Department of

Electrical & Electronics Engineering

Abdullah Gül University

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Project Report 1

EE1100 Computation and Analysis (COMA) Capsule

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OBJECTIVE

The purpose of this project is to understand what Archimedean spiral is otherwise known as an arithmetic spiral and what calculations we need to illustrate the spiral. In this paper we will learn to calculate position of angular velocity by using Cartesian coordinate system to understand and show how the spiral works. We aim to support these calculations with physical experiments.

BACKGROUND

The Archimedean Spiral was created by Archimedes in the third century B.C. It was used to make a better method to measure the area of a circle. Nevertheless, this was found not to be an appropriate way to measure the area when Archimedes determined a more accurate value of Pi. The spiral can be used to convert uniform angular motion into uniform linear motion. The Archimedean Spiral has different real-world applications such as scroll compressors and digital light processing (DLP).

ANALYTICAL AND SIMULATION PROCEDURES

Physics Part

The Archimedean Spiral is a spiral caused by a point moving away from a fixed point with constant speed along a line that turns with constant angular velocity. A classic example of this spiral in action in real life can be seen in vinyl players.

It can be described in polar coordinates by equation of

$$r = a + b\varphi$$

Equation of Archimedean Spiral in polar coordinates (eq 1)

with real numbers a and b.

Changing the parameters, a moves the center point of the spiral away from origin and b controls the distance between loops.

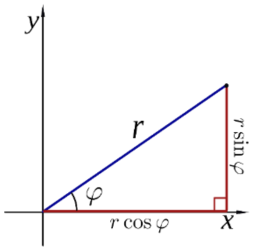
To derive mathematical equations for *x(t)* and *y(t)*, which are coordinates equations, and besides $v_x(t)$ and $v_y(t)$, velocity components for Archimedean spiral,

We can start with the cartesian coordinate system and polar coordinate system.

Polar Coordinate system in brief,

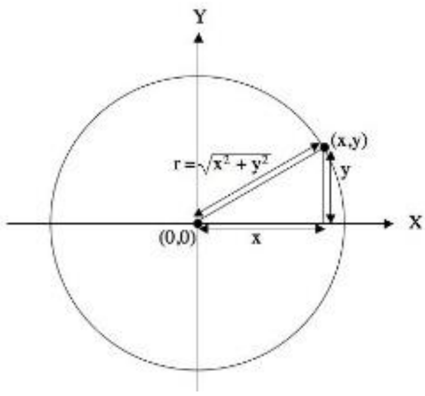
The polar coordinate system is a two-dimensional coordinate system in which each point on a plane is defined by a distance from a reference point and an angle from a reference direction.

Convert a Cartesian coordinate to polar coordinate,



In Cartesian a point has (x,y) coordinates, convert to polar we need to get distance between point and origin as r and angle that point makes with x-axis.

Relationship between Cartesian and polar coordinate systems (pic 2)



Magnitude of r (pic 2)

The position r is calculated by

$$r^2 = x^2 + y^2$$
$$r = \sqrt{x^2 + y^2}$$

And also, we can write $\sin\varphi$, $\cos\varphi$ and $\tan\varphi$ values as,

$$\sin\varphi = \frac{y}{r}$$

$$\cos\varphi = \frac{x}{r}$$

$$\tan\varphi = \frac{y}{x}$$

Now we can convert our polar equation of Archimedean spiral to cartesian equation as follows,

Step 1

First solve for x,

$$r = a + b\varphi$$

Let a be 0 to make our spiral start from origin

$$r = b\varphi$$

Step 2

We do algebraic manipulation by using the relationship between polar coordinates and Cartesian coordinates which was defined above,

$$r^2 = b^2\varphi^2$$

We can substitute r^2 with $x^2 + y^2$,

$$x^2 + y^2 = b^2\varphi^2$$

$$x^2 = b^2\varphi^2 - y^2$$

We can find y^2 by using trigonometric value

$$\sin\varphi = \frac{y}{r}$$

$$\text{r}\sin\varphi = y$$

$$r^2 \ (\sin\varphi)^2 = y^2$$

We know r^2 is equal to $b^2\varphi^2$ as well, then,

$$x^2 = b^2\varphi^2 - r^2 \ (\sin\varphi)^2$$

$$x^2 = b^2\varphi^2 - b^2\varphi^2 \ (\sin\varphi)^2$$

$$x^2 = b^2\varphi^2(1 - \ (\sin\varphi)^2)$$

We know $(\sin\varphi)^2 \underset{\text{Eq 3}}{+} (\cos\varphi)^2 = 1$ by using trigonometry

So, then we can substitute $(1 - \ (\sin\varphi)^2)$ with $(\cos\varphi)^2$

$$x^2 = b^2\varphi^2 \ (\cos\varphi)^2$$

$$x = |b\varphi\cos\varphi|$$

$$x = b\varphi\cos\varphi$$

At last, we found us x component in terms of cartesian coordinate system

Step 3

Solve for y,

$$r = b\varphi$$

$$r^2 = b^2\varphi^2$$

$$x^2 + y^2 = b^2\varphi^2$$

$$y^2 = b^2\varphi^2 - x^2$$

We can find x^2 by using trigonometric value

$$\cos\varphi = \frac{x}{r}$$

$$\mathrm{r}\cos\varphi = x$$

$$r^2(\cos\varphi)^2 = x^2$$

$$y^2 = b^2\varphi^2 - r^2(\cos\varphi)^2$$

$$y^2 = b^2\varphi^2 - b^2\varphi^2(\cos\varphi)^2$$

$$y^2 = b^2\varphi^2(1 - (\cos\varphi)^2)$$

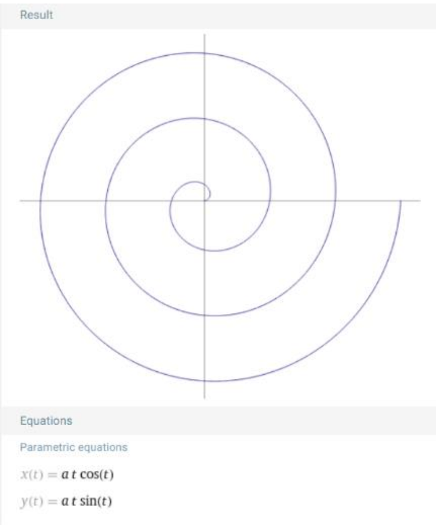
$$y^2 = b^2\varphi^2\,(\sin\varphi)^2$$

$$y = |b\varphi\sin\varphi|$$

$$y = b\varphi\sin\varphi$$

Finally, we found that y component in terms of Cartesian coordinate system

By using Wolfram Alpha, a computational machine which helps to plot functions, we can see our findings make this shape in cartesian coordinate system :



Solution in cartesian coordinate system pic 3

About velocity vectors, we can find them effortlessly by getting derivative of position vector, which means,

$$\vec{r} = (b\varphi\cos\varphi)\vec{\imath} + (b\varphi\sin\varphi)\vec{\jmath}$$

$$\frac{d\vec{r}}{d\varphi} = \vec{v} = (b\cos\varphi - b\varphi\sin\varphi)\vec{\imath} + (b\sin\varphi + \ b\varphi\cos\varphi)\vec{\jmath}$$

$$v_x = b\cos\varphi - b\varphi\sin\varphi$$

$$v_y = b\sin\varphi + b\varphi\cos\varphi$$

To write them with respect to time, we can use this method,

Imagine a particle moving on spiral, while it moves it rotates at certain angular displacement and it has an angular velocity which is ω,

$$\omega t = \ \varphi$$

Here φ is the angle that we scan, we can write as,

$$x(t) = b\omega t\cos\omega t$$

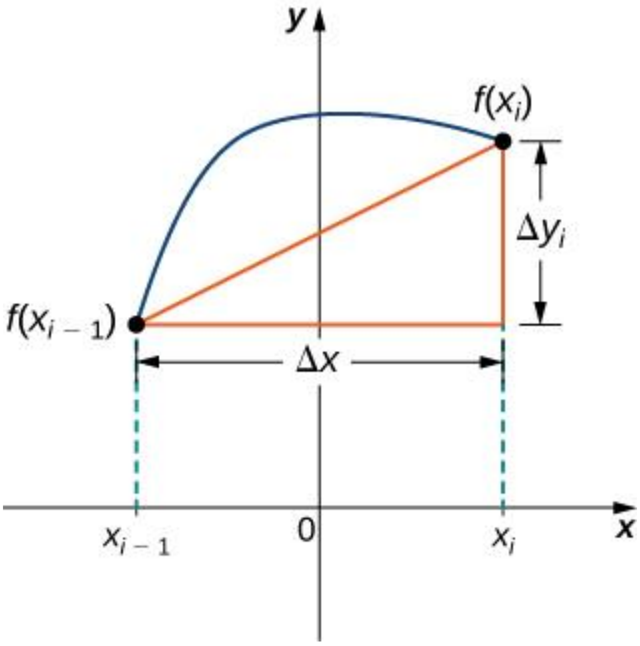
$$y(t) = b\omega t\sin\omega t$$

$$v_x(t) = b\cos\omega t - b\omega t\sin\omega t$$

$$v_y(t) = b\sin\omega t + b\omega t\cos\omega t$$

Arc Length

We can use this method to find arc length of Archimedean Spiral,



Describing arc length of function pic 4

From the Pythagorean Theorem, line part between $[x_{i-1}, x_i]$ is equal to

$$\sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}$$

By Mean Value Theorem, on the function there is a point, $x^* \in [x_{i-1}, x_i]$,

$$f'(x^*) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}} = \frac{\Delta y_i}{\Delta x_i},$$

Separate $(\Delta x_i)^2$ $as\ a\ factor,$

$$\sqrt{1 + \frac{(\Delta y_i)^2}{(\Delta x_i)^2}}\, \Delta x_i$$

Convert the equation above into,

$$\sqrt{1 + (f'(x^*))^2}\, \Delta x_i$$

Now adding up all lengths of all parts, we have

$$\sum_{i=1}^{n} \sqrt{1 + (f'(x^*))^2}\, \Delta x_i$$

By taking limit of this sum, while n goes to infinity, this is Reimann sum,

$$\lim_{n\to\infty}\sum_{i=1}^{n}\sqrt{1+(f'(\,x^{*}))^{2}}\ \Delta x_i$$

$$\downarrow$$

Finally, we found arc length formula : $\int_a^b \sqrt{1+\left(f'(x)\right)^2}\ dx$

So far, we have made calculations on normal equations, but our formula of Archimedean Spiral is parametric so we should manipulate our way into parametric equations as follows,

Step 1

Before to start, to find arc length of Archimedean Spiral we can use coordinate functions with respect to phi,

$$x = b\varphi\cos\varphi$$

$$x = g(\varphi)$$

$$y = b\varphi\sin\varphi$$

$$y = h(\varphi)$$

Now we can write them like,

$$y = h(\varphi) = f\big(g(\varphi)\big)$$

Take derivative of this equation with respect to t,

$$\frac{dy}{d\varphi} = h'(\varphi) = f'\big(g(\varphi)\big)g'(\varphi) \qquad (1)$$

We know that,

$$dx = g'(\varphi)\,d\varphi \qquad (2)$$

Use equation (2) into equation (1),

$$\frac{dy}{dt} = h'(\varphi) = f'\big(g(\varphi)\big)\frac{dx}{d\varphi}$$

We know this,

$$L = \int_a^b \sqrt{1+\left(f'(x)\right)^2}\ dx \quad (3)$$

Let us do the manipulation,

$$L = \int_{a}^{b} \sqrt{1 + \left(f'(x)\right)^2}\ dx$$

From this equation $\frac{dy}{d\varphi} = h'(\varphi) = f'(g(\varphi))\,\frac{dx}{d\varphi}$ *we can write* $f'(x) = f'\big(g(\varphi)\big)$

and also, we can write $dx = g'(\varphi)\ d\varphi$

By using these we get,

$$L = \int_{c}^{d} \sqrt{1 + \Big(f'\big(g(\varphi)\big)\Big)^2}\ g'(\varphi)\ d\varphi \quad \text{where } c \le \varphi \le d$$

Put g'(φ) into root,

$$L = \int_{c}^{d} \sqrt{(g'(\varphi))^2 + \Big(f'\big(g(\varphi)\big)\ g'(\varphi)\Big)^2}\ d\varphi$$

Replace g'(φ) with $\frac{dx}{d\varphi}$ *and* $f'\big(g(\varphi)\big)\ g'(\varphi)$ *with* $\frac{dy}{d\varphi}$ *(from (1))*

$$L = \int_{c}^{d} \sqrt{\left(\frac{dx}{d\varphi}\right)^2 + \left(\frac{dy}{d\varphi}\right)^2}\ d\varphi \quad (4)$$

Step 2

So now put all appropriate equation in (4) to find arc length of Archimedean Spiral

Like as follows,

$$x = b\varphi\cos\varphi$$

$$x = g(\varphi)$$

$$y = b\varphi\sin\varphi$$

$$y = h(\varphi)$$

$$\frac{dx}{d\varphi} = g'(t) = b\cos\varphi - bt\sin\varphi$$

$$\frac{dy}{d\varphi} = h'(\varphi) = b \sin\varphi + b\varphi \cos\varphi$$

$$\left(\frac{dx}{d\varphi}\right)^2 = b^2(\cos\varphi)^2 + b^2\varphi^2(\sin\varphi)^2 - 2b^2\varphi\cos\varphi\sin\varphi$$

$$\left(\frac{dy}{d\varphi}\right)^2 = b^2(\sin\varphi)^2 + b^2\varphi^2(\cos v)^2 + 2b^2\varphi\cos\varphi\sin\varphi$$

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = b^2(1+\varphi^2)$$

$$L = \int b\sqrt{1+\varphi^2}\ dt$$

$$L = b\int \sqrt{1+\varphi^2}\ dt$$

Now we just need to calculate the integral,

Step 1

$$\textit{Let us take}\ \ \varphi = tan\alpha$$

$$d\varphi = (\sec\alpha)^2 d\alpha$$

$$1+\varphi^2 = 1 + (\tan\alpha)^2 = (\sec\alpha)^2$$

$$b\int \sqrt{(\sec\alpha)^2}\ (\sec\alpha)^2 d\alpha$$

$$b\int (\sec\alpha)^3\ d\alpha$$

After now, we use the method that is integration by parts,

(Let us put b aside, and multiply the integral with b at the end of the calculation, we can do that just because b is just a constant)

$$\int (\sec\alpha)^3\ d\alpha$$

$$u = \sec\alpha \qquad\qquad dv = (\sec\alpha)^2$$

$$dv = \sec\alpha\tan\alpha \qquad\qquad v = \tan\alpha$$

$$\int (\sec\alpha)^3\ d\alpha\ = \sec\alpha\tan\alpha - \int \sec\alpha\ (\tan\alpha)^2 d\alpha$$

$$\int (\sec\alpha)^3\, d\alpha \;\; = \sec\alpha\tan\alpha - \int \sec\alpha\;\; ((\sec\alpha)^2 - 1)^2 d\alpha$$

$$\int (\sec\alpha)^3\, d\alpha \;\; = \sec\alpha\tan\alpha - \int (\sec\alpha)^3 - \sec\alpha\, d\alpha$$

$$\int (\sec\alpha)^3\, d\alpha \;\; = \sec\alpha\tan\alpha - \int (\sec\alpha)^3 d\alpha + \int \sec\alpha\, d\alpha$$

$$2\int (\sec\alpha)^3\, d\alpha \;\; = \sec\alpha\tan\alpha + \int \sec\alpha\, d\alpha$$

Right now, we need to calculate this integral $\int \sec\alpha\, d\alpha$ *to get the answer,*

$$\int \sec\alpha\, d\alpha$$

$$\int \sec\alpha\; \frac{\sec\alpha + \tan\alpha}{\sec\alpha + \tan\alpha} d\alpha$$

$$\int \frac{(\sec\alpha)^2 + \sec\alpha\tan\alpha}{\sec\alpha + \tan\alpha} d\alpha$$

$$\sec\alpha + \tan\alpha = k$$

$$((\sec\alpha)^2 + \sec\alpha\tan\alpha) = dk$$

$$\int \frac{1}{k}\, dk$$

And that is equal to,

$$\int \frac{1}{k} dk = \; \ln|k| + C$$

$$\int \sec\alpha\, d\alpha = \ln|\sec\alpha + \tan\alpha| + C$$

So, we can use this in $2\int (\sec\alpha)^3\, d\alpha \;\; = \sec\alpha\tan\alpha + \int \sec\alpha\, d\alpha$

$$L = \int (\sec\alpha)^3\, d\alpha = \frac{1}{2}(\sec\alpha\tan\alpha + \ln|\sec\alpha + \tan\alpha|)$$

From any right triangle we can found sec α and tan α values,

We wrote this $t = tan\alpha$ *already,*

And sec α = $\sqrt{1 + \varphi^2}$

Step 2

The length of the arc: r = bφ, is given

$$L(\theta) = \left[\frac{1}{2}b\left[\sqrt{1+\varphi^2}\,\varphi + \ln\left|\sqrt{1+\varphi^2}+\varphi\right|\right]\right]\begin{matrix}\theta_1\\ \theta_2\end{matrix}$$

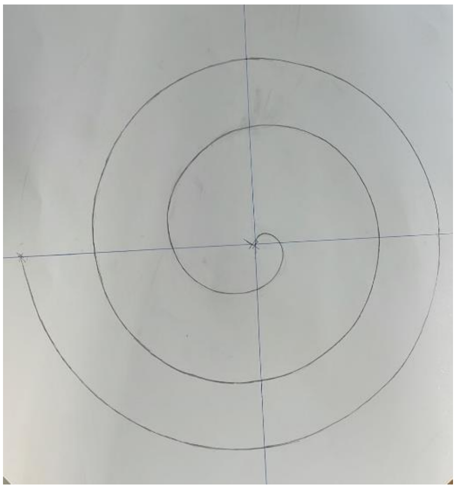
(From θ_1 to a θ_1)

This is our arc length formula for the Archimedean Spiral.

About comparing results of experimental and theoretical length of the Archimedean Spiral,

We can find the length of Archimedean Spiral from to 0 to 2 π by using this formula,

$L(\theta) = \left[\frac{1}{2}b\left[\sqrt{1+\varphi^2}\,\theta + \ln\left|\sqrt{1+\varphi^2}+\varphi\right|\right]\right]$ *(1)*



But we should find b as well,

It can be found from $r = a + b\varphi$,

As seen from the picture a = 1 because it starts at the origin,

For b, first turn to $\frac{\pi}{2}$; $r = 1.9$

$$1.9 = b\frac{\pi}{2}$$

So then, $b = 1{,}209577$

Now if we put the 2π into the formula (1),

We found *25,711122 cm theoretically.*

Picture of our Archimedean Spiral pic 5

For the experimental part, we used a cable to measure the length, and we found 27 cm.

About the percentage error of this experiment,

$$\text{Percentage error} = \frac{|V_{true} - V_{observed}|}{V_{true}} \times 100$$

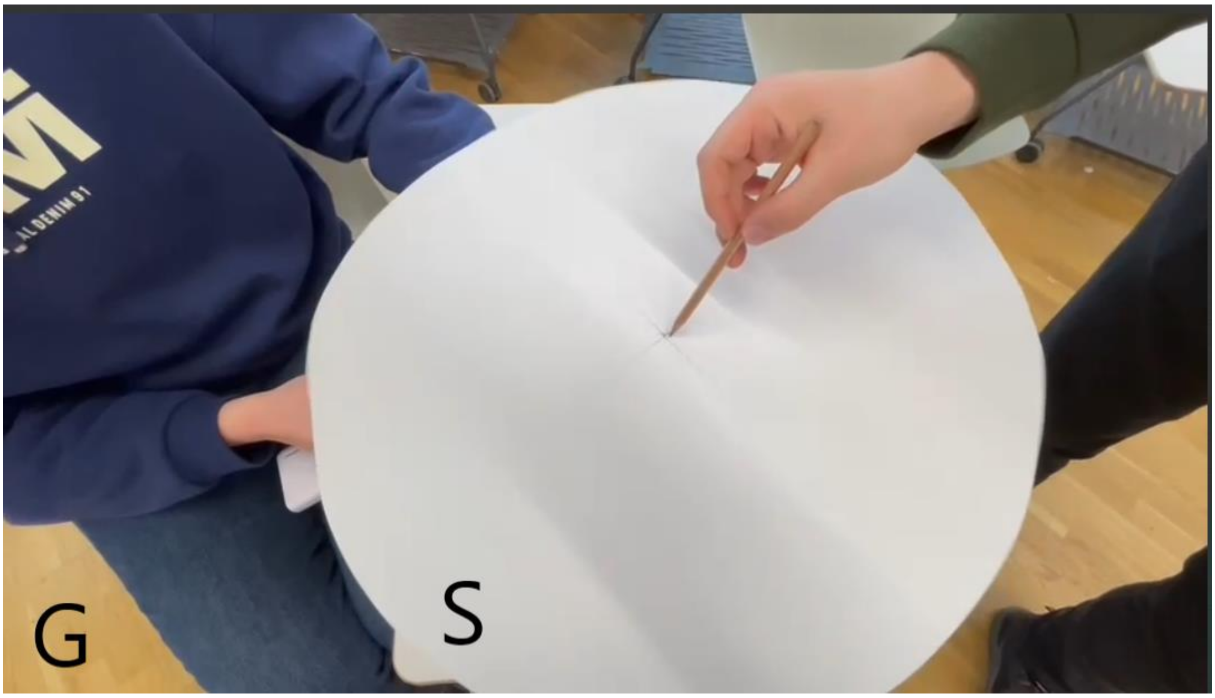
$$\frac{|27 - 25{,}711122|}{27} \times 100$$

$= 4{,}77362$ This is the percentage error.

Calculating arc lengh of spiral pic 6

Motion in different frame of reference on Archimedean Spiral

Starting by labeling different frame of references



Showing different frames from spectator view pic 7

G - Ground frame of reference

S – Archimedean Disk frame of reference

Suppose a camera mounted in table which is spinning, and a spectator watching from G frame of reference. From S frame, the table is not spinning. While the pencil goes in straight line from G frame, camera sees the pen as spinning from S frame, it is like at the same time intervals the pencil passing through camera view.

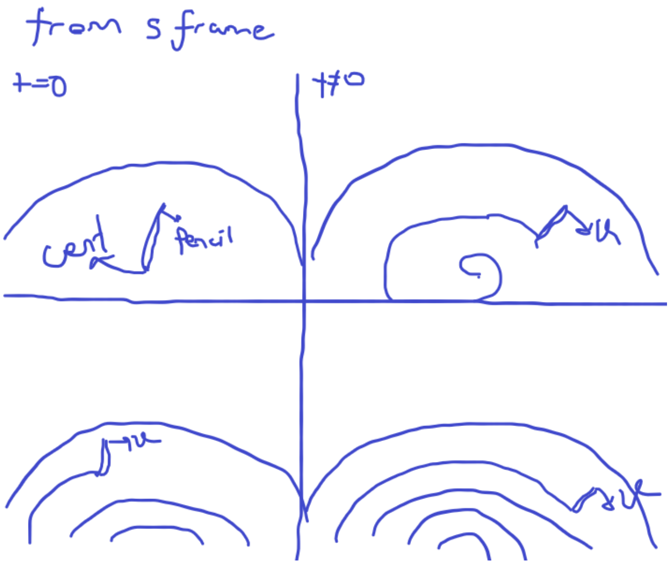
We can say something about shape of trajectories,

From G frame, pencil's trajectory is a straight line

from G frame

pencil's trajectory → straight line

From S frame, disk is stationary. However, pencil is spinning,



(cent – center)

Linear Algebra Part

Part 1

We can write the equations which are,

$$x(t) = b\omega t \cos\omega t$$

$$y(t) = b\omega t \sin \omega t$$

$$v_x(t) = b \cos \omega t - b\omega t \sin \omega t$$

$$v_y(t) = b \sin \omega t + b\omega t \cos \omega t$$

as in matrix equation form (Ax = b).

$$\begin{bmatrix} \cos\omega t & 0 \\ -\sin \omega t & \cos\omega t \end{bmatrix} \begin{bmatrix} b\omega t \\ b \end{bmatrix} = \begin{bmatrix} x \\ v_x \end{bmatrix}$$

$$\begin{bmatrix} \sin \omega t & 0 \\ \cos\omega t & \sin \omega t \end{bmatrix} \begin{bmatrix} b\omega t \\ b \end{bmatrix} = \begin{bmatrix} y \\ v_y \end{bmatrix}$$

$$\begin{bmatrix} \cos\omega t & 0 \\ -\sin \omega t & \cos\omega t \\ \sin \omega t & 0 \\ \cos\omega t & \sin \omega t \end{bmatrix} \begin{bmatrix} b\omega t \\ b \end{bmatrix} = \begin{bmatrix} x \\ v_x \\ y \\ v_y \end{bmatrix}$$

Part 2

Lengths of resultant vectors for coordinate and velocity can be found at $\omega t = \frac{\pi}{2}$ as following,

We know ωt= φ, substitute ωt with φ,

For coordinate,

$$x \rightarrow \frac{b\pi}{2}\cos\frac{\pi}{2} = 0$$

$$y \rightarrow \frac{b\pi}{2}\sin\frac{\pi}{2} = b\frac{\pi}{2}$$

$$|\vec{r_c}| = \sqrt{\left(\frac{b\pi}{2}\right)^2} = \frac{b\pi}{2}$$

For velocity,

$$v_x \rightarrow b\cos\frac{\pi}{2} - \frac{b\pi}{2}\sin\frac{\pi}{2} = \frac{-b\pi}{2}$$

$$v_y \rightarrow b\sin\frac{\pi}{2} + \frac{b\pi}{2}\cos\frac{\pi}{2} = b$$

$$|\vec{r_v}| = \sqrt{\left(\frac{-b\pi}{2}\right)^2 + b^2} = \frac{b}{2}\sqrt{5+\pi^2}$$

Part 3

Evaluating the system in terms of having solution,

$$\begin{bmatrix} \cos\omega t & 0 \\ -\sin\omega t & \cos\omega t \end{bmatrix} \begin{bmatrix} b\omega t \\ b \end{bmatrix} = \begin{bmatrix} x \\ v_x \end{bmatrix}$$

Write augmented matrix for this system,

$$\begin{bmatrix} \cos\omega t & 0 & x \\ -\sin\omega t & \cos\omega t & v_x \end{bmatrix}$$

For purposes of evaluation, we should create an upper triangular matrix as follows, our l_{21} is $\frac{-\sin\omega t}{\cos\omega t}$

$$\begin{bmatrix} \cos\omega t & 0 & x \\ -\sin\omega t & \cos\omega t & v_x \end{bmatrix}$$

$$\begin{bmatrix} \cos\omega t & 0 & x \\ 0 & \cos\omega t & v_x + x \tan \omega t \end{bmatrix}$$

Now back substitution,

$$b \cos \omega t = v_x + x \tan \omega t$$

We know what v_x is,

$$b \cos \omega t = b \cos \omega t - b\omega t \sin \omega t + x \tan \omega t$$

$$b\omega t \sin \omega t = x \frac{\sin \omega t}{\cos \omega t}$$

Finally, we cancel $\sin \omega t$ then we have,

$$b\omega t \cos \omega t = x$$

We also know what x is, $b\omega t\cos\omega t$,

$$b\omega t \cos \omega t = b\omega t \cos \omega t$$

$$1 = 1$$

System has infinitely many solutions. Another matrix which is $\begin{bmatrix} \sin \omega t & 0 \\ \cos\omega t & \sin \omega t \end{bmatrix} \begin{bmatrix} b\omega t \\ b \end{bmatrix} = \begin{bmatrix} y \\ v_y \end{bmatrix}$ has infinitely many solutions as well.

RESULTS AND DISCUSSION

In the project, we have calculated equations regarding the Archimedean Spiral. We have understood the nature of Archimedean Spiral.

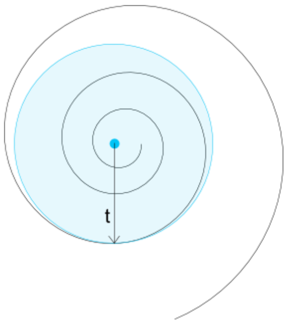
On Theoretical Part

For instance,

For uniform circular motion, we know that,

$$\omega = \frac{2\pi}{T}$$

T is period, how much time take to get one revolution,



So, for the Archimedean Spiral, we do not have constant radius for example,

For every point on this spiral, they have unique circle that makes different situations.

Let us look at this,

In terms of radius, we know that $2\pi r$ circumference of a circle, on the Archimedean spiral we have,

(r is radius)

$$r = 2\pi b\varphi$$

(General formula for radius of every circle on the Archimedean Spiral)

Additionally,

$$\omega = \frac{v}{r}$$

$$\omega = \frac{|v|}{r}$$

We know $|v|$ is equal to $\sqrt{b^2 + b\varphi^2}$ from previous calculations and also we know $r = b\varphi$

$$\omega = \frac{\sqrt{b^2 + b\varphi^2}}{b\varphi}$$

Limit of this equation when φ goes to infinity, it approaches to 1, so there is not big change in angular velocity, then we are able to assume that angular velocity as almost constant.

From this $|v| = \sqrt{b^2 + b\varphi^2}$ we can understand velocity is increasing, that means for every $\frac{\pi}{2}$ part will be passed different time values.

About the arc length for general when ($r = a + b\varphi$) a is equal to 0 and b is equal to 1.

φ	Arc Length	Velocity	Angular Velocity	Time
$\frac{\pi}{2}$	2.0792	1.8621	1.1854	1,116
π	6.1099	3.2969	1.0494	1,853
$\frac{3\pi}{2}$	12.4778	4.8173	1.0222	2,590
2π	21.2563	6.3623	1.0125	3,340
$\frac{5\pi}{2}$	32.4706	7.9174	1.0080	4,101

Previously we supposed that angular velocity would be almost constant.

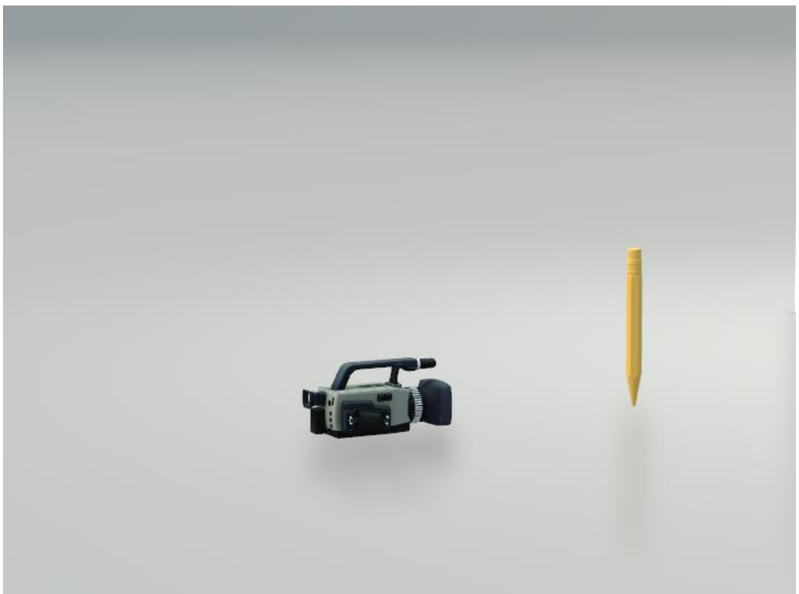
On Linear Algebra Part

In the project Ax=b for is wanted. So, while we are doing this we will use a matrix multiplication. As we know in matrix multiplication first we start the multiplication from row one and after we finish the multiplication of row one with each column we pass the other rows and make the same operations on other rows. Moreover, when we want to write Ax=b matrix form we will use this matrix multiplication rule but first to write matrix in this form we have to find the x part. About the part 2, we calculated resultant vectors for both coordinate and velocity at $\varphi = \frac{\pi}{2}$.

Finally, we evaluated our matrix system. First we made augmented matrix. Then by using elimination method to procure upper triangular matrix. Thirdly, with back substitution we found $b\cos\omega t = v_x + x\tan\omega t$. By using algebra, previously known values of x and v_x, we concluded that this matrix system has infinitely many solutions. For the another matrix, we concluded the same result.

On Motion Part

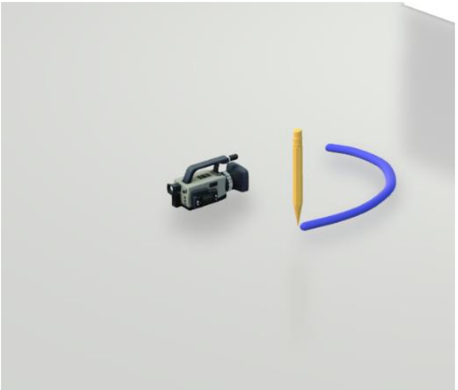
To understand the motion on the Archimedean Disc, we created a Thought Experiment. We supposed a camera mounted on the disc recording a video, while disc spinning.



From the disc frame, for the camera the ground would be disc itself. So, the camera is recording the video which has a view from stationary disc. We could understand disc is not spinning from disc frame.

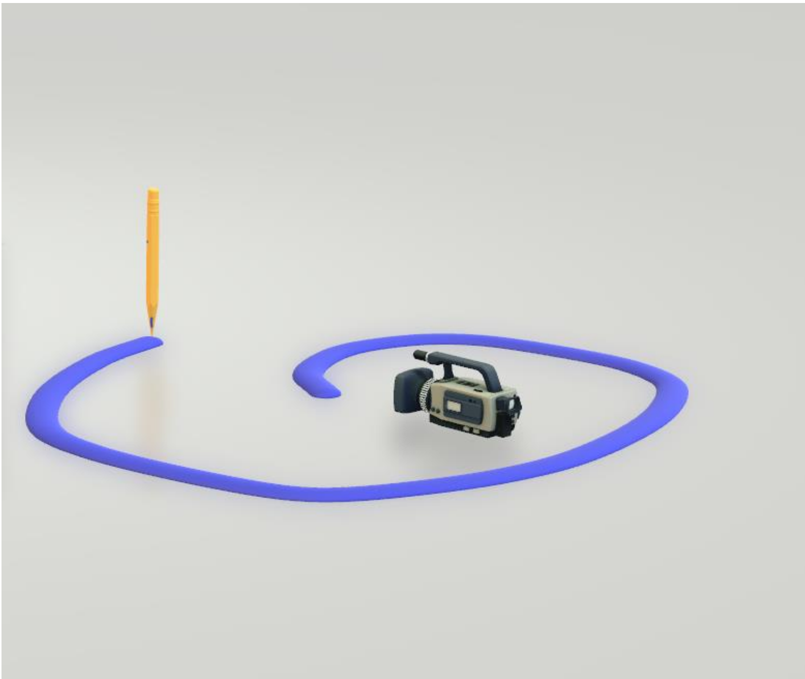
Then, the pencil started to move.

(Ground is disc)(pic1)



From the disc frame, the camera saw the pencil was moving.

(pic2)



After a while, from the disc frame, the camera saw first the pen was lost. Then, the pencil came from the other side. As a result, from the disc frame camera saw the trajectory of pencil was spiral.

However, from spectator frame, results are different.

(pic3)

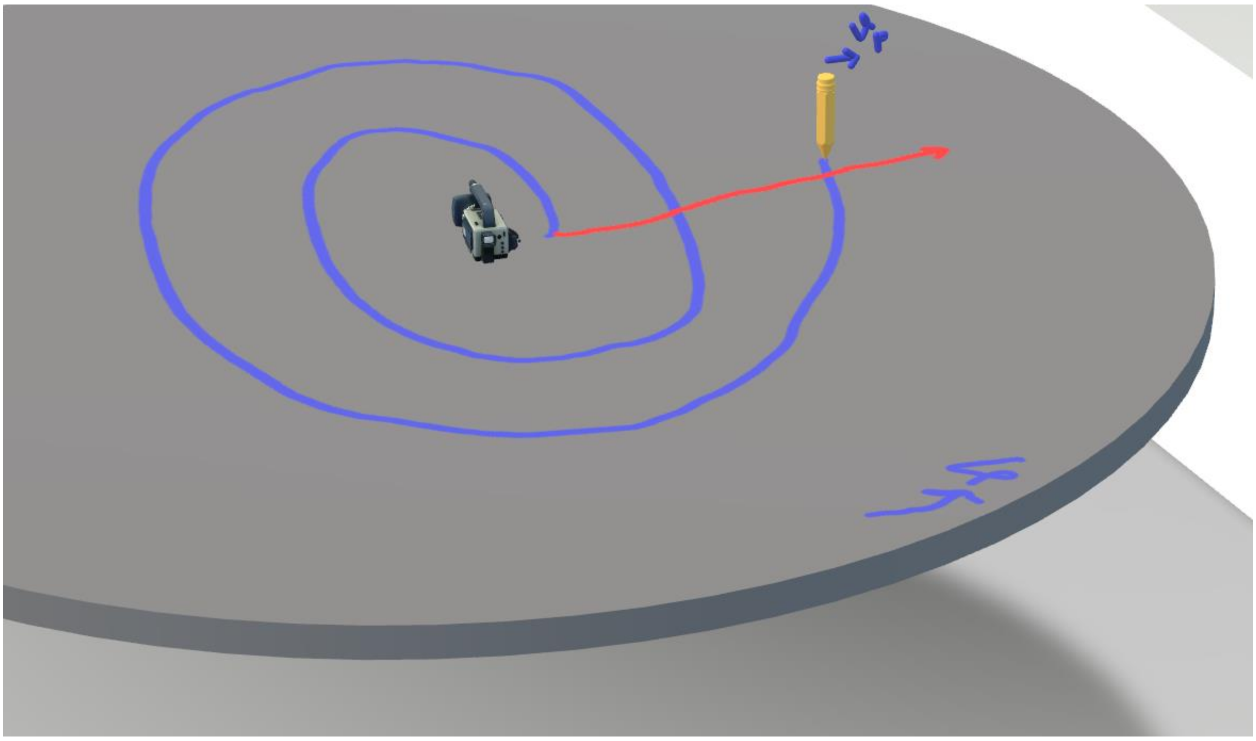
From ground view, a Spectator watched the camera and the pen. The spectator was not on the disc.

So, the view is like this,

From the ground view, spectator saw this system like on picture.

Pen was driven on a straight line, so pencil's trajectory is straight line

(pic4)



(pic5)

While both pencil was moving and disc were spinning, spectators saw their trajectories like in the picture 5.

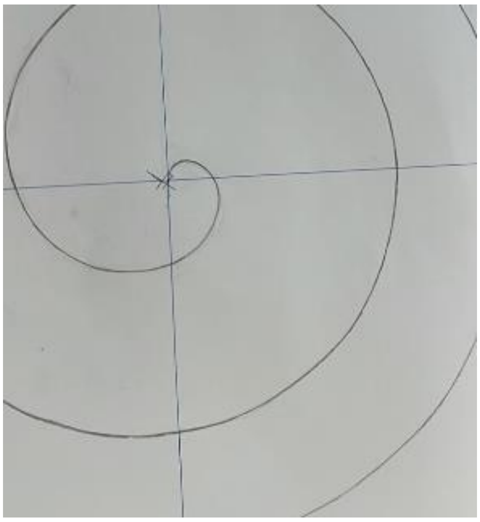
On Experimental Part

Process of preparation

We need some essentials to draw an Archimedean spiral. A round piece of cardboard to draw on, a pencil, a motor that we know has constant velocity, a thread that is long enough to measure the length of the spiral, and a battery to power the motor.

Firstly, we draw a Cartesian coordinate system on the board. To draw it correctly, we used two rulers, placed them perpendicular next to each other, and drew x-y lines. We need to glue the motor to our cardboard, and we need to make sure it is exactly in the center of it. After we stabilized the motor, all we needed to do was draw a straight line on the board and record it to put it in our project as requested. After we started the engine, we drew a straight line starting from the center while the motor was working, which means the board was turning at a constant speed. As it was asked in our project, we took a thread (we used wire) to measure the length of the Archimedean spiral we just drew. Finally, we took the thread and measured it with a ruler.

About comparison between the theoretical and experimental results,



We wanted our spiral to start from origin, so from the formula of $r = a + b\varphi$ we could understand a is equal to 0. Then we tried to find the value of b.

We measured the distance from the origin to the intersection point which is the first intersection with the y-axis because when spiral made it's the first right angle it interested with y-axis.

Then from the formula of

$$r = b\varphi$$

b was *1,209577.*

By using arc length formula of the Archimedean Spiral, we found theoretically (from 0 to 2π) *25,711cm.*

Nevertheless, the experimental result was *27 cm.* There is 4,77 % error.

Many reasons may create this error percentage. For instance, while drawing we should have used much stronger cardboard because when pencil was driven on, it made a pressure on cardboard. So, the pressure caused the cardboard to curve downward. As a result, it altered to result. Another instance, drawer could not forward the pencil properly, it can be seen on the video that attached to the reference part.

CONCLUSIONS

In this project we compared the experimental and theoretical information. As we realized the calculations we made do not match our experiment results. There is always an error when you compare results of physical experiments made in real life and theoretical calculations. It was a learning project for our group. We developed our skills such as working as a group and doing research.

REFERENCES

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