VECTORS AND SCALARS

1-) Vector and Scalar Quantities

Scalar Quantity: Magnitude + Unit F.E.: Speed, Mass, Temperature

Vector Quantity: Magnitude + Unit + Direction F.E.: Acceleration, Force, Velocity

Unit and Base Vectors

$\boxed{\begin{array}{l}\vec{A_x} = A.\cos\theta \\ \vec{A_y} = A.\sin\theta\end{array}}$ → Horizontal Component / → Vertical Component

$\vec{A_x} = A_x.\hat{i}$
$\vec{A_y} = A_y.\hat{j}$
$\vec{A_z} = A_z.\hat{k}$

$A = A_x + A_y + A_z$

$A^2 = A_x^2 + A_y^2 + A_z^2$

$\vec{A} = A_x.\hat{i} + A_y.\hat{j} + A_z.\hat{k}$

A_x and A_y are components, not vectors.

Multiplication of Vectors

i-) Scalar (Dot) Product

The scalar product $C = \vec{A}\cdot\vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ is a scalar quantity. It can be expressed in terms of the magnitudes of $\vec{A}$ and $\vec{B}$ and the angle θ btwn the two vectors, or in terms of the components of $\vec{A}$ and $\vec{B}$. The scalar product is commutative; $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$ ① It is a scalar (dot) product.

① $\vec{A}\cdot\vec{B} = A.B.\cos\theta = |\vec{A}|.|\vec{B}|.\cos\theta$

$\vec{A}\cdot\vec{B} = A_x.B_x + A_y.B_y + A_z.B_z$

* (Magnitude of $\vec{A}$) = A: $|\vec{A}|$ = Always positive.
* If two vectors are vertical each other, these vectors' scalar product is 0.

Ex: $\vec{A}: 4\hat{i} + 5\hat{j} - 3\hat{k}$
$\vec{B}: 2\hat{i} - 3\hat{j} + 2\hat{k}$

a-) $\vec{C} = \vec{A} - \vec{B}$
b-) $\vec{A}\cdot\vec{B} = ?$
c-) Angle btwn $\hat{A}$ and $\hat{B} = ?$

Sol: a-) $\vec{C} = \vec{A} - \vec{B} \Rightarrow (A_x - B_x).\hat{i} + (A_y - B_y).\hat{j} + (A_z - B_z).\hat{k} = 2\hat{i} + 8\hat{j} - 5\hat{k}$ $|\vec{C}| = [(2^2 + 8^2 + (-5)^2]^{\frac{1}{2}} = \dots$

b-) $\vec{A}.\vec{B} \Rightarrow A_x.B_x + A_y.B_y + A_z.B_z = (4.2) + (5.(-3)) + ((-3).(2)) \Rightarrow \vec{A}.\vec{B} = -13$

c-) $A = |\vec{A}| = (4^2 + 5^2 + (-3)^2)^{\frac{1}{2}} = 7.07$
$B = |\vec{B}| = (2^2 + (-3)^2 + 2^2)^{\frac{1}{2}} = 4.12$

$\vec{A}.\vec{B} = A.B.\cos\theta$ $-13 = 7.07 \times 4.12 \times \cos\theta$

$\cos\theta = -0{,}44$ $\theta = \arccos(-0{,}44) = 116{,}5°$

ii-) Vector (Cross) Product

$\vec{A} \times \vec{B} = \vec{C}$
$\vec{B} \times \vec{A} = -\vec{C}$
$\vec{C} = \vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

$|\vec{C}| = C = A.B.\sin\theta$
$C_x = A_y B_z - A_z B_y$
$C_y = A_z B_x - A_x B_z$
$C_z = A_x B_y - A_y B_x$

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$
$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$\vec{C} = (A_y B_z - A_z B_y).\hat{i} - (A_x B_z - A_z B_x)\hat{j} + (A_x B_y - A_y B_x).\hat{k}$

* If two vectors are parallel or antiparallel each other, these vectors' vector (cross) product is 0.

* Scalar Product → Max   Vector Product → Min
Scalar Product → Min   Vector Product → Max

Ex: $\vec{r}: 3\hat{i} - 2\hat{j} + 5\hat{k}$ (m)
$\vec{F}: 5\hat{i} + 3\hat{j} - 4\hat{k}$ (n)
$\tau: r.F.\sin\theta$

a-) $\vec{\tau} = \vec{r} \times \vec{F}$
b-) $\tau = |\vec{\tau}|$ → Magnitude of Torque
c-) Find the angle btwn $\vec{r}$ and $\vec{F}$ vectors

Sol: $\vec{\tau} = \vec{r} \times \vec{F} \Rightarrow \vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 5 \\ 5 & 3 & -4 \end{vmatrix} = (8-15)\hat{i} - (-12-25)\hat{j} + (9+10)\hat{k} = -7\hat{i} + 37\hat{j} + 19\hat{k}$, $\tau = |\vec{\tau}| = \sqrt{7^2 + 37^2 + 19^2} = \sqrt{1779} = 42{,}18$

$|\vec{r}| = \sqrt{9+4+25} = \sqrt{38} = 6.16$
$|\vec{F}| = \sqrt{25+9+16} = \sqrt{50} = 7.07$

$\tau = r.F.\sin\theta \Rightarrow 42{,}18 = 6{,}16 . 7{,}07 . \sin\theta$

$\dfrac{42{,}18}{6{,}16 . 7{,}07} = \sin\theta \Rightarrow \sin\theta = 0{,}96$ $\theta = 73.7$

MOTION ALONG A STRAIGHT LINE

POSITION, DISPLACEMENT, and AVERAGE VELOCITY
$(\vec{x})$ $(\Delta\vec{x} = \vec{x}_F - \vec{x}_i)$ $\vec{v}_{ave}$

$$\vec{v}_{ave} : \frac{\vec{x}_F - \vec{x}_i}{t_F - t_i} = \frac{\Delta \vec{x}}{\Delta t} \ (m/s) = \tan\theta$$

Instantaneus Velocity $\Rightarrow \vec{v} = \lim_{\Delta t \to 0} \frac{\vec{x}_F - \vec{x}_i}{t_F - t_i} = \frac{d\vec{x}}{dt} \ (m/s)$ → Position depends on time.

Ex: The position of a particle moving along the X-axis varies in time according to expression $X = 3t^2 + 2$ (m);
$x(t) = 3t^2 + 2$ (m)

a-) Find v_{ave} btwn $t_i = 3$ sec and $t_F = 6$ sec

b-) Find v at fourth second.

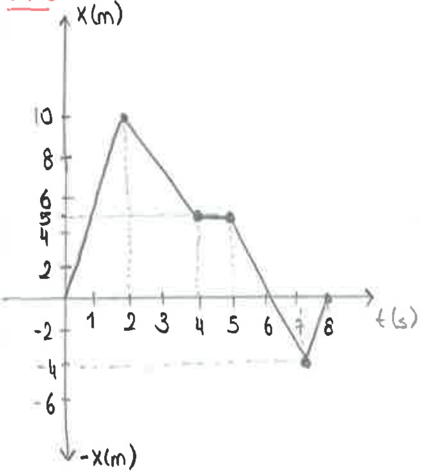
Sol: $\vec{v}_{ave} = \frac{\vec{x}_F - \vec{x}_i}{t_F - t_i} = \frac{110-29}{6-3} = \frac{81}{3} = 27$ m/s

$\vec{x}_F\ (t_F = 6s) = 3.6^2 + 2 = 110$ m

$\vec{x}_i\ (t_i = 3s) = 3.3^2 + 2 = 29$ m

$\vec{v}_{ins} = \frac{d\vec{x}}{dt} = 6t$ (m/s) $\quad v(t=4s) = 6.4 = 24$ m/s

Ex:



Find the $\vec{v}_{av}$ in time interval

a-) (0-2)s → $\vec{v}_{ave} = \frac{10-0}{2-0} = 5$ m/s

b-) (0-4)s → $\vec{v}_{ave} = \frac{5-0}{4-0} = 1.25$ m/s

c-) (2-4)s → $\vec{v}_{ave} = \frac{5-10}{4-2} = -2,5$ m/s

d-) (4-7)s → $\vec{v}_{ave} = \frac{-4-5}{7-4} = -3$ m/s

e-) (0-8)s → $\vec{v}_{ave} = 0$

AVERAGE and INSTANTANEUS ACCELERATION

$$\vec{a}_{ave} = \tan\theta = \frac{\vec{v}_F - \vec{v}_i}{t_F - t_i} = \frac{\Delta\vec{v}}{\Delta t} \ (m/s^2)$$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{v}_F - \vec{v}_i}{t_F - t_i} = \frac{d\vec{v}}{dt} \ (m/s^2)$$

Ex: The velocity of a particle moving along x-axis varies in time according to $v(t) = 30 - 4t^2$ (m/s)

a-) Find the $\vec{a}_{ave}$ in the interval (0-2)s

b-) Find the $\vec{a}$ at $t = 2s$

Sol: a-) $\vec{a}_{ave} = \frac{\vec{v}_F - \vec{v}_i}{t_F - t_i} \Rightarrow \vec{v}_F\ (t=2s) = 30 - 4.2^2$ (m/s) $= 14$ m/s $\quad \vec{v}_i\ (t=0s) = 30 - 4.0^2$ (m/s) $= 30$ m/s

$\vec{a}_{ave} = \frac{14-30}{2-0} = -8$ m/s²

b-) $\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow a(t) = -8t$ (m/s²) $\quad t = 2s \Rightarrow a(t=2s) = -8.2 = -16$ m/s²

NOTE-1 : Area under $\vec{a}-t$ graph gives the change in velocity. ($\Delta\vec{v}$)

NOTE-2 : Area under $\vec{v}-t$ graph gives displacement. (Δx)

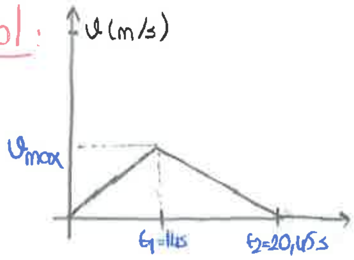
$$x_F = x_i + v_i \cdot t \mp \frac{1}{2} \cdot a \cdot t^2 \ (m) \qquad \vec{v}_F = \vec{v}_i \mp a \cdot t \ (m/s) \qquad \vec{v}_F^{\,2} = v_i^2 \mp 2 \cdot a \cdot \Delta x \ (m^2/s^2)$$

Ex : An electron with $v_i = 1{,}5 \times 10^5$ m/s enters a region $\underbrace{1 \text{ cm}}_{}$ (Δx) long where it is accelerated. Electron leaves this region with $\vec{v}_F = 5{,}7 \times 10^6$ m/s. What is its acceleration?

Sol: $v_F^2 = v_i^2 + 2 \cdot a \cdot \Delta x \Rightarrow (5{,}7 \times 10^6 \text{ m/s})^2 = (1{,}5 \times 10^5 \text{ m/s})^2 + 2 \cdot a \cdot 10^{-2} \text{ m} \qquad \vec{a} = 1{,}62 \times 10^{15} \text{ m/s}^2$

Ex : A subway train starts motion from a station. Initially, it accelerates with 1,6 m/s² during 14 seconds. Later, it continues its motion by decelerating with 3,5 m/s² during 6,45 seconds. Find the taken way train.

Sol :

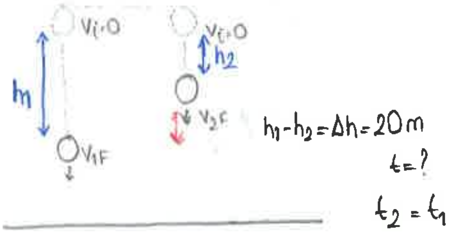


$$\text{Area} = \frac{a_1 \cdot t_1 \cdot t}{2} = \frac{1{,}6 \text{ m/s}^2 \cdot 14 s \cdot 20{,}45 s}{2} = 228{,}48 \text{ m}$$

Freely Falling Bodies

$$v_F^2 = v_i^2 \mp 2 \cdot g \cdot \Delta h \qquad v_F = v_i + g \cdot t \qquad h_F = h_i + v_i \cdot t \mp \frac{1}{2} \cdot g \cdot t^2$$

Ex : Two objects are left freely 1 second apart. How many seconds later the difference btwn objects gets 20 m? (Take $g \cong 10$ m/s²)



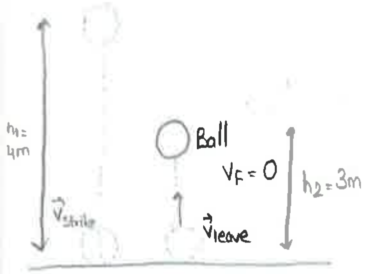
Sol : $h_1 = \cancel{v_i \cdot t_1}^{\,v_i = 0} + \frac{1}{2} \cdot g \cdot t_1^2 = \frac{1}{2} \cdot g \cdot t_1^2$

$h_2 = \frac{1}{2} \cdot g \cdot t_2^2 \Rightarrow \frac{1}{2} \cdot g \cdot (t_1 - 1)^2$

$h_1 - h_2 = \frac{1}{2} \cdot g \cdot t_1^2 - \frac{1}{2} \cdot g \cdot (t_1 - 1)^2 \Rightarrow 20 = 5t_1^2 - 5t_1^2 + 10t_1 - 5$

$t_1 = 2{,}5$ s

Ex : A ball is left freely from 4m height. It bounces back to the 3m after it strikes to the ground. If the action time btwn ball and ground is 0,02 second, find the $\vec{a}_{ave}$ during this action time?



Sol: $\vec{a}_{ave} = \frac{\vec{v}_F - \vec{v}_i}{t_F - t_i} = \frac{\vec{v}_{leave} - \vec{v}_{strike}}{t_F - t_i} = \frac{7{,}7 \text{ m/s} - (-8{,}9 \text{ m/s})}{0{,}02 \text{ s}} = 830 \text{ m/s}^2$

$v_{strike}^2 = \underbrace{v_i^2}_{0} + 2 \cdot g \cdot h_1 = 0 + (2 \times 10 \text{ m/s}^2 \times 4 \text{m})^{1/2} = 8{,}9$ m/s downward (-) ↓

$v_F^2 = v_{leave}^2 - 2 \cdot g \cdot h_2 \Rightarrow 0 = V_{leave}^2 - 2 \cdot 10 \text{ m/s}^2 \cdot 3\text{m} \Rightarrow \vec{v}_{leave} = 7{,}7$ m/s (+) ↑ upward

⍺ Direction is very important.

Ex: A student throws a set of keys upward to her sister at 4m above in a window. Sister catches keys 1.5 s later.

a-) V_i of keys? b-) Velocity of key set just before they were caught?

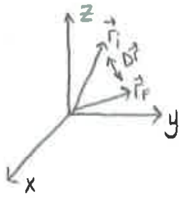
Sol: a-) $h = V_i \cdot t - \frac{1}{2} \cdot g \cdot t^2 \Rightarrow 4m = V_i \cdot 1,5\,s - \frac{1}{2} \cdot 10 \cdot (1,5s)^2 \Rightarrow \vec{V_i} \cong 10,17\ m/s$

b-) $V_F = V_i - g \cdot t = 10,17\ m/s - 10\,m/s^2 \cdot 1,5s = -4,83\ m/s$ "−"; means sister caught them when they fell down.

CHAPTER-3

MOTION in TWO and THREE DIMENSIONS

3.1. Position and Velocity Vectors



$$\vec{r} = x \cdot \hat{\imath} + y \cdot \hat{\jmath} + z \cdot \hat{k}$$

$$\Delta\vec{r} = (x_F - x_i)\hat{\imath} + (y_F - y_i)\hat{\jmath} + (z_F - z_i)\hat{k}\ (m)$$

$$\vec{V} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k} = V_x \hat{\imath} + V_y \hat{\jmath} + V_z \hat{k}\quad (m/s)$$

$$\vec{a} = a_x \cdot \hat{\imath} + a_y \cdot \hat{\jmath} + a_z \cdot \hat{k}\quad (m/s^2)$$

Ex: $\vec{r}(t) = 18t\,\hat{\imath} + (4t - 4,9t^2)\hat{\jmath}\ (m)$ for an object.

a-) $V(t) = ?$ b-) $\vec{a}(t) = ?$ c-) X and Y coordinates of object at $t = 3s$?

d-) $V(t=3s) = ?$ e-) $\vec{a}(t=3\,s) = ?$

Sol: a-) $\vec{V}(t) = \frac{d\vec{r}}{dt} = \vec{V}_x \cdot \hat{\imath} + \vec{V}_y \hat{\jmath} + \underbrace{\vec{V}_z \hat{k}}_{0} = 18\hat{\imath} + (4 - 9,8t)\hat{\jmath}\ m/s$

b-) $\vec{a}(t) = \frac{d\vec{V}}{dt} = 0\,\hat{\imath} + (-9,8)\hat{\jmath}\ m/s^2$

c-) $X(t=3s) \Rightarrow 18 \times 3 = 54\ m$ x coordinates

$y(t=3\,s) \Rightarrow (4 \times 3 - 4,9 \times 3^2) = -32,1\ m$ y coordinates

d-) $\vec{V}(t=3s) = 18\hat{\imath} - 25,4\hat{\jmath}\ m/s$

e-) $\vec{a}(t=3s) = -9,8\ m/s^2$ $\vec{a}$ is constant because it is not depend on t.

Ex: A fish has $\vec{v}_i = 4\hat{i} + \hat{j}$ m/s in the ocean and $\vec{r}_i = 10\hat{i} - 4\hat{j}$ (m) relative to a stationary rock at the shore. A fish swims with a constant acceleration for 20 s, its $\vec{v}_F = 20\hat{i} - 5\hat{j}$ (m/s)

a-) Find a_x and a_y ? b-) Direction of $\vec{a}$? c-) When $t = 25$ s find $\vec{r}_F = ?$

Sol: a-) $\vec{a} = \dfrac{\vec{V}_F - \vec{V}_i}{\Delta t} \Rightarrow \vec{a}_x = \dfrac{\vec{V}_{Fx} - \vec{V}_{ix}}{\Delta t} = \dfrac{20-4}{20} = 0{,}8\ m/s^2$

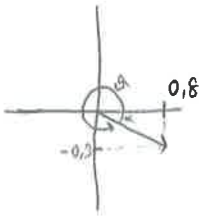
$\vec{a}_y = \dfrac{\vec{V}_{Fy} - \vec{V}_{iy}}{\Delta t} = \dfrac{-5-1}{20} = -0{,}3\ m/s^2$

$v_i = 4\hat{i} + \hat{j}$

$V_{ix} = 4 \quad V_{iy} = 1$

$V_{Fx} = 20 \quad V_{Fy} = -5$

b-) $\vec{a} = a_x \cdot \hat{i} + a_y \cdot \hat{j}$ $\qquad \vec{a} = 0{,}8\hat{i} - 0{,}3\hat{j}\ m/s^2$



$\tan\alpha = \dfrac{-0{,}3}{0{,}8} \Rightarrow \alpha = \arctan\left(\dfrac{-0{,}3}{0{,}8}\right) = -20{,}55°$

$\theta = 360° - 20{,}55° = 339{,}45°$

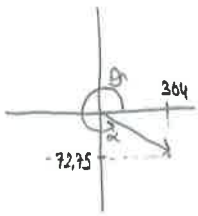
c-) $\vec{r}_F = x_F \cdot \hat{i} + y_F \cdot \hat{j} = ?$ Direction ?

$x_F = x_i + v_{ix} \cdot t + \frac{1}{2} \cdot a_x \cdot t^2$ $\qquad y_F = y_i + V_{iy} \cdot t + \frac{1}{2} \cdot a_y \cdot t^2$

$x_F = 10m + 4\ m/s \cdot 25s + \frac{1}{2} \cdot 0{,}8\ m/s^2 \cdot (25s)^2$ $\qquad y_F = -4m + 1\ m/s \cdot 25s - \frac{1}{2} \cdot 0{,}3\ m/s \cdot (25s)^2 = -72{,}75m$

$= 364\ m$

$\vec{r}_F = 364\hat{i} - 72{,}75\hat{j}$ (m)



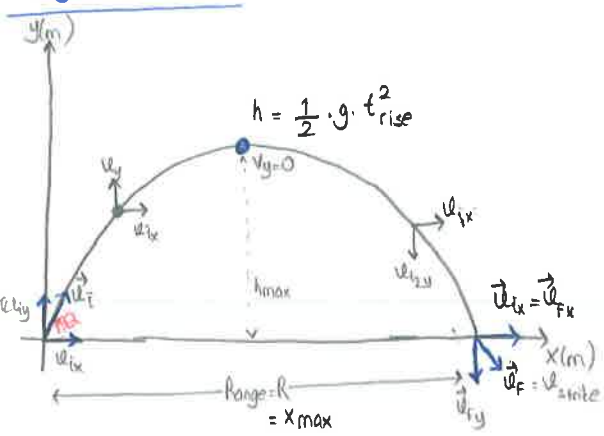
$\tan\alpha = \dfrac{364\ m}{-72{,}75m} \Rightarrow \theta = 348{,}7°$

*** For direction we need to calculate the angle that starts from $+a_x$ axis in counterclockwise direction.

$\vec{a}$ means; a_x, a_y and angle

Magnitude a ; $a^2 = |a_x^2 + a_y^2| \Rightarrow ((0{,}8)^2 + (-0{,}3)^2)^{1/2} = 0{,}85\ m/s^2$

Projectile Motion



→ Projectile motion is the combination free fall in vertical and steady motion ($\vec{a} = 0$) in horizontal.

$\upsilon_{ix} = \upsilon_i . \cos\theta \qquad \upsilon_{iy} = \upsilon_i . \sin\theta$

→ When object reaches to the hmax its vertical velocity gets zero.

$t_{rise} = \dfrac{\upsilon_i . \sin\theta}{g}$

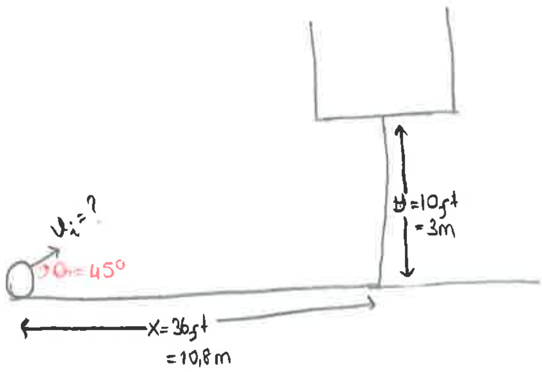
$t_{rise} = t_{drop} = \dfrac{t_{flight}}{2}$ (s) , $t_{flight} = \dfrac{2 . \upsilon_i . \sin\theta}{g}$ (s) , $h_{max} = \dfrac{\upsilon_i^2 . \sin^2\theta}{2g}$ (m) , $R = \dfrac{\upsilon_i^2 . \sin 2\theta}{g}$ (m)

$X = V_{ix} . t \Rightarrow t = \dfrac{x}{\upsilon_i . \cos\theta}$

$y = \upsilon_{iy} . t - \frac{1}{2} . g . t^2 \Rightarrow y = x . \tan\theta - \frac{1}{2} . g . \dfrac{x^2}{\upsilon_i^2 . \cos^2\theta}$

Equation of Trajection

Ex :



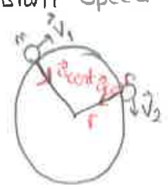
What must be initial if ball is to just clear the bar? ($g = 10 \ m/s^2$, $1 ft = 0,3 m$)

Sol : $y = x . \tan\theta - \frac{1}{2} . g . \dfrac{x^2}{\upsilon_i^2 . \cos^2\theta} \Rightarrow$

$3m = 10,8 \ m \times 1 - \frac{1}{2} . \dfrac{(10,8)^2}{\upsilon_i^2 (\cos 45°)^2} \Rightarrow \upsilon_i = 12,23 \ m/s$

Uniform Circular Motion (Düzgün Dairesel Hareket)

→ Constant Speed



$|\vec{V}_1| = |\vec{V}_2| \Rightarrow$ Magnitude equal

$\vec{\upsilon}_1 \neq \vec{\upsilon}_2$ → Direction changes

$a_{cent} = a_{rad} = \dfrac{\upsilon^2}{r}$ (m/s^2)

$\vec{\upsilon}$ = tangential velocity

$T = \dfrac{2 . \pi . r}{\upsilon}$ (s) (Period) $\qquad T . f = 1$

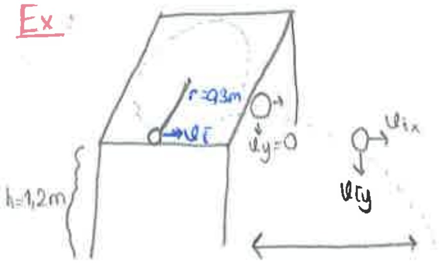
Ex : An object makes 100 rotations per minute (frequency) at constant rate in a circular orbit has 20 cm radius. ⇓ 0,2 m

Find the acent ? ($\pi = 3$ rad)

Sol : $a_{cent} = \dfrac{\upsilon^2}{r} \Rightarrow \dfrac{(2 m/s)^2}{0,2 m} = 20 \ m/s^2$

$\upsilon = \dfrac{2 . \pi . r}{T} = 2 . \pi . r . f = 2 \times 3 \ rad \times 0,2 \ m \times \dfrac{100}{60} \dfrac{rot}{s} = 2 \ m/s$

Ex :



An object were doing uniform circular motion when the rope was cut.

Find $\vec{a}_{cent}$?

Sol : $a_{cent} = \dfrac{v_i^2}{r}$

$$a_{cent} = \frac{(4{,}08\ m/s)^2}{0{,}3m}$$

$$= 55{,}55\ m/s^2$$

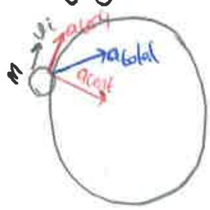
$R = v_i \cdot t_{drop}$

$h = \frac{1}{2} \cdot g \cdot t_d^2 \Rightarrow t_d = \left(\frac{2h}{g}\right)^{1/2}$

$R = v_i \cdot \left(\frac{2h}{g}\right)^{1/2} \Rightarrow 2m = v_i \cdot \left(\frac{2 \times 1{,}2m}{10m/s}\right)^{1/2} \Rightarrow v_i = 4{,}08\ m/s$

Nonuniform Circular Motion

→ Changing speed

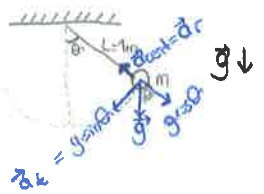


$\vec{a}_{cent} \Rightarrow$ due to change of direction $\vec{v}$

$\vec{a}_{tang} \Rightarrow$ change at the magnitude of $\vec{v}$
(every moment)

$\vec{a}_{total} = \vec{a}_{cent} + \vec{a}_{tang}\ (m/s^2) \Rightarrow a_{total} = (a^2_{cent} + a^2_{tang})^{1/2}\ (m/s^2)$

Ex :



The speed of object is 3m/s when $\theta = 30°$

a-) Find the $a_r = a_{cent} = ?$

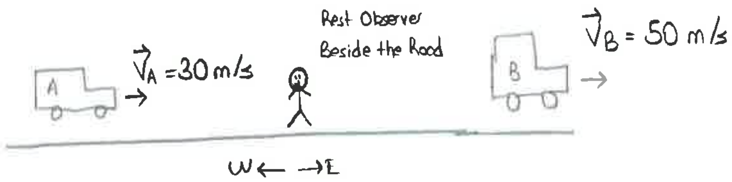
b-) Find the $a_{tot} = ?$

a-). $a_{cent} = a_r = \dfrac{v^2}{r} = \dfrac{(3m/s)^2}{1m} = 9m/s^2$

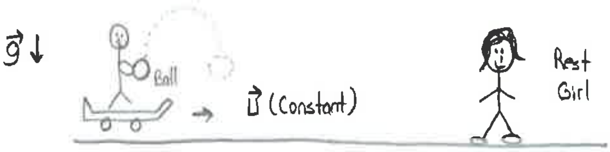
b-) $a_{tang} = g \cdot \sin\theta = 10\ m/s^2 \times \sin 30 = 5m/s^2$

$\Rightarrow \vec{a}_{total} = \vec{a}_{cent} + \vec{a}_{tan} \Rightarrow a_{tan} = (a_{cent}^2 + a_{tan}^2)^{1/2} = ((9m/s^2)^2 + (5\ m/s^2)^2)^{1/2} = 10.3\ m/s^2$

Relative Motion in Two Dimensions



The observer A observes the velocity of car B as it goes with 20 m/s toward East.
〃 〃 B 〃 〃 〃 〃 〃 A 〃 〃 〃 〃 20 m/s toward West.



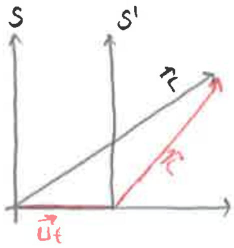
$a = a' = g$

According to Boy the ball makes free fall in vertical.
〃 〃 Girl 〃 〃 〃 projectile motion.

Let's define two reference systems

S = fixed (rest) ref. system of girl

S' = ref. system of boy that goes with constant ($\vec{U}$)



$\vec{r} = r' + \vec{U}t$ (I)

If we take the time derivative of eqn.

$$\frac{d\vec{r}}{dt} = \frac{dr'}{dt} + \frac{d(\vec{V}t)}{dt} \Rightarrow \vec{V} = \vec{V}' + \vec{U} \quad (\text{II})$$

Now take the derivative of eqn. 2

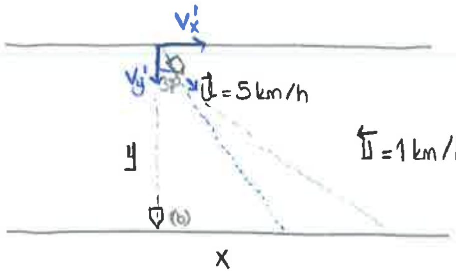
$$\frac{d\vec{V}}{dt} = \frac{d\vec{V}'}{dt} + \frac{d\vec{U}}{dt} \Rightarrow \vec{a} = a' \quad (\text{III})$$

Ex: A sailboat sails for 2h at 5 km/h by heading 37° East of South. The boat also exposures to a current has 1 km/h speed in the West direction.

a-) Find the distance from starting point when boat reacher to other side of river?

b-) What must be the heading angle for boat to reach directly to other side of river?

Sol: a-)



$\vec{u} = 1$ km/h toward West

$x' = v_x' \cdot t = 2$ km/h $\times 2$h $= 4$ km

$y' = v_y \cdot t = 4$ km/h $\times$ 2h $= 8$ km

$(x'^2 + y'^2)^{1/2} = r' = \underline{8.94 \text{ km}}$

$\vec{v} = \vec{v}' + \vec{u}$

$\vec{v}' = \vec{v} - \vec{u} \Rightarrow \vec{v}' = (v_x - u_x)\hat{\imath} + \underbrace{(v_y - u_y)}_{0}\hat{\jmath}$

$\vec{v}' = \vec{v}_x' + \vec{v}_y$

$\vec{v} = \vec{v}_x + \vec{v}_y$

$\vec{u} = \vec{u}_x$

$v_x = v \cdot \sin 37 = 5$ km/h $\times 0.6 = 3$ km/h

$v_y = v \cdot \cos 37 = 5$ km/h $\times 0{,}8 = 4$ km/h

$u_x = 1$ km/h

$\rightarrow (3-1)$ km/h $\hat{\imath} + 4$ km/h $\hat{\jmath} \Rightarrow v' = \underbrace{2 \text{ km/h}}_{v_x'}\hat{\imath} + \underbrace{4 \text{ km/h}}_{v_y'}\hat{\jmath}$

b-) For boat to reach other side directly the $v_x' = 0$ (Must be)

$v' = \underbrace{\vec{v}_x'}_{0} + \vec{v}_y$ $\qquad v_x' = v_x - u_x$

$0 = v_x - 1$ km/h $\Rightarrow v_x = 1$ km/h $\Rightarrow v \sin\theta = 1$ km/h $\Rightarrow 5$ km/h $\cdot \sin\theta = 1$ km/h

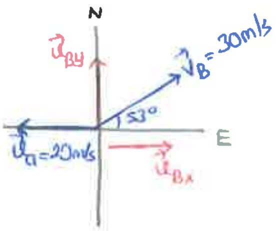
$\theta = \arcsin\left(\frac{1}{5}\right) \cong \underline{11.54°}$

Ex: Two cars A and B begin motion at the same point at the same time. Car A goes in westward direction at 20 m/s while Car B goes in a 53° North of East direction at 30 m/s.

a-) What is the velocity of A with respect to B, $\vec{V}_{BA} = ?$

b-) How long does it take for them to have 50m distance apart?

Sol: a-)



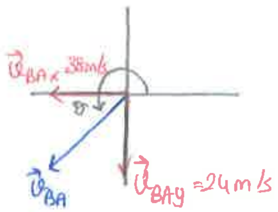
$v_{Bx} = v_B \cdot \cos 53° = 30$ m/s $\times 0{,}6 = 18$ m/s (toward E)

$v_{By} = v_B \cdot \sin 53° = 30$ m/s $\times 0{,}8 = 24$ m/s (toward N)

$v_{BA} = (V_{BAx}^2 + V_{BAy}^2)^{1/2} = \underline{45 \text{ m/s}}$

$\vec{v}_{BA} = ?$ $\qquad \tan\beta = \frac{v_{BAy}}{v_{BAx}} \Rightarrow \beta = 32.3°$ South of West

$\theta = 32.3° + 180° = \underline{212.3°}$



b-) $X = v_{BA} \cdot t$

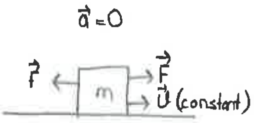
$50\text{m} = 45 \text{m/s} \times t$

$t = \underline{1.11 \text{s}}$

CHAPTER - 4

NEWTON'S LAWS of MOTION

4.1. Newton's I. Law

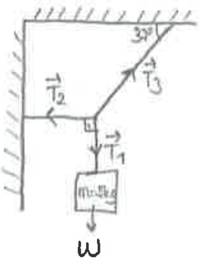


"If the net force acting on an object is zero then acceleration of object is zero."

$\vec{F}_{net} = 0$ Keeps its stable state. (inertia)

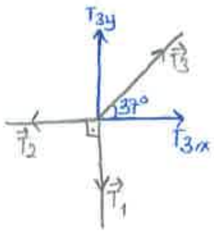
$\vec{a} = 0$ Eylemsizlik

Ex:



The object is an equililarium (at rest, in stable state). Find the tensions T_1, T_2, and T_3. (take $g \cong 10 \frac{N}{kg}$)

Sol: $\vec{F}_{net} = 0$ $\vec{F}_{net,x} + \vec{F}_{net,y} = 0$ so $F_{net,x} = 0$, $F_{net,y} = 0$



$T_1 = W$

$T_1 = m.g$

$= 5.10$

$= 50N$

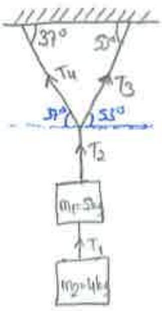
$T_2 = T_{3x} = T_3 . \cos 37°$ (Horizontal)

$T_1 = T_{3y} = T_3 . \sin 37°$ (Vertical)

$50N = T_3 \times 0,6 \Rightarrow T_3 \cong 83,33N$

$T_2 = 83,33N \times 0,8 \cong 66,66N$

Ex:



System is in equilibrium. Find the tensions T_1, T_2, T_3, and T_4.

Sol: $T_1 = 4kg . 10\frac{N}{kg} = 40N$

$T_2 = (4+5)kg . 10\frac{N}{kg} = 90N$

$T_3 . \cos 53° = T_4 . \cos 37° \longrightarrow T_3 . \frac{3}{\cancel{5}} = T_4 . \frac{4}{\cancel{5}} \Rightarrow 3T_3 = 4T_4$

$T_3 . \sin 53° + T_4 . \sin 37° = 90N$

$T_3 . \frac{4}{5} + T_4 . \frac{3}{5} = 90N$

$3/ \quad 4T_3 + 3T_4 = 450N$

$4/ \quad 3T_3 - 4T_4 = 0N$

$25T_4 = 1350 \quad T_4 = 54N \quad T_3 = 72N$

4.2. Newton's II. Law

$\vec{F}_{net} = m \cdot \vec{a}$ (N) "If there is a $\vec{F}_{net}$ acting on an object then that object accelerates"

Ex:

a-) Find the acceleration of objects (a_1 and a_2 or a_{system})

b-) Find the tension in the rope

Sol: a-) in Horizontal acting forces on m_1

$\boxed{m_1} \rightarrow \vec{T} = \vec{F}_{net,m_1}$ $\quad T = m_1 \cdot a_1 \quad (I)$

in Horizontal acting forces on m_2

$\vec{T} \leftarrow \boxed{m_2} \rightarrow \vec{F}$ $\quad F_{net,m_2} = F - T = m_2 \cdot a_2 \quad (II)$

Lets add eqns I and II side by side

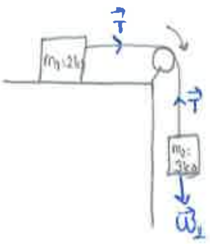
$$T = m_1 \cdot a_1$$
$$F - T = m_2 \cdot a_2$$
$$+$$

$a_1 = a_2 = a_{sys}$

$\cancel{T} + F - \cancel{T} = m_1 \cdot a_1 + m_2 \cdot a_2 \Rightarrow F = (m_1 + m_2) \times a_{sys} \Rightarrow 240\,N = 12\,kg \times a_{sys} \Rightarrow a_{sys} = 20\,N/kg = a_1 = a_2$

b-) $T = m_1 \cdot a_1 = 8\,kg \cdot 20\,N/kg = 160\,N$

Ex:



$\vec{g} \downarrow$

a-) If the system is left free, $a_{sys} = ?$

b-) $T = ?$

c-) Taken way by masses in 3 seconds ?

Sol: a-) $\vec{F}_{net} = m \cdot \vec{a}$

$$W_2 - T = m_2 \cdot a \quad (I)$$
$$T = m_1 \cdot a \quad (II)$$
$$+$$

We add eqns I and II side by side

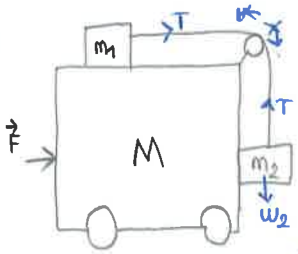
$$W_2 = (m_1 + m_2) \cdot a$$

$30\,N = 5\,kg \cdot a \Rightarrow a = 6\,\frac{N}{kg} = 6\,m/s^2 = a_{sys}$

b-) $T = m_1 \cdot a = 2\,kg \cdot 6\,\frac{N}{kg} = 12\,N$

c-) $h = \frac{1}{2} \cdot a \cdot t^2 = \frac{1}{2} \cdot 6\,\frac{m}{s^2} \cdot (3s)^2 = 27\,m$

Ex :



What horizontal force must be applied to the cart in order that the blocks remain stationary relative to cart? Assume surfaces are frictionless.

For m_2 to stay at rest in vertical

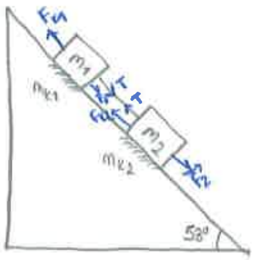
Sol: $W_2 = T$

$$\left.\begin{aligned} m_2 \cdot g &= T \\ m_1 \cdot a &= T \end{aligned}\right\} m_1 \cdot a = m_2 \cdot g$$

$a = \frac{m_2}{m_1} \cdot g = a_{sys}$ toward right

$F = (m_1+m_2+M) \cdot a_{sys} = (m_1+m_2+M) \cdot \frac{m_2}{m_1} \cdot g$ (N)

Ex :



$m_1 = 2$ kg , $m_2 = 4$ kg

$\mu_{k1} = 0.3$, $\mu_{k2} = 0.1$

a-) Find the acceleration of system ?

b-) Tension in the rope ?

Sol: on m_1 : $T + F_1 - f_{k1} = m_1 \cdot a$

on m_2 : $F_2 - T - f_{k2} = m_2 \cdot a$

$$F_1 + F_2 - f_{k1} - f_{k2} = (m_1+m_2) \cdot a$$

$16N + 32N - 3.6N - 2.4N = 6 \cdot a$

$F_1 = m_1 \cdot g \cdot \sin 53° = 2 \times 10 \times 0.8 = 16N$

$F_2 = m_2 \cdot g \cdot \sin 53° = 4 \times 10 \times 0.8 = 32\ N$

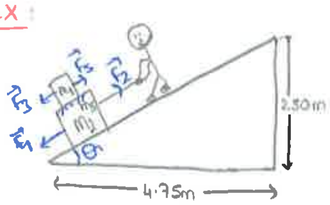
$f_{k1} = \mu_{k1} \cdot N_1 = \mu_{k1} \cdot m_1 \cdot g \cdot \cos 53 = 3.6\ N$

$f_{k2} = \mu_{k2} \cdot N_2 = \mu_{k2} \cdot m_2 \cdot g \cdot \cos 53 = 2.4\ N$

a-) $a_{sys} = 7\ N/kg = 7\ m/s^2$

b-) $T + 16N - 3.6N = 2\ kg \cdot 7\ N/kg \Rightarrow T = 1.6\ N$

Ex :



You are lowering two boxes, one on top of the other, down the ramp shown in figure by pulling on a rope parallel to the surface of ramp. Both boxes move together at a constant speed of 15.0 cm/s.

m_1 : 32 kg , m_2 : 48 kg

μ_s : 0.8 , μ_k : 0.44

a-) What force do you need to exert to accomplish this ? ($\vec{F_2} = ?$)

b-) What are the magnitude and direction of the function force on the upper box ?

Sol: a-) $\tan\theta = \frac{2.50\,m}{4.75\,m} \Rightarrow \theta = \arctan \frac{2.5}{4.75}$ $\theta = 27.8°$

b-) $F_3 = f_s = \mu_s \cdot N_1 = \mu_s \cdot m_1 \cdot g \cdot \cos 27.8° = \underline{225.28\ N}$ upward

Assume two boxes as single object that has resultant $m = m_1 + m_2 = 80$ kg

$F_1 - f_k = F_2$ then two boxes can move down with constant speed.

$F_{net} = m_{TOT} \cdot a_{sys} = (m_1+m_2) \cdot \underbrace{a_{sys}}_{0}$ $F_1 - f_k - F_2 = 0$

$F_1 = m \cdot g \cdot \sin\theta = 80\ kg \cdot 10 \frac{N}{kg} \cdot \sin 27.8° \Rightarrow F_1 = 373.1\ N$

$f_k = \mu_k \cdot N = \mu_k \cdot m \cdot g \cdot \cos 27.8° \Rightarrow f_k = 314\ N$

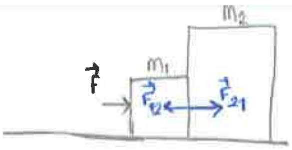
$F_2 = 373.1 - 314$

$F_2 = 59.1\ N$

4.3. Newton's III. Law

If an object (m_1) acts a force an another object (m_2), the same amount of force is reacted on m_1 by m_2 in reverse direction. This law is also known as action-reaction law.

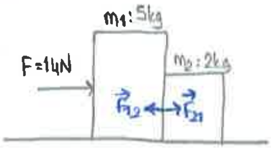
$\vec{F}_{action} = -\vec{F}_{reaction}$, $F_{action} = F_{reaction}$



$\vec{F}_{21}$: acting force on m_2 by m_1

$\vec{F}_{12}$: reacting " " m_1 " m_2

Ex :



a-) Find the force between blocks

b-) Compare this force when $\vec{F}$ acts from right at 2kg object

Sol : a-) $F_{21} = m_2 . a$

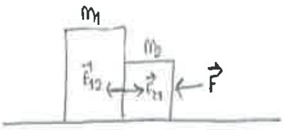
$F_{21} = 2\,\cancel{kg} . 2\,N/\cancel{kg}$

$F_{21} = 4N = F_{12}$

Lets find the $a_{sys} = a_1 = a_2$

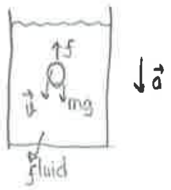
$F = (m_1 + m_2) . a_{sys} \Rightarrow 14N = 7\,kg \times a_{sys}$ $a_{sys} = 2\,N/kg = a_1 = a_2$

b-)



$F_{12} = m_1 . a_1 = 5\,\cancel{kg} \times 2\,N/\cancel{kg} = 10N = F_{21}$

4.4. Fluid Resistance and Terminal Speed



$\vec{f}$ = fluid resistance

$f = k.\upsilon \rightarrow$ Its magnitude at low speeds constant

$m.g = k.\upsilon_{terminal}$
$= k.\upsilon_{limit}$

Object accelerates until $m.\vec{g} = -\vec{f}$ then $\vec{a} = 0$

where $\upsilon_{terminal} = \upsilon_{limit}$ $\upsilon_{terminal} = \frac{m.g}{k} \left(\frac{m}{s}\right)$

$\upsilon(t) = \upsilon_{terminal} . \left[1 - e^{\frac{-k.t}{m}}\right] (m/s)$ where $e = 2.718 \rightarrow$ Euler's number

Ex : A spherical object is dropped into a beaker filled by oil freely. $m_{object} = 5g$ and $k = 600\,\frac{g}{s}$

a-) Find the $\upsilon_{terminal} = ?$ b-) Find the needed time for object to reach the 80% of υ_t ?

Sol : a-) $\upsilon_{term} = \frac{m.g}{k} = \frac{5\cancel{g} . 10\,m/s^2}{600\,\cancel{g}/s} = 0.083\,m/s = 8.3\,cm/s$

b-) $0.8 \times \upsilon_{term} = \upsilon_{term} \times \left[1 - e^{\frac{-k.t}{m}}\right] \Rightarrow e^{\frac{-600.t}{5}} = 1 - 0.8 \Rightarrow \ln e^{-120.t} = \ln 0.2$

$\Rightarrow \cancel{-}120.t = \cancel{-}1.6$

$t = 1.33 \times 10^{-3}\,s$

Terminal Speed In the Air

In the air $\vec{f}$ is proportional with v^2

$$f_{air} = \frac{1}{2} \times C \times \rho_{air} \times A \times v^2 \ (N)$$

where
C = coefficient
ρ_{air} = density of air = 1.2 kg/m³
A = effective cross-sectional area perpendicul $\vec{v}$.

when $f_{air} = m.g$

$$v_{term} = \left(\frac{2.m.g}{C.\rho_{air}.A}\right)^{1/2} \ (m/s)$$

Ex: A raindrop has 2mm radius foll from a clouod at 2000m height above the ground. Think that droplet is spherical. ($C = 0.6$, $\rho_{water} = 1000$ kg/m³, $\rho_{air} = 1.2$ kg/m³)

a-) $v_{terminal} = ?$ b-) What would have been the speed just before strikes to the ground, if there were no $\vec{f}_t$-?

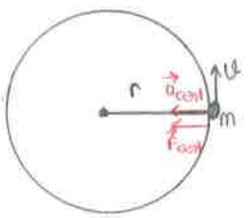
Sol: a-) $m_{droplet} = \rho_{water} . V = \rho_{water} \times \frac{4 \times \pi \times r^3}{3} = 1000 \frac{kg}{m^3} \times \frac{4 \times 3.14 \times (2\times10^{-3} m)}{3} = 3.35 \times 10^{-5}$ kg

$A = \pi.r^2 = 3.14 \times (2\times10^{-3} m)^2 = 1.25 \times 10^{-5} m^2$

$$v_{term} = \left(\frac{2 \times 3.35 \times 10^{-5} kg \times 10 \ m/s^2}{0.6 \times 1.2 \frac{kg}{m^3} \times 1.25 \times 10^{-5} m^2}\right) \cong 8.6 \ m/s$$

b-) $v_f^2 = v_i^2 + 2.g.h = 0 + 2 \times 10 \frac{m}{s^2} \times 2000 \ m \Rightarrow v_f = 200 \ m/s = v_{strike}$

4.5. Dynamics of Circular Motion



$$F_{centripital} = F_{radial} = m.a_{cent} = \frac{m.v^2}{r} \ (N)$$

Application of Fcent

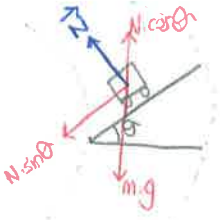
i-) Rounding a Curve in a Car

$f_s = F_{cent}$

$$\mu_s.N = \mu_s.\cancel{m}.g = \frac{\cancel{m}.v_{max}^2}{r} \Rightarrow v_{max} = \sqrt{\mu_s.g.r} \ \left(\frac{m}{s}\right)$$

→ If car goes with a velocity greater than v_{max}, then: it slides out of road.

ii-) Banked Road without Friction



$$N.\cos\theta = m.g \quad (I) \qquad \Rightarrow N = \frac{m.g}{\cos\theta}$$

$$N.\sin\theta = F_{cent} = \frac{m.V_{max}^2}{r} \quad (II) \qquad \longrightarrow \frac{\cancel{m}.g}{\cos\theta}.\sin\theta = \frac{\cancel{m}.V_{max}^2}{r}$$

$$V_{max} = \sqrt{g.r.\tan\theta} \quad (m/s)$$

Ex: A circular highway is designed for traffic moving with maximum 60 km/h.

a-) If r = 150 m, what is the correct angle to bank the road?

b-) If the road was not banked, what would be minumum M_s to keep traffic from skidding?

Sol: a-) $V_{max} = \sqrt{g.r.\tan\theta} \Rightarrow (\tan\theta)^2 = \dfrac{60000\,m}{3600\,s \times 10\,m/s^2 \times 150\,m} \Rightarrow \theta = 10.5°$

b-) $V_{max} = \sqrt{M_s.g.r} \Rightarrow \dfrac{60000\,m}{3600\,s} = \sqrt{M_s 10 \frac{m}{s} \times 150\,m} \Rightarrow M_s = 0.185$

Ex: An airplane is flying in a horizontal circle at a speed of 480 km/h. What is the radius of circle in which the plane is flying. Assume that required force is provided by aerodynamic lift, that is perpendicular to the wing surface. ($\tan 40° = 0.84$)

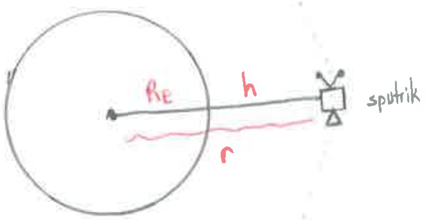
Sol: Plane can be thought like a car on a banked road.

$$V_{max} = (g.r.\tan\theta)^{1/2}$$

$$\frac{480000\,m}{3600\,s} = \left(10\,\frac{m}{s^2} \times r \times 0.84\right)^{1/2} \Rightarrow r = 211534\,m$$

Note: If there is friction on the banked. $N.\sin\theta + f_s.\cos\theta = \dfrac{m.V^2}{r}$

iii-) Satellite Motion



$$F_{cent} = F_{gravitational} \Rightarrow \frac{\cancel{M_S}.V^2}{\cancel{r}} = \frac{G.M_E.\cancel{M_S}}{r^{\cancel{2}}}$$

$R_E : 6.37 \times 10^6\,m$

$M_E : 5.98 \times 10^{24}\,kg$

$G = 6.672 \times 10^{-11} \left(\dfrac{N.m^2}{kg^2}\right)$

$$V = \left(\frac{G.M_E}{r}\right)^{1/2}$$

where

$r = R_E + h$

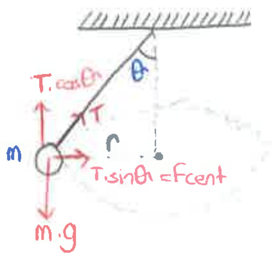
Ex : Determine the period of satellite which moves $\underset{h}{400\ km}$ apart above the Earth surface.

Sol : $r = R_E + h = 6.37\times10^6 m + 0.4\times10^6 m = 6.77\times10^6 m$

$$v = \left(\frac{G.m_E}{r}\right)^{1/2} = \left(\frac{6.672\times10^{-11}\ \frac{N.m^2}{kg^2}\times5.98\times10^{24}\ kg}{6.77\times10^6 m}\right)^{1/2} = 7677\ m/s$$

$$T = \frac{2.\pi.r}{v} = \frac{2\times3.14\times6.77\times10^6 m}{7677\ m/s} = 5538\ s$$

iv-) Conical Pendulum



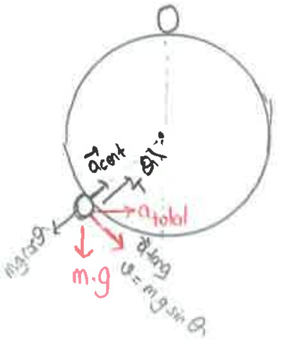
$$T.\cos\theta = m.g \quad (I) \quad \Rightarrow T = \frac{m.g}{\cos\theta}$$

$$T.\sin\theta = F_{cent} = m.\frac{V^2}{r} \quad (II)$$

$$\frac{\cancel{m}.g}{\cos\theta}.\sin\theta = \frac{\cancel{m}.V^2}{r} \Rightarrow v = (g.r.\tan\theta)^{1/2}\ m/s$$

The period of conical pendulum $\Rightarrow T = \frac{2.\pi.r}{v} = 2\pi.\sqrt{\frac{r}{g.\tan\theta}}$ (s)

4.6. Nonuniform Circular Motion



$\vec{g}\downarrow$

$$\cancel{m}.g.\sin\theta = \cancel{m}.a_t \quad (I) \quad \Rightarrow a_t = g.\sin\theta \quad \text{(Tangential acceleration)}$$

$$T - m.g.\cos\theta = F_{cent} = m.a_{cent} = \frac{m.V^2}{r} \Rightarrow T = m.g.\cos\theta + \frac{m.V^2}{r} \quad (N)$$

Two-special position

i-) When object is at top ; $\theta = 180° \rightarrow \cos 180° = -1$ $\quad T_{top} = T_{min} = \frac{m.V^2}{r} - m.g$ (N)

ii-) When object is at bottom ; $\theta = 0° \rightarrow \cos 0° = 1$ $\quad T_{bottom} = T_{max} = \frac{m.V^2}{r} + m.g$ (N)

Ex : A 50 kg child sits in a conventional swing of length 3 m, supported by two chains. T on each chain is 400 N,

a-) Find the child's speed at lowest point ?

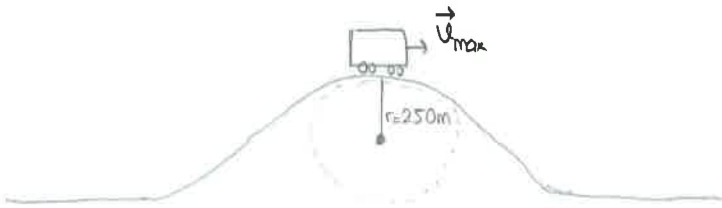
b-) Find normal force exerted by seat on the child ?

Sol : a-) $T_{bottom} = \frac{m \cdot V_{bottom}^2}{r} + m \cdot g \Rightarrow 2 \times 400\ N = 800\ N = \frac{50\ kg \times v_{bottom}^2}{3m} + 50 \times 10\ N \Rightarrow v_{bottom} = \underline{4.24\ m/s}$

b-) $F_{action} = T = F_{reaction} = N = \underline{800\ N}$

Ex : A man drives a car over a circular hill of radius 250 m. What is the greatest speed at which he can drive without the car leaving the road at the top of the hill ?

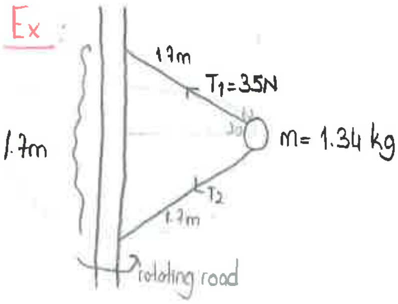
Sol :



$0 = N_{min} = \frac{m \cdot v_{top}^2}{r} - m \cdot g$ (N)

$\cancel{m} \cdot g = \frac{\cancel{m} \cdot V_{critical}^2}{r}$ $\quad v_{critical} = \sqrt{r \cdot g} = \underline{50\ m/s}$

Ex :



Uniform circular motion in Horizontal.

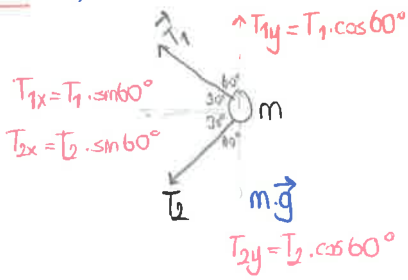
a-) Draw the free-body diagram for m. (That is, draw the acting forces on m)

b-) $T_2 = ?$

c-) What is F_{net} on m= ?

d-) $v = ?$

Sol : a-)



b-) Object is in equilibrium in vertical , $\sum y = 0$

$T_1 \cdot \cos 60° - T_2 \cdot \cos 60° - m \cdot g = 0$

$35\ N \times 0.5 - T_2 \times 0.5 - 1.34 \times 10 = 0 \Rightarrow \underline{T_2 = 8.2\ N}$

c-) $F_{net} = \sum F_x = T_{1x} + T_{2x} = T_1 \times \sin 60° + T_2 \times \sin 60° = 35N \times 0.86 + 8.2N \times 0.86 \Rightarrow \underline{F_{net} = 37.4N}$

d-) $F_{cent} = \frac{m \cdot V^2}{r} = F_{net} \Rightarrow 37.4\ N = \frac{1.34\ kg \times v^2}{1.47m} \Rightarrow \underline{v = 6.4\ m/s}$