

Department of Electrical & Electronics Engineering Abdullah Gül University

Project Report

EE1200 Electronic System Design (ESD) Capsule

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OBJECTIVE

In this project, we are required to fabricate cylindrical and toroidal inductors. For the cylindrical inductors, wind one around a dielectric material and the other around a ferromagnetic material. Similarly, for the toroidal inductors, wind one around a dielectric material and the other around a ferromagnetic material. We took clear photographs of the fabricated inductors and provide information such as winding number, length, diameter, and an educated guess of the relative permeability of the core material. Next, design circuits to measure the current passing through both types of inductors and report the measured values. Additionally, we designed a circuits to measure the inductance values of the solenoids and toroid's and report those values as well. Then, we compare the theoretical inductance values with the experimental results obtained in the experimental part. In a hypothetical part, we are given a solenoid wound around a core material with a time-dependent relative magnetic permeability and wire density. We calculate the total inductance of the solenoid as a function of time and determine the voltage on the inductor for a given current waveform. Also, we calculate the voltage at an arbitrary point on the solenoid as a function of position and time, analyzing its critical points. Lastly, we consider a finite solenoid and compute the magnetic field along its central axis at a specific point, given the relevant parameters. We determine the point at which the magnetic field has the maximum value. Additionally, we provide a comprehensive report detailing our findings, including photographs, experimental measurements, theoretical calculations, and analysis of discrepancies, as well as the mathematical analysis of the hypothetical scenarios.

BACKGROUND

In this project, we fabricate and make analysis of cylindrical and toroidal inductors are explored. For the cylindrical inductors, known as solenoids, we used magnet wire in order to wind around cardboard(dielectric material) cylinder and metal(ferromagnetic material) cylinder to provide solenoid. On the other hand metal washer are used and attached together to give toroid thickness for the ferromagnetic toroid we and for the dielectric toroid we used plastic ring which has a thickness as metal washer. Also, in our experiments we used 10 ohm resistor in order to make our circuit.

ANALYTICAL AND SIMULATION PROCEDURES

Part 1 – Fabrication of cylindrical inductors

For this task, we fabricated two different solenoid inductors. We used insulator-coated conductive wires, cardboard and ferrous material. We wound materials with wires.



Figure 1: non-Ferro solenoid



Figure 2 : Ferro Solenoid

Their properties are as follows,

Type	Winding	Diameter	μ_r	length
Non- Ferro	90	2 cm		10 cm
Ferro	270	1.8 cm		11cm

To find relative permeability of materials that we used, we calculated their inductance by using LCR meter, and we compared their experimental values and theoretical values.



Figure 3 and 4 for measuring inductance of ferro solenoid



Figure 5 and 6 for measuring inductance of non-ferro solenoid

For the theoretical value of inductance of solenoids, we can use the following formula,

$$L = \frac{(\mu_r)\mu_0 N^2 A}{l}$$
 1.1

Туре	Theoretical Value of Inductance	Experimental Value of Inductance
Ferro	0.00021192373 H	3.002 mH
Non-Ferro	0.00003197751 H	12.60 μΗ

Part 2 – Fabrication of toroidal inductors

For this task, we fabricated two different toroid inductors. We used insulator-coated conductive wires, a plastic circular material and ferrous material as circular . We wound materials with wires.



Figure 7: non-Ferro toroid



Figure 8 : Ferro toroid

Their properties are as follows,

Type	Winding	Inner Radius	Outer Radius	μ_r	Thickness
	number				
Non-Ferro	208	1 cm	1.5 cm		0.4cm
Ferro	30	0.2 cm	2.3cm		0.5cm

To find relative permeability of materials that we used for the fabrication of toroids, we calculated their inductance by using LCR meter, and we compared their experimental values and theoretical values.



Figure 9 and 10 for measurement of non-ferro toroid

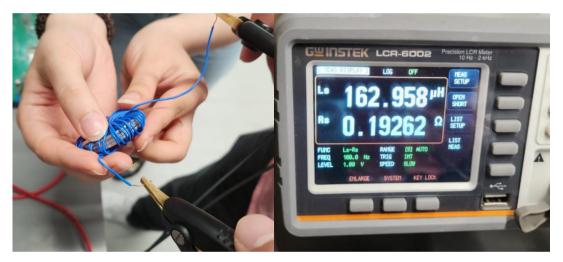


Figure 11 and 12 for measurement of ferro toroid

For the theoretical value of inductance of toroids, we can derive the formula,

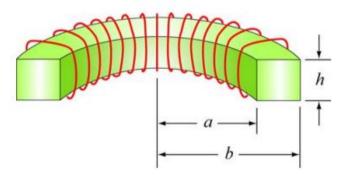


Figure 13: A toroid

To find self-inductance, first we should look at its magnetic field. Due to the symmetry, magnetic field must be circular. Thus applying Ampere's Law,

$$\oint \overrightarrow{B} \cdot d\vec{s} = \mu_r \mu_0 I_{enc}$$
2.1

$$B(2\pi r) = \mu_r \mu_0 NI$$
 2.2

This gives us,

$$B = \frac{\mu_r \mu_0 NI}{2\pi r}$$
 2.3

The magnetic flux of one turn can be obtain by integrating over the cross section, with dA = hdr

$$\phi_B = \int \overrightarrow{B} \cdot d\overrightarrow{A}$$
 3.1

$$\phi_B = \int \frac{\mu_r \mu_0 NI}{2\pi r} h dr$$
 3.2

$$\phi_B = \frac{\mu_r \mu_0 NIh}{2\pi} \int_a^b \frac{1}{r} dr$$
 3.3

As a result we get,

$$\phi_B = \frac{\mu_r \mu_0 NIh}{2\pi} \ln\left(\frac{b}{a}\right)$$
 3.4

And the total inductance for N turn is $N\phi_B$. The self-inductance is,

$$\frac{N\phi_B}{I} = \frac{\mu_r \mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right)$$
 3.5

$$L = \frac{(\mu_r)\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right)$$
 3.6

Type	Theoretical Value of Inductance	Experimental Value of Inductance
Non-Ferro	14.01753 uH	36.72 μH
Non-Perio	14.01733 u11	30.72 μΠ
Ferro	2,82743 uH	162.9 μΗ

Part 3

Sub-Section A – Measurement of current passing through inductors

To measure the current passing through inductors, we use the method that is giving sine signal through pure inductive circuit and measuring maximum current via AC Ammeter.

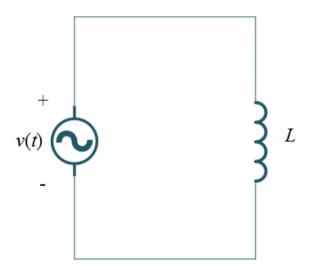


Figure 14: Pure inductive circuit

Assume that $v(t) = V_m \sin \omega t$, applying KVL we get,

$$V_m \sin \omega t - L \frac{di}{dt} = 0 4.1$$

$$V_m \sin \omega t = L \frac{di}{dt}$$
 4.2

$$\frac{di}{dt} = \frac{V_m}{L} \sin \omega t \tag{4.3}$$

$$\int di = \int \frac{V_m}{L} \sin \omega t \, dt \tag{4.4}$$

$$i(t) = \frac{V_m}{\omega L} (-\cos \omega t)$$
4.5

We can write $\sin \omega t - \frac{\pi}{2} = -\cos \omega t$

$$i(t) = \frac{V_m}{\omega L} \sin \omega t - \frac{\pi}{2}$$
4.6

Here maximum current is,

$$I_m = \frac{V_m}{\omega L} \tag{4.7}$$

And ωL is inductive reactance as X_L .

In the Lab, we found something interesting. First we directly applied sine signal to our inductors, and we measured the current by using AC Ammeter. The AC Ammeter measures the RMS value of current. So that our RMS value of voltage is nearly 0.707 V.



Figure 15: Properties of signal generator for the circuit

First we measured the current passing through the ferro solenoid. From the formula 4.7 we should have seen that a current which is, (100 Hz frequency and our X_L is 1.89 Ω)

$$I = \frac{0.707}{1.89}$$
 5.1

$$I = 0.374 A$$
 5.2

But we did not see this value on the ammeter.

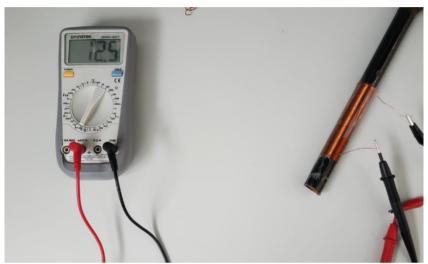


Figure 16: Current measurement of Ferro solenoid

From this suspicion we decided to simulate the same circuit on the Proteus.

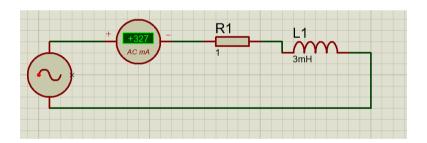


Figure 3

We should have seen this figure on the AC Ammeter. But in Proteus we saw 327mA. Then we consider inner resistance as 50 ohm. Then we change the resistance, we saw this figure

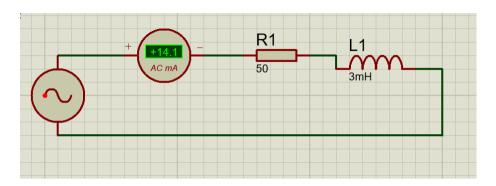


Figure 17: Simulation representation of current measurement

As we saw close number.

Other measurements are as follows,

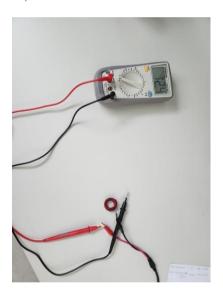


Figure 18

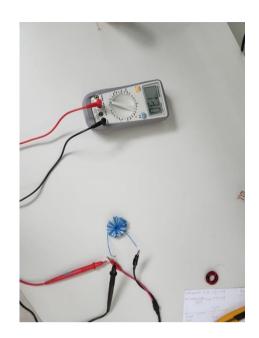


Figure 19

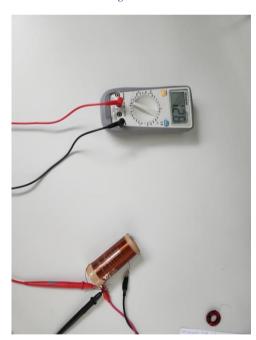


Figure 20

Square CH2 Square CH1 Square

Square CH2 Square CH1 Freq

1000Vpp

500.0mVdc

H-50.0%-N

CH1 Waveform Load: Hi-2

Prequency 10,000 000Hz

Phase

They

1,000Vpp

Phase 0,0

Offset

Chylage

Offset

O

To measure the inductances, we use transient analysis in small segments of square wave signals.

Figure 21: Square wave signal used for measuring

After doing experiments, measurements on the oscilloscope are as follows:

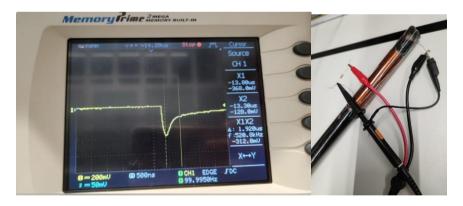


Figure 22 and 23

For the ferro solenoid we used 1.5k resistor (its real value is 1489). And our graph value increased with one time constant, we know that

$$\tau = \frac{L}{R} \tag{6.1}$$

$$1.9us = \frac{L}{1489}$$
 6.2

As a result the inductance value is 2.8mH

If we do the same process for the others, we get

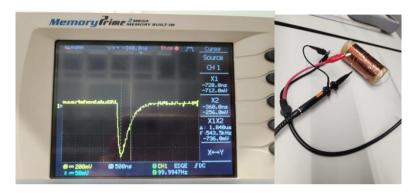


Figure 24 and 25: Measurement of inductance with 550 ohm resistor

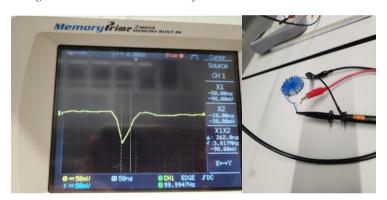


Figure 26 and 27 Measurement of inductance with 1.5kohm resistor

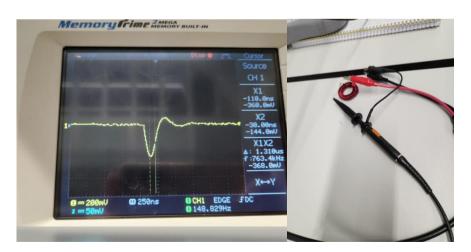


Figure 28 and 29 : Measurement of inductance with 550 ohm resistor

We used the same value of resistance. From time constants,

Туре	Time constant	Resistance	Inductance
Ferro Solenoid	1.9us	1489Ω	2.8mH
Non-Ferro Solenoid	1.84us	550Ω	1mH
Ferro Toroid	262ns	1489Ω	390uH
Non-Ferro Toroid	1.310us	550Ω	720uH

Part 5A

Sub-Section A - Computing Inductance of a Solenoid

The inductance per unit length of a solenoid with a varying relative magnetic permeability can be expressed as:

$$L(z,t) = \frac{\mu_0 \mu_r N^2 A}{I}$$
 7.1

where μ_0 is the vacuum permeability $(4\pi \times 10^{-4})$ T·m/A), $\mu r(t)$ is the relative magnetic permeability as a function of time, A is the cross-sectional area, and l is the length of the solenoid.

We can calculate the total inductance by integrating over the length of the solenoid. However, the number of turns N should be represented as a total number along the solenoid length, not a density. Given N(z,t)=2z as a density, we can express total number of turns as from your given, the relative magnetic permeability μr is a function of space and time given by $\mu r(z,t)=4t+5z$ and the density of the wires (number of turns per unit length) N(z,t)=2z

$$N_T = \int_0^L N(z, t) dz 7.2$$

$$N_T = \int_0^L 2z dz 7.3$$

$$N_T = L^2 7.4$$

The total inductance $L_t(t)$ as a function of time t is given by:

$$L_t(t) = \int_0^l L(z, t) dz$$
 7.2

Integration over dz can be explain as this,

The integration over z (represented as z=0 to z=L) in the calculation of the total inductance is performed because we are considering the contribution of each small segment of the solenoid along its length.

In the context of a solenoid, the inductance per unit length L(z,t) is a function that can vary with position z along the solenoid. By integrating L(z,t) over the length of the solenoid, we are essentially summing up the contributions of these small segments to obtain the total inductance.

The integral in 4.7 is performed to add up the inductance contributions from z = 0 (the starting point of the solenoid) to z = L (the end point of the solenoid). This integration takes into account the varying inductance values at different positions along the solenoid, which arise from the function L(t, z) or $\mu_r(t)$ at each position z.

Substituting the expression for L(t), we have:

$$L_t(t) = \int_0^l \frac{\mu_0 \mu_r N^2 A}{l} dz$$
 7.3

Since the density of wires N(z,t) = 2z, we can express the cross-sectional area A as:

$$A = 2z\left(\pi \frac{d^2}{4}\right) \tag{7.4}$$

Now, we can calculate the total inductance:

$$L_t(t) = \int_0^l \frac{\mu_0(4t + 3z)L^4\left(\pi \frac{d^2}{4}\right)}{l} dz$$
 7.5

$$L_t(t) = \frac{\pi \mu_0 d^2 L^4}{4l} \int_0^l (4t + 3z) 2z \, dz$$
 7.6

$$L_t(t) = \frac{\pi \mu_0 d^2 L^4}{4l} \left(4tz^2 + \frac{6z^3}{3} \right) \begin{vmatrix} l \\ 0 \end{vmatrix}$$
 7.7

Evaluating the definite integral, we have:

$$L_t(t) = \frac{\pi \mu_0 d^2 L^4}{4l} (4tl^2 + 2l^3)$$
 7.8

Sub-Section B - If the current passing through this circuit is i(t) = 3, what is the voltage on the inductor?

We know that the relation between voltage and current on the Inductor,

$$V_L = L \frac{di(t)}{dt}$$
 8.1

After arranging we get,

$$V_L = L \frac{d}{dt} (i(t))$$
8.2

$$V_L = \frac{d}{dt}Li(t)$$
8.3

We know L and i(t),

$$V_L = \frac{d}{dt} \frac{3\pi\mu_0 d^2 L^3}{4} (4tL^2 + 2L^3)$$
 8.4

$$V_L = 6\pi\mu_0 d^2 L^5$$
 8.5

Simplifying we get voltage on inductor,

$$V_L = 6\pi\mu_0 d^2 L^5$$
 8.6

Sub-Section C - Calculate the voltage at an arbitrary point on the inductor as a function z and t. Analyze its critical points as the functions of z and t.

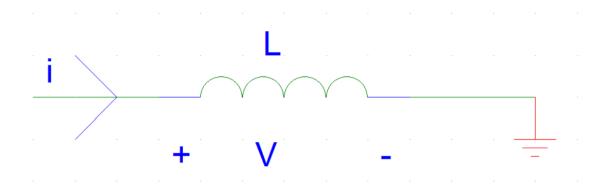


Figure 30: Inductor

To obtain the inductance formula with respect to time and space, you would integrate the inductance per unit length over the spatial domain of time and space as multiple integrals. To express the inductance formula, you would integrate L(t, z) over the desired range of positions. The integration with respect to space can be expressed as: (the reason for integral border is 0 to z is thinking that inductor's end is at origin and other end is at the arbitrary point on z)

$$L(z,t) = \int_0^t \int_0^z \mu_0(4t + 3z) L^3\left(\pi \frac{d^2}{4}\right) dz dt$$
 9.1

$$L(z,t) = \int_0^t tz\mu_0\pi d^2L^3 + \frac{3}{8}\mu_0\pi d^2L^3z^2dt$$
9.2

After calculation we get,

$$L(z,t) = \frac{t^2 z \mu_0 \pi d^2 L^3}{2} + \frac{3}{8} t \mu_0 \pi d^2 L^3 z^2$$
9.3

From figure 17, we can think that the z axis is parallel to the inductor, it is like placed on z axis. An arbitrary point on inductor will divide inductor two different part.

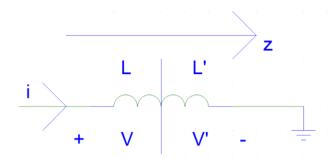


Figure 31: Inductor on z axis, and demonstration of voltage division

We know that the total Voltage is $V_L = 3\pi\mu_0 d^2 l$. And the total inductance is $L_T = L + L'$ Now we can write the voltage on arbitrary point as,

$$V - V_z = L \frac{di(t)}{dt} 9.4$$

Arranging and simplifying we get,

$$6\pi\mu_0 d^2 L^5 - V_z = \frac{dLi(t)}{dt}$$
 9.5

$$3\pi\mu_0 d^2 l - V_z = \frac{d}{dt} \frac{t^2 z \mu_0 \pi d^2 L^3}{2} + \frac{3}{8} t \mu_0 \pi d^2 L^3 z^2$$
 9.6

$$3\pi\mu_0 d^2l - V_z = tz\mu_0 \pi d^2L^3 + \frac{3}{8}\mu_0 \pi d^2L^3z^2$$
9.7

As a result the voltage on arbitrary point on inductor,

$$V(z,t) = 3\pi\mu_0 d^2l - tz\mu_0 \pi d^2L^3 + \frac{3}{8}\mu_0 \pi d^2L^3z^2$$
9.8

Finding its critical values we should look at first derivatives,

$$V_z = \frac{3}{2}\mu_0\pi d^2L^3z$$

$$V_t = -z\mu_0\pi d^2L^3$$

$$9.8$$

9.8

We can see that the derivative with respect to t cannot be zero or cannot fail to exist. So that it does not have critical points.

Part 5B

Solenoids are commonly used in experimental research requiring magnetic fields. There are some factors which affect the magnetic field that produced by the solenoid like consisting of N turns of wire tightly wound over a length L. A current I is flowing along the wire of the solenoid.

In our question we have a wire, and this wire is wound over a L which is length. So we will produce a current we show it as

$$dI = \frac{NI}{L} \, dy \tag{10.1}$$

 $\frac{N}{I}$ =The number of turns per unit length.

We used dy because the number of turns in an infinitesimal length dy are (N/L)dy turns.

We need to calculate the magnetic field at the P point. Firstly we know that the center of solenoid is equal to our P point.

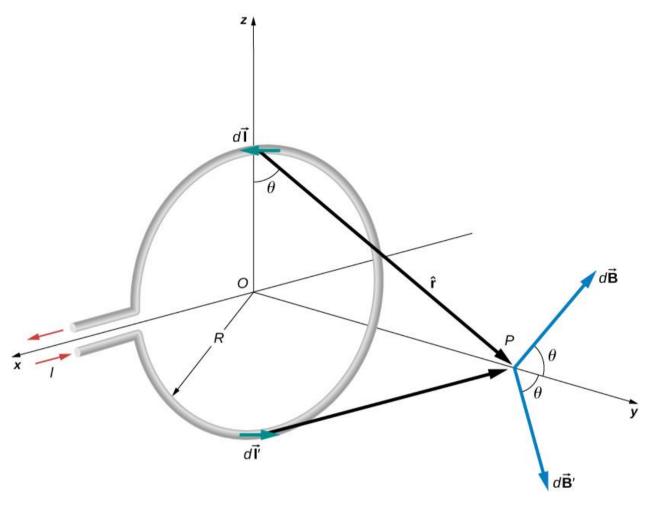


Figure 32

Let P be a distance y from the center of the loop. From the right-hand rule, the magnetic field dBat P, produced by the current element $I \ d \ l$ is directed at an angle θ above the y-axis as shown.

P point we know that because its infinitely long solenoid.

We know that dB is equal to:

$$dB = \frac{\mu}{4\pi} \cdot \frac{IdLxr}{r^2}$$
 10.2

$$r = y^2 + R^2$$

$$10.3$$

We know r is a unit vector so we can write it as 1 and according to the cross product we get:

$$dB = \frac{\mu}{4\pi} \cdot \frac{Idlsin\theta}{r^2}$$
 10.4

$$dB = \frac{\mu}{4\pi} \cdot \frac{Idlsin\theta}{y^2 + R^2}$$
 10.5

$$dB = \frac{\mu}{4\pi} \cdot \frac{Idl}{y^2 + R^2}$$
 10.6

Now we need to consider magnetic field dB due to the current element IdI so dB will be:

$$B = j \int_0^{2\pi} dB \cdot \cos\theta$$
 10.7

$$j\,\mu \frac{I}{4\pi} \int \frac{\cos\theta dl}{y^2} + R^2 \qquad \qquad 10.8$$

But we need to consider about one more thing that is the angle:

$$\cos\theta = \frac{R}{\sqrt[2]{y^2 + R^2}}$$
 10.9

So we get

$$B = j \,\mu \frac{IR}{4\pi^{\frac{3}{2}} \sqrt{y^2 + R^2}} \int dl$$
 10.10

$$\int dl = 2\pi r$$
10.11

Also we know that if we have a closed loop a magnetic dipole of moment

$$\mu = IAn$$
 10.12

$$A=\pi R^2$$
 and $j=n$
10.13

So according to those information magnetic field on the P will be

$$B = \mu 0\mu \frac{j}{2\pi (y^2 + R^2)^{\frac{3}{2}}}$$
10.14

We will use those equations to change the dl .When we put the μ in this equation we get

$$B = \mu 0 \frac{jIR^2N}{2L(y^2 + R^2)^{\frac{3}{2}}} dy$$
10.15

This integral by changing the independent variable from y to θ . From inspection of the figure which is blow , we have:

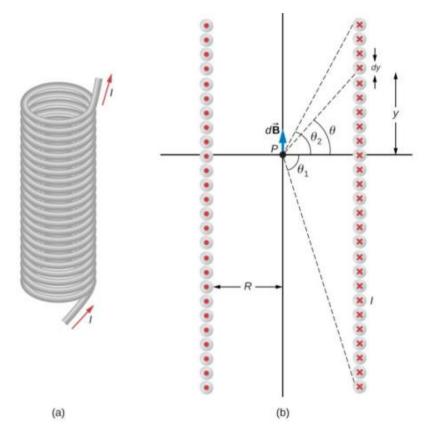


Figure 34

$$\sin \theta = \frac{y}{\sqrt{y^2 + R^2}}$$
10.16

Now we will take the derivative of both side and we get:

$$\cos \theta = -\frac{y^2}{y^2 + R^{2\frac{3}{2}}} + \frac{1}{\sqrt{y^2 + R^2}} dy$$
10.17

$$=\frac{R^2dy}{(y^2+R^2)^{\frac{3}{2}}}$$
10.18

When this is substituted into the equation for dB we will get:

$$B = \mu \frac{I0N}{2L} \int_{Q1}^{Q2} \cos\theta d\theta =$$
 10.19

$$B = \mu \frac{I0N}{2L} \cdot (\sin(\theta 2) - \sin(\theta 1))j$$
10.20

Behavior of the solenoid when **L** goes to *infinity* we can easily think that L \ge R. In that case according to this information we can say that $Q1 = -\frac{\pi}{2}$ and $Q2 = \frac{\pi}{2}$

So equation will be

$$B = \mu \frac{I0N}{2L} \cdot \left(\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right) \right)$$
j 10.21

$$B = \mu 0. \, n. \, I. \, j$$

RESULTS AND DISCUSSION

Part 1, Part 2 and Part 4)

In these parts of the project, we fabricated two solenoids and two toroids inductors. As it is known, fabrication of inductor is difficult in terms of winding process, so we were careful about it. After getting experimental value of the inductances, we calculated the inductance theoretically by using the formulas which is for the solenoid inductance and toroid inductance.

Types	Theoretic value of inductances with educated guess	Experimental value of inductances
Ferromagnetic solenoid	21,19mH	3.002 mH

Non-ferromagnetic solenoid	32.60 μΗ	12.60 μΗ
Ferromagnetic toroid	2,82743 uH	162.9 μΗ
Non-ferromagnetic toroid	14.01753 uH	36.72 μΗ

For the fabrication of ferromagnetic solenoid, we paid attention about the material we will use as a core should be ferromagnetic. So, we used iron bar, and it was attracted by the magnet. However, we are thinking that it is not a pure iron bar.

As it can be seen, there are differences between the theoretic and experimental values of inductances. For the ferromagnetic solenoid, the difference may be due to the structure of iron bar. The iron bar we used was painted and a little rusty. In the sources, the relative permeability of iron is $20 * 10^5$ but it has 99.95% pure. Since we did not think the bar is not pure iron, we made an educated guess, and it is nearly 100. By taking this value and write, we calculated theoretic value of inductance. It is a ferrite material but most probably it is mixture with some materials such as nickel zinc. This situation is also valid for ferromagnetic toroid. For the fabrication of toroid, we used scale which is attracted by the magnet. We determined its μr as nearly 1500 with educated guess and we calculated theoretically.

For the nonferromagnetic solenoid and toroid, since these are non-ferromagnetic material, our μr value is nearly 1.

These differences also may depend on windings irregularity and radius, length measurement margin of error.

Part 3)

In this part of the project, we were asked to design a circuit in order to measure the inductance values of solenoids and toroids and to measure the current passing through inductors. To measure current, by giving sine signal and applying KVL to the circuit, we obtained current equation.

First, we applied sine signal to pure inductive circuit that has AC ammeter in series. Then we saw the figure which is 12.5 mA. Then, from theoretical figure which is 374 mA, we get the susception that function generator has a resistance as 50 ohms. Then, from Figure 17, we designed a circuit on proteus to prove the signal generator has a resistance. We proved 0 ohms resistance in front of the inductor, and we applied sine signal to it. Then we saw the figure which is 12.5 mA. Our susception was correct.

After that, we obtained other current values. The current values are nearly close to each other because the impedance of the circuit is nearly 50 ohms.

To measure the inductance values, we generated RL circuits with different resistor values. We used oscilloscope to obtain our τ values by using cursors. Then we calculated L values.

Part 5A)

For this of the project, we were asked to calculate total inductance of a solenoid that has changing parameters such as relative magnetic permeability and density of the wires as function of space and time. First, we considered that to find total winding number we should integrate the function of the density of the wires through inductor (0 to L). after that the total inductance L(t) as a function of time we integrated inductance formula over z direction . Because we thought the contribution of each small segment.

Secondly, to find the voltage on inductor we used the formula 8.1. In a standard circuit with a constant inductor, Ohm's law takes the form $V = L \operatorname{di/dt}$, where V is the voltage across the inductor, L is the inductance, and $\operatorname{di/dt}$ is the rate of change of the current with respect to time. However, the inductance is not constant and instead changes with time so that we must arrange the formula. And we take both derivatives of the inductance and current as a function of time.

Finally, to calculate the voltage at an arbitrary point on the inductor as a space of time to obtain the inductance formula with respect to time and space we integrated the inductance over domain of time and space as multiple integrals. To express inductance formula we integrated L(z,t) over the desired range.

CONCLUSIONS

In conclusion, the fabrication and analysis of inductors, specifically cylindrical and toroidal configurations, have been explored in this project. The cylindrical inductors and toroidal inductors are components used in a wide range of electronic systems. The choice of core material, such as dielectric or ferromagnetic, significantly impacts the inductance and magnetic properties of the inductors. Through the design and fabrication process, important parameters including winding number, length, diameter, and core material permeability were considered. Experimental measurements of current and inductance were performed to evaluate the performance of the fabricated inductors. The comparison between theoretical and experimental results in the analysis of inductors allowed for the identification of any discrepancies and potential sources of error. Factors such as variations in wire thickness or core material properties could be evaluated to understand their impact on the performance of the inductors. Theoretical analysis using mathematical formulas provided insights into the behavior of inductors and facilitated the calculation of expected inductance values.

Having a theoretical foundation served as a reference point for comparison with the experimental measurements, enabling a deeper understanding of how the fabricated components performed. Hypothetical scenarios were also considered, which involved exploring the behavior of inductors under specific conditions.

By engaging in this project, a comprehensive understanding of inductor fabrication, measurement techniques, theoretical calculations, and analysis of hypothetical scenarios was achieved. This knowledge can be utilized to optimize the design and performance of inductors for various applications in the field of electronics.

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