Applied Mechanics 1 Formulae

Rules of Cosine and Sine

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Linear Motion

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{s} = \frac{\vec{u} + \vec{v}}{2}t$$

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v}^2 = \vec{u}^2 + 2\vec{a} \cdot \vec{s}$$

where:

- $\vec{u}$: Initial velocity
- $\vec{v}$: Final velocity
- $\vec{s}$: Displacement
- $\vec{a}$: Acceleration
- t: Time

Angular Motion

$$\omega_2 = \omega_1 \mp \alpha t$$

$$\theta = \frac{\omega_1 + \omega_2}{2} t$$

$$\theta = \omega_1 t \mp \frac{1}{2} \alpha t^2$$

$$\omega_2^2 = \omega_1^2 \mp 2\alpha\theta$$

where:

- ω_1: Initial angular velocity (rad/s)
- ω_2: Final angular velocity (rad/s)
- θ Angular displacement (rad)
- α: Angular acceleration (rad/s^2)
- t: Time (s)

Relation Between Linear and Angular Motion

The relationship between linear and angular motion is described by the following equations:

$s = r\theta$ (linear displacement s and angular displacement θ).

$v = r\omega$ (linear velocity v and angular velocity ω),

$a = r\alpha$ (linear acceleration a and angular acceleration α).

Conditions of Equilibrium for Simple Beams

Sum of Horizontal Forces	Sum of Vertical Force	Sum of Moments
$\sum F_x = 0$	$\sum F_y = 0$	$\sum M = 0$

Centre of Gravity

$$\bar{x} = \frac{\sum Moments\ of\ Weights}{\sum Weights} \qquad \bar{y} = \frac{\sum Moments\ of\ Weights}{\sum Weights}$$

Centroid

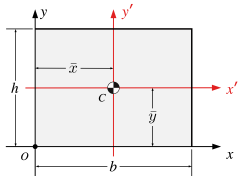
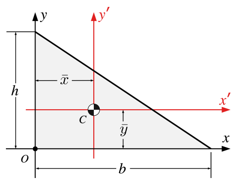
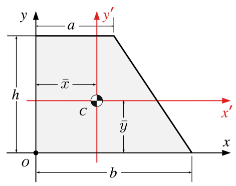
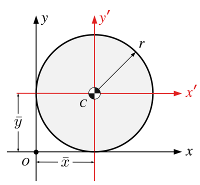
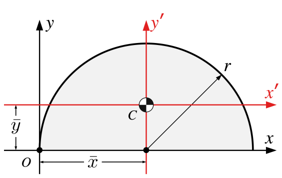
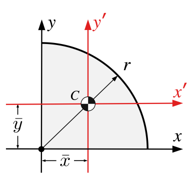
$$\bar{x} = \frac{\sum \bar{x}_i\ A_i}{\sum A_i} \qquad \bar{y} = \frac{\sum \bar{y}_i\ A_i}{\sum A_i}$$

Parallel Axis Theorem

To find the moment of inertia about an axis parallel to the centroidal axis:

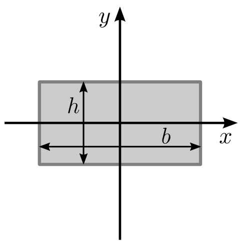
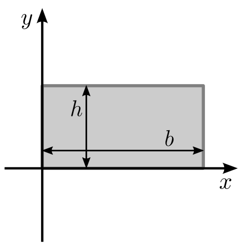
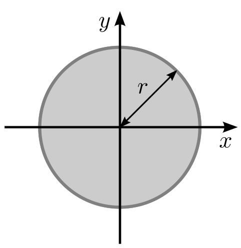
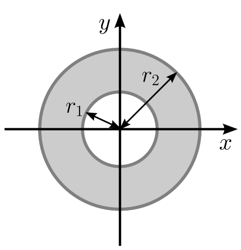
$$I = I_c + Ad^2$$

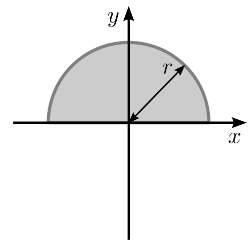
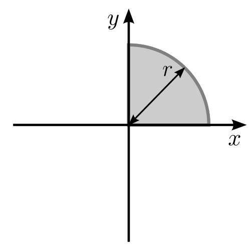
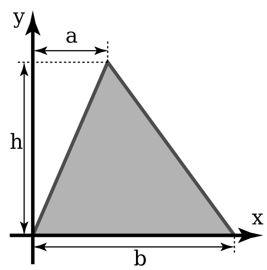
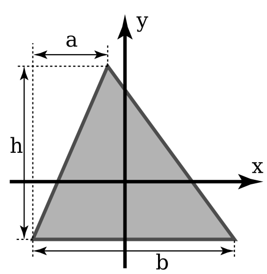
Table 2: Centroids of Common Shapes

Shape	Area	$\bar{x}$	$\bar{y}$
	$A = bh$	$b/2$	$h/2$
	$\dfrac{bh}{2}$	$b/3$	$h/3$
	$\dfrac{(a+b)h}{2}$	$\dfrac{a^2+ab+b^2}{3(a+b)}$	$\dfrac{h(2a+b)}{3(a+b)}$
	πr^2	r	r
	$\dfrac{\pi r^2}{2}$	r	$\dfrac{4r}{3\pi}$
	$\dfrac{\pi r^2}{4}$	$\dfrac{4r}{3\pi}$	$\dfrac{4r}{3\pi}$

Second Moments of Common Shapes

Table 3: Second moments

Shape	Second moment (I_x)	Second moment (I_y)
	$I_x = \frac{1}{12}bh^3$	$I_y = \frac{1}{12}b^3h$
	$I_x = \frac{1}{3}bh^3$	$I_y = \frac{1}{3}b^3h$
	$I_x = \frac{\pi}{4}r^4$	$I_y = \frac{\pi}{4}r^4$
	$I_x = \frac{\pi}{4}(r_2^4 - r_1^4)$	$I_y = \frac{\pi}{4}(r_2^4 - r_1^4)$

Shape	Second moment (I_x)	Second moment (I_y)
	$I_x = \frac{\pi}{8} r^4$	$I_y = \frac{\pi}{8} r^4$
	$I_x = \frac{\pi}{16} r^4$	$I_y = \frac{\pi}{16} r^4$
	$I_x = \frac{1}{12} b h^3$	
	$I_x = \frac{1}{36} b h^3$	