EE 628
Deep Learning
Fall 2019

Lecture 9

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Overview

- Last lecture we covered
  - Modern Convolutional Neural Networks

- Today, we will cover
  - Text Processing
  - Recurrent Neural Networks

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- For instance, the words written here are in sequence, and would be hard to understand if you permute randomly.
- Likewise, image frames in a video, the audio signal in a conversation, or the browsing behavior on a website, all follow sequential order.
- Thus we need specialized models for such data.

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- While CNNs process spatial information, RNNs are designed to better handle sequential information.
- These networks introduce state variables to store past information and, together with the current input, determine the current output.
- Many of the examples for using RNNs are based on text data. Hence, we will emphasize language models in this lecture.

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- Let's denote the prices by $x_t \geq 0$.
- For traders to do well in the stock market on day t, they want to predict x_t via

$$x_t \sim p(x_t | x_{t-1}, \ldots x_1).$$

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  2. Keep some summary h_t of the past observations around and update that in addition to the actual prediction. This leads to models that estimate $p(x_t, x_{t-1}, h_{t-1})$ and moreover updates of the form $h_t = g(h_{t-1}, x_{t-1})$. These models are called latent autoregressive models. LSTMs and GRUs are examples of this.

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- A common assumption is that while the specific values of x_t might change, at least the dynamics of the time series itself won't.
- Statisticians call dynamics that don't change stationary.
- Regardless of what we do, we will thus get an estimate of the entire time series via

$$p(x_1, \ldots x_T) = \prod_{t=1}^{T} p(x_t | x_{t-1}, \ldots x_1).$$

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- Such models are particularly nice since dynamic programming can be used to compute values along the chain exactly.

$$p(x_{t+1} | x_{t-1}) = \sum_{x_t} p(x_{t+1} | x_t) p(x_t | x_{t-1}).$$

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- In many cases, however, there exists a natural direction for the data, namely going forward in time.
- It is clear that future events cannot influence the past.

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  3. Builds a vocabulary for these tokens to map them into numerical indices
  4. Maps all tokens in the data into indices to facilitate to feed into models

Open notebook
IN_CLASS Text_Data_Example.ipynb

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- Language models are incredibly useful. For instance, an ideal language model would be able to generate a natural text just on its own by drawing one word at a time

$$w_t \sim p(w_t | w_{t-1}, \ldots w_1).$$

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- For example, the probability of a text sequence containing four tokens consisting of words and punctuation would be given as:

$$p(\text{Statistics}, \text{is}, \text{fun}, .) = p(\text{Statistics})p(\text{is}|\text{Statistics})p(\text{fun}|\text{Statistics}, \text{is})p(.|\text{Statistics}, \text{is}, \text{fun})$$

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- In order to compute the language model, we need to calculate the probability of a word given the previous few words.

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- Moving on, we could attempt to estimate $\hat{p}(\text{is}|\text{Statistics}) = \frac{n(\text{Statistics is})}{n(\text{Statistics})}.$
- Estimating the probability of a word pair is somewhat difficult, since the occurrences of 'Statistics is' are a lot less frequent. Things get worse for 3 word combinations.

Estimating a language model

- Laplace Smoothing: add a small constant to all counts.
- Unfortunately, models like this get unwieldy rather quickly:
  - we need to store all counts
  - This entirely ignores the meaning of the words

Markov Models and n-grams

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- Higher orders correspond to longer dependencies.
- This leads to a number of approximations that we could apply to a model sequence:

$$p(w_1, w_2, w_3, w_4) = p(w_1)p(w_2)p(w_3)p(w_4) \longrightarrow \text{unigram}$$

$$p(w_1, w_2, w_3, w_4) = p(w_1)p(w_2|w_1)p(w_3|w_2)p(w_4|w_3) \longrightarrow \text{bigram}$$

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Open IN_CLASS_Natural_Language_Statistics.ipynb

Open IN_CLASS_Training_Data_Preparation.ipynb

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- However, the number of parameters would also increase with exponentially with it.
- Hence, rather than modeling $p(x_t|x_{t-1}, \ldots, x_{t-n+1})$, it is preferable to use a latent variable model in which we have

$$p(x_t|x_{t-1}, \ldots x_1) \approx p(x_t|x_{t-1}, h_t).$$

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- Recurrent neural networks are neural networks with hidden states.

Recurrent Neural Networks with Hidden States

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- Since the hidden state uses the same definition of the previous time step in the current time step, the computation of the equation above is recurrent, hence the name recurrent neural network (RNN).

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- RNNs always use these model parameters, even for different time steps. Therefore, the number of RNN model parameters does not grow as the number of time steps increases.

Recurrent Neural Networks with Hidden States

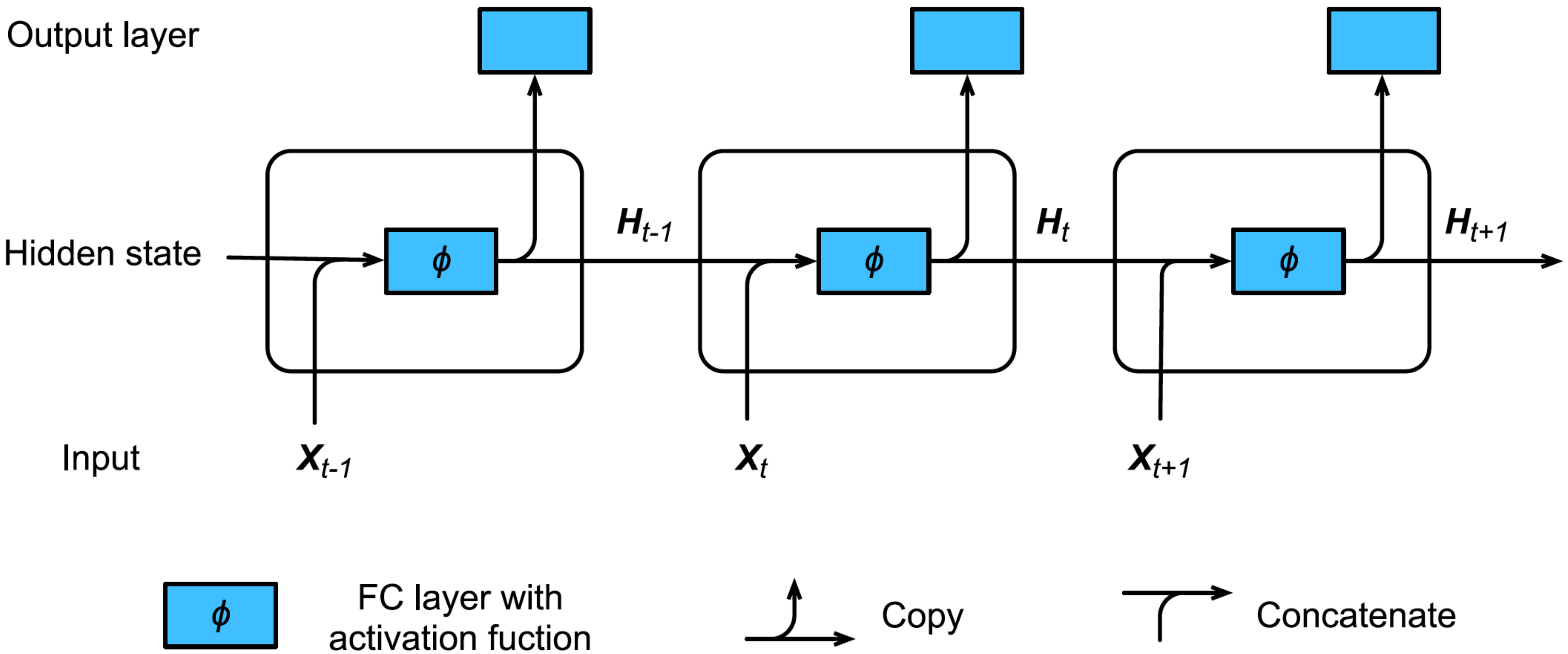

Fig. 10.4.1: An RNN with a hidden state.

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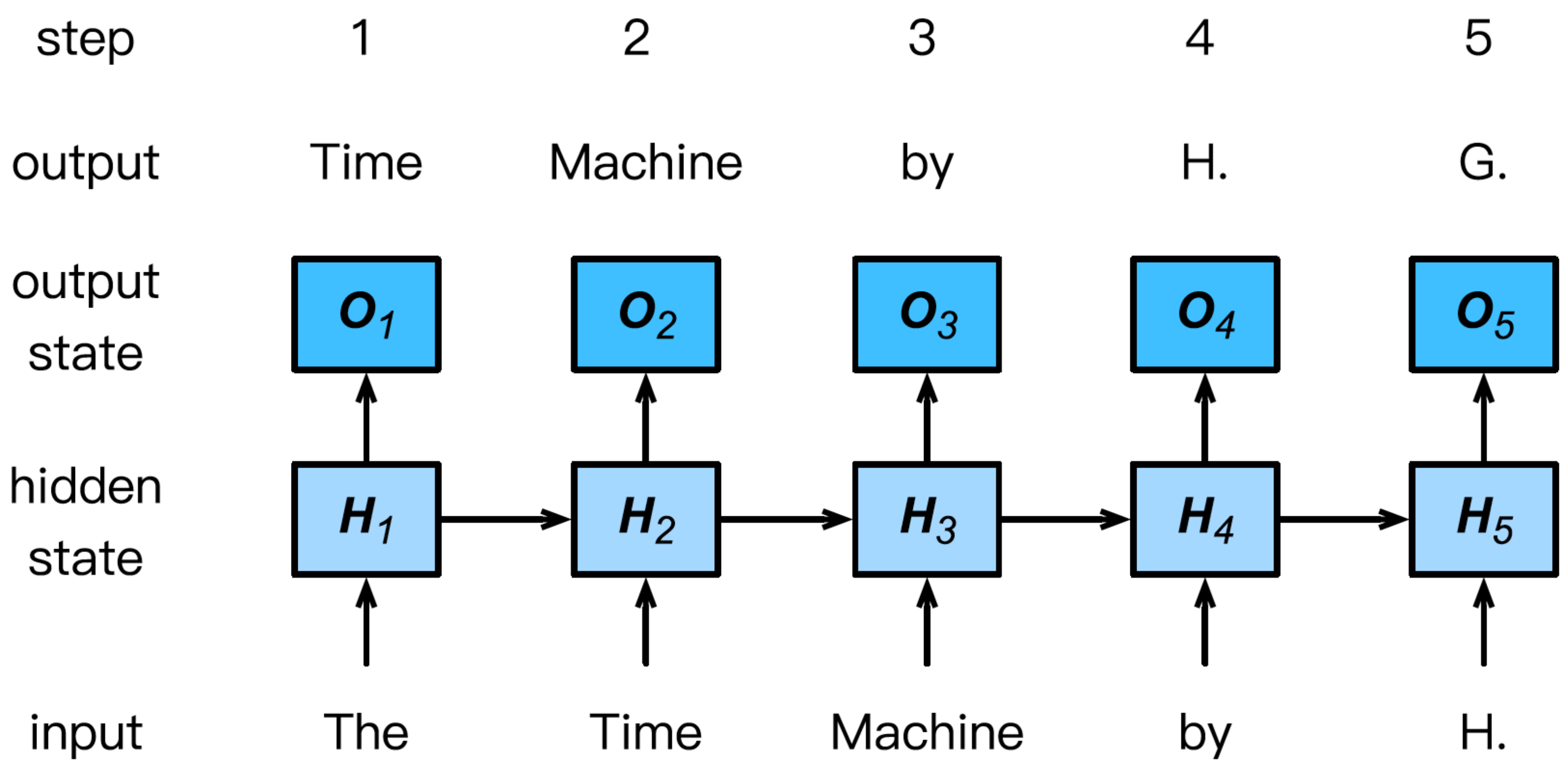

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- One way is to check how surprising the text is.
- A good language model is able to predict with high accuracy what we will see next.
- Consider the following continuations of the phrase *It is raining*, as proposed by different language models:

1. It is raining outside
2. It is raining banana tree
3. It is raining piouw;kcj pwepoiut

Perplexity

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- What are the best, worst and baseline cases?

Open notebook IN_CLASS_implementation_of_RNNs_from_scratch