Coulomb's Law

$$F_{12} = k \cdot \frac{|q_1| \cdot |q_2|}{r^2} \quad (N)$$

$$k = 9 \times 10^9 \ \frac{N.m^2}{C^2}$$

$$k = \frac{1}{4.\pi.\varepsilon_0}$$

$$F_{net} = \left[F_{12}^2 + F_{21}^2 + 2.F_1.F_2 \cos\theta\right]^{1/2} \ (N)$$

Electric Field ($\vec{E}$)

$$E = k \cdot \frac{|q|}{r^2} \quad \left(\frac{N}{C}\right)$$

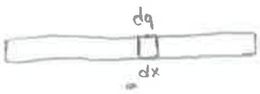
$$F_q = q_0 . E$$

Uniform Charge Distribution ($\vec{E}$)

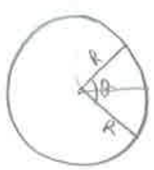
1-) Linear Charge Distribution

$$\lambda_{total} = \frac{q_{total}}{L_{total}} \quad \text{(Linear charge Density)}$$

$$dq = \lambda . dx \ , \quad dE_p = \frac{dq}{r^2}$$



Ring



$S = R.\theta$

$ds = R.d\theta$

(Çevreye göre integral)

2-) Surface Charge Distribution

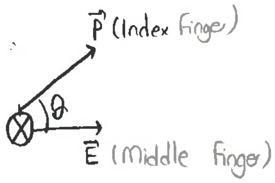
$$\sigma = \frac{q_{total}}{A_{total}} \Rightarrow dq = \sigma . dA$$

(Yarıçapa göre integral)

Electric Dipole

$$p = q.d \quad (C.m)$$

$$\vec{\tau} = \vec{p} \times \vec{E} = p.E.\sin\theta \ (N.m)$$



$$U_p = -\vec{p}.\vec{E} = -p.E.\cos\theta \ (J)$$

Electric Flux (Φ_E)

$$\Phi_E = \vec{E}.\vec{A} = E.A.\cos\theta \left(\frac{N}{C} m^2\right)$$

$$\Phi_{total} = \oint E.d\vec{A}$$

Gauss's Law

$$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

Spherical Surface Area $= 4.\pi.r^2$

Application of Gauss's Law

1-) Spherical Symmetry

For $r<a$ $\quad \Phi_E = \frac{q.r^3}{\varepsilon_0.a^3} \left(\frac{N}{C} m^2\right)$

$$E = k.\frac{q.r}{a^3} \quad \text{(Inside)}$$

For $a<r$ $\quad \Phi_E = \frac{q}{\varepsilon_0}$

(Outside) $\quad E = k.\frac{q}{r^2} \left(\frac{N}{C}\right)$

2-) Cylindrical Symmetry

$$\lambda = \frac{q_{total}}{L_{total}} \Rightarrow dq = \lambda.ds$$

$$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{\lambda.L_{total}}{\varepsilon_0}$$

$A = 2.\pi.r.L$

3-) Planar Symmetry

a-) Nonconducting (Insulator) Sheet

$$E = \frac{\sigma}{2.\varepsilon_0} \left(\frac{N}{C}\right)$$

b-) Conducting Sheet

$$E = \frac{\sigma}{\varepsilon_0} \left(\frac{N}{C}\right)$$

ELECTRIC POTENTIAL

$$\Delta V = -\int_{r_i}^{r_f} \vec{E}.d\vec{r} \ (V) = E.dr.\cos\theta \ (V)$$

Nonconducting Solid Sphere;

$$E_{outside} = k.\frac{q}{r^2} \qquad E_{inside} = k.\frac{q.r}{a^3}$$

In inside; $\quad V = -\int_{\infty}^{r} \vec{E}\, d\vec{r} = -\left[\int_{\infty}^{a} E_{out}.dr + \int_{a}^{r} E_{in}.dr\right]$

Point Charges

$$V_A = k.\frac{q}{r} \ (V) \qquad V_B = -k.\frac{q}{r} \ (V)$$

$$V_A = \frac{U}{q_0} \Rightarrow U = k.\frac{q.q_0}{r} \ (J)$$

Continuous Charge Distribution (V)

1-) Line of Charge

$$\lambda = \frac{q_{total}}{L_{total}} \qquad dE = k.\frac{dq}{r^2}$$

$$dV = k.\frac{dq}{r}$$

2-) Surface Charge Dist.

$$\sigma = \frac{q_{total}}{A_{total}} \qquad dV = k.\frac{dq}{r}$$

$$E = -\frac{dV}{dr} \left(\frac{N}{C}\right)$$

CHAPTER-21 17.02.2015

Tuesday (I)

ELECTRIC CHARGE AND ELECTRIC FIELD

(Elektrik Yükü ve Elektrik Alanı)

21.1 Electric Charge

→ Electrons are (-) charge carriers

→ Protons are (+) charge carriers

→ If the charge carriers mainly electrons stays at rest or move very slowly in a material then the term "electrostatic" is used.

→ Electricaly neutral object means number of negative charge carriers is equal to number of positive charge carriers.

→ For an isolated system the magnitude of charge is constant, this case is known as "conservation of charge".

Ex: $^{238}_{92}U \longrightarrow ^{234}_{90}Th + ^{4}_{2}He$

Ex: $e^- + e^+ \rightarrow \gamma + \gamma$ → annihilation

electron, positron, gamma ray

Ex: $\gamma \rightarrow e^- + e^+$ → pair production

Ex: $^{12}_{6}C + ^{1}_{1}H \rightarrow ^{13}_{7}N$ (Nitrogen)

→ Charge is a quantised quantity.

$e^- = -1.6 \times 10^{-19}$ C

$p = +1.6 \times 10^{-19}$ C

→ Materials are classified into three categories according to their conducting property.

1-) Conductors: Electrons can move freely in conductors. Ex: Metals, Tap water, Human body, ______. (iletken)

2-) Insulators: Electrons can not move freely. Ex: dry wood, plastic, glass, pure water, ______. (Yalıtkan)

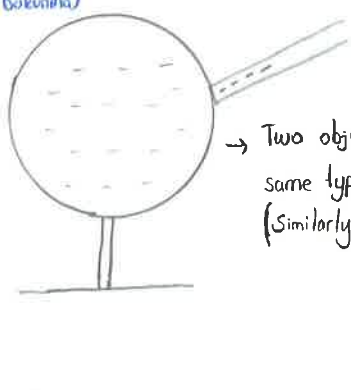
3-) Semiconductors: Electrons and holes can move under suitable conditions. Ex: Si, Ge, Ga, As, InP ... (Yarı iletken)

21.2 Types of Electrification

1-) by rubbing: When a plastic rod is rubbed with fur then some electrons pass from fur to rod. Hence rod gets negatively charged and fur gets positively charged.
(Sürtünme)

→ Objects are oppositely charged.

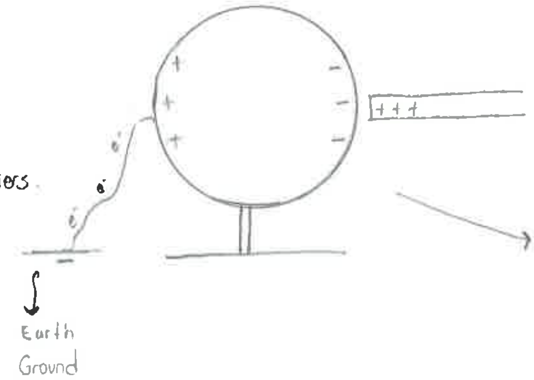
2-) by touching:
(Dokunma)

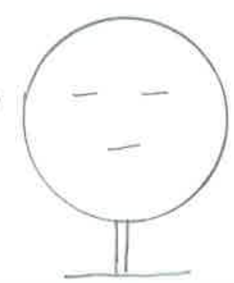


→ Two objects have same type of charge carriers. (Similarly positive)

3-) by induction

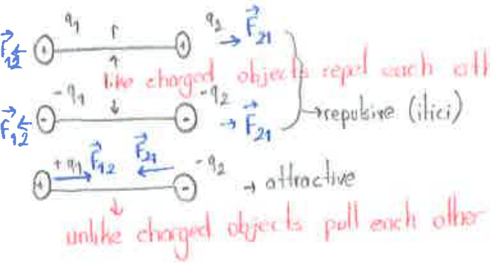
When e⁻ transfer is completed ground and rod are removed from the sphere. Hence sphere gets negatively charged.





24.02.2015
Tuesday

21.3 Coulomb's Law



$\vec{F}_{12}$: Force on q_1 produced by q_2

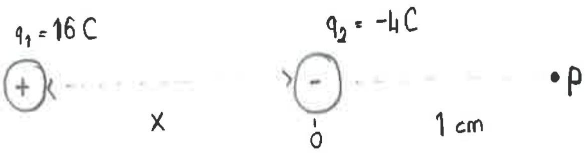
$\vec{F}_{21}$: " " q_2 " " q_1

$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$, $|\vec{F}_{12}| = |\vec{F}_{21}|$

$$F_{12} = F_{21} = k.\frac{|q_1|\,|q_2|}{r^2} \quad (N) \qquad , \; k = 9\times10^9 \; \frac{N.m^2}{C^2}$$

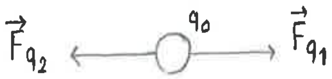
Coulomb's Law

Ex :



What must be the x to have zero resultant force on proton.
(bileşke kuvvet)

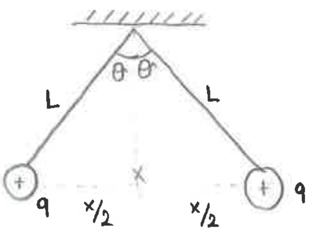
Sol :



$$F_{q_2} = F_{q_1} \Rightarrow \frac{\cancel{k}.\cancel{|q_0|}\,|q_2|}{|1\,cm|^2} = \frac{\cancel{k}.\cancel{|q_0|}.|q_1|}{(1+x)^2} \Rightarrow \frac{4\,C}{1\,cm^2} = \frac{16\,C}{(1+x)^2}$$

$x = 1\,cm$

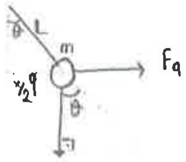
Ex :



Two identical objects have mass m and charge q. Assume that θ is so small for system in equilibrium that $\tan\theta$ can be replaced by its approximate equal $\sin\theta$. Show that $x = \left(\frac{q^2.L}{2.\pi.\varepsilon_0.m.g}\right)^{1/3}$ where x is seperation b/wn objects.

(Hint: $k = \frac{1}{4.\pi.\varepsilon_0} = 9\times10^9\,\frac{N.m^2}{C^2}$
ε_0: Electric permitivity of free space)

Proof : for small angles $\tan\theta \cong \sin\theta$



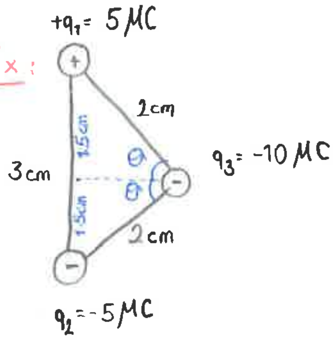
$\tan\theta = \frac{F_q}{mg}$, $\sin\theta = \frac{x}{2L}$

$$\frac{F_q}{mg} = \frac{x}{2L} \Rightarrow k.\frac{|q|.|q|}{x^2.mg} \Rightarrow \frac{x}{2L} = \frac{1}{4.\pi.\varepsilon_0}.\frac{q^2}{x^2.mg}$$

$$x^3 = \frac{2L.q^2}{4.\pi.\varepsilon_0.mg} \Rightarrow$$

$$x = \left(\frac{q^2.L}{2.\pi.\varepsilon_0.mg}\right)^{1/3}$$

Ex :



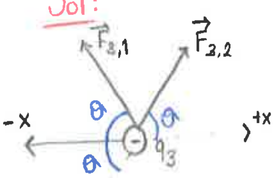
Three charges are at the corners of an isosceles triangle. Find the magnitude and direction of resultant force on q_3.

$\tan\theta = \frac{1.5\,cm}{2\,cm} = 0.75$

$\theta = 36.87$

$\mu = 10^{-6}$, micro

Sol :



$\vec{F}_{3,1}$ is the exerted force on q_3 by q_1

$\vec{F}_{3,2}$ " " " " " " q_2

$$F_{3,1} = k.\frac{|q_3|.|q_1|}{(2\times10^{-2}m)^2} = 9\times10^9\,\frac{N.m^2}{C^2}.\frac{10\times10^{-6}C\times5\times10^{-6}C}{(2\times10^{-2}m)^2}$$

$$F_{3,1} = \frac{450\times10^9\times10^{-12}}{4\times10^{-4}}\,N = \frac{4500\,N}{4} = 1125\,N$$

$$F_{3,2} = k.\frac{|q_3|.|q_1|}{(2\times10^{-2}m)^2} = 9\times10^9\,\frac{N.m^2}{C^2}.\frac{10\times10^{-6}C\times5\times10^{-6}C}{(2\times10^{-2}m)^2}$$

$= 1125\,N$

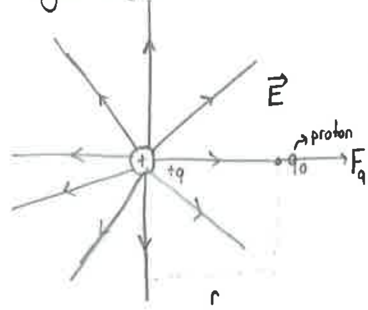
$\downarrow$

$\alpha = 180° - 2\theta = 180 - 2 \times 36.87 = 106.26°$

$F_{res} = \left[F_{3,1}^{2} + F_{3,2}^{2} + 2 . F_{3,1} . F_{3,2} . \cos\alpha \right]^{1/2} = \underline{1822}$ N (upward +y)

21.4 Electric Field

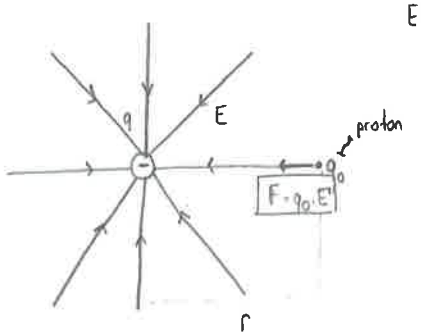
Charged objects exert Coulomb force on each other via electric field, $\vec{E}$.



Electric field lines extend away from positive charges.
(genişletmek)

$F_q = k . \frac{|q| . |q_0|}{r^2} = q_0 . E$

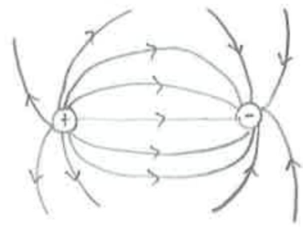
$$\boxed{E = k . \frac{|q|}{r^2} \left(\frac{N}{C}\right)}$$



Electric field lines are thrown $-q$.

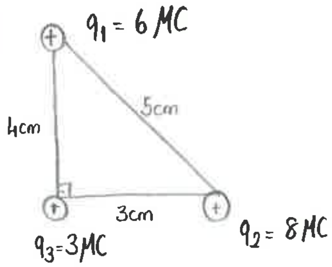
$$\boxed{E = k . \frac{|-q|}{r^2} \left(\frac{N}{C}\right)}$$

Electric Dipole



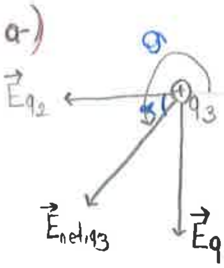
Note: $\vec{E}$ is a force field like gravitational and magnetic fields.

Ex:



Three charges are placed at the corners of a right triangle. a-) Find the magnitude and direction of $\vec{E}_{net}$ on q_3. b-) Find the magnitude and direction of $\vec{F}_{net}$ on q_3.

Sol: a-)



$E_{q_2} = k . \frac{|q_2|}{r^2} = 9 \times 10^9 \frac{N.m^2}{C^2} . \frac{8 \times 10^{-6} C}{(3 \times 10^{-2} m)^2} = 8 \times 10^7 \frac{N}{C}$

$E_{q_1} = k . \frac{|q_1|}{r^2} = 9 \times 10^9 \frac{N.m^2}{C^2} . \frac{6 \times 10^{-6} C}{(4 \times 10^{-2} m)^2} = \frac{54}{16} \times 10^7 \frac{N}{C}$

$E_{net} = \left[E_{q_1}^{2} + E_{q_2}^{2} + 2 . E_{q_1} . E_{q_2} . \underbrace{\cos 90°}_{0} \right]^{1/2} = \quad \frac{N}{C}$

$\theta = 180° + \alpha°$

$\tan\alpha = \frac{E_{q_1}}{E_{q_2}} = \frac{\frac{54}{16} \times 10^7 \frac{N}{C}}{8 \times 10^7 \frac{N}{C}}$

$\alpha =$

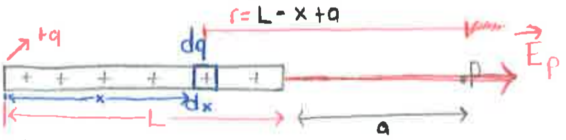
b-) $F_{net,q_3} = q_3 . E_{net,q_3}$

=

Direction is θ again.

21.5 Electric Field Produced By Objects that have Uniform Charge Distribution

i-) Objects that have linear charge distribution



insulating rod has total charge q and length L.

Lets find $\vec{E}_p$ produced by rod that has uniform charge distribution.

λ = linear charge density, $\lambda_{total} = \dfrac{q_{total}}{L_{total}}$

Solution: $\lambda = \dfrac{q}{L} = \dfrac{dq}{dx} \Rightarrow \boxed{dq = \lambda . dx}$

$$dE_p = k . \frac{dq}{r^2} = k . \frac{dq}{(L-x+a)^2} = k . \lambda . \frac{dx}{(L-x+a)^2}$$

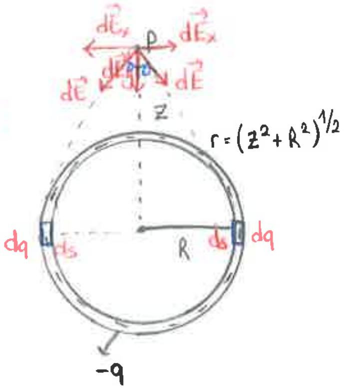
$$\int dE_p = k . \lambda \int_0^{L \text{ until P}} \frac{dx}{(L-x+a)^2} \Rightarrow E_p = k . \lambda . (L-x+a)^{-1} \Big|_0^L \Rightarrow$$

First Points

$$= k . \lambda . \frac{1}{(L-x+a)} \Big|_0^L \Rightarrow E_p = k . \lambda \left[\frac{1}{a} - \frac{1}{L-a} \right] =$$

$$\boxed{E_p = k . \lambda \left[\frac{L}{a . (L-a)} \right] \frac{N}{C}}$$

Ex:



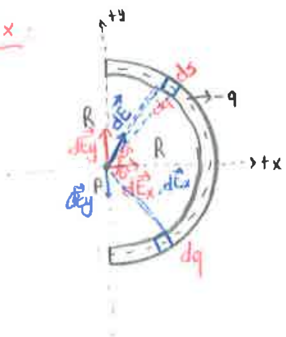
Ring has uniform charge distribution. Find $\vec{E}_p$

Solution: $dE_{net,p} = dE_y$ since dE_x components cancel each other

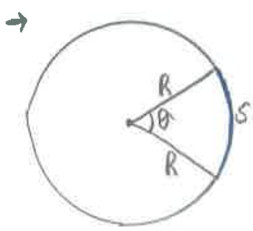
$$dE_{net,p} = dE_y = dE . \cos\theta = k . \frac{dq}{r^2} . \frac{z}{r} = k . \frac{dq}{(z^2+R^2)} . \frac{z}{(z^2+R^2)^{1/2}} = \frac{k . \lambda . z}{(z^2+R^2)^{3/2}} . ds$$

$$\int dE_{net,p} = \frac{k . \lambda . z}{(z^2+R^2)^{3/2}} \int_0^{2.\pi.r} ds \Rightarrow E_{net,p} = \frac{k . \lambda . z}{(z^2+R^2)^{3/2}} . S \Big|_0^{2\pi r} \Rightarrow \boxed{E_{net,p} = \frac{k . \lambda . z . 2 . \pi . R}{(z^2+R^2)^{3/2}} \ \frac{N}{C} \ \text{Downward}}$$

Ex:



Find the $\vec{E}_p$.



$S = R . \theta$

$ds = R . d\theta$

Solution: Since dE_y components cancel each other the resultant electric field because of dE_y components

$$dE_p = dE_x = dE . \cos\theta = k . \frac{dq}{r^2} . \cos\theta$$

$$= k . \frac{\lambda . R}{R^2} . \cos\theta . d\theta$$

$$= k . \frac{\lambda}{R} . \cos\theta . d\theta$$

$\lambda = \dfrac{dq}{ds} \Rightarrow dq = \lambda . d$

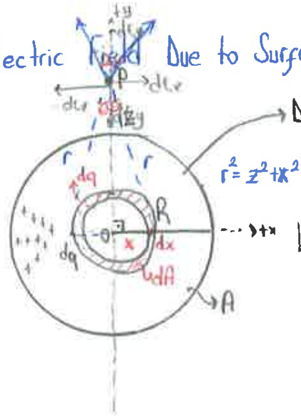
$= \lambda . R . d\theta$

03.03.2015
Tuesday

→ to find the resultant E_p, we should integrate both sides.

$$\int dE_p = \int_0^{\pi/2} 2.k.\frac{\lambda}{R}.\cos\theta\, d\theta = 2k\frac{\lambda}{R}\int_0^{\pi/2}\cos\theta\, d\theta = 2k.\frac{\lambda}{R}.\sin\theta\Big|_0^{\pi/2} \Rightarrow E_p = 2k.\frac{\lambda}{R}\left[\sin\frac{\pi}{2}-\sin 0\right] = 2k.\frac{\lambda}{R}\ \left(\frac{N}{C}\right)$$

ii-) Electric Field Due to Surface Charge Distribution



Disc has uniform surface charge distribution, $\sigma = \frac{q_{total}}{A_{total}} = \frac{q}{A} = \frac{dq}{dA} \Rightarrow dq = \sigma.dA = \sigma.2\pi.x.dx$

$r^2 = z^2 + x^2$

Lets find $\vec{E_p}$

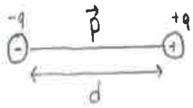
Solution: $dE_p = dE_y = dE\cos\theta = \frac{k.dq}{r^2}.\frac{z}{r} = k.z.\frac{dq}{(x^2+z^2)^{3/2}} = k.z.\sigma.2\pi.\frac{x.dx}{(x^2+z^2)^{3/2}}$

$$\boxed{\rightarrow \int\frac{x.dx}{(x^2+z^2)^{3/2}} = \frac{-1}{(x^2+z^2)^{1/2}}}$$

$$E_p = k.z.\sigma.2.\pi\int_0^R\frac{x.dx}{(x^2+z^2)^{3/2}} = k.z.\sigma.2.\pi\left(-\frac{1}{(x^2+z^2)^{1/2}}\Big|_0^R\right) =$$

$$= k.z.\sigma.2.\pi\left[\frac{1}{z}-\frac{1}{(R^2+z^2)^{1/2}}\right]\left(\frac{N}{C}\right) \text{ upward}$$

21.6 Electric Dipole

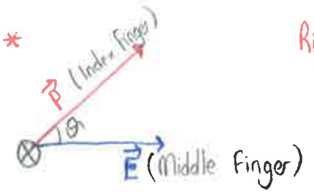


Electric dipole system

Electric dipole moment, $\vec{p}$ → it's direction is from -q to +q

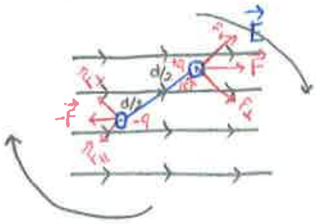
$\boxed{p = |\vec{p}| = qd}$ (c.m)

* Right-Hand Rule



→ If an electric dipole is placed in an uniform $\vec{E}$

* $\tau = r.F.\sin\theta$
$\vec{\tau} = \vec{r}\times\vec{F}$



$F_\perp$: perpendicular component of F

Acting torque on electric dipole

$$\tau = \frac{d}{2}.F_\perp + \frac{d}{2}.F_\perp = dF_\perp = d.F.\sin\theta$$

$$\tau = \underbrace{d.q}_{p}.E.\sin\theta = p.E.\sin\theta\ (N.m)$$

$\boxed{\tau = p.E.\sin\theta}$ → Scaler Eqn.

(Cross Product) $\boxed{\vec{\tau} = \vec{p}\times\vec{E}}$ → Vector eqn.

Index Middle Finger

→ When dipole is left free, it begins to rotate. This means there must be some magnitude of electric potential energy, U_p.

$$\boxed{U_p = -\underbrace{\vec{p}.\vec{E}}_{\text{Dot Product}} = -p.E.\cos\theta\ (J)}$$

05.03.2015
Thursday

Ex : For an electric dipole $q_1 = -4.5\ nC$, $q_2 = +4.5\ nC$ and $d = 3.1\ mm$
($n = 10^{-9}$)

a-) Find the magnitude of $\vec{P}$.
b-) The acting torque on dipole is 7.2×10^{-9} Nm when $\vec{P}$ and $\vec{E}$ makes $36.9°$. Find the magnitude of $\vec{E}$.

Solution : a-) $p = q.d = 4.5 \times 10^{-9}\ C \times 3.1 \times 10^{-3}\ m \cong 12 \times 10^{-12}\ C.m.$

The direction of p $(- \rightarrow +)$

b-) $\tau = p.E.\sin\theta \Rightarrow 7.2 \times 10^{-9}\ N.m. = 12 \times 10^{-12}\ C.m. \times E \times \underbrace{\sin 36.9°}_{0.6}$ $\quad E = 860\ \frac{N}{C}$

Ex : For an electric dipole $q_1 = +3.2 \times 10^{-19}\ C$, $q_2 = -3.2 \times 10^{-19}\ C$ and $d = 0.78\ nm$ Dipole exposures to an $E = 3.4 \times 10^6\ \frac{N}{C}$.

Calculate the magnitude of torque on dipole when a-) $\vec{p} \parallel \vec{E}$ b-) $\vec{p} \perp \vec{E}$ and c-) $\vec{p}$ antiparallel to $\vec{E}$.
d-) Calculate the U_p's for (a),(b),(c) cases.

Solution : a-) $\vec{p} \parallel \vec{E}$, $\tau = p.E.\sin\theta = q.d.E.\overbrace{\sin 0°}^{0} = \underline{\underline{0}}$

b-) $\vec{p} \perp \vec{E}$, $\tau = q.d.E.\sin 90° = 3.2 \times 10^{-19}\ \cancel{C} \times 0.78 \times 10^{-19}\ m \times 3.4 \times 10^6\ \frac{N}{\cancel{C}} = \underline{\underline{8.5 \times 10^{-22}\ Nm}}$

c-) $\vec{p}$ antiparallel $\vec{E}$, $\tau = q.d.E.\overbrace{\sin 180°}^{0} = \underline{\underline{0}}$

d-) $\vec{p} \parallel \vec{E}$, $U_p = -\vec{p}.\vec{E} = -p.E.\cos\theta = -q.d.E.\overbrace{\cos 0°}^{1} = \underline{\underline{-8.5 \times 10^{-22}}}\ J$ (Minumum)

$\vec{p} \perp \vec{E}$, $U_p = -q.d.E.\overbrace{\cos 90°}^{0} = \underline{\underline{0}}$

$\vec{p}$ antiparallel to $\vec{E}$, $U_p = -q.d.E.\underbrace{\cos 180°}_{-1} = q.d.E = \underline{\underline{+8.5 \times 10^{-22}}}\ J$ (Maximum)

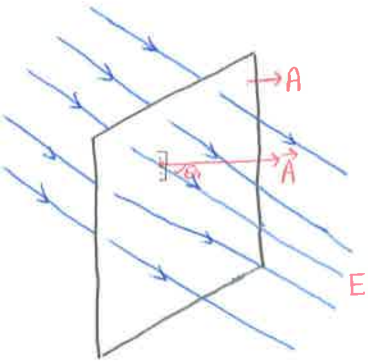
CHAPTER-22
GAUSS LAW

22.1 Electric Flux (Elektrik Akısı)

infinitesmall = very small
penetrating = içine işleyen, giren

Number of electric field lines penetrating through a surface is called as electric flux, Φ_E.

$$\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot\vec{A}\cdot\cos\theta^\circ \left(\frac{N}{C}\cdot m^2\right)$$



$\vec{A}$ is surface vector

$E.\cos\theta$ perpendicular of $\vec{E}$

→ If the surface has random shape then

flux passing through ΔA_i

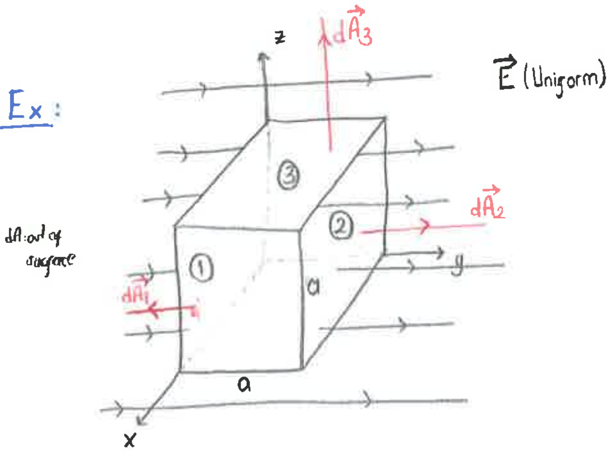
$$\Delta\Phi_i = \vec{E}\cdot\Delta\vec{A}_i$$

to find the total flux through the surface

$$\Phi_{total} = \lim_{\Delta A_i \to 0} \sum \vec{E}\cdot\Delta\vec{A}_i = \int \vec{E}\cdot d\vec{A}$$

If the surface is a closed surface, $\Phi_{total} = \oint \vec{E}\cdot d\vec{A}$

Ex:



Find the electric flux through the cube.

Solution: $\Phi_E = \oint \vec{E}\cdot d\vec{A} \left(\frac{N}{C}\cdot m^2\right)$

Cube has closed surfaces

$$\Phi_E = \int \vec{E}\cdot d\vec{A}_1 + \int \vec{E}\cdot d\vec{A}_2 + \int \vec{E}\cdot d\vec{A}_3 + \int \vec{E}\cdot d\vec{A}_4 + \int \vec{E}\cdot d\vec{A}_5 + \int \vec{E}\cdot d\vec{A}_6$$

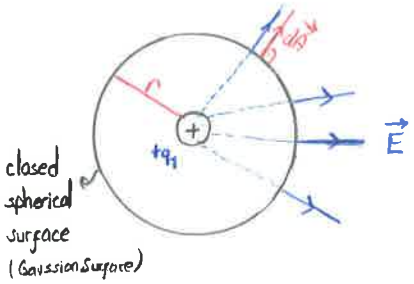
$$\Phi_E = \int E.dA_1.\underbrace{\cos 180^\circ}_{=-1} + \int E.dA_2.\underbrace{\cos 0^\circ}_{1} + \int E.dA_3.\underbrace{\cos 90^\circ}_{=0} + \int E.dA_4.\underbrace{\cos 90^\circ}_{=0} + \int E.dA_5.\underbrace{\cos 90^\circ}_{=0} + \int E.dA_6.\underbrace{\cos 90^\circ}_{=0}$$

$$= -E\underbrace{\int dA_1}_{=a^2} + E\underbrace{\int dA_2}_{=a^2} + 0 + 0 + 0 + 0 = -Ea^2 + Ea^2 = 0$$

Note: Since the number $\vec{E}$ lines entering to closed surface is equal to number $\vec{E}$ lines leaving this closed surface, the resultant electric flux ($\Phi_{E,net}$) is zero

10.03.2015
Tuesday

22.2 Gauss's Law



Volume = $\frac{4}{3}\cdot\pi\cdot r^3$

Area = $4\cdot\pi\cdot r^2$

Electric flux through the sphere.

$$\Phi_E = \oint \vec{E}\cdot d\vec{A}$$

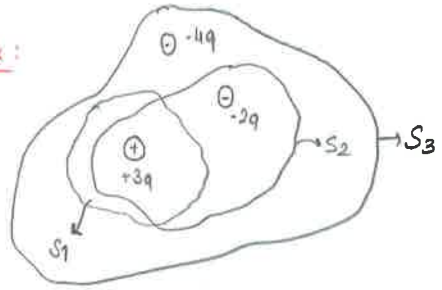
$E = k\cdot\frac{q}{r^2}$ where $k = \frac{1}{4\cdot\pi\cdot\varepsilon_0}$ ↳ Electric permity free space

$$\Phi_E = E\underbrace{\int dA}_{\text{Area of sphere}} = k\cdot\frac{q}{\cancel{r^2}}\cdot 4\cdot\pi\cdot\cancel{r^2} = \frac{1}{4\cdot\pi\cdot\varepsilon_0}\cdot q\cdot 4\pi = \frac{q}{\varepsilon_0}$$

$$\boxed{\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}}$$

Note : We can apply the Gauss's Law to find the Φ_E produced not just by point charges, also by 3 dimensional objects.

Ex :



S_1, S_2 and S_3 are closed surfaces. Find the Φ_E through each closed surface.

Solution : $\Phi_{E,S_1} = \frac{q_{enc}}{\varepsilon_0} = \frac{+3q}{\varepsilon_0}\left(\frac{N}{C}\cdot m^2\right)$

$$\Phi_{E,S_2} = \frac{q_{enc}}{\varepsilon_0} = \frac{3q-2q}{\varepsilon_0} = \frac{q}{\varepsilon_0}\left(\frac{N}{C}\cdot m^2\right)$$

$$\Phi_{E,S_3} = \frac{3q-2q-4q}{\varepsilon_0} = \frac{-3q}{\varepsilon_0}\left(\frac{N}{C}\cdot m^2\right)$$

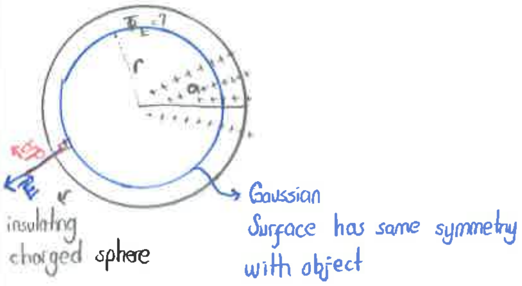
22.3 Application of Gauss's Law

Gauss law gives good results to find the Φ_E that is produced by charged objects that have well-known symmetry.

i-) Charged Objects with Spherical Symmetry

Lets find the Φ_E produced by spherical insulating object.

a-) for $r<a$, Inside the sphere



$$\frac{4}{3}\cdot\pi\cdot a^3 \quad \times \quad +q$$
$$\frac{4}{3}\cdot\pi\cdot r^3 \quad \times \quad q_{enc}$$

$$q_{enc} = \frac{\frac{4}{3}\cdot\pi\cdot r^3}{\frac{4}{3}\cdot\pi\cdot a^3}\cdot q = \frac{q\cdot r^3}{a^3}$$

Note: For insulating objects charges are present not just on the outer surface, but also every point inside the object.

$\Phi_E = \frac{q_{enc}}{\varepsilon_0}$, here we should specify the magnitude of q_{enc}

$$\boxed{\Phi_E = \frac{q\cdot r^3}{\varepsilon_0\cdot a^3}\left(\frac{N}{C}\cdot m^2\right)}$$

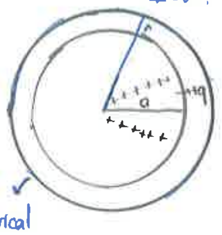
We can even calculate the $\vec{E}$

$$\Phi_E = \boxed{\oint \vec{E}.d\vec{A} = \frac{q_{enc}}{\varepsilon_0}}$$

$$= \oint E.dA.\cos 0° = \frac{q_{enc}}{\varepsilon_0}$$

$$E \oint dA = \frac{q_{enc}}{\varepsilon_0} \Rightarrow E.4.\pi.\cancel{r^2} = \frac{q.r^{\cancel{3}}}{\varepsilon_0.a^3} \Rightarrow \boxed{E = \frac{q.r}{4\pi\varepsilon_0 a^3} = k.\frac{q.r}{a^3}} \left(\frac{N}{C}\right)$$

b-) $r > a$, outside the sphere



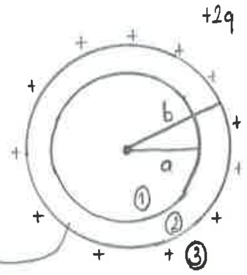
$$\Phi_E = \frac{q_{enc}}{\varepsilon_0} = \frac{+q}{\varepsilon_0} \left(\frac{N}{C}.m^2\right)$$

$$\oint \vec{E}.d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E \oint dA = \frac{q}{\varepsilon_0} \Rightarrow E.4\pi r^2 = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2} \Rightarrow \boxed{E = k.\frac{q}{r^2} \left(\frac{N}{C}\right)}$$

for outside object behaves like point charge

Ex:

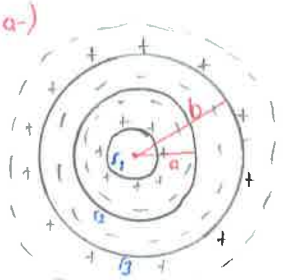


A metal solid sphere that has charge +q is placed radius C $(c < a)$ and total charge +q is placed concentric with shell.

a-) Find the Φ_E and E values for regions 1, 2, and 3.

b-) Determine the surface charge densities for inner and outer surfaces of shell.

Solution: a-)



$$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

1. Region $(c < r_1 < a)$

$$\Phi_{E_1} = \frac{q_{enc}}{\varepsilon_0} = \frac{+q}{\varepsilon_0} \left(\frac{N}{C}.m^2\right) \Rightarrow \oint \vec{E_1}.dA = \frac{q_{enc}}{\varepsilon_0} \Rightarrow E_1 \underbrace{\oint dA}_{4.\pi.r_1^2} = \frac{+q}{\varepsilon_0}$$

$$E_1.4.\pi.r_1^2 = \frac{q}{\varepsilon_0} =$$

$$E_1 = \frac{q}{4.\pi.r_1^2} = \frac{k.q}{r_1^2} \left(\frac{N}{C}\right)$$

2. Region $(a < r_2 < b)$

$$\Phi_{E_{1,2}} = \frac{q_{enc}}{\varepsilon_0} = \frac{+q-q}{\varepsilon_0} = 0$$

$E_2 = 0$ $\quad \Phi_{E_{1,2}} = 0$, $E_2 = 0$ — Since region is inside the conductor

3. Region $(r_3 > b)$

$$\Phi_{E,3} = \frac{q_{enc}}{\varepsilon_0} = \frac{+3q-q+q}{\varepsilon_0} = \frac{+3q}{\varepsilon_0} \left(\frac{N}{C}.m^2\right)$$

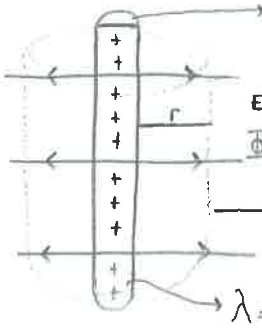
$$\oint = \vec{E_3}.d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E_3 \oint dA = \frac{+3q}{\varepsilon_0} \Rightarrow E_3 = 3k.\frac{q}{r_3^2} \left(\frac{N}{C}\right)$$

b-) $\sigma = \dfrac{q_{total}}{A_{total}}$ → Surface charge density

for inner surface of shell, $\sigma_{inner} = \dfrac{-q}{4.\pi.a^2} \left(\dfrac{C}{m^2}\right)$ $\sigma_{outer} = \dfrac{+3q}{4.\pi.b^2} \left(\dfrac{C}{m^2}\right)$

ii-) Application of Gauss's Law for Cylindrical Symmetry



→ Very long cylindrical rod

$E = ?$

$\Phi = ?$

→ Cylindrical gaussian surface

→ $\lambda = \dfrac{q_{total}}{L_{total}}$ => $q_{total} = L.\lambda = q_{enc}$

Since thin cylindrical rod is very long mas Φ_E passes through the side surface of cylindrical gaussian surface.

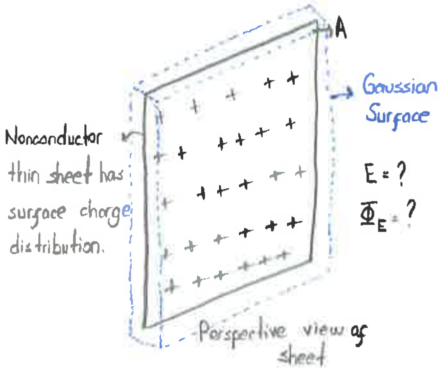
$$\Phi_E = \frac{q_{enc}}{\varepsilon_0} = \frac{\lambda.L}{\varepsilon_0} \left(\frac{N}{C}.m^2\right) \Rightarrow \oint \vec{E}.d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

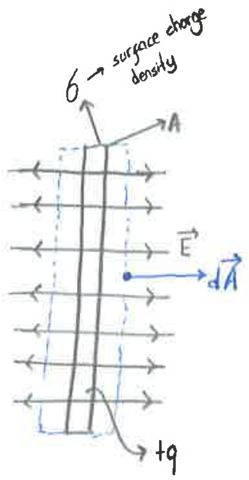
$$\Rightarrow E.\underbrace{\oint d\vec{A}}_{\text{Area of side surface}} = \frac{\lambda.L}{\varepsilon_0} \Rightarrow E.2\pi r\cancel{L} = \frac{\lambda.\cancel{L}}{\varepsilon_0} \Rightarrow E = \frac{1}{2.\pi.\varepsilon_0}.\frac{\lambda}{r} = \left(\frac{2.k.\lambda}{r}\right)\left(\frac{N}{C}\right)$$

17.03.2015
Tuesday

iii-) Planar Symmetry

a-) Nonconducting (Insulator) Sheet





$$\Phi_E = \oint \vec{E}\,d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\sigma = \frac{q_{total}}{A_{total}} = \frac{q}{A}\ \ q_{enc}$$

$$\Phi_E = \frac{\sigma A}{\varepsilon_0}\left(\frac{N}{C}\,m^2\right)$$

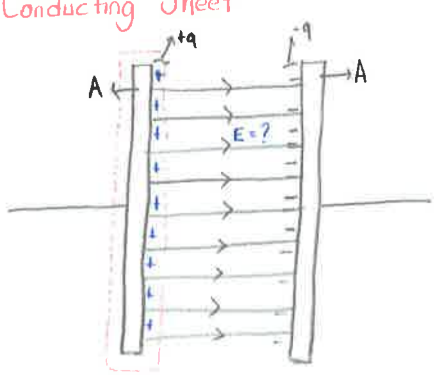
$$\oint \vec{E}\,.d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\vec{E}\underbrace{\oint d\vec{A}}_{=2A} = \frac{\sigma A}{\varepsilon_0} \Rightarrow E.2\cancel{A} = \frac{\sigma.\cancel{A}}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}\left(\frac{N}{C}\right)$$
→ for insulating sheet

*Note: If the plate is very large then at reasonable distances the magnitude of $\vec{E}$ is constant, that is $\vec{E}$ uniform.

b-) Conducting Sheet



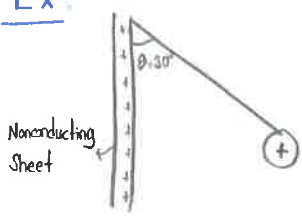
$$\Phi = \frac{q_{enc}}{\varepsilon_0} = \frac{\sigma.A}{\varepsilon_0}$$

$$\oint \vec{E}\,.d\vec{A} = \frac{\sigma.A}{\varepsilon_0}$$

$$E\underbrace{\oint dA}_{=A} = \frac{\sigma.\cancel{A}}{\varepsilon_0} \Rightarrow E.\cancel{A} = \frac{\sigma\cancel{A}}{\varepsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{\varepsilon_0}}\left(\frac{N}{C}\right)$$

↳ Uniform field produced by conducting sheet

Ex:



$m = 1mg = 10^{-6}\,kg$
$q = 2\times10^{-8}\,C$

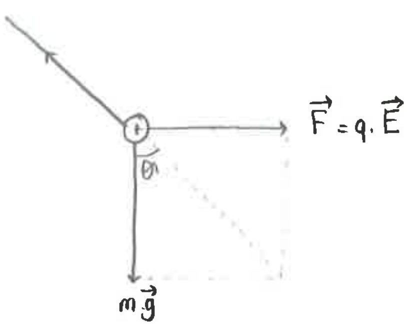
Charged object is in static equilibrium as seen in figure. Find the σ.

$(\tan 30° = 0.58)$

$\varepsilon_0 = 8.85\times10^{-12}\,\frac{C^2}{N.m^2}$

$g \cong 10\,\frac{N}{kg}$

Solution:



$$\tan\theta = \frac{F}{m.g} = \frac{q.E}{m.g}$$

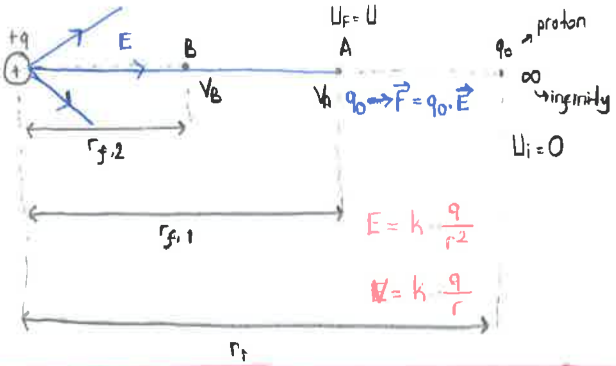
$$\tan 30° = \frac{q.\sigma}{m.g.2\varepsilon_0} \Rightarrow \sigma = 5.13\times10^{-9}\left(\frac{C}{m^2}\right)$$

CHAPTER - 23

17.03.2015
Tuesday

ELECTRIC POTENTIAL

23.1 Electric Potential and Electric Potential Difference



When q_0 is brought from infinity to a near position of q, then work done by external agent is stored as electric potential energy (U) in the system.

The stored potential energy per unit charge (q_0) is called as electric potential (V)

$$V = \frac{U}{q_0} \left(\frac{J}{C}\right) \Rightarrow V = \frac{-W_\infty}{q_0} = -\frac{1}{q_0} \int \vec{F} \cdot d\vec{r}$$

$$\boxed{\Delta V = V_F - V_i = V_B - V_A = \frac{\Delta U}{q_0} = \frac{-W_{A \to B}}{q_0} = -\int_{r_A}^{r_B} \vec{E} \cdot d\vec{r} \ (V)}$$

→ Electric Potential Difference

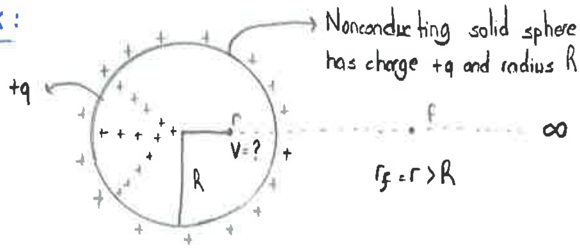
$$\boxed{\begin{aligned} \Delta U &= -W \\ U_F - U_i &= -W \\ U &= -W_\infty \end{aligned}}$$

$$\boxed{\begin{aligned} U_B &> U_A \\ V_B &> V_A \end{aligned}}$$

$$\Rightarrow \boxed{V = -\int_{\infty}^{r_F} \frac{\cancel{q_0} \cdot \vec{E} \cdot d\vec{r}}{\cancel{q_0}} = -\int_{r_i}^{r_F} \vec{E} \cdot d\vec{r} \ (V)}$$

→ Electric Potential

Ex:



→ Nonconducting solid sphere has charge $+q$ and radius R.

a-) Find the electric potential at a point outside the sphere.

b-) " " " " " " " inside " "

$$E_{outside} = k \cdot \frac{q}{r^2} \qquad E_{inside} = k \cdot \frac{q \cdot r}{a^3}$$

Solution : a-)

$$V = -\int_{\infty}^{r_f = r} \vec{E} \cdot d\vec{r} = -\int_{\infty}^{r} E \cdot dr \cdot \cos 0° \Rightarrow V = -\int_{\infty}^{r} E \cdot dr = -\int_{\infty}^{r} k \cdot \frac{q}{r^2} \cdot dr \Rightarrow V = -k \cdot q \cdot \int_{\infty}^{r} \frac{dr}{r^2} = -k \cdot q \left(-\frac{1}{r} \Big|_{\infty}^{r} \right) \Rightarrow$$

$$V = k \cdot q \cdot \left(\frac{1}{r} - \underbrace{\cancel{\frac{1}{\infty}}}_{=0} \right) = k \cdot \frac{q}{r} \ (V)$$

b-)

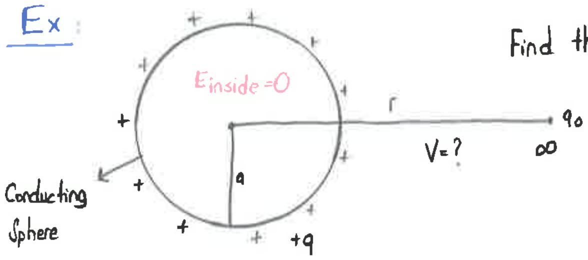
$$V = -\int_{\infty}^{r} \vec{E} \cdot d\vec{r} = -\left[\int_{\infty}^{a} E_{outside} \cdot dr + \int_{a}^{r} E_{inside} \cdot dr \right] \Rightarrow$$

$$V = -\left[\int_{\infty}^{a} k \cdot \frac{q}{r^2} \cdot dr + \int_{a}^{r} k \cdot \frac{q \cdot r}{a^3} \cdot dr \right] = -\left[k \cdot q \left(\frac{-1}{r} \right) \Big|_{\infty}^{a} + k \cdot \frac{q}{a^3} \cdot \frac{r^2}{2} \Big|_{a}^{r} \right] \Rightarrow$$

$$V = -\left[k \cdot q \left(\frac{-1}{a} + \frac{1}{\infty} \right) + \frac{k \cdot q}{2 \cdot a^3} \left(r^2 - a^2 \right) \right] = \frac{k \cdot q}{2a^3} \left(3a^2 - r^2 \right)$$

19.03.2015
Thursday

Ex:



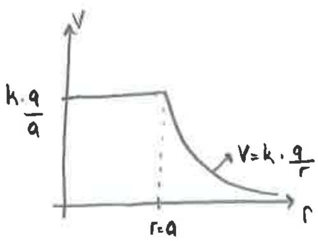
Find the V for a-) $r>a$, outside
b-) $r<a$, inside

Solution: a-)

$$V=-\int_{\infty}^{r} E_{outside}\cdot dr = -\int_{\infty}^{r} k\cdot\frac{q}{r^2}\cdot dr = k\cdot\frac{q}{r} \quad (V)$$

b-) $r<a$, inside

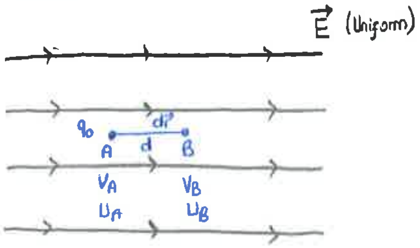
$$V=-\int_{\infty}^{r}\vec{E}\cdot d\vec{r} = -\left[\int_{\infty}^{a} E_{outside}\cdot dr + \cancel{\int_{a}^{r} \underbrace{E_{inside}}_{0}\cdot dr}\right] \Rightarrow V=-\int_{\infty}^{a} k\cdot\frac{q}{r^2}\cdot dr = -\left(-k\cdot\frac{q}{r}\Big|_{\infty}^{a}\right) = k\cdot\frac{q}{a} - \underbrace{\cancel{k\cdot\frac{q}{\infty}}}_{0} \Rightarrow$$



$$= V = k\cdot\frac{q}{a} = V_{surface}$$

inside the conducting sphere V is constant and equal to $V_{surface}$.

23.2 Electric Potential Inside the Uniform Fields



When q_0 is left free when it moves from A to B.
The potential difference btwn point B and A

$$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{r} = -\int_A^B E.dr.\cos 0° = -E.\underbrace{\int_A^B dr}_{=d} = -E.d$$

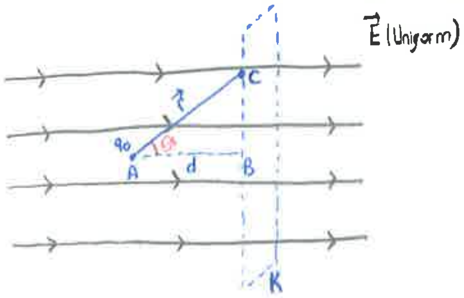
$$\boxed{V=\frac{U}{q_0} \Rightarrow U=V.q_0}$$

*** Note: Uniform $\vec{E}$ means the magnitude of field is the same at every point and field lines are parallel.

$$\Delta V = V_B - V_A = -E.d \ (Volt)$$

$\rightarrow V_B < V_A$
$U_B < U_A$

Equipotential surface, every point on it, has same potential.



q moves from A to C.

$$V_C - V_A = -\int_A^C \vec{E}.d\vec{r} = -\int_A^C E.dr.\cos\theta = -E.\cos\theta \underbrace{\int_A^C dr}_{=r} = -E.\underbrace{r.\cos\theta}_{d} = -E.d \ (V) = V_B - V_A$$

$\Downarrow$

$V_B = V_C = V_E = V_K$

24.03.2015
Tuesday

23.3 Electric Potential and Electric Potential Energy Due to Group of Point Charges

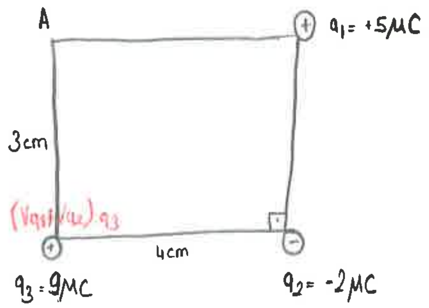
$$V_A - V_\infty = -\frac{W_\infty}{q_0} = -\int_\infty^r \vec{E}\cdot d\vec{r}$$

$$\boxed{V_A = k\cdot\frac{q}{r}}\ (V) \qquad \text{if charge is negative then} \qquad \boxed{V_A = -k\cdot\frac{q}{r}}\ (V)$$

$$V_A = \frac{U}{q_0} \Rightarrow \boxed{U = k\cdot\frac{q\cdot q_0}{r}}\ (J)$$

Energy gained by system.

Ex:



a-) Find the resultant potential at point A.

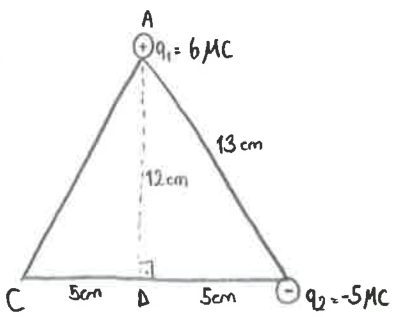
b-) If another charge $q_4 = 3\mu C$ is brought from infinity to point A, what is the potential energy of system?

Sol: a-)
$$V_{net,A} = V_{q_1} + V_{q_2} + V_{q_3} = k\cdot\frac{q_1}{r_1} + k\cdot\frac{q_2}{r_2} + k\cdot\frac{q_3}{r_3} = 9\times10^9\ \frac{N.m^2}{C^2}\left(\frac{5\times10^{-6}C}{4\times10^{-2}m} - \frac{2\times10^{-6}C}{5\times10^{-2}m} + \frac{9\times10^{-6}C}{3\times10^{-2}m}\right) \Rightarrow$$

$$V_{net,A} = 9\times10^9 \times 4.65\times10^{-4} = \underline{41.85\times10^5}\ (V)$$

b-)
$$U_{system} = \underbrace{k\cdot\frac{q_1\cdot q_2}{r_{1,2}}}_{V_{q_1,C}} + \underbrace{k\cdot\frac{q_1\cdot q_3}{r_{1,3}}}_{V_{q_1,D}} + \underbrace{k\cdot\frac{q_2\cdot q_3}{r_{2,3}}}_{V_{q_2,D}} + k\cdot\frac{q_1\cdot q_4}{r_{1,4}} + k\cdot\frac{q_2\cdot q_4}{r_{2,4}} + k\cdot\frac{q_3\cdot q_4}{r_{3,4}} \Rightarrow U_{system} = k\cdot(\ldots)\ J$$

Ex:



a-) What is the $V_C - V_A$ = ?

b-) If another charge $q_3 = -2\mu C$ is brought from infinity to C, how much does the potential energy of system increase?

c-) What is the potential energy of system when $q_3 = -2\mu C$ is brought from infinity?

Sol: a-)
$$V_C = k\cdot\frac{q_1}{r_{1,C}} + k\cdot\frac{q_2}{r_{2,C}} = 9\times10^9\ \frac{N.m^2}{C^2}\left(\frac{6\times10^{-6}C}{13\times10^{-2}m} - \frac{5\times10^{-6}C}{10\times10^{-2}m}\right) = -3.6\times10^4\ V$$

$$V_A = k\cdot\frac{q_2}{r_{2,a}} = 9\times10^9\ \frac{N.m^2}{C^2}\cdot\left(-\frac{5\times10^{-6}C}{13\times10^{-2}m}\right) = -3.46\times10^4\ V, \quad V_C - V_A = (-3.6-(-3.46))\times10^4\ V \Rightarrow \underline{V_C - V_A = -1.4\times10^4\ V}$$

$$V_D = k\cdot\frac{q_1}{r_{1,D}} + k\cdot\frac{q_2}{r_{2,D}} = 9\times10^9\ \frac{N.m^2}{C^2}\left(\frac{6\times10^{-6}C}{12\times10^{-2}m} - \frac{5\times10^{-6}C}{5\times10^{-2}m}\right) = \quad V_C - V_D = \underline{-3.6\times10^4\ V} -$$

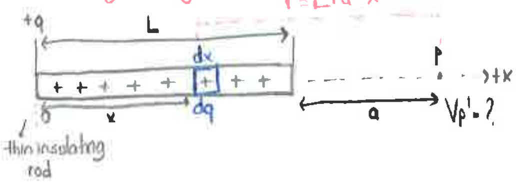
b-)
$$\Delta U_{system} = q_3\cdot V_C = -2\times10^{-6}C\times(-3.6\times10^4\ V) = \underline{7.2\times10^{-2}\ J}$$

increament of potential energy of system.

c-)
$$U_{system} = k\cdot\frac{q_1\cdot q_2}{r_{1,2}} + k\cdot\frac{q_1\cdot q_3}{r_{1,3}} + k\cdot\frac{q_2\cdot q_3}{r_{2,3}} =$$

2.3.4 Electric Potential Due to Continuous Charge Distribution

i-) Line of Charge



What is the $V_p = ?$

$$\lambda = \frac{q_{total}}{L_{total}} = \frac{dq}{dx} \Rightarrow dq = \lambda . dx$$

$$dE = k . \frac{dq}{r^2}$$

$$dV = k . \frac{dq}{r}$$

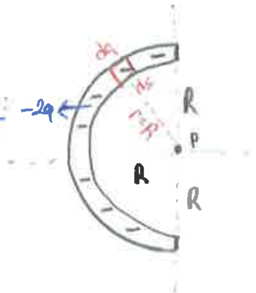
$$dV_p = k . \frac{dq}{r} = k . \frac{\lambda . dx}{L+a-x} \Rightarrow V_p = k . \lambda . \int_0^L \frac{dx}{L+a-x}$$

$$= k . \lambda \left[-\ln(L+a-x) \Big|_0^L \right] \Rightarrow$$

$$= k . \lambda \left[\ln(L+a) - \ln a \right] \Rightarrow$$

$$= k . \lambda . \ln \frac{(L+a)}{a} \ (V)$$

Ex: $V_p = ?$



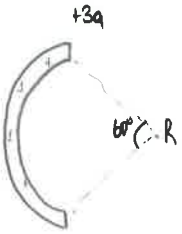
Sol: $dV_p = k . \frac{dq}{r} = \frac{k . \lambda . ds}{R}$

$$V_p = \frac{k . \lambda}{R} \int_0^{\pi . R} ds = \frac{k . \lambda . \pi . R}{R} = -2 . k . \frac{q}{r}$$

$$\lambda = \frac{q_{total}}{L_{total}} = \frac{-2q}{\pi . r} \Rightarrow$$

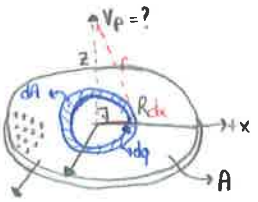
$$= \pi . \lambda . R = -2q$$

Ex: $V_p = ?$



Sol: $V_p = 3k . \frac{q}{R}$

ii-) Surface Charge Distribution



Disk has uniform surface distribution, σ.

Lets find potential produced by disk at point P

$$dV_p = k . \frac{dq}{r} = k . \frac{\sigma . dA}{(x^2+z^2)^{1/2}} = k . \sigma . 2\pi . \frac{x . dx}{(x^2+z^2)^{1/2}} \Rightarrow$$

$$\sigma = \frac{q_{total}}{A_{total}} = \frac{dq}{dA} \Rightarrow dq = \sigma . dA = \sigma . 2 . \pi . x . dx$$

$$V_p = k . \sigma \, 2\pi \underbrace{\int_0^R \frac{x . dx}{(x^2+z^2)^{1/2}}}_{\text{Solution from integral table}} = k . \sigma \, 2\pi \, (x^2+z^2)^{1/2} \Big|_0^R$$

$$\boxed{V_p = k . \sigma . 2\pi . \left[(R^2+z^2)^{1/2} - z \right]} \ (V) \text{ here } k = \frac{1}{4 . \pi . \varepsilon_0}$$

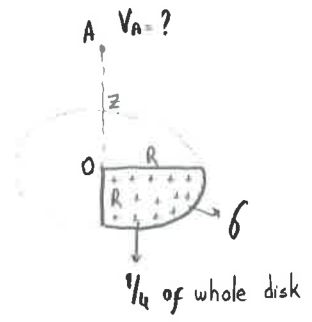
or

$$V_p = \frac{\sigma . 2 . \pi}{4 . \pi . \varepsilon_0} \left[\quad \right]$$

$$\boxed{V_p = \frac{\sigma}{2 . \varepsilon_0} \left[(R^2+z^2)^{1/2} - z \right]}$$

26.03.2015
Thursday

Ex



Solution : $V_A = \frac{V_P}{4} = \frac{6}{8\varepsilon_0}\left[(R^2+z^2)^{1/2} - z\right] (V)$

23.5 Calculating the $\vec{E}$ by using V equation

$$dV = -\vec{E}.d\vec{r} = -E.dr.\cos 0^\circ = -E.dr$$

$$\boxed{E = -\frac{dV}{dr}\left(\frac{N}{C}\right)}$$

$$\vec{r} = x.\hat{i} + y.\hat{j} + z.\hat{k}$$

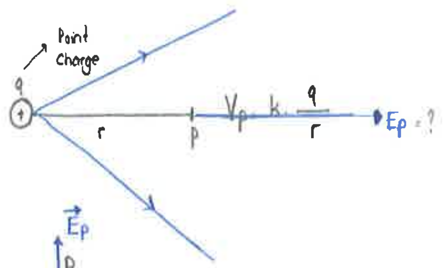
in 3 dimensions

$$E_x = \frac{-dV}{dx}, \quad E_y = \frac{-dV}{dy}, \quad E_z = \frac{-dV}{dz}$$

$$\vec{E} = -\left(\hat{i}.\frac{dV}{dx} + \hat{j}.\frac{dV}{dy} + \hat{k}.\frac{dV}{dz}\right) = -\vec{\nabla} V$$

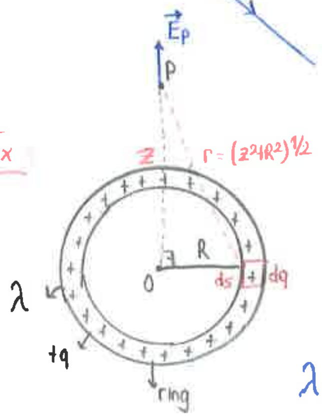
del operator.
It is a derivative operator.

Ex :



$$E_P = \frac{-dV_P}{dr} = -k.q.\frac{d(r^{-1})}{dr} = -k.q.(-).r^{-2} \Rightarrow E_P = k.\frac{q}{r^2}\left(\frac{N}{C}\right)$$

Ex



a-) Find $V_P = ?$

b-) Obtain the $\vec{E}_P$ equation by using V_P equation

$$\lambda = \frac{dq}{ds} \Rightarrow dq = \lambda.ds$$

Solution : a-) $dV_P = k.\frac{dq}{r} = \frac{k.\lambda.ds}{(z^2+R^2)^{1/2}}$

$$\int dV_P = \frac{k.\lambda}{(z^2+R^2)^{1/2}}.\int_0^{2.\pi.R} ds$$

$$V_P = \frac{k.\lambda.2.\pi.R}{(z^2+R^2)^{1/2}} \; (V)$$

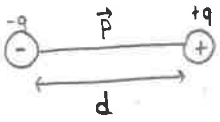
b-) $E_P = \frac{-dV_P}{dz} = -k.\lambda.2.\pi.R.\frac{d}{dz}.(z^2+R^2)^{-1/2}$

$$= -k.\lambda.2.\pi.R.\left(\frac{-1}{\cancel{2}}\right)(z^2+R^2)^{-\frac{1}{2}-1}.\cancel{2}z$$

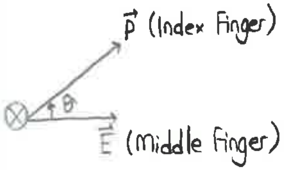
$$E_P = \frac{k.\lambda.2.\pi.R.z}{(z^2+R^2)^{3/2}}\left(\frac{N}{C}\right)$$

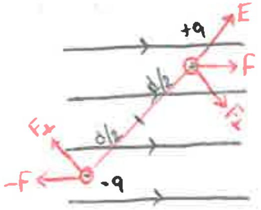
We have obtained their result at Chp 21.

Electric Dipole



→ Direction is from -q to +q.





qE

$$\tau = \frac{d}{2} . F_{\perp} + \frac{d}{2} . F_{\perp} = d . F_{\perp}$$

$$\tau = \underbrace{d . q}_{p} . E . \sin\theta = p . E . \sin\theta \quad (N.m)$$

$$\boxed{p = q . d} \; (c.m)$$

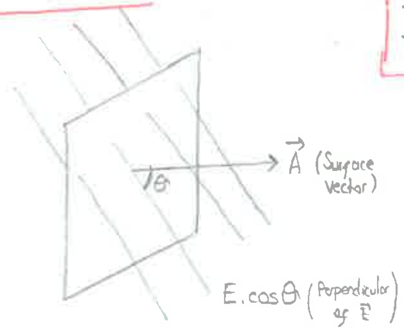
$$\tau = p . E . \sin\theta$$

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

$$U_p = -\vec{p} . \vec{E} = -p . E . \cos\theta \quad (J)$$

Electric Flux

$$\boxed{\Phi_E = \vec{E} . \vec{A} = \vec{E} . \vec{A} . \cos\theta} \; \left(\frac{N}{C} . m^2\right)$$

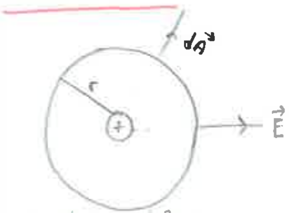


Random Shape; $\Delta\Phi_i = \vec{E} \; \Delta\vec{A}_i$

$$\Phi_{total} = \int \vec{E} \, d\vec{A}$$

Closed Surface: $\Phi_{total} = \oint E . d\vec{A}$

Gauss's Law

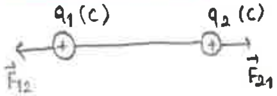


$$\boxed{A = 4 . \pi . r^2}$$

$E = k . \frac{q_0}{r^2}$ where $k = \frac{1}{4 . \pi . \varepsilon_0}$, $\Phi = \oint E . d\vec{A}$

$$\boxed{\Phi_E = \oint \vec{E} . d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}} \; \left(\frac{N}{C} \, m^2\right)$$

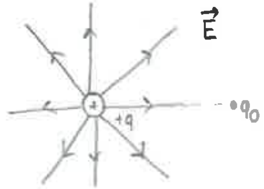
Coulomb's Law



$$F_{12} = F_{21} = k . \frac{|q_1| . |q_2|}{r^2} \; (N), \quad k = 9 \times 10^9 \; \frac{N . m^2}{C^2}$$

$$F_{res} = \left[F_{1,2}^{\,2} + F_{2,1}^{\,2} + 2 . F_{1,2} . F_{2,1}\right]^{1/2} \; \ldots \; N$$

Electric Field

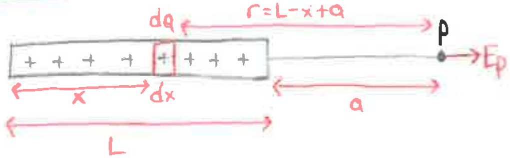


$$F_q = k . \frac{|q_0| . |q|}{r^2} \qquad F_q = q_0 . E$$

$$E = \frac{k . |q|}{r^2} \; \left(\frac{N}{C}\right)$$

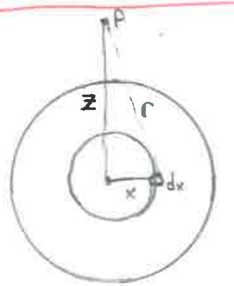
Electric Field Produced By Objects that have Uniform Charge Distribution

i-) Objects that have linear charge distribution



$$\lambda_{total} = \frac{q_{total}}{L_{total}} \; \text{(Linear charge Density)}$$

ii-) Electric Field Due to Surface Charge Distribution



$$\sigma = \frac{q_{total}}{A_{total}} \Rightarrow dq = \sigma . dA \; \text{(surface charge distribution)}$$

ELECTRIC POTENTIAL AND ELECTRIC POTENTIAL DIFFERENCE

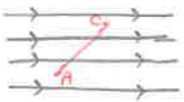
$$\boxed{\Delta V = -\int_{r_A}^{r_B} \vec{E}\cdot d\vec{r} \quad (V)}$$

$$\boxed{V = -\int_{r_i}^{r_f} \vec{E}\cdot d\vec{r} \quad (V)}$$

Nonconducting solid sphere ; $E_{outside} = k.\dfrac{q}{r^2}$ $\quad E_{inside} = k.\dfrac{q.r}{a^3}$

ELECTRIC POTENTIAL INSIDE THE UNIFORM FIELDS

$$\boxed{V = \frac{U}{q_0}}$$



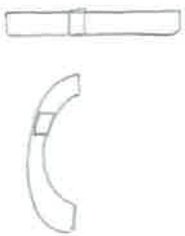
$$V_C - V_A = -\int_A^C \vec{E}.d\vec{r} = -\int_A^C E.dr.\cos\theta$$

ELECTRIC POTENTIAL and ELECTRIC POTENTIAL ENERGY DUE TO GROUP OF POINT CHARGES

$$V = k.\frac{q}{r} \ (V) \qquad U = k.\frac{q.q_0}{r} = V_A.q_0 \quad (j)$$

ELECTRIC POTENTIAL DUE TO CONTINUOUS CHARGE DISTRIBUTION

i-) Line of Charge



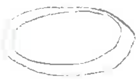
$$\lambda = \frac{q_{total}}{L_{total}}$$

$$dq = \lambda.dx$$

$$dE = k.\frac{dq}{r^2}$$

$$dV = k.\frac{dq}{r}$$

ii-) Surface Charge Distribution



DISC

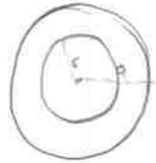
$$\sigma = \frac{q_{total}}{A_{total}}$$

$$dq = \sigma.dA$$

$$dV = \frac{k.dq}{r}$$

Application of Gauss's Law

i-) Charged Objects with Spherical Symmetry



for $a > r$

$$q_{enc} = \frac{\frac{4}{3}.\pi.r^3}{\frac{4}{3}.\pi.a^3}.q = \frac{q.r^3}{a^3}, \quad \Phi_E = \oint \vec{E}.d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$\boxed{\Phi_E = \frac{q.r^3}{\varepsilon_0.a^3} \left(\frac{N}{C}.m^2\right)}$$

$$\boxed{E = \frac{q.r}{4\pi.\varepsilon_0.a^3} = k.\frac{q.r}{a^3}}$$ inside

for $r > a$
(Outside the sphere)

$$\boxed{\Phi_E = \frac{q_{enc}}{\varepsilon_0} = \frac{q}{\varepsilon_0}\left(\frac{N}{C}.m^2\right)}$$

$$\boxed{E = k.\frac{q}{r^2}\left(\frac{N}{C}\right)}$$

for outside object behaves like point charge

ii-) Application of Gauss's Law for Cylindrical Symmetry

$$\lambda = \frac{q_{total}}{L_{total}} \Rightarrow q_{total} = q_{enc} = \lambda.L_{total}$$

$$A_{cylindrical} = 2.\pi.r.L$$

$$\Phi_E = \frac{q_{enc}}{\varepsilon_0} = \frac{\lambda.L_{total}}{\varepsilon_0} = \oint \vec{E}.d\vec{A}$$

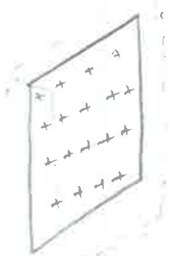
Calculating the $\vec{E}$ by using V equation

$$\vec{E} = -\frac{dV}{dr}\left(\frac{N}{C}\right) \qquad \vec{r} = x.\hat{i} + y.\hat{j} + z.\hat{k}$$

$$\vec{E} = -\left(\hat{i}.\frac{dV}{dx} + \hat{j}.\frac{dV}{dy} + \hat{k}.\frac{dV}{dz}\right)$$

iii-) Planar Symmetry

a-) Nonconducting (Insulator) Sheet



A = 2A

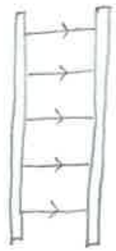
$$\Phi_E = \oint \vec{E}.d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$q_{enc} = q_{total} = \sigma.A_{total}$$

$$E.2A = \frac{\sigma.A}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2.\varepsilon_0}}\left(\frac{N}{C}\right)$$

b-) Conducting Sheet



A = A

$$\Phi = \frac{q_{enc}}{\varepsilon_0} = \frac{\sigma.A}{\varepsilon_0}$$

$$E.A = \frac{\sigma.A}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}\left(\frac{N}{C}\right)$$