Applied Mechanics

Workbook

Serhat Beyenir

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Preface

This workbook presents a collection of lecture notes on Applied Mechanics, designed to provide learners with succinct yet essential insights into key topics covered in class. Each chapter is accompanied by a problem set to facilitate comprehension and reinforce understanding.

Chapter 1: The International System of Units (SI) is the globally accepted standard for measurement. Established to provide a consistent framework for scientific and technical measurements, SI units facilitate clear communication and data comparison across various fields and countries. The system is based on seven fundamental units: the meter for length, the kilogram for mass, the second for time, the ampere for electric current, the kelvin for temperature, the mole for substance, and the candela for luminous intensity.

Chapter 2: In mathematics and physics, a **scalar** is a quantity with only magnitude (size), whereas a **vector** has both magnitude and direction. Examples of scalar quantities include numbers, mass, speed, temperature, volume, and time. In contrast, examples of vector quantities include velocity, acceleration, and forces like weight and friction.

1 International System of Units

1.1 Objectives

- Recall the based and derived units.
- Practice the application of unity fraction.

1.2 SI Units

The International System of Units (SI) is the globally accepted standard for measurement. Established to provide a consistent framework for scientific and technical measurements, SI units facilitate clear communication and data comparison across various fields and countries. The system is based on seven fundamental units: the meter for length, the kilogram for mass, the second for time, the ampere for electric current, the kelvin for temperature, the mole for substance, and the candela for luminous intensity.

Table 1.1: Base SI units.

Physical Quantity	SI Base Unit	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric Current	Ampere	A
Temperature	Kelvin	K

Physical Quantity	SI Base Unit	Symbol
Amount of Substance	Mole	mol
Luminous Intensity	Candela	cd

Table 1.2: Derived SI units.

Physical Quantity	Derived SI Unit	Symbol
Area	Square meter	m^2
Volume	Cubic meter	m^3
Speed	Meter per second	m/s
Acceleration	Meter per second squared	m/s^2
Force	Newton	N
Pressure	Pascal	Pa
Energy	Joule	J
Power	Watt	W
Electric Charge	Coulomb	C
Electric Potential	Volt	V
Resistance	Ohm	Ω
Capacitance	Farad	F
Frequency	Hertz	Hz
Luminous Flux	Lumen	lm
Illuminance	Lux	lx
Specific Energy	Joule per kilogram	J/kg
Specific Heat Capacity	Joule per kilogram Kelvin	$J/(kg \cdot K)$

Table 1.3: Common multiples and submultiples for SI units.

Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M

Factor	Prefix	Symbol
10^3	kilo	k
10^2	hecto	h
10^1	deca	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	µ

1.3 Unity Fraction

The **unity fraction** method, or **unit conversion using unity fractions**, is a systematic way to convert one unit of measurement into another. This method relies on multiplying by fractions that are equal to one, where the numerator and the denominator represent the same quantity in different units. Since any number multiplied by one remains the same, unity fractions allow for seamless conversion without changing the value.

1.3.1 How Unity Fraction Works

The principle of unity fractions is based on:

1. **Setting up equal values**: Write a fraction where the numerator and denominator are equivalent values in different units, so the fraction equals one. For example, $\frac{1km}{1000m}$ is a unity fraction because 1 km equals 1000 m.
2. **Multiplying by unity fractions**: Multiply the initial quantity by the unity fraction(s) so that the undesired units cancel out, leaving only the desired units.

1.3.2 Example of Unity Fraction in Action

Suppose we want to convert 5 kilometers to meters.

1. Start with 5 kilometers:
$$5\,\mathrm{km}$$
2. Multiply by a unity fraction that cancels kilometers and introduces meters. We use $(\frac{1000\,\mathrm{m}}{1\,\mathrm{km}}), since\, 1\,\mathrm{km} = 1000\,\mathrm{m}$:
$$5\,\mathrm{km} \times \frac{1000\,\mathrm{m}}{1\,\mathrm{km}} = 5000\,\mathrm{m}$$
3. The kilometers km cancel out, leaving us with meters m:
$$5\,\mathrm{km} = 5000\,\mathrm{m}$$

This step-by-step approach illustrates how the unity fraction cancels the undesired units and achieves the correct result in meters.

Unity fractions can be extended by using multiple conversion steps. For example, converting hours to seconds would require two unity fractions: one to convert hours to minutes and another to convert minutes to seconds. This approach ensures accuracy and is widely used in science, engineering, and other fields that require precise unit conversions.

1.4 Problem Set

Instructions:

1. Use unity fractions to convert between derived SI units.
2. Show each step of your work to ensure accuracy.

3. Simplify your answers and include correct units.

1.4.1 Example Problem

Convert 15 m/s to km/h.

Solution:

1. Start with 15 m/s.
2. To convert meters to kilometers, multiply by $\frac{1\,\text{km}}{1000\,\text{m}}$.
3. To convert seconds to hours, multiply by $\frac{3600\,\text{s}}{1\,\text{h}}$.

$$15\,\text{m/s} \times \frac{1\,\text{km}}{1000\,\text{m}} \times \frac{3600\,\text{s}}{1\,\text{h}} = 54\,\text{km/h}$$

The meters and seconds cancel out, leaving kilometers per hour: 54 km/h.

1.4.2 Practice Problems

Convert each of the following using unity fractions. Show all steps!

1. **Speed**
   Convert 72 km/h to m/s.

2. **Force**
   Convert 980 N (newtons) to $\text{kg} \cdot \text{m/s}^2$.

3. **Energy**
   Convert 2500 J (joules) to kJ.

4. **Power**
   Convert 1500 W (watts) to kW.

5. **Pressure**
   Convert 101325 Pa (pascals) to kPa.

6. **Volume Flow Rate**
   Convert $3\,\mathrm{m}^3/\mathrm{min}$ to L/s.

7. **Density**
   Convert $1000\,\mathrm{kg/m}^3$ to $\mathrm{g/cm}^3$.

8. **Acceleration**
   Convert $9.8\,\mathrm{m/s}^2$ to $\mathrm{cm/s}^2$.

9. **Torque**
   Convert $50\,\mathrm{N\cdot m}$ to $\mathrm{kN\cdot cm}$.

10. **Frequency**
    Convert 500 Hz (hertz) to kHz.

1.4.3 Challenge Problems

1. **Work to Energy Conversion**
   A force of 20 N moves an object 500 cm. Convert the work done to joules.

2. **Kinetic Energy Conversion**
   Calculate the kinetic energy in kilojoules of a 1500 kg car moving at 72 km/h.

3. **Power to Energy Conversion**
   A machine operates at 2 kW for 3 hours. Convert the energy used to megajoules.

4. **Pressure to Force Conversion**
Convert a pressure of $200\,\mathrm{kPa}$ applied to an area of $0.5\,\mathrm{m}^2$ to force in newtons.

5. **Density to Mass Conversion**
Convert $0.8\,\mathrm{g/cm}^3$ for an object with a volume of $250\,\mathrm{cm}^3$ to mass in grams.

1.4.4 Answer Key

1. $72\,\mathrm{km/h} = 20\,\mathrm{m/s}$
2. $980\,\mathrm{N} = 980\,\mathrm{kg}\cdot\mathrm{m/s}^2$
3. $2500\,\mathrm{J} = 2.5\,\mathrm{kJ}$
4. $1500\,\mathrm{W} = 1.5\,\mathrm{kW}$
5. $101325\,\mathrm{Pa} = 101.325\,\mathrm{kPa}$
6. $3\,\mathrm{m}^3/\mathrm{min} = 50\,\mathrm{L/s}$
7. $1000\,\mathrm{kg/m}^3 = 1\,\mathrm{g/cm}^3$
8. $9.8\,\mathrm{m/s}^2 = 980\,\mathrm{cm/s}^2$
9. $50\,\mathrm{N}\cdot\mathrm{m} = 0.5\,\mathrm{kN}\cdot\mathrm{cm}$
10. $500\,\mathrm{Hz} = 0.5\,\mathrm{kHz}$

Challenge Problems

1. $20\,\mathrm{N} \times 5\,\mathrm{m} = 100\,\mathrm{J}$
2. Kinetic energy $= 1500\,\mathrm{kg} \times (20\,\mathrm{m/s})^2/2 = 300\,\mathrm{kJ}$
3. $2\,\mathrm{kW} \times 3\,\mathrm{hours} = 21.6\,\mathrm{MJ}$
4. $200\,\mathrm{kPa} \times 0.5\,\mathrm{m}^2 = 100,000\,\mathrm{N}$
5. $0.8\,\mathrm{g/cm}^3 \times 250\,\mathrm{cm}^3 = 200\,\mathrm{g}$

1.5 Further Reading

Introduction in Russell, Jackson, and Embleton (2021) and SI units in Bolton (2021) for additional information.

2 Scalar and Vector Quantities

2.1 Objectives

- Distinguish between scalar and vector quantities.
- Define equilibrium of an object.
- Practice vector problems.

2.2 Definitions

In mathematics and physics, a **scalar** is a quantity with only magnitude (size), whereas a **vector** has both magnitude and direction. Examples of scalar quantities include numbers, mass, speed, temperature, volume, and time. In contrast, examples of vector quantities include velocity, acceleration, and forces like weight and friction.

A vector is represented as an arrow, where the length denotes its magnitude, and the arrowhead indicates its direction. Vector diagrams are useful for visually conveying information about vector relationships, allowing us to analyze effects like the combination of forces or the movement of objects in two or three dimensions. In such diagrams, vectors are often drawn to scale and placed with respect to an origin or another reference point.

2.2.1 Coplanar Forces

Coplanar forces are forces that act within the same plane. This means they lie along a single, flat surface, and their lines of action do not extend out of that plane. Coplanar forces are often analyzed together in physics and engineering because their combined effects can be resolved within two dimensions.

2.2.2 Space Diagrams

The space diagram is an illustration of the system of forces.

2.2.3 Vector Diagrams

The vector diagram is a diagram drawn to scale with the vectors joined end to end. Vector diagrams are used to analyze and combine forces to find the resultant.

2.2.4 Resultant

The resultant is a single force that represents the combined effect of two or more forces acting on an object. It has the same effect as applying all the original forces together and is found by adding the individual forces, taking both their magnitudes and directions into account. The resultant gives the overall direction and magnitude of the combined forces.

2.2.5 Equilibrium

Equilibrium of an object occurs when all the forces acting on it are balanced, so the object remains at rest or moves at a constant speed in a straight line. In equilibrium, there is no net force or acceleration, meaning the object is in a stable state without any change in its motion.

Condition of equilibrium:

1. Net force must be zero: $\sum F = 0$
2. Net torque must be zero: $\sum T = 0$

2.3 Problem Set

1. Calculate the magnitude and direction of the resultant of the 1.7 kN and 2.9 kN forces, which are aligned along the same line and act in the same direction.
2. Calculate the magnitude and direction of the resultant of the 457 N and 583 N forces, which are aligned along the same line but act in opposite directions.
3. Use the triangle of forces method to find the magnitude and direction of the resultant force from a 14 N force acting at 0° and a 23 N force acting at 35°.

2.3.1 Answer Key

1. Magnitude of the resultant: 4.6 kN. Direction: Same as the direction of the individual forces.

2. Magnitude of the resultant: 126 N. Direction: Same as the direction of the 583 N force.
3. The magnitude of the resultant force is approximately 35.39 N, and its direction is approximately 21.80° above the positive x-axis.

2.4 Further Reading

Chapter 1 in Russell, Jackson, and Embleton (2021) and Chapter 2 in Bolton (2021) for additional information.

3 Summary

In summary, we have used several books by Ahrens (2022), Russell, Jackson, and Embleton (2021), Bolton (2021), Polya and Conway (2014), Bird and Ross (2020) and Bird (2021)

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