

ALGORITMA ANALİZİ

FINAL ÇALIŞMA

Ex: $5x^3 + 2x^2 + 3 \leq c \cdot g(x)$

$$|5x^3 + 2x^2 + 3| \leq |5x^3| + |2x^2| + |3x^3|$$

$$f(x) \leq 10 \cdot x^3$$

$$\Rightarrow c = 10$$

$$O(x^3)$$

$$g(x) = x^3$$

$$c \cdot g'(x) \leq f(x) \Rightarrow c = 5$$

$$5 \cdot x^3$$

$$g'(x) = x^3$$

Ex: $\frac{n \cdot (n+1)}{2} \leq c \cdot g(x) \Rightarrow n^2 + n \leq n^2 + n^2 \quad n \leq n^2$

Approximate Solving = Ya parametre yok, Ya da değişkenlerin ortalaması. Tahmin var.

Exact Solving = Matematiksel formül ile ifade edilir.

Algoritmalar nasıl tasarlanır?

- Iterative mi Recursive mi karar verilir.
- Approximate solving mi exact solving mi karar verilir.

Algoritmanın Doğruluğunun İspatı

- Her durumda doğru sonuca vermeli
- Algoritmanın verdiği değerler ispat edilmeli (Matris formunda)

! Hata payı düşük olmalı - Parametre verilmeli - Ortalama - Kesin sonuca var mı? - Doğruluğu ispatlanıyor mu?

Spanning Tree

* Tree has no cycles. * Spanning bütün köşelere uğrar, cycle oluşturmaz.

Prim Algoritması = Loop durumuna bakılır, The vertex bir kez seçilmeli, Yol hesabı yok.

Brute Force

- Problem statement & Definitions of the concepts (Durum & Tanım) - Ex: Bubble Sort - Selection Sort

Selection Sort

Minimum değeri bul, başa getir:
for $i \leftarrow 0$ to $n-2$ do
 $\min \leftarrow i$
 for $j \leftarrow i+1$ to $n-1$ do
 if $A[j] < A[\min]$ $\min \leftarrow j$
 swap $A[i]$ and $A[\min]$

Bubble Sort

- İlk indisten başla, bir sonraki indisten büyüğe yer değiştir: for $i \leftarrow 0$ to $n-2$ do
for $j \leftarrow 0$ to $n-2-i$ do
if $A[j+1] < A[j]$
swap $A[j]$ and $A[j+1]$

Ex :	Job 1	Job 2	Job 3	Job 4	
Person 1	9	2	7	8	$\langle 1, 2, 3, 4 \rangle = 1+2+4+6 = 13$
Person 2	6	4	3	7	
Person 3	5	8	1	8	
Person 4	7	6	9	4	

Insertion Sort

- İkili kontrol yapar, bir sonraki indisten büyüğe yer değiştirir. Küçük en sola gelir.

for $i \leftarrow 1$ to $n-1$ do
 $u \leftarrow A[i]$
 $j \leftarrow i-1$
 while $j > 0$ and $A[j] > u$ do
 $A[j+1] \leftarrow A[j]$
 $j \leftarrow j-1$
 $A[j+1] \leftarrow u$

2) Find the big-O notation of the algorithm given by $f(n) = 100n + 5$

a) For all $n \geq 5$

$$f(x) \leq c \cdot g(x)$$
$$100n + 5 \leq 100n + n$$

$$100n + 5 \leq 101n$$

$$g(x) = O(n)$$

$$c = 101$$

b) For all $n \geq 1$

$$f(x) \leq c \cdot g(x)$$

$$100n + 5 \leq 100n + 5n$$

$$100n + 5 \leq 105n$$

$$g(x) = O(n)$$

$$c = 105$$

Merge Sort

if $n > 1$

copy $A[0 \dots (n/2)-1]$ to $B[0 \dots (n/2)-1]$

copy $A[(n/2) \dots (n-1)]$ to $C[0 \dots (n/2)-1]$

Mergesort($B[0 \dots (n/2)-1]$)

Mergesort($C[0 \dots (n/2)-1]$)

Merge(B, C, A)

Transform & Conquer

Problem's \Rightarrow Simpler Instance

\Rightarrow Solution

Instance \Rightarrow Another Representation

or
Another Problem's Instance

Greedy Algorithms

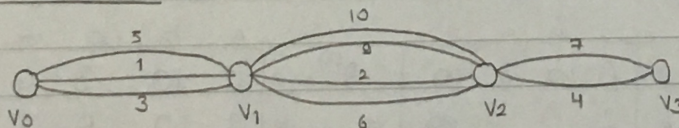
Change Making

$d_1 = 25$ (quarter), $d_2 = 10$ (dime), $d_3 = 5$ (nickel), $d_4 = 1$ (penny)

$$48 = 25 + 10 + 10 + 1 + 1 + 1 = \textcircled{6}$$

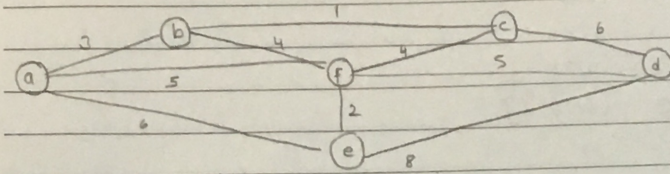
$$d_1 = 20 \quad d_2 = 19 \quad d_3 = 5 \quad d_4 = 1 \quad 24 = 20 + 1 + 1 + 1 + 1 = \textcircled{5}$$

Graph Problems

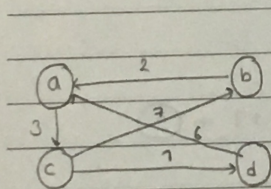


$$1 + 2 + 4 = 7$$

PRIM



Tree Vertices	Remaining Vertices	
$a(-, -)$	$b(a, 3), c(-, \infty), d(-, \infty), e(a, 6), f(a, 5)$	$b(a, 3)$
$b(a, 3)$	$c(b, 1), d(-, \infty), e(a, 6), f(b, 4)$	$d(b, 8)$
$c(b, 1)$	$d(c, 6), e(a, 6), f(b, 4)$	$c(b, 7)$
$f(b, 4)$	$d(c, 6), e(f, 2)$	$e(d, 9)$
$e(f, 2)$	$d(f, 5)$	



	a	b	c	d		a	b	c	d
$D^{(0)} = a$	0	∞	3	∞	$D^{(1)} = a$	0	∞	3	∞
$\rightarrow b$	2	∞	∞	∞	$\Rightarrow b$	2	0	5	∞
c	∞	7	0	1	c	∞	7	0	1
d	6	∞	∞	0	d	6	∞	9	0

	a	b	c	d		a	b	c	d
$D^{(2)} = a$	0	∞	3	∞	$D^{(3)} = a$	0	10	3	4
b	2	0	5	∞	$\rightarrow b$	2	0	5	6
c	9	7	0	1	c	9	7	0	1
d	6	∞	9	0	d	6	16	9	0