Angular Displacement: $\Delta\theta = \theta_f - \theta_i$ (rad) (x)

$(m)\ x \to \theta$ (rad)
$v \to w$
$a \to \alpha$

Average Angular Velocity: $w = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{t_f - t_i}$ (rad/s) ($v$)

Instantaneus Angular Velocity: $w = \frac{d\theta}{dt}$ (rad/s)

For one complete Revolution: $w = \frac{2\pi}{T} = 2.\pi.f$ (rad/s)

Average Angular Acceleration: $\alpha_{ave} = \frac{\Delta w}{\Delta t} = \frac{w_f - w_i}{t_f - t_i}$ (rad/s²) (a)

Instantaneus Angular Acceleration: $\alpha = \frac{dw}{dt}$ (rad/s²)

Rotation with Constant Angular Acceleration; $v_f = v_i \mp a.t \longrightarrow w_f = w_i \mp \alpha.t$

$x_f = x_i \mp v_i\, t \mp \frac{1}{2}.a.t^2 \to \theta_f = \theta_i \mp w_i\, t \mp \frac{1}{2}\alpha t^2$

$v_f^2 = v_i^2 \mp 2.a.\Delta x \longrightarrow w_f^2 = w_i^2 \mp 2.\alpha.\Delta\theta$

Linear;
Speed of Rotating Rigid Body: $S = r.\theta$ $\quad v_t = r.w = r.\frac{d\theta}{dt}$ (m/s)

→ Every point on the rigid body has same $w$. Different positions have different $v_t$.

Acceleration of Rotating Rigid Body: $a_t = r.\alpha$ (m/s²)

$a_{cent} = \frac{v_t^2}{r} = r.w^2$

$a_{total}^2 = a_{cent}^2 + a_{tang}^2 \Rightarrow a_{total} = (r^2.w^4 + r^2.\alpha^2)^{1/2}$

Rotational Kinetic Energy: $KE_{rot} = \frac{1}{2}.I.w^2$ (J) for Ball $I = m.r^2$

I: rotational inertia (kg.m²) $\quad KE_{rot} = \frac{1}{2}.m.V^2$ (J) $\quad V = r.w$ (m/s)

WORK: $W = F.\Delta x.\cos\theta$ (N.m)

KE: $\frac{1}{2}.m.V^2$ $\quad W_F = \Delta KE = KE_f - KE_i$

SPRING FORCE: $F_s(x) = -k.\Delta x$ $\quad W_{F_s} = \frac{1}{2}.k.x_i^2 - \frac{1}{2}.k.x_f^2$

POWER: $P_{ave} = \frac{\Delta W}{\Delta t}$ (watt), $P = \vec{F}.\vec{v}$

POTENTIAL ENERGY: $U = m.g.h$ (J) $\quad \Delta U = U_f - U_i = -W_{mg}$

MOMENTUM: $\vec{P} = m \times \vec{v}$ $(kg.\frac{m}{s})$ $\quad \vec{F} = \frac{d\vec{P}}{dt}$

IMPULSE: $\underbrace{\vec{P}_f - \vec{P}_i}_{\Delta P} = \vec{F} \times dt = I$ $\quad \to F_{net} = 0 \quad \vec{P}_i = \vec{P}_f$

ONE DIMENSION;
ELASTIC COLLISION: KE and P are conserved. $\quad KE_i = KE_f$, $\vec{P}_i = \vec{P}_f$

(If $V_{2i} = 0$) $\Rightarrow \vec{V}_{1f} = \frac{m_1 - m_2}{m_1 + m_2}.\vec{V}_{1i}$, $\quad \vec{V}_{2f} = \frac{2.m_1}{m_1 + m_2}.\vec{V}_{1i}$

INELASTIC COLLISION: P is conserved, but KE is not conserved.

→ Objects may stick to each other and move with same magnitude of speed. Completely Inelastic Collision

$\sum\vec{P}_i = \sum\vec{P}_f$ $\quad m_1.\vec{V}_{1i} + m_2.\vec{V}_{2i} = (m_1 + m_2).\vec{V}_f$

TWO DIMENSION; $\sum\vec{P}_{x,i} = \sum\vec{P}_{x,f}$ $\quad$ Glancing Collision: Two Dimension Collision

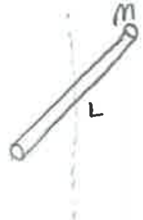
$\sum\vec{P}_{y,i} = \sum\vec{P}_{y,f}$ $\quad m_1 = m_2 \longrightarrow$ Angle is 90°

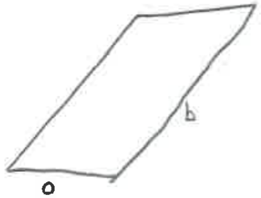
CENTER OF MASS; $x_{c.m.} = \frac{m_1.x_1 + m_2.x_2}{m_1 + m_2}$, $\quad y_{c.m.} = \frac{m_1.y_1 + m_2.y_2}{m_1 + m_2}$

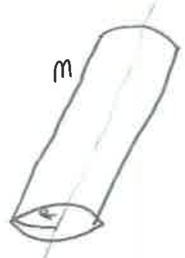
for Rigid Bodies; $x_{cm} = \frac{1}{m}.\int x.dm$ $\quad y_{cm} = \frac{1}{m}.\int y.dm$

$z_{cm} = \frac{1}{m}.\int z.dm$ $\quad \vec{r}_{cm} = x_{cm}.\hat{i} + y_{cm}.\hat{j} + z_{cm}.\hat{k}$

## Parallel-Axis Theorem for Rotational Inertia





$I_{rod} = \frac{1}{12} \cdot m \cdot L^2$

$I_{\substack{rectangular \\ plate}} = \frac{1}{12} \cdot m \cdot (a^2+b^2)$

$I_{\substack{solid \\ cylindrical}} = \frac{1}{2} \cdot m \cdot R^2$

$I_{\substack{solid \\ sphere}} = \frac{1}{5} \cdot m \cdot R^2$

$I = I_{cm} + m \cdot R^2$ $\left\{ \begin{matrix} m \cdot D^2 \\ \\ L^2 \end{matrix} \right.$ Distance btwn

wheel $\cdot$ $\frac{1}{2} \cdot I \cdot w^2$

## Torque

$\vec{\tau} = \vec{r} \times \vec{F}$

$r \times F = I \cdot \alpha$

$\tau = r \cdot F \cdot \sin\theta$

$r \times F = I \cdot \frac{a}{r}$

$\tau = I \cdot \alpha \ (N \cdot m)$

$\boxed{I = m \cdot r^2 \text{(Ball)}}$

$V_F^2 = V_i^2 + 2 \cdot a \cdot \Delta X$

$V = w \cdot r$

## Rigid Body Rotation Arond a Moving Axis

$$KE_{total} = \frac{1}{2} \cdot m \cdot V_{cm}^2 + \frac{1}{2} \cdot I_{cm} W^2 \ (J)$$

$$KE_{total} = \frac{1}{2} \cdot m \cdot R^2 \cdot W^2 + \frac{1}{2} \cdot I_{cm} \cdot W^2 = \frac{1}{2} (m \cdot R^2 + I_{cm}) \cdot W^2$$

$$KE_{total} = \frac{1}{2} \cdot I_p \cdot W^2 \ (J)$$

## MOTION ALONG A STRAIGHT LINE

$$\vec{v}_{ave} = \frac{\vec{x}_F - \vec{x}_i}{t_F - t_i} = \frac{\Delta \vec{x}}{\Delta t} \quad (m/s)$$

Instantaneus Velocity $\Rightarrow$ $\vec{v} = \lim_{\Delta t \to 0} \frac{\vec{x}_F - \vec{x}_i}{t_F - t_i} = \frac{d\vec{x}}{dt}$ $(m/s)$

$$\vec{a}_{ave} = \tan\theta = \frac{\vec{v}_F - \vec{v}_i}{t_F - t_i} = \frac{\Delta \vec{v}}{\Delta t} \quad (m/s^2)$$

Instantaneus Acceleration $\Rightarrow$ $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_F - \vec{v}_i}{t_F - t_i} = \frac{d\vec{v}}{dt} = (m/s^2)$

→ Area under $\vec{a} - t$ graph gives the change in velocity. $(\Delta \vec{v})$

→ Area under $\vec{v} - t$ graph gives displacement. $(\Delta \vec{x})$

$$X_F = X_i + v_i \cdot t + \frac{1}{2} \cdot a \cdot t^2 \ (m)$$

$$v_F = v_i + a \cdot t \ (m/s)$$

$$v_F^2 = v_i^2 + 2 \cdot a \cdot \Delta X \ (m^2/s^2)$$

## FREELY FALLING BODIES

$$h_F = h_i + v_i \cdot t + \frac{1}{2} \cdot g \cdot t^2 \ (m)$$

$$v_F = v_i + g \cdot t \ (m/s)$$

$$v_F^2 = v_i^2 + 2 \cdot g \cdot \Delta h \ (m^2/s^2)$$

## VECTORS

### MULTIPLICATION of VECTORS

### SCALAR (DOT) PRODUCT

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

$$\vec{A} \cdot \vec{B} = A B . \cos\theta = |\vec{A}| . |\vec{B}| . \cos\theta$$

$$\vec{A} \cdot \vec{B} = A_x . B_x + A_y . B_y + A_z . B_z$$

$$A_x . B_x + A_y . B_y + A_z . B_z = |\vec{A}| \cdot |\vec{B}| . \cos\theta$$

$|\vec{A}| = A =$ Always positive . .

$\vec{A} = x_i + y_j + z_k \Rightarrow |\vec{A}| = \sqrt{x^2 + y^2 + z^2}$

### VECTOR (CROSS) PRODUCT

$$\vec{A} \times \vec{B} = \vec{C}$$

$$\vec{B} \times \vec{A} = -\vec{C}$$

$$|\vec{C}| = C = A . B . \sin\theta$$

$\vec{A} = A_x . i + A_y \hat{j} + A_z k$

$\vec{B} = B_x . \hat{\imath} + B_y . \hat{j} + B_z . k$

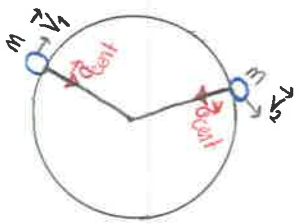
$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{C} = (A_y B_z - A_z B_y) . \hat{\imath} - (A_x B_z - A_z B_x) . \hat{j} + (A_x B_y - A_y B_x) . \hat{k}$$

$$(A_y B_z - A_z B_y)^2 + (A_x B_z - A_z B_x)^2 + (A_x B_y - A_y B_x)^2 = (A . B . \sin\theta)^2$$

# Uniform Circular Motion

→ Constant Speed




$|\vec{v}_1| = |\vec{v}_2|$ ⇒ Magnitude equal

$\vec{v}_1 \neq \vec{v}_2$ → Direction changes

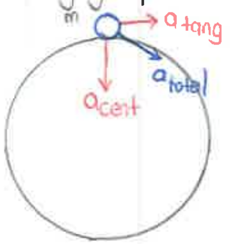
$$\vec{a}_{cent} = \vec{a}_{rad} = \frac{v^2}{r} \quad (m/s^2)$$

$v$ → Tangential Velocity

$$T = \frac{2.\pi.r}{V} \quad (s) \qquad T.f = 1$$

# Nonuniform Circular Motion

→ Changing Speed




$\vec{a}_{cent}$ = due to change of direction $\vec{v}$

$\vec{a}_{tang}$ = change at the magnitude of $\vec{v}$ (Every moment)

$$a_{total} = \left(a^2_{cent} + a^2_{tang}\right)^{1/2} \quad (m/s^2)$$

# Relative Motion in Two Dimensions

# Motion in Two and Three Dimensions

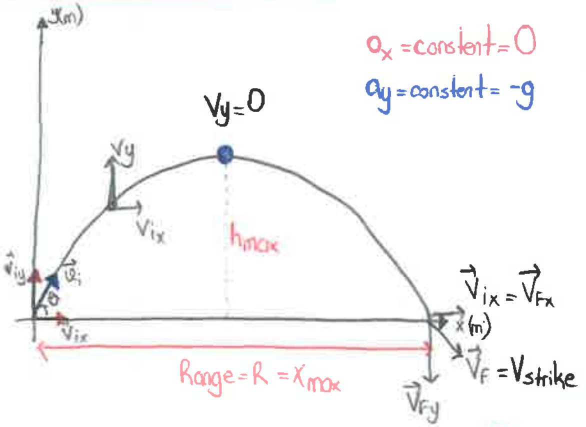
$$\vec{r} = x.\hat{i} + y.\hat{j} + z.\hat{k} \quad (m)$$

$$\Delta\vec{r} = (x_F - x_i).\hat{i} + (y_F - y_i).\hat{j} + (z_F - z_i).\hat{k} \quad (m)$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}.\hat{i} + \frac{dy}{dt}.\hat{j} + \frac{dz}{dt}.\hat{k} = V_x.\hat{i} + V_y.\hat{j} + V_z.\hat{k} \quad (m/s)$$

$$\vec{a} = a_x.\hat{i} + a_y.\hat{j} + a_z.\hat{k} \quad (m/s^2)$$

# Projectile Motion




$a_x$ = constant = 0

$a_y$ = constant = $-g$

→ $V_{ix} = V_i.\cos\theta$ $\qquad$ $V_{iy} = V_i.\sin\theta$

→ When object reaches to the hmax its vertical velocity gets zero. ($V_y = 0$)

$$\underbrace{V_{Fy}}_{0} = V_{iy} - g.t_{rise}$$

$$t_{rise} = \frac{V_i.\sin\theta}{g}$$

→ $t_{rise} = t_{drop} = \dfrac{t_{flight}}{2}$

→ $\underbrace{h_F - \overset{0}{h_i}}_{\Delta h = h_{max}} = \dfrac{V_i^2.\sin^2\theta}{2.g}$

→ $V_x = V_{ix}$

$X_F = X_i + V_{ix}.t$

$\Delta X = V_{ix}.t$

$\Delta X = V_i.\cos\theta.t$

→ $V_{Fy} = V_{iy} - g.t$

$y_F = y_i + V_{iy}.t - \frac{1}{2}.g.t^2$

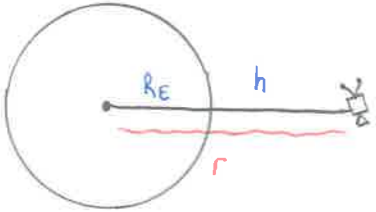
$\underbrace{\Delta y}_{h_{max}} = V_i.\sin\theta.t - \frac{1}{2}.g.t^2$

Equation of Trajection ⇒ $y = (\tan\theta).x - \frac{1}{2}.g.\dfrac{x^2}{v_i^2.\cos^2\theta}$

→ Herhangi bir anda cismin başlangıça n. uzaklığı; $r = \sqrt{x^2+y^2}$

→ " " " " hızı; $V = \sqrt{V_x^2+V_y^2}$

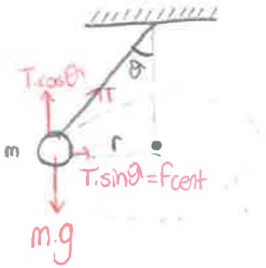
### iii-) Satellite Motion




$$v = \left(\frac{\overset{\text{Sbt}}{G}.\overset{\text{Sbt}}{M_E}}{r}\right)^{1/2}$$

$$r = \overset{\text{Sbt}}{R_E} + h$$

### iv-) Conical Pendulum



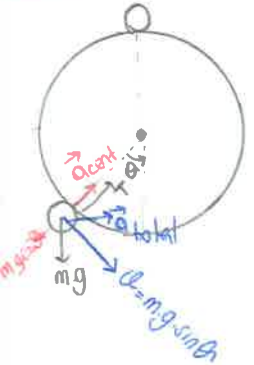

$$T.\cos\theta = m.g$$

$$T.\sin\theta = \frac{m.V^2}{r}$$

$$v = (g.r.\tan\theta)^{1/2} \quad (m/s)$$

$$T = \frac{2.\pi.r}{v}$$

## NONUNIFORM CIRCULAR MOTION




$\vec{g} \downarrow$

$$\cancel{m}.g.\sin\theta = \cancel{m}.a_{tang} \Rightarrow a_{tang} = g.\sin\theta$$

$$T - m.g.\cos\theta = F_{cent} = \frac{m.V^2}{r}$$

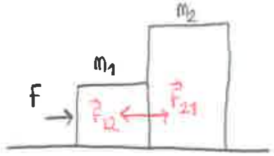
$$T = m.g.\cos\theta + \frac{m.V^2}{r} \quad (N)$$

Two special position

1-) When object is at top; $\theta = 180° \to \cos 180° = -1$   $T_{top} = T_{min} = \frac{m.V^2}{r} - m.g$ (N)

2-) When object is at bottom; $\theta° = 0° \to \cos 0° = 1$   $T_{bottom} = T_{max} = \frac{m.V^2}{r} + m.g$ (N)

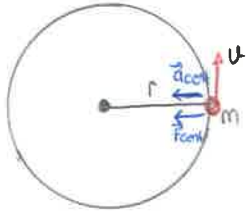
## NEWTON'S LAWS of MOTION




$\vec{F}_{21}$ : acting force on $m_2$ by $m_1$   $m_2.a$

$\vec{F}_{12}$ : acting force on $m_1$ by $m_2$   $m_1.a$

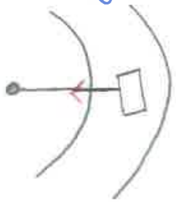
## Dynamics of Circular Motion




$$F_{centripital} = F_{radial} = m.a_{cent} = \frac{m.V^2}{r} \quad (N)$$

## Application of Fcent

### i-) Rounding a Curve in a Car

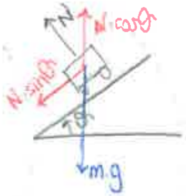


$F_{cent} = f_s$

$$v_{max} = \sqrt{\mu_s.g.r} \quad (m/s)$$

⇒ Not banked

### ii-) Banked Road without Friction




$$N.\cos\theta = m.g \qquad N = \frac{m.g}{\cos\theta}$$

$$N.\sin\theta = F_{cent} = \frac{m.v_{max}^2}{r}$$

$$v_{max} = \sqrt{g.r.\tan\theta} \quad (m/s)$$

⇒ Highway, Airplane

Note: If there is friction on the banked.

$$N.\sin\theta + f_s.\cos\theta = \frac{m.V^2}{r}$$