Department of

Electrical & Electronics Engineering

Abdullah Gül University

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Project Report 2

EE1100 Computation and Analysis (COMA) Capsule

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OBJECTIVE

The objective of this project is to study the motion of a ballistic pendulum and to analyze the physical principles governing its behavior. A ballistic pendulum is a simple mechanical system consisting of a rigid rod or bar that is suspended from a pivot point, and a mass that is suspended from the end of the rod. When the mass is released, it swings back and forth under the influence of gravity, and the pendulum's motion can be described by a set of simple differential equations. By studying the motion of the pendulum, we can learn about the principles of conservation of energy and momentum, as well as the role of friction and other forces in determining the pendulum's behavior. Through experimentation and analysis, we will seek to understand how the pendulum's motion is influenced by factors such as the mass of the pendulum, the length of the rod, the initial release angle, and the presence of external forces such as air resistance

BACKGROUND

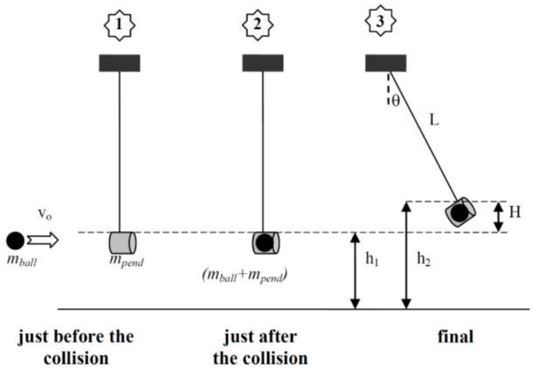
A ballistic pendulum is a device used to measure the velocity of a projectile, such as a bullet. It consists of a heavy mass suspended from a fixed point by a low-friction support, such as a wire or pivot. When a projectile is fired into the pendulum, it becomes embedded in the mass and causes it to swing upward. By measuring the height to which the pendulum swings and the time it takes to complete its motion, the velocity of the projectile can be calculated using the equations of motion.

ANALYTICAL AND SIMULATION PROCEDURES

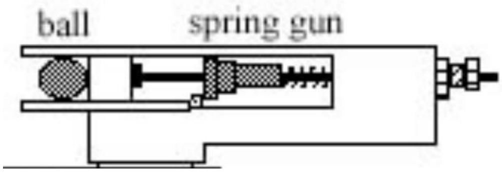
Physics Part

Due to the fact that the collision is inelastic, we cannot accurately derive the kinetic energy of the pendulum after the collision with the kinetic energy of the ball before the swing. Nevertheless, momentum is conserved in all collision types. Then we know the relation between spring gun and ball, so we can know the momentum of ball before the collision. So, we are able to determine the initial velocity.

Part 1 : Step of the collision



Picture 1



To give the initial velocity to the ball, we use a spring gun. We can use the principle of conservation of energy. The ball gains some kinetic energy due to elastic potential energy from the spring gun.

Picture 2

Suppose spring has k as spring constant and there is no friction inside of the gun.

Our elastic potential energy formula with respect to different compression level as x in meters and the ball has a mass as m in kilograms:

$$U_P = \frac{1}{2}kx^2 \tag{1.0}$$

This will be converted into kinetic energy of the ball:

$$U_P = K_{ball} \tag{1.1}$$

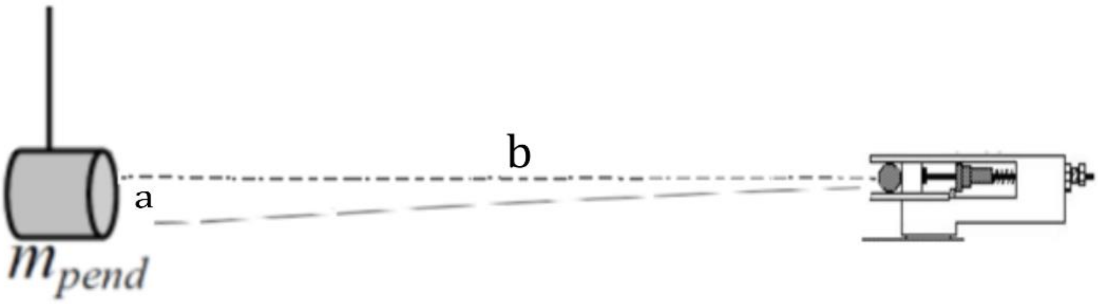
$$\frac{1}{2}kx^2 = \frac{1}{2}m_{ball}{V_0}^2 \tag{1.2}$$

V_i is initial velocity. We can write initial velocity as follows,

$$V_0 = \sqrt{\frac{kx^2}{m_{ball}}}$$

(1.3)

Now we can focus on the travel of the ball to the ballistic pendulum. During the travel, due to the gravitational pull may cause deviation from the original course to the pendulum, suppose the wire has no mass and there is no air friction.



Picture 3

Let us call deviation distance a and distance between gun and pendulum b. Initially the mechanical energy of the ball is

$$E_i = K_{ball} + U_{ball} = \frac{1}{2} m_{ball} {V_0}^2 + m_{ball} g h_1$$

(2.0)

Before the moment of collision, due to the deviation mechanical energy will be,

$$E_f = \frac{1}{2} m_{ball} {V_0}^2 + m_{ball} g (h_1 - a)$$

(2.1)

The ball has velocity in only one direction which is x axis, so it makes horizontal projectile motion, we can derive the time,

$$b = V_0 t$$

(2.2)

$$\frac{b}{V_0} = t$$

(2.3)

To achieve conservation of the mechanical energy, we should keep the distance b as short as possible because deviation a is equal to $a = \frac{1}{2}gt^2$ when b goes to 0 $\left(\frac{b}{V_0} = t\right)$ t will approach to 0, so the deviation will be equal to 0.

So then initial and final mechanical energy,

$$E_i = E_f$$

(2.4)

Furthermore, the initial momentum of the ball is:

$$P_i = m_{ball} V_0$$

(3.0)

And we can suppose final momentum of the ball (after the moment of the collision)

$$P_f = (m_{ball} + m_{pend}) V_x$$

(V_x is the velocity that is result from the collision)

(3.1)

While the ball is travelling to the pendulum, gravitation exerts a force that may make impulse, as a result initial and final may not be equal,

$$J = P_f - P_i$$

(3.2)

$$J = F\Delta t = mg\left(\frac{b}{V_0}\right)$$

(3.3)

In addition, if we keep the distance between ball and pendulum very short as well, impulse will be negligible, so we can now write the conservation of the momentum

$$J = P_f - P_i$$

(3.4)

$$0 = P_f - P_i$$

(3.5)

$$P_i = P_f$$

(3.6)

$$m_{ball}V_0 = (m_{ball} + m_{pend})V_x$$

(3.7)

$$V_x = \frac{m_{ball}}{(m_{ball} + m_{pend})}V_0$$

(3.8)

After the very moment of the collision, we can the conservation of mechanical energy as follows,

$$K_i = \frac{1}{2}m_{ball}{V_0}^2$$

(4.0)

At the same time ball embedded in the pendulum and the pendulum gains velocity V_x

$$K_{i(pend)} = \frac{1}{2}(m_{pend} + m_{ball}){V_x}^2$$

(4.1)

$$K_{i(pend)} = \frac{1}{2}(m_{pend} + m_{ball})\left(\frac{m_{ball}}{(m_{ball} + m_{pend})}\right)^2 V_0{}^2$$

(4.2)

The ratio of final to initial kinetic energy is,

$$\frac{K_{i(pend)}}{K_i} = \frac{m_{ball}}{(m_{ball} + m_{pend})}$$

(4.3)

The collision in which the total kinetic energy after the collision is less than before the collision is called an inelastic collision. The right side is always less than unity because the denominator is always greater than the numerator. During the collision, kinetic energy cannot be conserved.

Part 2 : Step of the deviation of the pendulum from the equilibrium to the height H

During the levitation to the point of H, the mechanical energy of the embedded mass, ball and pendulum, will be conserved due to the fact that forces acting on this system are conservative energies.

After the very moment of the collision, we can write the mechanical energy,

$$E_i = \frac{1}{2}(m_{pend} + m_{ball})V_x{}^2 + (m_{pend} + m_{ball})gh_1$$

(5.0)

Then at the moment of H, velocity of the system will be zero, then kinetic energy will be zero. System has only potential energy.

$$E_f = (m_{pend} + m_{ball})gh_2$$

(5.1)

Due to the principle of conservation of energy, E_i and E_f will equal.

$$E_i = E_f$$

(5.2)

$$\frac{1}{2}(\mathrm{m_{pend}} + \mathrm{m_{ball}}){\mathrm{V_x}}^2 + (\mathrm{m_{pend}} + \mathrm{m_{ball}})gh_1 = (\mathrm{m_{pend}} + \mathrm{m_{ball}})gh_2$$

(5.3)

$$\frac{1}{2}(\mathrm{m_{pend}} + \mathrm{m_{ball}}){\mathrm{V_x}}^2 = (\mathrm{m_{pend}} + \mathrm{m_{ball}})g(h_2 - h_1)$$

($h_2 - h_1$= H) (5.4)

$$V_x = \sqrt{2gH}$$

(5.5)

Now we can write H in terms of x, which is displacement, we know V_x from the equation (3.8)

$$\mathrm{V_x} = \frac{\mathrm{m_{ball}}}{(\mathrm{m_{ball}} + \mathrm{m_{pend}})}\mathrm{V_0}$$

$$H = \frac{1}{2g}\left(\frac{\mathrm{m_{ball}}}{(\mathrm{m_{ball}} + \mathrm{m_{pend}})}\mathrm{V_0}\right)^2$$

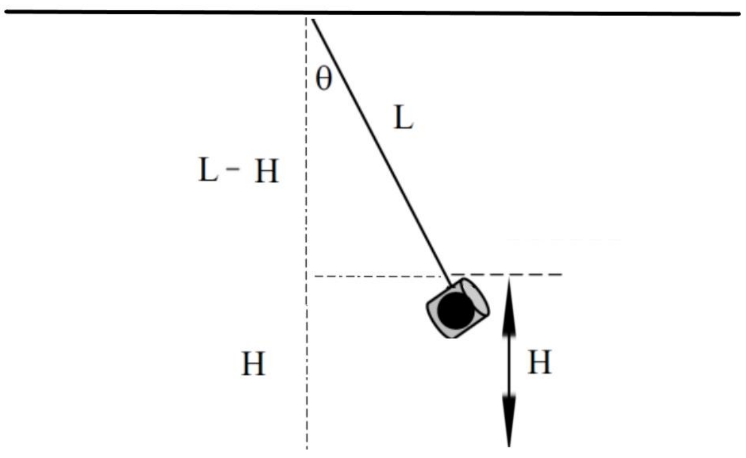
(5.6)

We can also write $\mathrm{V_0}$ in terms of H,

$$\mathrm{V_0} = \frac{\sqrt{2gH}(\mathrm{m_{ball}} + \mathrm{m_{pend}})}{\mathrm{m_{ball}}}$$

(5.7)

Additionally, we can write H and find angle θ,



Picture 4

We can write cosine θ as follows,

$$\cos\theta = \frac{L - H}{L}$$

(6.0)

We can arrange this expression,

$$H = L - L\cos\theta$$

(6.1)

From the statement (5.8) we can write θ as a function of the displacement of spring gun,

$$\theta = \cos^{-1}(\frac{L - H}{L})$$

(6.2)

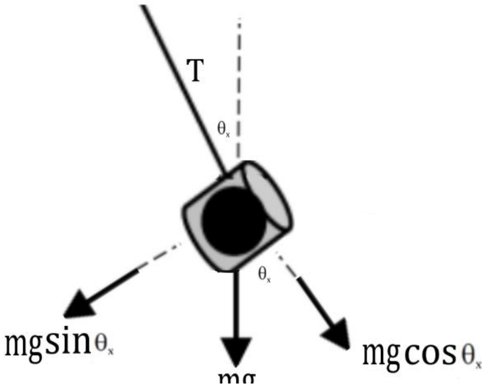
We know H from,

$$\theta = \cos^{-1}(1 - \frac{\left[\frac{1}{2g}\left(\frac{\mathrm{m_{ball}}}{(\mathrm{m_{ball}}+\mathrm{m_{pend}})}\mathrm{V_0}\right)^2\right]}{L})$$

(6.3)

About the conservation of momentum, during the travel to the equilibrium of height H, due to the fact that there are some forces acting on system. Now we can start for the conservation of momentum by examining system from the view of free body diagram at any time,

For any system, the forces that the particles of the system exert on each other are called internal forces. Forces exerted on any part of the system by some object outside it are called external forces. For the system shown in picture 5, tension of the wire is in equilibrium with the $mg\cos\theta_x$, but for the other axis $mg\sin\theta_x$ is a force which is external force, and there is no an opposite to $mg\sin\theta_x$ as a result the momentum is not conserved. (m is the total mass of the system)



Picture 5

Furthermore, it could have been seen from initial and final momenta of the system we can use equations from *(3.1)* (this statement turns out to be initial momentum for the system after the collision)

$$P_i = (\mathrm{m_{ball}} + \mathrm{m_{ball}})\mathrm{V_x}$$

(7.0)

And the velocity of system at the point H will be zero, so final momentum is zero,

$$P_f = 0$$

(7.1)

Final and initial momenta are not equal, so the momentum is not conserved.

During the deviation, $mg\sin\theta_x$ exerts a force , which varies with time, on the system, that makes the impulse.

$$\mathrm{J} = \mathrm{P_f} - \mathrm{P_i}$$

(7.2)

$$\mathrm{J} = 0 - \mathrm{P_i}$$

(7.3)

$$\mathrm{J} = -\mathrm{P_i}$$

(7.4)

As a result, during the deviation of the pendulum from the equilibrium to the height H, momentum is not conserved.

Linear Algebra Part

Construct the matrix Ax=b to calculate H

Initially, we use the formula (5.5),

$$2gH = \left(\frac{\mathrm{m_{ball}}}{(\mathrm{m_{ball}} + \mathrm{m_{pend}})}\mathrm{V_0}\right)^2$$

$$\frac{\mathrm{m_{ball}}}{(\mathrm{m_{ball}} + \mathrm{m_{pend}})} = \frac{1}{3}$$

$$2gH = \frac{1}{9}(\mathrm{V_0})^2$$

$$18gH = (\mathrm{V_0})^2$$

The linear system,

$$\begin{bmatrix} 18g & 0 & 0 \\ 0 & 18g & 0 \\ 0 & 0 & 18g \end{bmatrix}\begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = \begin{bmatrix} {V_1}^2 \\ {V_2}^2 \\ {V_3}^2 \end{bmatrix}$$

$({V_1}^2 = 17{,}9682$, ${V_2}^2 = 9{,}9029, {V_3}^2 = 14{,}5608$ and 2g = 19,6)

$$\begin{bmatrix} 176{,}4 & 0 & 0 \\ 0 & 176{,}4 & 0 \\ 0 & 0 & 176{,}4 \end{bmatrix}\begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = \begin{bmatrix} 17{,}9682 \\ 9{,}9029 \\ 14{,}5608 \end{bmatrix}$$

To check the matrix A is invertible or not, we can use the method to check the determinant of the matrix A

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a \bullet \det\begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \bullet \det\begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \bullet \det\begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

$$det\begin{bmatrix} 176{,}4 & 0 & 0 \\ 0 & 176{,}4 & 0 \\ 0 & 0 & 176{,}4 \end{bmatrix} = 176{,}4\ det\begin{bmatrix} 176{,}4 & 0 \\ 0 & 176{,}4 \end{bmatrix} - 0\ det\begin{bmatrix} 0 & 0 \\ 0 & 176{,}4 \end{bmatrix} + 0\ det\begin{bmatrix} 0 & 176{,}4 \\ 0 & 0 \end{bmatrix}$$

$$det\begin{bmatrix}176,4 & 0 & 0\\ 0 & 176,4 & 0\\ 0 & 0 & 176,4\end{bmatrix} = 176,4\ det\begin{bmatrix}176,4 & 0\\ 0 & 176,4\end{bmatrix}$$

$$176,4(176,4\,.\,176,4-0\,.\,0)=6859,216$$

$$det\begin{bmatrix}176,4 & 0 & 0\\ 0 & 176,4 & 0\\ 0 & 0 & 176,4\end{bmatrix} = 5451776,64$$

As a result of $5451776,64 > 0$, the matrix A is invertible.

About null space of matrix A, due to the fact that matrix A is invertible, its null space zero vector.

$$Ax = 0$$

It has only the trivial solution x = 0

$$Ax = 0$$

$$A^{-1}Ax = A^{-1}0$$

$$x = 0$$

Moreover, the column space of matrix A, C(A), consists of the vectors such,

$$H_1\begin{bmatrix}176,4\\ 0\\ 0\end{bmatrix} + H_2\begin{bmatrix}0\\ 176,4\\ 0\end{bmatrix} + H_3\begin{bmatrix}0\\ 0\\ 176,4\end{bmatrix} \qquad H_1, H_2, H_3 \in \mathbb{R}$$

Column space of matrix A is the whole 3-dimensional plane.

Furthermore, the matrix A has 3 pivots for its each columns, it can be seen from its reduced row echelon form. $\begin{bmatrix}176,4 & 0 & 0 & 17,9682\\ 0 & 176,4 & 0 & 9,9029\\ 0 & 0 & 176,4 & 14,5608\end{bmatrix}$ So its complete solution,

$$\begin{bmatrix}H_1\\ H_2\\ H_3\end{bmatrix} = \begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$X_C = \begin{bmatrix}0\\ 0\\ 0\end{bmatrix} + \begin{bmatrix}0,101862\\ 0,05614\\ 0,082545\end{bmatrix}$$

Because there are no free variables.

Construct a matrix C and show its LU factorization and reduced echelon form.

It is the first column consisting of three different momentum of the ball, and second row consisting of three different potential energy and last row consisting of various heights.

$$\begin{bmatrix} P_1 & P_2 & P_3 \\ PE_1 & PE_2 & PE_3 \\ H_1 & H_2 & H_3 \end{bmatrix}$$

$P = m_{ball}.V$

$P_1 = 0{,}005 . 3{,}87096 = 0{,}0193548$

$P_2 = 0{,}005. 2{,}56410 = 0{,}0128205$

$P_1 = 0{,}005 . 3{,}41463 = 0{,}01707315$

$PE = (m_{ball} + m_{pend})gH$

$PE_1 = 0{,}015 . 9{,}8 . 0{,}11 = 0{,}01617$

$PE_2 = 0{,}015 . 9{,}8 . 0{,}050 = 0{,}00735$

$PE_3 = 0{,}015 . 9{,}8 . 0{,}092 = 0{,}013524$

$H_1 = 0{,}11$ m

$H_2 = 0{,}050$ m

$H_3 = 0{,}092$ m

$$C = \begin{bmatrix} 0{,}0193548 & 0{,}0128205 & 0{,}01707315 \\ 0{,}01617 & 0{,}00735 & 0{,}013524 \\ 0{,}11 & 0{,}050 & 0{,}092 \end{bmatrix}$$

LU decomposition,

$$\begin{bmatrix} 0{,}0193548 & 0{,}0128205 & 0{,}01707315 \\ 0{,}01617 & 0{,}00735 & 0{,}013524 \\ 0{,}11 & 0{,}050 & 0{,}092 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1183}{1416} & 1 & 0 \\ \frac{8292}{1459} & \frac{1000}{147} & 1 \end{bmatrix} \begin{bmatrix} \frac{3223}{166522} & \frac{12820}{999961} & \frac{2007}{117553} \\ 0 & \frac{-13}{3868} & \frac{-41}{55421} \\ 0 & 0 & 0 \end{bmatrix}$$

C=LU

Reduced row echelon form,

$$\begin{bmatrix} 1 & 0 & 0,736311 \\ 0 & 1 & 0,220116 \\ 0 & 0 & 0 \end{bmatrix}$$

RESULTS AND DISCUSSION

To calculate the variables which are H, V_0 and Mechanical energy experimentally, we can do some experiment.

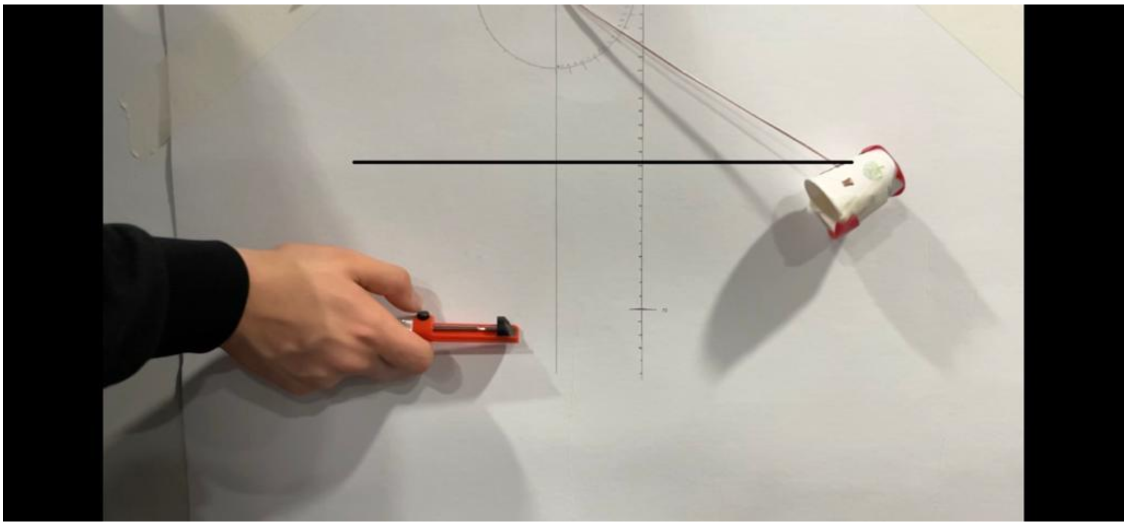
The length of the wire is 24 cm, the mass of the ball is 5 gr., and the mass of the pendulum is 10gr.(approximate values)

To find H

We know the formula to find H

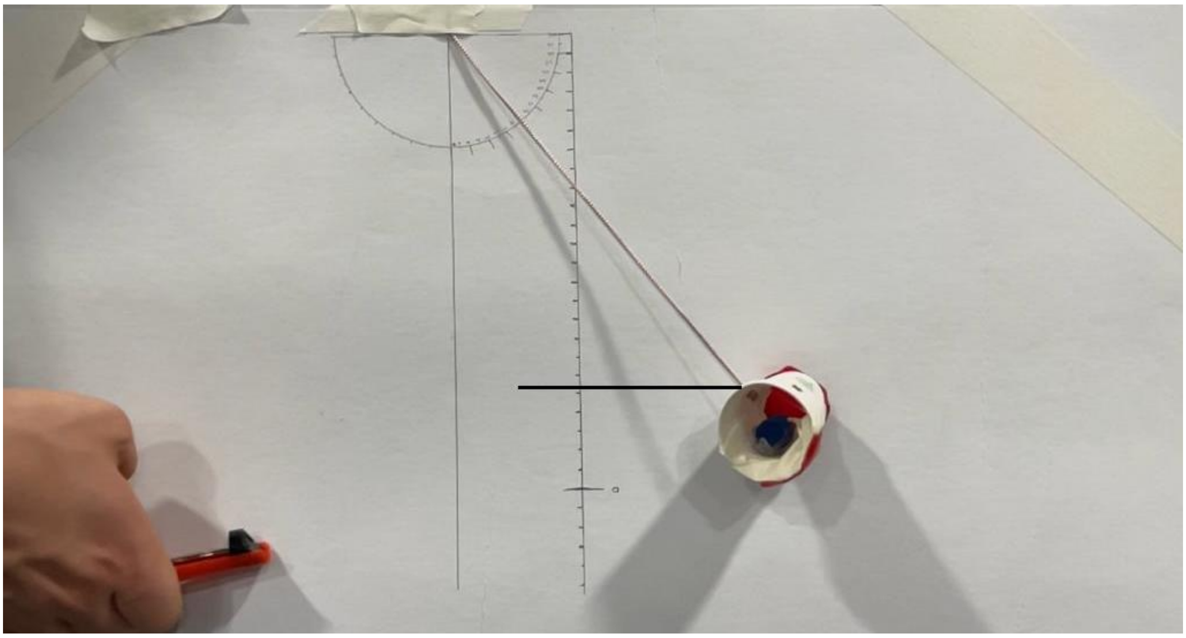
$$H = L - L\cos\theta$$

We did three trials to measure the height of the pendulum,



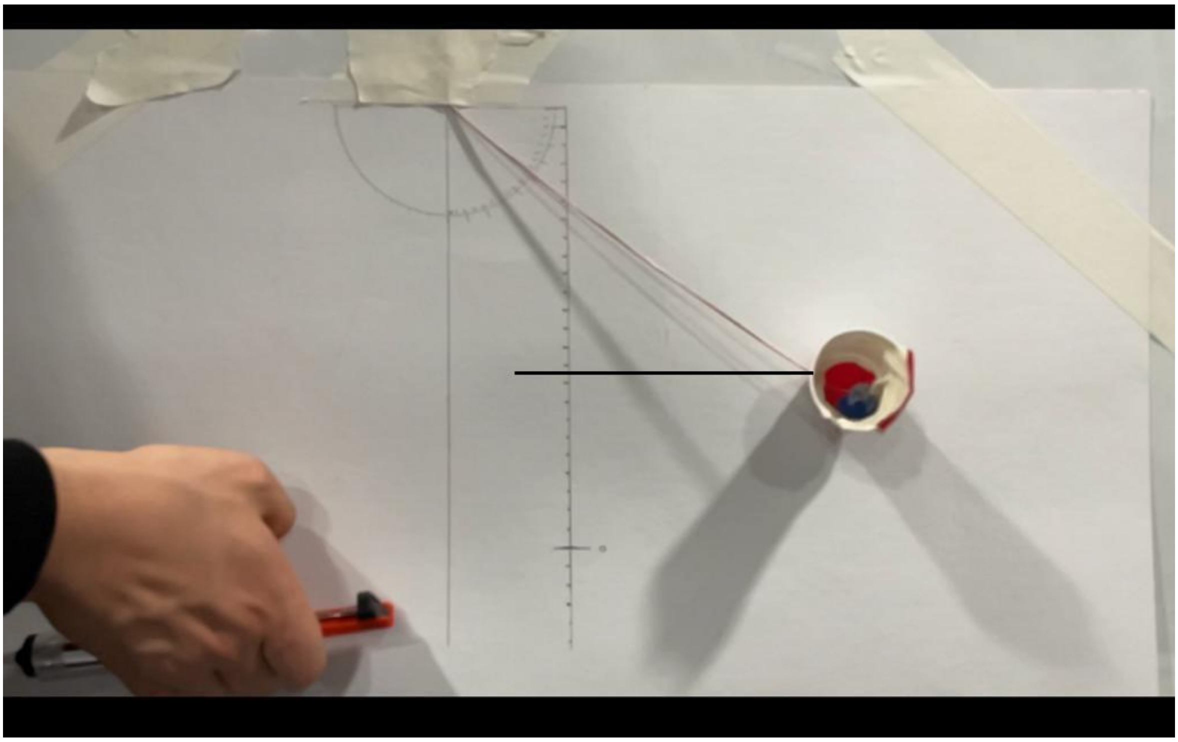
Picture 6 Trial 1

Experimentally H was 11 cm, as can be seen from the picture three, and θ was 55°(approximate values)



Picture 7 Trial 2

Experimentally H was 5.1 cm, as can be seen from the picture three, and θ was 36°(approximate values)



Picture 8 Trial 3

Experimentally H was 9.2 cm, as can be seen from the picture three, and θ was 49°(approximate values)

Trial Number	Theoretically calculated value of H	Experimentally calculated value of H	Error Percentage
1	0,101862 m	0,11 m	7,3981
2	0,05614 m	0,050 m	10,07843
3	0,082545 m	0,092 m	10,27717

To find V_0

We know the formula to find V_0 with respect to H (used theoretical H)

$$V_0 = \sqrt{2gH}\frac{(\mathrm{m_{ball}} + \mathrm{m_{pend}})}{\mathrm{m_{ball}}}$$

We did three experiment to measure the initial velocity of the ball,

We used the method of slow motion. We observed the ball during its motion.

Trial 1

Displacement of string was 0,041 m

Picture 9 Trial 1 for Velocity

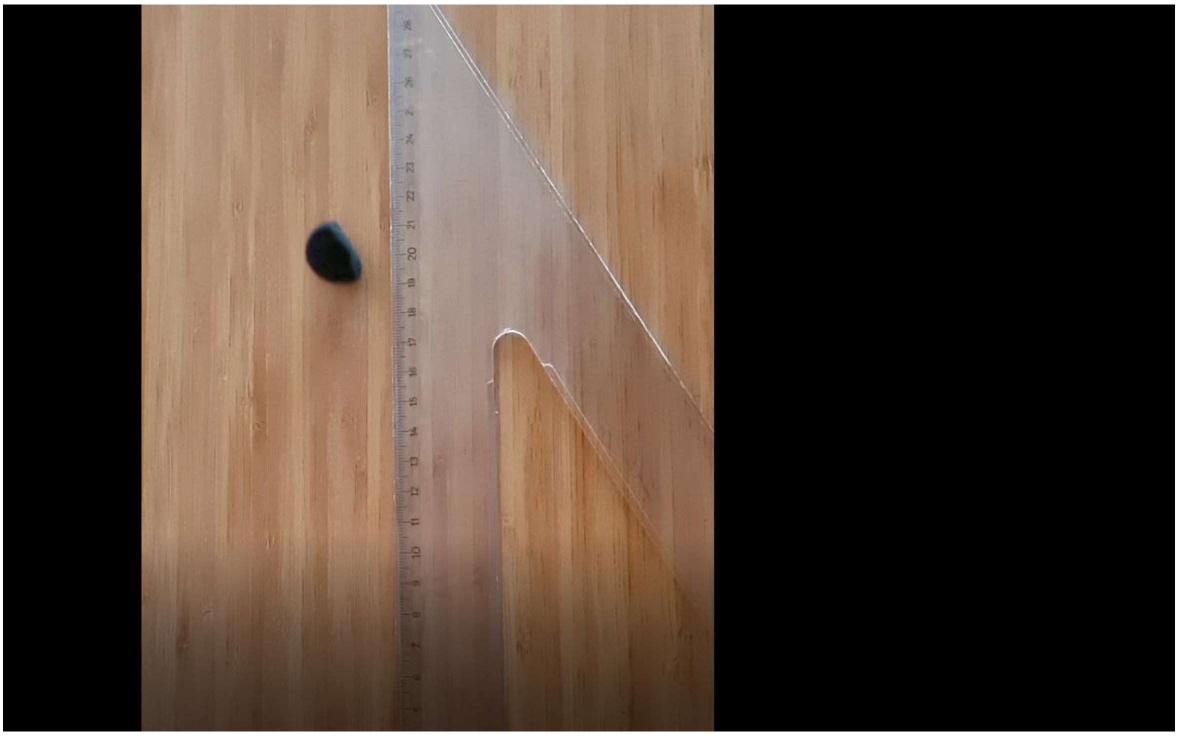
Picture 10 Trial 1 for Velocity

The ball started passing from 0,08 m to 0,20 m in 0,031 seconds, as a result its velocity is experimentally 3,87096 m/s.

Trial 2

Displacement of string was 0,02 m

Picture 11 Trial 2 for Velocity

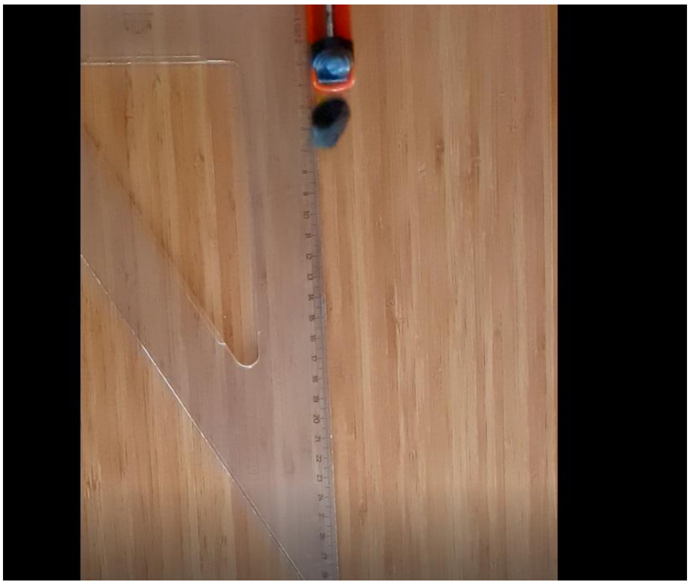


Picture 12 Trial 2 for Velocity

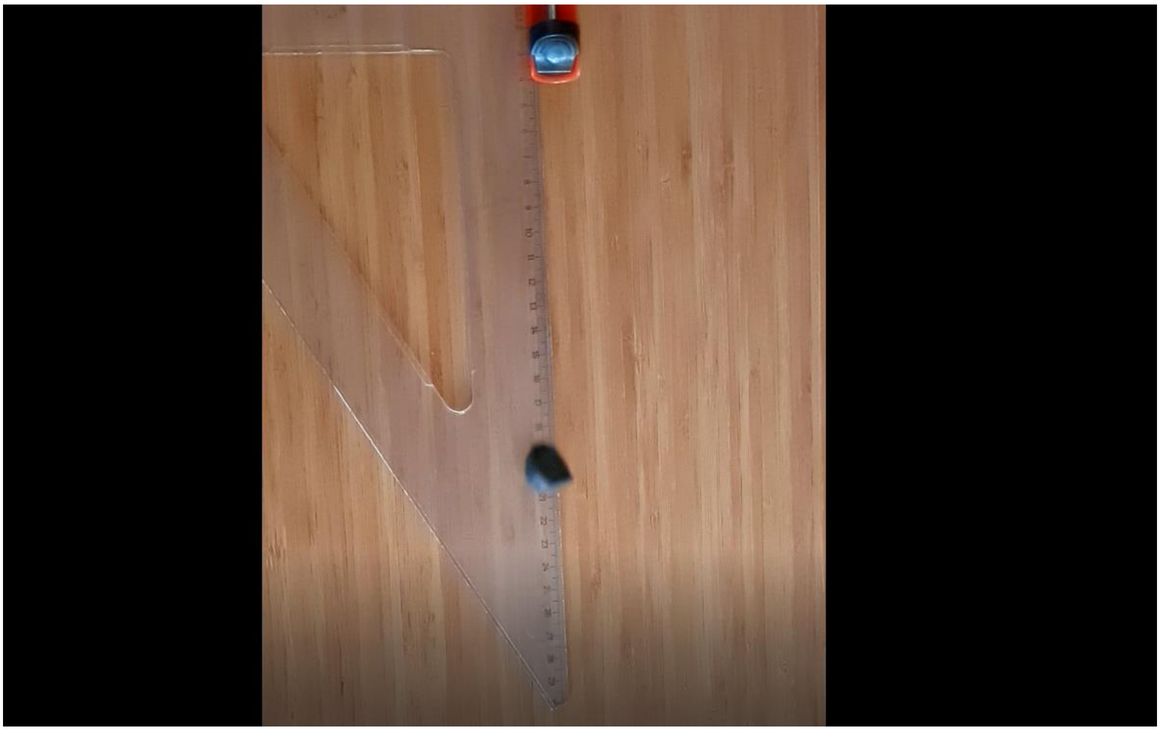
The ball started passing from 0,10 m to 0,20 m in 0,039 seconds, as a result its velocity is experimentally 2,56410 m/s.

Trial 3

Displacement of string was 0,03 m



Picture 13 Trial 3 for Velocity



Picture 14 Trial 3 for Velocity

The ball started passing from 0,07 m to 0,21 m in 0,041 seconds, as a result its velocity is experimentally 3,87096 m/s.

Trial Number	Value of H	Theoretically calculated value of V_0	Experimentally calculated value of V_0	Error Percentage
1	0,11 m	4,2389 m/s	3,87096 m/s	8,6800
2	0,050 m	3,1469 m/s	2,56410 m/s	18,5198
3	0,092 m	3,81587 m/s	3,41463 m/s	10,5150

To find Mechanical Energy

Final mechanical energy is just potential energy at the height of H,

$$E_f = (\mathrm{m_{pend}} + \mathrm{m_{ball}})gH$$

(If reference point is considered from h_2, it will be H)

Initial mechanical energy is just first kinetic energy, given by the spring,

$$E_i = \frac{1}{2} \mathrm{m_{ball}} \mathrm{V_0}^2$$

The ratio of final to initial,

$$\frac{E_f}{E_i} = \frac{(\mathrm{m_{pend}} + \mathrm{m_{ball}})gH}{\frac{1}{2}\mathrm{m_{ball}}\mathrm{V_0}^2}$$

(H and V values are the values that were calculated experimentally)

Trial Number	Final mechanical energy	Initial mechanical energy	The average % of the lost for mechanical energy
1	0,01617	0,03746	43,1660
2	0,00735	0,01643	44,7352
3	0,013524	0,02914	46,41043

Mechanical energy decreased virtually 50 percent. We can tell some reasons responsible for this consequence. Firstly, due to the inelastic collision, during the collision system lost its initial mechanical energy to heating, deformation, etc. Secondly, air friction did a work against the motion, it made some loss as well. Finally, our system was not perfect, sometimes it made a contact with the whiteboard, and it caused a huge amount of friction.

CONCLUSIONS

In conclusion, the ballistic pendulum is a simple mechanical system that exhibits rich and complex behavior. Through our experimentation and theorical analysis, we have gained a thorough understanding of the physical principles governing the pendulum's motion, including the role of conservation of energy and momentum, as well as the influence of friction and other external forces. We also gained a knowledge of spring gun, and learnt how to forecast pendulum’s velocity, angle and height.

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