NN's - 2016 - Final

① Define the differences between Artificial Neural Networks (ANNs) and Cellular Neural Networks (CNNs).

② For a Cellular Neural Network with $r = 1$ neighborhood, give the state equations for all cells and define the nonlinear differential equation of this neural network in vector-matrix form.

$$T = \begin{Bmatrix} 0 & b & 0 \\ e & a & e \\ 0 & b & 0 \end{Bmatrix}$$

→ Amaç, merkezdeki a harfini sırasıyla tüm yerlere yerleştirip, çözüm elde etmektir

$$\Rightarrow \begin{Bmatrix} a & e & 0 \\ b & 0 & 0 \\ 0 & 0 & 0 \end{Bmatrix} \rightarrow \dot{x}_1 \Rightarrow -x_1 + a.y(x_1) + e.y(x_2) + b.y(x_4) + V_1$$

$$\Rightarrow \begin{Bmatrix} e & a & e \\ 0 & b & 0 \\ 0 & 0 & 0 \end{Bmatrix} \rightarrow \dot{x}_2 \Rightarrow -x_2 + e.y(x_1) + a.y(x_2) + e.y(x_3) + b.y(x_5) + V_2$$

$$\Rightarrow \begin{Bmatrix} 0 & e & a \\ 0 & 0 & b \\ 0 & 0 & 0 \end{Bmatrix} \rightarrow \dot{x}_3 \Rightarrow -x_3 + e.y(x_2) + a.y(x_3) + b.y(x_6) + V_3$$

$$\Rightarrow \begin{Bmatrix} b & 0 & 0 \\ a & e & 0 \\ b & 0 & 0 \end{Bmatrix} \rightarrow \dot{x}_4 \Rightarrow -x_4 + b.y(x_1) + a.y(x_4) + e.y(x_5) + b.y(x_7) + V_4$$

$$\Rightarrow \begin{Bmatrix} 0 & b & 0 \\ e & a & e \\ 0 & b & 0 \end{Bmatrix} \rightarrow \dot{x}_5 \Rightarrow -x_5 + b.y(x_2) + e.y(x_4) + a.y(x_5) + e.y(x_6) + b.y(x_8) + V_5$$

$$\dot{x}_6 \Rightarrow -x_6 + b.y(x_3) + e.y(x_5) + a.y(x_6) + b.y(x_9) + V_6$$

$$\dot{x}_7 \Rightarrow -x_7 + b.y(x_4) + a.y(x_7) + e.y(x_8) + V_7$$

$$\dot{x}_8 \Rightarrow -x_8 + b.y(x_5) + e.y(x_7) + a.y(x_8) + e.y(x_9) + V_8$$

$$\dot{x}_9 \Rightarrow -x_9 + b.y(x_6) + e.y(x_8) + a.y(x_9) + V_9$$

Vector-Matrix Form

$$\dot{x} = -x + A.y(x) + u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \\ x_9 \end{bmatrix} = - \begin{bmatrix} -x_1 \\ -x_2 \\ -x_3 \\ -x_4 \\ -x_5 \\ -x_6 \\ -x_7 \\ -x_8 \\ -x_9 \end{bmatrix} + \begin{bmatrix} a & e & 0 & b & 0 & 0 & 0 & 0 & 0 \\ e & a & e & 0 & b & 0 & 0 & 0 & 0 \\ 0 & e & a & 0 & 0 & b & 0 & 0 & 0 \\ b & 0 & 0 & a & e & 0 & b & 0 & 0 \\ 0 & b & 0 & e & a & e & 0 & b & 0 \\ 0 & 0 & b & 0 & e & a & 0 & 0 & b \\ 0 & 0 & 0 & b & 0 & 0 & a & e & 0 \\ 0 & 0 & 0 & 0 & b & 0 & e & a & e \\ 0 & 0 & 0 & 0 & 0 & b & 0 & e & a \end{bmatrix} \cdot \begin{bmatrix} y(x_1) \\ y(x_2) \\ y(x_3) \\ y(x_4) \\ y(x_5) \\ y(x_6) \\ y(x_7) \\ y(x_8) \\ y(x_9) \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \\ u_8 \\ u_9 \end{bmatrix}$$

③ Consider the Hopfield Neural Network that described by the following set of differential equations:

$$\dot{x}(t) = -A.x(t) + W.f(x(t)) + W'f(x(t-\tau)) + I$$

a) Define the equilibrium equation for this neural network model.

$$0 = -A.x^* + Wf(x^*) + W'f(x^*) + I \quad \left(\begin{array}{c} x(t) \\ \text{ve} \\ x(t-\tau) \end{array} \text{ yerine} \rightarrow x^* \text{ yazılır.} \right)$$

b) Shift the equilibrium point of the neural network system to the origin.

$$z(t) = x(t) - x^* \longrightarrow x(t) = z(t) + x^*$$

$$\dot{z}(t) = \dot{x}(t) \longrightarrow I = Ax^* - Wf(x^*) - W'f(x^*)$$

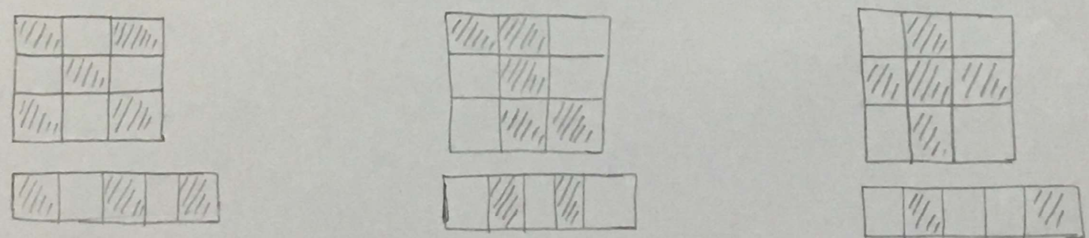
$$\dot{x}(t) = -A.x(t) + W.f(x(t)) + Wf(x(t-\tau)) + I$$

$$\dot{z}(t) = -A.(z(t) + x^*) + W.f(z(t) + x^*) + W'f(z(t-\tau) + x^*) + Ax^* - Wf(x^*) - W'f(x^*)$$

$$\dot{z}(t) = -Az(t) - \cancel{Ax^*} + W\underbrace{\left[f(z(t)+x^*) - f(x^*)\right]}_{g(z(t))} + W'\underbrace{\left[f(z(t-\tau)+x^*) - f(x^*)\right]}_{g(z(t-\tau))} + \cancel{Ax^*}$$

$$\dot{z}(t) = -Az(t) + W(g(z(t))) + W'(g(z(t-\tau)))$$

④ Consider the BAM Neural Network that trained by the following set of pattern pairs.

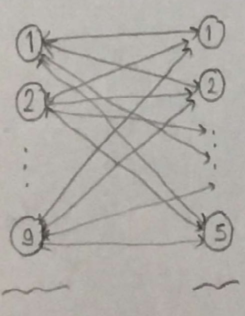


Pair (1) Pair (2) Pair (3)

a) Give the general architectural graph of this BAM Neural Network

$a_i : 1, 2, \ldots, n \rightarrow n = 9 \quad \rightarrow$ Number of neurons of Layer A

$b_i : 1, 2, \ldots, m \rightarrow m = 5 \quad \rightarrow$ Number of neurons of Layer B



→ Simplified Graph

Layer A Layer B

b) Find the weight matrix of the network after storage phase

Pair (1) : $a^1 = [1\ -1\ 1\ -1\ 1\ -1\ 1\ -1\ 1]^T$ $\quad b^1 = [1\ -1\ 1\ -1\ 1]^T$

Pair (2) : $a^2 = [1\ 1\ -1\ -1\ 1\ -1\ -1\ 1\ 1]^T$ $\quad b^2 = [-1\ 1\ -1\ 1\ -1]^T$

Pair (3) : $a^3 = [-1\ 1\ -1\ 1\ 1\ 1\ -1\ 1\ -1]^T$ $\quad b^3 = [-1\ 1\ -1\ -1\ 1]^T$

$\cdot\ W = \sum_{i=1}^{P} a^{(i)}.(b^{(i)})^T \longrightarrow W = a^1.(b^1)^T + a^2.(b^2)^T + a^3.(b^3)^T$

$$a^1.(b^1)^T = \underbrace{\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}}_{9\times1} . \underbrace{\begin{bmatrix} 1 & -1 & 1 & -1 & 1 \end{bmatrix}}_{1\times5} = \underbrace{\begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & 1 \end{bmatrix}}_{9\times5}$$

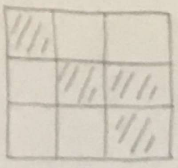
$$a^2.(b^2)^T = \underbrace{\begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 1 \\ -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}}_{9\times1} . \underbrace{\begin{bmatrix} -1 & 1 & -1 & 1 & -1 \end{bmatrix}}_{1\times5} = \underbrace{\begin{bmatrix} -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 & -1 \end{bmatrix}}_{9\times5}$$

$$\Rightarrow \underset{\substack{\downarrow \\ \text{weight} \\ \text{Matrix}}}{W} = \underbrace{\begin{bmatrix} 1 & -1 & 1 & 1 & -1 \\ -3 & 3 & -3 & 1 & -1 \\ 3 & -3 & 3 & -1 & 1 \\ -1 & -1 & -1 & -1 & 1 \\ -1 & 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & -1 & 1 \\ 3 & -1 & 3 & -1 & 1 \\ -3 & 3 & -3 & 1 & -1 \\ 1 & -1 & 1 & 1 & -1 \end{bmatrix}}_{9\times5}$$

$$a^3.(b^3)^T = \underbrace{\begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \\ 1 \\ 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}}_{9\times1} . \underbrace{\begin{bmatrix} -1 & 1 & -1 & -1 & 1 \end{bmatrix}}_{1\times5} = \underbrace{\begin{bmatrix} 1 & -1 & 1 & 1 & -1 \\ -1 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 & -1 \\ -1 & 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 & -1 \\ -1 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 & -1 \end{bmatrix}}_{9\times5}$$

$$\Rightarrow W^T = \begin{bmatrix} 1 & -3 & 3 & -1 & -1 & -1 & 3 & -3 & 1 \\ -1 & 3 & -3 & -1 & 1 & 1 & -1 & 3 & -1 \\ 1 & -3 & 3 & -1 & -1 & -1 & 3 & -3 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \end{bmatrix}$$

c) Process the retrieval phase by showing the steps of retrieval of pairs for the following key pattern:



$$\Rightarrow a^1 = \underbrace{\begin{bmatrix} 1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 \end{bmatrix}^T}_{1\times 9} \rightarrow \text{Key pattern}$$

$$b^2 = \operatorname{sgn}(W^T . a^1)$$

$(5\times 9) . (9\times 1) \Rightarrow [5\times 1]$

$$= \begin{bmatrix} 1 & -3 & 3 & -1 & -1 & -1 & 3 & -3 & 1 \\ -1 & 3 & -3 & -1 & 1 & 1 & -1 & 3 & -1 \\ 1 & -3 & 3 & -1 & -1 & -1 & 3 & -3 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \\ 1 \\ 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \\ -1 \end{bmatrix} \longrightarrow b^2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$a^3 = \operatorname{sgn}(W . b^2)$$

$(9\times 5)\times(5\times 1) \Rightarrow [9\times 1]$

$$= \begin{bmatrix} 1 & -1 & 1 & 1 & -1 \\ -3 & 3 & -3 & 1 & -1 \\ 3 & -3 & 3 & -1 & 1 \\ -1 & -1 & -1 & -1 & 1 \\ -1 & 1 & -1 & -1 & 1 \\ -1 & 1 & -1 & -1 & 1 \\ 3 & -1 & 3 & -1 & 1 \\ -3 & 3 & -3 & -1 & 1 \\ 1 & -1 & 1 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 5 \\ -7 \\ 7 \\ -3 \\ -5 \\ -5 \\ 5 \\ -7 \\ 5 \end{bmatrix} \longrightarrow a^3 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

$$b^4 = \operatorname{sgn}(W^T . a^3)$$

$(5\times 9) . (9\times 1) = (5\times 1)$

$$= \begin{bmatrix} 1 & -3 & 3 & -1 & -1 & -1 & 3 & -3 & 1 \\ -1 & 3 & -3 & -1 & 1 & 1 & -1 & 3 & -1 \\ 1 & -3 & 3 & -1 & -1 & -1 & 3 & -3 & 1 \\ 1 & 1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \\ -1 \\ -1 \\ 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 17 \\ -13 \\ +17 \\ 1 \\ -3 \end{bmatrix} \rightarrow b^4 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$\begin{tikzcd} b^2 \arrow[r] \arrow[rr, bend right, "\text{eşit}"'] & a^3 \arrow[r] \arrow[rr, bend left, "\text{eşit olur}"] & b^4 \arrow[r] & a^5 \end{tikzcd}$$

$$\Rightarrow b^2 = b^4 \Rightarrow a^3 = a^5$$

↳ 2. örüntü çiftini key pattern olarak çağırdı

① ANN

- ANN is a mathemetical or computational model that is inspired by the structure and/or functional aspects of biological neural networks.
- In an ANN, simple artificial nodes, variously called "neurons", are connected together to from a network of nodes mimicking the biological networks.

CNN

- CNN is an array of cells with local connections only. The communication is allowed btwn neighbouring units.
- Basic circuit unit of CNN is called a "cell"
- Each cell has an input, a state and an output.
- Each cell interacts directly only with the cells within its radius of neighborhood.
- Cells are multiple-input / single-output processors.
- In order to calculate the state of the cells, it is necessary to define the "template" of the network.
- It uses piece-wise linear function.

OR :

No	Inputs	Desired Outputs
1	$x^1 = [1\ 0\ 1]^T$	-1
2	$x^2 = [0\ -1\ -1]^T$	1
3	$x^3 = [-1\ -0.5\ -1]^T$	1

PERCEPTION L.R.

$f(.) = \mathrm{sgn}(.)$

$c = 0.1$

$W^1 = [1\ -1\ 0]^T$

X = Input , W = Weight , r = Learning Signal , d = Desired Output

$r_i = d_i - y_i$, $\Delta W_i^k = c.r_i.\ x^k$, $W_i^{k+1} = W_i^k + \Delta W^k$, $\mathrm{sgn}(.) = \begin{cases} 1 & \text{if } x>0 \\ 0 & \text{if } x=0 \\ -1 & \text{if } x<0 \end{cases}$

k = 1

$y^1 = f(\langle W^1, x^1\rangle) = \mathrm{sgn}(1) = 1 \neq -1 \Leftarrow d_1$

$= \underbrace{[1\ -1\ 0]}_{W^1} . \underbrace{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}_{x^1} = 1$

$\Delta W^1 = c.r_i.\ x^k = c.(d_1 - y_1).\ x^k$

$= \underbrace{(0.1).(-1-1)}_{(-0.2)} . \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0 \\ -0.2 \end{bmatrix}$

$W_i^{k+1} = W_i^k + \Delta W^k \Rightarrow W^2 = W^1 + \Delta W^1$

$= \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.2 \\ 0 \\ -0.2 \end{bmatrix} = \begin{bmatrix} 0.8 \\ -1 \\ -0.2 \end{bmatrix}$

k = 2

$y^2 = f(\langle W^2, x^2\rangle) = \mathrm{sgn}(1.2) = 1 = 1 \Leftarrow d_2$

$= [0.8\ -1\ -0.2] \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} \cdot 1.2$

$y^2 = d^2$

$W^3 = W^2$

k = 3

$y^3 = (\langle W^3, x^3\rangle) = \mathrm{sgn}(-0.1) = -1 \neq 1 \Leftarrow d_3$

$= [0.8\ -1\ -0.2] \begin{bmatrix} -1 \\ -0.5 \\ -1 \end{bmatrix} = -0.1$

$\Delta W^3 = c.(d_3 - y_3).\ x^3 = \underbrace{(0.1).(1+(-1))}_{0.2} \begin{bmatrix} -1 \\ -0.5 \\ -1 \end{bmatrix} = \begin{bmatrix} -0.2 \\ -0.1 \\ -0.2 \end{bmatrix}$

$W^3 = W^2 + \Delta W^3 = \begin{bmatrix} 0.8 \\ -1 \\ -0.2 \end{bmatrix} + \begin{bmatrix} -0.2 \\ -0.1 \\ -0.2 \end{bmatrix} = \begin{bmatrix} 0.6 \\ -1.1 \\ -0.4 \end{bmatrix}$

k=1

$y^4 = (\langle W^4, X^1 \rangle) = sgn(0.2) = 1 \neq -1 = d_1$

$= \begin{bmatrix} 0.6 & -1.1 & -0.4 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 0.2$

$\Delta W^4 = c \cdot (d_4 - y_4) \; x^4 = \underbrace{(0.1)(-1-1)}_{-0.2} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0 \\ -0.2 \end{bmatrix}$

$W^5 = W^4 + \Delta W^4 = \begin{bmatrix} 0.6 \\ -1.1 \\ -0.4 \end{bmatrix} \begin{bmatrix} -0.2 \\ 0 \\ -0.2 \end{bmatrix} = \begin{bmatrix} 0.4 \\ -1.1 \\ -0.6 \end{bmatrix}$

k=2

$y^5 = (\langle W^5, X^2 \rangle) = sgn(1.7) = 1 = 1 \Leftarrow d_2$

$= \begin{bmatrix} 0.4 & -1.1 & -0.6 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = 1.7$

$y^5 = d^5$

$W^6 = W^5$

k=3

$y^6 = (\langle W^6, X^3 \rangle) = sgn(0.75) = 1 = 1 \Leftarrow d_3$

$= \begin{bmatrix} 0.4 & -1.1 & -0.6 \end{bmatrix} \begin{bmatrix} -1 \\ -0.5 \\ -1 \end{bmatrix} = (-0.4 + 0.55 + 0.6) = 0.75$

$y^6 = d^6$

$W^7 = W^6 = W^5$

k=1

$y^7 = (\langle W^7, X^1 \rangle) = sgn(-0.2) = -1 = -1 \Leftarrow d^1$

$= \begin{bmatrix} 0.4 & -1.1 & -0.6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = -0.2$

$y^7 = d^7$

$\boxed{W^8 = W^7 = W^6 = W^5}$

Types of Activation Function

$\tanh h \Rightarrow f(x) = \dfrac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$

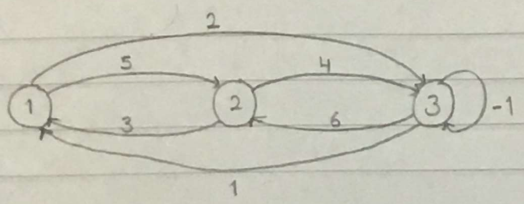
$Sgn(.) \Rightarrow f(x) = \begin{cases} 1 & , x>0 \\ 0 & , x=0 \\ -1 & , x<0 \end{cases}$

Sigmoid (Bipolar) $\Rightarrow f(x) = \dfrac{1-e^{-x}}{1+e^{-x}}$

Sigmoid (Unipolar) $\Rightarrow f(x) = \dfrac{1}{1+e^{-x}}$

Step $\Rightarrow f(x) = \begin{cases} 1 & , x \geq 0 \\ 0 & , x<0 \end{cases}$

Ex :



$x_1 = 0.1$

$x_2 = 0.5$

$x_3 = -0.2$

$u_j = \sum_{i=1}^{n} w_{ji} \cdot x_i$

For neuron (1) $j=1$ $\quad u_1 = W_{11}.x_1 + W_{12}.x_2 + W_{13}.x_3 = 0 \times (0.1) + 3 \times (0.5) + 1.(-0.2) = 1.3$

For neuron (2) $j=2$ $\quad u_2 = W_{21}.x_1 + W_{22}.x_2 + W_{23}.x_3 = 5 \times (0.1) + 0 \times (0.5) + 6.(-0.2) = -0.7$

For neuron (3) $j=3$ $\quad u_3 = W_{31}.x_1 + W_{32}.x_2 + W_{33}.x_3 = 2.4$

$U = W.X \Rightarrow \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} W \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ $\qquad W = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{bmatrix}$

PERCEPTION L.R.

$r_i = d_i - y_i$

$\Delta w_i^{k} = c.r_i.x^{k}$

$W_i^{k+1} = W_i^{k} + \Delta W^{k}$

DELTA L.R.

$r = [d-y].y'$

$y = f(\langle w, x \rangle)$

$y' = f'(x)$

WIDROW-HOFF LR

$r = [d - W.x]$

CORRELATION LR

$r = d$

HEBBIAN L.R.

$r = y$

$\Delta W^{k} = c.y^{k}.x^{k}$

NEURAL NETWORK

adder = soma = cell body

$$\underbrace{u_k}_{\text{net input of k neuron}} = \sum_{j=1}^{n} \underbrace{w_{kj}}_{\text{Synaptic weights}} \cdot \underbrace{x_j}_{\text{Inputs}}$$

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Ex = $n = 3$

$$u_1 = \sum_{i=1}^{3} w_{1i} \cdot x_i \Rightarrow u_1 = w_{11} \cdot x_1 + w_{12} \cdot x_2 + w_{13} \cdot x_3 = 0 \cdot x_1 + 3 \cdot x_2 + 1 \cdot x_3 = 1.3$$

$$u_2 = \sum_{i=1}^{3} w_{2i} \cdot x_i \Rightarrow u_2 = w_{21} \cdot x_1 + w_{22} \cdot x_2 + w_{23} \cdot x_3 = 5 \cdot x_1 + 0 \cdot x_2 + 6 \cdot x_3 = -0.7$$

$$u_3 = \sum_{i=1}^{3} w_{3i} \cdot x_i \Rightarrow u_3 = w_{31} \cdot x_1 + w_{32} \cdot x_2 + w_{33} \cdot x_3 = 2 \cdot x_1 + 4 \cdot x_2 + (-1 \cdot x_3) = 2.4$$

$x_1 = 0.1 \quad x_2 = 0.5 \quad x_3 = -0.2 \qquad y_1 = f(u_1) \quad y_2 = f(u_2) \quad y_3 = f(u_3)$

$$W = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{32} \end{bmatrix} = \begin{bmatrix} 0 & 3 & 1 \\ 5 & 0 & 6 \\ 2 & 4 & -1 \end{bmatrix} \qquad U = W \cdot x \qquad \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} W \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Hw = -1 ile 1 arasında = x değeri Activation Function

MATHEMATICAL REVIEW for NNs

System is a collection of elements or components that are organized for a common purpose.

INPUT ⟶ $\boxed{\text{SYSTEM}}$ ⟶ OUTPUT

- Systems have structure, defined by components/elements and their composition. (=ilişki)
- Systems have behaviour, which involves (=contain) inputs, processing and outputs of material, energy, information or data
- Systems have interconnectivity, the various parts of a system have functional as well as structural relationships to each other.

LINEAR SYSTEM

A linear system is a mathematical model of a system based on the use of a linear operator.

INPUT ⟶ $\boxed{L(.)}$ ⟶ OUTPUT

Linear Operator

Linear systems satisfy the properties of superposition: Additivity (=Toplamsallık)
Homogeneity

Homogeinity: $\mathcal{L}(u_1) = y_1$ $\quad \mathcal{L}(\alpha_1 . u_1) = \alpha_1 . y_1$ $\quad f(\alpha . x) = \alpha . f(x)$

$\mathcal{L}(u_2) = y_2$ $\quad \mathcal{L}(\alpha_2 u_2) = \alpha_2 . y_2$ $\quad \Rightarrow$ Superposition Principle

Additivity: $\mathcal{L}(u_1 + u_2) = y_1 + y_2 = \mathcal{L}(u_1) + \mathcal{L}(u_2)$ $\quad f(x+y) = f(x) + f(y)$

* $\xrightarrow{\alpha_1 u_1 + \alpha_2 u_2} \boxed{\mathcal{L}(.)} \xrightarrow{\alpha_1 y_1 + \alpha_2 y_2}$

NONLINEAR SYSTEM: A nonlinear system is mathematical model of a system that does not satisfy the superposition principle.

Ex: $\int (\alpha_1 y_1 + \alpha_2 y_2) . dy = \alpha_1 \int y_1 dy + \alpha_2 \int y_2 dy$ $\Rightarrow$ LINEAR

$\cos(\alpha_1 x_1 + \alpha_2 x_2) \neq \alpha_1 . \cos x_1 + \alpha_2 . \cos x_2$ $\Rightarrow$ NONLINEAR

STATE SPACE: To provide convenient (=uygun, müsait) way to model and analyze systems with multiple inputs and outputs, one needs to represent the state space of the system.

Sistemi karakterize eden parametre

The state is the present condition of a system and can be represented as a vector within that space.

State $\leftarrow x(t) \rightarrow$ state changes by time

State variables are the parameters that can represent the entire system.

$$\frac{dx(t)}{dt} = A . x(t) + u$$ → No time dependency ⇒ Time Invariant System

A: System Matrix (Numerical values of physical components of the system)

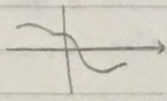
$x(t) = [x_1(t) \dots x_n(t)]^T$

$$\frac{dx(t)}{dt} = A(t)\, x(t) + u$$ ⇒ Time Variant System

$0 = x_i(t)' = f(x(t))$

FUNCTIONS

* Continuous Functions: $\lim_{x \to c} f(x) = f(c)$

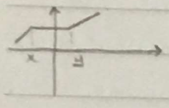


=> Continuous

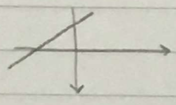
× Bütün aktivasyon fonksiyonları continuous'dur.

* Increasing Functions: * if $y > x \Rightarrow f(y) \geq f(x) \Rightarrow f(.)$ is Monotonically increasing (non-decreasing)

if $y > x \Rightarrow f(y) > f(x) \Rightarrow f(.)$ is Strictly Increasing



=> Monoton



=> Strictly

* Decreasing Functions: * if $y > x \Rightarrow f(y) \leq f(x) \Rightarrow f(.)$ is Monotonically Decreasing

if $y > x \Rightarrow f(y) < f(x) \Rightarrow f(.)$ is Strictly Decreasing

* Derivative of Functions: * if $\frac{df(x)}{d(x)} > 0$, $\forall x \Rightarrow f(.)$ is Strictly Increasing

if $\frac{df(x)}{d(x)} < 0$, $\forall x \Rightarrow f(.)$ is Strictly Decreasing

if $\frac{df(x)}{d(x)} = 0 \Rightarrow$ Stationary Points (Local/Global Max -
(Local/Global Min.)

if $d''f(x)$ contains x parameter Local Min./Max.

if $d''f(x)$ does not contain X parameter Global Min./Max.

if $d''f(x) > 0$ Local/Global Min.

if $d''f(x) < 0$ Local/Global Max.

Ex: $f(x) = x^3 - 3x^2$

$\frac{df(x)}{d(x)} = 3x^2 - 6x = 3\underbrace{x}_{0}(\underbrace{x-2}_{2})$ } stationary Points

$d''f(x) = 6x - 6$

$\boxed{x=0} \Rightarrow d''f(x) = -6 < 0 \Rightarrow$ Local Max

$\boxed{x=2} \Rightarrow d''f(x) = 6 > 0 \Rightarrow$ Local Min.

* Bounded Functions: If there exists a real number $M < \infty$ such that $|f(x)| \leq M$, as $|x| \to \infty$
$f(.)$ Bounded

* stimulated: Dışarıdan yeni bir girdi verilmesi
undergoes changes:
responds in a new way: parametre değişikliğinden sonra değişir
learning paradigm: ağı eğitirken kullanılan yapı
learning alg.: Sinaptik ağırlıkl değişt. kullanılan formüller
teacher: eğitmen; ağa sunulan çıkış bilgisi (verilen bilgiye göre çıktı beklenir)
W: Sinaptik ağırlıkları tutan matris
Aynı bilgi üretilene kadar döngü devam eder. (y ve d birbirine eşit olana kadar)
training: eğitim

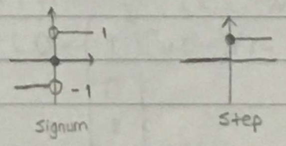
PERCEPTRON (PERCEPTION) L.R

* Supervised

x_i: inputs
w_i: weights
u_i : Net input $= \langle w_i, x_i \rangle$
$f(u_i) = y_i$: Outputs

$c > 0$: Learning Rate
r_i : Learning Signal
d_i : Desired Outputs

$r_i = d_i - y_i \Rightarrow$ Only works for threshold activation function (signum/step)
$\Delta w_i^k = c \cdot r_i \cdot x^k \longrightarrow = c \cdot (d_i^k - y_i^k) \cdot x_i^k$
$w_i^{k+1} = w_i^k + \Delta w^k$



ÖR:

No	Inputs (x)	Desired Outputs (d)
1	$x^1 = [1\ 0\ 1]^T$	-1
2	$x^2 = [0\ -1\ -1]^T$	1
3	$x^3 = [-1\ -0.5\ -1]^T$	1

$f(.) = \text{sgn}(.)$
$c = 0.1$
$W^1 = [1\ -1\ 0]^T$

$\{(x^1, d^1), (x^2, d^2), (x^3, d^3)\}$

k = 1

$y^1 = f(\langle w^1, x^1 \rangle) = \text{sgn}(1) = 1 \neq -1$

$[1\ -1\ 0] \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 1$

$\Delta w^1 = c \cdot (d^1 - y^1) \cdot x^1$

$= 0.1(-1-1) \cdot \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0 \\ -0.2 \end{bmatrix}$

$W^2 = W^1 + \Delta W^1$

$W^2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} -0.2 \\ 0 \\ -0.2 \end{bmatrix} = \begin{bmatrix} 0.8 \\ -1 \\ -0.2 \end{bmatrix}$

k=2

$y^2 = f(\langle W^2, x^2 \rangle) = sgn(1.2) = 1$

$$[0.8 \ -1 \ -0.2] \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = 1.2 \qquad y^2 = d^2 \qquad W^3 = W^2$$

k=3

$y^3 = f(\langle W^3, x^3 \rangle) = sgn(-0.1) = -1 \neq 1$ $\qquad \Delta W^3 = 0.1\,(1-(-1)) \begin{bmatrix} -1 \\ -0.5 \\ -1 \end{bmatrix} = \begin{bmatrix} -0.2 \\ -0.1 \\ -0.2 \end{bmatrix}$

$$[0.8 \ -1 \ 0.2] \begin{bmatrix} -1 \\ -0.5 \\ -1 \end{bmatrix} = -0.1$$

$W^4 = W^3 + \Delta W^3$

$$W^4 = \begin{bmatrix} 0.8 \\ -1 \\ -0.2 \end{bmatrix} + \begin{bmatrix} -0.2 \\ -0.1 \\ -0.2 \end{bmatrix} = \begin{bmatrix} 0.6 \\ -1.1 \\ -0.4 \end{bmatrix}$$

k=4

$y^1 = f(\langle W^4, x^1 \rangle) = sgn(0.2) = 1 \neq -1$ $\qquad \Delta W^4 = 0.1\,(-1-1) \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.2 \\ 0 \\ -0.2 \end{bmatrix}$

$$= [0.6 \ -1.1 \ -0.4] \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = 0.2$$

$$W^5 = W^4 + \Delta W^4 = \begin{bmatrix} 0.6 \\ -1.1 \\ -0.4 \end{bmatrix} + \begin{bmatrix} -0.2 \\ 0 \\ -0.2 \end{bmatrix} = \begin{bmatrix} 0.4 \\ -1.1 \\ -0.6 \end{bmatrix}$$

k=5

$y^2 = f(\langle W^5, x^2 \rangle) = sgn(1.7) = 1 = d^2 = 1 \qquad W^6 = W^5$

$$= [0.4 \ -1.1 \ -0.6] \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} = 1.7$$

k=6

$y^3 = f(\langle W^6, x^3 \rangle) = sgn(0.75) = 1 = d^3 = 1 \qquad W^7 = W^6 = W^5$

$$[0.4 \ -1.1 \ -0.6] \begin{bmatrix} -1 \\ -0.5 \\ -1 \end{bmatrix} = 0.75$$

$k=7$

$y^1 = f(\langle w^7, x^1 \rangle) = sgr(-0.2) = -1 = d^1 = -1$ $\quad \boxed{W^8 = W^7 = W^6 = W^5}$

$$\begin{bmatrix} 0.4 & -1.1 & -0.6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = -0.2$$

DELTA L.R.

$r = [d-y] \cdot y'$ $\quad y = f(\langle w, x \rangle)$ $\quad y' = \frac{df(x)}{dx}$

WIDROW-HOFF L.R.

$r = [d - W.x]$

CORRELATION L.R.

$r = d$

HEBBIAN L.R.

$r = f(\langle w, x \rangle)$ $\quad r = y$

$\Delta w^k = c.y^k.x^k$

ASSIGNMENT-3

1. For an ANN with one neuron, process four steps for Hebbian L.R.

$w^1 = [-1 \ \ 1]^T$, $c = 1$

$x^1 = [1 \ \ -2]^T$ $\quad x^2 = [0 \ \ 1]^T$ $\quad x^3 = [2 \ \ 3]^T$ $\quad x^4 = [1 \ \ -1]^T$

a-) $f(u) = \frac{1}{1+e^{-u}}$ $\quad$ b-) $f(u) = sgn(u) \begin{cases} +1, & u>0 \\ -1, & u<0 \end{cases}$

2. Use Perceptron L.R. for training network

$w^1 = [0 \ 1 \ 0]^T$

$c = 1$ $\quad x^1 = [-1 \ \ 1 \ \ 1]^T$ $\quad d^1 = -1$

$f(u) = sgn(u)$ $\quad x^2 = [-1 \ -1 \ \ 0]^T$ $\quad d^2 = 1$

LEARNING TASK

1) Örüntü Çagrışımı $\quad$ 3) Fonksiyon Yaklaştırma $\quad$ 2) Örüntü Tanıma

assoc. m = çagrışımlı bellekler

First: Ag egitimi; Her paterne göre kurulur → karşılık ait olan sınıf (kategori)

Later: Agın daha önce görmediği patern

Finally: Yeni paternin ait oldugu kategoriyi verir $\quad$ 17. slayt y ve d degişecek

-VİZE SONRASI-

14/11/2017 Tuesday

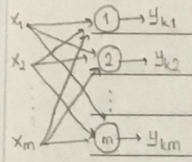
MEMORY

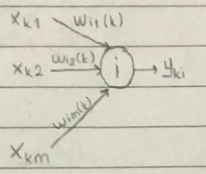
- When a particular pattern is learned, it is stored in the memory where it can be recalled later when required
- Learning from the association between the patterns X_k and y_k.

$X_k \rightarrow y_k$ $\quad$ $X_k = [x_1\ x_2 \cdots x_m]^T$ $\quad$ $y_k = [y_1\ y_2 \cdots y_m]^T$ } same dimensionality

k = no of pattern

n = no of neurons





k = Pattern

i th neuron of y_k

$y_k = W(k) \cdot X_k \quad , k = 1, \ldots, q$

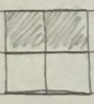
$$\boxed{M = \sum_{k=1}^{q} W(k)}$$ M: Memory Matrix

$$\boxed{M = \sum_{k=1}^{q} \underbrace{y_k \cdot x_k^T}_{W(k)}} \longrightarrow W(1) + W(2) + \ldots + W(q)$$

$m\times m = (m\times 1)\ (1\times m)$

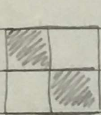
$$\begin{bmatrix} y_{11} & y_{21} & \cdots \\ \vdots & & \\ y_{1m} & y_{2m} & \cdots\ y_{qm} \end{bmatrix}_{m\times q} \begin{bmatrix} x_1 \rightarrow \\ x_2 \rightarrow \\ \vdots \\ x_q \rightarrow \end{bmatrix}_{q\times m} = \begin{bmatrix} M \end{bmatrix}_{m\times m}$$

Ex: $a_1 =$



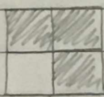
$b_1 = a_1$

$a_2 =$



$b_2 = a_2$

$a_3 =$



$b_3 = a_3$

$M = ?\ (4\times 4)$

$k = 1,2,3$

$q = 3$ (max)

$a_1 = [1\ 1\ 0\ 0]^T$

$a_2 = [1\ 0\ 0\ 1]^T$

$a_3 = [1\ 1\ 0\ 1]^T$

$m = 4$

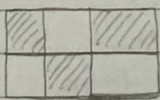
$$M = \underbrace{b_1 \cdot a_1^T}_{W(1)} + \underbrace{b_2 \cdot a_2^T}_{W(2)} + \underbrace{b_3 \cdot a_3^T}_{W(3)} =$$

$$= \begin{bmatrix}1\\1\\0\\0\end{bmatrix}\begin{bmatrix}1&1&0&0\end{bmatrix} + \begin{bmatrix}1\\0\\0\\1\end{bmatrix}\begin{bmatrix}1&0&0&1\end{bmatrix} + \begin{bmatrix}1\\1\\0\\1\end{bmatrix}\begin{bmatrix}1&1&0&1\end{bmatrix}$$

$$= \begin{bmatrix}1&1&0&0\\1&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} + \begin{bmatrix}1&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&1\end{bmatrix} + \begin{bmatrix}1&1&0&1\\1&1&0&1\\0&0&0&0\\1&1&0&1\end{bmatrix}$$

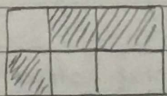
$$= \underbrace{\begin{bmatrix}3&2&0&2\\2&2&0&1\\0&0&0&0\\2&1&0&2\end{bmatrix}}_{M}$$

Assignment = For $X_1 =$



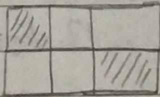
$y_1 = X_1$

$X_2 =$



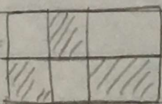
$y_2 = X_2$

$X_3 =$



$y_3 = X_3$

$X_4 =$



$y_4 = X_4$

Find Memory Matrix (Correlation Matrix) ?

▨ = 1

□ = -1

EX-1 : $A = [-1 \quad 1]^T$ $\quad W = A A^T + B B^T$ $\quad w_{11}, w_{22} = 0$

$B = [1 \quad -1]^T$ $\quad A A^T + B.B^T - p.I$ ②

①

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix} \qquad - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

③

$w_{11} = (-1).(-1) + (1)(1) = 2$ $\qquad w_{21} = -2$ $\qquad w_{11} = 0$ $\qquad \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$

$w_{12} = (-1).(1) + (1)(-1) = -2$ $\qquad w_{22} = 2$

EX-2 : $A = [-1 \quad -1 \quad 1]^T$

$B = [1 \quad -1 \quad -1]^T$

$C = [-1 \quad 1 \quad 1]^T$

$$W = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{bmatrix}$$

$w_{11} = 0$

$w_{12} = (-1).(-1) + (1)(-1) + (-1)(1) = -1$

$w_{13} = (-1).(1) + (1).(-1) + (-1).(1) = -3$

$w_{21} = -1$ $\qquad w_{31} = -3$

$w_{22} = 0$ $\qquad w_{32} = 1$

$w_{23} = (-1)(1) + (-1)(-1) + (1)(1) = 1$ $\qquad w_{33} = 0$

$$W = \begin{bmatrix} 0 & -1 & -3 \\ -1 & 0 & 1 \\ -3 & 1 & 0 \end{bmatrix}$$ Store Phase

Recalling Phase

$$\left.\begin{matrix} A = [1 \quad -1 \quad 1] \\ B = [-1 \quad 1 \quad -1] \end{matrix}\right\} \quad W = \begin{bmatrix} 0 & -2 & 2 \\ -2 & 0 & -2 \\ 2 & -2 & 0 \end{bmatrix}$$

$x(0) = [-1 \quad -1 \quad 1]^T \rightarrow$ Key Vector

0 gelince 1 öncekine bakılır.

$$x(1) = sgn(w.x(0)) = sgn\left(\begin{bmatrix} 0 & -2 & 2 \\ -2 & 0 & -2 \\ 2 & -2 & 0 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$x(2) = x(1) \}$ x fixed = y

$$x(2) = sgn(w.x(1)) = sgn\left(\begin{bmatrix} 0 & -2 & 2 \\ -2 & 0 & -2 \\ 2 & -2 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Distance'a bakılırsa Key Vector gibi A'yı getirir.

Hopfield Neural Network (Feedback N.N. / Recurrent N.N.)

$$\frac{dx(t)}{dt} = -a_i . x_i (t) + \sum_{j=1}^{n} w_{ij} . g_j (x_j(t)) + I_i \quad , i = 1, 2, \dots, n$$

$$\dot{x}(t) = -A . x(t) + W . g(x(t)) + I$$

Pure Delayed Hopfield NNs

$$\frac{dx_i(t)}{d(t)} = -a_i x_i (t) + \sum_{j=1}^{n} w_{ij} . g_j (x_j(t-z)) + I_i \quad , i = 1, 2, \dots, n$$

$$\dot{x}(t) = -A x(t) + W . g(x(t-z)) + I$$

Hybrid Hopfield NNs

$$\frac{dx_i(t)}{d(t)} = -a_i x_i (t) + \sum_{j=1}^{n} w_{ij} . g_j (x_j(t)) + \sum_{j=1}^{n} v_{ij} \, g_j (x_j (t-z)) + I_i \quad , i = 1, 2, \dots, n$$

$$\dot{x}(t) = -A . x(t) + W . g(x(t)) + V . g(x(t-z)) + I$$

Some Terminologies (Feedback NN's)

* The output of each neuron is a binary number in $\{-1, 1\}$
* The output vector is the state vector.
* Starting from an initial state (given as the input vector) the state of the network changes from one to another like automation.
* If the state converges, the point to which it converges is called the "equiblirum point" or "attractor"

Feedback NNs are defined by the following differential equation.

$$\frac{dx}{dt} = -a_i \cdot x_i(t) + \sum_{i=1}^{n} W_{ij} \cdot g_j(x_j(t)) + I_i \quad , i = 1, 2, \ldots, n$$

$g(.)$ = Activation Function n = Number of Neurons

$a_i > 0$ Determines the convergence rate

W_{ij} = Interconnection weights

I = External Input

Vector - Matrix Forms

$X = -A \cdot x(t) + W \cdot g(x(t)) + I \longrightarrow x' = 0$ Desired Solution

$x(t) = [x_1(t) \quad x_2(t) \quad x_3(t) \quad \ldots \quad x_n(t)]$: State Vector

$A = \text{diag}\{a_i > 0\}_{n \times n}$ $W = \{W_{ij}\}_{n \times n}$: Interconnection Matrix $I = [I_1, I_2, \ldots I_n]^T$

$g(x(t)) = [g(x_1(t_1)) \quad g(x_2(t_2)) \quad \ldots \quad g(x_n(t_n))]^T$ $g(.) \in S \rightarrow$ Sigmoid Function

X^* : Equilibrum Point

$0 = -A \cdot x^* + W \cdot g(x^*) + I$

Equilibrum Equation

- Equilibrum Point might be eigter stable or unstable.
- This is determined by W-Interconnection Matrix

Lyapunon Stability Theorems

It gives information about the behaviour of system without the need of solving the system.

1- First, we define the Lyapunon Function belonging to the system. Lyapunon function should satisfy the following properties. * $V(x) > 0 \, , \forall x \neq 0$ * $V(x) = 0$ only at $x = 0$

2- Then we analyze the derivative of the Lyapunon Function.

If, * $\dot{V}(x) \leq 0 \, , \forall x \in R^n$ then $x = 0$ is stable (Multiple eq. point)

* $\dot{V}(x) < 0 \, , \forall x \neq 0$

$\dot{V}(x) = 0$, only at $x = 0$ then, $x = 0$ is asympotically stable (Unique eq. point)

For feedback Neural Networks : $\dot{X} = -A.x(t) + W.g(x(t)) + I$

$X^* =$ Eq. Point

$0 = -A.x^* + W.g(x^*) + I \rightarrow$ Eq. Equation

- Before applying Lyapunov Theorem, we need to shift the eq. point to the origin: $Z = X - X^*$

$Z(t) = x(t) - x^*$

$\dot{Z} = \dot{X}$

$$\dot{Z}(t) = -A.(Z(t)+x^*) + W.g(Z(t)+x^*) + \overbrace{(A.x^* + W.g(x^*))}^{I}$$

$$\dot{Z}(t) = -A.Z(t) - A.x^* + W[\underbrace{g(Z(t)+x^*) - g(x^*)}_{f(Z(t))}] - A.x^* \quad , f \in S$$

$\dot{Z}(t) = -A.Z(t) + W.f(Z(t)) \longrightarrow$ *The Lyapunov Function is defined for this system.

The eq. point of this system is origin *Then, the derivative of the function is analyzed.

$\dot{Z}(t) = -A.Z(t) + W.f(Z(t))$

$\ast\ V(Z(t)) = \sum_{i=0}^{n} \int_{0}^{Z_i} f(\emptyset).d\emptyset \quad (W = W^T)$ $\qquad V(Z) > 0$

$V(Z) = 0$ only at $Z = 0$

İlk adımda Lyapunov Function belirledik. Sistemin enerjisini belirleyen, sonra bu fonksiyonun davranışını belirlemek.

$\ast\ \dot{V}(Z(t)) = \sum_{i=n}^{n} f_i(Z_i(t)).\dot{Z}_i(t)$ (Skaler) $\qquad \dot{V}(Z(t)) = f^T(Z(t)).\dot{Z}(t)$ (Vector-Matrix)

$\rightarrow f^T(Z(t)).[-A.Z(t) + W.f(Z(t))]$

$\dot{V}(Z(t)) = -f^T(Z(t)).A.Z(t) + f^T(Z(t)).W.f'(Z(t))$

Amacımız Lyapunov fonksiyonunun türevini negatif yaparak koşulları elde etmek $\} \underbrace{-f^T(Z(t)).(-W).f^T(Z(t))}_{+}$

$-W \geq 0$ olmalıdır

$\dot{V}(Z(t)) \leq -f^T(Z(t)).A.Z(t)$

$\dot{V}(Z(t)) = 0 \ , \ Z(t) = 0$

$\dot{V}(Z(t)) < 0 \ , \ \forall_Z(t) \neq 0$

Stability Condition: $-W \geq 0$

Bu sistem için elde ettiğimiz kararlı koşul oldu. ($-W$ positive semidefinite matrix olmalı) $\qquad -W \rightarrow$ Positive semidefinete matrix

$\dot{z}(t) = -A \cdot z(t) + W \cdot f(z(t))$

$* \; V(z) = 2 \cdot \sum_{i=1}^{n} p_i \cdot \int_0^{z_i} f(\phi) \, d\phi \quad , \quad p_i > 0 \quad , \; i = 1, 2, \ldots, n$

$* \; \dot{V}(z) = 2 \cdot f^T(z) \cdot P \cdot \dot{z}$

$\dot{V}(z) = 2 \cdot f^T(z) \cdot P \cdot [-A \cdot z + W \cdot f(z)]$

$\dot{V}(z) = -2 \cdot f^T(z) \cdot PA \cdot z + 2 \cdot f^T(z) \cdot P \cdot W \cdot f(z)$

$\dot{V}(z) = -2 \cdot f^T(z) \cdot PA \cdot z + f^T(z) \cdot (PW + W^T P^T) \cdot f(z)$

$- f^T(z) \cdot (P(-W) + (-W)^T \cdot P^T) \cdot f(z) \Big\} \rightarrow \geq 0$ ise yazılır.

$\dot{V}(z) \leq -2 \cdot f^T(z) \cdot P \cdot A \cdot z$

$\dot{V}(z) = 0$, only at $z = 0$ Stability Condition $P(-W) + (-W)^T \cdot P \geq 0$

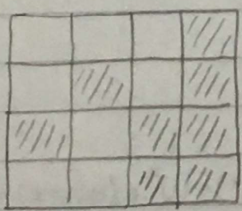
$\dot{V}(z) < 0 \quad , \quad \forall z \neq 0$

Öyle bir P matrisi bulman gereklik diagonally semi-stable.

BAM - Neural Networks

$a = [\;\;]_{16\times 1} \quad , \quad b = [\quad]_{1\times 7} = [\; W \;]_{16\times 7}$

Örneği yap getir.

$a =$ 

$b^2 = \Gamma(W^T \cdot a^1)$

$(7\times 16) \; (16\times 1) = (7\times 1)$

$a^3 = \Gamma(W \cdot b^2)$

$b^4 = \Gamma(W^T \cdot a^3)$