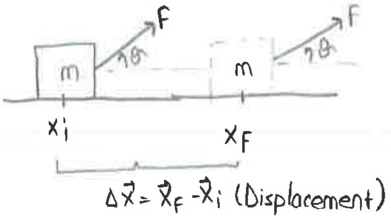
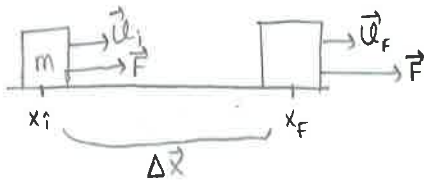
CHAPTER-6 (WORK and KINETIC ENERGY)

6.1 Work



$$W = \vec{F} \cdot \Delta\vec{X} = F \cdot \Delta X \cdot \cos\theta \quad (N.m)$$

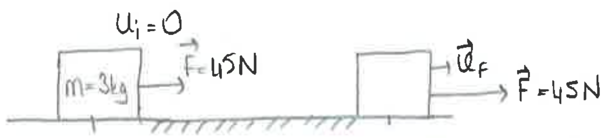
6.2 KE and Work-KE Relation



$$KE_i = \frac{1}{2} \cdot m \cdot v_i^2 \qquad KE_F = \frac{1}{2} \cdot m \cdot v_F^2$$

$$W_F = \Delta KE = KE_F - KE_i \quad \text{(Work-KE Relation)}$$

Ex :



$X_i = 0$ $\mu_K = 0.2$ $X_F = 30$ m in +X direction

a-) Find the $v_F = ?$

b-) Find the $W_F = ?$

c-) Find the $W_{F_K} = ?$

d-) Find the $W_{F_{net}} = ?$

Sol : a-) $F_{net} = F - f_K = m.a$

$= F - \mu_K . m . g = m.a \quad \Rightarrow 45\,N - 0.2 \times 3kg \times 10\frac{m}{s^2} = 3kg . a \qquad a = 13\ m/s^2$

$v_F^2 = v_i^2 + 2.a.\Delta X \quad \Rightarrow v_F^2 = 0^2 + 2 \times 13 m/s^2 \times 30m = 27.92\ m/s$

b-) $W_F = \vec{F} . \Delta\vec{X} = 45N . 30m . \cos 0° \Rightarrow W_F = 1350\ N.m$

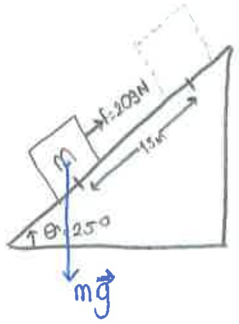
c-) $W_{F_K} = f_K . \Delta X . \cos 180° = 6N . 30m . -1 = -180\,Nm$

d-) $F_{net} = F - f_K = 45\,N - 6N = 39\,N \qquad W_{F_{net}} = \vec{F}_{net} \times \Delta\vec{X} = 39N \times 30m \times 1 = 1170\ Nm$

$W_{F_{net}} = W_F + W_{F_K} = 1170\ N.m$

$W_{F_{net}} = KE_F - KE_i = \frac{1}{2} m . v_F^2 - \frac{1}{2} m . \underbrace{v_i^2}_{0} = \frac{1}{2} \times 3kg \times (27.92 m)^2 \Rightarrow W_{F_{net}} = 1170\ Nm$

Ex-)



$m = 25\ kg$

As the crate slides 1.5m up, how much work is done on the crate?

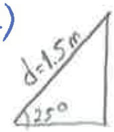
a-) by F = ?

b-) by weight = ?

c-) what is the total work?

Sol: a-) $W_F = \vec{F} \times \vec{d} = 209\ N \times 1.5\ m \times \cos 0° = \underline{313.5\ Nm}$

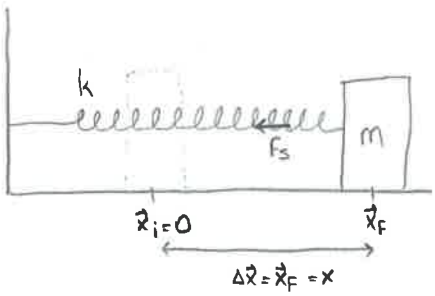
b-)



$y = d.\sin 25° = 1.5m \times 0.42 = 0.63m$ $\qquad W_{mg} = m\vec{g}.y.\cos 180° = -250\ N \times 0.63m = \underline{-158.5}\ Nm$

c-) $W_{Total} = W_F + W_{mg} = 313.5\ Nm - 158.5\ Nm = \underline{155}\ Nm$

6.3 Work Done By Varying Force



$$\underbrace{F_s}_{\text{Restoring Force of Spring}}(x) = -\underbrace{k}_{\text{Spring Constant}}.\Delta\vec{x} = -k.\underbrace{\vec{x}}_{\text{Displacement}}$$

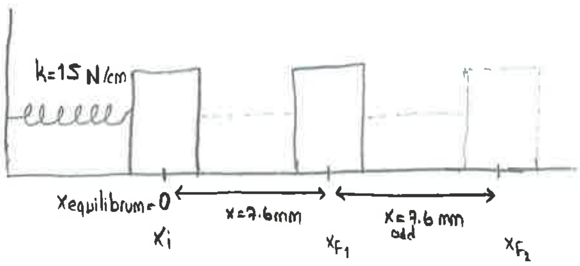
$$W_{F_S} = \frac{1}{2}.k.x_i^2 - \frac{1}{2}.k.x_F^2 \quad (Nm)$$

Ex: A spring with $k = 15\ N/cm$ is attached to an object.

a-) How much work does the F_s do an object if the spring is streched from its relaxed position by 7.6 mm?

b-) How much additional work is done by F_s if spring is streched by an aditional 7.6 mm?

Sol:

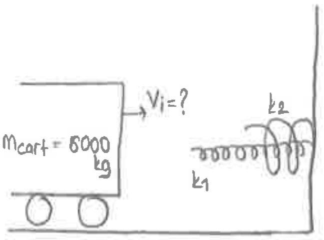


a-) $W_{F_S} = \frac{1}{2}.k.x_i^2 - \frac{1}{2}.k.x_{F_1}^2 = 0 - \frac{1}{2} \times \frac{15}{10^{-2}} \frac{N}{cm} \times (7.6\ mm \times 10^{-3})^2 = -0.043\ Nm$

b-) $W_{F_S} = \frac{1}{2}.k.x_{F_1}^2 - \frac{1}{2}.k.x_{F_2}^2 = \frac{1}{2} \times \frac{15}{10^{-2}} \frac{N}{cm} \times (7.6\ mm \times 10^{-3})^2 - \frac{1}{2} \times \frac{15}{10^{-2}} \frac{N}{cm} \times (15.2 \times 10^{-3})^2 =$

$W_{F_S} = -0.13\ Nm$

Ex-)



$k_1 = 1600$ N/m $k_2 = 3400$ N/m

After first spring compresses 30 cm, cart gets in contact with second spring and stops again after 20 cm more compression of two springs. $V_i = ?$

Sol: $\boxed{W_{F_S} = \Delta KE} \rightarrow$ work-KE relation

Total compression for 1. spring is 50 cm
" " " 2. spring " 20 "

$$W_{F_{S1}} + W_{F_{S2}} = KE_f - KE_i$$

$$\frac{1}{2} \cdot k_1 \cdot \overset{0}{\cancel{X_{i_1}^2}} - \frac{1}{2} \cdot k_1 X_{F_1}^2 + \frac{1}{2} \cdot k_2 \cdot \overset{0}{\cancel{X_{i_2}^2}} - \frac{1}{2} \cdot k_2 \cdot X_{F_2}^2 = \frac{1}{2} \cdot m \cdot \overset{0}{\cancel{V_f^2}} - \frac{1}{2} \cdot m \cdot V_i^2$$

$$0 - \cancel{\tfrac{1}{2}} \times 1600\,\text{N/m} \times (0.5\,\text{m})^2 - \cancel{\tfrac{1}{2}} \times 3400\,\text{N/m} \times (0.2\,\text{m})^2 = -\cancel{\tfrac{1}{2}} \times 6000\,\text{kg} \times V_i^2 \qquad \underline{v_i = 0.3\,\text{m/s}}$$

B.4. Power

Time rate of energy transfer or time rate of work is called as power, P.

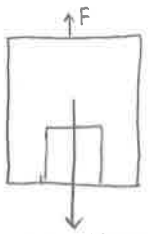
$P_{ave} = \dfrac{\Delta W}{\Delta t} \left(\dfrac{Nm}{s} = \text{Watt} \right)$ } Average Power

$P = \lim\limits_{\Delta t \to 0} \dfrac{dw}{dt}$ (watt) } Instentaneus Power

$P = \vec{F} \times \vec{v}$ (Watt)
↳ Instentaneus Velocity

Ex-) An elevator has $m = 1000$ kg and can carry max. 800 kg load. A constant friction force of 4000N retards its motion upward.
a-) What is the P_{min} of a motor to lift the elevator with $a = 0$ m/s^2 and $v = 3$ m/s constant speed?
b-) with $a = 1$ m/s^2 ?

Sol:



$mg = 18000$ N
$f = 4000$ N

a-) $P_{min} = \vec{F} \times \vec{V} = F.V. \cos 0° = 22000\text{N} \times 3\text{m/s} \times 1 = 66000$ watt

$$F_{net} = m_{total} \times \underset{0}{a} = 0 \Rightarrow F - m_{total} \cdot g - f = 22000$$

b-) $m_{tot} \times a = F - m_{total} \times g - f \Rightarrow F = m_{total} \times a + m_{total} \times g \times f$

$F = 1800\,\text{kg} \times 1\,\text{m/s}^2 + 18000\,\text{N} + 4000\,\text{N} =$ 23800N

$P_{min} = \vec{F} . \vec{v} = 23800$ N.v (Watts)

Ex : When its 75 kW engine is generating full power, an airplane with mass 700 kg gains altitude at a rate of 2.5 m/s. What fraction of the engine power is being used to make the airplane climb ?

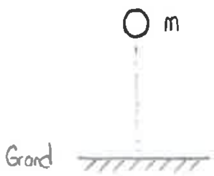
Sol : $P = \vec{F}.\vec{V} = mg.V = 700\ kg \times 10\ \frac{N}{kg} \times 2.5\ m/s = 17500\ (N.\frac{m}{s} = watt)$ $\qquad \frac{P}{P_{max}} = \frac{17500}{75000} = 0.23$

CHAPTER-7

POTENTIAL ENERGY AND ENERGY CONSERVATION

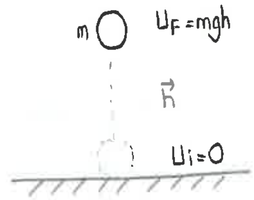
7.1. Gravitational Potential Energy

The energy of an object because of its position is called as potential energy, U.



$U = mgh\ (J) \rightarrow$ Gravitational Potential Energy

There is a relation between the work done by gravitational attraction force (weight) and change in the potential energy. $\Delta U = U_F - U_i \qquad W_{mg} = -\Delta U$

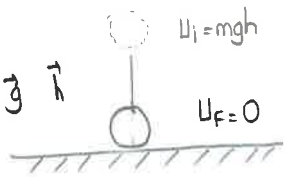


Two cases are possible. 1. Case : Object moves upward

$$W_{mg} = m\vec{g}.\vec{h} = m.g.h.\underbrace{\cos 180^\circ}_{-1} = -(U_F - U_i)$$

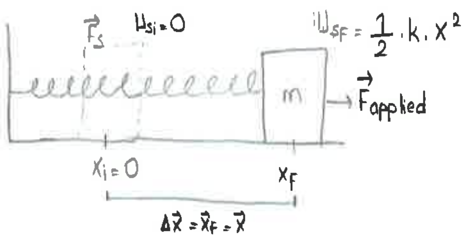
2. Case : Object moves downward

$$W_{mg} = m.\vec{g}.\vec{h} = m.g.h.\underbrace{\cos 0^\circ}_{1} = -(U_F - U_i)$$



Like weight, the restoring force $(\vec{F}_s)$ in the spring does work and the relation between work done by $\vec{F}_s$ and ΔU is

$W_{F_s} = -\Delta U$



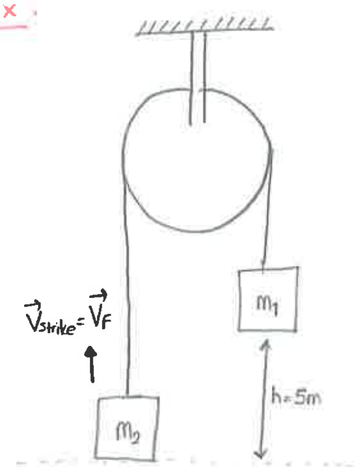
$U_s = \frac{1}{2}.k.x^2 \rightarrow$ Elastic Potential Energy in the spring

$$\underbrace{\frac{1}{2}.k.x_i^2}_{0} - \frac{1}{2}.k.x_F^2 = -(U_{s_F} - \underbrace{U_{s_i}}_{0}) \qquad -\frac{1}{2}.k.x_F^2 = -\left(\frac{1}{2}.k.x^2\right)$$

The common characteristic property of weight and F_s is that, they are conservative forces.
So, the general form of relation $W_{F_{conservative}} = -\Delta U$

$$-\Delta U = \Delta KE \qquad U_i + KE_i = KE_F + U_F$$

Ex :



m_1: 10 kg

m_2: 4 kg

The system is left free. Find the strike velocity of m_1 to the grand. ?

Sol : $\boxed{E_i = E_F}$ → Conservation of Total Mechanical Energy

$KE_i + U_i = KE_F + U_F$

$0 + m_1 \cdot g \cdot h = \frac{1}{2}(m_1 + m_2) \cdot V_{strike}^2 + m_2 \cdot g \cdot h$

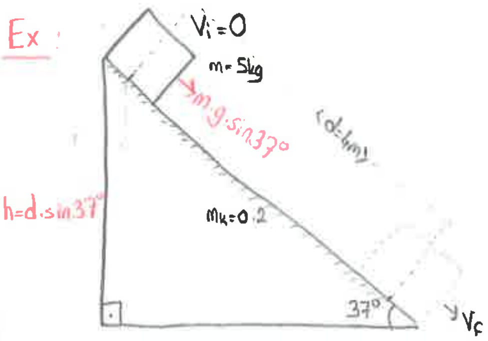
$10\ kg \times 10\ \frac{N}{kg} \times 5m = \frac{1}{2} \times 14\ kg \times V_F^2 + 4\ kg \times 10\ \frac{N}{kg} \times 5m$ $\quad V_F = 6.55\ m/s$

7.2. Work Done By Nonconservative Forces

If there is work done by nonconservative force (friction force) in the system then total mechanical energy (E) of system is not conserved.

$$W_F = W_{NC} = \Delta E = E_F - E_i \quad (J)$$

Ex :



Object is left free on the top of inclined plane. Find its speed when it reaches the bottom of inclined plane ?

I. Way

Sol : $W_{F_K} = E_F - E_i$

$-f_K \cdot d \cdot \cos 180° = KE_F - U_i$

$+\mu_K \cdot m \cdot g \cdot \cos 37° \cdot d = \frac{1}{2} \cdot m \cdot V_F^2 - m \cdot g \cdot \underset{\sim}{h}$ (d.sin37°)

$0.2 \times 50N \times 0.8 \times 4m = \frac{1}{2} \times 5kg \times V_F^2 - 50N \times 4m \times 0.6$

$V_F = 5.9\ m/s$

II. Way

$F_{net} = m \times a$

$m \cdot g \cdot \sin\theta - f_K = m \times a$

$\cancel{m} \cdot g \cdot \sin\theta - \mu_K \cdot \cancel{m} \cdot g = \cancel{m} \cdot a$

$a = \ldots$

$V_F^2 = V_i^2 + 2 \cdot a \cdot \Delta X$

$V_F^2 = 2 \times a \times 4m$

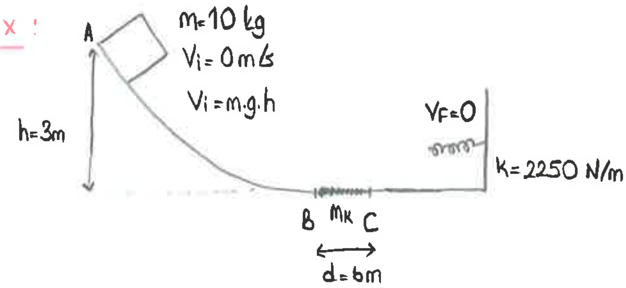
$V_F = \ldots\ m/s$

$W_{F_{net}} = \Delta KE_i = KE_F - KE_i$

$W_C = -\Delta U = -(U_F - U_i)$ → $F_S = -k \cdot x$; $W = m \cdot g$

$E_i = E_F \Rightarrow KE_i + U_i = KE_F + U_F$

Ex :



The block travels down compresses the spring 0.3 m after passing a frictional region of 6m. Find the M_K = ?

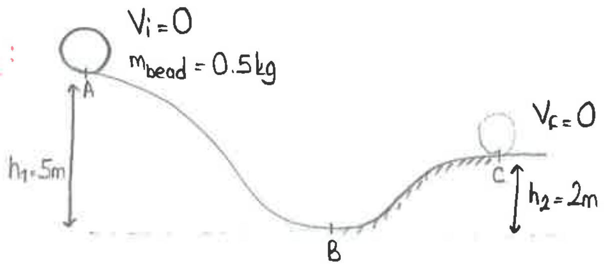
Sol : $W_{FK} = E_f - E_i$

$-f_K \cdot d = \frac{1}{2} \cdot k \cdot x^2 - m \cdot g \cdot h \Rightarrow -M_K \cdot m \cdot g \cdot d = \frac{1}{2} \cdot k \cdot x^2 - m \cdot g \cdot h$

$\Rightarrow -M_K \times 10\,kg \times 10\,\frac{N}{kg} \times 6m = \frac{1}{2} \cdot 2250\,\frac{N}{m} \times (0.3\,m)^2 - 10\,kg \times 10\,\frac{N}{kg} \times 3m$

$M_K = 0.33$

Ex :



Segment AB is frictionless but segment BC has friction. Bead stops at point C.

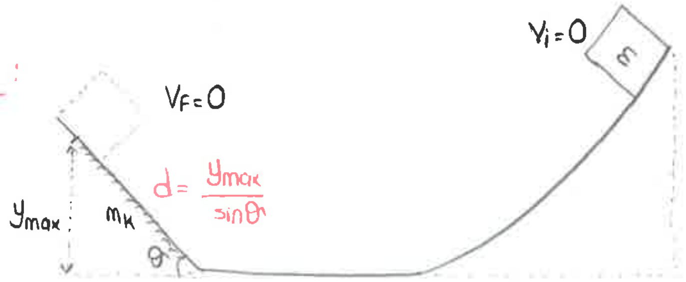
a-) Find the energy lost due to segment BC ?

b-) Find the velocity at the point B, V_B = ?

Sol : a-) $\Delta E_{lost} = E_f - E_i = m \cdot g \cdot h_2 - m \cdot g \cdot h_1 \Rightarrow \Delta E_{lost} = 0.5\,kg \times 10\,\frac{N}{kg} \times (2m - 5m) \Rightarrow \Delta E_{lost} = -15\,J$

b-) $E_f - E_i = 0$ $\quad \frac{1}{2} \cdot \cancel{m} \cdot V_B^2 = \cancel{m} \cdot g \cdot h_1$ $\quad V_B = 10\,m/s$

Ex :

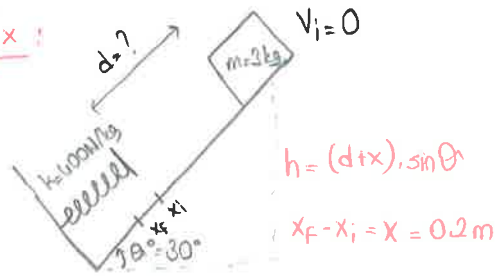


Use energy method to show that the max. height reached by block is $y_{max} = \frac{h}{1 + M_K \cdot \cot\theta}$

Sol : $W_{F_K} = E_F - E_i$ $\qquad f_K = M_K \cdot N = M_K \cdot m \cdot g \cdot \cos\theta$

$-f_K \cdot d = m \cdot g \cdot y_{max} - m \cdot g \cdot h \Rightarrow -M_K \cdot \cancel{m} \cdot \cancel{g} \cdot \cos\theta \cdot \frac{y_{max}}{\sin\theta} = \cancel{m} \cdot \cancel{g} \cdot y_{max} - \cancel{m} \cdot \cancel{g} \cdot h$

Ex :



The mass slides an additional distance X=0.2 m as it is brought to rest by compressing the spring Find d= ?

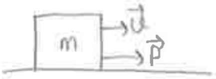
Sol : $E_i = E_F$

$m \cdot g \cdot h = \frac{1}{2} \cdot k \cdot x^2 \qquad m \cdot g \cdot (d+x) \cdot \underbrace{\sin 30°}_{0.5} = \frac{1}{2} \times 400\,\frac{N}{kg} \times (0.2\,m)^2 \qquad d = 0.33m$

CHAPTER-8

MOMENTUM, IMPULSE and COLLISIONS

8.1 Momentum and Impulse



$$\underbrace{\vec{P} = m \times \vec{v}}_{\text{Momentum of Object}} \quad \left(kg \cdot \frac{m}{s}\right)$$

$\vec{F} = \dfrac{d\vec{P}}{dt}$ } => Net force acting on the object equals time rate of change of velocity.

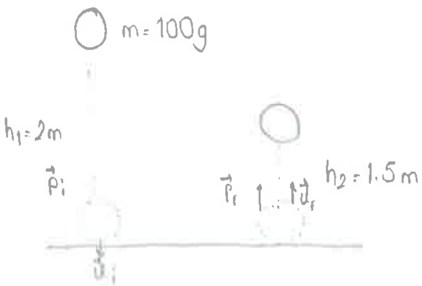
$$d\vec{P} = \vec{F} \times dt$$

$$\underbrace{\vec{P}_F - \vec{P}_i}_{\Delta\vec{P}} = \vec{F} \times dt = I = \text{Impulse}$$

$$\Delta\vec{P} = I$$

Ex: A ball of mass 100g is dropped from 2m height. It rebounces vertically to a height of 1.5 m after colliding with floor. Determine the F_{ave} exerted by floor on ball by assuming the time of collision is 10^{-2} sec.

Sol: $\Delta\vec{P} = \vec{I} = \vec{F}_{ave} \cdot \Delta t \qquad \vec{F}_{ave} = \dfrac{\Delta\vec{P}}{\Delta t}$



$V_f^2 = V_i^2 + 2 \cdot a \cdot \Delta X$

$V_f^2 = V_i^2 + 2 \cdot g \cdot h_1 \qquad 0 = V_f^2 - 2 \cdot g \cdot h_2$

$V_i^2 = 0 + 2 \times 10\, m/s^2 \times 2m \qquad V_f^2 = 2 \cdot g \cdot h_2 \Rightarrow 2 \times 10\, m/s^2 \times 1.5m$

$V_i = 6.32\, m/s \qquad V_f = 5.47\, m/s$

$$\vec{F}_{ave} = \frac{\vec{P}_F - \vec{P}_i}{\Delta t} = \frac{m \cdot \vec{v}_f - m \cdot \vec{v}_i}{\Delta t} = \frac{0.1\,kg \times 5.47\, m/s\, \hat{j} - 0.1\,kg \times 6.32\, m/s\, \hat{j}}{10^{-2}\, s} = 118\, \hat{j}\, N \quad \left(\text{Direction is upward}\right)$$

→ If the F_{net} acting on an object is zero then $\vec{P}$ is conserved.

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} \Rightarrow 0 = d\vec{P} = \vec{P}_f - \vec{P}_i = \Delta\vec{P}$$

$$\underbrace{\vec{P}_i = \vec{P}_F}_{\text{Conservation of Momentum}}$$

8.2 Collisions

i-) Elastic Collisions in one Dimension

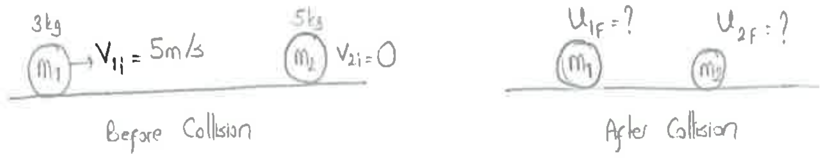
If the KE and momentum of a system are conserved, after the collision then this kind of collision is called as elastic collision.

$KE_{i_{system}} = KE_{F_{system}}$ $\qquad \frac{1}{2}\cdot m_1 \cdot V_{1_i}^2 + \frac{1}{2}\cdot m_2 \cdot V_{2_i}^2 = \frac{1}{2}\cdot m_1 \cdot V_{F_1}^2 + \frac{1}{2}\cdot m_2 \cdot V_{F_2}^2 \quad (I)$

$\vec{P}_{i_{system}} = \vec{P}_{F_{system}}$ $\qquad m_1 \cdot \vec{V}_{1_i} + m_2 \cdot \vec{V}_{2_i} = m_1 \cdot \vec{V}_{1_F} + m_2 \cdot \vec{V}_{2F} \quad (II)$

$$V_{1F} = \frac{m_1 - m_2}{m_1 + m_2} \cdot V_{1i} \qquad V_{2F} = \frac{2 \cdot m_1}{m_1 + m_2} \cdot V_{1i}$$

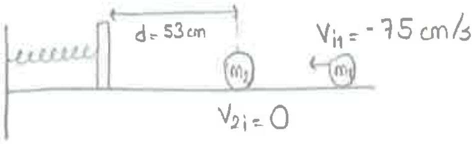
Ex:



Two objects make elastic collision. Find $\vec{U}_{1f}$ and $\vec{U}_{2F}$ = ?

Sol: $V_{1F} = \frac{3\,kg - 5\,kg}{3\,kg + 5\,kg} \cdot 5\,m/s \cdot \hat{i} = -1.25 \cdot \hat{i}\ m/s \qquad V_{2F} = \frac{2 \cdot 3\,kg}{3\,kg + 5\,kg} \cdot 5\,m/s \cdot \hat{i} = 3.75 \cdot \hat{i}\ m/s$

Ex:



$m_1 = 590g$
$m_2 = 350g$

m_1 collides to the rest m_2 elastically m_2 rebounds elastically from a spring and meets with incident m_2 for second times. How far from wall does the second collision occur?

Sol: $V_{1F} = \frac{(590-350)g}{(590+350)g} \cdot (-75\,cm/s) = -19.2\ cm/s$

$V_{2F} = \frac{2 \cdot 590\,g}{(590+350)g} \cdot (-75\,cm/s) = -94.2\ cm/s$

$X = d - U_{1f} \cdot t$ → Needed time for second collision

Total taken distance by two objects is equal to 2d when 2.collision occurs.

$2d = |V_{2f}| \cdot t + |V_{1f}| \cdot t$

$2 \times 53\,cm = |-94.2\,cm/s| \cdot t + |-19.2\,cm/s| \cdot t \qquad t = 0.93\ sec$

$X = d - U_{1f} \cdot t$

$X = 53\,cm - |-19.2\,cm/s| \times 0.93\,s \qquad \underline{X = 35\,cm}$

ii-) Inelastic Collisions

If two objects make type of collision then total momentum $(\sum P)$ of system is conserved but total KE is not conserved. After an inelastic collision colliding objects may stick to each other and move with same magnitude of speed. This type of collision is called as completely inelastic collision.

Ex:

After collision two objects stick to each other and move together. Find V_F ?

Sol: $\sum \vec{P}_i = \sum \vec{P}_F$, $m_1 . \vec{V}_{1i} + m_2 . \vec{V}_{2i} = (m_1 + m_2) . \vec{V}_F$

$$3kg \times 5 m/s + 8 kg \times -10 m/s = (3kg + 8kg) . \vec{V}_F$$

$$V_F = \frac{-65}{11} m/s$$

Ex:

$m_1 : 2kg$, $m_2 : 5kg$, $V_{1i} = 10 m/s$, $V_{2i} = 3 m/s$, $k = 1120 N/m$

Hint: $1N = 1 kg . \frac{m}{s^2}$

What is the max. compression in the spring?

(Hint: When max compression occurs, two objects behave like single object)

Sol:

$\sum \vec{P}_i = \sum \vec{P}_F$, $m_1 . \vec{V}_{1i} + m_2 . \vec{V}_{2i} = m_T . \vec{V}_F$

$$2kg \times 10 m/s + 5kg \times 3 m/s = 7 kg \times \vec{V}_F \qquad \vec{V}_F = 5 m/s$$

Since collision is inelastic type then $\sum KE$ is not conserved.

Lost in total $\sum KE$ is because of W_{Fs}.

$\Delta KE_{system} = W_{Fs}$ $\qquad KE_F - KE_i = W_{Fs}$

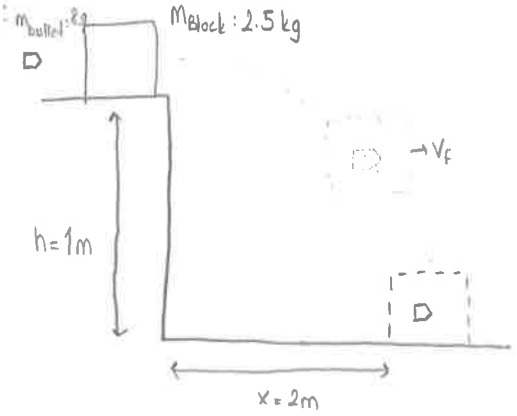
$$\cancel{\tfrac{1}{2}} (m_1 + m_2) . V_F^2 - \left(\cancel{\tfrac{1}{2}} m_1 . V_{1i}^2 + \cancel{\tfrac{1}{2}} . m_2 . V_{1i}^2 \right) = \cancel{\tfrac{1}{2}} . k . X_i^2 - \cancel{\tfrac{1}{2}} . k . X_f^2$$

here $X_i = 0$, $X_F = X_{max}$

$$7 kg . (5 m/s)^2 - \left(2 kg . (10 m/s)^2 + 5 kg . (3 m/s)^2 \right) = -1120 \frac{kg}{m} \frac{m}{s^2} . X_{max}^2$$

$$X_{max} = 0.25 m$$

Ex:



Determine the initial speed of bullet.

Sol: $\sum \vec{P}_i = \sum \vec{P}_F$, $m_b \cdot \vec{V}_{ib} = (m_b + m) \cdot \vec{V}_F$

$8g \cdot \vec{V}_{ib} = (2508g) \times 4.474\ m/s$

$\vec{V}_{ib} = 1402.6\ m/s$

$X = V_F \cdot t$

$2m = V_F \times 0.447s$

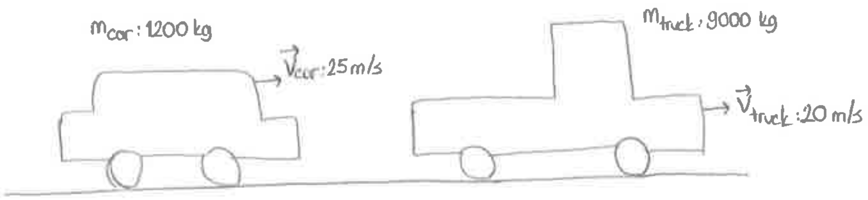
$V_F = 4.474 m/s$

, $h = \frac{1}{2} \cdot g \cdot t^2$

$1m = \frac{1}{2} \cdot 10 \frac{m}{s^2} \cdot t^2$

$t = 0.447s$

Ex:



After crash, the velocity of car is 18 m/s.

a-) V_{truck} after crash?

b-) How much mechanical energy is lost?

Sol: a-) $\sum \vec{P}_i = \sum \vec{P}_F$ → Conservation of Momentum

$m_{car} \cdot \vec{V}_{car,i} + m_{truck} \cdot \vec{V}_{truck,i} = m_{car} \cdot \vec{V}_{car,f} + m_{truck} \cdot \vec{V}_{truck,f}$

$1200\ kg \times 25 m/s + 9000\ kg \times 20\ m/s = 1200\ kg \times 18\ m/s + 9000\ kg \times V_{truck,f}$

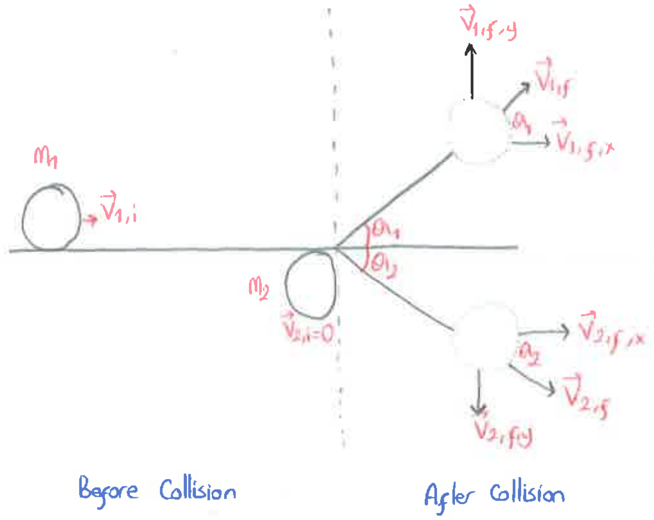
$V_{truck,f} = 20.93\ m/s$ after crash

b-) $\Delta KE = \sum KE_f - \sum KE_i$

$\Delta KE = \frac{1}{2} \cdot m_{car} \cdot V_{car,f}^2 + \frac{1}{2} \cdot m_{truck} \cdot V_{truck,f}^2 - \left(\frac{1}{2} \cdot m_{car} \cdot V_{car,i}^2 + \frac{1}{2} \cdot m_{truck} \cdot V_{truck,i}^2 \right) = -9308\ J$

This energ is converted to thermal (heat) energy.

8.3 Collisions in Two Dimensions



If two dimensional collision is elastic type then both $\Sigma\vec{P}$ and ΣKE are conserved.

In Horizontal, $\Sigma\vec{P}_{x,i} = \Sigma\vec{P}_{x,f}$

$$m_1 \cdot V_{1,i,x} + \underbrace{m_2 \cdot V_{2,i,x}}_{0} = m_1 \cdot V_{1,f,x} + m_2 \cdot V_{2,f,x} \quad (\mathrm{I})$$

In Vertical, $\Sigma\vec{P}_{y,i} = \Sigma\vec{P}_{y,f}$

$$\underbrace{m_1 \cdot V_{1,i,y}}_{0} + \underbrace{m_2 \cdot V_{2,i,y}}_{0} = m_1 \cdot V_{1,f,y} + m_2 \cdot V_{2,f,y} \quad (\mathrm{II})$$

$$0 = m_1 \cdot V_{1,f,y} + m_2 \cdot V_{2,f,y}$$

$$V_{1,f,x} = V_{1,f} \cdot \cos\theta_1 \qquad V_{1,f,y} = V_{1,f} \cdot \sin\theta_1$$

$$V_{2,f,x} = V_{2,f} \cdot \cos\theta_2 \qquad V_{2,f,y} = V_{2,f} \cdot \sin\theta_2$$

$$\Sigma KE_i = \Sigma KE_f$$

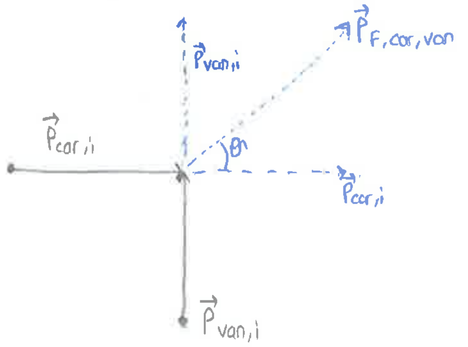
$$\frac{1}{2} \cdot m_1 \cdot V_{1,i}^2 + \frac{1}{2} \cdot m_2 \cdot V_{2,i}^2 = \frac{1}{2} \cdot m_1 \cdot V_{1,f}^2 + \frac{1}{2} \cdot m_2 \cdot V_{2,f}^2 \quad (\mathrm{III})$$

NOTE: If two objects make inelastic type of collision in two dimensions then just $\Sigma\vec{P}$ is conserved in two dimensions.

Ex: A 1500 kg traveling east with a speed of 25 m/s makes complete inelastic collision with a 2500 kg van travelling north at a speed of 20 m/s.

Find the magnitude and direction of common speed of after collision?

Sol:



$\vec{P}_{F,car,van} = \vec{P}_{car,i} + \vec{P}_{van,i}$ (lets take square of both sides)

$$\vec{P}_{F,car,van}^2 = \vec{P}_{car,i}^2 + \vec{P}_{van,i}^2 + 2 \cdot \vec{P}_{car,i} \cdot \vec{P}_{van,i} \cdot \underbrace{\cos\theta_i}_{(\cos 90^\circ = 0)}$$

$$m_{car,van}^2 \cdot V_{F,car,van}^2 = m_{car}^2 \cdot V_{car,i}^2 + m_{van}^2 \cdot V_{van,i}^2$$

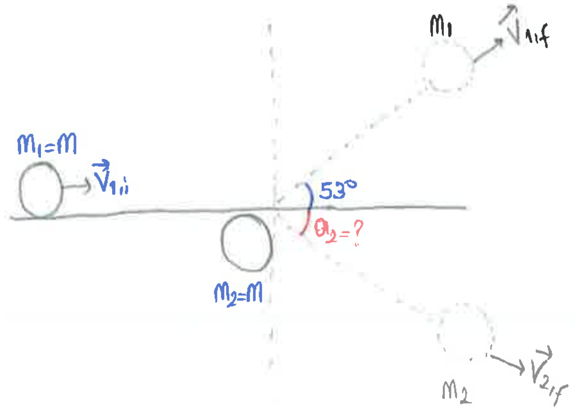
$$(4000\ kg)^2 \cdot V_{f,car,van}^2 = (1500\ kg)^2 \cdot (25\ m/s)^2 + (2500\ kg)^2 \cdot (20\ m/s)^2$$

$$V_{F,car,van} = 15.6\ m/s$$

$$\tan\theta = \frac{P_{van,i}}{P_{car,i}} = \frac{m_{van} \cdot V_{van,i}}{m_{car} \cdot V_{car,i}} = \frac{2500\ kg \cdot 20\ m/s}{1500\ kg \cdot 25\ m/s} = 1.33$$

$$\theta = \arctan(1.33) = 53^\circ \text{ North of East}$$

Ex:



Two billiard balls make elastic type of glancing collision.
(two dimension collision)

Find $\theta_2 = ?$

Sol: $\sum KE_i = \sum KE_f$

$$\cancel{\tfrac{1}{2}} \cdot \cancel{m_1} \cdot V_{1,i}^2 + 0 = \cancel{\tfrac{1}{2}} \cdot \cancel{m_1} \cdot V_{1,f}^2 + \cancel{\tfrac{1}{2}} \cdot \cancel{m_2} \cdot V_{2,f}^2 \qquad \text{since } m_1 = m_2 = m$$

$$V_{1,i}^2 = V_{1,f}^2 + V_{2,f}^2 \quad (\mathrm{I})$$

$$\sum \vec{P}_i = \sum \vec{P}_f$$

$$\cancel{m_1} \cdot \vec{V}_{1,i} = \cancel{m_1} \cdot \vec{V}_{1,f} + \cancel{m_2} \cdot \vec{V}_{2,f} \qquad \text{since } m_1 = m_2 = m$$

$$\vec{V}_{1,i} = \vec{V}_{1,f} + \vec{V}_{2,f} \quad (\mathrm{II}) \quad \text{take the square of both sides of equation}$$

$$V_{1,i}^2 = V_{1,f}^2 + V_{2,f}^2 + 2 \cdot V_{1,f} \cdot V_{2,f} \cdot \cos\theta \Rightarrow \cancel{V_{1,f}^2} + \cancel{V_{2,f}^2} = \cancel{V_{1,f}^2} + \cancel{V_{2,f}^2} + 2 \cdot V_{1,f} \cdot V_{2,f} \cdot \cos\theta \Rightarrow \cos\overset{90°}{\theta} = 0$$

$$\theta = 90° \qquad 53° + \theta_2 = 90° \qquad \underline{\theta_2 = 37°}$$

Note: If two equal mass make elastic type of glancing collision then angle between masses after collision is always $90°$.

Ex: Same above question but two billiard balls make elastic type of glancing collision. Find the final velocities after collision.
$V_{1,i,x} = 5\,m/s$

Sol: $\sum \vec{P}_{i,x} = \sum \vec{P}_{f,x}$

in Horizontal, $\cancel{m_1} \cdot V_{1,i,x} = \cancel{m_1} \cdot V_{1,f,x} + \cancel{m_2} \cdot V_{2,f,x}$

$$5\,m/s = V_{1,f} \cdot \cos 53° + V_{2,f} \cdot \cos 37°$$

$$5\,m/s = V_{1,f} \times 0.6 + V_{2,f} \times 0.8 \quad (\mathrm{I})$$

in Vertical, $\sum \vec{P}_{i,y} = \sum \vec{P}_{f,y}$

$$0 = \cancel{m_1} \cdot V_{1,f,y} - \cancel{m_2} \cdot V_{2,f,y}$$

$$0 = V_{1,f} \cdot \sin 53° - V_{2,f} \cdot \sin 37°$$

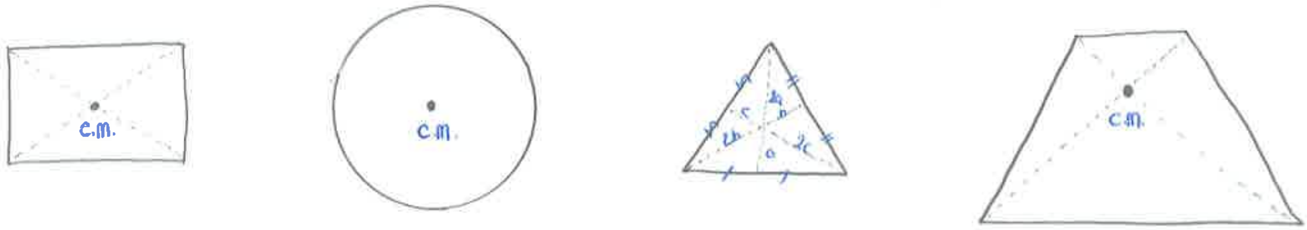
$$V_{1,f} \times 0.8 = V_{2,f} \times 0.6 \quad (\mathrm{II})$$

$$V_{1,f} = 3\,m/s \;//$$

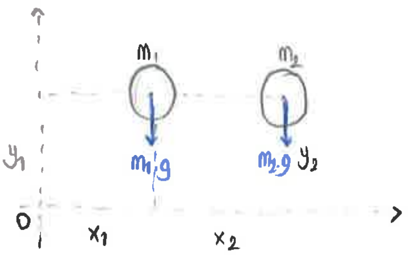
$$V_{2,f} = 4\,m/s \;//$$

8.4 Center of Mass

Center of mass is a point on the objects and when object is suspended from that point, it does not rotate.



Assume a system that consists of two point like objects.

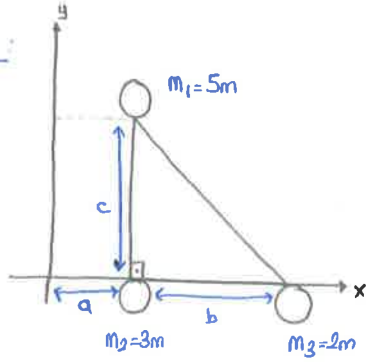


$$\Sigma \vec{\tau} = 0$$

$$X_{c.m.} = \frac{m_1 . g . X_1 + m_2 . g . X_2}{(m_1+m_2).g} = \frac{m_1.X_1 + m_2.X_2}{m_1+m_2}$$

$$y_{c.m.} = \frac{m_1.y_1 + m_2.y_2}{m_1+m_2}$$

Ex :



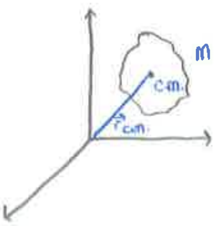
Determine the $\vec{r}_{c.m.} = X_{c.m.} . \hat{i} + y_{c.m.} . \hat{j}$?

Sol : $X_{c.m.} = \dfrac{m_1 . x_1 + m_2 . x_2 + m_3 . x_3}{m_1+m_2+m_3} = \dfrac{5m.a + 3m.a + 2m.(a+b)}{5m+3m+2m} = \dfrac{10m.a}{10m} + \dfrac{2m.b}{10m} = a + \dfrac{b}{5}$ (m)

$y_{c.m.} = \dfrac{m_1.y_1 + m_2.y_2 + m_3.y_3}{m_1+m_2+m_3} = \dfrac{5m.c + 3m.0 + 2m.0}{5m+3m+2m} = \dfrac{5m.c}{10m} = \dfrac{c}{2}$ (m)

$$\vec{r}_{cm} = \left(a + \frac{b}{5}\right).\hat{i} + \left(\frac{c}{2}\right).\hat{j} \text{ (m)}$$

Center of mass for rigid bodies



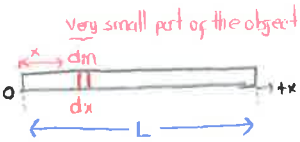
Limit of summation is integral

$$X_{cm} = \frac{\lim\limits_{\Delta m \to 0} \sum\limits_{i=1} \Delta m_i . x_i}{m} = \frac{1}{m} \int x.dm$$

$$y_{c.m.} = \frac{1}{m} \int y.dm \quad , \quad Z_{c.m.} = \frac{1}{m} \int z.dm$$

$$\vec{r}_{c.m.} = X_{c.m.}.\hat{i} + y_{c.m.}.\hat{j} + Z_{cm}.\hat{k}$$

Ex :

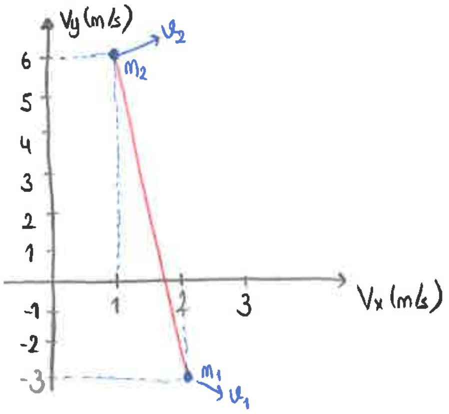


Thin rod has homogenous mass distribution. Find the X_{cm} ?

Sol : $\rho = \dfrac{m}{L}$ (kg/m) $= \dfrac{dm}{dx} \Rightarrow dm = \dfrac{m}{L} . dx$

$$X_{c.m.} = \frac{1}{m} \int x.dm = \frac{1}{\cancel{m}} . \int x . \frac{\cancel{m}}{L} . dx = \frac{1}{L} . \int_0^L x.dx = \frac{1}{L} . \left.\frac{x^2}{2}\right|_0^L = \frac{L}{2}$$

Ex:



$m_1: 2$ kg has velocity $\vec{v}_1 = 2\hat{i} - 3\hat{j}$ (m/s)

$m_2: 3$ kg " " $\vec{v}_2 = \hat{i} + 6\hat{j}$ (m/s)

a-) Find $\vec{v}_{cm}$ for the system?

b-) $\vec{P}_{total} = ?$

Sol: a-) $\vec{v}_{cm} = \vec{v}_{cm,x} + \vec{v}_{cm,y}$

$\vec{v}_{cm} = \vec{v}_{cm,x}\hat{i} + \vec{v}_{cm}\hat{j}$

$$v_{cm,x} = \frac{m_1 . V_{1,x} + m_2 . V_{2,x}}{m_1 + m_2} = \frac{2\,kg \times 2\,m/s + 3\,kg \times 1\,m/s}{2\,kg + 3\,kg} \Rightarrow \underline{v_{cm,x} = 1.4\ m/s} \qquad \underline{v_{cm,y} = 2.4\ m/s}$$

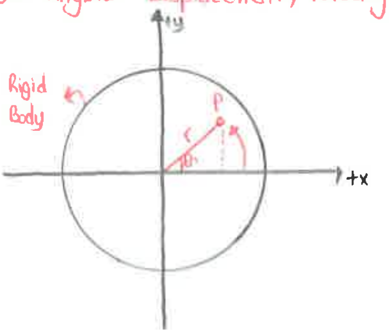
$\underline{v_{cm} = 1.4\hat{i} + 2.4\hat{j}}$

b-) $P_{total} = \vec{P}_1 + \vec{P}_2 = m_1 . \vec{V}_1 + m_2 . \vec{V}_2 = 2\,kg \times (2\hat{i} - 3\hat{j}) + 3\,kg \times (\hat{i} + 6\hat{j}) = \underline{7\hat{i} + 12\hat{j}\ (kg.m/s)}$

CHAPTER-9

ROTATION OF RIGID BODIES

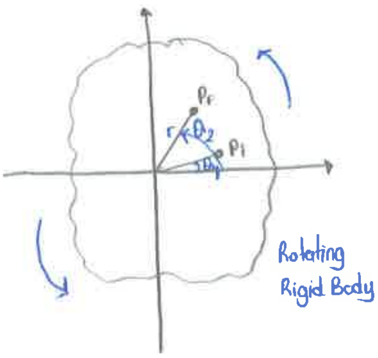
9.1 Angular Displacement, Velocity and Acceleration



θ: Angular position is the angle from $+x$ in counterclockwise direction.

$$\theta\,(rad) = \frac{\pi}{180}\ \theta\,(degree)$$

π: 3.14 radians $\qquad$ 1 rad: 57.3° $\qquad$ 1 complete revolution: 2π rad = 360°



Angular Displacement, $\underline{\Delta\theta = \theta_f - \theta_i}$ (rad)

Average Angular Velocity, $\omega = \dfrac{\Delta\theta}{\Delta t} = \dfrac{\theta_f - \theta_i}{t_f - t_i}$ (rad/s)

Instantaneous Angular Velocity, $\omega = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\theta}{\Delta t} = \dfrac{d\theta}{dt}$ (rad/s)

For one complete revolution, $\omega = \dfrac{2\pi}{T} = 2.\pi f$

Average Angular Acceleration, $\alpha_{ave} = \dfrac{\Delta\omega}{\Delta t} = \dfrac{\omega_f - \omega_i}{t_f - t_i}$ (rad/s²)

Instantaneous Angular Acceleration, $\alpha = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\omega}{\Delta t} = \dfrac{d\omega}{dt}$ (rad/s²)

Ex: The angular position of rim of a rotating wheel is given as $\theta = t^3 - 3.t^2 + 4t$ (rad)

a-) What are the angular velocities at $t_1 = 2s$ and at $t_2 = 4s$?

b-) What is the W_{ave} between t_1 and t_2?

c-) What are the instantaneus accelerations at t_1 and t_2?

Sol: a-) $W = \frac{d\theta}{dt} = 3.t^2 - 6t + 4$ (rad/s)

$W_1 (t_1 = 2s) = 3.2^2 - 6.2 + 4 = 4$ rad/s

$W_2 (t_2 = 4s) = 3.4^2 - 6.4 + 4 = 28$ rad/s

b-) $W_{ave} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$, $\theta_1 (t_1 = 2s) = 2^3 - 3.2^2 + 4.2 = 4$ rad

$\theta_2 (t_2 = 4s) = 4^3 - 3.4^2 + 4.4 = 32$ rad

$W_{ave} = \frac{32 \text{ rad} - 4 \text{ rad}}{4s - 2s} = 14$ rad/s

c-) $\alpha = \frac{dw}{dt} = \frac{d^2\theta}{dt^2} = 6t - 6$ (rad/s²)

$\alpha_1 (t_1 = 2s) = 6 \times 2 - 6 = 6$ rad/s²

$\alpha_2 (t_2 = 4s) = 6 \times 4 - 6 = 18$ rad/s²

9.2 Rotation with Constant Angular Acceleration

Comparision of linear and rotational motions by formulae;

$\vec{v}_F = \vec{v}_i \mp a.t \longrightarrow \vec{W}_F = \vec{W}_i \mp \vec{\alpha}.t$

$\vec{X}_F = \vec{X}_i \mp \vec{v}_i.t \mp \frac{1}{2}.\vec{a}.t^2 \longrightarrow \theta_F = \theta_i \mp \vec{W}_i.t \mp \frac{1}{2}.\vec{\alpha}.t^2$

$v_F^2 = v_i^2 \mp 2.a.(\Delta X) \longrightarrow W_F^2 = W_i^2 \mp 2.\alpha.(\Delta\theta)$

Ex: An electric motor rotates a wheel at a rate of 100 rev/min is switch off. Assume that it decelerates with 2 rad/s².

a-) How long will it take for wheel to stop? b-) How many radians does it turn until stops?

Sol:

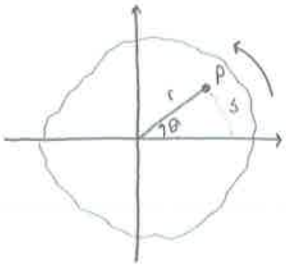
a-) $W_i = 2.\pi.f = 2 \times 3.14 \text{ rad} \times \frac{100}{60} . \frac{rev}{s} = 10.46$ rad/s

$W_F = W_i - \alpha.t \Rightarrow 0 = 10.46 \text{ rad/s} - 2 \text{ rad/s}^2 . t \Rightarrow t = 5.23s$

b-) $W_F^2 = W_i^2 - 2.\alpha.(\Delta\theta) \Rightarrow 0 = (10.46 \text{ rad/s})^2 - 2.2 \text{ rad/s} . \Delta\theta \Rightarrow \Delta\theta = 27.51$ rad

9.3 Relation Between Linear and Angular Variables

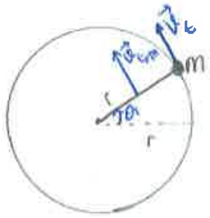
i-) Speed of Rotating Rigid Body



$S = r.\theta$ (Take the derivative of this eqn.)

$$\frac{ds}{dt} = \frac{d(r\theta)}{dt} = r.\frac{d\theta}{dt}$$

$$\boxed{v_t = r.w} \ (m/s)$$



$$m \to v_t.m = \frac{2\pi.r}{T}$$

$$m \to v_t.m = \frac{2\pi.d}{T}$$

$$v_t = \frac{ds}{dt}$$

Note-1: Radian is not real unit. It can be neglected when it is not needed.

Note-2: Every point on the rigid body has same w. But the points that have different positions to the rotation axis, have different linear speed (or v_t).

ii-) Acceleration of Rotating Rigid Body

$\vec{v}_t = r.w$ (take the derivative)

$$\frac{dv_t}{dt} = r.\frac{dw}{dt}$$

$$\boxed{a_t = r.\alpha}$$

If angular velocity of body increases.

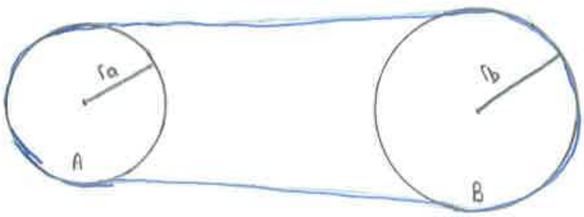
$$\vec{a}_{total} = \vec{a}_{cent} + \vec{a}_{tang}$$

$$a^2_{total} = a^2_{cent} + a^2_{tang}$$

$$\boxed{a_{cent} = \frac{v_t^2}{r} = \frac{r^2.w^2}{r} = r.w^2}$$

$$a_{total} = (r^2.w^4 + r^2.\alpha^2)^{1/2}$$

Ex:



$r_A = 10$ cm, $r_B = 25$ cm wheel A increases its angular speed from rest at a uniform of 1.6 rad$/s^2$. Find the time for wheel B to reach frequency of $100 \frac{rev}{min}$.

(Hint: Linear speeds at rims of two wheels are equal)

Sol: $v_{t,A} \overset{\text{at rims}}{=} v_{t,B}$

$$w_B = 2.\pi.f_B = 2.(3.14 \text{ rad}).\frac{100}{60}.\frac{rev}{s} = \frac{62.8}{6}.\frac{rad}{s}$$

$$r_A.w_A = r_B.w_B$$

$$10 \text{ cm}.w_A = 25 \text{ cm} \times \frac{62.8}{6}\frac{rad}{s}$$

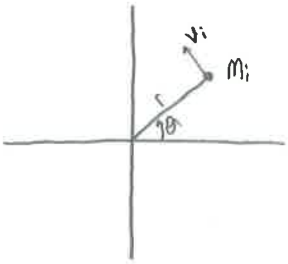
$$w_A = 26.17 \frac{rad}{s}$$

$$w_{A,f} = w_{A,i} + \alpha_A.t$$

$$26.17 \frac{rad}{s} = 0 + 1.6 \frac{rad}{s^2} \times t$$

$$t = 16.35 \text{ s}$$

9.4 Rotational Kinetic Energy

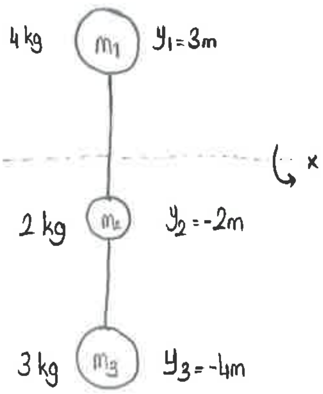


The KE of m_i is $KE_i = \frac{1}{2} \cdot m_i \cdot V_i^2 = \frac{1}{2} \cdot m_i \cdot r_i^2 \cdot w_i^2$

The total KE of rigid body $KE = \sum_{i=1}^{n} \frac{1}{2} \cdot m \cdot V_i^2 = \frac{1}{2} \underbrace{\left(\sum_{i=1}^{n} m_i \cdot r_i^2 \right)}_{I, \text{Moment of Inertia}} w_i^2 \rightarrow$ Since $w_1 = w_2 = w_3 = w$

$$\boxed{KE_{rot} = \frac{1}{2} \cdot I \cdot w^2}$$

Ex:



If the system rotates around X-axis with $w = 2$ rad/s

a-) $KE_{rot,total} = ?$

b-) Linear speed of each particle and KE_{total} evoluated from $\sum_{i=0} \frac{1}{2} \cdot m_i \cdot v_i^2 = ?$

Sol: a-) $KE_{rot} = \frac{1}{2} \cdot I \cdot w^2$

$$KE_{rot,tot} = \frac{1}{2} \cdot I_1 \cdot w^2 + \frac{1}{2} \cdot I_2 \cdot w^2 + \frac{1}{2} \cdot I_3 \cdot w^2 = \frac{1}{2} \cdot I_{total} \cdot w^2$$

$$= \frac{1}{2} \cdot \left[m_1 \cdot y_1^2 + m_2 \cdot y_2^2 + m_3 \cdot y_3^2 \right] \cdot w^2$$

$$= \frac{1}{2} \cdot \left[4kg \cdot (3m)^2 + 2kg \cdot (-2m)^2 + 3kg \cdot (-4m)^2 \right] \cdot \left(2 rad/s \right)^2 = \underline{184 J}$$

b-) $V = r \cdot w$

$= y_1 \cdot w = 3m \cdot 2 rad/s = 6 m/s$

$y_2 \cdot w = 2m \cdot 2 rad/s = 4 m/s$

$y_3 \cdot w = 4m \cdot 2 rad/s = 8 m/s$

$$KE_{total} = \frac{1}{2} \cdot m_1 \cdot V_1^2 + \frac{1}{2} \cdot m_2 \cdot V_2^2 + \frac{1}{2} \cdot m_3 \cdot V_3^2$$

$$= \frac{1}{2} \cdot 4kg \cdot (6m/s)^2 + \frac{1}{2} \cdot 2kg \cdot (4m/s)^2 + \frac{1}{2} \cdot 3kg \cdot (8m/s)^2$$

$$= \underline{184 J}$$

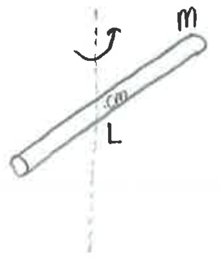
Ex: Calculate the rotational inertia (moment of inertia) of a wheel has, KE rot $= 24400 J$ when rotating at 602 rev/min.

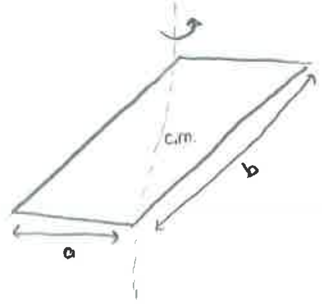
Sol: $KE_{rot} = \frac{1}{2} \cdot I \cdot w^2 \Rightarrow w = 2 \cdot \pi \cdot f = 2 \cdot (3.14 rad) \cdot \frac{602 rev}{60 s} = \underline{63 rad/s}$

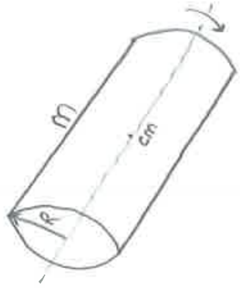
$$24400 = \frac{1}{2} \cdot I \cdot (63 rad/s)^2 \qquad \underline{I = 12.3 kg \cdot m^2}$$

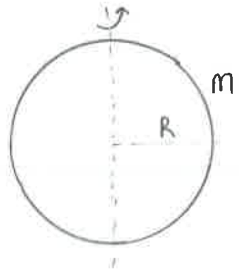
9.5 Parallel-Axis (Steiner's) Theorem for Rotational Inertia

The moment of inertia of some objects when they rotate about their center of mass are given in Table 9.2





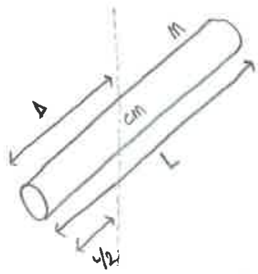




$I_{rod} = \frac{1}{12} \cdot m \cdot L^2$ $\quad I_{\substack{rectangular \\ plate}} = \frac{1}{12} m \cdot (a^2+b^2)$ $\quad I_{\substack{solid \\ cylindrican}} = \frac{1}{2} \cdot m \cdot R^2$ $\quad I_{\substack{solid \\ sphere}} = \frac{1}{5} \cdot m \cdot R^2$

If the objects rotate around on x-axis which is parallel to the axis that passes through the center of mass then rotational inertia of objects is calculated by parallel-axis theorem.

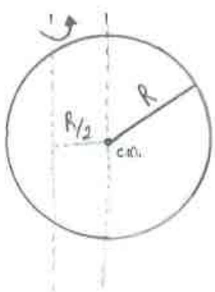
Ex:



Sol: $\boxed{I = I_{cm} + M.\Delta^2}$ → Parallel-axis theorem $\quad \Delta$ = Distance between two paralel-axis

$$I_{rod} = \frac{1}{12} \cdot M.L^2 + m.\left(\frac{L}{2}\right)^2 = \frac{1}{12} m.L^2 + m.\frac{L^2}{\underset{(3)}{4}} = \frac{1}{3} \cdot m.L^2$$

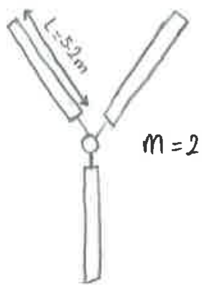
Ex:



Sol: $I_{sphere} = I_{sphere,cm} + \frac{m.R^2}{4}$

$$I_{sphere} = \frac{1}{\underset{(4)}{5}} m.R^2 + \frac{m.R^2}{\underset{(5)}{4}} = \frac{9}{20} \cdot m.R^2$$

Ex:



m = 240 kg

$I_{rod,cm} = \frac{1}{12} \cdot m.L^2$

The rotor is rotating at 350 rev/min. Find $KE_{rot,total}$ = ?

Sol: $KE_{rot} = \frac{1}{2} \cdot I_{rotor,total} \cdot W^2 \qquad I_{\substack{rotor \\ blade}} = I_{rod,cm} + m.\Delta^2$

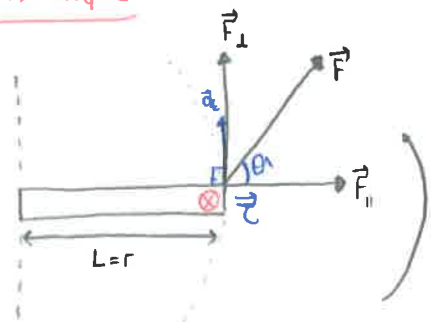
$KE_{rot} = \frac{1}{2} \cdot (3 \cdot I_{\substack{rotor \\ blade}}) \cdot W^2 \qquad I_{\substack{roter \\ blade}} = \frac{1}{12} m.L^2 + \frac{m.L^2}{\underset{(3)}{4}} = \frac{1}{3} \cdot m.L^2$

$$KE_{rot} = \frac{1}{2} \cdot \left(\frac{3}{3} \cdot m.L^2\right) \cdot w^2 = \frac{1}{2} \cdot 240\,kg \cdot (5{,}2\,m)^2 \cdot 2 \cdot (3{,}14) \cdot \frac{350}{60} \cdot \frac{rev}{s} = 4{,}35 \times 10^6\,J$$

CHAPTER-10

DYNAMICS of ROTATIONAL MOTION

10.1 Torque

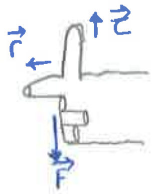


Torque is turning effect of F

$\tau = r.F.\sin\theta = r.F_\perp$ (N.m)

$= r.F_{tang}$ (N.m)

$\vec{\tau} = \vec{r} \times \vec{F}$



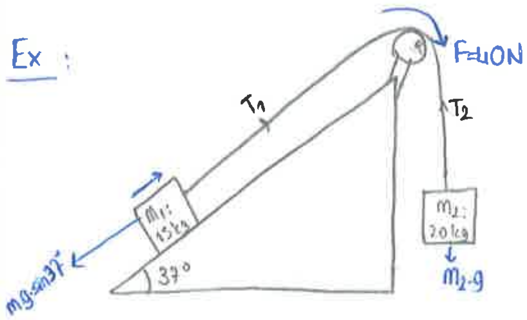
Right-hand rule

θ = angle between $\vec{r}$ and $\vec{F}$

$\tau = r.F_{sin} = r.m.a_t = r.m.r.\alpha = \underbrace{m.r^2}_{I}.\alpha$

$\boxed{\tau = I.\alpha}$ (N.m)

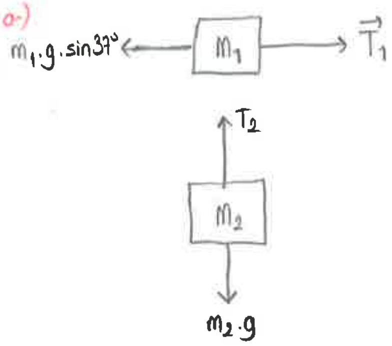
Ex :



a-) m_1 and m_2 move with $a_1 = a_2 = a_{sys} = 2\ m/s^2$. Find the tension T_1 and T_2 = ?

b-) Find the moment of inertia (rotational inertia) of pulley, $R_{pulley} = 0.25$ m

Sol : a-)



$F_{net} = m_1.a_1$

$T_1 - m_1.g.\sin 37 = m_1.a_1$

$T_1 = 15\,kg.2\,m/s^2 + 15\,kg.10\,m/s^2.0{,}6 = \underline{120\ N}$

$F_{net} = m_2.a_2 \Rightarrow m_2.g - T_2 = m_2.a_2$

$T_2 = m_2.g - m_2.a_2 = 20\,kg.10\,N/kg - 20\,kg.2\,m/s$

$\underline{T_2 = 160\ N}$

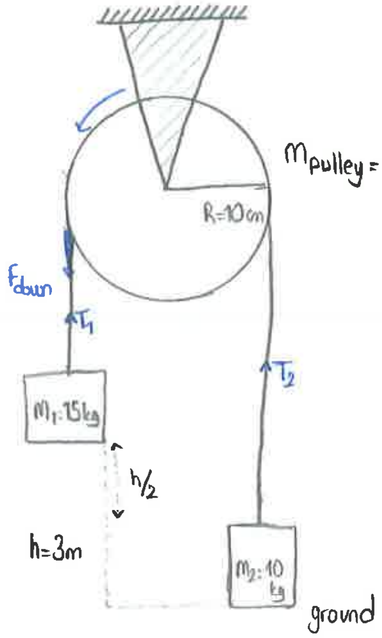
b-) $T = T_2 - T_1 = 40\ N = F \rightarrow$ It is used to rotate the pulley which has inertia.

$\tau = I \times \alpha \qquad a = R.\alpha \Rightarrow \alpha = \frac{a}{R}$

$F.R = I_{pulley}.\alpha = I_{pulley}.\frac{a}{R} \Rightarrow 0.25 \times 40\ N = I_{pulley} \times \frac{2\,m/s^2}{0.25} \Rightarrow \underline{I_{pulley} = 1.25\ kg.m^2}$

α

Ex :



$$I_{pulley} = \frac{1}{2} \cdot m_{pulley} \cdot R^2$$

Determine the speeds of two masses as they pass each other ?

Sol : I. Way

$$F_{net} = m_{tot} \cdot a_{sys}$$

$$m_1 \cdot g - T_1 = m_1 \cdot a_1 \quad (I)$$

$$+ \quad T_2 - m_2 \cdot g = m_2 \cdot a_2 \quad (II)$$

$$m_1 \cdot g - m_2 \cdot g - \underbrace{(T_1 - T_2)}_{F} = (m_1 + m_2) \cdot a_{sys}$$

$$\tau = r \cdot F \qquad R \cdot F = I \cdot \alpha$$

$$\tau = I \cdot \alpha \qquad R \cdot F = I_{pulley} \cdot \frac{a}{R}$$

$$a = r \cdot \alpha$$

$$F \cdot \cancel{R} = I \cdot \alpha = \frac{1}{2} \cdot M_{pulley} \cdot R^{\cancel{2}} \cdot \alpha$$

$$F = \frac{1}{2} \cdot M_{pulley} \cdot R \cdot \alpha = \frac{1}{2} \cdot M_{pulley} \cdot a_{sys}$$

$$m_1 \cdot g - m_2 \cdot g - \frac{1}{2} \cdot M_{pulley} \cdot a_{sys} = (M_1 + M_2) \cdot a_{sys} \quad \Rightarrow \boxed{a_{sys} = 1.89 \ m/s^2}$$

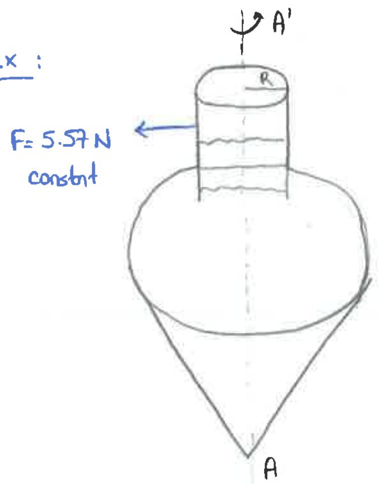
$$V_f^2 = V_i^2 + 2 \cdot a_{sys} \cdot h/2 \qquad V_f = (0 + 2 \times 1.89 \ m/s^2 \times 1.5 \ m)^{1/2} \qquad \boxed{V_f = 2.38 \ m/s^2}$$

II. Way

$$\Sigma E_i = \Sigma E_f$$

$$m_1 \cdot g \cdot h = m_1 \cdot g \cdot h/2 + m_2 \cdot g \cdot h/2 + \frac{1}{2} \cdot m_1 \cdot V^2 + \frac{1}{2} \cdot m_2 \cdot V^2 + \frac{1}{2} \cdot I_{pulley} \cdot W^2 \qquad \boxed{V = 2.38 \ m/s^2}$$

Ex :



The top has $I_{top} = 4 \times 10^{-4}$ $kg.m^2$ initially at rest and free to rotate around AA' axis. What is the angular speed of top after 80 cm of string was pulled of ?

Sol : $\tau = I_{top} \cdot \alpha$

$$R \cdot F = I_{top} \cdot \frac{a}{R}$$

$$a = \frac{R^2 \cdot F}{I_{top}}$$

$$V_f^2 = \overset{0}{\widehat{V_i^2}} + 2 \cdot a \cdot \Delta X$$

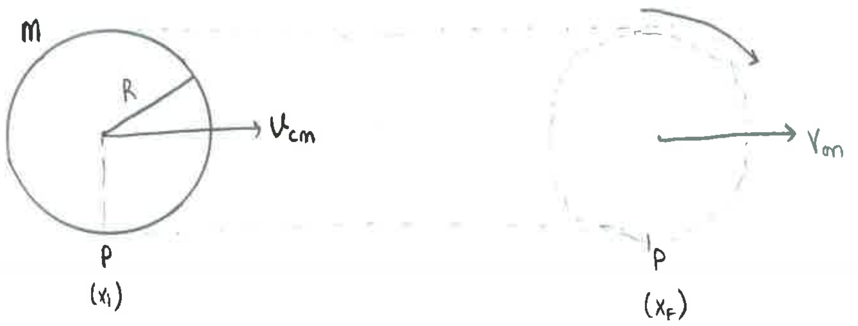
$$R^2 \cdot W^2 = 2 \cdot \frac{R^2 \cdot F}{I_{top}} \cdot \Delta X$$

$$V = R \cdot W$$

$$W = \left(2 \times \frac{5.57 \ N}{4 \times 10^{-4} \ kg m^2} \times 0.8 \ m \right)^{1/2}$$

$$\boxed{W = 149.3 \ rad/s}$$

10.2 Rigid Body Rotation Around a Moving Axis



The rolling disk has two types of kinetic energy, $KE_{translation}$ and $KE_{rotation}$.

$$\boxed{KE_{total} = \frac{1}{2} \cdot m \cdot V_{cm}^2 + \frac{1}{2} \cdot I_{cm} \cdot w^2} \quad (J)$$

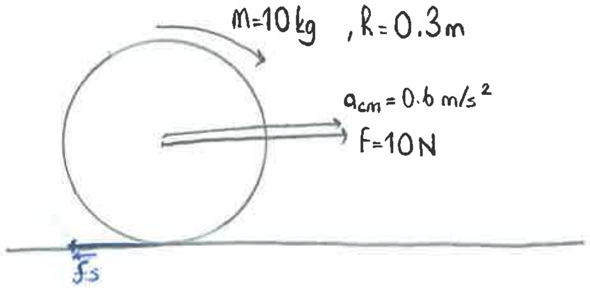
$V_{cm} = R \cdot w$

$$KE_{total} = \frac{1}{2} \cdot m \cdot R^2 \cdot w^2 + \frac{1}{2} \cdot I_{cm} \cdot w^2 = \frac{1}{2} \cdot (m \cdot R^2 + I_{cm}) \cdot w^2$$

$$\boxed{KE_{total} = \frac{1}{2} \cdot I_P \cdot w^2 \quad (J)}$$

$I_P \rightarrow$ Inertia with respect to P

Ex: $M = 10\,kg$, $R = 0.3\,m$



a-) $f_s = ?$ b-) $I_{cm} = ?$ c-) Find the KE_{total} 3 s later?

Wheel rolls without sliding. The $a_{cm.} = 0.6\,m/s^2$.

Sol: a-) $F_{net} = m \cdot a_{c.m.} \Rightarrow F - F_s = m \cdot a_{c.m.} \Rightarrow 10N - f_s = 10\,kg \cdot (0.6\,m/s^2)$ $\quad \underline{F_s = 4N}$

b-) Torque is produced by $\vec{f_s}$ with respect to center of mass.

$$f_s \cdot R = I_{c.m.} \cdot \alpha = I_{c.m.} \cdot \frac{a_{cm}}{R} \Rightarrow I_{cm} = \frac{f_s \cdot R^2}{a} = \frac{4N \cdot (0.3m)^2}{0.6\,m/s^2} \qquad \underline{I_{cm} = 0.6\,kg \cdot m^2}$$

c-) $KE_{total} = \frac{1}{2} \cdot I_{cm} \cdot w^2 + \frac{1}{2} \cdot m \cdot v_{cm}^2$

$V_{cm} = a_{cm} \cdot t = (0.6\,m/s^2) \cdot 3s = 1.8\,m/s$

$V_{cm} = R \cdot w \Rightarrow w = \frac{V_{cm}}{R} = \frac{1.8\,m/s}{0.3\,m}$ $\quad w = 6\,rad/s$

$$KE_{total} = \frac{1}{2} \cdot (0.6\,kg \cdot m^2) \cdot (6\,rad/s)^2 + \frac{1}{2} \cdot 10\,kg \cdot (1.8\,m/s)^2$$

$\underline{KE_{total} = 27\,J}$