$V \to$ Vector set

$x \in V,\ y \in V,\ z \in V,\ \alpha, \beta \in F$

1-) $x + y \in V$

2-) $x + y = y + x$

3-) $x + (y+z) = (x+y) + z$

4-) $\alpha\,(x+y) = \alpha . x + \alpha . y$

5-) $(\alpha + \beta) . x = \alpha . x + \beta x$

6-) $\alpha(\beta x) = (\alpha \beta) x$

7-) $\alpha . x \in V$

8-) $-x \in V$

9-) $1 . x \in V$

10-) $0 + x \in V$

Basis

$S = \{U_1, U_2, \dots, U_n\}$

S is linearly independent

$V = \text{span}\ (S)$

$U_0 \in V$

$U_0 = \alpha_1 U_1 + \alpha_2 . U_2 + \dots + \alpha_n . U_n$

$$U_0 = [\alpha_1\ \alpha_2\ \dots\ \alpha_n] \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{bmatrix}$$

$U_0 = \alpha^T . U$

$\alpha^T = \{\alpha_1\ \alpha_2\ \dots\ \alpha_n\}$ is the representation of U_0 w/ respect to the basis $U = \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{bmatrix}$

$$U_0 = \alpha_1 . U_1 + \alpha_2 . U_2 + \dots + \alpha_n . U_n = \sum_{i=1}^{n} \alpha_i . U_i$$

$$U_0 = \beta_1 . U_1 + \beta_2 . U_2 + \dots + \beta_n . U_n = \sum_{i=1}^{n} \beta_i . U_i$$

$$0 = \underbrace{(\alpha_1 - \beta_1)}\, U_1 + \dots + \underbrace{(\alpha_n - \beta_n)} = \sum_{i=1}^{n} (\alpha_i - \beta_i) . U_i$$

$\alpha_1 = \beta_1\ ,\ \alpha_2 = \beta_2\ ,\ \alpha_n = \beta_n$

Standart Basis

$$S = \{e_1\ e_2\ \dots\ e_n\}\ ,\ e_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix},\ e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix},\ \dots,\ e_n = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$\alpha_1 . e_1 + \alpha_2 . e_2 + \dots + \alpha_n . e_n = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$S = \{e_1\ e_2\ \dots\ e_n\}$

$U = \{U_1\ U_2\ \dots\ U_n\}$

$$\alpha_1 . e_1 + \alpha_2 . e_2 + \dots + \alpha_n . e_n = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix}$$

$$U_1 . e_1 + U_2 . e_2 + \dots + U_n . e_n = \begin{bmatrix} U_1 \\ U_2 \\ \vdots \\ U_n \end{bmatrix}$$

Bir vektörün

Standart Baze göre ①

vektörün kendine eşittir

$\alpha = U$ representation ②

Midterm's Questions Answers

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$$\begin{bmatrix}1&1&1\\0&1&1\\0&0&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}3\\2\\1\end{bmatrix}$$

$\Delta=1$ $\begin{bmatrix}3&1&1\\2&1&1\\1&0&1\end{bmatrix}$ $\Delta_1=3-1\cdot1=1$ $x_1=\frac{\Delta_1}{\Delta}=\frac{1}{1}=1$ $\begin{bmatrix}1&1&3\\0&1&2\\0&0&1\end{bmatrix}$

$\begin{bmatrix}1&3&1\\0&2&1\\0&1&1\end{bmatrix}$ $\Delta_2=1$ $x_2=\frac{\Delta_2}{\Delta}=\frac{1}{1}=1$ $x_3=\frac{\Delta_3}{\Delta}=\frac{1}{1}=1$

$\begin{bmatrix}1&1&1\\0&1&1\\0&0&1\end{bmatrix}$

$m_{11}=1$ $m_{21}=1$ $m_{31}=0$
$m_{12}=0$ $m_{22}=1$ $m_{32}=1$
$m_{13}=0$ $m_{23}=0$ $m_{33}=1$

$c_{11}=1$ $c_{21}=-1$ $c_{31}=0$
$c_{12}=0$ $c_{22}=1$ $c_{32}=-1$
$c_{13}=0$ $c_{23}=0$ $c_{33}=1$

$C=\begin{bmatrix}1&0&0\\-1&1&0\\0&-1&1\end{bmatrix}$

$$A^{-1}=C^T=\begin{bmatrix}1&-1&0\\0&1&-1\\0&0&1\end{bmatrix}$$

$\alpha_1,\alpha_2,\alpha_3$

Q-2-) $X=\begin{bmatrix}a\\b\\c\end{bmatrix}$ $y=\begin{bmatrix}a\\b\\b\end{bmatrix}$ $Z=\begin{bmatrix}a\\c\\c\end{bmatrix}$, $a\neq b\neq c$

$\alpha_1X+\alpha_2y+\alpha_3Z=0$ (ise lineer bağımsız) $\alpha_1\begin{bmatrix}a\\b\\c\end{bmatrix}+\alpha_2\begin{bmatrix}a\\b\\b\end{bmatrix}+\alpha_3\begin{bmatrix}a\\c\\c\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

$\alpha_1.a+\alpha_2.a+\alpha_3.a=0$
$\alpha_1.b+\alpha_2 b+\alpha_3 c=0$
$\alpha_1 c+\alpha_2 b+\alpha_3 c=0$

$\Rightarrow\begin{bmatrix}a&a&a\\b&b&c\\c&b&c\end{bmatrix}\begin{bmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$

Eğer det 0'dan farklıysa lineer bağımsız (Tersi alınır)

$\Delta=a\cdot(\cancel{bc-bc})-a(cb-c^2)+a(b^2-cb)$

$=-abc+ac^2+ab^2-abc=a(b^2+c^2-2bc)=a\underbrace{(b-c)^2}_{>0}$

• Lineer bağımsız olabilmesi için a 0'dan farklı olması gerekir.

Q-3-) $\alpha\in\mathbb{R}$ $\det(\alpha A)=\alpha^2.\det(A)$ $A=n\times n$

$\begin{bmatrix}\alpha.a_{11}&\alpha a_{12}&\cdots&\alpha.a_{1n}\\ \\ \alpha.a_{n1}&&&\alpha.a_{nn}\end{bmatrix}$ $\alpha.a_{11}.C_{11}+\alpha.a_{12}.C_{12}+\ldots+\alpha.a_{1n}.C_{1n}$

$$(\alpha).(A)=\begin{bmatrix}\alpha.a_{11}&\alpha.a_{12}&\cdots&\alpha.a_{1n}\\\alpha a_{21}&&&\\\vdots&&&\\\alpha.a_{n1}&\cdots&&\alpha.a_{nn}\end{bmatrix}=(\alpha.I).A=\begin{bmatrix}\alpha&0&\cdots&0\\0&\alpha&&\vdots\\\vdots&&\ddots&0\\0&&&\alpha\end{bmatrix}.A=\det((\alpha.I).A)=\underbrace{\det(\alpha I)}_{\alpha^n}.\det(A)$$

$\alpha^n.\det(A)$

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Dimension in Vector Spaces

$\{v_1\ v_2\ \dots\ v_n\} \rightarrow$ basis

$\dim(V) = n \Rightarrow$ number of the max vectors

$+v_{n+1} \rightarrow \{v_1\ v_2\ \dots\ v_n\ v_{n+1}\}$ linear independent ?

$\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n + \alpha_{n+1} \cdot v_{n+1} = 0$ (Lineer Bağımlı)

—

$v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \quad v_3 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\alpha_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \alpha_2 \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \alpha_3 \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\alpha_1 + 2\alpha_2 + 2\alpha_3 = 0$
$\alpha_1 + 2\alpha_2 + \alpha_3 = 0$

$\alpha_1 = -2\alpha_2$

$\{v_1, v_3\}$
$\{v_2, v_3\}$

—

$S = \{v_1, v_2, \dots, v_n\}$ in V $\xrightarrow{\text{Finite-dimension}}$ Basis, Lineer Independent sets

$\dim(V) = n$

$\hat{S} = \{\hat{v}_1, \hat{v}_2, \dots, \hat{v}_m\}$ $m > n$ ise linear bağımlı olur. (İSPATI)

*Her bir vektör basis'in lineer kombinasyonu olabilir. (Her bir eleman)

$\hat{v}_1 = a_{11} \cdot v_1 + a_{12} \cdot v_2 + \dots + a_{1n} \cdot v_n$

$\hat{v}_2 = a_{21} \cdot v_1 + \dots + a_{2n} \cdot v_n$

$\vdots$

$\hat{v}_m = a_{m1} \cdot v_1 + \dots + a_{mn} \cdot v_n$

$$\begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \\ \vdots \\ \hat{v}_m \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & & & \\ | & & & \\ a_{m1} & & \text{———} & a_{mn} \end{bmatrix} \begin{bmatrix} v_1 \\ | \\ v_n \end{bmatrix}$$

$\underline{\hat{V} = A.V}$

$\beta = [\beta_1\ \beta_2\ \beta_3\ \dots\ \beta_m]^T \neq 0$

$\beta_1 \cdot \hat{v}_1 + \beta_2 \cdot \hat{v}_2 + \dots + \beta_m \cdot \hat{v}_m = 0$

$\hat{V} = \begin{bmatrix} \hat{v}_1 \\ \hat{v}_2 \\ \vdots \\ \hat{v}_m \end{bmatrix} \quad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{bmatrix}$

$\beta^T \cdot \hat{V} = \beta^T \cdot A \cdot V = 0$

$V^T \cdot (A^T \cdot \beta) = 0 \qquad A^T \cdot \beta = 0$

$$\begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & & & \\ \vdots & & & \\ a_{1n} & \cdots & & a_{mn} \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ | \\ \beta_m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$m > n$ (Bilinmeyen > Denklem sayısı)

Infinite Solutions

→

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{m1} & \cdots & & a_{mn} \end{bmatrix}$$

$R(A) \rightarrow$ row space of A

$S = \{r_1\ r_2 \cdots r_m\}$

$R(A) = \text{span}(S)$

$$C_1 = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} \quad C_2 = \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} \cdots \quad C_n = \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix}$$

$C(A) =$ column space

$S = \{C_1\ C_2 \cdots C_n\}$

$C(A) = \text{span}(S)$

$r_1 = [a_{11}\ a_{12} \cdots a_{1n}]$

$r_2 = [a_{21}\ a_{22} \cdots a_{2n}]$

$\vdots$

$r_n = [a_{m1}\ a_{m2} \cdots a_{mn}]$

$A.x = 0$ iken NULL $N(A)$

x'in tüm çözümleri Null space olur.

$S = \{C_1\ C_2 \ldots C_n\}$ Rank = # of max. column vector that is in S linearly independent.

$\dim(C(A))$ $r(A) \rightarrow$ rank of A

m>n
Linear dependent
∞ solutions

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad C_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad C_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad C_3 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad r(A) = 2$$

m denklem, n bilinmeyen
m>n

n>m 0 olmayan x vektörü var Linearly Independent x

Rank max column sayısı linearly independent → rank n değil

Elem. row Op → 0 olan satır sayı bakılır → 0 satır yoksa rank = n-m

k adet çıkarsa m-k kadar 0 olmayan var

k≠0 ; n-m+k Keyfi değiş. seç.

Lineer Bağımsız yaz. → Toplam d. − Keyfi d. = Column Say. (Birb. bağımsız)

$\underline{m-k}$

ÖR: $n > m$

* En az bir. değ. keyfi seç.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 \end{bmatrix} \quad m{=}3 \left\{ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \right\} k{=}2$$

$n = 4 \rightarrow$ Sütun
$m = 3 \rightarrow$ Satır

El. Row Op.
$k = 2$

$n - m = 4 - 3 = \underline{1}$

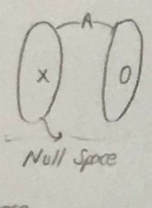
Bilinmeyen −

$n - m + k = 3$

Bağ.(-) seç. d. sayısı

$r(A) \leq m, n\ (m.n)$

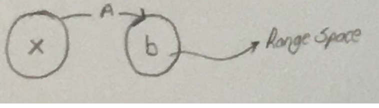
* $N(A)$
$M(A)$



$A.x = 0$

$n = M(A) + r(A)$

* *Range Space*



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Eigenvalues and Eigenvectors

$A . X_i = \lambda_i X_i$ $\qquad A = \{a_{ij}\}_{n \times n}$ $\qquad \lambda_i$ is a complex number, representatio of A matrix

$\lambda_i . x_i - A . X_i = 0 \quad , \quad X_i \neq 0$ $\qquad \det(\lambda_i I - A) = 0$ $\qquad$ n dif. solutions for previous eq. $(\lambda_1 . \lambda_2 \dots \lambda_n)$

$(\lambda_i . I - A) . X_i = 0$ $\qquad = (\lambda - \lambda_1)(\lambda - \lambda_2) \dots (\lambda - \lambda_n) = 0$ $\qquad$ Özdeğer (Eigenvalue)

$X_i \neq 0$

Ex :

$A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ $\qquad n = 2 \quad (\lambda_1 , \lambda_2)$

$\lambda . I - A = \begin{bmatrix} \lambda - 1 & -1 \\ 1 & \lambda - 1 \end{bmatrix} \Rightarrow \det(\lambda I - A) = (\lambda - 1)^2 + 1 = 0$

$(\lambda - 1)^2 = -1 = j^2$

$\lambda - 1 = \mp j \Rightarrow \lambda = 1 \mp j$

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$\left.\begin{aligned} \lambda_1 &= 1 + j \\ \lambda_2 &= 1 - j \end{aligned}\right\}$ Eigenvalues

$P_A(\lambda) = \det(\lambda I - A) = \lambda^n + C_{n-1} \lambda^{n-1} + C_{n-2} \lambda^{n-2} \dots + c_1 . \lambda + C_0 = \lambda^n + \sum_{k=1}^{n} C_{n-k} . \lambda^{n-k}$ $\quad , n \times n$ $\quad$ = Karakteristik Polinom

$\left.\begin{matrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{matrix}\right\}$ Çözüm sayısı = Eigenvalues

→ $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $\qquad \lambda . I - A = \begin{bmatrix} \lambda - a_{11} & -a_{12} \\ -a_{21} & \lambda - a_{22} \end{bmatrix} = (\lambda - a_{11})(\lambda - a_{22}) - a_{12} . a_{21} = \lambda^2 - (a_{11} + a_{22}) . \lambda + \underline{a_{11} . a_{22} - a_{12} . a_{21}}$

→ $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ $\qquad P_A(\lambda) = \lambda^3 - (a_{11} + a_{22} + a_{33}) . \lambda^2 + (a_{11} a_{22} + a_{12} . a_{21} + a_{11} . a_{33} - a_{13} a_{31} + a_{22} . a_{33} - a_{32} . a_{13}) \lambda$

$- (a_{11} a_{22} a_{33} - a_{11} a_{23} a_{32} - a_{22} a_{13} a_{31} - a_{33} a_{12} a_{21} + a_{12} a_{23} a_{31} + a_{13} a_{21} a_{32})$

$C_0 = (-1)^n . \det(A)$

Eigenvectors

$\lambda_1 , \lambda_2 \dots \lambda_n \longrightarrow X_1 , X_2 , \dots , X_n$ $\qquad (\lambda - \lambda_1)(\lambda - \lambda_2) \dots (\lambda - \lambda_n)$ $\qquad$ * Bir matrisin öz değ. top. = diagonal el. top. (*)

$A . x_i = \lambda_i . X_i$ $\qquad (\lambda_i . I - A) . X_i = 0$ $\qquad X^n - (\lambda_1 + \lambda_2 + \dots + \lambda_n) . \lambda^{n-1}$ $\qquad \text{trace}(A) = \sum_{i=1}^{n} \lambda_i = \sum_{i=1}^{n} a_{ii}$

$A \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ $\qquad \lambda_1 + \lambda_2 = 4$ * $\qquad \lambda_1 = 1$

$\lambda_1 . \lambda_2 = 3$ det $\qquad \lambda_2 = 3$

* Simetrik mat. tüm öz değ. reeldir.

→ $\lambda_1 \quad \lambda_2 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$(\lambda_1 . I - A) X_1 = 0$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\qquad \underset{-\infty}{x_{11}} + \underset{\propto}{x_{12}} = 0 \Rightarrow X_1 = \begin{bmatrix} -\propto \\ \propto \end{bmatrix}$

→ Linearly Independent ?

$$k_1\begin{bmatrix}-\alpha\\ \alpha\end{bmatrix}+k_2\begin{bmatrix}\beta\\ \beta\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix} \Rightarrow \begin{bmatrix}-\alpha & \beta\\ \alpha & \beta\end{bmatrix}\begin{bmatrix}k_1\\k_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix} \quad \det(A)=-2\alpha\beta\neq 0 \quad \left.\begin{matrix}k_1=0\\k_2=0\end{matrix}\right\} \text{Linearly Independent}$$

→ $x^T.A\,x=\lambda.x.x^T$ (Transpoze alınırsa)

$\cancel{x^T}A^T.x=\cancel{x^T}(\lambda^*).x$

$A^T.x=\lambda^*.x \quad \Rightarrow \lambda_1=1-j,\ \lambda_2=1+j$

$A.x=\lambda x \quad \Rightarrow \lambda_1=1+j$

$\lambda_2=1-j$

→ $A \quad \lambda_1,\lambda_2\ldots\lambda_n \quad A^{-1}$

$A^{-1}A.x=A^{-1}.\lambda.x$

$I.x=A^{-1}.\lambda.x$

$\dfrac{1}{\lambda}.x=A^{-1}.x$

$\lambda.x=A.x$

22.04.2016 Friday

COFACTOR

$A=(a_{ij})_{n\times n}$

The cofactor of $a_{ij} \Rightarrow (-1)^{i+j}.M_{ij}=c_{ij}$

$C=(c_{ij})_{n\times n}$ the cofactor of A

INVERSE MATRICES

$A=(a_{ij})_{n\times n}$, $B=(b_{ij})_{n\times n}$

If $A.B=I$, then B is inverse of A is denoted by A^{-1}

$A.A^{-1}=I \qquad A^{-1}=\dfrac{1}{\det(A)}.C^T=\dfrac{1}{\det(A)}.\text{adj}(A) \qquad \text{adj}(A)=C^T$

Ex: $A=\begin{bmatrix}1&1&1\\2&3&4\\1&0&-1\end{bmatrix}$

a) $|A|=?$
b) $\text{adj}(A)=?$
c) $A^{-1}=?$

a-) $\det(A)=1.\underbrace{\begin{vmatrix}1&1\\3&4\end{vmatrix}}_{1}+(-1)\underbrace{\begin{vmatrix}1&1\\2&3\end{vmatrix}}_{1}=0 \Rightarrow$ Singular matrix

b-) $C^T=\text{adj}(A)$

$c_{11}=(-1)^2\begin{vmatrix}3&4\\0&-1\end{vmatrix}=-3 \qquad c_{21}=(-1)^3\begin{vmatrix}1&1\\0&-1\end{vmatrix}=1 \qquad c_{31}=(-1)^4\begin{vmatrix}1&1\\3&4\end{vmatrix}=1$

$c_{12}=(-1)^3\begin{vmatrix}2&4\\1&-1\end{vmatrix}=6 \qquad c_{22}=(-1)^4\begin{vmatrix}1&1\\1&-1\end{vmatrix}=-2 \qquad c_{32}=(-1)^5\begin{vmatrix}1&1\\2&4\end{vmatrix}=-2$

$c_{13}=(-1)^4\begin{vmatrix}2&3\\1&0\end{vmatrix}=-3 \qquad c_{23}=(-1)^5\begin{vmatrix}1&1\\1&0\end{vmatrix}=1 \qquad c_{33}=(-1)^6\begin{vmatrix}1&1\\2&3\end{vmatrix}=1$

$C=\begin{bmatrix}-3&6&-3\\1&-2&1\\1&-2&1\end{bmatrix} \qquad \text{adj}(A)=C^T=\begin{bmatrix}-3&1&1\\6&-2&-2\\-3&1&1\end{bmatrix}$

c-) $A^T=\dfrac{1}{\det(A)}.\text{adj}(A) \qquad A^T$ does not EXIST $\quad \det(A)=0$

Ex:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix} \qquad \det(A) = ?$$

$$\det(A) = a_{11} \cdot \underbrace{M_{11}}_{a_{22} \cdot \underbrace{M_{22}}_{a_{33} \cdot a_{44}}} = a_{11} \cdot a_{22} \cdot a_{33} \cdot a_{44}$$

$$\det(A) = \prod_{i=1}^{n} a_{ii} \longrightarrow \text{for Triangular Matrices}$$

Ex:

$$\left.\begin{aligned} 2 \cdot x_1 &= 5 + x_2 \\ 3 + 2 \cdot x_2 + 3 \cdot x_1 &= 0 \end{aligned}\right\} \text{Use Cramer's Rule to solve the system}$$

$$2x_1 - x_2 = 5$$
$$3x_1 + 2x_2 = -3$$

$$\underbrace{\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 5 \\ -3 \end{bmatrix}}_{b}$$

$\det(A) = 4 - (-3) = 7 \rightarrow$ non-singular matrix $\neq 0$

$$x_1 = \frac{\begin{vmatrix} 5 & -1 \\ -3 & 2 \end{vmatrix}}{\det(A)} = \frac{7}{7} = 1$$

$$x_2 = \frac{\begin{vmatrix} 2 & 5 \\ 3 & -3 \end{vmatrix}}{\det(A)} = \frac{-21}{7} = -3$$

Ex: $ab \neq 0$

$$\left.\begin{aligned} ax_1 - 2bx_2 &= c \\ 3ax_1 - 5bx_2 &= 2c \end{aligned}\right\} \text{Solve system by Cramer's Rule}$$

$$\underbrace{\begin{bmatrix} a & -2b \\ 3a & -5b \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} c \\ 2c \end{bmatrix}}_{b}$$

$\det(A) = -5ab + 6ab = ab \neq 0 \rightarrow$ non-singular matrix

$$x_1 = \frac{\begin{vmatrix} c & -2b \\ 2c & -5b \end{vmatrix}}{\det(A)} = \frac{-bc}{ab} = \frac{-c}{a}, \quad x_2 = \frac{\begin{vmatrix} a & c \\ 3a & 2c \end{vmatrix}}{\det(A)} = \frac{-ac}{ab} = \frac{-c}{b} \Rightarrow \text{Solution Set} = \left\{ \frac{-c}{a}, \frac{-c}{b} \right\}$$

Ex: $\left.\begin{aligned} 2x+y-z&=3 \\ x+y+z&=1 \\ x-2y-3z&=4 \end{aligned}\right\}$ Solve system by Cramer's Rule

$$\underbrace{\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -2 & -3 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}}_{b}$$

$\det(A)=5$

$$x=\frac{\begin{vmatrix} 3 & 1 & -1 \\ 1 & 1 & 1 \\ 4 & -2 & -3 \end{vmatrix}}{5}=$$

$$y=\frac{\begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & 1 & -3 \end{vmatrix}}{5}=$$

$$z=\frac{\begin{vmatrix} 2 & 1 & 3 \\ 1 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix}}{5}=$$

LINEAR DEPENDENCE - INDEPENDENCE

Let $S=\{v_1, v_2, \ldots, v_n\}$ be a vector set

For the below equation

$$k_1 \cdot v_1 + k_2 v_2 + \ldots + k_n \cdot v_n = 0 \quad , \quad k_i \in \mathbb{R} \;, i=1,2,\ldots,n$$

if the only solution is $k_1=0, k_2=0, \ldots, k_n=0$ then the vectors $v_1, v_2, \ldots v_n$ are linearly independent

If there exist $k_1, k_2, \ldots, k_n$ not all of them 0 then the vectors $v_1, v_2, \ldots, v_n$ are linearly dependent.

$v_3=0 \rightarrow k_3=\alpha$ linear bağımsız $\qquad v\neq 0 \quad k \cdot v=0 \quad k=0$ linear bağımsız.

Ex: $\left.\begin{aligned} v_1&=[2 \;\; -1 \;\; 0]^T \\ v_2&=[1 \;\; 2 \;\; 5]^T \\ v_3&=[7 \;\; -1 \;\; 5]^T \end{aligned}\right\}$ Determine whether the vectors are linearly independent?

$$\boxed{k_1 \cdot v_1 + k_2 \cdot v_2 + k_3 \cdot v_3 = 0}$$

Tringular → Only one sol.

$$k_1 \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} + k_3 \begin{bmatrix} 7 \\ -1 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow \left.\begin{aligned} 2k_1+k_2+7k_3&=0 \\ -k_1+2k_2-k_3&=0 \\ 5k_2+5k_3&=0 \end{aligned}\right\} L_2 \to L_1 + 2k_2$$

Esalon form

$$\left\{\begin{aligned} 2k_1+k_2+7k_3&=0 \\ 5k_2+5k_3&=0 \\ \cancel{5k_2+5k_3=0} \end{aligned}\right\} L_2 = L_3$$

↓

∞ sayıda çözüm var.

$\left.\begin{aligned} k_3&=\alpha \neq 0 \\ k_2&=-\alpha \neq 0 \\ k_1&=-3\alpha \neq 0 \end{aligned}\right\}$ Solution set $\Rightarrow \alpha$ is any number

↳ 0'dan farklı değerler aldığından dolayı lineer bağımlı

Leading Variable $=k_1, k_2$ $\qquad$ Free Variable $=k_3$

$$V = \begin{array}{c} \begin{bmatrix} 2 & -1 & 7 \\ 1 & 2 & -1 \\ 0 & 5 & 5 \end{bmatrix} \\ \uparrow \ \ \uparrow \ \ \uparrow \\ V_1 \ \ V_2 \ \ V_3 \end{array} \qquad \det(V) = 5 \cdot \begin{vmatrix} 2 & 7 \\ 1 & -1 \end{vmatrix} + 5 \cdot \begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} = 0$$

$\det(V) = 0 \rightarrow$ Singular Vector $\Rightarrow V_1, V_2, V_3 \rightarrow$ Linearly dependent

Ex : Let the vectors u, v, w are linearly independent. Are the vectors $(u-v), (2u+v-w), (v+3w)$ linearly independent ?

$k_1(u-v) + k_2(2u+v-w) + k_3(v+3w) = 0$ $\qquad \boxed{k_1 = k_2 = k_3 = 0}$ old. dolayı linear bagımsız dır.

$k_1 u - k_1 v + 2k_2 u + u k_2 - k_2 w + k_3 v + k_3 3w$

$\underbrace{(k_1 + 2k_2)}_{\alpha_1 = 0} u + \underbrace{(-k_1 + k_2 + k_3)}_{\alpha_2 = 0} v + \underbrace{(-k_2 + 3k_3)}_{\alpha_3 = 0} w = 0$ $\rightarrow$ u, v, w = linear bagımsız old. dolayı

$k_1 + 2k_2 = 0$

$-k_1 + k_2 + k_3 = 0 \quad \rightarrow k_2 = 0, k_1 = 0, k_3 = 0$

$-k_2 + 3k_3 = 0$

EIGENVALUES - EIGENVECTORS

$A = (a_{ij})_{n \times n}$

$A \cdot v = \lambda \cdot v \qquad v \neq 0$ vector

$\lambda \downarrow$ scalar

$(A - \lambda I) \cdot v = 0$

λ: eigenvalue, v: eigenvectors

$\det(\lambda I - A) = 0$ Characteristic Equation

$\boxed{n=2}$ C.E $\Rightarrow \lambda^2 - \overbrace{(\Delta_1 + \Delta_2)}^{\text{1. order principal minor}} \lambda + \Delta = 0$

$\boxed{n=3}$ C.E. $\Rightarrow \lambda^3 - (\Delta_1 + \Delta_2 + \Delta_3)\lambda^2 + \overbrace{(\Delta_{12} + \Delta_{21} + \Delta_{13})}^{\text{second order}} \lambda - \Delta = 0$

$\boxed{n=4}$ C.E. $\Rightarrow \lambda^4 - (\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4)\lambda^3 + (\Delta_{12} + \Delta_{13} + \Delta_{14} + \Delta_{23} + \Delta_{24} + \Delta_{34})\lambda^2$

$- \underbrace{(\Delta_{123} + \Delta_{124} + \Delta_{134} + \Delta_{234})}_{\text{3. order}} \lambda + \Delta = 0$

$\sum_{i=1}^{n} \lambda_i = tr(A) \quad , \ tr(A) = \sum_{i=1}^{n} a_{ii}$

Ex:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

λ_1 λ_2

Bulunduğu tamsatır-sütun $= 0$

$$CE \Rightarrow \lambda^2 + 1 = 0$$

$$\lambda^2 = -1$$

$$\lambda_2 = i$$

$$\lambda_3 = -i$$

Ex:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}$$

$$\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = a_{11} + a_{22} + a_{33} + a_{44} = \text{trace}(A) = \text{diagonal}$$

$$\underbrace{\lambda_1}_{a_{11}} \cdot \underbrace{\lambda_2}_{a_{22}} \cdot \underbrace{\lambda_3}_{a_{33}} \cdot \underbrace{\lambda_4}_{a_{44}} = a_{11} \cdot a_{22} \cdot a_{33} \cdot a_{44}$$

$$\det(\lambda I - A) = 0$$

$$\begin{bmatrix} \lambda - a_{11} & -a_{12} & -a_{13} & -a_{14} \\ 0 & \lambda - a_{22} & -a_{23} & -a_{24} \\ 0 & 0 & \lambda - a_{33} & -a_{34} \\ 0 & 0 & 0 & \lambda - a_{44} \end{bmatrix}$$

$$\det(\lambda I - A) = \underbrace{(\lambda - a_{11})}_{\lambda_1 \cdot a_{11}} \underbrace{(\lambda - a_{22})}_{\lambda_2 \cdot a_{22}} \underbrace{(\lambda - a_{33})}_{\lambda_3 \cdot a_{33}} \underbrace{(\lambda - a_{44})}_{\lambda_4 \cdot a_{44}} = 0$$

Ex:

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 7 \end{bmatrix} \qquad \lambda_1 = 2, \ \lambda_2 = 7$$

$\lambda_1 = 2, \ V_1$

$A \cdot V_1 = \lambda_1 \cdot V_1$

$$\begin{bmatrix} 2 & 1 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix}$$

$\lambda_2 = 7, \ V_2$

$A \cdot V_2 = \lambda_2 \cdot V_2$

$$\begin{bmatrix} 2 & 1 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = 7 \cdot \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$V_2 = \begin{bmatrix} \ \end{bmatrix}$ for $\lambda_2 = 7$

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$\rightarrow A \quad \lambda_1, \lambda_2, \ldots, \lambda_n$

$A = A^T$

$A.X_i = \lambda_i.X_i$

$X_i^T.A.X_i = \lambda_i.X_i^T.X_i$

$\overline{X_i}.\vec{A}.\overline{X_i}^T = \overline{\lambda_i}.\overline{X_i}^T.\overline{X_i}$

$\overline{X_i}.A.\cancel{\overline{X_i}^T} = \overline{\lambda_i}.\cancel{\overline{X_i}^T}.\overline{X_i}$

$A.\overline{X_i} = \overline{\lambda_i}.\overline{X_i}$

$\overline{\lambda_i} = \lambda_i \quad \Rightarrow \lambda_i = a + jb \quad \overline{\lambda_i} = a - jb \quad \Rightarrow b = 0$ (Simetrik matrisin öz değerleri reeldir.)

$\overline{X_i}$ = Hem transpoz + eşlenik $\Rightarrow \bar{x} = (x^T)^*$

$\rightarrow A^T.A \rightarrow \lambda_i^*.\lambda_i$

$\left.\begin{array}{l} a+jb = \lambda_1 \\ a-jb = \lambda_2 \end{array}\right\rangle A \qquad \left.\begin{array}{l} a-jb = \lambda_1 \\ a+jb = \lambda_2 \end{array}\right\rangle A^T$

$A^T \downarrow \lambda_i^* \qquad A \downarrow \lambda_i$

Her zaman simetrik matristir.

ISPAT : $A.x_i = \lambda.x_i$

$(A.x_i)^T.A.x_i = \cancel{x_i^T}.A^T.A.X$

$(x_i.x_i)^T.A.x_i = x_i^T.\lambda_i^*.\lambda_i.x_i$

$= \lambda_i.\lambda_i^*.\cancel{x_i^T}.x_i$

$A^T.A.x_i = \lambda.\lambda_i^*.x_i$

$\lambda_1 \neq \lambda_2 \neq \ldots \neq \lambda_n \rightarrow$ Set is basis Linearly independent

$\rightarrow$ Özvektör 0 olmaz.

$\rightarrow A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad \lambda_1 = \lambda_2 = 1, \quad \lambda_3 = 2$

$v_{13} = 0$

$-v_{13} = 0$

$\underset{v_1}{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}} \underset{v_2}{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}} \underset{v_3}{\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}}$

$\lambda_1.I - A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \\ v_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} \alpha & 0 & -\gamma \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{bmatrix} \Rightarrow \det(A) = \alpha.\beta.\gamma$ = Linear Independent

$\lambda_3.I - A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_{31} \\ v_{32} \\ v_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\underset{-1}{\underbrace{v_{31}}} + \underset{1}{\underbrace{v_{33}}} = 0$

$\rightarrow$ $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{bmatrix}$ $\lambda_1 = \lambda_2 = 1$, $\lambda_3 = 2$

$$\lambda_1 I - A = \begin{bmatrix} 0 & -1 & 2 \\ 0 & 0 & -3 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{12} \\ u_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$u_{13} = 0$

$u_{12} = 0$

$$\begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \beta \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 5\gamma \\ 3\gamma \\ \gamma \end{bmatrix}$$

$$\lambda_3 I - A = \begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{31} \\ u_{32} \\ u_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$u_{32} = 3u_{33}$

$u_{31} - u_{32} - 2\underbrace{u_{33}}_{\gamma} = 0$

α Öz değer farklı ise Lineer Bağımsız

$\rightarrow \det(A) = 0 \rightarrow$ Lineer Bağımsız set yok.

Lineer Bağımlı

Norm

$f(x, y) = x^T . y = x^T . x$

$\varphi(x) \quad = \varphi(x) = \sum_{i=1}^{n} x_i^{n}$

$y = x$

$\sqrt{f(x,y)} = \sqrt{x^T . x} = \varphi(x)$

real valued function of x : $\varphi(x)$

1-) $\varphi(x+y) \leq \varphi(x) + \varphi(y)$

2-) $\varphi(\alpha . x) = |\alpha| . \varphi(x)$

3-) $\varphi(x) = 0$, iff $x = 0$, and $\varphi(x) > 0 \quad \forall x \neq 0$

$$\sqrt{(x_1+y_1)^2 + (x_2+y_2)^2 + \ldots + (x_n+y_n)^2}$$

$x_1^2 + 2.x_1.y_1 + y_1^2 \quad => 2.x_1.y_1 \leq x_1^2 + y_1^2$

$\leq x_1^2 + y_1^2 + x_1^2 + y_1^2$

$2.(x_1^2 + y_1^2) + 2.(x_2^2 + y_2^2)$

$$2.\underbrace{(x_1^2 + x_2^2 \ldots + x_n^2)}_{a} + 2.\underbrace{(y_1^2 + y_2^2 + \ldots + y_n^2)}_{b}$$

$\sqrt{} \quad \leq \sqrt{a} + \sqrt{b}$

$x = [x_1 \ x_2 \ \ldots \ x_n]^T$

$\rightarrow \varphi(x) = ||x||_p = (|x_1|^p + |x_2|^p + \ldots + |x_n|^p)^{1/p} \longrightarrow$ Formül

$$||x||_1 = |x_1| + |x_2| + \ldots + |x_n| = \sum_{i=1}^{n} |x_i|$$

$$||x||_2 = \sqrt{x_1^2 + x_2^2 + \ldots + x_n^2} = \sqrt{\sum_{i=1}^{n} x_i^2}$$

$$||x||_\infty = \max_{i=1,2,\ldots,n} |x_i|$$

MATRICE NORM

supreme = Least Upper Bound (En küçük üst sınır)

$||x||$

$A = \{a_{ij}\}_{n \times n}$ $\quad ||A||_p = \sup \frac{||Ax||_p}{||x||_p} = \max ||Ax||_p \leqslant$

$||x||_p = 1$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ & & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \rightarrow y = A.x = \begin{bmatrix} \sum_{j=1}^{n} a_{1j} \cdot x_j \\ \sum_{j=1}^{n} a_{2j} \cdot x_j \\ \vdots \end{bmatrix} \qquad y_i = \sum_{j=1}^{n} a_{ij} \cdot x_j$$

$||A||_\infty$

$$|y_i| = \left| \sum_{j=1}^{n} a_{ij} \cdot x_j \right| \leqslant \sum_{j=1}^{n} |a_{ij} \cdot x_j| = \sum_{j=1}^{n} |a_{ij}||x_j|$$

$$\leqslant \left(\sum_{j=1}^{n} |a_{ij}| \right) ||x||_\infty$$

$$\alpha = \max_{i=1,2,\ldots,n} \left\{ \sum_{j=1}^{n} |a_{ij}| \right\} = ||A||_\infty$$

α'nın 1 normunu ispatla
Satırlar incele
Mutlak d. incele
sonsuz normun tersi

$$||y||_\infty \leqslant \alpha ||x||_\infty$$

$$\underbrace{||Ax||_\infty \leqslant \alpha ||x||_\infty}_{\text{Least Upper Bound}} = ||A||_\infty$$

$||A||_2 = \sqrt{\lambda_{max}(A^T.A)}$ (2 Norm)

$\times$ $||A.x|| \leqslant ||A||\,||x||$ $\qquad ||A+B|| \leqslant ||A|| + ||B||$

$\rightarrow$ $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $\quad x = [x_1 \; x_2 \; \ldots \; x_n]^T \quad A = \{a_{ij}\}_{n \times n}$ quadratic

$$x^T.A.x = \sum_{i=1}^{n} \sum_{j}^{n} a_{ij} \cdot x_i \cdot x_j$$

$x = [x_1 x_2]^T$

$x \neq 0$

$$= a_{11} \cdot x_1^2 + a_{12} \cdot x_1 \cdot x_2 + a_{21} \cdot x_2 \cdot x_1 + a_{22} \cdot x_2^2$$

$$= a_{11} x_1^2 + a_{22} \cdot x_2^2 + 2 \cdot a_{12} \cdot x_1 \cdot x_2 > 0 \text{ (Kosul degerlendirilmesi)}$$

$2ab \leqslant a^2 + b^2$
$x > -|x|$

$$= (\sqrt{a_{11}}|x_1| - \sqrt{a_{12}} \cdot |x_2|)^2 + 2\sqrt{a_{11} \cdot a_{22}}\, |x_1||x_2| + 2 \cdot a_{12} \cdot x_1 \cdot x_2 \geqslant \underbrace{(\sqrt{a_{11}} \cdot |x_1| - \sqrt{a_{22}}|x_2|)^2}_{\geqslant 0} + \underbrace{2\sqrt{a_{11} \cdot a_{22}}\, |x_1||x_2| - 2|a_{12}||x_1||x_2|}_{2(\sqrt{a_{11}a_{22}} - |a_{12}|)|x_1||x_2|}$$

$\hookrightarrow a_{11} > 0$
$a_{22} > 0$

Simetrik bir a mat. için quadra > 0 a mat. btn öz degerleri 0'dan büyükse sağlar. = Positive definite matrix
(Sadece simetrik matris için kullanılır)

$$\underbrace{x^T.A.x}_{\text{quadratic form}} = \lambda_i . \underbrace{x^T . x}_{x\text{'in 2. normunu karesi} > 0}$$

$\hookrightarrow \lambda_i > 0$

$\sqrt{a_{11}a_{22}} - |a_{12}| > 0$

$\sqrt{a_{11} \cdot a_{22}} > |a_{12}| \Rightarrow a_{11} \cdot a_{22} > a_{12}^2$

$\lambda_1 \lambda_2 = a_{11} \cdot a_{22} - a_{12}^2 > 0$

$a_{11} + a_{22} = \lambda_1 + \lambda_2$

Quantile

3x3 Matrix

1.Norm
2.Norm
∞ Norm
En önemli 3 Norm

benefit of the doubt

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$\rightarrow A = A^T$, $\lambda_i(A) > 0$, $A \rightarrow$ Pozitive D.

$\lambda_i(A) \geq 0$ PSD $\rightarrow$ Positive Semi Definite

$A = \begin{bmatrix} 0 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(1.NORM)
$\|A\|_1 = 2$ = Eleman toplamları (Sütun)

$\|A\|_\infty = 2$ = Mutlak değer t. (Satır)

$\sqrt{\text{Max Öz değerinin}}$

$\|A\|_2 = \sqrt{\lambda_{max}(A)}$

$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

$\lambda^2 - 3\lambda + 1 = 0 \rightarrow \Delta = 5$

$\frac{3+\sqrt{5}}{2}$ = Öz değer $\rightarrow \sqrt{\frac{3+\sqrt{5}}{2}} = \|A\|_2$

$\rightarrow X = [1, 2, -4]$

$\|x\|_1 = 7$, $\|x\|_2 = \sqrt{21}$, $\|x\|_\infty = 4$

Mutlak değer top. Karelerinin top. $\sqrt{\ }$ En byk elemanın Mutlak d.

Özdeğer ?
Norm ?

$\rightarrow A > 0$ PD Btn öz d. Pozitif

$A \geq 0$ PSP

$\alpha > 0$ $\alpha . I + A > 0$

$\rightarrow$ $Re\{\lambda_i(A)\} > 0$ Stable Matrix

$Re\{\lambda_i(A)\} > 0$ Semi-Stable Matrix

$P = P^T > 0$

$PA + A^T.P > 0$

$x^T(PA + A^TP)x > 0$

Stable Matrix, $A \in H$

$P = P^T > 0$, Diagonal

$P.A + A^T.P > 0 \rightarrow$ Diagonally Stable

$A \in D$

$PA + A^TP \geq 0$

Diagonally semi-stable

$A \in D_0$

$\rightarrow A = \{a_{ij}\}_{n \times n}$

Comparison matrix of $A \rightarrow S$

$\left. \begin{array}{l} s_{ii} = a_{ii} > 0 \\ s_{ij} = -|a_{ij}| \end{array} \right\}$ $S \in D \Rightarrow A \in C$

$Re\{\lambda_i(S)\} > 0, A \in C$

$Re\{\lambda_i(S)\} \geq 0, A \in C_0$

Ex:

$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $S = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}$

$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & P_1 \\ 0 & 0 \end{bmatrix}$

P . A

$PA + A^TP = \begin{bmatrix} 0 & P_1 \\ P_1 & 0 \end{bmatrix}$ $P_1 > 0$

$A \notin D_0$

$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $S = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$, $A \notin C_0$

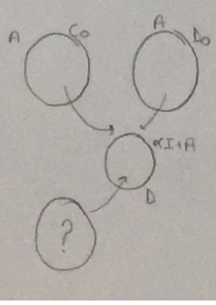
$A + A^T = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $A \in D_0$

$\rightarrow C_0, D_0 \in A$

$\alpha > 0$

$(\alpha I . A) \in D$

additively diagonally stablematrix



$A = \begin{bmatrix} 0 & 1 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ $S = \begin{bmatrix} 0 & -1 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

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EXAM'S PRACTICE

$$A=\begin{bmatrix}1 & 0 & -1\\ 0 & 1 & 1\\ 1 & -1 & 1\end{bmatrix}$$

$\|A\|_1 =$ Mutlak değer toplamı (Sütun) $= 3$

$\|A\|_\infty = 3 =$ (Satır)

$$\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & -1\\ -1 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & -1\\ 0 & 1 & 1\\ 1 & -1 & 1\end{bmatrix}=\begin{bmatrix}2 & -1 & 0\\ -1 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}$$

$A^T \cdot A$

$\lambda_1 = 1$

$\lambda_2 = \lambda_3 = 3$

$\|A\|_2 = \sqrt{\lambda_{max} \cdot A^T \cdot A}$

$= \sqrt{3}$

$$\lambda \cdot I - A = \begin{bmatrix}\lambda-2 & 1 & 0\\ 1 & \lambda-2 & 0\\ 0 & 0 & \lambda-3\end{bmatrix} \Rightarrow \underbrace{(\lambda-3)}_{\lambda=3}\left[(\lambda-2)^2-1\right]=0$$

→ Complex sayıya göre öz değer, lineer bağımlı/bağımsız, öz vektör ? = SINAV SORUSU

$$A=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$$

$\lambda_1+\lambda_2 = 0 \Rightarrow trace(A) =$ diagonal el. top.

$\lambda_1 \cdot \lambda_2 = 1 \Rightarrow det(A)$

$\lambda_1 = j$

$\lambda_2 = -j$

$$\lambda_1 I - A = \begin{bmatrix}j & -1\\ 1 & j\end{bmatrix}\begin{bmatrix}u_{11}\\ u_{12}\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$j / \; j \cdot u_{11} - u_{12} = 0$

$+ \quad u_{11} + j \cdot u_{12} = 0$

$0 \, u_{11} = 0$

$u_{11} = \alpha$

$u_{12} = j \cdot \alpha$

$$\lambda_2 \cdot I - A = \begin{bmatrix}-j & -1\\ 1 & j\end{bmatrix}\begin{bmatrix}u_{21}\\ u_{22}\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$-j \cdot u_{11} - u_{22} = 0$

$u_{21} - j \cdot u_{22} = 0$

$u_{22} = \beta$

$u_{21} = j \cdot \beta$

* $det(A) = 0 \Rightarrow$ Lineer Bağımlı
* Öz Vektörler 0'dan farklı olmak zorunda (α ve β)

→ 3'e 3 olanını çöz.

$$u_1 = \begin{bmatrix}\alpha\\ j \cdot \alpha\end{bmatrix}$$

$$u_2 = \begin{bmatrix}j\beta\\ \beta\end{bmatrix}$$

$k_1 \cdot u_1 + k_2 \cdot u_2 = 0$

$$k_1 \cdot \begin{bmatrix}\alpha\\ j\alpha\end{bmatrix} + k_2 \cdot \begin{bmatrix}j\beta\\ \beta\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

$-j(k_1 \cdot \alpha + j \cdot k_2 \cdot \beta) = 0$

$+ \quad j \cdot k_1 \cdot \alpha + k_2 \cdot \beta = 0$

$-j^2 = 1 = -i^2$

$2k_2 \cdot \beta = 0, \; \beta \neq 0 \Rightarrow k_2 = 0, \; k_1 = 0$ (Linearly Independent)

→ $$A=\begin{bmatrix}1 & 0 & 5\\ -2 & -4 & -3\\ 3 & 6 & 0\end{bmatrix}$$ Öz değerlerini bulun.

Karakteristik Polinom $P(\lambda) = \lambda^3 + \underbrace{3\lambda^2}_{\text{1. Der.}} \underbrace{-\lambda}_{\text{2. Der}} \underbrace{-18}_{det(A)}$

1. Der. Principal Minor top.

$\rightarrow (\lambda - a)(\lambda^2 + b\lambda + c) = \lambda^3 + 3\lambda^2 - \lambda - 18$

$\lambda_1 + \lambda_2 + \lambda_3 = -3 \rightarrow \lambda_2 + \lambda_3 = -5$

$\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = 18 \rightarrow \lambda_2 \cdot \lambda_3 = 9$

$\lambda_1 = 2$

$> \lambda^2 + 5\lambda + 9 = 0$

$\Delta = -11$

$$\frac{-5 \mp j\sqrt{11}}{2}$$

$$A = \begin{bmatrix} -1 & -2 & -7 & 11 \\ 1 & 2 & 4 & 5 \\ -1 & -2 & -6 & 9 \end{bmatrix} \qquad \alpha_1 \begin{bmatrix} -1 \\ -2 \\ -7 \\ 11 \end{bmatrix} + \alpha_2 \begin{bmatrix} 1 \\ 2 \\ 4 \\ 5 \end{bmatrix} + \alpha_3 \begin{bmatrix} -1 \\ -2 \\ -6 \\ 9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Denklem > Bilinmeyen
S. S.

$m = 3$, $n = 4$

$\max(\text{rank}) = 3$

$$-\alpha_1 + \alpha_2 - \alpha_3 = 0 \quad \times$$
$$-2\alpha_1 + 2\alpha_2 - 2\alpha_3 = 0$$
$$-7\alpha_1 + 4\alpha_2 - 6\alpha_3 = 0$$
$$11\alpha_1 + 5\alpha_2 + 9\alpha_3 = 0$$

$$\begin{bmatrix} -1 & 1 & -1 \\ -7 & 4 & -6 \\ 11 & 5 & 9 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-1.(66) - 1(-3) - 1(-79) \neq 0 \neq \det(A) \qquad \underline{\text{Rank} = 3}$$

→ $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ $\quad \|x\|_\infty$ ile $\|x\|_1$ arasındaki ilişki ?

$$\|x_1\| = |x_1| + |x_2| + \ldots + |x_n| = \sum_{i=1}^{n} |x_i| \leq \sum_{i=1}^{n} \|x\|_\infty = \|x\|_\infty \sum_{i=1}^{n} 1 \qquad , \|x\|_\infty \leq \|x\|_1$$

$$= n\|x\|_\infty$$

→ SINAV KONULARI:

3x3 Matris

- Özdeğer
- Özvektör
- Lineer bağımlılık / bağımsızlık
- Norm
- Minors
- Cramer
- Matris Tersi
- Null Space / Range Space / Rank