Department of

Electrical & Electronics Engineering

Abdullah Gül University

PROJECT REPORT OF RC CIRCUITS, CAPACITORS, CAPACITOR DESIGN, CONVERGENCE OF DISCRETE TIME SERIES AND MACLAURIN SERIES

EE1200 Electronic System Design (ESD) Capsule

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Grade: / 100

OBJECTIVE

The project aims to enhance the understanding of basic electronic circuits and capacitors, and to develop practical skills in circuit analysis, simulation, design, and fabrication. The specific project tasks and their objectives are as follows:

Calculate the lamp-on time and capacitor charging time: The objective of this task is to determine how long the lamp stays on each time the capacitor discharges, and how long it takes to charge the capacitor after the initial cycle. This requires knowledge of the circuit components, the voltage and resistance values, and the capacitor behavior.

Simulate the circuit and plot the capacitor voltage: The objective of this task is to use a simulation tool to model the relaxation oscillator circuit and plot the voltage across the capacitor over time. This allows for a better understanding of the circuit behavior and the effects of different parameters on the capacitor voltage.

Design and fabricate cylindrical and parallel-plate capacitors: The objective of this task is to design and fabricate homemade capacitors with a target capacitance of 6 μF using different materials and configurations.

Compare the theoretical and experimental capacitance values: The objective of this task is to measure the capacitance of the homemade capacitors using a multimeter and compare the experimental values with the theoretical values calculated from the capacitor design equations. This allows for a validation of the design and fabrication processes and an assessment of the accuracy and precision of the measurements.

Investigate the convergence properties of the capacitor voltage series: The objective of this task is to discretize the capacitor voltage function over time using a fixed time step of $\Delta t = 0.01$ ms and investigate the convergence properties of the resulting series. This allows for a numerical analysis of the capacitor behavior and the approximation of the continuous-time function.

Write the Maclaurin series expansion for $V_C(t)$: The objective of this task is to use the Maclaurin series expansion to approximate the capacitor voltage function as a polynomial of increasing degree around $t = 0$. This allows for a mathematical analysis of the capacitor behavior and the derivation of approximate formulas for the capacitor voltage and other properties.

BACKGROUND

Paper, multimeter that is used to measure the capacitance, aluminum folio, wire, glue, plastic used as dielectric material, water and salt used for creating conducting material, band, PSpice that is used to simulate the voltage of capacitor.

ANALYTICAL AND SIMULATION PROCEDURES

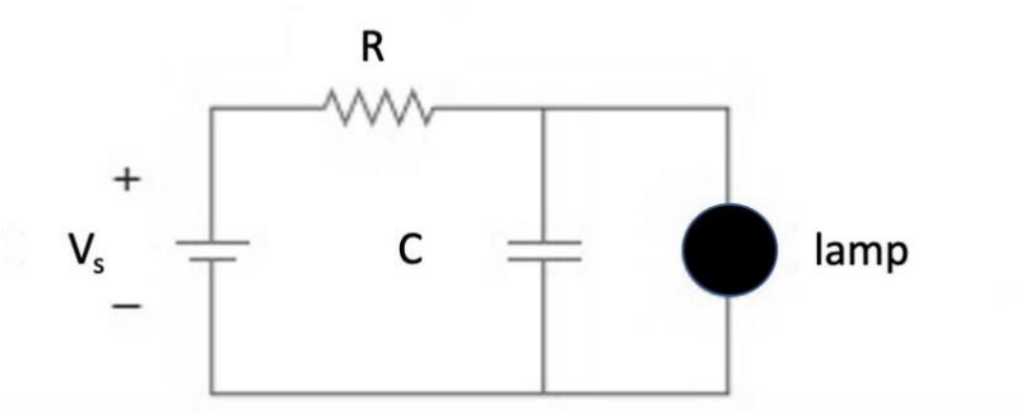

Figure 1: A diagram of simple relaxation oscillator circuit

Given above a circuit diagram, which is relaxation oscillator (an oscillator in which sharp, sometimes aperiodic oscillations result from the rapid discharge of a capacitor) circuit, to understand this mechanism we will analyze the circuit with this conditions, as follows

$$V_s = 120V$$

$$R = 4M\Omega$$

$$C = 6\mu F$$

$$The\ lamp\ fires\ when\ its\ voltage\ reaches\ 80V\ and\ turns\ off\ when\ its\ voltage\ drops\ to\ 40V.$$

$$The\ resistance\ of\ lamp\ is\ 120\Omega\ \text{when on, when off its resistance infinitely high.}$$

$$Assume\ initial\ voltage\ of\ capacitor\ is\ zero.$$

Question 1 : How long is the lamp on each time the capacitor discharges?

Given the circuit diagram from Figure 1, first we think initially charging cycle since the initial voltage of capacitor that results in off condition for the lamp.

1- Firstly, since the resistance of the lamp is infinitely high, there will be no current flow, so that we can ignore this part of the circuit, so we have

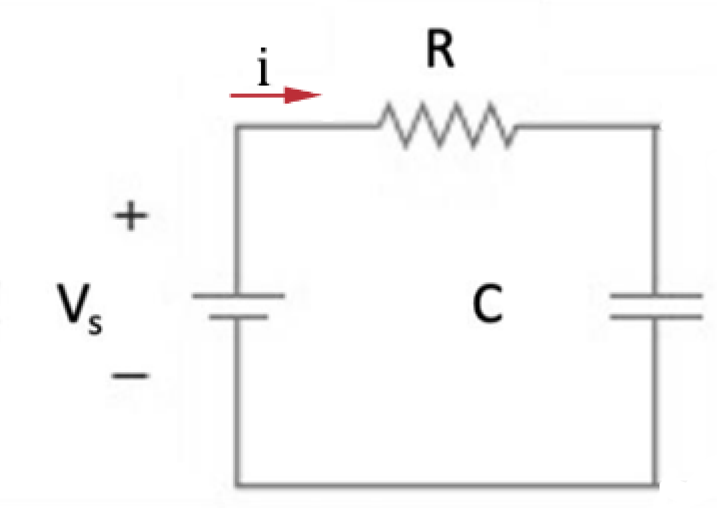

Figure 2: The circuit part for charging cycle

Our objective is to determine the circuit response, we assume to be the voltage $V(t)$ across the capacitor. Since the capacitor is initially empty, we can assume that at time t = 0, the initial voltage is

$$V(0) = 0$$

Then we assign a current as i, and write KVL through the circuit,

$$0 = -120 \ + i.4M\Omega + V(t)$$

By definition, we know that $i = C\frac{dV}{dt}$, then if we arrange this equation,

$$0 = -120 + \ C\frac{dV}{dt}.4M\Omega + V(t)$$

$$0 = -120 + 6.10^{-6}\frac{dV}{dt}.4.10^{6} + V(t)$$

$$0 = -120 + V(t) + 24\frac{dV}{dt}$$

$$\frac{120 - V(t)}{24} = -\frac{dV}{dt}$$

This is a first-order differential equation, to solve this we can rearrange,

$$\frac{dV}{120 - V(t)} = -\frac{dt}{24}$$

By integrating both sides, we get

$$\ln 120 - V(t) = -\frac{t}{24} + \ln A$$

$$\ln \frac{120 - V(t)}{A} = -\frac{t}{24}$$

Thus, take e powers for both sides, then we have

$$\frac{120 - V(t)}{A} = e^{-\frac{t}{24}}$$

$$120 - V(t) = Ae^{-\frac{t}{24}}$$

$$V(t) = 120 - Ae^{-\frac{t}{24}}$$

We already have the first condition, which is $V(0) = 0$, so that A will be 120.

$$V(t) = 120 - 120e^{-\frac{t}{24}}$$

This shows us that the voltage of the RC circuit is an exponential increase.

We can also define the time constant, as Greek latter tau (τ).

$$\tau = RC$$

So within these conditions, our time constant is 24.

To find out how long is the lamp on each time the capacitor discharges, we consider that before $V(t)$ is equal to 80V, lamp will not be on. Thus we must find the time when the capacitor voltage will be 80V. Say that V(t) is 80V,

$$80 = 120 - 120e^{-\frac{t}{24}}$$

$$40 = 120e^{-\frac{t}{24}}$$

$$\frac{1}{3} = e^{-\frac{t}{24}}$$

$$\ln\frac{1}{3} = -\frac{t}{24}$$

$$\ln\frac{1}{3} = -\frac{t}{24}$$

$$t = 24.\ln 3$$

$$t = 26{,}3666949\ s$$

Thus, the time takes for the capacitor voltage increases to 80V is $26{,}3666949\ s$. Then the condition for the lamp holds, so its resistance dramatically drops to 120Ω. As a result, the discharge cycle begins.

2- Secondly, since the resistance of the lamp is 120Ω after its voltage reaches 80V, there will be a current flow through the lamp, because of the discharging of the capacitor.

While the capacitor discharging, we can kill the voltage source as short circuit. Then we get this diagram,

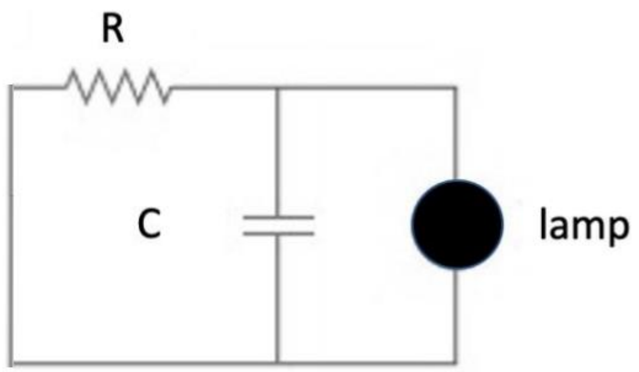

Figure 3: The circuit part of discharging cycle

The lamp and resistor are parallel to each other, we can write equivalent resistance and arranged circuit as follows,

$$R_{eq} = \frac{4000000\Omega * 120\Omega}{4000120\Omega}$$

$$R_{eq} = 119{,}996\Omega$$

So the arranged circuit,

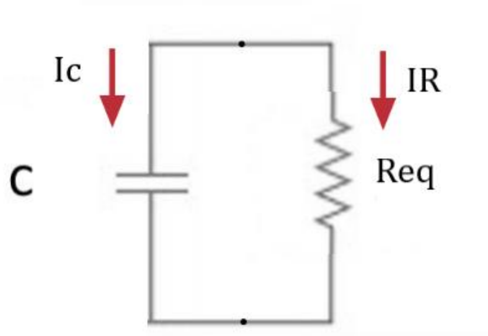

Figure 4: Rearranged circuit

Since the capacitor is initially charged after charging cycle, we can assume that at time t = 0, the initial voltage is $V(0) = 80V$

Then we can apply KCL to the top node, then we get

$$i_c + i_R = 0$$

By definition, $i_c = C\frac{dv}{dt}$ and $i_R = \frac{V}{R}$, Therefore,

$$\frac{dv}{dt} + \frac{V}{RC} = 0$$

This is also a first-order differential equation, to solve it, we can arrange it as follows,

$$\frac{dv}{V} = -\frac{dt}{RC}$$

By integration both sides,

$$\ln V = -\frac{t}{RC} + \ln A$$

ln A is a constant,

$$\ln\frac{V}{A} = -\frac{t}{RC}$$

Taking powers of e produces

$$\frac{V}{A} = e^{-\frac{t}{RC}}$$

So finally we have,

$$V(t) = Ae^{-\frac{t}{RC}}$$

From the initial condition $V(0) = 80V$,

$$V(t) = 80e^{-\frac{t}{RC}}$$

And also out time constant ($\tau = RC$) will be,

$$\tau = 0{,}000719976$$

This shows us that the voltage of capacitor of the RC circuit is an exponential decay. Not because of any voltage source, since the voltage response is due to initial energy stored, and it is called natural response.

To find out how long the lamp will be on, after the voltage passes the 80V threshold. Afterwards capacitor discharges very rapidly. We can calculate like this,

When its voltage is 80V it will turn on, when its voltage is 40V is turn off. So its voltage decays 80V to 40V, following calculations are as follows,

$$40 = 80e^{-\frac{t}{\tau}}$$

$$\ln\frac{1}{2} = \frac{-t}{\tau}$$

$$t = \ln 2 * \tau$$

$$t = 0.000499\ s$$

Thus, the time takes for the capacitor voltage drops from 80V to 40V is 499 μs. That means during every discharging cycle lamp on within 499 μs.

Question 2 : Calculate the time it takes to charge the capacitor after the initial cycle

After the initial cycle, which is $26{,}3666949\ s$ charging and 499 μs discharging, the voltage of capacitor will oscillate between 80V and 40V back and forth. So we can assign two time variables t_1 and t_2, for 80V and 40V which are time for voltage reaches 80V and 40V from 0, respectively. We should find $t_2 - t_1$

$$V(t) = 120 - 120e^{-\frac{t}{24}}$$

$$V(t_1) = 80V$$

$$V(t_2) = 40V$$

So our t_1 and t_2 will be,

$$t_1 = 26{,}3666949\ s$$

$$t_2 = 9{,}73116259\ s$$

As a result the time taking to charge the capacitor after the initial cycle will be 16,6355323 s

Question 3 : Using a simulation tool plot the voltage across the capacitor V(t). Show your findings on a plot representing how the system works by marking the important time and voltage values.

After much research to choose the best spice simulators for plotting the voltage graph across the capacitor, we decided to use Pspice. But first we could not create a circuit proving the conditions, because there is no control on the resistance changing of lamp during time, but we find a way that behaves as if it behaves on theoretical part.

So first we design a circuit like this,

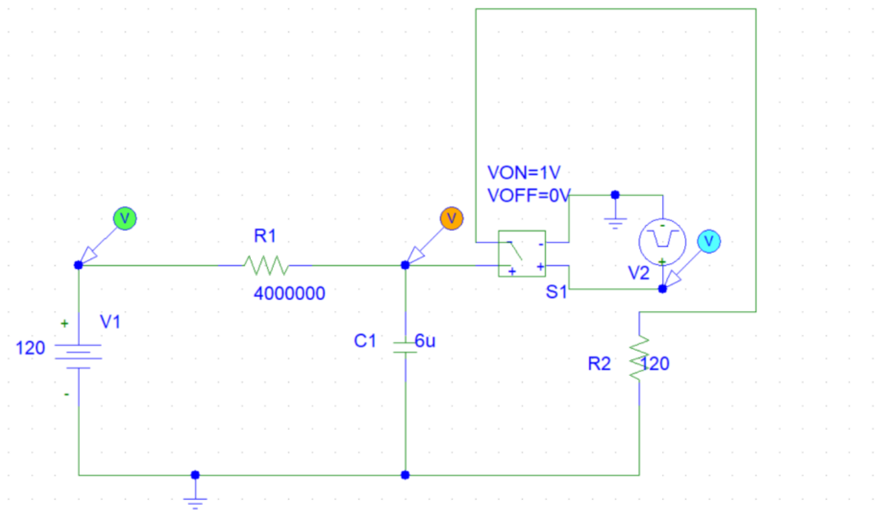

Figure 5: The circuit diagram of relaxation oscillator in Pspice

We use voltage controlled switch and pulse generator for turn on/off the switch.

Properties of pulse generator are as follows,

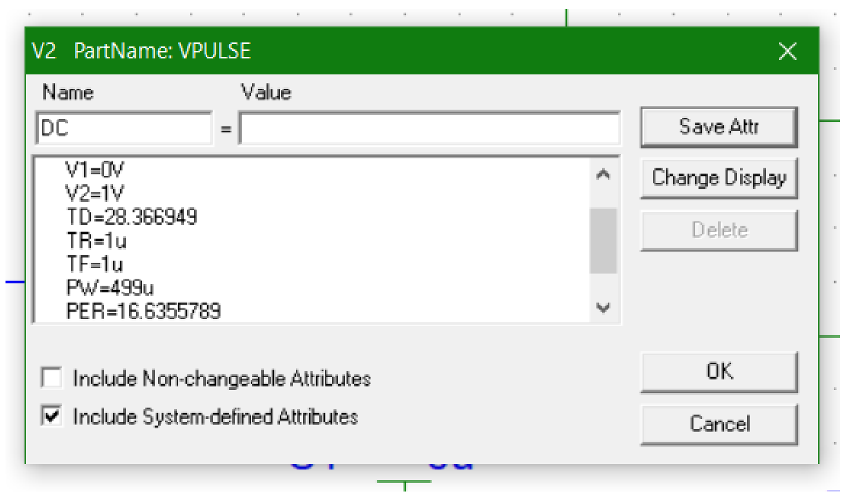

Figure 6: Properties of Pulse generator for voltage controlled switch

The reason of doing this to get the same behavior as theoretical. First switch is off until capacitor charged to 80V. Already calculated the time for initial charging, it will take 26,3666949 seconds. So first the pulse will be delayed with 26,3666949 s.

Then it closes the switch within 499 μs. After that it will on and off the switch with the period of 16,6355323s because after initial cycle, capacitor will be charged 40V to 80V periodically. As a result, we get this graph,

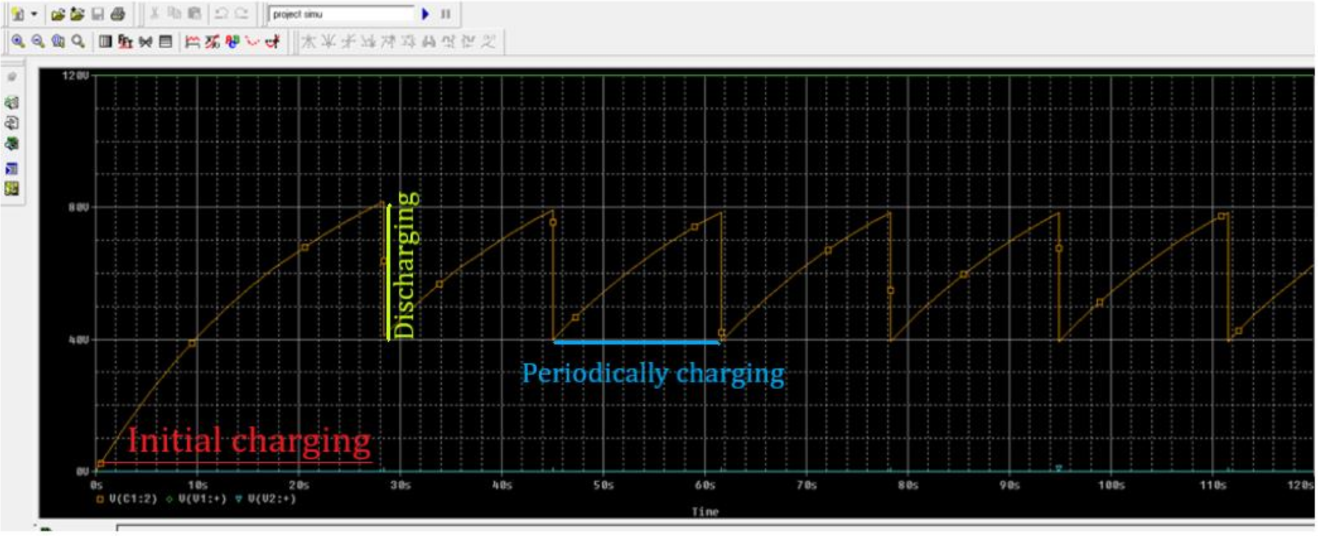

Figure 7 : The graph of voltage of capacitor in Pspice

The first voltage reaches 80V and it decreases very radically to 40V then within 16,6355323s periods it will create this graph.

Question 4 : Design a cylindrical capacitor using a dielectric material of your choice such that the capacitance becomes 6 µF

To design a capacitor using a dielectric material, this formula is used:

$$C = \varepsilon_r * C_0$$

$C_0 \rightarrow$ the value of unfilled cylindrical capacitor

$\varepsilon r \rightarrow$ dielectric constant

C $\rightarrow$ the value of filled cylindrical capacitor with a material having electric permittivity εr

In this case, C should be $6\mu F$ to design a $6\mu F$ capacitor. The value of εr is 80.2 which is dielectric constant of water at 20 degrees.

$$6\ \mu F = 80.2 * C_0 \quad \rightarrow \quad C_0 = \ 7.48 * 10^{-2}\ \mu F = 7.48 * 10^{-8}\ F$$

Calculating C_0

R_a → the radius of cylinder inner

R_b → the radius of cylinder outer

L → the height of cylinders

$+\,Q$ → the total charge of inner radius

$-\,Q$ → the total charge of outer radius

Firstly, E field which is between two cylinders and potential difference V should be found. Then, C is calculated by using the relationship between Q, V and C.

To calculate E field, using Gauss Law, appropriate Gauss surface is chosen. A cylindrical shell that has L height and "r" radius is the Gauss surface. "r" should be Ra < r < Rb because if "r" is smaller than Ra, E field will be zero since there is no charge enclosed. If "r" is greater than Rb, E field will be zero since the sum of total charges is 0.

So, Ra < r < Rb:

$$\oint \vec{E} * \overrightarrow{ds} = \frac{Qenc}{\varepsilon 0}$$

$$\underbrace{\oint \vec{E} * \overrightarrow{ds}}_{\textit{Bottom surface}} \quad + \quad \underbrace{\oint \vec{E} * \overrightarrow{ds}}_{\textit{Top surface}} \quad + \quad \underbrace{\oint \vec{E} * \overrightarrow{ds}}_{\textit{Side surfaces}}$$

Since the normal of bottom and top is perpendicular to E field direction, E field is 0.

So, $\oint E\ .\hat{r} * \overrightarrow{ds}$ for side surfaces should be considered.

$$\oint E\ .\hat{r} * dr.\hat{r}$$

$$2 * \pi * r * L * \vec{E} = \frac{Q}{\varepsilon 0}$$

$$\vec{E} = \frac{Q}{2 * \pi * r * \varepsilon 0 * L} * \hat{r}$$

The relationship between V and E:

$$\Delta V = -\int \vec{E} * \overline{dl}$$

$$Vab = -\int_{Rb}^{Ra} \vec{E} * \overrightarrow{dl} = -\int_{Rb}^{Ra} \frac{Q}{2 * \pi * r * \varepsilon 0 * L} * \hat{r} * \hat{r} * dr$$

$$= -\frac{Q}{2 * \pi * \varepsilon 0 * L} \int_{Rb}^{Ra} \frac{dr}{r}$$

$$Vab = -\frac{Q}{2 * \pi * \varepsilon 0 * L}(lnRa - lnRb) = \frac{Q * ln(^{Rb}/_{Ra})}{2 * \pi * \varepsilon 0 * L}$$

The relationship between Q, C and V:

$$Q = V * C_0 \quad \rightarrow \quad C_0 = \frac{Q}{V}$$

$$C_0 = \frac{Q}{\frac{Q * \ln(^{Rb}/_{Ra})}{2 * \pi * \varepsilon 0 * L}} \quad \rightarrow \quad C_0 = \frac{2 * \pi * \varepsilon 0 * L}{ln\frac{Rb}{Ra}}$$

$$C_0 = 7.48 * 10^{-8} = \frac{2 * \pi * \varepsilon 0 * L}{ln\frac{Rb}{Ra}}$$

Rb = 0.1001 m

Ra = 0.1 m

$\varepsilon_0 = 8.85 * 10^{-12}$ F.m^{-1}

L can be calculated.

$$C_0 = 7.48 * 10^{-8} = \frac{2 * \pi * 8.85 * 10^{-12} * L}{ln \frac{0.1001}{0.1}}$$

$$= 7.48 * 10^{-8} = \frac{2 * \pi * 8.85 * 10^{-12} * L}{9.9 * 10^{-4}}$$

From this equation L is equal to 1.246 m.

As a result, if we have a cylindrical capacitor that has 0.1001 m outer radius, 0.1 m inner radius and 1.246 m height, and if we use water as a dielectric material, the C value becomes 6 μF.

Question 5 : Design a parallel plate capacitor using a dielectric material of your choice such that the capacitance becomes 6 μF

For parallel plate capacitor, we can use this equation,

$$C = \varepsilon_r \varepsilon_0 \frac{A}{d}$$

$$C - Capacitance\ of\ parellel\ plate\ capacitor = 6\mu F$$

$$\varepsilon_r - relative\ permittivity\ of\ paper = 3.780$$

$$\varepsilon_0 - dielectric\ permittivity\ of\ vacuum = 8.85 * 10^{-12}\ F * m^{-1}$$

$$A - Area\ of\ one\ plate$$

$$d - the\ distance\ between\ plates\ which\ is\ thickness\ of\ paper = 0.00005\ m$$

To design a parallel plate capacitor, we chose paper as dielectric material. From formula of capacitance of parallel plate, we get value of A as follows

$$6 * 10^{-6} = 3.780 * 8.85 * 10^{-12} \frac{A}{0.00005}$$

$$A = 8{,}96780\ m^2$$

So we need 8,96780 m^2 parallel plate to create capacitor having 6 μF capacitance.

Question 6 : Fabricate a cylindrical capacitor using the materials that you can find at home. What is the theoretical capacitance of your capacitor? Measure the capacitance of your capacitance in laboratory and compare with your theoretical prediction.

To fabricate cylindrical capacitor, we used one sheet of aluminum foil, a plastic bottle, band and copper cable, water and salt. We wrapped the plastic bottle with aluminum foil. Then, we created a conducting area with water and salt. And using copper cable we intended to reach water from outside of the bottle. Between water and aluminum foil, as a dielectric material, there is just plastic bottle itself.

According to cylindrical capacitor formula,

$$C = \frac{2 * \pi * \varepsilon 0 * L * \varepsilon r}{ln \frac{Rb}{Ra}}$$

We measured L, rb and ra

$$L = 0.15\ m$$

$$Ra = 4.35\ cm$$

$$Rb = 4.45\ cm$$

And $\varepsilon r\ is\ 3$.

Then we get the capacitance value as,

$$1{,}1003\ nF$$

For the experimental part, we found the capacitance 1.3 nF by using multimeter.

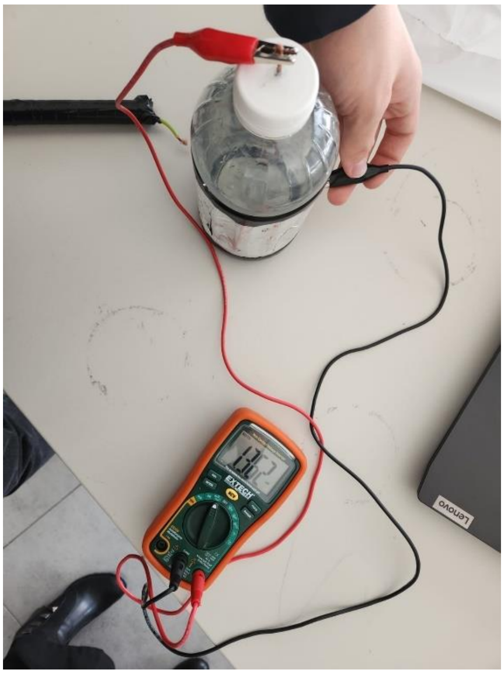

Figure 8: Capacitance measurement

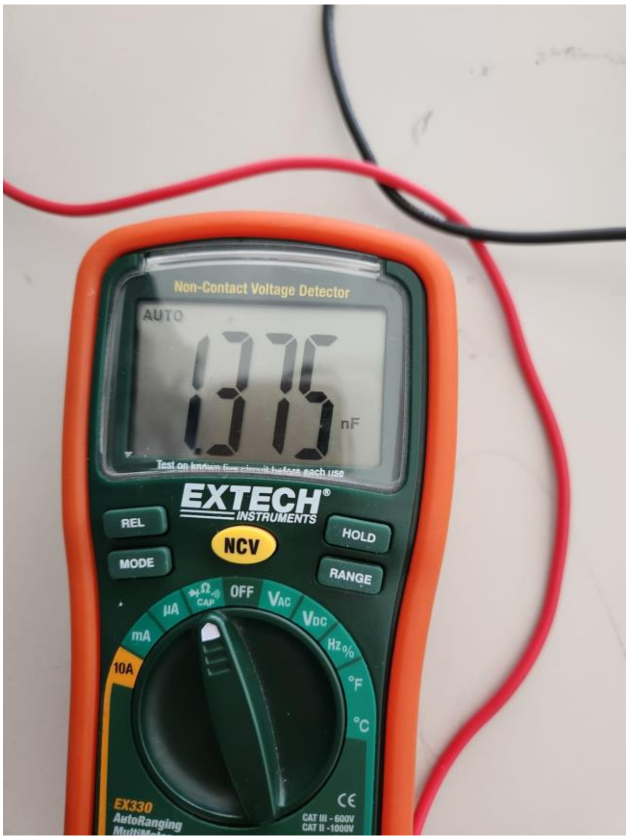

Figure 9: Capacitance measurement by using multimeter

Question 7 : Fabricate a parallel-plate capacitor using the materials that you can find at home. What is the theoretical capacitance of your capacitor? Measure the capacitance of your capacitance in laboratory and compare with your theoretical prediction.

To fabricate parallel plate capacitor, we used two sheet of aluminum foil and two sheet of PVC that are used for book covers.

The areas of foils are nearly 0.06 m^2 and thickness of PVC is 0.0005m

So according to formula we get, 0.000003186 F.0

For the experimental part, in lab by using multimeter, we calculated capacitance as 0.000000558 F.

The experimental result is more nearly two times than theoretical result, because we rolled the capacitor so that area doubled.

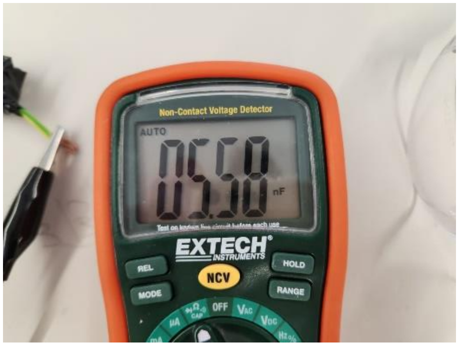

Figure 10: Capacitance measurement

Figure 11: Capacitance measurement by using multimeter

Question 8 : Consider a discrete time step Δt = 0.01 ms and write the discrete-time series for capacitor voltage V(t). Investigate the convergence properties of the series

A discrete time series is a sequence of data points measured at fixed time intervals. The term "discrete" means that the time intervals are separated by fixed periods of time, and the term "time series" refers to a set of observations collected over a period of time.

We can write discrete time series for V(t) which consist of two different function.

1- First we have discharging formula,

$$V_d(t) = 80e^{-\frac{t}{\tau}}, \tau = \ 0{,}000719976$$

The sequence of ordered pairs representing the voltage can then be written as:

$$(t_0, V_0), (t_1, V_1), (t_2, V_2), \ldots \ldots, (t_{n-1}, V_{n-1})$$

Where n is the sample that collected during the sample interval. If we apply this to discharging with $\Delta t = 0.01ms$ we have,

$$(0, 80V), (0.01, 78{,}8964V), (0.02, 77{,}80823V), \ldots \ldots, (t_{n-1}, V_{n-1})$$

We can write this series with sigma notation,

$$\sum_{n=0}^{\infty} 80e^{-\frac{n}{\tau}}, \qquad \tau = \ 0{,}000719976$$

$$\sum_{n=0}^{\infty} V_d[n]$$

Then we can use unit sample sequence to

$$\delta[n-k] = \begin{cases} 0, & n-k \neq 0 \\ x, & n-k = 0 \end{cases}$$

k is the step of discrete time series,

$$k = \Delta t = 0.01ms,$$

Then we have,

$$\sum_{n=0}^{\infty} V_d[n] \ \delta[n-k]$$

We can graph this series in MATLAB

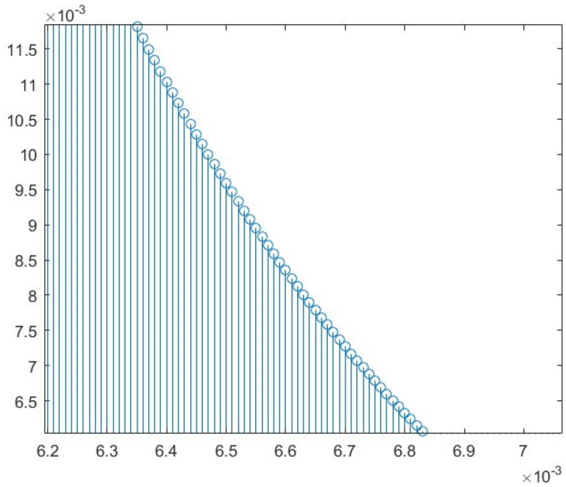

Figure 12: Discrete time series of discharging formula

Because τ is very small, our graph very dramatically drops.

2- Second we have charging formula,

$$V_c(t) = 120 - 120e^{-\frac{t}{24}}$$

The sequence of ordered pairs representing the voltage can then be written as:

$$(t_0, V_0), (t_1, V_1), (t_2, V_2), \dots \dots, (t_{n-1}, V_{n-1})$$

Where n is the sample that collected during the sample interval. If we apply this to discharging with $\Delta t = 0.01ms$ we have,

$$(0, 0V), (0.01, 0.00499V), (0.02, 0.000999\ V), \dots \dots, (t_{n-1}, V_{n-1})$$

We can write this series with sigma notation,

$$\sum_{n=0}^{\infty} 120 - 120e^{-\frac{n}{24}}$$

$$\sum_{n=0}^{\infty} V_c[n]$$

Then we can use unit sample sequence to

$$\delta[n-k] = \begin{cases} 0, & n-k \neq 0 \\ x, & n-k = 0 \end{cases}$$

k is the step of discrete time series,

$$k = \Delta t = 0.01ms,$$

Then we have,

$$\sum_{n=0}^{\infty} V_c[n]\ \delta[n-k]$$

We can graph this series in MATLAB

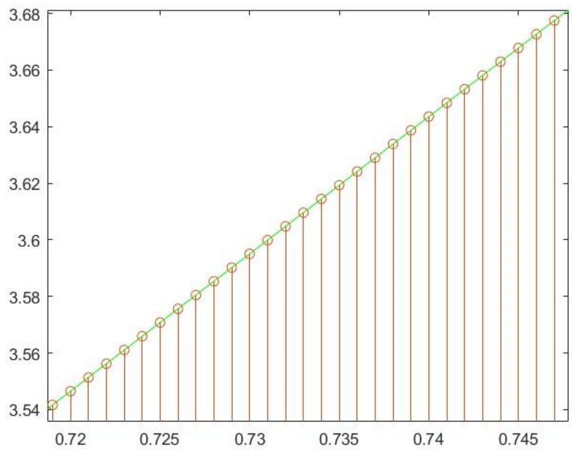

Figure 13: Discrete time series of charging formula, green is function itself

About properties of convergency of discrete time series, we can first analyze discharging discrete time series,

$$\sum_{n=0}^{\infty} 80e^{-\frac{n}{\tau}}\ \delta[n-k], \qquad \tau = \ 0{,}00071997$$

$\delta[n-k]$ this part of the series makes it discrete, so if we analyze the part $\sum_{n=0}^{\infty} 80e^{-\frac{n}{\tau}}$ by ratio test

Suppose we have the series $\sum a_n$, Define

$$L = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|$$

If,

1- L<1 the series is absolutely convergent (and thus convergent)
2- L=1 the series is divergent
3- L>1 inconclusive

So we can check for $\sum_{n=0}^{\infty} 80e^{-\frac{n}{\tau}}$

$$L = \lim_{n\to\infty}\left|\frac{80e^{-\frac{n+1}{\tau}}}{80e^{-\frac{n}{\tau}}}\right|$$

$$L = \lim_{n\to\infty}\left|\frac{\frac{80}{e^{-\frac{n+1}{\tau}}}}{\frac{80}{e^{-\frac{n}{\tau}}}}\right|$$

$$L = \lim_{n\to\infty}\left|\frac{e^{-\frac{n}{\tau}}}{e^{-\frac{n+1}{\tau}}}\right|$$

$$L = \lim_{n\to\infty}\left|e^{\frac{1}{\tau}}\right|$$

So that,

$$L < 1$$

As a result discharging discrete time series is convergent.

Then we can apply same procedure on charging discrete time series

$$\sum_{n=0}^{\infty} 120 - 120e^{-\frac{n}{24}}$$

$$L = \lim_{n\to\infty}\left|\frac{120-120e^{-\frac{n+1}{24}}}{120-120e^{-\frac{n}{24}}}\right|$$

$$\lim_{n\to\infty}\left|\frac{120-120e^{-\frac{n+1}{24}}}{120-120e^{-\frac{n}{24}}}\right| =_{\mathbf{L'H\hat{o}pital}} \lim_{n\to\infty}\left|\frac{5e^{-\frac{n+1}{24}}}{5e^{-\frac{n}{24}}}\right|$$

$$\lim_{n\to\infty}\left|\frac{\frac{1}{e^{-\frac{n+1}{24}}}}{\frac{1}{e^{-\frac{n}{24}}}}\right|$$

$$\lim_{n\to\infty}\left|\frac{e^{\frac{n}{24}}}{e^{\frac{n+1}{24}}}\right|$$

$$L \ = \lim_{n\to\infty}\left|e^{-\frac{1}{24}}\right|$$

So that,

$$L < 1$$

As a result charging discrete time series is convergent as well.

Question 9 : Write the Maclaurin series expansion for V(t)

As we stated functions at question 1 and 2, we have two different equation for charging and discharging.

For discharging formula,

$$V_d(t) = 80e^{-\frac{t}{\tau}}, \tau = \ 0{,}000719976$$

Then we can write Maclaurin series of $V_d(t)$ by using Maclaurin series of $f(t) = e^t$

$$f(t) = e^t$$

$$Maclaurin\ series = f(t) = f(0) + f'(0)t + \frac{f''(0)t^2}{2!} + \cdots + \frac{f^n(0)t^n}{n!} + \cdots$$

$$f(0) = f'(0) = f''(0) = f^n(0) = 1$$

$$e^t \ = 1 + t + \frac{t^2}{2!} + \cdots + \frac{t^n}{n!} + \cdots = \sum_{n=0}^{\infty}\frac{t^n}{n!}$$

$$V_d(t) = 80e^{-\frac{t}{\tau}}, \tau = \ 0{,}000719976$$

$$V_d(t) = 80\sum_{n=0}^{\infty}\frac{(-1)^n(\frac{t}{\tau})^n}{n!}$$

For charging formula,

$$V_c(t) = 120 - 120e^{-\frac{t}{24}}$$

Then we can write Maclaurin series of $V_c(t)$

$$Maclaurin\ series = f(t) = f(0) + f'(0)t + \frac{f''(0)t^2}{2!} + \cdots + \frac{f^n(0)t^n}{n!} + \cdots$$

$$f(0) = 0$$

$$f'(0) = 5$$

$$f''(0) = -\frac{5}{24}$$

$$f'''(0) = \frac{5}{24^2}$$

$$0 + 5t - \frac{5t^2}{24\ 2!} + \frac{5t^3}{24^2\ 3!} + \cdots$$

Then we have,

$$V_c(t) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{5^n}{24^{n-1}} \frac{(t)^n}{n!}$$

RESULTS AND DISCUSSION

a) In the given RC circuit including a voltage source and a lamb, we analyzed the circuit. The lamp fires at 80 V, and it is off at 40V. According to this, we calculated the charging and discharging times with using KVL and KCL and first-order differential equation rules. We found the time takes charging is equal to $26{,}3666949\ s$. In this circuit, the value of the time constant which is represented τ equals to 24 and τ = R.C. We can consider that if we have higher C or higher R which means τ is greater than 24, the time for charging would be greater than $26{,}3666949\ s$. That means, in the RC circuit, while higher time constant results in a slow response, smaller time constant produces a quick response. So, we can consider that we need to set τ value according to different RC circuits we want to use in many areas for different purposes.
In addition, we computed the time as 499 μS which takes for the capacitor voltage decreases from 80V to 40V. To do this, we used R_{eq} since the resistor and lamp are in parallel.

To analyze the effect of R, let's assume we have 1 MΩ instead of 4 MΩ. To see the difference, we can generate the table:

For the resistor 4 MΩ:	$R_{eq} = 119{,}996\Omega$	$\tau = 0{,}000719976$	The time : 499 μs

For the resistor 0.5MΩ	$R_{eq} = 119{,}971\Omega$	$\tau = 0{,}000719826$	The time : 498.8 μs

As it can be seen lower R leads to have lower R_{eq} and lower τ value and lower time.

b) For this part, we computed the time takes to charge after the initial cycle. By using formula $V(t) = 120 - 120e^{-\frac{t}{24}}$, since the lamp fires at 80 V and it is off at 40V, we calculated times accordingly. The result we found is 16,6355323 s. We can also associate this situation to τ. If τ is smaller than 24, the time is smaller than 16,6355323. So, we would get a fast-acting RC circuit.

c) From the graph, the time value for initial charging $(26{,}3666949s)$, the time takes discharging $(499\ \mu s)$ can be seen. After initial cycling, the bases of the waves in the graph shows us the time interval taking to charge the capacitor after the initial cycle as we calculate and it is 16,6355323s. The time will be the same while the voltage oscillates from 80V to 40V.
Also, we can say that the relationship between the period and RC is proportional.

d) For this question, it was asked to design a cylindrical capacitor 6 μF with a dielectric material and to design this, $C = \varepsilon_r * C_0$ is used. To calculate C_0, we used the relationship between V and E and to find E, chosen Gauss surface should be proper. So, we realized that in the case of Ra < r and in the case of r < Rb, E field will be zero. Thus, we took the radius of Gauss cylinder "r" as Ra < r < Rb.

In the equation C_0, we calculated the necessary height of cylindrical capacitor by using assumed Ra and Rb values. Then, using water as a dielectric material, we get the cylindrical capacitor 6 μF.

In addition, the units had to be corrected while making these calculations. We assigned the unit values correctly.
The equation is:

$$C = \varepsilon_r * \frac{2 * \pi * \varepsilon 0 * L}{ln \frac{Rb}{Ra}}$$

So , we can write:

$$F = {}^{F}/_{m} * m$$

$$F \rightarrow unit\ of\ capacitance$$

$${}^{F}/_{m} \rightarrow unit\ of\ \varepsilon_0$$

$$m \rightarrow unit\ of\ length$$

It can be clearly seen that there is unit equality on both sides of the equation.

e) To design a parallel plate capacitor 6 μF with a dielectric material, we used the formula $C = \varepsilon_r * C_0$ where the C_0 equals to $\varepsilon_0\, {}^{A}/_{d}$. By using the paper as a dielectric material which has

3.780 permittivity, we determined "d" and we found A value to design a parallel plate capacitor. The equation used:

$$C = \varepsilon_r \varepsilon_0 \frac{A}{d}$$

We can write:

$$F = {}^{F}\!/_{m} * \frac{m^2}{m}$$

$$F \rightarrow unit\ of\ capacitor$$

$${}^{F}\!/_{m} \rightarrow unit\ of\ \varepsilon_0$$

$$m^2 \rightarrow unit\ of\ area$$

$$m \rightarrow unit\ of\ length$$

f) During the fabrication of cylindrical capacitor, we used salty water as conductor material inside the plastic bottle. We thought that salty water behaved like conductor, as inside of it there is no electric field, so that between water's surface and aluminum foil electric field was created. Due to electric field, voltage difference occurred. As a result, the capacitance is valid.

The difference between theoretical and experimental results is caused by temperature, imprecise dielectric permittivity constant, imprecise measurements.

g) During the fabrication of parallel plate capacitor, we used two sheet of aluminum foil and two piece of plastic. We glued them together, then we rolled it. After rolling, we measured its capacitance as nearly 5.5nF. However the theoretical result was much lower than the experimental result.

By rolling the capacitor, we are increasing the area that creates the capacitance. We can explain this as follows,

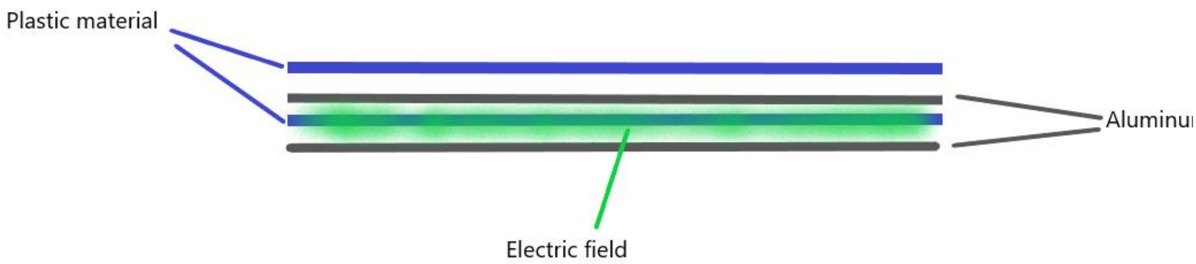

Figure 14: Initial condition of parallel plate capacitor

The first condition is this, the electric field will just occur between the surfaces looking at each other. But think, after 1 fold, the situation will change.

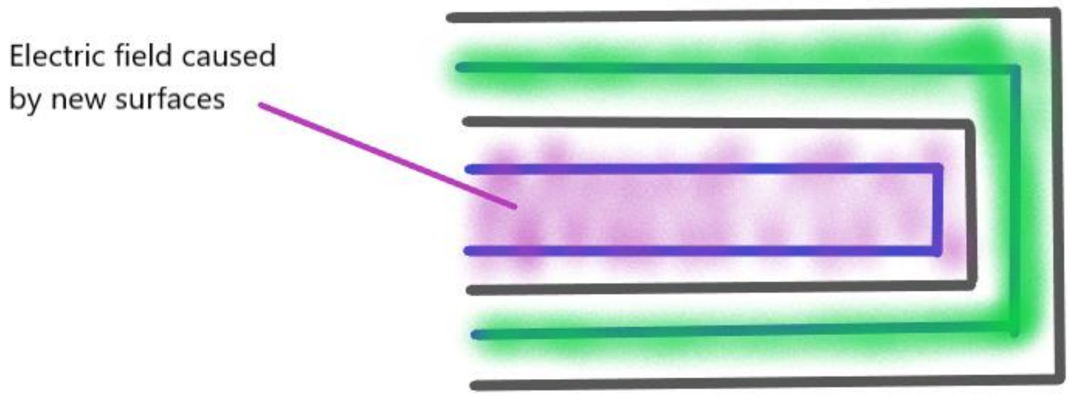

Figure 15: Condition of parallel plate capacitor after 1 folding

The situation after 1 folding, is above. There will be a new electric field caused by new surfaces that are now looking at each other. However, by rolling we bring outer surface of one next to outer surface of other. It is like folding but it is limit of folding. So we are doubling area. According to formula, Capacitance is directly proportional to area. So if we double the area, we doble the capacitance. As a result, our experimental result is nearly as twice as much than theoretical result.

i) In this part, the expectation is writing the Maclaurin series expansion for $V_c(t)$. By using both equations for charging and discharging $V_d(t), V_c(t)$, Maclaurin series were written. To get correct sigma notation for this series, we assigned the correct values to obtain each term.

CONCLUSIONS

In conclusion, RC circuit analysis including a voltage source and a lamp was done and the times take for charging and discharging were found using KVL and KCL rules, and voltage behavior on each time was shown graphically on PSpice. To design capacitors, equations and calculations related to capacitors were made. Cylindrical and parallel plate capacitors were created with materials that can be found at home, and it was understood how to measure these capacitors with a multimeter. In addition, the discrete-time series for capacitor voltage was written and the Maclaurin series was written about the obtained equations $V_d(t)$ and $V_c(t)$.

REFERENCES

[1] Discrete-time Signals and Systems by Sanjoy Mahajan and Dennis Freeman

[2] Calculating the Relative Permittivity Constants of Various Dielectric Materials Using a Parallel Plate Capacitor, Jonathan Markell, Jingbo Ye

[3] Time Series Models for Discrete Data, Iain Lachie MacDonald

[4]https://www.ti.com/lit/an/snoa998/snoa998.pdf?ts=1682328310196&ref_url=https%253A%252F%252Fwww.google.com%252F

[5]https://courses.engr.illinois.edu/ece110/fa2021/content/labs/Experiments/C_RC_SquareWave_analysis.pdf

[6] https://www.electrical4u.com/relaxation-oscillator/

[7] https://en.wikipedia.org/wiki/Relaxation_oscillator

[8] https://en.wikipedia.org/wiki/Pearson%E2%80%93Anson_effect

[9] https://en.wikipedia.org/wiki/Capacitor

[10] http://hyperphysics.phy-astr.gsu.edu/hbase/Electronic/relaxo.html

[11] https://en.wikipedia.org/wiki/Relative_permittivity