Applied Mechanics 1 Formulae

Rules of Cosine and Sine

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Linear Motion

$$\vec{v} = \vec{u} + \vec{a}t$$

$$\vec{s} = \frac{\vec{u} + \vec{v}}{2}t$$

$$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$$\vec{v}^2 = \vec{u}^2 + 2\vec{a} \cdot \vec{s}$$

where:

- $\vec{u}$: Initial velocity
- $\vec{v}$: Final velocity
- $\vec{s}$: Displacement
- $\vec{a}$: Acceleration
- t: Time

Angular Motion

$$\omega_2 = \omega_1 \mp \alpha t$$

$$\theta = \frac{\omega_1 + \omega_2}{2}t$$

$$\theta = \omega_1 t \mp \frac{1}{2}\alpha t^2$$

$$\omega_2^2 = \omega_1^2 \mp 2\alpha\theta$$

where:

- ω_1: Initial angular velocity (rad/s)
- ω_2: Final angular velocity (rad/s)
- θ Angular displacement (rad)
- α: Angular acceleration (rad/s^2)
- t: Time (s)

Relation Between Linear and Angular Motion

The relationship between linear and angular motion is described by the following equations:

$s = r\theta$ (linear displacement s and angular displacement θ).

$v = r\omega$ (linear velocity v and angular velocity ω),

$a = r\alpha$ (linear acceleration a and angular acceleration α).

Centre of Gravity

$$\bar{x} = \frac{\sum Moments\ of\ Weights}{\sum Weights} \qquad \bar{y} = \frac{\sum Moments\ of\ Weights}{\sum Weights}$$

Centroid

$$\bar{x} = \frac{\sum \bar{x}_i\ A_i}{\sum A_i} \qquad \bar{y} = \frac{\sum \bar{y}_i\ A_i}{\sum A_i}$$

Parallel Axis Theorem

To find the moment of inertia about an axis parallel to the centroidal axis:

$$I = I_c + Ad^2$$

Radius of Gyration

$$k = \sqrt{\frac{I}{A}} \quad \text{(for area)} \quad \text{or} \quad k = \sqrt{\frac{I}{m}} \quad \text{(for mass)},$$

where:

- I: Moment of inertia about the axis
- A: Area of the cross-section (for area calculations)
- m: Mass of the body (for mass calculations)

Rectangle (about its centroidal axis)

- Dimensions: (b) (breadth), (h) (height)
- Radius of gyration about the centroidal x-axis:

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{\frac{1}{12}bh^3}{bh}} = \frac{h}{\sqrt{12}}$$

Circle (about its centroidal axis)

- Radius: (r)
- Radius of gyration:

$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi r^4}{4}}{\pi r^2}} = \frac{r}{\sqrt{2}}$$

Beam Calculations

Sum of Horizontal Forces	Sum of Vertical Force	Sum of Moments
$\sum F_x = 0$	$\sum F_y = 0$	$\sum M = 0$

Load Type	**Shear Diagram Shape**	**Moment Diagram Shape**
Point Load	Rectangular (constant)	Triangular
Uniformly Distributed Load (UDL)	Triangular	Parabolas (second degree)

Dynamics

Linear momentum

$$Linear\ momentum = m\vec{v}$$

Where:

- Linear momentum is in $\text{kg} \cdot \text{m/s}$.
- m is the mass of the object in kilograms.
- $\vec{v}$ is the velocity of the object in meters per second.

Angular momentum

$$\text{Angular momentum} = I\omega$$

Where:

- I is the moment of inertia in m^4.
- ω is the angular velocity in rad/s.

Moment of inertia

$$I = mk^2$$

Where:

- I is the moment of inertia in m^4.
- m is the mass in kg.
- k is the radius of gyration in m.

Turning moment

$$\tau = I\alpha$$

Where:

- τ is the torque in Nm.
- I is the moment of inertia in m^4.
- α: Angular acceleration in rad/s^2.

Power by Torque

$$P = \tau \cdot \omega$$

Where:

P is the power in watts (W),

τ is the torque in newton-meters (Nm), and

ω is the angular velocity in radians per second (rad/s).

Kinetic Energy of Rotation

$$\text{Rotational K.E.} = \frac{1}{2} I \omega^2$$

Where:

- I is the moment of inertia in m^4.
- ω is the angular velocity in radians per second (rad/s).

Stress and Strain

Stress

$$\sigma = \frac{F}{A}$$

$$\tau = \frac{F}{A}$$

Where:

- σ is the stress (Pa),
- τ is the shearing stress (Pa),
- F is the shearing force (N),
- A is the cross-sectional area (m^2).

Strain

$$\varepsilon = \frac{\Delta L}{L_0}$$

Where:

- ε is the strain (unitless),
- ΔL is the change in length,
- L_0 is the original length.

Hooke's Law

$$E = \frac{\sigma}{\varepsilon}$$

Where:

- σ is the stress (Pa).
- ε is the strain (unitless).
- E: Young's modulus (Pa), a material property (modulus of elasticity).

Factor of Safety (FOS)

$$\text{FOS} = \frac{\text{Breaking Stress}}{\text{Working Stress}}$$

Hydrodynamics

Volume Flow

$$\dot{v} = A \cdot C$$

Where:

$\dot{v}$: Volume flow rate, m^3/s

A: Cross-sectional area of the flow, m^2

C: Mean (average) velocity of the fluid, m/s

Mass Flow

$$\dot{m} = \rho \cdot \dot{v}$$

Where:

$\dot{m}$: mass flow rate, kg/s

ρ: density, kg/m^3

$\dot{v}$: volume flow, m^3/s

Specific Weight

$$\gamma = g \cdot \rho$$

Where:

γ: specific weight, N/m^3

g: gravitational acceleration, m/s^2

ρ: density, kg/m^3

Continuity Equation

$$A_1 \cdot C_1 = A_2 \cdot C_2$$

Energy Equation

$$Z_1 + \frac{C_1^2}{2g} + \frac{P_1}{g\rho_1} = Z_2 + \frac{C_2^2}{2g} + \frac{P_2}{g\rho_2}$$

$$Z_1 + \frac{C_1^2}{2g} + \frac{P_1}{\gamma_1} = Z_2 + \frac{C_2^2}{2g} + \frac{P_2}{\gamma_2}$$

Each term has units of m, therefore:

- Potential energy Z is known as the elevation head.
- Kinetic energy $\frac{c^2}{2g}$ is known as the velocity head.
- Pressure energy $\frac{P}{\gamma}$ is known as the pressure head.

$$Total\ Head = Elevation\ Head + Velocity\ Head + Pressure\ Head$$

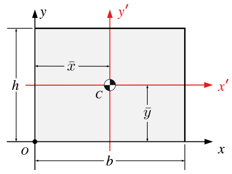
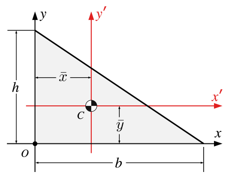
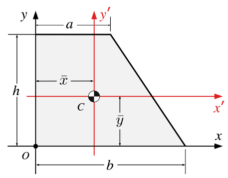
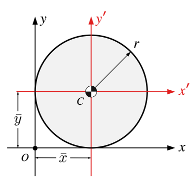
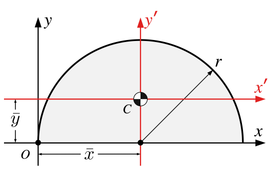
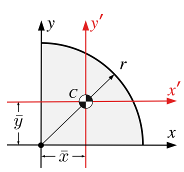
Bernoulli's Equation

$$P_1 + \frac{1}{2}\rho C_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho C_2^2 + \rho g h_2$$

Where:

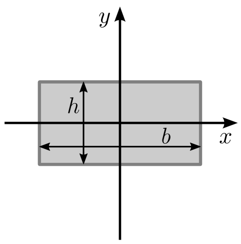
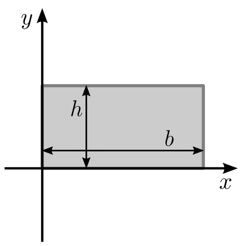
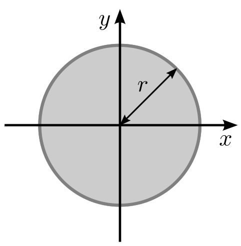
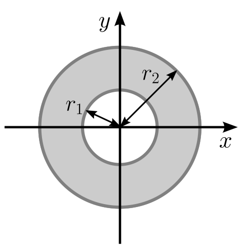
- P_1 and P_2 are the pressures at points 1 and 2, respectively.
- ρ is the density of the fluid.
- C_1 and C_2 are the velocities of the fluid at points 1 and 2, respectively.
- g is the acceleration due to gravity.
- h_1 and h_2 are the heights of the fluid at points 1 and 2, respectively.

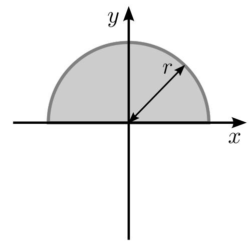
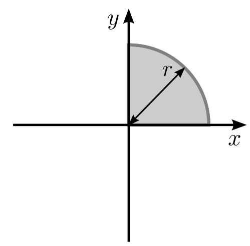
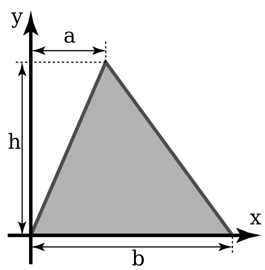
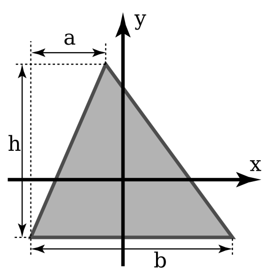
Table 3: Centroids of Common Shapes

<table>
<tr><th>Shape</th><th>Area</th><th>$\bar{x}$</th><th>$\bar{y}$</th></tr>
<tr><td>
</td><td>$A = bh$</td><td>$b/2$</td><td>$h/2$</td></tr>
<tr><td>
</td><td>$\dfrac{bh}{2}$</td><td>$b/3$</td><td>$h/3$</td></tr>
<tr><td>
</td><td>$\dfrac{(a+b)h}{2}$</td><td>$\dfrac{a^2+ab+b^2}{3(a+b)}$</td><td>$\dfrac{h(2a+b)}{3(a+b)}$</td></tr>
<tr><td>
</td><td>πr^2</td><td>r</td><td>r</td></tr>
<tr><td>
</td><td>$\dfrac{\pi r^2}{2}$</td><td>r</td><td>$\dfrac{4r}{3\pi}$</td></tr>
<tr><td>
</td><td>$\dfrac{\pi r^2}{4}$</td><td>$\dfrac{4r}{3\pi}$</td><td>$\dfrac{4r}{3\pi}$</td></tr>
</table>

Second Moments of Common Shapes

Table 4: Second moments

Shape	Second moment (I_x)	Second moment (I_y)
	$I_x = \frac{1}{12}bh^3$	$I_y = \frac{1}{12}b^3h$
	$I_x = \frac{1}{3}bh^3$	$I_y = \frac{1}{3}b^3h$
	$I_x = \frac{\pi}{4}r^4$	$I_y = \frac{\pi}{4}r^4$
	$I_x = \frac{\pi}{4}(r_2^4 - r_1^4)$	$I_y = \frac{\pi}{4}(r_2^4 - r_1^4)$

Shape	Second moment (I_x)	Second moment (I_y)
	$I_x = \frac{\pi}{8} r^4$	$I_y = \frac{\pi}{8} r^4$
	$I_x = \frac{\pi}{16} r^4$	$I_y = \frac{\pi}{16} r^4$
	$I_x = \frac{1}{12} b h^3$	
	$I_x = \frac{1}{36} b h^3$	