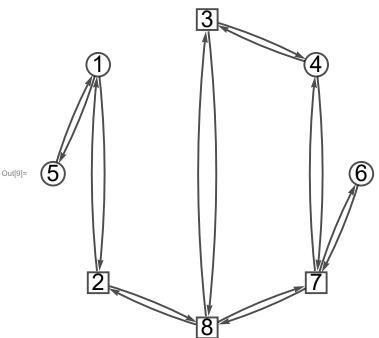
```
In[1]:= ClearAll["Global`*"]
     SetDirectory[NotebookDirectory[]];
    Needs["FlowSolver`"]
In[4]:= readGraph2[file_, dir_] := Module[{
         fn = FileNameJoin[{dir, file}],
         stream, imod, umod, u, b
         stream = OpenRead[fn];
         imod = Read[stream, {Word, Number}][[2]];
         umod = Read[stream, {Word, Number}][[2]];
       u = \left( \{ \#_{\llbracket 1 \rrbracket} \leftrightarrow \#_{\llbracket 2 \rrbracket}, \#_{\llbracket 2 \rrbracket} \leftrightarrow \#_{\llbracket 1 \rrbracket} \} \& /@ ReadList[stream, Expression, umod] \right) // Flatten;
       b = ConstantArray[0, imod];
          (b[[Read[StringToStream[StringTake[#1, {5, -3}]], Number]]] = #2) &@@@
        ReadList[stream, {Word, Expression}, imod];
       {Graph[u, VertexSize -> Medium, VertexLabels → Placed["Name", Center],
          VertexStyle → Directive[White],
          VertexShapeFunction \rightarrow \{xx\_ \Rightarrow If[SameQ[b[[xx]], x], "Square", "Circle"]\},
          VertexLabelStyle -> Directive[Black, 24], GraphLayout -> "CircularEmbedding"], b}]
In[8]:=
     {g, b} = readGraph2["gr3.txt", NotebookDirectory[]];
    GraphPlot[g, EdgeStyle → Directive[Black, Thick],
      VertexStyle → Directive[EdgeForm[Thick], White], MultiedgeStyle → .05]
```



```
log[10] = balanceEqs = ((Total[x_{\#} \& /@ EdgeList[g, \_ \leftrightarrow \#]] - Total[x_{\#} \& /@ EdgeList[g, \# \leftrightarrow \_]])) = 0
                  MapIndexed[#1 /. x \rightarrow x_{\#2[[1]]} \&, b][[#]] & /@ VertexList[g];
          balanceEqs //
            forma
Out[11]//TableForm=
          - X_{1,2} - X_{1,5} + X_{2,1} + X_{5,1} = 0
          X_{1,2} - X_{2,1} - X_{2,8} + X_{8,2} = X_2
          x_{1,5} - x_{5,1} = 0
          X_{2,8} + X_{3,8} + X_{7,8} - X_{8,2} - X_{8,3} - X_{8,7} = X_{8}
          -X_{3,4}-X_{3,8}+X_{4,3}+X_{8,3}=X_3
          x_{3,4} - x_{4,3} - x_{4,7} + x_{7,4} = 0
          X_{4,7} + X_{6,7} - X_{7,4} - X_{7,6} - X_{7,8} + X_{8,7} = X_{7}
          -x_{6,7} + x_{7,6} = 0
  In[14]:= M = \{8\};
          Print["M = ", M];
          M = \{8\}
   In[⊕]:= (*Do[inclist=EdgeList[g,u→_];
            Do[p_v=1/Length[inclist];, \{v,inclist\}];, \{u,VertexList[g]\}]*)
   In[*]:= (*p#&/@EdgeList[g]*)
  In[16]:= (*incL=
           \label{lem:decomp} Delete Cases [Delete Duplicates [Cases [Incidence List[g, \#], i\_ \leftrightarrow j\_ \leftrightarrow \{i,j\}] / / Flatten],
          incL = (IncidenceList[g, #] & /@M) // Flatten
  Out[16]= \{2 \rightarrow 8, 8 \rightarrow 2, 3 \rightarrow 8, 8 \rightarrow 3, 7 \rightarrow 8, 8 \rightarrow 7\}
```

```
In[17]:= (*Do[If[MemberQ[M,j[1]],b[j[2]]]+=fj,b[j[1]]-=fj],{j,incL}]*)

\[
\overline{b} = Fold[If[MemberQ[M, #2[1]], ReplacePart[#, #2[2]] → #[#2[2]] - f#2],

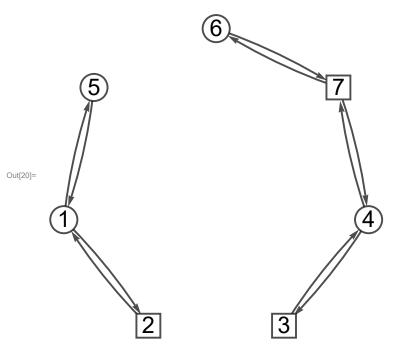
ReplacePart[#, #2[1]] → #[#2[1]]] + f#2]] &, b, incL];

\[
\overline{b} = \overline{b}[[Range[g // VertexCount] ~ Complement ~ M]];

\[
\overline{ng} = VertexDelete[g, M];

GraphPlot[\overline{ng}, EdgeStyle → Directive[Black, Thick],

VertexStyle → Directive[EdgeForm[Thick], White], MultiedgeStyle → .05]
\[
\overline{b}
\]
```



$$\text{Out}[\text{21}] = \left\{ \text{0, } x + f_{2 \mapsto 8} - f_{8 \mapsto 2} \text{, } x + f_{3 \mapsto 8} - f_{8 \mapsto 3} \text{, 0, 0, 0, } x + f_{7 \mapsto 8} - f_{8 \mapsto 7} \right\}$$

 $ii_i^{+} \ [g_{\_}] := Cases[IncidenceList[g, i], u_{\_} \leftrightarrow v_{\_}/; u = i : \rightarrow v]$ 

In[24]:= M+ = CC[g, M]

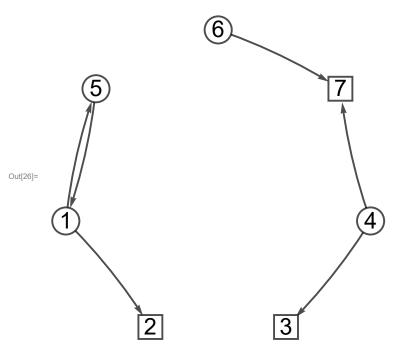
Out[24]=  $\{2 \leftrightarrow 8, 3 \leftrightarrow 8, 7 \leftrightarrow 8\}$ 

 $In[25]:= \overline{b1} = Fold[$ 

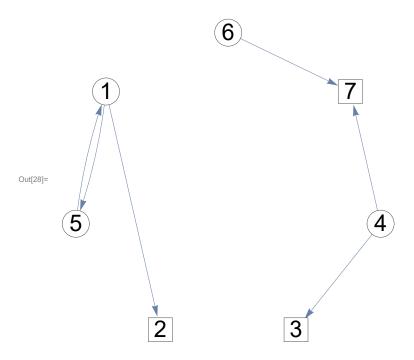
 $\begin{aligned} &\text{Module} \big[ \{ bb = \text{#1, i} = \text{#2}_{[[1]]}, \, k = \text{#2}_{[[2]]} \}, \, \left( \text{Fold} \big[ \text{Module} \big[ \{ bbb = \text{#1, jj} = \text{#2} \}, \, \text{ReplacePart} \big[ \\ & bbb, \, \left( \left( \left\{ jj \rightarrow bbb_{[[jj]]} - \frac{p_{i \rightarrow jj}}{p_{i \rightarrow k}} \, f_{i \rightarrow k}, \, i \rightarrow bbb_{[[i]]} + \frac{p_{i \rightarrow jj}}{p_{i \rightarrow k}} \, f_{i \rightarrow k} \right\} \right) \right) \big) \, // \\ & \text{Flatten} \big] \, \&, \, bb, \, ii_i^* \big[ \overline{ng} \big] \, \Big) \big] \, \&, \, \overline{b}, \, M^* \big] \end{aligned}$ 

$$\begin{array}{l} \text{Out} [25] = \\ \Big\{ -\frac{f_{2 \to 8} \; p_{2 \to 1}}{p_{2 \to 8}} \; , \; x + f_{2 \to 8} - f_{8 \to 2} + \frac{f_{2 \to 8} \; p_{2 \to 1}}{p_{2 \to 8}} \; , \; x + f_{3 \to 8} - f_{8 \to 3} + \frac{f_{3 \to 8} \; p_{3 \to 4}}{p_{3 \to 8}} \; , \\ \\ -\frac{f_{3 \to 8} \; p_{3 \to 4}}{p_{3 \to 8}} \; -\frac{f_{7 \to 8} \; p_{7 \to 4}}{p_{7 \to 8}} \; , \; \emptyset \; , \; -\frac{f_{7 \to 8} \; p_{7 \to 6}}{p_{7 \to 8}} \; , \; x + f_{7 \to 8} - f_{8 \to 7} + \frac{f_{7 \to 8} \; p_{7 \to 4}}{p_{7 \to 8}} + \frac{f_{7 \to 8} \; p_{7 \to 6}}{p_{7 \to 8}} \Big\} \\ \end{array}$$

 $\label{eq:continuous_continuous$ 



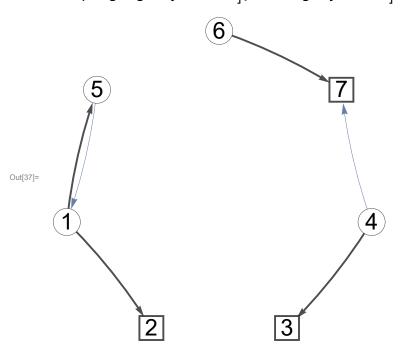
 $\label{eq:continuous} \begin{array}{ll} \ln[27] \coloneqq \overline{g1} = \operatorname{Fold} \left[ \operatorname{EdgeDelete} \left[ \#1, \, u_{-} \leftrightarrow v_{-} \, /; \, u == \#2 \right] \, \&, \, \overline{ng}, \, \#_{[1]} \, \& \, /@ \, M^{+} \right]; \\ & \operatorname{GraphPlot} \left[ \overline{g1}, \, \operatorname{MultiedgeStyle} \to .05 \right] \end{array}$ 

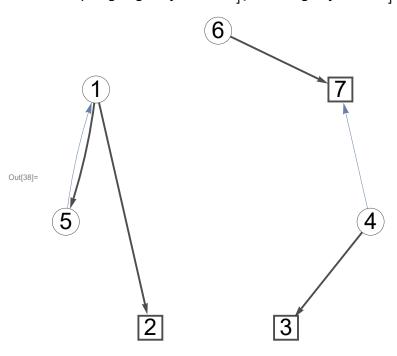


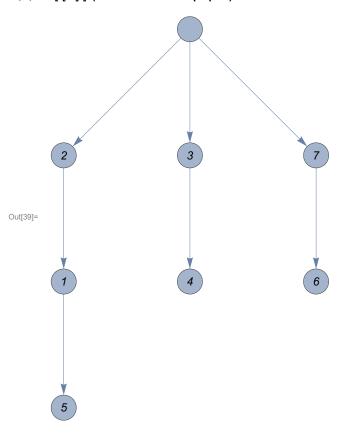
 $\label{eq:linear_line$ 

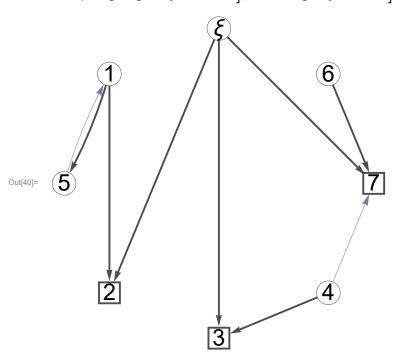
```
ln[30]:= \lambda = SparseArray
            \left(\left\{\left(i \leftrightarrow jf\right) \rightarrow 1\text{, }\left(i \leftrightarrow \text{\#}\right) \rightarrow -\frac{p_{i \to \text{\#}}}{p_{i \to jf}}\right\}\right) \& \ / @ \ \text{Icur[[2 ;;]]}\right)\right] \& \ / @ \ \text{II}_{\text{rem}}\text{, }1\right]\right),
              \_ \leftrightarrow \_ \rightarrow 0, 2]]
 In[31]:= Grid[\lambda]
         1 - \frac{p_{1 \mapsto 5}}{2} = 0 = 0
  In[32]:= g = \overline{g1};
         b = \overline{b1};
  ln[34]:= II^* = Cases[MapIndexed[{#1, #2} &, b],
              \{el\_, i\_\} /; MemberQ[el, x] || SameQ[el, x] \Rightarrow i] // Flatten
 Out[34]= \{2, 3, 7\}
  In[35]:= buildt = Timing[{t, g} = buildTree[g, II*];][[1]]
         TableForm[t[1;; 4]],
          TableHeadings → {{"pred", "dir", "depth", "d"}, t // pred // Length // Range}]
 Out[35]= 0.015625
Out[36]//TableForm=
         dir
                     2
         depth
```

```
In[37]:= GraphPlot[HighlightGraph[ Fold[HighlightGraph[#1, Style[u_ \leftrightarrow v_ /; u == #2, White]] &, \overline{ng}, #_{[[1]]} & /@ M^+], {Style[u_ /; VertexQ[g, u] && pred[t][[u]] == root[t], EdgeForm[Thick]], Style[u_ \leftrightarrow v_ /; (pred[t][[u]] == v && dir[t][[u]] == -1) || (pred[t][[v]] == u && dir[t][[v]] == 1), Directive[Black, Thick]]}, GraphHighlightStyle \rightarrow None], MultiedgeStyle \rightarrow .05]
```









```
In[41]:= AppendTo[b, -Total[b]];
                               b = Simplify[b /. x \rightarrow 0]
      \text{Out} [42] = \left\{ -\frac{f_{2 \mapsto 8} \ p_{2 \mapsto 1}}{p_{2 \mapsto 8}} \text{, } -f_{8 \mapsto 2} + f_{2 \mapsto 8} \ \left( 1 + \frac{p_{2 \mapsto 1}}{p_{2 \mapsto 8}} \right) \text{, } -f_{8 \mapsto 3} + f_{3 \mapsto 8} \ \left( 1 + \frac{p_{3 \mapsto 4}}{p_{3 \mapsto 8}} \right) \text{, } -\frac{f_{3 \mapsto 8} \ p_{3 \mapsto 4}}{p_{3 \mapsto 8}} - \frac{f_{7 \mapsto 8} \ p_{7 \mapsto 4}}{p_{7 \mapsto 8}} \text{, 0, } \right\} 
                                   -\frac{f_{7 \leftrightarrow 8} \; p_{7 \leftrightarrow 6}}{p_{7 \leftrightarrow 6}} \text{, } -f_{8 \leftrightarrow 7} + \frac{f_{7 \leftrightarrow 8} \; \left(p_{7 \leftrightarrow 4} + p_{7 \leftrightarrow 6} + p_{7 \leftrightarrow 8}\right)}{p_{7 \leftrightarrow 8}} \text{, } -f_{2 \leftrightarrow 8} - f_{3 \leftrightarrow 8} - f_{7 \leftrightarrow 8} + f_{8 \leftrightarrow 2} + f_{8 \leftrightarrow 3} + f_{8 \leftrightarrow 7} \right\}
        log[43] balanceEqs = (Total[x_{\#} \& /@ EdgeList[g, _ <math>\rightarrow \#]] - Total[x_{\#} \& /@ EdgeList[g, \# \rightarrow _]]) /.
                                                                    root[t] \rightarrow \xi = b[[#]] & /@ VertexList[g];
                               balanceEqs //
                                    forma
Out[44]//TableForm=
                               -X_{1,2}-X_{1,5}+X_{5,1} = -\frac{f_{2,8}p_{2,1}}{}
                              x_{1,2} + x_{\xi,2} = -f_{8,2} + f_{2,8} \left(1 + \frac{p_{2,1}}{p_{2,8}}\right)
                               x_{1,5} - x_{5,1} = 0
                              x_{4,3} + x_{\xi,3} = -f_{8,3} + f_{3,8} \left(1 + \frac{p_{3,4}}{p_{3,8}}\right)
                              -X_{4,3} - X_{4,7} = -\frac{f_{3,8} p_{3,4}}{p_{3,8}} - \frac{f_{7,8} p_{7,4}}{p_{7,8}}
X_{4,7} + X_{6,7} + X_{\xi,7} = -f_{8,7} + \frac{f_{7,8} (p_{7,4} + p_{7,6} + p_{7,8})}{p_{7,8}}
                               -x_{6,7} = -\frac{f_{7,8}p_{7,6}}{}
                               -x_{\xi,2}-x_{\xi,3}-x_{\xi,7}=-f_{2,8}-f_{3,8}-f_{7,8}+f_{8,2}+f_{8,3}+f_{8,7}
        ln[45]:= ps = partSolve[g, -b, t, \tilde{x}];
                              ps // forma
Out[46]//TableForm=
                              \tilde{X}_{1,2} \rightarrow \frac{f_{2,8} \, p_{2,1}}{}
                               \tilde{x}_{1,5} \rightarrow 0
                               \tilde{X}_{4,3} \rightarrow \frac{f_{3,8} p_{3,4}}{f_{7,8} p_{7,4}} + \frac{f_{7,8} p_{7,4}}{f_{7,8} p_{7,4}}
                               \tilde{x}_{4,7} \rightarrow 0
                               \widetilde{x}_{5,1} \to 0
                               \tilde{X}_{6.7} \rightarrow \frac{f_{7.8} p_{7.6}}{}
                             \begin{split} \widetilde{X}_{8,2} &\to -f_{8,2} + f_{2,8} \, \left(1 + \frac{p_{2,1}}{p_{2,8}}\right) - \frac{f_{2,8} \, p_{2,1}}{p_{2,8}} \\ \widetilde{X}_{8,3} &\to -f_{8,3} + f_{3,8} \, \left(1 + \frac{p_{3,4}}{p_{3,8}}\right) - \frac{f_{3,8} \, p_{3,4}}{p_{3,8}} - \frac{f_{7,8} \, p_{7,4}}{p_{7,8}} \\ \widetilde{X}_{8,7} &\to -f_{8,7} - \frac{f_{7,8} \, p_{7,6}}{p_{7,8}} + \frac{f_{7,8} \, (p_{7,4} + p_{7,6} + p_{7,8})}{p_{7,8}} \end{split}
        log[47]:= Simplify (balanceEqs /. \{x \to \tilde{x}, \xi \to root[t]\}) /. ps]
     Out[47]= {True, True, True, True, True, True, True, True}
        In[48]:= matrt = Timing[\deltaMatr = \delta1[g, t]];
                               roott = VertexCount[g];
                               TableForm \left[\delta \text{Matr, TableHeadings} \rightarrow \left\{\text{uNb}\left[g, t\right], \delta_{\left[\#_{\left[2\right]}\right]} \#_{\left[1\right]} \#_{\text{roott}} \& /@ \text{EdgeList}\left[g\right]\right\}\right] // \text{ formal formal formal substitution of the substitution 
Out[50]//TableForm=
```

 $ln[51]:= \lambda = SparseArray[\lambda, {Length[\lambda], Length[\lambda[[1]]] + Length[II*]}];$ 

 $(*\lambda=\lambda[[;;-2]]*)$ 

```
ln[52]:= dopEq = # == 0 & /@ Flatten \left[\lambda.\left\{x_{\#} \& /@ EdgeList[g]\right\}^{\intercal}\right];
             dopEq // forma
Out[53]//TableForm=
             x_{1,2} - \frac{p_{1,5} x_{1,5}}{2} = 0
             X_{4,3} - \frac{p_{4,7} X_{4,7}}{p_{4,3}} = 0
   In[54]:= \Lambda = \lambda \cdot (\delta Matr)^{\mathsf{T}};
             "cicle det's:"
            Λ// forma
  Out[55]= cicle det's:
Out[56]//TableForm=
             _ p<sub>1⊷5</sub>
              p_{1 \boldsymbol{\longleftrightarrow} 2}
                              -\,1\,-\,\tfrac{p_{4\! \rightarrow 7}}{\phantom{-}}
   In[57]:= MatrixRank[Λ]
  Out[57]= 2
   In[62]:= "U_c ="
            U_c = \{1, 2\}
            "Unc="
            U_{nc} = \{\}
  Out[62]= U_c =
  Out[63]= \{1, 2\}
  Out[64]= U_{nc}=
  Out[65]= { }
   In[70]:= \Lambda C = \Lambda [[All, U_c]];
             \Lambdanc = \Lambda[[All, U<sub>nc</sub>]];
             ^{"}\Lambda_{c}="
             Λc // MatrixForm
  Out[72]= \Lambda_{\mathbf{C}}=
Out[73]//MatrixForm=
   In[74]:= "det (\Lambda_c) ="
             Simplify[det = Det[\Lambdac]] // forma
  Out[74]= det(\Lambda_c) =
Out[75]//TableForm=
              p_{1,5}\ (p_{4,3}+p_{4,7})
                     p_{1,2} p_{4,3}
   In[76]:= "UT="
             utind = Cases[t[[6]], \xi_{-}/; \xi \neq 0];
             U<sub>T</sub> = EdgeList[g][[utind]]
  Out[76]= U_T=
  Out[78]= \{1 \leftrightarrow 2, 8 \leftrightarrow 2, 8 \leftrightarrow 3, 4 \leftrightarrow 3, 1 \leftrightarrow 5, 6 \leftrightarrow 7, 8 \leftrightarrow 7\}
```

$$\begin{aligned} &\text{Im}_{(79)} = \text{"U}_{Nb} = \text{UNb}[g,t] \\ &\text{Out}_{(79)} = \text{U}_{Nb} = \text{UNb}[g,t] \\ &\text{Out}_{(79)} = \text{U}_{Nb} = \\ &\text{Out}_{(80)} = \{5 \leftrightarrow 1, 4 \leftrightarrow 7\} \\ &\text{Im}_{(81)} = A = -\lambda. \{\tilde{\chi}_{H} \& / \Theta \text{EdgeList}[g]\}^{T} / \cdot ps; \\ &\text{"A="} \\ &A // \text{MatrixForm} \\ &\text{Out}_{(83)} \text{(MatrixForm)} \\ &\text{Out}_{(83)} \text{(MatrixForm)} \\ &- \frac{f_{2ad}}{f_{2ad}} \frac{f_{2ad}}{f_{2ad}} \frac{f_{2ad}}{f_{2ad}} \\ &- \frac{f_{2ad}}{f_{2ad}} \frac{f_{2ad}}{f_{2ad}} \frac{f_{2ad}}{f_{2ad}} \\ &- \frac{f_{2ad}}{f_{2ad}} \frac{f_{2ad}}{f_{2ad}} \frac{f_{2ad}}{f_{2ad}} \frac{f_{2ad}}{f_{2ad}} \\ &- \frac{f_{2ad}}{f_{2ad}} \frac{f_{2ad}}{f_{2ad}} \frac{f_{2ad}}{f_{2ad}} \frac{f_{2ad}}{f_{2ad}} \frac{f_{2ad}}{f_{2ad}} \\ &- \frac{f_{2ad}}{f_{2ad}} \frac{f_{2ad}$$

```
In[95]:= "общее решение:"
               xsol = ((s /. xcp) \sim Join \sim xcp);
               xsol /. \{\xi_{u_{\rightarrow v_{-}}} \rightarrow \xi_{u,v}\} // Simplify // TableForm
   Out[95]= общее решение:
Out[97]//TableForm=
               X_{1,2} \to \, \tfrac{f_{2,8}\,p_{2,1}}{}
               X_{1,5} \rightarrow \frac{f_{2,8} p_{1,2} p_{2,1}}{f_{2,8} p_{1,2} p_{2,1}}
                               p<sub>1,5</sub> p<sub>2,8</sub>
               X_{4,3} \rightarrow {}^{p_{4,7}} (f_{7,8} p_{3,8} p_{7,4} + f_{3,8} p_{3,4} p_{7,8})
                                  p_{3,8} (p_{4,3}+p_{4,7}) p_{7,8}
               x_{6,7} \rightarrow \frac{f_{7,8} p_{7,6}}{f_{7,8} p_{7,6}}
                               p<sub>7,8</sub>
               x_{8,2} \rightarrow f_{2,8} - f_{8,2}
               x_{8,3} \to \tfrac{-f_{7,8}\,p_{3,8}\,p_{4,7}\,p_{7,4} + (-f_{8,3}\,p_{3,8}\,(p_{4,3} + p_{4,7}) + f_{3,8}\,(p_{3,4}\,p_{4,3} + p_{3,8}\,(p_{4,3} + p_{4,7})\,)\,)\,p_{7,8}}{}
                                                                       p_{3,8}\ (p_{4,3}+p_{4,7})\ p_{7,8}
               x_{8,7} \rightarrow \tfrac{-(f_{3,8}\,p_{3,4}\,p_{4,3}+f_{8,7}\,p_{3,8}\,(p_{4,3}+p_{4,7})\,)\,p_{7,8}+f_{7,8}\,p_{3,8}\,(p_{4,3}\,p_{7,8}+p_{4,7}\,(p_{7,4}+p_{7,8})\,)}{}
                                                                     p_{3,8} (p_{4,3} + p_{4,7}) p_{7,8}
               x_{5,1} \to \, \tfrac{f_{2,8}\,p_{1,2}\,p_{2,1}}{}
                              p<sub>1,5</sub> p<sub>2,8</sub>
               X_{4,7} \rightarrow \ ^{p_{4,3}} \ (f_{7,8} \ p_{3,8} \ p_{7,4} + f_{3,8} \ p_{3,4} \ p_{7,8})
                                     p_{3,8} (p_{4,3}+p_{4,7}) p_{7,8}
    In[98]:= "eq test:"
               Simplify[balanceEqs /. \xi \rightarrow \text{root[t]} /. s /. xcp]
               Simplify[(dopEq /. s) /. xcp]
   Out[98]= eq test:
   Out[99]= {True, True, True, True, True, True, True, True}
 Out[100]= {True, True}
```