Data Import

In[ ]:=
```
SetDirectory[
  "C:/Users/serha/OneDrive/Masaüstü/MyRepo/master_thesis_MMT003/210324_disc_time_windows
    _and_OR_model"];
```

In[ ]:=
```
Get["../algoritm_packages/SingleNetworks-algorithm-package.wl"]
(* ?SingleNetworks`* *)
```

In[ ]:=
```
datafull = Import["../data/ccm_manipulated_396096.csv", HeaderLines → 1];
```

In[ ]:=
```
data = TakeList[datafull,
   {39871, 39696, 38791, 40063, 39620, 39311, 39795, 39264, 39974, 39711}];
```

In[ ]:=
```
<< IGraphM`
```

Out[ ]=
```
IGraph/M 0.5.1 (October 12, 2020)
Evaluate IGDocumentation[] to get started.
```

Investigation of Constraints Impact in Time Windows

Defining Fixed Step Size and Fixed Bucket Size

Girvan & Newman Modularity

M.E.J.Newman, Modularity and community structure in networks (2006).

$$Q = \frac{1}{4\,m}\sum_{i\,j}\left(A_{i\,j} - \frac{k_i\,k_j}{2\,m}\right)s_i\,s_j \quad , \quad B_{i\,j} = A_{i\,j} - \frac{k_i\,k_j}{2\,m}$$

$$Q = \frac{1}{4\,m}\,s^T\,B\,s = \frac{1}{4\,m}\sum_{i=1}^{n}\left(u_i^T\,.\,s\right)^2\,\beta_i$$

$$\Delta\,Q = \frac{1}{4\,m}\,s^T\,B^{(g)}\,s \quad , \quad B_{i\,j}^{(g)} = B_{i\,j} - \delta_{i\,j}\sum_{k\in g}B_{i\,k}$$

```
GirvanNewmanmodularity[xx_] :=
 Module[{network, m, Aij, B, u, β, s, Q, div1, div2, moditer, r1,
   r11, r12, r2, r21, r22, subfinal1Q, subfinal2Q, finalQ},
  network = xx;
  m = EdgeCount@network;
  Aij = AdjacencyMatrix@network;
  B = N@(Aij - Outer[Times, VertexDegree@network, VertexDegree@network] / (2 * m));
  u = Eigenvectors@B;
  β = Eigenvalues@B;
  s = (u /. x_ /; x < 0 → -1) /. x_ /; x ≥ 0 → 1;
  Q = (Total@(Table[(u[[i]].Flatten@s[[Flatten@Position[β, Max@β][[1]]]])^2,
        {i, VertexList@network}] * β)) / (4 * m);
  div1 = Flatten@Position[Flatten@s[[Flatten@Position[β, Max@β][[1]]]], _?Negative];
  div2 = Flatten@Position[Flatten@s[[Flatten@Position[β, Max@β][[1]]]], _?Positive];
  moditer[pos_, B1_] := Module[{Bg, ug, βg, sg, deltaQ, divv1, divv2, Bg1, Bg2},
    Bg = (Table[B1[[i, j]] - KroneckerDelta[i, j] * Total@B1[[i, pos]],
        {i, Length@B1}, {j, Length@B1}])[[pos, pos]];
    ug = Eigenvectors@Bg;
    βg = Eigenvalues@Bg;
    sg = (ug /. x_ /; x < 0 → -1) /. x_ /; x ≥ 0 → 1;
    deltaQ =
     (Total@(Table[(ug[[i]].Flatten@sg[[Flatten@Position[βg, Max@βg][[1]]]])^2,
           {i, Range@Length@pos}] * βg)) / (4 * m);
    divv1 = Flatten@Position[Flatten@sg[[Flatten@Position[βg, Max@βg][[1]]]],
       _?Negative];
    divv2 = Flatten@Position[Flatten@sg[[Flatten@Position[βg, Max@βg][[1]]]],
       _?Positive];
    {deltaQ, divv1, divv2, Bg}];
  r1 = moditer[div1, B];
  r11 = If[r1[[1]] > 0.001, moditer[r1[[2]], r1[[4]]], {0, 0, 0, 0}];
  r12 = If[r1[[1]] > 0.001, moditer[r1[[3]], r1[[4]]], {0, 0, 0, 0}];
  r2 = moditer[div2, B];
  r21 = If[r2[[1]] > 0.001, moditer[r2[[2]], r2[[4]]], {0, 0, 0, 0}];
  r22 = If[r2[[1]] > 0.001, moditer[r2[[3]], r2[[4]]], {0, 0, 0, 0}];
  subfinal1Q = If[r1[[1]] > 0.001, If[r11[[1]] > 0.001,
     If[r12[[1]] > 0.001, r12[[1]] + r11[[1]] + r1[[1]], r11[[1]] + r1[[1]]],
     If[r12[[1]] > 0.001, r12[[1]] + r1[[1]], r1[[1]]]], 0];
  subfinal2Q = If[r2[[1]] > 0.001, If[r21[[1]] > 0.001,
     If[r22[[1]] > 0.001, r22[[1]] + r21[[1]] + r2[[1]], r21[[1]] + r2[[1]]],
     If[r22[[1]] > 0.001, r22[[1]] + r2[[1]], r2[[1]]]], 0];
  finalQ = Q + subfinal1Q + subfinal2Q]
```

Assortativity Modularity (Wolfram)

$$Q = \frac{1}{2\,m} \sum_{i\,j} \left(A_{i\,j} - \frac{k_i\,k_j}{2\,m} \right) \delta_{C_i\,C_j}$$

where m is the number of edges,
A is the adjacency matrix,
k_i is the degree of node i,
c_i is the community that node i belongs to,
$\delta_{i\,j}$ is the Kronecker δ symbol.

For weighted graphs,
A is the weighted adjacency matrix,
k_i are the sum of weights of edges incident on node i,
m is the sum of all weights.

```
GraphAssortativity[graph, communities, "Normalized" → False]
```

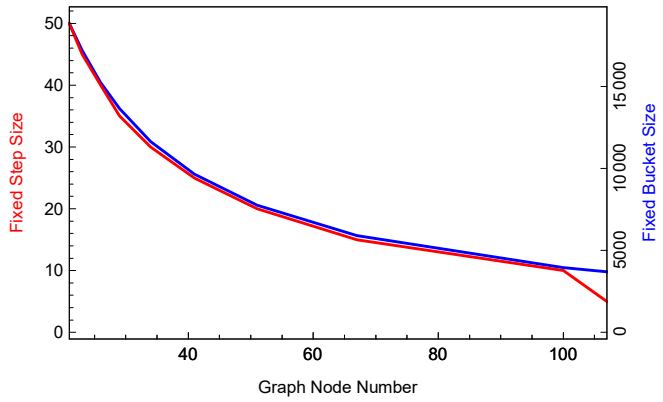
Modularity Change Across Varying Step & Bucket Sizes

```
In[ ]:= AbsoluteTiming[widthdatavaryingFixedstep =
          Table[snetworkdatabinned[9, i, "datafull"], {i, Range[5, 50, 5]}];]
        graphsandnodenumbers = Table[snetworkgraph[widthdatavaryingFixedstep[[i]][[1]],
           widthdatavaryingFixedstep[[i]][[2]], 2, 7, 400, Green], {i, Range@10}];
        GNmodularityvalues = Table[GirvanNewmanmodularity[graphsandnodenumbers[[i]][[1]]],
          {i, Length@graphsandnodenumbers}];
        modularityvalues = Table[N@GraphAssortativity[graphsandnodenumbers[[i]][[1]],
            FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
            "Normalized" → False], {i, Length@graphsandnodenumbers}];

Out[ ]= {302.106, Null}

In[ ]:= AbsoluteTiming[widthdatavaryingFixedbucket =
          Table[snetworkdatafxdbucket[9, i, "datafull"], {i, graphsandnodenumbers[[All, 2]]}];]
        graphsandnodenumbersFixedbucket = Table[snetworkgraph[widthdatavaryingFixedbucket[[i]][[
            1]], widthdatavaryingFixedbucket[[i]][[2]], 2, 7, 400, Green], {i, Range@10}];
        GNmodularityvaluesFixedbucket = Table[GirvanNewmanmodularity[
           graphsandnodenumbersFixedbucket[[i]][[1]]],
          {i, Length@graphsandnodenumbersFixedbucket}];
        modularityvaluesFixedbucket = Table[
          N@GraphAssortativity[graphsandnodenumbersFixedbucket[[i]][[1]],
            FindGraphCommunities[graphsandnodenumbersFixedbucket[[i]][[1]]],
            "Normalized" → False], {i, Length@graphsandnodenumbersFixedbucket}];

Out[ ]= {120.951, Null}

In[ ]:= stepamounts = Range[5, 50, 5]
        nodenumbers = graphsandnodenumbers[[All, 2]]
        bucketamounts = Table[(Values@Counts@widthdatavaryingFixedbucket[[i, 1]])[[1]], {i, 10}]

Out[ ]= {5, 10, 15, 20, 25, 30, 35, 40, 45, 50}

Out[ ]= {107, 100, 67, 51, 41, 34, 29, 26, 23, 21}
```

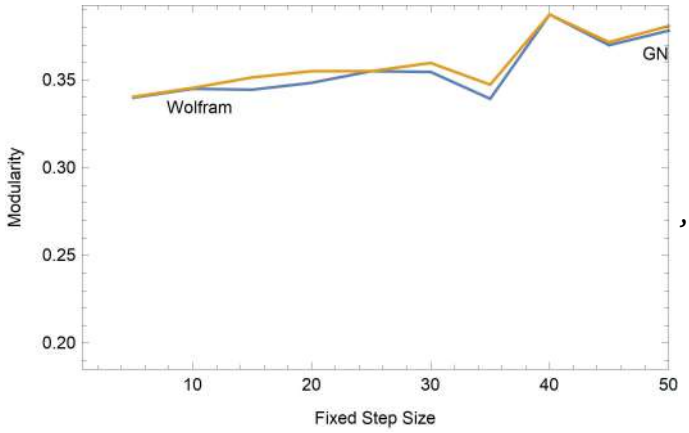
Out[•]= {3702, 3961, 5912, 7767, 9661, 11 650, 13 659, 15 235, 17 222, 18 862}

In[•]:=
```
tickvalues = {{0, 0}, {5000, 5000}, {10 000, 10 000}, {15 000, 15 000}};
ticks = MapAt[Rotate[#, Pi / 2] &, tickvalues, {All, -1}];
{Overlay[{ListLinePlot[Thread[{nodenumbers, bucketamounts}], Frame → True,
    ImagePadding → 35, FrameTicks → {{None, ticks}, {All, None}}, FrameLabel →
     {{None, Style["Fixed Bucket Size", Blue]}, {None, None}}, PlotStyle → Blue,
    ImageSize → 350, PlotRange → {{First@nodenumbers, Last@nodenumbers}, All}],
   ListLinePlot[Thread[{nodenumbers, stepamounts}], Frame → True,
    ImagePadding → 35, FrameTicks → {{All, None}, {All, None}},
    FrameLabel → {"Graph Node Number", Style["Fixed Step Size", Red]}, PlotStyle → Red,
    ImageSize → 350, PlotRange → {{First@nodenumbers, Last@nodenumbers}, All}]}],
 ListLinePlot[{Thread[{stepamounts, GNmodularityvalues}],
   Thread[{stepamounts, modularityvalues}]}, Frame → True,
  FrameLabel → {"Fixed Step Size", "Modularity"}, ImageSize → 350,
  PlotRange → {{First@stepamounts, Last@stepamounts},
    {Min[(MinMax@GNmodularityvalues)[[1]], (MinMax@GNmodularityvaluesFixedbucket)[[
        1]]] - 0.005, Max[(MinMax@GNmodularityvalues)[[2]],
       (MinMax@GNmodularityvaluesFixedbucket)[[2]]] + 0.005}},
  PlotLabels → Placed[{"GN", "Wolfram"}, {Scaled[1], Below}]](*,
 ListLinePlot[Thread[{Range[5,50,5],modularityvalues}],Frame→True,
  FrameLabel→{"Fixed Step Size","Modularity (Wolfram)"},
  ImageSize→350,PlotRange→Automatic]*),
 ListLinePlot[{Thread[{bucketamounts, GNmodularityvaluesFixedbucket}],
   Thread[{bucketamounts, modularityvaluesFixedbucket}]}, Frame → True,
  FrameLabel → {"Fixed Bucket Size", "Modularity"}, ImageSize → 350,
  PlotRange → {{First@bucketamounts, Last@bucketamounts},
    {Min[(MinMax@GNmodularityvalues)[[1]], (MinMax@GNmodularityvaluesFixedbucket)[[
        1]]] - 0.005, Max[(MinMax@GNmodularityvalues)[[2]],
       (MinMax@GNmodularityvaluesFixedbucket)[[2]]] + 0.005}},
  PlotLabels → Placed[{"GN", "Wolfram"}, {Scaled[1], Above}]](*,
 ListLinePlot[Thread[{graphsandnodenumbers[[All,2]],modularityvaluesFixedbucket}],
  Frame→True,FrameLabel→{"Fixed Bucket Size","Modularity (Wolfram)"},
  ImageSize→350,PlotRange→Automatic]*)}
```

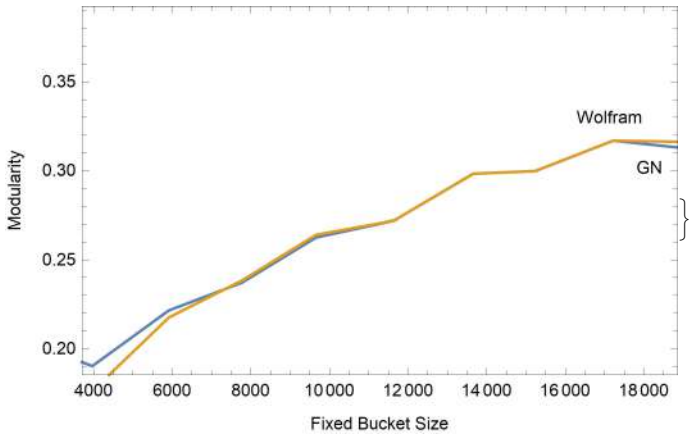
Out[•]=


Fixed Step Size
Fixed Bucket Size
Graph Node Number

,


Modularity
Wolfram
GN
Fixed Step Size

,


Modularity
Wolfram
GN
Fixed Bucket Size

In[•]:=
```
AbsoluteTiming[thicknessdatavaryingFixedstep =
   Table[snetworkdatabinned[10, i, "datafull"], {i, Range@10}];]
graphsandnodenumbers = Table[snetworkgraph[thicknessdatavaryingFixedstep[[i]][[1]],
    thicknessdatavaryingFixedstep[[i]][[2]], 1.5,
    7, 400, RGBColor[0.1, 0.5, 1.]], {i, Range@10}];
GNmodularityvalues = Table[GirvanNewmanmodularity[graphsandnodenumbers[[i]][[1]]],
   {i, Length@graphsandnodenumbers}];
modularityvalues = Table[N@GraphAssortativity[graphsandnodenumbers[[i]][[1]],
     FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
     "Normalized" → False], {i, Length@graphsandnodenumbers}];
```

Out[•]= {93.5853, Null}

In[•]:=
```
AbsoluteTiming[thicknessdatavaryingFixedbucket =
   Table[snetworkdatafxdbucket[10, i, "datafull"], {i, graphsandnodenumbers[[All, 2]]}];]
graphsandnodenumbersFixedbucket = Table[snetworkgraph[
    thicknessdatavaryingFixedbucket[[i]][[1]], thicknessdatavaryingFixedbucket[[i]][[
     2]], 1.5, 7, 400, RGBColor[0.1, 0.5, 1.]], {i, Range@10}];
GNmodularityvaluesFixedbucket = Table[GirvanNewmanmodularity[
    graphsandnodenumbersFixedbucket[[i]][[1]]],
   {i, Length@graphsandnodenumbersFixedbucket}];
modularityvaluesFixedbucket = Table[
   N@GraphAssortativity[graphsandnodenumbersFixedbucket[[i]][[1]],
     FindGraphCommunities[graphsandnodenumbersFixedbucket[[i]][[1]]],
     "Normalized" → False], {i, Length@graphsandnodenumbersFixedbucket}];
```

Out[•]= {818.97, Null}

In[•]:=
```
stepamounts = Range@10
nodenumbers = graphsandnodenumbers[[All, 2]]
bucketamounts =
 Table[(Values@Counts@thicknessdatavaryingFixedbucket[[i, 1]])[[1]], {i, 10}]
```

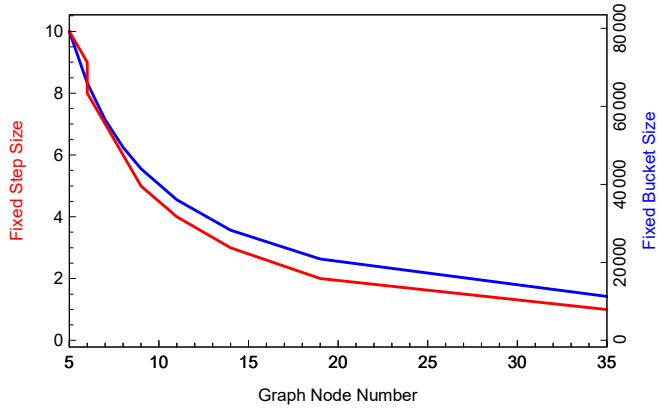
Out[•]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Out[•]= {35, 19, 14, 11, 9, 8, 7, 6, 6, 5}

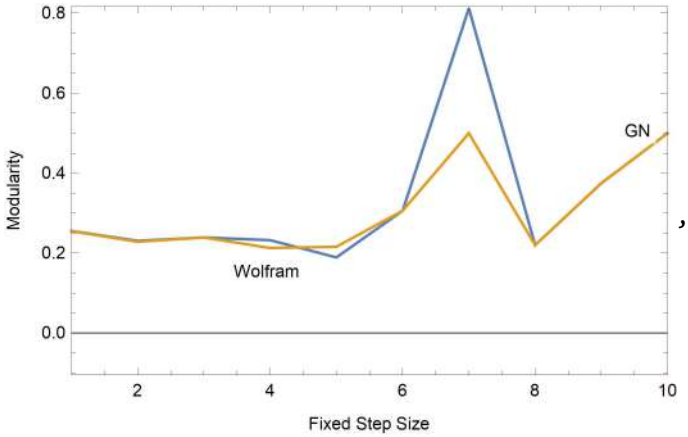
Out[•]= {11 284, 20 832, 28 287, 36 006, 44 008, 49 512, 56 580, 66 016, 66 016, 79 216}

In[ ]:=
```
tickvalues = {{0, 0}, {20 000, 20 000}, {40 000, 40 000}, {60 000, 60 000}, {80 000, 80 000}};
ticks = MapAt[Rotate[#, Pi / 2] &, tickvalues, {All, -1}];
{Overlay[{ListLinePlot[Thread[{nodenumbers, bucketamounts}], Frame → True,
    ImagePadding → 35, FrameTicks → {{None, ticks}, {All, None}}, FrameLabel →
     {{None, Style["Fixed Bucket Size", Blue]}, {None, None}}, PlotStyle → Blue,
    ImageSize → 350, PlotRange → {{First@nodenumbers, Last@nodenumbers}, All}],
   ListLinePlot[Thread[{nodenumbers, stepamounts}], Frame → True,
    ImagePadding → 35, FrameTicks → {{All, None}, {All, None}},
    FrameLabel → {"Graph Node Number", Style["Fixed Step Size", Red]}, PlotStyle → Red,
    ImageSize → 350, PlotRange → {{First@nodenumbers, Last@nodenumbers}, All}]}],
 ListLinePlot[{Thread[{stepamounts, GNmodularityvalues}],
   Thread[{stepamounts, modularityvalues}]}, Frame → True,
  FrameLabel → {"Fixed Step Size", "Modularity"}, ImageSize → 350,
  PlotRange → {{First@stepamounts, Last@stepamounts},
    {Min[(MinMax@GNmodularityvalues)[[1]], (MinMax@GNmodularityvaluesFixedbucket)[[
        1]]] - 0.005, Max[(MinMax@GNmodularityvalues)[[2]],
       (MinMax@GNmodularityvaluesFixedbucket)[[2]]] + 0.005}},
  PlotLabels → Placed[{"GN", "Wolfram"}, {Scaled[1], Below}]],
 ListLinePlot[{Thread[{bucketamounts, GNmodularityvaluesFixedbucket}],
   Thread[{bucketamounts, modularityvaluesFixedbucket}]}, Frame → True,
  FrameLabel → {"Fixed Bucket Size", "Modularity"}, ImageSize → 350,
  PlotRange → {{First@bucketamounts, Last@bucketamounts},
    {Min[(MinMax@GNmodularityvalues)[[1]], (MinMax@GNmodularityvaluesFixedbucket)[[
        1]]] - 0.005, Max[(MinMax@GNmodularityvalues)[[2]],
       (MinMax@GNmodularityvaluesFixedbucket)[[2]]] + 0.005}},
  PlotLabels → Placed[{"GN", "Wolfram"}, {Scaled[1], Above}]]}
```

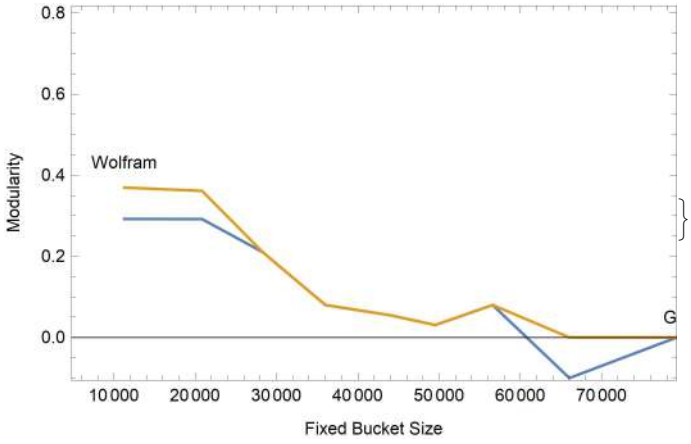
Out[•]=


Fixed Step Size
Fixed Bucket Size
Graph Node Number

,


Modularity
GN
Wolfram
Fixed Step Size

,


Modularity
Wolfram
Fixed Bucket Size

In[•]:=
```
AbsoluteTiming[lengthdatavaryingFixedstep =
   Table[snetworkdatabinned[12, i, "datafull"], {i, Range[400, 4000, 400]}];]
graphsandnodenumbers = Table[snetworkgraph[lengthdatavaryingFixedstep[[i]][[1]],
    lengthdatavaryingFixedstep[[i]][[2]], 2, 7,
    400, RGBColor[1., 0.53, 0.2]], {i, Range@10}];
GNmodularityvalues = Table[GirvanNewmanmodularity[graphsandnodenumbers[[i]][[1]]],
   {i, Length@graphsandnodenumbers}];
modularityvalues = Table[N@GraphAssortativity[graphsandnodenumbers[[i]][[1]],
     FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
     "Normalized" → False], {i, Length@graphsandnodenumbers}];
```

Out[•]= {186.879, Null}

In[•]:=
```
AbsoluteTiming[lengthdatavaryingFixedbucket =
   Table[snetworkdatafxdbucket[12, i, "datafull"], {i, graphsandnodenumbers[[All, 2]]}];]
graphsandnodenumbersFixedbucket = Table[snetworkgraph[
    lengthdatavaryingFixedbucket[[i]][[1]], lengthdatavaryingFixedbucket[[i]][[2]],
    2, 7, 400, RGBColor[1., 0.53, 0.2]], {i, Range@10}];
GNmodularityvaluesFixedbucket = Table[GirvanNewmanmodularity[
    graphsandnodenumbersFixedbucket[[i]][[1]]],
   {i, Length@graphsandnodenumbersFixedbucket}];
modularityvaluesFixedbucket = Table[
   N@GraphAssortativity[graphsandnodenumbersFixedbucket[[i]][[1]],
     FindGraphCommunities[graphsandnodenumbersFixedbucket[[i]][[1]]],
     "Normalized" → False], {i, Length@graphsandnodenumbersFixedbucket}];
```

Out[•]= {89.5098, Null}

In[•]:=
```
stepamounts = Range[400, 4000, 400]
nodenumbers = graphsandnodenumbers[[All, 2]]
bucketamounts =
 Table[(Values@Counts@lengthdatavaryingFixedbucket[[i, 1]])[[1]], {i, 10}]
```

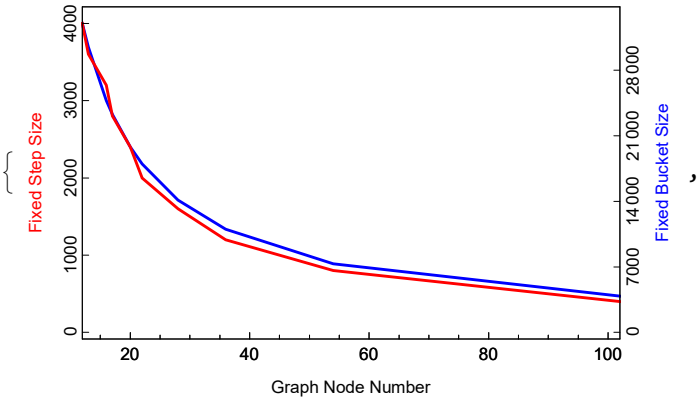
Out[•]= {400, 800, 1200, 1600, 2000, 2400, 2800, 3200, 3600, 4000}

Out[•]= {102, 54, 36, 28, 22, 20, 17, 16, 13, 12}

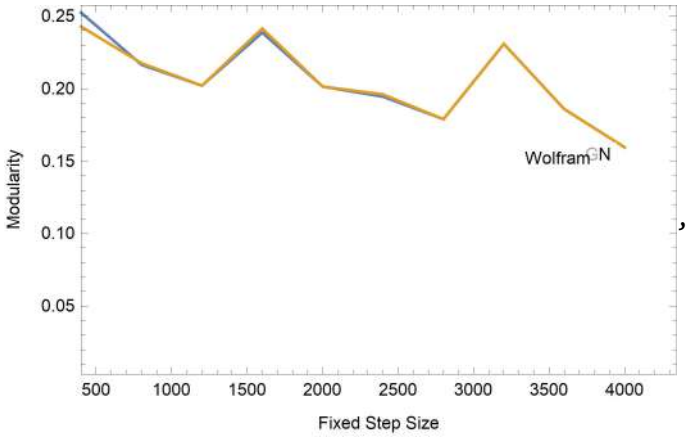
Out[•]= {3884, 7336, 11 003, 14 147, 18 005, 19 805, 23 300, 24 756, 30 469, 33 008}

In[ ]:=
```
tickvalues =
  {{0, 0}, {7000, 7000}, {14000, 14000}, {21000, 21000}, {28000, 28000}, {35000, 35000}};
ticks = MapAt[Rotate[#, Pi / 2] &, tickvalues, {All, -1}];
tickvaluesst = {{0, 0}, {1000, 1000}, {2000, 2000}, {3000, 3000}, {4000, 4000}};
ticksst = MapAt[Rotate[#, Pi / 2] &, tickvaluesst, {All, -1}];
{Overlay[{ListLinePlot[Thread[{nodenumbers, bucketamounts}], Frame → True,
    ImagePadding → 35, FrameTicks → {{None, ticks}, {All, None}}, FrameLabel →
     {{None, Style["Fixed Bucket Size", Blue]}, {None, None}}, PlotStyle → Blue,
    ImageSize → 350, PlotRange → {{First@nodenumbers, Last@nodenumbers}, All}],
   ListLinePlot[Thread[{nodenumbers, stepamounts}], Frame → True,
    ImagePadding → 35, FrameTicks → {{ticksst, None}, {All, None}},
    FrameLabel → {"Graph Node Number", Style["Fixed Step Size", Red]}, PlotStyle → Red,
    ImageSize → 350, PlotRange → {{First@nodenumbers, Last@nodenumbers}, All}]}],
 ListLinePlot[{Thread[{stepamounts, GNmodularityvalues}],
   Thread[{stepamounts, modularityvalues}]}, Frame → True,
  FrameLabel → {"Fixed Step Size", "Modularity"}, ImageSize → 350,
  PlotRange → {{First@stepamounts, Last@stepamounts},
    {Min[(MinMax@GNmodularityvalues)[[1]], (MinMax@GNmodularityvaluesFixedbucket)[[
        1]]] - 0.005, Max[(MinMax@GNmodularityvalues)[[2]],
       (MinMax@GNmodularityvaluesFixedbucket)[[2]]] + 0.005}},
  PlotLabels → Placed[{"GN", "Wolfram"}, {Scaled[1], Below}]],
 ListLinePlot[{Thread[{bucketamounts, GNmodularityvaluesFixedbucket}],
   Thread[{bucketamounts, modularityvaluesFixedbucket}]}, Frame → True,
  FrameLabel → {"Fixed Bucket Size", "Modularity"}, ImageSize → 350,
  PlotRange → {{First@bucketamounts, Last@bucketamounts},
    {Min[(MinMax@GNmodularityvalues)[[1]], (MinMax@GNmodularityvaluesFixedbucket)[[
        1]]] - 0.005, Max[(MinMax@GNmodularityvalues)[[2]],
       (MinMax@GNmodularityvaluesFixedbucket)[[2]]] + 0.005}},
  PlotLabels → Placed[{"GN", "Wolfram"}, {Scaled[1], Above}]]}
```

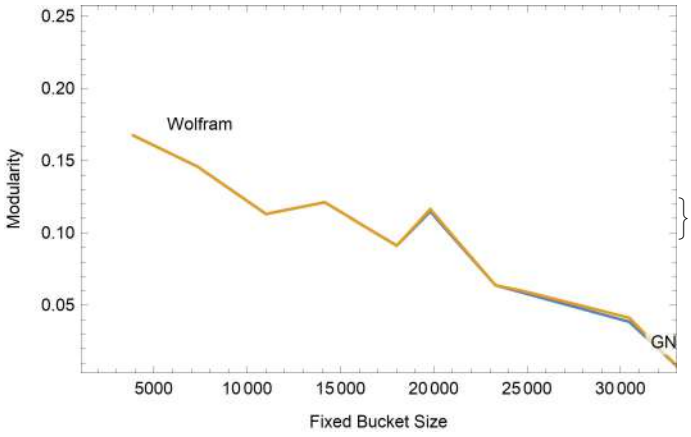
Out[•]=


Fixed Step Size
Fixed Bucket Size
Graph Node Number
0
1000
2000
3000
4000
7000
14000
21000
28000
20
40
60
80
100

,


Modularity
Fixed Step Size
Wolfram
GN
0.05
0.10
0.15
0.20
0.25
500
1000
1500
2000
2500
3000
3500
4000

,


Modularity
Fixed Bucket Size
Wolfram
GN
0.05
0.10
0.15
0.20
0.25
5000
10000
15000
20000
25000
30000

In[ ]:=
```
AbsoluteTiming[weightdatavaryingFixedstep =
   Table[snetworkdatabinned[11, i, "datafull"], {i, Range[200, 2000, 200]}];]
graphsandnodenumbers = Table[snetworkgraph[weightdatavaryingFixedstep[[i]][[1]],
    weightdatavaryingFixedstep[[i]][[2]], 2, 7, 400, Red], {i, Range@10}];
GNmodularityvalues = Table[GirvanNewmanmodularity[graphsandnodenumbers[[i]][[1]]],
   {i, Length@graphsandnodenumbers}];
modularityvalues = Table[N@GraphAssortativity[graphsandnodenumbers[[i]][[1]],
     FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
     "Normalized" → False], {i, Length@graphsandnodenumbers}];
```

Out[ ]= {227.761, Null}

In[ ]:=
```
AbsoluteTiming[weightdatavaryingFixedbucket =
   Table[snetworkdatafxdbucket[12, i, "datafull"], {i, graphsandnodenumbers[[All, 2]]}];]
graphsandnodenumbersFixedbucket = Table[snetworkgraph[weightdatavaryingFixedbucket[[i]][[
     1]], weightdatavaryingFixedbucket[[i]][[2]], 2, 7, 400, Red], {i, Range@10}];
GNmodularityvaluesFixedbucket = Table[GirvanNewmanmodularity[
    graphsandnodenumbersFixedbucket[[i]][[1]]],
   {i, Length@graphsandnodenumbersFixedbucket}];
modularityvaluesFixedbucket = Table[
   N@GraphAssortativity[graphsandnodenumbersFixedbucket[[i]][[1]],
     FindGraphCommunities[graphsandnodenumbersFixedbucket[[i]][[1]]],
     "Normalized" → False], {i, Length@graphsandnodenumbersFixedbucket}];
```

Out[ ]= {91.914, Null}

In[ ]:=
```
stepamounts = Range[200, 2000, 200]
nodenumbers = graphsandnodenumbers[[All, 2]]
bucketamounts =
 Table[(Values@Counts@weightdatavaryingFixedbucket[[i, 1]])[[1]], {i, 10}]
```

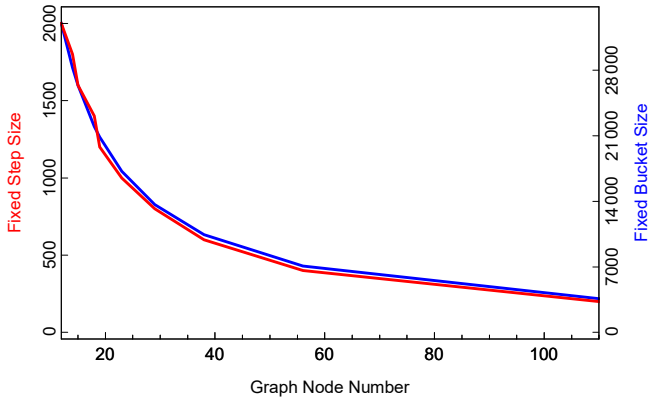
Out[ ]= {200, 400, 600, 800, 1000, 1200, 1400, 1600, 1800, 2000}

Out[ ]= {110, 56, 38, 29, 23, 19, 18, 15, 14, 12}

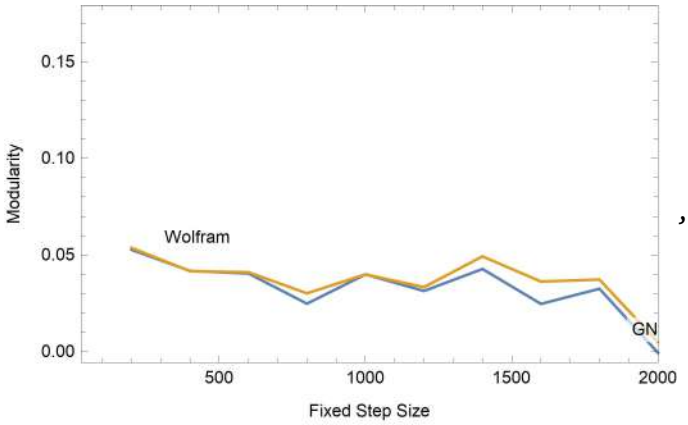
Out[ ]= {3601, 7074, 10 424, 13 659, 17 222, 20 848, 22 006, 26 407, 28 293, 33 008}

In[ ]:=
```
tickvalues =
  {{0, 0}, {7000, 7000}, {14000, 14000}, {21000, 21000}, {28000, 28000}, {35000, 35000}};
ticks = MapAt[Rotate[#, Pi / 2] &, tickvalues, {All, -1}];
tickvaluesst = {{0, 0}, {500, 500}, {1000, 1000}, {1500, 1500}, {2000, 2000}};
tickssst = MapAt[Rotate[#, Pi / 2] &, tickvaluesst, {All, -1}];
{Overlay[{ListLinePlot[Thread[{nodenumbers, bucketamounts}], Frame → True,
    ImagePadding → 35, FrameTicks → {{None, ticks}, {All, None}}, FrameLabel →
     {{None, Style["Fixed Bucket Size", Blue]}, {None, None}}, PlotStyle → Blue,
    ImageSize → 350, PlotRange → {{First@nodenumbers, Last@nodenumbers}, All}],
   ListLinePlot[Thread[{nodenumbers, stepamounts}], Frame → True,
    ImagePadding → 35, FrameTicks → {{tickssst, None}, {All, None}},
    FrameLabel → {"Graph Node Number", Style["Fixed Step Size", Red]}, PlotStyle → Red,
    ImageSize → 350, PlotRange → {{First@nodenumbers, Last@nodenumbers}, All}]}],
 ListLinePlot[{Thread[{stepamounts, GNmodularityvalues}],
   Thread[{stepamounts, modularityvalues}]}, Frame → True,
  FrameLabel → {"Fixed Step Size", "Modularity"}, ImageSize → 350,
  PlotRange → {{First@stepamounts, Last@stepamounts},
    {Min[(MinMax@GNmodularityvalues)[[1]], (MinMax@GNmodularityvaluesFixedbucket)[[
        1]]] - 0.005, Max[(MinMax@GNmodularityvalues)[[2]],
       (MinMax@GNmodularityvaluesFixedbucket)[[2]]] + 0.005}},
  PlotLabels → Placed[{"GN", "Wolfram"}, {Scaled[1], Above}]],
 ListLinePlot[{Thread[{bucketamounts, GNmodularityvaluesFixedbucket}],
   Thread[{bucketamounts, modularityvaluesFixedbucket}]}, Frame → True,
  FrameLabel → {"Fixed Bucket Size", "Modularity"}, ImageSize → 350,
  PlotRange → {{First@bucketamounts, Last@bucketamounts},
    {Min[(MinMax@GNmodularityvalues)[[1]], (MinMax@GNmodularityvaluesFixedbucket)[[
        1]]] - 0.005, Max[(MinMax@GNmodularityvalues)[[2]],
       (MinMax@GNmodularityvaluesFixedbucket)[[2]]] + 0.005}},
  PlotLabels → Placed[{"GN", "Wolfram"}, {Scaled[1], Top}]]}
```

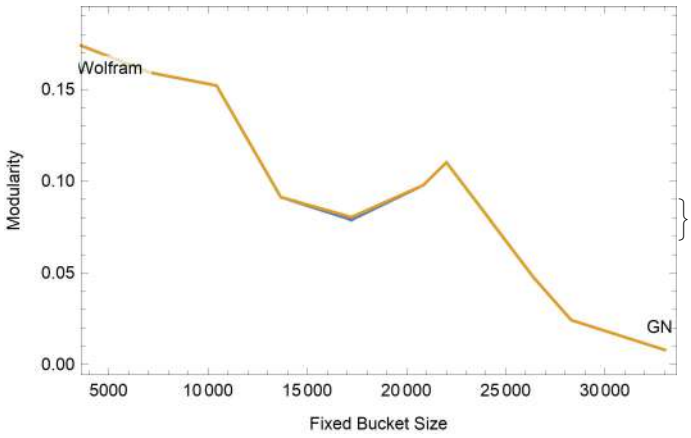
Out[ ]=



,



,

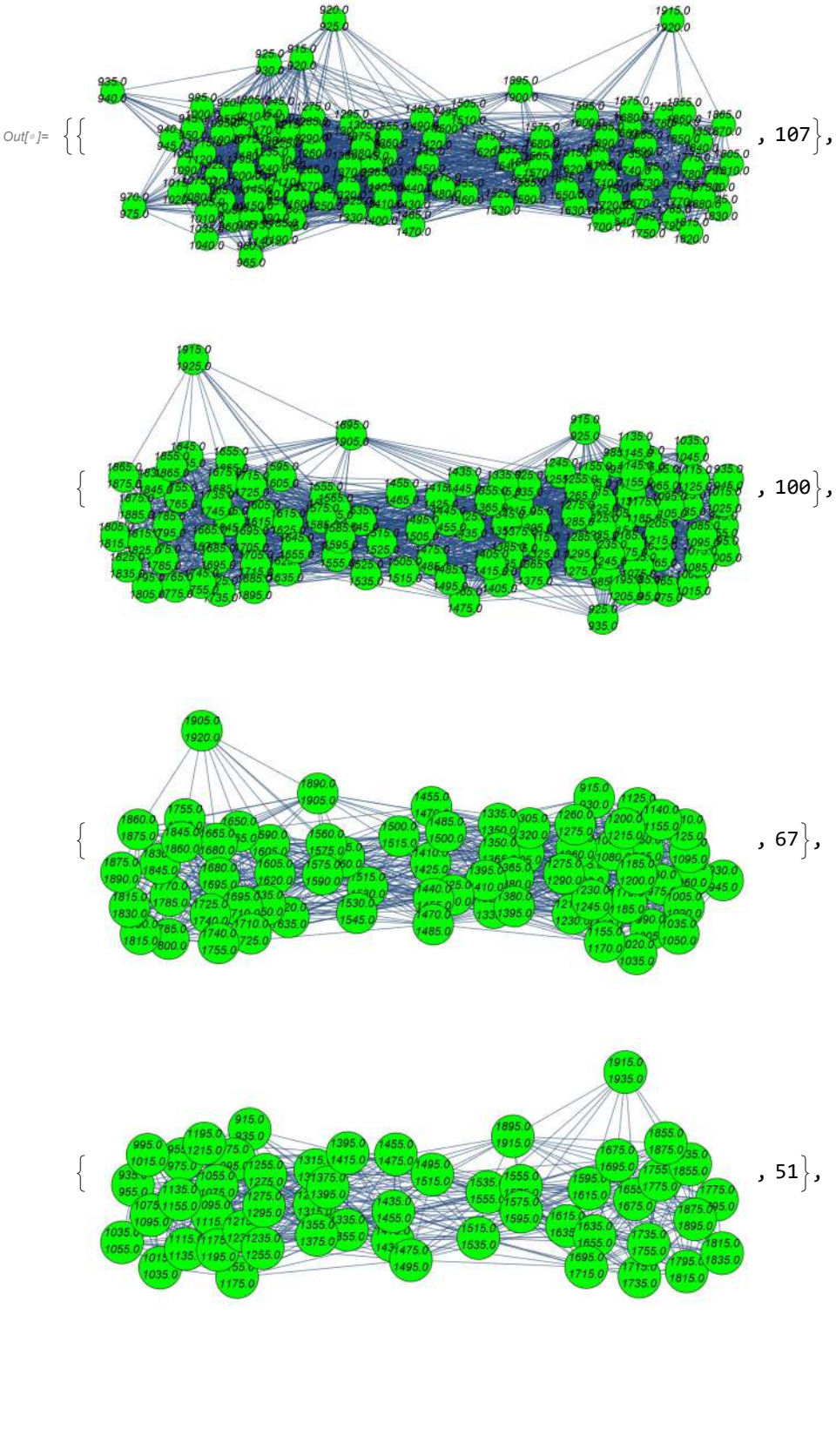


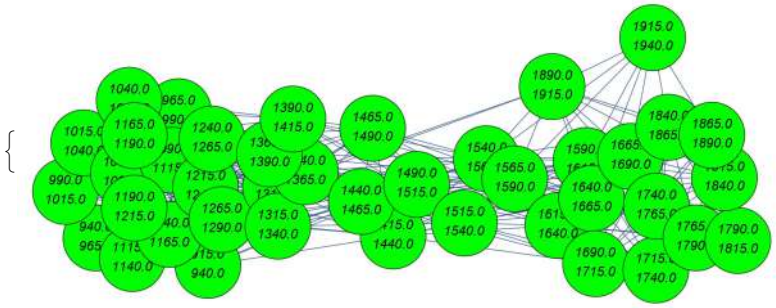
}

Width Graphs for Varying Fixed Step Sizes and Fixed Bucket Sizes

In[ ]:= **graphsandnodenumbers**

Out[•]=

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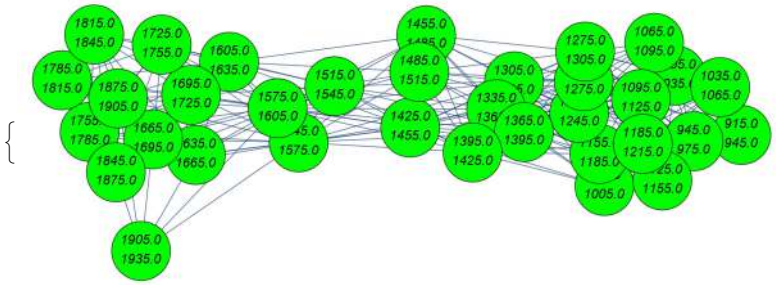
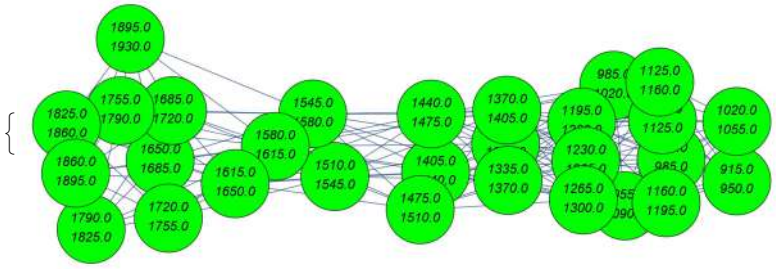
1940.0
1890.0
1915.0
1040.0
1165.0
1190.0
1240.0
1265.0
1390.0
1415.0
1465.0
1490.0
1840.0
1865.0
1890.0
1015.0
990.0
1015.0
1190.0
1215.0
1265.0
1290.0
1315.0
1340.0
1440.0
1465.0
1490.0
1515.0
1540.0
1565.0
1590.0
1640.0
1665.0
1690.0
1715.0
1740.0
1765.0
1790.0
1815.0
940.0
1140.0

, 41},


1815.0
1845.0
1725.0
1755.0
1605.0
1635.0
1785.0
1815.0
1875.0
1905.0
1695.0
1725.0
1575.0
1605.0
1515.0
1545.0
1455.0
1485.0
1515.0
1425.0
1455.0
1305.0
1335.0
1365.0
1395.0
1425.0
1275.0
1305.0
1065.0
1095.0
1035.0
1065.0
1245.0
1125.0
1185.0
1215.0
945.0
975.0
915.0
945.0
1005.0
1155.0
1905.0
1935.0

, 34},


1895.0
1930.0
1825.0
1860.0
1755.0
1790.0
1685.0
1720.0
1650.0
1685.0
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1895.0
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1615.0
1545.0
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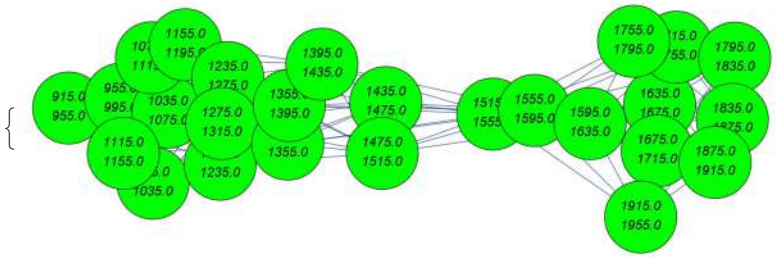
1615.0
1650.0
1510.0
1545.0
1440.0
1475.0
1370.0
1405.0
1405.0
1440.0
1475.0
1510.0
1335.0
1370.0
1195.0
1230.0
1265.0
1300.0
985.0
1020.0
1125.0
1160.0
1125.0
1160.0
1195.0
1020.0
1055.0
915.0
950.0

, 29},


1155.0
1195.0
1235.0
1275.0
1315.0
1355.0
1395.0
1435.0
1035.0
1075.0
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1395.0
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1875.0
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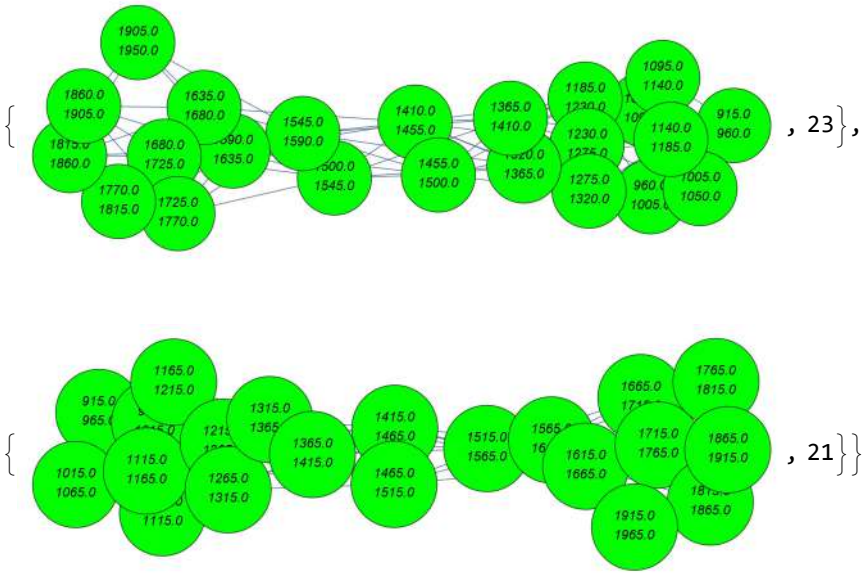
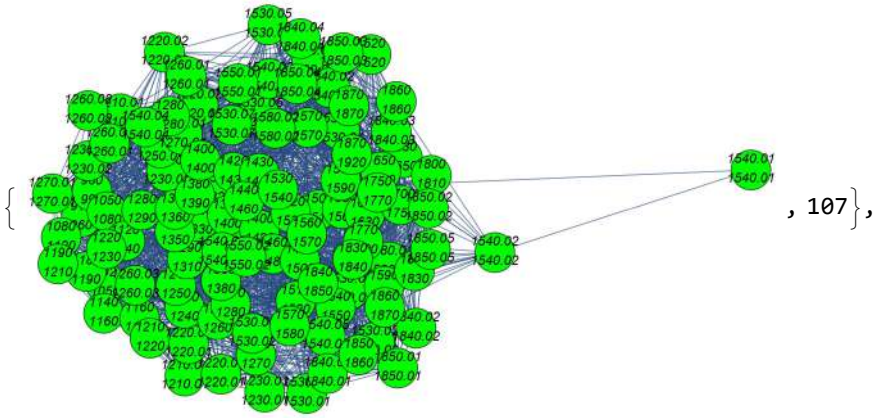
1915.0
1955.0

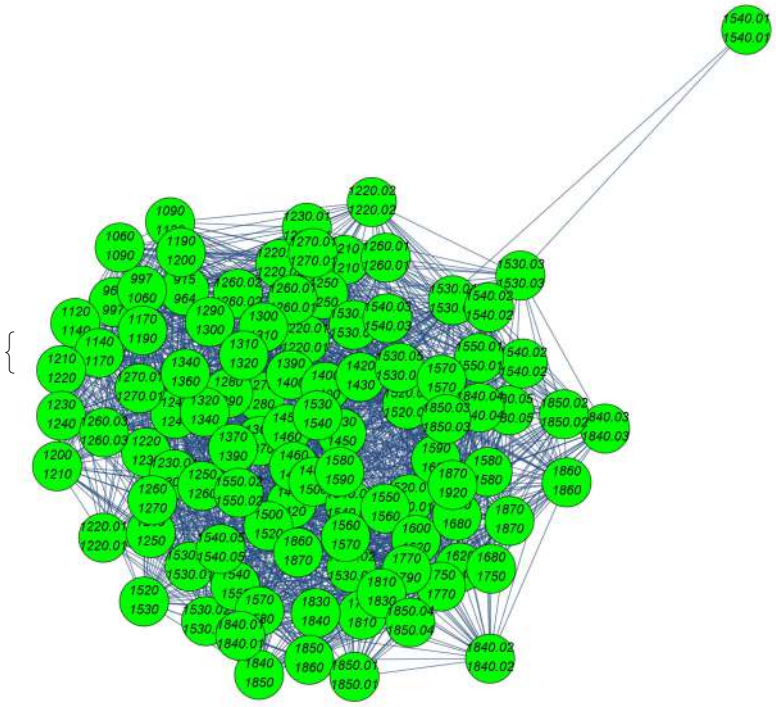
, 26},



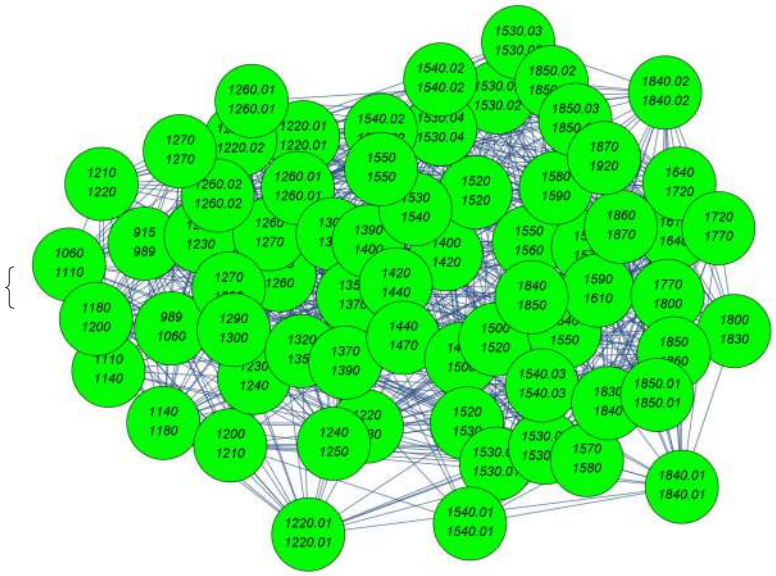
In[ ]:= **graphsandnodenumbersFixedbucket**

Out[ ]=

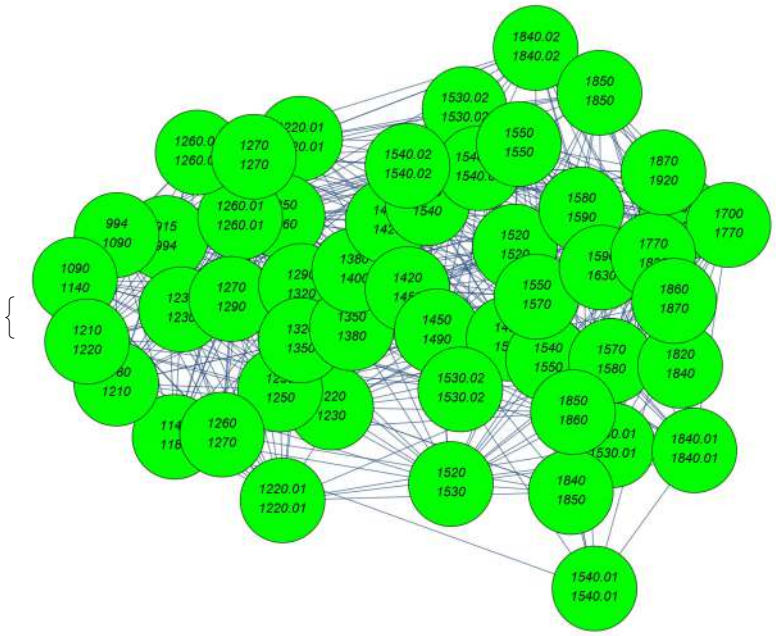


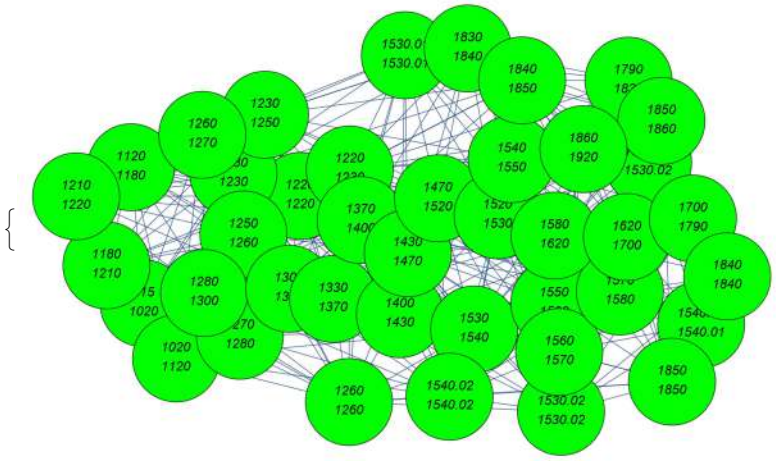


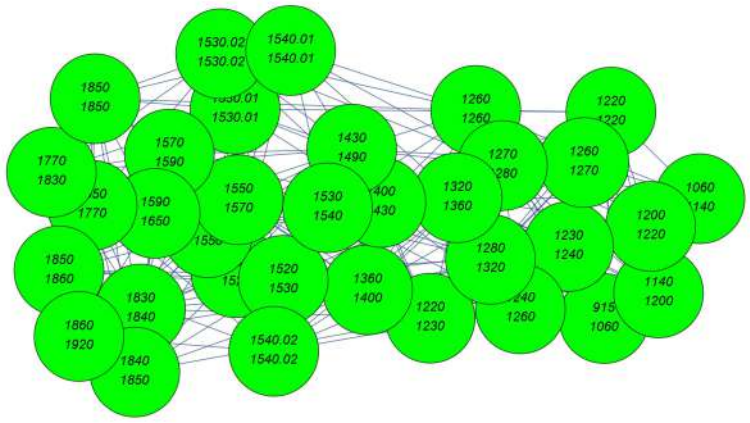
, 100},

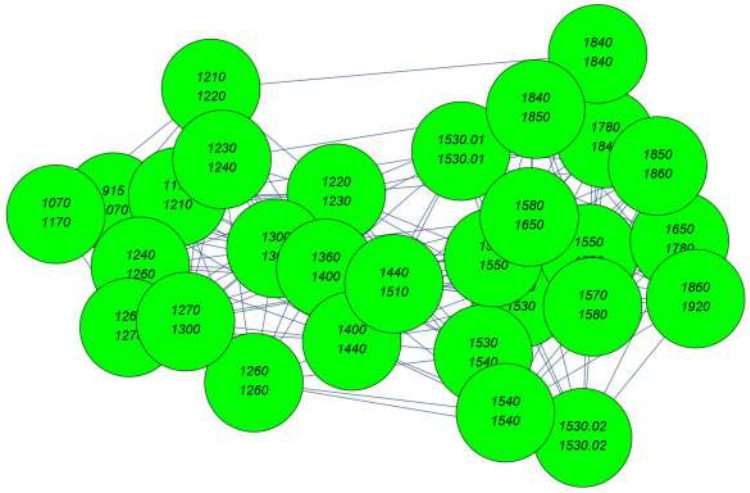


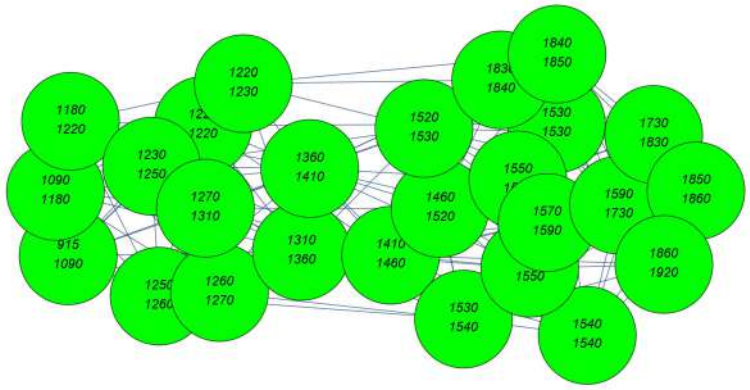
, 67},

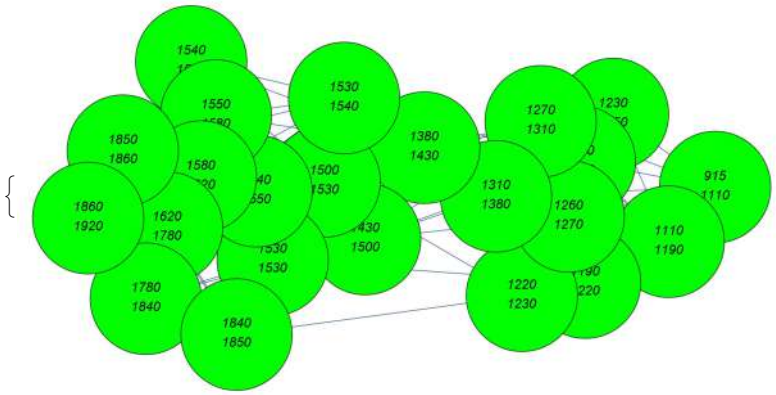
{  , 51},

{  , 41},

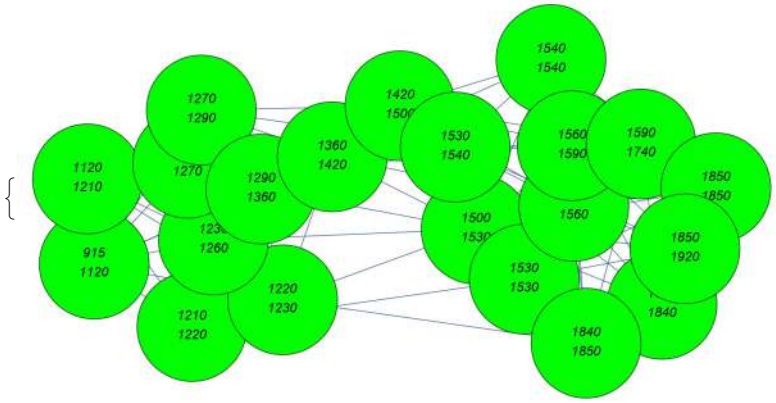
{  , 34},

{  , 29},

{  , 26},



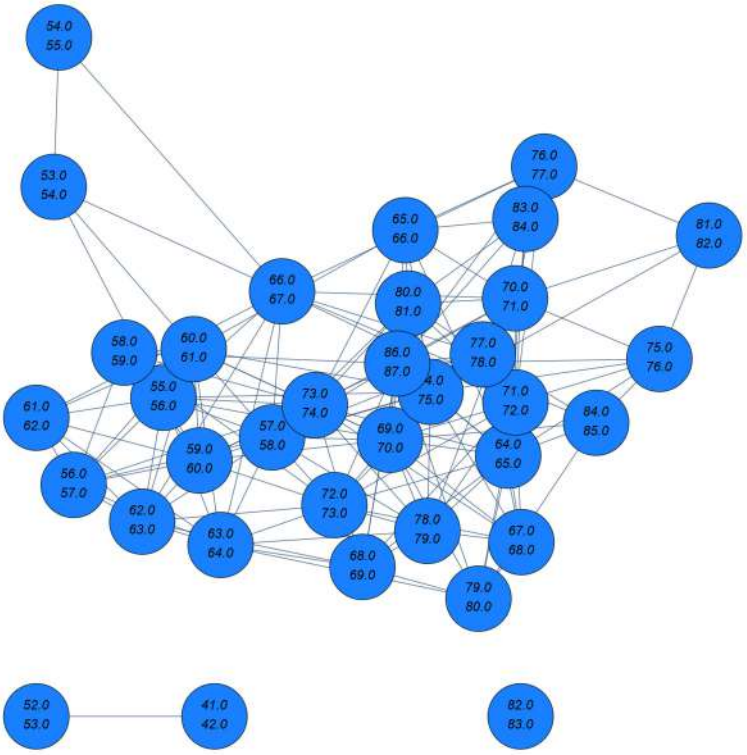
, 23},



, 21}}

Thickness Graphs for Varying Fixed Step Sizes and Fixed Bucket Sizes

In[ ]:= **graphsandnodenumbers**

Out[•]= {{ 
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66.0
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82.0
66.0
67.0
80.0
81.0
70.0
71.0
58.0
59.0
60.0
61.0
86.0
87.0
77.0
78.0
75.0
76.0
55.0
56.0
73.0
74.0
4.0
75.0
71.0
72.0
84.0
85.0
61.0
62.0
57.0
58.0
69.0
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63.0
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83.0
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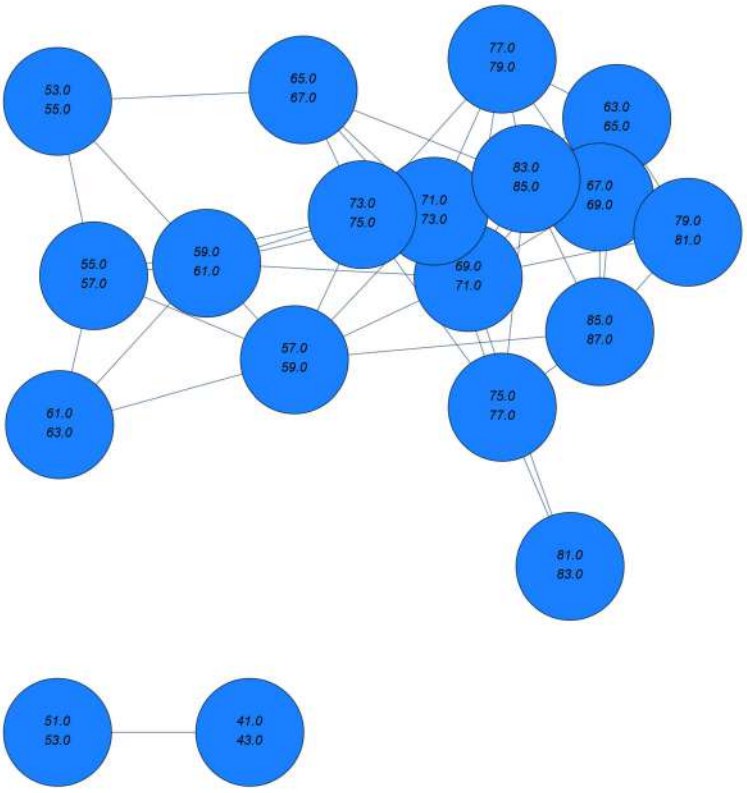
{ 
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79.0
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73.0
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73.0
67.0
69.0
79.0
81.0
55.0
57.0
59.0
61.0
69.0
71.0
85.0
87.0
57.0
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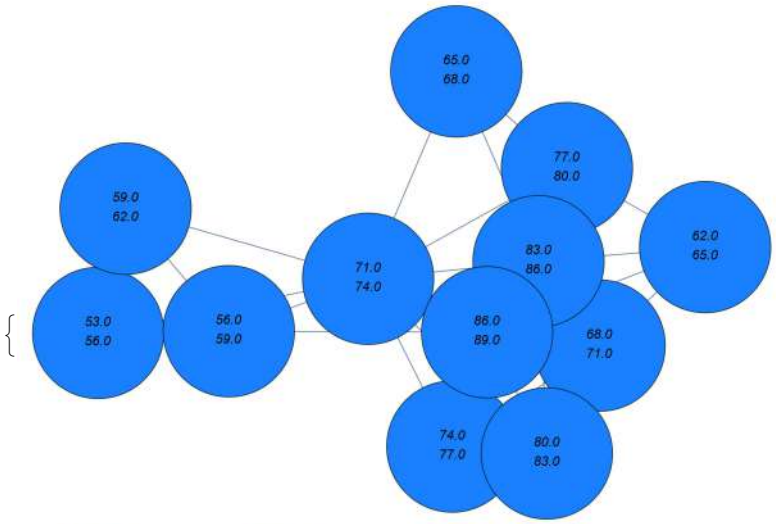
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80.0
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62.0
62.0
65.0
71.0
74.0
83.0
86.0
53.0
56.0
56.0
59.0
86.0
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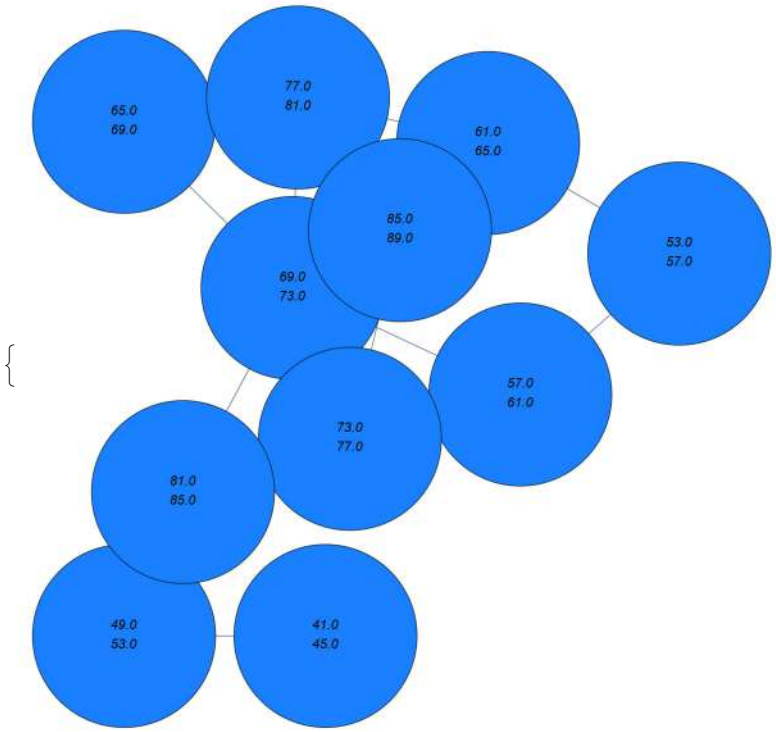
71.0
74.0
77.0
80.0
83.0
50.0
53.0
41.0
44.0

{ , 14},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85.0
89.0
53.0
57.0
69.0
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57.0
61.0
73.0
77.0
81.0
85.0
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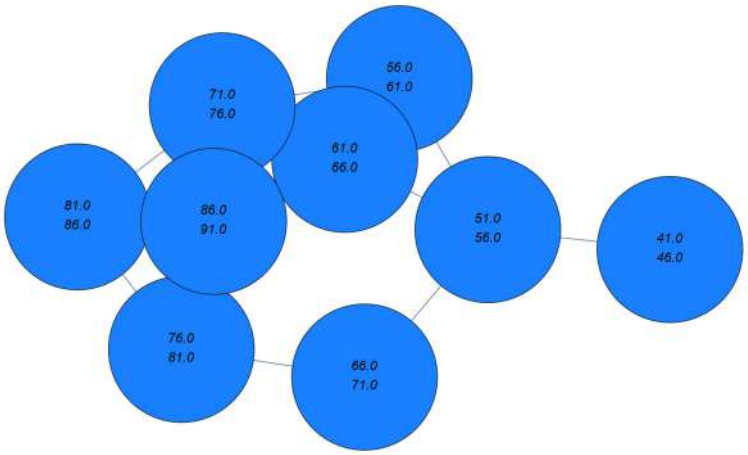
53.0
41.0
45.0

{ , 11},

{ 
71.0
76.0
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66.0
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71.0
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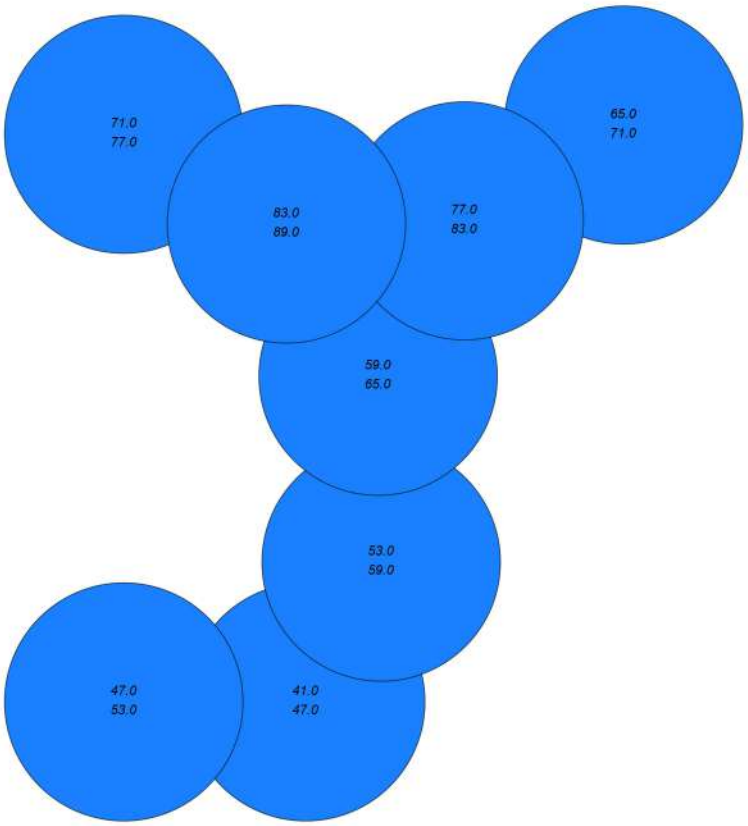
{ 
71.0
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65.0
71.0
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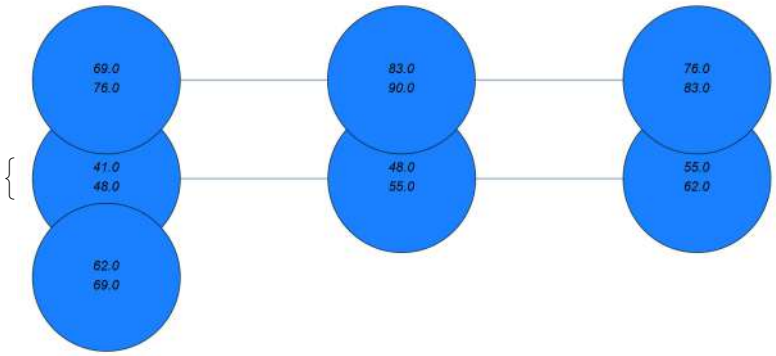
83.0
59.0
65.0
53.0
59.0
47.0
53.0
41.0
47.0
, 8},


69.0
76.0
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83.0
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48.0
48.0
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55.0
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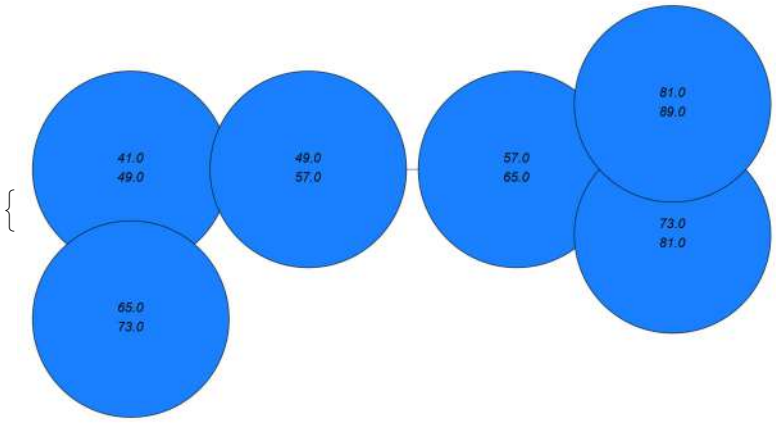
62.0
69.0

{ , 7},


41.0
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49.0
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57.0
65.0
81.0
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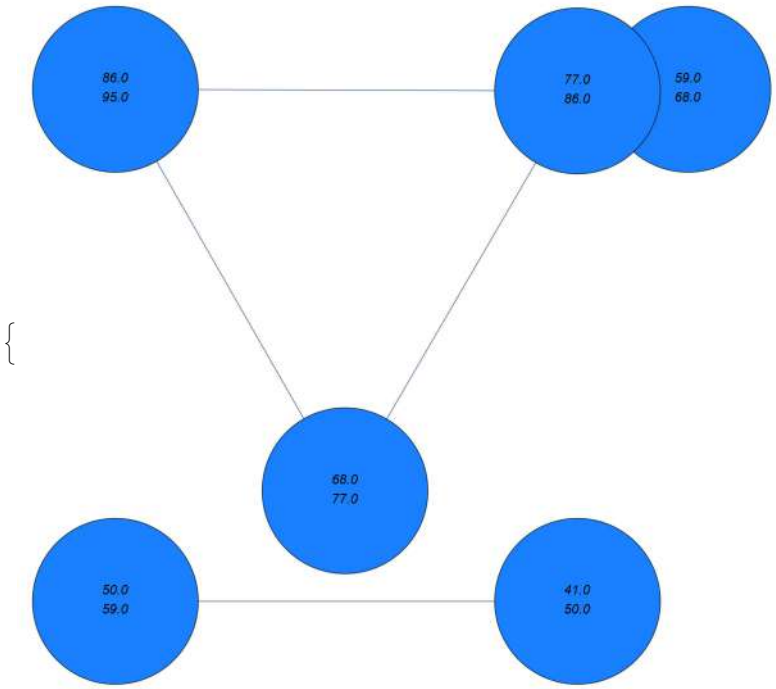
73.0
81.0
65.0
73.0

{ , 6},


86.0
95.0
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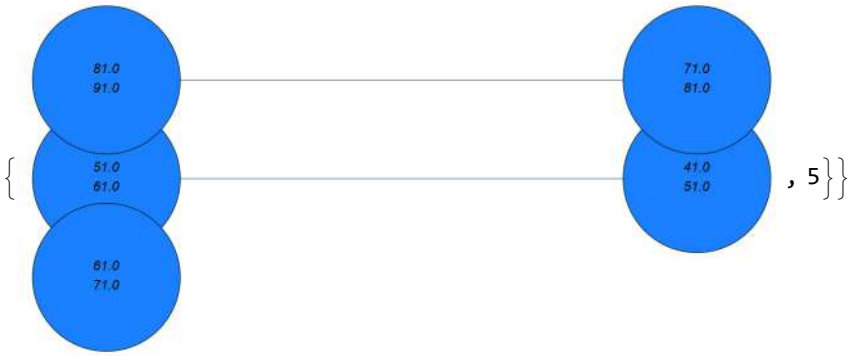
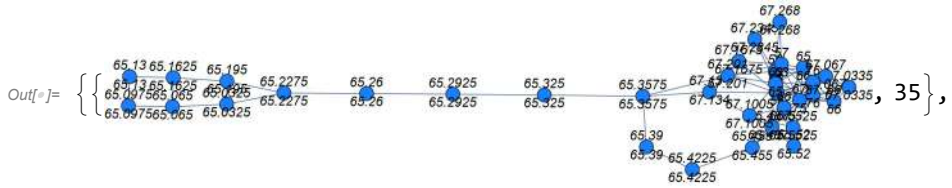
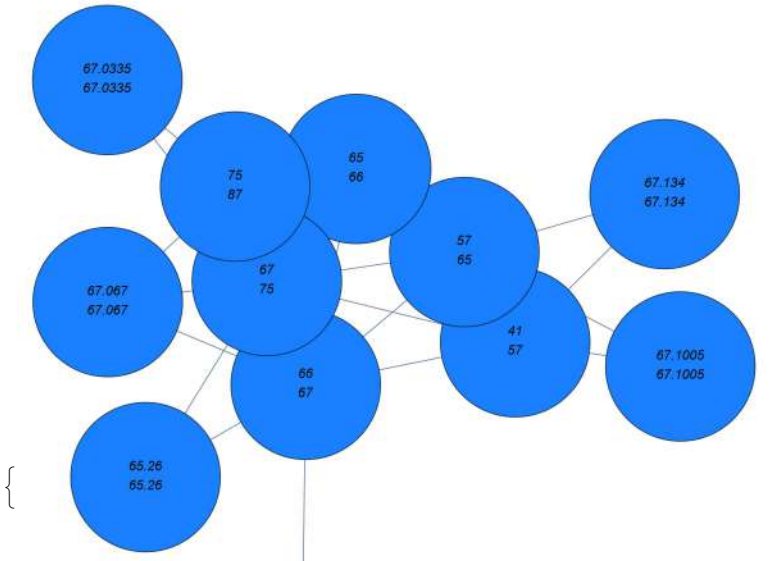
68.0
68.0
77.0
50.0
59.0
41.0
50.0

{ , 6},



In[ ]:= **graphsandnodenumbersFixedbucket**

Out[ ]= 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67.1005
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65.26

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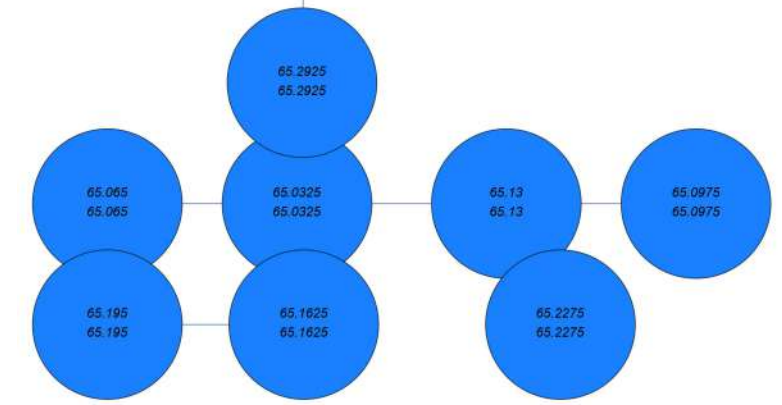
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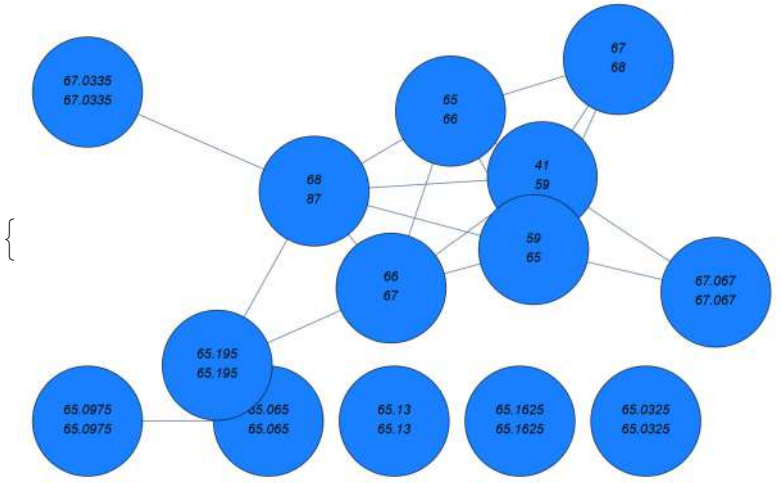
65.2275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65.0325

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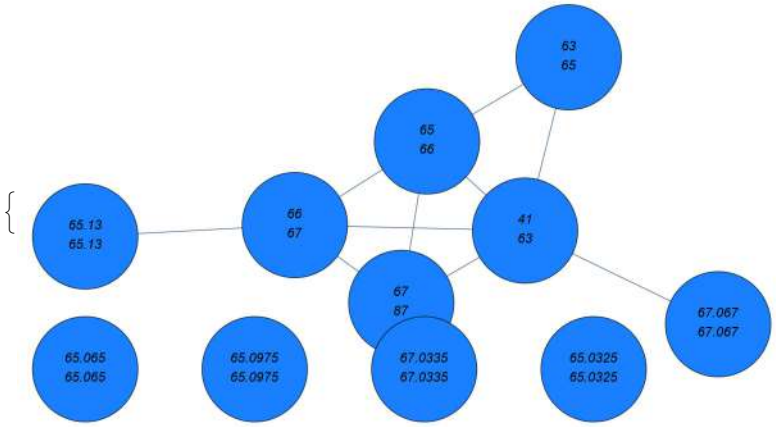
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65.0325

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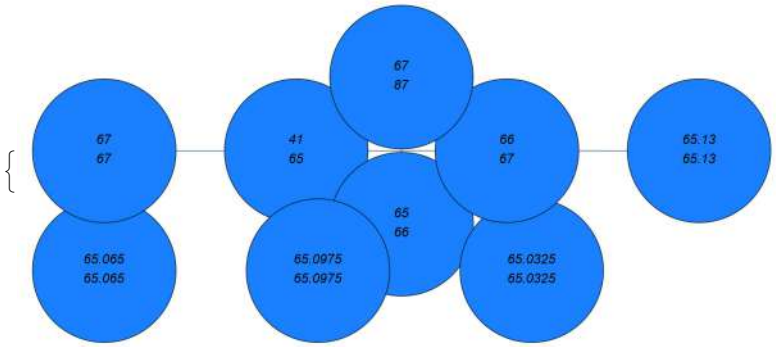
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65.0325

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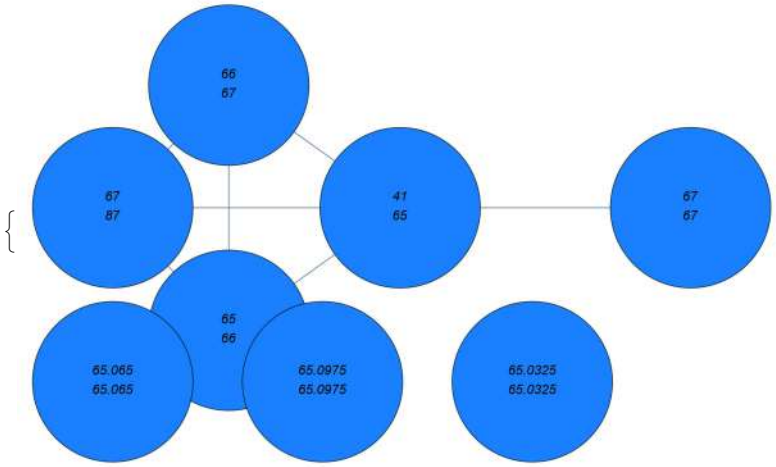
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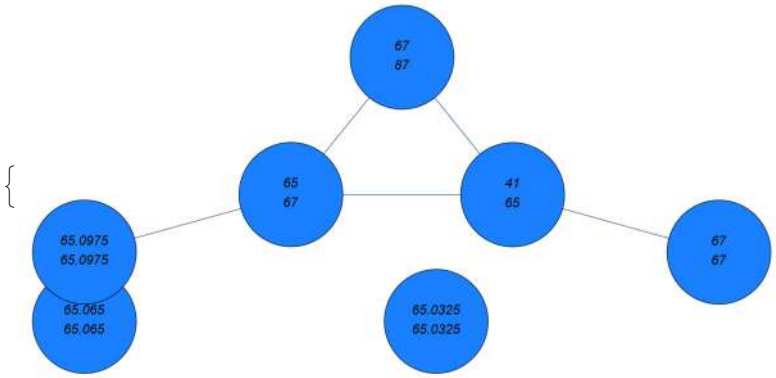
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65.0325

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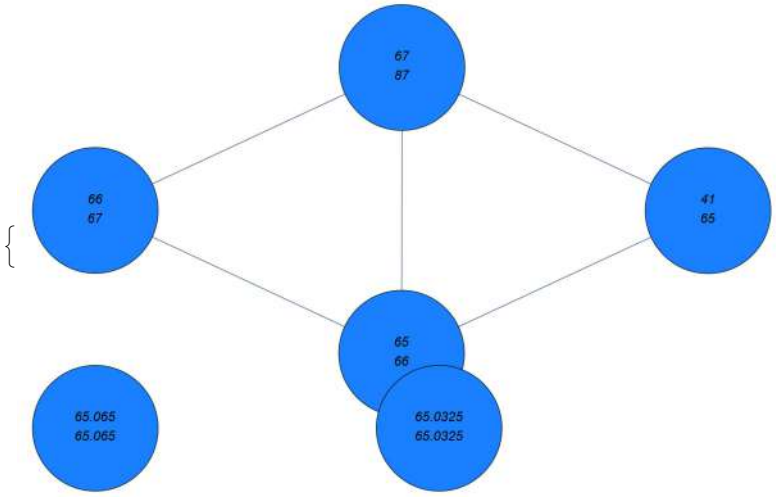
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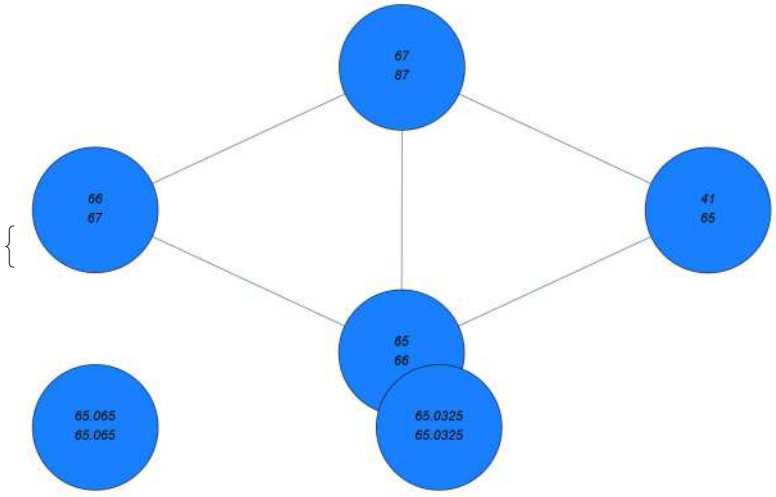
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65.0325

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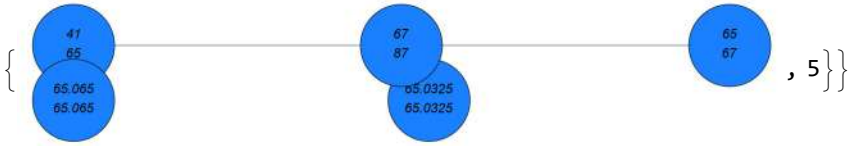

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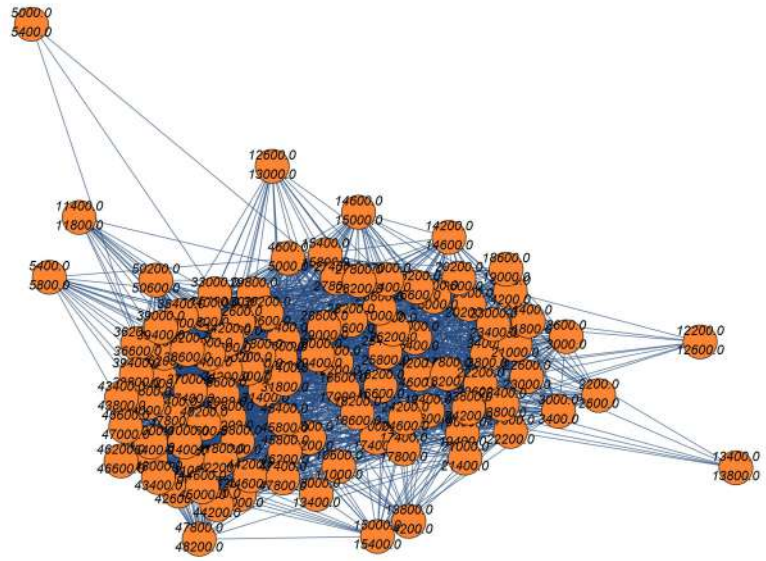
Length Graphs for Varying Fixed Step Sizes and Fixed Bucket Sizes

In[ ]:= **graphsandnodenumbers**

Out[ ]= {{



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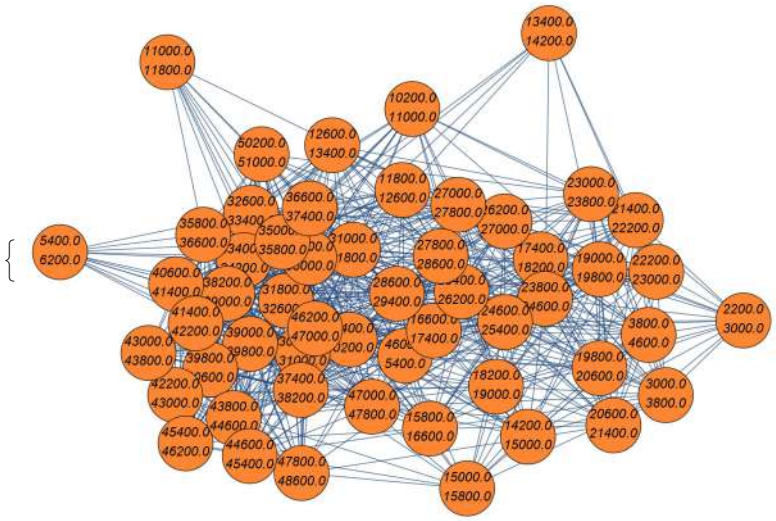

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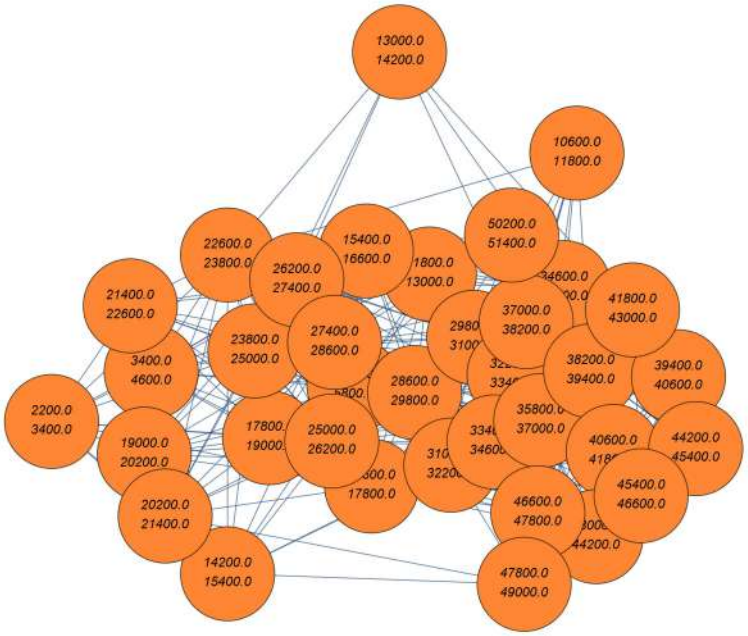
{ 
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19000.0
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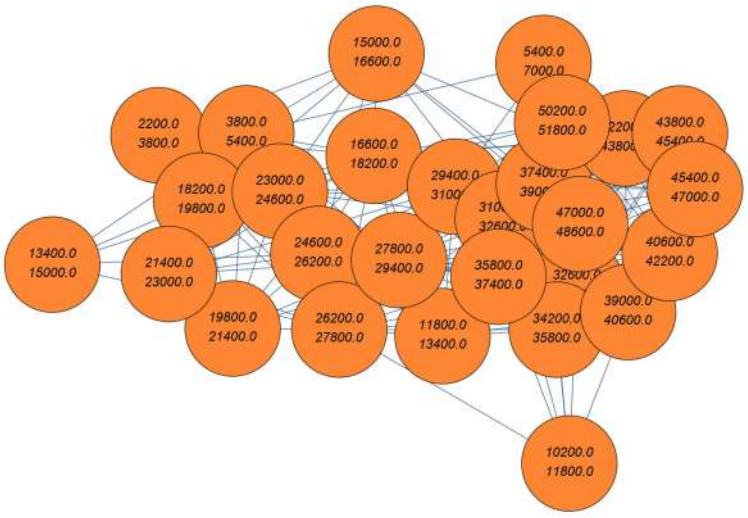
{ 
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19800.0
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24600.0
45400.0
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15000.0
21400.0
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37400.0
39000.0
40600.0
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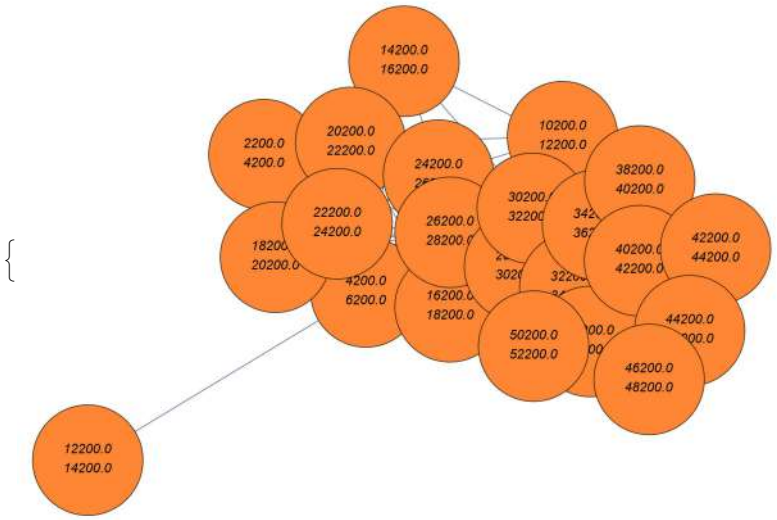
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42200.0
4200.0
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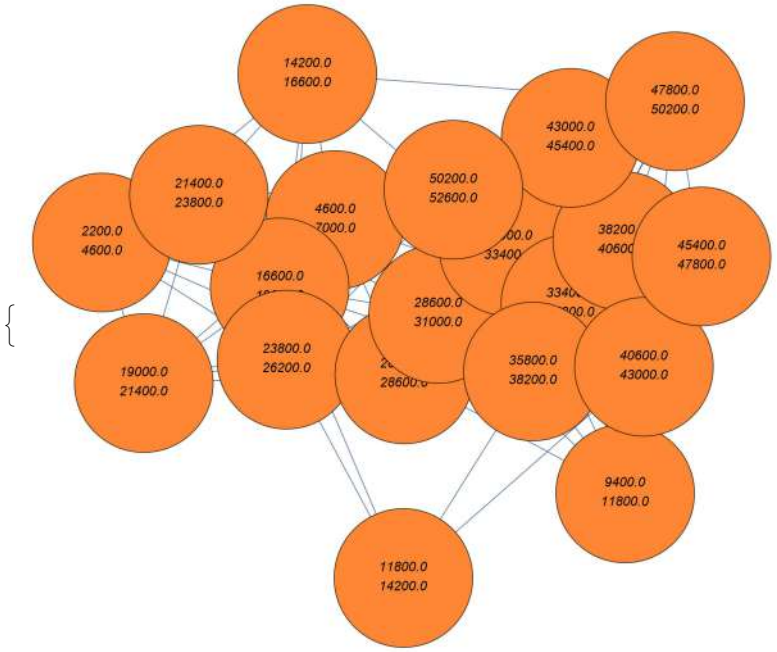
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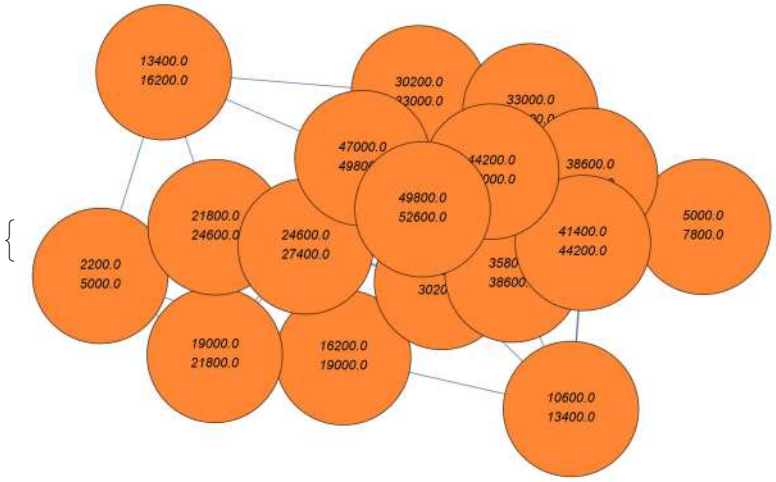
{ 
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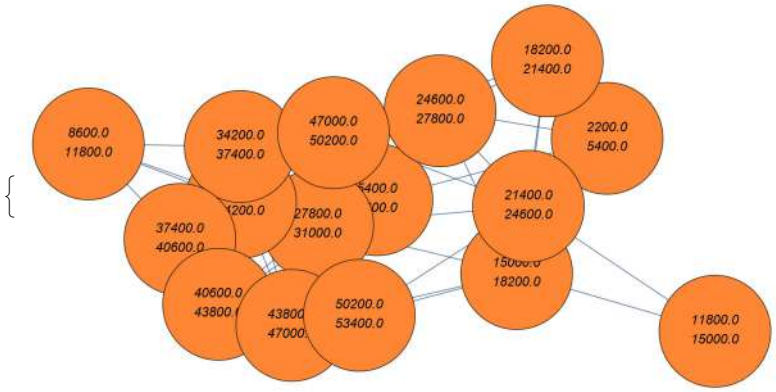
{ 
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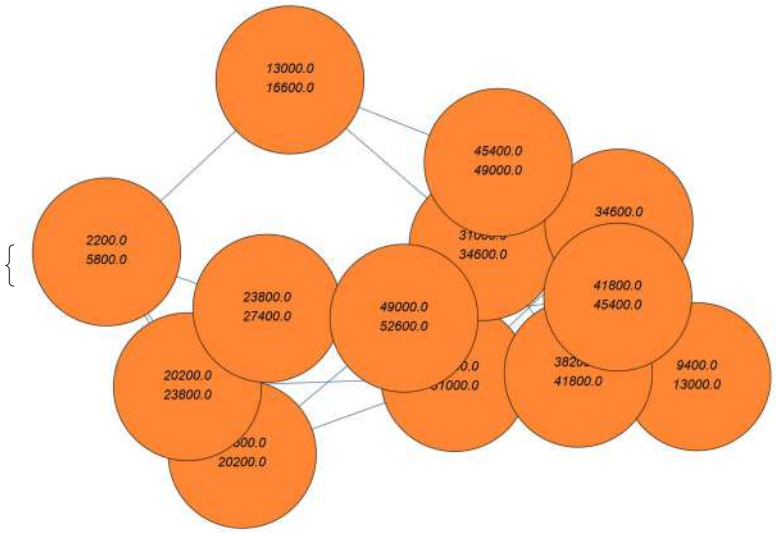
{ 
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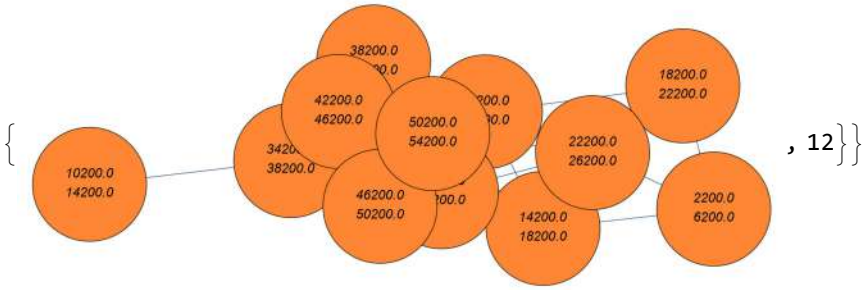

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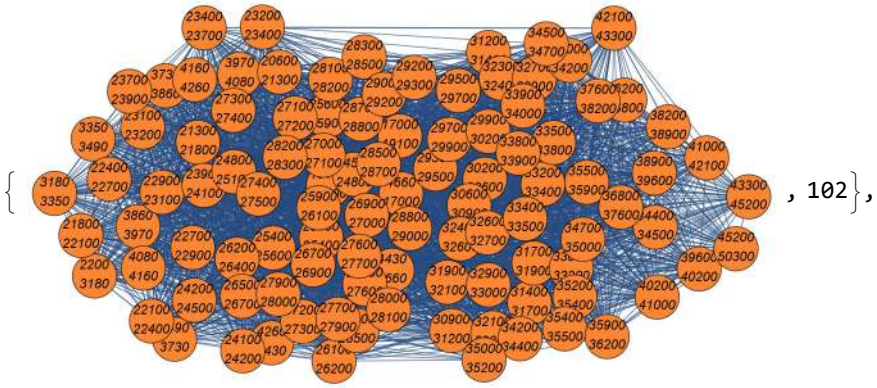
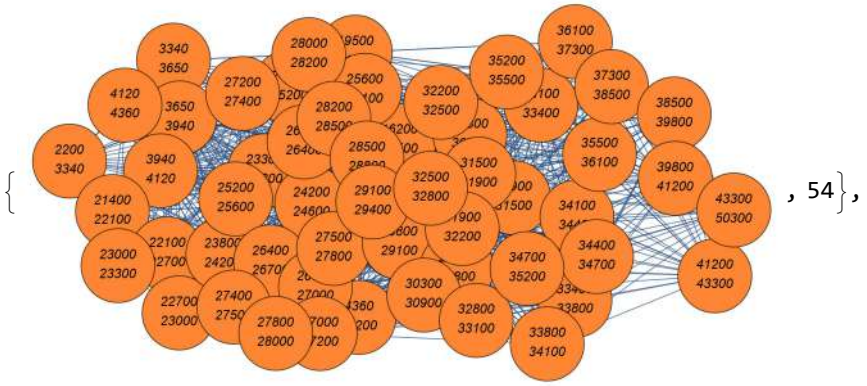
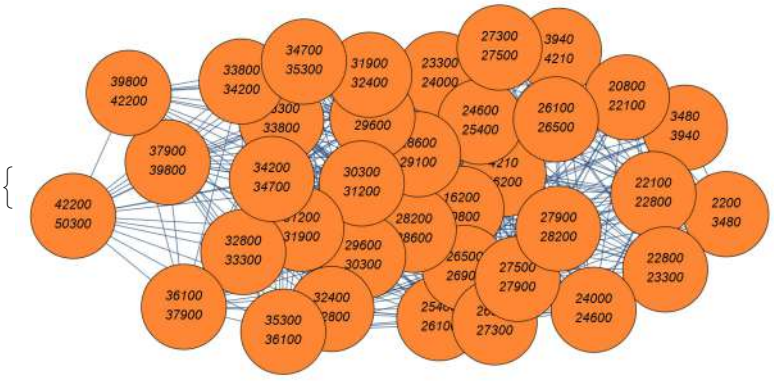
Out[ ]= 

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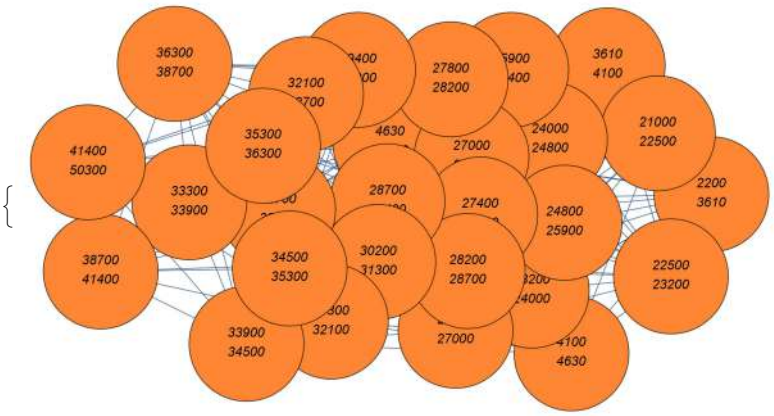
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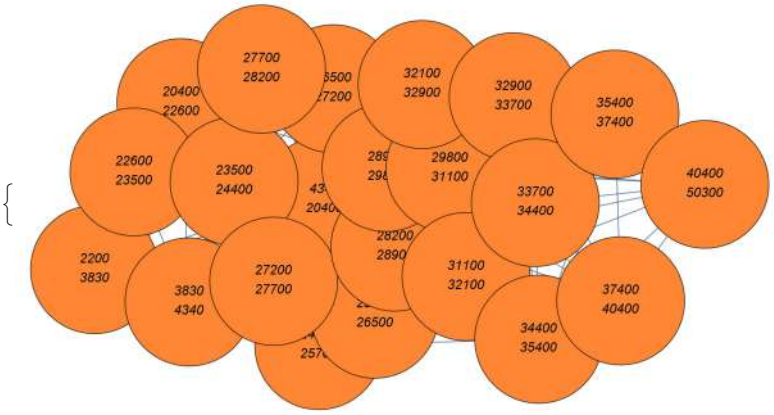
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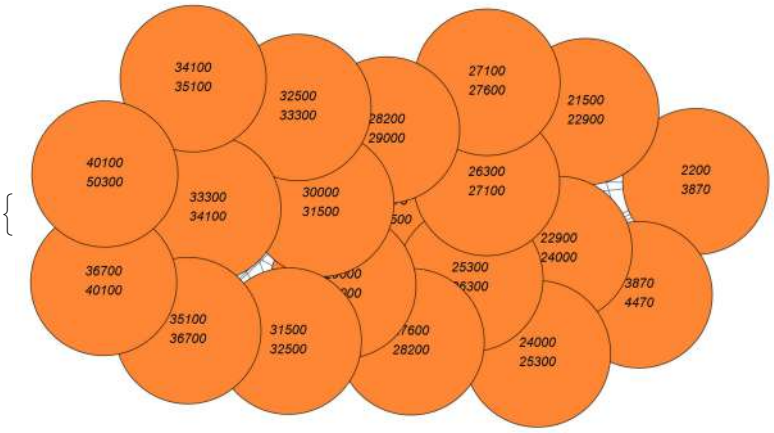
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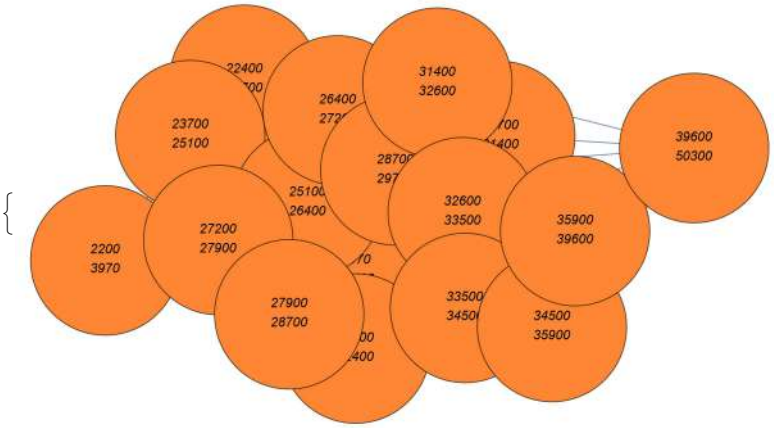
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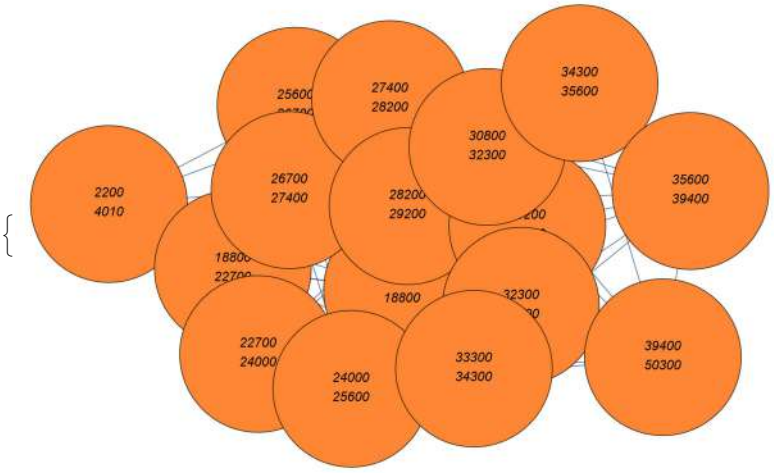
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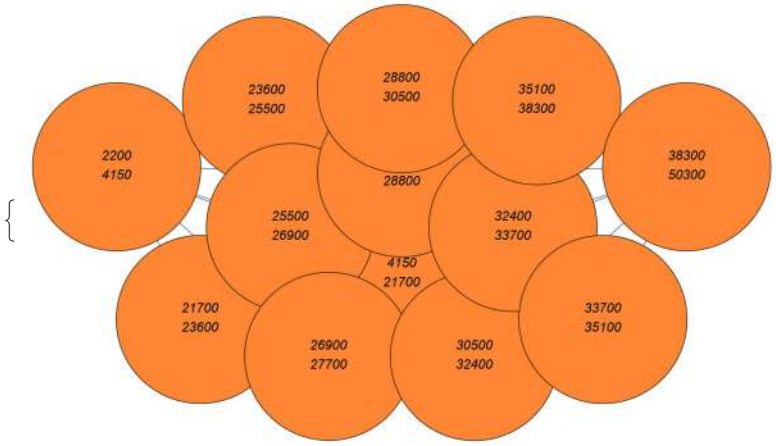

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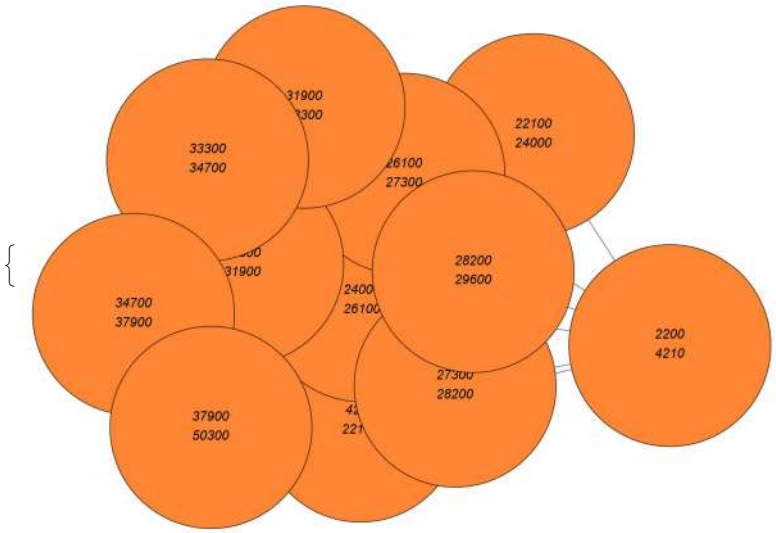

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Weight Graphs for Varying Fixed Step Sizes and Fixed Bucket Sizes

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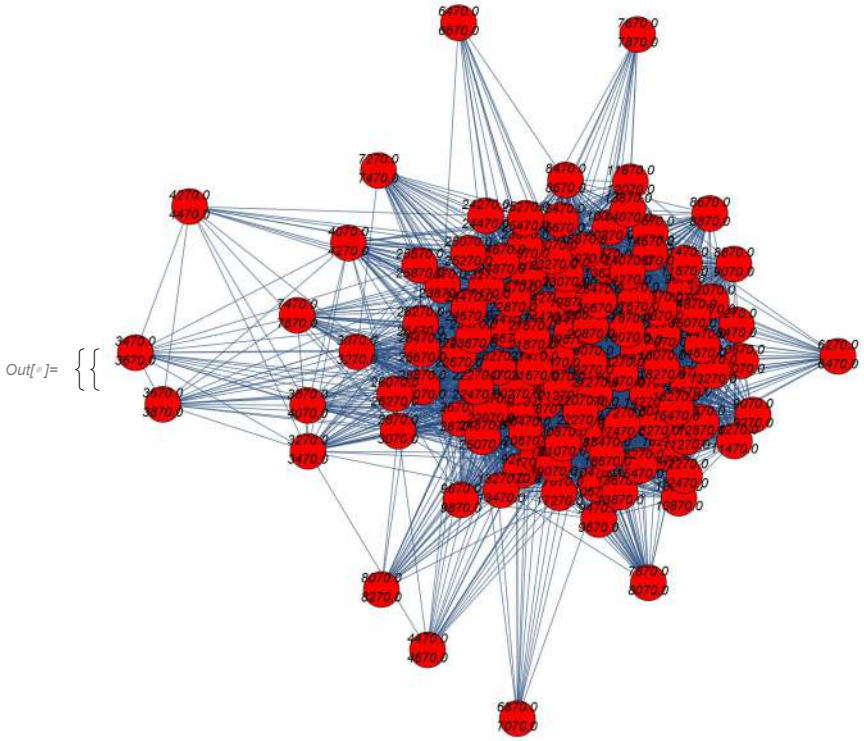
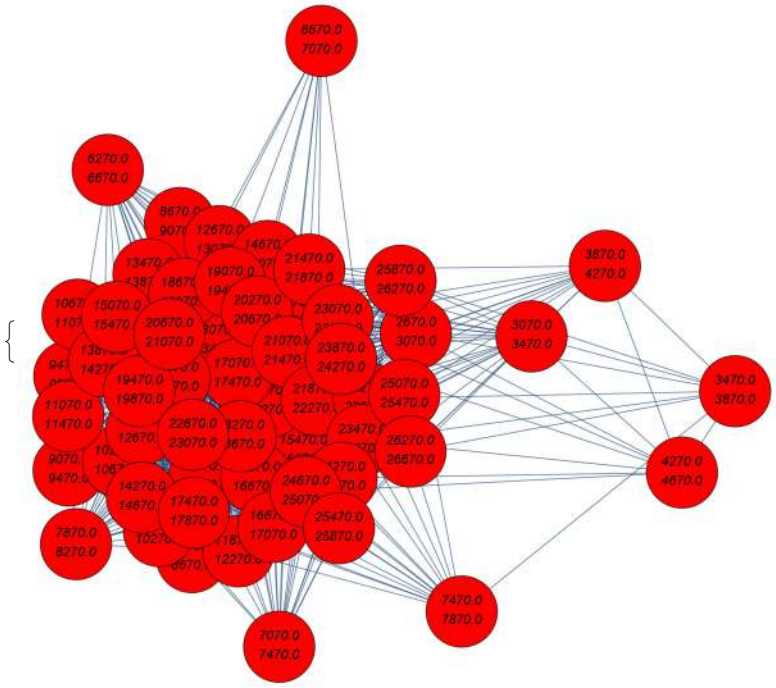
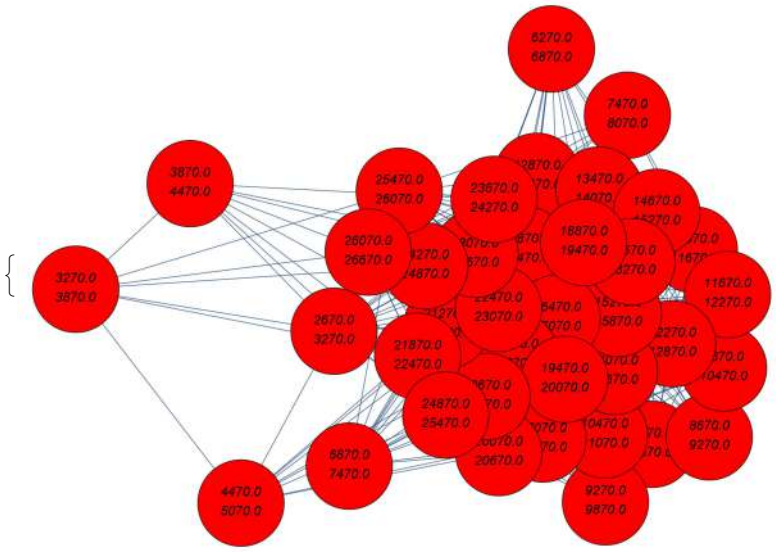
Out[•]=  , 110},

 , 56},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23670.0
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14670.0
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26670.0
18870.0
19470.0
3270.0
3870.0
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2670.0
3270.0
21870.0
22470.0
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9870.0

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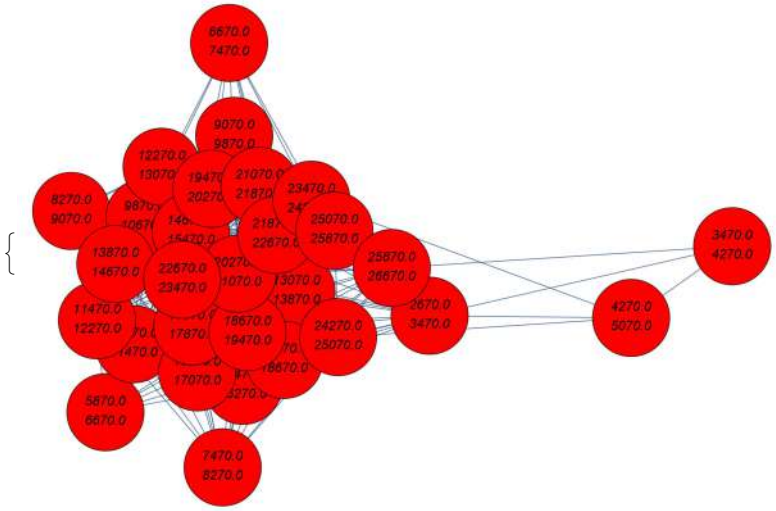
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8270.0

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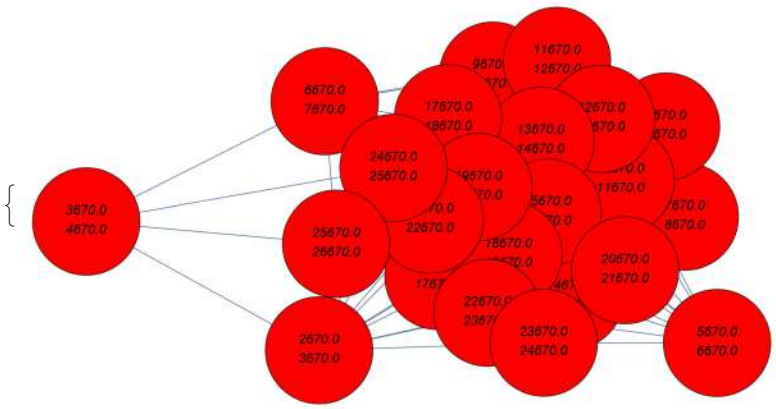
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6670.0

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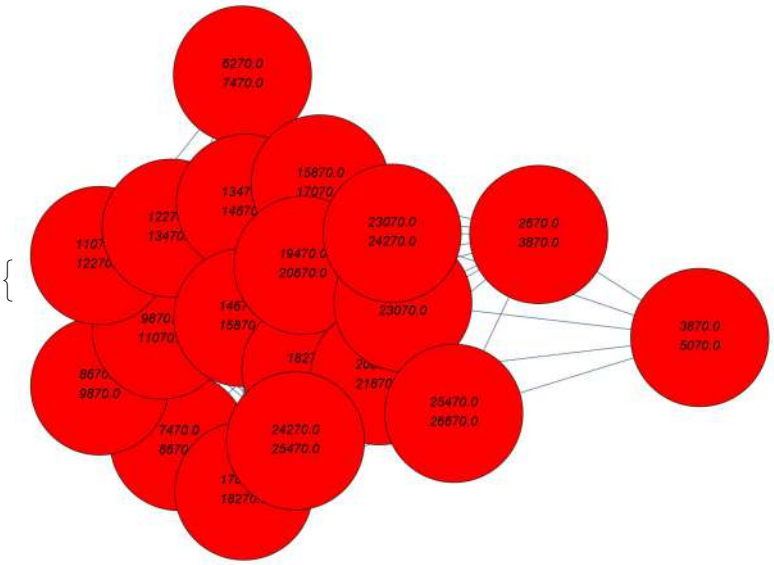
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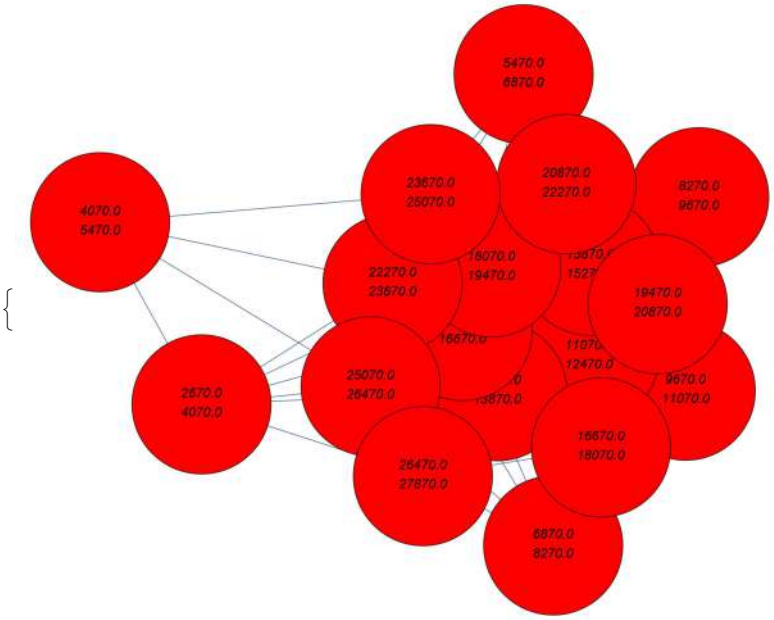
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8270.0

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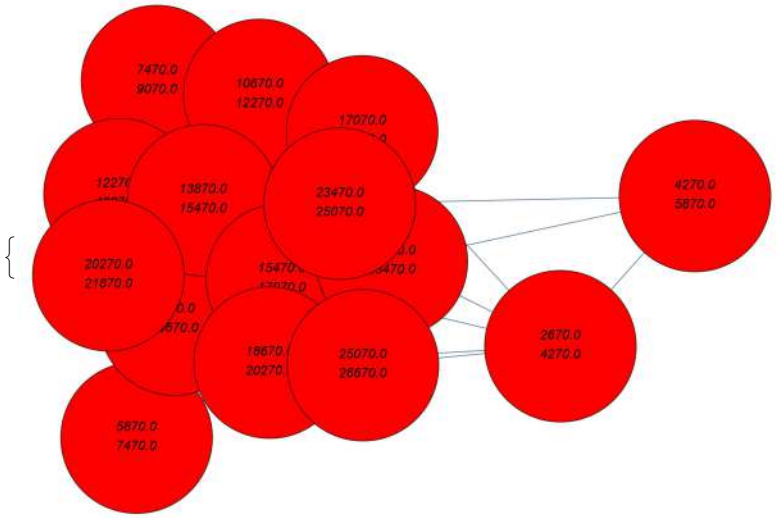
{ 
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7470.0
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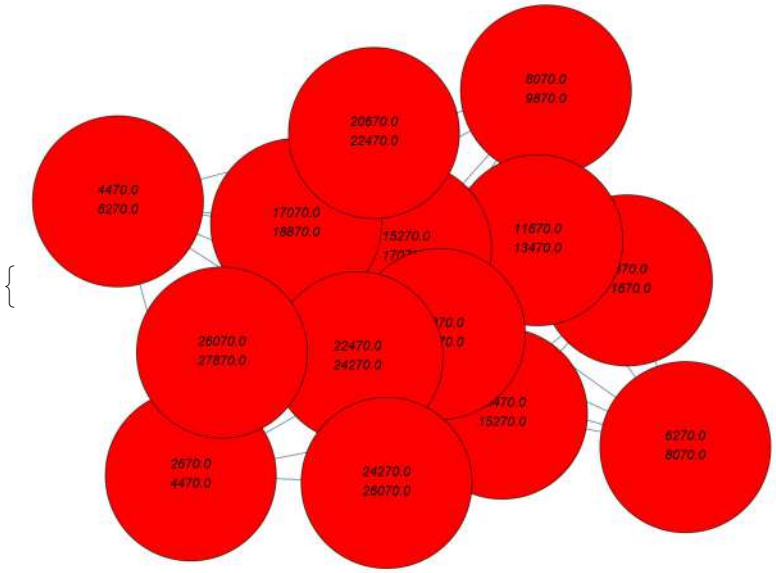
{ 
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11670.0
13470.0
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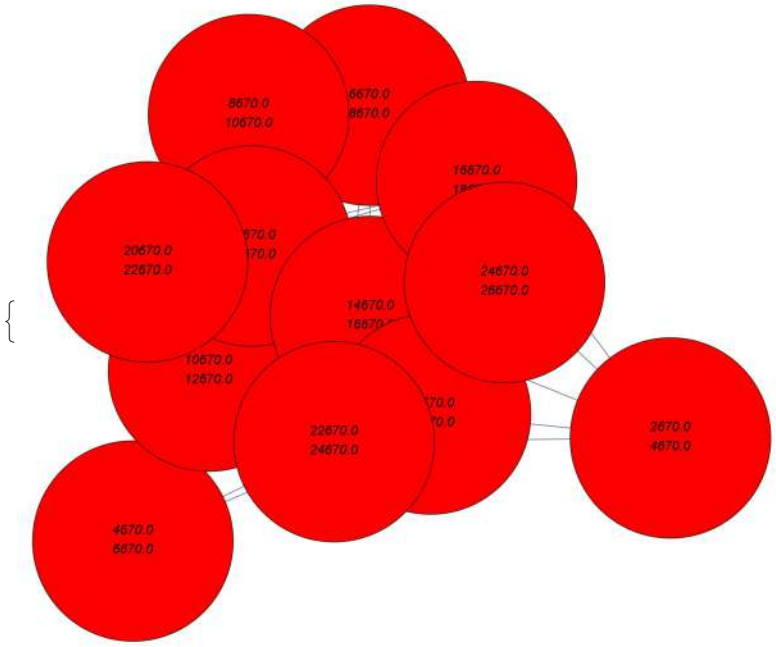
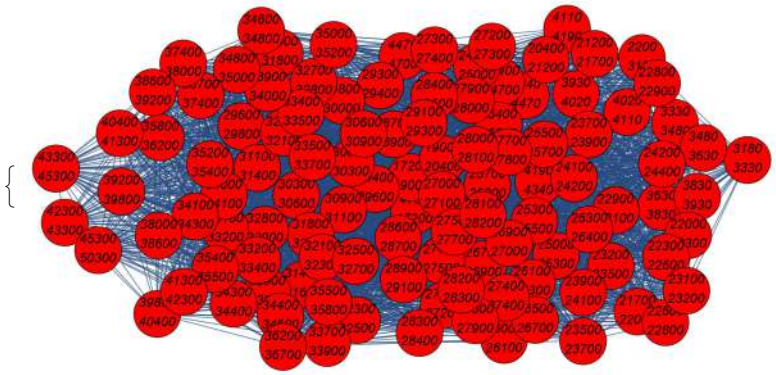
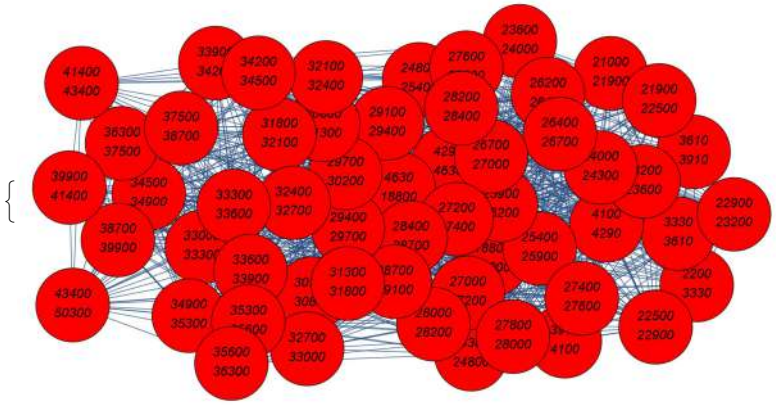


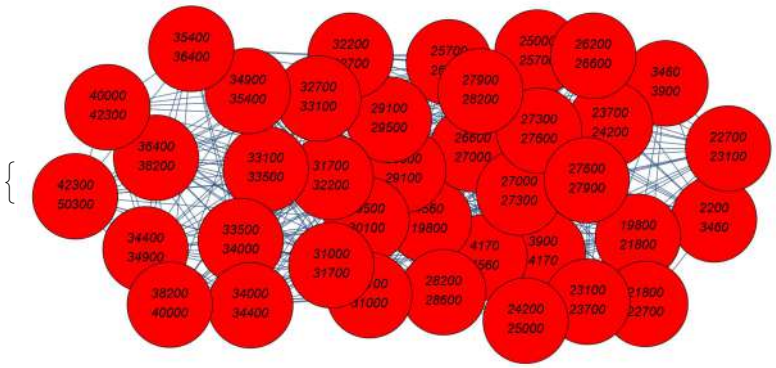
, 12}}

In[ ]:= **graphsandnodenumbersFixedbucket**

Out[ ]= {{  , 110},



{, 56},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33500
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31000
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19800
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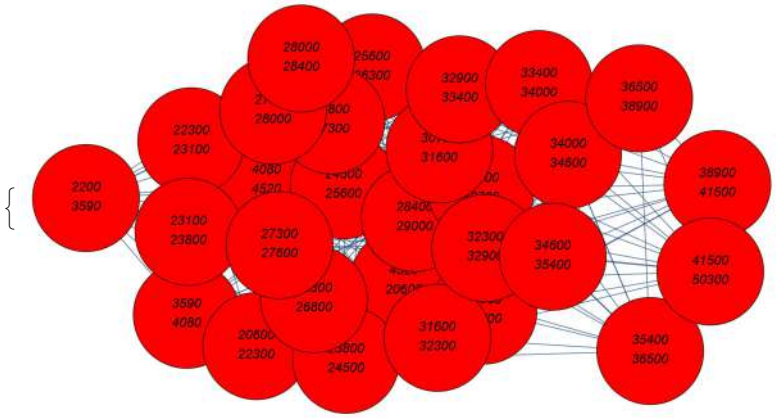
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23100
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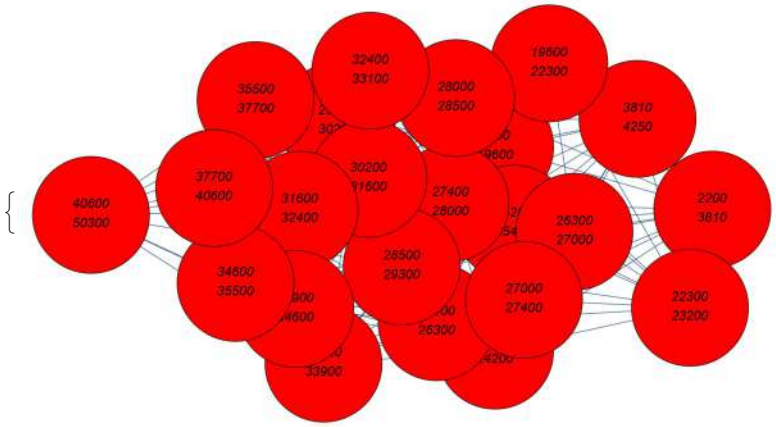
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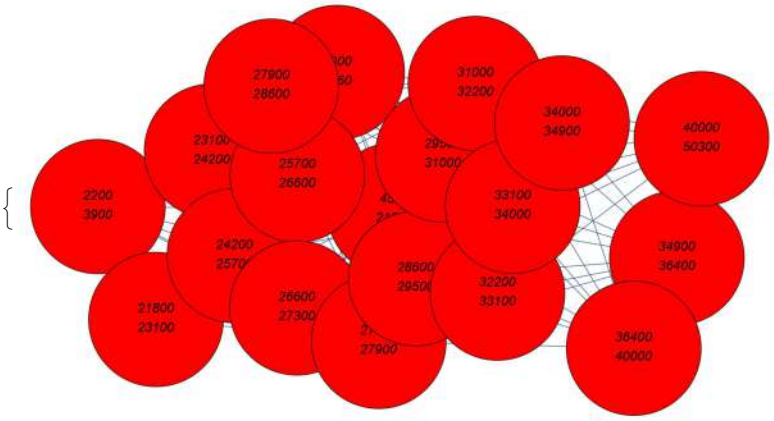
27400
22300
23200
33900
26300

{ , 23},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40000
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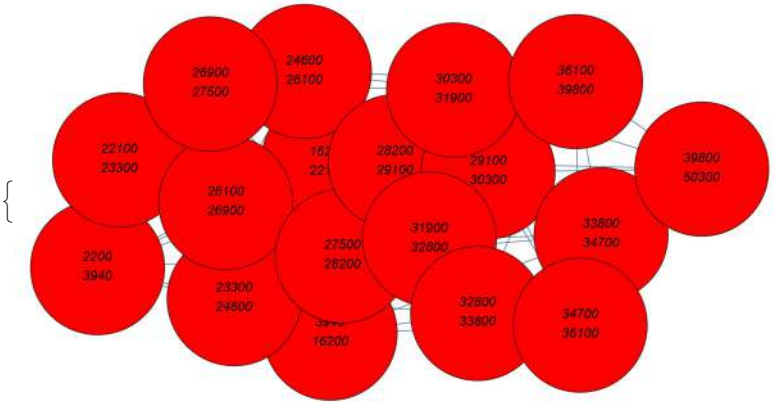
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16200

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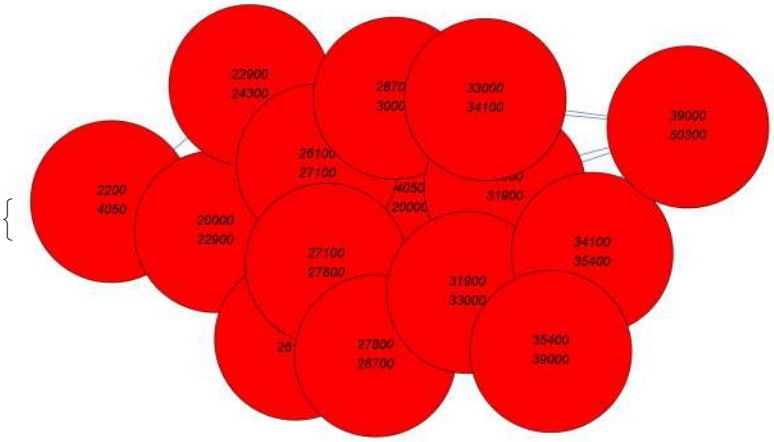
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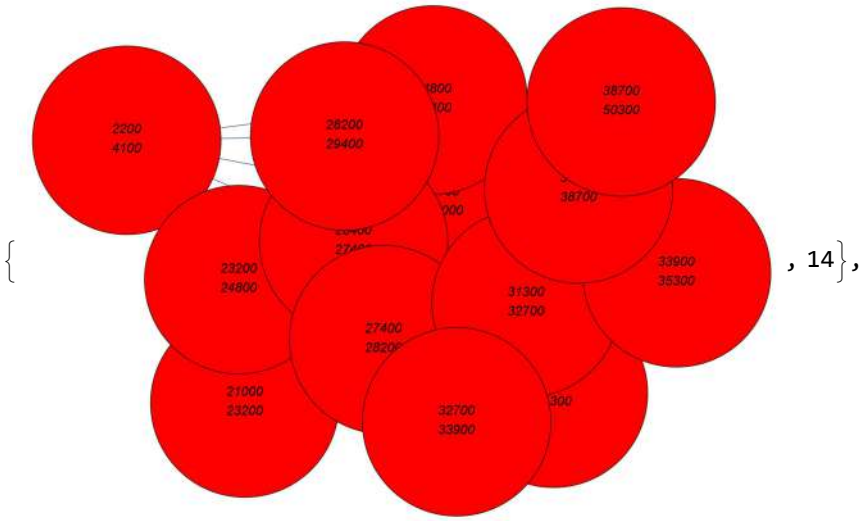
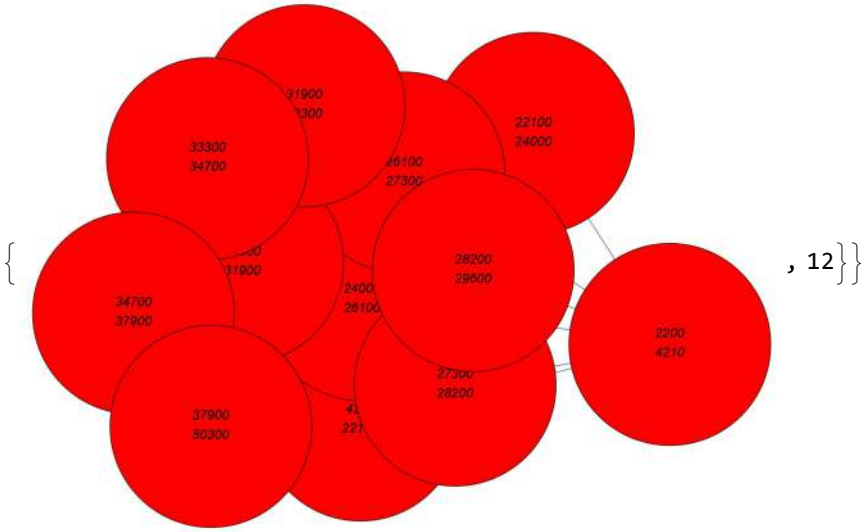


, 14},



, 12}}

Plots Together

Fixed Step Size Networks

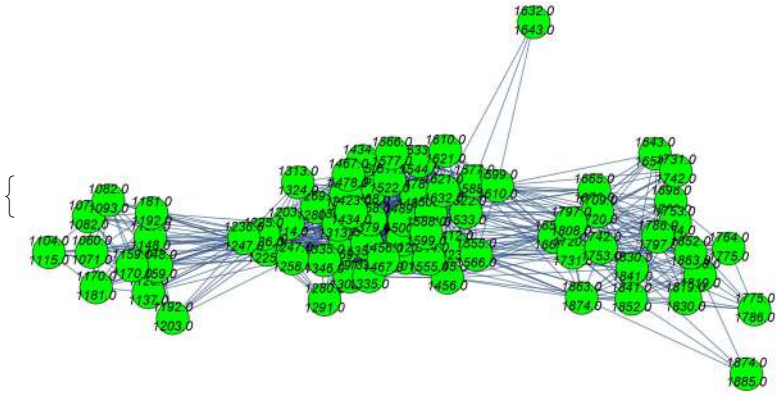
Width Feature

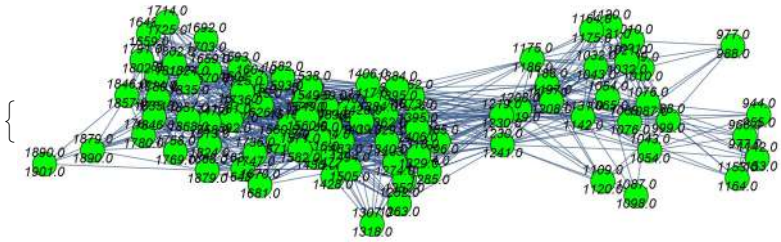
In[ ]:=
```
AbsoluteTiming[
 widthdataintimewindowsFixedstep = snetworkdatabinnedintimewindows[9, 11, 10];]
```

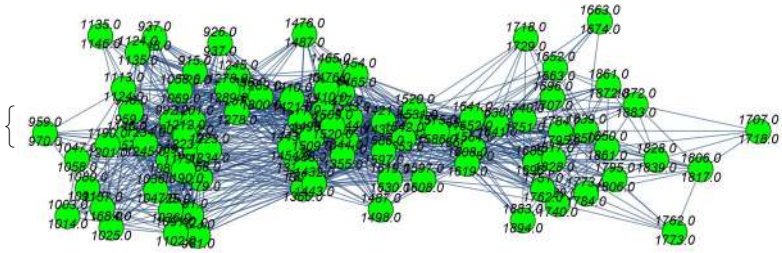
Out[ ]= {48.4787, Null}

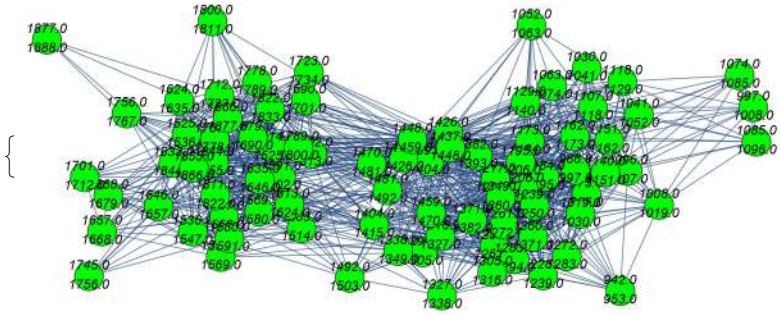
In[ ]:=
```
graphsandnodenumbers = Table[snetworkgraph[widthdataintimewindowsFixedstep[[1]][[i]],
    widthdataintimewindowsFixedstep[[2]][[i]], 2, 7, 400, Green], {i, Range@10}];
```

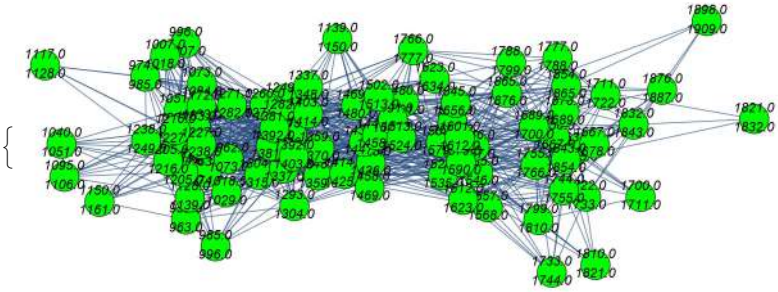
In[ ]:=
```
graphsandnodenumbers
```

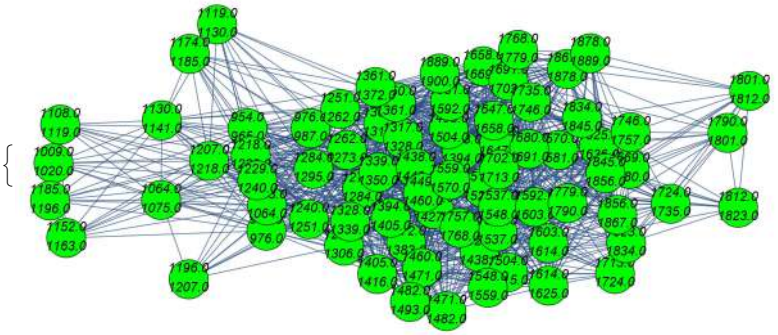
Out[•]= {{  , 71},

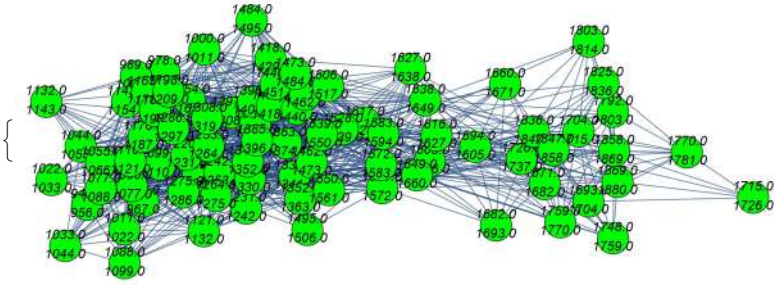
{  , 85},

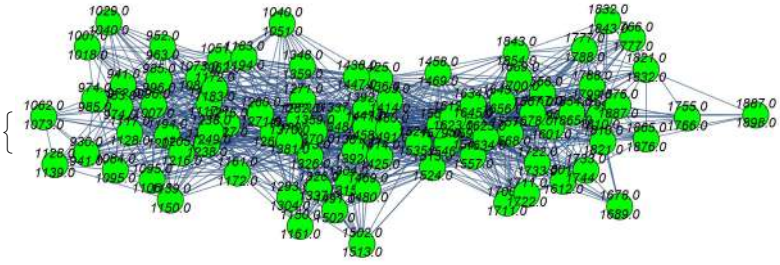
{  , 88},

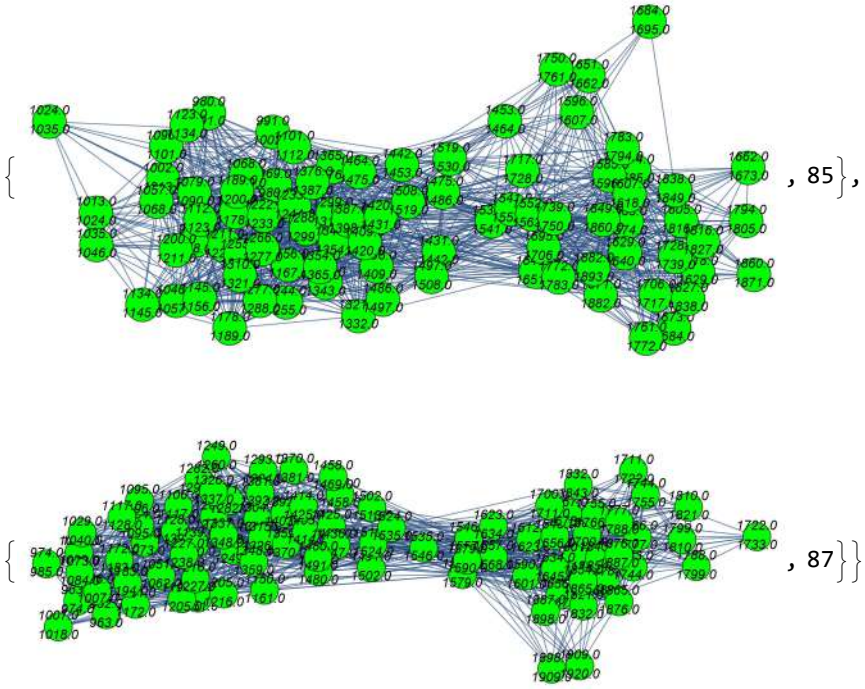
{  , 83},


, 83},


, 76},


, 81},


, 86},



In[ ]:=
```
ABCvalues = Table[Mean@BetweennessCentrality[graphsandnodenumbers[[i]][[1]]],
    {i, Length@graphsandnodenumbers}];
modularityvalues = Table[N@GraphAssortativity[graphsandnodenumbers[[i]][[1]],
      FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
      "Normalized" → False], {i, Length@graphsandnodenumbers}];
degreevalues = Table[N@Mean@VertexDegree[graphsandnodenumbers[[i]][[1]]],
    {i, Length@graphsandnodenumbers}];
```

In[ ]:=
```
singlerandomgraphserdren = Table[
    RandomGraph[{VertexCount[i], EdgeCount[i]}], {i, graphsandnodenumbers[[All, 1]]}];
singlerandomerdrenmodularityvalues =
  Table[N@GraphAssortativity[singlerandomgraphserdren[[i]],
      FindGraphCommunities[singlerandomgraphserdren[[i]]], "Normalized" -> False],
    {i, Length@singlerandomgraphserdren}];
singlerandomgraphsdegreesfxd = Table[IGDegreeSequenceGame[Total[AdjacencyMatrix@i],
Method -> "VigerLatapy"], {i, graphsandnodenumbers[[All, 1]]}];
singlerandomdegreesfxdmodularityvalues =
  Table[N@GraphAssortativity[singlerandomgraphsdegreesfxd[[i]],
      FindGraphCommunities[singlerandomgraphsdegreesfxd[[i]]], "Normalized" -> False],
    {i, Length@singlerandomgraphsdegreesfxd}];
singlerandomgraphscomm = Table[randomizedgraphamongcommunities[i],
    {i, graphsandnodenumbers[[All, 1]]}];
singlerandomcommmodularityvalues = Table[N@GraphAssortativity[
      singlerandomgraphscomm[[i]], FindGraphCommunities[singlerandomgraphscomm[[i]]],
      "Normalized" -> False], {i, Length@singlerandomgraphscomm}];
```

```
In[ ]:= AbsoluteTiming[ZscoresmodularityWolf =
          Table[randomnessfunctionformodularity[i, "Wolf"], {i, graphsandnodenumbers[[All, 1]]}]]
```

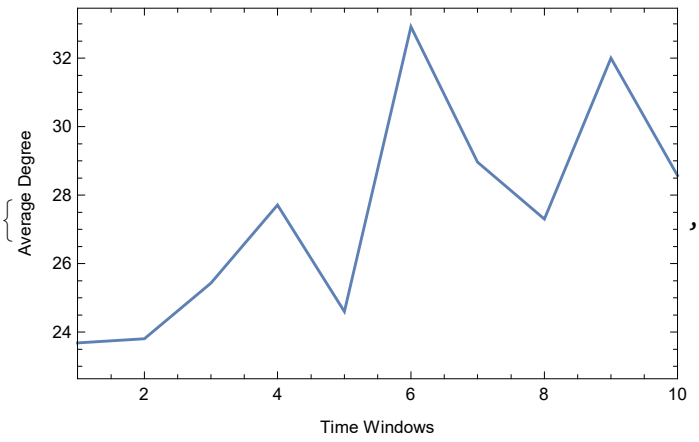
```
Out[ ]= {244.235, {{33.57, 40.9554, -19.8872}, {40.4218, 47.9585, -50.5403},
          {39.0145, 45.3088, -56.4996}, {43.6673, 49.1992, -28.4451},
          {41.7119, 47.7857, -37.8848}, {22.2202, 28.5683, -63.0805}, {35.7747, 41.8342, -58.0281},
          {39.7642, 47.8633, -39.7476}, {52.156, 60.1734, -34.2295}, {55.885, 59.8185, -23.5193}}}
```

```
In[ ]:= Table[Length@FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
         {i, Length@graphsandnodenumbers}]
```

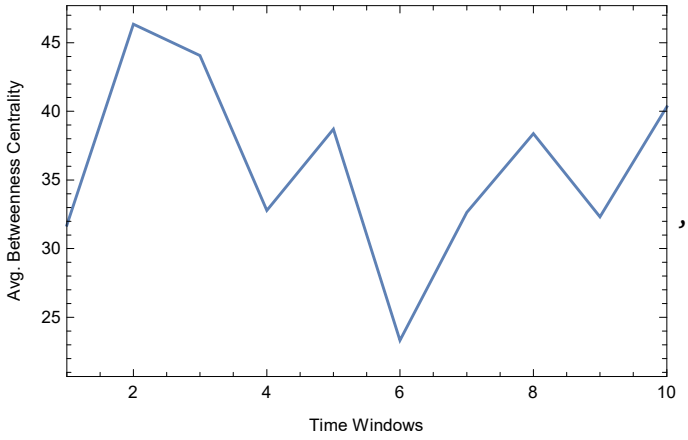
```
Out[ ]= {3, 3, 3, 3, 2, 2, 3, 2, 2, 2}
```

```
In[ ]:= {ListLinePlot[Thread[{Range@10, degreevalues}], Frame → True,
          FrameLabel → {"Time Windows", "Average Degree"}, ImageSize → 350,
          PlotRange → {{1, 10}, All}], ListLinePlot[Thread[{Range@10, ABCvalues}],
          Frame → True, FrameLabel → {"Time Windows", "Avg. Betweenness Centrality"},
          ImageSize → 350, PlotRange → {{1, 10}, All}],
         ListLinePlot[Thread[{Range@10, modularityvalues}], Frame → True,
          FrameLabel → {"Time Windows", "Modularity"}, ImageSize → 350, PlotRange → All],
         ListLinePlot[{Thread[{Range@10, singlerandomerdrenmodularityvalues}],
           Thread[{Range@10, singlerandomdegreesfxdmodularityvalues}],
           Thread[{Range@10, singlerandomcommmodularityvalues}]}, Frame → True,
          FrameLabel → {"Time Windows", "Modularity (Single Random Graphs)"},
          ImageSize → 350, PlotRange → {{1, 10}, All}, PlotLabels →
           Placed[{"Erdös-Renyi", "Degrees Fxd.", "Communities"}, {Scaled[1], Below}]],
         ListLinePlot[{Thread[{Range@10, ZscoresmodularityWolf[[All, 1]]}],
           Thread[{Range@10, ZscoresmodularityWolf[[All, 2]]}],
           Thread[{Range@10, ZscoresmodularityWolf[[All, 3]]}]},
          Frame → True, FrameLabel → {"Time Windows", "Z-Score for Modularity"},
          ImageSize → 350, PlotRange → All, PlotLabels →
           Placed[{"Erdös Renyi", "Degrees Fxd.", "Communities"}, {Scaled[1], Above}]]}
```

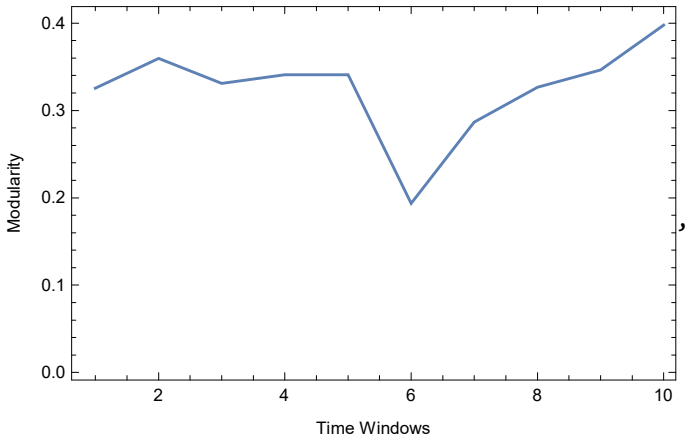
```
Out[ ]=
```



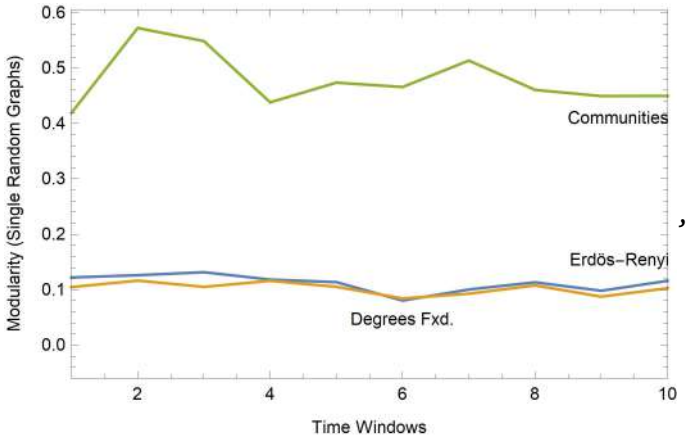
,


Avg. Betweenness Centrality
45
40
35
30
25
2
4
6
8
10
Time Windows

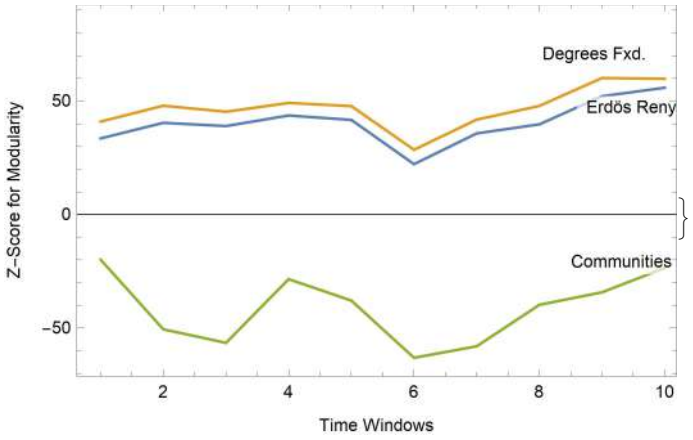
,


Modularity
0.4
0.3
0.2
0.1
0.0
2
4
6
8
10
Time Windows

,


Modularity (Single Random Graphs)
0.6
0.5
0.4
0.3
0.2
0.1
0.0
Communities
Erdös–Renyi
Degrees Fxd.
2
4
6
8
10
Time Windows

,



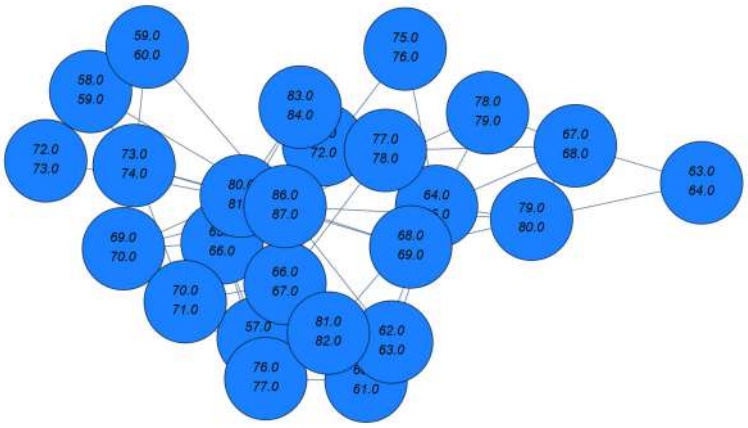
Thickness Feature

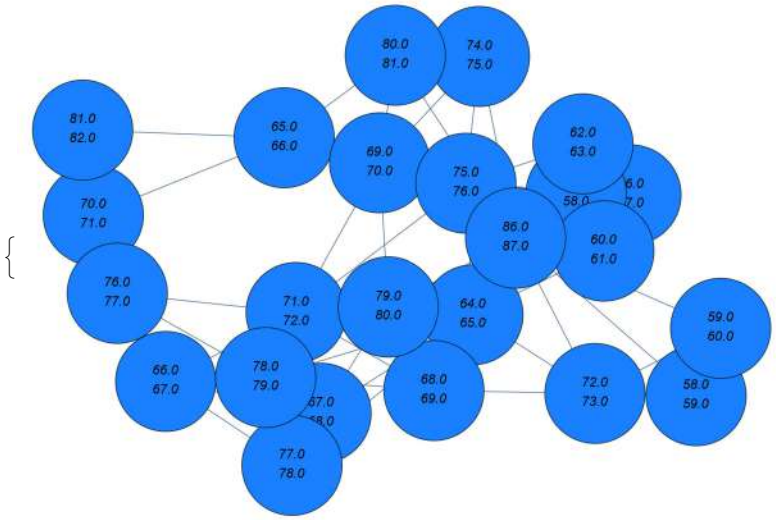
In[ ]:=
```
AbsoluteTiming[
 thicknessdataintimewindowsFixedstep = snetworkdatabinnedintimewindows[10, 1, 10];]
```

Out[ ]= {13.7807, Null}

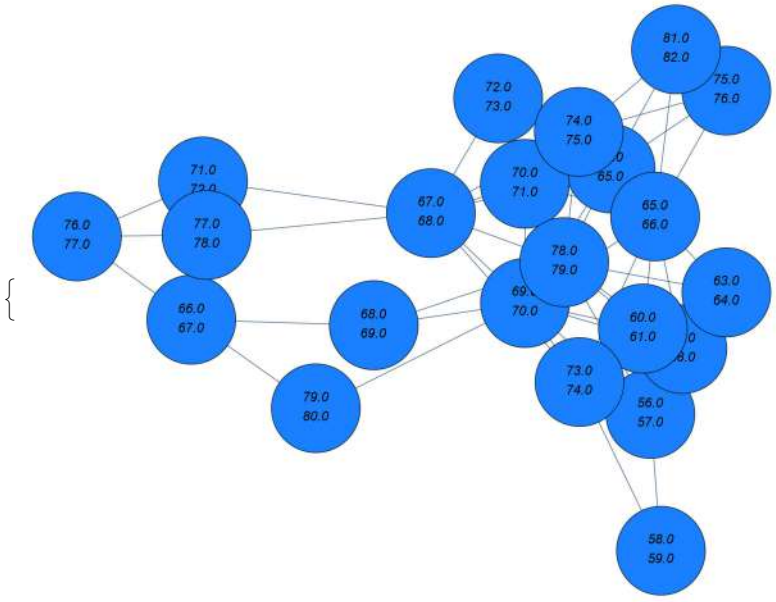
In[ ]:=
```
graphsandnodenumbers = Table[snetworkgraph[thicknessdataintimewindowsFixedstep[[1]][[i]],
    thicknessdataintimewindowsFixedstep[[2]][[i]],
    2, 7, 400, RGBColor[0.1, 0.5, 1.]], {i, Range@10}];
```

In[ ]:=
```
graphsandnodenumbers
```

Out[ ]= {{  , 25},


80.0
81.0
74.0
75.0
81.0
82.0
65.0
66.0
69.0
70.0
62.0
63.0
75.0
76.0
58.0
70.0
71.0
86.0
87.0
60.0
61.0
76.0
77.0
71.0
72.0
79.0
80.0
64.0
65.0
59.0
60.0
66.0
67.0
78.0
79.0
68.0
69.0
72.0
73.0
58.0
59.0
77.0
78.0

, 24},


81.0
82.0
72.0
73.0
75.0
76.0
74.0
75.0
71.0
70.0
71.0
65.0
67.0
68.0
65.0
66.0
76.0
77.0
77.0
78.0
78.0
79.0
63.0
64.0
66.0
67.0
68.0
69.0
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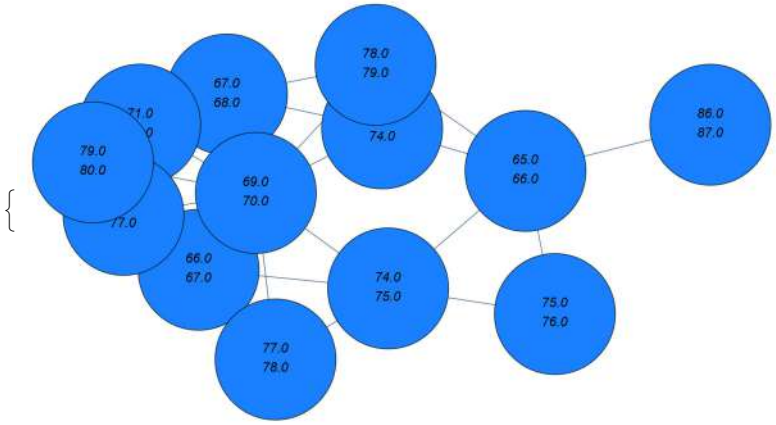
80.0
56.0
57.0
58.0
59.0

, 22},


78.0
79.0
67.0
68.0
74.0
86.0
87.0
65.0
66.0
79.0
80.0
69.0
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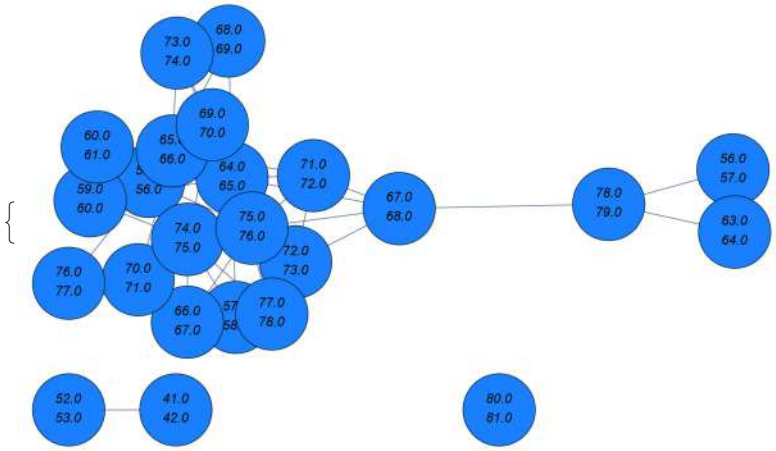
67.0
74.0
75.0
75.0
76.0
77.0
78.0

{ , 13},


73.0
74.0
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70.0
60.0
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72.0
67.0
68.0
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76.0
77.0
70.0
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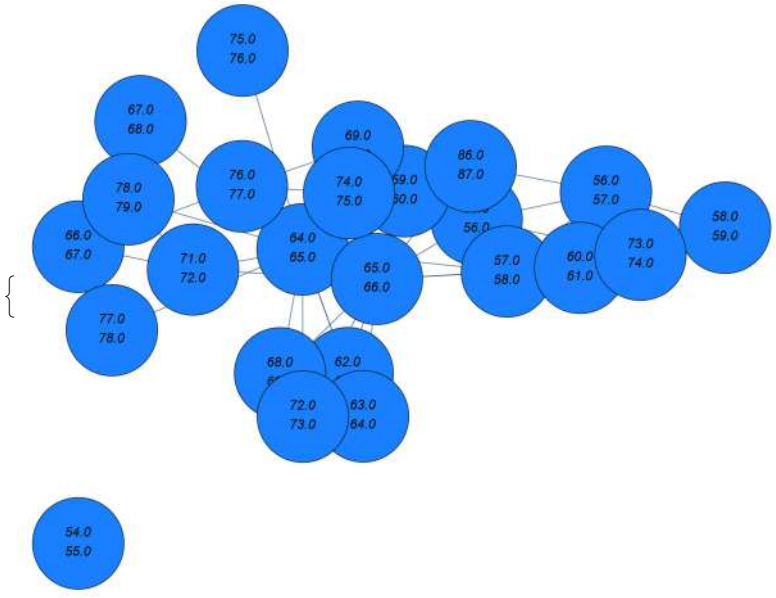
41.0
42.0
80.0
81.0

{ , 24},


75.0
76.0
67.0
68.0
69.0
86.0
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63.0
64.0
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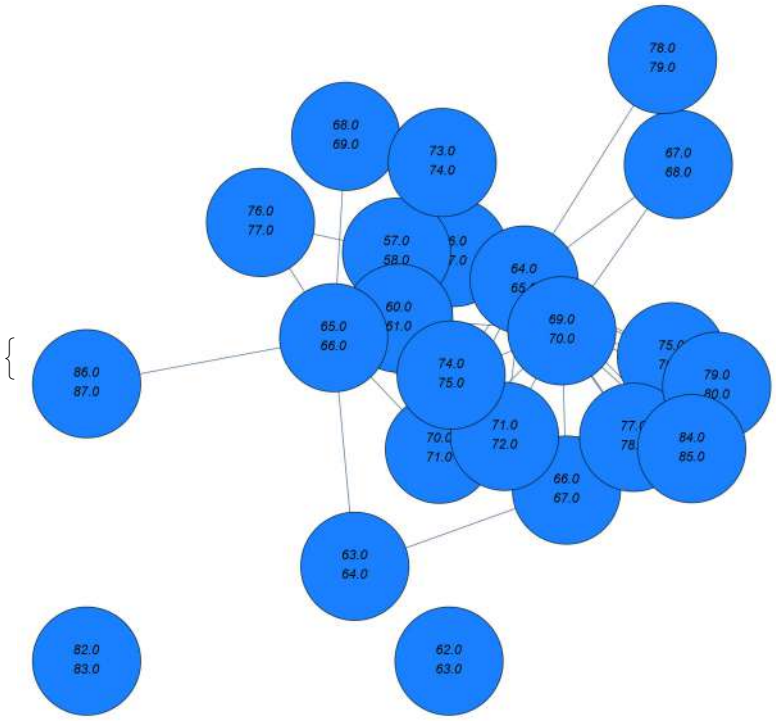
55.0

{ , 24},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79.0
68.0
69.0
73.0
74.0
67.0
68.0
76.0
77.0
57.0
58.0
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65.0
60.0
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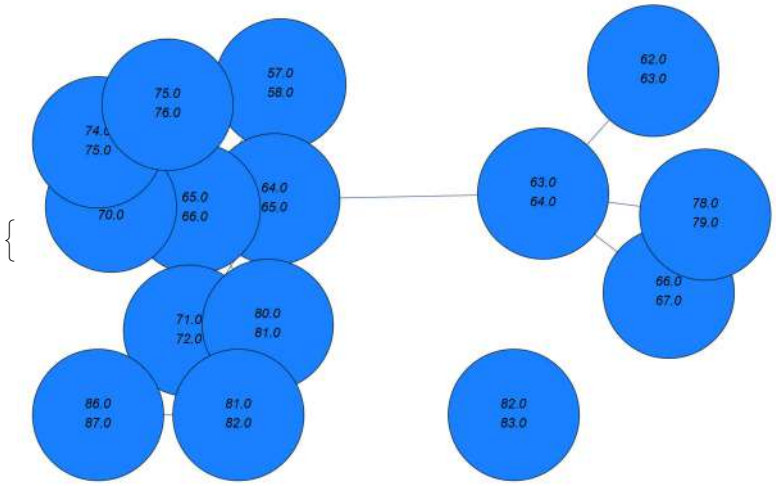
74.0
75.0
79.0
80.0
70.0
71.0
71.0
72.0
84.0
85.0
66.0
67.0
63.0
64.0
86.0
87.0
82.0
83.0
62.0
63.0

, 23},


57.0
58.0
75.0
76.0
74.0
75.0
64.0
65.0
65.0
66.0
70.0
62.0
63.0
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64.0
78.0
79.0
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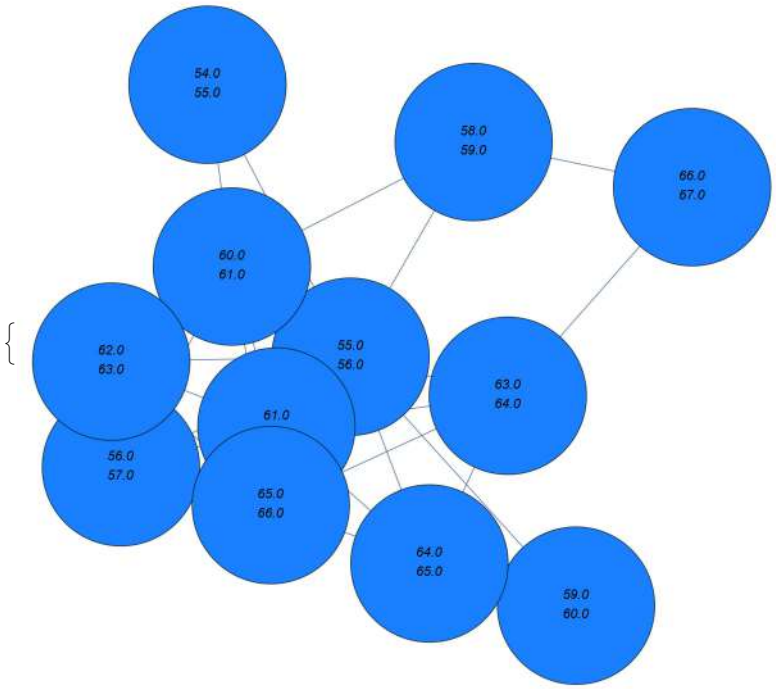
87.0
81.0
82.0
82.0
83.0

, 15},


54.0
55.0
58.0
59.0
66.0
67.0
60.0
61.0
62.0
63.0
55.0
56.0
63.0
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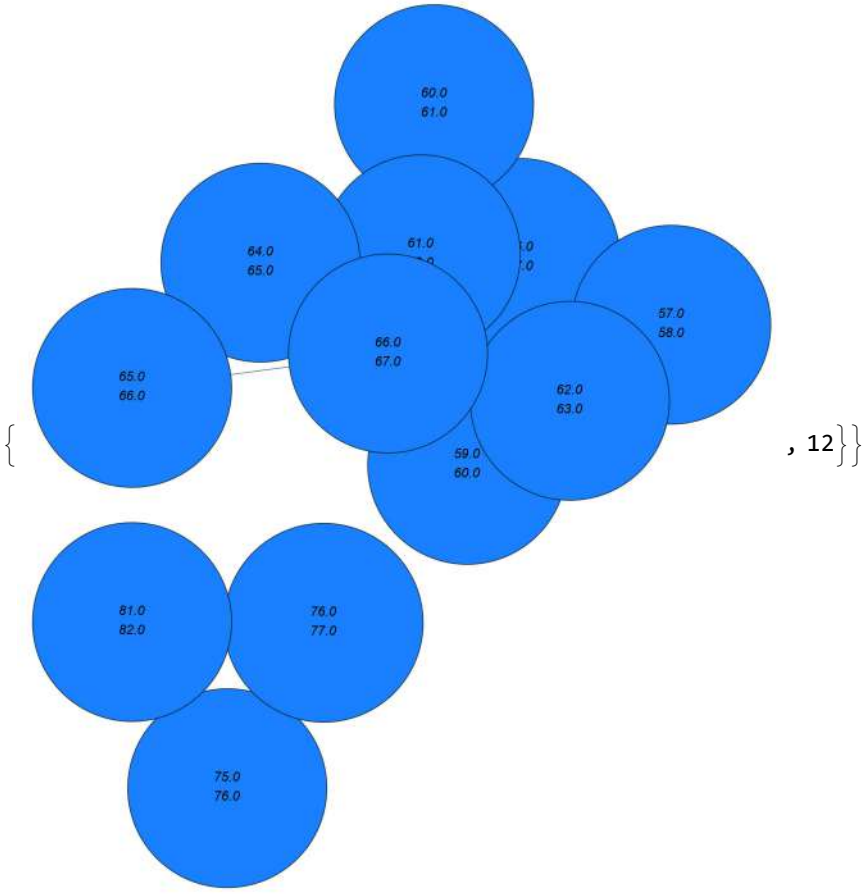
56.0
57.0
65.0
66.0
64.0
65.0
59.0
60.0

{ , 12},



{ , 12}}

In[ ]:=
```
ABCvalues = Table[Mean@BetweennessCentrality[graphsandnodenumbers[[i]][[1]]],
    {i, Length@graphsandnodenumbers}];
modularityvalues = Table[N@GraphAssortativity[graphsandnodenumbers[[i]][[1]],
      FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
      "Normalized" → False], {i, Length@graphsandnodenumbers}];
degreevalues = Table[N@Mean@VertexDegree[graphsandnodenumbers[[i]][[1]]],
    {i, Length@graphsandnodenumbers}];
```

In[ ]:=
```
singlerandomgraphserdren = Table[
    RandomGraph[{VertexCount[i], EdgeCount[i]}], {i, graphsandnodenumbers[[All, 1]]}];
singlerandomerdrenmodularityvalues =
  Table[N@GraphAssortativity[singlerandomgraphserdren[[i]],
     FindGraphCommunities[singlerandomgraphserdren[[i]]], "Normalized" -> False],
   {i, Length@singlerandomgraphserdren}];
singlerandomgraphsdegreesfxd = Table[IGDegreeSequenceGame[Total[AdjacencyMatrix@i],
Method -> "VigerLatapy"], {i, graphsandnodenumbers[[All, 1]]}];
singlerandomdegreesfxdmodularityvalues =
  Table[N@GraphAssortativity[singlerandomgraphsdegreesfxd[[i]],
     FindGraphCommunities[singlerandomgraphsdegreesfxd[[i]]], "Normalized" -> False],
   {i, Length@singlerandomgraphsdegreesfxd}];
singlerandomgraphscomm = Table[randomizedgraphamongcommunities[i],
   {i, graphsandnodenumbers[[All, 1]]}];
singlerandomcommmodularityvalues = Table[N@GraphAssortativity[
     singlerandomgraphscomm[[i]], FindGraphCommunities[singlerandomgraphscomm[[i]]],
     "Normalized" -> False], {i, Length@singlerandomgraphscomm}];
```

In[ ]:=
```
AbsoluteTiming[ZscoresmodularityWolf =
  Table[randomnessfunctionformodularity[i, "Wolf"], {i, graphsandnodenumbers[[All, 1]]}]]
```

Out[ ]=
```
{93.2888,
 {{2.50502, 4.54607, -6.06845}, {4.11302, 5.20171, -6.38714}, {4.3782, 6.11277, -4.85},
  {0.716177, 1.36241, -3.21859}, {0.134648, ComplexInfinity, -4.1646},
  {<<1>>}, {-0.474878, ComplexInfinity, -6.80042},
  {0.585053, ComplexInfinity, 30 Sqrt[1110] (0.34814 + <<1>>/1000)/Sqrt[<<1>>]},
  {-0.897705, 0.446371, -3.28261}, {1.67402, 1.84314, -1.75217}}}
```

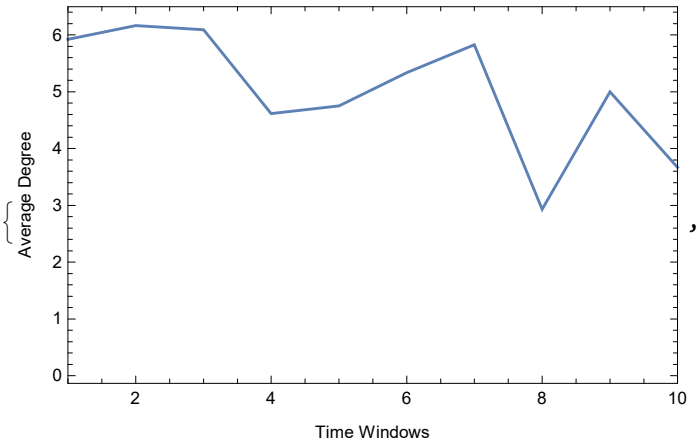
large output | show less | show more | show all | set size limit...

In[ ]:=
```
Table[Length@FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
 {i, Length@graphsandnodenumbers}]
```

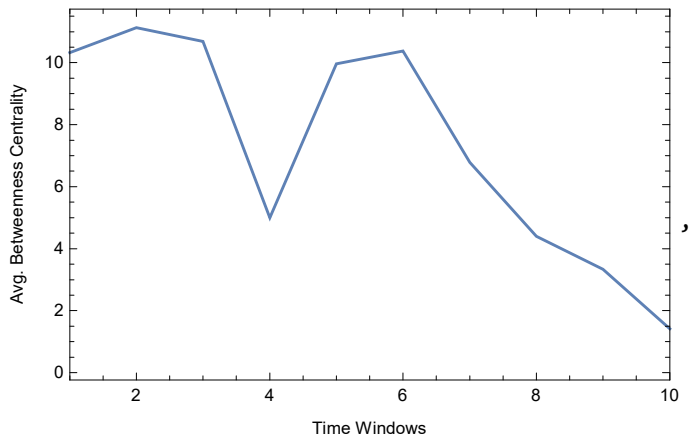
Out[ ]=
```
{4, 4, 3, 3, 5, 5, 5, 4, 2, 3}
```

In[ ]:=
```
{ListLinePlot[Thread[{Range@10, degreevalues}], Frame → True,
  FrameLabel → {"Time Windows", "Average Degree"}, ImageSize → 350,
  PlotRange → {{1, 10}, All}], ListLinePlot[Thread[{Range@10, ABCvalues}],
  Frame → True, FrameLabel → {"Time Windows", "Avg. Betweenness Centrality"},
  ImageSize → 350, PlotRange → {{1, 10}, All}],
 ListLinePlot[Thread[{Range@10, modularityvalues}], Frame → True,
  FrameLabel → {"Time Windows", "Modularity"}, ImageSize → 350, PlotRange → All],
 ListLinePlot[{Thread[{Range@10, singlerandomerdrenmodularityvalues}],
   Thread[{Range@10, singlerandomdegreesfxdmodularityvalues}],
   Thread[{Range@10, singlerandomcommmodularityvalues}]}, Frame → True,
  FrameLabel → {"Time Windows", "Modularity (Single Random Graphs)"},
  ImageSize → 350, PlotRange → {{1, 10}, All}, PlotLabels →
   Placed[{"Erdös-Renyi", "Degrees Fxd.", "Communities"}, {Scaled[1], Below}]],
 ListLinePlot[{Thread[{Range@10, ZscoresmodularityWolf[[All, 1]]}],
   Thread[{Range@10, ZscoresmodularityWolf[[All, 2]]}],
   Thread[{Range@10, ZscoresmodularityWolf[[All, 3]]}]},
  Frame → True, FrameLabel → {"Time Windows", "Z-Score for Modularity"},
  ImageSize → 350, PlotRange → All, PlotLabels →
   Placed[{"Erdös Renyi", "Degrees Fxd.", "Communities"}, {Scaled[1], Above}]]}
```

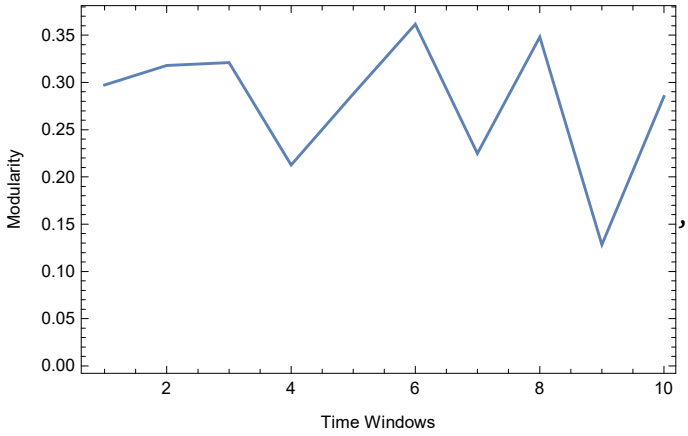
Out[ ]=

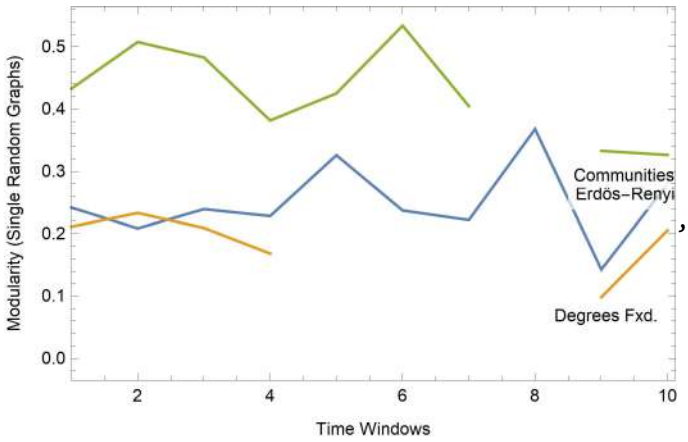


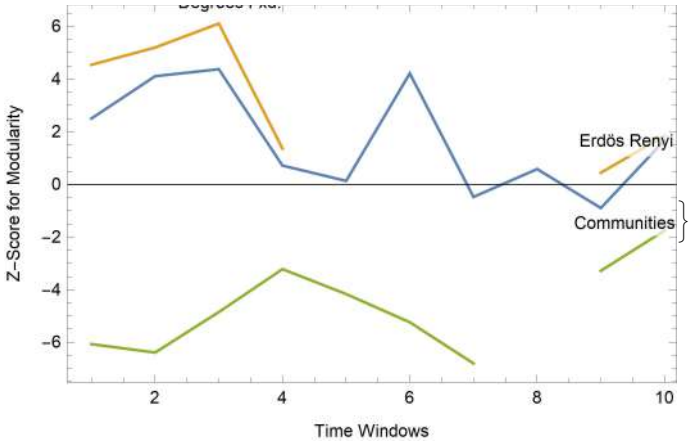
,



,







Length Feature

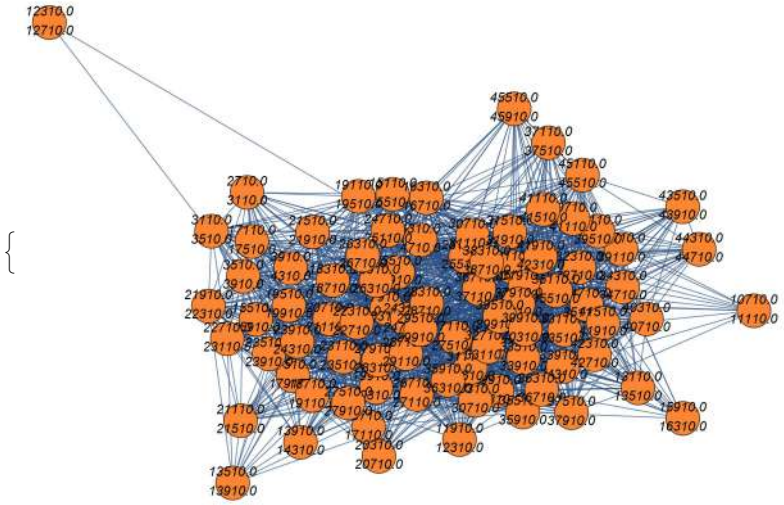
In[ ]:=
```
AbsoluteTiming[
 lengthdataintimewindowsFixedstep = snetworkdatabinnedintimewindows[12, 400, 10];]
```

Out[ ]= {61.2188, Null}

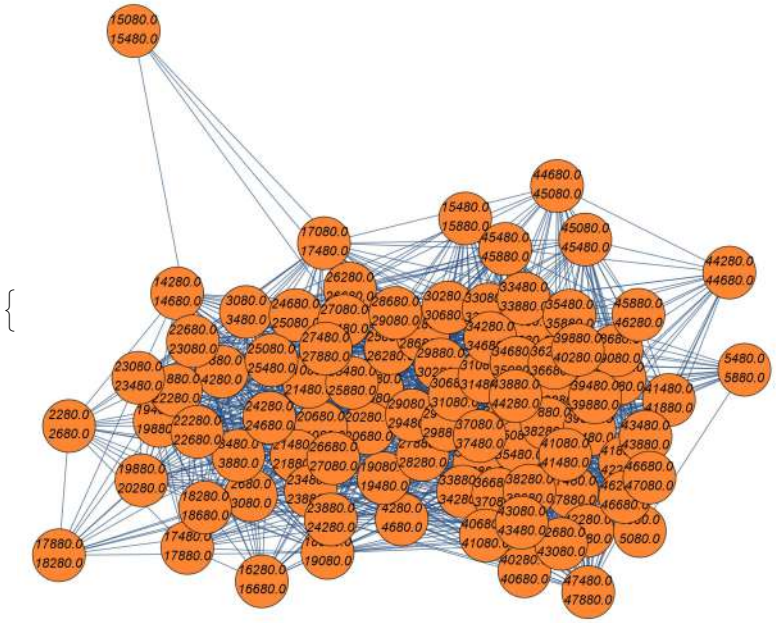
In[ ]:=
```
graphsandnodenumbers = Table[snetworkgraph[lengthdataintimewindowsFixedstep[[1]][[i]],
    lengthdataintimewindowsFixedstep[[2]][[i]], 2,
    7, 400, RGBColor[1., 0.53, 0.2]], {i, Range@10}];
```

In[ ]:=
```
graphsandnodenumbers
```

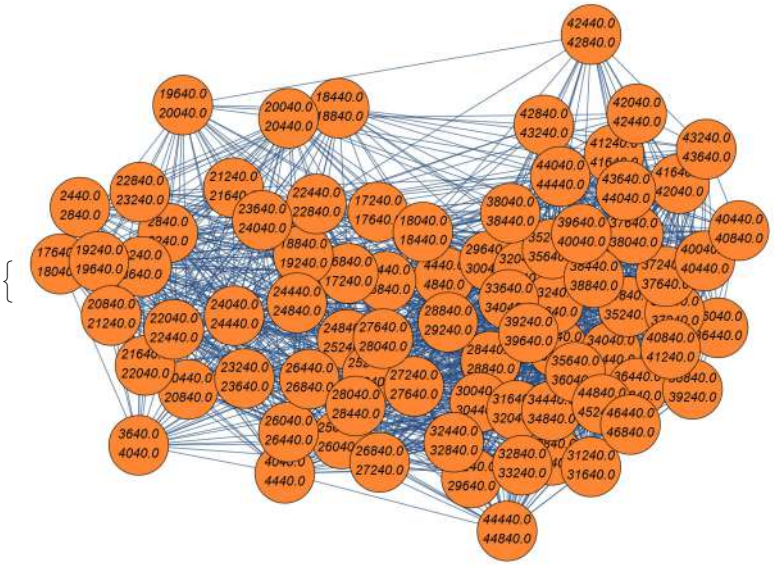
Out[ ]=



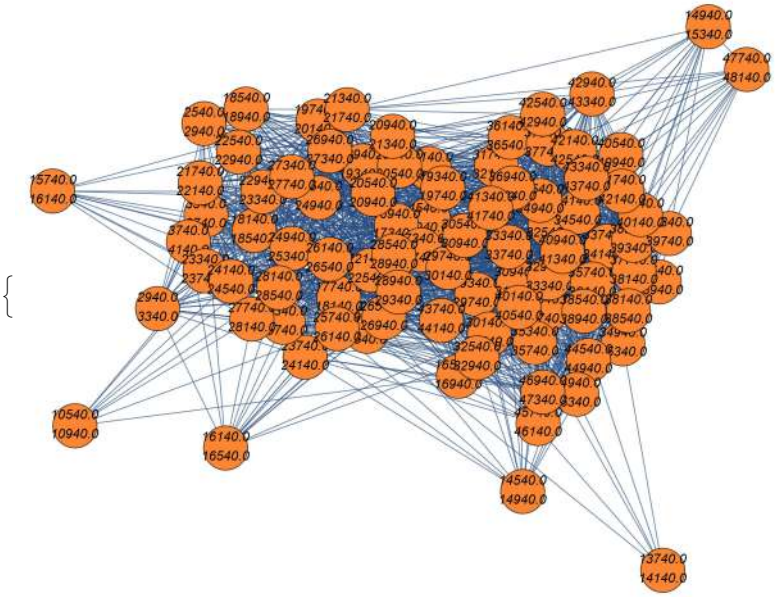
, 84},



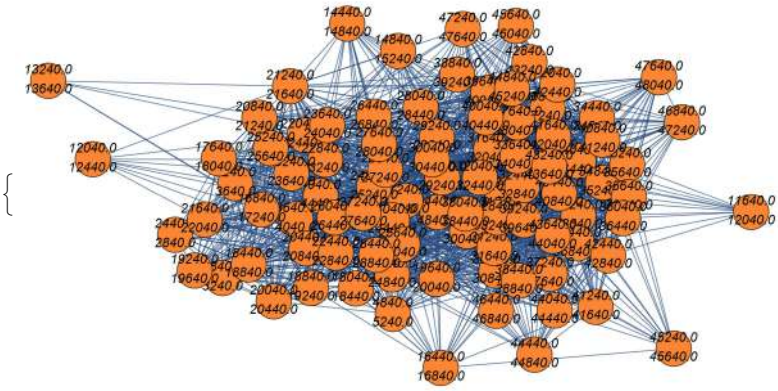
, 89},



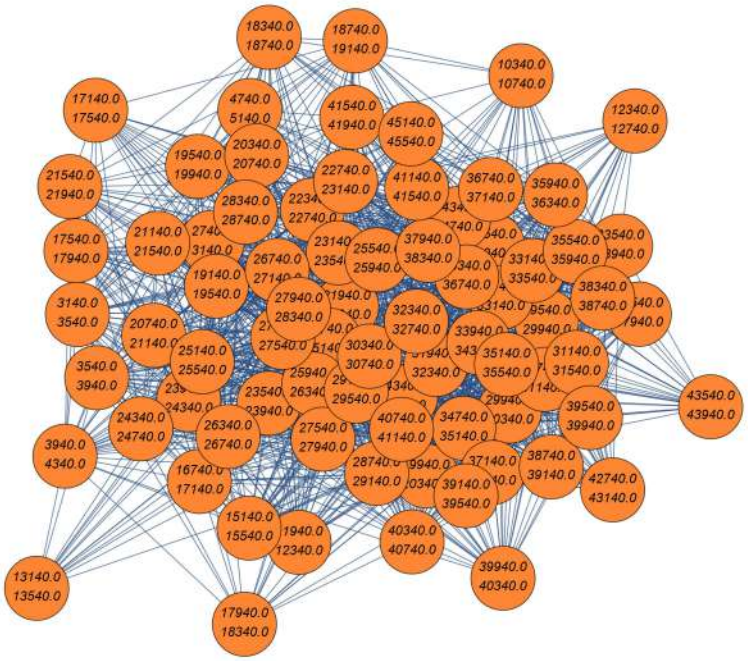
, 79},



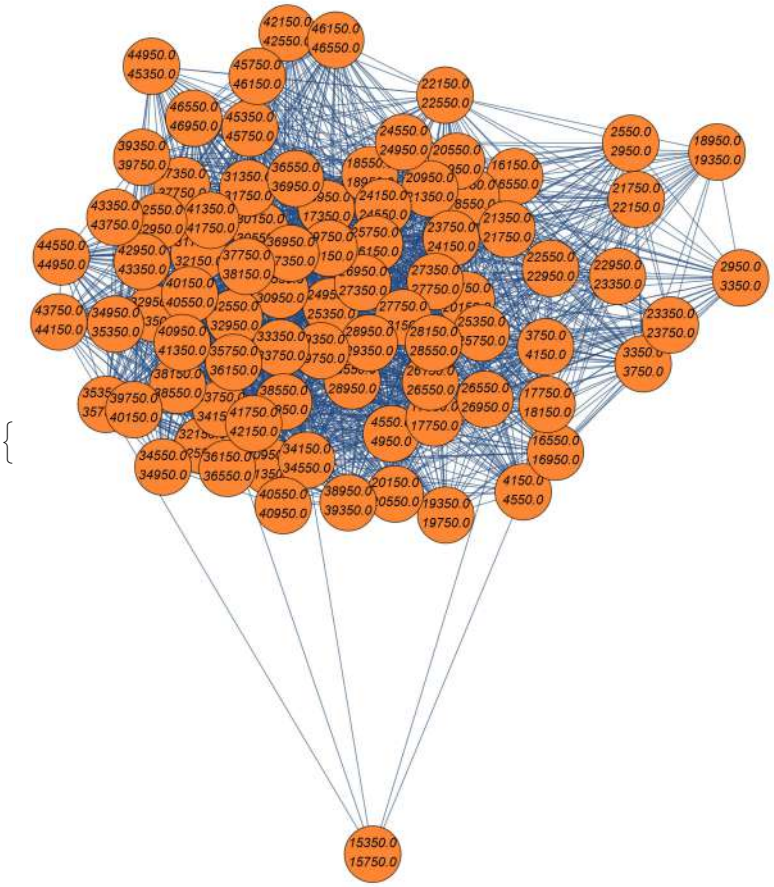
, 87},



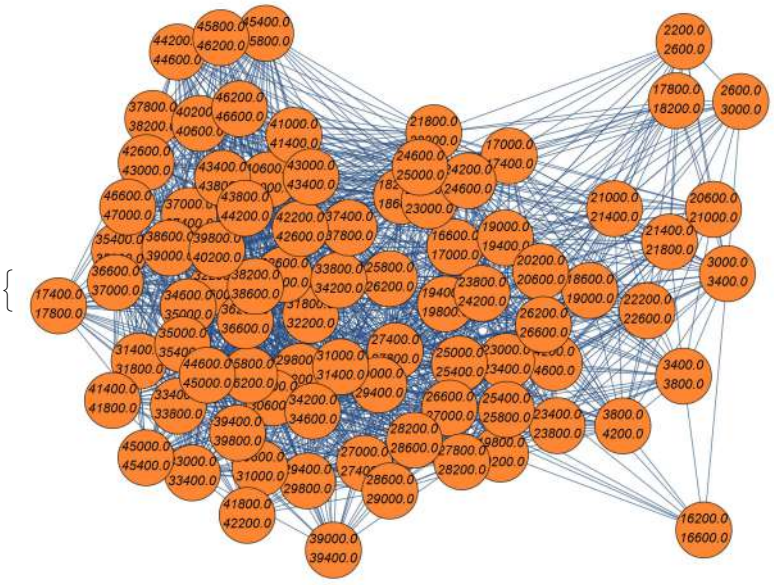
{ , 90},



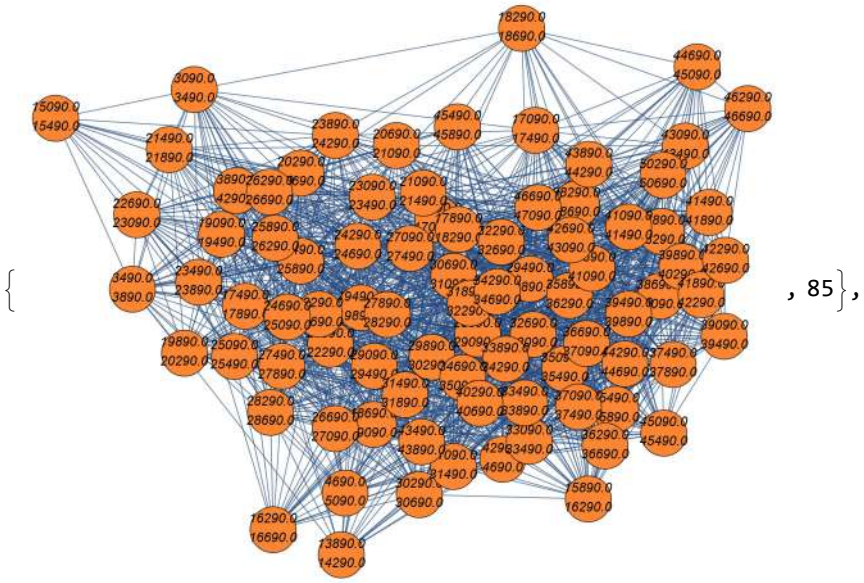
{ , 77},

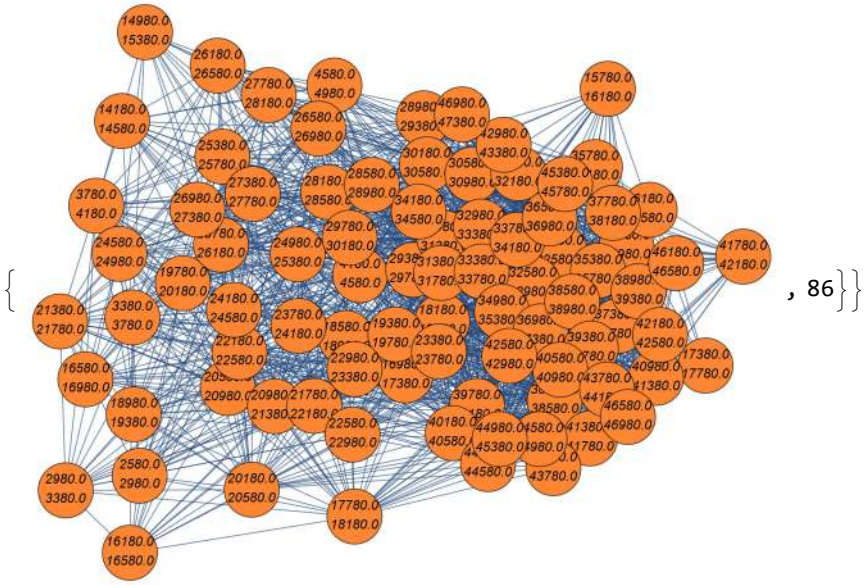


, 83},



, 83},



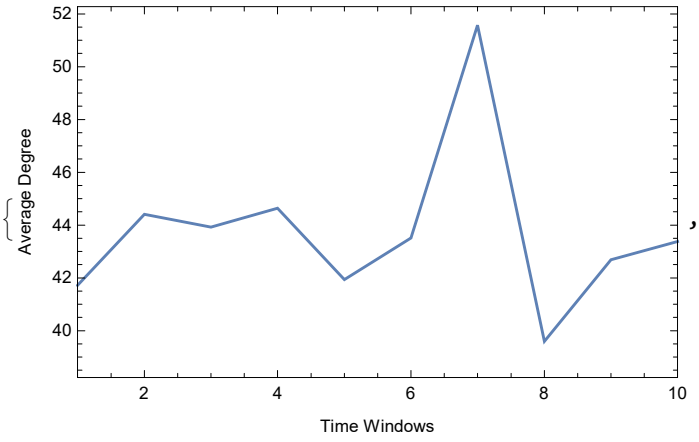


In[ ]:=
```
ABCvalues = Table[Mean@BetweennessCentrality[graphsandnodenumbers[[i]][[1]]],
    {i, Length@graphsandnodenumbers}];
modularityvalues = Table[N@GraphAssortativity[graphsandnodenumbers[[i]][[1]],
      FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
      "Normalized" → False], {i, Length@graphsandnodenumbers}];
degreevalues = Table[N@Mean@VertexDegree[graphsandnodenumbers[[i]][[1]]],
    {i, Length@graphsandnodenumbers}];
```

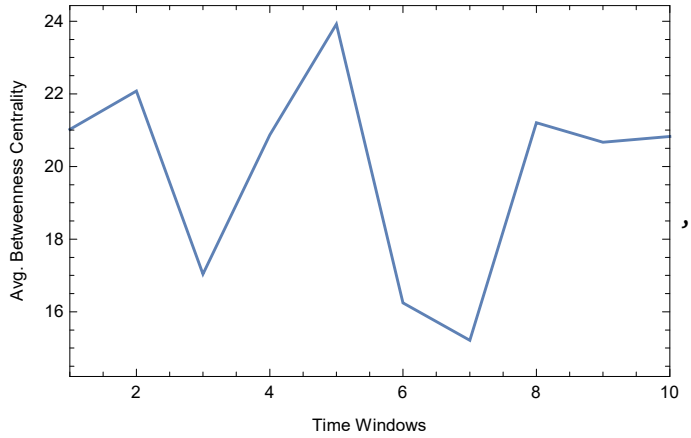
```
In[ ]:= singlerandomgraphserdren = Table[
          RandomGraph[{VertexCount[i], EdgeCount[i]}], {i, graphsandnodenumbers[[All, 1]]}];
        singlerandomerdrenmodularityvalues =
          Table[N@GraphAssortativity[singlerandomgraphserdren[[i]],
              FindGraphCommunities[singlerandomgraphserdren[[i]]], "Normalized" -> False],
            {i, Length@singlerandomgraphserdren}];
        singlerandomgraphsdegreesfxd = Table[IGDegreeSequenceGame[Total[AdjacencyMatrix@i],
        Method -> "VigerLatapy"], {i, graphsandnodenumbers[[All, 1]]}];
        singlerandomdegreesfxdmodularityvalues =
          Table[N@GraphAssortativity[singlerandomgraphsdegreesfxd[[i]],
              FindGraphCommunities[singlerandomgraphsdegreesfxd[[i]]], "Normalized" -> False],
            {i, Length@singlerandomgraphsdegreesfxd}];
        singlerandomgraphscomm = Table[randomizedgraphamongcommunities[i],
            {i, graphsandnodenumbers[[All, 1]]}];
        singlerandomcommmodularityvalues = Table[N@GraphAssortativity[
              singlerandomgraphscomm[[i]], FindGraphCommunities[singlerandomgraphscomm[[i]]],
              "Normalized" -> False], {i, Length@singlerandomgraphscomm}];

In[ ]:= AbsoluteTiming[ZscoresmodularityWolf =
          Table[randomnessfunctionformodularity[i, "Wolf"], {i, graphsandnodenumbers[[All, 1]]}]]

Out[ ]= {530.131, {{35.7081, 45.2461, -78.1516}, {41.6224, 48.2656, -78.9717},
          {33.8486, 37.5899, -75.6403}, {48.8782, 55.501, -79.2671}, {39.6166, 47.0115, -76.2427},
          {24.1346, 30.4739, -84.9721}, {24.3581, 29.614, -107.266}, {30.2289, 35.8713, -71.9001},
          {33.6259, 36.8646, -79.0163}, {35.8483, 40.8516, -73.0956}}}

In[ ]:= Table[Length@FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
          {i, Length@graphsandnodenumbers}]

Out[ ]= {2, 2, 2, 2, 2, 2, 2, 2, 3, 2}
```

In[ ]:=
```
{ListLinePlot[Thread[{Range@10, degreevalues}], Frame → True,
  FrameLabel → {"Time Windows", "Average Degree"}, ImageSize → 350,
  PlotRange → {{1, 10}, All}], ListLinePlot[Thread[{Range@10, ABCvalues}],
  Frame → True, FrameLabel → {"Time Windows", "Avg. Betweenness Centrality"},
  ImageSize → 350, PlotRange → {{1, 10}, All}],
 ListLinePlot[Thread[{Range@10, modularityvalues}], Frame → True,
  FrameLabel → {"Time Windows", "Modularity"}, ImageSize → 350, PlotRange → All],
 ListLinePlot[{Thread[{Range@10, singlerandomerdrenmodularityvalues}],
   Thread[{Range@10, singlerandomdegreesfxdmodularityvalues}],
   Thread[{Range@10, singlerandomcommmodularityvalues}]}, Frame → True,
  FrameLabel → {"Time Windows", "Modularity (Single Random Graphs)"},
  ImageSize → 350, PlotRange → {{1, 10}, All}, PlotLabels →
   Placed[{"Erdös-Renyi", "Degrees Fxd.", "Communities"}, {Scaled[1], Below}]],
 ListLinePlot[{Thread[{Range@10, ZscoresmodularityWolf[[All, 1]]}],
   Thread[{Range@10, ZscoresmodularityWolf[[All, 2]]}],
   Thread[{Range@10, ZscoresmodularityWolf[[All, 3]]}]},
  Frame → True, FrameLabel → {"Time Windows", "Z-Score for Modularity"},
  ImageSize → 350, PlotRange → All, PlotLabels →
   Placed[{"Erdös Renyi", "Degrees Fxd.", "Communities"}, {Scaled[1], Above}]]}
```

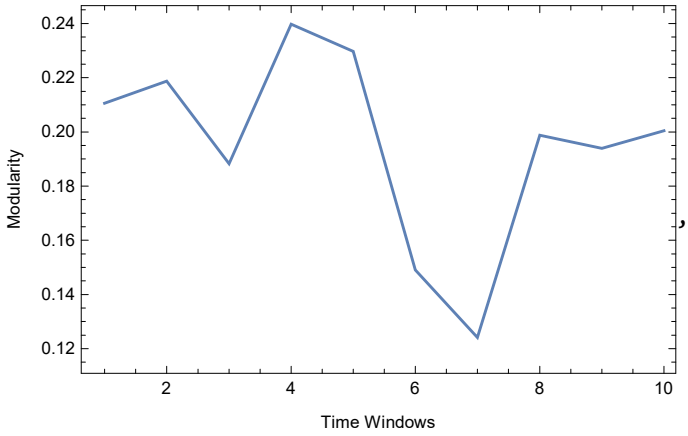
Out[ ]=



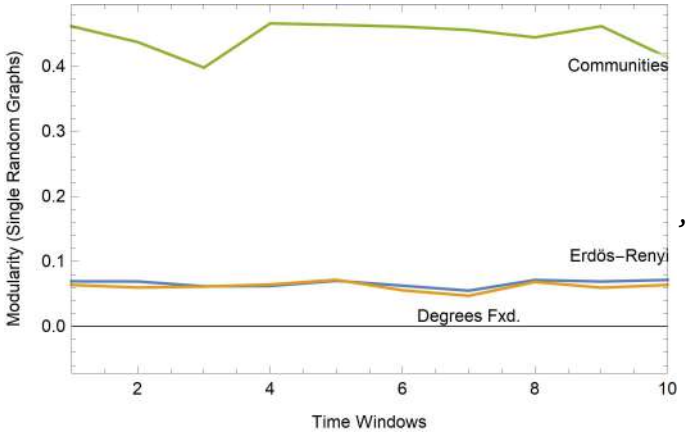
,



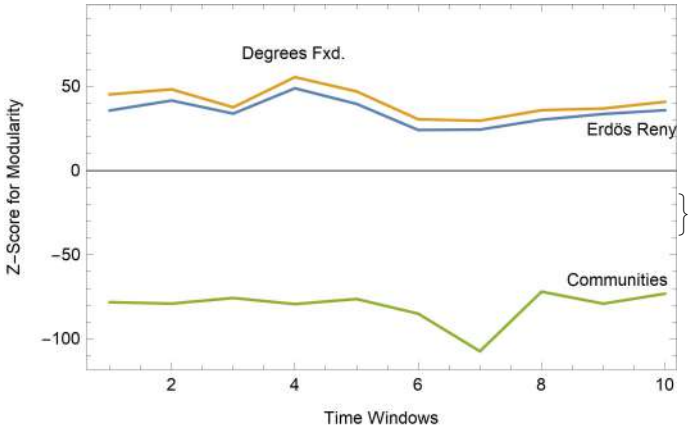
,



,



,



}

Weight Feature

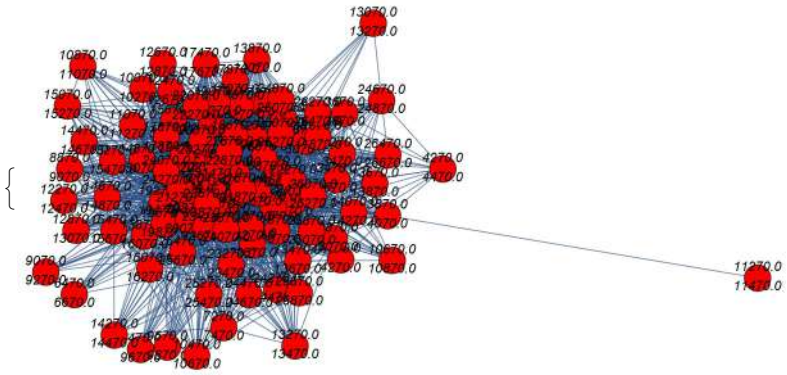
In[ ]:=
```
AbsoluteTiming[
 weightdataintimewindowsFixedstep = snetworkdatabinnedintimewindows[11, 200, 10];]
```

Out[ ]= {77.0267, Null}

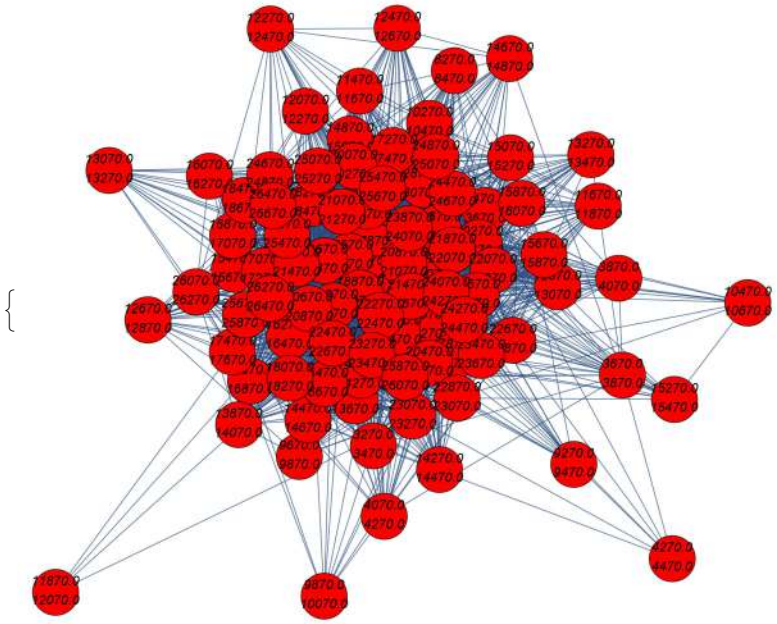
In[ ]:=
```
graphsandnodenumbers = Table[snetworkgraph[weightdataintimewindowsFixedstep[[1]][[i]],
    weightdataintimewindowsFixedstep[[2]][[i]], 2, 7, 400, Red], {i, Range@10}];
```

In[ ]:=
```
graphsandnodenumbers
```

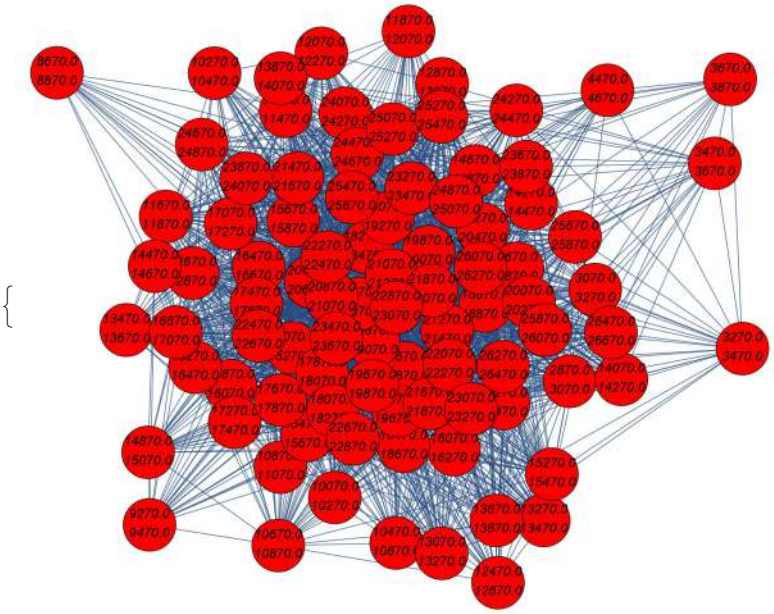
Out[ ]=



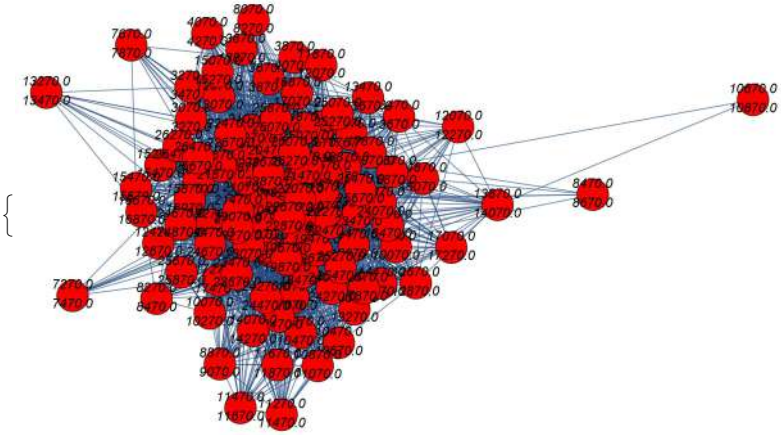
, 94},



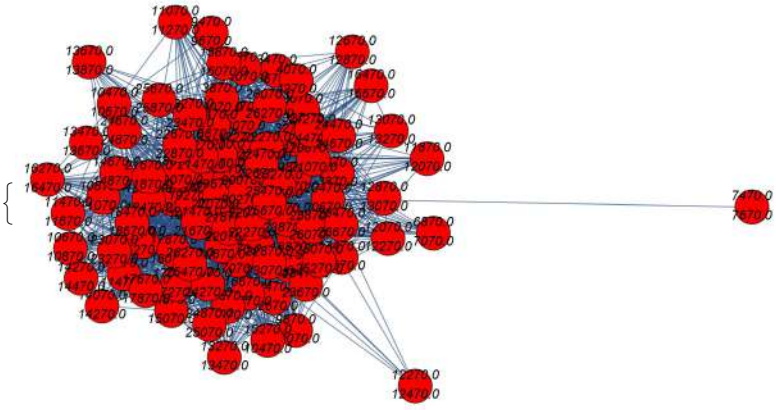
, 90},



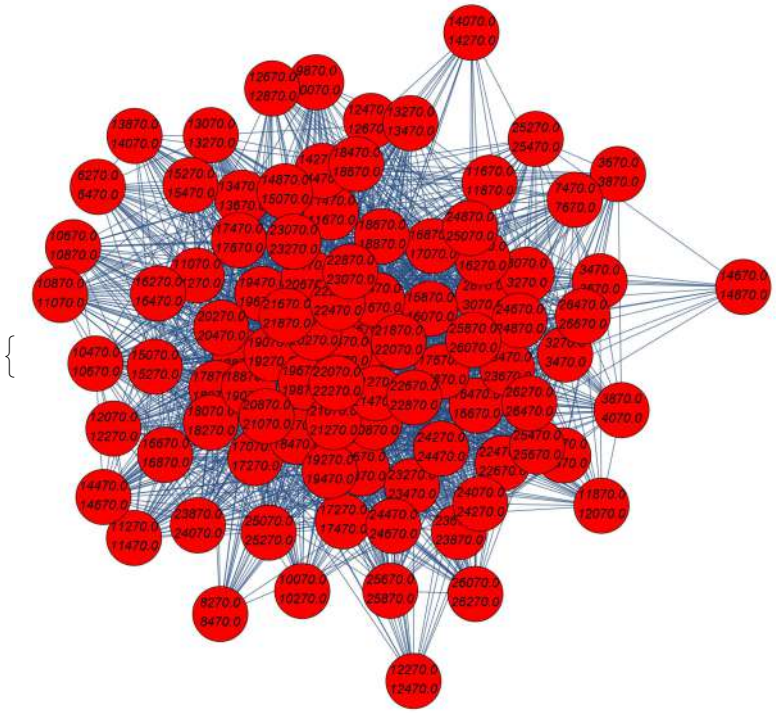
{ , 89},



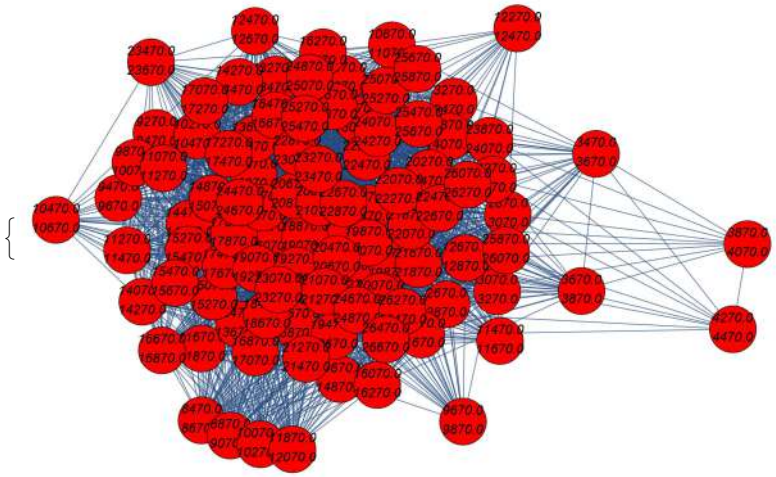
{ , 97},



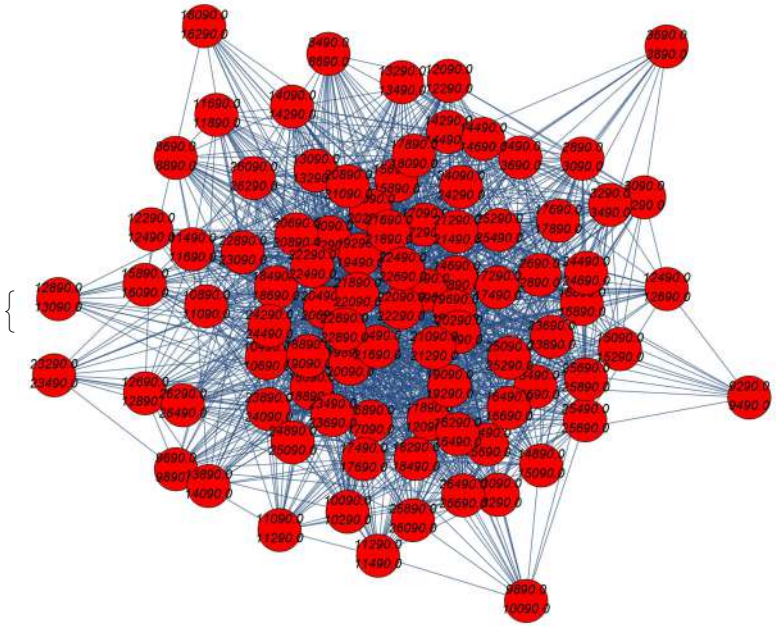
{ , 91},



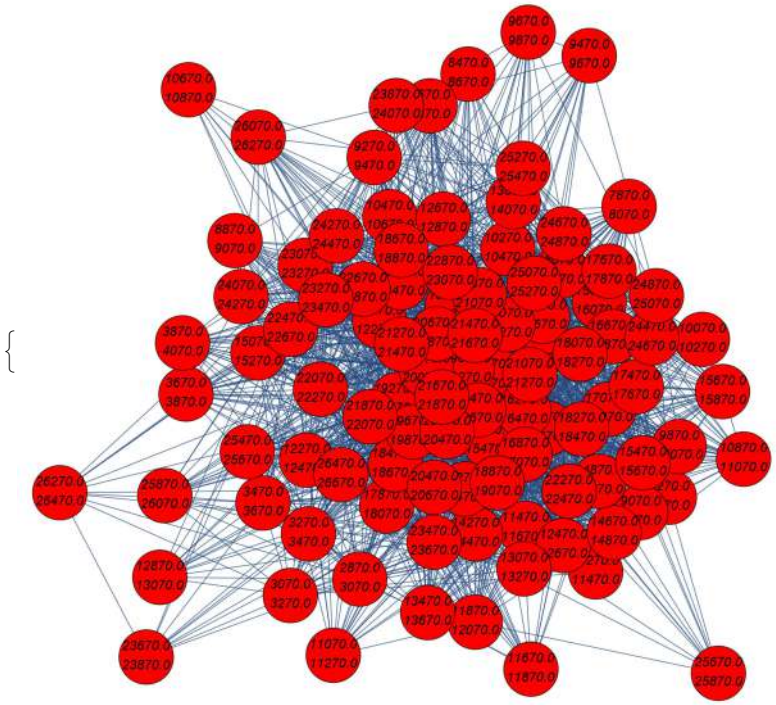
, 92},



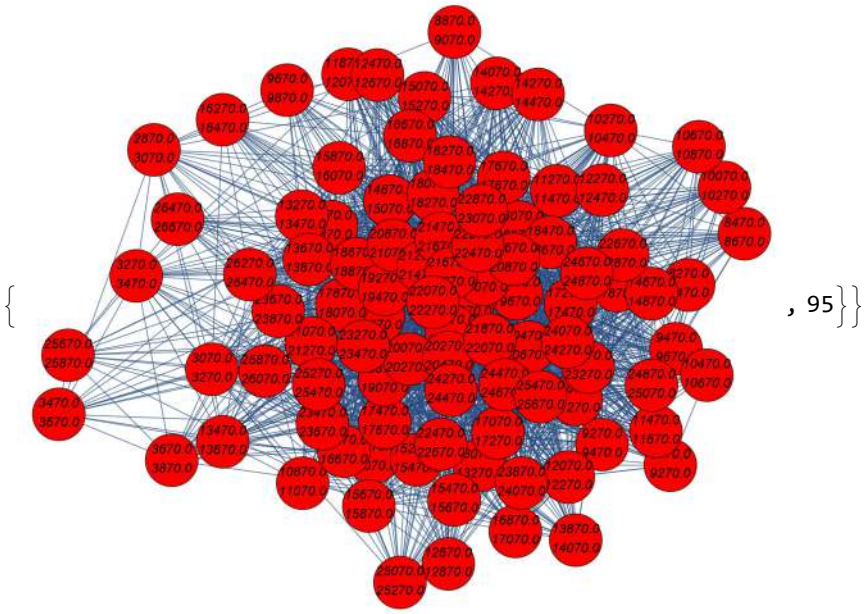
, 96},



, 89},



, 100},



In[ ]:=
```
ABCvalues = Table[Mean@BetweennessCentrality[graphsandnodenumbers[[i]][[1]]],
    {i, Length@graphsandnodenumbers}];
modularityvalues = Table[N@GraphAssortativity[graphsandnodenumbers[[i]][[1]],
      FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
      "Normalized" → False], {i, Length@graphsandnodenumbers}];
degreevalues = Table[N@Mean@VertexDegree[graphsandnodenumbers[[i]][[1]]],
    {i, Length@graphsandnodenumbers}];
```

In[ ]:=
```
singlerandomgraphserdren = Table[
    RandomGraph[{VertexCount[i], EdgeCount[i]}], {i, graphsandnodenumbers[[All, 1]]}];
singlerandomerdrenmodularityvalues =
  Table[N@GraphAssortativity[singlerandomgraphserdren[[i]],
      FindGraphCommunities[singlerandomgraphserdren[[i]]], "Normalized" -> False],
    {i, Length@singlerandomgraphserdren}];
singlerandomgraphsdegreesfxd = Table[IGDegreeSequenceGame[Total[AdjacencyMatrix@i],
Method -> "VigerLatapy"], {i, graphsandnodenumbers[[All, 1]]}];
singlerandomdegreesfxdmodularityvalues =
  Table[N@GraphAssortativity[singlerandomgraphsdegreesfxd[[i]],
      FindGraphCommunities[singlerandomgraphsdegreesfxd[[i]]], "Normalized" -> False],
    {i, Length@singlerandomgraphsdegreesfxd}];
singlerandomgraphscomm = Table[randomizedgraphamongcommunities[i],
    {i, graphsandnodenumbers[[All, 1]]}];
singlerandomcommmodularityvalues = Table[N@GraphAssortativity[
      singlerandomgraphscomm[[i]], FindGraphCommunities[singlerandomgraphscomm[[i]]],
      "Normalized" -> False], {i, Length@singlerandomgraphscomm}];
```

In[ ]:=
```
AbsoluteTiming[ZscoresmodularityWolf =
  Table[randomnessfunctionformodularity[i, "Wolf"], {i, graphsandnodenumbers[[All, 1]]}]]
```

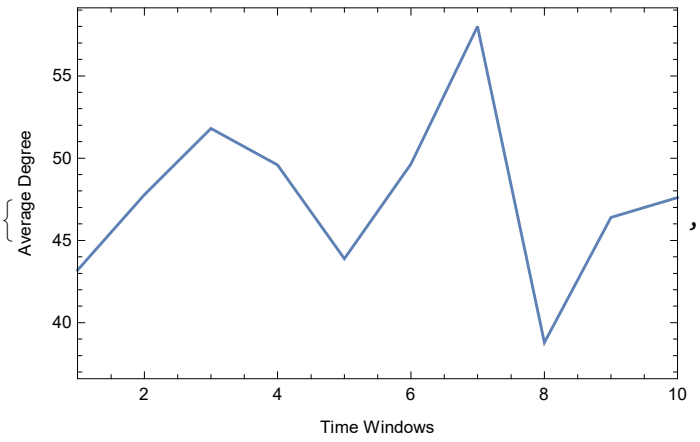
Out[•]= {674.48, {{7.62639, 14.3356, -82.984}, {1.14625, 5.60766, -109.218},
{3.50924, 7.04036, -114.502}, {8.94567, 14.6903, -120.16}, {7.66123, 14.1205, -99.8434},
{12.782, 18.6146, -111.619}, {7.75521, 10.747, -138.651}, {3.42677, 6.94266, -80.2143},
{3.2996, 7.69011, -94.6969}, {8.27245, 12.244, -117.493}}}

In[•]:=
```
Table[Length@FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
 {i, Length@graphsandnodenumbers}]
```

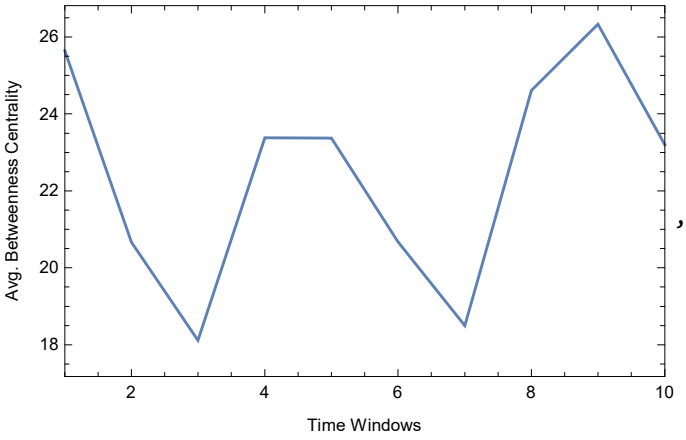
Out[•]= {3, 3, 3, 3, 3, 2, 3, 3, 3, 3}

In[•]:=
```
{ListLinePlot[Thread[{Range@10, degreevalues}], Frame → True,
  FrameLabel → {"Time Windows", "Average Degree"}, ImageSize → 350,
  PlotRange → {{1, 10}, All}], ListLinePlot[Thread[{Range@10, ABCvalues}],
  Frame → True, FrameLabel → {"Time Windows", "Avg. Betweenness Centrality"},
  ImageSize → 350, PlotRange → {{1, 10}, All}],
 ListLinePlot[Thread[{Range@10, modularityvalues}], Frame → True,
  FrameLabel → {"Time Windows", "Modularity"}, ImageSize → 350, PlotRange → All],
 ListLinePlot[{Thread[{Range@10, singlerandomerdrenmodularityvalues}],
   Thread[{Range@10, singlerandomdegreesfxdmodularityvalues}],
   Thread[{Range@10, singlerandomcommmodularityvalues}]}, Frame → True,
  FrameLabel → {"Time Windows", "Modularity (Single Random Graphs)"},
  ImageSize → 350, PlotRange → {{1, 10}, All}, PlotLabels →
   Placed[{"Erdös-Renyi", "Degrees Fxd.", "Communities"}, {Scaled[1], Above}]],
 ListLinePlot[{Thread[{Range@10, ZscoresmodularityWolf[[All, 1]]}],
   Thread[{Range@10, ZscoresmodularityWolf[[All, 2]]}],
   Thread[{Range@10, ZscoresmodularityWolf[[All, 3]]}]},
  Frame → True, FrameLabel → {"Time Windows", "Z-Score for Modularity"},
  ImageSize → 350, PlotRange → {All, {0, 20}}, PlotLabels →
   Placed[{"Erdös Renyi", "Degrees Fxd.", "Communities"}, {Scaled[1], Below}]],
 ListLinePlot[{Thread[{Range@10, ZscoresmodularityWolf[[All, 1]]}],
   Thread[{Range@10, ZscoresmodularityWolf[[All, 2]]}],
   Thread[{Range@10, ZscoresmodularityWolf[[All, 3]]}]},
  Frame → True, FrameLabel → {"Time Windows", "Z-Score for Modularity"},
  ImageSize → 350, PlotRange → {All, {-150, 0}}, PlotLabels →
   Placed[{"Erdös Renyi", "Degrees Fxd.", "Communities"}, {Scaled[1], Below}]]}
```

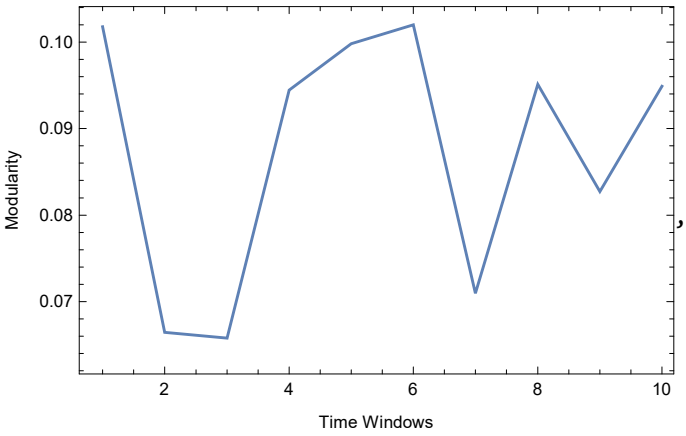
Out[•]=



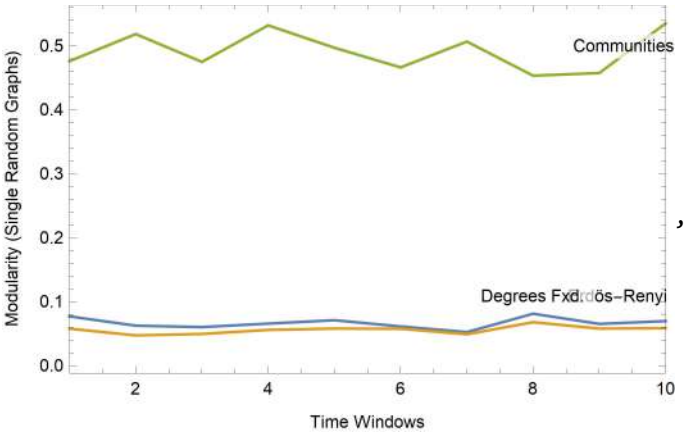
,


Avg. Betweenness Centrality
26
24
22
20
18
2
4
6
8
10
Time Windows

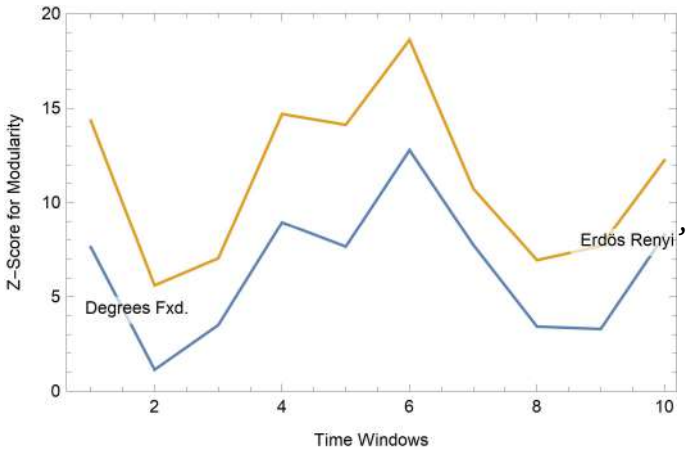
,

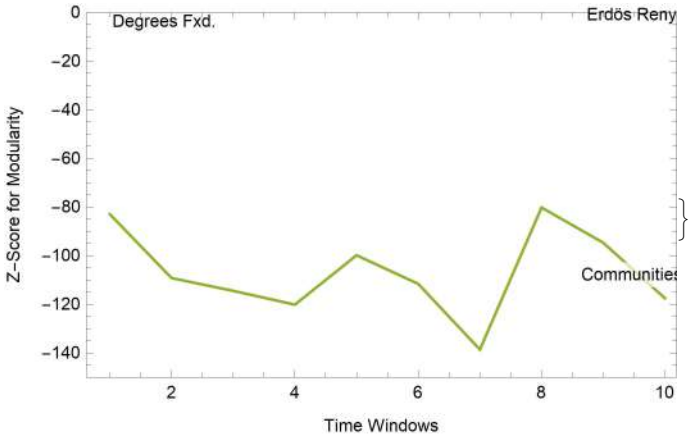

Modularity
0.10
0.09
0.08
0.07
2
4
6
8
10
Time Windows

,


Modularity (Single Random Graphs)
0.5
0.4
0.3
0.2
0.1
0.0
Communities
Degrees Fxd. Erdös–Renyi
2
4
6
8
10
Time Windows

,

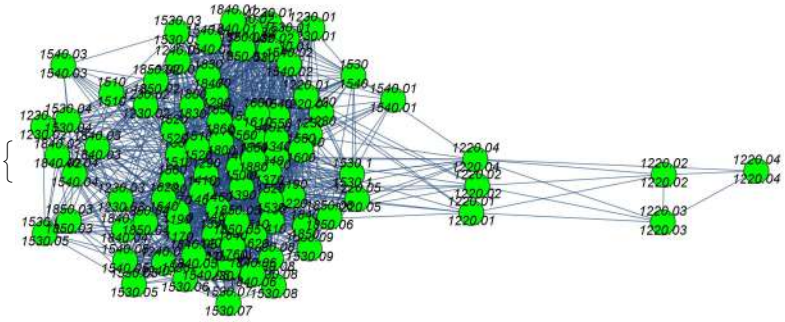


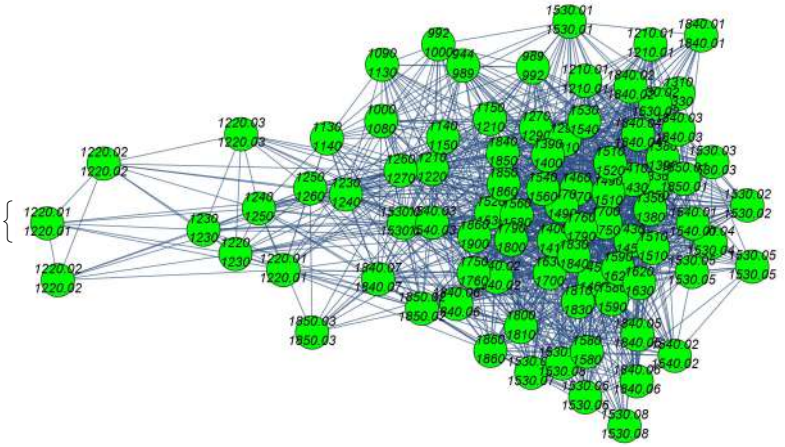


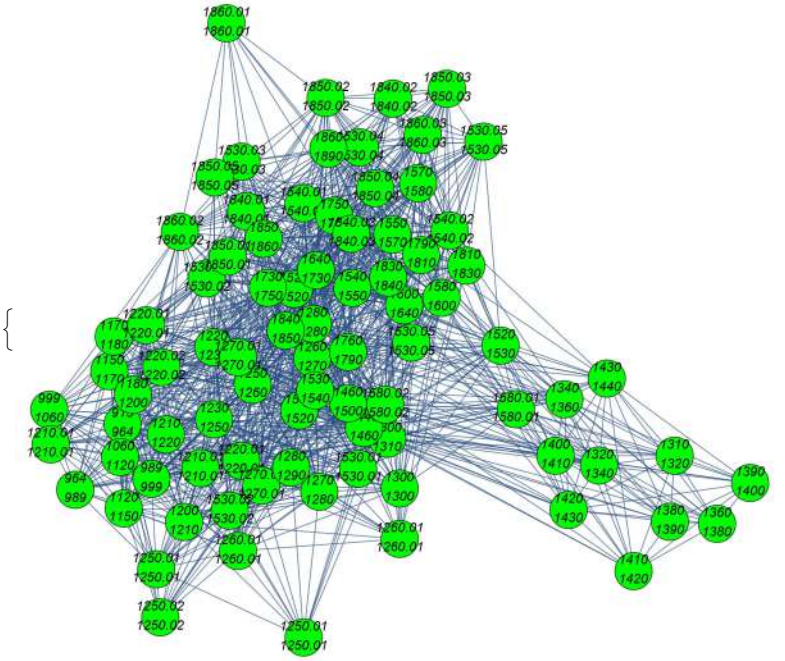
Fixed Bucket Size Networks

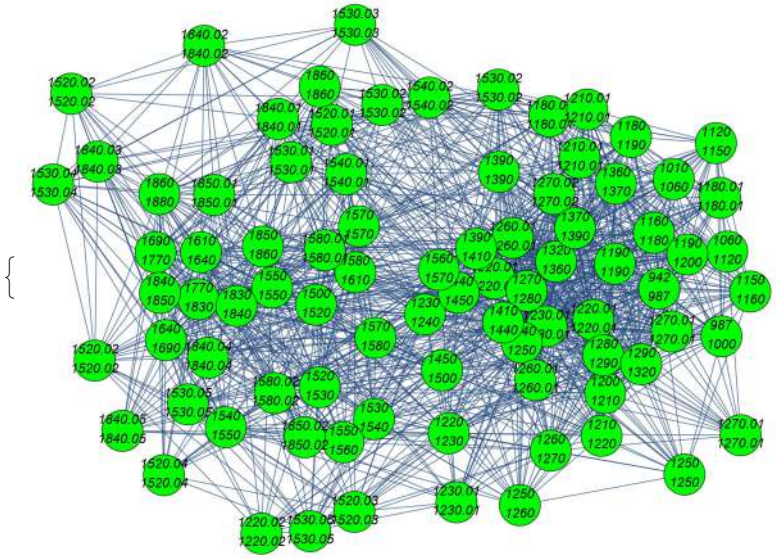
Width Feature

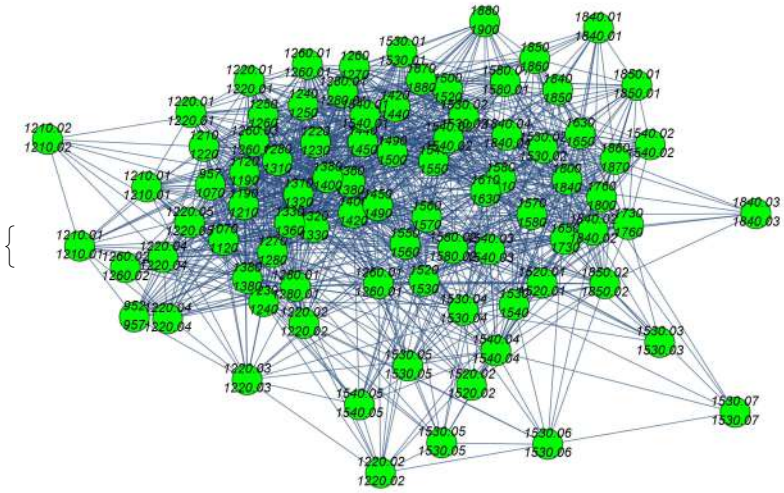
```
In[ ]:= AbsoluteTiming[
         widthdataintimewindowsFixedbucket = snetworkdatafxdbucketintimewindows[9, 85, 10];]
Out[ ]= {5.36296, Null}

In[ ]:= graphsandnodenumbers = Table[snetworkgraph[widthdataintimewindowsFixedbucket[[1]][[i]],
          widthdataintimewindowsFixedbucket[[2]][[i]], 1.5, 7, 400, Green], {i, Range@10}];

In[ ]:= graphsandnodenumbers
```

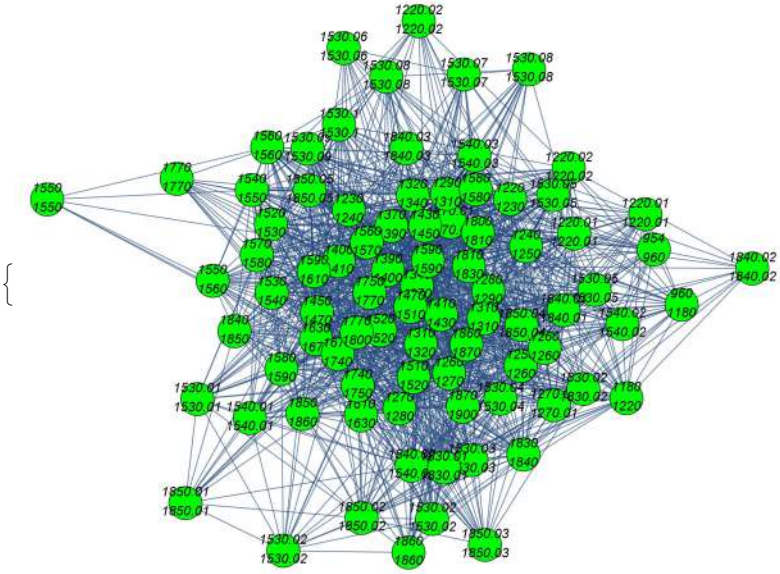
Out[•]=  , 85},

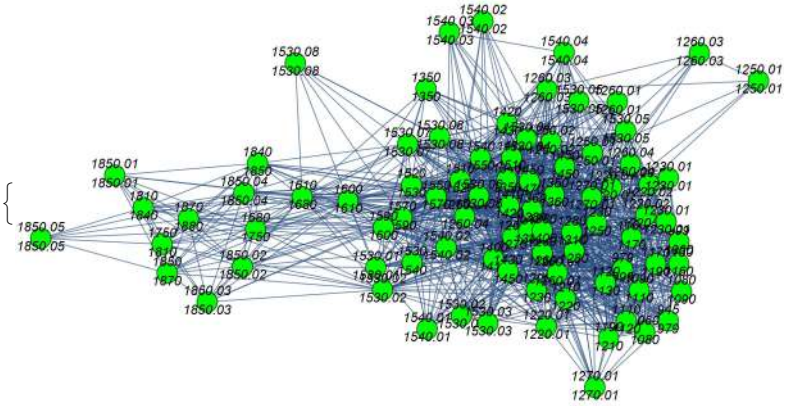
 , 85},

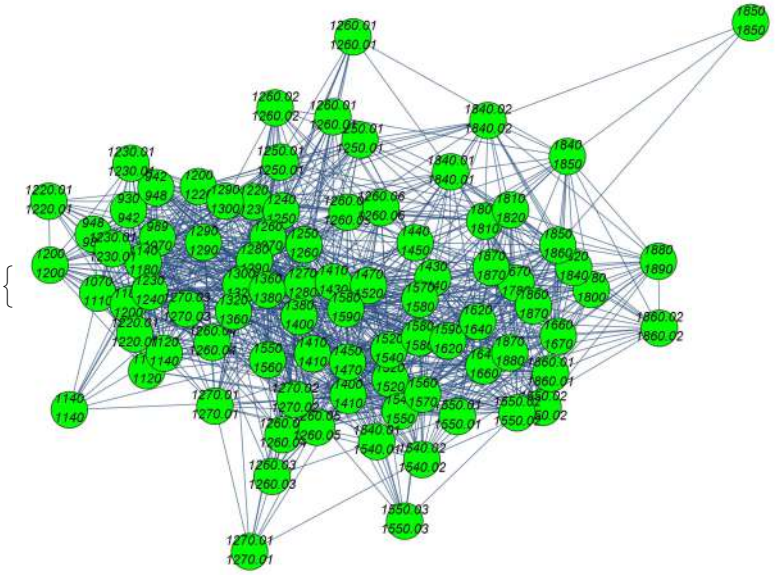
 , 85},

{  , 85},

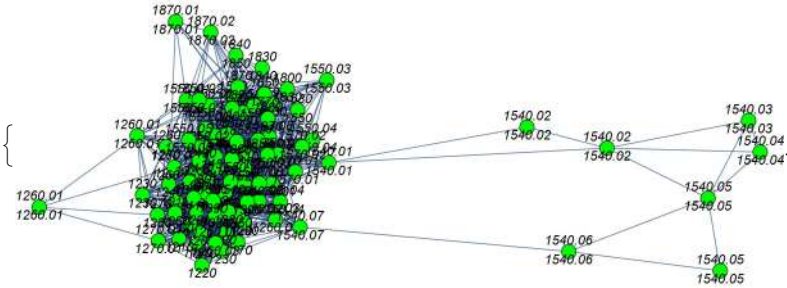
{  , 85},

{  , 85},

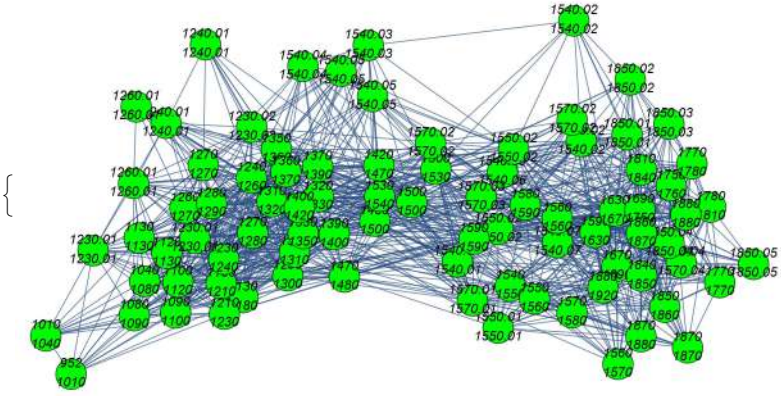
{  , 85},



, 85},



, 85},



, 85}}

In[•]:=
```
ABCvalues = Table[Mean@BetweennessCentrality[graphsandnodenumbers[[i]][[1]]],
    {i, Length@graphsandnodenumbers}];
modularityvalues = Table[N@GraphAssortativity[graphsandnodenumbers[[i]][[1]],
      FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
      "Normalized" → False], {i, Length@graphsandnodenumbers}];
degreevalues = Table[N@Mean@VertexDegree[graphsandnodenumbers[[i]][[1]]],
    {i, Length@graphsandnodenumbers}];
```

In[•]:=
```
singlerandomgraphserdren = Table[
    RandomGraph[{VertexCount[i], EdgeCount[i]}], {i, graphsandnodenumbers[[All, 1]]}];
singlerandomerdrenmodularityvalues =
  Table[N@GraphAssortativity[singlerandomgraphserdren[[i]],
      FindGraphCommunities[singlerandomgraphserdren[[i]]], "Normalized" -> False],
    {i, Length@singlerandomgraphserdren}];
singlerandomgraphsdegreesfxd = Table[IGDegreeSequenceGame[Total[AdjacencyMatrix@i],
Method -> "VigerLatapy"], {i, graphsandnodenumbers[[All, 1]]}];
singlerandomdegreesfxdmodularityvalues =
  Table[N@GraphAssortativity[singlerandomgraphsdegreesfxd[[i]],
      FindGraphCommunities[singlerandomgraphsdegreesfxd[[i]]], "Normalized" -> False],
    {i, Length@singlerandomgraphsdegreesfxd}];
singlerandomgraphscomm = Table[randomizedgraphamongcommunities[i],
    {i, graphsandnodenumbers[[All, 1]]}];
singlerandomcommmodularityvalues = Table[N@GraphAssortativity[
      singlerandomgraphscomm[[i]], FindGraphCommunities[singlerandomgraphscomm[[i]]],
      "Normalized" -> False], {i, Length@singlerandomgraphscomm}];
```

In[•]:=
```
AbsoluteTiming[ZscoresmodularityWolf =
  Table[randomnessfunctionformodularity[i, "Wolf"], {i, graphsandnodenumbers[[All, 1]]}]]
```

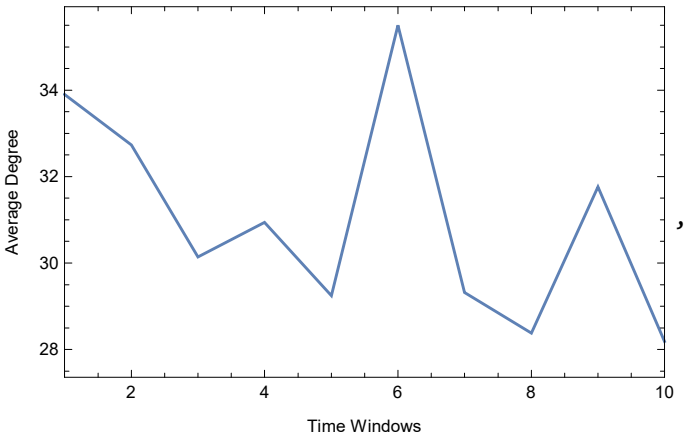
Out[•]=
```
{281.521, {{18.1208, 22.7519, -65.7198}, {18.4316, 23.9744, -50.6182},
  {21.7818, 24.2696, -53.3967}, {36.0531, 40.7257, -44.7595}, {26.9589, 33.176, -48.7689},
  {12.3565, 16.4717, -82.235}, {19.0099, 25.0379, -63.0921}, {32.3239, 37.1309, -45.9318},
  {22.5486, 32.0465, -62.4136}, {50.035, 55.7312, -34.7445}}}
```

In[•]:=
```
Table[Length@FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
 {i, Length@graphsandnodenumbers}]
```

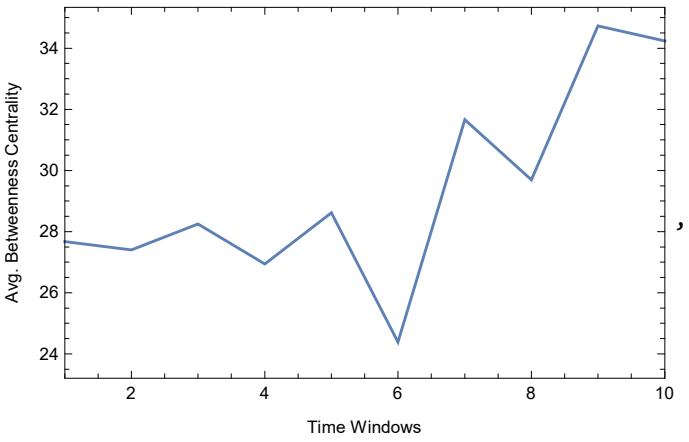
Out[•]=
```
{2, 3, 2, 2, 2, 3, 3, 2, 3, 3}
```

In[ ]:=
```
{ListLinePlot[Thread[{Range@10, degreevalues}], Frame → True,
  FrameLabel → {"Time Windows", "Average Degree"}, ImageSize → 350,
  PlotRange → {{1, 10}, All}], ListLinePlot[Thread[{Range@10, ABCvalues}],
  Frame → True, FrameLabel → {"Time Windows", "Avg. Betweenness Centrality"},
  ImageSize → 350, PlotRange → {{1, 10}, All}],
 ListLinePlot[Thread[{Range@10, modularityvalues}], Frame → True,
  FrameLabel → {"Time Windows", "Modularity"}, ImageSize → 350, PlotRange → All],
 ListLinePlot[{Thread[{Range@10, singlerandomerdrenmodularityvalues}],
   Thread[{Range@10, singlerandomdegreesfxdmodularityvalues}],
   Thread[{Range@10, singlerandomcommmodularityvalues}]}, Frame → True,
  FrameLabel → {"Time Windows", "Modularity (Single Random Graphs)"},
  ImageSize → 350, PlotRange → {{1, 10}, All}, PlotLabels →
   Placed[{"Erdös-Renyi", "Degrees Fxd.", "Communities"}, {Scaled[1], Above}]],
 ListLinePlot[{Thread[{Range@10, ZscoresmodularityWolf[[All, 1]]}],
   Thread[{Range@10, ZscoresmodularityWolf[[All, 2]]}],
   Thread[{Range@10, ZscoresmodularityWolf[[All, 3]]}]},
  Frame → True, FrameLabel → {"Time Windows", "Z-Score for Modularity"},
  ImageSize → 350, PlotRange → All, PlotLabels →
   Placed[{"Erdös Renyi", "Degrees Fxd.", "Communities"}, {Scaled[1], Above}]]}
```

Out[ ]=



,



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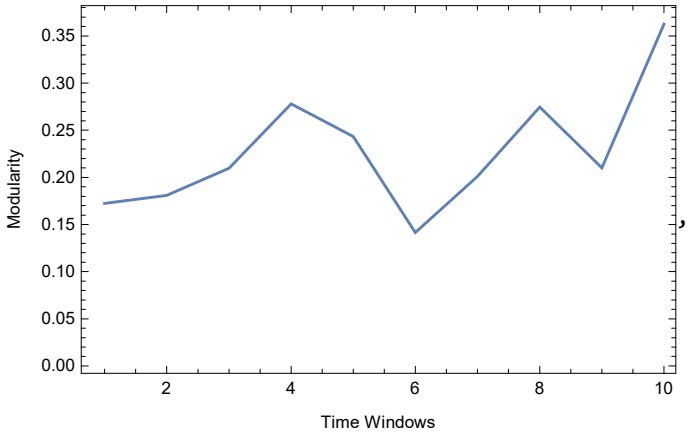
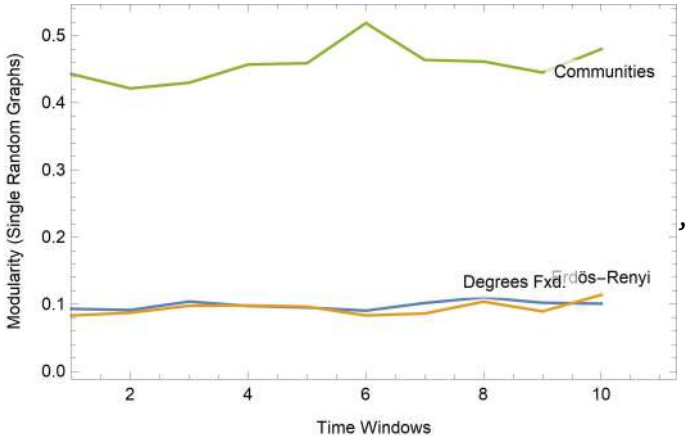
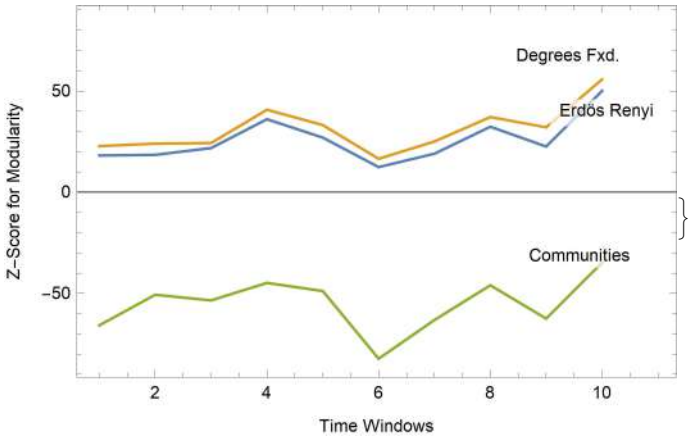






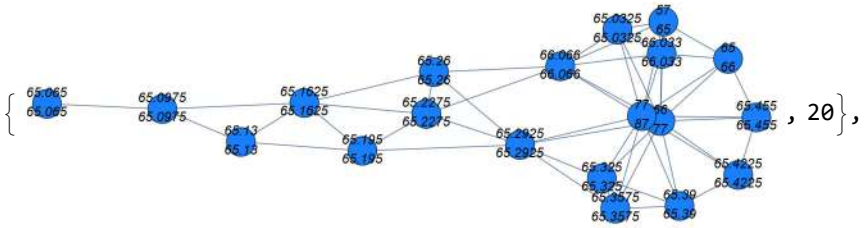
Thickness Feature

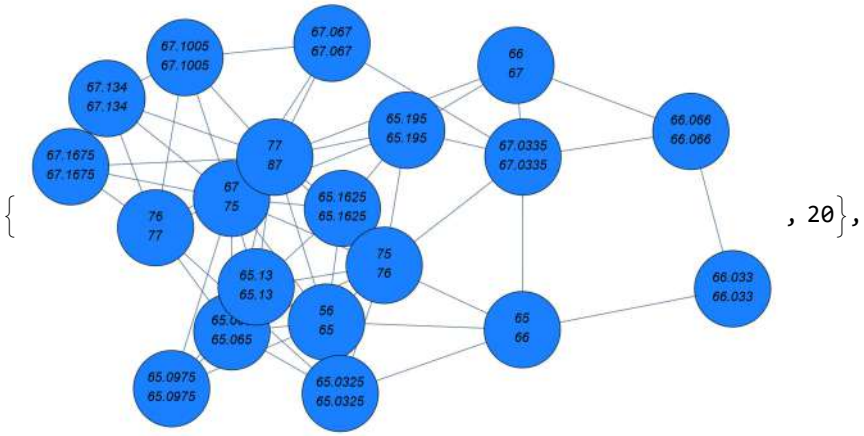
```
In[ ]:= AbsoluteTiming[
          thicknessdataintimewindowsFixedbucket = snetworkdatafxdbucketintimewindows[10, 20, 10];]
Out[ ]= {5.16073, Null}
```

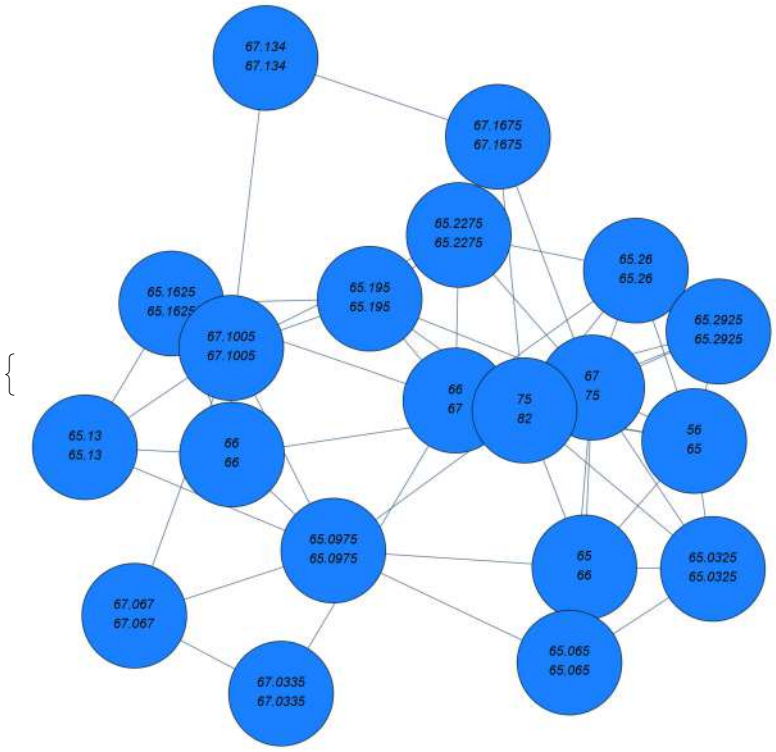
In[ ]:=
```
graphsandnodenumbers =
  Table[snetworkgraph[thicknessdataintimewindowsFixedbucket[[1]][[i]],
    thicknessdataintimewindowsFixedbucket[[2]][[i]],
    1.5, 7, 400, RGBColor[0.1, 0.5, 1.]], {i, Range@10}];
```

In[ ]:=
```
graphsandnodenumbers
```

Out[ ]=



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67.1675
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65.2275
65.26
65.26
65.1625
65.1625
65.195
65.195
67.1005
67.1005
65.2925
65.2925
66
67
75
82
67
75
65.13
65.13
66
66
56
65
65.0975
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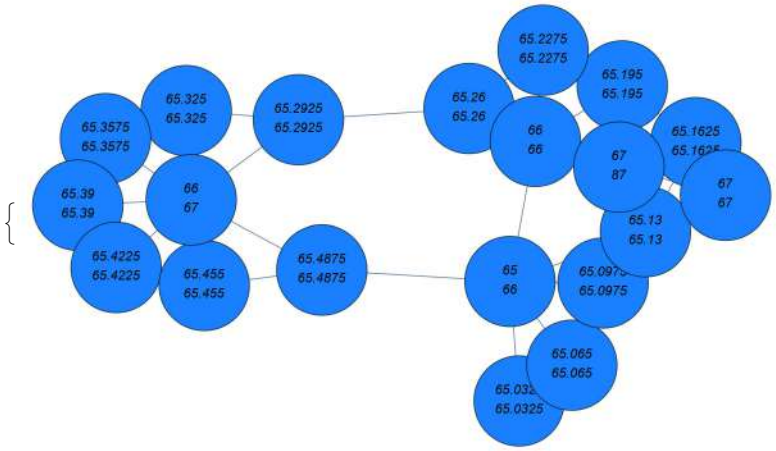
65.065
67.0335
67.0335

, 20},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65.3575
66
66
65.1625
65.1625
66
67
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65.4225
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65.065
65.065
65.0325
65.0325

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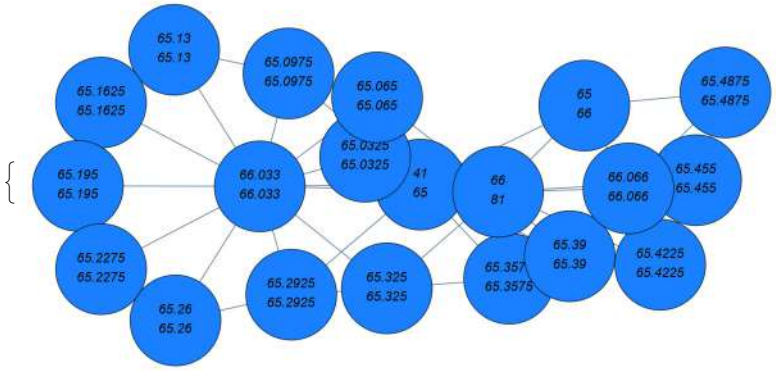
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65
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66.033
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65
66
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66.066
65.455
65.455
65.39
65.39
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65.26

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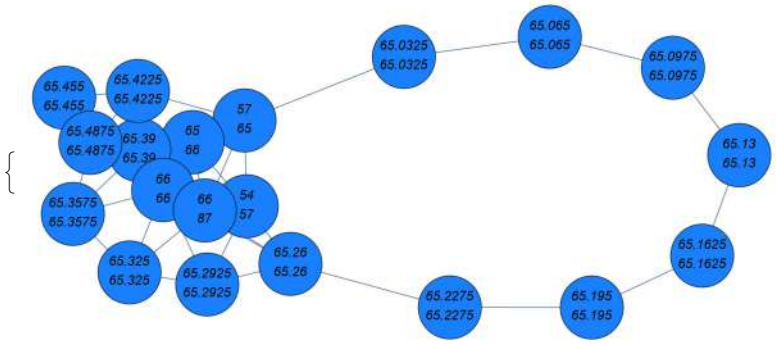
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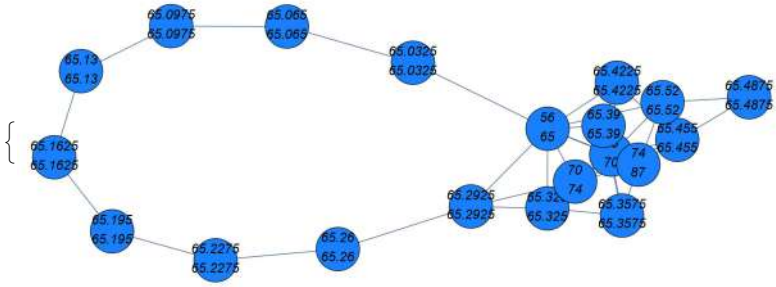
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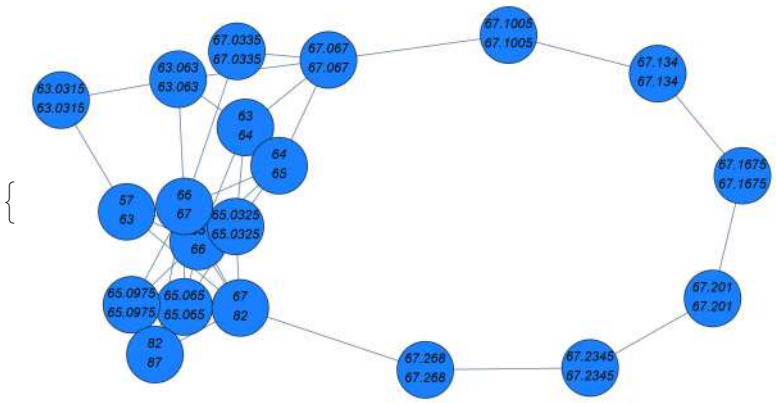
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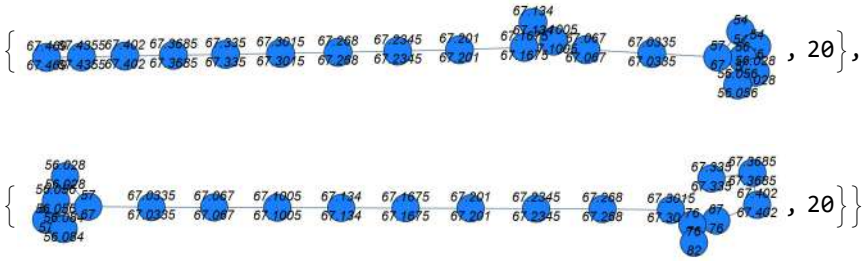


In[ ]:=
```
ABCvalues = Table[Mean@BetweennessCentrality[graphsandnodenumbers[[i]][[1]]],
   {i, Length@graphsandnodenumbers}];
modularityvalues = Table[N@GraphAssortativity[graphsandnodenumbers[[i]][[1]],
     FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
     "Normalized" → False], {i, Length@graphsandnodenumbers}];
degreevalues = Table[N@Mean@VertexDegree[graphsandnodenumbers[[i]][[1]]],
   {i, Length@graphsandnodenumbers}];
```

In[ ]:=
```
singlerandomgraphserdren = Table[
   RandomGraph[{VertexCount[i], EdgeCount[i]}], {i, graphsandnodenumbers[[All, 1]]}];
singlerandomerdrenmodularityvalues =
  Table[N@GraphAssortativity[singlerandomgraphserdren[[i]],
     FindGraphCommunities[singlerandomgraphserdren[[i]]], "Normalized" -> False],
   {i, Length@singlerandomgraphserdren}];
singlerandomgraphsdegreesfxd = Table[IGDegreeSequenceGame[Total[AdjacencyMatrix@i],
Method -> "VigerLatapy"], {i, graphsandnodenumbers[[All, 1]]}];
singlerandomdegreesfxdmodularityvalues =
  Table[N@GraphAssortativity[singlerandomgraphsdegreesfxd[[i]],
     FindGraphCommunities[singlerandomgraphsdegreesfxd[[i]]], "Normalized" -> False],
   {i, Length@singlerandomgraphsdegreesfxd}];
singlerandomgraphscomm = Table[randomizedgraphamongcommunities[i],
   {i, graphsandnodenumbers[[All, 1]]}];
singlerandomcommmodularityvalues = Table[N@GraphAssortativity[
     singlerandomgraphscomm[[i]], FindGraphCommunities[singlerandomgraphscomm[[i]]],
     "Normalized" -> False], {i, Length@singlerandomgraphscomm}];
```

In[ ]:=
```
AbsoluteTiming[ZscoresmodularityWolf =
  Table[randomnessfunctionformodularity[i, "Wolf"], {i, graphsandnodenumbers[[All, 1]]}]]
```

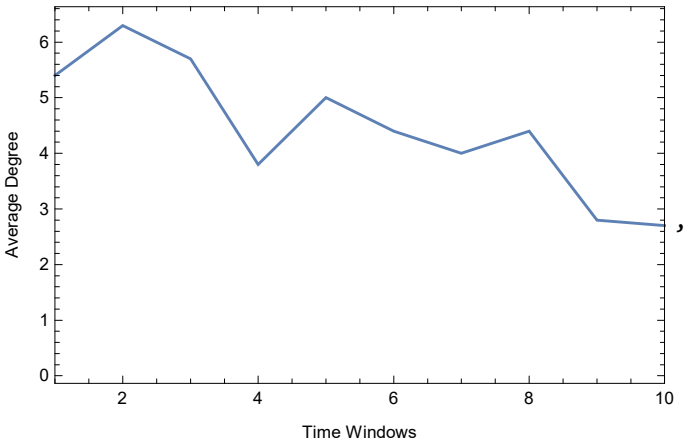
Out[ ]=
```
{65.1567, {{4.07445, 5.50428, -4.0838}, {2.68955, 4.14274, -5.86804},
  {1.83862, 2.54654, -3.07004}, {5.26397, 4.6218, -0.125558}, {3.0181, 4.33807, -3.21577},
  {0.924623, 1.94833, -3.23749}, {0.213684, 0.962007, -2.55683},
  {1.64527, 2.95783, -1.56844}, {4.68317, 5.01019, 1.85761}, {4.64162, 4.59406, 1.34961}}}
```

In[ ]:=
```
Table[Length@FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
 {i, Length@graphsandnodenumbers}]
```

Out[ ]=
```
{3, 3, 3, 3, 3, 3, 3, 3, 4, 4}
```

In[ ]:=
```
{ListLinePlot[Thread[{Range@10, degreevalues}], Frame → True,
  FrameLabel → {"Time Windows", "Average Degree"}, ImageSize → 350,
  PlotRange → {{1, 10}, All}], ListLinePlot[Thread[{Range@10, ABCvalues}],
  Frame → True, FrameLabel → {"Time Windows", "Avg. Betweenness Centrality"},
  ImageSize → 350, PlotRange → {{1, 10}, All}],
 ListLinePlot[Thread[{Range@10, modularityvalues}], Frame → True,
  FrameLabel → {"Time Windows", "Modularity"}, ImageSize → 350, PlotRange → All],
 ListLinePlot[{Thread[{Range@10, singlerandomerdrenmodularityvalues}],
   Thread[{Range@10, singlerandomdegreesfxdmodularityvalues}],
   Thread[{Range@10, singlerandomcommmodularityvalues}]}, Frame → True,
  FrameLabel → {"Time Windows", "Modularity (Single Random Graphs)"},
  ImageSize → 350, PlotRange → {{1, 10}, All}, PlotLabels →
   Placed[{"Erdös-Renyi", "Degrees Fxd.", "Communities"}, {Scaled[1], Below}]],
 ListLinePlot[{Thread[{Range@10, ZscoresmodularityWolf[[All, 1]]}],
   Thread[{Range@10, ZscoresmodularityWolf[[All, 2]]}],
   Thread[{Range@10, ZscoresmodularityWolf[[All, 3]]}]},
  Frame → True, FrameLabel → {"Time Windows", "Z-Score for Modularity"},
  ImageSize → 350, PlotRange → All, PlotLabels →
   Placed[{"Erdös Renyi", "Degrees Fxd.", "Communities"}, {Scaled[1], Above}]]}
```

Out[ ]=



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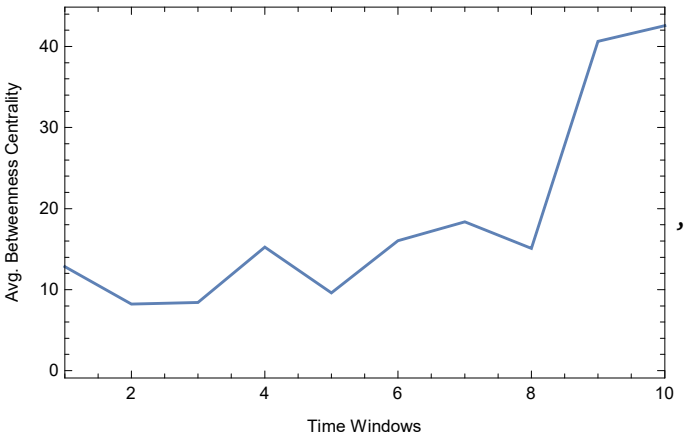

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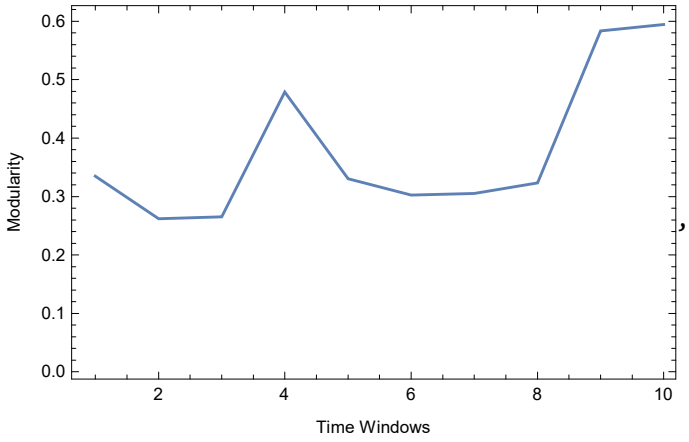

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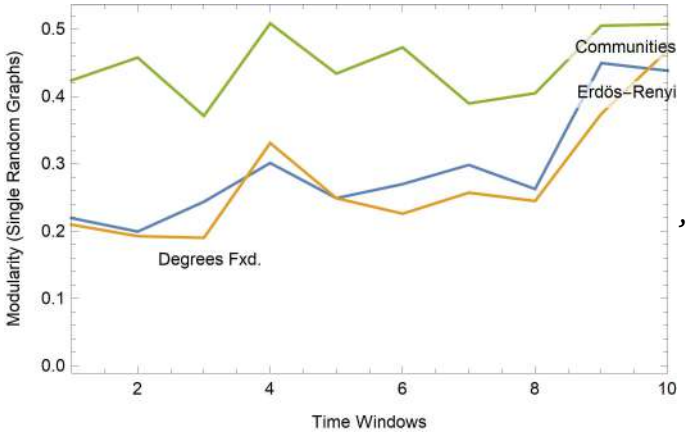

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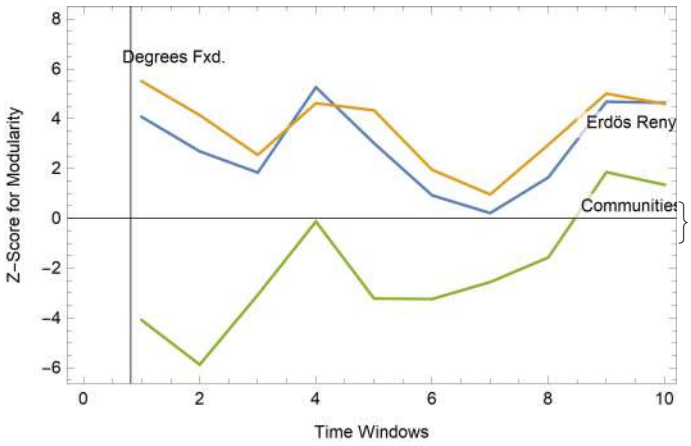


Length Feature

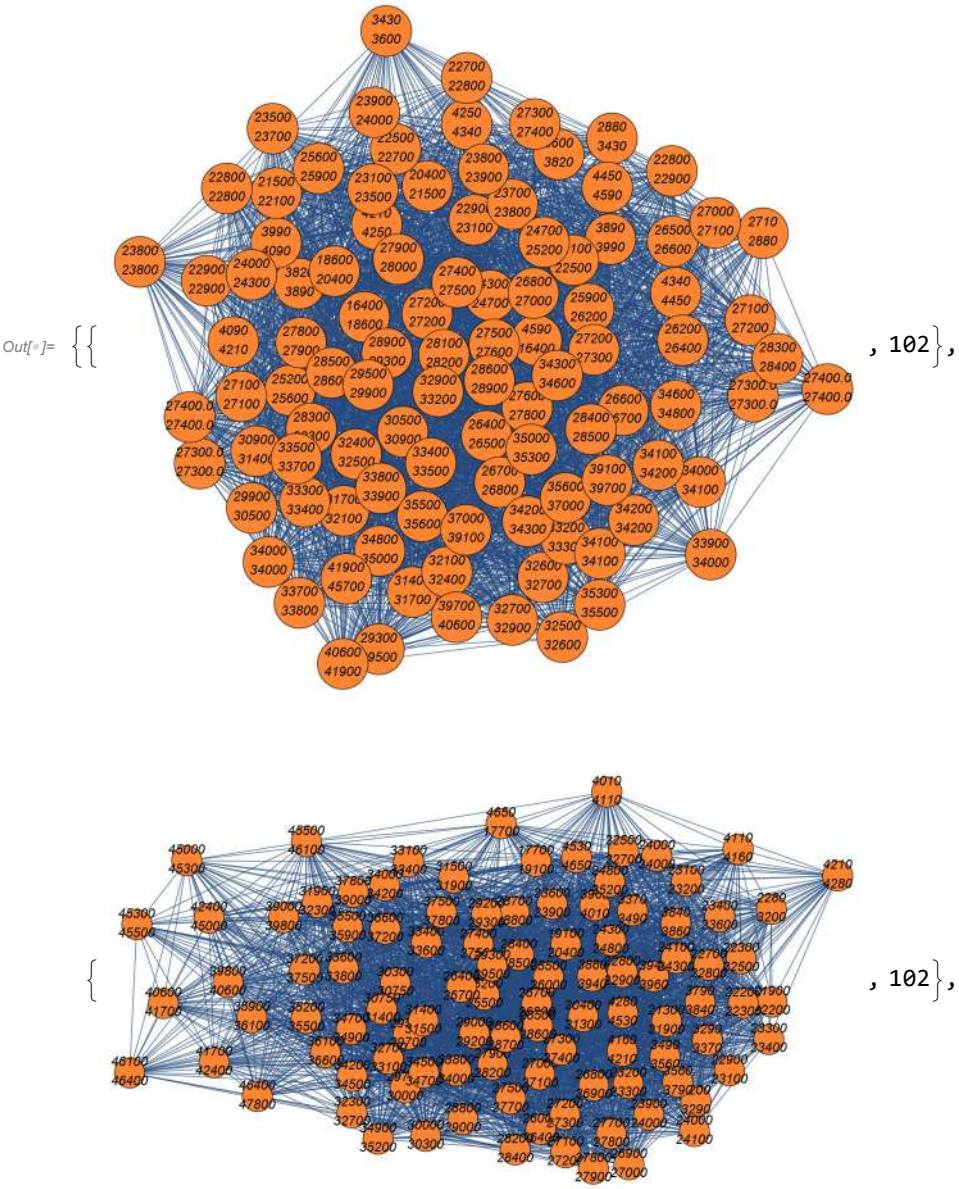
In[ ]:=
```
AbsoluteTiming[
 lengthdataintimewindowsFixedbucket = snetworkdatafxdbucketintimewindows[12, 102, 10];]
```

Out[ ]= {6.6219, Null}

In[ ]:=
```
graphsandnodenumbers = Table[snetworkgraph[lengthdataintimewindowsFixedbucket[[1]][[i]],
    lengthdataintimewindowsFixedbucket[[2]][[i]], 1.5,
    7, 400, RGBColor[1., 0.53, 0.2]], {i, Range@10}];
```

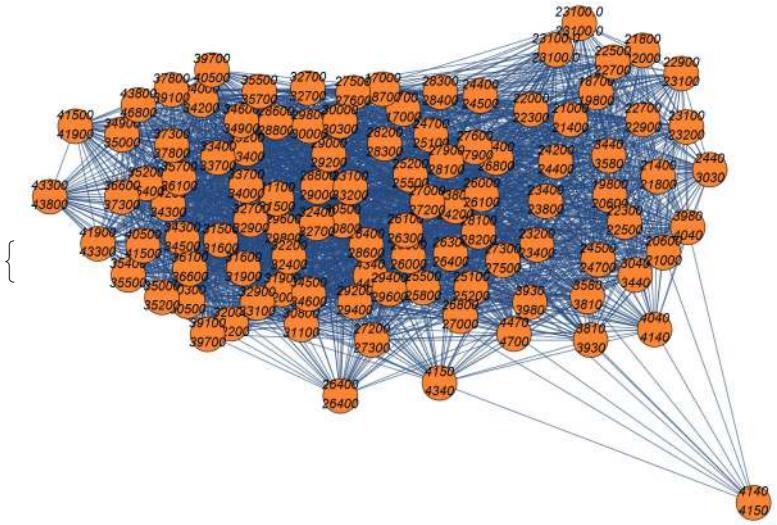
In[ ]:=
```
graphsandnodenumbers
```

Out[ ]=

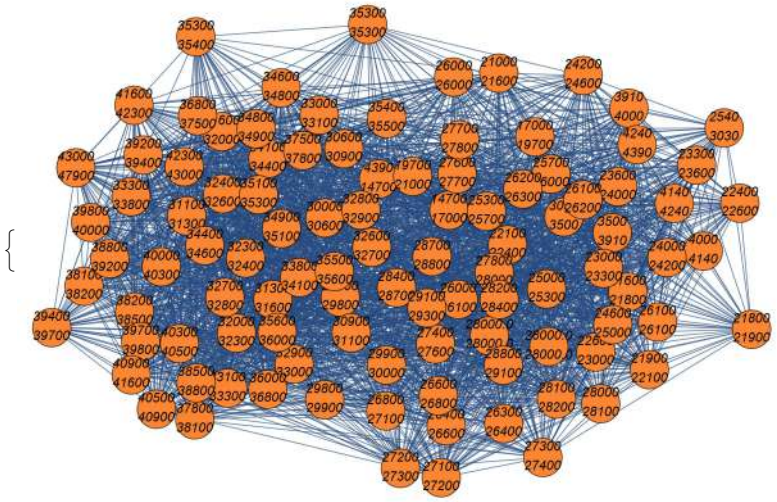


, 102},

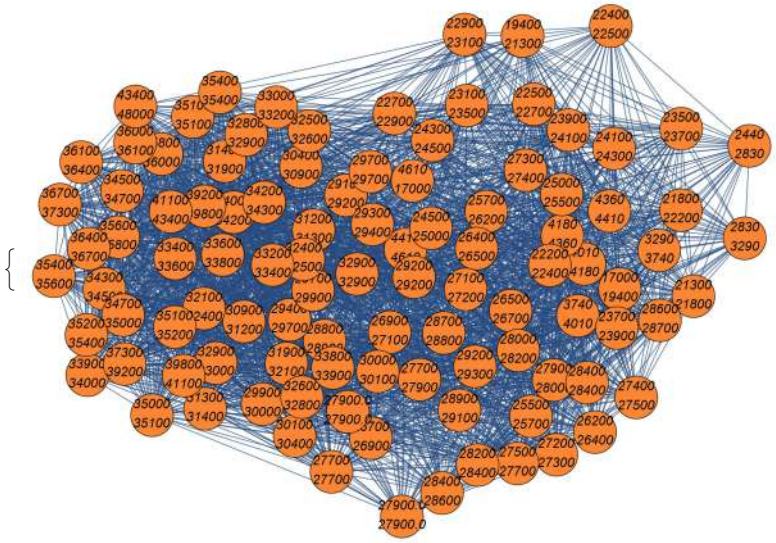
, 102},



, 102},



, 102},



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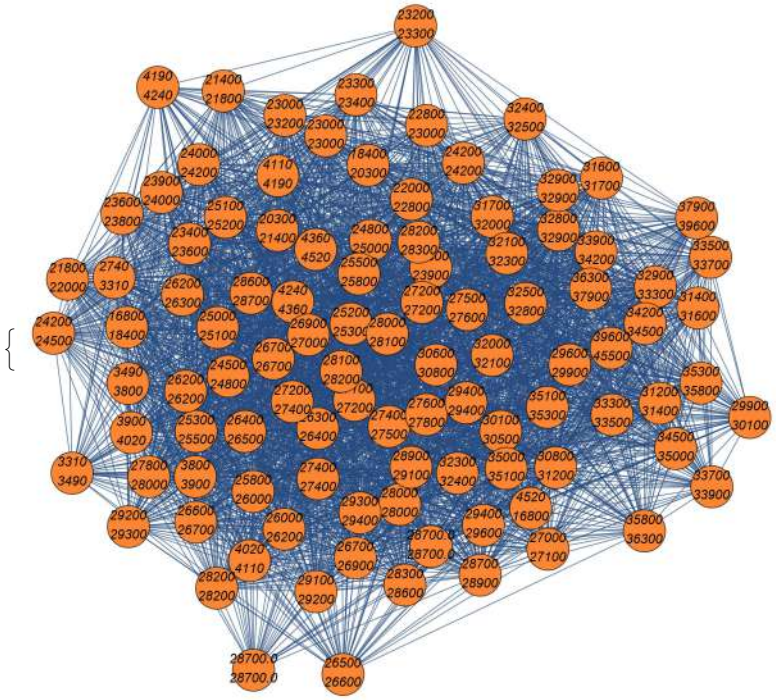

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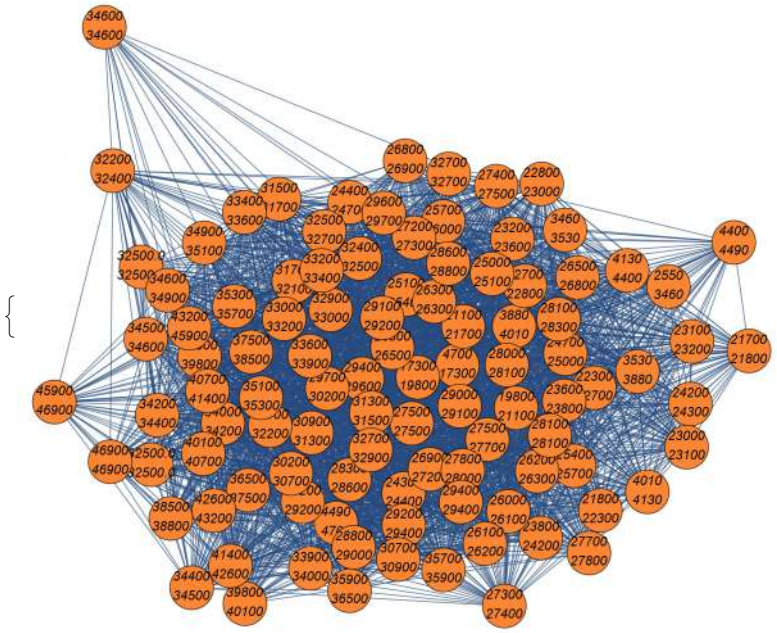

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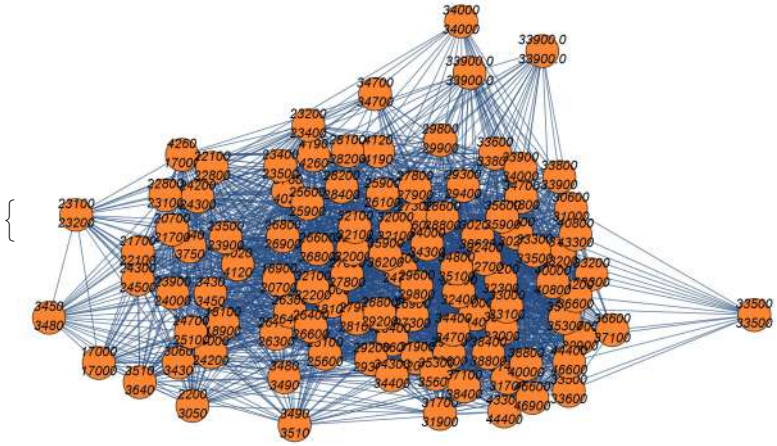
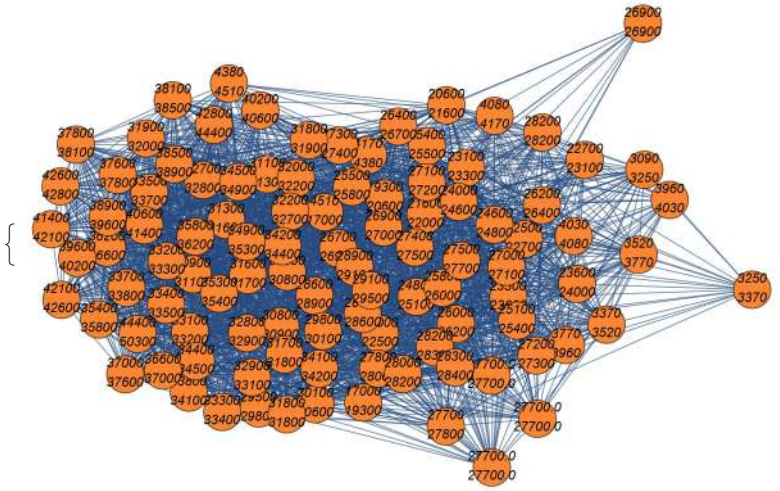
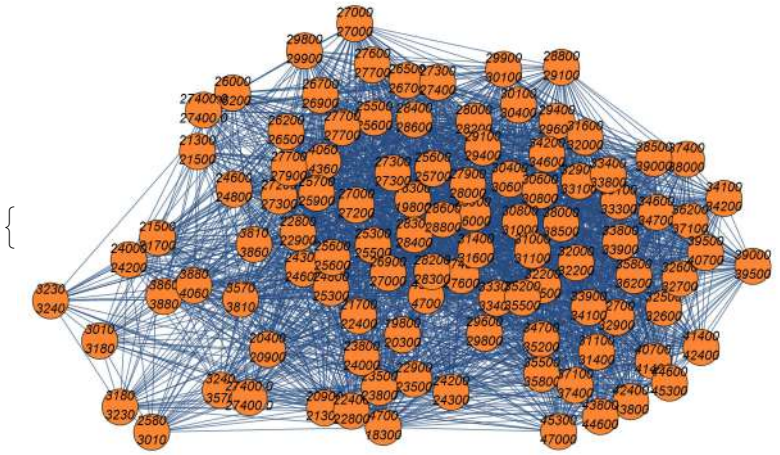


, 102},



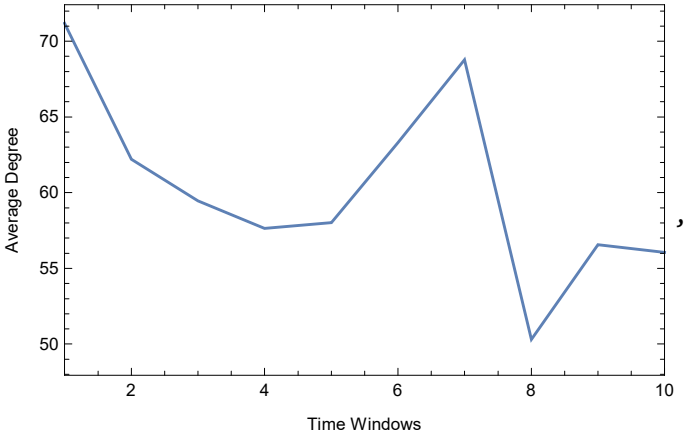


In[ ]:=
```
ABCvalues = Table[Mean@BetweennessCentrality[graphsandnodenumbers[[i]][[1]]],
    {i, Length@graphsandnodenumbers}];
modularityvalues = Table[N@GraphAssortativity[graphsandnodenumbers[[i]][[1]],
      FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
      "Normalized" → False], {i, Length@graphsandnodenumbers}];
degreevalues = Table[N@Mean@VertexDegree[graphsandnodenumbers[[i]][[1]]],
    {i, Length@graphsandnodenumbers}];
```

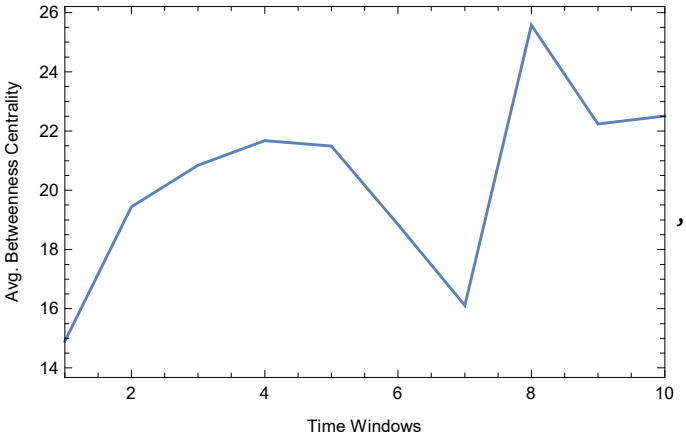
```
In[•]:= singlerandomgraphserdren = Table[
          RandomGraph[{VertexCount[i], EdgeCount[i]}], {i, graphsandnodenumbers[[All, 1]]}];
        singlerandomerdrenmodularityvalues =
          Table[N@GraphAssortativity[singlerandomgraphserdren[[i]],
              FindGraphCommunities[singlerandomgraphserdren[[i]]], "Normalized" -> False],
            {i, Length@singlerandomgraphserdren}];
        singlerandomgraphsdegreesfxd = Table[IGDegreeSequenceGame[Total[AdjacencyMatrix@i],
        Method -> "VigerLatapy"], {i, graphsandnodenumbers[[All, 1]]}];
        singlerandomdegreesfxdmodularityvalues =
          Table[N@GraphAssortativity[singlerandomgraphsdegreesfxd[[i]],
              FindGraphCommunities[singlerandomgraphsdegreesfxd[[i]]], "Normalized" -> False],
            {i, Length@singlerandomgraphsdegreesfxd}];
        singlerandomgraphscomm = Table[randomizedgraphamongcommunities[i],
            {i, graphsandnodenumbers[[All, 1]]}];
        singlerandomcommmodularityvalues = Table[N@GraphAssortativity[
              singlerandomgraphscomm[[i]], FindGraphCommunities[singlerandomgraphscomm[[i]]],
              "Normalized" -> False], {i, Length@singlerandomgraphscomm}];

In[•]:= AbsoluteTiming[ZscoresmodularityWolf =
          Table[randomnessfunctionformodularity[i, "Wolf"], {i, graphsandnodenumbers[[All, 1]]}]]

Out[•]= {1229.7, {{18.9852, 19.9745, -163.711}, {46.3861, 53.5962, -133.914},
          {60.6419, 63.9224, -116.304}, {59.2833, 63.7099, -108.884}, {53.4703, 56.04, -114.006},
          {30.6836, 31.0555, -151.224}, {27.8457, 32.9705, -158.79}, {41.9923, 47.0041, -103.988},
          {47.0245, 51.2727, -117.141}, {43.1295, 49.4626, -123.508}}}

In[•]:= Table[Length@FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
          {i, Length@graphsandnodenumbers}]

Out[•]= {2, 2, 2, 2, 3, 2, 3, 2, 2, 2}
```

In[ ]:=
```
{ListLinePlot[Thread[{Range@10, degreevalues}], Frame → True,
  FrameLabel → {"Time Windows", "Average Degree"}, ImageSize → 350,
  PlotRange → {{1, 10}, All}], ListLinePlot[Thread[{Range@10, ABCvalues}],
  Frame → True, FrameLabel → {"Time Windows", "Avg. Betweenness Centrality"},
  ImageSize → 350, PlotRange → {{1, 10}, All}],
 ListLinePlot[Thread[{Range@10, modularityvalues}], Frame → True,
  FrameLabel → {"Time Windows", "Modularity"}, ImageSize → 350, PlotRange → All],
 ListLinePlot[{Thread[{Range@10, singlerandomerdrenmodularityvalues}],
   Thread[{Range@10, singlerandomdegreesfxdmodularityvalues}],
   Thread[{Range@10, singlerandomcommmodularityvalues}]}, Frame → True,
  FrameLabel → {"Time Windows", "Modularity (Single Random Graphs)"},
  ImageSize → 350, PlotRange → {{1, 10}, All}, PlotLabels →
   Placed[{"Erdös-Renyi", "Degrees Fxd.", "Communities"}, {Scaled[1], Above}]],
 ListLinePlot[{Thread[{Range@10, ZscoresmodularityWolf[[All, 1]]}],
   Thread[{Range@10, ZscoresmodularityWolf[[All, 2]]}],
   Thread[{Range@10, ZscoresmodularityWolf[[All, 3]]}]},
  Frame → True, FrameLabel → {"Time Windows", "Z-Score for Modularity"},
  ImageSize → 350, PlotRange → All, PlotLabels →
   Placed[{"Erdös Renyi", "Degrees Fxd.", "Communities"}, {Scaled[1], Above}]]}
```

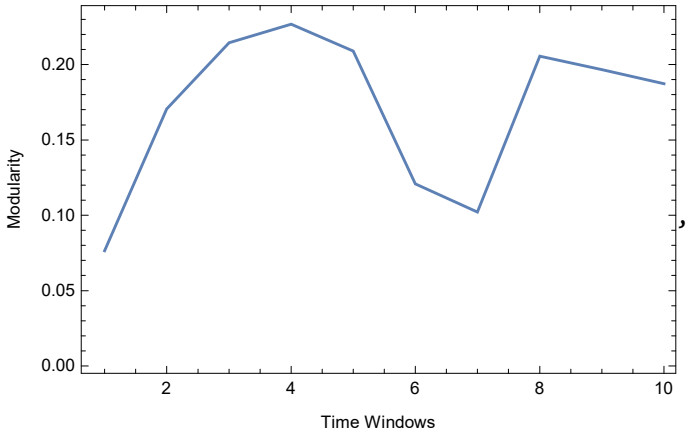
Out[ ]=



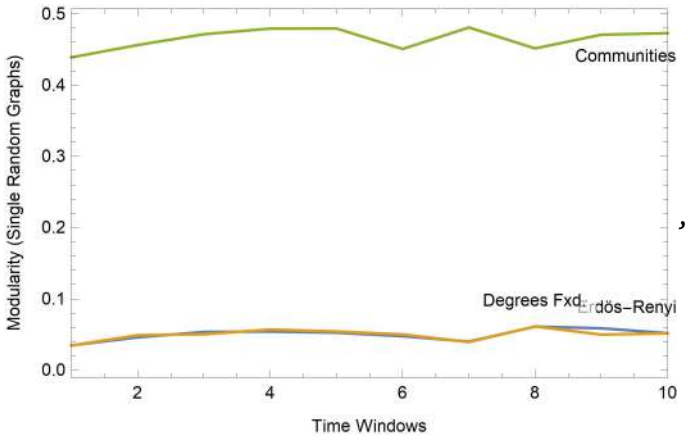
,



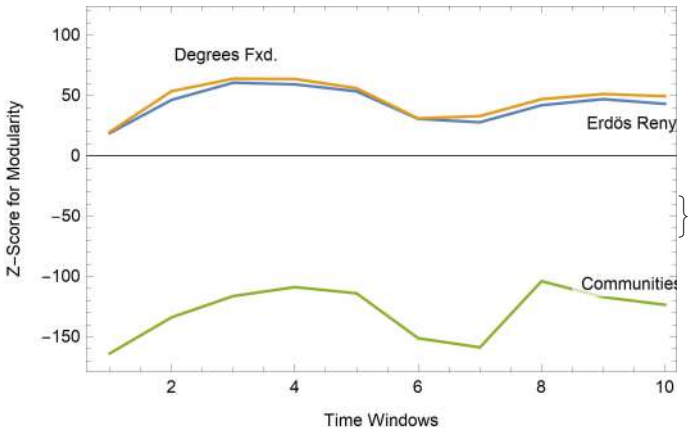
,



,



,



}

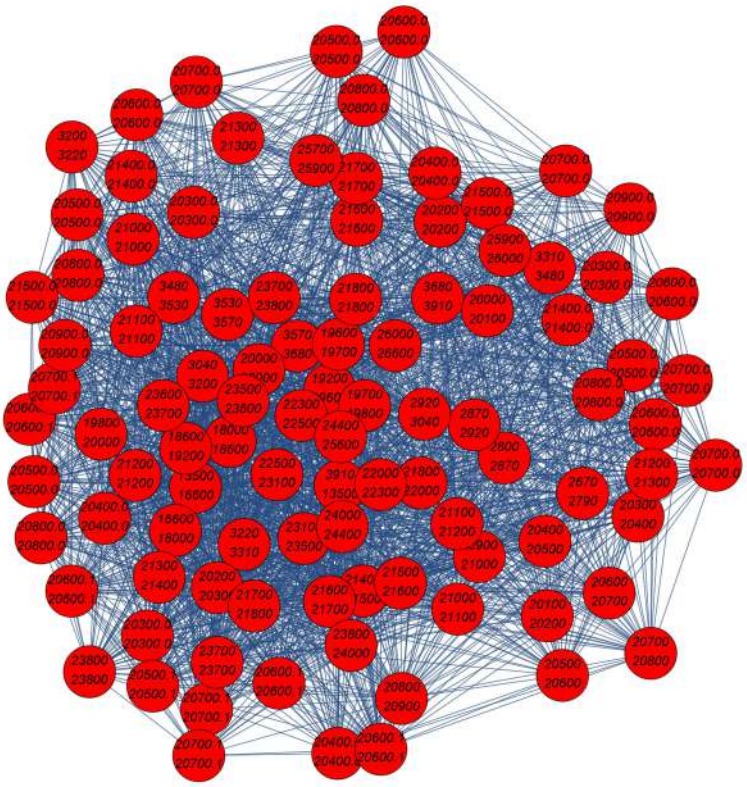
Weight Feature

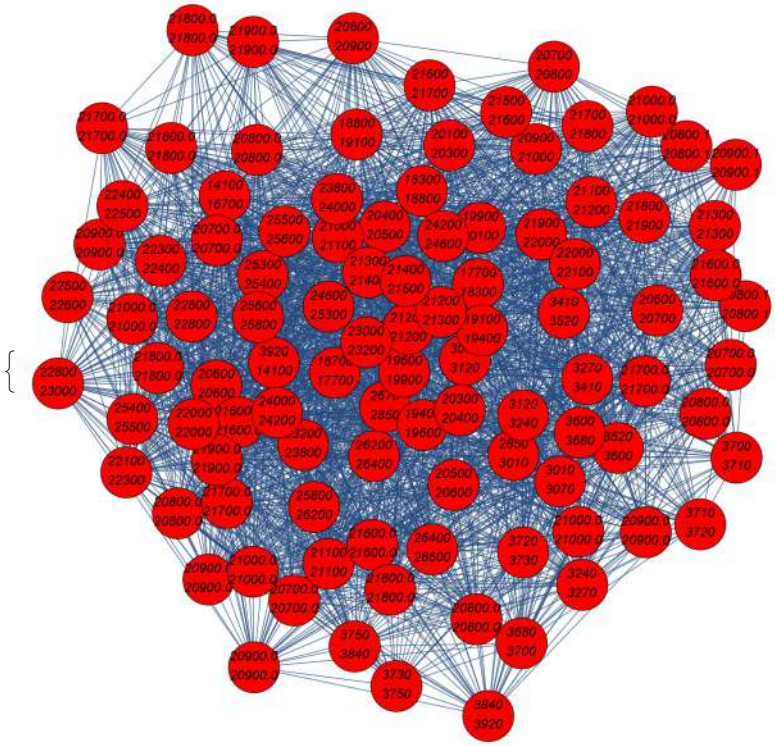
In[ ]:=
```
AbsoluteTiming[
 weightdataintimewindowsFixedbucket = snetworkdatafxdbucketintimewindows[11, 100, 10];]
```

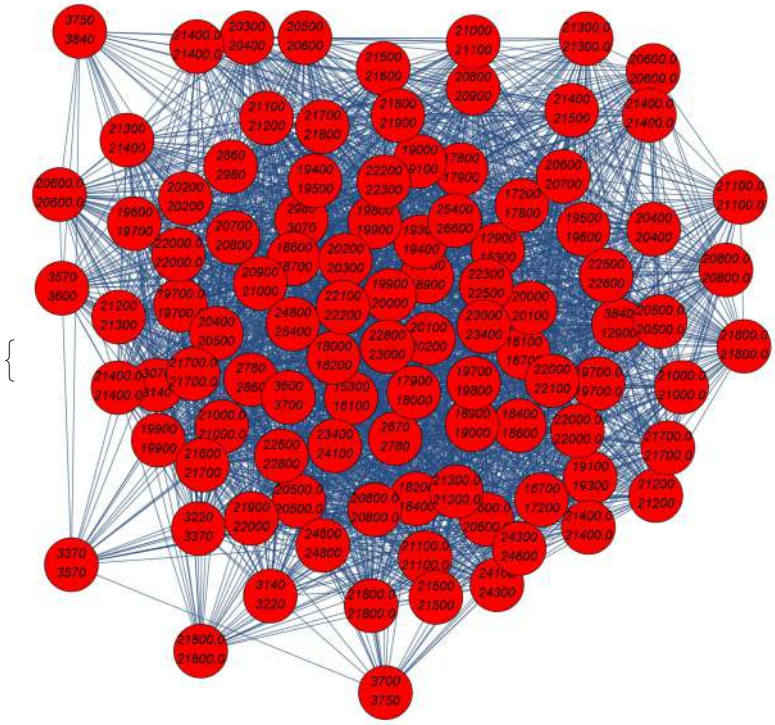
Out[ ]= {6.95992, Null}

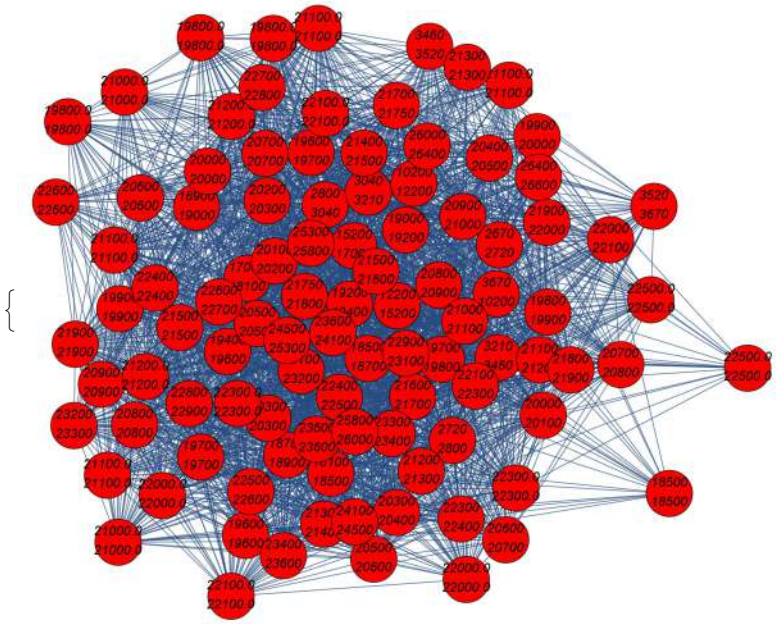
In[ ]:=
```
graphsandnodenumbers = Table[snetworkgraph[weightdataintimewindowsFixedbucket[[1]][[i]],
    weightdataintimewindowsFixedbucket[[2]][[i]], 1.5, 7, 400, Red], {i, Range@10}];
```

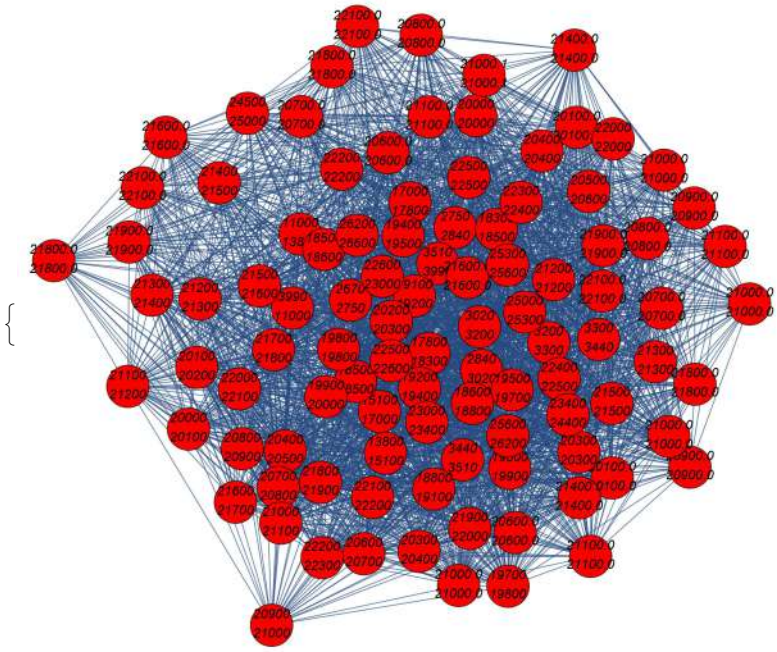
In[ ]:=
```
graphsandnodenumbers
```

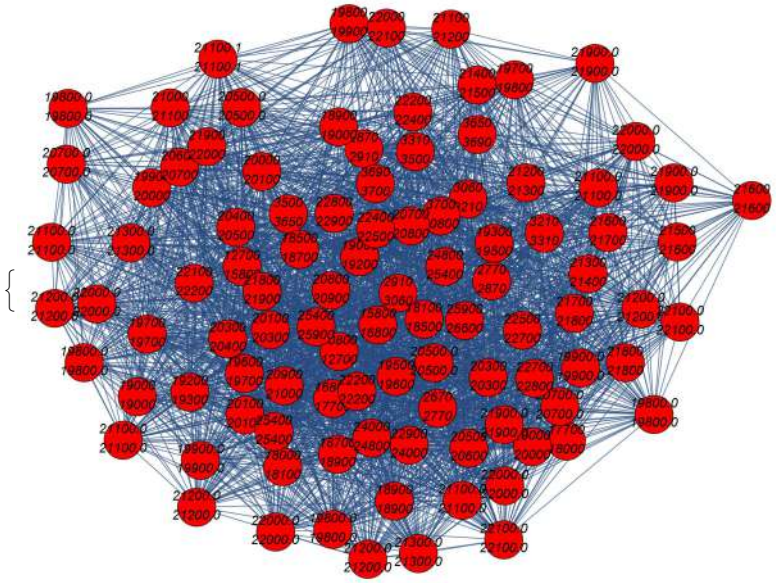
Out[ ]=



, 100},

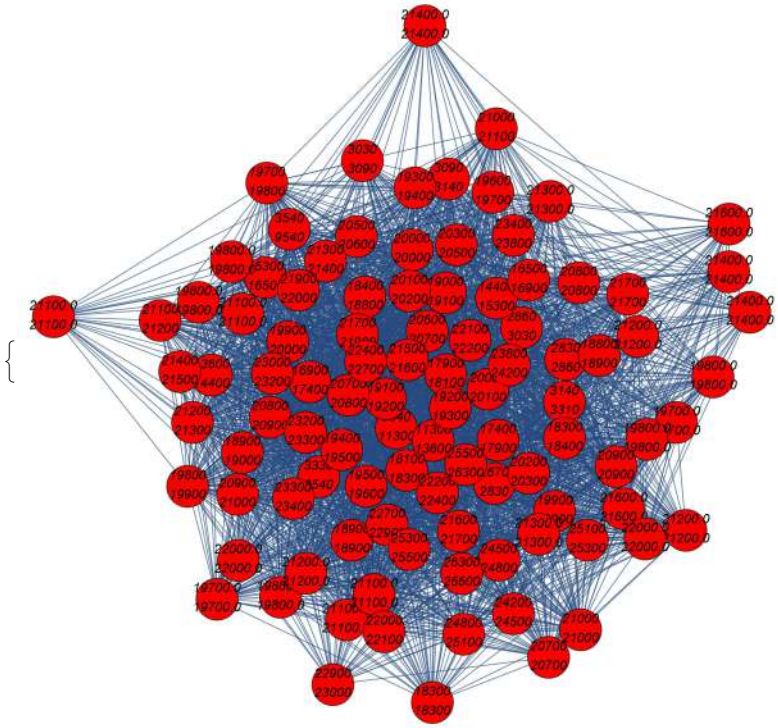

, 100},

{  , 100},

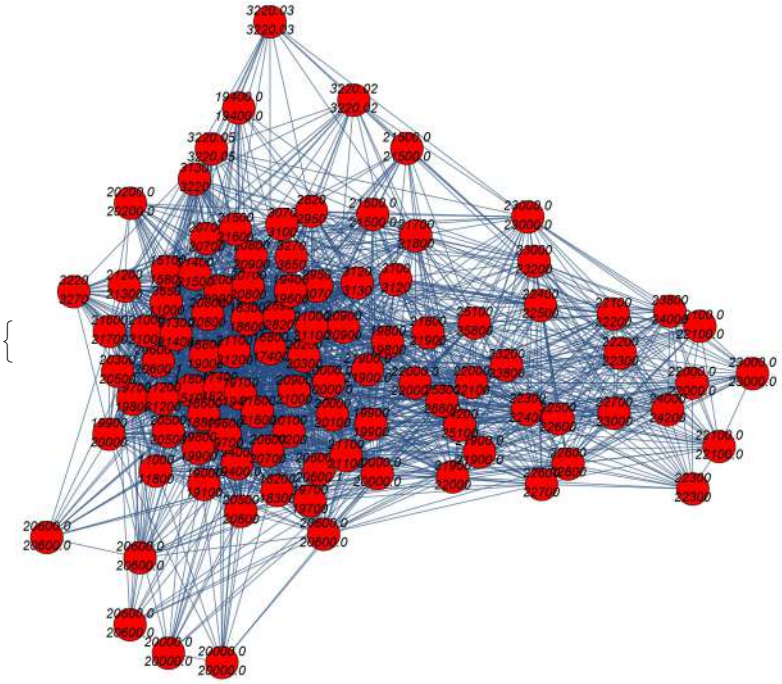
{  , 100},



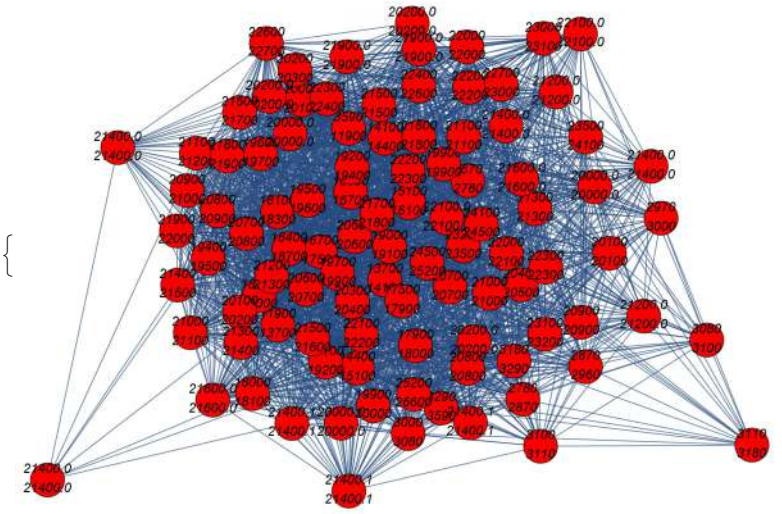
, 100},



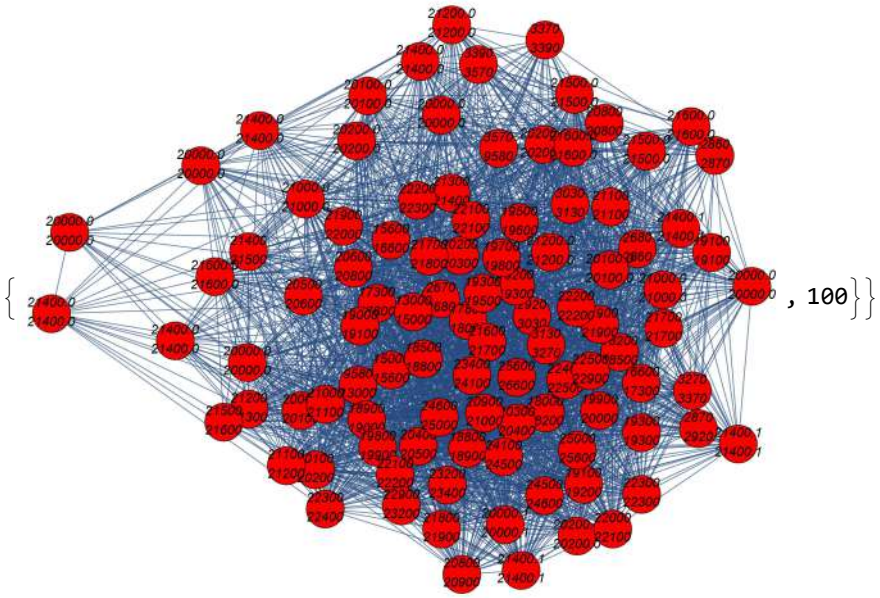
, 100},



{ , 100},



{ , 100},



In[ ]:=
```
ABCvalues = Table[Mean@BetweennessCentrality[graphsandnodenumbers[[i]][[1]]],
   {i, Length@graphsandnodenumbers}];
modularityvalues = Table[N@GraphAssortativity[graphsandnodenumbers[[i]][[1]],
     FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
     "Normalized" → False], {i, Length@graphsandnodenumbers}];
degreevalues = Table[N@Mean@VertexDegree[graphsandnodenumbers[[i]][[1]]],
   {i, Length@graphsandnodenumbers}];
```

In[ ]:=
```
singlerandomgraphserdren = Table[
   RandomGraph[{VertexCount[i], EdgeCount[i]}], {i, graphsandnodenumbers[[All, 1]]}];
singlerandomerdrenmodularityvalues =
  Table[N@GraphAssortativity[singlerandomgraphserdren[[i]],
     FindGraphCommunities[singlerandomgraphserdren[[i]]], "Normalized" -> False],
   {i, Length@singlerandomgraphserdren}];
singlerandomgraphsdegreesfxd = Table[IGDegreeSequenceGame[Total[AdjacencyMatrix@i],
Method -> "VigerLatapy"], {i, graphsandnodenumbers[[All, 1]]}];
singlerandomdegreesfxdmodularityvalues =
  Table[N@GraphAssortativity[singlerandomgraphsdegreesfxd[[i]],
     FindGraphCommunities[singlerandomgraphsdegreesfxd[[i]]], "Normalized" -> False],
   {i, Length@singlerandomgraphsdegreesfxd}];
singlerandomgraphscomm = Table[randomizedgraphamongcommunities[i],
   {i, graphsandnodenumbers[[All, 1]]}];
singlerandomcommmodularityvalues = Table[N@GraphAssortativity[
     singlerandomgraphscomm[[i]], FindGraphCommunities[singlerandomgraphscomm[[i]]],
     "Normalized" -> False], {i, Length@singlerandomgraphscomm}];
```

In[ ]:=
```
AbsoluteTiming[ZscoresmodularityWolf =
  Table[randomnessfunctionformodularity[i, "Wolf"], {i, graphsandnodenumbers[[All, 1]]}]]
```

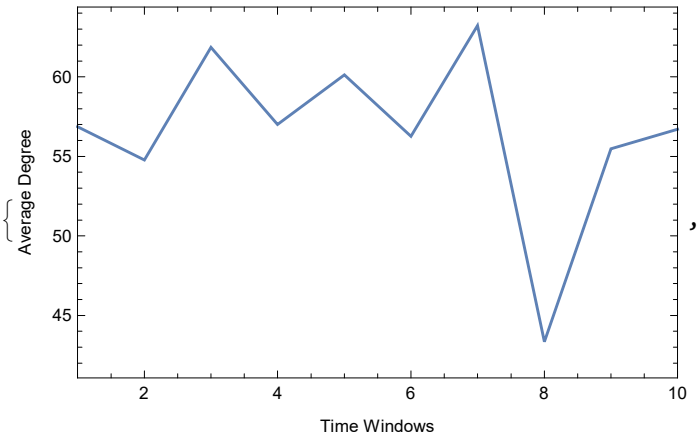
Out[•]= {855.782, {{34.8401, 39.9894, -123.48}, {28.4128, 32.2426, -120.603},
{22.9095, 26.2783, -152.214}, {17.1835, 18.729, -139.01}, {18.5607, 18.9559, -145.308},
{26.775, 28.4406, -127.48}, {11.8857, 13.5977, -158.882}, {31.1421, 37.6489, -76.5715},
{9.14574, 11.3237, -119.686}, {15.8447, 19.4654, -128.804}}}

In[•]:=
```
Table[Length@FindGraphCommunities[graphsandnodenumbers[[i]][[1]]],
 {i, Length@graphsandnodenumbers}]
```

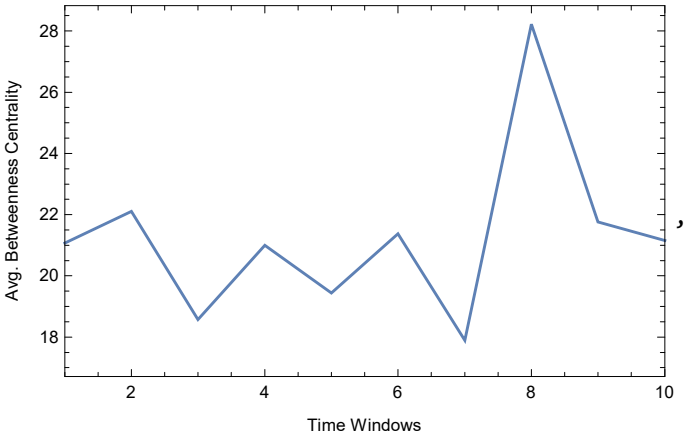
Out[•]= {2, 2, 2, 2, 3, 3, 3, 3, 3, 2}

In[•]:=
```
{ListLinePlot[Thread[{Range@10, degreevalues}], Frame → True,
  FrameLabel → {"Time Windows", "Average Degree"}, ImageSize → 350,
  PlotRange → {{1, 10}, All}], ListLinePlot[Thread[{Range@10, ABCvalues}],
  Frame → True, FrameLabel → {"Time Windows", "Avg. Betweenness Centrality"},
  ImageSize → 350, PlotRange → {{1, 10}, All}],
 ListLinePlot[Thread[{Range@10, modularityvalues}], Frame → True,
  FrameLabel → {"Time Windows", "Modularity"}, ImageSize → 350, PlotRange → All],
 ListLinePlot[{Thread[{Range@10, singlerandomerdrenmodularityvalues}],
   Thread[{Range@10, singlerandomdegreesfxdmodularityvalues}],
   Thread[{Range@10, singlerandomcommmodularityvalues}]}, Frame → True,
  FrameLabel → {"Time Windows", "Modularity (Single Random Graphs)"},
  ImageSize → 350, PlotRange → {{1, 10}, All}, PlotLabels →
   Placed[{"Erdös-Renyi", "Degrees Fxd.", "Communities"}, {Scaled[1], Above}]],
 ListLinePlot[{Thread[{Range@10, ZscoresmodularityWolf[[All, 1]]}],
   Thread[{Range@10, ZscoresmodularityWolf[[All, 2]]}],
   Thread[{Range@10, ZscoresmodularityWolf[[All, 3]]}]},
  Frame → True, FrameLabel → {"Time Windows", "Z-Score for Modularity"},
  ImageSize → 350, PlotRange → All, PlotLabels →
   Placed[{"Erdös Renyi", "Degrees Fxd.", "Communities"}, {Scaled[1], Below}]]}
```

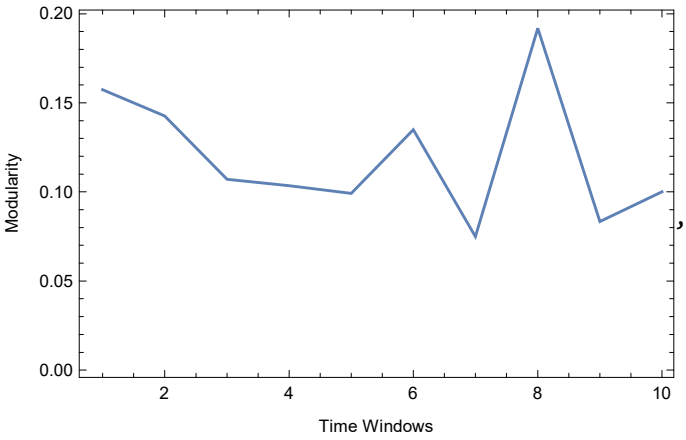
Out[•]=



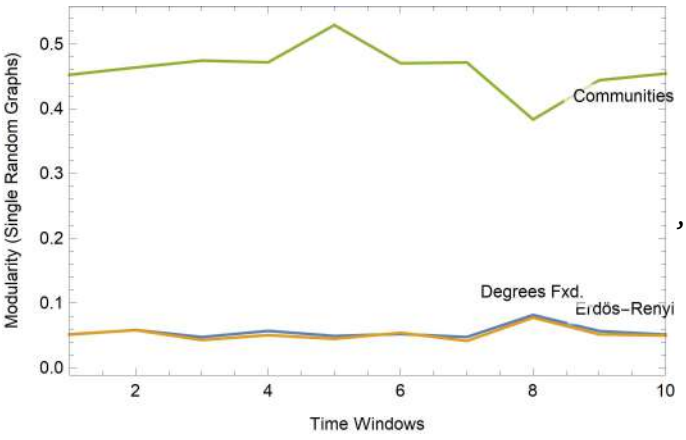
,


Avg. Betweenness Centrality
28
26
24
22
20
18
2
4
6
8
10
Time Windows

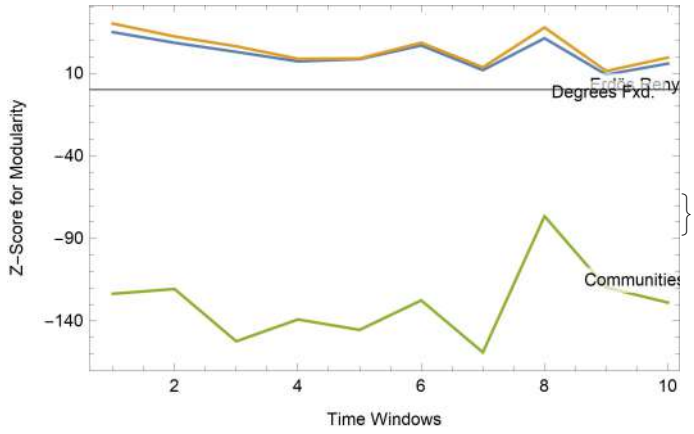
,


Modularity
0.20
0.15
0.10
0.05
0.00
2
4
6
8
10
Time Windows

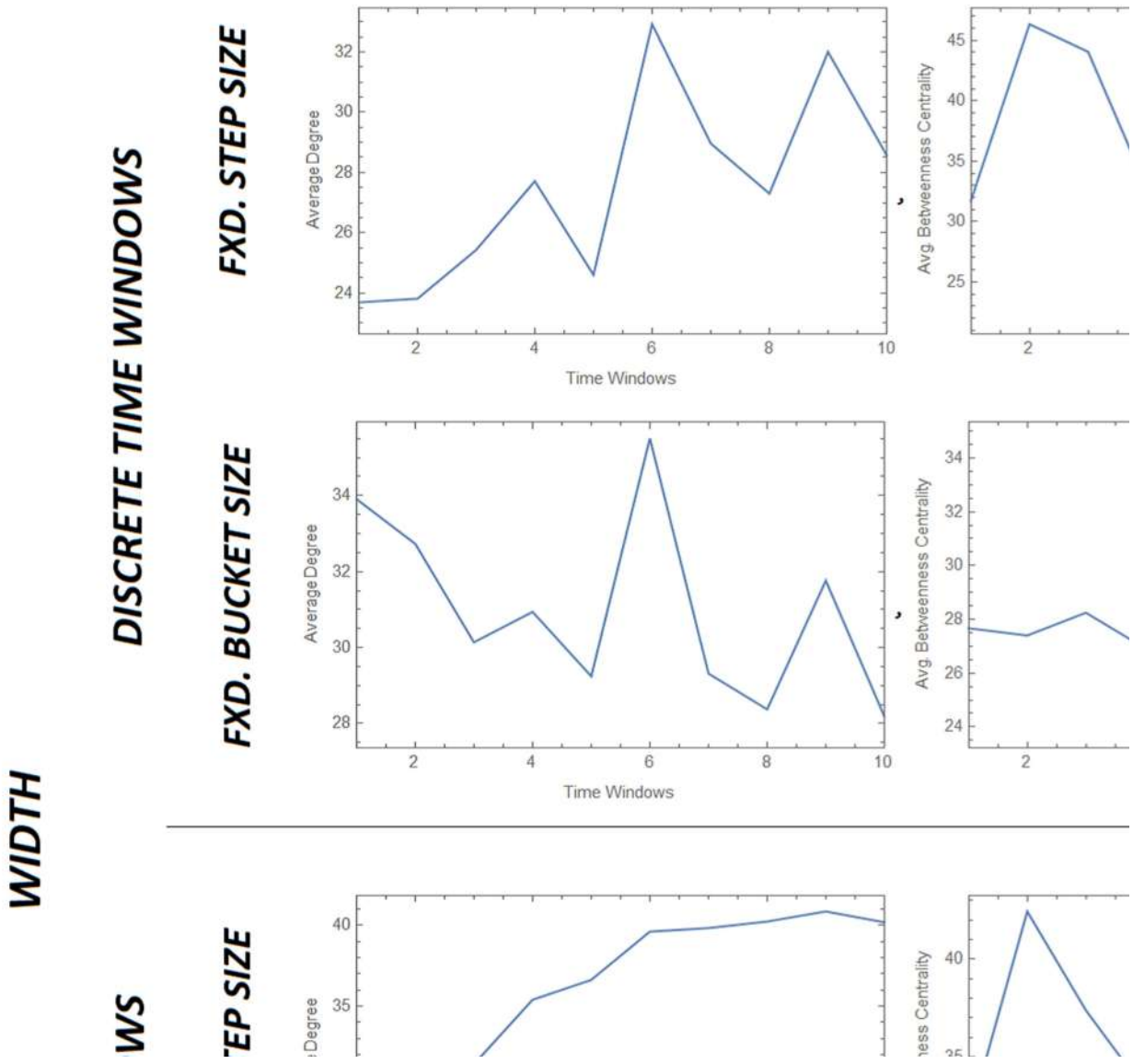
,


Modularity (Single Random Graphs)
0.5
0.4
0.3
0.2
0.1
0.0
Communities
Degrees Fxd.
Erdös–Renyi
2
4
6
8
10
Time Windows

,

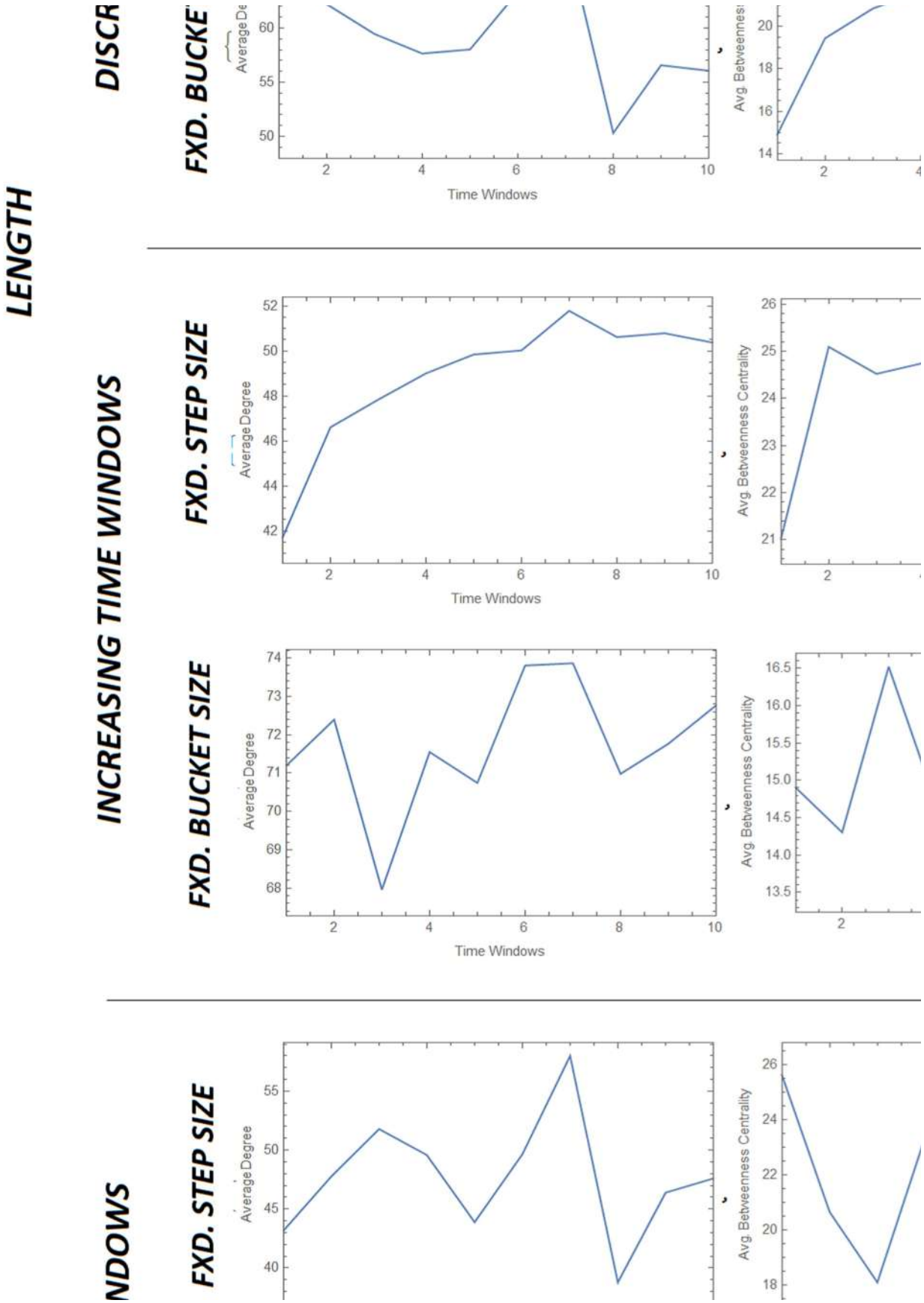


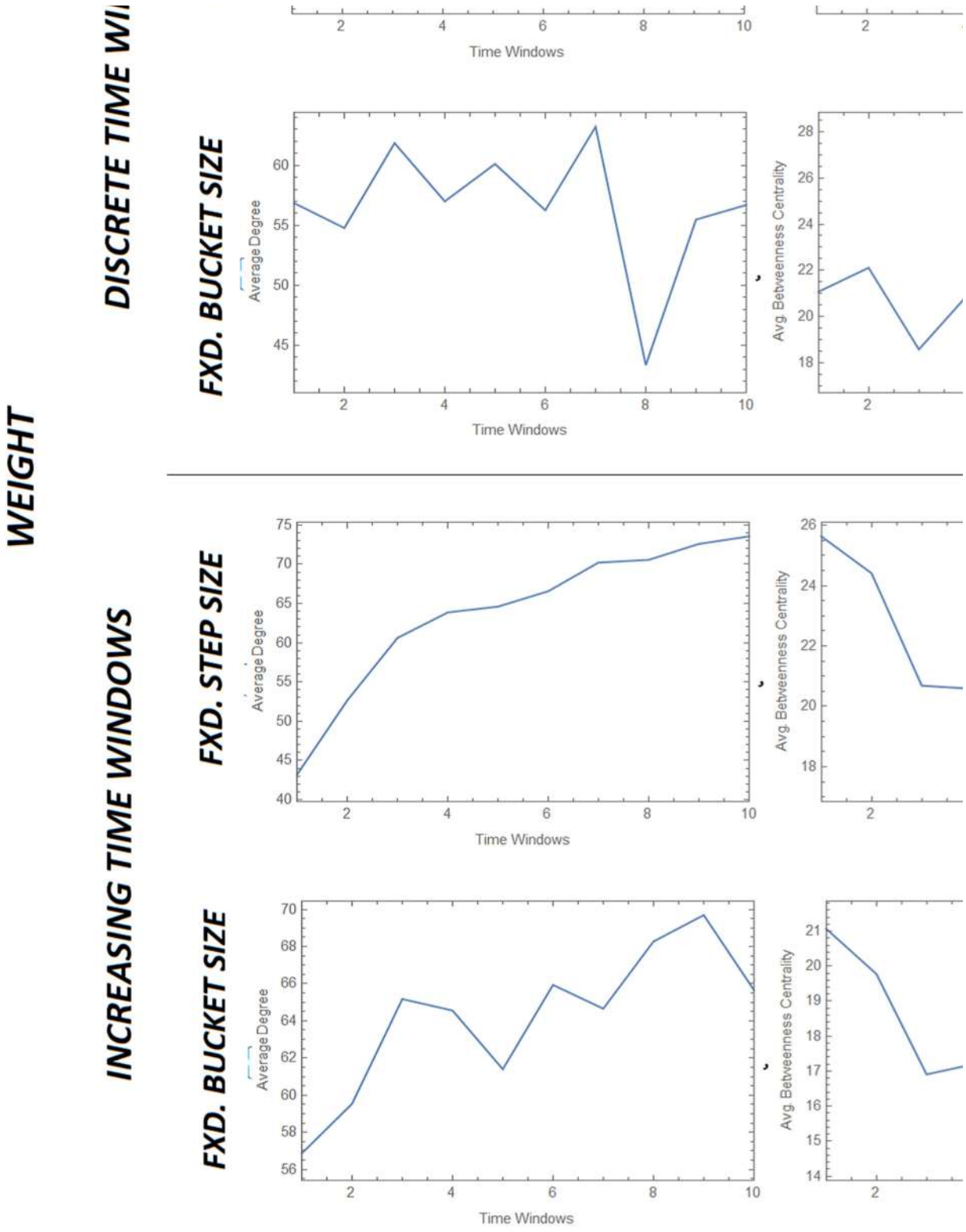
Plots for Network Metrics and Z-scores



INCREASING TIME WINDO
FXD. ST
Average
Time Windows
Avg. Betweenn
FXD. BUCKET SIZE
Average Degree
Time Windows
Avg. Betweenness Centrality
DISCRETE TIME WINDOWS
FXD. STEP SIZE
Average Degree
Time Windows
Avg. Betweenness Centrality
FXD. BUCKET SIZE
Average Degree
Avg. Betweenness Centrality

THICKN
INCREASING TIME WINDOWS
FXD. STEP SIZE
FXD. BUCKET SIZE
ETE TIME WINDOWS
T SIZE
Time Windows
Average Degree
Avg. Betweenness Centrality


LENGTH
DISCR
FXD. BUCKE
Average De
Time Windows
Avg Betweenness
INCREASING TIME WINDOWS
FXD. STEP SIZE
Average Degree
Time Windows
Avg Betweenness Centrality
FXD. BUCKET SIZE
Average Degree
Time Windows
Avg Betweenness Centrality
NDOWS
FXD. STEP SIZE
Average Degree
Avg Betweenness Centrality



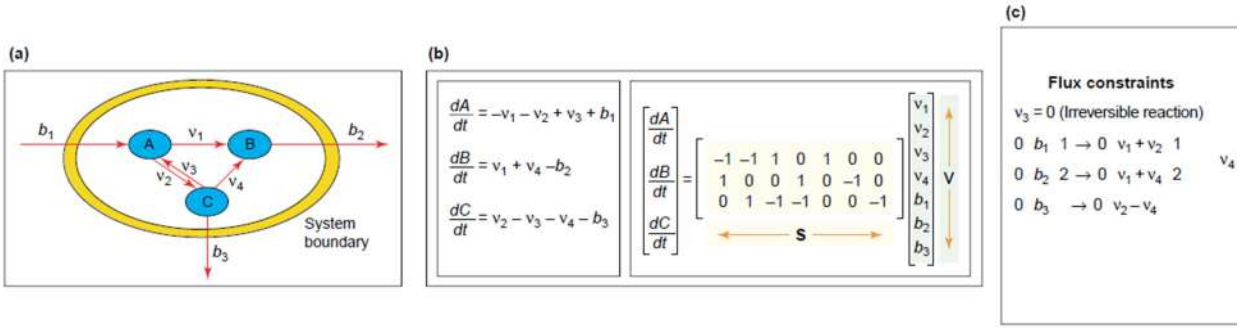
Modeling Works with OR Methods

FBA Model Reproducing from Kauffman et al. 2003

In[ ]:=
```
stochmatrix = {{-1, -1, 1, 0, 1, 0, 0}, {1, 0, 0, 1, 0, -1, 0}, {0, 1, -1, -1, 0, 0, -1}};
objectivefunc1 = {2, 0, 0, 1, 0, 0, 0};
objectivefunc2 = {1, 0, 0, 2, 0, 0, 0};
boundaries = {{0, Infinity}, {0, Infinity}, {0, 0}, {0, Infinity}, {0, 1}, {0, 2}, {0, 0}};
steadystatevector = {{0, 0}, {0, 0}, {0, 0}};
Vvector = {v1, v2, v3, v4, b1, b2, b3};
```

stochmatrix . Vvector = steadystatevector ， -objectivefunc ∈ solution space within boundaries

A, B, and C are metabolites. v_1, v_2, v_3, and v_4 are reactions. b_1, b_2, and b_3 are flux exchanges.

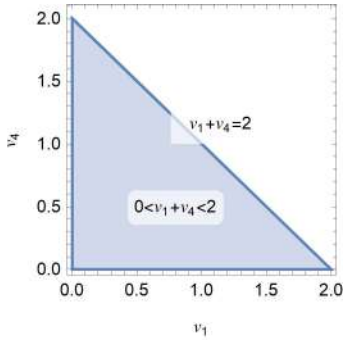



```
Print["1st objective function solution vector: ", Vvector1 =
  LinearProgramming[-objectivefunc1, stochmatrix, steadystatevector, boundaries]]
Print["2nd objective function solution vector: ", Vvector2 =
  LinearProgramming[-objectivefunc2, stochmatrix, steadystatevector, boundaries]]
```

```
1st objective function solution vector: {1, 0, 0, 0, 1, 1, 0}
2nd objective function solution vector: {0, 1, 0, 1, 1, 1, 0}
```

In[ ]:=
```
Show[RegionPlot[0 < v1 + v4 < 2, {v1, 0, 2}, {v4, 0, 2}, Frame -> True,
  PlotLabels → Placed["0<v1+v4<2", {0.8, 0.5}], FrameLabel → {"v1", "v4"}],
 Plot[-v1 + 2, {v1, 0, 1.2}, Frame -> True, PlotLabels → Placed["v1+v4=2", {1.1, 1}]],
 PlotRange → All, ImageSize -> Small]
```

Out[ ]=



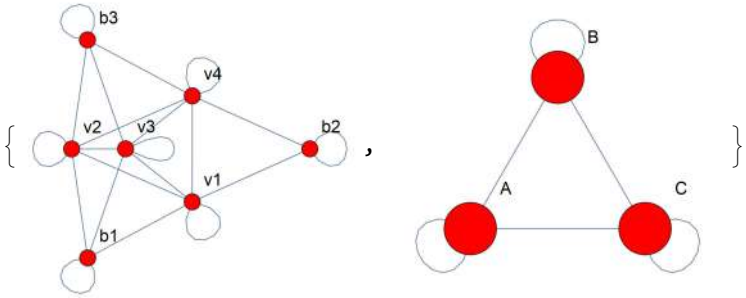
In[ ]:=
```
FindMaximum[{v4 + 2 v1, -v1 - v2 + v3 + b1 == 0 && v1 + v4 - b2 == 0 && v2 - v3 - v4 - b3 == 0 &&
    0 ≤ v1 + v2 ≤ 1 && 0 ≤ v1 + v4 ≤ 2 && v2 - v4 == 0 && 0 ≤ b1 ≤ 1 && 0 ≤ b2 ≤ 2 &&
    b3 == 0 && v1 ≥ 0 && v2 ≥ 0 && v3 == 0 && v4 ≥ 0}, {v1, v2, v3, v4, b1, b2, b3}]
```

Out[ ]=
```
{2., {v1 → 1., v2 → 0., v3 → 0., v4 → 0., b1 → 1., b2 → 1., b3 → 0.}}
```

In[ ]:=
```
AdjmatR = Transpose[stochmatrix].stochmatrix;
NormAdjmatR = AdjmatR /. x_ /; x ≠ 0 → 1;
AdjmatM = stochmatrix.Transpose[stochmatrix];
NormAdjmatM = AdjmatM /. x_ /; x ≠ 0 → 1;
```

In[ ]:=
```
{AdjacencyGraph[NormAdjmatR,
   {DirectedEdges → False, VertexSize → 0.3, VertexStyle → Red, VertexLabels →
     {1 -> "v1", 2 -> "v2", 3 -> "v3", 4 -> "v4", 5 -> "b1", 6 -> "b2", 7 -> "b3"}}],
 AdjacencyGraph[NormAdjmatM, {DirectedEdges → False, VertexSize → 0.3,
   VertexStyle → Red, VertexLabels → {1 -> "A", 2 -> "B", 3 -> "C"}}]}
```

Out[ ]=



Implementation Trials

```
stoichioforhomosapiens =
  Drop[Import["iAT_PLT_636_stoichiomat.csv", HeaderLines → 1], None, {1}];
SparseArray@stoichioforhomosapiens
```

SparseArray[ Specified elements: 4006
Dimensions: {738, 1008}
Default: 0.
Density: 0.00539
Elements:
{1, 12} → −1.
{1, 14} → 1.
{1, 345} → −1.
{1, 439} → −1.
⋮]

```
stoichiometricmatrix = stoichioforhomosapiens;
objectivefunctionamount = 200;
metabolites = 738;
fluxexchanges = 1008;
zerosforobj = Floor[0.7 * fluxexchanges, 1];
steadystatevector = ConstantArray[{0, 0}, metabolites];
SeedRandom[6];
boundaries = RandomChoice[{0.7, 0.3} → {{0, 500}, {0, 0}}, fluxexchanges];
SeedRandom[5];
objj = Table[RandomReal[{-2, 2}, fluxexchanges], objectivefunctionamount];
obj = Table[ReplacePart[i,
    Partition[RandomSample[Range@fluxexchanges, zerosforobj], 1] → 0.], {i, objj}];
obj // MatrixForm;
Solutionvectors = Table[LinearProgramming[-obj[[i]],
   stoichiometricmatrix, steadystatevector, boundaries], {i, Length@obj}]
```

In[ ]:=
```
Dimensions@Solutionvectors
```

Out[ ]=
```
{200, 1008}
```

In[ ]:=
```
nonzerocolumnsinsolutionvectorswithstoichioforhomosapiens =
  Position[Table[Count[Solutionvectors[[All, i]], x_ /; x ≠ 0.], {i, fluxexchanges}],
   objectivefunctionamount];
```

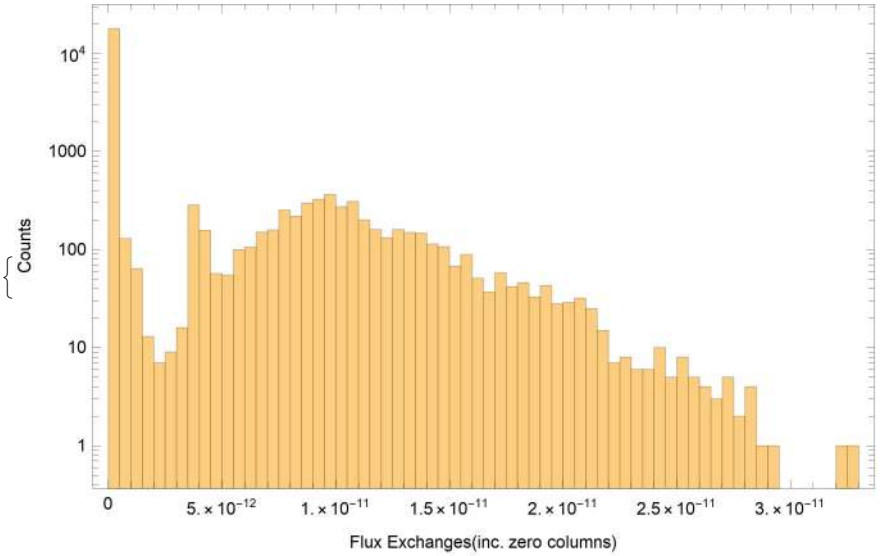
In[ ]:=
```
strial = stoichioforhomosapiens[[All,
   Flatten@nonzerocolumnsinsolutionvectorswithstoichioforhomosapiens]];
Dimensions@
 strial
```

Out[ ]=
```
{738, 396}
```

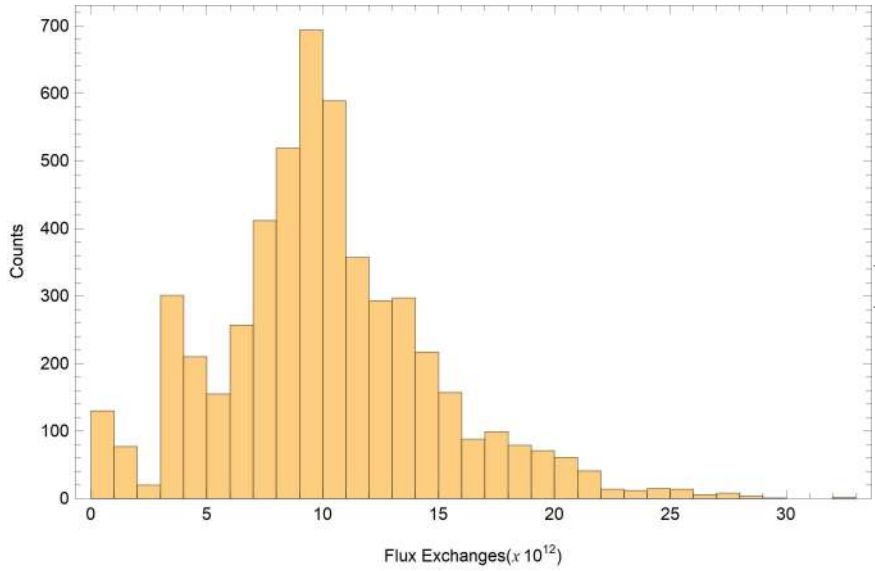
```
stoichiometricmatrix = strial;
objectivefunctionamount = 200;
metabolites = 738;
fluxexchanges = 396;
zerosforobj = Floor[0.7 * fluxexchanges, 1];
steadystatevector = ConstantArray[{0, 0}, metabolites];
SeedRandom[6];
boundaries = RandomChoice[{0.7, 0.3} → {{0, 500}, {0, 0}}, fluxexchanges];
SeedRandom[5];
objj = Table[RandomReal[{-2, 2}, fluxexchanges], objectivefunctionamount];
obj = Table[ReplacePart[i,
    Partition[RandomSample[Range@fluxexchanges, zerosforobj], 1] → 0.], {i, objj}];
obj // MatrixForm;
Solutionvectors = Table[LinearProgramming[-obj[[i]],
   stoichiometricmatrix, steadystatevector, boundaries], {i, Length@obj}]
```

```
In[ ]:= nonzerocolumnsinsolutionvectorswithstrial =
          Position[Table[Count[Solutionvectors[[All, i]], x_ /; x ≠ 0.], {i, fluxexchanges}],
           objectivefunctionamount];

In[ ]:= Dimensions@nonzerocolumnsinsolutionvectorswithstrial

Out[ ]= {116, 1}

In[ ]:= strial2 = strial[[All, Flatten@nonzerocolumnsinsolutionvectorswithstrial]];
        Dimensions@strial2

Out[ ]= {738, 116}

In[ ]:= stoichiometricmatrix = strial2;
        objectivefunctionamount = 200;
        metabolites = 738;
        fluxexchanges = 116;
        zerosforobj = Floor[0.7 * fluxexchanges, 1];
        steadystatevector = ConstantArray[{0, 0}, metabolites];
        SeedRandom[6];
        boundaries = RandomChoice[{0.7, 0.3} → {{0, 500}, {0, 0}}, fluxexchanges];
        SeedRandom[5];
        objj = Table[RandomReal[{-2, 2}, fluxexchanges], objectivefunctionamount];
        obj = Table[ReplacePart[i,
            Partition[RandomSample[Range@fluxexchanges, zerosforobj], 1] → 0.], {i, objj}];
        obj // MatrixForm;
        Solutionvectors = Table[LinearProgramming[-obj[[i]],
           stoichiometricmatrix, steadystatevector, boundaries], {i, Length@obj}]

        (* with 3rd trial, solution vectors resulted as zeros *)

In[ ]:= nonzerocolumnsinsolutionvectorswithstrial2 =
          Position[Table[Count[Solutionvectors[[All, i]], x_ /; x ≠ 0.], {i, fluxexchanges}],
           objectivefunctionamount];
        nonzerocolumnsinsolutionvectorswithstrial2

Out[ ]= {{22}, {23}, {25}, {26}, {39}, {40}, {51}, {53}, {54}, {55}, {56}, {62}, {63},
         {67}, {73}, {79}, {81}, {84}, {85}, {86}, {87}, {90}, {91}, {105}, {108}, {109}}

In[ ]:= networkfeaturesdata =
          Solutionvectors[[All, Flatten@nonzerocolumnsinsolutionvectorswithstrial2]] * 10^12;

In[ ]:= {Histogram[Flatten@Solutionvectors, Frame → True, ScalingFunctions → "Log",
          PlotRange → All, FrameLabel → {"Flux Exchanges(inc. zero columns)", "Counts"},
          ImageSize → 450], Histogram[Flatten@networkfeaturesdata, Frame → True,
          PlotRange → All, FrameLabel → {"Flux Exchanges (x 10^12)", "Counts"}, ImageSize → 450]}
```

Out[•]=



,



In[•]:=
```
Get["../algoritm_packages/SingleNetworks-algorithm-package.wl"]
(* ?SingleNetworks`* *)
```

In[•]:=
```
data = networkfeaturesdata;
Dimensions@data
```

Out[•]= {200, 26}

```
snetworkgraphsinglenodes[data[[]]]

(* snetworkgraphsinglenodes[aim_,campaign_,
 vertexsize_,vertexlabelsize_,imagesize_,vertexcolor_] *)
```