

Applied Mechanics Workbook  
Supplementary Exercises for In-Class Learning

Serhat Beyenir

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# Preface

This workbook presents a collection of lecture notes on Applied Mechanics, designed to provide learners with succinct yet essential insights into key topics covered in class. Each chapter is accompanied by a problem set to facilitate comprehension and reinforce understanding.

Chapter 1: The International System of Units (SI) is the globally accepted standard for measurement. Established to provide a consistent framework for scientific and technical measurements, SI units facilitate clear communication and data comparison across various fields and countries. The system is based on seven fundamental units: the meter for length, the kilogram for mass, the second for time, the ampere for electric current, the kelvin for temperature, the mole for substance, and the candela for luminous intensity.

Chapter 2: In mathematics and physics, a **scalar** is a quantity with only magnitude (size), whereas a **vector** has both magnitude and direction. Examples of scalar quantities include numbers, mass, speed, temperature, volume, and time. In contrast, examples of vector quantities include velocity, acceleration, and forces like weight and friction.

Chapter 3: A **moment** is a mathematical concept that represents the product of a distance and a physical quantity, such as force. Moments are typically defined relative to a fixed reference point and pertain to physical quantities situated at some distance from this point. For instance, the moment of force, commonly referred to as torque, is the product of the force applied to an object and the distance from the reference point to the object.

Chapter 4: Review section



# Chapter 1

## International System of Units

### 1.1 Objectives

- Recall the base and derived units.
- Practice the application of unity fraction.

### 1.2 SI Units

The International System of Units (SI) is the globally accepted standard for measurement. Established to provide a consistent framework for scientific and technical measurements, SI units facilitate clear communication and data comparison across various fields and countries. The system is based on seven fundamental units: the meter for length, the kilogram for mass, the second for time, the ampere for electric current, the kelvin for temperature, the mole for substance, and the candela for luminous intensity.

Table 1.1: Base SI units.

Physical Quantity	SI Base Unit	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric Current	Ampere	A
Temperature	Kelvin	K
Amount of Substance	Mole	mol
Luminous Intensity	Candela	cd

Table 1.2: Derived SI units.

Physical Quantity	Derived SI Unit	Symbol
Area	Square meter	m <sup>2</sup>
Volume	Cubic meter	m <sup>3</sup>
Speed	Meter per second	m/s
Acceleration	Meter per second squared	m/s <sup>2</sup>
Force	Newton	N
Pressure	Pascal	Pa
Energy	Joule	J
Power	Watt	W
Electric Charge	Coulomb	C
Electric Potential	Volt	V
Resistance	Ohm	$\Omega$
Capacitance	Farad	F
Frequency	Hertz	Hz
Luminous Flux	Lumen	lm
Illuminance	Lux	lx
Specific Energy	Joule per kilogram	J/kg
Specific Heat Capacity	Joule per kilogram Kelvin	J/(kg · K)

Table 1.3: Common multiples and submultiples for SI units.

Factor	Prefix	Symbol
$10^9$	giga	G
$10^6$	mega	M
$10^3$	kilo	k
$10^2$	hecto	h
$10^1$	deca	da
$10^{-1}$	deci	d
$10^{-2}$	centi	c
$10^{-3}$	milli	m
$10^{-6}$	micro	$\mu$

1.3 SI System Rules and Common Mistakes

Using the SI system correctly is crucial for clear communication in science and engineering. Below are common mistakes in using the SI system, examples of incorrect usage, and how to correct them.

Table 1.4: SI system rules and common mistakes

Concept	Mistake	Correct Usage	Notes
Use of SI Unit Symbols	m./s	m/s	Use the correct format without additional punctuation.
Spacing Between Value & Unit	10kg	10 kg	Always leave a space between the number and the unit symbol.
Incorrect Unit Symbols	sec, hrs, °K	s, h, K	Use the proper SI symbols; symbols are case-sensitive.
Abbreviations for Units	5 kilograms (kgs)	5 kilograms (kg)	Avoid informal abbreviations like “kgs”; adhere to standard symbols.
Multiple Units in Expressions	5 m/s/s, 5 kg/meter <sup>2</sup>	5 m/s <sup>2</sup> , 5 kg/m <sup>2</sup>	Use compact, standardized formats for derived units.

Concept	Mistake	Correct Usage	Notes
<b>Incorrect Use of Prefixes</b>	0.0001 km	100 mm	Choose prefixes to keep numbers in the range (0.1 x < 1000).
<b>Misplaced Unit Symbols</b>	5/s, kg10	5 s <sup>-1</sup> , 10 kg	Symbols must follow numerical values, not precede them.
<b>Degrees Celsius vs. Kelvin</b>	300°K	300 K	Kelvin is written without “degree”
<b>Singular vs. Plural Units</b>	5 kgs, 1 meters	5 kg, 1 meter	Symbols do not pluralize; full unit names follow grammar rules.
<b>Capitalization of Symbols</b>	Kg, S, Km, MA	kg, s, km, mA	Symbols are case-sensitive; use uppercase only where specified (e.g., N, Pa).
<b>Capitalization of Unit Names</b>	Newton, Pascal, Watt	newton, pascal, watt	Unit names are lowercase, even if derived from a person’s name, unless starting a sentence.
<b>Prefix Capitalization</b>	MilliMeter, MegaWatt	millimeter, megawatt	Prefixes are lowercase for (10 <sup>-1</sup> ) to (10 <sup>-9</sup> ), uppercase for (10 <sup>6</sup> ) and larger (except k for kilo).
<b>Formatting in Reports</b>	5, Temperature: 300	5 kg, Temperature: 300 K	Always specify units explicitly.

## 1.4 Unity Fraction

The **unity fraction** method, or **unit conversion using unity fractions**, is a systematic way to convert one unit of measurement into another. This method

relies on multiplying by fractions that are equal to one, where the numerator and the denominator represent the same quantity in different units. Since any number multiplied by one remains the same, unity fractions allow for seamless conversion without changing the value.

The principle of unity fractions is based on:

1. **Setting up equal values:** Write a fraction where the numerator and denominator are equivalent values in different units, so the fraction equals one. For example,  $\frac{1\text{ km}}{1000\text{ m}}$  is a unity fraction because 1 km equals 1000 m.
2. **Multiplying by unity fractions:** Multiply the initial quantity by the unity fraction(s) so that the undesired units cancel out, leaving only the desired units.

## 1.5 Classwork

**Example 1.1.** Suppose we want to convert 5 kilometers to meters.

1. Start with 5 kilometers:

$$5 \text{ km}$$

2. Multiply by a unity fraction that cancels kilometers and introduces meters.  
We use  $(\frac{1000\text{ m}}{1\text{ km}})$ , since 1 km = 1000 m:

$$5 \text{ km} \times \frac{1000 \text{ m}}{1 \text{ km}} = 5000 \text{ m}$$

3. The kilometers km cancel out, leaving us with meters m:

$$5 \text{ km} = 5000 \text{ m}$$

This step-by-step approach illustrates how the unity fraction cancels the undesired units and achieves the correct result in meters.

Unity fractions can be extended by using multiple conversion steps. For example, converting hours to seconds would require two unity fractions: one to convert hours to minutes and another to convert minutes to seconds. This approach ensures accuracy and is widely used in science, engineering, and other fields that require precise unit conversions.

**Example 1.2.** Convert 15 m/s to km/h.

1. Start with 15 m/s.
2. To convert meters to kilometers, multiply by  $\frac{1\text{ km}}{1000\text{ m}}$ .
3. To convert seconds to hours, multiply by  $\frac{3600\text{ s}}{1\text{ h}}$ .

$$15 \text{ m/s} \times \frac{1 \text{ km}}{1000 \text{ m}} \times \frac{3600 \text{ s}}{1 \text{ h}} = 54 \text{ km/h}$$

The meters and seconds cancel out, leaving kilometers per hour: 54 km/h.

## 1.6 Problem Set

### Instructions:

1. Use unity fraction to convert between derived SI units.
  2. Show each step of your work to ensure accuracy.
  3. Simplify your answers and include correct units.
- 

1. **Speed**

Convert 72 km/h to m/s.

2. **Force**

Convert 980 N (newtons) to  $\text{kg} \cdot \text{m/s}^2$ .

3. **Energy**

Convert 2500 J (joules) to kJ.

4. **Power**

Convert 1500 W (watts) to kW.

5. **Pressure**

Convert 101325 Pa (pascals) to kPa.

6. **Volume Flow Rate**

Convert  $3 \text{ m}^3/\text{min}$  to L/s.

7. **Density**

Convert  $1000 \text{ kg/m}^3$  to  $\text{g/cm}^3$ .

8. **Acceleration**

Convert  $9.8 \text{ m/s}^2$  to  $\text{cm/s}^2$ .

9. **Torque**

Convert  $50 \text{ N} \cdot \text{m}$  to  $\text{kN} \cdot \text{cm}$ .

10. **Frequency**

Convert 500 Hz (hertz) to kHz.

11. **Work to Energy Conversion**

A force of 20 N moves an object 500 cm. Convert the work done to joules.

12. **Kinetic Energy Conversion**

Calculate the kinetic energy in kilojoules of a 1500 kg car moving at 72 km/h.



**13. Power to Energy Conversion**

A machine operates at 2 kW for 3 hours. Convert the energy used to megajoules.

**14. Pressure to Force Conversion**

Convert a pressure of 200 kPa applied to an area of  $0.5 \text{ m}^2$  to force in newtons.

**15. Density to Mass Conversion**

Convert  $0.8 \text{ g/cm}^3$  for an object with a volume of  $250 \text{ cm}^3$  to mass in grams.

---

**1.6.1 Answer Key**

1.  $72 \text{ km/h} = 20 \text{ m/s}$
2.  $980 \text{ N} = 980 \text{ kg} \cdot \text{m/s}^2$
3.  $2500 \text{ J} = 2.5 \text{ kJ}$
4.  $1500 \text{ W} = 1.5 \text{ kW}$
5.  $101325 \text{ Pa} = 101.325 \text{ kPa}$
6.  $3 \text{ m}^3/\text{min} = 50 \text{ L/s}$
7.  $1000 \text{ kg/m}^3 = 1 \text{ g/cm}^3$
8.  $9.8 \text{ m/s}^2 = 980 \text{ cm/s}^2$
9.  $50 \text{ N} \cdot \text{m} = 5 \text{ kN} \cdot \text{cm}$
10.  $500 \text{ Hz} = 0.5 \text{ kHz}$
11.  $20 \text{ N} \times 5 \text{ m} = 100 \text{ J}$
12. Kinetic energy  $= 1500 \text{ kg} \times (20 \text{ m/s})^2 / 2 = 300 \text{ kJ}$
13.  $2 \text{ kW} \times 3 \text{ hours} = 21.6 \text{ MJ}$
14.  $200 \text{ kPa} \times 0.5 \text{ m}^2 = 100,000 \text{ N}$
15.  $0.8 \text{ g/cm}^3 \times 250 \text{ cm}^3 = 200 \text{ g}$

**1.7 Further Reading**

Introduction in Russell, Jackson, and Embleton (2021) and SI units in Bolton (2021) for additional information.



## Chapter 2

# Scalar and Vector Quantities

### 2.1 Objectives

- Distinguish between scalar and vector quantities.
- Define equilibrium of an object.
- Practice vector problems.

### 2.2 Concepts

In mathematics and physics, a **scalar** is a quantity with only magnitude (size), whereas a **vector** has both magnitude and direction. Examples of scalar quantities include numbers, mass, speed, temperature, volume, and time. In contrast, examples of vector quantities include velocity, acceleration, and forces like weight and friction.

A vector is represented as an arrow, where the length denotes its magnitude, and the arrowhead indicates its direction. Vector diagrams are useful for visually conveying information about vector relationships, allowing us to analyze effects like the combination of forces or the movement of objects in two or three dimensions. In such diagrams, vectors are often drawn to scale and placed with respect to an origin or another reference point.

### 2.3 Coplanar Forces

Coplanar forces are forces that act within the same plane. This means they lie along a single, flat surface, and their lines of action do not extend out of that

plane. Coplanar forces are often analyzed together in physics and engineering because their combined effects can be resolved within two dimensions.

## 2.4 Addition of Vectors by Nose-to-tail Method

The “nose-to-tail” method, also known as the “head-to-tail”, is a visual way to add vectors in physics or engineering. Here’s a step-by-step explanation of how it works:

1. **Draw the First Vector:** Start by drawing the first vector on your graph or paper. This vector should have its tail (starting point) at the origin or any chosen starting point. Make sure you draw it with the correct direction and length to scale.
2. **Place the Second Vector:** Take the second vector and place its tail at the “head” (or “tip”) of the first vector. The direction and length should be accurately represented here as well.
3. **Continue with Additional Vectors:** If there are more vectors to add, repeat the process by placing each vector’s tail at the head of the previous one.
4. **Draw the Resultant Vector:** The resultant vector, or the sum of all the vectors, is the vector that goes directly from the “tail” of the first vector to the “head” of the last vector in the chain. Draw this vector as a straight line connecting the starting point to the endpoint of the last vector.
5. **Measure the Resultant:** The length and direction of this resultant vector represent the combined effect of all the original vectors.

## 2.5 Addition of Vectors by Parallelogram Method

The parallelogram method is another visual technique for adding two vectors, specifically used when you have two vectors that start from the same point. Here’s how it works:

1. **Draw Both Vectors:** Start by drawing the two vectors with their tails (starting points) at the same origin point. The direction and length of each vector should be accurate and to scale.
2. **Complete the Parallelogram:** Using the two vectors as adjacent sides, imagine a parallelogram where each vector is duplicated to form the opposite sides. Draw lines parallel to each vector from the head (tip) of the other vector to form a closed parallelogram.
3. **Draw the Resultant Vector:** The resultant vector, or the sum of the two vectors, is represented by the diagonal of the parallelogram that starts

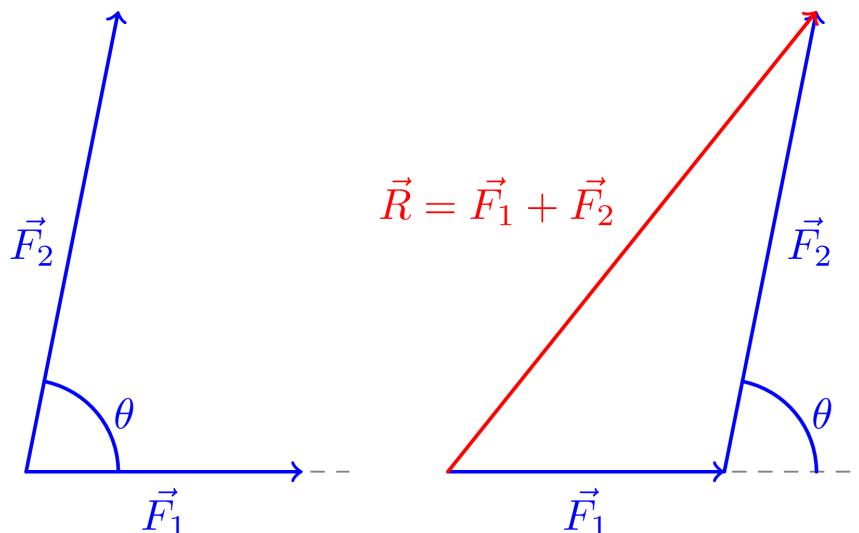


Figure 2.1: Addition of Vectors by Nose-to-tail Method

from the common origin point of the two vectors and goes to the opposite corner of the parallelogram.

4. Measure the Resultant: The length and direction of this diagonal vector represent the combined effect of the two vectors.

## 2.6 Cartesian Coordinates

In a two-dimensional Cartesian coordinate system, a point is represented as  $(x, y)$ , where:

- $x$  is the horizontal distance from the origin (*the  $x$  – coordinate*).
- $y$  is the vertical distance from the origin (*the  $y$  – coordinate*).

The distance  $r$  of a point  $(x, y)$  from the origin can be calculated using:

$$r = \sqrt{x^2 + y^2} \quad (2.1)$$

The angle  $\theta$  between the positive  $(x)$  – *axis* and the line joining the origin to the point can be found as:

$$\theta = \tan^{-1} \left( \frac{y}{x} \right) \quad (2.2)$$

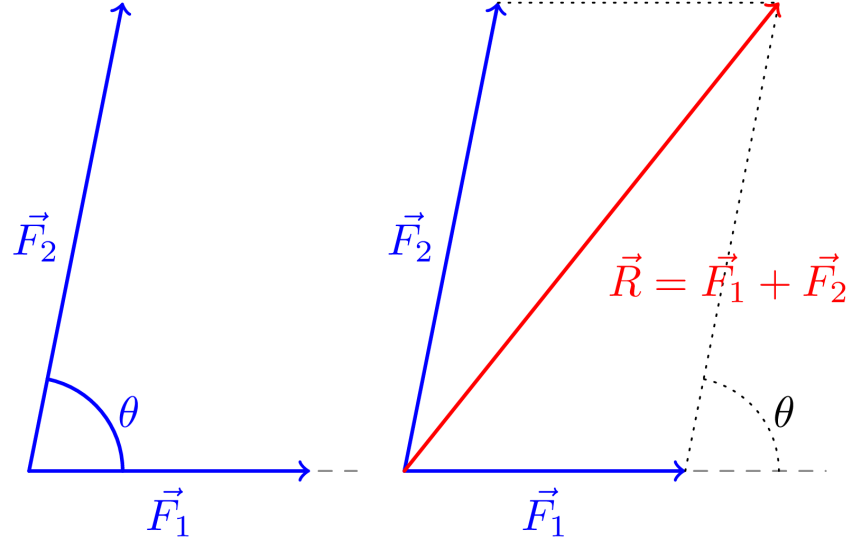


Figure 2.2: Addition of Vectors by Parallelogram Method

## 2.7 Polar Coordinates

In a two-dimensional polar coordinate system, a point is represented as  $(r, \theta)$ , where:

- $r$  is the radial distance from the origin.
- $\theta$  is the angle between the positive (x)-axis and the line joining the origin to the point (measured in radians or degrees).

The relationships between polar and Cartesian coordinates are:

$$x = r \cos \theta, \quad y = r \sin \theta \quad (2.3)$$

Conversely, given Cartesian coordinates  $((x, y))$ , the polar coordinates can be obtained as:

$$r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1} \left( \frac{y}{x} \right) \quad (2.4)$$

## 2.8 Cosine Rule

The Cosine Rule is used to relate the lengths of the sides of a triangle as shown in Figure 2.3 to the cosine of one of its angles:

$$c^2 = a^2 + b^2 - 2ab \cos \gamma \quad (2.5)$$

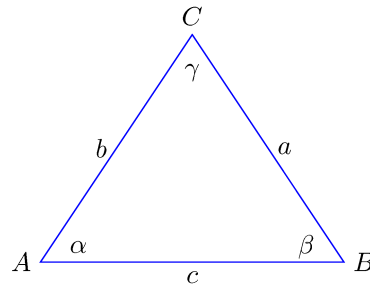


Figure 2.3: Rules of Cosine and Sine

Where:

- $a, b, c$  are the sides of the triangle.
- $\gamma$  is the angle opposite side  $c$ .

## 2.9 Sine Rule

The Sine Rule relates the sides and angles of a triangle in Figure 2.3:

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} \quad (2.6)$$

Where:

- $\alpha, \beta, \gamma$  are the angles of the triangle.
- $a, b, c$  are the sides of the triangle opposite to angles  $\alpha, \beta, \gamma$  respectively.

## 2.10 Space Diagrams

The space diagram is an illustration of the system of forces.

## 2.11 Vector Diagrams

The vector diagram is a diagram drawn to scale with the vectors joined end to end. Vector diagrams are used to analyze and combine forces to find the resultant.

### 2.11.1 Resultant

The resultant is a single force that represents the combined effect of two or more forces acting on an object. It has the same effect as applying all the original

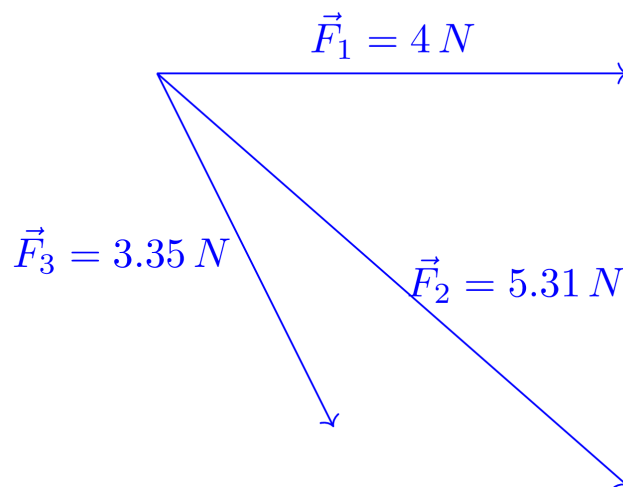


Figure 2.4: Space Diagram

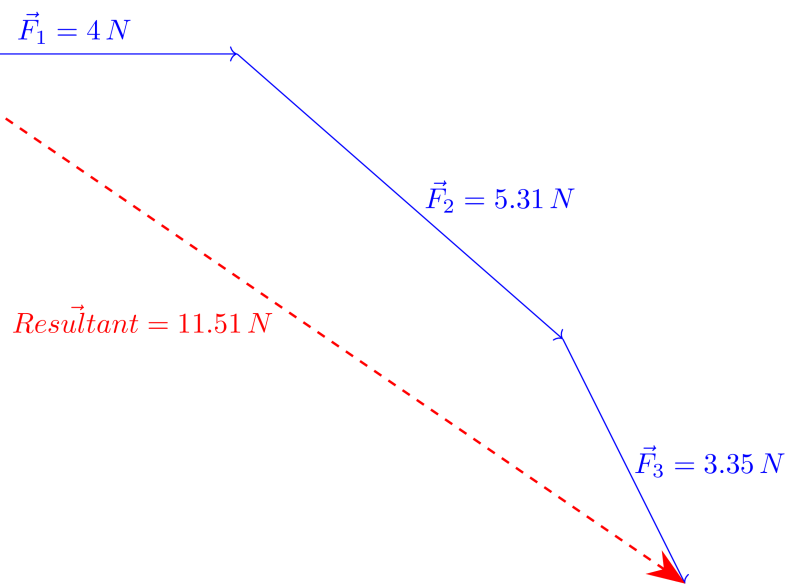


Figure 2.5: Vector Diagram



forces together and is found by adding the individual forces, taking both their magnitudes and directions into account. The resultant gives the overall direction and magnitude of the combined forces as shown in Figure 2.5.

### 2.11.2 Equilibrium

Equilibrium of an object occurs when all the forces acting on it are balanced, so the object remains at rest or moves at a constant speed in a straight line. In equilibrium, there is no net force or acceleration, meaning the object is in a stable state without any change in its motion.

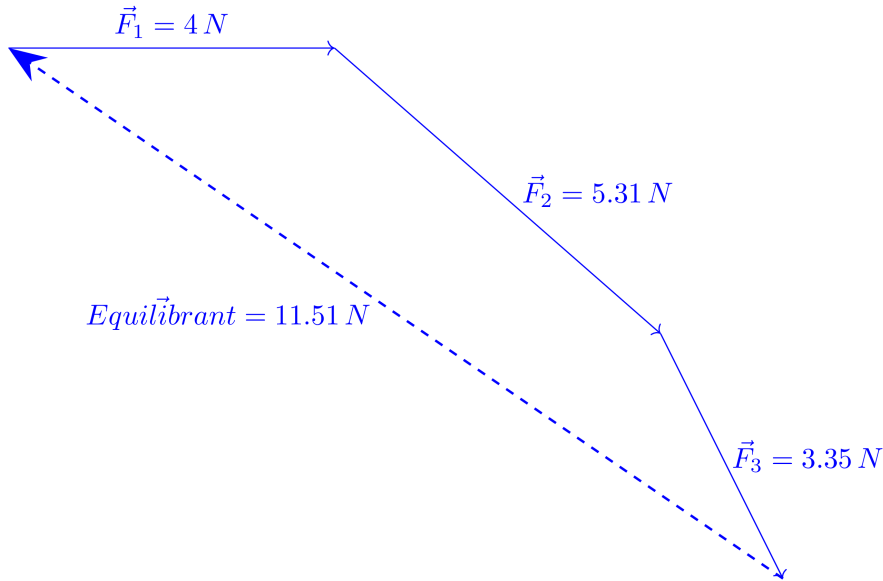


Figure 2.6: Equilibrant

Conditions of equilibrium:

1. Net force must be zero:

$$\sum_k \vec{F}_k = \vec{0} \quad (2.7)$$

2. Net torque must be zero:

$$\sum_k \vec{\tau}_k = \vec{0} \quad (2.8)$$

## 2.12 Bow's Notation

Bow's Notation is a graphical method employed in structural engineering to label the forces acting on a truss or structural framework. The notation was developed by British mathematician Robert Bow in the 19th century and continues to be a practical tool for visualizing and solving force equilibrium problems in structures.

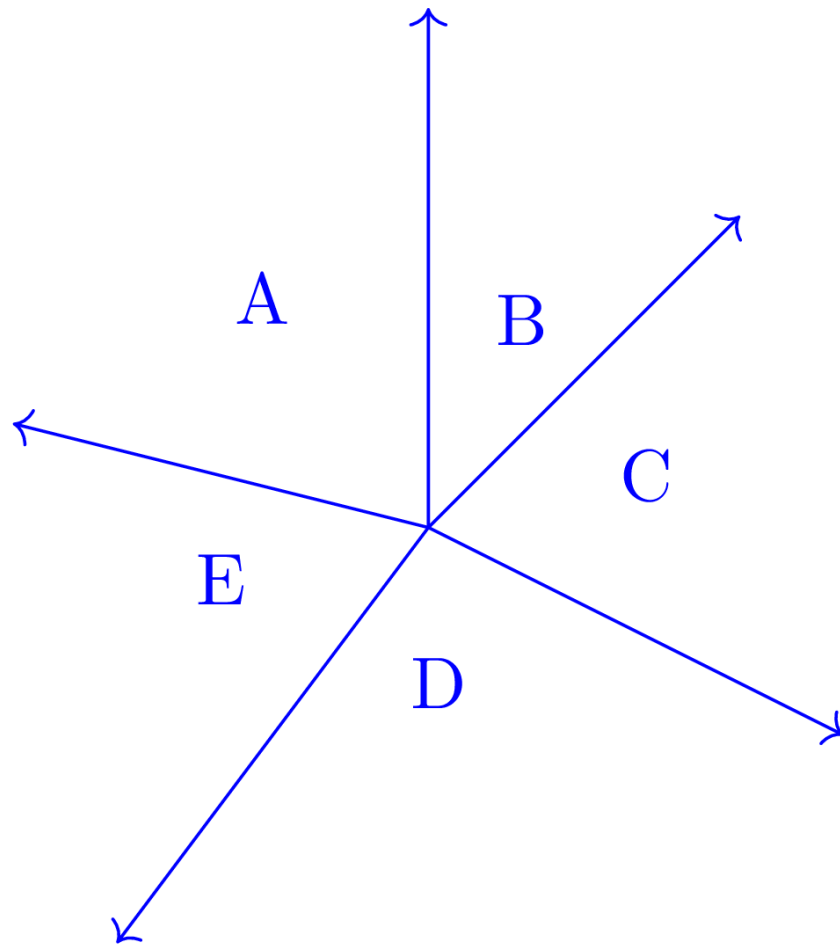


Figure 2.7: Bow's Notation Space Diagram

### 2.12.1 Key Concepts of Bow's Notation

1. Labeling Spaces Between Forces:

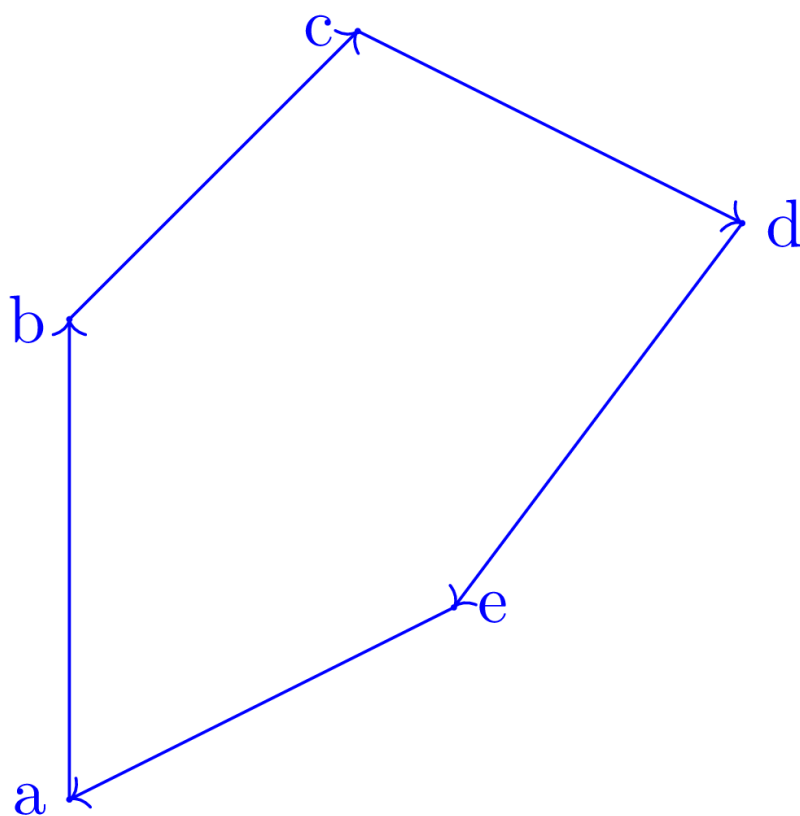


Figure 2.8: Bow's Notation Vector Diagram

- Each space between external forces (such as loads and reactions) and each internal member of a truss is designated with a unique letter (e.g., A, B, C, etc.).
  - These labels are assigned sequentially around the structure in a clockwise or counterclockwise direction.
2. Force Polygon:
    - Each labeled space corresponds to a point on a force polygon, which is a closed polygon representing the equilibrium of forces.
    - The lengths and directions of the sides of the force polygon reflect the magnitudes and directions of the forces in each member.
  3. Member Notation:
    - Each truss member is identified by the two letters representing the spaces it separates. For instance, a member between spaces A and B is labeled as AB.
  4. Utilizing Bow's Notation in Analysis:
    - After labeling the spaces, equilibrium equations and graphical methods (such as constructing a force polygon) can be employed to determine the magnitude and direction of each force in the truss.
    - The notation simplifies calculations by visually connecting the forces and facilitating the identification of how different forces interact within the structure.

### 2.12.2 Example of Bow's Notation Application

In a simple triangular truss, the spaces surrounding the triangle may be labeled A, B, and C. If the members connect these spaces, they would be named as AB, BC, and CA. By employing Bow's Notation, engineers can construct a force polygon, analyze it for equilibrium, and determine the unknown forces in each member of the truss.

## 2.13 Slings

A sling is a device or assembly of ropes, cables, or straps used to support and lift loads. Slings play a crucial role in rigging operations, allowing objects to be lifted, lowered, or moved safely and efficiently. Slings are arranged to distribute the load evenly across their length, reducing stress points and ensuring stability. In multi-leg slings (e.g., two-leg or four-leg), the load is shared among the sling legs, which helps stabilize and balance the load.

## 2.14 Jib Cranes

A simple jib crane has a vertical post, a jib, and a tie. The jib is hinged at its lower end to the post, and the tie connects the top of the jib to the base of the post, forming the crane head where the tie and jib meet.

When a load is hung directly from the crane head, solving for forces involves a simple triangle of forces. In other cases, the crane may have a pulley at the head, with a rope running over it to a winch, creating a system with more than three forces.

## 2.15 Classwork

**Example 2.1.** A vertical lifting force of 95 N is applied to a body, and simultaneously, a horizontal force of 135 N pulls on it. Determine the magnitude and direction of the resulting force.

To solve for the magnitude and direction of the resultant force, we can use vector addition.

Given:

- Vertical force,  $F_v = 95$  N
- Horizontal force,  $F_h = 135$  N

Step 1: Calculate the Magnitude of the Resultant Force

The resultant force  $F_r$  is the vector sum of the vertical and horizontal forces. Using the Pythagorean theorem:

$$F_r = \sqrt{F_v^2 + F_h^2}$$

Substituting the values:

$$F_r = \sqrt{95^2 + 135^2}$$

Calculating further:

$$F_r = \sqrt{9025 + 18225} = \sqrt{27250}$$

Thus,

$$F_r \approx 165.07 \text{ N}$$

Step 2: Determine the Direction of the Resultant Force

The direction  $\theta$  of the resultant force with respect to the horizontal can be found using the tangent function:

$$\theta = \arctan\left(\frac{F_v}{F_h}\right)$$

Substituting the values:

$$\theta = \arctan\left(\frac{95}{135}\right)$$

Calculating  $\theta$ :

$$\theta \approx 35.1341^\circ$$

### Final Answer

The magnitude of the resultant force is approximately 165.07N, and its direction is  $35.1341^\circ$  above the horizontal.

**Example 2.2.** Two forces act upon a body. One exerts a horizontal force to the right with a magnitude of 25 Newtons, while the other exerts a vertical force downward with a magnitude of 20 Newtons. Determine the magnitude and direction of a third force that would counteract the combined effects of the other two forces.

To determine the magnitude and direction of the third force that counteracts the combined effects of the two forces, we first need to find the resultant force of the two given forces.

Given Forces:

- Horizontal force to the right:  $F_h = 25 \text{ N}$
- Vertical force downward:  $F_v = 20 \text{ N}$

Step 1: Calculate the Resultant Force

The resultant force  $F_r$  can be found using the Pythagorean theorem since the forces are perpendicular to each other.

$$F_r = \sqrt{F_h^2 + F_v^2}$$

Substituting the values:

$$F_r = \sqrt{25^2 + 20^2}$$

Calculating:

$$F_r = \sqrt{625 + 400} = \sqrt{1025}$$

Thus,

$$F_r \approx 32.02 \text{ N}$$

Step 2: Determine the Direction of the Resultant Force

The direction  $\theta$  of the resultant force can be found using the tangent function:

$$\theta = \arctan\left(\frac{F_v}{F_h}\right)$$

Substituting the values:

$$\theta = \arctan\left(\frac{20}{25}\right)$$

Calculating  $\theta$ :

$$\theta \approx 38.66^\circ$$

This angle is measured from the horizontal axis (to the right) downward.

Step 3: Determine the Third Force

To counteract the resultant force, the third force  $F_3$  must have the same magnitude as  $F_r$  but in the opposite direction. Therefore, its magnitude is:

$$F_3 = F_r \approx 32.02 \text{ N}$$

The direction of the third force will be opposite to the direction of the resultant force, which means it will be directed at an angle of:

$$\theta + 180^\circ \approx 38.66^\circ + 180^\circ \approx 218.66^\circ$$

### Final Answer

The magnitude of the third force is approximately 32.02N , and its direction is approximately 218.66° (measured counterclockwise from the positive x-axis or horizontal right).

**Example 2.3.** Determine the magnitude and direction of the equilibrium force resulting from the combination of two forces: a horizontal pull of 15 N and a pull of 25 N at an angle of 55 degrees with respect to the 15 N force.

To find the magnitude and direction of the equilibrium force resulting from the combination of two forces (15 N horizontally and 25 N at an angle of 55 degrees), we can use vector addition.

Step 1: Resolve the Forces into Components

1. Force  $F_1 = 15 \text{ N}$  (Horizontal):

$$F_{1x} = 15 \text{ N}$$

$$F_{1y} = 0 \text{ N}$$

2. Force  $F_2 = 25 \text{ N}$  at  $55^\circ$ :

$$F_{2x} = F_2 \cdot \cos(55^\circ) = 25 \cdot \cos(55^\circ)$$

$$F_{2y} = F_2 \cdot \sin(55^\circ) = 25 \cdot \sin(55^\circ)$$

Step 2: Calculate Components of  $F_2$

Using  $\cos(55^\circ) \approx 0.5736$  and  $\sin(55^\circ) \approx 0.8192$ :

$$F_{2x} = 25 \cdot 0.5736 \approx 14.34 \text{ N}$$

$$F_{2y} = 25 \cdot 0.8192 \approx 20.48 \text{ N}$$

Step 3: Find the Resultant Components

Now, we sum the components in the x and y directions:

- Resultant x-component:

$$R_x = F_{1x} + F_{2x} = 15 + 14.34 \approx 29.34 \text{ N}$$

- Resultant y-component:

$$R_y = F_{1y} + F_{2y} = 0 + 20.48 \approx 20.48 \text{ N}$$

Step 4: Calculate the Magnitude of the Resultant Force

The magnitude of the resultant force  $R$  can be found using the Pythagorean theorem:

$$R = \sqrt{R_x^2 + R_y^2}$$

Calculating  $R$  :

$$R = \sqrt{(29.34)^2 + (20.48)^2} \approx \sqrt{861.64 + 419.04} \approx \sqrt{1280.68} \approx 35.8 \text{ N}$$

Step 5: Determine the Direction of the Resultant Force

The direction (angle  $\theta$  of the resultant force can be found using the tangent function:

$$\tan(\theta) = \frac{R_y}{R_x}$$



$$\theta = \arctan\left(\frac{20.48}{29.34}\right)$$

Calculating  $\theta$ :

$$\theta \approx \arctan(0.698) \approx 34.9159^\circ$$

### Final Answer

Magnitude of the Equilibrium Force  $\approx 35.8 \text{ N}$

Direction of the Equilibrium Force  $\approx 34.92^\circ$  above the horizontal (in the direction of the 15 N force).

**Example 2.4.** Calculate the resultant of the forces 0.7 kN at  $147^\circ$  and 1.3 kN at  $-71^\circ$  by using the cosine and sine rules.

Given the following forces:

- Force 1,  $F_1 = 0.7 \text{ kN}$ , at  $147^\circ$ ,
- Force 2,  $F_2 = 1.3 \text{ kN}$ , at  $-71^\circ$ .

The angle between the two forces is:

$$\theta = 147^\circ - (-71^\circ) = 147^\circ + 71^\circ = 218^\circ$$

Since  $(218^\circ > 180^\circ)$ , we subtract from  $360^\circ$  to get the smaller angle:

$$\theta = 360^\circ - 218^\circ = 142^\circ$$

Step 1: Using the Cosine Rule to Find the Resultant Magnitude

The magnitude of the resultant  $R$  is given by the cosine rule:

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2 \cos(\theta)}.$$

Substituting the known values:

$$R = \sqrt{(0.7)^2 + (1.3)^2 + 2(0.7)(1.3) \cos(142^\circ)}.$$

We calculate  $\cos 142^\circ$ :

$$\cos(142^\circ) = \cos(180^\circ - 38^\circ) = -\cos(38^\circ) \approx -0.788.$$

Now substitute into the formula for ( R ):

$$R = \sqrt{0.49 + 1.69 - 1.432}.$$

$$R = \sqrt{0.748} \approx 0.865 \text{ kN}.$$

Step 2: Using the Sine Rule to Find the Direction

The direction of the resultant  $\alpha$  relative to  $F_1$  is found using the sine rule:

$$\frac{\sin(\alpha)}{F_2} = \frac{\sin(\theta)}{R}.$$

We calculate  $\sin 142^\circ$ :

$$\sin(142^\circ) = \sin(180^\circ - 142^\circ) = \sin(38^\circ) \approx 0.616.$$

Now apply the sine rule:

$$\sin(\alpha) = \frac{1.3 \times 0.616}{0.865} \approx 0.926.$$

Thus,  $\alpha \approx 67.9^\circ$ .

The direction of the resultant relative to the positive x-axis is:

$$\text{Direction} = 147^\circ - 67.9^\circ = 79.1^\circ.$$

**Example 2.5.** A ship is heading in a direction N  $30^\circ$  E at a speed which in still water would be 20 knots. It is carried off course by a current of 5 knots in a direction of E  $60^\circ$  S. Calculate the ship's actual speed and direction.

**Solution:**

We need to resolve both the ship's velocity in still water and the current into their vector components (x and y) and then compute the resultant speed and direction.

**1. Ship's velocity in still water (N  $30^\circ$  E at 20 knots):**

- Direction: N  $30^\circ$  E means  $30^\circ$  clockwise from North.
- Speed: 20 knots.

The **x** (East-West) component is:

$$v_{x1} = 20 \times \cos(30^\circ) = 20 \times \frac{\sqrt{3}}{2} \approx 17.32 \text{ knots}$$

The **y** (North-South) component is:

$$v_{y1} = 20 \times \sin(30^\circ) = 20 \times \frac{1}{2} = 10 \text{ knots}$$

**2. Current's velocity (E 60° S at 5 knots):**

- Direction: E 60° S means 60° South of East.
- Speed: 5 knots.

The **x** (East-West) component is:

$$v_{x2} = 5 \times \cos(60^\circ) = 5 \times \frac{1}{2} = 2.5 \text{ knots}$$

The **y** (North-South) component is:

$$v_{y2} = 5 \times \sin(60^\circ) = 5 \times \frac{\sqrt{3}}{2} \approx 4.33 \text{ knots}$$

Since the current is pushing the ship southward, the y-component will be **negative**:

$$v_{y2} = -4.33 \text{ knots}$$

**3. Add the components:**

Total **x** component:

$$v_x = v_{x1} + v_{x2} = 17.32 + 2.5 = 19.82 \text{ knots}$$

Total **y** component:

$$v_y = v_{y1} + v_{y2} = 10 - 4.33 = 5.67 \text{ knots}$$

**4. Calculate the actual speed and direction:**

- The **actual speed** is the magnitude of the total velocity vector:

$$\text{Speed} = \sqrt{v_x^2 + v_y^2} = \sqrt{19.82^2 + 5.67^2} = \sqrt{393.39 + 32.14} = \sqrt{425.53} \approx 20.62 \text{ knots}$$

- The **actual direction** is the angle relative to East (measured counter-clockwise from the positive x-axis):

$$\theta = \tan^{-1} \left( \frac{v_y}{v_x} \right) = \tan^{-1} \left( \frac{5.67}{19.82} \right) \approx \tan^{-1}(0.286) \approx 16.0^\circ$$

So, the ship's actual direction is **16.0° North of East**.

Final Answer, see Figure 2.9:

- **Speed:** 20.62 knots
- **Direction:** N 16.0° E

**Example 2.6.** Given the space diagram in Figure 2.10, estimate the resultant vector using a graphical method.

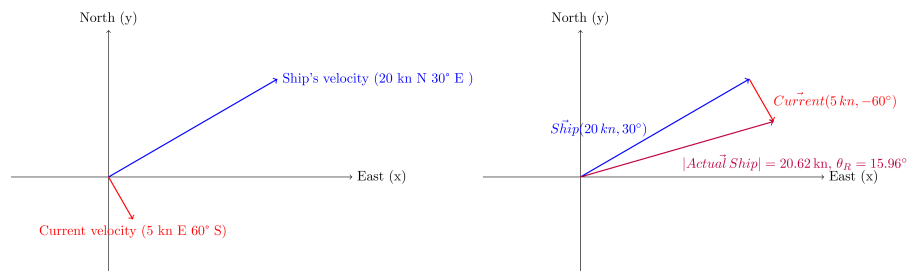


Figure 2.9: Ship vs Current diagram

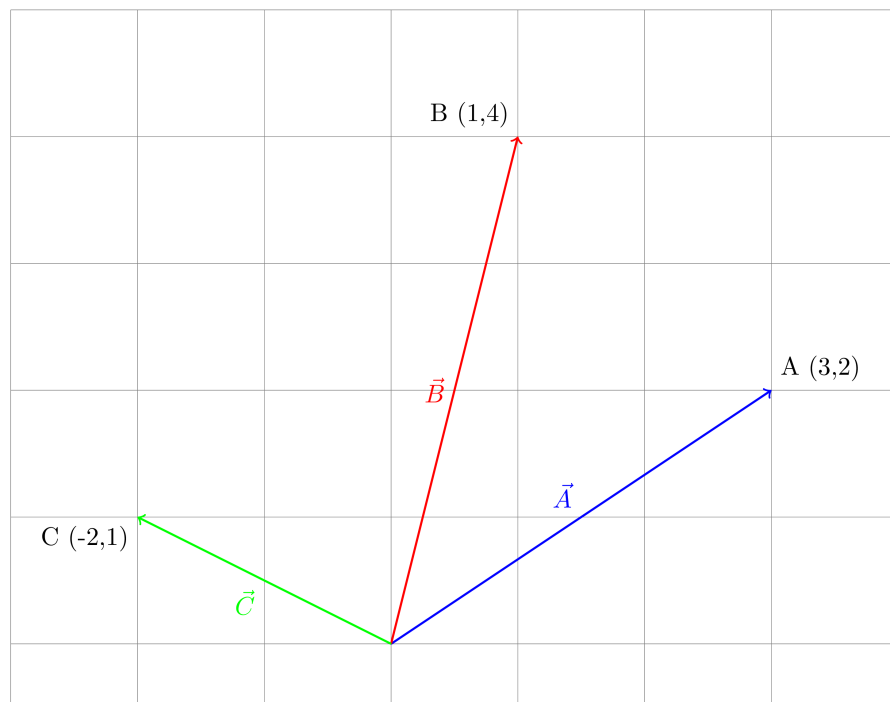


Figure 2.10: Vector Cartesian coordinates

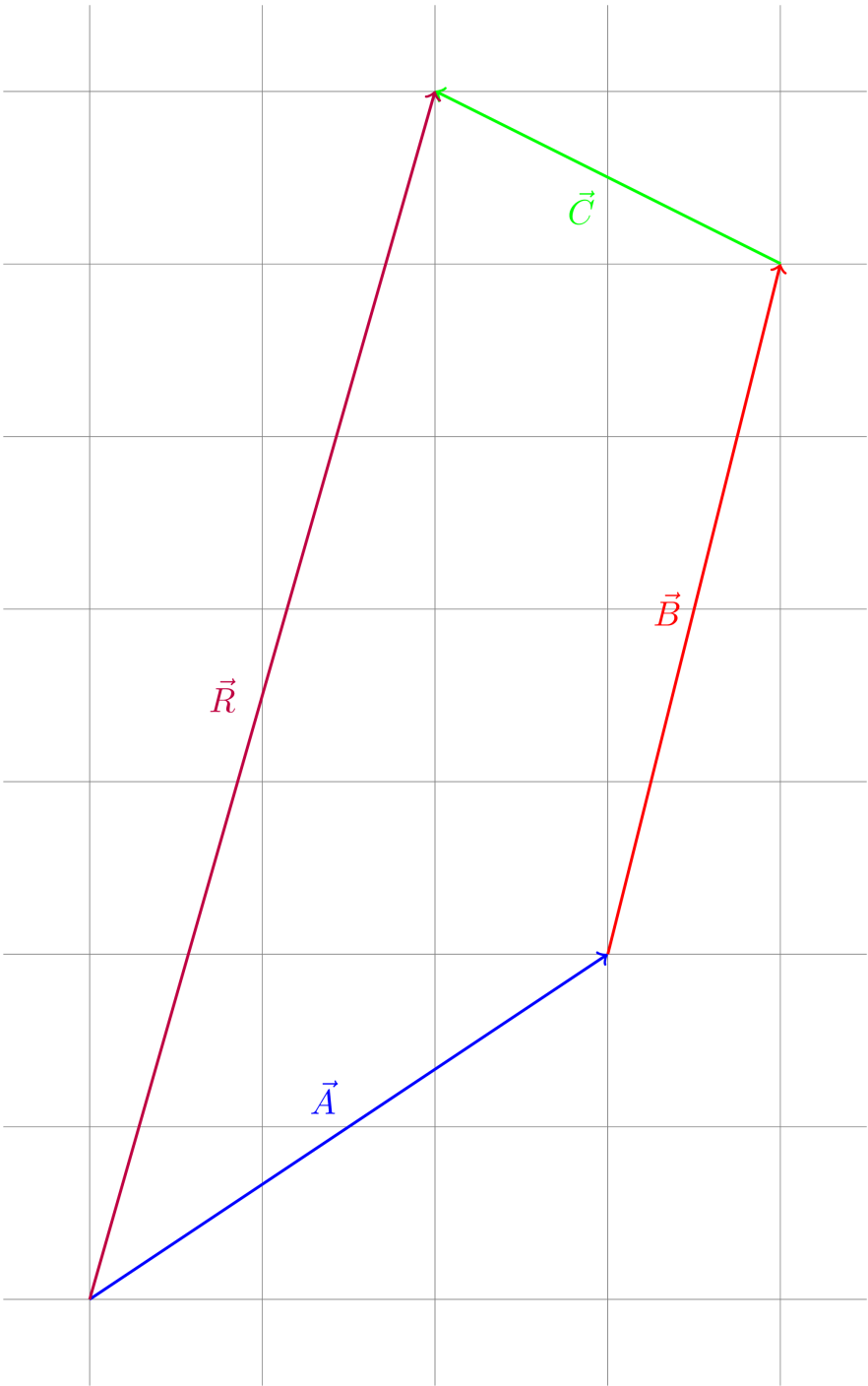


Figure 2.11: Solution

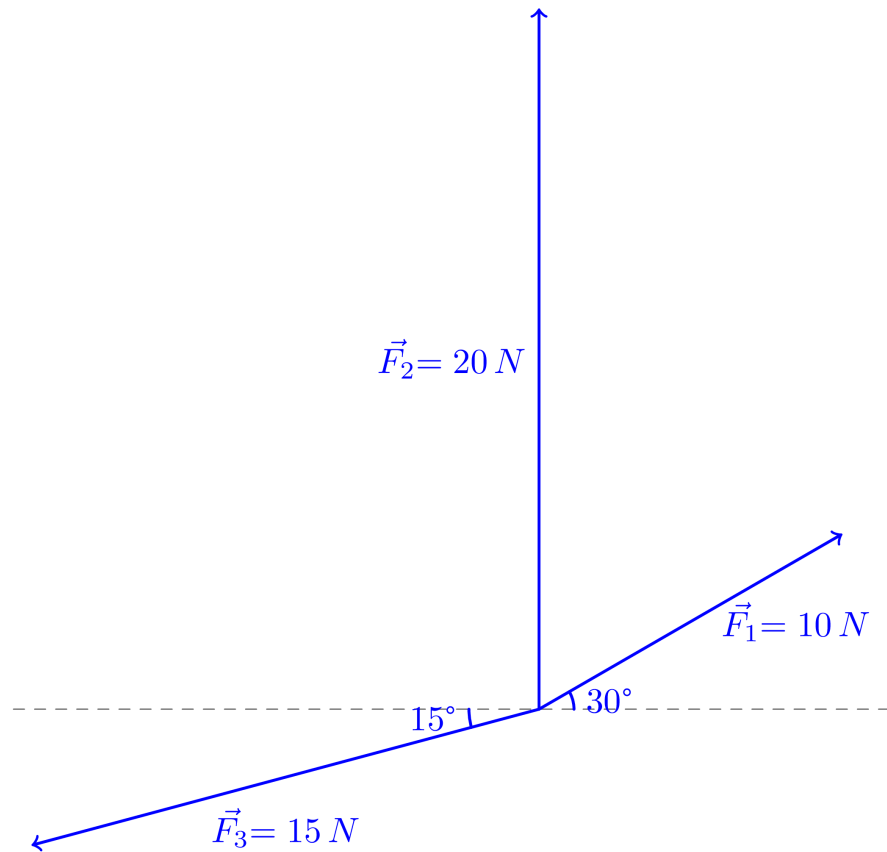


Figure 2.12: Vector polar coordinates

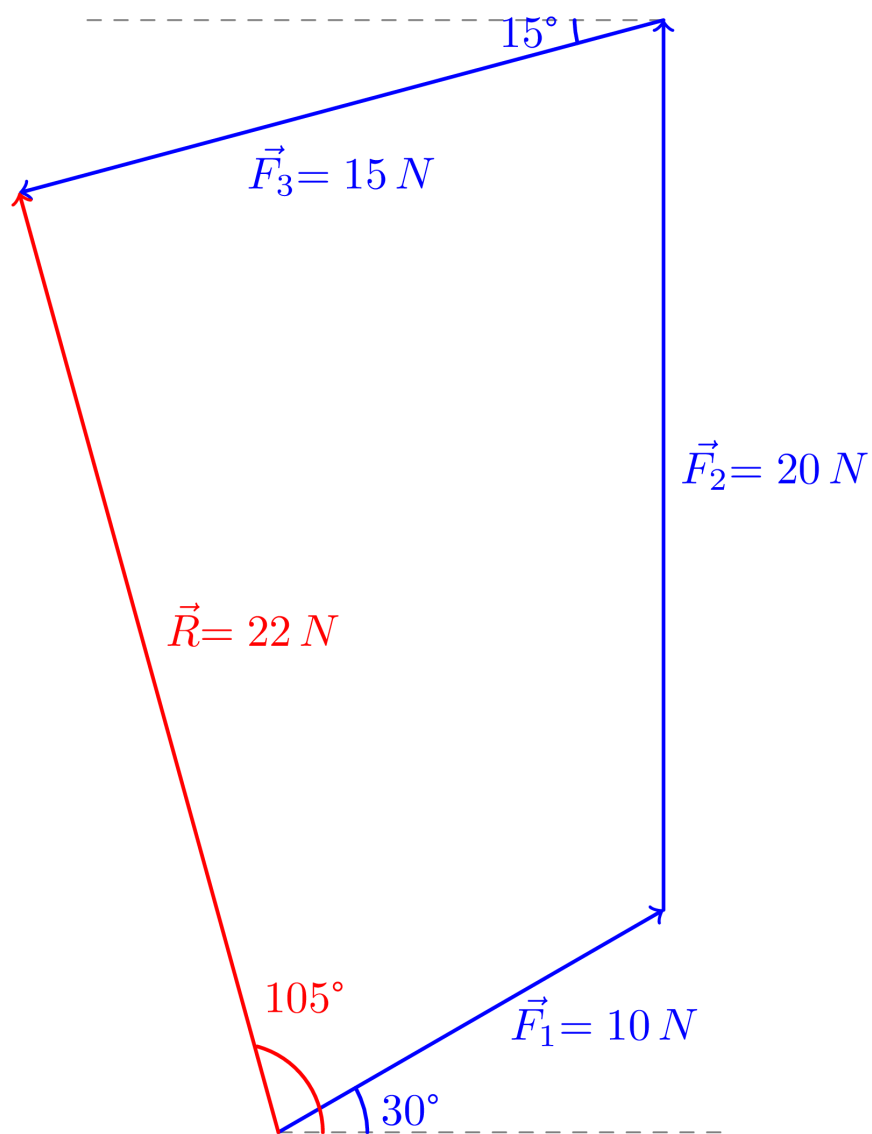


Figure 2.13: Solution

**Example 2.7.** Given the space diagram in Figure 2.12, estimate the resultant vector using a graphical method.

**Example 2.8.** Three forces of 5 N, 8 N and 13 N act as shown in Figure 2.14. Calculate the magnitude and direction of the resultant force.

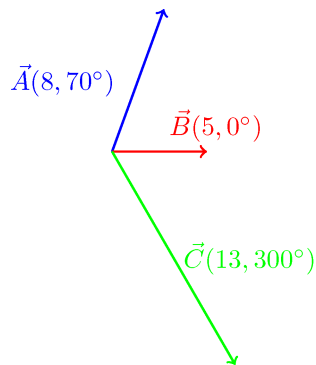


Figure 2.14

Given the polar coordinates, solution is shown in Figure 2.15,

**Example 2.9.** If velocity  $v_1 = 25$  m/s at  $60^\circ$  and  $v_2 = 15$  m/s at  $330^\circ$ , calculate the magnitude and direction of resultant velocity.

Graphical solution is shown in Figure 2.16.

**Example 2.10.** Calculate the magnitude and direction of the resultant vector of the force system shown in Figure 2.17

Solution is shown in Figure 2.18,

**Example 2.11.** Calculate the magnitude and direction of the resultant vector of the system shown Figure 2.19

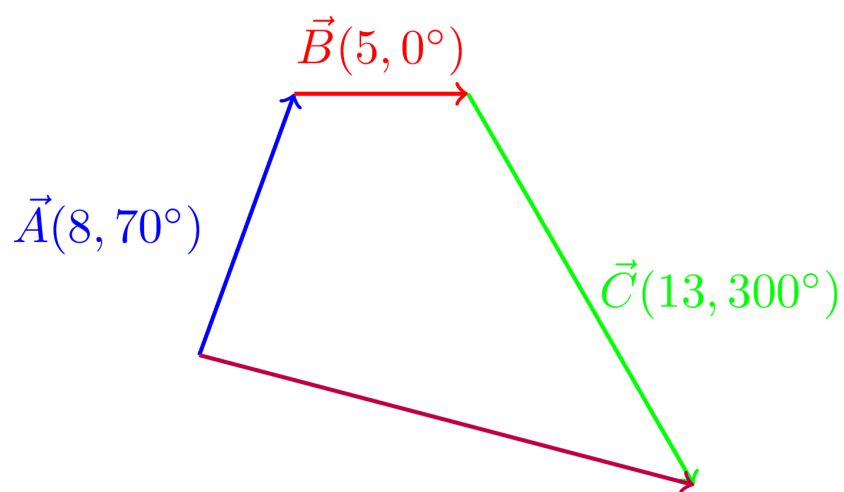
Solution shown in Figure 2.20,

**Example 2.12.** A ship heads in a direction of E  $20^\circ$  S at a speed of 20 knots while the current is 4 knots in a direction of N  $30^\circ$  E. Determine the speed and actual direction of the ship. Solution shown in Figure 2.21,

**Example 2.13.** A 200 kg mass is suspended as shown in Figure 2.22. Rope A is attached to a beam in two places and is passed through a ring that rest naturally at the centre of rope A. Rope B is attached to the bottom of the ring, and to the 200 kg mass.

- What is the tension in rope B?
- Draw a free body diagram of the forces exerted on the ring.
- What is the tension in rope A?





$$|\vec{R}| = 14.71947, \theta_R = -14.72276^\circ$$

Figure 2.15: Solution

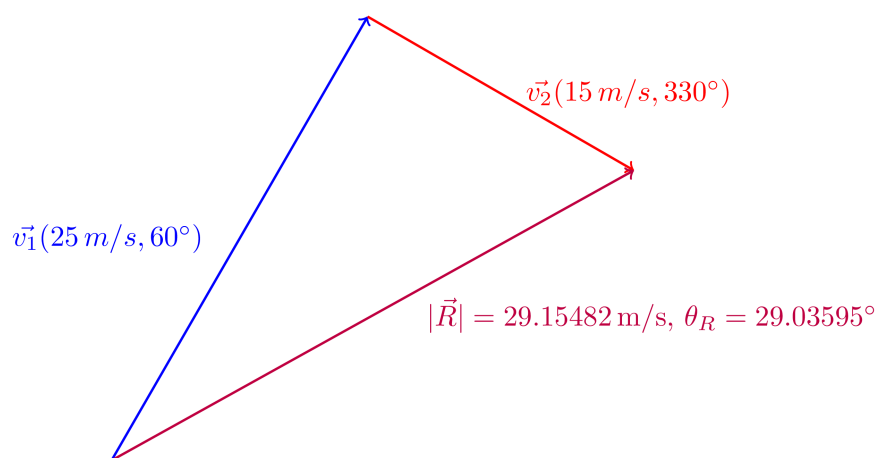


Figure 2.16: Graphical solution.

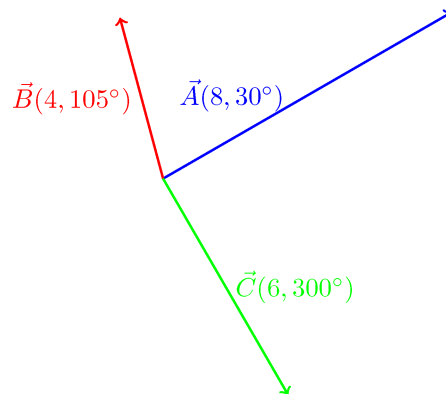


Figure 2.17

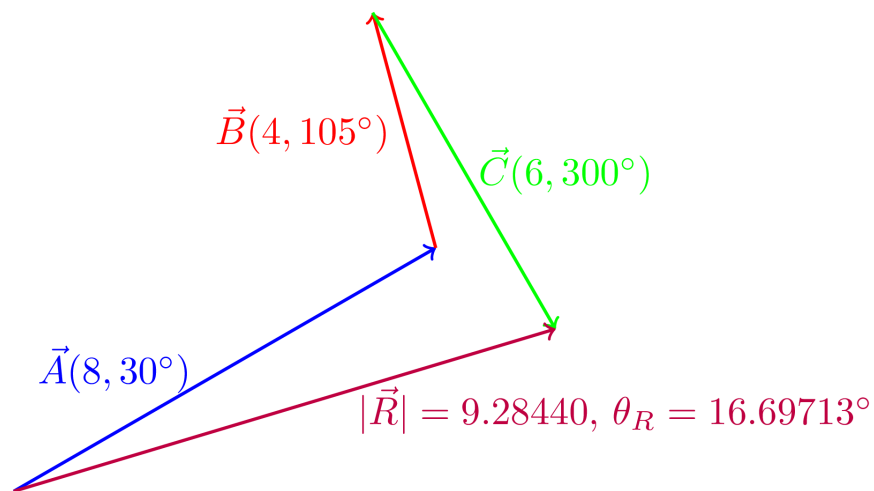


Figure 2.18: Solution

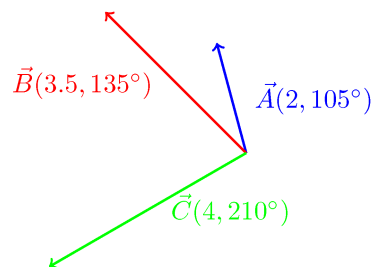


Figure 2.19: Vectors

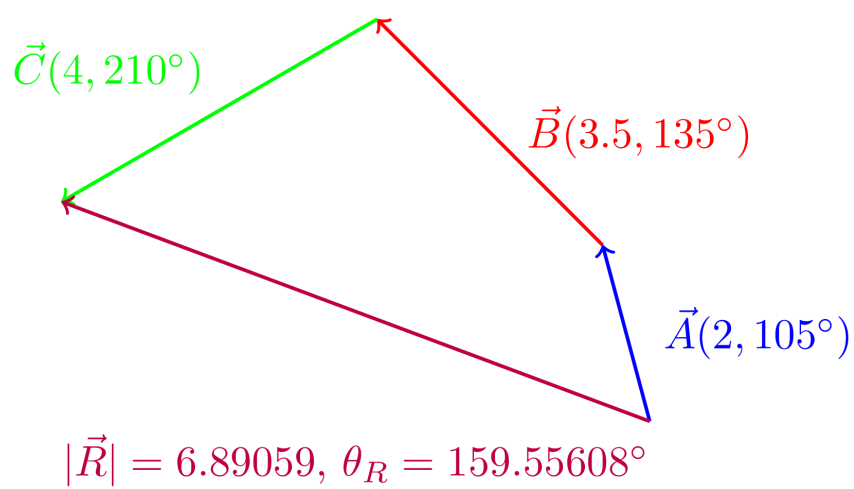


Figure 2.20: Solution

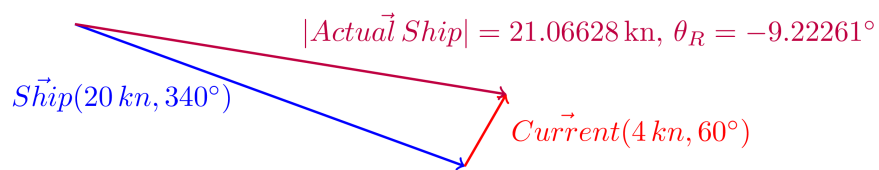


Figure 2.21: Solution

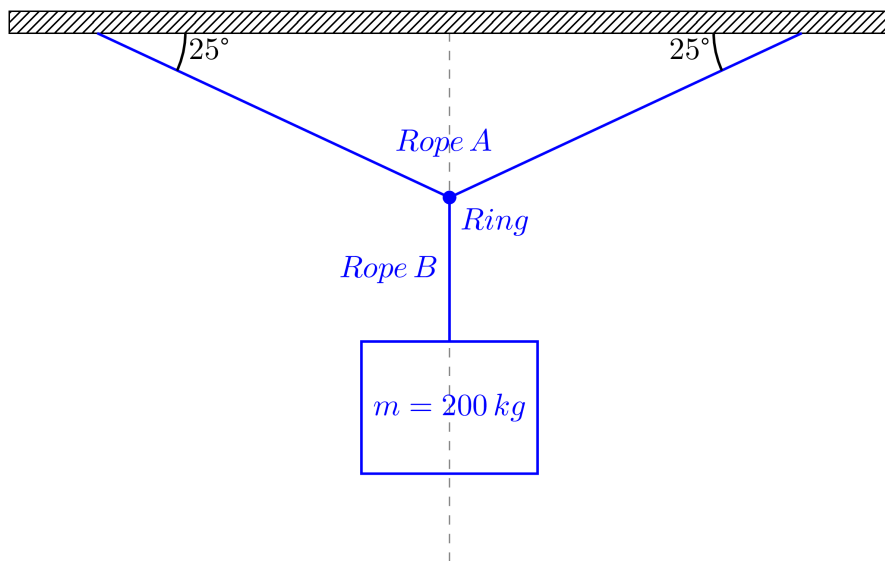


Figure 2.22: A 200 kg suspended mass.

Given Information

- Mass  $m = 200 \text{ kg}$
- Gravitational acceleration  $g = 9.81 \text{ m/s}^2$
- Angle  $\theta = 25^\circ$

a. Tension in Rope B

The tension  $T_B$  in Rope B must support the entire weight of the mass. Therefore,

$$T_B = m \cdot g = 200 \times 9.81 = 1962 \text{ N}$$

So, **the tension in Rope B is 1962 N.**

b. Free Body Diagram of the Ring

The ring is in equilibrium, meaning the net force acting on it is zero. Here's a breakdown of the forces:

- **Tension**  $T_B$ : Acts downward, equal to the weight of the 200 kg mass.
- **Tensions**  $T_{A1}$  and  $T_{A2}$ : These are the tensions in each side of Rope A. Since the ring is at the center and the setup is symmetrical,  $T_{A1} = T_{A2} = T_A$ .

Since the ring is in equilibrium, the vertical components of  $T_{A1}$  and  $T_{A2}$  must balance the downward force from  $T_B$ .

## c. Tension in Rope A

Since the angle between each side of Rope A and the horizontal is  $\theta = 25^\circ$ , we can use trigonometry to find  $T_A$ .

The equilibrium condition is:

$$2 \cdot T_A \cdot \sin(\theta) = T_B$$

Solving for  $T_A$ :

$$T_A = \frac{T_B}{2 \sin(\theta)}$$

Substitute the values:

$$T_A = \frac{1962}{2 \cdot \sin(25^\circ)}$$

Calculating:

$$T_A \approx 2321.24 \text{ N}$$

So, the tension in Rope A is approximately **2321.24 N**.

## 2.16 Problem Set

1. Calculate the magnitude and direction of the resultant of the 1.7 kN and 2.9 kN forces, which are aligned along the same line and act in the same direction.
2. Calculate the magnitude and direction of the resultant of the 457 N and 583 N forces, which are aligned along the same line but act in opposite directions.
3. Use the triangle of forces method to find the magnitude and direction of the resultant force from a 14 N force acting at  $0^\circ$  and a 23 N force acting at  $35^\circ$ .

---

### 2.16.1 Answer Key

1. Magnitude of the resultant: 4.6 kN. Direction: Same as the direction of the individual forces.
2. Magnitude of the resultant: 126 N. Direction: Same as the direction of the 583 N force.

3. The magnitude of the resultant force is approximately 35.39 N, and its direction is approximately  $21.80^\circ$  above the positive x-axis.

## 2.17 Further Reading

Read Chapter 1 in Russell, Jackson, and Embleton (2021), Chapter 1 in Hannah and Hillier (1995) and Chapter 2 in Bolton (2021) for additional exercises.

## Chapter 3

# Moment of a Force

### 3.1 Objectives

- Recall.
- Practice the application.

### 3.2 Concepts

### 3.3 Classwork

### 3.4 Problem Set

### 3.5 Further Reading

Read Chapter 7 in Russell, Jackson, and Embleton (2021), Chapter 1 in Hannah and Hillier (1995) and Chapter 2 in Bolton (2021) for additional information.





## Chapter 4

# Review

We have used several books by Ahrens (2022), Russell, Jackson, and Embleton (2021), Bolton (2021), Polya and Conway (2014), Bird and Ross (2020) and Bird (2021). These sources have helped you understand complex concepts.

Chapter 1:

- Purpose of SI Units: Provide a consistent framework for scientific and technical measurements.
- Advantages of SI Units: Facilitate clear communication and data comparison across various fields and countries.
- Fundamental Units of SI: Meter, kilogram, second, ampere, kelvin, mole, and candela.
- Method Name: Unity fraction method.
- Purpose: Converting one unit of measurement into another.
- Methodology: Multiplying by fractions equal to one, where the numerator and denominator represent the same quantity in different units.

Chapter 2:

- Scalar Definition: A quantity with only magnitude (size).
- Vector Definition: A quantity with both magnitude and direction.
- Scalar Examples: Numbers, mass, speed, temperature, volume, and time.
- Vector Examples: Velocity, acceleration, weight, and friction.

Chapter 3:

- Definition of Moment: A mathematical concept representing the product of a distance and a physical quantity.

- Application of Moment: Used to analyze the effect of forces acting on objects at a distance from a reference point.
- Example of Moment: Torque, the moment of force, is the product of force and distance.

As you near the final exam, remember that your knowledge and skills will help you succeed in your future courses. Stay confident, trust your preparation, and be composed. You have put in a lot of effort; now's the time to show what you know.

We wish you the best on the exam.

# References

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