<u>Formula Sheet – Project 2</u>

Definition 3.9 – Negative Binomial Probability Distribution

A random variable Y is said to have a negative binomial probability distribution if and only if

$$p(y) = \binom{y-1}{r-1} p^r q^{y-r}, \quad y = r,\ r+1,\ r+2. \dots$$

Theorem 3.9 – Negative Binomial mean and variance

If Y is a random variable with a negative binomial distribution,

$$\mu = E(Y) = \frac{r}{p} \quad \text{and} \quad \sigma^2 = V(Y) = \frac{r(1-p)}{p^2}$$

Definition 3.10 – Hypergeometric Probability Distribution

A random variable Y is said to have a hypergeometric probability distribution if and only if

$$p(y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}$$

where y is an integer 0 to n, subject to the restrictions y <= r and n-y <= N-r.

Theorem 3.10 – Hypergeometric mean and variance

If Y is a random variable with a hypergeometric distribution,

$$\mu = E(Y) = \frac{nr}{N} \quad \text{and} \quad \sigma^2 = V(Y) = n * \frac{r}{N} * \frac{N-r}{N} * \frac{N-n}{N-1}$$

Definition 3.11 – Poisson Probability Distribution

A random variable Y is said to have a Poisson probability distribution if and only if

$$p(y) = \frac{\lambda^y}{y!} e^{-\lambda}, \quad y = 0, 1, 2, \dots, \lambda > 0.$$

Theorem 3.11 – Poisson mean and variance

If Y is a random variable possessing a Poisson distribution with parameter λ, then

$$\mu = E(Y) = \lambda \quad \text{and} \quad \sigma^2 = V(Y) = \lambda$$

Theorem 3.14 – Tchebysheff's Theorem

Let Y be a random variable with mean μ and finite variance σ^2. Then, for any constant $k > 0$,

$$P(|Y - \mu| < k\sigma) \geq 1 - \frac{1}{k^2}$$

Definition 4.3 – Probability Density Function

Let F(y) be the distribution function for a continuous random variable Y . Then f (y), given by

$$f(y) = \frac{dF(y)}{dy} = F'(y)$$

wherever the derivative exists, is called the probability density function for the random variable Y.

Theorem 4.3 – Probability of Y on a PDF

If the random variable Y has density function f (y) and $a < b$, then the probability that Y falls in the interval [a, b] is

$$P(a \leq Y \leq b) = \int_a^b f(y)\, dy$$

Definition 4.5 – Expected Values for Continuous Random Variables

The expected value of a continuous random variable Y, provided the integral exists, is

$$E(Y) = \int_{-\infty}^{\infty} y f(y) dy$$

Variance for Continuous Random Variables

The variance for a continuous random variable Y would be

$$E(Y^2) = \int_{-\infty}^{\infty} y^2 * f(y)\, dy$$

$$\sigma^2 = V(Y) = E(Y^2) - E(Y)^2$$

Definition 4.6 – Uniform Probability Distribution

If $\theta_1 < \theta_2$, a random variable Y is said to have a continuous uniform probability distribution on the interval (θ_1, θ_2) if and only if the density function of Y is

$$f(y) = \begin{cases} \dfrac{1}{\theta_2 - \theta_1}, & \theta_1 \leq y \leq \theta_2 \\ 0, & elsewhere \end{cases}$$

Theorem 4.6 – Uniform mean and variance

If $\theta_1 < \theta_2$ and Y is a random variable uniformly distributed on the interval (θ_1, θ_2), then

$$\mu = E(Y) = \frac{\theta_1 + \theta_2}{2} \quad \text{and} \quad \sigma^2 = V(Y) = \frac{(\theta_2 - \theta_1)^2}{12}$$

Definition 4.8 – Normal Probability Distribution

A random variable Y is said to have a normal probability distribution if and only if, for $\sigma > 0$ and $-\infty < \mu < \infty$, the density function of Y is

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(y-\mu)^2}{2\sigma^2}}$$

Definition 4.9 – Exponential Distribution

A random variable Y is said to have an exponential distribution with parameter $\beta > 0$ if and only if the density function of Y is

$$f(y) = \begin{cases} \dfrac{1}{\beta} e^{-\frac{y}{\beta}}, & 0 \leq y < \infty \\ 0, & elsewhere \end{cases}$$

Definition 4.10 – Exponential mean and variance

If Y is an exponential random variable with parameter β, then

$$\mu = E(Y) = \beta \quad \text{and} \quad \sigma^2 = V(Y) = \beta^2$$

Bivariate and Multivariate Probability

$P(a_1 \leq Y_1 \leq a_2, b_1 \leq Y_2 \leq b_2)$ is equal to $\int_{b_1}^{b_2} \int_{a_1}^{a_2} f(y_1, y_2) dy_1 dy_2$.

Definition 5.4 – Marginal Probability

Let Y_1 and Y_2 be jointly discrete random variables with probability function $p(y_1, y_2)$. Then the marginal probability functions of Y_1 and Y_2, respectively, are given by

$$p_1(y_1) = \sum_{all\ y_2} p(y_1, y_2)$$

$$p_2(y_2) = \sum_{all\ y_1} p(y_1, y_2)$$

Let Y_1 and Y_2 be jointly continuous random variables with joint density function $f(y_1, y_2)$. Then the marginal density functions of Y_1 and Y_2, respectively, are given by

$$f_1(y_1) = \int_{-\infty}^{\infty} f(y_1, y_2) dy_2$$

$$f_2(y_2) = \int_{-\infty}^{\infty} f(y_1, y_2) dy_1$$

Definition 5.5 – Conditional Discrete Probability Function

If Y_1 and Y_2 are jointly discrete random variables with joint probability function $p(y_1, y_2)$ and marginal probability functions $p_1(y_1)$ and $p_2(y_2)$, respectively, then the conditional discrete probability function of Y_1 given Y_2 is

$$p(y_1|y_2) = P(Y_1 = y_1|Y_2 = y_2) = \frac{P(Y_1 = y_1,\ Y_2 = y_2)}{P(Y_2 = y_2)} = \frac{p(y_1, y_2)}{p_2(y_2)}$$

Definition 5.7 – Conditional Density

Let Y_1 and Y_2 be jointly continuous random variables with joint density $f(y_1, y_2)$ and marginal densities $f_1(y_1)$ and $f_2(y_2)$, respectively. For any y_2 such that $f_2(y_2) > 0$, the conditional density of Y_1 given $Y_2 = y_2$ is given by

$$f(y_1|y_2) = \frac{f(y_1, y_2)}{f_2(y_2)}$$