Department of

Electrical & Electronics Engineering

Abdullah Gül University

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Project Report

EE1100 Computation and Analysis (COMA) Capsule

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Submitted by: Serkan KIYAKLI 2111011078

Group Number/Name: ASH

Group Partner: Aliye BİRGÜL 2111011070

Hasbi Mert BEYAZ 2111011020

Grade: / 100

OBJECTIVE

In this project we will try to explain how anti-gravity works exactly and effects of the influence of the angle of inclination of the bases on the time of rolling. Also we will determine which equations should we use to illustrate the effects. Also, in this project we will learn the motion of center of mass of double cone. We aim to support these calculations with physical experiments which has two stick, one double cone, cylindrical shell and object which has a thickness. When we put the cylindrical shell at the top of the sticks it goes downward also when we put it at the bottom side of sticks it keeps it position. On the other hand, when we put the double cone if angle and the height is true it starts to go up from bottom of the bottom side but if the angle and incline is not true double cone moves are starting to match up with cylindrical shell. By studying the motion of double cone, we can learn about the principles.

BACKGROUND

2 funnels which are used to generate double cone, glue, two sticks used to form the rail along which double cone will move, a few books used to arrange the height of the set-up, ruler, calculator, timekeeper for measuring velocity, protractor. In addition, camera is used for the video shooting.

ANALYTICAL AND SIMULATION PROCEDURES

PHYSICS PART

The double-cone ascending on inclined V-rail, which is known as anti-gravity paradox, is a great phenomenon to demonstrate an illusion which is counter-intuitive in between physical intuition and real world. To explain this counter-intuitive phenomena we will make dynamical and mathematical calculations.

Question 1 : Explanation of The Paradox

As can be seen the double cone kept at the lower end of the inclined V shaped rails moves by its own and reaches the higher end, the remarkable idea coming to our mind is that this is a clear breaching of the law of conservation of energy. The center of mass is moving from a higher point to lower point. We will demonstrate that the phenomena are perfectly consistent with the laws of physics.

A body in the presence of the gravitational force moves towards the minimum of the gravitational potential energy from maximum gravitational potential energy. For the system consisting of double cone and diverging inclined plane, in fact it is seen as moving upward but its center of mass moving down. With the aim of understanding its motion, we should look at its horizontal displacement and the motion of its center of mass.

Consider a Cartesian coordinate system with x horizontal displacement, y in the plane perpendicular to x, z vertical displacement and consider the angle between rails 2φ and the angle of inclined plane θ.

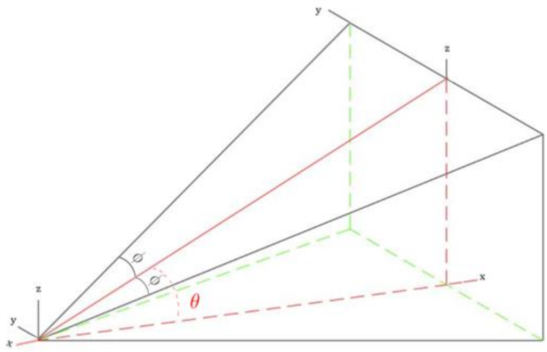

Figure 1: Inclined plane with Cartesian coordinates

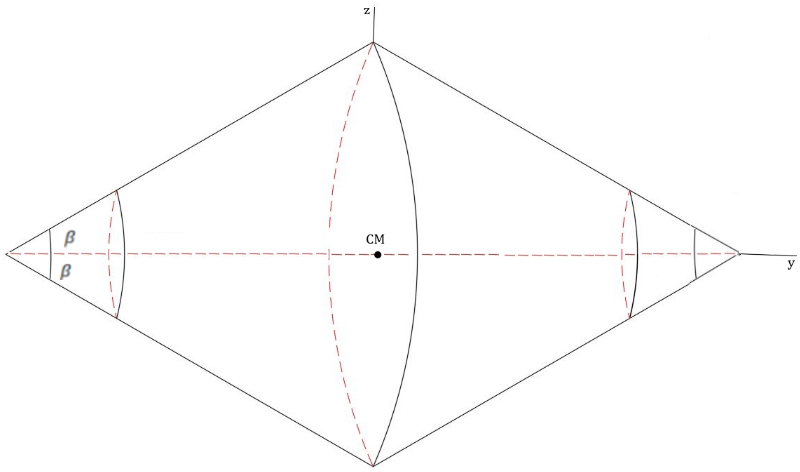

Figure 2: Double cone with radius R and height h

Firstly, in order to understand its motion from a dynamical perspective , let us look at the connection between torque and motion of center of mass. Figure 2 shows the picture of double cone with its coordinates, apex angle β and its center of mass (CM). Any solid on the inclined ramp consisting of two part will contact at two points. Assume that there is a solid cylinder on the ramp connected with two points.

If the angle of the ramp is zero (Figure 3), the cylinder will make a connection with one point and its center of mass will be on the axis which goes through that point. Due to the fact that the center of mass goes through axis of rotation, there will be no resultant torque because of $\sin 90$, and as a result cylinder will not rotate.

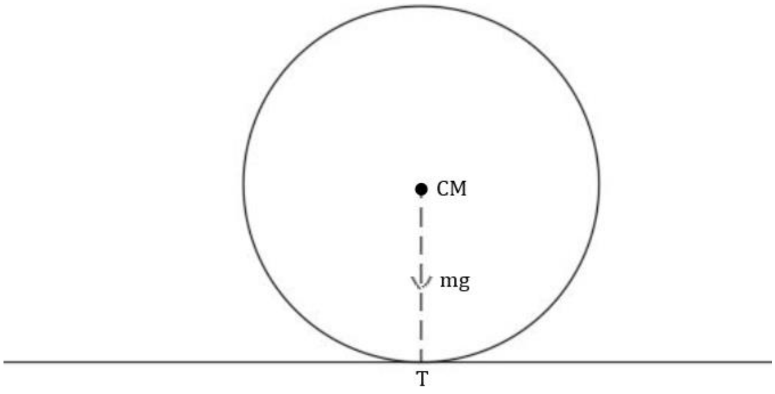

Figure 3: Lateral section of cylinder

$$\vec{\tau} = \vec{F} \, x \, \vec{r}$$

$$\tau = F.r.sin\alpha$$

Because of $\sin 90$, torque will be zero.

$$\tau = 0$$

As a result, the cylinder will not rotate. However, assume the ramp inclines with an angle which is a (Figure 4). It makes two connection with two point at the inclined angle of the ramp. This points make a line, and this line identifies instantaneous axis of rotation. The torque about this axis will be due to gravity since the axis of center of mass will make an angle with this axis.

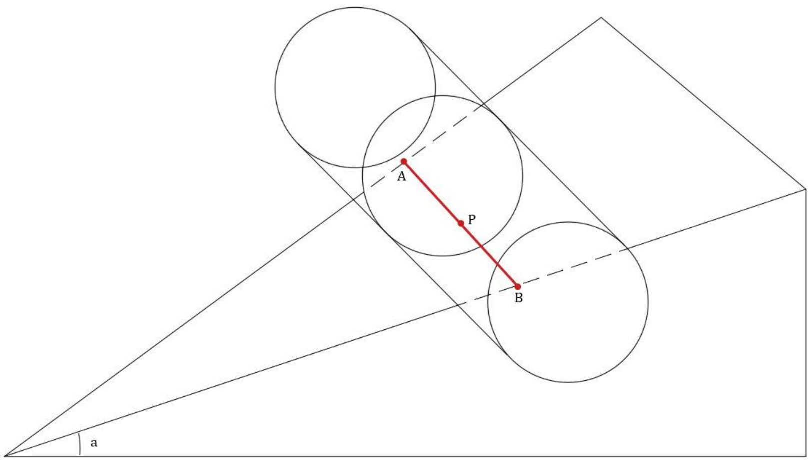

Figure 4: A solid cone on the inclined ramp

For this cylinder placed on the ramp (Figure 4), the angle between the weight and the vector that comes from the point of P, which enters the torque, is determined by the inclination a of the ramp. Thus the torque resultant of the weight, $-mg\ R \sin a$, always tends the cylinder to roll down the ramp (Figure 5)

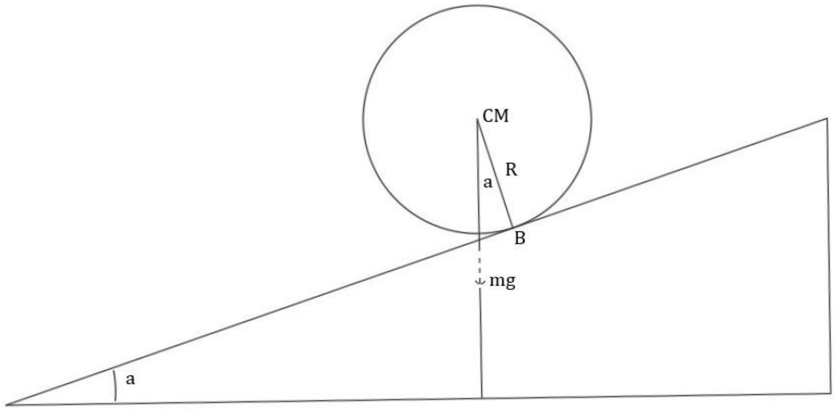

Figure 5: Lateral section of the cylinder on the inclined ramp

The situation for the double-cone is different. The position of center of mass of double cone at initial is different from at the top of the ramp. Suppose, for purpose of observing, double cone is, at initial, at the bottom of the ramp. Its center of mass (G) has height of $h + r$. However, at top of the ramp the center of mass has height of $h\ +\ k$. As can be seen from Figure 6, because of $k < r$, the center of mass of double cone decreased in height.

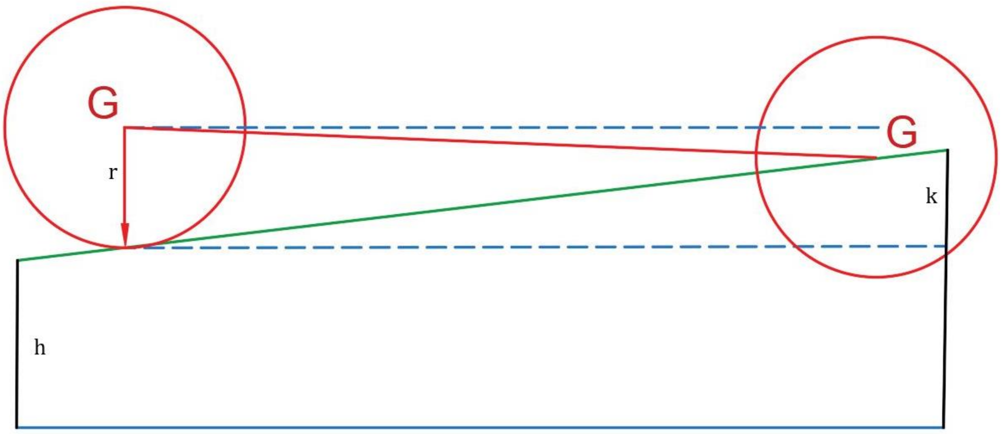

Figure 6: Displacement of center of mass

As a result, its last potential energy U_f is lower than its first potential energy U_i . It was explained earlier that a body in the presence of the gravitational force moves towards the minimum of the gravitational potential energy from maximum gravitational potential energy. The conclusion for this matter will be that its center of mass drops although we see it rises.

$$U_i = mg(h + r)$$

$$U_f = mg(h + k)$$

Because of $k < r$,

$$U_f < U_i$$

So from the perspective of center of mass, its movement will be like this,

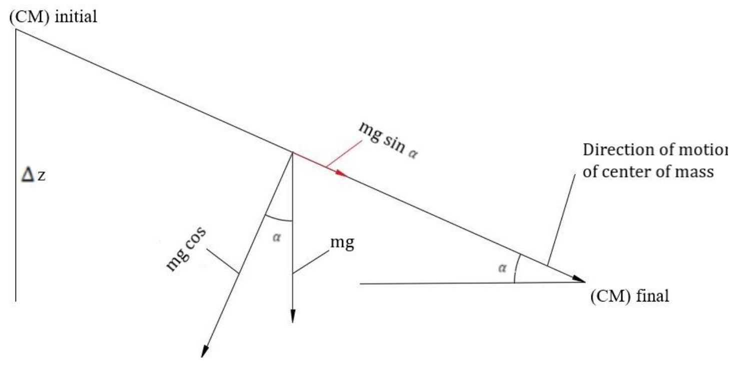

Figure 7: Motion of center of mass from the reference of center of mass

Due to the downward motion, a force should be present in the direction of motion. As observed previously, the height of center of mass falls from highest to lowest. During the motion, it vertically falls with vertical displacement Δz with the angle of α (Figure 7). Force parallel to the path taken by the center of mass accelerates the double cone.

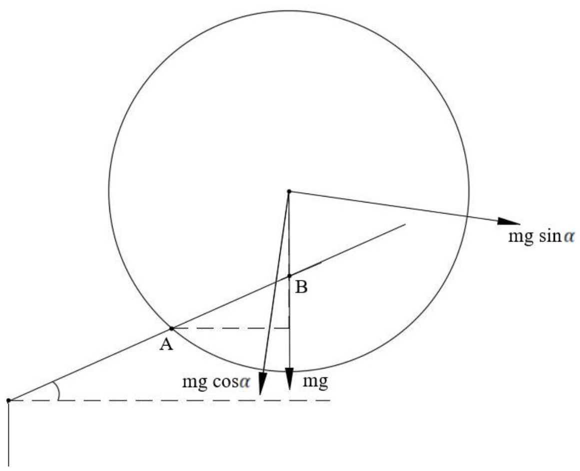

Figure 8: Demonstration of forces from the perspective of the ramp

With the information that was given previously about solid cylinder, the connection point between double cone and the ramp should be in between A and B (Figure 8). Thus, $mg \sin \alpha$ will create torque that tends to rotate double cone to uphill.

Question 2 : Derivation of the moment of inertia of double cone

To calculate the moment of inertia of solid double cone, we can follow the following calculation.

$$dm = \rho \;\; dv$$

$$dI \;=\; dm\, r2$$

$$\int dI = \int dm\, r^2$$

$$\boldsymbol{I \;=\; \int r^2\, dm}$$

For solid double cone, we used the solid disc part to integrate,

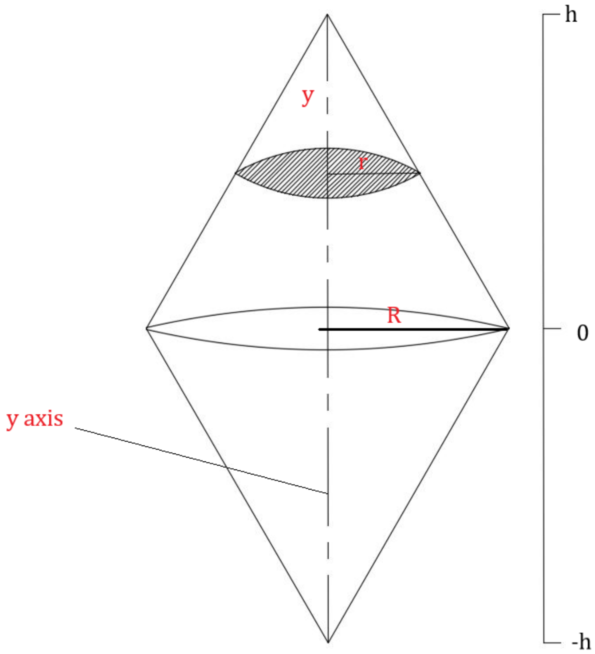

Figure 9: Solid double cone and solid disc part

$$dI \;= \frac{1}{2}\, dm\, r2$$

To find elemental mass dm,

$$\frac{dm}{\pi r^2 dy} \;= \frac{M}{\frac{2}{3}\pi R^2 h} \quad \rightarrow \; dm \;= \frac{3M}{2R^2 h} r^2 dy$$

$$dI = \frac{1}{2}\frac{3M}{2R^2h}\ r2\,r2\,dy$$

Take the integral of dI

$$I = \int_{-h}^{h} \frac{1}{2}\ \frac{3M}{2R^2h} r^4\,dy$$

$$I = \frac{3M}{4R^2h}\int_{-h}^{h} r^4\ dy$$

The ratio related to r can be written as

$$\frac{r}{y} = \frac{R}{h} \ \rightarrow\ r = \frac{Ry}{h}$$

$$I = \frac{3M}{4R^2h}\int_{-h}^{h} \frac{R^4y^4}{h^4}\ dyI = \frac{3M}{2R^2h}\ \frac{R^4}{h^4}\ \int_{-h}^{h} y^4\ dy$$

$$I = \left(\frac{3M}{4R^2h}\ \frac{R^4}{h^4}\ \frac{y^5}{5}\right)\Big|_{-h}^{h}$$

$$I = \frac{3M}{4R^2h}\ \frac{R^4}{h^4}\ \left[\frac{h^5}{5} - \left(-\frac{h^5}{5}\right)\right]$$

$$Moment\ of\ inertia\ of\ solid\ double\ cone \rightarrow I = \frac{3}{10}MR^2$$

To calculate the moment of inertia of hollow double cone, we can follow the following calculation.

$$dI = dm\,r^2$$

$$dm = \rho\ dV$$

ρ is density of hollow double cone.

For hollow double cone, we used the ring part to integrate,

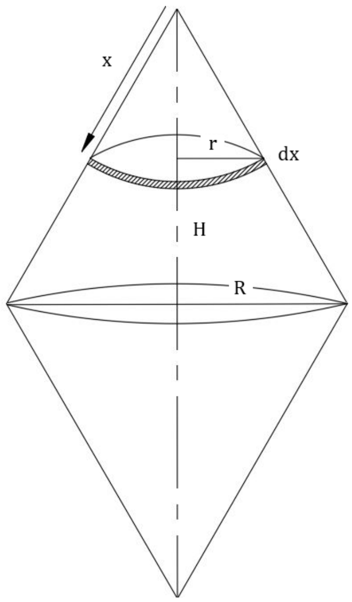

Figure 10: Hollow double cone and ring part

$$dV = 2\pi r \, da$$

$$dm = \rho \, 2\pi r \, da$$

$$dI = \rho \, 2\pi \, r^3 \, da$$

We are going to integrate all the little slices from -L to L, or we can do it from the halfway point to the top L and double it.

$$\rho \, 2\pi \, 2 \int_0^L r^3 \, da$$

To integrate this integrand we should find relation between r and a,

$$\frac{r}{a} = \frac{R}{L}$$

$$r = \frac{R}{L} \, a$$

Then we have,

$$\rho \, 4\pi \, \frac{R^3}{L^3} \int_0^L a^3 \, da$$

We can write ρ as following,

$$\rho = \frac{M}{2\pi RL} = \frac{M}{\pi RL}$$

Finally,

$$\frac{M}{2\pi RL} 4\pi \frac{R^3}{L^3} \frac{L^4}{4}$$

$$Moment\ of\ inertia\ of\ hollow\ double\ cone \rightarrow I\ = \frac{1}{2} MR^2$$

The moment of inertia that was given in the article that was shared with us is calculated for solid double cone.

Question 3 : Observing angles in which double cone make the motion.

To determine if the center of mass declines or not, we should look at a change of the height of the double-cone axis with respect to the horizontal plane. Use the cartesian coordinates that have been already defined in Figure 1.

First let us look at horizontal displacement x from the top view.

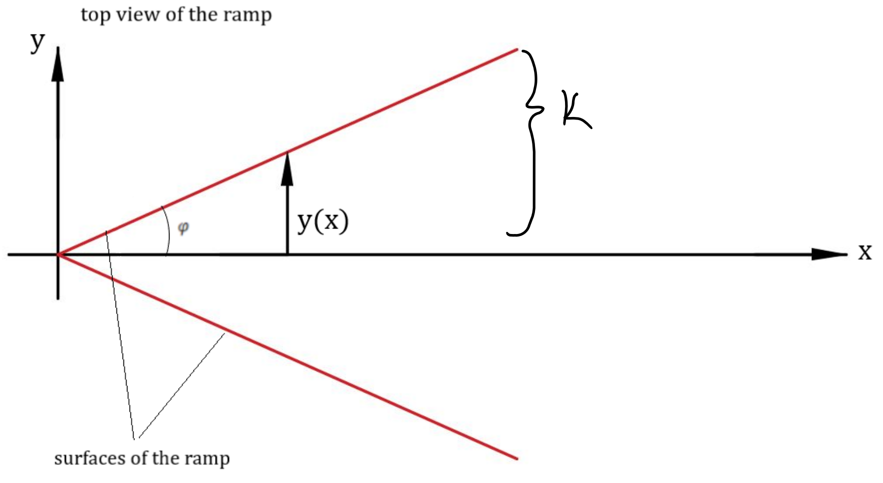

Figure 11: Top view of the ramp.

The distance y(x) is between the contacts of the double cone and the ramp. With an infinitesimal displacement dx in the direction of x, which is uphill, distance between contact points will increase. We have,

$$dy = y(x + dx) - y(x)$$

If we look at its $\tan\varphi$,

$$\tan\varphi = \frac{dy}{dx}$$

$$\tan\varphi \, dx = dy$$

While it is moving in the direction of x, it is moving vertically, in the direction of z.

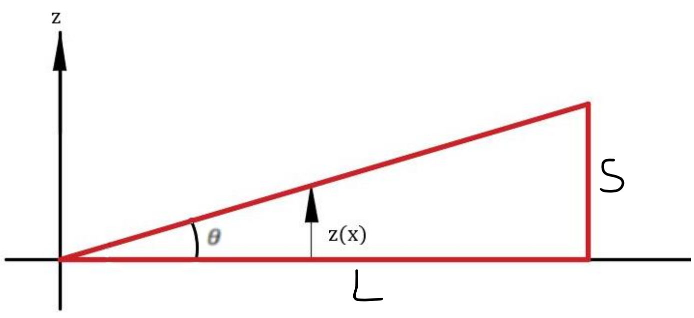

Figure 12: Side view of the ramp

The distance z(x) is the height of double cone. As double cone is moving in the x direction with dx, it rises with infinitesimal vertical displacement dz and we have that,

$$dz = z(x + dx) - z(x)$$

As we are trying to understand its motion with respect to angles, let us look at $\tan\theta$

$$\tan\theta = \frac{dz}{dx}$$

$$\tan\theta \, dx = dz$$

It was already mentioned that the center of mass of double cone falls as the whole body is ascending. So again and last, we should look at motion of center of mass with respect to angles. From the double cone view its motion is like this,

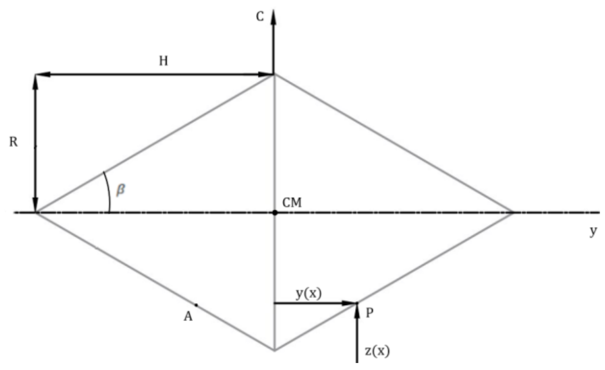

Figure 13: Front view of double cone and connection point with rail A and P

C axis goes through the center of mass. While the double cone goes from vertex to top of the ramp, with respect to C axis center of mass falls. So the level difference between the double-cone axis and the contact points decreases by the following amount,

$$dC = C(y + dy) - C(y)$$

And as angle ratio,

$$\tan\beta = \frac{dC}{dy}$$

$$\tan\beta\, dy = dC$$

We know what dy is from $\tan\varphi\, dx$ so that

$$\tan\beta\tan\varphi\, dx = dC$$

As a result we got $-dC$ as the amount of falling displacement of center of mass, and dz as the amount of vertical displacement of the whole body. Because if center of mass falls much more than ascending vertical displacement to create the motion, so following condition must be true,

$$dz\ - dC < 0$$

$$dz < dC$$

$$\tan\theta <\ \tan\beta\tan\varphi$$

When dC > dz, even though the contact points between the double cone and the ramps shift upwards, the CM of the double cone goes lower, and the gravitational potential energy diminishes. This results in the seeming contradiction of the double cone moving higher on the rails.

Furthermore, to find what parameters the critical angle of inclination depends on, so that we should look at the geometrical shape of the double cone. We can use the values of $\tan\beta \; and \tan\varphi$.

$$\tan\beta = \frac{R}{H}$$

$$\tan\varphi = \frac{\mathrm{K}}{\mathrm{L}}$$

$$\tan\theta < \frac{R}{H}\frac{\mathrm{K}}{\mathrm{L}}$$

LINEAR ALGEBRA PART

Question 1 : Create a matrix, and make explanations about consequences of changing variables

Let us create a matrix Ax=b, by using equations that we found in physics part.

$$\begin{bmatrix} \frac{3}{10}MR & \frac{3}{2\pi R^2 H} \\ \frac{1}{2}MR & \frac{1}{2\pi RL} \end{bmatrix} \begin{bmatrix} R \\ M \end{bmatrix} = \begin{bmatrix} I_{dc} + \rho_{dc} \\ I_{hc} + \rho_{hc} \end{bmatrix}$$

dc – solid double cone

hc – hollow double cone

ρ – for density

First let us look at the determinant of the matrix: To find the determinant

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$(a.d) - (b.c) =$ determinant.

So determinant is:

$$\frac{3M}{20\pi L} - \frac{3M}{2\pi RH} = \frac{3\mathrm{M}(10\mathrm{RH} - \mathrm{L})}{20\pi\mathrm{HRL}}$$

Now let us make a comment about what happens if we change the changing variables

- M is directly proportional with determinant if we increase it the determinant will increase but if we decrease it determinant also decrease.
- L is inversely proportional with the determinant if we increase it determinant will decrease but if we decrease the l determinant will increase.
- R is directly proportional with determinant, but we have two cases if 0<R<5 and R>5 for the 0<R<5 if we increase the R determinant will decrease but for the R>5 situation if we increase the r determinant will be negative and for the negative number it will decrease also.

Secondly let us look at the inverse of matrix. To find the inverse of matrix in $2x2$ matrix we use

$$\frac{1}{det}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

So our inverse matrix is

$$A^{-1}=\frac{1}{\frac{3\mathrm{M}(10\mathrm{RH}-\mathrm{L})}{20\pi\mathrm{HRL}}}\begin{bmatrix} \frac{3MR}{10} & \frac{3}{2\pi R^2 H} \\ \frac{1}{2}MR & \frac{1}{2\pi \mathrm{RL}} \end{bmatrix}$$

$$A^{-1}=\begin{bmatrix} \frac{3\mathrm{M}(10\mathrm{RH}-\mathrm{L})}{40\pi^2\mathrm{HR}^2\mathrm{L}^2} & \frac{3\mathrm{M}(10\mathrm{RH}-\mathrm{L})}{20\pi^2\mathrm{H}^2\mathrm{R}^3\mathrm{L}} \\ -\frac{3\mathrm{M}^2(10\mathrm{RH}-\mathrm{L})}{40\pi\mathrm{HL}} & \frac{9\mathrm{M}^2(10\mathrm{RH}-\mathrm{L})}{200\pi\mathrm{HL}} \end{bmatrix}$$

•To find the inverse of the matrix we need determinant and if our R is equal to L/10H we cannot say inverse of $2x2$ matrix because if matrix determinant is zero it means matrix is singular and we do not have any inverse matrix. Otherwise, we can say we have inverse matrix.

Thirdly let us look at the column space and row space

To find the column space first we need to find the pivots of the matrix. In order to find the pivot we will use reduced echelon form

$$R_2-\left(\frac{5R_1}{3}\right)$$

Matrix will be

$$\begin{bmatrix} \frac{3}{10}MR & \frac{3}{2\pi R^2 H} \\ 0 & \left(\frac{RH-5L}{\pi LHR^2}\right) \end{bmatrix}$$

In that case we have two columns the first and second columns are our pivot columns. So column spaces are:

$$C1=\begin{bmatrix} \frac{3MR}{10} \\ \frac{1}{2}MR \end{bmatrix} \quad C2=\begin{bmatrix} \frac{3}{2\pi R^2 H} \\ \frac{1}{2\pi \mathrm{RL}} \end{bmatrix}$$

In the column space part if H is equal to L and R is 5. In that case we will just have the one column space because the number of the pivot columns decrease from 2 to 1. In our matrix a_{22} will be zero and to say that is a pivot it has to be 1 otherwise we cannot take it as a pivot column. So, we will have just one column space and that is the C1.

Rows spaces are:

$$R1=\begin{bmatrix} \frac{3MR}{10} & \frac{3}{2\pi R^2 H} \end{bmatrix}$$

$$R2=\begin{bmatrix} 0 & \frac{RH-5L}{\pi LHR^2} \end{bmatrix}$$

For the Row space part if H is equal to L and R is 5. We will have the one row space since the number of the pivot columns decrease from 2 to 1. In our matrix a_{22} will be zero and to say that is a pivot it has to be 1 otherwise we cannot take it as a pivot column. Thus, we will have one row space and that is the R1.

To write the transpose of the matrix we just need to change the row and column places. So, how we do this first row will be our first column and column will become the row. Matrix will be like that:

$$A^T=\begin{bmatrix} \frac{3MR}{10} & \frac{1}{2}MR \\ \frac{3}{2\pi R^2 H} & \frac{1}{2\pi \mathrm{RL}} \end{bmatrix}$$

Changing variables doesn't affect the transpose of the matrix because matrix is a rectangular array of numbers, and we get the transpose matrix by swapping the rows and columns of the matrix.

Question 2 : Construct a vector a, and find an orthogonal vector to vector a

We start to construct vector a with $\omega, \Delta z$ and r which are already given in the article.

$$a = \begin{bmatrix} \sqrt{\frac{20g\,\Delta z}{3r^2}} \\ \frac{\omega^2\,3r^2}{20g} \\ r \end{bmatrix}$$

To find an orthogonal vector, let us say vector c, to vector a we can use principles of orthogonality such as the dot product of two orthogonal vectors is zero.

Let us say for vector c,

$$c = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Now take the dot product of these two vectors,

$$a\,.\,c \;=\; a^T\,c = \;0$$

$$\begin{bmatrix} \sqrt{\frac{20g\,\Delta z}{3r^2}} & \frac{\omega^2\,3r^2}{20g} & r \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$x\sqrt{\frac{20g\,\Delta z}{3r^2}} + y\,\frac{\omega^2\,3r^2}{20g} + zr = 0$$

If we choose x, y and z respectively as following,

$$x = -\sqrt{\frac{20g\,\Delta z}{3r^2}}$$

$$y = \frac{400g}{\omega^2 9r^2}$$

$$z = \frac{20(1-\Delta z)}{3r^3}$$

$$-\sqrt{\frac{20g\,\Delta z}{3r^2}}\sqrt{\frac{20g\,\Delta z}{3r^2}} + \frac{400g}{\omega^2 9r^2}\,\frac{\omega^2\,3r^2}{20g} + \frac{20(\Delta z - 1)}{3r^3}r = 0$$

$$-\frac{20g\ \Delta z}{3r^2} + \frac{20g}{3r^2} + \frac{20(\Delta z - 1)}{3r^2} = 0$$

$$-\frac{20(1 - \Delta z)}{3r^2} + \frac{20(1 - \Delta z)}{3r^2} = 0$$

$$0 = 0$$

As a result, our orthogonal vector c,

$$c = \begin{bmatrix} -\sqrt{\dfrac{20g\ \Delta z}{3r^2}} \\ \dfrac{400g}{\omega^2 9r^2} \\ \dfrac{20(1 - \Delta z)}{3r^3} \end{bmatrix}$$

Question 3 : Finding the projection p of vector b onto vector a

We have vector b and vector a,

$$b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad a = \begin{bmatrix} \sqrt{\dfrac{20g\ \Delta z}{3r^2}} \\ \dfrac{\omega^2\ 3r^2}{20g} \\ r \end{bmatrix}$$

And we will find the projection p of vector b onto vector a which was previously found. We can use projection matrix,

$$p = P.b$$

$$P = \frac{a\ a^T}{a^T\ a}$$

$$P = \frac{\begin{bmatrix} \sqrt{\frac{20g\ \Delta z}{3r^2}} \\ \frac{\omega^2\ 3r^2}{20g} \\ r \end{bmatrix} \begin{bmatrix} \sqrt{\frac{20g\ \Delta z}{3r^2}} & \frac{\omega^2\ 3r^2}{20g} & r \end{bmatrix}}{\begin{bmatrix} \sqrt{\frac{20g\ \Delta z}{3r^2}} & \frac{\omega^2\ 3r^2}{20g} & r \end{bmatrix} \begin{bmatrix} \sqrt{\frac{20g\ \Delta z}{3r^2}} \\ \frac{\omega^2\ 3r^2}{20g} \\ r \end{bmatrix}}$$

$$P = \frac{1}{\frac{20g\ \Delta z}{3r^2} + \frac{\omega^4 9r^4}{400g^2} + r^2} \begin{bmatrix} \frac{20g\ \Delta z}{3r^2} & \sqrt{\frac{\Delta z\ \omega^4 3r^2}{20g}} & \sqrt{\frac{20g\ \Delta z}{3r^2}} \\ \sqrt{\frac{\Delta z\ \omega^4 3r^2}{20g}} & \frac{\omega^4 9r^4}{400g^2} & \frac{\omega^2\ 3r^3}{20g} \\ \sqrt{\frac{20g\ \Delta z}{3r^2}} & \frac{\omega^2\ 3r^3}{20g} & r^2 \end{bmatrix}$$

Projection matrix

$$p = \frac{1}{\frac{20g\ \Delta z}{3r^2} + \frac{\omega^4 9r^4}{400g^2} + r^2} \begin{bmatrix} \frac{20g\ \Delta z}{3r^2} & \sqrt{\frac{\Delta z\ \omega^4 3r^2}{20g}} & \sqrt{\frac{20g\ \Delta z}{3r^2}} \\ \sqrt{\frac{\Delta z\ \omega^4 3r^2}{20g}} & \frac{\omega^4 9r^4}{400g^2} & \frac{\omega^2\ 3r^3}{20g} \\ \sqrt{\frac{20g\ \Delta z}{3r^2}} & \frac{\omega^2\ 3r^3}{20g} & r^2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$p = \begin{bmatrix} \dfrac{\dfrac{20g\,\Delta z}{3r^2} + 2\sqrt{\dfrac{\Delta z\,\omega^4 3r^2}{20g}} + 3\sqrt{\dfrac{20g\,\Delta z}{3r^2}}}{\dfrac{20g\,\Delta z}{3r^2} + \dfrac{\omega^4 9r^4}{400g^2} + r^2} \\ \dfrac{\sqrt{\dfrac{\Delta z\,\omega^4 3r^2}{20g}} + \dfrac{\omega^4 9r^4}{200g^2} + \dfrac{\omega^2\,9r^3}{20g}}{\dfrac{20g\,\Delta z}{3r^2} + \dfrac{\omega^4 9r^4}{400g^2} + r^2} \\ \dfrac{\sqrt{\dfrac{20g\,\Delta z}{3r^2}} + \dfrac{\omega^2\,3r^3}{10g} + 3r^2}{\dfrac{20g\,\Delta z}{3r^2} + \dfrac{\omega^4 9r^4}{400g^2} + r^2} \end{bmatrix}$$

Projection p

And we can write the error by using $e = b - p$

$$e = \begin{bmatrix} 1 - \dfrac{\dfrac{20g\,\Delta z}{3r^2} + 2\sqrt{\dfrac{\Delta z\,\omega^4 3r^2}{20g}} + 3\sqrt{\dfrac{20g\,\Delta z}{3r^2}}}{\dfrac{20g\,\Delta z}{3r^2} + \dfrac{\omega^4 9r^4}{400g^2} + r^2} \\ 2 - \dfrac{\sqrt{\dfrac{\Delta z\,\omega^4 3r^2}{20g}} + \dfrac{\omega^4 9r^4}{200g^2} + \dfrac{\omega^2\,9r^3}{20g}}{\dfrac{20g\,\Delta z}{3r^2} + \dfrac{\omega^4 9r^4}{400g^2} + r^2} \\ 3 - \dfrac{\sqrt{\dfrac{20g\,\Delta z}{3r^2}} + \dfrac{\omega^2\,3r^3}{10g} + 3r^2}{\dfrac{20g\,\Delta z}{3r^2} + \dfrac{\omega^4 9r^4}{400g^2} + r^2} \end{bmatrix}$$

RESULTS AND DISCUSSION

Discussion 1 : Observing Angles experimentally

As theoretically calculated the connection between in apex angle, inclination and angle between two rails with the height of the cone and its radius, we can make some predictions, we know that,

$$\tan\theta < \tan\beta \tan\varphi$$

$$\theta - inclination$$

$$\beta - half\ of\ apex\ angle$$

$$\varphi - half\ of\ angle\ between\ two\ rails$$

For the experiment, we used a double cone of radius 4.3 cm, of height 12.5. From inverse tangent, we got the half of apex angle β

$$\tan^{-1}\frac{4.3}{12.5} = 18.98$$

$$\beta = 18.98°$$

First experiment shows that with a constant angle of inclination $0{,}034$, by changing the angle of φ we observed that 5° ($\tan\varphi = 0.0874$) does not cause double cone to go uphill.

Figure 14: Demonstration of the angle between two rails

From the inequality of $\tan\theta < \tan\beta\tan\varphi$

$$0{,}098 > \tan\varphi$$

As we expected, double will not go uphill.

Figure 15: Demonstration of double cone does not go up

Second experiment shows that with a constant angle of inclination $0{,}034$ too, by changing the angle of φ we observed that $15°$ ($\tan\varphi = 0.2679$) cause double cone to go uphill.

From the inequality of $\tan\theta < \tan\beta\tan\varphi$

$$0{,}098 < \tan\varphi$$

Figure 16: Demonstration of the angle between two rails

As we expected, double will go uphill. From Figure 17, double cone stays at its first position, and will not go down.

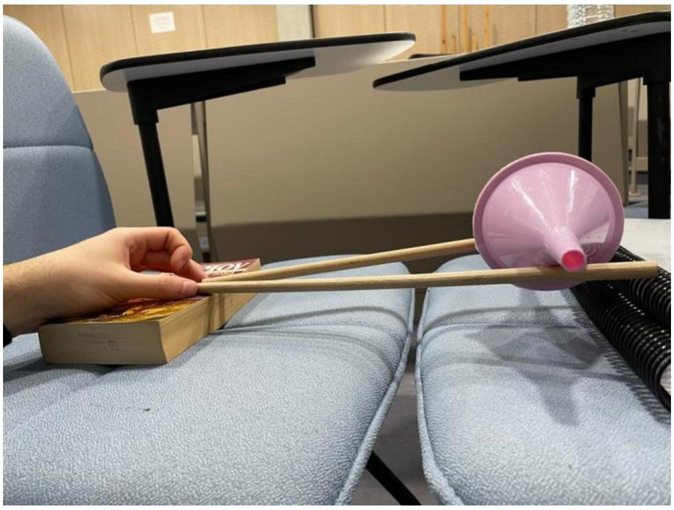

Figure 17: Demonstration of double cone go up, does not go down.

Discussion 2 : The critical angle of inclination that causes double go up

From the inequality $\tan\theta < \frac{R}{H}\frac{\mathrm{K}}{\mathrm{L}}$, with a constant angle of φ $(\frac{\mathrm{K}}{\mathrm{L}})$ which is 10 degrees,

$$\tan\theta < 0{,}031$$

If this inequality is correct, double cone go up. With constant angle between two rails,

First experiment,

Figure 18: Inclination of first experiment

3.6 cm is vertical height and 33 is horizontal distance.

$$\tan \theta = \frac{3.6}{33} = 0{,}109$$

And it does not correct so, double cone will not go up.

Figure 19: Result of first experiment, double cone does not go up.

Second experiment,

Figure 20: Inclination of second experiment

3.1 cm is vertical height and 33 cm is horizontal distance

$$\tan \theta = \frac{3.1}{33} = 0{,}093$$

And it does not correct so, double cone will not go up

Figure 21: Result of second experiment, double cone does not go up.

Third experiment,

As we add some more pages to change the angle of inclination, at last double cone starts to move up,

Figure 22: Inclination of third experiment

1.7 cm is vertical height and 33 cm is horizontal distance,

$$\tan\theta = \frac{1.7}{33} = 0{,}042$$

Value of 0,042 is not smaller than 0,031 but it is close to it, however double cone starts to move up. Because of calculation mistakes and not precise measurements tools, there seems to be a wrong application. But it is negligible.

Discussion 3 : Check experimentally the equation of angular velocity

Let us look at the angular velocity of double cone. By using the principle of energy conservation, we can write,

$$mg\Delta z = \frac{1}{2} I_{cm}\omega^2$$

As Δz is vertical displacement of center of mass and I_{cm} (MR^2) is for hollow double cone that we used in the experiment. The reason for there is no translational energy is because during the motion radius goes to 0. So that from $V = \omega R$, V goes to 0 and at the and we have just got rotational energy. So that ω comes like this,

$$\omega = \sqrt{\frac{4g\Delta z}{R^2}}$$

With constant angle of inclination θ and constant half of the angle between two rails φ, vertical displacement of center of mass is 0.5 cm. Consider that in our system movement is pure rolling thus this prevents the work by friction. So we got,

$$\omega = \sqrt{\frac{4g\Delta z}{R^2}} \approx 10\, rad\ \mathrm{s}^{-1} \approx 1{,}6\, revolution\ \mathrm{s}^{-1}$$

Double cone when its angular velocity is maximum, will make nearly 2 revolutions, The following pictures were obtained by slow motion video recording.

Figure 23: First position of hollow double cone.

Figure 24: Position of hollow double cone after a half turn.

Figure 25: Position of hollow double cone after a one turn.

Figure 26: Position of hollow double cone after a one and nearly half turn.

As we saw from the Figure 25 and 26, double cone revolved nearly half turn in almost 0.4s, like 3/8 turn if we divide double cone as equal 1/8 sections, so that we are able to make this estimation. As a result experimental revolution will be 0,9375 $revolution\ \mathrm{s}^{-1}$. The reason that error is high is because we observed and made an experiment with not very precise measurements, not precise time evaluation and existence of friction.

CONCLUSIONS

In conclusion, we studied ascending double cone on inclined V-rail, which is known as anti-gravity paradox, showed a great phenomenon to demonstrate an illusion which is counter-intuitive in between physical intuition and real world, and explained its nature within principle of dynamics and center of mass. We understood that some actions which may be opposite to our intuition, but they can be explained within basic principles of physics.

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