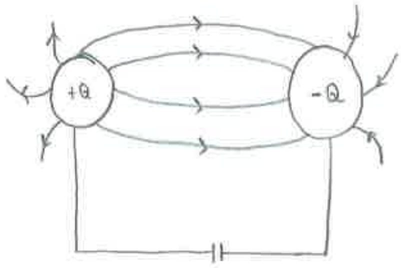
CHAPTER 24

Capacitors and Dielectrics

24.1 Capacitors and Capacitance

(Sigacılar)



$\Delta V = V_+ - V_-$

Capacitors consists of two charged conductors isolated from each other and their surroundings.

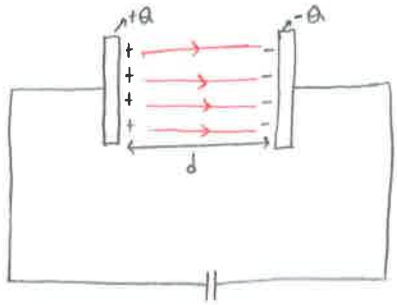
A capacitor has a limit called as capacitance (C).

Capacitance is measure of how much charge must be put on the conductors to maintain the potential difference constant. Capacitor are used in electronic circuits mainly as energy storing units.

$$\underbrace{Q}_{C} = \underbrace{C}_{\text{Farad (F)}} \cdot \underbrace{\Delta V}_{V}$$

24.2 Calculating the Capacitance of Capacitors

i-) Capacitor (Parallel-Plate)



Battery $\Delta V = V_+ - V_-$

$$C_{p-p} = \frac{Q}{\Delta V} = \frac{Q}{V_+ - V_-} \Rightarrow C_{pp} = \frac{\varepsilon_0 . A}{d} \text{ Farad (F)}$$

$$V_- - V_+ = -\int_+^- \vec{E} . d\vec{r} \Rightarrow -\int_+^- E . dr . \cos 0° = -\int_+^- \frac{Q}{\varepsilon_0 . A} . dr$$

$$= \frac{-Q}{\varepsilon_0} \int_+^- dr \Rightarrow \frac{-Q . d}{\varepsilon_0}$$

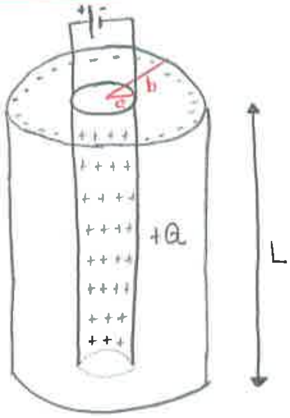
Ex : A parallel-plate capacitor has circular plates of 8.2 cm radius and 1.3 mm seperation. a-) $C_{p.p} = ?$ b-) $Q = ?$ to $\Delta V = 120$ V

Solution : a-) $C_{pp} = \dfrac{\varepsilon_0 . A}{d} = \dfrac{\varepsilon_0 \, \pi r^2}{d} \Rightarrow 8.85 \times 10^{-12} \times 3.14 \times \dfrac{(8.2 \times 10^{-2} m)^2}{1.3 \times 10^{-3} m} = 1.44 \times 10^{-10}$ F

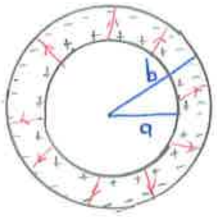
b-) $Q = C_{pp} . \Delta V$

$= 1.44 \times 10^{-10}$ F $\times 120$ V $= 1.73 \times 10^{-8}$ C

ii-) Cylindrical Capacitor



Top view



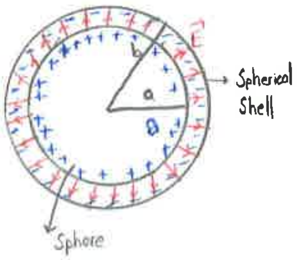
$$C_{cyclindrical} = \frac{Q}{\Delta V} = \frac{Q}{V_+ - V_-} \Rightarrow 2.\pi.\varepsilon_0.\ell \cdot \frac{1}{\ln\frac{b}{a}}$$

$$V_- - V_+ = -\int_{+}^{-} E.dr = -\int_{a}^{b} E.dr$$

$$= -\int_{a}^{b} \frac{Q}{2.\pi.\varepsilon_0.r.L}.dr = \frac{-Q}{2.\pi.\varepsilon_0.L}.\ln r \Big|_{a}^{b}$$

$$= \frac{-Q}{2.\pi.\varepsilon_0.\ell}.\ln\frac{b}{a}$$

iii-) Spherical Capacitor



$$C_{spherical} = \frac{Q}{\Delta V} = \frac{Q}{V_+ - V_-} = \frac{Q}{k.Q\left(\frac{b-a}{a.b}\right)} = \frac{1}{k}.\frac{ab}{(b-a)} = \underline{4.\pi.\varepsilon_0.\frac{a.b}{(b-a)}} \quad (F)$$

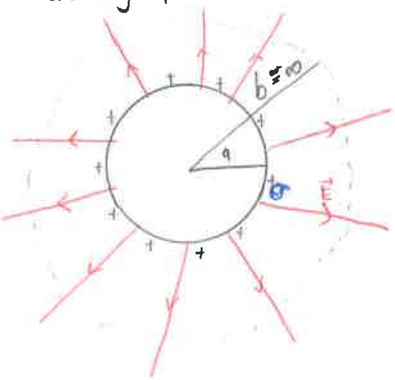
$$V_- - V_+ = -\int_{+}^{-} \vec{E}.d\vec{r} = -\int_{a}^{b} E.dr = -\int_{a}^{b} k.\frac{Q}{r^2}.dr = -k.Q\int_{a}^{b}\frac{dr}{r^2} = -k.Q\left(\frac{-1}{r}\Big|_{a}^{b}\right) = k.Q\left(\frac{a-b}{ab}\right)$$

$$k = \frac{1}{4.\pi.\varepsilon_0}$$

$$\underline{V_+ - V_- = k.Q\left(\frac{b-a}{ab}\right)}$$

iv-) Isolated Sphere

An isolated conducting sphere is also accepted as a capacitor. We assume that outer spherical shell is missing with respect to ordinary spherical capacitor.



$$C_{\substack{isolated\\sphere}} = 4.\pi.\varepsilon_0.\frac{a.\cancel{b}}{\cancel{b}.(1-\underbrace{\tfrac{a}{b}}_{0})} = \underline{4.\pi.\varepsilon_0.a}$$

NOTE: The capacitance of capacitors depends on dimensions.

14.04.2015
Tuesday

Ex: The plates of a spherical capacitor have radius 38 mm and 40 mm.

a-) Calculate its capacitance

b-) What must be area of a p-p capacitor to have same capacitance and seperation ?

Sol: a-)

$$C_{spherical} = \frac{1}{k} \cdot \frac{a.b}{(b-a)} = \frac{1}{9 \times 10^9 \frac{N.m^2}{C^2}} \cdot \frac{(38 \times 10^{-3} m) \times (40 \times 10^{-3} m)}{2 \times 10^{-3} m} \Rightarrow \underline{C_{spherical} = 8.5 \times 10^{-11} \text{ F}}$$

b.)

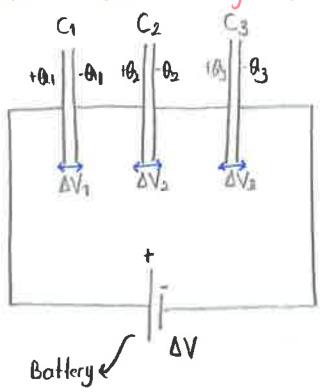
$$C_{p-p} = \varepsilon_0 \cdot \frac{A}{d} \qquad C_{p-p} = C_{spherical} \quad , d = b-a$$

$$C_{spherical} = \frac{1}{k} \cdot \frac{ab}{(b-a)} \qquad \varepsilon_0 \cdot \frac{A}{\cancel{d}} = \frac{1}{k} \cdot \frac{ab}{\cancel{(b-a)}} \Rightarrow A = \frac{a.b}{\cancel{\varepsilon_0} \cdot \frac{1}{4.\pi.\cancel{\varepsilon_0}}} = a.b.4.\pi = 4 \times 3.14 \times 38 \times 10^{-3} m \times 40 \times 10^{-3} m$$

$$\underline{A = 0.0192 \text{ m}^2}$$

24.3 Combination of Capacitors

i-) Series Combination of Capacitors

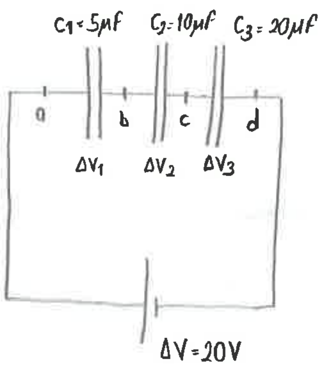


- The stored charges on each capacitor are equal. $\boxed{Q = Q_1 = Q_2 = Q_3 = \dots}$
- The potential difference of battery is equal to summation of potential differences of capacitors

$$\boxed{\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 + \dots}$$

$$\frac{Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3} + \dots \Rightarrow \boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_2} + \dots}$$

Ex:



a-) Find the C_{eq}.

b-) Stored charges on each capacitor ?

c-) $\Delta V_{ac} = V_a - V_c = ?$

Hint: $\mu F \times V = \mu C$, $Q_1 = C_1 . \Delta V_1 \Rightarrow \Delta V_1 = \frac{Q_1}{C_1}$

Solution: a-)

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{5\mu F} + \frac{1}{10\mu F} + \frac{1}{20\mu F} \Rightarrow \underline{C_{eq} = \frac{20\mu F}{7}}$$

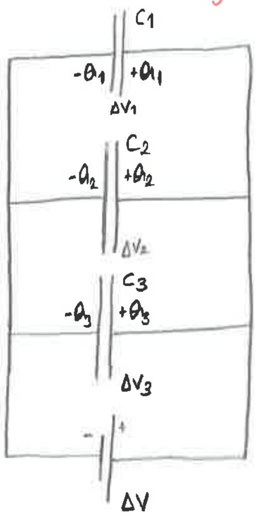
b-)

$$Q = Q_1 = Q_2 = Q_3 \quad , Q = C_{eq} . \Delta V = \frac{20}{7}\mu F \times 20\,V = \underline{\frac{400}{7}\mu C} = Q_1 = Q_2 = Q_3$$

c-)

$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 \quad , \Delta V_{AC} = V_a - V_c = \underbrace{\Delta V_1}_{\Delta V_{ab}} + \underbrace{\Delta V_2}_{\Delta V_{bc}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} = \frac{\frac{400}{7}\mu C}{5\mu F} + \frac{\frac{400}{7}\mu C}{10\mu F} = \underline{17.14 \text{ V}}$$

ii-) Parallel Combination of Capacitors



— Potential difference of each capacitor are equal.

$$\boxed{\Delta V = \Delta V_1 = \Delta V_2 = \Delta V_3 = \ldots}$$

— The Resultant stored charge (Q) is equal to sum of stored charges on each capacitor.

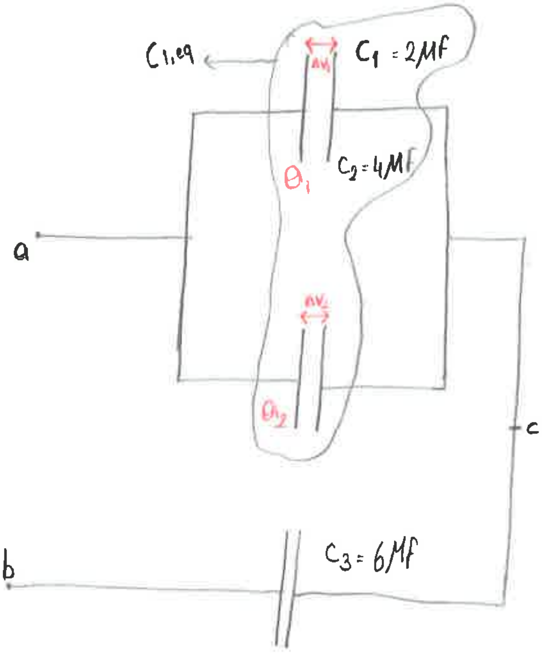
$$\boxed{Q = Q_1 + Q_2 + Q_3 + \ldots}$$

$$C_{eq} \cdot \cancel{\Delta V} = C_1 \cdot \cancel{\Delta V_1} + C_2 \cdot \cancel{\Delta V_2} + C_3 \cdot \cancel{\Delta V_3} \ldots$$

$$\boxed{C_{eq} = C_1 + C_2 + C_3 + \ldots}$$

Equivalent capacitance of parallel combination

Ex:



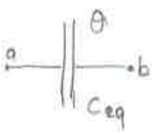
a-) $C_{eq} = ?$

b-) $\Delta V_{ab} = 20\ V$ is applied. Find the stored charge on each capacitor.

Solution. a-) $C_{1,eq} = C_1 + C_2 = 2\mu F + 4\mu F = 6\mu F$

$$\frac{1}{C_{eq}} = \frac{1}{C_{1,eq}} + \frac{1}{C_3} = \frac{1}{6\mu F} + \frac{1}{6\mu F} \Rightarrow 3\mu F = C_{eq}$$

b-)



$Q = C_{eq} \cdot \Delta V = 3\mu F \times 20\ V$

$Q = 60\ \mu C = Q_{1,eq} = Q_3$

$Q_1 = C_1 \cdot \Delta V_1 = C_1 \cdot \Delta V_{ac} = C_1 (\Delta V_{ab} - \Delta V_{cd}) = C_1 (\Delta V_{ab} - \Delta V_3)$

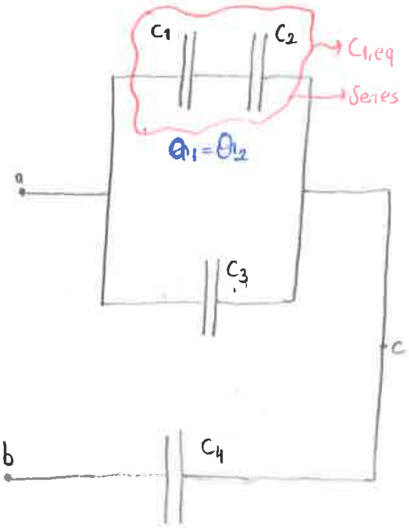
$= C_1 \left(\Delta V_{ab} - \frac{Q_3}{C_3}\right) = 2\mu F \cdot \left(20V - \frac{60\mu C}{6\mu F}\right) = 2\mu F \times 10V = 20\mu C$

$Q_2 = C_2 \cdot \Delta V_2 = C_2 \cdot \Delta V_1 = 4\mu F \cdot 10\ V = 40\mu C$

II. way to find Q_2

$Q_{1,eq} = Q_1 + Q_2 \Rightarrow 60\mu C = 20\mu C + Q_2 \Rightarrow Q_2 = 40\mu C$

Ex:



Each capacitor has $4\mu F$ of capacitance.

a-) $C_{eq} = ?$

b-) $\Delta V_{ab} = 28\ V$ is applied. Find the stored charge on each capacitor.

c-) Find the stored energies each capacitor.

Sol. a-) $\frac{1}{C_{1,eq}} = \frac{1}{C_1} + \frac{1}{C_2} = 2\mu F$ $\quad C_{2,eq} = C_{1,eq} + C_3 = 6\mu F$

$$\frac{1}{C_{eq}} = \frac{1}{C_{2,eq}} + \frac{1}{C_4} \Rightarrow C_{eq} = 2.4\mu F$$

c-) $U_{C_1} = \frac{Q_1^2}{2C_1}$

b-) $Q = C_{eq} \cdot \Delta V_{ab} = 2.4\mu F \times 28\ V = 67.2\ \mu C = Q_{2,eq} = Q_4$

$Q_3 = C_3 \cdot \Delta V_3 = C_3\ \Delta V_{ac} = C_3\ (\Delta V_{ab} - \Delta V_{cb}) = C_3 \cdot (\Delta V_{ab} - \Delta V_4) = C_3 \cdot \left(\Delta V_{ab} - \frac{Q_4}{C_4}\right)$

$4\mu F \times \left(28V - \frac{67.2\ \mu C}{4\ \mu F}\right) \Rightarrow Q_3 = 44.8\ \mu C$

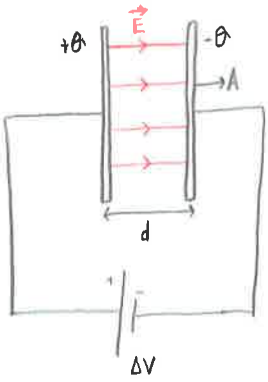
$Q_1 = Q_2 = Q_{1,eq}$

$Q_{2,eq} = Q_{1,eq} + Q_3 \Rightarrow 67.2\mu C = Q_{1,eq} + 44.8\ \mu C$

$Q_{1,eq} = 22.4\ \mu C = Q_1 = Q_2$

16.04.2015
Thursday

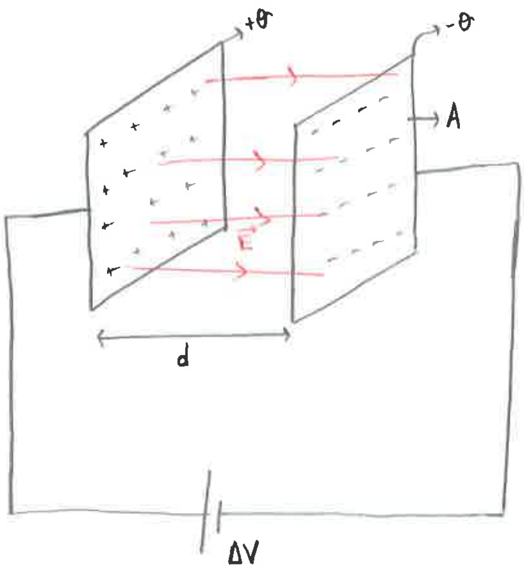
24.4 Stored Potential Energy in the Capacitors



When the capacitor is fully charged the stored electric potential energy.

$$\boxed{U = \frac{Q^2}{2C} = \frac{C.(\Delta V)^2}{2} = \frac{Q.\Delta V}{2}} \ (J)$$

NOTE: We learnt that charges are stored mainly on the inner surfaces of plates. As position of stored energy the $\vec{E}$ btwn plates is assigned.



$$\boxed{U = \frac{Q^2}{2C} = \frac{Q.\Delta V}{2} = \frac{C.(\Delta V)^2}{2} \ (J)}$$

If capacitor is a parallel-plate capacitor

$$\boxed{U_{p.p} = \frac{C_{p.p}.(\Delta V)^2}{2} = \frac{\varepsilon_0.A}{d} \cdot \frac{E^2.d^2}{2} = \frac{1}{2}.\varepsilon_0.A\,d.E^2 \ (J)}$$

$$\boxed{u = \frac{U_{p.p}}{A.d} = \frac{1}{2} \cdot \frac{\varepsilon_0.A.d.E^2}{A.d} = \frac{1}{2}.\varepsilon_0.E^2 \left(\frac{J}{m^3}\right)}$$

→ Energy density for a p.p capacitor.

We assume that the electric potential energy is stored in the region btwn plates. The volume of this region is A.d. Then we can calculate the energy density, $u\left(\frac{J}{m^3}\right)$

Ex: A p.p capacitor has $A = 40\ cm^2$, $d = 1\ mm$ and exposures to $\Delta V = 600\ V$. Find a.) $C_{p.p}$ b.) Q c.) U d.) E e.) u

Solution: a.) $C_{p.p} = \frac{\varepsilon_0.A}{d} = 8.85\times10^{-12}\,\frac{F}{m} \cdot \frac{40\times10^{-4}\,m^2}{10^{-3}\,m} = \underline{35.4\times10^{-12}\,F}$

b.) $Q = C_{p.p}.\Delta V = 35.4\times10^{-12}\,F \times 600\,V = \underline{21.4\times10^{-9}\,C}$

c.) $U = \frac{Q^2}{2C_{p.p}} = \frac{(21.4\times10^{-9}\,C)^2}{2\times35.4\times10^{-12}\,F} = \underline{6.37\times10^{-6}\,J}$

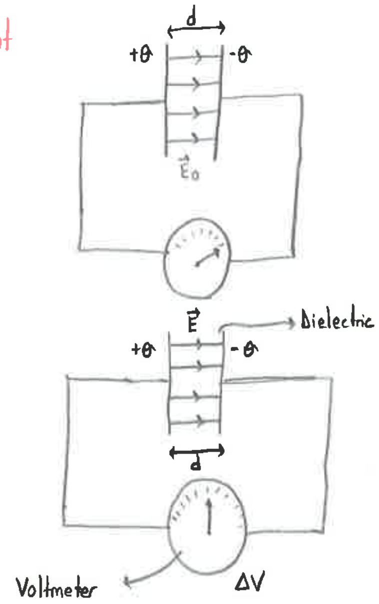
d.) $\Delta V = E.d \Rightarrow E = \frac{600\,V}{10^{3}\,m} = \underline{6\times10^{9}}\,\frac{V}{m} = \frac{N}{C}$

e.) $u_{p.p} = \frac{\varepsilon_0.E^2}{2} = 8.85\times10^{-12}\,\frac{F}{m} \cdot \frac{(6\times10^{5}\,\frac{N}{C})^2}{2} = \underline{1.6\,\frac{J}{m^3}}$

24.5 Dielectrics

Dielectrics are mainly insulating materials which we use to increase the capacitance of a capacitor.

An easy experiment



We have a fully charged capacitor without dielectric.

$$C_0 = \varepsilon_0 . \frac{A}{d}$$

$$\Delta V_0 = E_0 . d \qquad E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon . A}$$

Note : The region between plates must be filled by dielectric completely.

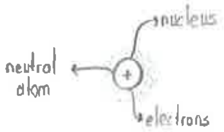
Dielectric constant, it has no dimension

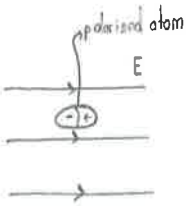
$$C = K . C_0 = K . \varepsilon_0 . \frac{A}{d} \uparrow$$

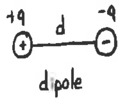
$$\Delta V = \frac{\Delta V_0}{K} \downarrow$$

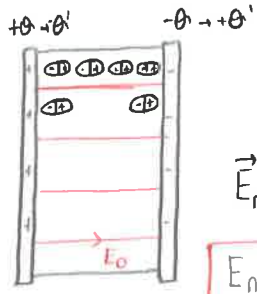
$$E = \frac{E_0}{K} = \frac{Q}{K . \varepsilon_0 . A} \downarrow$$

Lets analyse the reason to observe smaller resultant field E when region between plates is filled by dielectric material.









$$\vec{E}_{net} = \vec{E}_0 + \vec{E}_{op}$$

Produced by dielectric

$$\boxed{E_{net} = E_0 - E_{op}}$$

Q' = induced charge on the surface of dielectric

$$\oint \vec{E} . d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$$E \oint dA = \frac{Q - Q'}{\varepsilon_0}$$

$$E . A = \frac{Q - Q'}{\varepsilon_0}$$

$$\boxed{E = \frac{Q - Q'}{\varepsilon_0 A}}$$

Ex : A capacitor with dielectric has $A = 100\ cm^2$, $\theta = 8.9 \times 10^{-7}\ C$ and $E = 1.4 \times 10^6\ \frac{V}{m}$

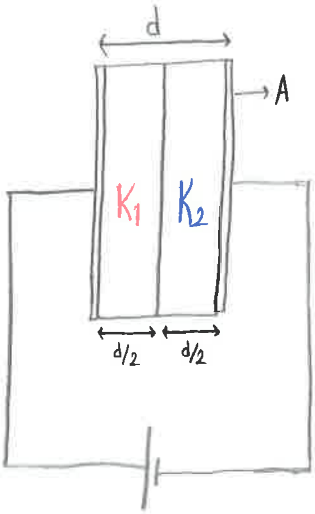
a-) Find the K

b-) " " θ'

Solution : a-) $E = \frac{\theta}{K.\varepsilon_0.A} \Rightarrow K = \frac{8.9 \times 10^{-7}\ C}{1.4 \times 10^6\ \frac{V}{m} \times 8.85 \times 10^{-12}\ \frac{F}{m} \times 100 \times 10^{-4}\ m^2} = 7.18$

b-) $\frac{\theta}{K} = \theta - \theta' \Rightarrow \theta' = \theta - \frac{\theta}{K} = \theta\left(\frac{K-1}{K}\right) = 8.9 \times 10^{-7}\ C\left(\frac{7.18-1}{7.18}\right)$ $\theta' = 7.65 \times 10^{-7}\ C < \theta$

Ex



Find the C_{eq} for this capacitor.

Solution. $C_1 = K_1 . \varepsilon_0 . \frac{A}{\frac{d}{2}} = 2.K_1.\varepsilon_0 . \frac{A}{d}$

$C_2 = K_2 . \varepsilon_0 . \frac{A}{\frac{d}{2}} = 2.K_2.\varepsilon_0 \frac{A}{d}$

We accept two capacitors are in series combination. $\Rightarrow$ $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2.K_1.\varepsilon_0.\frac{A}{d}} + \frac{1}{2.K_2.\varepsilon_0.\frac{A}{d}} \Rightarrow$

(K_2) (K_1)

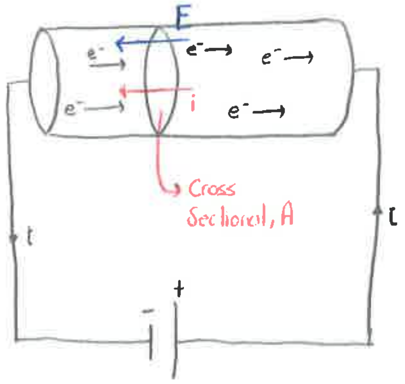
$$\boxed{C_{eq} = \frac{2.K_1.K_2}{(K_1+K_2)} \cdot \frac{\varepsilon_0.A}{d}}$$ (F)

28.04.2015
Tuesday

CHAPTER - 25

CURRENT, RESISTANCE AND ELECTROMOTIVE FORCE

25.1 Electric Current



Electric current is the net flow of charge carriers through certain area per unit time

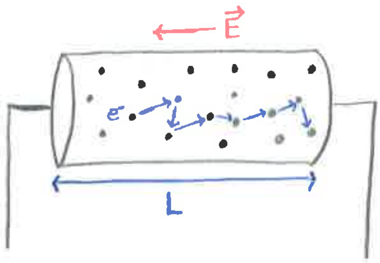
$$i = \frac{dq}{dt} \quad \left(\frac{C}{s} = \text{Ampere} = A \right)$$

*** Direction of current is accepted in reverse direction of moving electrons in the conductor. This case is a convention.

→ Current has an accepted direction, a magnitude, and a unit. However, it is not a vector quantity. As a vector quantity, the current density ($\vec{J}$) is defined as current per unit area.

$$\boxed{J = \frac{i}{A} \left(\frac{A}{m^2} \right)} \quad \text{or} \quad \boxed{i = \int \vec{J} \cdot d\vec{A} = \int J \cdot dA \cdot \cos\theta \quad (A)}$$

Drift Velocity ($\vec{V}_d$): In the materials the charge carriers can not move linearly due to collisions of the atoms of material. The collisions cause deviations thus charge carriers do zig zag motion. Their average velocity in the material is called as Drift Velocity ($\vec{V}_d$).



zig zag motion

$$\boxed{V_d = \frac{L}{t} \left(\frac{m}{s} \right)} \qquad \boxed{V_d = \frac{i}{n \cdot q \cdot A} \left(\frac{m}{s} \right)}$$

n: charge carrier concentration per unit volume $\left(\frac{\#}{m^3} \right)$

q: magnitude of charge of carriers

$$\boxed{V_d = \frac{J}{n \cdot q} \left(\frac{m}{s} \right)}$$

Note: The electrons move very fast btwn two collisions ($\sim 10^6 \frac{m}{s}$) but due to zig zag motion the net velocity that is V_d is very small ($\sim 10^{-4} \frac{m}{s}$)

$$n = N_a \cdot \frac{d}{m} \left(\frac{\#}{m^3} \right)$$

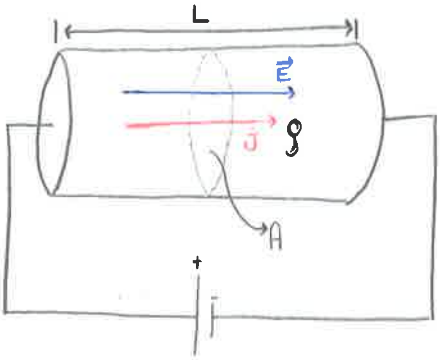
Ex How long does it takes electrons to get from a car battery to the starting motor? Assume that $i = 300\ A$ and electrons travel through a copper wire with $A = 0.21\ cm^2$ and length $0.85\ m$ ($N_A = 6.02 \times 10^{23}\ \frac{\#}{mole}$, $d_{cu} = 9\ \frac{g}{cm^3}$, $M_{cu} = 64\ \frac{g}{mole}$, $|q_e| = |-1.6 \times 10^{-19} C|$)

Avogadro's Number

Solution : $t = \dfrac{L}{v_d} = \dfrac{L}{\dfrac{i}{n \cdot |q_e| \cdot A}}$ where $n = \dfrac{N_A \cdot d_{cu}}{M_{cu}} = \dfrac{(6.02 \times 10^{23}\ \frac{\#}{mole}) \times (9\ \frac{g}{cm^3})}{64\ \frac{g}{mole}} = 8.46 \times 10^{28}\ \frac{\#}{m^3}$

$$v_d = \frac{0.85\,m}{\dfrac{300\ A}{(8.46 \times 10^{28}\ \frac{\#}{m^3}) \times |-1.6 \times 10^{-19} C| \times (0.21 \times 10^{-4}\ m^2)}} = 806\ s = 13\ min\ 26\ s$$

25.2 Resistance, Resistivity with Ohm's Law



$\Delta V = E \cdot L$

The resistance of material (R) against electric current is

$$\boxed{R = \rho \cdot \frac{L}{A}\ (\Omega)}$$

Resistivity: Characteristic property according to type of material

Table 25.1 Resistivities at 20° C in Book

$$\boxed{E = \rho \cdot J \quad \left(\frac{N}{C} = \frac{V}{m}\right)}$$ Ohm's Law

The reciprocal of resistivity is called as conductivity (σ)

$$\sigma = \frac{1}{\rho}\ (\Omega m)^{-1}$$

$$\boxed{\vec{J} = \sigma \cdot \vec{E}\ \left(\frac{A}{m^2}\right)}$$

$$\frac{i}{A} = \frac{1}{\rho} \cdot \frac{\Delta V}{L} \Rightarrow i \cdot \underbrace{\frac{\rho \cdot L}{A}}_{R} = \Delta V \Rightarrow \boxed{\Delta V = i \cdot R}$$

Result of Ohm's Law

Ex A wire has 4m length, r = 3mm, and resistance 15 mΩ. A potential difference 23 V is applied btwn the ends

a-) i = ?   b-) j = ?   c-) ρ = ?

Solution : a-) $i = \frac{\Delta V}{R} = \frac{23\,V}{15\times10^{-3}\,\Omega} = 1533.33\ A$

b-) $j = \frac{i}{A} = \frac{1533.33\ A}{(3.14)\times(3\times10^{-3}m)^2} = 5.42\times10^{6}\ \frac{A}{m^2}$

c-) $\rho = R\cdot\frac{A}{L} = \frac{(15\times10^{-3}\Omega)\times(3.14)\times(3\times10^{-3}m)^2}{4m} = 1.05\times10^{-9}\,\Omega m$

25.3 Temperature Variation Resistivity and Resistance

When the temperature of substance (or medium) change the resistance against the current varies.

$$\boxed{R = R_0\left[1+\alpha(T-T_0)\right]}$$

$$\boxed{\rho = \rho_0\left[1+\alpha(T-T_0)\right]}$$

R_0 = resistance of material at 20° C

ρ_0 = resistivity " " " " "

T_0 = 20°C or T_0 = 293° K

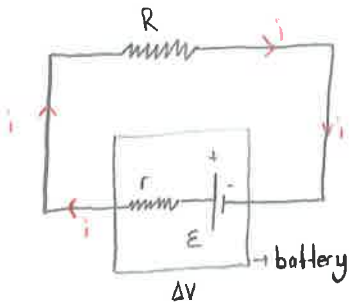
α = temperature coefficient of resistivity $\left(\frac{1}{°C}, \frac{1}{K}\right)$

Ex : At what temperature would be resistance of a copper wire be double its resistance at 20° C ? $\left(\alpha_{Cu} = 4.3\times10^{-3}\frac{1}{K}\right)$

$2\cancel{R_0} = \cancel{R_0}\left[1+\alpha(T-T_0)\right] \Rightarrow 2-1 = (4.3\times10^{-3}\frac{1}{K})(T-293)K \Rightarrow T = 525.5\ K$

25.4 Electromotive Force and Circuits

Every battery (or power supply) has certain magnitude of internal resistance (r). Because of this internal resistance battery can provide a lower potential then its maximum potential that is called electromotive force (ε).



r : internal resistance of battery

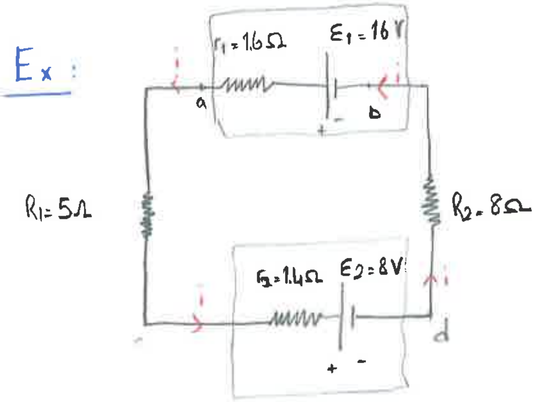
R : external resistance

ε : electromotive force (emf)

ΔV : terminal voltage of battery

$\Delta V = \varepsilon - i.r$ (V)
$i.R = \varepsilon - i.r$ (V)
$i = \frac{\varepsilon}{r+R}$ (A)

Ex :



a-) $i = ?$

b-) Terminal voltage of find battery $\Delta V_{ab} = ?$

c-) Find the potential difference btwn ports a and d ; $\Delta V_{ad} = ?$

d-) Draw a graph which showns potential increases or decreases in the circent.

Solution: a-) $i = \dfrac{\varepsilon_{net}}{R_{eq}} = \dfrac{\varepsilon_1 - \varepsilon_2}{r_1 + r_2 + R_1 + R_2} = \dfrac{16V - 8V}{(1.6 + 1.4 + 5 + 8)\Omega} = 0.5\ A$

b-) $\Delta V_{a,b} = \varepsilon_1 - i.r_1 = 16\ V - (0.5A \times 1.6\Omega) = 15.2\ V$

I. WAY

c-) $V_a - i.R_1 - i.r_2 - \varepsilon_2 = V_d \Rightarrow V_a - V_d = i.R_1 + i.r_2 + \varepsilon_2$

$\Delta V_{ad} = (0.5\ A \times 5\Omega) + (0.5A \times 1.4\Omega) + 8V = 11.2\ V$

$V_a = 15.2\ V$

$V_d = 4\ V$

$V_a - V_d = 11.2\ V$

II. WAY

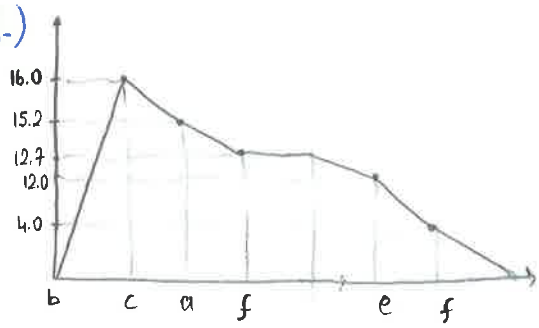
$V_a + i.r_1 - \varepsilon_1 + i.R_2 = V_d$

$V_a - V_d = \varepsilon_1 - i.r_1 - i.R_2$

$\Delta V_{ad} = 16\ V - (0.5\ A \times 1.6\Omega) - (0.5A \times 8\Omega)$

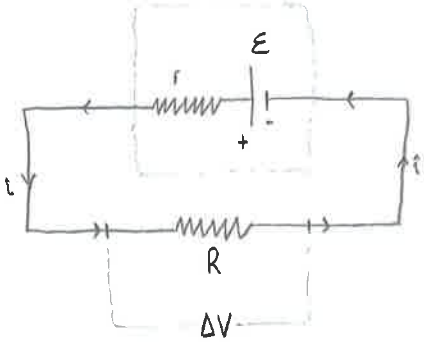
$\Delta V_{ad} = 11.2\ V$

d-)



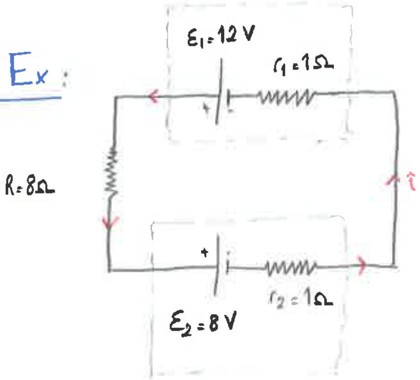
25.5 Power and Energy in Electric Circuits

Battery (or power supply) provides energy to the circuit elements



$P_{\mathcal{E}} = i \cdot \mathcal{E}$ (Watt)	→ Power delivered by battery to the circuit
$P_r = i^2 \cdot r$ (watt)	→ dissipated (lost) power by internal resistor
$P_R = i^2 \cdot R = \dfrac{\Delta V^2}{R}$ (Watt)	→ dissipated (lost) power by external resistor

Ex:



a-) i = ?

b-) Which battery delivery energy to the circuit, at what rate ?

c-) Which battery stores energy, at what rate ?

d-) Dissipated power by resistors ?

e-) Show that $P_{delivered} = P_{lost}$

Solution: a-) $i = \dfrac{\mathcal{E}_1 - \mathcal{E}_2}{R_{eq}} = \dfrac{\mathcal{E}_1 - \mathcal{E}_2}{r_1 + r_2 + R} = \dfrac{12\text{ V} - 8\text{ V}}{10\Omega} = 0.4$ A

b-) $P_{\mathcal{E}_1} = i \cdot \mathcal{E}_1 = (0.4\text{ A}) \times (12\text{ V}) = 4.8$ Watts → $\mathcal{E}_1$ delivers power to the circuit

c-) $P_{\mathcal{E}_2} = i \cdot \mathcal{E}_2 = (0.4\text{ A}) \times (8\text{ V}) = 3.2$ Watts → $\mathcal{E}_2$ stores the some part of electrical energy provided by $\mathcal{E}_1$

d-) $P_{r_1} = i^2 \cdot r_1 = (0.4\text{ A})^2 \times (1\Omega) = 0.16$ Watt

$P_{r_2} = i^2 \cdot r_2 = (0.4\text{ A})^2 \times (1\Omega) = 0.16$ Watt

$P_R = i^2 \cdot R = (0.4\text{ A})^2 \times (8\Omega) = 1.28$ Watt

e-) $\underbrace{P_{\mathcal{E}_1}}_{\text{Provider}} = \underbrace{P_{\mathcal{E}_2} + P_{r_1} + P_{r_2} + P_R}_{\text{consumers}}$

4.8 Watts = (3.2 + 0.16 + 0.16 + 1.28) watts

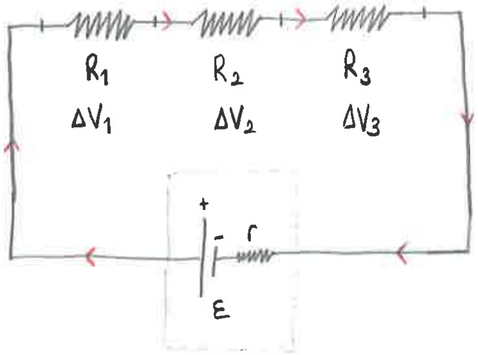
4.8 Watts = 4.8 Watts

CHAPTER-26

DIRECT CURRENT CIRCUITS

26.1 Resistors in Series and Parallel

a-) Resistors in Series



$\Delta V \rightarrow$ Terminal Voltage of Battery

$\Delta V = \mathcal{E} - i.r$

- Currents through each resistor are equal

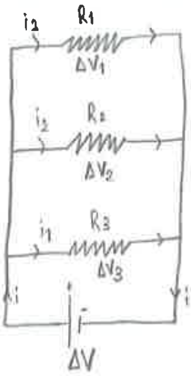
$\boxed{i = i_1 = i_2 = i_3}$

- Algebric sum of potential differences of each resistor is equal to ΔV of battery.

$\boxed{\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 + \ldots}$

$\boxed{R_{eq} = R_1 + R_2 + R_3 + \ldots}$

b-) Resistors in Parallel

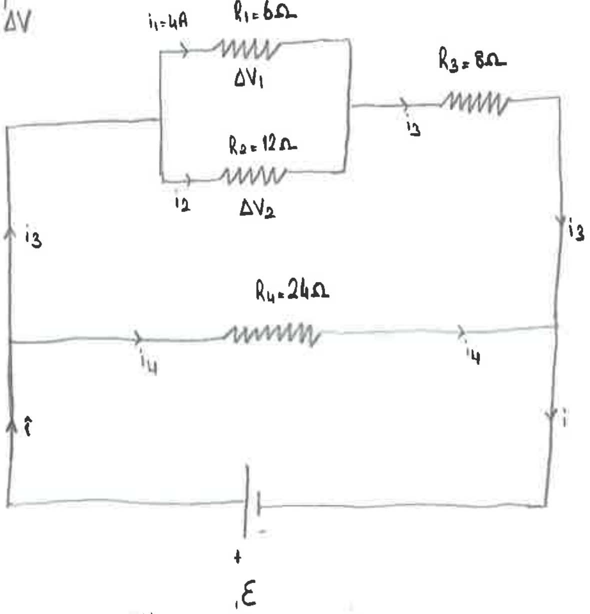


$\boxed{i = i_1 + i_2 + i_3}$

$\boxed{\Delta V = \Delta V_1 = \Delta V_2 = \Delta V_3}$

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

Ex:



a-) Find R_{eq} of circuit ?

b-) Calculates currents through each resistor ?

c-) Power delivered by battery ?

d-) " dissipated " resistors ?

Sol: a-) $\frac{1}{R_{1,eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{\underset{(2)}{6\Omega}} + \frac{1}{12\Omega} \Rightarrow R_{1,eq} = 4\Omega$

$R_{2,eq} = R_{1,eq} + R_3 = (4+8)\Omega = 12\Omega$

$\frac{1}{R_{eq}} = \frac{1}{R_{2,eq}} + \frac{1}{R_4} = \frac{1}{\underset{(2)}{12\Omega}} + \frac{1}{24\Omega} \Rightarrow R_{eq} = 8\Omega$

b-) $\Delta V_1 = \Delta V_2$

$i_1.R_1 = i_2.R_2$

$4A.6\Omega = i_2.12\Omega$

$i_2 = 2A$

$i_1 + i_2 = i_3$

$4A + 2A = i_3$

$i_3 = 6A$

$\Delta V_{2,eq} = i_3.R_{2,eq} = 6A \times 12\Omega = 72\,V = \Delta V_4 = i_4.R_4 \Rightarrow$

$i_4 = \frac{72\,V}{24\Omega} = 3A$

c-) $P_\varepsilon = i.\varepsilon = (i_3+i_4).\varepsilon$

$P_\varepsilon = 9A \times 72\,V = \underline{648\ \text{Watts}}$

Checkyourself

$P_\varepsilon = P_1+P_2+P_3+P_4$

648 Watts = 648 Watts

d-) $P_{R_1} = i_1^2.R_1 = \dfrac{(\Delta V_1)^2}{R_1} = \dfrac{(24\,V)^2}{6\Omega} = \underline{96\ \text{Watts}}$

$P_{R_2} = i_2^2.R_2 = \dfrac{(\Delta V_2)^2}{R_2} = \dfrac{(24V)^2}{12\Omega} = \underline{48\ \text{Watts}}$

$P_{R_3} = i_3^2.R_3 = (6A)^2.(8\Omega) = \underline{288\ \text{Watts}}$

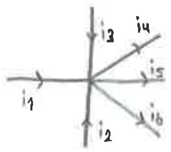
$P_{R_4} = i_4^2.R_4 = (3A)^2.(24\Omega) = \underline{216\ \text{Watts}}$

26.2 Kirchhoff's Rules

We need to apply the Kirchhoff's Rules when there are two or more batteries which combined as mixed.

i-) Junction (Point) Rule

Algebraic sum of currents incident to a junction is equal to
" " " " leaving that junction.



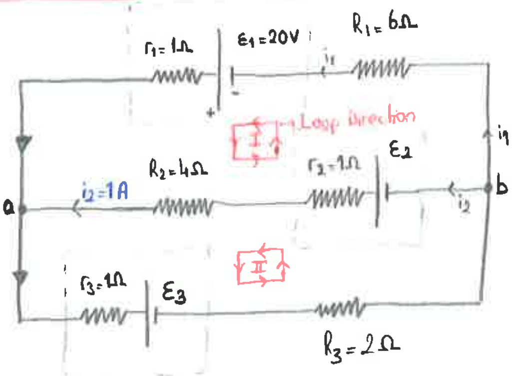
$$\boxed{i_1+i_2+i_3... = i_4+i_5+i_6...}$$

ii-) Loop Rule

In a closed loop the summation of potential changes due to circuits elements is equal to zero.

$$\boxed{\Sigma \Delta V = 0}$$

Ex :



a-) Find current through each resistor ?

b-) Find ε_2 and ε_3 = ?

c-) ΔV_{ab} = ?

Solution : a-) $i_1+i_2=i_3$

$1A + i_2 = 2A$

$i_2 = 1A$

$i_1 = 1A$

$i_2 = 1A$

$i_3 = 2A$

b-) I. Loop: $\Sigma\Delta V=0$

$-i_1.R_1 + \varepsilon_1 - i_1.r_1 + i_2.R_2 + i_2.r_2 - \varepsilon_2 = 0$

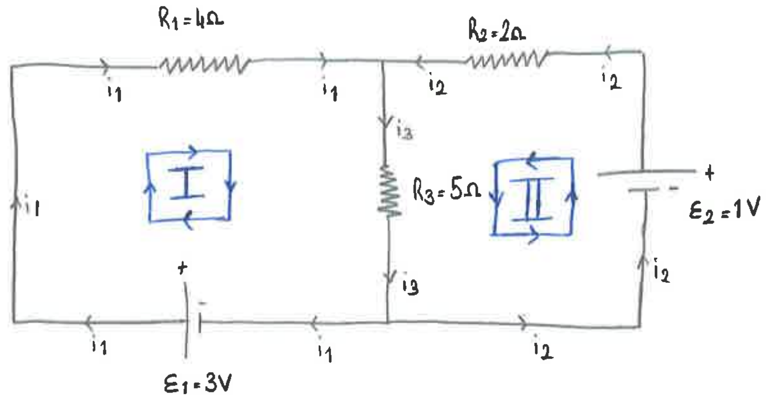
II. Loop: $\Sigma\Delta V=0$

$\varepsilon_2 - i_2.r_2 - i_2.R_2 - i_3.r_3 - \varepsilon_3 - i_3.R_3 = 0$

c-) $V_a + i_2.R_2 + i_2.r_2 - \varepsilon_2 = V_b$

$\Delta V_{ab} = V_a - V_b = \varepsilon_2 - i_2.R_2 - i_2.r_2 =$

Ex :



a-) What is the rate at which energy is lost in R_1, R_2 and R_3?

b-) Which battery provides power to the circuit ?

c-) Which battery consumes power ?

Solution :

a-) We should specify flowing directions of currents.

$\boxed{i_1 + i_2 = i_3}$ → from junction rule (1)

1. Loop : $\Sigma \Delta V = 0$

$+\varepsilon_1 - i_1.R_1 - i_3.R_3 = 0$

$3V - i_1.4\Omega - i_3.5\Omega = 0$ (2)

2. Loop : $\Sigma \Delta V = 0$

$+\varepsilon_2 - i_2.R_2 - i_3.R_3 = 0$

$1V - i_2.2\Omega - i_3.5\Omega = 0$

$\boxed{i_1 = i_3 - i_2}$

$3V - \underset{i_3 - i_2}{i_1}.4\Omega - i_3.5\Omega = 0$

$1V - i_2.2\Omega - i_3.5\Omega = 0$

$3V - i_3.9\Omega + i_2.4\Omega = 0$

$2/ 1V - i_2.2\Omega - i_3.5\Omega = 0$

$5V - i_3.9\Omega - i_3.10\Omega = 0$

$5V = 19\Omega . i_3 \Longrightarrow i_3 = \frac{5V}{19\Omega} = 0.26A \Rightarrow 3V - (0.26A \times 9\Omega) + i_2.4\Omega = 0$

$i_2 = -0.16A$

the real direction of i_2 in circuit is reverse

$i_1 = i_3 - i_2$

$= [0.26 - (-0.16)] A$

$i_1 = 0.42 A$

$P_{R_1} = i_1^2 \cdot R_1 = (0.42\ A)^2 \cdot 4\Omega = 0.7$ watts

$P_{R_2} = i_2^2 \cdot R_2 = (0.16\ A)^2 \cdot 2\Omega = 0.051$ watts

$P_{R_3} = i_3^2 \cdot R_3 = (0.26\ A)^2 \cdot 5\Omega = 0.34$ watts

b-) $P_{\varepsilon_1} = i_1 \cdot \varepsilon_1 = (0.42\ A) \times 3V = 1.26$ watts $\longrightarrow$ ε_1 provides

c-) $P_{\varepsilon_2} = i_2 \cdot \varepsilon_2 = (0.16\ A) \times 1V = 0.16$ watts $\longrightarrow$ ε_2 consumes

Checkyourself

$$\boxed{P_{Provided} = P_{consumed}}$$

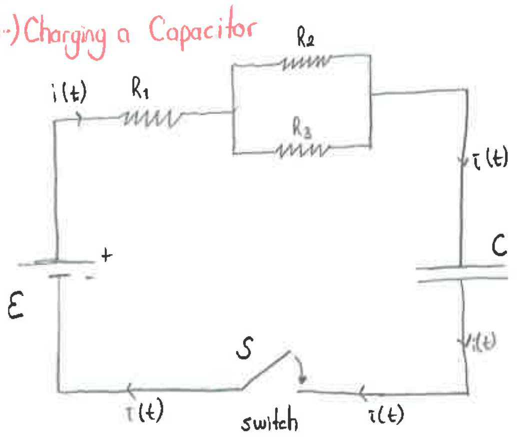
$$P_{\varepsilon_1} = P_{\varepsilon_2} + P_{R_1} + P_{R_2} + P_{R_3}$$

$$1.26\ W = (0.16 + 0.7 + 0.051 + 0.34)\ W$$

12.05.2015
Tuesday

26.3 RC Circuits

i-) Charging a Capacitor



Just after the S is closed the current begins to flow in the circuit by charging capacitor. The initial magnitude of current $(t_0=0)$ is $i_0 = \frac{\mathcal{E}}{R}$ and it reaches to zero when charging process finishes.

The time dependent $i(t)$ and $q(t)$ eqns in the charging process are

$$\boxed{i(t) = I_0 . e^{\frac{-t}{R_{eq}.C}} = \frac{\mathcal{E}}{R} . e^{\frac{-t}{\tau}}} \quad (A)$$

$$\boxed{q(t) = C.\mathcal{E}.\left[1 - e^{\frac{-t}{R_{eq}.C}}\right] = Q\left[1 - e^{\frac{-t}{\tau}}\right]} \quad (C)$$

where $\tau = R_{eq}.C \rightarrow$ time constant since it has unit of time.

$e = 2.718 \rightarrow$ Euler's Number

Example : A $3M\Omega$ resistor, $1\mu F$ capacitor and $\mathcal{E}=4V$ battery are connected in series. 1 sec. later, what are the parameters

a-) $q_C = ?$ b-) $U_C = ?$ c-) $P_R = ?$ d-) $P_{\mathcal{E}} = ?$

Solution :

a-) $q(t=1s) = C.\mathcal{E}.\left[1 - e^{\frac{-t}{R.C}}\right]$

$q(t=1s) = (1\times10^{-6}F)\times4V\times\left[1 - e^{-\frac{1}{3\times10^6\Omega\times10^{-6}F}}\right]$

$q(t=1s) = 1.13\times10^{-6}\,C$

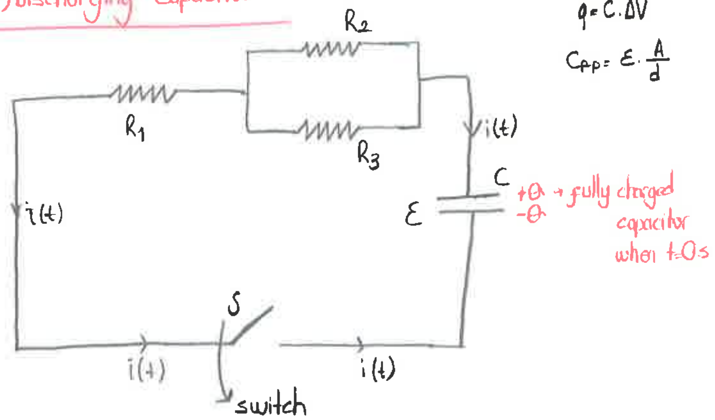
b-) $U_C = \frac{q^2}{2C} = \frac{(1.13\times10^{-6}C)^2}{2\times(10^{-6}F)} = 0.64\times10^{-6}\,J$

c-) $P_R = i^2.R = (9.57\times10^{-7}A)^2\times(3\times10^6\Omega) = 2.75\times10^{-7}$ Watt $\rightarrow$ dissipated (lost) power

$i(t=1s) = \frac{4V}{3\times10^6\Omega} . e^{-\frac{1s}{3\times10^6\Omega\times10^{-6}F}} = 9.57\times10^{-7}\,A$

d-) $P_{\mathcal{E}} = i.\mathcal{E} = (9.57\times10^{-7}A)\times4V = 3.8\times10^{-6}$ Watt

ii-) Discharging Capacitor



$q = C.\Delta V$

$C_{pp} = \varepsilon . \frac{A}{d}$

The time dependent i(t) and q(t) eqns. due to discharging process are

$$i(t) = I_0 . e^{\frac{-t}{R_{eq}.C}} = \frac{\varepsilon}{R} . e^{-\frac{t}{\tau}} \quad (A)$$

$$q(t) = C.\varepsilon . e^{\frac{-t}{R_{eq}.C}} = Q . e^{\frac{-t}{R_{eq}.C}} \quad (C)$$

Ex : A capacitor with initial $\varepsilon = 100$ V is discharged through a resistor. When $t = 10$ s the $\Delta V = 1$ V

a-) What is $\tau = ?$ b-) $\Delta V = ?$ when $t = 17$ s.

Solution :

a-) $\tau = R_{eq}.C$

$$q(t) = C.\varepsilon . e^{-\frac{t}{R_{eq}.C}}$$

$$C.\Delta V(t = 10s) = C.\varepsilon . e^{\frac{-10s}{\tau}}$$

$$1 V = 100 V . e^{\frac{-10s}{\tau}} = \ldots \Rightarrow \tau = 2.17 \text{ sec}$$

b-) $\Delta V = \varepsilon . e^{\frac{-t}{\tau}}$

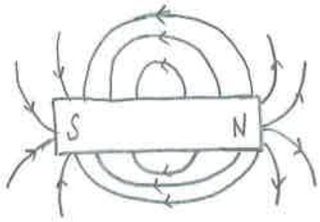
$$\Delta V(t = 17s) = 100 V\, e^{\frac{-17s}{2.17s}}$$

$$\Delta V(t = 17s) = 3.9 \times 10^{-2} V$$

CHAPTER-27

MAGNETIC FIELD

27.1 Magnetic field



→ Magnetic field ($\vec{B}$) lines of a bar magnet.

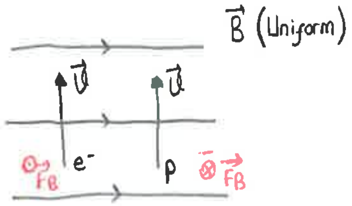
N: North Pole

S: South Pole

→ Same poles repel each other

Opposite poles attract " "

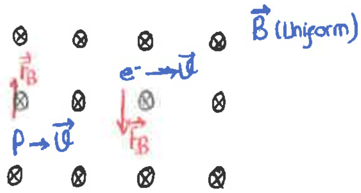
Magnetic field ($\vec{B}$) exerts force on moving carge carriers.



⊗ = toward board (page)

⊙ = out of page

e⁻ = electron, p = proton



→ Magnitude of charge

$$\boxed{F_B = |q| \cdot \vec{v} \times B} \text{ (N)}$$ → Magnetic force acting on moving charge carrier (its velocity is $\vec{v}$)

$$\boxed{F_B = q \cdot v \cdot B \cdot \sin\varnothing} \text{ (N)}$$ → $\varnothing$: angle between $\vec{v}$ and $\vec{B}$ vectors

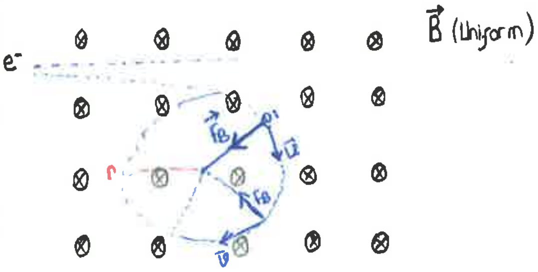
B.P → FB ↑

i.P → V →

O.P → B ⊗

When Object is negative [illegible] direction is reversed

27.2 Circulating Charged Particle in Uniform Magnetic Field



↳ Here $\vec{F}_{cent} = \vec{F_B}$

$$m \cdot \frac{v^{\cancel{2}}}{r} = q \cdot \cancel{v} \cdot B \cdot \sin\theta^{\,90°}$$

$$\boxed{r = \frac{m \cdot v}{B \cdot q}} \text{ (m)}$$ → radius of circular orbit

the period of circular motion, T

$$T = \frac{2 \cdot \pi \cdot r}{v} = \frac{2 \cdot \pi \cdot m \cdot \cancel{v}}{\cancel{v} \cdot q \cdot B} =$$

$$\Rightarrow \boxed{T = \frac{2 \cdot \pi \cdot m}{q \cdot B}} \text{ (s)}$$

the frequency $= \boxed{\dfrac{1}{T} = \dfrac{q \cdot B}{2 \cdot \pi \cdot m}}$ (Hz)

The angular frequency $w = 2 \cdot \pi \cdot f = \dfrac{\cancel{2 \cdot \pi} \cdot q \cdot B}{\cancel{2 \cdot \pi} \cdot m} = \dfrac{q \cdot B}{m}$

Ex: An e- with $K.E = 1,2$ KeV circles in a plane perpendicular to a uniform $\vec{B}$. The radius of circular orbit is 25 cm.

a-) Find $V = ?$ b-) $B = ?$ c-) $f = ?$ d-) $T = ?$

($e = 1eV = 1,6 \times 10^{-19}$ J , $q_e = -1.6 \times 10^{-19}$ C , $m_e = 9.1 \times 10^{-21}$ kg)

Solution:

a-) $V = \dfrac{q.r.B}{m_e} \Rightarrow KE = \dfrac{1}{2}.m.V^2$

$$(1.2 \times 10^3 eV) \times (1.6 \times 10^{-19} J) = \frac{1}{2}.(9.1 \times 10^{-21} kg).V \qquad V = 2 \times 10^7 \text{ m/s}$$

b-) $B = \dfrac{m.V}{q.r} = \quad = 4.55 \times 10^4$ Tesla (T)

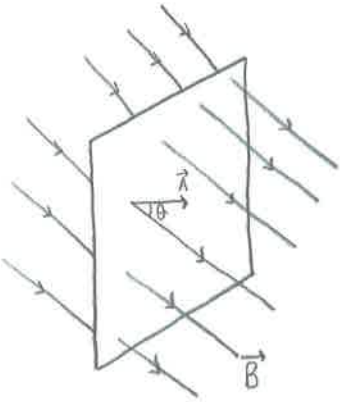
c-) $f = \dfrac{q.B}{2.\pi.m} = \quad = 12.74 \times 10^6 \ \dfrac{1}{s}$

d-) $T = \dfrac{1}{f} = 7.85 \times 10^{-8}$ s

27.3 Magnetic Flux

14.05.2015
Thursday

Magnetic flux (Φ_B) is equal to number of magnetic field ($\vec{B}$) lines passing through some surface.

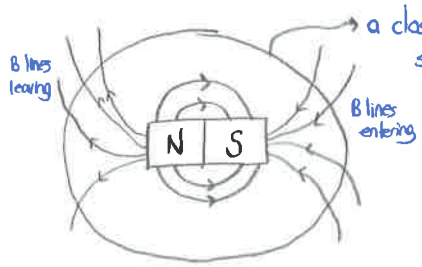


$$\boxed{\Phi_B = \vec{B}.\vec{A} = B.A.\cos\theta} \quad (T.m^2 = Weber = Web)$$

If the surface has random shape

$$\boxed{\Phi_B = \int \vec{B}.d\vec{A}} \quad (Wb)$$

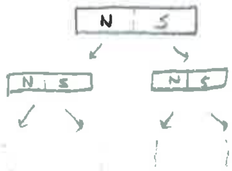
The magnetic flux through a closed surface is always equals to zero since # of field lines entering to surface = # of field lines leaving the surface.



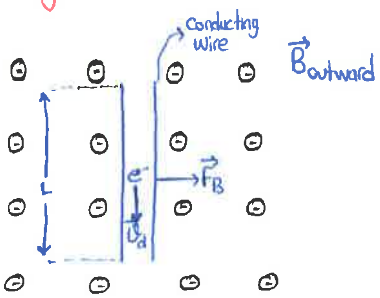
$$\boxed{\Phi_B = \oint \vec{B}.d\vec{A} = 0}$$

→ Gauss Law for magnetism

→ It proves that an isolated monopole of a magnet does not exist in the nature.



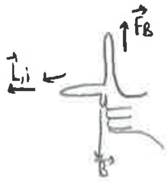
27.4 Magnetic Force on a Current Carrying Conducting Wire



$\vec{F}_B = q_e . \vec{v} \times \vec{B}$

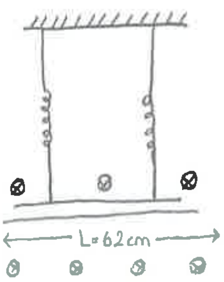
$\vec{F}_B = e . \vec{v}_d \times \vec{B}$ here $e = i.t = i.\dfrac{L}{v_d}$

$\vec{F}_B = i.\dfrac{L}{\cancel{v_d}} . \cancel{v_d} . B . \sin\theta = i.L.B.\sin\theta$



$\boxed{F_B = B.i.L}$ (N) where $\vec{L}$ is in the same direction of i.

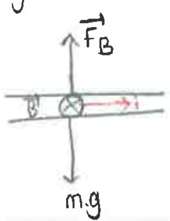
Ex:



$B = 0.44\ T$

The wire has $L = 62$ cm and $m = 13$ g. What must be the direction and magnitude of current through the wire to have zero tension in the springs?

Solution: $g = 10\ N/kg$



$F_B = m.g$ to have zero tension

θ is angle btwn i and $\vec{B}$

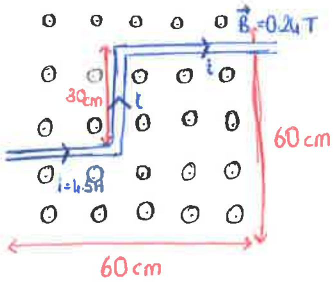
↓

$i.L\ B.\sin\theta = m.g$

$i \times (0.62\ m) \times (0.441) \times (\sin 90^\circ) = 13 \times 10^{-3}\ kg . 10\ N/kg$ $\quad i = 0.47\ A$ toward right

18.05.2015
Monday

Ex :

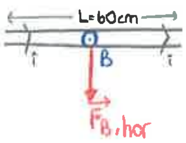


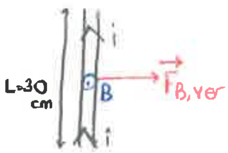
Find the magnitude and direction the net magnetic force on wire.

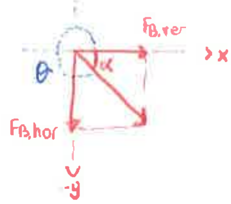
$F_{net} = ?$

Solution : $\vec{F}_{B,net} = \vec{F}_{B,ver} + \vec{F}_{B,hor} \Rightarrow F_{B,net} = \left(F_{B,hor}^{\,2} + F_{B,ver}^{\,2}\right)^{1/2} = \left[(0.628N)^2 + (0.314N)^2\right]^{1/2} = 0.723\ N$

Horizontal Parts







$\theta = 360 - \alpha \rightarrow$ Direction

$\tan\alpha = \dfrac{F_{B,hor}}{F_{B,ver}} = \dfrac{0.628\,N}{0.314\,N}$

$\alpha = \arctan 2 = 63.43°$

$\theta = 360° - 63.43° = 296.57°$

$F_{B,hor} = i.L_{hor}.B.\sin 90°$

$F_{B,hor} = 4.5\,A \times 0.6\,m \times 0.24\,T = 0.628\ N$

$F_{B,ver} = i.L_{ver}.B.\sin 90°$

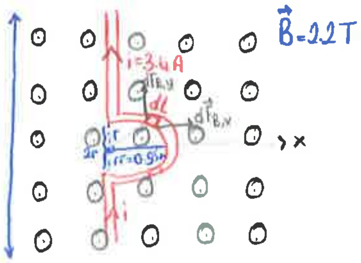
$F_{B,ver} = 4.5\,A \times 0.3\,m \times 0.24\,T = 0.314\,N$

→ If the conducting wire is not straight then magnetic force can be found by taking the integral of

$d\vec{F}_B = i.d\vec{L}.\vec{B}$, $dF_B = i.d.L.\sin\phi$

$$\boxed{\vec{F}_B = i\int d\vec{L}\times\vec{B}}\ (N)$$

Ex :



Find the magnitude and direction the net magnetic force on wire.

Solution : $\vec{F}_{B,net} = \vec{F}_{B,cir} + \vec{F}_{B,straight}$

Circular Part

$d\vec{F}_{B,y,net} = 0$ since $d\vec{F}_{B,y}$ component cancel each other.

$dF_{B,net} = dF_{B,x} = dF_B.\sin\theta = i.dL.B.\sin\underbrace{\phi}_{90°}.\sin\theta = i.dL.B.\sin\theta$

$F_{B,net} = i.B.\int dL.\sin\theta$

$F_{B,net,cr} = i.B.\int r.\sin\theta.d\theta$

$F_{B,net,cr} = i.r.B.\int_{\theta_i=0}^{\theta_f=\pi} \sin\theta.d\theta = i.r.B.\left(-\cos\theta\,\Big|_0^{\pi}\right) = i.r.B.\left[\cos 0° - \cos\pi\right] = i.2r.B \Rightarrow F_{B,net,cir} = 2\times 3.4A \times 0.95\,m \times 2.2\,T = 14.212\ N$ (toward right)

$F_{B,straight,net} = i.L_{str}.B = 1\times(3m - 2m)\times B$

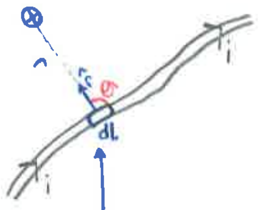
$F_{B,net,straight} = 3.4\,A\times 1.1\,m \times 2.2\,T = 8.228\ N$ (toward right)

$F_{B,net} = F_{B,net,cir} + F_{B,net,straight} = 22.440\ N$ (toward right)

CHAPTER-28

SOURCES OF MAGNETIC FIELD

28.1 Magnetic Field of a Current Element



→ randomly shaped current carrying conductor

$\vec{r} = r.\hat{r}$ ($\hat{r}$ ↳ unit vector)

infinitesmall current element, $i \parallel d\vec{L}$

$$\boxed{d\vec{B} = \frac{\mu_0 . i}{4\pi} . \frac{d\vec{L} \times \hat{r}}{r^2} = \frac{\mu_0 . i}{4\pi} . \frac{d\vec{L} \times \vec{r}}{r^3}} \quad (T)$$

↳ Biot-Savartz Law

in scalar form here μ_0 = magnetic permeability of free space

$|d\vec{L} \times \hat{r}| = dL \times 1 \times \sin\theta$ (θ: angle btwn $d\vec{L}$ and $\hat{r}$ (or $\vec{r}$))

$$\boxed{dB = \frac{\mu_0 . i}{4\pi} . \frac{dL . \sin\theta}{r^2}} \quad (T)$$

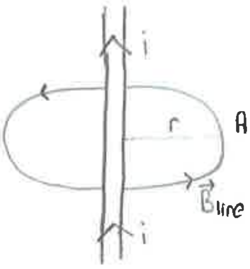
$$\boxed{\mu_0 = 1.26 \times 10^{-6} \ \frac{T.m}{A}}$$

right-hand rule-3

closing direction of four fingers shows the $d\vec{B}$.

28.2 Applications of Biot-Savartz Law

i-) Magnetic Field Produced By Long, Straight Current Carrying Wire



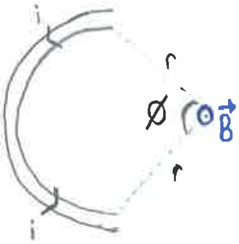
The magnitude of $\vec{B}$ at point A can be calculated by the expression.

$$\boxed{B = \frac{\mu_0 . i}{2\pi . r}} \quad (T)$$

→ Magnetic field produced by long and straight wire.

Expression is derived by applying Biot-Savartz Law (Look At Book)

ii-) Magnetic Field Produced By Circularly Bent Conducting Wire



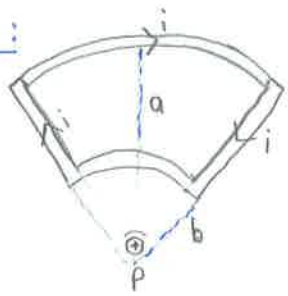
The magnetic field at the center of circularly bent wire is

$$\boxed{B_{circular} = \frac{\mu_0 . i}{4\pi . r} . \emptyset} \quad (T)$$

→ Substitude the angle in terms of radian

It is derived by applying the Biot-Swarte Law again.

Ex :



Find the magnetic field at point P (center of curvature)

Solution : Here, just circular parts have contribution at P.

$$\vec{B}_P = \vec{B}_{\substack{upper\\circular\\part}} + \vec{B}_{\substack{lower\\circular\\part}} = \frac{\mu_0 . i}{4\pi . a} . \emptyset \otimes + \frac{\mu_0 . i}{4\pi . b} \emptyset \odot$$

$$\vec{B}_P = \frac{\mu_0 . i}{4\pi} . \emptyset \left[\frac{1}{b} - \frac{1}{a} \right] = \frac{\mu_0 . i}{4\pi} . \emptyset . \left(\frac{a-b}{a.b} \right) \odot$$

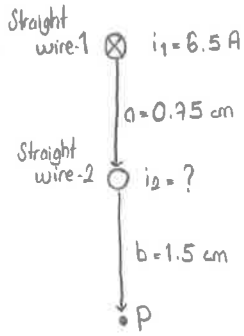
21.05.2015
Thursday

(Last Lecture)

$$B_{\substack{\text{straight}\\\text{wire}}} = \frac{\mu_0 . i}{2.\pi.r}$$

$$B_{\substack{\text{circular}\\\text{wire}}} = \frac{\mu_0 . i}{4.\pi.r} . \phi \xrightarrow{} \text{in units of radian}$$

Ex :



What are the direction and magnitude of i_2 to have zero resultant magnetic field at P ?

Solution :

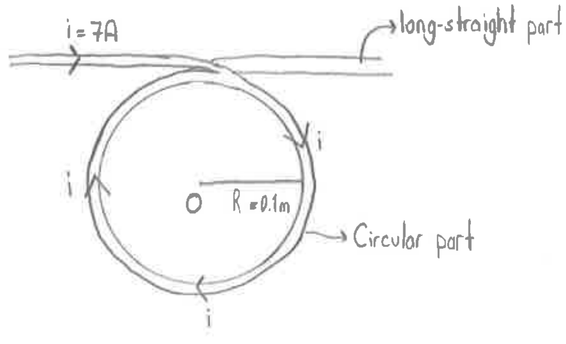
$\vec{B}_1 = -\vec{B}_2$

i_2 is outward. $B_1 = B_2$

$$\frac{\cancel{\mu}_0 . i_1}{\cancel{2.\pi}.(a+b)} = \frac{\cancel{\mu}_0 . i_2}{\cancel{2.\pi}.b} \longrightarrow \frac{i_1}{a+b} = \frac{i_2}{b} \Rightarrow \frac{6.5\,A}{2.25\cancel{cm}} = \frac{i_2}{1.5\cancel{cm}}$$

$$i_2 = 4.33\,A \quad \odot$$

Ex :



Find the resultant magnetic field at the center of circular part.

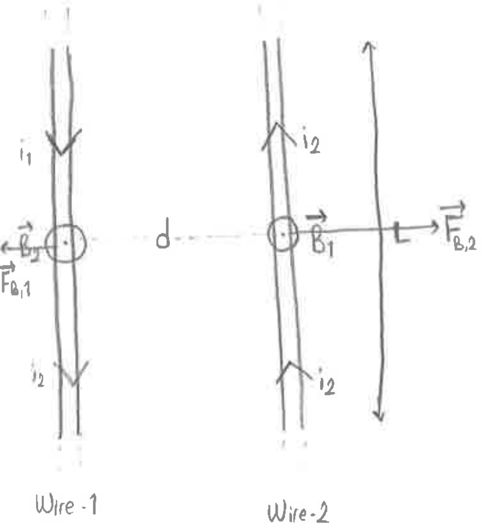
Solution : $\vec{B}_{net,O} = \vec{B}_{straight} + \vec{B}_{circular}$

$$\vec{B}_{net,O} = B_{straight} \otimes + B_{circular} \otimes$$

$$B_{net,O} = \frac{\mu_0 . i}{2.\pi.R} + \frac{\mu_0 . i}{4.\pi.r} . \underbrace{2\pi}_{\text{Complete Circle}}$$

$$\vec{B}_{net,O} = 5.81 \times 10^{-5}\,T \quad \otimes$$

29.3 Force Between Two Parallel Conductors



Two long and straight lines.

$$\vec{B}_1 = \frac{\mu_0 . i_1}{2.\pi.d} \quad \odot$$ → Magnetic field produced by wire -1 on wire -2

$$\vec{F}_{B,2} = i_2 . \vec{L} \times \vec{B}_1$$

$$F_{B,2} = i_2 . L . B_1 . \sin 90°$$

$$F_{B,2} = i_2 . L . \frac{\mu_0 . i_1}{2.\pi.d} \Rightarrow \boxed{F_{B,2} = \frac{\mu_0 . i_1 . i_2 . L}{2.\pi.d}} \quad (N)$$

↳ Magnetic Force on wire-2.

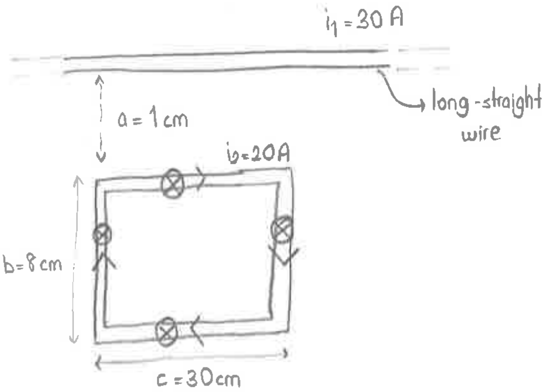
$$\vec{B}_2 = \frac{\mu_0 . i_2}{2.\pi.d} \quad \odot$$ → Magnetic field produced by wire-2 on wire-1

$$\vec{F}_{B,1} = i_1 . \vec{L} \times \vec{B}_2$$

$$\boxed{F_{B,1} = i_1 . L . B_2 . \sin 90° = \frac{\mu_0 . i_1 . i_2 . L}{2\pi d}} \quad (N)$$

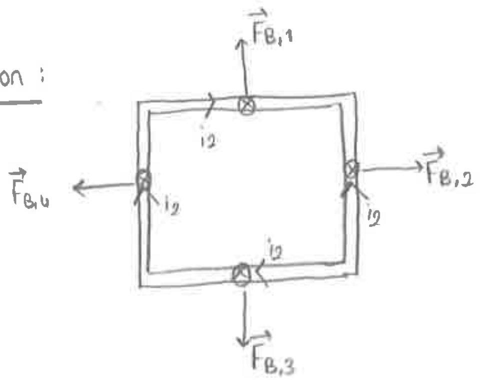
$$\boxed{\vec{F}_{B,1} = -\vec{F}_{B,2}}$$

Ex :



Find the resultant force ($\vec{F}_{B,net}$) on rectangular current loop.

Solution :



$F_{B,1} > F_{B,3}$ thus resultant force in vertical is different than zero.

$$F_{B,net} = F_{B,1} - F_{B,3} = \frac{\mu_0 . i_1 . i_2 . C}{2.\pi.a} - \frac{\mu_0 . i_1 . i_2 . C}{2.\pi.(a+b)} = 3.2 \times 10^{-3}\ N$$

(Upward)

$$\boxed{\vec{F}_{B,2} = -\vec{F}_{B,4}}$$

↝ in horizontal resultant force is zero.

Electromotive Force

$\Delta V = \mathcal{E} - i.r$ (V)

$i.R = \mathcal{E} - i.r$ (V)

$i = \frac{\mathcal{E}}{r+R}$ (A) $= \frac{\mathcal{E}_{net}}{R_{eq}}$

r: internal resistance of battery
$\mathcal{E}$: electromotive force
R: external resistance
ΔV: terminal voltage of battery

Power and Energy

$P_{\mathcal{E}} = i.\mathcal{E}$ (Watt) (Power delivered by battery to the circuit)

$P_r = i^2.r$ (Watt) (lost power by internal resistance)

$P_R = i^2.R = \frac{\Delta V^2}{R}$ (external resistor)

Resistors in Series

Currents are equal $i = i_1 = i_2 = i_3$

Pot. Diff. = $\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$

$R_{eq} = R_1 + R_2 + R_3$

Resistors in Parallel

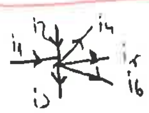
Pot. Diff are equal = $\Delta V = \Delta V_1 = \Delta V_2 = \Delta V_3$

Currents: $i = i_1 + i_2 + i_3$

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

Kirchhoff's Rules

i-) Junction (Point) Rule



$i_1 + i_2 + i_3 = i_4 + i_5 + i_6$

ii-) Loop Rule

$\sum \Delta V = 0$

I RC Circuits

i-) Charging a Capacitor

$i(t) = I_0.e^{\frac{-t}{R_{eq}.C}} = \frac{\mathcal{E}}{R}.e^{\frac{-t}{\tau}}$ $\qquad e = 2.718$

$q(t) = C.\mathcal{E}.\left[1 - e^{\frac{-t}{R_{eq}.C}}\right] = Q\left[1 - e^{\frac{-t}{\tau}}\right]$ C

$U = \frac{q^2}{2C}$ $\qquad \tau = R_{eq}.C$ $\qquad Q = C.\mathcal{E}$

ii-) Discharging Capacitor

$i(t) = \frac{\mathcal{E}}{R}.e^{\frac{-t}{\tau}}$ (A) $\qquad q(t) = C.\mathcal{E}^{\frac{-t}{R_{eq}.C}} = Q.e^{\frac{-t}{\tau}}$ (C)

$\Delta V = \mathcal{E}.e^{\frac{-t}{\tau}}$

MAGNETIC FIELD

$F_B = B.|q|.\vec{V}$ (N) → Magnetic force acting on moving charge carrier

$F_B = B.q.V.\sin\varnothing$ (N) → $\varnothing$: Angle btwn $\vec{V}$ and $\vec{B}$ vectors

Baş P. → FB ↑
İş. P → V →
O.P → B ⊗

When Object is neg. charge carriers direction is reversed.

$r = \frac{m.V}{B.q}$ (m) → Radius of Circular Orbit $\qquad (\vec{F}_{cent} = \vec{F}_B)$

$T = \frac{2.\pi.m}{B.q}$ (s) → Period of Circular Motion $\qquad T.f = 1$ → frequency (Hz)

$W = 2.\pi.f$ (Angular frequency) $= \frac{B.q}{m}$

Magnetic Flux

$\Phi_B = \vec{B}.\vec{A} = B.A.\cos\theta$ $(T.m^2) = $ (Weber = Web)

If the surface has random shape = › $\Phi_B = \int \vec{B}.d\vec{A}$ (Wb)

Closed Surface is equal to 0.
$= 0$

Mag. Force on a Current Carrying Conducting Wire

$F_B = B.i.L$ (N) where L is in the same direction of i.

$= B.i.L.\sin\theta$

is not Straight

$\vec{F}_B = i\int d\vec{L} \times B$ (N)

Force Btwn Two Parallel Conductors

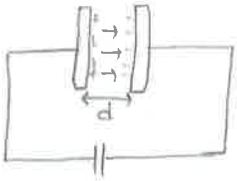
$F_{B_{1,2}} = \frac{\mu_0.i_1.i_2.L}{2.\pi.d}$

Capacitors and Capacitance

$$\underset{C}{Q} = \underset{F}{C} \cdot \underset{V}{\Delta V} \quad , \quad \Delta V = E \cdot d \ (V)$$

Calculating the Capacitance of Capacitors

i-) Capacitor (Parallel-Plate)



$$C_{p\text{-}p} = \frac{Q}{\Delta V} \quad , \quad C_{p.p} = \frac{\varepsilon_0 . A}{d} \ (F)$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \left(\frac{F}{m}\right), \ k = 9 \times 10^9 \ \frac{N.m^2}{C^2}$$

ii-) Cylindrical Capacitor

$$C_{cyc} = 2.\pi.\varepsilon_0 . l . \frac{1}{\ln \frac{b}{a}}$$

iii-) Spherical Capacitor

$$C_{sph} = \frac{1}{k} \cdot \frac{a.b}{(b-a)} \quad , \quad k = \frac{1}{4.\pi.\varepsilon_0}$$

iv-) Isolated Sphere

$$C_{is} = 4.\pi.\varepsilon_0 . a = \frac{a}{k}$$

Stored Potential Energy in the Capacitors

$$U = \frac{Q^2}{2C} = \frac{C^2 (\Delta V)^2}{2C} = \frac{Q . \Delta V}{2} \ (J)$$

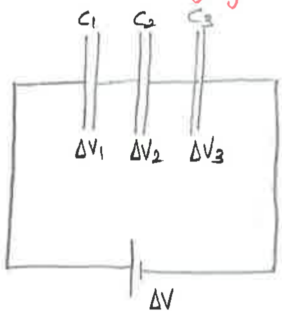
Parallel-Plate

$$U_{p\text{-}p} = \frac{C_{p\text{-}p} (\Delta V)^2}{2} = \frac{1}{2} . \varepsilon_0 . A . d . E^2 \ (J)$$

$$U_{p\text{-}p} = \frac{\varepsilon_0 . E^2}{2} \left(\frac{J}{m^3}\right) \qquad \Delta V = E . d$$

Combination of Capacitors

i-) Series Combination of Capacitors

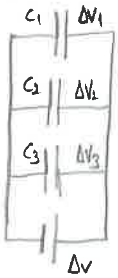


Stored Charges $= Q = Q_1 = Q_2 = Q_3$

Potential Diff. $= \Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

ii-) Parallel Combination of Capacitors



Stored Charges $Q = Q_1 + Q_2 + Q_3$

Potential Diff. $= \Delta V = \Delta V_1 = \Delta V_2 = \Delta V_3$

$$C_{eq} = C_1 + C_2 + C_3$$

Dielectrics

$$C = \kappa . \varepsilon_0 . \frac{A}{d}$$

$$E = \frac{Q}{\kappa . \varepsilon_0 . A}$$

$$\frac{Q}{\kappa} = Q - Q'$$

Sources of Magnetic Field

i-) Magnetic Field Produced By Long, Straight Current Carry Wire

$$B = \frac{\mu_0 . i}{2.\pi.r} \ (T)$$

Electric Current

$$\vec{J} = \frac{i}{A} \left(\frac{A}{m^2}\right) \rightarrow \text{The Current Density}$$

$$i = \int \vec{J} . d\vec{A} = \int J . dA . \cos\theta$$

Drift Velocity

$$V_d = \frac{L}{t} \left(\frac{m}{s}\right)$$

$$V_d = \frac{i}{n.q.A} \left(\frac{m}{s}\right)$$

n: charge carrier concentration per unit volume $\left(\frac{\#}{m^3}\right)$

q: magnitude of charge carriers

$$V_d = \frac{J}{n.q} \left(\frac{m}{s}\right)$$

$$n = N_a \cdot \frac{d}{m} \left(\frac{\#}{m^3}\right)$$

Resistance

$$\boxed{R = \rho \cdot \frac{L}{A} \ (\Omega)}$$ (The resistance of material)

ρ → Resistivity

$$E = \rho . J \left(\frac{N}{C}\right)$$

$$\vec{J} = \sigma . \vec{E} \left(\frac{A}{m^2}\right)$$ σ → Conductivity

$$\boxed{\Delta V = i . R}$$ (Result's of Ohm's Law)

Temperature Variation Resistivity and Resistance

$T_0 = 293°K$

$$R = R_0 \left[1 + \alpha (T - T_0)\right]$$

$$\rho = \rho_0 \left[1 + \alpha . (T - T_0)\right]$$

$$\boxed{\mu_0 = 1.26 \times 10^{-6} \ \frac{T.m}{A}}$$

ii-) Circularly Bent Conducting Wire

$$B_{circular} = \frac{\mu_0 . i}{4.\pi.r} . \phi \ (T)$$

Substitude the angle in terms of radians

DIRECT CURRENT CIRCUITS

i-) Resistors in Series

$i = i_1 = i_2 = i_3$

$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$

$R_{eq} = R_1 + R_2 + R_3$

ii-) Resistors in Parallel

$i = i_1 + i_2 + i_3$

$\Delta V = \Delta V_1 = \Delta V_2 = \Delta V_3$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

KIRCHHOFF'S RULES

i-) Junction (Point) Rule

$$\underbrace{i_1 + i_2 + i_3}_{\text{incident}} = \underbrace{i_4 + i_5 + i_6}_{\text{leaving}}$$

ii-) Loop Rule

$$\sum \Delta V = 0$$

RC Circuits

i-) Charging a Capacitor

$$i(t) = \frac{\varepsilon}{R} \cdot e^{\frac{-t}{\tau}} \quad (A)$$

$$q(t) = Q \cdot \left[1 - e^{\frac{-t}{\tau}}\right] \quad (C)$$

$e = 2.718$

$Q = \varepsilon . C$

$\tau = R_{eq} . C$

ii-) Discharging Capacitor

$$i(t) = \frac{\varepsilon}{R} \cdot e^{\frac{-t}{\tau}} \quad (A)$$

$$q(t) = Q . e^{\frac{-t}{\tau}} \quad (C)$$

$q = C . \Delta V$

Magnetic Field

$F_B = B . |q| . \vec{V}$ (N) → Magnetic Force acting on moving charge carrier

$F_B = B . q . V . \sin\varnothing$ (N) → $\varnothing$: Angle btwn $\vec{V}$ and $\vec{B}$ vectors

Baş P. : F_B , İşaret P. : V , Avuç içi : B (It is changeable ac. to direction)

$$r = \frac{m . V}{B . q} \text{ (m)}$$ → Radius of Circular Orbit

$$T = \frac{2 . \pi . m}{B . q} \text{ (s)}$$ → Period of Circular Motion

$T . f = 1$ (f → Frequency (Hz))

$W = 2 . \pi . f$ → Angular Frequency

Magnetic Flux

$\Phi_B = \vec{B} . \vec{A} = B . A . \cos\theta$ ($T.m^2$ = Weber = Web)

If the Surface is Random shape : $\Phi_B = \int \vec{B} . d\vec{A}$ (Web)

" " Closed Surface $\Phi_B = 0$

Magnetic Force on a Current Carrying Conducting Wire

$F_B = B . i . L$ (N)

If the conducting wire is not straight : $\vec{F_B} = i \int d\vec{L} \times \vec{B}$ (N)

SOURCES OF MAGNETIC FIELD

$$B_{\substack{\text{straight}\\\text{wire}}} = \frac{\mu_0 . i}{2 . \pi . r} \quad , \quad B_{\substack{\text{circular}\\\text{wire}}} = \frac{\mu_0 . i}{4 . \pi . r} . \varnothing$$

($\varnothing$ → in units of radians)

$\mu_0 = 1.26 \times 10^{-6} \ \frac{T . m}{A}$

Force Between Two Parallel Conductors

$$F_{B_1} = -F_{B_2} = \frac{\mu_0 . i_1 . i_2 . L}{2 . \pi . d}$$

Capacitors and Capacitance

$Q = C \cdot \Delta V$
(c) (F) (V)

$\varepsilon_0 = 8.85 \times 10^{-12} \frac{F}{m}$

Calculating the Capacitance of Capacitors

i-) Paralel-Plate

$C_{p-p} = \frac{\varepsilon_0 . A}{d}$ (F)

$k = 9 \times 10^9 \frac{N.m^2}{c^2}$

ii-) Cylindrical

$C_{cyl} = 2.\pi.\varepsilon_0.L.\frac{1}{\ln \frac{b}{a}}$

$k = \frac{1}{4.\pi.\varepsilon_0}$

iii-) Spherical

$C_{sph} = \frac{1}{k} \cdot \frac{a.b}{(b-a)}$

iv-) Isolated Sphere

$C_{iso_{sph}} = \frac{a}{k} = 4\pi\varepsilon_0 . a$

Combination of Capacitors

i-) Series Combination

→ Stored charges are equal

$Q = Q_1 = Q_2 = Q_3$

→ Pot. Dif $\Rightarrow \Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3$

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

ii-) Parallel Combination

→ Stored charges $\Rightarrow Q = Q_1 + Q_2 + Q_3$

→ Pot. dif are equal

$\Delta V = \Delta V_1 = \Delta V_2 = \Delta V_3$

$C_{eq} = C_1 + C_2 + C_3$

Stored Potential Energy in the Capacitor

$U = \frac{Q^2}{2C}$ (J) , $\Delta V = E.d$, $\underbrace{U_{p-p}}_{\text{The energy density}} = \frac{\varepsilon_0 . E^2}{2}$

Dielectrics

$C = K . C_0 = K . \frac{\varepsilon_0 . A}{d}$, $\Delta V = \frac{\Delta V_0}{K}$

$E = \frac{E_0}{K} = \frac{Q}{K.\varepsilon_0.A}$

$1\,mF = 10^{-3}\,F$ (Mili)

$1\,\mu F = 10^{-6}\,F$ (Micro)

$1\,nF = 10^{-9}\,F$ (Nano)

Ferhat Mercan

Electric Current

$i = \frac{dq}{dt}$ $\left(\frac{C}{s} = \text{Amper}\right)$

$\underbrace{\vec{J}}_{\text{The Current Density}} = \frac{i}{A}$ $\left(\frac{A}{m^2}\right)$ $\quad i = \int \vec{J}.dA \cos\theta$ (A)

Drift Velocity $(\vec{V_d})$ = $V_d = \frac{L}{t}$ $\left(\frac{m}{s}\right)$

$V_d = \frac{\vec{J}}{n.q}$ $\left(\frac{m}{s}\right)$ $= \frac{i}{A.n.q}$

n: charge carrier concentration per unit volume

q: magnitude of charge of carriers

$n = \frac{N_A . d}{m}$ $\left(\frac{\#}{m^3}\right)$

Resistance, Resistivity, Ohm's Law

$\underbrace{R}_{\text{Resistance}} = \underbrace{\rho}_{\text{Resistivity}} \cdot \frac{L}{A}$ (Ω)

$E = \rho . \vec{J}$ $\left(\frac{N}{C} = \frac{V}{m}\right)$

$\vec{J} = \underbrace{\sigma}_{\text{Conductivity}} . \vec{E}$ $\left(\frac{A}{m^2}\right)$

$\Delta V = i.R$

Temperature Variation Resistivity and Resistance

$R = R_0\,[1 + \alpha\,(T - T_0)]$ $\quad T_0 = 293°K$

$\rho = \rho_0\,[1 + \alpha\,(T - T_0)]$

α : temp. coefficient of resistivity $\left(\frac{1}{°K}\right)$

Electromotive Force and Circuits

$\Delta V = \varepsilon - i.r$ (V)

$i.R = \varepsilon - i.r$ (V)

$i = \frac{\varepsilon_{net}}{R_{eq}} = \frac{\varepsilon}{r+R}$ (A)

r: internal resistance of battery

ε: electromotive force

R: external resistance

ΔV: terminal voltage of battery

Power and Energy in Electric Circuits

$P_\varepsilon = i.\varepsilon$ (Watt) → Power delivered by battery to the circuit

$P_r = i^2.r$ (Watt) → Dissipated (Lost) power by internal resistor

$P_R = i^2.R = \frac{\Delta V^2}{R}$ → Dissipated (Lost) power by external resistor