Stress and Strain

Stress (σ)

Stress is defined as the internal force per unit area within a material that arises due to externally applied forces. It is given by:

$$\sigma = \frac{F}{A}$$

Where:

- σ is the stress (Pa),
- F is the applied force (N),
- A is the cross-sectional area (m^2).

In the case of direct tensile or compressive forces, the area carrying the force is the cross-section in the plane of the material normal (i.e. perpendicular) to the direction of the force.

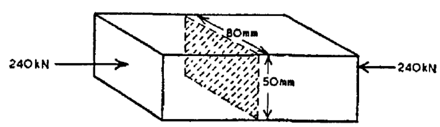

Figure 1: Compressive force

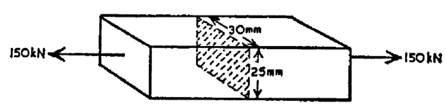

Figure 2: Tensile force

$$\tau = \frac{F}{A}$$

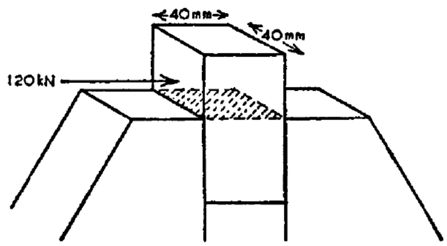

Figure 3: Shearing force

Where:

- τ is the shearing stress (Pa),
- F is the shearing force (N),
- A is the cross-sectional area (m^2).

Strain (ε)

Strain is a measure of deformation representing the relative change in the material's dimensions under stress. It is dimensionless since it is a ratio. Strain is defined as:

$$\varepsilon = \frac{\Delta L}{L_0}$$

Where:

- ε is the strain (unitless),
- ΔL is the change in length,
- L_0 is the original length.

Hooke's Law

For linear elastic materials, stress is proportional to strain:

For elastic materials:

$$\sigma = E\varepsilon$$

$$E = \frac{\sigma}{\varepsilon}$$

Where:

- σ is the stress (Pa).
- ε is the strain (unitless).
- E: Young's modulus (Pa), a material property (modulus of elasticity).

Factor of Safety (FOS)

The **Factor of Safety (FOS)** is a critical concept in engineering design and analysis. It provides a measure of the margin of safety in a structure or material under load. The FOS is defined as the ratio of the material's ultimate strength (breaking stress) to the actual applied stress (working stress).

$$\text{FOS} = \frac{\text{Breaking Stress}}{\text{Working Stress}}$$

- **Breaking Stress**: This refers to the maximum stress that a material can withstand before failure. For tensile stresses, this is called the tensile strength; for compressive stresses, it is the compressive strength.
- **Working Stress**: The stress experienced by the material under the given loading conditions.

Importance of FOS

1. **Safety**: A higher FOS ensures that the structure or material has a greater margin before failure, accounting for uncertainties in loading conditions, material properties, or fabrication methods.
2. **Economic Design**: While a higher FOS improves safety, it may lead to over-design and increased material costs. Engineers balance safety and cost efficiency to determine an appropriate FOS for each application.

Tensile Test

The **tensile test** is a mechanical test used to determine the mechanical properties of a material under axial tensile loading. The test involves pulling a specimen apart until it fractures while measuring the applied force and the resulting elongation.

Procedure

1. Prepare a standardized specimen with a known cross-sectional area, A_0, and gauge length, L_0.
2. Mount the specimen in the testing machine and apply an axial tensile force, F.
3. Record the force and elongation throughout the test until the specimen fractures.

Key Parameters

- **Stress (σ)**: Defined as the applied force per unit area:

  $$\sigma = \frac{F}{A_0}$$

  where:

  - F is the applied force.
  - A_0 is the initial cross-sectional area.

- **Strain (ϵ)**: Defined as the relative elongation of the specimen:

  $$\epsilon = \frac{\Delta L}{L_0}$$

  where:

  - ΔL is the change in length.
  - L_0 is the initial gauge length.

Stress-Strain Curve

The stress-strain curve Figure 4 illustrates the material's response to the applied force. Important points on the curve include:

1. **Elastic Region**: The material deforms elastically and returns to its original shape when the load is removed. Hooke's Law applies:

   $$\sigma = E\epsilon$$

   where E is the Young's modulus.

2. **Yield Point**: The stress at which the material begins to deform plastically.

3. **Ultimate Tensile Strength (UTS)**: The maximum stress the material can withstand.

4. **Fracture Point**: The point at which the material fails.

Ultimate Tensile Strength (UTS)

Ultimate Tensile Strength (UTS) is the maximum stress a material can endure before breaking. It is determined during a tensile test.

Formula:

$$\text{UTS} = \frac{F_{\text{max}}}{A_0}$$

Where:

- F_{max} is the maximum applied force (N),
- A_0 is the original cross-sectional area (m^2).

Key Points:

- **Units**: Measured in Pascals (Pa), commonly in MPa or GPa.
- **Ductile Materials**: Undergo significant plastic deformation after UTS.
- **Brittle Materials**: Break shortly after reaching UTS.

Below is a typical stress-strain curve showing the UTS point:

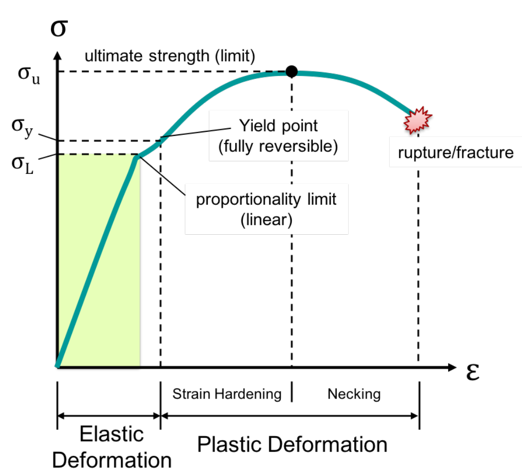

Figure 4: Stress–strain curve