Math 113 Assignment 5

The due date for this assignment is Friday December 26,2014.

1. (3 points) If it is exist find the local maximum and local minimum values of following functions:

   (a) $f(x) = 3x^4 - 8x^3 - 6x^2 + 24x + 1$

   (b) $g(x) = \cos(x) - \sin(x)$

   (c) $h(x) = \tan(x)$

2. (3 points) If it is exist find the absolute maximum and absolute minimum values of following functions in the given interval.

   (a) $f(x) = 2x^3 - 15x^2 + 24x + 2$ on $[0, 2]$

   (b) $h(x) = 2x + 3$ on $[-1, 4]$

   (c) $t(x) = \cos(x)$ on $[-\frac{\pi}{2}, \frac{5\pi}{2}]$

3. (3 points) Determine the intervals where the following functions are concave up and concave down:

   (a) $f(x) = x^4 + 4x^3 - 18x^2 + 25x + 33$

   (b) $g(x) = \arctan(\frac{2}{x})$

   (c) $h(x) = \frac{x}{x^2+2}$

4. (9 points) Sketch the graph of following functions by getting informations from derivatives and asypmtotes.

   (a) $f(x) = \frac{2x^2-3x}{x-2}$

   (b) $g(x) = \frac{x^3-4x}{x^2-1}$

   (c) $h(x) = \frac{x^3}{x-1}$

5. (2 points) Let the functions f and g be differantiable on (a, b) such that $f'(x) = g(x)$ and $g'(x) = f(x)$ for all $x \in (a, b)$, furthermore $f(x_0) = 1$ and $g(x_0) = 0$ for some $x_0 \in (a, b)$. Show that

$$f^2(x) - g^2(x) = 1 \text{ for all } x \in (a, b)$$

1) a) $f(x) = 3x^4 - 8x^3 - 6x^2 + 24x + 1$

$f'(x) = 12x^3 - 24x^2 - 12x + 24 = 0 \Rightarrow x^3 - 2x^2 - x + 2 = 0$

$\Rightarrow x^2(x-2) - 1(x-2) = 0 \Rightarrow (x^2-1)(x-2) = 0$

$\Rightarrow x = \mp 1 \quad , x = 2$

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b) $g(x) = \cos x - \sin x$

$g'(x) = -\sin x - \cos x = 0 \Rightarrow \sin x = -\cos x \Rightarrow \dfrac{\sin x}{\cos x} = -1$

$\Rightarrow \tan x = -1$

$\Rightarrow x = \dfrac{3\pi}{4} + k.\pi \quad , k = \mp 1, \mp 2, \mp 3 \ldots$

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c) $h(x) = \tan x$

$h'(x) = 1 + \tan^2 x = 0 \Rightarrow \tan^2 x = -1 \Rightarrow \dfrac{\sin^2 x}{\cos^2 x} = -1$

$\Rightarrow \sin^2 x = -\cos^2 x$

$\Rightarrow \sin^2 x + \cos^2 x = 0$ ↯

$h'(x) = 1 + \tan^2 x$ denklemi hiçbir zaman 0'a eşit olmaz.

2)

a) $f(x) = 2x^3 - 15x^2 + 24x + 2$, $[0,2]$

$f'(x) = 6x^2 - 30x + 24 = 0 \Rightarrow x^2 - 5x + 4 = 0 \Rightarrow x_1 = 0$
$x_2 = 1$

$f(0) = 2 \Rightarrow x_1 = 0$ local min.

$f(1) = 13 \Rightarrow x_2 = 1$ local max.

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b) $h(x) = 2x + 3$, $[-1, 4]$

$x = 4$ local max

$x = -1$ local min.

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c) $t(x) = \cos x$, $\left[-\frac{\pi}{2}, \frac{5\pi}{2}\right]$

$t'(x) = -\sin x = 0 \Rightarrow \sin x = 0 \Rightarrow x = k \cdot \pi$, $k = 0, \mp 1, \mp 2, \ldots$

3)

a) $f(x) = x^4 + 4x^3 - 18x^2 + 25x + 33$

$f'(x) = 4x^3 + 12x^2 - 36x + 25 = 0 \Rightarrow$

$f''(x) = 12x^2 + 24x - 36 = 0 \Rightarrow x^2 + 2x - 3 = 0 \Rightarrow \begin{matrix} x_1 = -3 \\ x_2 = 1 \end{matrix}$

		-3		1	
$f''(x)$	+		−		+

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b) $g(x) = \arctan\left(\frac{2}{x}\right)$

$g'(x) = \dfrac{-2 \cdot x^{-2}}{1 + \frac{4}{x^2}} = \dfrac{-2 \cdot x^{-4}}{x^2 + 4} = \dfrac{-2}{x^6 + 4 \cdot x^4}$

$g''(x) = -2 \cdot \left[\dfrac{-(6x^5 + 16x^3)}{(x^6 + 4x^4)^2} \right] = \dfrac{12 \cdot x^5 + 32x^3}{x^{12} + 8 \cdot x^{10} + 16x^8} = \dfrac{12 \cdot x^2 + 32}{x^9 + 8 \cdot x^7 + 16x^5} = 0$

$\Rightarrow 12x^2 + 32 = 0 \Rightarrow 3x^2 + 8 = 0 \Rightarrow x^2 = -\frac{8}{3} \Rightarrow$ x'ler yok

$x^5(x^4 + 8x^2 + 16) \Rightarrow \boxed{x_1 = 0}$, $(x^2+4)^2 = 0$'dan x'ler gelmez.

		0	
$f^4(x)$	−	○	+

c) $h(x) = \frac{x}{x^2+2}$

$$h'(x) = \frac{(x^2+2) - x(2x)}{(x^2+2)^2} = \frac{2-x^2}{x^4+4x^2+4}$$

$$h''(x) = \frac{-2x(x^4+4x^2+4) - (2-x^2)(4x^3+8x)}{(x^4+4x^2+4)^2} = 0$$

$$\frac{2x\cdot(x^4-4x^2-16)}{(x^4+4x^2+4)} = 0 \Rightarrow 2x\cdot(x^4-4x^2-8) = 0$$

$$\Rightarrow \begin{array}{l} 2x = 0 \\ x^4-4x^2-8 = 0 \\ x^4+4x^2+4 = 0 \end{array} \Rightarrow \begin{array}{l} x_1 = 0 \\ (x^2-2)^2 = 12 \\ (x^2+2)^2 = 0 \end{array} \Rightarrow \begin{array}{l} x_1 = 0 \\ x^2 = \mp 2 + 2\sqrt{3} \\ \text{tek gelmez.} \end{array} \Rightarrow \begin{array}{l} x_1 = 0 \\ x_2 = -\sqrt{2\sqrt{3}-2} \\ x_3 = \sqrt{2\sqrt{3}-2} \\ x_4 = -\sqrt{2\sqrt{3}+2} \\ x_5 = \sqrt{2\sqrt{3}+2} \end{array}$$

		x_4		x_2		0		x_3		x_5	
$f''(x)$	$-$		$+$		$-$		$+$		$-$		$+$
	$\cap$		$\cup$		$\cap$		$\cup$		$\cap$		$\cup$

4) a) $f(x) = \frac{2x^2-3x}{x-2}$

$\lim_{x\to-\infty} \frac{2x^2-3x}{x-2} = -\infty$, $\lim_{x\to\infty} \frac{2x^2-3x}{x-2} = \infty$

$x-2 \neq 0 \Rightarrow x \neq 2$

$\lim_{x\to2^+} \frac{2x^2-3x}{x-2} = +\infty$ $\qquad \lim_{x\to2^-} \frac{2x^2-3x}{x-2} = -\infty$

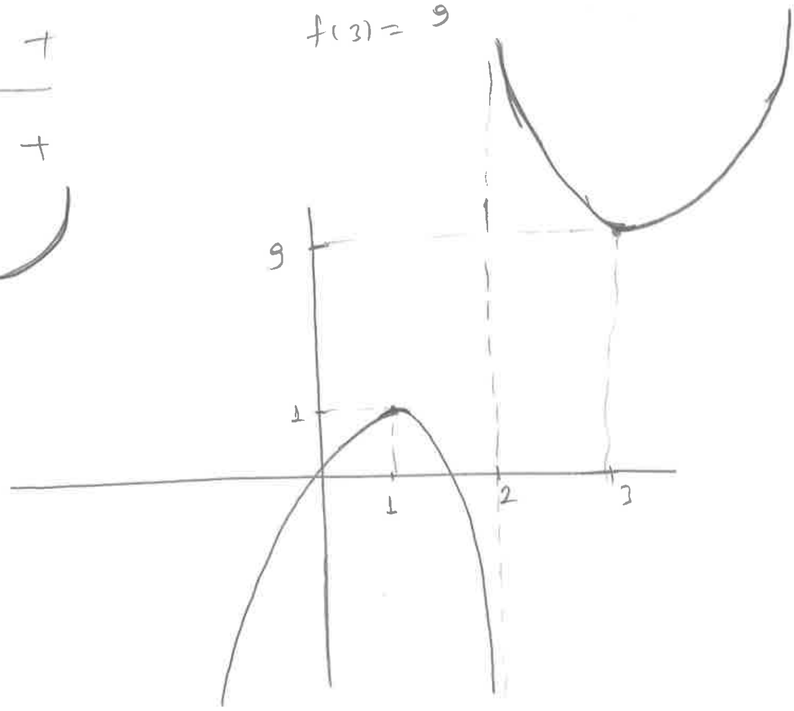
$$f'(x) = \frac{(4x-3)(x-2) - 2x^2+3x}{(x-2)^2} = \frac{2x^2-8x+6}{x^2-4x+4} = \frac{2.(x-3)(x-1)}{(x-2)^2} = 0 \Rightarrow \begin{matrix} x_1 = 2 \\ x_2 = 3 \\ x_3 = 1 \end{matrix}$$

$$f''(x) = \frac{(4x-8)(x^2-4x+4) - (2x-4)(2x^2-8x+6)}{(x-2)^4} = \frac{2.(x^2+4x+5)}{(x-2)^3} \Rightarrow x_1 = 2$$

		1		2		3	
f'	+	0	−	0	−	0	+
f''	−		−	0	+		+

$f(1) = 1$

$f(3) = 9$



5)

$f'(x)=g(x)$ ve $g'(x)=f(x)$, $f(x_0)=1$ ve $g(x_0)=0$ ise

$f^2(x)-g^2(x)=1$ olduğunu gösteriniz.

$f^2(x)-g^2(x)=1$

$\Rightarrow 2 f'(x) \cdot f(x) - 2 \cdot g'(x) \cdot g(x) = 0$

$\Rightarrow f'(x) \cdot f(x) = g'(x) \cdot g(x)$, 1) $f'(x)=g(x)$ ise

$g(x) \cdot f(x) = g'(x) \cdot g(x)$

$f(x)=g'(x)$ olur.

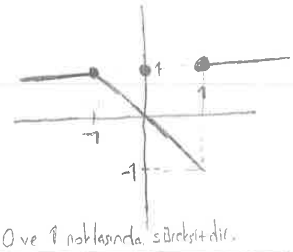
2) $g'(x)=f(x)$ ise

$f'(x) \cdot \cancel{f(x)} = \cancel{f(x)} \cdot g(x)$

$f'(x)=g(x)$ olur.

$f^2(x)-g^2(x)=1 \iff (f'(x)=g(x)) \wedge (g'(x)=f(x))$

ÖDEV – 2 Mate113 : Analiz – I

Sorular	Cevaplar
1. Aşağıdaki fonksiyonunun grafiğini çiziniz. $f(x) = \begin{cases} 1, & x \le -1 \\ -x, & -1 < x < 0 \\ 1, & x = 0 \\ -x, & 0 < x < 1 \\ 1, & x \ge 1. \end{cases}$ Daha sonra, detaylı olarak $x = -1, 0, 1$ noktalarında f'nin limitini, tek-taraflı limitini, sürekliliğini ve tek-taraflı sürekliliğini inceleyiniz. Bu noktalardan herhangi birinde kaldırılabilir süreksizlik var mıdır? Açıklayınız.	 0 ve 1 noktalarında süreksizdir. $\lim_{x\to -1} f(x) = 1$ $\lim_{x\to -1^-} f(x) = 1$, $\lim_{x\to -1^+}$ 1 $\lim_{x\to 0^-} f(x) = 0$ $\lim_{x\to 1^-} f(x) = -1$ $\lim_{x\to 0^+} f(x) = 0$ $\lim_{x\to 1^+} f(x) = 1$ $\lim_{x\to 0} f(x) = 0$ $\lim_{x\to 1} f(x) =$ There is no limit
2. $\lim_{x\to 0} f(x) = 1/2$ ve $\lim_{x\to 0} g(x) = \sqrt{2}$ olmak üzere, her x için $f(x)$ ve $g(x)$ tanımlı olsun. $x \to 0$ iken aşağıdaki fonksiyonların limitlerini bulunuz. a. $-g(x) = -\sqrt{2}$ b. $g(x)\cdot f(x) = \frac{\sqrt{2}}{2}$ c. $f(x) + g(x) = \frac{1}{2} + \sqrt{2} = \frac{2\sqrt{2}+1}{2}$ d. $1/f(x) = 2$ e. $x + f(x) = x + \frac{1}{2} = \frac{1}{2}$ f. $\frac{f(x)\cdot \cos x}{x-1} = \frac{\frac{1}{2}\cdot \cos(0)}{x-1} = \frac{\frac{1}{2}}{-1} = -1/2$	a) $-\sqrt{2}$ c) $\frac{2\sqrt{2}+1}{2}$ f) $-\frac{1}{2}$ b) $\frac{\sqrt{2}}{2}$ d) 2 e) $1/2$
3. verilen limit değerlerini sağlayacak şekilde $\lim_{x\to 0} g(x)$ değerini bulunuz. $$\lim_{x\to 0}\left(\frac{4-g(x)}{x}\right) = 1$$	$\frac{4-g(x)}{x} = 1$ $4 - g(x) = x$ $g(x) = 4 - x$ $\lim_{x\to 0}\left(\frac{4-g(x)}{x}\right) = 1 \Rightarrow g(0) = 4 - 0 = 4$ //
4. Aşağıdaki fonksiyonlar hangi aralıklarda süreklidir? a. $f(x) = \tan x$ b. $g(x) = \csc x$ c. $h(x) = \frac{\cos x}{x - \pi}$ d. $k(x) = \frac{\sin x}{x}$ a) $\mathbb{R} \setminus \{$ $x \ne \pi$ $\tan \frac{\pi}{2} = \infty$ $\frac{\sin x}{\cos x}$ $\frac{1}{\sin x}$ $\frac{\sin x}{\cos x} = \infty$ $\cos x = 90$ $0, \pi, 2\pi$	a) $\mathbb{R} - \{k\frac{\pi}{2}\}$, $k = \mp 1, \mp 2, \mp 3, \ldots \Rightarrow \ldots (-\frac{3\pi}{2}, -\frac{\pi}{2}) \cup (-\frac{\pi}{2}, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \frac{3\pi}{2})$ b) $\mathbb{R} - \{k\pi\}$, $k = 0, \mp 1, \mp 2, \ldots \Rightarrow \ldots \cup (-\pi, \pi) \cup (\pi, 2\pi) \cup \ldots$ c) $\mathbb{R} - \{\pi\}$. d) $\mathbb{R} - \{0\}$
Limiti bulunuz, eğer limit yoksa nedenini açıklayınız.	
5. $\lim \frac{x^2 - 4x + 4}{x^3 + 5x^2 - 14x}$ a) $x \to 0$ için. b) $x \to 2$ için.	$\Rightarrow \frac{0^2 - 4.0 + 4}{0^3 + 5.0^2 - 14.0}$ $\frac{4}{0}$ (?) $\Rightarrow \frac{(x-2)^2}{x.(x^2+5x-14)} = \frac{(x-2)^{\cancel{2}}}{x.(x+7)\cancel{(x-2)}} = \frac{x-2}{x.(x+7)} = \frac{-2}{0}$ (? $\Rightarrow \frac{2^2 - 4.2 + 4}{2^3 + 5.2^2 - 14.2} = \frac{4-8+4}{8+20-28} = \frac{0}{0}$ $\downarrow \frac{0}{2.(2+7)} = 0$
6. $\lim \frac{x^2 + x}{x^5 + 2x^4 + x^3}$ a) $x \to 0$ için. b) $x \to -1$ için.	$\Rightarrow \frac{0}{0} \rightarrow \frac{x(x+1)}{x^3(x^2+2x+1)} = \frac{\cancel{x}.\cancel{(x+1)}}{x^{\cancel{3}}(x+1)^{\cancel{2}}} = \frac{1}{x^2.(x+1)} = ?$ ∞ X $\hookrightarrow \frac{x(x+1)}{x^3(x+1)^2} = \frac{1}{x^2(x+1)}$, $\lim_{x\to -1} \frac{1}{x^2(x+1)} = \frac{1}{0} \rightarrow ?$
7. $$\lim_{x\to a} \frac{x^2 - a^2}{x^4 - a^4}$$ $\frac{0}{0} \rightarrow$	$\frac{(x-a)(x+a)}{(x-a)(x+a)(x^2+a^2)} = \frac{1}{x^2+a^2} = \frac{1}{2.a^2}$ //

8. $\lim_{x\to 0} \dfrac{\frac{1}{2+x}-\frac{1}{2}}{x} \to \dfrac{0}{0}$	$\dfrac{\frac{1}{2+x}-\frac{1}{2}}{x} = \dfrac{\frac{2-2-x}{4+2x}}{x} = \dfrac{-x}{4+2x}\cdot\dfrac{1}{x} \Rightarrow \dfrac{-x}{x.(4+2x)} = -\dfrac{1}{4}$ //
9. $\lim_{x\to 0} \dfrac{(2+x)^3-8}{x} \to \dfrac{0}{0}$	$\dfrac{(2+x)^3-2^3}{x} = \dfrac{(2+x-2).((2+x)^2+(2+x)\cdot 2+4)}{x} = \dfrac{(4+x).(\cancel{4}+2x+x^2-\cancel{4}-2x+x)}{x}$ $(x^2 4)(4+x)$ / x ; 12 //
10. $\lim_{x\to 1} \dfrac{x^{1/3}-1^3}{\sqrt{x}-1^3} \to \dfrac{\sqrt[3]{x}-1^3}{\sqrt{x}-1^3} \to \dfrac{0}{0}$	$\dfrac{(\sqrt{x}-1)((\sqrt{x})^2+\sqrt{x}+1)}{(\sqrt{x}-1)} = \dfrac{\sqrt{x}+x+1}{1} = 3$ //
11. $\lim_{x\to 64} \dfrac{x^{2/3}-16}{\sqrt{x}-8}$ ($\sqrt[3]{x^2}$) $= \lim_{x\to 64}$	$\dfrac{(x^{1/3}-4)(x^{1/3}+4)}{(\sqrt{x}-8)} = \lim_{x\to 64} \dfrac{(x^{1/3}-4)(x^{1/3}+4)(x^{1/2}+8)}{x-64}$; $\boxed{a^3-b^3=(a-b)(a^2+ab+b^2)}$; $\lim_{x\to 64} \dfrac{(x^{1/3}+4)(x^{1/2}+8)(x-64)}{(x-64)(x^{2/3}+4.x^{1/3}+16)} = \lim_{x\to 64}\dfrac{(x^{1/3}+4)(x^{1/2}+8)}{(x^{2/3}+4x^{1/3}+16)} = \dfrac{8.16}{16+4.4+16} = \dfrac{8}{3}$ //
12. $\lim_{x\to 0} \dfrac{\tan(2x)}{\tan(\pi x)} \to \dfrac{0}{0}$	$\to \dfrac{2x}{\pi.x} = \dfrac{2}{\pi}$ // ; $\lim_{x\to 0} \dfrac{\frac{\tan(2x)}{2x}.2x}{\frac{\tan(\pi x)}{\pi x}.\pi x} = \dfrac{2x}{\pi x} = \dfrac{2}{\pi}$
13. $\lim_{x\to\pi} \sin\left(\dfrac{x}{2}+\sin x\right)$	$\sin(90+\sin 180) \to \cos\left(\dfrac{\sin x}{\pi}\right) \to 1$ //
14. $\lim_{x\to 0} \dfrac{8x}{3\sin x - x} \to \dfrac{0}{0}$	$\dfrac{\sin x}{x}\to 1$; $\lim_{x\to 0} \dfrac{1}{\frac{3}{8}\frac{\sin x}{x}-\frac{1}{8}} = \lim_{x\to 0} \dfrac{1}{\frac{3}{8}-\frac{1}{8}} = \lim_{x\to 0} 4 = 4$ //
15. $\lim_{x\to 0} \dfrac{\cos 2x - 1}{\sin x} = \dfrac{0}{0}$	$\cos 2x = 1-2\sin^2 x \Rightarrow \lim_{x\to 0} \dfrac{1-2\sin^2 x-1}{\sin x} = -2 \lim_{x\to 0} \sin x = -2.0 = 0$ //

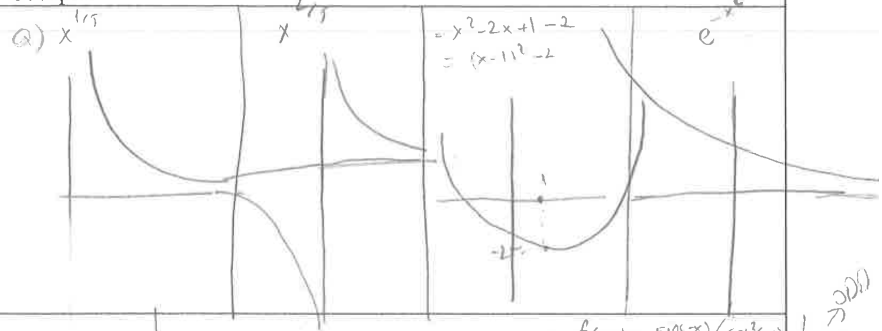
$g(x)$ fonksiyonunun (belirtilen x değerleri için) limitini bulunuz.

16. $\lim_{x\to 0^+} (4g(x))^{1/3} = 2$	$\lim_{x\to 0^+} (4.(g(x)))^{1/3} = 2 \Rightarrow \lim_{x\to 0^+} 4.g(x) = 8 \Rightarrow 4.\lim_{x\to 0^+} g(x) = 8$; $\Rightarrow \boxed{\lim_{x\to 0^+} g(x) = 2}$
17. $\lim_{x\to\sqrt{5}} \dfrac{1}{x+g(x)} = 2$	$\lim_{x\to\sqrt{5}} \dfrac{1}{x+g(x)} = \dfrac{\lim_{x\to\sqrt{5}} 1}{\lim_{x\to\sqrt{5}} x+g(x)} = 2 \Rightarrow \lim_{x\to\sqrt{5}} x + \lim_{x\to\sqrt{5}} g(x) = \dfrac{1}{2}$; $\Rightarrow \lim_{x\to\sqrt{5}} g(x) = \dfrac{1}{2}+\sqrt{5} = \dfrac{1-2\sqrt{5}}{2}$
18. $\lim_{x\to -2} \dfrac{5-x^2}{\sqrt{g(x)}} = g(x) = (x+2)^2$; $(x+2)^2$; $(x+2)^{-1}$	$\{\ n \in \mathbb{Z}^+ \ \vert\ g(x) = (x+2)^{-n}\ \}$

19. $f(x) = x^3 - x - 1$ olsun. a. Ara Değer Teoremi'ni kullanarak f'nin -1 ve 2 aralığında bir kökü olduğunu gösteriniz.	$f(-1) = -1+1-1 = -1$; $f(2) = 8-2-1 = 3$; $f(x) = 0 \Rightarrow f(-1) < f(x) < f(2) \Rightarrow -1<x<2$
20. $f(\theta) = \theta^3 - 2\theta + 2$ olsun. Ara Değer Teoremi'ni kullanarak f'nin -2 ve 0 aralığında bir kökü olduğunu gösteriniz.	

$\sqrt[5]{x}$

Ö D E V - I (Mate 113: Analiz – I)

Sorular	Cevaplar
1. Aşağıdaki fonksiyonların y-eksenine, orijine göre simetrik olup olmadığını belirleyin. a) $f(x) = x^{1/5}$ b) $f(x) = x^{2/5}$ c) $f(x) = x^2 - 2x - 1$ d) $f(x) = e^{-x^2}$	
2. Aşağıda verilen fonksiyonların tek, çift yada ne tek ne de çift olup olmadığını belirleyin. a) $f(x) = \sec(x)\tan(x)$ b) $f(x) = x\cos(x)$ c) $f(x) = x^5 - x^3 - x$ d) $f(x) = \frac{x^4+1}{x^3-2x}$	$f(x)=f(-x)$ çift $f(x)=-f(-x)$ tek a) $f(x) = \sec(x)\tan(x) = \sin x/\cos^2 x \Rightarrow f(-x) = \sin(-x)/\cos^2(-x)$, $f(-x) = -\sin x/\cos^2(x)$ } Tek → ODD b) $f(x) = x.\cos x \Rightarrow f(-x) = -x.\cos(-x) = -x.\cos x \Rightarrow$ Tek → ODD c) $f(x) = x^5 - x^3 - x \Rightarrow f(-x) = -x^5 + x^3 + x \Rightarrow$ Tek → ODD d) $f(x) = \frac{x^4+1}{x^3-2x} \Rightarrow f(-x) = \frac{x^4+1}{-x^3+2x} \Rightarrow$ ODD f is odd
3. Eğer $f(a-x) = f(a+x)$ Ise o zaman $g(x) = f(x+a)$ Bir çift fonksiyondur, gösterin. $f(x+a) = f(-x-a)$	if $f(x)$ is even, then $f(x)=f(-x)$. if $f(x+a)$ is even, then $f(x+a) = f(-x-a)$ $f(-x-a) = f(a-x) \Rightarrow -x-a = a-x \Rightarrow a=-a$
4. Aşağıdaki fonksiyonların tanım kümesini bulunuz: domain a) $f(x) = \|x\| - 2$ b) $f(x) = \sqrt{16-x^2}$ c) $f(x) = x^{2/5}$ $-\infty < x <$ $\sqrt[5]{x^2}$ d) $f(x) = -1 + \sqrt[3]{2-x}$ $x \le 2$	a) $-\infty < x < +\infty$ b) $\sqrt{16-x^2} \geq 0$ $16 - x^2 \geq 0$ $16 \geq x^2 \Rightarrow 4 \geq x \geq -4$
5. Aşağıdaki fonksiyonların değer kümesini bulunuz: range a) $f(x) = \|x\| - 2$ b) $f(x) = \sqrt{16-x^2}$ c) $f(x) = x^{2/5}$ d) $f(x) = -1 + \sqrt[3]{2-x}$	a) $-\infty < x < +\infty$ $0 \le \|x\| < \infty$ $-2 \le \|x\| - 2 < \infty$ $-2 \le f(x) < \infty$ b) $-\infty < x < \infty$ $0 \le x^2 < \infty$ $-\infty < -x^2 \le 0$ $-\infty < 16 - x^2 \le 16$ $\boxed{0 \le 16 - x^2 \le 4}$ $\boxed{-4 \le f(x) \le 4}$ c) $-\infty < x < +\infty$ $0 \le x^2 < +\infty$ $0 \le x^{2/5} < \infty$ $\boxed{0 \le f(x) < \infty}$ d) $-\infty < x < +\infty$ $\infty > -x > -\infty$ $\infty > 2-x > -\infty$ $\infty > \sqrt[3]{2-x} \geq 0$ $\infty > \sqrt[3]{2-x} - 1 \geq -1$

Sorular	Cevaplar
6. Aşağıdaki fonksiyonların tanım kümesini bulunuz: a) $f(x) = 2e^{-x} - 3$ b) $f(x) = 3^{2-x} + 1$ c) $f(x) = 2\sin(3x + \pi) - 1$ d) $f(x) = \ln(x - 3) + 1$ $(1,\infty)$ $x-3>0$ $x>3$ $(3,\infty)$	a) $-\infty < x < +\infty$ b) $-\infty < x < +\infty$ c) $-\infty < x < +\infty$ d) $x \in \mathbb{R}\setminus\{3\}$, $\ln(x-3) \Rightarrow x-3 \neq 0 \Rightarrow x \neq 3$
7. Aşağıdaki fonksiyonların değer kümesini bulunuz: a) $f(x) = 2e^{-x} - 3$ b) $f(x) = 3^{2-x} + 1$ c) $f(x) = 2\sin(3x + \pi) - 1$ d) $f(x) = \ln(x - 3) + 1$	a) $-\infty < x < \infty$ $\infty > -x > -\infty$ $e^{\infty} > e^{-x} > e^{-\infty}$ $e^{\infty} > e^{-x} > 0$ $e^{\infty} > 2e^{-x} > 0$ $e^{\infty} > 2e^{-x} - 3 > -3$ $e^{\infty} > f(x) \geq -3$ b) $\infty > -x > -\infty$ $\infty > 2-x > -\infty$ $3^{\infty} > 2-x > 3^{-\infty}$ $3^{\infty} > 3^{2-x} \geq 0$ $3^{\infty} > 3^{2-x} + 1 \geq 1$ $3^{\infty} > f(x) \geq 1$ d-)
8. $f(x) = \begin{cases} x+1, & -2 \leq x \leq 0 \\ x-1, & 0 < x \leq 2 \end{cases}$ ve $g(x) = \begin{cases} -x-2, & -2 \leq x \leq -1 \\ x, & -1 < x \leq 1 \\ -x+2, & 1 < x \leq 2 \end{cases}$ $f(g(x))$ parçalı-tanımlı fonksiyonları verilmektedir. Buna gore *fog* ve *gof* bileşke fonksiyonlarını bulunuz.	$fog(x) = \begin{cases} -x-1, & -2 \leq x \leq -1 \\ x+1, & -1 < x \leq 0 \\ x-1, & 0 < x \leq 1 \\ -x+1, & 1 < x \leq 2 \end{cases}$ Domain of $f(x)$ $[-2,0] \cup (0,2]$ $g(x)$ $[-2,-1] \cup (-1,1] \cup (1,2]$ $gof(x) = \begin{cases} -x-3, & -2 \leq x \leq -1 \\ x+1, & -1 < x \leq 0 \\ x-1, & 0 < x \leq 1 \\ -x+3, & 1 < x \leq 2 \end{cases}$
9. Aşağıdaki fonksiyonların artan olduğu en geniş aralığı bulunuz: a) $f(x) = \lvert x - 2\rvert + 1$ b) $f(x) = (x + 1)^4$ c) $f(x) = (3x - 1)^{1/3}$ d) $f(x) = \sqrt{2x - 1}$	a) $[2, \infty) \Rightarrow x \geq 2 \rightarrow f(x)$ is increasing b) $[-1, \infty) \Rightarrow$ c) $[\frac{1}{3}, \infty)$ d) $[\frac{1}{2}, \infty) \Rightarrow x \geq \frac{1}{2}$ $f(x)$ is increasing
10. Aşağıda parçalı tanımlı fonksiyonun tanım ve değer kümesini bulunuz. 1 0 -1 $f(x) = \begin{cases} -x-2, & -2 \leq x \leq -1 \\ x, & -1 < x \leq 1 \\ -x+2, & 1 < x \leq 2 \end{cases}$	Tanım: $-2 \leq x \leq 2$ (Domain) Değer: $[-1, 1]$ (Range) D: $[-2,-1] \cup (-1,1] \cup (1,2]$ R: $f([-2,-1]) \cup f((-1,1]) \cup f((1,2])$ $= [-1,0] \cup (-1,1] \cup [0,1)$ $= [-1, 1]$

<table>
<tr><th>Sorular</th><th>Cevaplar</th></tr>
<tr><td>11. $f(x) = 2 - x$ ve $g(x) = \sqrt[3]{x+1}$ ise aşağıdaki değerleri hesaplayın.

a) $(fof)(-1)$ $= f(f(-1)) \Rightarrow 2-(2-x)$
b) $(gof)(2)$ $= g(f(2)) \Rightarrow \sqrt[3]{(2-x)+1}$
c) $(fof)(x)$ $= x$
d) $(gog)(x)$ $= g(g(x)) = g\sqrt[3]{(2-x)+1}$</td><td>$= 2-(2-(-1)) = 1$
$= \sqrt[3]{3-x} = 1$

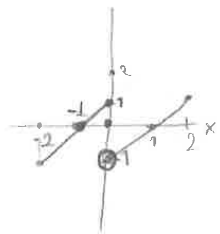
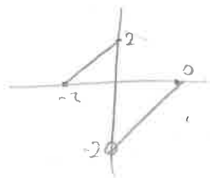
$= \sqrt{3-x}$</td></tr>
<tr><td>12. $f(x) = 2 - x^2$ ve $g(x) = \sqrt{x+2}$ ise aşağıda verilen bfonksiyon bileşke-lerinin tanım ve değer kümelerini belirleyin:

a) $(fof)(x)$
b) $(gof)(x)$
c) $(fog)(x)$
d) $(gog)(x)$ $\sqrt{\sqrt{x+2}+2}$</td><td>$(fof)(x) = f(f(x)) = 2-(2-x^2)^2 = 2-(4-4x^2+x^4) = -x^4+4x^2-2$ D: $\mathbb{R}$ R: $\mathbb{R}$
TK DK
$(-\infty,\infty)$ $(-2,\infty)$
$(gof)(x) = g(f(x)) = \sqrt{(2-x^2)+2} = \sqrt{-x^2+4} =$ $[-2,+2]$ TK ← D
$[0,4]$ DK ← R
$f(g(x)) = 2-(\sqrt{x+2})^2 = 2-\lvert x+2\rvert \Rightarrow$ $x < -2$ $(-\infty,-2)$ $(-\infty,2)$
$x \geq -2$ $2-y$ $(-\infty,-2]$
$x \geq -2$ $[-2,\infty)$ $[\sqrt{2},\infty)$ $(-2,\infty)$</td></tr>
<tr><td>13. $g(x) = \sqrt{x}$ olsun. Buna göre aşağıdaki işlemler altında elde edilen fonksiyonun grafiklerini çiziniz :

a) Dikey 4 çarpanı ile genişlet → $x.4$
b) Dikey 4 çarpanı ile daralt → $x/4$
c) Yatay 4 çarpanı ile genişlet → $y.4$
d) Yatay 4 çarpanı ile daralt → $x/4$</td><td>a-) $g(x) = \sqrt{x}$
↳ $y = 4 . g(x) = 4.\sqrt{x}$ stretch vertically 4

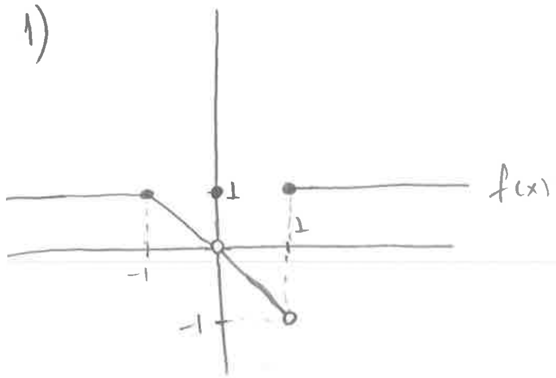
d-) $y = g(4x) = \sqrt{4x}$ Compress horizontally 4</td></tr>
<tr><td>14. $g(x) = \sqrt{x}$ olsun. Aşağıdaki öteleme(kaydırma) ve yansıma işlemleri altında elde edilen fonksiyon grafiklerini çizin:
reflecting
shift up right
a) $\frac{1}{2}$ birim yukarı, 3birim sağa ötele
b) 2 birim aşağı, $\frac{2}{3}$ birim sola kaydır.
c) x-eksenine göre yansıt (reflect)
d) y-eksenine göre yansıt.</td><td>a) $y - \frac{1}{2} = \sqrt{x-3}$
b) $y + 2 = \sqrt{x + \frac{2}{3}}$
c) $y = -(\sqrt{x})$ $y = \sqrt{-x}$
d) $y = \sqrt{-x}$ $y = -\sqrt{x}$</td></tr>
<tr><td>15. $y = f(x)$ fonksiyonun grafiğini kullanılarak aşağıdaki fonksiyonların grafiğinin eldeedilmesi için hangi işlemleri yapmak gerekir.

a) $y = f(-3x)$
b) $y = f(2x+1)$
c) $y = f\left(\frac{x}{3}\right) - 4$
d) $y = -3f(x) + \frac{1}{4}$</td><td>a) $x = -3x$ → Stretch horizontally by a factor of 3 and then reflect about x axis
b) $x = 2x+1$ ↳ Shift 1/2 unit to the left and then compress horizontally by factor of 2
c) $x = \frac{x}{3}$, $y = y+4$
d) $y = -\frac{1}{3}\left(y - \frac{1}{4}\right)$</td></tr>
</table>

Sorular	Cevaplar
16. Aşağıdaki trigonometric fonksiyonların peryotlarını belirleyin: a) $y = \cos(2x)$ b) $y = \sin(\frac{x}{2})$ c) $y = 2\cos(x - \frac{\pi}{3})$ d) $y = 1 + \sin(x + \frac{\pi}{4})$	$\sin 0 = 0$ $\cos 0 = 1$ $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\sin\frac{\pi}{2} = 1$ $\cos\frac{\pi}{2} = 0$ $\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$ $\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$ $\sin\pi = 0$ $\cos\pi = -1$ $\sin -\frac{\pi}{4} = -\frac{\sqrt{2}}{2}$ $\cos -\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\sin -\frac{\pi}{2} = -1$ $\cos -\frac{\pi}{2} = 0$ $\sin -\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$ $\cos -\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$ a) $y = \cos(2x) \Rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$ b) $y = \sin(\frac{x}{2}) \Rightarrow [-2\pi, 2\pi]$ c) $y = 2\cos(x - \frac{\pi}{3}) \Rightarrow [-\frac{2\pi}{3}, \frac{4\pi}{3}]$ d) $y = 1 + \sin(x + \frac{\pi}{4}) \Rightarrow [-\frac{5\pi}{4}, \frac{3\pi}{4}]$
17. $f(x) = \sin(x)$ ve $g(x) = \cos(x)$ Trigonometric fonksiyonların grafiklerini kullanarakaşağıdaki fonksiyonların grafiklerini çizin : a) $y = \cos(2x)$ b) $y = \sin(\frac{x}{2})$ c) $y = 2\cos(x - \frac{\pi}{3})$ d) $y = 1 + \sin(x + \frac{\pi}{4})$	$y = a \cdot f(b \cdot (x + c)) + d$ $y = \cos(2x) \Rightarrow$ $a = 0$, $b = 2$, $c = 0$, $d = 0$ $y = \sin(\frac{x}{2}) \Rightarrow$ $a = 0$, $b = 1/2$, $c = 0$, $d = 0$ $y = 2 \cdot \cos(x - \frac{\pi}{3}) \Rightarrow$ $a = 2$, $b = 1$, $c = -\frac{\pi}{3}$, $d = 0$ $2 \cdot \cos(\frac{3x - \pi}{3}) \Rightarrow$ $a = 1$, $b = 1$, $c = \frac{\pi}{4}$, $d = 1$
18. 0°, 30°, 45°, 60° ve 90° deki trigonometric değerlerini kullanarak aşağıdaki değerleri hesaplayın: a) $sin(22{,}5°)$ b) $cos(75°)$ c) $tan(15°)$ d) $sec(330°)$ $\Rightarrow \frac{1}{\cos(330°)} = \frac{2}{\sqrt{3}}$ $\sec(330°) = \sec(0° - 30°) =$	$\cos^2 x + \sin^2 x = 1$ $\cos 2x = 1 - 2\sin^2 x$ $\cos 2x = 2\cos^2 x - 1$ $\Rightarrow$ a.) $\sin^2(\frac{\pi}{8}) = \frac{1 - \cos(2 \cdot \frac{\pi}{8})}{2} \Rightarrow \frac{1 - \cos(\frac{\pi}{4})}{2} = \frac{1 - \frac{\sqrt{2}}{2}}{2} = \frac{2 - \sqrt{2}}{4}$ b.) $\cos(75°) = \cos(45° + 30°) = \cos 45 \cdot \cos 30 - \sin 45 \cdot \sin 30$ $= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2} = \frac{\sqrt{6} - \sqrt{2}}{2}$ c.) $\tan(45° - 30°) = \frac{\tan 45° - \tan 30°}{1 + \tan 45 \cdot \tan 30} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3} + 1} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{4 - 2\sqrt{3}}{4}$ $(\sqrt{3} - 1)$ $\cos(0° - 30°) = \cos 0 \cdot \cos 30 + \sin 0 \cdot \sin 30 = \frac{\sqrt{3}}{2}$
19. ABC is dik üçgenin C açısı diktir.. A, B, C açılarına karşılık gelen kenarlar sırasıyla a, b, c olsun. Bu durumda aşağıdakileri hesaplayın : a) a ve b eğer $c = 2$ ve $B = \pi/3$ ise b) 'a'yı 'A ve c' cinsinden c) 'a'yı 'B ve b' cinsinden d) $sin(A)$ yı 'a ve c' cinsinden	B, 60°, 2, 30°, $a = 1$, C, $\sqrt{3}$, A $\cos(\frac{\pi}{3}) = \frac{1}{2} = \frac{a}{c} = \frac{a}{2} \Rightarrow \frac{a}{2} = \frac{1}{2} \Rightarrow a = 1$ $\sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2} = \frac{b}{c} \Rightarrow$ $b = \sqrt{3}$, $c = 2$ $\frac{a}{c} = \cos(A) \Rightarrow a = \cos(A) \cdot c$ $a \cdot b = \frac{b}{a}$
20. $f(x) = \begin{cases} x + 1, & -2 \le x \le 0 \\ x - 1, & 0 < x \le 2 \end{cases}$ parçalı tanımlı fonksiyon verilmektedir. Buna göre f ve fof fonksiyonların grafiklerini çizin.	 $fof = \begin{cases} x + 2 & -2 \le x \le 0 \\ x - 2 & 0 < x \le 2 \end{cases}$ 

24.10.2014 Friday

1)



$$f(x) = \begin{cases} 1, & x \leq -1 \\ -x, & -1 < x < 0 \\ 1, & x = 0 \\ -x, & 0 < x < 1 \\ 1, & x \geq 1 \end{cases}$$

$x = 0$, f is removable dis-contunity.

$x = 1$ f is dis-contunity.

$$\lim_{x \to -1^-} f(x) = 1 = \lim_{x \to -1^+} f(x)$$

$$\lim_{x \to 0^-} f(x) = 0 = \lim_{x \to 0^+} f(x) \quad , \lim_{x \to 0} f(x) = 1$$

$$\left.\begin{array}{l} \lim_{x \to 1^-} f(x) = -1 \quad , \quad \lim_{x \to 1^+} f(x) = 1 \end{array}\right\} \text{does not exist limit.}$$

5) $$\lim_{x \to 0} \frac{x^2 - 4x + 4}{x^3 + 5x^2 - 14x} = \lim_{x \to 0} \frac{(x-2)^2}{x.(x-2)(x+7)} = \lim_{x \to 0} \frac{x-2}{x.(x+7)} = \frac{-2}{0.7} = -\infty$$

$$\lim_{x \to 2} \frac{x^2 - 4x + 4}{x^3 + 5x^2 - 14x} = \lim_{x \to 2} \frac{x-2}{x(x+7)} = \frac{0}{2.9} = 0$$

6) $$\frac{x^2 + x}{x^5 + 2x^4 + x^3} = \frac{x(x+1)}{x^3(x+1)^2} = \frac{1}{x^2} \cdot \frac{1}{(x+1)}$$

$$\left.\begin{array}{l} \lim_{x \to -1^-} \frac{1}{x^2.(x+1)} = \frac{1}{(-1)^2.(-1^- + 1)} = -\infty \\ \lim_{x \to -1^+} \frac{1}{x^2(x+1)} = \frac{1}{(-1)^2(-1^+ + 1)} = \infty \end{array}\right\} \text{does not exist}$$

$$\lim_{x \to 0} \frac{x^2 + x}{x^5 + 2x^4 + x^3} = \lim_{x \to 0} \frac{1}{x^2} \cdot \frac{1}{x+1} = \frac{1}{0} = \infty$$

$$\lim_{x \to -1} \frac{x^2 + x}{x^5 + 2x^4 + x^3} = \lim_{x \to -1} \frac{1}{x^2} \cdot \frac{1}{x+1} = \frac{1}{0} = \infty$$

10-) $\lim\limits_{x\to 1} \dfrac{x^{1/3}-1^3}{\sqrt{x}-1^3} \begin{cases} a^2-b^2 \\ a^3-b^3 \end{cases} = \lim\limits_{x\to 1} \dfrac{\cancel{(x^{1/6}-1)}(x^{1/6}+1)}{\cancel{(x^{1/6}-1)}(x^{2/6}+x^{1/6}+1)} = \dfrac{2}{3}$ //

11-) $\lim\limits_{x\to 64} \dfrac{\overbrace{x^{2/3}-16}^{a^2-b^2}}{\underbrace{x^{1/2}-8}_{a^3-b^3}} = \dfrac{(x^{2/6}-4)(x^{2/6}+4)}{(x^{1/6}-2)(x^{2/6}+2x^{1/6}+4)} = \dfrac{\cancel{(x^{1/6}-2)}(x^{1/6}+2).(x^{1/3}+4)}{\cancel{(x^{1/6}-2)}.(x^{1/3}+2x^{1/6}+4)} = \dfrac{(2+2).(4+4)}{(4+2.2+4)}$

$= \dfrac{4.8}{12} = \dfrac{8}{3}$ //

18-) $\lim\limits_{x\to -2} \dfrac{5-x^2}{\sqrt{g(x)}} = 0$

$\dfrac{\lim\limits_{x\to -2} 5-x^2}{\lim\limits_{x\to -2}} = \dfrac{1}{\sqrt{\lim\limits_{x\to -2} g(x)}} = 0 \Rightarrow \lim\limits_{x\to -2} g(x) = \infty$ //

Derivative of the Trigonometric Functions

sin(x)

$f(x) = \sin x \quad f'(x) = \cos x$

$\sin(x+h) = \sin x \cdot \cosh + \sinh \cdot \cos x$

$f(x) = \sin x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h} = \lim_{h\to 0} \frac{\sin x.\cosh + \sinh.\cos x - \sin x}{h} =$$

$$= \lim_{h\to 0} \frac{\sin x(\cosh - 1) + \sinh.\cos x}{h} = \lim_{h\to 0} \underbrace{\left(\sin x.\frac{\cosh - 1}{h}\right)}_{0} + \lim_{h\to 0} \left(\cos x.\underbrace{\frac{\sinh}{h}}_{1}\right) =$$

$$= \sin x.0 + \cos x.1 = \underline{\cos x}$$

cos(x)

$\cos(x+h) = \cos x.\cosh - \sin x.\sinh \qquad f(x) = \cos x \quad f'(x) = -\sin x$

$f(x) = \cos x$

$$f'(x) = \lim_{h\to 0} \frac{\cos(x+h) - \cos(x)}{h} = \lim_{h\to 0} \frac{\cos x.\cosh - \sin x.\sinh - \cos x}{h} = \lim_{h\to 0} \frac{\cos x(\cosh - 1) - \sin x.\sinh}{h}$$

$$= \lim_{h\to 0} \cos x.\underbrace{\frac{\cosh - 1}{h}}_{0} - \lim_{h\to 0} \sin x.\underbrace{\frac{\sinh}{h}}_{1} = \cos x.0 - \sin x.1 = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2.x \qquad \frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x.\tan x \qquad \frac{d}{dx}(\mathrm{cosec}\, x) = -\mathrm{cosec}\, x.\cot x$$

$$\frac{d}{dx}(\tan x) = \tan' x = 1 + \tan^2 x$$

tan(x)

$\tan x = \frac{\sin x}{\cos x}$

$$\frac{d}{dx}\left(\frac{\sin x}{\cos x}\right) = \frac{\cos x \cdot \frac{d}{dx}(\sin x) - \sin x.\frac{d}{dx}(\cos x)}{\cos^2 x} = \frac{\cos x.\cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

Kapalı Fonksiyonlarda Türev

$$\frac{-F_x}{F_y}$$

Ters Fonksiyonunun Türevi

$$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}, \; x \neq 0$$

Karekök fonksiyonun Türevi

$$\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

Slope

$\frac{\Delta y}{\Delta t} = \frac{\text{Degisim Orani}}{\text{Kiris Egimi}} = \frac{f(x_1+h) - f(x_1)}{h}$ $\quad h \Rightarrow$ Teget Egimi

$y - y_0 = h.(x - x_0) \rightarrow$ Teget Denklemi

Functions

$f(x) = f(-x) \Rightarrow$ EVEN $\rightarrow$ Symmetric y axis

$f(x) = -f(-x) \Rightarrow$ ODD $\rightarrow$ Symmetric origin

Trigonometric Identities

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A . \tan B}$$

Double Angle

$\sin 2\alpha = 2.\sin\alpha.\cos\alpha$ $\qquad \cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$$\tan 2\alpha = \frac{2.\tan\alpha}{1-\tan^2\alpha}$$

Half Angle

$\cos 2X = 2.\cos^2 X - 1$

$= 1 - 2\sin^2 X$

$$y = \underset{\sim}{a}.f(b(x+c)) + \underset{\sim}{d}$$

Vertical Stretch or Compress	Horizontal Stretch or Compress	Horizontal Shift	Vertical Shift
$a>1$			
a 1 $\uparrow$ (Stretch)	$b>1 \leftarrow$ (Compress)	$c>0 \leftarrow$	$d>0 \uparrow$
$\frac{1}{a}$ $\downarrow$ (Compress)	$b<1 \rightarrow$ (Stretch)	$c<0 \longrightarrow$	$d<0 \downarrow$
$a=-1$ $y=-f(x)$ (x - reflect)			
$y=f(-x)$ (y-reflect)			

Limit

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$

Extra

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{x\to\infty} \frac{a_n x^n + .. + a_1 x + a_0}{b_m x^m + .. + b_1 x + b_0} = \begin{cases} \infty & n>m \\ \frac{a_n}{b_n} & n=m \\ 0 & n<m \end{cases}$

$\cos 2x = 1 - 2\sin^2 x$

$\lim_{x\to \mp\infty} \frac{1}{x} = 0$

Horizontal Asymptote ; $\lim_{x\to+\infty} f(x) = a$ $\lim_{x\to-\infty} f(x) = a$ $y = a$ (Horizontal Asy.)

Vertical Asymptote ; $\lim_{x\to 0^+} \frac{1}{x} = \mp\infty$ $\lim_{x\to 0^-} \frac{1}{x} = \mp\infty$ x (Vertical Asy.)

Oblique Asymptote; if $\frac{a_n x^n}{b_m x^m}$ → $n \geq m+1$ There are oblique asymptote.

→ Bir fonk. tersini bulmak için x ile y'nin yerleri değiştirilir.

→ $u = \sqrt{3x+1}$ ⟹ $\frac{du}{dx} = \frac{1}{2} \cdot (3x+1)^{-1/2} \cdot 3 = \frac{3}{2\sqrt{3x+1}}$

→ $\frac{d}{dx}(\sec^{-1} u) = \frac{1}{|u|\sqrt{u^2-1}} \cdot \frac{du}{dx} = \cos u^{-1} \cdot \frac{1}{x}$

→ $\frac{d}{dx}(\mathrm{cosec}^{-1} u) = \frac{-1}{|u|\sqrt{u^2-1}} \cdot \frac{du}{dx} = \sin u^{-1} \cdot \frac{1}{x}$

Derivative of the Trigonometric Functions

$f(x) = \sin x$ $f'(x) = \cos x$ $f(x) = \cos x$ $f'(x) = -\sin x$ $f(x) = -\sin x$ $f'(x) = -\cos x$

$\frac{d}{dx}(\tan x) = \sec^2 x = 1 + \tan^2 x$ $\frac{d}{dx}(\cot x) = \mathrm{cosec}^2 x$

$\frac{d}{dx}(\sec x) = \sec(x).\tan(x)$ $\frac{d}{dx}(\mathrm{cosec}\, x) = -\mathrm{cosec}(x).\cot(x)$

Ters Fonksiyonun Türevi

$\frac{d}{dx}\left(\frac{1}{x}\right) = \frac{-1}{x^2}$

Karekök Fonksiyonun Türevi

$\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$, $\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = \frac{-1}{2.\sqrt{x^3}}$

$y = \ln x$ 'in Türevi

→ $\ln(e) = 1$ → $\frac{d}{dx}\ln u = \frac{u'}{u}$ → $\ln x = \frac{1}{x}$

Logaritmik Türev

→ Her iki taraf $\ln$'e göre türevlenir, sonuç fonk. ile çarpılır.

e^x Türevi

$\frac{d}{dx} e^x = e^x$ → $\frac{d}{dx} e^u = e^u \cdot \frac{du}{dx}$ → $e^{\ln x} = x$, $\ln(e^x) = x$

→ $a^x = e^{x.\ln a} = a^x . \ln a . (x)'$

→ $\frac{d}{dx}(\log_a u) = \frac{1}{\ln a} \cdot \frac{1}{u} \cdot \frac{du}{dx}$

→ $\frac{d}{dx}(\sin^{-1} u) = \frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dx}$ → $\frac{d}{dx}(\tan^{-1} u) = \frac{1}{1+u^2} \cdot \frac{du}{dx}$

→ $\frac{d}{dx}(\cos^{-1} u) = \frac{-1}{\sqrt{1-u^2}}$ → $\frac{d}{dx}(\cot^{-1} u) = \frac{-1}{1+u^2} \cdot \frac{du}{dx}$

08.12.2014
Monday
(D-206)

LIMITS of INDETERMINATE FORMS

In general, $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0.\infty$, $\infty-\infty$ are named as indeterminate form in limiting process.

Also 1^{∞}, 0^{0}, ∞^{0} are exponential type indeterminate forms.

* L' Hospital's Rule

Assume functions $f(x)$ and $g(x)$ are continious and differentiable on (a,b). Assume $f(a)=g(a)=0$ but then

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$ provided that both limits above exist

Note that the derivative process in L'Hospital's rule is not same as in quatient rule for derivative

EXP; $\lim_{x\to -2}\frac{x+2}{x^2-4}\to\frac{0}{0}$ case

$$\lim_{x\to -2}\frac{(x+2)'}{(x^2-4)'}=\lim_{x\to -2}\frac{1}{2x}=\frac{-1}{4}$$

EXP; $\lim_{x\to 0}\frac{\sin 5x}{x}\to\frac{0}{0}$ case

$$\lim_{x\to 0}\frac{(\sin 5x)'}{(x)'}=\lim\frac{\cos \overset{1}{\widehat{5x}}.5}{1}=5$$

EXP; $\lim_{\theta\to\pi/2}\frac{1-\sin\theta}{1+\cos 2\theta}\to\frac{0}{0}$ case

$$\lim_{\theta\to\pi/2}\frac{(1-\sin\theta)'}{(1+\cos 2\theta)'}=\lim_{\theta\to\pi/2}\frac{-\cos\theta}{-2.\sin 2\theta}\to\frac{0}{0}\text{ case}$$

$$\lim_{\theta\to\pi/2}\frac{-\sin\theta}{4.\cos 2\theta}=\frac{-1}{4.(-1)}=\frac{1}{4}$$

If we have $\frac{\infty}{\infty}$ indeterminate form then we can apply the L'Hospital's rule again.

EXP: $\lim_{x\to\infty}\frac{\ln x}{2\sqrt{x}}\to\frac{\infty}{\infty}$ form

$$\lim_{x\to\infty}\frac{\ln x}{2\sqrt{x}}=\lim_{x\to\infty}\frac{\frac{1}{x}}{2.\frac{1}{2\sqrt{x}}}=\lim_{x\to\infty}\frac{\sqrt{x}}{x}=\lim_{x\to\infty}\frac{1}{\sqrt{x}}=0$$

$\sqrt{x}'=\frac{1}{2\sqrt{x}}$

$(\ln x)'=\frac{1}{x}$

$(e^x)'=e^x$

$\left(\frac{1}{x}\right)'=\frac{-1}{x^2}$

$\left(\frac{1}{x-1}\right)'=$

$\left(\frac{1}{\ln x}\right)'=$

EXP: $\lim_{t\to\infty}\frac{e^t+t^2}{e^t-t}\to\frac{\infty}{\infty}$ (I)

$$\lim_{t\to\infty}\frac{e^t+t^2}{e^t-t}=\lim_{t\to\infty}\underbrace{\frac{e^t+2t}{e^t-1}}_{\frac{\infty}{\infty}\ (\mathrm{II})}=\lim_{t\to\infty}\underbrace{\frac{e^t+2}{e^t}}_{\frac{\infty}{\infty}\ (\mathrm{III})}=\lim_{t\to\infty}\frac{e^t}{e^t}=1$$

EXP: $\lim_{x\to\infty}x.\sin\left(\frac{1}{x}\right)\to\infty.0$ form

$$\lim_{x\to\infty}\frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}}\to\frac{0}{0}$$

$$\lim_{x\to\infty}\frac{\sin\left(\frac{1}{x}\right)'}{\left(\frac{1}{x}\right)'}=\lim_{x\to\infty}\frac{\frac{-1}{x^2}.\cos\left(\frac{1}{x}\right)}{\frac{-1}{x^2}}=\lim_{x\to\infty}\cos\left(\frac{1}{x}\right)=1$$

EXP: $\lim_{x\to 1^+}\left[\frac{1}{x-1}-\frac{1}{\ln x}\right]\to\infty-\infty$

$$\lim_{x\to 1^+}\frac{\ln x-x+1}{(x-1)(\ln x)}\to\frac{0}{0}$$

$$=\lim_{x\to 1^+}\underbrace{\frac{\frac{1}{x}-1}{\ln x+\frac{x-1}{x}}}_{\frac{0}{0}}=\lim_{x\to 1^+}\frac{\frac{-1}{x^2}}{\frac{1}{x}+\frac{1}{x^2}}=\frac{-1}{1+1}=\frac{-1}{2}$$

EXP: $\lim_{x\to 0^+} (\sin x . \ln x) \to 0.\infty$

$\sin x . \ln x \equiv \dfrac{\ln x}{\frac{1}{\sin x}} \to \dfrac{\infty}{\infty}$ OR $\sin x . \ln x \equiv \dfrac{\sin x}{\frac{1}{\ln x}} \to \dfrac{0}{0}$

$\lim_{x\to 0^+} \sin x . \ln x = \lim_{x\to 0^+} \dfrac{\ln x}{\frac{1}{\sin x}} \to \dfrac{\infty}{\infty} \Rightarrow \lim_{x\to 0^+} \dfrac{1/x}{-\operatorname{cosec} x . \cot x} = \lim_{x\to 0^+} \dfrac{-1/x}{\operatorname{cosec} x . \cot x} \to \dfrac{\infty}{\infty}$

EXP: $\lim_{x\to\infty} \left(\dfrac{x^2+1}{x+2}\right)^{1/x} \to \infty^0$

$\lim_{x\to\infty} \left(\dfrac{x^2+1}{x+2}\right)^{1/x} = A \Rightarrow \ln\left(\lim_{x\to\infty} \left(\dfrac{x^2+1}{x+2}\right)^{1/x}\right) = \ln A$

$= \lim_{x\to\infty} \left[\ln\left(\dfrac{(x^2+1)}{(x+2)}\right)^{1/x}\right] = \ln A$

$= \lim_{x\to\infty} \left[\dfrac{1}{x} . \ln\left(\dfrac{x^2+1}{x+2}\right)\right] = \ln A$

$= \lim_{x\to\infty} \underbrace{\left[\dfrac{\ln\left(\frac{x^2+1}{x+2}\right)}{x}\right]}_{\frac{\infty}{\infty}} = \ln A$

$= \lim_{x\to\infty} \left(\dfrac{\frac{2x(x+2)-(x^2+1).1}{(x+2)^2}}{\frac{x^2+1}{x+2}}\right) = \ln A$

$= \lim_{x\to\infty} \dfrac{x^2+4x-1}{(x+2)^2(x^2+1)} = 0 = \ln A \Rightarrow A = 1$

$\boxed{\frac{1}{\ln x} = ?}$

If the $\lim_{x\to a} f(x) = L$ exists then

$\ln\left(\lim_{x\to a} f(x)\right) = \ln L$ whenever the $f(x)$ is continuous then it can be written that

$\lim_{x\to a} \ln f(x) = \ln L$

EXP: $\lim_{x\to 0^+} (1+x)^{1/x} \to 1^\infty$

$\ln\left(\lim_{x\to 0^+} (1+x)^{1/x}\right) = \ln L \Rightarrow \lim_{x\to 0^+} \underbrace{\left[\ln (1+x)^{1/x}\right]}_{\frac{1}{x}.\ln(1+x)} = \ln L \Rightarrow \lim_{x\to 0^+} \underbrace{\frac{1}{x} . \ln(1+x)}_{\frac{\ln(1+x)}{x} \to \frac{0}{0}} = \ln L$

$\lim_{x\to 0^+} \dfrac{\frac{1}{1+x}}{1} = \ln L \Rightarrow 1 = \ln L \qquad e^1 = e^{\ln L} = L$

$\lim_{x\to 0^+} (1+x)^{1/x} = e$

EXP: $\lim_{x\to 0^+} x^x \to 0^0 \to$ Define $\lim_{x\to 0^+} x^x = L$

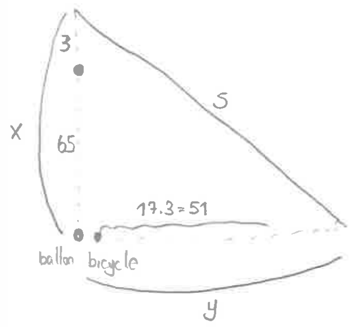
$\boxed{x . \ln x = \frac{\ln x}{\frac{1}{x}}}$

$\ln\left(\lim_{x\to 0^+} x^x\right) = \ln L \Rightarrow \lim_{x\to 0^+} (\ln x^x) = \ln L \Rightarrow \lim_{x\to 0^+} (x . \ln x) = \ln L$

$\lim_{x\to 0^+} \underbrace{\dfrac{\ln x}{\frac{1}{x}}}_{\frac{\infty}{\infty}} = \lim_{x\to 0^+} \dfrac{1/x}{-1/x^2} = \lim_{x\to 0^+} (-x) = 0 \qquad \ln L = 0$

$e^{\ln L} = L = e^0 = 1$

③ A balloon and a bicycle , A balloon is rising vertical above at a constant rate of 1 ft/sec just when the balloon is 65 ft above the ground, a bicycle moving at a constant rate of 17 ft/sec passes under it. How fast is the distance S(t) between balloon and bicycle increasing 3 sec later?



$S^2 = x^2+y^2 \Rightarrow S^2 = \underset{17.4}{(68)^2} + \underset{12.3}{(51)^2} \Rightarrow 17.5 = 85$

$2.S.\boxed{\frac{dS}{dt}} = 2.X.\frac{dx}{dt} + 2.y.\frac{dy}{dt}$

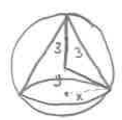
$\frac{dx}{dt} = 1$ $\qquad X = 68 \quad , y = 51 \, , \, S = 85$

$\frac{dy}{dt} = 17$ $\qquad 85.\frac{dS}{dt} = 68.1 + 51.17$

$\frac{dS}{dt} = \frac{68 + 51.17}{85} = \, =$

Optimization

① Find the volume of a largest cone that can be inscribed in a sphere of radius 3.



$V = \frac{\pi.r^2.h}{3}$

$V = \frac{\pi.x^2.(y+3)}{3}$

$x^2+y^2 = 9 \Rightarrow x^2 = 9-y^2$

$f(y) = \frac{\pi.(9-y^2).(y+3)}{3}$

$f'(y) = \frac{\pi}{3}.\left(-2y(y+3) + (9-y^2).(1)\right)$

$f'(y) = \frac{\pi}{3}.\left(\overbrace{-2y^2-6y+9-y^2}^{-3y^2-6y+9}\right)$

$f'(y) = \frac{\pi}{3}.(-3).(y^2+2y-3)$

$f'(y) = 0$

$f'(y) = -\pi.(y+3).(y-1) = 0$

$y_1 = -3 \quad , y_2 = 1$

-3		1	
−	+		−

when $y=1$ $\quad f(y) =$ is max

$f(1) = \frac{\pi.(9-1)(1+3)}{3} = \frac{32\pi}{3}$

Related Rates

1-) If $L = \sqrt{x^2+y^2}$, $\frac{dx}{dt} = -1$ $\quad \frac{dy}{dt} = 3$, Find $\frac{dL}{dt}$ when $\begin{matrix} x=5 \\ y=12 \end{matrix}$

$L = (x^2+y^2)^{1/2}$

$\frac{dL}{dt} = \frac{1}{2}.(x^2+y^2)^{-1/2}.\left[2x(x)' + 2y(y)'\right]$ $\qquad x' = \frac{dx}{dt} = -1 \, , \, y' = \frac{dy}{dt} = 3$

$\frac{dL}{dt} = \frac{1}{2}(x^2+y^2)^{-1/2}\left[2x.\frac{dx}{dt} + 2y.\frac{dy}{dt}\right]$

$\frac{dL}{dt} = \frac{1}{2\sqrt{169}}.\left[2.5.(-1) + 2.12(3)\right] = \frac{1}{2.13}.2.(31) = \frac{31}{13}$

2-) If the original 24 m edge length X of a cube decreases at the rate of 5m/min when X=3 what rate does the cube's

a-) Surface area change?

b-) Volume change?

$S = 6x^2 \Rightarrow \frac{dS}{dt} = 6.2.X.\frac{dx}{dt} \Rightarrow \frac{dS}{dt} = 6.2.3.(-5) =$

$V = x^3 \Rightarrow \frac{dV}{dt} = 3.x^2.\frac{dx}{dt} \Rightarrow 3.(3^2).(-5) = \ldots$

$X = 3$

$\frac{dx}{dt} = -5$

② Find a positive number for which ~~the~~ the sum of its reciprocal and 4X its square is the smallest possible.

$f(x) = \frac{1}{x} + 4x^2 \qquad f'(x) = \frac{-1}{x^2} + 8x$

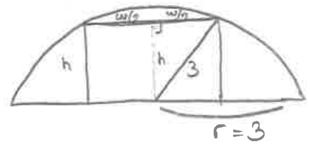
$f'(x) = 0 \qquad \frac{-1}{x^2} + 8x = 0 \qquad \frac{-1+8x^3}{x^2} = 0 \qquad 8x^3 - 1 = 0 \qquad x^3 = \frac{1}{8}$

$x = \frac{1}{2}$

	$1/2$	
$-$		$+$
↘		↗

$f(1/2) = \frac{1}{\frac{1}{2}} + 4 \cdot \left(\frac{1}{2}\right)^2 = \underline{\underline{3}}$

③



$S = h \cdot w$

$9 = h^2 + \left(\frac{w}{2}\right)^2$

$h = \sqrt{9 - \frac{w^2}{4}}$

$S = \sqrt{9 - \frac{w^2}{4}} \cdot w$

$f(w) = \left(9 - \frac{w^2}{4}\right)^{1/2} \cdot (w^2)^{1/2}$

$f(w) = \left(9w^2 - \frac{w^4}{4}\right)^{1/2}$

$f'(w) = \frac{1}{2} \cdot \left(9w^2 - \frac{w^4}{4}\right)^{-1/2} \cdot \left[18w - \frac{4 \cdot w^3}{4}\right]$

$f'(w) = \frac{18w - w^3}{2 \cdot \sqrt{9w^2 - \frac{w^4}{4}}}$

$f'(w) = 0 \qquad w^3 - 18w = 0$

$w(w^2 - 18) = 0$

$(w - \sqrt{18})(w + \sqrt{18})$

$w_1 = 0$

$w_2 = 3\sqrt{2}$

$w_3 = -\cancel{8}\sqrt{2}$

	$-3\sqrt{2}$		0		$3\sqrt{2}$	
$+$		$-$		$+$		$-$
↗		↘		↗		↘

$\underline{\underline{w = 3\sqrt{2}}}$

Questions	Answers
In exercises 1-8 find the derivative of y with respect to appropriate variable	
1. $y = \ln(\sec^2\theta)$	$y' = \frac{(\sec^2\theta)'}{\sec^2\theta} = \frac{2.\sec\theta.\sec\theta.\tan\theta}{\sec^2\theta} = 2.\tan\theta$
2. $y = 9^{2t}$	$y' = (9^{2t})' = (81^t)' = 81^t.\ln 81 = 81^t.\ln 3^4 = 4.81^t.\ln 3$
3. $y = 2(\ln x)^{x/2}$	$\frac{y}{2} = (\ln x)^{x/2}$ $\ln\frac{y}{2} = \ln(\ln x)^{x/2} = \frac{x}{2}.\ln(\ln x)$ $\left(\ln\frac{y}{2}\right)' = \left(\frac{x}{2}.\ln(\ln x)\right)'$ $\frac{\frac{1}{2}y'}{\frac{1}{2}y} = \frac{1}{2}.\ln(\ln x) + \frac{x}{2}.\frac{(\ln x)'}{\ln x} = \frac{1}{2}.\ln(\ln x) + \frac{\cancel{x}}{2}.\frac{1}{\cancel{x}}.\frac{1}{\ln x}$ $\to y' = y.\left(\frac{1}{2}.\ln(\ln x) + \frac{1}{2\ln x}\right)$
4. $y = (x+2)^{x+2}$	$\ln y = \ln(x+2)^{x+2} = (x+2).\ln(x+2)$ $\frac{y'}{y} = ((x+2).\ln(x+2))' = 1.\ln(x+2) + \frac{1}{x+2}.(x+2) = \ln(x+2)+1$ $y' = y.(\ln(x+2)+1) = (x+2)^{x+2}.(\ln(x+2)+1)$
5. $y = z\cos^{-1}z - \sqrt{1-z^2}$	$y' = 1.\cos^{-1}z + z.\left(\frac{-1}{\sqrt{1-z^2}}\right) - \frac{(-2z)}{2.\sqrt{1-z^2}}$ $y' = \cos^{-1}z + \frac{(-2z)}{2(\sqrt{1-z^2})} + \frac{2z}{2\sqrt{1-z^2}} = \arccos z$
6. $y = (1+t^2)\cot^{-1}2t$	$y' = 2t.\cot^{-1}2t + (1+t^2).\frac{-2}{1+4t^2}$
7. $y = 2\sqrt{x-1}\sec^{-1}\sqrt{x}$	$y' = 2.\frac{1}{2\sqrt{x-1}}.\sec^{-1}\sqrt{x} + 2\sqrt{x-1}.\frac{1}{2\sqrt{x}}.\frac{1}{x\sqrt{1-\frac{1}{x}}} = \frac{1}{\sqrt{x-1}}.\sec^{-1}\sqrt{x} + \frac{\sqrt{x-1}}{x.\sqrt{x-1}} = \frac{\sec^{-1}\sqrt{x}}{\sqrt{x-1}} + \frac{1}{x}$
8. $y = (1+x^2)e^{\tan^{-1}x}$	$y' = 2x.e^{\tan^{-1}x} + (1+x^2).(\tan^{-1}x)'.e^{\tan^{-1}x}$ $= 2x.e^{\tan^{-1}x} + \cancel{(1+x^2)}.\frac{1}{\cancel{x^2+1}}.e^{\tan^{-1}x} \Rightarrow (2x+1).e^{\tan^{-1}x}$
In exercises 9-12 use logarithmic differentiation to find the derivative of y with respect to the appropriate variable	
9. $y = \sqrt[10]{\frac{3x+4}{2x-4}}$ $\frac{y'}{y} = \frac{-2}{(2x-4).(3x+4)}$	$\ln y = \ln\left(\frac{3x+4}{2x-4}\right)^{1/10} = \frac{1}{10}.\ln\left(\frac{3x+4}{2x-4}\right) \Rightarrow \frac{y'}{y} = \frac{1}{10}.\frac{\left(\frac{3x+4}{2x-4}\right)'}{\frac{3x+4}{2x-4}} = \frac{1}{10}.\frac{\frac{3.(2x-4)-2.(3x+4)}{(2x-4)^2}}{\frac{3x+4}{2x-4}}$ $\to \frac{6x-12-6x-8}{2x+4}.\frac{1}{3x+4} = \frac{-20}{(2x+4)(3x+4)}.\frac{1}{10} = \frac{-1}{(x+2)(3x+4)}.\sqrt[10]{\frac{3x+4}{2x-4}}$
10. $y = \left(\frac{(t+1)(t-1)}{(t-2)(t+3)}\right)^5,\ t > 2$ $y' = \left(\frac{t^2-1}{t^2+t-6}\right)^5.\frac{(t^2-10t+1)}{(t^2+t-6)(t^2+1)}$	$y = \left(\frac{t^2-1}{t^2+t-6}\right)^5$ $\ln y = \ln\left(\frac{t^2-1}{t^2+t-6}\right)^5 = 5.\ln\left(\frac{t^2-1}{t^2+t-6}\right)$ $\frac{y'}{y} = \frac{5.\left(\frac{t^2-1}{t^2+t-6}\right)'}{\frac{t^2-1}{t^2+t-6}} = 5.\ \frac{2t.(t^2+t-6)-(2t+1)(t^2-1)}{}$
11. $y = (\sin\theta)^{\sqrt{\theta}}$	$\ln y = \ln(\sin\theta)^{\sqrt{\theta}} = \sqrt{\theta}.\ln\sin\theta$ $\frac{y'}{y} = \frac{1}{2\sqrt{\theta}}.\ln\sin\theta + \sqrt{\theta}.\frac{(\sin\theta)'}{\sin\theta}$ $\to y' = (\sin\theta)^{\sqrt{\theta}}.\left(\frac{\ln\sin\theta}{2\sqrt{\theta}} + \frac{\sqrt{\theta}.\cos\theta}{\sin\theta}\right)$ $\sqrt{\theta}.\cot\theta$
12. $y = (\ln x)^{1/(\ln x)}$	$\ln y = \frac{1}{\ln x}.\ln(\ln x) \Rightarrow \frac{y'}{y} = \frac{-1}{(\ln x)^2}.\frac{1}{x}.\ln(\ln x) + \frac{1}{\ln x}.\frac{1}{\ln x}.\frac{1}{x}$

$\frac{y'}{y} = \frac{1}{x}.\frac{1}{\ln x^2}.(1-\ln(\ln x)) \Rightarrow y' = (\ln x)^{1/(\ln x)}.\left[\frac{1}{x}.\frac{1}{\ln x^2}.(1-\ln(\ln x))\right]$

Solve the equations for y	
13. $4^{-y} = 3^{y+2}$	$\ln 4^{-y} = \ln 3^{y+2}$ $-y.\ln 4 = (y+2).\ln 3 = y\ln 3 + 2.\ln 3$ $-2\ln 3 = y.(\ln 3 + \ln 4)$ $y = \frac{\ln 3+\ln 4}{-2.\ln 3} = \frac{\ln 12}{\ln 1/9}$
14. $\ln(10\ln y) = \ln 5x$	$10\ln y = 5x$ $\ln y = \frac{5x}{10} = \frac{x}{2}$ $e^{\ln y} = e^{x/2}$ $y = e^{x/2}$
In exercises 15-20 use L'Hopital's rule to find the limits	
15. $\lim_{y\to 0^+} e^{-1/y}\ln y$	$\lim_{y\to 0^+} \frac{\ln y}{e^{1/y}} = \frac{\infty}{\infty}$ by L'H $\Rightarrow \lim_{y\to 0^+} \frac{\frac{1}{y}}{e^{1/y}.\frac{-1}{y^2}} = \lim_{y\to 0^+} \frac{-y}{e^{1/y}} = \frac{0}{\infty} = 0$
16. $\lim_{x\to 0^+}\left(1+\frac{3}{x}\right)^x$	$f(x) = \left(1+\frac{3}{x}\right)^x \Rightarrow \ln f(x) = x.\ln\left(1+\frac{3}{x}\right)$ $\lim_{x\to 0^+} \ln f(x) = \lim_{x\to 0^+} \frac{\ln\left(1+\frac{3}{x}\right)}{\frac{1}{x}} = \frac{\infty}{\infty}$ $x' = \frac{1}{1+\frac{3}{x}} \cdot \frac{-3}{x^2} \Rightarrow \frac{1}{\frac{x+3}{x}} \cdot \frac{-3}{x^2} = \frac{x}{x+3} \cdot \frac{-3}{x^2} = \frac{-3}{x^2+3x}$ $\to$ by L'H $\Rightarrow \lim_{x\to 0^+} \frac{\frac{3}{x^2+3x}}{\frac{1}{x^2}} = \lim_{x\to 0^+} \frac{3x^2}{x^2+3x} = \frac{0}{0}$ $\lim_{x\to 0^+} \frac{3x^2}{x^2+3x} = \lim_{x\to 0^+} \frac{6x}{2x+3} = 0$ $\lim_{x\to 0^+} \ln f(x) = 0$ $\lim_{x\to 0^+} e^{\ln f(x)} = \lim_{x\to 0^+} f(x) = e^0 = 1$
17. $\lim_{\theta\to 0}\frac{3^\theta - 1}{\theta} \to \frac{1-1}{0} = \frac{0}{0}$	by L'H $\Rightarrow \lim_{\theta\to 0} \frac{3^\theta - 1}{\theta} = \lim_{\theta\to 0} \frac{3^\theta \ln 3}{1} = \frac{3^0 \ln 3}{1} = \ln 3$
18. $\lim_{x\to 0^+}\sqrt{x}\sec x$	$\lim_{x\to 0^+} \frac{\sqrt{x}}{\cos x} = \frac{\sqrt{0}}{\cos 0} = \frac{0}{1} = 0$
19. $\lim_{x\to 0}\left(\frac{1}{x^4}-\frac{1}{x^2}\right)$	$\lim_{x\to 0}\left(\frac{x^2-x^4}{x^6}\right) \to \frac{0}{0}$ by L'H $\Rightarrow \lim_{x\to 0} \frac{2x-4x^3}{6x^5} \to \frac{0}{0}$ by L'H $\Rightarrow \lim_{x\to 0} \frac{2-12x^2}{30x^4} = \frac{2}{0} = \infty$
20. $\underbrace{\lim_{x\to\infty}\left(\frac{e^x+1}{e^x-1}\right)^{\ln x}}_{A}$	$\ln A = \ln x.\ln\left(1+\frac{2}{e^x-1}\right)$ $\lim_{x\to\infty} \ln A = \underbrace{\lim_{x\to\infty} \frac{\ln x}{x}}_{B} . \underbrace{\lim_{x\to\infty} x.\ln\left(1+\frac{2}{e^x-1}\right)}_{C}$

$B = \lim_{x\to\infty} \frac{\ln x}{x} \to \frac{\infty}{\infty} \Rightarrow$ by L'H $\Rightarrow B = \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$

$C = \lim_{x\to\infty} x.\ln x \underbrace{\left(\frac{e^x+1}{e^x-1}\right)}_{\infty}$