

# Let's play with Regular Expressions

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# Outline

Introduction

Regular Expressions refresher

Glushkov Automaton Construction

Laziness allows matching strictly more expressive languages

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Functional Pearl by Sebastian Fischer, Frank Huch and  
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- ▶ Regular expressions are usually taken for granted and are implemented in C

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  - ▶ Have a trie data structure, want all entries where prefix matches a regex. Can do this naively but will have to rerun matching process for each prefix
  - ▶ Compiling via GHCJS and can't link to C libraries
- ▶ Can do symbolic manipulation on regular expression AST - just for fun

# Regular Expressions refresher

## Definitions

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String concatenation is denoted with  $\mathbin{\text{++}}$ :  $aaaaaa \mathbin{\text{++}} b = aaaaaab$

$$\forall x : x \mathbin{\text{++}} \varepsilon = \varepsilon \mathbin{\text{++}} x = x$$

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Alphabet - set of all characters, denoted  $\Sigma$ . E.g.  $\Sigma = \{a, b, c\}$ .

# Regular Expressions refresher

## Introducing Haskell ADT for regexps

Regular expressions are defined inductively

```
data Sigma = A | B | C
```

```
    deriving (Show, Eq, Ord, Enum, Bounded)
```

```
type Str a = [a]
```

```
data Regex a
```

```
    = Eps                                -- Empty regex
```

```
    | Sym a                             -- Singleton regex
```

```
    | Seq (Regex a) (Regex a)  -- Sequence
```

```
    | Alt (Regex a) (Regex a)  -- Alternatives
```

```
    | Rep (Regex a)  -- Repetition
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    deriving (Show, Eq)
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# Regular Expressions refresher

## Example

Consider regular expression  $a^* \cdot b \cdot (c \mid \varepsilon)$

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$$L(a^* \cdot b \cdot \varepsilon \mid c) = \{b, bc, ab, abc, aab, aabc, aaab, aaabc, \dots\}$$

In Haskell,

$re = Seq (Rep (Sym A)) (Seq (Sym B) (Alt Eps (Sym C)))$

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Or, with less parens

$re = Rep (Sym A) 'Seq' Sym B 'Seq' Alt Eps (Sym C)$

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Or, with less parens

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```

Expected output

```
accept re [B] ⇒ True
```

```
accept re [B, C] ⇒ True
```

```
accept re [A, A, A, B] ⇒ True
```

```
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accept re [A, A, A, B, C] ⇒ True
```

# Regular Expressions refresher

## Defining regexp semantics

For any regex ( $A$ ), notation  $L(A)$  denotes all strings that regex matches.

Let's define regexp semantics in Haskell by defining regexp matching function:

$$\text{accept} :: \text{Eq } a \Rightarrow \text{Regex } a \rightarrow \text{Str } a \rightarrow \text{Bool}$$
$$\text{accept } \text{Eps} \quad s = \text{acceptEps } s$$
$$\text{accept } (\text{Sym } c) \quad s = \text{acceptSym } c \ s$$
$$\text{accept } (\text{Seq } x \ y) \quad s = \text{acceptSeq } x \ y \ s$$
$$\text{accept } (\text{Alt } x \ y) \quad s = \text{acceptAlt } x \ y \ s$$
$$\text{accept } (\text{Rep } x) \quad s = \text{acceptRep } x \ s$$

# Regular Expressions refresher

## Empty string regex

Empty string -  $\epsilon$  is a regular expression that matches the empty string.

$$L(\epsilon) = \{\epsilon\}$$

*acceptEps :: Str a → Bool*

*acceptEps s = null s*

# Regular Expressions refresher

## Singleton character regex

Atoms - all symbols from  $\Sigma$  are regular expressions that match themselves.

$$\forall x \in \Sigma : L(x) = \{x\}$$

Haskell semantics:

```
acceptSym :: Eq a  $\Rightarrow$  a  $\rightarrow$  Str a  $\rightarrow$  Bool  
acceptSym c s = s == [c]
```

# Regular Expressions refresher

## Concatenation

Concatenation - if A and B are regular expressions, so is  $A \cdot B$

$$L(A \cdot B) = \{x \mathbin{\dot{+}} y \mid x \in L(A), y \in L(B)\}$$

Haskell semantics:

```
acceptSeq :: Eq a  $\Rightarrow$  Regex a  $\rightarrow$  Regex a  $\rightarrow$  Str a  $\rightarrow$  Bool  
acceptSeq r1 r2 s =  
  or [ accept r1 s1 && accept r2 s2 | (s1, s2)  $\leftarrow$  split s ]
```

```
split :: Str a  $\rightarrow$  [(Str a, Str a)]  
split xs = map ( $\lambda(x, y) \rightarrow$  (reverse x, y)) (go [] xs)
```

**where**

```
go p []      = [(p, [])]  
go p (c : cs) = (p, c : cs) : go (c : p) cs
```

```
split "abc"  $\Rightarrow$  [("", "abc"), ("a", "bc"), ("ab", "c"), ("abc", "")]
```



# Regular Expressions refresher

## Alternatives

Alternative - if A and B are regular expressions, so is  $A \mid B$

$$L(A \mid B) = L(A) \cup L(B)$$

Haskell semantics:

```
acceptAlt :: Eq a => Regex a -> Regex a -> Str a -> Bool  
acceptAlt r1 r2 s = accept r1 s || accept r2 s
```



# Regular Expressions refresher

What is not a regular expression

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- ▶ Generally, we want an automaton since it's very fast

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- ▶ Some popular implementations allow “regexps” with backreferences, like  $(a|b|c)\backslash 1$
- ▶  $(.*)\backslash 1$  describes non-regular language. Therefore it cannot be converted to finite automaton
- ▶ Generally, we want an automaton since it's very fast
- ▶ We don't include backreferences in our regexps. Regexps without them have enough expressive power

# Regular Expressions refresher

## Drawbacks

- ▶ Although usefull for defining semantics, *accept* function is not appropriate for practical applications.

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- ▶ Reliance on function *parts* means exponential worst-case runtime in length of string being matched



# Glushkov Automaton Construction

Glushkov proposed Algorithm for generating Nondeterministic Finite Automaton, NFA, from regular expression.

NFA is a good way to run regexps because

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- ▶ Size of NFA is linear in size of regular expression
- ▶ Given NFA with  $m$  states we can run it on string of  $n$  characters in  $O(m \cdot n)$  time
- ▶ DFA is simpler than NFA but can have  $\Omega(2^m)$  states of states for an NFA with  $m$  states.

# Glushkov Automaton Construction

## Example

We're going to fuse translation of regexp to NFA and execution to NFA.

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Glushkov observation: every NFA states corresponds to a position within regexp. Position is a place where a symbol occurs.

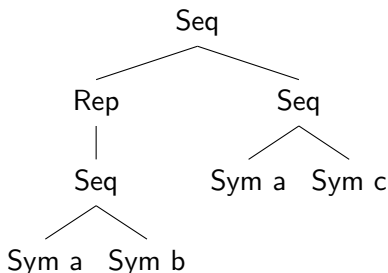
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Let's match  $(a \cdot b)^* \cdot a \cdot c$  against "ababac"



# Glushkov Automaton Construction

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"a|babac"



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"a babac"

$(a \cdot b)^* \cdot a \cdot c$

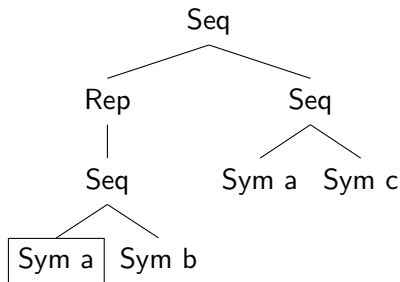
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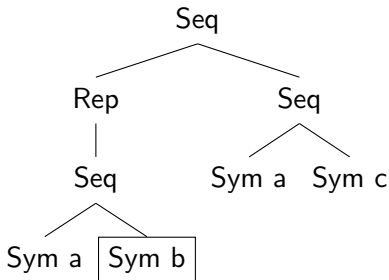
# Glushkov Automaton Construction

## Example - step 2

Let's match  $(a \cdot b)^* \cdot a \cdot c$  against "ababac"

"a**b**abac"

$(a \cdot \boxed{b})^* \cdot a \cdot c$



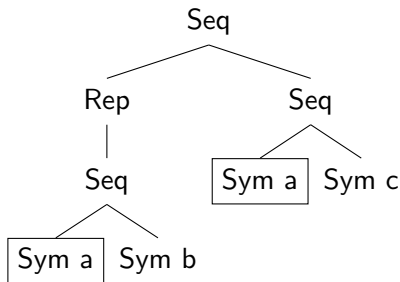
# Glushkov Automaton Construction

## Example - step 3

Let's match  $(a \cdot b)^* \cdot a \cdot c$  against "ababac"

"ab**a**bac"

$(\boxed{a} \cdot b)^* \cdot \boxed{a} \cdot c$



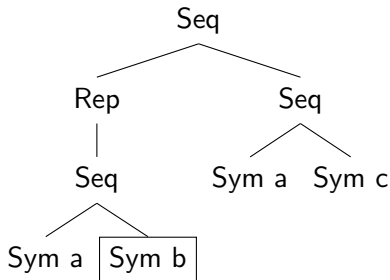
# Glushkov Automaton Construction

## Example - step 4

Let's match  $(a \cdot b)^* \cdot a \cdot c$  against "ababac"

"aba**b**ac"

$(a \cdot \boxed{b})^* \cdot a \cdot c$



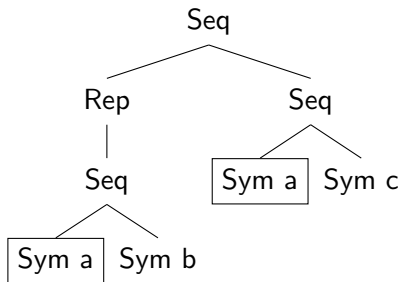
# Glushkov Automaton Construction

## Example - step 5

Let's match  $(a \cdot b)^* \cdot a \cdot c$  against "ababac"

"ababac"

$(\boxed{a} \cdot b)^* \cdot \boxed{a} \cdot c$



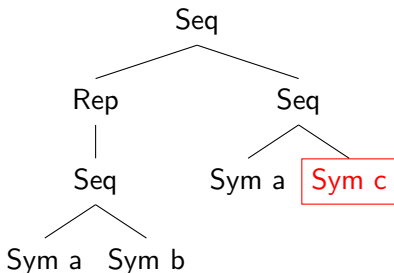
# Glushkov Automaton Construction

## Example - step 6

Let's match  $(a \cdot b)^* \cdot a \cdot c$  against "ababac"

"ababac"

$(a \cdot b)^* \cdot a \cdot$ c



# Glushkov Automaton Construction

## Haskell implementation

```
data Regex' a
  = Eps'                                -- Empty regex
  | Sym' Bool a                          -- Singleton regex
  | Seq' (Regex' a) (Regex' a)         -- Sequence
  | Alt' (Regex' a) (Regex' a)         -- Alternatives
  | Rep' (Regex' a)                      -- Repetition
deriving (Show, Eq)

-- Convenient constructor
sym' :: a → Regex' a
sym' c = Sym' False c
```



# Glushkov Automaton Construction

## Moving mark

$shift :: Eq\ a \Rightarrow Bool \rightarrow Regex'\ a \rightarrow a \rightarrow Regex'\ a$   
 $shift\ \_ \ Eps' \quad \quad \_ = Eps'$   
 $shift\ m\ (Sym'\ \_ \ c)\ c' = Sym'\ (m\ \&\&\ c == c')\ c$   
 $shift\ m\ (Alt'\ p\ q)\ c = Alt'\ (shift\ m\ p\ c)\ (shift\ m\ q\ c)$   
 $shift\ m\ (Seq'\ p\ q)\ c =$   
 $\quad Seq'\ (shift\ m\ p\ c)\ (shift\ (m\ \&\&\ empty\ p\ ||\ final\ p)\ q\ c)$   
 $shift\ m\ (Rep'\ p)\ c = Rep'\ (shift\ (m\ ||\ final\ p)\ p\ c)$

$empty :: Regex'\ a \rightarrow Bool$   
 $empty\ Eps' \quad \quad = True$   
 $empty\ (Sym'\ \_ \ \_) = False$   
 $empty\ (Alt'\ p\ q) = empty\ p\ ||\ empty\ q$   
 $empty\ (Seq'\ p\ q) = empty\ p\ \&\&\ empty\ q$   
 $empty\ (Rep'\ p)\ \quad = True$

# Glushkov Automaton Construction

Checking if mark is in the final position

$final :: Regex' a \rightarrow Bool$

$final\ Eps' = False$

$final\ (Sym'\ m\ \_) = m$

$final\ (Alt'\ p\ q) = final\ p \parallel final\ q$

$final\ (Seq'\ p\ q) = final\ p \ \&\&\ empty\ q \parallel final\ q$

$final\ (Rep'\ p) = final\ p$

# Glushkov Automaton Construction

## Matching

$accept' :: Eq\ a \Rightarrow Regex'\ a \rightarrow [a] \rightarrow Bool$

$accept' re [] = empty\ re$

$accept' re (c : cs) = final\ (foldl\ (shift\ False)\ (shift\ True\ re\ c)\ cs)$

# Glushkov Automaton Construction

## Matching

$accept' :: Eq\ a \Rightarrow Regex'\ a \rightarrow [a] \rightarrow Bool$   
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Let's try our previous test regexp  $a^* \cdot b \cdot (\epsilon \mid c)$

$re = Rep'\ (sym'\ A)\ 'Seq'\ sym'\ B\ 'Seq'\ Alt'\ Eps'\ (sym'\ C)$

Expected output

$accept'\ re\ [B] \Rightarrow True$

$accept'\ re\ [B, C] \Rightarrow True$

$accept'\ re\ [A, A, A, B] \Rightarrow True$

$accept'\ re\ [A, A, A, C] \Rightarrow False$

$accept'\ re\ [A, A, A, B, C] \Rightarrow True$

# Glushkov Automaton Construction

## Improving efficiency

It's costly to recompute *final* and *empty* every time. We can memoize them in expressions

```
data RegexMemo a
  = EpsMemo           -- Empty regex
  | SymMemo Bool a    -- Singleton regex
  | SeqMemo (R a) (R a) -- Sequence
  | AltMemo (R a) (R a) -- Alternatives
  | RepMemo (R a)      -- Repetition
deriving (Show, Eq)

data R a = R
  { rEmpty :: Bool
  , rFinal  :: Bool
  , rRegex  :: RegexMemo a
  } deriving (Show, Eq)
```

# Glushkov Automaton Construction

## Improving efficiency

Having defined *RegexMemo* and *R*, we can define smart constructors and we'll only need to replace all explicit constructors with smart constructors in function *shift* to get the benefit.

E.g.

$$reEps :: R\ a$$
$$reEps = R$$
$$\begin{aligned} &\{ rEmpty = True \\ &\quad , rFinal = False \\ &\quad , rRegex = EpsMemo \\ &\} \end{aligned}$$
$$reAlt :: R\ a \rightarrow R\ a \rightarrow R\ a$$
$$reAlt\ p\ q = R$$
$$\begin{aligned} &\{ rEmpty = rEmpty\ p \parallel rEmpty\ q \\ &\quad , rFinal = rFinal\ p \parallel rFinal\ q \\ &\quad , rRegex = AltMemo\ p\ q \\ &\} \end{aligned}$$

## One cool laziness trick

Laziness allows matching strictly more expressive languages. Even some context-sensitive ones

To make our regex lazy we'll have to add another field that tracks, whether the expression was active. If it wasn't, we can avoid forcing it. This would allow us to operate infinite regexps, which can match some context-free languages and even some context-sensitive ones.

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Let's match language of parentheses  $\{\epsilon, (), (()), ()(), (())(), \dots\}$

Our alphabet is  $\Sigma = \{ (, ) \}$

```
data Parens = LParen | RParen deriving (Show, Eq)
```

Now make regex that matches this language

```
let re = (symL LParen 'seqL' re 'seqL' symL RParen) 'seqL' re in re
```



# One cool laziness trick

Sample run

```
let re = (symL LParen 'seqL' re 'seqL' symL RParen) 'seqL' re in re
```

# One cool laziness trick

## Sample run

**let** *re* = (*symL LParen 'seqL' re 'seqL' symL RParen*) *'seqL' re* **in** *re*

Try it out

*acceptLazy re [LParen, RParen] ⇒ True*

*acceptLazy re [LParen, RParen, LParen, RParen] ⇒ True*

*acceptLazy re [LParen, RParen, LParen, RParen, RParen] ⇒ False*

Thank you!

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Questions?