Applied Mechanics

Workbook

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Preface

This workbook presents a collection of lecture notes on Applied Mechanics, designed to provide learners with succinct yet essential insights into key topics covered in class. Each chapter is accompanied by a problem set to facilitate comprehension and reinforce understanding.

Chapter 1: The International System of Units (SI) is the globally accepted standard for measurement. Established to provide a consistent framework for scientific and technical measurements, SI units facilitate clear communication and data comparison across various fields and countries. The system is based on seven fundamental units: the meter for length, the kilogram for mass, the second for time, the ampere for electric current, the kelvin for temperature, the mole for substance, and the candela for luminous intensity.

Chapter 2: In mathematics and physics, a **scalar** is a quantity with only magnitude (size), whereas a **vector** has both magnitude and direction. Examples of scalar quantities include numbers, mass, speed, temperature, volume, and time. In contrast, examples of vector quantities include velocity, acceleration, and forces like weight and friction.

1 International System of Units

1.1 Objectives

- Recall the based and derived units.
- Practice the application of unity fraction.

1.2 SI Units

The International System of Units (SI) is the globally accepted standard for measurement. Established to provide a consistent framework for scientific and technical measurements, SI units facilitate clear communication and data comparison across various fields and countries. The system is based on seven fundamental units: the meter for length, the kilogram for mass, the second for time, the ampere for electric current, the kelvin for temperature, the mole for substance, and the candela for luminous intensity.

Table 1.1: Base SI units.

Physical Quantity	SI Base Unit	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric Current	Ampere	A
Temperature	Kelvin	K

Physical Quantity	SI Base Unit	Symbol
Amount of Substance	Mole	mol
Luminous Intensity	Candela	cd

Table 1.2: Derived SI units.

Physical Quantity	Derived SI Unit	Symbol
Area	Square meter	m^2
Volume	Cubic meter	m^3
Speed	Meter per second	m/s
Acceleration	Meter per second squared	m/s^2
Force	Newton	N
Pressure	Pascal	Pa
Energy	Joule	J
Power	Watt	W
Electric Charge	Coulomb	C
Electric Potential	Volt	V
Resistance	Ohm	Ω
Capacitance	Farad	F
Frequency	Hertz	Hz
Luminous Flux	Lumen	lm
Illuminance	Lux	lx
Specific Energy	Joule per kilogram	J/kg
Specific Heat Capacity	Joule per kilogram Kelvin	$J/(kg \cdot K)$

Table 1.3: Common multiples and submultiples for SI units.

Factor	Prefix	Symbol
10^9	giga	G
10^6	mega	M

Factor	Prefix	Symbol
10^{3}	kilo	k
10^{2}	hecto	h
10^{1}	deca	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	µ

1.3 Unity Fraction

The **unity fraction** method, or **unit conversion using unity fractions**, is a systematic way to convert one unit of measurement into another. This method relies on multiplying by fractions that are equal to one, where the numerator and the denominator represent the same quantity in different units. Since any number multiplied by one remains the same, unity fractions allow for seamless conversion without changing the value.

1.3.1 How Unity Fraction Works

The principle of unity fractions is based on:

1. **Setting up equal values**: Write a fraction where the numerator and denominator are equivalent values in different units, so the fraction equals one. For example, $\frac{1km}{1000m}$ is a unity fraction because 1 km equals 1000 m.

2. **Multiplying by unity fractions**: Multiply the initial quantity by the unity fraction(s) so that the undesired units cancel out, leaving only the desired units.

1.3.2 Example of Unity Fraction in Action

Suppose we want to convert 5 kilometers to meters.

1. Start with 5 kilometers:
$$5\,\mathrm{km}$$
2. Multiply by a unity fraction that cancels kilometers and introduces meters. We use $(\frac{1000\,\mathrm{m}}{1\,\mathrm{km}}), since\ 1\,\mathrm{km} = 1000\,\mathrm{m}$:
$$5\,\mathrm{km} \times \frac{1000\,\mathrm{m}}{1\,\mathrm{km}} = 5000\,\mathrm{m}$$
3. The kilometers km cancel out, leaving us with meters m:
$$5\,\mathrm{km} = 5000\,\mathrm{m}$$

This step-by-step approach illustrates how the unity fraction cancels the undesired units and achieves the correct result in meters.

Unity fractions can be extended by using multiple conversion steps. For example, converting hours to seconds would require two unity fractions: one to convert hours to minutes and another to convert minutes to seconds. This approach ensures accuracy and is widely used in science, engineering, and other fields that require precise unit conversions.

1.4 Problem Set

Instructions:

1. Use unity fractions to convert between derived SI units.
2. Show each step of your work to ensure accuracy.

3. Simplify your answers and include correct units.

1.4.1 Example Problem

Convert 15 m/s to km/h.

Solution:

1. Start with 15 m/s.
2. To convert meters to kilometers, multiply by $\frac{1\,\mathrm{km}}{1000\,\mathrm{m}}$.
3. To convert seconds to hours, multiply by $\frac{3600\,\mathrm{s}}{1\,\mathrm{h}}$.

$$15\,\mathrm{m/s} \times \frac{1\,\mathrm{km}}{1000\,\mathrm{m}} \times \frac{3600\,\mathrm{s}}{1\,\mathrm{h}} = 54\,\mathrm{km/h}$$

The meters and seconds cancel out, leaving kilometers per hour: 54 km/h.

1.4.2 Practice Problems

Convert each of the following using unity fractions. Show all steps!

1. **Speed**
   Convert 72 km/h to m/s.
2. **Force**
   Convert 980 N (newtons) to $\mathrm{kg} \cdot \mathrm{m/s}^2$.
3. **Energy**
   Convert 2500 J (joules) to kJ.

4. **Power**
   Convert 1500 W (watts) to kW.
5. **Pressure**
   Convert 101325 Pa (pascals) to kPa.
6. **Volume Flow Rate**
   Convert $3\,\mathrm{m}^3/\mathrm{min}$ to L/s.
7. **Density**
   Convert $1000\,\mathrm{kg/m}^3$ to $\mathrm{g/cm}^3$.
8. **Acceleration**
   Convert $9.8\,\mathrm{m/s}^2$ to $\mathrm{cm/s}^2$.
9. **Torque**
   Convert $50\,\mathrm{N}\cdot\mathrm{m}$ to $\mathrm{kN}\cdot\mathrm{cm}$.
10. **Frequency**
    Convert 500 Hz (hertz) to kHz.

1.4.3 Challenge Problems

1. **Work to Energy Conversion**
   A force of 20 N moves an object 500 cm. Convert the work done to joules.
2. **Kinetic Energy Conversion**
   Calculate the kinetic energy in kilojoules of a 1500 kg car moving at 72 km/h.
3. **Power to Energy Conversion**
   A machine operates at 2 kW for 3 hours. Convert the energy used to megajoules.

4. **Pressure to Force Conversion**
   Convert a pressure of $200\,\mathrm{kPa}$ applied to an area of $0.5\,\mathrm{m}^2$ to force in newtons.

5. **Density to Mass Conversion**
   Convert $0.8\,\mathrm{g/cm}^3$ for an object with a volume of $250\,\mathrm{cm}^3$ to mass in grams.

1.4.4 Answer Key

1. $72\,\mathrm{km/h} = 20\,\mathrm{m/s}$
2. $980\,\mathrm{N} = 980\,\mathrm{kg \cdot m/s}^2$
3. $2500\,\mathrm{J} = 2.5\,\mathrm{kJ}$
4. $1500\,\mathrm{W} = 1.5\,\mathrm{kW}$
5. $101325\,\mathrm{Pa} = 101.325\,\mathrm{kPa}$
6. $3\,\mathrm{m}^3/\mathrm{min} = 50\,\mathrm{L/s}$
7. $1000\,\mathrm{kg/m}^3 = 1\,\mathrm{g/cm}^3$
8. $9.8\,\mathrm{m/s}^2 = 980\,\mathrm{cm/s}^2$
9. $50\,\mathrm{N \cdot m} = 0.5\,\mathrm{kN \cdot cm}$
10. $500\,\mathrm{Hz} = 0.5\,\mathrm{kHz}$

Challenge Problems

1. $20\,\mathrm{N} \times 5\,\mathrm{m} = 100\,\mathrm{J}$
2. Kinetic energy $= 1500\,\mathrm{kg} \times (20\,\mathrm{m/s})^2/2 = 300\,\mathrm{kJ}$
3. $2\,\mathrm{kW} \times 3\,\mathrm{hours} = 21.6\,\mathrm{MJ}$
4. $200\,\mathrm{kPa} \times 0.5\,\mathrm{m}^2 = 100,000\,\mathrm{N}$
5. $0.8\,\mathrm{g/cm}^3 \times 250\,\mathrm{cm}^3 = 200\,\mathrm{g}$

1.5 Further Reading

Introduction in Russell, Jackson, and Embleton (2021) and SI units in Bolton (2021) for additional information.

2 Scalar and Vector Quantities

2.1 Objectives

- Distinguish between scalar and vector quantities.
- Define equilibrium of an object.
- Practice vector problems.

2.2 Definitions

In mathematics and physics, a **scalar** is a quantity with only magnitude (size), whereas a **vector** has both magnitude and direction. Examples of scalar quantities include numbers, mass, speed, temperature, volume, and time. In contrast, examples of vector quantities include velocity, acceleration, and forces like weight and friction.

A vector is represented as an arrow, where the length denotes its magnitude, and the arrowhead indicates its direction. Vector diagrams are useful for visually conveying information about vector relationships, allowing us to analyze effects like the combination of forces or the movement of objects in two or three dimensions. In such diagrams, vectors are often drawn to scale and placed with respect to an origin or another reference point.

2.2.1 Coplanar Forces

Coplanar forces are forces that act within the same plane. This means they lie along a single, flat surface, and their lines of action do not extend out of that plane. Coplanar forces are often analyzed together in physics and engineering because their combined effects can be resolved within two dimensions.

2.2.2 Space Diagrams

The space diagram is an illustration of the system of forces.

2.2.3 Vector Diagrams

The vector diagram is a diagram drawn to scale with the vectors joined end to end. Vector diagrams are used to analyze and combine forces to find the resultant.

2.2.4 Resultant

The resultant is a single force that represents the combined effect of two or more forces acting on an object. It has the same effect as applying all the original forces together and is found by adding the individual forces, taking both their magnitudes and directions into account. The resultant gives the overall direction and magnitude of the combined forces.

2.2.5 Equilibrium

Equilibrium of an object occurs when all the forces acting on it are balanced, so the object remains at rest or moves at a constant speed in a straight line. In equilibrium, there is no net force or acceleration, meaning the object is in a stable state without any change in its motion.

Condition of equilibrium:

1. Net force must be zero: $\sum F = 0$
2. Net torque must be zero: $\sum T = 0$

2.2.6 Bow's Notation

Bow's Notation is a way to label forces in a system by marking spaces in the diagram with capital letters (A, B, C, etc.). Each force is named after the two spaces it separates, like force AB or force BC. In the vector diagram, each force's vector is labeled at both ends with matching lowercase letters (e.g., ah, bc) in the direction of the arrow.

2.2.7 Slings

A sling is a device or assembly of ropes, cables, or straps used to support and lift loads. Slings play a crucial role in rigging operations, allowing objects to be lifted, lowered, or moved safely and efficiently. Slings are arranged to distribute the load evenly across their length, reducing stress points and ensuring stability. In multi-leg slings (e.g., two-leg or four-leg), the load is shared among the sling legs, which helps stabilize and balance the load.

2.2.8 Jib Cranes

A simple jib crane has a vertical post, a jib, and a tie. The jib is hinged at its lower end to the post, and the tie connects the top of the jib to the base of the post, forming the crane head where the tie and jib meet.

When a load is hung directly from the crane head, solving for forces involves a simple triangle of forces. In other cases, the crane may have a pulley at the head, with a rope running over it to a winch, creating a system with more than three forces.

2.2.9 Example 1

A vertical lifting force of 95 N is applied to a body, and simultaneously, a horizontal force of 135 N pulls on it. Determine the magnitude and direction of the resulting force.

To solve for the magnitude and direction of the resultant force, we can use vector addition.

Given:

- Vertical force, $F_v = 95\,\mathrm{N}$
- Horizontal force, $F_h = 135\,\mathrm{N}$

Step 1: Calculate the Magnitude of the Resultant Force

The resultant force F_r is the vector sum of the vertical and horizontal forces. Using the Pythagorean theorem:

$$F_r = \sqrt{F_v^2 + F_h^2}$$

Substituting the values:

$$F_r = \sqrt{95^2 + 135^2}$$

Calculating further:

$$F_r = \sqrt{9025 + 18225} = \sqrt{27250}$$

Thus,

$$F_r \approx 165.07, \mathrm{N}$$

Step 2: Determine the Direction of the Resultant Force

The direction θ of the resultant force with respect to the horizontal can be found using the tangent function:

$$\theta = \arctan\left(\frac{F_v}{F_h}\right)$$

Substituting the values:

$$\theta = \arctan\left(\frac{95}{135}\right)$$

Calculating θ:

$$\theta \approx 35.07^\circ$$

Final Answer

The magnitude of the resultant force is approximately 165.07N, and its direction is 35.07° above the horizontal.

2.2.10 Example 2

Two forces act upon a body. One exerts a horizontal force to the right with a magnitude of 25 Newtons, while the other exerts a vertical force downward with a magnitude of 20 Newtons. Determine the magnitude and direction of a third force that would counteract the combined effects of the other two forces.

To determine the magnitude and direction of the third force that counteracts the combined effects of the two forces, we first need to find the resultant force of the two given forces.

Given Forces:

- Horizontal force to the right: $F_h = 25\,\mathrm{N}$
- Vertical force downward: $F_v = 20\,\mathrm{N}$

Step 1: Calculate the Resultant Force

The resultant force F_r can be found using the Pythagorean theorem since the forces are perpendicular to each other.

$$F_r = \sqrt{F_h^2 + F_v^2}$$

Substituting the values:

$$F_r = \sqrt{25^2 + 20^2}$$

Calculating:

$$F_r = \sqrt{625 + 400} = \sqrt{1025}$$

Thus,

$$F_r \approx 32.02, \mathrm{N}$$

Step 2: Determine the Direction of the Resultant Force

The direction θ of the resultant force can be found using the tangent function:

$$\theta = \arctan\left(\frac{F_v}{F_h}\right)$$

Substituting the values:

$$\theta = \arctan\left(\frac{20}{25}\right)$$

Calculating θ:

$$\theta \approx 38.66^\circ$$

This angle is measured from the horizontal axis (to the right) downward.

Step 3: Determine the Third Force

To counteract the resultant force, the third force F_3 must have the same magnitude as F_r but in the opposite direction. Therefore, its magnitude is:

$$F_3 = F_r \approx 32.02, \mathrm{N}$$

The direction of the third force will be opposite to the direction of the resultant force, which means it will be directed at an angle of:

$$\theta + 180^\circ \approx 38.66^\circ + 180^\circ \approx 218.66^\circ$$

Final Answer

The magnitude of the third force is approximately 32.02N , and its direction is approximately 218.66° (measured counterclockwise from the positive x-axis or horizontal right).

2.2.11 Example 3

Determine the magnitude and direction of the equilibrium force resulting from the combination of two forces: a horizontal pull of 15 N and a pull of 25 N at an angle of 55 degrees with respect to the 15 N force.

To find the magnitude and direction of the equilibrium force resulting from the combination of two forces (15 N horizontally and 25 N at an angle of 55 degrees), we can use vector addition.

Step 1: Resolve the Forces into Components

1. Force $F_1 = 15\text{N}$ (Horizontal):

$F_{1x} = 15\,\text{N}$

$F_{1y} = 0\,\text{N}$

2. Force $F_2 = 25\text{N}$ at 55°:

$F_{2x} = F_2 \cdot \cos(55^\circ) = 25 \cdot \cos(55^\circ)$

$F_{2y} = F_2 \cdot \sin(55^\circ) = 25 \cdot \sin(55^\circ)$

Step 2: Calculate Components of F_2

Using $\cos(55^\circ) \approx 0.5736$ and $\sin(55^\circ) \approx 0.8192$:

$$F_{2x} = 25 \cdot 0.5736 \approx 14.34\,\text{N}$$

$$F_{2y} = 25 \cdot 0.8192 \approx 20.48\,\mathrm{N}$$

Step 3: Find the Resultant Components

Now, we sum the components in the x and y directions:

- Resultant x-component:

$$R_x = F_{1x} + F_{2x} = 15 + 14.34 \approx 29.34\,\mathrm{N}$$

- Resultant y-component:

$$R_y = F_{1y} + F_{2y} = 0 + 20.48 \approx 20.48\,\mathrm{N}$$

Step 4: Calculate the Magnitude of the Resultant Force

The magnitude of the resultant force R can be found using the Pythagorean theorem:

$$R = \sqrt{R_x^2 + R_y^2}$$

Calculating R :

$$R = \sqrt{(29.34)^2 + (20.48)^2} \approx \sqrt{861.64 + 419.04} \approx \sqrt{1280.68} \approx 35.8\,\mathrm{N}$$

Step 5: Determine the Direction of the Resultant Force

The direction (angle θ of the resultant force can be found using the tangent function:

$$\tan(\theta) = \frac{R_y}{R_x}$$

$$\theta = \tan^{-1}\left(\frac{20.48}{29.34}\right)$$

Calculating θ:

$$\theta \approx \tan^{-1}(0.698) \approx 55.2^{\circ}$$

Final Answer

Magnitude of the Equilibrium Force: $\approx 35.8\,\text{N}$

Direction of the Equilibrium Force: $\approx 55.2^{\circ}$ above the horizontal (in the direction of the 15 N force).

2.2.12 Problem Set

1. Calculate the magnitude and direction of the resultant of the 1.7 kN and 2.9 kN forces, which are aligned along the same line and act in the same direction.
2. Calculate the magnitude and direction of the resultant of the 457 N and 583 N forces, which are aligned along the same line but act in opposite directions.
3. Use the triangle of forces method to find the magnitude and direction of the resultant force from a 14 N force acting at 0° and a 23 N force acting at 35°.

2.2.13 Answer Key

1. Magnitude of the resultant: 4.6 kN. Direction: Same as the direction of the individual forces.

2. Magnitude of the resultant: 126 N. Direction: Same as the direction of the 583 N force.

3. The magnitude of the resultant force is approximately 35.39 N, and its direction is approximately 21.80° above the positive x-axis.

2.3 Further Reading

Chapter 1 in Russell, Jackson, and Embleton (2021) and Chapter 2 in Bolton (2021) for additional information.

3 Summary

In summary, we have used several books by Ahrens (2022), Russell, Jackson, and Embleton (2021), Bolton (2021), Polya and Conway (2014), Bird and Ross (2020) and Bird (2021)

Chapter 1: The International System of Units (SI) is the globally accepted standard for measurement. Established to provide a consistent framework for scientific and technical measurements, SI units facilitate clear communication and data comparison across various fields and countries. The system is based on seven fundamental units: the meter for length, the kilogram for mass, the second for time, the ampere for electric current, the kelvin for temperature, the mole for substance, and the candela for luminous intensity.

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