Functions

↳ If the period of $f(x)$ is p, then the period of $f(ax+b)$ is $\frac{\pi}{a}$,

the period of $a.f(x)+b$ is $a.\pi$..

$y = \cos(2x)$

Period of $\cos(x)$ is 2π

" " $\cos(2x)$ " $\frac{2\pi}{2} = \pi$

$y = 2.\cos\left(x - \frac{\pi}{3}\right)$ Period of y is $2\pi.2 = 4\pi$

Intermediate Value Theorem

$f(I) > 0$ $f(II) < 0$ There are value btwn $f(I)$ and $f(II)$.

$\sin^2\theta = \frac{1-\cos 2\theta}{2} \Rightarrow \cos 2\theta = 1 - 2\sin^2\theta$

$\cos^2\theta = \frac{1+\cos 2\theta}{2} \Rightarrow \cos 2\theta = 2\cos^2\theta - 1$

$\lim_{h\to 0} \frac{\cosh - 1}{h} = \frac{-2.\sin^2(h/2)}{h}$

$\ln_x y$ $x > 0$

e^x → Range > 0

Reflect (Yansıtma)

Limit

$\frac{\Delta y}{\Delta t} = \frac{f(x_1+h) - f(x_1)}{h}$

→ Eğer bir $f(x)$ fonksiyonunun sağdan ve soldan limiti eşiti ise o noktada limiti vardır.

→ $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$, $\cos 2x = 1 - 2\sin^2 x$

→ Tanım aralığı dışındaki değerde noktanın tek taraflı limiti $f(x)$'in değerine eşitse o noktada $f(x)$ süreklidir.

→ $\lim_{x\to\infty} \frac{1}{x} = 0$ $\lim_{x\to-\infty} \frac{1}{x} = 0$

→ $\lim_{x\to\infty} \frac{a_n x^n + \dots + a_1 x + a_0}{b_m x^m + \dots + b_1 x + b_0} = \begin{cases} +\infty & n > m \\ \frac{a_n}{b_n} & n = m \\ 0 & n < m \end{cases}$

→ Yatay Asimptot ; $\lim_{x\to\infty} f(x) = \underline{b}$, $\lim_{x\to-\infty} f(x) = \underline{b}$ $y = b$ (Yatay Asimptot)

→ Eğik Asimptot ; Payın derecesi paydanınkinden 1 derece büyükse vardır.

→ $\lim_{x\to 0^+} \frac{1}{x} = \infty$, $\lim_{x\to 0^-} \frac{1}{x} = -\infty$

→ Düşey Asimptot ; $\lim_{x\to 0^+} \frac{1}{x} = \mp\infty$, $\lim_{x\to 0^-} \frac{1}{x} = \mp\infty$

oblique : eğik

→ II. Türevin Geometrik Yorumu

$f''(x) > 0$ Konveks (Dış Bükey)

$f''(x) < 0$ Konkav (İç Bükey)

→ II. Türevi sıfır yapan köklerden sağındaki işaret ile solundaki işaret farklı olanlar dönüm (bükum) noktasının apsisidir.

Yatay Asimptot: $\lim_{x \to \infty} f(x) = b$, $\lim_{x \to -\infty} f(x) = b$ $\quad y = b$ (Yatay Asimptot)

Düşey Asimptot: $\lim_{x \to a^+} f(x) = \mp\infty$, $\lim_{x \to a^-} f(x) = \mp\infty$ $\quad x = a$ (Düşey Asimptot)

→ Paydayı 0 yapan değer payı da sıfır yapıyorsa düşey asimptot yoktur.

→ $f(x) = \sqrt{ax^2+bx+c}$ ⇒ Eğik Asimptotlar $y = \sqrt{a}\left|x + \frac{b}{2a}\right|$

Simetri Merkezi

1-) Polinomların büküm noktası (dönüm) simetri merkezidir.

2-) Fonk. simetri merkezi asimptotların kesim noktasıdır.

Grafik

1-) Grafiğin uç noktaları $\mp\infty$ için limit alınarak bulunur.

2-) Çift katlı köklerde grafiğin X eksenine teğet, tek katlı köklerde ise X eksenini keser.

Rasyonel Fonk. Grafiği

$\frac{f(x)}{g(x)}$

$g(x) = 0$ → Tek Kat Kelebek

$g(x) = 0$ → Çift Kat Baca

1-) Fonk. Tanım Kümesi bulunur.

2-) Fonk. eksenleri kestiği nokta varsa bulunur.

3-) $\mp\infty$ için fonk. limiti bulunur.

4-) Türev incelemesi yapılır.

5-) Asimptotlar bulunur.

$1^\infty, 0^0, \infty^0$ → ln kullanılır.

Bileşke Fonk. Türevi: → $(f \circ g)'(x) = g'(x) \cdot f'(g(x))$

→ $([f(x)]^n)' = n \cdot [f(x)]^{n-1} \cdot f'(x)$ $\qquad$ → $[\sec x]' = \sec x \cdot \tan x$, $[\operatorname{cosec} x]' = -\operatorname{cosec} x \cdot \cot x$

→ $[\sin(f(x))]' = f'(x) \cdot \cos f(x)$ $\qquad$ → $[\cos(f(x))]' = -f'(x) \cdot \sin f(x)$

→ $(\tan x)' = 1 + \tan^2 x = \sec^2 x$ $\qquad$ → $(\cot x)' = -\operatorname{cosec}^2 x = -(1+\cot^2 x)$

→ $[\arcsin f(x)]' = \frac{f'(x)}{\sqrt{1-f^2(x)}} = \frac{u'(x)}{\sqrt{1-u^2}}$ $\qquad$ → $[\arccos f(x)]' = \frac{-f'(x)}{\sqrt{1-f^2(x)}} = \frac{-u'(x)}{\sqrt{1-u^2(x)}}$

→ $[\arctan f(x)]' = \frac{f'(x)}{1+f^2(x)} = \frac{u'(x)}{1+u^2}$ $\qquad$ → $[\operatorname{arccot} f(x)]' = \frac{-f'(x)}{1+f^2(x)} = \frac{u'(x)}{1+u^2}$

→ $[a^{f(x)}]' = f'(x) \cdot a^{f(x)} \cdot \ln a$ $\qquad$ → $[e^{f(x)}]' = f'(x) \cdot e^{f(x)}$

→ $[\log_a f(x)]' = \frac{f'(x)}{f(x)} \cdot \log_a e$ $\qquad$ → $[\ln f(x)]' = \frac{f'(x)}{f(x)}$

→ $y = f(x)^{g(x)} \Rightarrow \ln y = \ln f(x)^{g(x)} \Rightarrow \ln y = g(x) \cdot \ln f(x) \Rightarrow$

$(\ln y)' = g'(x) \cdot \ln f(x) + g(x) \cdot [\ln f(x)]' = y' = y \cdot \left(g'(x) \cdot \ln f(x) + \frac{f'(x)}{f(x)} \cdot g(x)\right)$

L'Hospital: $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ veya $\frac{\infty}{\infty}$ ise $\lim_{x \to a} \frac{f'(x)}{g'(x)}$

$m_{teğet} = f'(a)$ $\qquad$ $m_{teğet} \cdot m_{normal} = -1$

→ Artan mı Azalan mı olduğu Türevinin işaretine göre değerlendirilir.

→ Yerel Maksimum ve Yerel Minimum (Ekstremum Değerler) → En Büyük ve En Küçük değerlerdir.

I. Türevin köklerinden bulunur. (Çift katlı kökler dikkate alınmaz)

Preliminaries

1-) Real Numbers and Real Line (Real Numbers, Sets, Interval, Inequality)

Real Numbers: We can show these numbers on the real line.

a. Naturel Numbers → $\mathbb{N}: \{0,1,2,\dots\}$

b. Integers → $\mathbb{Z}: \{0,\mp 1,\mp 2,\dots\}$

c. Rational Numbers → $\mathbb{Q}: \{\frac{m}{n}: m\in\mathbb{Z}, n\in\mathbb{N}\}$ F.E.; * $\frac{-3}{4}=-0,75$, $\frac{1}{3}=0,33$ → Either there is no repating or there are back repating.

* $\sqrt{2}: 1.414135\dots$ → This number is not rational numbers.

Inequalities

Ex: $2x-1<x+3$ => Solve the inequality

Sol: $x<4$ Sol. Set $=\{x\in\mathbb{R}, x<4\}=\{-\infty,4\}$

Ex: $\frac{6}{x-1}\geqslant 5$ => Solve the inequality

Sol: Multiply both side;

1. Situation: $x<1$

$\frac{6}{x-1}\geqslant 5 \Rightarrow 6\leqslant 5.(x-1) \Rightarrow x<1$ and $x\geqslant\frac{11}{5}$ so $\varnothing$ = Sol. Set.

2. Situation: $x\geqslant 1$

$\frac{6}{x-1}\geqslant 5 \Rightarrow 6\geqslant 5.(x-1) \Rightarrow x\leqslant\frac{11}{5}$ $x>1$ and $x\leqslant\frac{11}{5}$

$\Rightarrow (1,\frac{11}{5}]$ = Gen. Sol.

Absolute Value

$|x|=\begin{cases} x, & x\geqslant 0\\ -x, & x<0\end{cases}$ *That is absolute value of x is always positive.

* $|a|=|-a|$
* $|a.b|=|a|.|b|$
* $\left|\frac{a}{b}\right|=\frac{|a|}{|b|}$ $(b\neq 0)$
* $|a+b|\leqslant|a|+|b|$

Ex: $|2x-3|=7$ => Solve the equation

Sol: $|2x-3|=7$ $2x-3=0 \Rightarrow \frac{3}{2}$ either $x\geqslant\frac{3}{2}$ or $x<\frac{3}{2}$

1. $x\geqslant\frac{3}{2}$ then $|2x-3|=7$ $2x-3=7$ $x=5$
2. $x<\frac{3}{2}$ then $|2x-3|=7$ $-2x+3=7$ $x=-2$

Sol. Set $=\{-2,5\}$

Ex: $\left|5-\frac{2}{x}\right|<1$ => Solve the inequality

Sol: $\left|5-\frac{2}{x}\right|<1 \Rightarrow -1<5-\frac{2}{x}<1$ $-6<-\frac{2}{x}<-4$

$6>\frac{2}{x}>4$ $\frac{1}{6}>\frac{x}{2}>\frac{1}{4}$ $\frac{1}{3}>x>\frac{1}{2}$

Gen. Sol $=\left(\frac{1}{3},\frac{1}{2}\right)$

Ex: $|x-1|=1-x$ => Solve the equation

Sol: 1. $x\geqslant 1$ $|x-1|=1-x \Rightarrow x-1=1-x$ $2x=2$ $x=1$

2. $x<1$ $x-1=-1+x$ $0=0$ for each $x<1$

S.S. $=\{x\in\mathbb{R}: x<1\}$ General Sol. $=\{1\}\cup\{-\infty,1\}=(-\infty,1]$

Increments and Straight Lines

$\Delta x = x_2 - x_1 \rightarrow$ Run
$\Delta y = y_2 - y_1 \rightarrow$ Rise

Ex: In going from the point $A(4,-3)$ to the point $B(2,5)$ the increments in the xy coordinates are;

$\Delta x = x_2 - x_1 = 2 - 4 = -2$ $\qquad$ $\Delta y = y_2 - y_1 = 5 - (-3) = 8$

The Ratio $\Rightarrow$ $m = \dfrac{\text{Rise}}{\text{Run}} = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1} = \tan\alpha$

(I) Point-Slope Equation $\Rightarrow$ $y - y_1 = m.(x - x_1)$

(II) Two Points-Quation $\Rightarrow$ $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\Rightarrow$ $y - y_1 = m.(x - x_1)$ or equally $y - y_2 = m.(x - x_2)$

(III) Intercept-Slope Equation $\Rightarrow$ $y - y_1 = m.(x - x_1)$ $\Rightarrow$ $y - b = m(x - 0)$ $\qquad$ $y = mx + b$ slope, y-intercept

(IV) Intercept-Equation $\Rightarrow$ $\dfrac{x}{a} + \dfrac{y}{b} = 1$

(V) General Linear Equation $\Rightarrow$ $A.x + B.y + C = 0$

Ex: Write an equation of the line through the point $P(2,3)$ with slope $m = \dfrac{-3}{2}$

Sol: $y - y_1 = m.(x - x_1)$ $\Rightarrow$ $y - 3 = \dfrac{-3}{2}.(x - 3)$ $\Rightarrow$ $y = \dfrac{-3}{2}x + 6$ $\Rightarrow$ $(0,6)$ $(4,0)$

Ex: Write an equation for the line through the points $(-2,-1)$ and $(3,4)$

Sol: $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4+1}{3+2} = \dfrac{5}{5} = 1$ $\qquad$ $y - y_1 = m.(x - x_1)$ $\qquad$ $y + 1 = 1.(x + 2)$ $\qquad$ $y = x + 1$

Parallel and Perpendicular Lines

$\rightarrow L_1 \parallel L_2 \Leftrightarrow m_1 = m_2 \Leftrightarrow d_1 = d_2$ (Angle) $\qquad$ $\rightarrow L_1 \perp L_2 \Leftrightarrow m_1 . m_2 = 1 \Leftrightarrow d_2 - d_1 = 90° = \dfrac{\pi}{2}$

Distance and Circles in the Plane

$P_1(x_1, y_1)$
$P_2(x_2, y_2)$ $\qquad$ $d = |P_1P_2| = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\Rightarrow$ Distance

Ex: Find distance btwn the points $P(-1,2)$ and $Q(3,4)$;

Sol: $d = |PQ| = \sqrt{(3-(-1))^2 + (4-2)^2} = \sqrt{20} = 2\sqrt{5}$

Circles

Standard Equation of a Circle $\Rightarrow$ $\sqrt{(x-h)^2 + (y-k)^2} = a$ or $(x-h)^2 + (y-k)^2 = a^2$ $\qquad$ a = radius, C = center $\qquad$ $C(h,k)$

Ex: If a circle with radius 2 is centred out $(3,4)$ then write its equation;

Sol: $(x-3)^2 + (y-4)^2 = 2^2$ $\qquad$ $(x-3)^2 + (y-4)^2 = 4$

Ex: Find the radius and center of the circle $x^2 + y^2 + 4x - 6y - 3 = 0$;

Sol: $(x^2 + 4x) + (y^2 - 6y) = 3$

$\left(x^2 + 4x + \left(\frac{4}{2}\right)^2\right) + \left(y^2 - 6y + \left(\frac{6}{2}\right)^2\right) = 3 + \left(\frac{4}{2}\right)^2 + \left(\frac{6}{2}\right)^2$

$(x+2)^2 + (y-3)^2 = 16$ $\qquad$ $r = 4$ and centered at $(-2,3)$

Parabola = Graph of equation $y=ax^2+bx+c$ when $a \neq 0$ is called parabola.

→ A parabola takes smallest or largest value at vertex ;

$$x=\frac{-b}{2a} \qquad y=\frac{b^2-4ac}{4a} \qquad V=\left(\frac{-b}{2a}, \frac{b^2-4ac}{4a}\right)$$

$a>0$ brenchs are upward,
$a<0$ brenchs are downward.

Ex : Graph the equation $y=\frac{-1}{2}\cdot x^2 - x + 4$

Sol : Since $a=\frac{-1}{2}<0 \Rightarrow$ Brenches are downward.

$$V(x,y) \Rightarrow x=\frac{-b}{2a}=\frac{-(-1)}{2\cdot\left(\frac{-1}{2}\right)}=-1 \qquad y=\frac{-1}{2}\cdot(-1)^2-(-1)+4=\frac{9}{2} \qquad V=\left(-1,\frac{9}{2}\right)$$

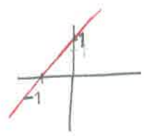
CHAPTER - 1 : FUNCTIONS

Ex :

$f(x)$	Domain of f	Range of f
x^2	$(-\infty,\infty)$	$[0,\infty)$
$\frac{1}{x}$	$R\setminus\{0\}$	$R\setminus\{0\}$
$\sqrt{x}$	$[0,\infty)$	$[0,\infty)$
$\sqrt{4-x}$	$(-\infty,4]$	$[0,\infty)$
$\sqrt{1-x^2}$	$[-1,1]$	$[0,1]$

Ex : $y=x+1$; Draw the graph of the function.

Sol : $x=0 \Rightarrow y=1$ $P_1(0,1)$
$y=0 \Rightarrow x=-1$ $P_2(-1,0)$

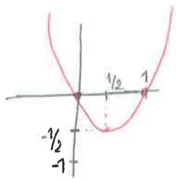


Ex : $y=2x^2-2x$; Draw the graph of the function

Sol : ① Since leading of $y=2x^2-2x \rightarrow 2>0$ Brenches are upward

② Vertex of this parabola $x=\frac{-b}{2a}=\frac{-(-2)}{2.2}=\frac{1}{2}$ $y=2\cdot\left(\frac{1}{2}\right)^2-2\cdot\left(\frac{1}{2}\right)=\frac{-1}{2}$ $V\left(\frac{1}{2},-\frac{1}{2}\right)$

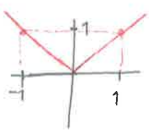
③ Finding y- intercepts of the parabola $2x^2-2x=0$ $2(x-1).x=0$ $x=0$
$x=1$



Function Graph : Vertical line intersects the graph at most once. (x)
Circles are not function graph.

Ex : $f(y)=|x|$ is a piecewise-defined function ;

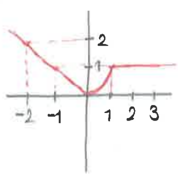
$$|x|=\begin{cases} x, & x\geq 0 \\ -x, & x<0 \end{cases}$$



$y=|x|$

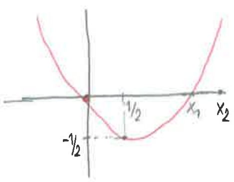
Ex : Draw the graph of the following piecewise-defined function ;

$$f(x)=\begin{cases} -x, & x<0 \\ x^2, & 0\leq x\leq 1 \\ 1, & x>1 \end{cases}$$

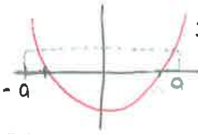


Increasing Function : $f(x_2)>f(x_1)$
Decreasing Function: $f(x_2)<f(x_1)$

Ex : $y=x+1$ is an increasing function on $R=\{-\infty,\infty\}$
$y=2x^2-2x$ is an increasing function on $I=\left[\frac{1}{2},\infty\right)$
is a decreasing function on $\left(-\infty,\frac{1}{2}\right]$



Even Function: $f(x)=f(-x)$ → Çift → "Symmetric" → y-axis
Odd Function: $-f(x)=f(-x)$ → Tek → "Symmetric" → origin

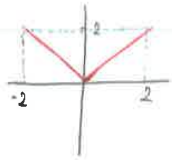


Ex: Consider $f(x)=x^2$

Sol: $f(-x)=(-x)^2=x^2=f(x) \Rightarrow \forall x \in I=(-\infty,\infty)$

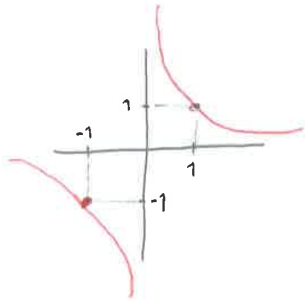
f is an even function, so its graph is symmetric with respect to y-axis.

Ex: $y=|x|$; Draw the graph of the function



Sol: $f(-x)=|-x|=|x|=f(x) \Rightarrow \forall x \in I=(-\infty,\infty)$

Ex: $y=f(x)=\frac{1}{x}$; Draw the graph of the function



Sol: Domain of $f=(-\infty,0)\cup(0,\infty)$ is symmetric.

$f(x)=\frac{1}{-x}\neq\frac{1}{x} \Rightarrow -\left(\frac{1}{x}\right)=-f(x) \quad \forall x \in I$

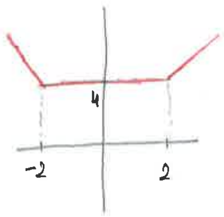
So f is an odd function therefore f is symmetric with respect to Origin.

EX: Complete the graph on the function $y(x)=|x-2|+|x+2|$ is its right-hand side is given as followers:

Sol: I: Domain of $f=(-\infty,\infty)$ is symmetric

$f(-x)=|-x-2|+|-x+2|=|-1(x+2)|+|1(x+2)|=|-1||(x+2)|+|-1||x-2|=|x-2||x+2|=f(x)$

$\forall x \in I$, $f(-x)=f(x)$ so f is even function. Hence graph of f is symmetric with respect to y-axis.



Types of Functions

1-) $f(x) = a_0 + a_1 \cdot x^1 + a_2 \cdot x^2 + \dots + a_n \cdot x^n$ => Polynomials

if $n=0$ then $f(x) = a_0$ Constant Function

if $n=1$ then $f(x) = a_0 + a_1 \cdot x^1$ Linear Function

if $n=2$ then $f(x) = a_0 + a_1 \cdot x^1 + a_2 \cdot x^2$ Quadretic Function

if $n=3$ then $f(x) = a_0 + a_1 \cdot x^1 + a_2 \cdot x^2 + a_3 \cdot x^3$ Cube Function

2-) $f(x) = x^a$ $a \in \mathbb{R}$ is called Power Function

$f(x) = \sqrt{x} = x^{1/2}$

3-) $f(x) = \dfrac{P(x)}{Q(x)}$ => Rational Function

4-) $f(x) = a^n$ => Exponential Function

$f(x) = 2^x$, $g(x) = e^x$

5-) $f(x) = \log_a x$ => Logaritmic Function

$g(x) = \log x = \log_{10} x$

6-) $\sin(x), \cos(x), \tan(x), \cot(x), \sec(x), \csc(x)$ => Trigonometric Function

Examples: Draw the graph of the following functions.

1-) $f(x) = \sqrt{x}$

2-) $f(x) = x^{2/3}$

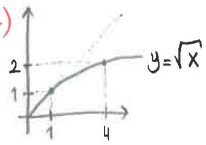
3-) $f(x) = x^{3/2}$

4-) $f(x) = x^3$

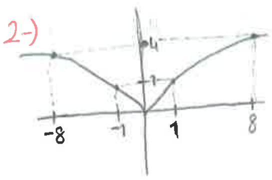
5-) $f(x) = x^{1/3}$

6-) $f(x) = \frac{1}{x}$

Sol.: 1-)



2-)



$f(x) = x^{2/3} = \sqrt[3]{x^2}$ Domain of $\mathbb{R}$

$f(-x) = \sqrt[3]{(-x)^2} = \sqrt[3]{x^2} = f(x)$ => $f(x)$ is even function

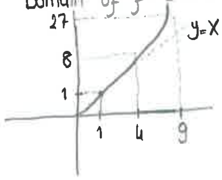
So, f is symmetric with respect to y-axis.

x	0	1	8
y	0	1	4

3-) $f(x) = x^{3/2} = x\sqrt{x}$

Domain of $f = [0, \infty)$

x	0	1	4	9
y	0	1	8	27

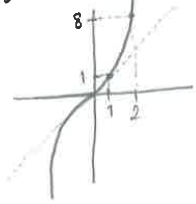


4-) $f(x) = x^3$ Domain of $f = \mathbb{R}$

$f(-x) = (-x)^3 = (-1)^3 \cdot x^3 = -1 \cdot x^3 = -f(x)$ => f is an odd function,

So, its symmetric with respect to origin.

x	0	1	2
y	0	1	8

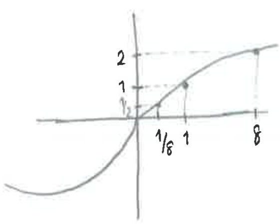


5-) $f(x) = x^{1/3}$ = its domain $= \mathbb{R}$

$f(-x) = \sqrt[3]{(-x)} = \sqrt[3]{-1} \cdot \sqrt[3]{x} = -1 \cdot \sqrt[3]{x} = -f(x)$

f is an odd function so it is symmetric with respect to origin.

x	0	1/8	1	8
y	0	1/2	1	2



$y = \sqrt[3]{x} = x^{1/3}$

Sums, Differences, Products and Quotients

1-) $h=f+g$ then for $x \in R$ $h(x)=(f+g)(x)=f(x)+g(x)$

2-) $h=f-g$ then for $x \in R$ $h(x)=(f-g)(x)=f(x)-g(x)$

3-) $h=f.g$ then for $x \in R$ $h(x)=(f.g)(x)=f(x).g(x)$

4-) $h=\frac{f}{g}$ then for $x \in R$ $h(x)=\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$ $(g(x)\neq 0)$

Ex: $f(x)=x^2-1$, $g(x)=x+1$ then;

$(f+g)(x)=(x^2-1)+(x+1)=x^2+x$

$(f-g)(x)=(x^2-1)-(x+1)=x^2-x-2$

$(f.g)(x)=(x^2-1).(x+1)=x^3+x^2-x-1$

$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{x^2-1}{x+1}=\frac{(x-1).\cancel{(x+1)}}{\cancel{(x+1)}}=x-1$ $\left(\frac{f}{g}\right)(x)=\begin{cases} x-1 & ,x\neq 1 \\ \text{undefined} & ,x=-1 \end{cases}$

Composition of Two Functions

$\Rightarrow (fog)(x)=f(g(x))$ $\Rightarrow (gof)(x)=g(f(x))$

Ex: Let $f(x)=\sqrt{x}$ and $g(x)=x+1$. Find $fof(x)$, $fog(x)$, $gof(x)$ and $gog(x)$

	Composition	Domain
$fof(x)$	$\sqrt{\sqrt{x}}=x^{1/4}$	$[0,\infty)$
$fog(x)$	$\sqrt{x+1}$	$[-1,\infty)$
$gof(x)$	$\sqrt{x}+1$	$[0,\infty)$
$gog(x)$	$x+2$	R

Shifting a graph of a function

$y-k=f.(x-h)$ $\updownarrow$ $|k|\rightarrow$ Vertically $\Rightarrow$ k is positive; upward, negative; downward

$\leftrightarrow$ $|h|\rightarrow$ Horizontally $\Rightarrow$ h is negative; "to the left", positive; "to the right"

Ex: By using the graph of the function $y=x^2$ draw the graph of the following functions

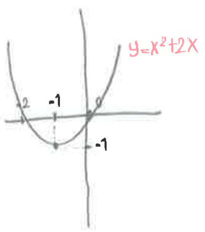
a-) $y=x^2+2.x$ b-) $y=x^2+1$ c-) $y=x^2-4x$ d-) $y=x^2-2x-2$

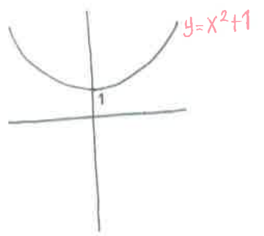
Sol.: a-) $y=(x^2+2x+1)-1=(x+1)^2-1 \Rightarrow y+1=(x+1)^2 \Rightarrow y-(-1)=(x-(-1))$ $x^2+2x=0$ $x.(x+2)=0$ $x=0,-2$

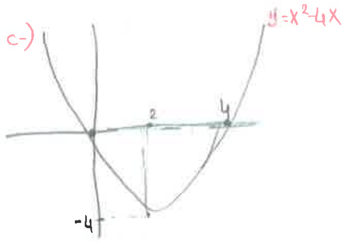
b-) $y=x^2+1 \Rightarrow y-1=(x-0)^2 \Rightarrow y-1=f(x-p)$ $k=1$ $y=0$

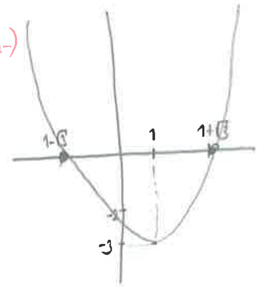
c-) $y=x^2-4x=(x^2-4x+4)-4 \Rightarrow y-(-4)=(x-(-2))^2=f(x-2)$ $k=-4$ $h=2$

d-) $y=x^2-2x-2=(x^2-2x+1)-3=y-(-3)=(x-(1))^2$ $k=-3$ $h=1$ $(x-1)^2-3=0$ $x=1\mp\sqrt{3}$

a-)


b-)


c-)


d-)


Vertical and Horizontal Scaling and Reflecting Formulas

Scaling for $c>1$;

$y=c\cdot f(x)$	Stretches the graph of f vertically by c factor. (c kadar dikey uzatır)
$y=\frac{1}{c}\cdot f(x)$	Compress the graph of f vertically by c factor. (c kadar dikey sıkıştırır)
$y=f(c\cdot x)$	Compress the graph of f horizontally by c factor. (c kadar yatay sıkıştırır)
$y=f\left(\frac{x}{c}\right)$	Stretches the graph of f horizontally by c factor. (c kadar yatay uzatır)

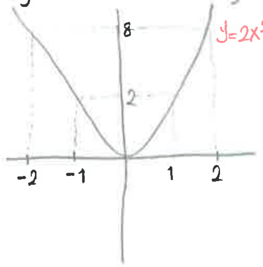
For $c=-1$

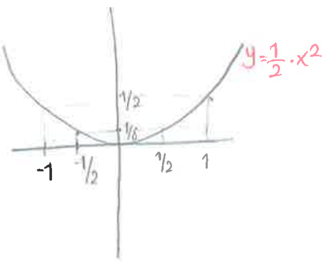
$y=-f(x)$	Reflect the graph of f across the x-axis. (x ekseninin diğer tarafına yansıtır)
$y=f(-x)$	Reflect the graph of f across the y-axis. (y ekseninin diğer tarafına yansıtır)

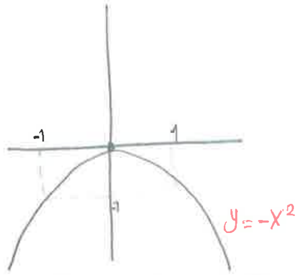
Ex: By using the graph all the parabola $y=x^2$, the following parabola;

a-) $f(x)=2x^2$ b-) $f(x)=\frac{1}{2}\cdot x^2$ c-) $f(x)=-x^2$

Sol.:

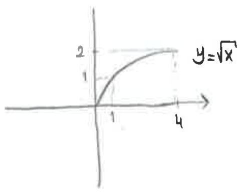






Ex: Find the graph of the function $f(x)=\sqrt{4-2x}$ by using the graph of the function $y=\sqrt{x}$

Sol:

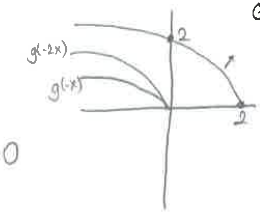


$g(x)=\sqrt{x}$

$f(x)=\sqrt{4-2x}=\sqrt{-2.(x-2)}=g(-2(x-2))$

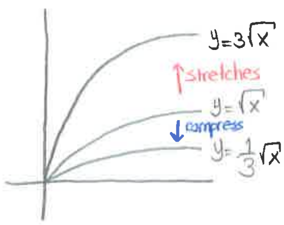
$=g(-1.2.(x-2))$

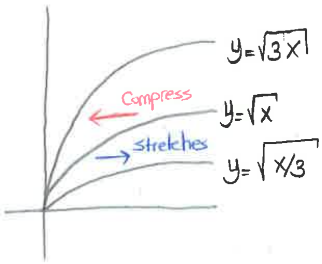
$c=-1,\ c=2,\ h=2>0$

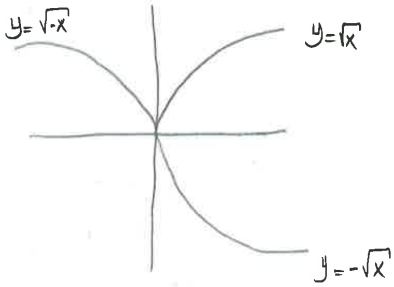


Graph of $g(-2(x-2))=f(x)$

Examples

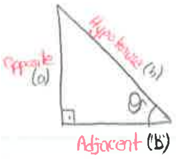






Trigonometric Functions

Angles: $\frac{\theta}{\pi} = \frac{\alpha}{180}$



$\text{Sin}\,\theta = \frac{\text{Opp.}}{\text{Hyp.}}$ $\text{Cos}\,\theta = \frac{\text{Adj.}}{\text{Hyp.}}$ $\text{Tan}\,\theta = \frac{\text{Opp.}}{\text{Adj.}}$ $\text{Cot}\,\theta = \frac{\text{Adj.}}{\text{Opp.}}$

$\text{Sec}\,\theta = \frac{\text{Hyp.}}{\text{Adj.}}$ $\text{Cosec}\,\theta = \frac{\text{Hyp}}{\text{Opp.}}$ $\text{Tan}\,\theta = \frac{\sin\theta}{\cos\theta}$ $\text{Cot}\,\theta = \frac{\text{Cos}\,\theta}{\text{Sin}\,\theta}$

$\text{Sec}\,\theta = \frac{1}{\cos\theta}$ $\text{Cosec}\,\theta = \frac{1}{\text{Sin}\,\theta}$ $\text{Tan}\,\theta \,.\, \text{Cot}\,\theta = 1$

Trigonometric Identities

1-) $\text{Sin}^2\theta + \text{Cos}^2\theta = 1 \Rightarrow \left(\frac{a}{h}\right)^2 + \left(\frac{b}{h}\right)^2 = \frac{a^2+b^2}{h^2} \Rightarrow h^2 = a^2+b^2 = \frac{a^2+b^2}{a^2+b^2} = 1$

2-) $1 + \tan^2\theta = \sec^2\theta \Rightarrow 1 + \left(\frac{a}{b}\right)^2 = 1 + \frac{a^2}{b^2} = \frac{a^2+b^2}{b^2} = \frac{1}{\frac{b^2}{a^2+b^2}} = \frac{1}{\frac{b}{\sqrt{a^2+b^2}}} = \frac{1}{\cos^2\theta} = \sec^2\theta$

3-) $1 + \cot^2\theta = \text{coséc}^2\theta \Rightarrow 1 + \left(\frac{b}{a}\right)^2 = \frac{a^2+b^2}{a^2} = \frac{1}{\frac{a^2}{a^2+b^2}} = \frac{1}{\frac{a}{\sqrt{a^2+b^2}}} = \frac{1}{\sin^2\theta} = \text{cosec}^2\theta$

Addition Formulas

1-) $\text{Sin}\,(A+B) = \text{Sin}\,A.\text{Cos}\,B + \text{Sin}\,B.\text{Cos}\,A$

2-) $\text{Cos}\,(A+B) = \text{Cos}\,A.\cos B - \text{Sin}\,A.\sin B$

* Sin. Function is an Odd Function. $\Rightarrow \text{Sin}(-A) = -\sin(A)$

Cos. Function is an Even Function. $\Rightarrow \text{Cos}(-A) = \cos(A)$

3-) $\text{Sin}(A-B) = \sin(A+(-B)) = \sin A.\cos(-B) + \sin(-B).\cos(A) = \sin(A).\cos(B) - \sin(B).\cos(A)$

4-) $\text{Cos}(A-B) = \cos(A+(-B)) = \cos(A).\cos(-B) - \sin(A).\sin(-B) = \cos(A).\cos(B) + \sin(A).\sin(B)$

* $\text{Tan}\,(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A.\cos B + \sin B.\cos A}{\cos A.\cos B - \sin A.\sin B}$ => divide bothside by $\text{Cos}\,A$ and $\text{Cos}\,B$ $\Rightarrow \frac{\frac{\sin A.\cancel{\cos B}}{\cos A.\cancel{\cos B}} + \frac{\sin B.\cancel{\cos A}}{\cancel{\cos A}.\cos B}}{\frac{\cancel{\cos A.\cos B}}{\cancel{\cos A.\cos B}} - \frac{\sin A.\sin B}{\cos A.\cos B}} = \frac{\tan(A)+\tan(B)}{1-\tan A.\tan B}$

Double Angle Formulas

1-) $\sin 2A = \sin(A+A) = \sin A.\cos B + \sin B.\cos A \rightarrow 2.\sin A.\cos A$

2-) $\cos 2A = \cos(A+A) = \cos A.\cos B - \sin A.\sin B \Rightarrow \cos^2 A - \sin^2 A$

3-) $\tan 2A = \frac{\tan(A)+\tan(B)}{1-\tan(A).\tan(B)} \Rightarrow \frac{2.\tan(A)}{1-\tan^2(A)}$

Ex: Find $\sin\left(\frac{\pi}{8}\right)$, $\sin\left(\frac{5\pi}{6}\right)$, $\sin\left(\frac{2\pi}{3}\right)$

Sol: $\cos^2\theta+\sin^2\theta=1$
$\cos^2\theta-\sin^2\theta=\cos 2\theta$
$\Big\}\ \sin^2\theta=\frac{1-\cos(2\theta)}{2}$, $\cos^2\theta=\frac{1+\cos(2\theta)}{2}$

$$\sin^2\left(\frac{\pi}{8}\right)=\frac{1-\cos\left(2\cdot\frac{\pi}{8}\right)}{2}\Rightarrow\frac{1-\cos\left(\frac{\pi}{4}\right)}{2}=\frac{1-\frac{\sqrt{2}}{2}}{2}=\frac{2-\sqrt{2}}{4}\ //$$

$$\sin\left(\frac{5\pi}{6}\right)=\sin\left(\frac{\pi}{2}+\frac{\pi}{3}\right)=\sin\frac{\pi}{2}\cdot\cos\frac{\pi}{3}+\sin\frac{\pi}{3}\cdot\cos\frac{\pi}{2}=1\cdot\frac{1}{2}+0\cdot\frac{\sqrt{3}}{2}=\frac{1}{2}\ //$$

$$\sin\left(\frac{2\pi}{3}\right)=\sin\left(2\cdot\frac{\pi}{3}\right)=2\cdot\sin\left(\frac{\pi}{3}\right)\cdot\cos\left(\frac{\pi}{3}\right)=2\cdot\frac{\sqrt{3}}{2}\cdot\frac{1}{2}=\frac{\sqrt{3}}{2}\ //$$

Ex: Find the values of the following examples;

Sol: $\tan(315^\circ)=\tan\left(2\pi-\frac{\pi}{4}\right)=-\tan\left(\frac{\pi}{4}\right)=-1$ $\qquad$ $\sin(270^\circ)=\sin\left(\pi+\frac{\pi}{2}\right)=-\sin\left(\frac{\pi}{2}\right)=-1$

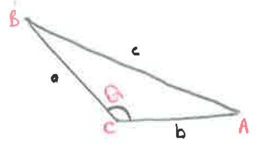
$\cot(240^\circ)=\cot\left(\pi+\frac{\pi}{3}\right)=\cot\left(\frac{\pi}{3}\right)=\frac{1}{\sqrt{3}}$ $\qquad$ $\sec(300^\circ)=\sec\left(2\pi-\frac{\pi}{3}\right)=\sec\left(\frac{\pi}{3}\right)=2$

Half Angle Formulas

$\cos^2\theta=\frac{1+\cos 2\theta}{2}$ $\qquad$ $\cos^2\left(\frac{\theta}{2}\right)=\frac{1+\tan\theta}{2}$

$\sin^2\theta=\frac{1-\cos 2\theta}{2}$ $\qquad$ $\sin^2\left(\frac{\theta}{2}\right)=\frac{1-\cot\theta}{2}$

Law of Cosinus



$$c^2=a^2+b^2-2.a.b.\cos\theta$$

$$\cos\theta=\frac{a^2+b^2-c^2}{2.a.b}$$

Ex: In a triangle ABC, sides are given like $a=5, b=6, c=7$ units then find in $(B)=?$

Sol: $\cos(B^\circ)=\frac{a^2+c^2-b^2}{2.a.c}=\frac{5^2+7^2-6^2}{2.5.7}=\frac{19}{35}$ $\qquad$ $\frac{\pi}{3}>B>\frac{\pi}{6}$

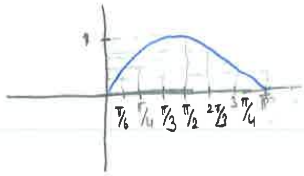
Graphs of Trigonometric Functions

Ex: Graph of sin Function let $y=f(x)=\sin(x)$

Sol: STEP-1: Constrast its table of values

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	$3\pi/4$	$5\pi/6$	π
$\sin(x)$	0	$1/2$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$1/2$	0

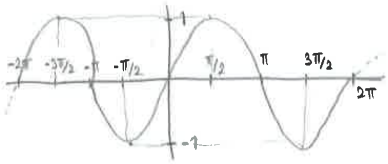
* cos graph is even function. (2π) respect to y-axis
* tan graph is odd function. (π) respect to origin.



STEP-2: Since $f(-x)=\sin(-x)=-\sin x=-f(x)$

f is odd function. So its graph is symmetric respect to origin. (2π)

STEP-3: Copy this graph by using its periodicity.



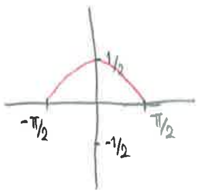
Transformations of Trigonometric Graphs

$$y=a.f(b(x+c))+d$$

Vertical Stretches or Compress — Horizontal Stretch or Compress — Horizontal Shift — Vertical Shift

Ex: Draw graph of the functions $f(x)=\cos^2 x$

Sol: $y=g(x)=\cos^2 x=\dfrac{1+\cos(2x)}{2}=\underbrace{\frac{1}{2}}_{a}.\cos(\underbrace{2}_{b}.(x+\underbrace{0}_{c}))+\underbrace{\frac{1}{2}}_{d}$



$a=\frac{1}{2}$ says that Compress the graph $f(x)$ vertically by the factor 2.

$b=2$ says that Compress the graph $\frac{1}{2}f(x)$ horizontally by the factor 2.

$\Rightarrow$ If the period of $f(x)$ is p then the period of $f(ax+b)$ is $\frac{\pi}{a}$, period of function $a.f(x)+b$ is $\pi.a$