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Tema lab 6

1. Compute the autocorrelation function for the signal $x(t)=A\sin(\omega_0 t+\varphi)$

$$\varphi_x(\tau) = \frac{1}{T_0}\int_0^{T_0} x(t)\cdot x(t-\tau)\,dt = \frac{1}{T_0}\int_0^{T_0} A\sin(\omega_0 t+\varphi)\cdot A\sin(\omega_0(t-\tau)+\varphi)\,dt$$

$$= \frac{A^2}{T_0}\int_0^{T_0}\sin(\omega_0 t+\varphi)\cdot\sin(\omega_0 t+\varphi-\omega_0\tau)\,dt =$$

$$= \frac{A^2}{T_0}\int_0^{T_0}\frac{\cos(-\omega_0\tau)-\cos(2\omega_0 t+2\varphi-\omega_0\tau)}{2}\,dt =$$

$$= \frac{A^2}{2T_0}\cdot\cos(\omega_0\tau)\cdot T_0 - \frac{A}{2T_0}\int_0^{T_0}\cos(2\omega_0 t+2\varphi-\omega_0\tau)\,dt =$$

$$= \frac{A^2}{2}\cos(\omega_0\tau) - \frac{A^2}{2T_0}\cdot\left.\frac{\sin(2\omega_0 t+2\varphi-\omega_0\tau)}{2\omega_0}\right|_0^{T_0} =$$

$$= \frac{A^2}{2}\cos(\omega_0\tau) - \frac{A^2}{4T_0\omega_0}\left(\sin(2\omega_0 T_0+2\varphi-\omega_0\tau)-\sin(2\varphi-\omega_0\tau)\right)$$

We know that $T_0=\frac{2\pi}{\omega_0} \Rightarrow \varphi_x(\tau)=\frac{A^2}{2}\cos(\omega_0\tau)-\frac{A^2}{8\pi}(\sin(4\pi+2\varphi-\omega_0\tau)-$
$-\sin(2\varphi-\omega_0\tau))$

$$\Rightarrow \varphi_x(\tau) = \frac{A^2}{2}\cos(\omega_0\tau)-\frac{A^2}{8\pi}\Big(\underbrace{\cancel{\sin(4\pi)\cos(2\varphi-\omega_0\tau)}}_{0}+\underbrace{\cos(4\pi)\sin(2\varphi-\omega_0\tau)}_{1}-$$
$$\cancel{-\sin(2\varphi-\omega_0\tau)}\Big)$$

$$\Rightarrow \varphi_x(\tau) = \frac{A^2}{2}\cos(\omega_0\tau)-\frac{A^2}{8\pi}\cdot 0 \Rightarrow \varphi_x(\tau)=\frac{A^2}{2}\cdot\cos(\omega_0\tau)$$