Applied Mechanics Workbook

Supplementary Exercises for In-Class Learning

Serhat Beyenir

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Preface

This workbook presents a collection of lecture notes on Applied Mechanics, designed to provide learners with succinct yet essential insights into key topics covered in class. Each chapter is accompanied by a problem set to facilitate comprehension and reinforce understanding.

Chapter 1: The International System of Units (SI) is the globally accepted standard for measurement. Established to provide a consistent framework for scientific and technical measurements, SI units facilitate clear communication and data comparison across various fields and countries. The system is based on seven fundamental units: the meter for length, the kilogram for mass, the second for time, the ampere for electric current, the kelvin for temperature, the mole for substance, and the candela for luminous intensity.

Chapter 2: In mathematics and physics, a **scalar** is a quantity with only magnitude (size), whereas a **vector** has both magnitude and direction. Examples of scalar quantities include numbers, mass, speed, temperature, volume, and time. In contrast, examples of vector quantities include velocity, acceleration, and forces like weight and friction.

Chapter 3: Kinematic equations describe the motion of objects under constant acceleration. They relate displacement (x), initial velocity (v_0), final velocity (v), acceleration (a), and time (t).

Chapter 4: A **moment** is a mathematical concept that represents the product of a distance and a physical quantity, such as force. Moments are typically defined relative to a fixed reference point and pertain to physical quantities situated at some distance from this point. For instance, the moment of force, commonly referred to as torque, is the product of the force applied to an object and the distance from the reference point to the object.

Chapter 5: Review section

Chapter 1

International System of Units

1.1 Objectives

- Recall the base and derived units.
- Practice the application of unity fraction.

1.2 SI Units

The International System of Units (SI) is the globally accepted standard for measurement. Established to provide a consistent framework for scientific and technical measurements, SI units facilitate clear communication and data comparison across various fields and countries. The system is based on seven fundamental units: the meter for length, the kilogram for mass, the second for time, the ampere for electric current, the kelvin for temperature, the mole for substance, and the candela for luminous intensity.

Table 1.1: Base SI units.

Physical Quantity	SI Base Unit	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric Current	Ampere	A
Temperature	Kelvin	K
Amount of Substance	Mole	mol
Luminous Intensity	Candela	cd

Table 1.2: Derived SI units.

Physical Quantity	Derived SI Unit	Symbol
Area	Square meter	m^2
Volume	Cubic meter	m^3
Speed	Meter per second	m/s
Acceleration	Meter per second squared	m/s^2
Force	Newton	N
Pressure	Pascal	Pa
Energy	Joule	J
Power	Watt	W
Electric Charge	Coulomb	C
Electric Potential	Volt	V
Resistance	Ohm	Ω
Capacitance	Farad	F
Frequency	Hertz	Hz
Luminous Flux	Lumen	lm
Illuminance	Lux	lx
Specific Energy	Joule per kilogram	J/kg
Specific Heat Capacity	Joule per kilogram Kelvin	$J/(kg \cdot K)$

Table 1.3: Common multiples and submultiples for SI units.

Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^2	hecto	h
10^1	deca	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	µ

1.3 SI System Rules and Common Mistakes

Using the SI system correctly is crucial for clear communication in science and engineering. Below are common mistakes in using the SI system, examples of incorrect usage, and how to correct them.

Table 1.4: SI system rules and common mistakes

Concept	**Mistake**	**Correct Usage**	**Notes**
Use of SI Unit Symbols	`m./s`	`m/s`	Use the correct format without additional punctuation.
Spacing Between Value & Unit	`10kg`	`10 kg`	Always leave a space between the number and the unit symbol.
Incorrect Unit Symbols	`sec`, `hrs`, `°K`	`s`, `h`, `K`	Use the proper SI symbols; symbols are case-sensitive.
Abbreviations for Units	`5 kilograms (kgs)`	`5 kilograms (kg)`	Avoid informal abbreviations like “kgs”; adhere to standard symbols.
Multiple Units in Expressions	`5 m/s/s`, `5 kg/meter²`	`5 m/s²`, `5 kg/m²`	Use compact, standardized formats for derived units.

Concept	Mistake	Correct Usage	Notes
Incorrect Use of Prefixes	`0.0001 km`	`100 mm`	Choose prefixes to keep numbers in the range (0.1 x < 1000).
Misplaced Unit Symbols	`5/s`, `kg10`	`5 s` ^{1}, `10 kg`	Symbols must follow numerical values, not precede them.
Degrees Celsius vs. Kelvin	`300°K`	`300 K`	Kelvin is written without "degree"
Singular vs. Plural Units	`5 kgs`, `1 meters`	`5 kg`, `1 meter`	Symbols do not pluralize; full unit names follow grammar rules.
Capitalization of Symbols	`Kg`, `S`, `Km`, `MA`	`kg`, `s`, `km`, `mA`	Symbols are case-sensitive; use uppercase only where specified (e.g., `N`, `Pa`).
Capitalization of Unit Names	`Newton`, `Pascal`, `Watt`	`newton`, `pascal`, `watt`	Unit names are lowercase, even if derived from a person's name, unless starting a sentence.
Prefix Capitalization	`MilliMeter`, `MegaWatt`	`millimeter`, `megawatt`	Prefixes are lowercase for (10^{-1}) to (10^{-9}), uppercase for (10^6) and larger (except `k` for kilo).
Formatting in Reports	`5`, `Temperature: 300`	`5 kg`, `Temperature: 300 K`	Always specify units explicitly.

1.4 Unity Fraction

The **unity fraction** method, or **unit conversion using unity fractions**, is a systematic way to convert one unit of measurement into another. This method relies on multiplying by fractions that are equal to one, where the numerator and the denominator represent the same quantity in different units. Since any number multiplied by one remains the same, unity fractions allow for seamless conversion without changing the value.

The principle of unity fractions is based on:

1. **Setting up equal values**: Write a fraction where the numerator and denominator are equivalent values in different units, so the fraction equals one. For example, $\frac{1km}{1000m}$ is a unity fraction because 1 km equals 1000 m.
2. **Multiplying by unity fractions**: Multiply the initial quantity by the unity fraction(s) so that the undesired units cancel out, leaving only the desired units.

1.5 Classwork

Example 1.1. Suppose we want to convert 5 kilometers to meters.

1. Start with 5 kilometers:
$$5\,\mathrm{km}$$
2. Multiply by a unity fraction that cancels kilometers and introduces meters. We use $(\frac{1000\,\mathrm{m}}{1\,\mathrm{km}})$, *since* $1\,\mathrm{km} = 1000\,\mathrm{m}$:
$$5\,\mathrm{km} \times \frac{1000\,\mathrm{m}}{1\,\mathrm{km}} = 5000\,\mathrm{m}$$
3. The kilometers km cancel out, leaving us with meters m:
$$5\,\mathrm{km} = 5000\,\mathrm{m}$$

This step-by-step approach illustrates how the unity fraction cancels the undesired units and achieves the correct result in meters.

Unity fractions can be extended by using multiple conversion steps. For example, converting hours to seconds would require two unity fractions: one to convert hours to minutes and another to convert minutes to seconds. This approach ensures accuracy and is widely used in science, engineering, and other fields that require precise unit conversions.

Example 1.2. Convert $15\,\mathrm{m/s}$ to km/h.

1. Start with $15\,\mathrm{m/s}$.
2. To convert meters to kilometers, multiply by $\frac{1\,\mathrm{km}}{1000\,\mathrm{m}}$.
3. To convert seconds to hours, multiply by $\frac{3600\,\mathrm{s}}{1\,\mathrm{h}}$.

$$15\,\mathrm{m/s} \times \frac{1\,\mathrm{km}}{1000\,\mathrm{m}} \times \frac{3600\,\mathrm{s}}{1\,\mathrm{h}} = 54\,\mathrm{km/h}$$

The meters and seconds cancel out, leaving kilometers per hour: 54 km/h.

1.6 Problem Set

Instructions:

1. Use unity fraction to convert between derived SI units.
2. Show each step of your work to ensure accuracy.
3. Simplify your answers and include correct units.

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1. **Speed**
   Convert 72 km/h to m/s.
2. **Force**
   Convert 980 N (newtons) to $kg \cdot m/s^2$.
3. **Energy**
   Convert 2500 J (joules) to kJ.
4. **Power**
   Convert 1500 W (watts) to kW.
5. **Pressure**
   Convert 101325 Pa (pascals) to kPa.
6. **Volume Flow Rate**
   Convert $3\,m^3/min$ to L/s.
7. **Density**
   Convert $1000\,kg/m^3$ to g/cm^3.
8. **Acceleration**
   Convert $9.8\,m/s^2$ to cm/s^2.
9. **Torque**
   Convert $50\,N \cdot m$ to $kN \cdot cm$.
10. **Frequency**
    Convert 500 Hz (hertz) to kHz.
11. **Work to Energy Conversion**
    A force of 20 N moves an object 500 cm. Convert the work done to joules.
12. **Kinetic Energy Conversion**
    Calculate the kinetic energy in kilojoules of a 1500 kg car moving at 72 km/h.

13. **Power to Energy Conversion**
A machine operates at $2\,\mathrm{kW}$ for 3 hours. Convert the energy used to megajoules.

14. **Pressure to Force Conversion**
Convert a pressure of $200\,\mathrm{kPa}$ applied to an area of $0.5\,\mathrm{m}^2$ to force in newtons.

15. **Density to Mass Conversion**
Convert $0.8\,\mathrm{g/cm}^3$ for an object with a volume of $250\,\mathrm{cm}^3$ to mass in grams.

1.6.1 Answer Key

1. $72\,\mathrm{km/h} = 20\,\mathrm{m/s}$
2. $980\,\mathrm{N} = 980\,\mathrm{kg \cdot m/s}^2$
3. $2500\,\mathrm{J} = 2.5\,\mathrm{kJ}$
4. $1500\,\mathrm{W} = 1.5\,\mathrm{kW}$
5. $101325\,\mathrm{Pa} = 101.325\,\mathrm{kPa}$
6. $3\,\mathrm{m}^3/\mathrm{min} = 50\,\mathrm{L/s}$
7. $1000\,\mathrm{kg/m}^3 = 1\,\mathrm{g/cm}^3$
8. $9.8\,\mathrm{m/s}^2 = 980\,\mathrm{cm/s}^2$
9. $50\,\mathrm{N \cdot m} = 5\,\mathrm{kN \cdot cm}$
10. $500\,\mathrm{Hz} = 0.5\,\mathrm{kHz}$
11. $20\,\mathrm{N} \times 5\,\mathrm{m} = 100\,\mathrm{J}$
12. Kinetic energy $= 1500\,\mathrm{kg} \times (20\,\mathrm{m/s})^2/2 = 300\,\mathrm{kJ}$
13. $2\,\mathrm{kW} \times 3\,\mathrm{hours} = 21.6\,\mathrm{MJ}$
14. $200\,\mathrm{kPa} \times 0.5\,\mathrm{m}^2 = 100,000\,\mathrm{N}$
15. $0.8\,\mathrm{g/cm}^3 \times 250\,\mathrm{cm}^3 = 200\,\mathrm{g}$

1.7 Further Reading

Introduction in Russell, Jackson, and Embleton (2021) and SI units in Bolton (2021) for additional information.

Chapter 2

Scalar and Vector Quantities

2.1 Objectives

- Distinguish between scalar and vector quantities.
- Define equilibrium of an object.
- Practice vector problems.

2.2 Concepts

In mathematics and physics, a **scalar** is a quantity with only magnitude (size), whereas a **vector** has both magnitude and direction. Examples of scalar quantities include numbers, mass, speed, temperature, volume, and time. In contrast, examples of vector quantities include velocity, acceleration, and forces like weight and friction.

A vector is represented as an arrow, where the length denotes its magnitude, and the arrowhead indicates its direction. Vector diagrams are useful for visually conveying information about vector relationships, allowing us to analyze effects like the combination of forces or the movement of objects in two or three dimensions. In such diagrams, vectors are often drawn to scale and placed with respect to an origin or another reference point.

2.3 Coplanar Forces

Coplanar forces are forces that act within the same plane. This means they lie along a single, flat surface, and their lines of action do not extend out of that plane. Coplanar forces are often analyzed together in physics and engineering because their combined effects can be resolved within two dimensions.

2.4 Addition of Vectors by Nose-to-tail Method

The "nose-to-tail" method, also known as the "head-to-tail", is a visual way to add vectors in physics or engineering. Here's a step-by-step explanation of how it works:

1. Draw the First Vector: Start by drawing the first vector on your graph or paper. This vector should have its tail (starting point) at the origin or any chosen starting point. Make sure you draw it with the correct direction and length to scale.

2. Place the Second Vector: Take the second vector and place its tail at the "head" (or "tip") of the first vector. The direction and length should be accurately represented here as well.

3. Continue with Additional Vectors: If there are more vectors to add, repeat the process by placing each vector's tail at the head of the previous one.

4. Draw the Resultant Vector: The resultant vector, or the sum of all the vectors, is the vector that goes directly from the "tail" of the first vector to the "head" of the last vector in the chain. Draw this vector as a straight line connecting the starting point to the endpoint of the last vector.

5. Measure the Resultant: The length and direction of this resultant vector represent the combined effect of all the original vectors.

2.5 Addition of Vectors by Parallelogram Method

The parallelogram method is another visual technique for adding two vectors, specifically used when you have two vectors that start from the same point. Here's how it works:

1. Draw Both Vectors: Start by drawing the two vectors with their tails (starting points) at the same origin point. The direction and length of each vector should be accurate and to scale.

2. Complete the Parallelogram: Using the two vectors as adjacent sides, imagine a parallelogram where each vector is duplicated to form the opposite sides. Draw lines parallel to each vector from the head (tip) of the other vector to form a closed parallelogram.

3. Draw the Resultant Vector: The resultant vector, or the sum of the two vectors, is represented by the diagonal of the parallelogram that starts

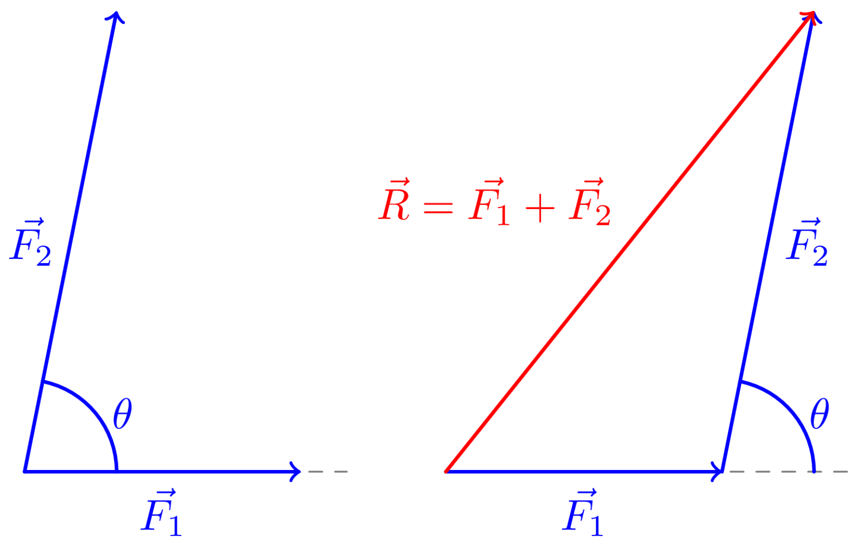


Figure 2.1: Addition of Vectors by Nose-to-tail Method

from the common origin point of the two vectors and goes to the opposite corner of the parallelogram.

4. Measure the Resultant: The length and direction of this diagonal vector represent the combined effect of the two vectors.

2.6 Cartesian Coordinates

In a two-dimensional Cartesian coordinate system, a point is represented as (x, y), where:

- x is the horizontal distance from the origin ($the\ x - coordinate$).
- y is the vertical distance from the origin ($the\ y - coordinate$).

The distance r of a point (x, y) from the origin can be calculated using:

$$r = \sqrt{x^2 + y^2} \tag{2.1}$$

The angle θ between the positive $(x) - axis$ and the line joining the origin to the point can be found as:

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) \tag{2.2}$$

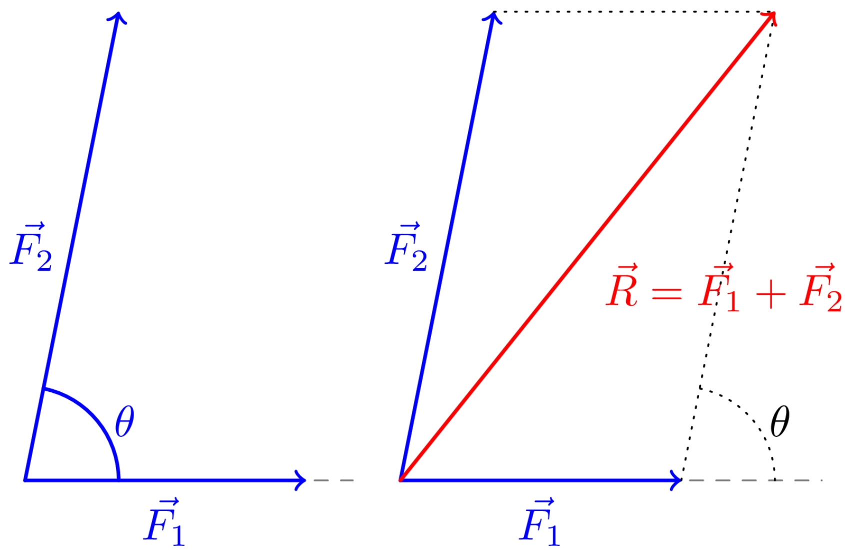


Figure 2.2: Addition of Vectors by Parallelogram Method

2.7 Polar Coordinates

In a two-dimensional Polar coordinate system, a point is represented as (r, θ), where:

- r is the radial distance from the origin.
- θ is the angle between the positive (x)-axis and the line joining the origin to the point (measured in radians or degrees).

The relationships between Polar and Cartesian coordinates are:

$$x = r\cos\theta, \quad y = r\sin\theta \tag{2.3}$$

Conversely, given Cartesian coordinates (x, y), the polar coordinates can be obtained as:

$$r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}\left(\frac{y}{x}\right) \tag{2.4}$$

2.8 Cosine Rule

The Cosine Rule is used to relate the lengths of the sides of a triangle as shown in Figure 2.3 to the cosine of one of its angles:

$$c^2 = a^2 + b^2 - 2ab\cos\gamma \tag{2.5}$$

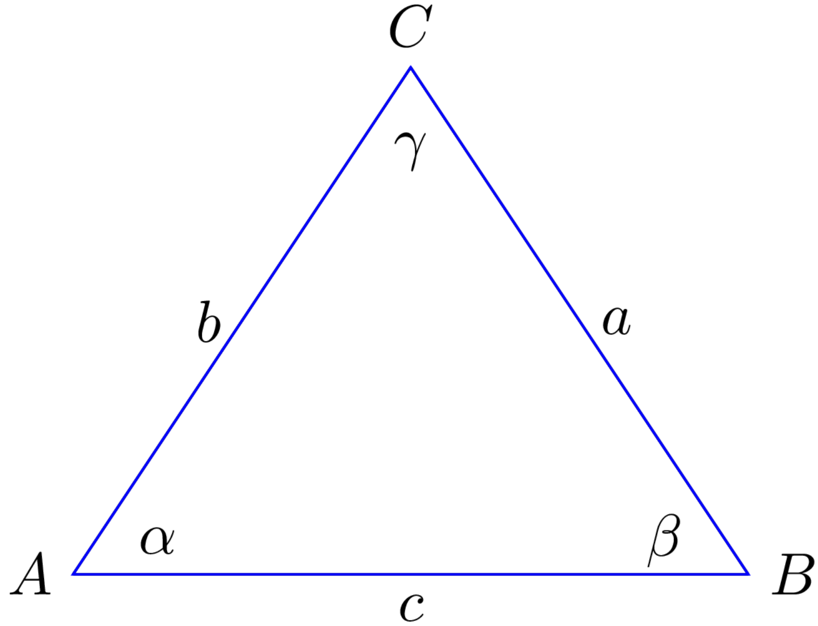


Figure 2.3: Rules of Cosine and Sine

Where:

- a, b, c are the sides of the triangle.
- γ is the angle opposite side c.

2.9 Sine Rule

The Sine Rule relates the sides and angles of a triangle in Figure 2.3:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \tag{2.6}$$

Where:

- α, β, γ are the angles of the triangle.
- a, b, c are the sides of the triangle opposite to angles α, β, γ respectively.

Example 2.1. Calculate the resultant of the forces 0.7 kN at 147° and 1.3 kN at -71° by using the cosine and sine rules.

Given the following forces:

- Force 1, $F_1 = 0.7$ kN, at 147°,
- Force 2, $F_2 = 1.3$ kN, $at - 71°$.

The angle between the two forces is:

$$\theta = 147^\circ - (-71^\circ) = 147^\circ + 71^\circ = 218^\circ$$

Since $(218^\circ > 180^\circ), we\ subtract\ from$ 360° to get the smaller angle:

$$\theta = 360^\circ - 218^\circ = 142^\circ$$

Step 1: Using the Cosine Rule to Find the Resultant Magnitude

The magnitude of the resultant R is given by the cosine rule:

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos(\theta)}.$$

Substituting the known values:

$$R = \sqrt{(0.7)^2 + (1.3)^2 + 2(0.7)(1.3)\cos(142^\circ)}.$$

We calculate $\cos 142^\circ$:

$$\cos(142^\circ) = \cos(180^\circ - 38^\circ) = -\cos(38^\circ) \approx -0.788.$$

Now substitute into the formula for (R):

$$R = \sqrt{0.49 + 1.69 - 1.432}.$$

$$R = \sqrt{0.748} \approx 0.865\,\text{kN}.$$

Step 2: Using the Sine Rule to Find the Direction

The direction of the resultant α relative to F_1 is found using the sine rule:

$$\frac{\sin(\alpha)}{F_2} = \frac{\sin(\theta)}{R}.$$

We calculate $\sin 142^\circ$:

$$\sin(142^\circ) = \sin(180^\circ - 142^\circ) = \sin(38^\circ) \approx 0.616.$$

Now apply the sine rule:

$$\sin(\alpha) = \frac{1.3 \times 0.616}{0.865} \approx 0.926.$$

Thus, $\alpha \approx 67.9^\circ$.

The direction of the resultant relative to the positive x-axis is:

$$\text{Direction} = 147^\circ - 67.9^\circ = 79.1^\circ.$$

2.10 Space Diagrams

The space diagram is an illustration of the system of forces.

2.11 Vector Diagrams

The vector diagram is a diagram drawn to scale with the vectors joined end to end. Vector diagrams are used to analyze and combine forces to find the resultant.

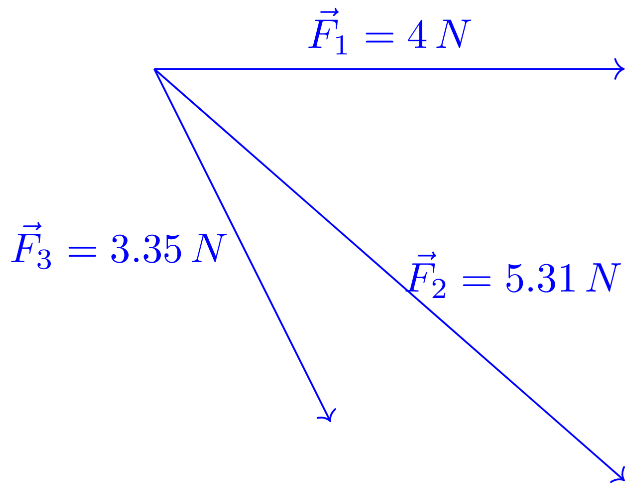


Figure 2.4: Space Diagram

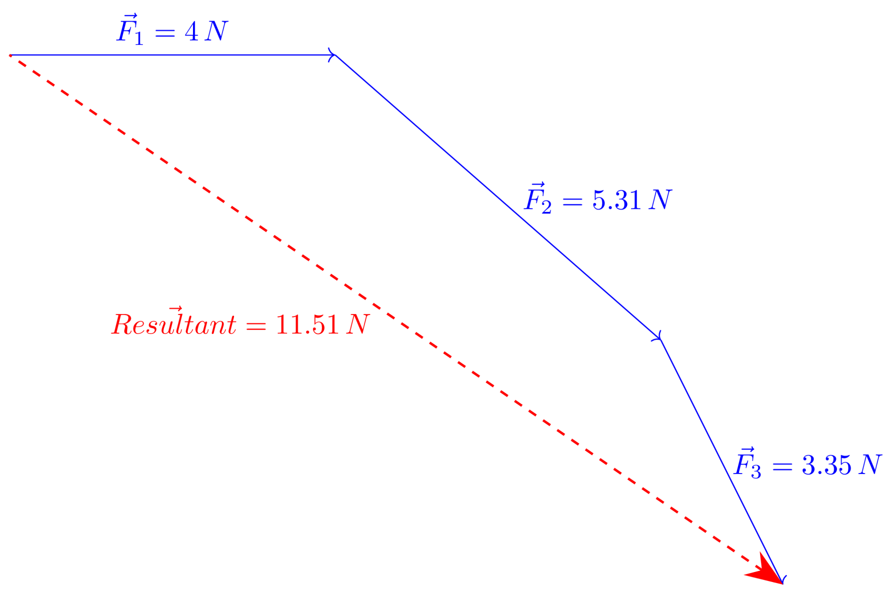


Figure 2.5: Vector Diagram

2.11.1 Resultant

The resultant is a single force that represents the combined effect of two or more forces acting on an object. It has the same effect as applying all the original forces together and is found by adding the individual forces, taking both their magnitudes and directions into account. The resultant gives the overall direction and magnitude of the combined forces as shown in Figure 2.5.

2.11.2 Equilibrium

Equilibrium of an object occurs when all the forces acting on it are balanced, so the object remains at rest or moves at a constant speed in a straight line. In equilibrium, there is no net force or acceleration, meaning the object is in a stable state without any change in its motion.

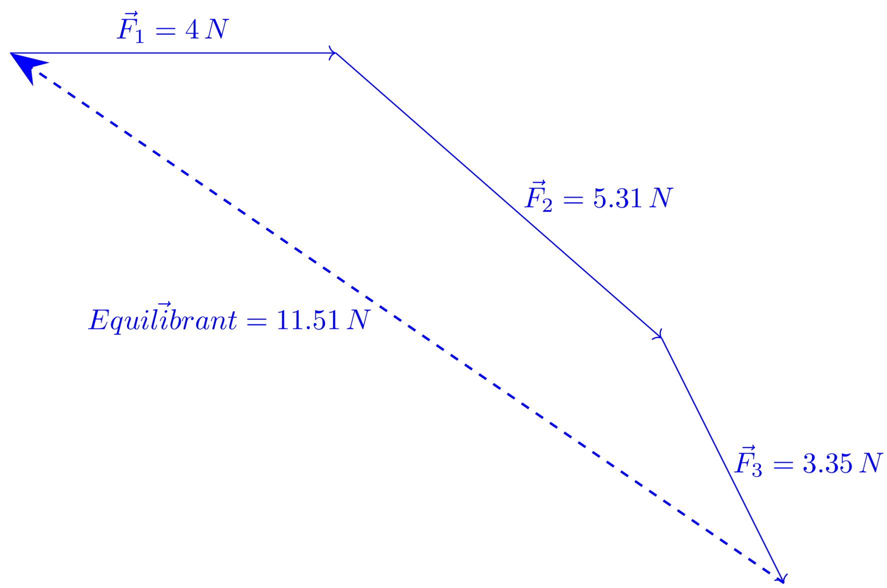


Figure 2.6: Equilibrant

Conditions of equilibrium:

1. Net force must be zero:

$$\sum_k \vec{F}_k = \vec{0} \tag{2.7}$$

2. Net torque must be zero:

$$\sum_k \vec{\tau}_k = \vec{0} \tag{2.8}$$

2.12 Bow's Notation

Bow's Notation is a graphical method employed in structural engineering to label the forces acting on a truss or structural framework. The notation was developed by British mathematician Robert Bow in the 19th century and continues to be a practical tool for visualizing and solving force equilibrium problems in structures.

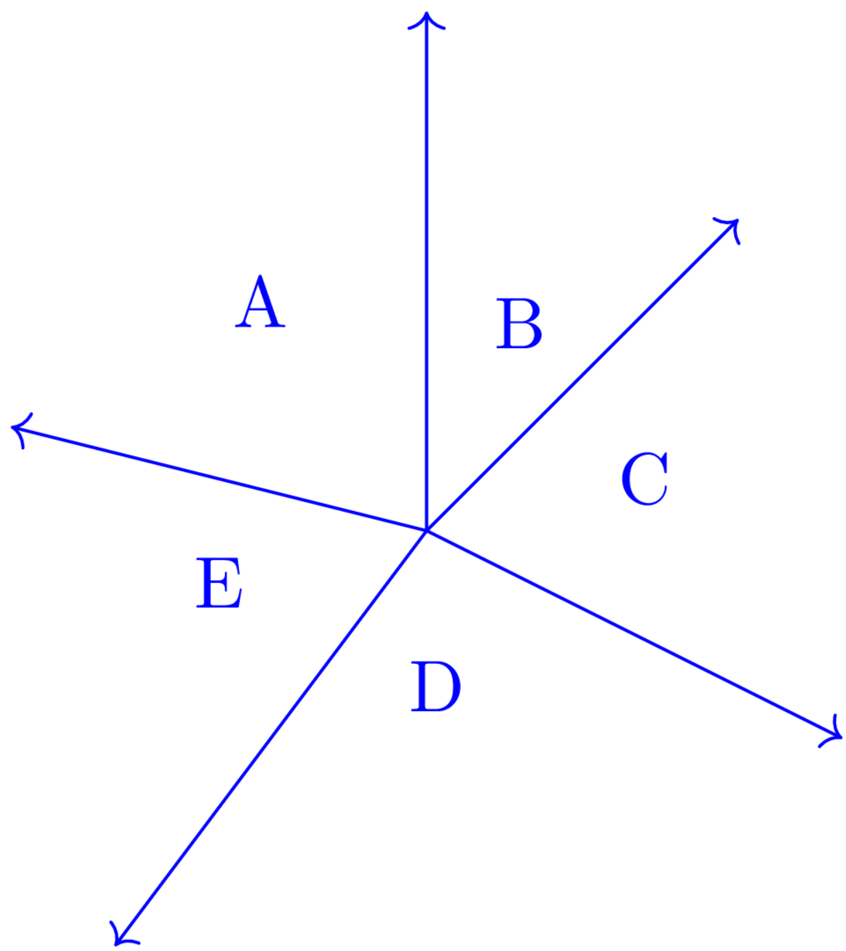


Figure 2.7: Bow's Notation Space Diagram

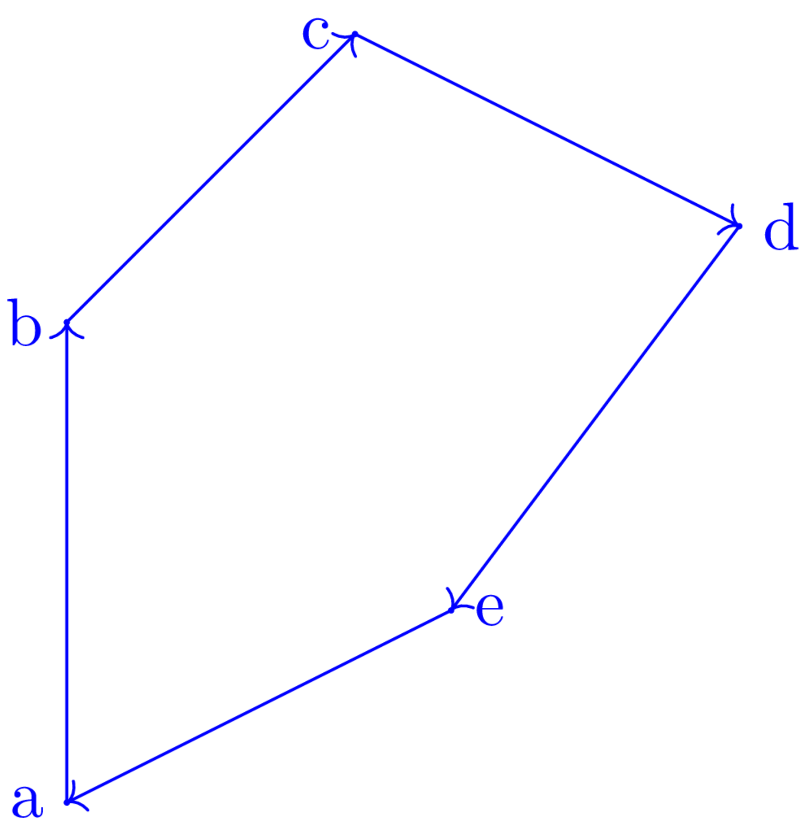


Figure 2.8: Bow's Notation Vector Diagram

2.12.1 Key Concepts of Bow's Notation

1. Labeling Spaces Between Forces:
   - Each space between external forces (such as loads and reactions) and each internal member of a truss is designated with a unique letter (e.g., A, B, C, etc.).
   - These labels are assigned sequentially around the structure in a clockwise or counterclockwise direction.
2. Force Polygon:
   - Each labeled space corresponds to a point on a force polygon, which is a closed polygon representing the equilibrium of forces.
   - The lengths and directions of the sides of the force polygon reflect the magnitudes and directions of the forces in each member.
3. Member Notation:
   - Each truss member is identified by the two letters representing the spaces it separates. For instance, a member between spaces A and B is labeled as AB.
4. Utilizing Bow's Notation in Analysis:
   - After labeling the spaces, equilibrium equations and graphical methods (such as constructing a force polygon) can be employed to determine the magnitude and direction of each force in the truss.
   - The notation simplifies calculations by visually connecting the forces and facilitating the identification of how different forces interact within the structure.

2.12.2 Example of Bow's Notation Application

In a simple triangular truss, the spaces surrounding the triangle may be labeled A, B, and C. If the members connect these spaces, they would be named as AB, BC, and CA. By employing Bow's Notation, engineers can construct a force polygon, analyze it for equilibrium, and determine the unknown forces in each member of the truss.

2.13 Slings

A sling is a device or assembly of ropes, cables, or straps used to support and lift loads. Slings play a crucial role in rigging operations, allowing objects to be lifted, lowered, or moved safely and efficiently. Slings are arranged to distribute the load evenly across their length, reducing stress points and ensuring stability. In multi-leg slings (e.g., two-leg or four-leg), the load is shared among the sling legs, which helps stabilize and balance the load.

Example 2.2. A 200 kg mass is suspended as shown in Figure 2.9. Rope A is attached to a beam in two places and is passed through a ring that rest naturally at the centre of rope A. Rope B is attached to the bottom of the ring, and to the 200 kg mass.

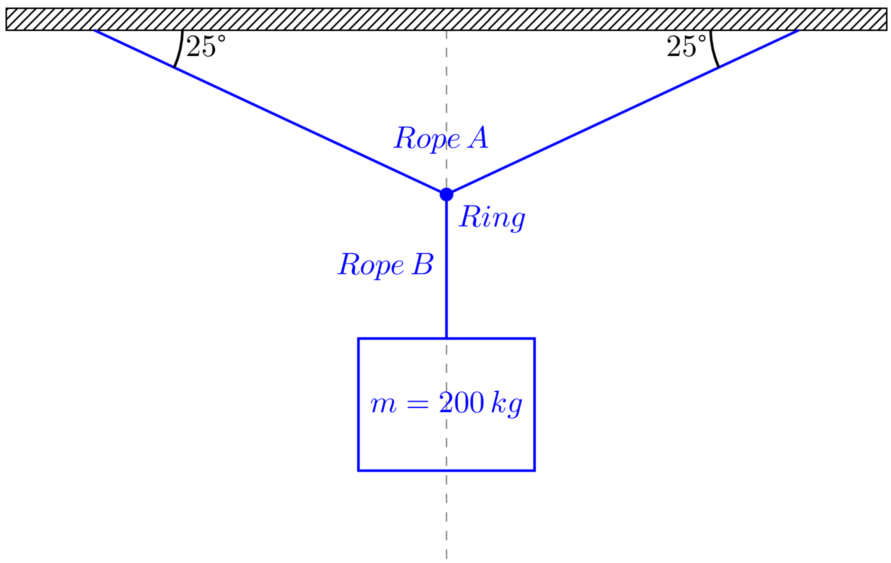


Figure 2.9: A 200 kg suspended mass.

a. What is the tension in rope B?
b. Draw a free body diagram of the forces exerted on the ring.
c. What is the tension in rope A?

Given Information

- Mass $m = 200\,\text{kg}$
- Gravitational acceleration $g = 9.81\,\text{m/s}^2$
- Angle $\theta = 25^\circ$

a. Tension in Rope B

The tension T_B in Rope B must support the entire weight of the mass. Therefore,

$$T_B = m \cdot g = 200 \times 9.81 = 1962\,\text{N}$$

So, **the tension in Rope B is 1962 N**.

b. Free Body Diagram of the Ring

The ring is in equilibrium, meaning the net force acting on it is zero. Here's a breakdown of the forces:

- **Tension** T_B: Acts downward, equal to the weight of the 200 kg mass.
- **Tensions** T_{A1} and T_{A2}: These are the tensions in each side of Rope A. Since the ring is at the center and the setup is symmetrical, $T_{A1} = T_{A2} = T_A$.

Since the ring is in equilibrium, the vertical components of T_{A1} and T_{A2} must balance the downward force from T_B.

c. Tension in Rope A

Since the angle between each side of Rope A and the horizontal is $\theta = 25^\circ$, we can use trigonometry to find T_A.

The equilibrium condition is:

$$2 \cdot T_A \cdot \sin(\theta) = T_B$$

Solving for T_A:

$$T_A = \frac{T_B}{2\sin(\theta)}$$

Substitute the values:

$$T_A = \frac{1962}{2 \cdot \sin(25^\circ)}$$

Calculating:

$$T_A \approx 2321.24\,\mathrm{N}$$

So, **the tension in Rope A is approximately 2321.24 N**.

2.14 Jib Cranes

A simple jib crane has a vertical post, a jib, and a tie. The jib is hinged at its lower end to the post, and the tie connects the top of the jib to the base of the post, forming the crane head where the tie and jib meet.

When a load is hung directly from the crane head, solving for forces involves a simple triangle of forces. In other cases, the crane may have a pulley at the head, with a rope running over it to a winch, creating a system with more than three forces.

2.15 Classwork

Example 2.3. A vertical lifting force of 95 N is applied to a body, and simultaneously, a horizontal force of 135 N pulls on it. Determine the magnitude and direction of the resulting force.

To solve for the magnitude and direction of the resultant force, we can use vector addition.

Given:

- Vertical force, $F_v = 95\,\mathrm{N}$
- Horizontal force, $F_h = 135\,\mathrm{N}$

Step 1: Calculate the Magnitude of the Resultant Force

The resultant force F_r is the vector sum of the vertical and horizontal forces. Using the Pythagorean theorem:

$$F_r = \sqrt{F_v^2 + F_h^2}$$

Substituting the values:

$$F_r = \sqrt{95^2 + 135^2}$$

Calculating further:

$$F_r = \sqrt{9025 + 18225} = \sqrt{27250}$$

Thus,

$$F_r \approx 165.07\,\mathrm{N}$$

Step 2: Determine the Direction of the Resultant Force

The direction θ of the resultant force with respect to the horizontal can be found using the tangent function:

$$\theta = \arctan\left(\frac{F_v}{F_h}\right)$$

Substituting the values:

$$\theta = \arctan\left(\frac{95}{135}\right)$$

Calculating θ:

$$\theta \approx 35.1341^\circ$$

Final Answer

The magnitude of the resultant force is approximately 165.07N, and its direction is 35.1341° above the horizontal.

Example 2.4. Two forces act upon a body. One exerts a horizontal force to the right with a magnitude of 25 Newtons, while the other exerts a vertical force downward with a magnitude of 20 Newtons. Determine the magnitude and direction of a third force that would counteract the combined effects of the other two forces.

To determine the magnitude and direction of the third force that counteracts the combined effects of the two forces, we first need to find the resultant force of the two given forces.

Given Forces:

- Horizontal force to the right: $F_h = 25\,\mathrm{N}$
- Vertical force downward: $F_v = 20\,\mathrm{N}$

Step 1: Calculate the Resultant Force

The resultant force F_r can be found using the Pythagorean theorem since the forces are perpendicular to each other.

$$F_r = \sqrt{F_h^2 + F_v^2}$$

Substituting the values:

$$F_r = \sqrt{25^2 + 20^2}$$

Calculating:

$$F_r = \sqrt{625 + 400} = \sqrt{1025}$$

Thus,

$$F_r \approx 32.02\,\mathrm{N}$$

Step 2: Determine the Direction of the Resultant Force

The direction θ of the resultant force can be found using the tangent function:

$$\theta = \arctan\left(\frac{F_v}{F_h}\right)$$

Substituting the values:

$$\theta = \arctan\left(\frac{20}{25}\right)$$

Calculating θ:

$$\theta \approx 38.66^\circ$$

This angle is measured from the horizontal axis (to the right) downward.

Step 3: Determine the Third Force

To counteract the resultant force, the third force F_3 must have the same magnitude as F_r but in the opposite direction. Therefore, its magnitude is:

$$F_3 = F_r \approx 32.02\,\mathrm{N}$$

The direction of the third force will be opposite to the direction of the resultant force, which means it will be directed at an angle of:

$$\theta + 180^\circ \approx 38.66^\circ + 180^\circ \approx 218.66^\circ$$

Final Answer

The magnitude of the third force is approximately 32.02N , and its direction is approximately 218.66° (measured counterclockwise from the positive x-axis or horizontal right).

Example 2.5. Determine the magnitude and direction of the equilibrium force resulting from the combination of two forces: a horizontal pull of 15 N and a pull of 25 N at an angle of 55 degrees with respect to the 15 N force.

To find the magnitude and direction of the equilibrium force resulting from the combination of two forces (15 N horizontally and 25 N at an angle of 55 degrees), we can use vector addition.

Step 1: Resolve the Forces into Components

1. Force $F_1 = 15\,\mathrm{N}$ (Horizontal):

$F_{1x} = 15\,\mathrm{N}$

$F_{1y} = 0\,\mathrm{N}$

2. Force $F_2 = 25\,\mathrm{N}$ at 55°:

$F_{2x} = F_2 \cdot \cos(55^\circ) = 25 \cdot \cos(55^\circ)$

$F_{2y} = F_2 \cdot \sin(55^\circ) = 25 \cdot \sin(55^\circ)$

Step 2: Calculate Components of F_2

Using $\cos(55^\circ) \approx 0.5736$ and $\sin(55^\circ) \approx 0.8192$:

$$F_{2x} = 25 \cdot 0.5736 \approx 14.34\,\mathrm{N}$$

$$F_{2y} = 25 \cdot 0.8192 \approx 20.48\,\mathrm{N}$$

Step 3: Find the Resultant Components

Now, we sum the components in the x and y directions:

- Resultant x-component:

$$R_x = F_{1x} + F_{2x} = 15 + 14.34 \approx 29.34\,\mathrm{N}$$

- Resultant y-component:

$$R_y = F_{1y} + F_{2y} = 0 + 20.48 \approx 20.48\,\mathrm{N}$$

Step 4: Calculate the Magnitude of the Resultant Force

The magnitude of the resultant force R can be found using the Pythagorean theorem:

$$R = \sqrt{R_x^2 + R_y^2}$$

Calculating R :

$$R = \sqrt{(29.34)^2 + (20.48)^2} \approx \sqrt{861.64 + 419.04} \approx \sqrt{1280.68} \approx 35.8\,\mathrm{N}$$

Step 5: Determine the Direction of the Resultant Force

The direction (angle θ of the resultant force can be found using the tangent function:

$$\tan(\theta) = \frac{R_y}{R_x}$$

$$\theta = \arctan\left(\frac{20.48}{29.34}\right)$$

Calculating θ:

$$\theta \approx \arctan(0.698) \approx 34.9159^\circ$$

Final Answer

Magnitude of the Equilibrium Force $\approx 35.8\,\mathrm{N}$

Direction of the Equilibrium Force $\approx 34.92^\circ$ above the horizontal (in the direction of the 15 N force).

Example 2.6. A ship is heading in a direction N 30° E at a speed which in still water would be 20 knots. It is carried off course by a current of 5 knots in a direction of E 60° S. Calculate the ship's actual speed and direction.

Solution:

We need to resolve both the ship's velocity in still water and the current into their vector components (x and y) and then compute the resultant speed and direction.

1. **Ship's velocity in still water (N 30° E at 20 knots):**

- Direction: N 30° E means 30° clockwise from North.
- Speed: 20 knots.

The **x** (East-West) component is:

$$v_{x1} = 20 \times \cos(30^\circ) = 20 \times \frac{\sqrt{3}}{2} \approx 17.32\,\text{knots}$$

The **y** (North-South) component is:

$$v_{y1} = 20 \times \sin(30^\circ) = 20 \times \frac{1}{2} = 10\,\text{knots}$$

2. **Current's velocity (E 60° S at 5 knots):**

- Direction: E 60° S means 60° South of East.
- Speed: 5 knots.

The **x** (East-West) component is:

$$v_{x2} = 5 \times \cos(60^\circ) = 5 \times \frac{1}{2} = 2.5\,\text{knots}$$

The **y** (North-South) component is:

$$v_{y2} = 5 \times \sin(60^\circ) = 5 \times \frac{\sqrt{3}}{2} \approx 4.33\,\text{knots}$$

Since the current is pushing the ship southward, the y-component will be **negative**:

$$v_{y2} = -4.33\,\text{knots}$$

3. **Add the components:**

Total **x** component:

$$v_x = v_{x1} + v_{x2} = 17.32 + 2.5 = 19.82\,\text{knots}$$

Total **y** component:

$$v_y = v_{y1} + v_{y2} = 10 - 4.33 = 5.67\,\text{knots}$$

4. **Calculate the actual speed and direction:**

- The **actual speed** is the magnitude of the total velocity vector:

$$\text{Speed} = \sqrt{v_x^2 + v_y^2} = \sqrt{19.82^2 + 5.67^2} = \sqrt{393.39 + 32.14} = \sqrt{425.53} \approx 20.62\,\text{knots}$$

- The **actual direction** is the angle relative to East (measured counter-clockwise from the positive x-axis):

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{5.67}{19.82}\right) \approx \tan^{-1}(0.286) \approx 16.0^\circ$$

So, the ship's actual direction is **16.0° North of East**.

Final Answer, see Figure 2.10:

- **Speed:** 20.62 knots
- **Direction:** N 16.0° E

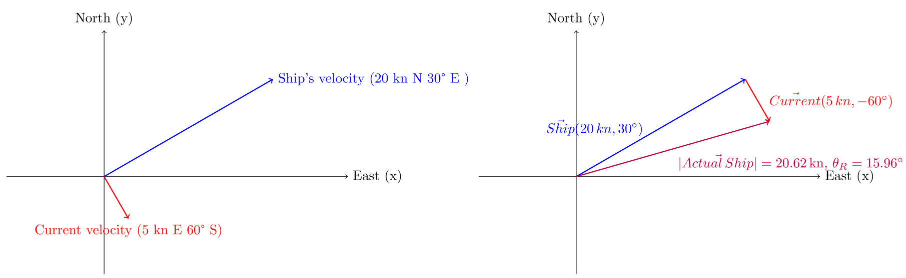


Figure 2.10: Ship vs Current diagram

Example 2.7. Given the space diagram in Figure 2.11, estimate the resultant vector using a graphical method.

Example 2.8. Given the space diagram in Figure 2.13, estimate the resultant vector using a graphical method.

Example 2.9. Three forces of 5 N, 8 N and 13 N act as shown in Figure 2.15. Calculate the magnitude and direction of the resultant force.

Given the polar coordinates, solution is shown in Figure 2.16,

Example 2.10. If velocity v_1= 25 m/s at 60° and v_2= 15 m/s at 330°, calculate the magnitude and direction of resultant velocity.

Graphical solution is shown in Figure 2.17.

Example 2.11. Calculate the magnitude and direction of the resultant vector of the force system shown in Figure 2.18

Solution is shown in Figure 2.19,

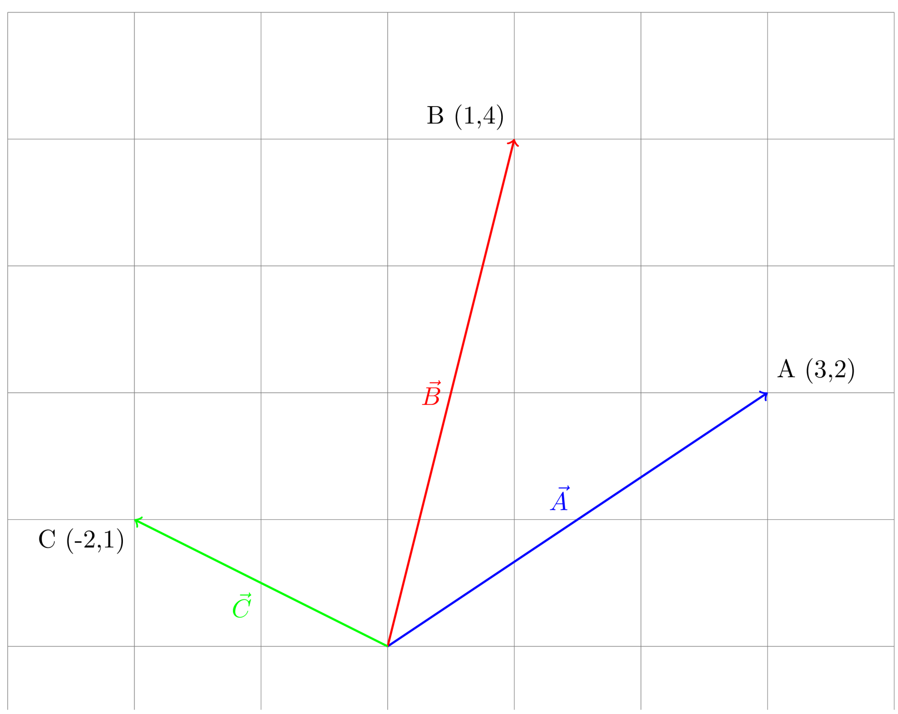


Figure 2.11: Vector Cartesian coordinates

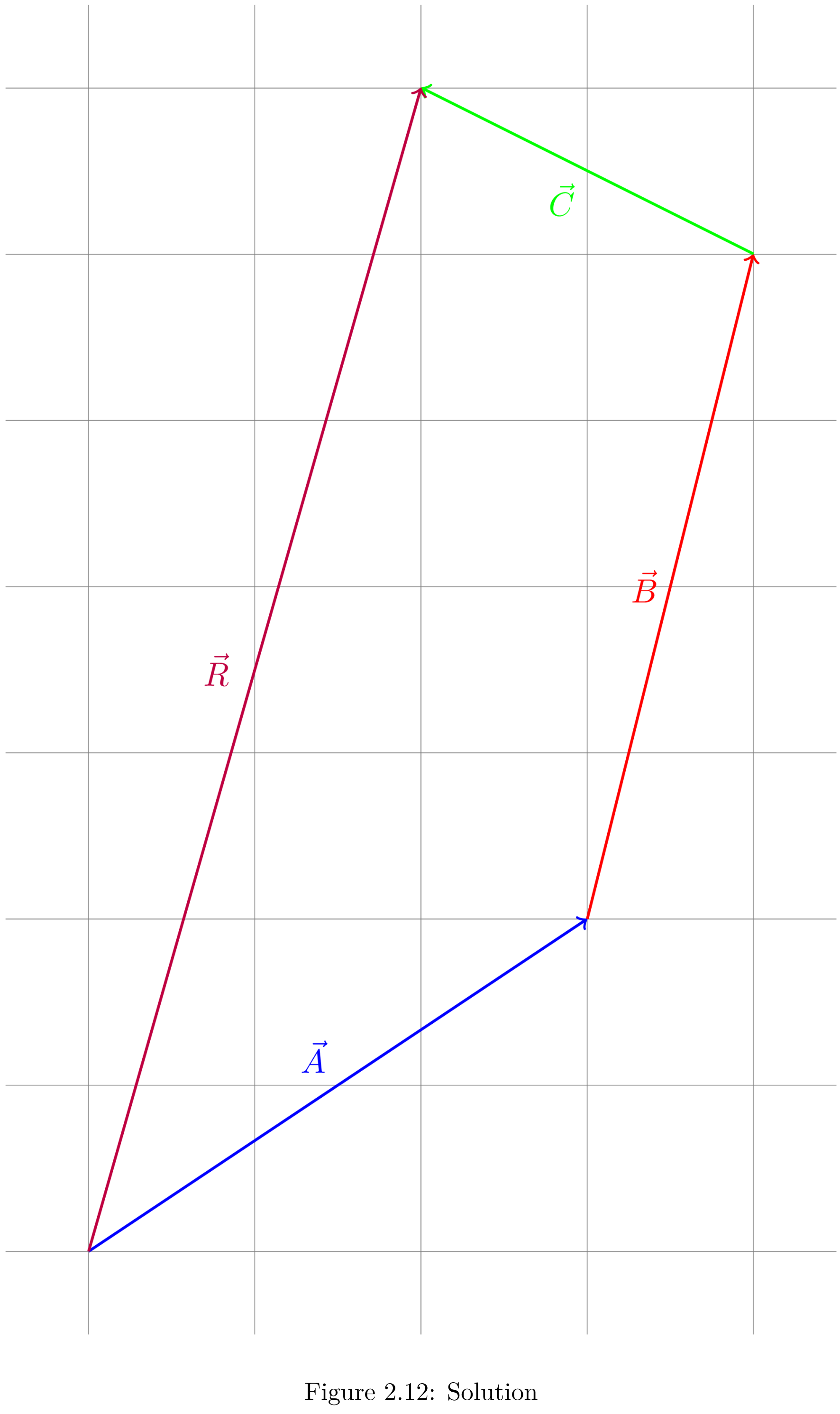


Figure 2.12: Solution

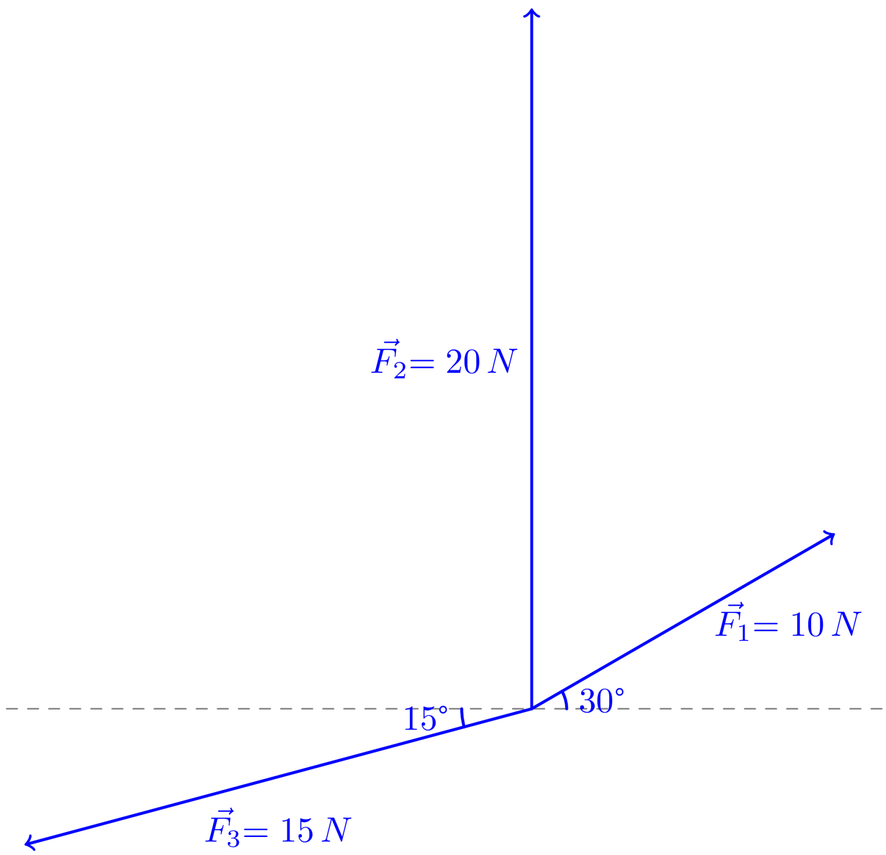


Figure 2.13: Vector polar coordinates

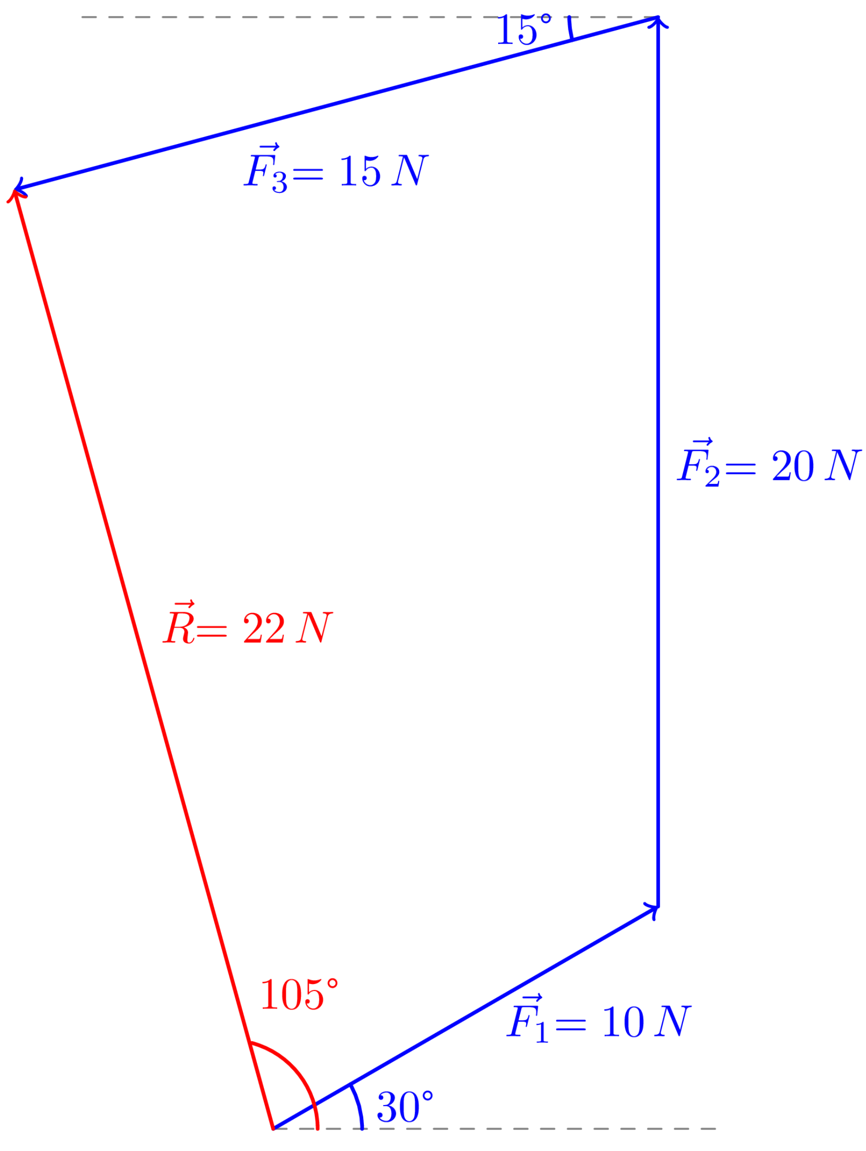


Figure 2.14: Solution

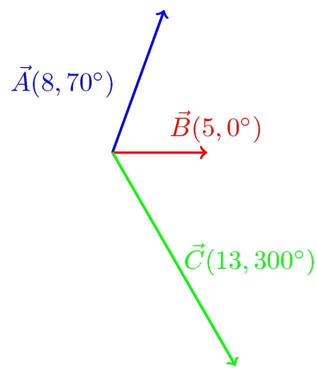


Figure 2.15

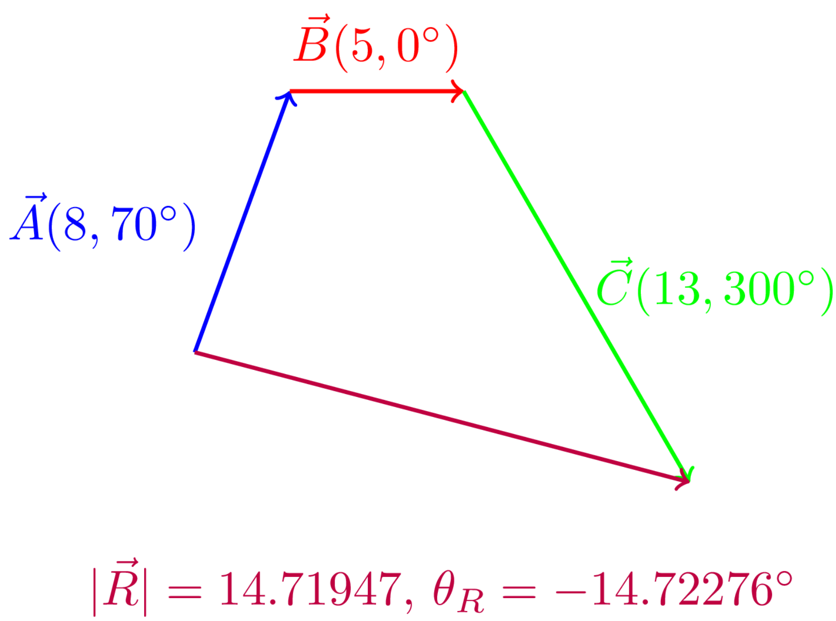


Figure 2.16: Solution

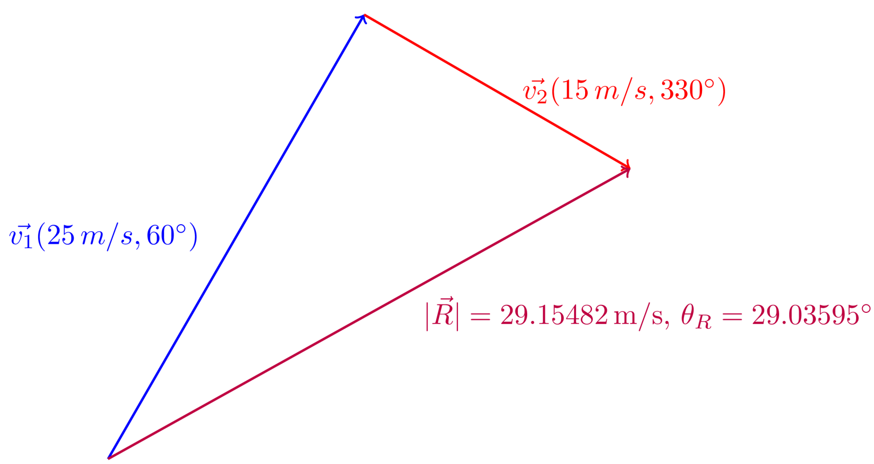


Figure 2.17: Graphical solution.

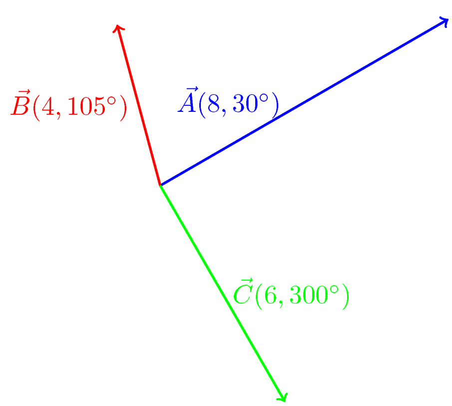


Figure 2.18

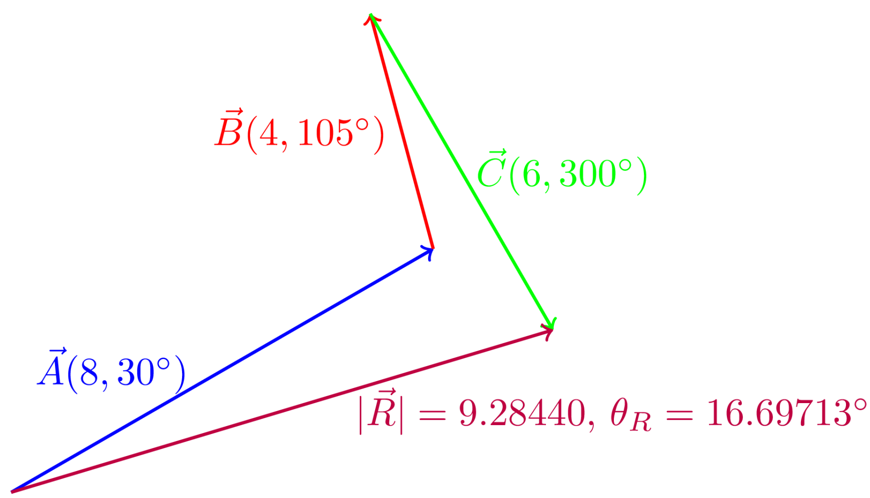


Figure 2.19: Solution

Example 2.12. Calculate the magnitude and direction of the resultant vector of the system shown Figure 2.20

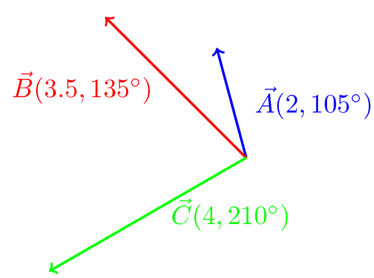


Figure 2.20: Vectors

Solution shown in Figure 2.21,

Example 2.13. A ship heads in a direction of E 20° S at a speed of 20 knots while the current is 4 knots in a direction of N 30° E. Determine the speed and actual direction of the ship. Solution shown in Figure 2.22,

Example 2.14. The angle between the jib and the vertical post of a jib crane is 42 degree, and between the tie and jib the angle is 36 degree. Find the forces in the jib and tie when 37.5 kN is suspended from the crane head.

Problem Setup:

1. Jib: 48° from the x-axis.

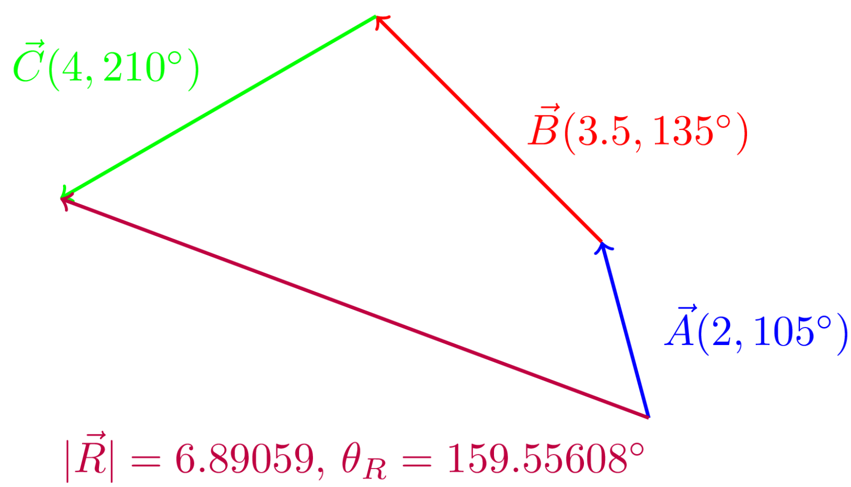


Figure 2.21: Solution

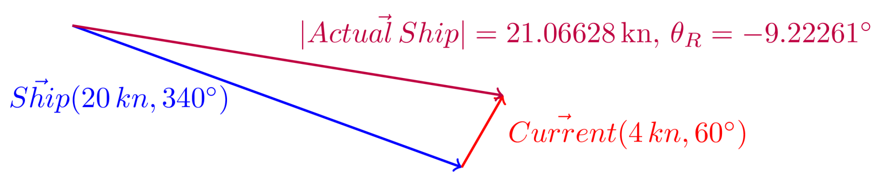


Figure 2.22: Solution

2. Tie: 192° from the x-axis.
3. Weight (W): 270° from the x-axis.
4. Load: W = 37.5 kN.

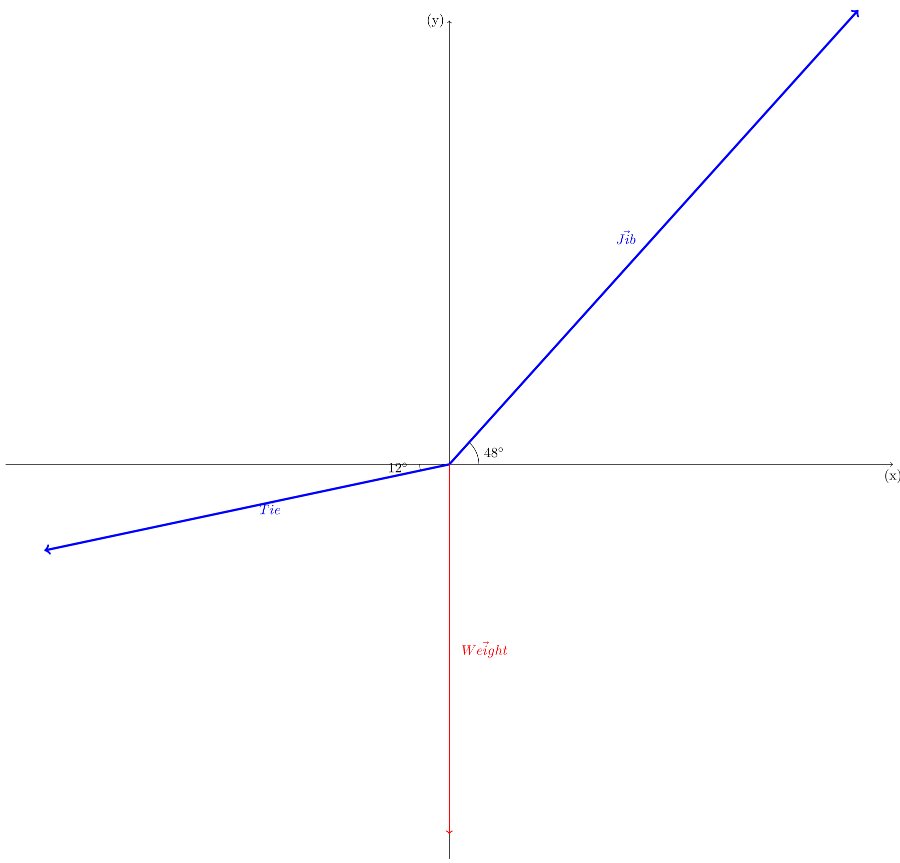


Figure 2.23: Jib

Step 1: Equilibrium Equations

Trigonometric Values:

- For the jib 48°:
  - $\cos(48^\circ) \approx 0.6691$,
  - $\sin(48^\circ) \approx 0.7431$.
- For the tie 192°:
  - $\cos(192^\circ) = \cos(180^\circ + 12^\circ) = -\cos(12^\circ) \approx -0.9781$,

  - $\sin(192^\circ) = \sin(180^\circ + 12^\circ) = -\sin(12^\circ) \approx -0.2079$.

- For the weight 270°:
  - $\cos(270^\circ) = 0$,
  - $\sin(270^\circ) = -1$.

Step 2: Force Components

Tie (T):

$T_x = T\cos(192^\circ)$, $T_y = T\sin(192^\circ)$.

Jib (J):

$J_x = J\cos(48^\circ), \quad J_y = J\sin(48^\circ)$.

Weight (W):

$W_x = 0, \quad W_y = -37.5, \mathrm{kN}$.

Step 3: Equations of Equilibrium

Horizontal (x-direction):

$T_x + J_x = 0$.

Substitute:

$T\cos(192^\circ) + J\cos(48^\circ) = 0$.

$T(-0.9781) + J(0.6691) = 0$.

Rearrange for T:

$T = \frac{J(0.6691)}{0.9781}$.

$T \approx 0.684\ J$.

Vertical (y-direction):

$T_y + J_y + W_y = 0$.

Substitute:

$T\sin(192^\circ) + J\sin(48^\circ) - 37.5 = 0$.

$T(-0.2079) + J(0.7431) = 37.5$.

Substitute T = 0.684 J:

$(0.684\ J)(-0.2079) + J(0.7431) = 37.5$.

Simplify:

$-0.142\ J + 0.7431\ J = 37.5$.

$0.6011\ J = 37.5$.

Solve for J:

$J = \frac{37.5}{0.6011} \approx 62.37, \text{kN}.$

Step 4: Solve for T

Using $T = 0.684\ J$:

$T = 0.684(62.37) \approx 42.67$ kN.

Final Results:

- Force in the jib (J) = 62.37 kN (compressive).
- Force in the tie (T) = 42.67 kN (tensile).

2.16 Problem Set

1. Calculate the magnitude and direction of the resultant of the 1.7 kN and 2.9 kN forces, which are aligned along the same line and act in the same direction.
2. Calculate the magnitude and direction of the resultant of the 457 N and 583 N forces, which are aligned along the same line but act in opposite directions.
3. Use the triangle of forces method to find the magnitude and direction of the resultant force from a 14 N force acting at 0° and a 23 N force acting at 35°.

2.16.1 Answer Key

1. Magnitude of the resultant: 4.6 kN. Direction: Same as the direction of the individual forces.

2. Magnitude of the resultant: 126 N. Direction: Same as the direction of the 583 N force.

3. The magnitude of the resultant force is approximately 35.39 N, and its direction is approximately 21.80° above the positive x-axis.

2.17 Further Reading

Read Chapter 1 in Russell, Jackson, and Embleton (2021), Chapter 1 in Hannah and Hillier (1995) and Chapter 2 in Bolton (2021) for additional exercises.

Chapter 3

Kinematics

3.1 Objectives

- Recall
- Practice

3.2 Linear Motion Definitions

- **Speed**(v): The scalar measure of the rate of change of distance. Mathematically,
$$v = \frac{d}{t}$$
where (d) is distance and (t) is time.
- **Velocity**($\vec{v}$): The vector measure of the rate of change of displacement. It is given by:
$$\vec{v} = \frac{\Delta \vec{x}}{\Delta t}$$
where $\Delta \vec{x}$ is the displacement vector and Δt is the time interval.
- **Acceleration**($\vec{a}$): The rate of change of velocity with respect to time. It is defined as:
$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

3.3 Equations of Motion (Constant Acceleration)

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{s} = \frac{\vec{u} + \vec{v}}{2}t$$

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

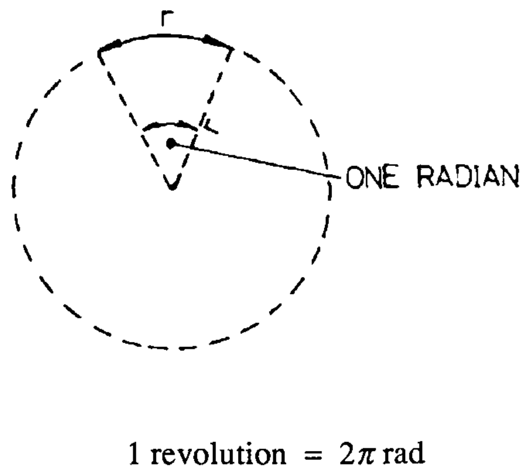
$$\vec{v}^2 = \vec{u}^2 + 2\vec{a} \cdot \vec{s}$$

where:

- $\vec{u}$: Initial velocity
- $\vec{v}$: Final velocity
- $\vec{s}$: Displacement
- $\vec{a}$: Acceleration
- t: Time

3.4 Angular Motion Definitions

- **Angular Displacement**(θ): The angle through which an object rotates, measured in radians.



- **Angular Velocity**(ω): The rate of change of angular displacement. Mathematically:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$\omega(rad/s) = 2\pi n \, where \, n = speed \, in \, rev/s$$

- **Angular Acceleration**(α): The rate of change of angular velocity with respect to time:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

3.5 Equations of Angular Motion (Constant Angular Acceleration)

$$\omega_2 = \omega_1 \mp \alpha t$$

$$\theta = \frac{\omega_1 + \omega_2}{2} t$$

$$\theta = \omega_1 t \mp \frac{1}{2}\alpha t^2$$

$$\omega_2^2 = \omega_1^2 \mp 2\alpha\theta$$

where:

- ω_1: Initial angular velocity (rad/s)
- ω_2: Final angular velocity (rad/s)
- θ Angular displacement (rad)
- α: Angular acceleration (rad/s^2)
- t: Time (s)

3.6 Relation Between Linear and Angular Motion

The relationship between linear and angular motion is described by the following equations:

$s = r\theta$ (linear displacement s and angular displacement θ).

$v = r\omega$ (linear velocity v and angular velocity ω),

$a = r\alpha$ (linear acceleration a and angular acceleration α),

3.6.1 Variables

- v: Linear velocity, the rate of change of linear displacement ($v = \frac{ds}{dt}$).
- a: Linear acceleration, the rate of change of linear velocity ($a = \frac{dv}{dt}$).
- s: Linear displacement, the distance moved along the circular path.
- r: Radius of the circular path.
- ω: Angular velocity, the rate of change of angular displacement ($\omega = \frac{d\theta}{dt}$).
- α: Angular acceleration, the rate of change of angular velocity ($\alpha = \frac{d\omega}{dt}$).
- θ: Angular displacement, the angle swept by the radius in radians.

3.7 Key Points

- Linear motion is directly proportional to angular motion, with the radius (r) acting as the proportionality constant.
- Units for the variables:
  - v: meters per second (m/s),
  - a: meters per second squared (m/s^2),
  - s: meters (m),
  - r: meters (m),
  - ω: radians per second (rad/s),
  - α: radians per second squared (rad/s^2),
  - θ: radians (rad).

3.8 Further Reading

Introduction in Russell, Jackson, and Embleton (2021) and SI units in Bolton (2021) for additional information.

Chapter 4

Moment of a Force

4.1 Objectives

- Recall.
- Practice the application.

4.2 Definitions

Centre of Gravity (CG): The point where the entire weight of an object acts, and at which gravity can be considered to apply. CG depends on the distribution of mass and gravity.

$$\bar{x} = \frac{\sum Moments\ of\ Weights}{\sum Weights} \qquad \bar{y} = \frac{\sum Moments\ of\ Weights}{\sum Weights} \tag{4.1}$$

$$\bar{x} = \frac{\sum Moments\ of\ Volumes}{\sum Volumes} \qquad \bar{y} = \frac{\sum Moments\ of\ Volumes}{\sum Volumes} \tag{4.2}$$

$$\bar{x} = \frac{\sum Moments\ of\ Areas}{\sum Areas} \qquad \bar{y} = \frac{\sum Moments\ of\ Areas}{\sum Areas} \tag{4.3}$$

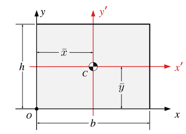
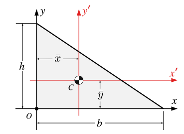
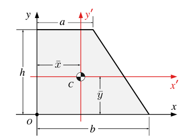
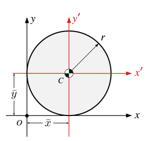
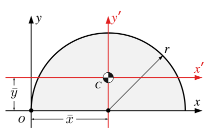
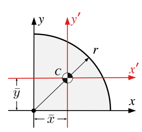
Centroid: The geometric center of a shape or object, determined purely by its geometry and independent of mass or weight distribution. The centroid of a uniform shape coincides with the center of gravity because the mass is evenly distributed.

$$\bar{x} = \frac{\sum \bar{x}_i\ A_i}{\sum A_i} \qquad \bar{y} = \frac{\sum \bar{y}_i\ A_i}{\sum A_i} \tag{4.4}$$

4.3 Properties of Common Shapes

In Table 4.1, all centroids are measured from the indicated origin. You must make the appropriate adjustments when the origin of your coordinate system is located elsewhere.

Table 4.1: Centroids of Common Shapes

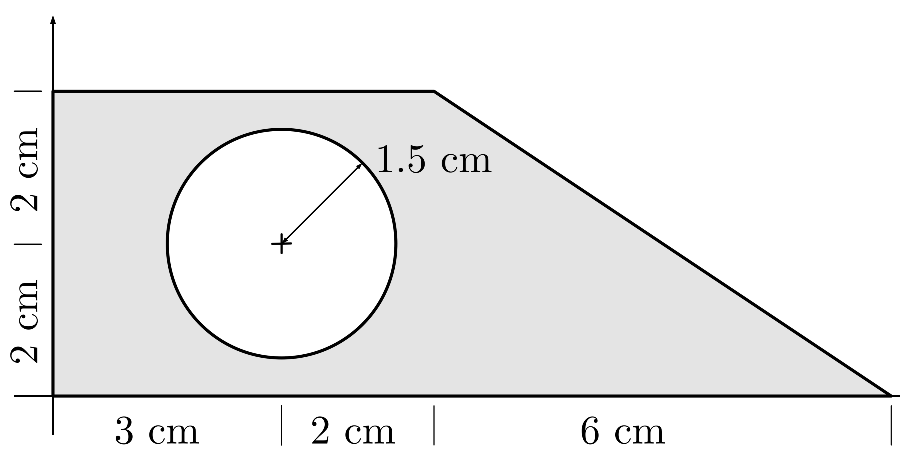
Shape	Area	$\bar{x}$	$\bar{y}$
	$A = bh$	$b/2$	$h/2$
	$\dfrac{bh}{2}$	$b/3$	$h/3$
	$\dfrac{(a+b)h}{2}$	$\dfrac{a^2+ab+b^2}{3(a+b)}$	$\dfrac{h(2a+b)}{3(a+b)}$
	πr^2	r	r
	$\dfrac{\pi r^2}{2}$	r	$\dfrac{4r}{3\pi}$
	$\dfrac{\pi r^2}{4}$	$\dfrac{4r}{3\pi}$	$\dfrac{4r}{3\pi}$

4.4 Centroids using Composite Parts

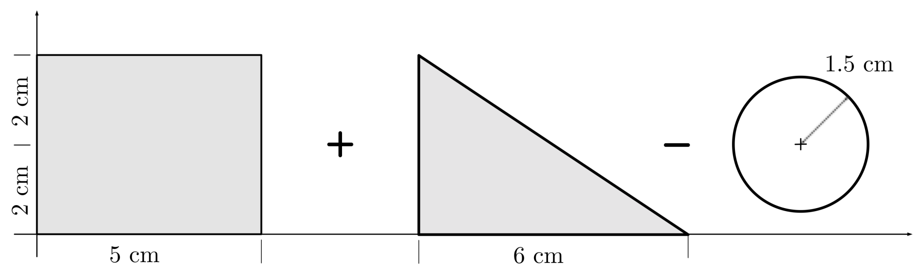
In this section we will discuss how to find centroids of two-dimensional shapes by first dividing them into pieces with known properties, and then combining the pieces to find the centroid of the original shape. This method will work when the geometric properties of all the sub-shapes are known or can be easily determined. The equations we will use for this approach are

$$\bar{x} = \frac{\sum \bar{x}_i \; A_i}{\sum A_i} \qquad \bar{y} = \frac{\sum \bar{y}_i \; A_i}{\sum A_i}$$

Consider the complex shape below.



There are often several ways to divide a shape, but it's best to use as few parts as possible to minimize your computations and opportunities for error. For example, you could choose to break this shape into either a rectangle, a right triangle, and an circular hole:



Once the complex shape has been divided into parts, the next step is to determine the area and centroidal coordinates for each part.

Part	A_i	$\bar{x}_i$	$\bar{y}_i$	$A_i\bar{x}_i$	$A_i\bar{y}_i$
1	20	2.5	2	50	40
2	12	7	4/3	84	16
3	-7.0685	3	2	-21.2057	-14.1371
Σ	24.9315	—	—	112.7943	41.8629

The last two columns of the table contain the first moments of area $Q_x = A_i\bar{y}_i$ and $Q_y = A_i\bar{x}_i$,, and are easily filled in by multiplying the values in columns two to four. Be sure to attend to positive and negative signs when multiplying. Note that the moment of area with respect to the x axis uses the distance from the x axis, which is $\bar{y}_i$,, and vice-versa.

$$\bar{x} = \frac{Q_y}{A} = \frac{112.8}{24.93} = 4.52\ cm$$

$$\bar{y} = \frac{Q_x}{A} = \frac{41.86}{24.93} = 1.692\ cm$$

Finally, plot the centroid $(\bar{x}, \bar{y})$ on the diagram. If you have made a calculation error it will usually be obvious, because the centroid location won't "feel right."

4.5 Second Moment Calculations

The **second moment**, or **moment of inertia**, measures how an area or mass is distributed about an axis. Below are the key concepts and formulas.

4.5.1 1. General Formula

The second moment of area about the (x)-axis or (y)-axis is:

$$I_x = \int_A y^2 \, dA, \quad I_y = \int_A x^2 \, dA$$

4.5.2 2. Polar Moment of Inertia

The polar moment of inertia about the origin is:

$$J = \int_A \left(x^2 + y^2\right) \, dA$$

4.5.3 3. Composite Areas

For a composite area, the total moment of inertia is the sum of the moments of its components:

$$I_{\text{total}} = \sum I_i$$

For each component:

$$I_i = I_{c,i} + A_i d_i^2$$

4.5.4 4. Parallel Axis Theorem

To find the moment of inertia about an axis parallel to the centroidal axis:

$$I = I_c + Ad^2$$

4.5.5 5. Moments of Common Shapes

4.5.5.1 Rectangle

For a rectangle with base (b) and height (h):

$$I_x = \frac{bh^3}{12}, \quad I_y = \frac{b^3 h}{12}$$

4.5.5.2 Circle

For a circle with radius (r):

$$I_x = I_y = \frac{\pi r^4}{4}, \quad J = \frac{\pi r^4}{2}$$

4.5.5.3 Triangle

For a triangle with base (b) and height (h):

$$I_x = \frac{bh^3}{36}, \quad I_y = \frac{hb^3}{36}$$

4.6 Derivation of the Moment of Inertia of a Rectangle About its Centroidal Axis

We calculate the **moment of inertia** of a rectangle about its **centroidal** x-axis. The centroidal axis passes horizontally through the centroid of the rectangle, located at $y = h/2$ (mid-height of the rectangle).

4.7 1. Moment of Inertia Formula

The formula for the moment of inertia is:

$$I = \int y^2 \, dA$$

Here:

- y: distance from the axis of rotation (centroidal axis in this case),
- dA: the infinitesimal area element.

4.8 2. Setup for the Rectangle

- The rectangle is centered on the centroidal x-axis, so the height ranges from $y = -h/2$ to $y = h/2$.
- The width of the rectangle is b.
- The infinitesimal area element is $dA = b \, dy$.

The integral becomes:

$$I_{\text{centroidal}} = \int_{-h/2}^{h/2} y^2 \, b \, dy$$

4.9 3. Solve the Integral

Factor b (constant width) outside of the integral:

$$I_{\text{centroidal}} = b \int_{-h/2}^{h/2} y^2 \, dy$$

The integral of y^2 is:

$$\int y^2 \, dy = \frac{y^3}{3}$$

Apply the limits $y = -h/2$ to $y = h/2$:

$$I_{\text{centroidal}} = b \left[\frac{y^3}{3} \right]_{-h/2}^{h/2}$$

Substitute the limits:

$$I_{\text{centroidal}} = b \left(\frac{(h/2)^3}{3} - \frac{(-h/2)^3}{3} \right)$$

Since $(-h/2)^3 = -(h/2)^3$, the terms simplify:

$$I_{\text{centroidal}} = b \left(\frac{(h/2)^3}{3} + \frac{(h/2)^3}{3} \right)$$

Combine the terms:

$$I_{\text{centroidal}} = b \left(\frac{2(h/2)^3}{3} \right)$$

Simplify $(h/2)^3 = \frac{h^3}{8}$:

$$I_{\text{centroidal}} = b \cdot \frac{2}{3} \cdot \frac{h^3}{8}$$

$$I_{\text{centroidal}} = b \cdot \frac{h^3}{12}$$

4.10 4. Final Result

The moment of inertia of a rectangle about its centroidal x-axis is:

$$I_{\text{centroidal}} = \frac{bh^3}{12}$$

This is the standard formula for the moment of inertia of a rectangle about its centroidal axis.

4.11 Difference Between First Moment and Second Moment

The **first moment** and **second moment** are concepts from mathematics and physics, describing how quantities are distributed around a reference point (such as the mean or origin). Below is a detailed explanation:

4.12 1. First Moment

- **Definition**: The first moment measures the mean or center of mass relative to a reference point.
- **Formula**:
$$M_1 = \sum (x_i \cdot f_i)$$
where x_i is a value, and f_i is its weight.
- **Physical Interpretation**: In mechanics, it determines the center of mass or centroid of an object.

4.13 2. Second Moment

- **Definition**: The second moment measures the spread or dispersion of values around a reference point.
- **Formula**:
$$M_2 = \sum (x_i^2 \cdot f_i)$$
where x_i is a value, and f_i is its weight.
- **Physical Interpretation**: In mechanics, the second moment of area (I) is used to describe an object's resistance to bending or torsion (e.g., the moment of inertia).

4.14 Key Differences

Feature	First Moment	Second Moment
Reference	Measures position relative to a point (e.g., centroid).	Measures spread relative to a point.
Physical Meaning	Indicates center or balance point.	Indicates resistance or spread.
Example (Physics)	Center of mass.	Moment of inertia.

In summary, the **first moment** tells you *where things are*, while the **second moment** tells you *how far things are spread.*Problem Set

4.15 Second Moment Calculations

The **second moment**, or **moment of inertia**, measures how an area or mass is distributed about an axis.

4.15.1 General Formula

The second moment of area about the (x)-axis or (y)-axis is:

$$I_x = \int y^2 \, dA, \quad I_y = \int x^2 \, dA$$

4.16 Derivation of the Moment of Inertia of a Rectangle About Its Base

We calculate the **moment of inertia** of a rectangle about its **base**, where the base is along the x-axis and the height extends from $y = 0$ to $y = h$.

4.16.1 1. Moment of Inertia Formula

The formula for the moment of inertia is:

$$I = \int y^2 \, dA$$

Here: - y: distance from the axis of rotation (the base of the rectangle in this case), - dA: the infinitesimal area element.

4.16.2 2. Setup for the Rectangle

- The rectangle has a width of b and height h.
- The height extends from $y = 0$ to $y = h$.

- The infinitesimal area element is $dA = b\,dy$.

The integral becomes:

$$I_{\text{base}} = \int_0^h y^2\, b\, dy$$

4.16.3 3. Solve the Integral

Factor b (constant width) outside of the integral:

$$I_{\text{base}} = b \int_0^h y^2\, dy$$

The integral of y^2 is:

$$\int y^2\, dy = \frac{y^3}{3}$$

Apply the limits $y = 0$ to $y = h$:

$$I_{\text{base}} = b \left[\frac{y^3}{3} \right]_0^h$$

Substitute the limits:

$$I_{\text{base}} = b \left(\frac{h^3}{3} - \frac{0^3}{3} \right)$$

Simplify:

$$I_{\text{base}} = b \cdot \frac{h^3}{3}$$

4.16.4 4. Final Result

The moment of inertia of a rectangle about its base is:

$$I_{\text{base}} = \frac{bh^3}{3}$$

4.17 Derivation of the Moment of Inertia of a Rectangle About its Centroidal Axis

We calculate the **moment of inertia** of a rectangle about its **centroidal** x-axis. The centroidal axis passes horizontally through the centroid of the rectangle, located at $y = h/2$ (mid-height of the rectangle).

4.18 1. Moment of Inertia Formula

The formula for the moment of inertia is:

$$I = \int y^2 \, dA$$

Here:

- y: distance from the axis of rotation (centroidal axis in this case),
- dA: the infinitesimal area element.

4.19 2. Setup for the Rectangle

- The rectangle is centered on the centroidal x-axis, so the height ranges from $y = -h/2$ to $y = h/2$.
- The width of the rectangle is b.
- The infinitesimal area element is $dA = b \, dy$.

The integral becomes:

$$I_{\text{centroidal}} = \int_{-h/2}^{h/2} y^2 \, b \, dy$$

4.20 3. Solve the Integral

Factor b (constant width) outside of the integral:

$$I_{\text{centroidal}} = b \int_{-h/2}^{h/2} y^2 \, dy$$

The integral of y^2 is:

$$\int y^2 \, dy = \frac{y^3}{3}$$

Apply the limits $y = -h/2$ to $y = h/2$:

$$I_{\text{centroidal}} = b\left[\frac{y^3}{3}\right]_{-h/2}^{h/2}$$

Substitute the limits:

$$I_{\text{centroidal}} = b\left(\frac{(h/2)^3}{3} - \frac{(-h/2)^3}{3}\right)$$

Since $(-h/2)^3 = -(h/2)^3$, the terms simplify:

$$I_{\text{centroidal}} = b\left(\frac{(h/2)^3}{3} + \frac{(h/2)^3}{3}\right)$$

Combine the terms:

$$I_{\text{centroidal}} = b\left(\frac{2(h/2)^3}{3}\right)$$

Simplify $(h/2)^3 = \frac{h^3}{8}$:

$$I_{\text{centroidal}} = b \cdot \frac{2}{3} \cdot \frac{h^3}{8}$$

$$I_{\text{centroidal}} = b \cdot \frac{h^3}{12}$$

4.21 4. Final Result

The moment of inertia of a rectangle about its centroidal x-axis is:

$$I_{\text{centroidal}} = \frac{bh^3}{12}$$

This is the standard formula for the moment of inertia of a rectangle about its centroidal axis.

4.22 Comparison with the Centroidal Moment of Inertia

The moment of inertia about the base is four times greater than that about the centroidal x-axis:

$$I_{\text{base}} = 4 \cdot I_{\text{centroidal}}$$

where:

$$I_{\text{centroidal}} = \frac{bh^3}{12}$$

4.23 Parallel Axis Theorem

To find the moment of inertia about an axis parallel to the centroidal axis:

$$I = I_c + Ad^2$$

4.24 Radius of Gyration

The radius of gyration of a body about an axis is a measure of the distribution of its area or mass relative to that axis. It is defined as the distance from the axis at which the total area or mass of the body could be concentrated without changing its moment of inertia.

Mathematically, the radius of gyration k is given by:

where:

- I : Moment of inertia about the axis
- A : Area of the cross-section (for area calculations)
- m : Mass of the body (for mass calculations)

4.25 Difference Between First Moment and Second Moment

The **first moment** and **second moment** are concepts from mathematics and physics, describing how quantities are distributed around a reference point (such as the mean or origin). Below is a detailed explanation:

4.26 First Moment

- **Definition**: The first moment measures the mean or center of mass relative to a reference point.
- **Formula**:
$$M_1 = \sum(x_i \cdot w_i)$$
where x_i is a value, and w_i is its weight.
- **Physical Interpretation**: In mechanics, it determines the center of mass or centroid of an object.

4.27 Second Moment

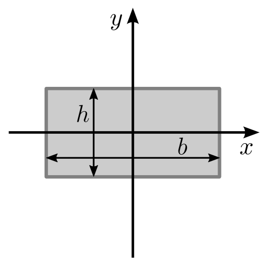
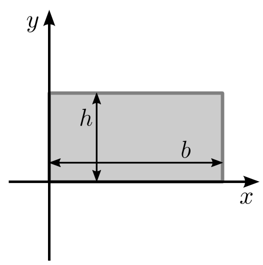
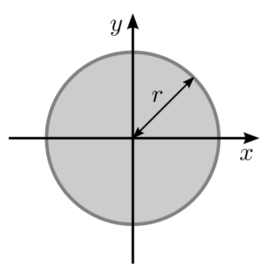
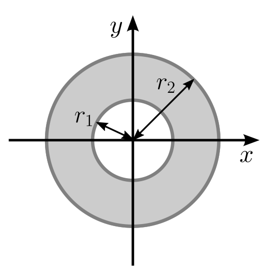
- **Definition**: The second moment measures the spread or dispersion of values around a reference point.
- **Formula**:
$$M_2 = \sum(x_i^2 \cdot w_i)$$
where x_i is a value, and w_i is its weight.
- **Physical Interpretation**: In mechanics, the second moment of area (I) is used to describe an object's resistance to bending or torsion (e.g., the moment of inertia).

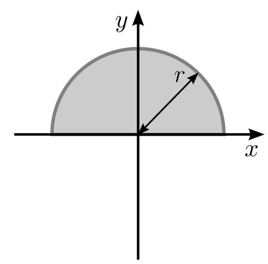
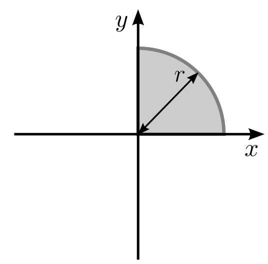
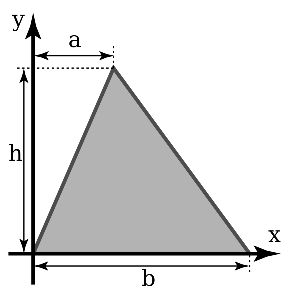
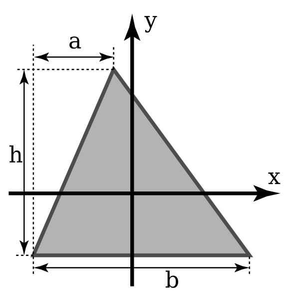
4.28 Key Differences

Feature	First Moment	Second Moment
Reference	Measures position relative to a point (e.g., centroid).	Measures spread relative to a point.
Physical Meaning	Indicates center or balance point.	Indicates resistance or spread.
Example (Physics)	Center of mass.	Moment of inertia.

4.29 Second Moments of Common Shapes

Table 4.5: Second moments

Shape	Second moment (I_x)	Second moment (I_y)
	$I_x = \frac{1}{12}bh^3$	$I_x = \frac{1}{12}b^3h$
	$I_x = \frac{1}{3}bh^3$	$I_x = \frac{1}{3}b^3h$
	$I_x = \frac{\pi}{4}r^4$	$I_y = \frac{\pi}{4}r^4$
	$I_x = \frac{\pi}{4}(r_2^4 - r_1^4)$	$I_y = \frac{\pi}{4}(r_2^4 - r_1^4)$

Shape	Second moment (I_x)	Second moment (I_y)
	$I_x = \frac{\pi}{8} r^4$	$I_y = \frac{\pi}{8} r^4$
	$I_x = \frac{\pi}{16} r^4$	$I_y = \frac{\pi}{16} r^4$
	$I_x = \frac{1}{12} b h^3$	
	$I_x = \frac{1}{36} b h^3$	

4.30 Further Reading

Read Chapter 7 in Russell, Jackson, and Embleton (2021), Chapter 1 in Hannah and Hillier (1995) and Chapter 2 in Bolton (2021) for additional information.

Chapter 5

Review

We have used several books by Ahrens (2022), Russell, Jackson, and Embleton (2021), Bolton (2021), Polya and Conway (2014), Bird and Ross (2020) and Bird (2021). These sources have helped you understand complex concepts.

Chapter 1:

- Purpose of SI Units: Provide a consistent framework for scientific and technical measurements.
- Advantages of SI Units: Facilitate clear communication and data comparison across various fields and countries.
- Fundamental Units of SI: Meter, kilogram, second, ampere, kelvin, mole, and candela.
- Method Name: Unity fraction method.
- Purpose: Converting one unit of measurement into another.
- Methodology: Multiplying by fractions equal to one, where the numerator and denominator represent the same quantity in different units.

Chapter 2:

- Scalar Definition: A quantity with only magnitude (size).
- Vector Definition: A quantity with both magnitude and direction.
- Scalar Examples: Numbers, mass, speed, temperature, volume, and time.
- Vector Examples: Velocity, acceleration, weight, and friction.

Chapter 4:

- Definition of Moment: A mathematical concept representing the product of a distance and a physical quantity.

- Application of Moment: Used to analyze the effect of forces acting on objects at a distance from a reference point.
- Example of Moment: Torque, the moment of force, is the product of force and distance.

As you near the final exam, remember that your knowledge and skills will help you succeed in your future courses. Stay confident, trust your preparation, and be composed. You have put in a lot of effort; now's the time to show what you know.

We wish you the best on the exam.

References

Ahrens, Sönke. 2022. *How to Take Smart Notes: One Simple Technique to Boost Writing, Learning and Thinking*. 2nd ed. edition. Sönke Ahrens.

Bird, J. O. 2021. *Bird's Engineering Mathematics*. Ninth edition. Abingdon, Oxon ; New York: Routledge.

Bird, J. O., and C. T. F. Ross. 2020. *Mechanical Engineering Principles*. Fourth edition. Abingdon, Oxon: Routledge.

Bolton, W. 2021. *Engineering Science*. Seventh edition. Abingdon, Oxon ; New York, NY: Routledge.

Hannah, J., and M. J. Hillier. 1995. *Applied Mechanics*. 3rd edition. Harlow: Longman Pub Group.

Polya, G., and John H. Conway. 2014. *How to Solve It: A New Aspect of Mathematical Method*. With a Foreword by John H. Con ed. edition. Princeton Oxford: Princeton University Press.

Russell, Paul A., Leslie Jackson, and William Embleton. 2021. *Applied Mechanics for Marine Engineers*. 7th edition. Reeds Marine Engineering and Technology Series, vol. 2. London: Reeds.

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