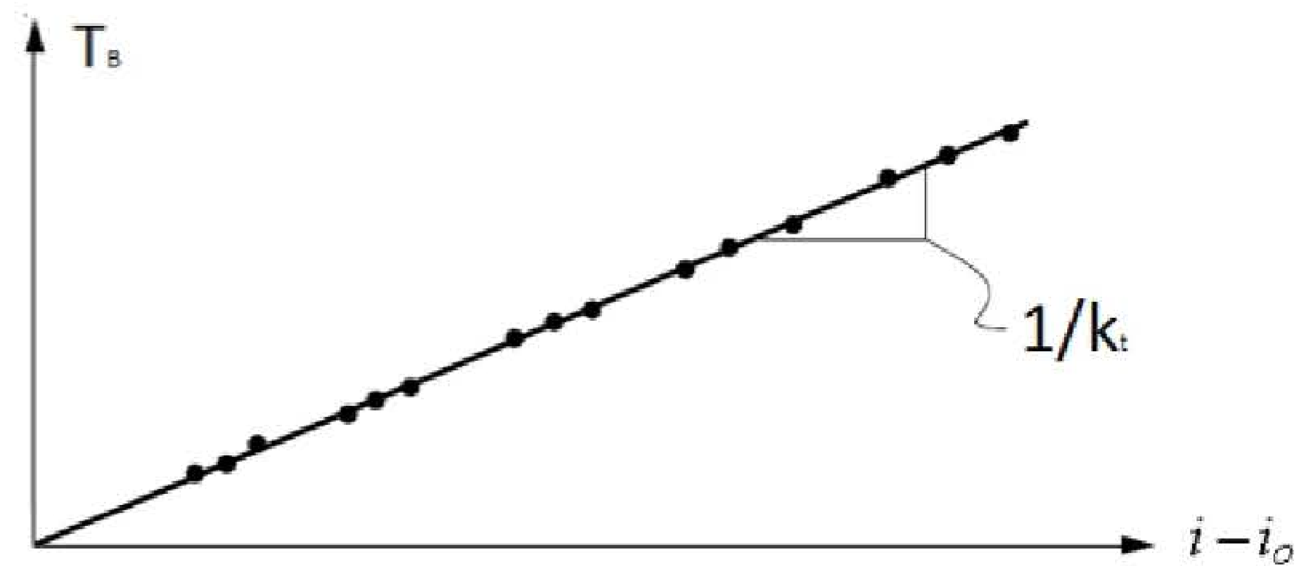
Motor resistance R

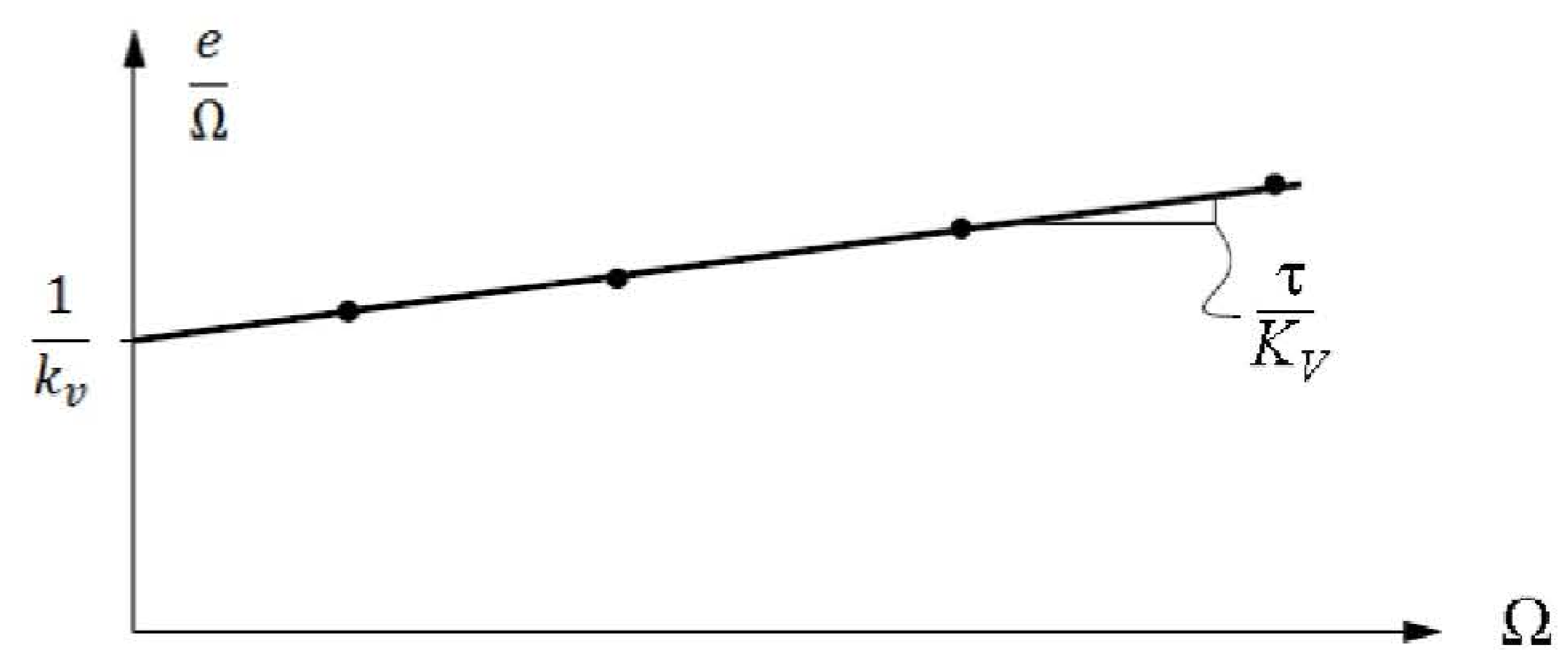
Zero load current i_0

Back emf voltage e

Voltage constant $k_e = \frac{\Omega}{e}$

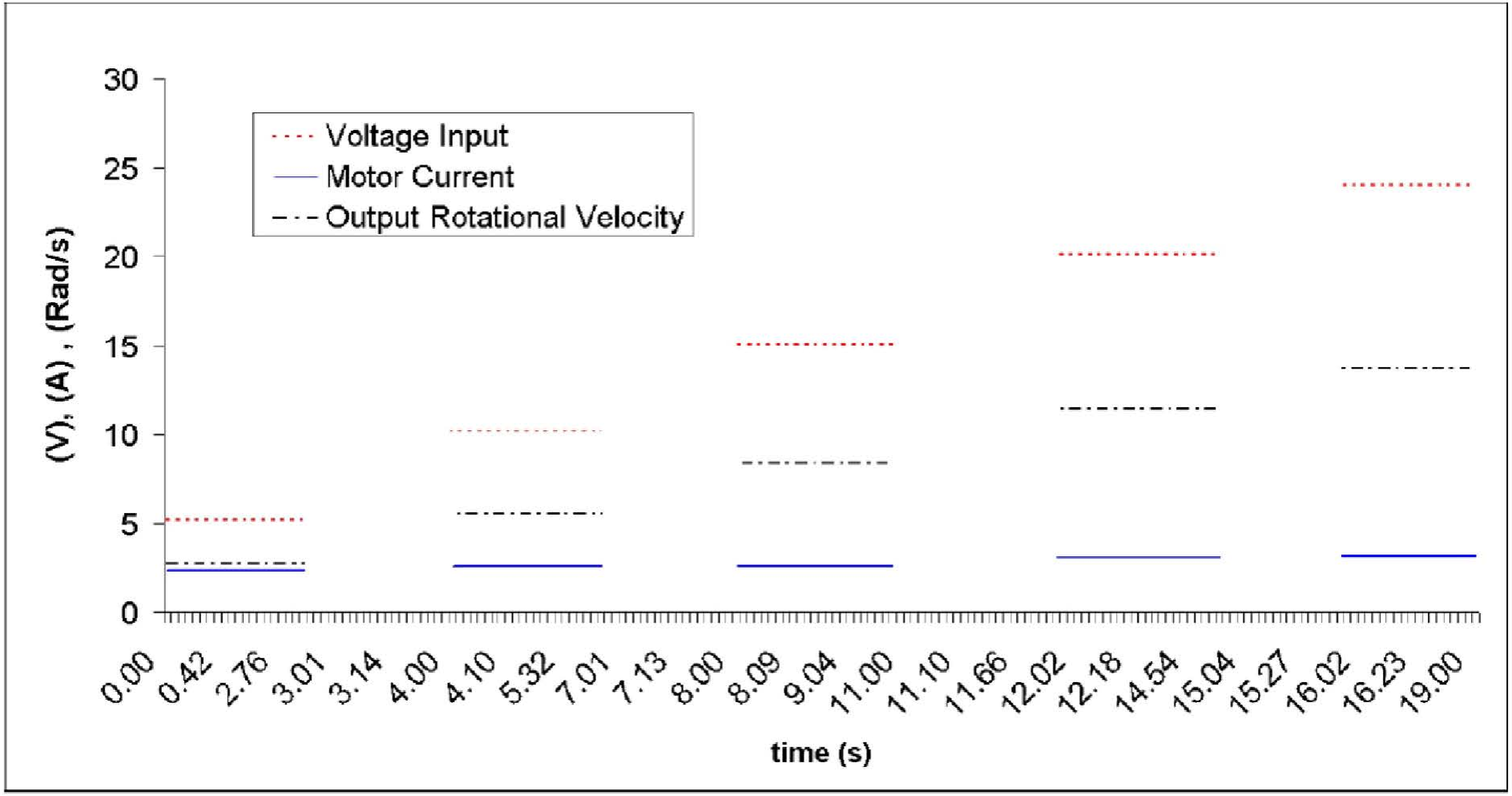
Torque constant, Ideal case $k_t = k_v$





$\tau - torque\ lag\ time\ constant$

DC motor response functions



Measured steady state values of PM DC geared motor.

DC motor with load

The motor exerts a torque, while supplied by voltages on the stator and on the rotor.

This torque acts on the mechanical structure, which is characterized by the rotor inertia **J** and the viscous friction coefficient **F**.

In any operating environment, a load torque is exerted on the motor; then, if $\mathbf{T_L}$ is the load torque, the following equation may be written:

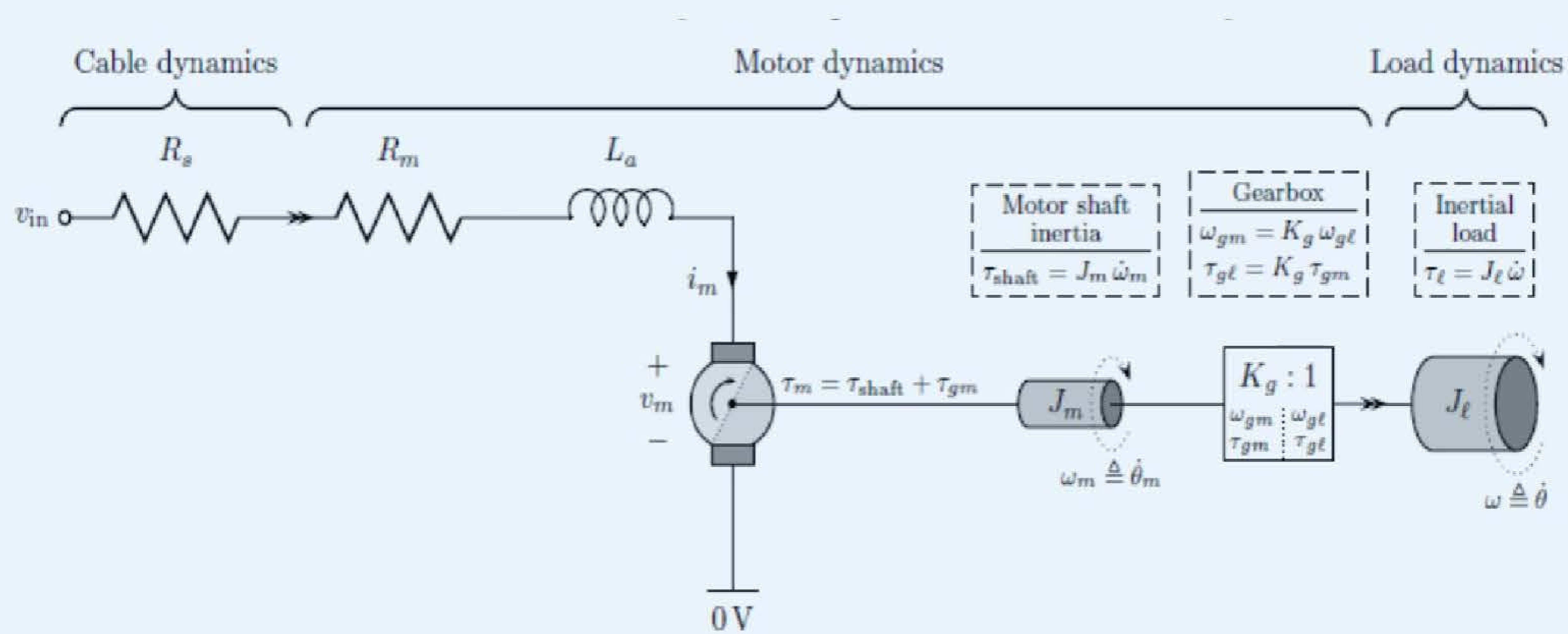
$$T_B - T_L = \frac{Jd\omega}{dt} + F\omega$$

The useful torque $\mathbf{T_B}$ would be equal to what the load would feel, $\mathbf{T_L}$ in the ideal case for which there is no friction and no self-inductance

Basic model for a DC motor in an application

In applications all four components must be considered:

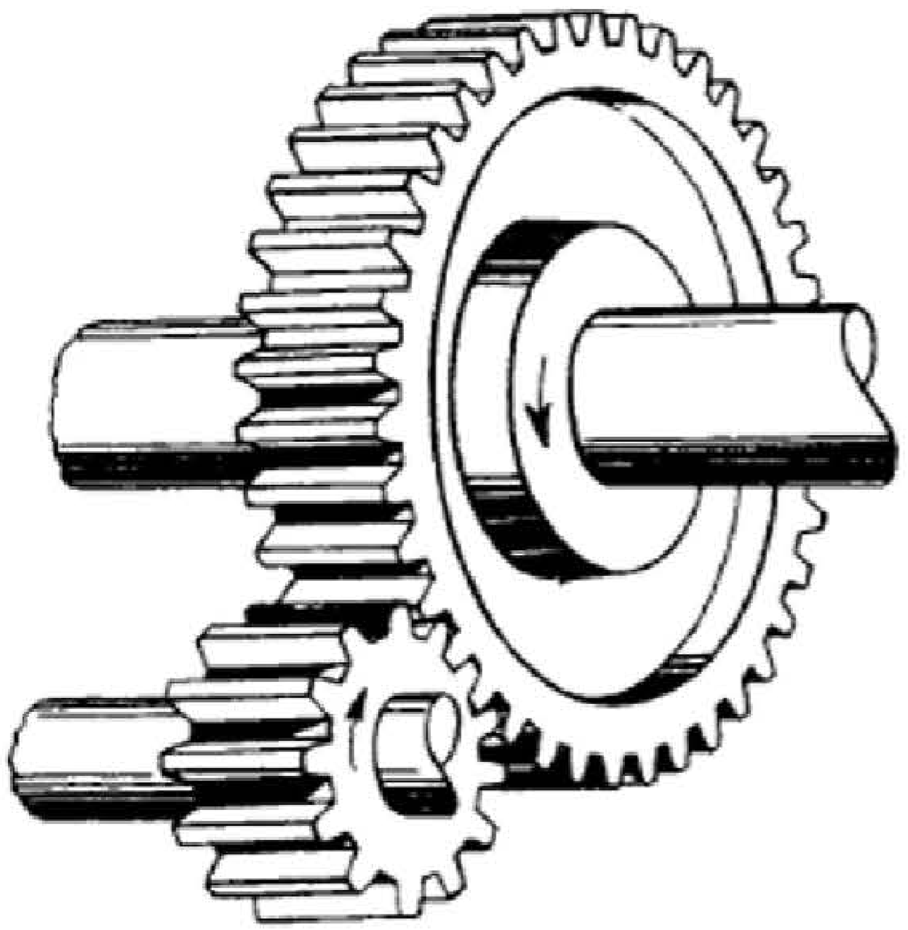
- Powering cable
- Motor
- Coupling with the load (gearing)
- Load



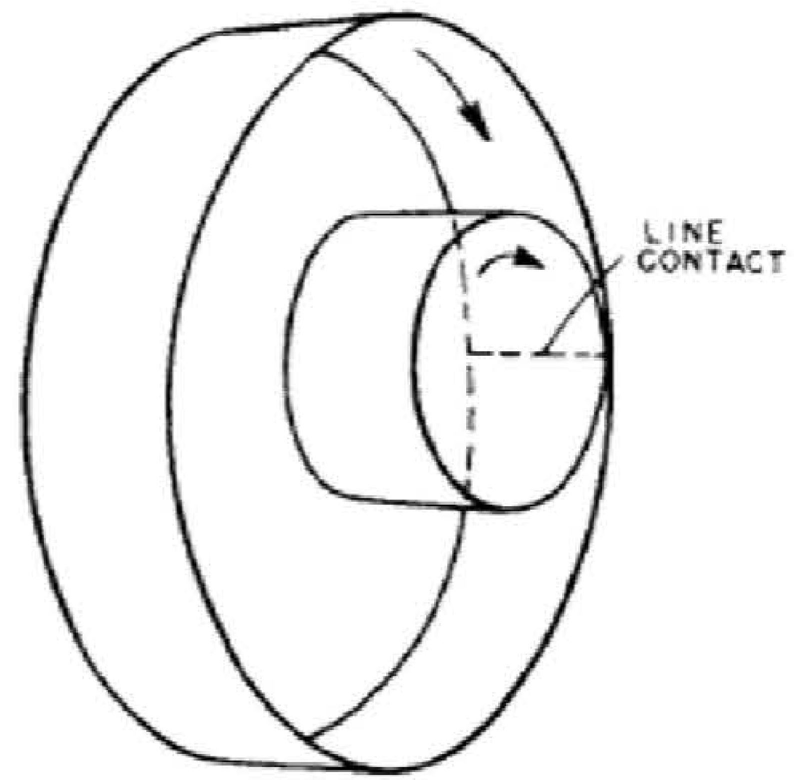
EQUATIONS

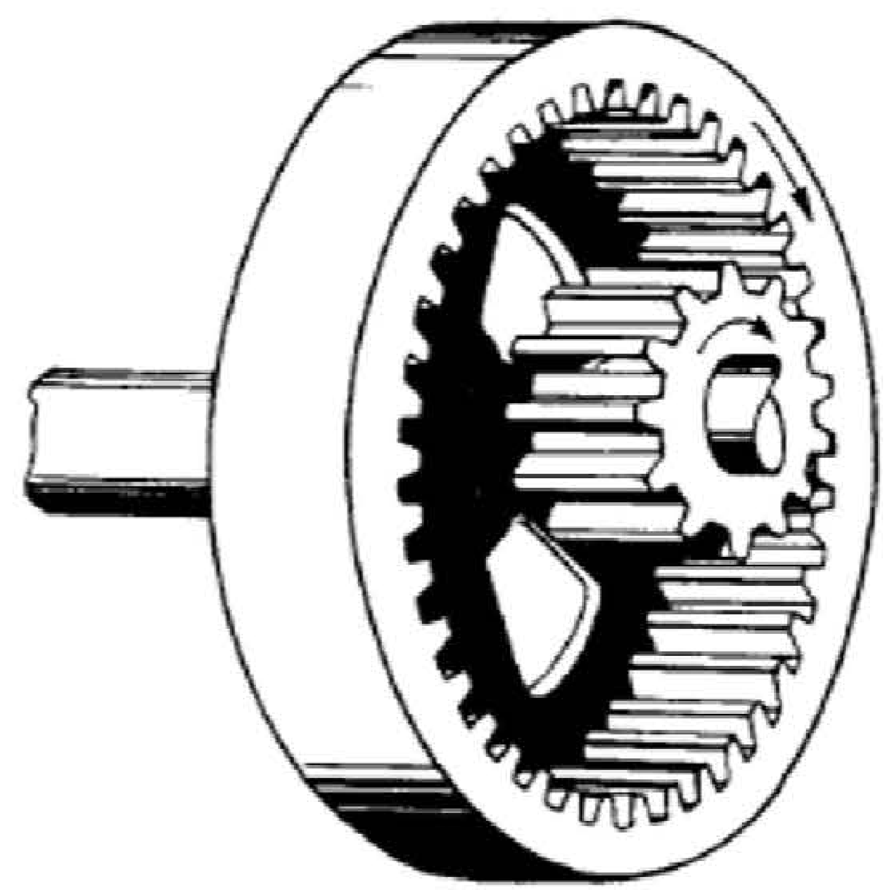
Motor power conservation	$i_m v_m = \tau_m \omega_m$
Motor shaft inertia	$T_{shaft} = J_m \dot{\omega}_m$
Gearbox	$\omega_{gm} = K_g \omega_{gl}$
	$\tau_{gl} = K_g \tau_{gm}$
Inertial load	$\tau_l = J_l \dot{\omega}$
Gear power conservation	$\tau_{gm} \omega_{gm} = \tau_{gl} \omega_{gl}$

Common gearing setups



Typical **external spur gear** set.





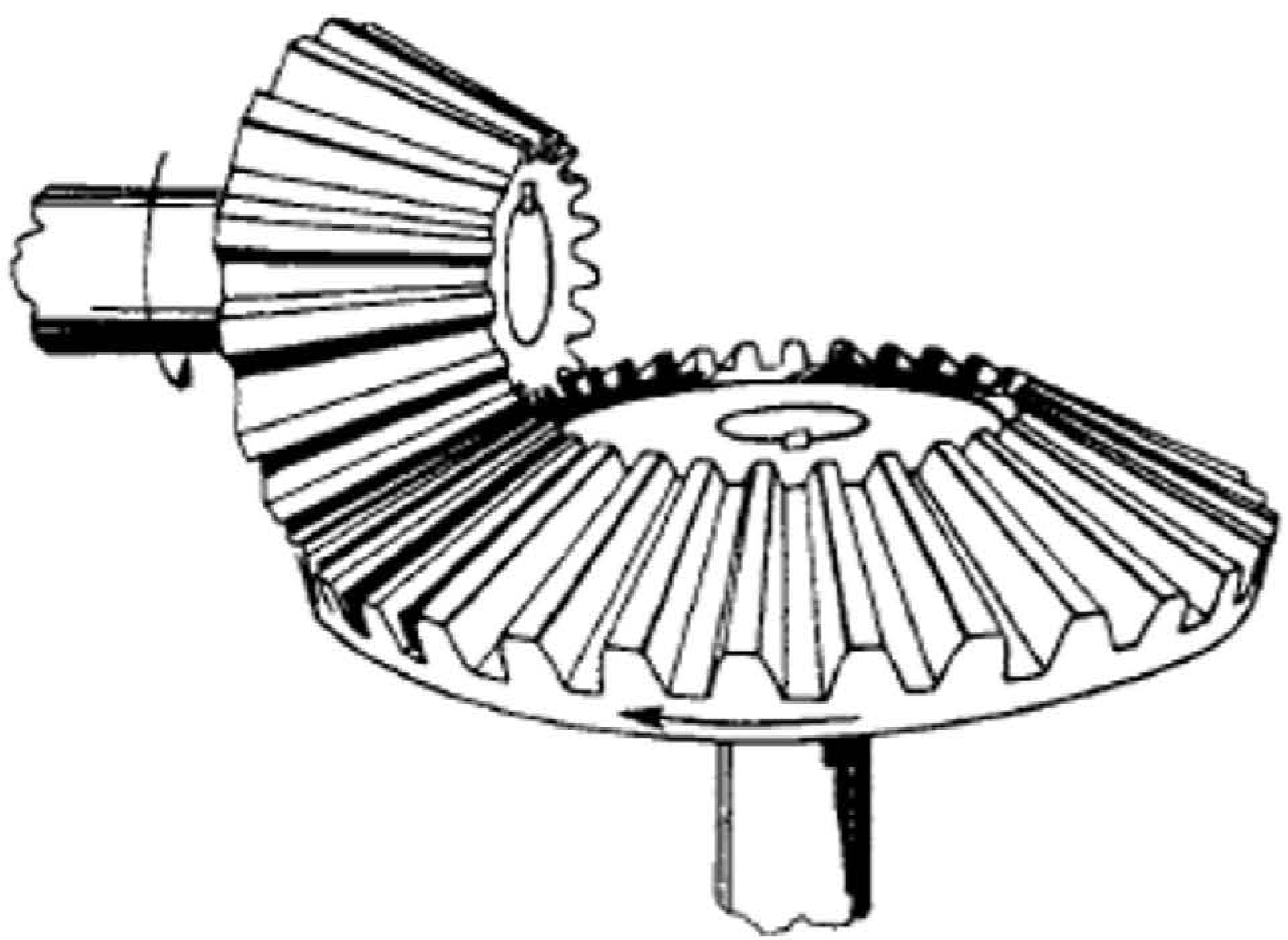
<u>Internal spur gears</u>

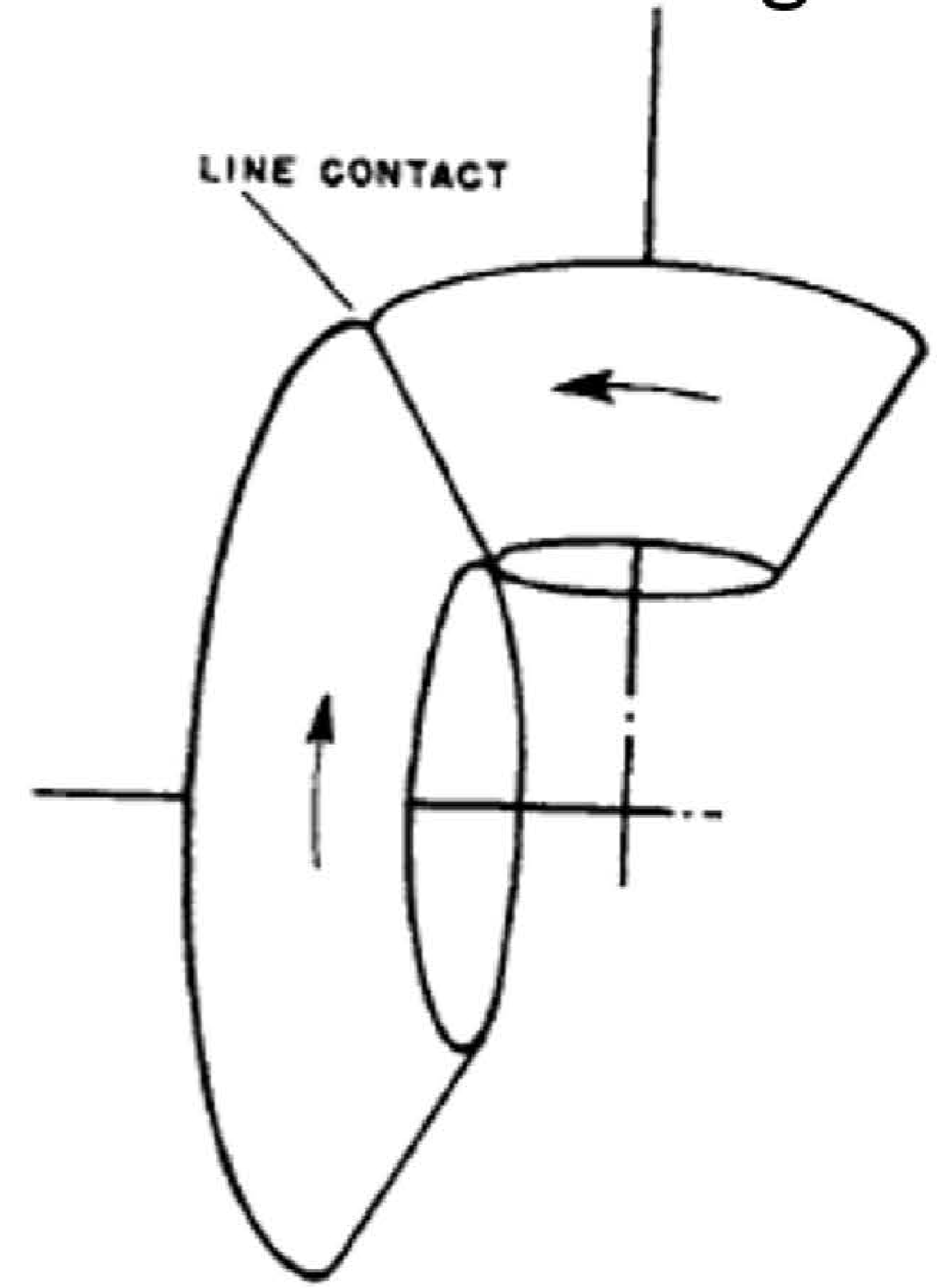
The pitch surfaces on friction wheels contact along a line (upper view) to transmit motion between parallel shafts. The teeth are developed on internal and external cylindrical pitch surfaces as shown in the lower view.

Common gearing setups

Bevel gear

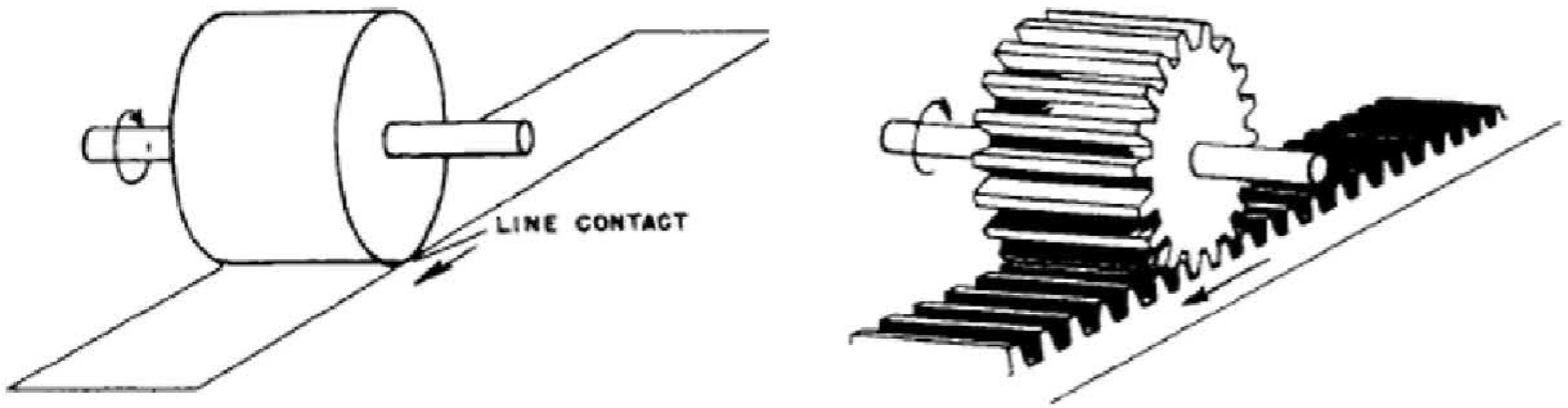
The shafts intersect at a right angle, although bevel gears may also be used between shafts that intersect at larger or smaller angles



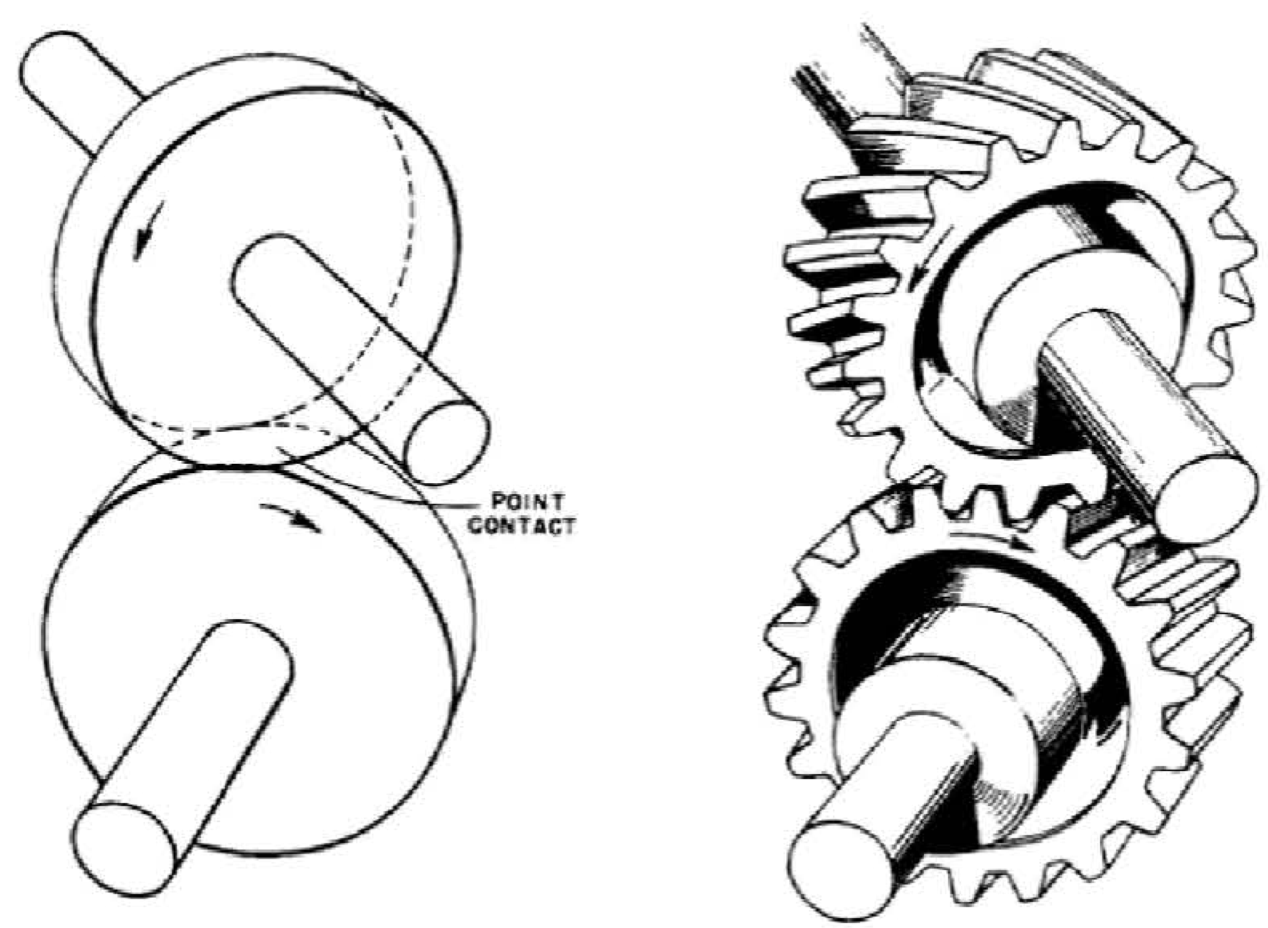


Common gearing setups

Rack-and-pinion gear set

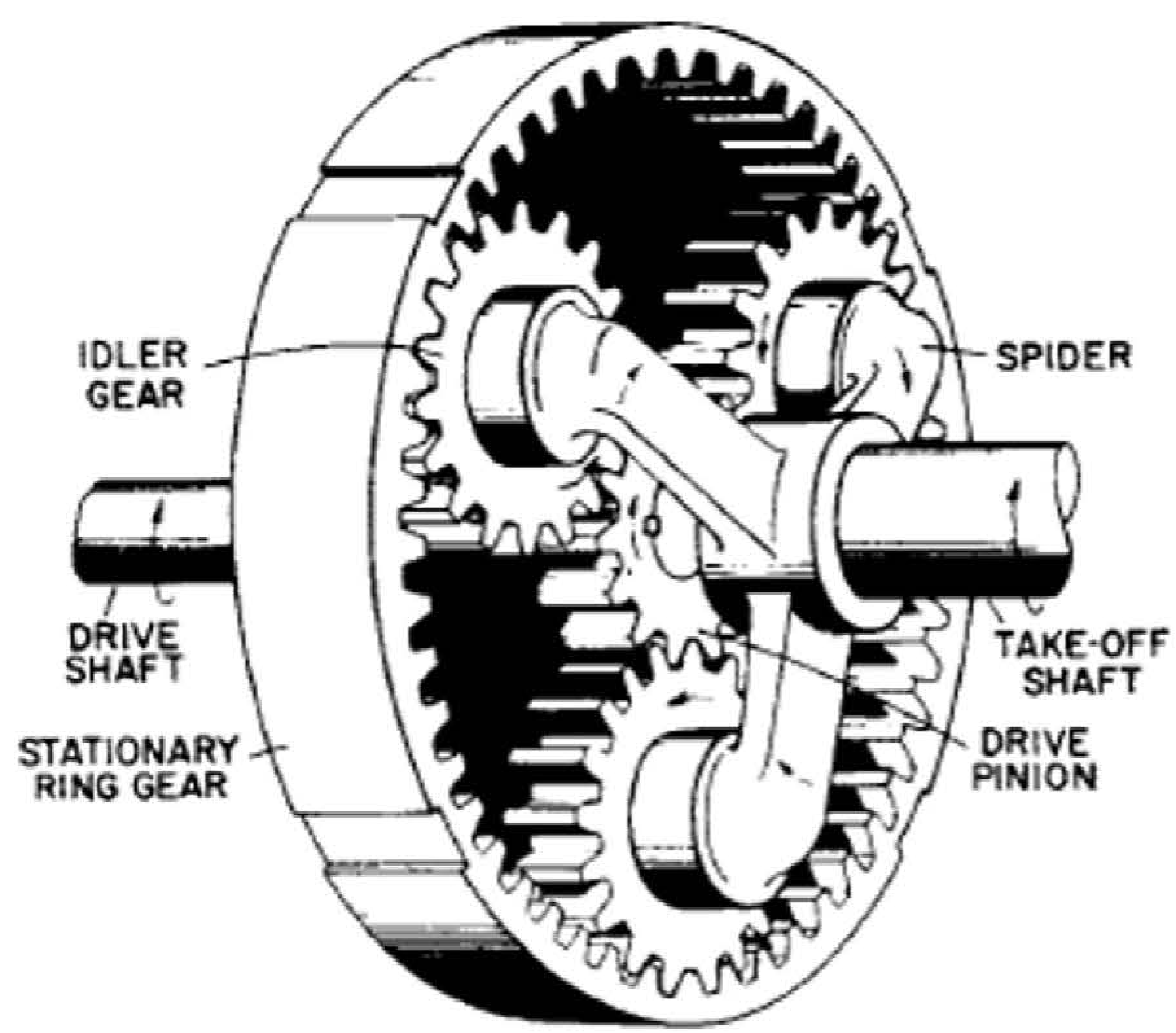


Principle of helical gearing



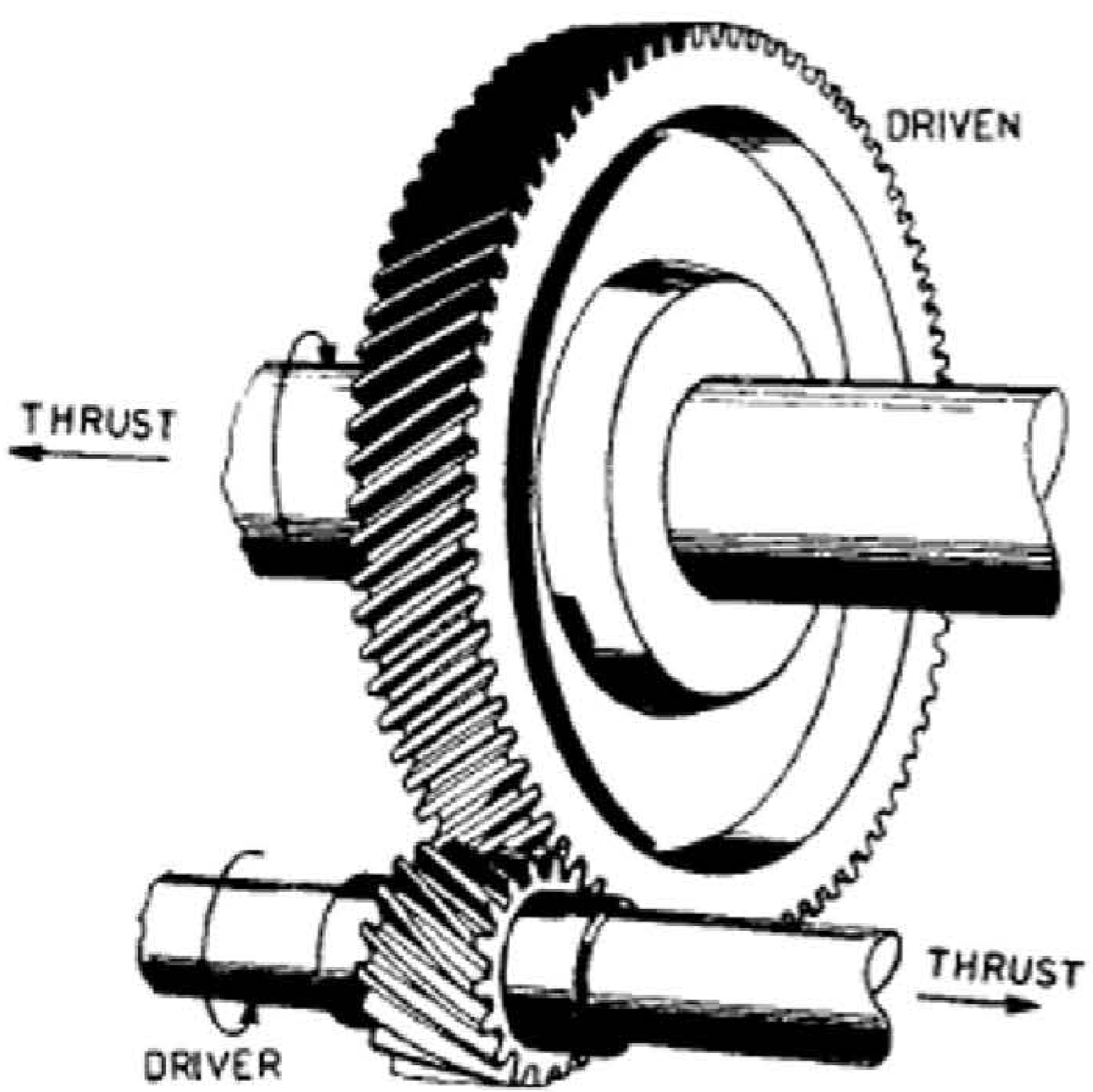
Common gearing setups

Epicyclic, or planetary gearing

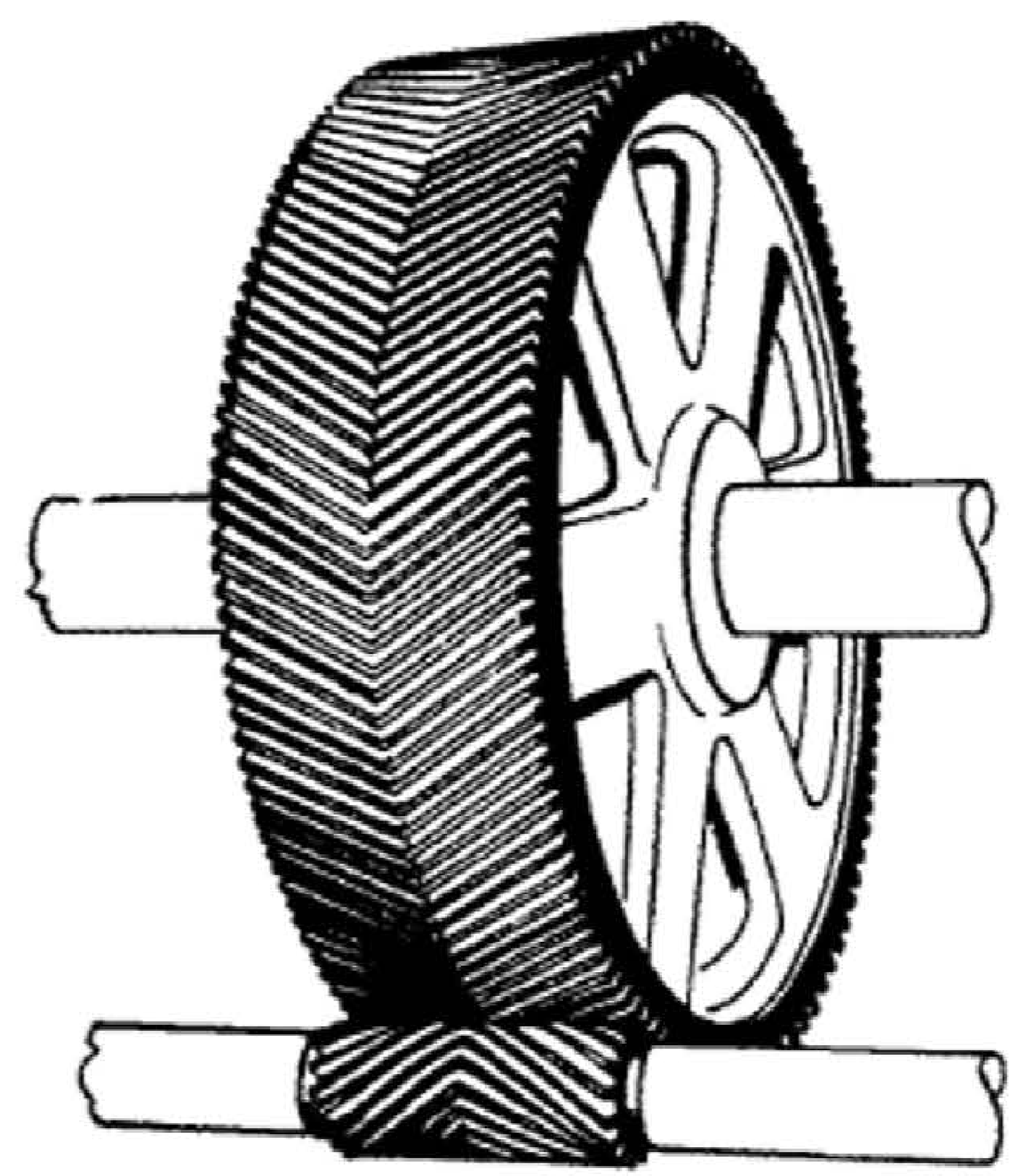


Common gearing setups

Helical gears

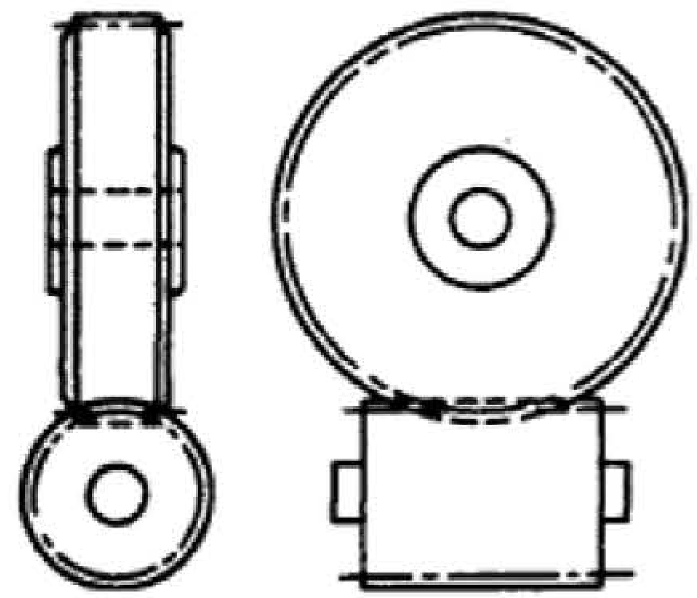


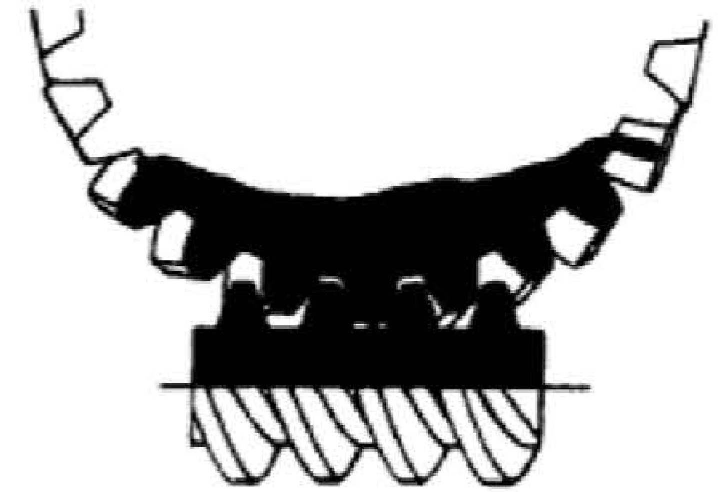
Double-helical or herringbone gear

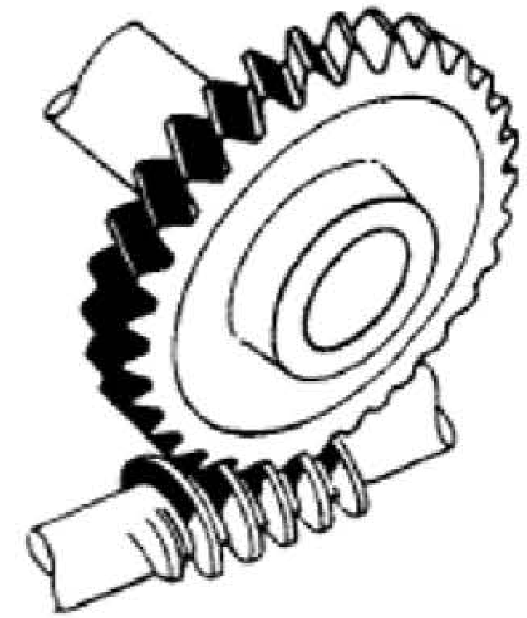


Common gearing setups

Nonthroated worm gear set. The teeth are straight and do not envelop the worm.

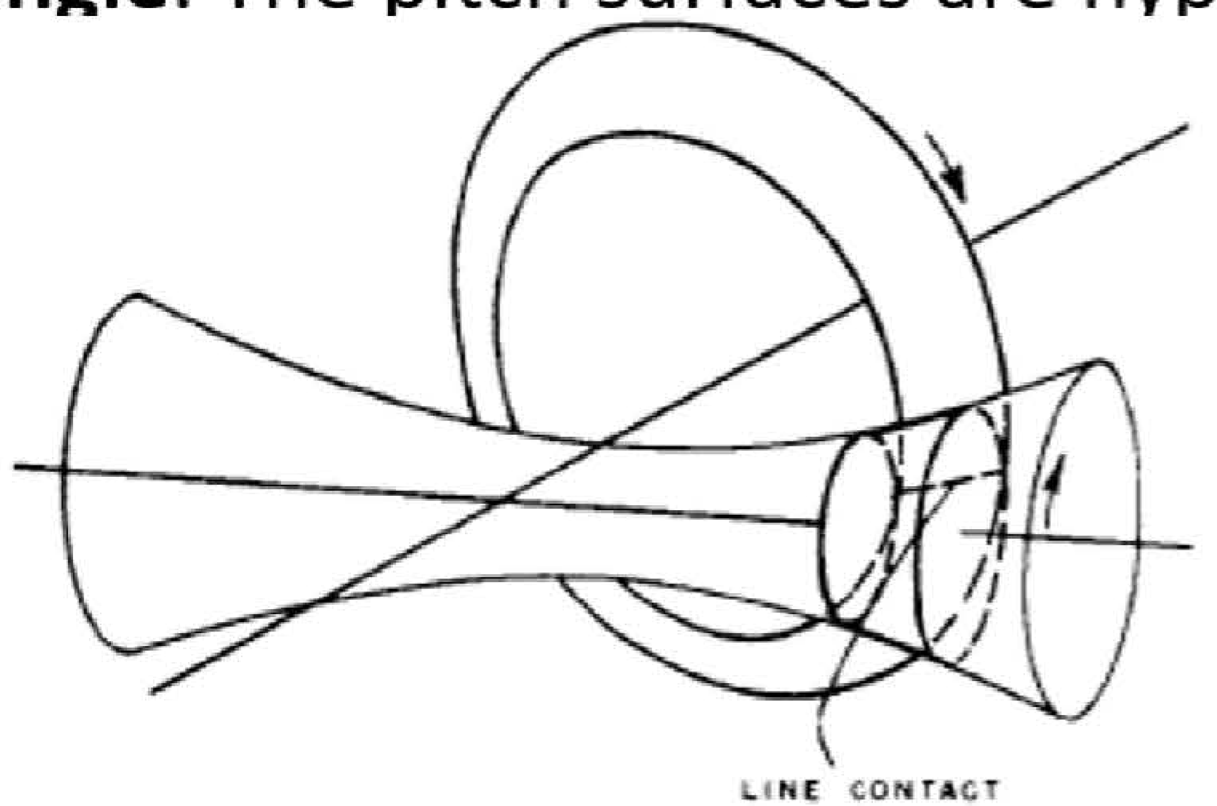


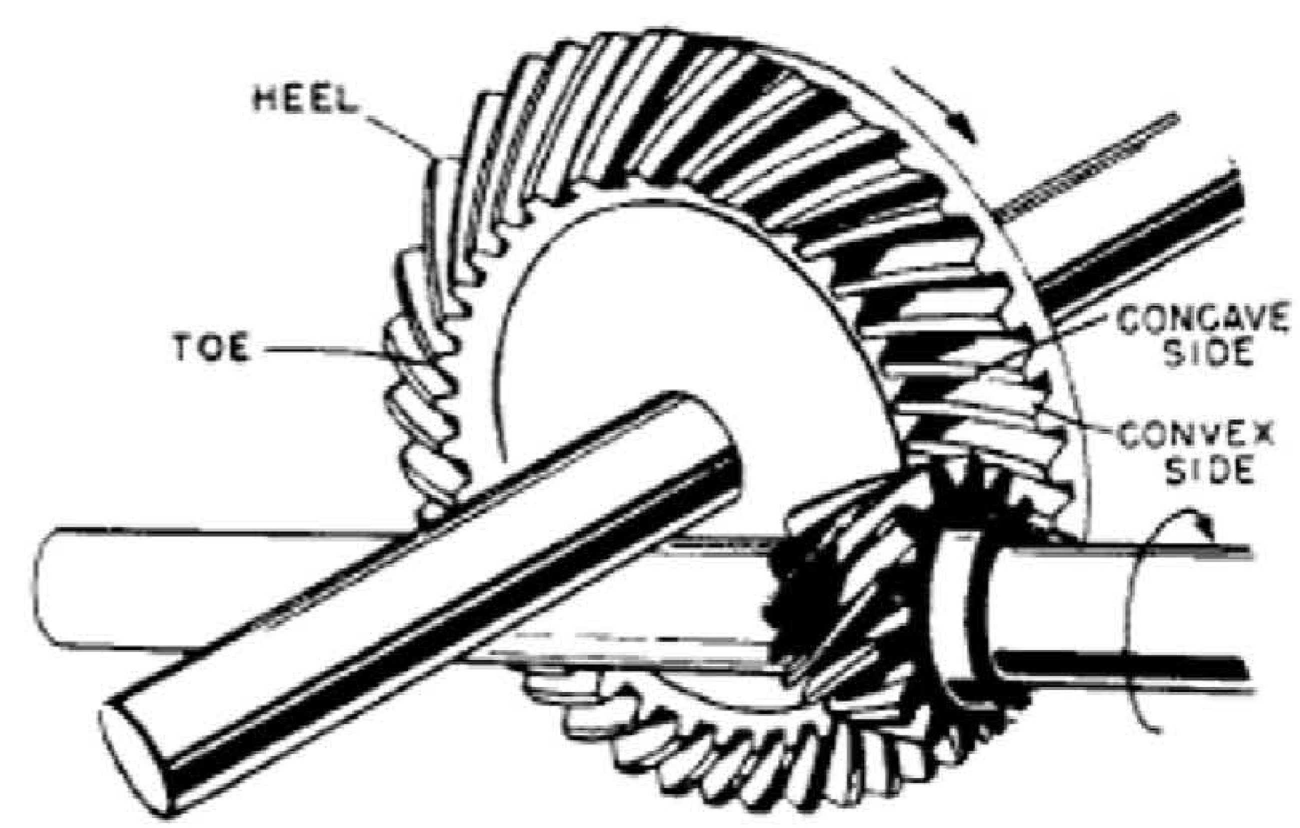




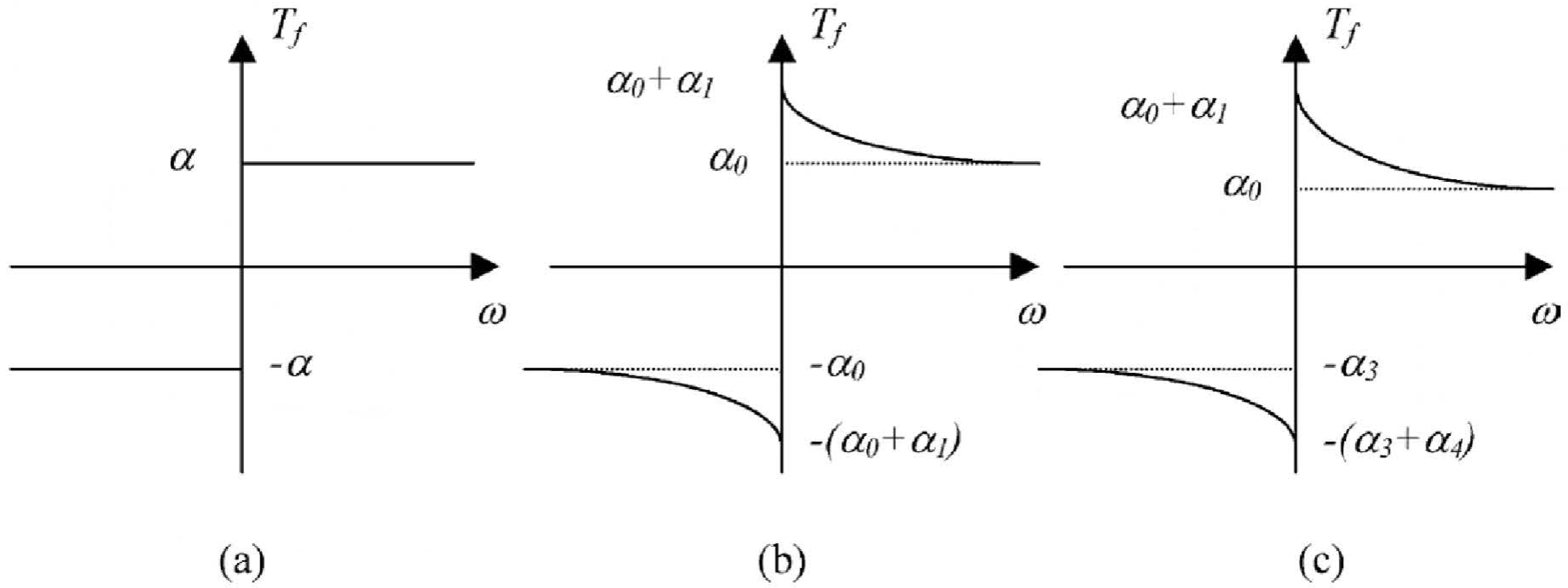
Hypoid gear and pinion

The gears transmit motion between **nonintersecting shafts crossing at a right angle**. The pitch surfaces are hyperbolic in form.





Possible improvements – Nonlinear friction torque



$$T_f(\omega) = \alpha\ sgn(\omega)$$

$$T_f(\omega) = \alpha_0\ sgn(\omega) + \alpha_1\ e^{-\alpha_2|\omega|} sgn(\omega)$$

$$sgn(\omega) = \begin{cases} 1 & \omega > 0 \\ 0 & \omega = 0 \\ -1 & \omega < 0 \end{cases}$$

$$sgn1(\omega) = \begin{cases} 1 & \omega \geq 0 \\ 0 & \omega < 0 \end{cases}$$

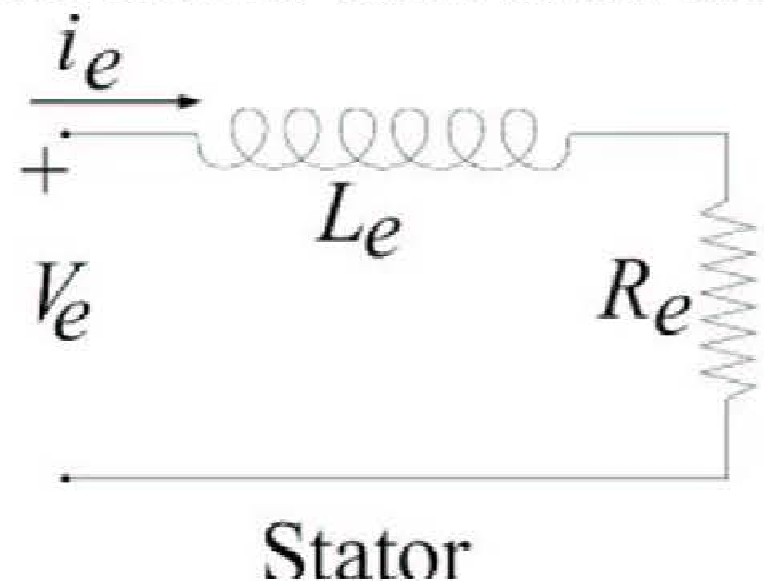
$$sgn2(\omega) = \begin{cases} 1 & \omega \geq 0 \\ -1 & \omega < 0 \end{cases}$$

$$T_f(\omega) = \left(\alpha_0 + \alpha_1\ e^{-\alpha_2|\omega|}\right) sgn1(\omega) + \left(\alpha_3 + \alpha_4 e^{-\alpha_5|\omega|}\right) sgn2(\omega)$$

DC motor with stator coil and rotor coil

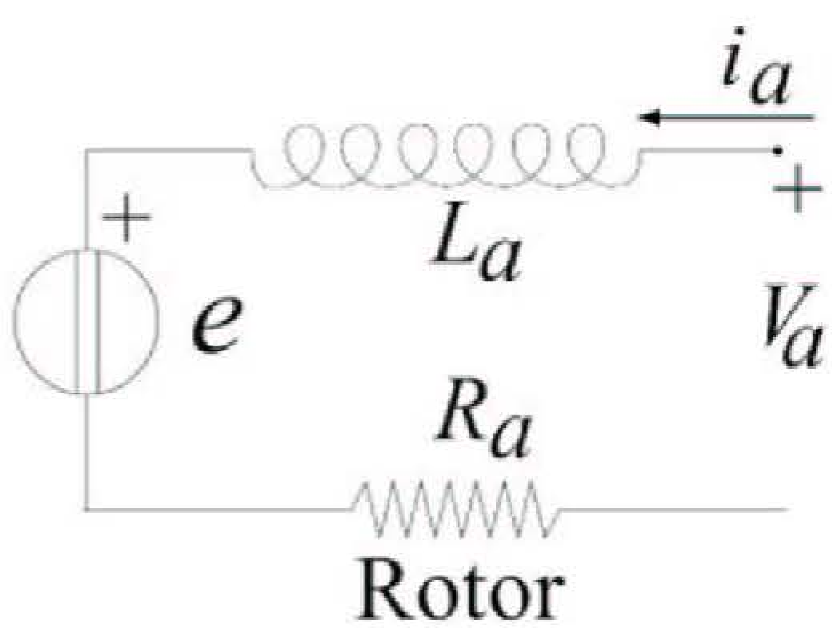
The stator of the motor will be assumed to have a single coil characterized by an inductance L_e due to the windings and a resistance $\mathbf{R_e}$ due to dispersions in the conductor . The equation associated with such an electric circuit is given by

$$v_e(t) = \frac{L_e di_e}{dt} + R_e i_e$$



The rotor is assumed to be a single coil characterized by inductance $\mathbf{L_a}$ and resistance $\mathbf{R_a}$ but it has to be taken into account the back EMF, **e**, of the motor. The equation associated with such an electric circuit is given by

$$v_a(t) = \frac{L_a(t) di_a}{dt} + R_a i_a + e$$

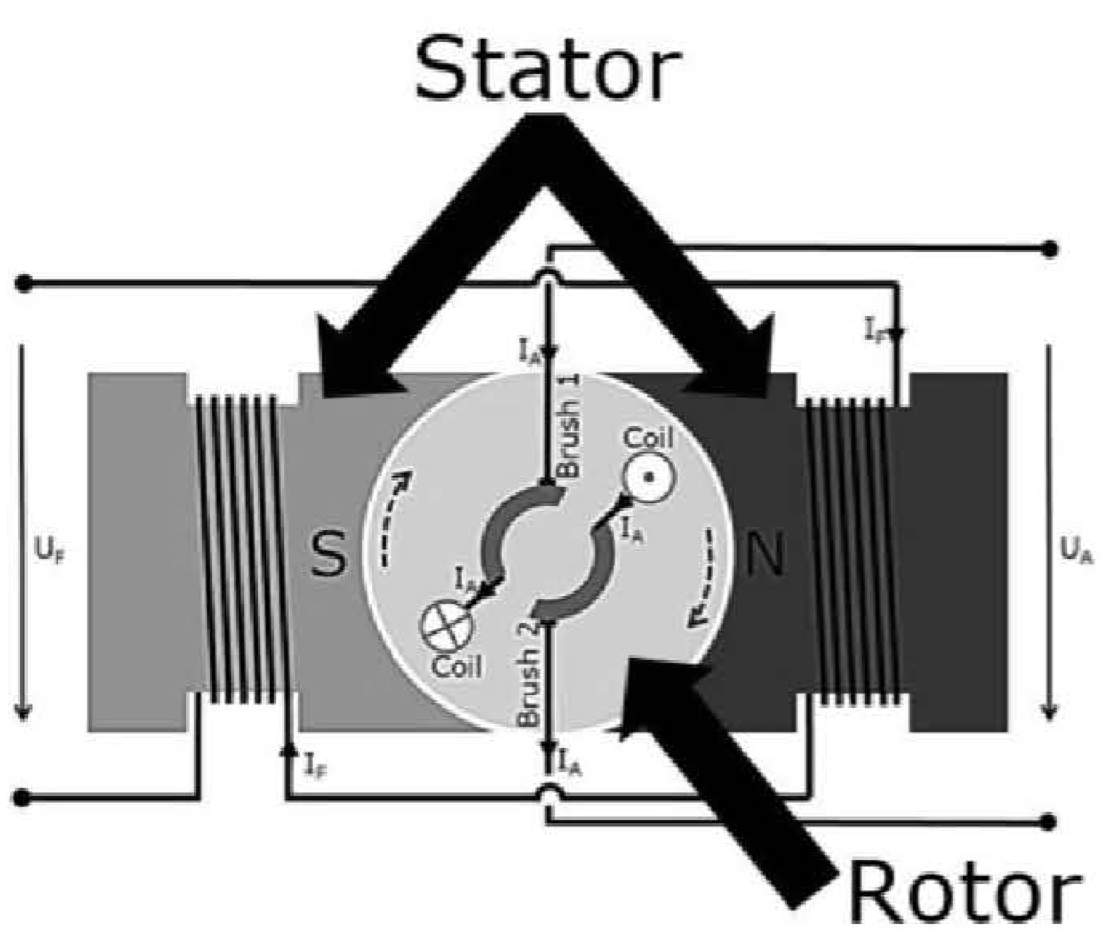


Remark: When modelling the permanent magnet DC motor, the permanent magnet excitation can be represented by an *equivalent electromagnetic excitation* with a constant excitation current.

DC motor with stator coil and rotor coil

The stator field is produced by dedicated DC windings, generating the required excitation field.

In contrast to permanent magnet based excitation this setup allows the excitation field to be regulated.
The rotor of the machine is also called an “armature” and the rotor current therefore is called *armature current* (i_a). Due to the structure of DC motors, the excitation field and the armature field are orthogonal to each other.
If both fields are independently adjustable such as in separately excited DC machines, *independent torque and flux control* can be applied.



DC motor with stator coil and rotor coil

Nonlinear model with two inputs (v_a, v_e), one output, one disturbance input (T_L)
Four state variables, related to:

- the energy stored in the inductance L_e;
- the energy stored in the inductance L_a;
- the kinetic energy of the rotor (related to J);
- the position ϑ of the rotor.

MECHANICAL DOMAIN

$$T_M - T_L = J\,\frac{\mathrm{d}\omega}{\mathrm{d}t} + F\,\omega$$

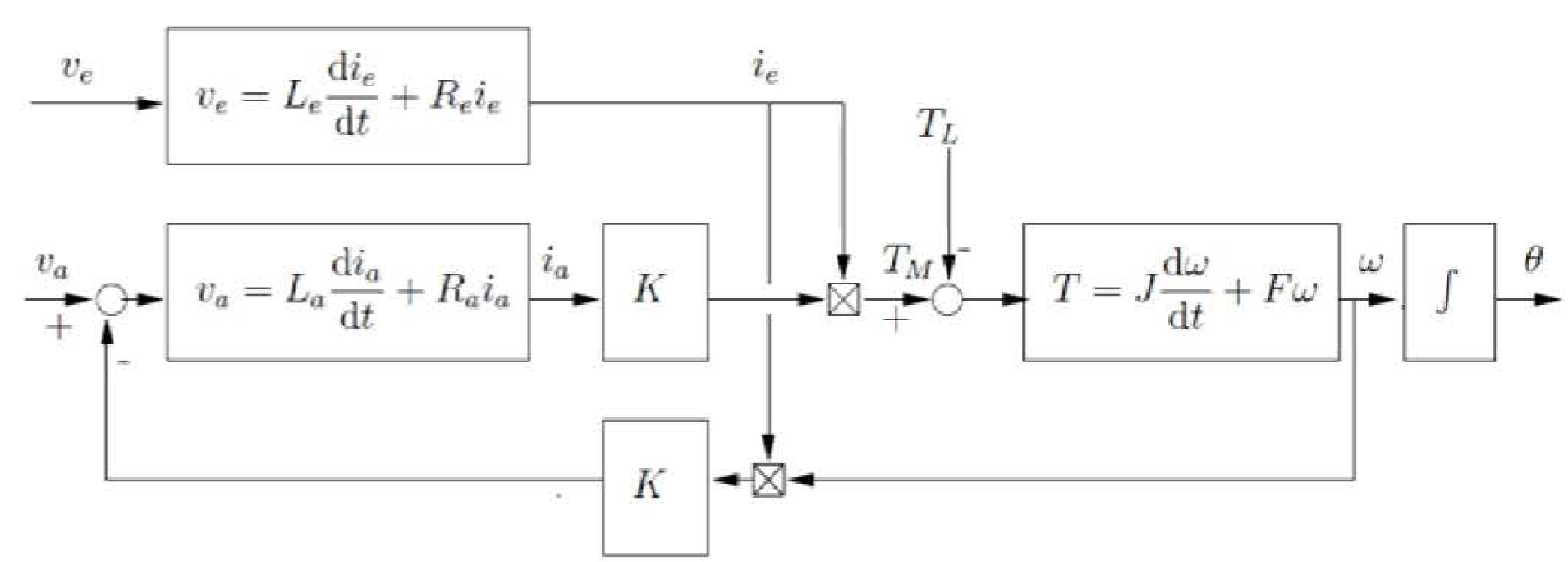
ELECTRICAL DOMAIN

$$v_e(t) = \frac{L_e di_e}{dt} + R_e i_e$$

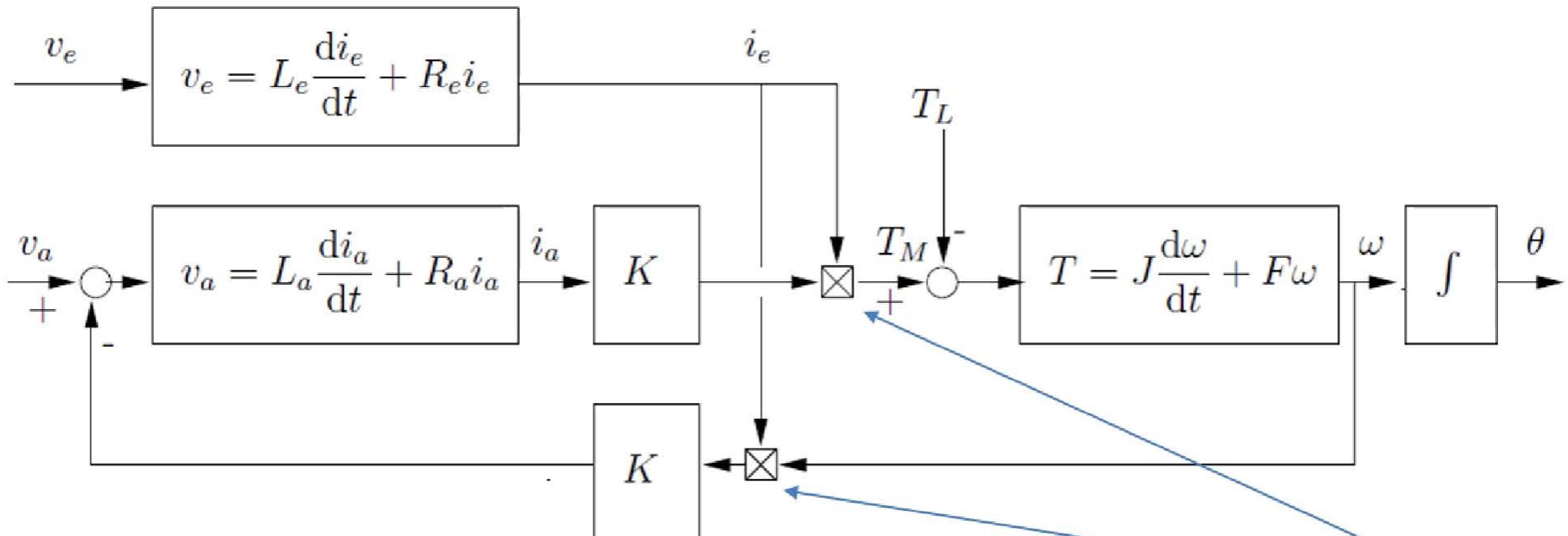
$$v_a(t) = \frac{L_a(t) di_a}{dt} + R_a i_a + e$$

$$T_M = K\,i_e\,i_a$$

$$e = K\,i_e\,\omega$$



Block diagram for modelling a DC motor



The model contains three **linear** equations between physical quantities. Due to the **multipliers**, which represent the system nonlinearities, the model of the motor is **nonlinear**.