Листинг 2

In[202]:=
```
ClearAll["Global`*"]
SetDirectory[NotebookDirectory[]];
Needs["FlowSolver`"]
```

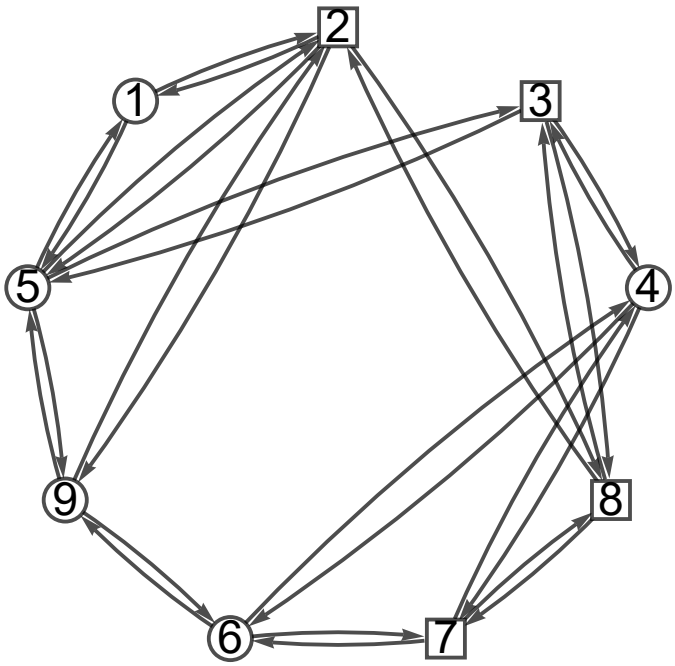
In[205]:=
```
readGraph2[file_, dir_] := Module[{
    fn = FileNameJoin[{dir, file}],
    stream, imod, umod, u, b
    },
    stream = OpenRead[fn];
    imod = Read[stream, {Word, Number}][[2]];
    umod = Read[stream, {Word, Number}][[2]];

  u = ({#[[1]] ↔ #[[2]], #[[2]] ↔ #[[1]]} & /@ ReadList[stream, Expression, umod]) // Flatten;
  b = ConstantArray[0, imod];
    (b[[Read[StringToStream[StringTake[#1, {5, -3}]], Number]]] = #2) &@@@
   ReadList[stream, {Word, Expression}, imod];
  {Graph[u, VertexSize -> Medium, VertexLabels → Placed["Name", Center],
    VertexStyle → Directive[White],
    VertexShapeFunction → {xx_ :> If[SameQ[b[[xx]], x], "Square", "Circle"]},
    VertexLabelStyle -> Directive[Black, 24], GraphLayout -> "CircularEmbedding"], b}]
```

In[206]:=
```
forma[ff_] := ((ff /. {ξ_u_↔v_ → ξu,v}) // TableForm)
```

In[859]:=
```
{g, b} = readGraph2["gr.txt", NotebookDirectory[]];
GraphPlot[g, EdgeStyle → Directive[Black, Thick],
 VertexStyle → Directive[EdgeForm[Thick], White], MultiedgeStyle → .05]
```

Out[860]=



In[861]:=
```
balanceEqs = ((Total[x_# & /@ EdgeList[g, _ ↔ #]] - Total[x_# & /@ EdgeList[g, # ↔ _]])) ==
      MapIndexed[#1 /. x → x_#2[[1]] &, b][[#]] & /@ VertexList[g];
balanceEqs //
 forma
```

Out[862]//TableForm=

$$-x_{1,2} - x_{1,5} + x_{2,1} + x_{5,1} == 0$$
$$x_{1,2} - x_{2,1} - x_{2,5} - x_{2,8} - x_{2,9} + x_{5,2} + x_{8,2} + x_{9,2} == x_2$$
$$x_{1,5} + x_{2,5} + x_{3,5} - x_{5,1} - x_{5,2} - x_{5,3} - x_{5,9} + x_{9,5} == 0$$
$$x_{2,8} + x_{3,8} + x_{7,8} - x_{8,2} - x_{8,3} - x_{8,7} == x_8$$
$$-x_{3,4} - x_{3,5} - x_{3,8} + x_{4,3} + x_{5,3} + x_{8,3} == x_3$$
$$x_{3,4} - x_{4,3} - x_{4,6} - x_{4,7} + x_{6,4} + x_{7,4} == 0$$
$$x_{4,7} + x_{6,7} - x_{7,4} - x_{7,6} - x_{7,8} + x_{8,7} == x_7$$
$$x_{4,6} - x_{6,4} - x_{6,7} - x_{6,9} + x_{7,6} + x_{9,6} == 0$$
$$x_{2,9} + x_{5,9} + x_{6,9} - x_{9,2} - x_{9,5} - x_{9,6} == 0$$

In[863]:=
```
M = {9, 8};
Print["M = ", M];
```

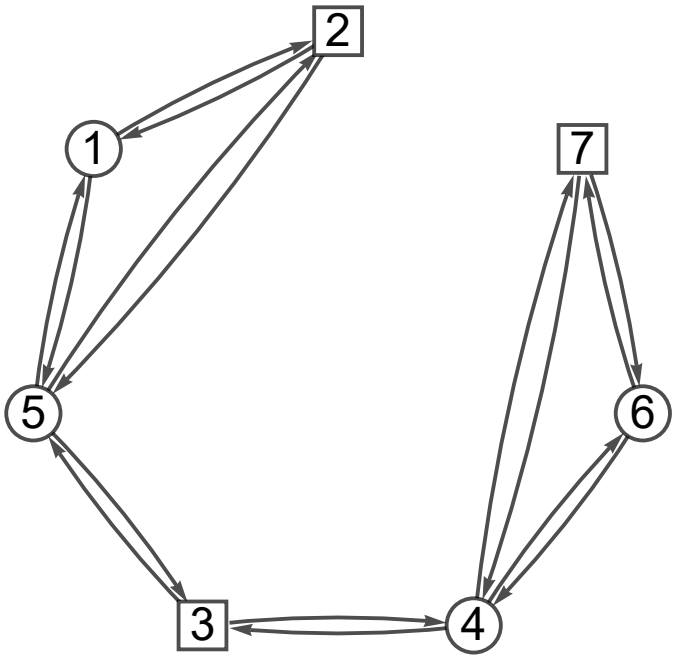
M = {9, 8}

In[865]:=
```
(*incL=
 DeleteCases[DeleteDuplicates[Cases[IncidenceList[g,#],i_↔j_:>{i,j}]//Flatten],
    v_/;v==#]&/@M*)
incL = (IncidenceList[g, #] & /@ M) // Flatten
```

Out[865]= {9 ↔ 6, 6 ↔ 9, 9 ↔ 5, 5 ↔ 9, 9 ↔ 2, 2 ↔ 9, 2 ↔ 8, 8 ↔ 2, 3 ↔ 8, 8 ↔ 3, 8 ↔ 7, 7 ↔ 8}

In[866]:=
```
(*Do[If[MemberQ[M,j[[1]]],b[[j[[2]]]]+=f_j,b[[j[[1]]]]-=f_j],{j,incL}]*)
b̄ = Fold[If[MemberQ[M, #2[[1]]], ReplacePart[#, #2[[2]] → #[[#2[[2]]]] + f_#2],
      ReplacePart[#, #2[[1]] → #[[#2[[1]]]] - f_#2]] &, b, incL];
n̄ḡ = VertexDelete[g, M];
b̄ = b̄[[Range[n̄ḡ // VertexCount]]];
GraphPlot[n̄ḡ, EdgeStyle → Directive[Black, Thick],
 VertexStyle → Directive[EdgeForm[Thick], White], MultiedgeStyle → .05]
b̄
```

Out[869]=



Out[870]= {0, x − f2↔8 − f2↔9 + f8↔2 + f9↔2, x − f3↔8 + f8↔3, 0, − f5↔9 + f9↔5, − f6↔9 + f9↔6, x − f7↔8 + f8↔7}

In[871]:=
```
CC[g_, M_] :=
 (DeleteDuplicates[Cases[IncidenceList[g, #], i_ ↔ j_ /; j == #]] & /@ M) // Flatten

ii_i^+[g_] := Cases[IncidenceList[g, i], u_ ↔ v_ /; u == i :> v]
```

In[873]:=
```
M⁺ = CC[g, M]
```

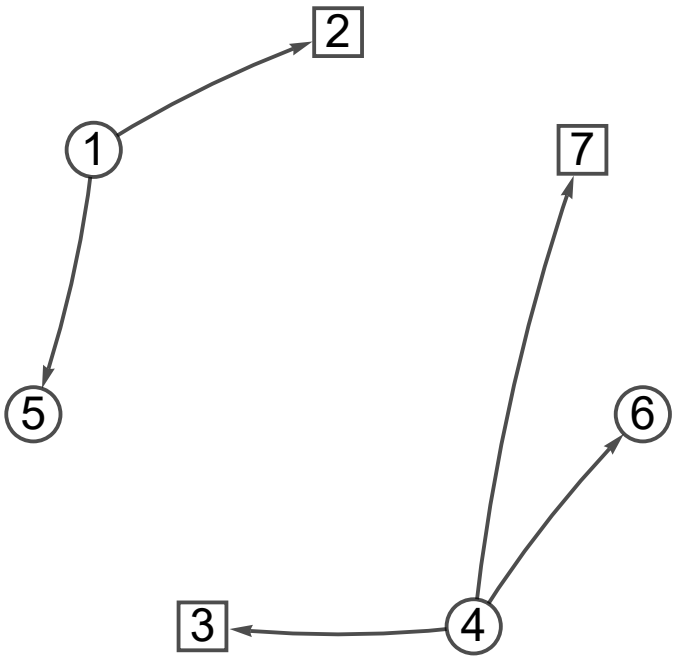
Out[873]=
$$\{6 \leftrightarrow 9,\ 5 \leftrightarrow 9,\ 2 \leftrightarrow 9,\ 2 \leftrightarrow 8,\ 3 \leftrightarrow 8,\ 7 \leftrightarrow 8\}$$

In[874]:=
```
b1 = Fold[Module[{bb = #1, i = #2[[1]], k = #2[[2]]},
    (ReplacePart[bb, ((({# → bb[[#]] + p_{i↔#}/p_{i↔k} f_{i↔k}, i → bb[[i]] - p_{i↔#}/p_{i↔k} f_{i↔k}}) &) /@ ii_i^+[ng]) //
      Flatten])] &, b, M⁺]
```

Out[874]=
$$\left\{\frac{f_{2\leftrightarrow 8}\, p_{2\leftrightarrow 1}}{p_{2\leftrightarrow 8}} + \frac{f_{2\leftrightarrow 9}\, p_{2\leftrightarrow 1}}{p_{2\leftrightarrow 9}} + \frac{f_{5\leftrightarrow 9}\, p_{5\leftrightarrow 1}}{p_{5\leftrightarrow 9}},\right.$$
$$x - f_{2\leftrightarrow 8} - f_{2\leftrightarrow 9} + f_{8\leftrightarrow 2} + f_{9\leftrightarrow 2} - \frac{f_{2\leftrightarrow 8}\, p_{2\leftrightarrow 1}}{p_{2\leftrightarrow 8}} - \frac{f_{2\leftrightarrow 9}\, p_{2\leftrightarrow 1}}{p_{2\leftrightarrow 9}} + \frac{f_{5\leftrightarrow 9}\, p_{5\leftrightarrow 2}}{p_{5\leftrightarrow 9}},$$
$$x - f_{3\leftrightarrow 8} + f_{8\leftrightarrow 3} - \frac{f_{3\leftrightarrow 8}\, p_{3\leftrightarrow 4}}{p_{3\leftrightarrow 8}} + \frac{f_{5\leftrightarrow 9}\, p_{5\leftrightarrow 3}}{p_{5\leftrightarrow 9}}, \frac{f_{3\leftrightarrow 8}\, p_{3\leftrightarrow 4}}{p_{3\leftrightarrow 8}} + \frac{f_{6\leftrightarrow 9}\, p_{6\leftrightarrow 4}}{p_{6\leftrightarrow 9}} + \frac{f_{7\leftrightarrow 8}\, p_{7\leftrightarrow 4}}{p_{7\leftrightarrow 8}},$$
$$- f_{5\leftrightarrow 9} + f_{9\leftrightarrow 5} + \frac{f_{2\leftrightarrow 8}\, p_{2\leftrightarrow 5}}{p_{2\leftrightarrow 8}} + \frac{f_{2\leftrightarrow 9}\, p_{2\leftrightarrow 5}}{p_{2\leftrightarrow 9}} + \frac{f_{3\leftrightarrow 8}\, p_{3\leftrightarrow 5}}{p_{3\leftrightarrow 8}} - \frac{f_{5\leftrightarrow 9}\, p_{5\leftrightarrow 1}}{p_{5\leftrightarrow 9}},$$
$$\left. - f_{6\leftrightarrow 9} + f_{9\leftrightarrow 6} - \frac{f_{6\leftrightarrow 9}\, p_{6\leftrightarrow 7}}{p_{6\leftrightarrow 9}} + \frac{f_{7\leftrightarrow 8}\, p_{7\leftrightarrow 6}}{p_{7\leftrightarrow 8}},\ x - f_{7\leftrightarrow 8} + f_{8\leftrightarrow 7} + \frac{f_{6\leftrightarrow 9}\, p_{6\leftrightarrow 7}}{p_{6\leftrightarrow 9}} - \frac{f_{7\leftrightarrow 8}\, p_{7\leftrightarrow 4}}{p_{7\leftrightarrow 8}}\right\}$$

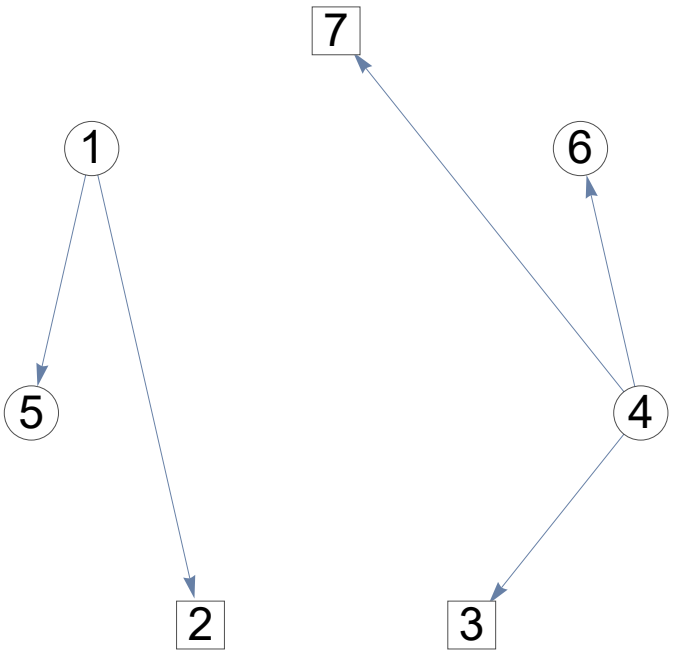
In[875]:=
```
GraphPlot[Fold[HighlightGraph[#1, u_ ↔ v_ /; u == #2, GraphHighlightStyle → "White"] &,
  ng, #[[1]] & /@ M⁺], EdgeStyle → Directive[Black, Thick],
 VertexStyle → Directive[EdgeForm[Thick], White], MultiedgeStyle → .05]
```

Out[875]=



In[876]:=
```
g1 = Fold[EdgeDelete[#1, u_ ↔ v_ /; u == #2] &, ng, #[[1]] & /@ M⁺];
GraphPlot[g1, MultiedgeStyle → .05]
```

Out[877]=



In[878]:=
```
IIrem = VertexList[g1] ~Complement~ (M⁺[[All, 1]])
```

Out[878]=
```
{1, 4}
```

In[879]:=
```
λ = SparseArray[
  Replace[(EdgeList[g1] /. # & /@ Flatten[Module[{i = #, jf, Icur},
      (Icur = ii[g1];
       jf = First[Icur];
       ({(i ↔ jf) → 1, (i ↔ #) → -pi↔#/pi↔jf}) & /@ Icur[[2 ;;]])] & /@ IIrem, 1]),
   _ ↔ _ → 0, 2]]
```

Out[879]=
```
SparseArray[ ⊞ Specified elements: 6
               Dimensions: {3, 5} ]
```

In[880]:=
```
Grid[λ]
```

Out[880]=

$$\begin{array}{ccccc} 1 & -\frac{p_{1\leftrightarrow 5}}{p_{1\leftrightarrow 2}} & 0 & 0 & 0 \\ 0 & 0 & 1 & -\frac{p_{4\leftrightarrow 7}}{p_{4\leftrightarrow 3}} & 0 \\ 0 & 0 & 1 & 0 & -\frac{p_{4\leftrightarrow 6}}{p_{4\leftrightarrow 3}} \end{array}$$

In[881]:=
```
g = g1;
b = b1;
```

In[883]:=
```
II* = Cases[MapIndexed[{#1, #2} &, b], {el_, i_} /; MemberQ[el, x] :> i] // Flatten
```

Out[883]=
```
{2, 3, 7}
```

In[884]:=
```
buildt = Timing[{t, g} = buildTree[g, II*];][[1]]
TableForm[t[[1 ;; 4]],
 TableHeadings → {{"pred", "dir", "depth", "d"}, t // pred // Length // Range}]
```

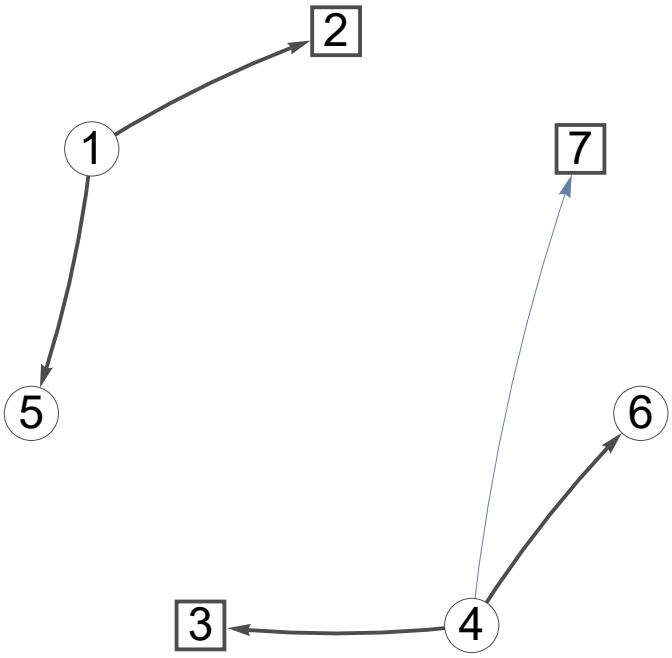
Out[884]= 0.015625

Out[885]//TableForm=

	1	2	3	4	5	6	7	8
pred	2	8	8	3	1	4	8	0
dir	−1	1	1	−1	1	1	1	0
depth	2	1	1	2	3	3	1	0
d	5	1	4	6	3	7	8	2

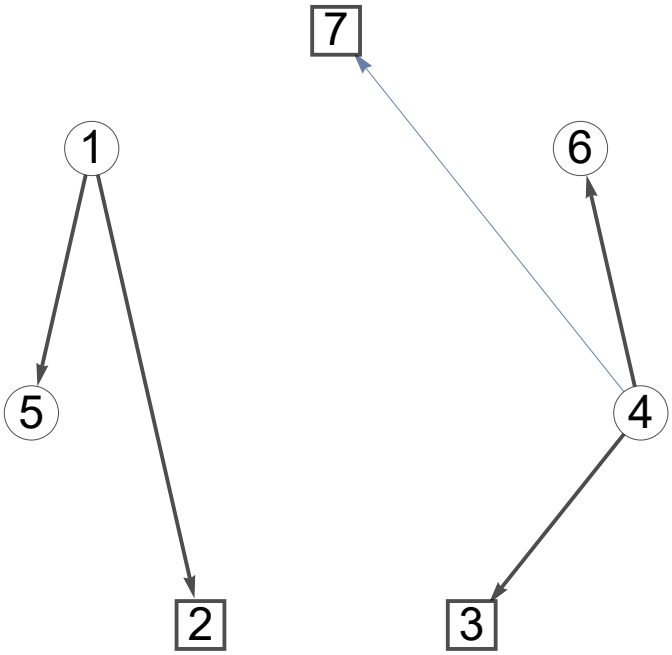
In[886]:=
```
GraphPlot[HighlightGraph[
  Fold[HighlightGraph[#1, Style[u_ ↔ v_ /; u == #2, White]] &, ng, #[[1]] & /@ M+],
  {Style[u_ /; VertexQ[g, u] && pred[t][[u]] == root[t], EdgeForm[Thick]],
   Style[u_ ↔ v_ /; (pred[t][[u]] == v && dir[t][[u]] == -1) ||
       (pred[t][[v]] == u && dir[t][[v]] == 1), Directive[Black, Thick]]},
  GraphHighlightStyle → None], MultiedgeStyle → .05]
```

Out[886]=



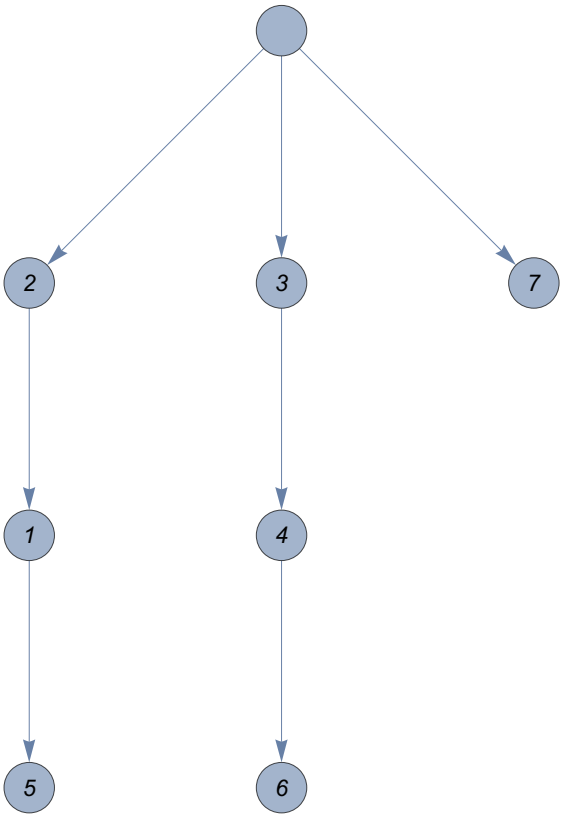
In[887]:=
```
GraphPlot[HighlightGraph[g1,
  {Style[u_ /; VertexQ[g, u] && pred[t][[u]] == root[t], EdgeForm[Thick]],
   Style[u_ ↔ v_ /; (pred[t][[u]] == v && dir[t][[u]] == -1) ||
       (pred[t][[v]] == u && dir[t][[v]] == 1), Directive[Black, Thick]]},
  GraphHighlightStyle → None], MultiedgeStyle → .05]
```

Out[887]=



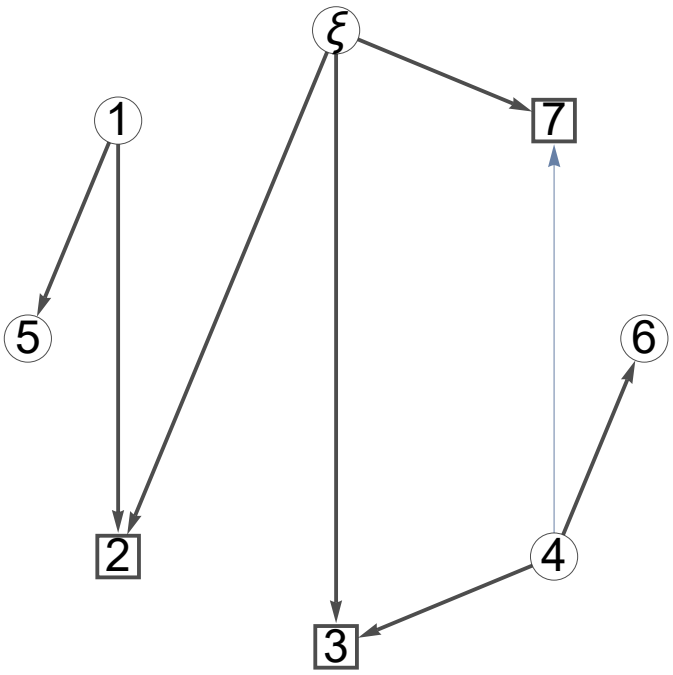
In[888]:=
```
t[[7]](*пометить на графе*)
```

Out[888]=



In[889]:=
```
(*GraphPlot[g,MultiedgeStyle→.05]*)
GraphPlot[HighlightGraph[g,
  {Style[u_ /; VertexQ[g, u] && pred[t][[u]] == root[t], EdgeForm[Thick]],
   Style[u_ ↔ v_ /; (pred[t][[u]] == v && dir[t][[u]] == -1) ||
       (pred[t][[v]] == u && dir[t][[v]] == 1), Directive[Black, Thick]]},
  GraphHighlightStyle → None], MultiedgeStyle → .05]
```

Out[889]=



In[890]:=
```
AppendTo[b, -Total[b]];
b = Simplify[b /. x → 0]
```

Out[891]=

$$\left\{\frac{f_{2\leftrightarrow 8}\,p_{2\leftrightarrow 1}}{p_{2\leftrightarrow 8}}+\frac{f_{2\leftrightarrow 9}\,p_{2\leftrightarrow 1}}{p_{2\leftrightarrow 9}}+\frac{f_{5\leftrightarrow 9}\,p_{5\leftrightarrow 1}}{p_{5\leftrightarrow 9}},\right.$$

$$f_{8\leftrightarrow 2}+f_{9\leftrightarrow 2}+f_{2\leftrightarrow 8}\left(-1-\frac{p_{2\leftrightarrow 1}}{p_{2\leftrightarrow 8}}\right)+f_{2\leftrightarrow 9}\left(-1-\frac{p_{2\leftrightarrow 1}}{p_{2\leftrightarrow 9}}\right)+\frac{f_{5\leftrightarrow 9}\,p_{5\leftrightarrow 2}}{p_{5\leftrightarrow 9}},$$

$$f_{8\leftrightarrow 3}+f_{3\leftrightarrow 8}\left(-1-\frac{p_{3\leftrightarrow 4}}{p_{3\leftrightarrow 8}}\right)+\frac{f_{5\leftrightarrow 9}\,p_{5\leftrightarrow 3}}{p_{5\leftrightarrow 9}},\ \frac{f_{3\leftrightarrow 8}\,p_{3\leftrightarrow 4}}{p_{3\leftrightarrow 8}}+\frac{f_{6\leftrightarrow 9}\,p_{6\leftrightarrow 4}}{p_{6\leftrightarrow 9}}+\frac{f_{7\leftrightarrow 8}\,p_{7\leftrightarrow 4}}{p_{7\leftrightarrow 8}},$$

$$f_{9\leftrightarrow 5}+\frac{f_{2\leftrightarrow 8}\,p_{2\leftrightarrow 5}}{p_{2\leftrightarrow 8}}+\frac{f_{2\leftrightarrow 9}\,p_{2\leftrightarrow 5}}{p_{2\leftrightarrow 9}}+\frac{f_{3\leftrightarrow 8}\,p_{3\leftrightarrow 5}}{p_{3\leftrightarrow 8}}+f_{5\leftrightarrow 9}\left(-1-\frac{p_{5\leftrightarrow 1}}{p_{5\leftrightarrow 9}}\right),$$

$$f_{9\leftrightarrow 6}+f_{6\leftrightarrow 9}\left(-1-\frac{p_{6\leftrightarrow 7}}{p_{6\leftrightarrow 9}}\right)+\frac{f_{7\leftrightarrow 8}\,p_{7\leftrightarrow 6}}{p_{7\leftrightarrow 8}},\ f_{8\leftrightarrow 7}+\frac{f_{6\leftrightarrow 9}\,p_{6\leftrightarrow 7}}{p_{6\leftrightarrow 9}}+f_{7\leftrightarrow 8}\left(-1-\frac{p_{7\leftrightarrow 4}}{p_{7\leftrightarrow 8}}\right),$$

$$f_{3\leftrightarrow 8}+f_{5\leftrightarrow 9}+f_{6\leftrightarrow 9}+f_{7\leftrightarrow 8}-f_{8\leftrightarrow 2}-f_{8\leftrightarrow 3}-f_{8\leftrightarrow 7}-f_{9\leftrightarrow 2}-f_{9\leftrightarrow 5}-f_{9\leftrightarrow 6}+f_{2\leftrightarrow 8}\left(1-\frac{p_{2\leftrightarrow 5}}{p_{2\leftrightarrow 8}}\right)+$$

$$\left.f_{2\leftrightarrow 9}\left(1-\frac{p_{2\leftrightarrow 5}}{p_{2\leftrightarrow 9}}\right)-\frac{f_{3\leftrightarrow 8}\,p_{3\leftrightarrow 5}}{p_{3\leftrightarrow 8}}-\frac{f_{5\leftrightarrow 9}\,p_{5\leftrightarrow 2}}{p_{5\leftrightarrow 9}}-\frac{f_{5\leftrightarrow 9}\,p_{5\leftrightarrow 3}}{p_{5\leftrightarrow 9}}-\frac{f_{6\leftrightarrow 9}\,p_{6\leftrightarrow 4}}{p_{6\leftrightarrow 9}}-\frac{f_{7\leftrightarrow 8}\,p_{7\leftrightarrow 6}}{p_{7\leftrightarrow 8}}\right\}$$

In[892]:=
```
balanceEqs = ((Total[x_# & /@ EdgeList[g, _ ↔ #]] - Total[x_# & /@ EdgeList[g, # ↔ _]]) /.
        root[t] → ξ) == b[[#]] & /@ VertexList[g];
balanceEqs //
 forma
```

Out[893]//TableForm=

$$-x_{1,2}-x_{1,5}==\frac{f_{2,8}\,p_{2,1}}{p_{2,8}}+\frac{f_{2,9}\,p_{2,1}}{p_{2,9}}+\frac{f_{5,9}\,p_{5,1}}{p_{5,9}}$$

$$x_{1,2}+x_{\xi,2}==f_{8,2}+f_{9,2}+f_{2,8}\left(-1-\frac{p_{2,1}}{p_{2,8}}\right)+f_{2,9}\left(-1-\frac{p_{2,1}}{p_{2,9}}\right)+\frac{f_{5,9}\,p_{5,2}}{p_{5,9}}$$

$$x_{1,5}==f_{9,5}+\frac{f_{2,8}\,p_{2,5}}{p_{2,8}}+\frac{f_{2,9}\,p_{2,5}}{p_{2,9}}+\frac{f_{3,8}\,p_{3,5}}{p_{3,8}}+f_{5,9}\left(-1-\frac{p_{5,1}}{p_{5,9}}\right)$$

$$x_{4,3}+x_{\xi,3}==f_{8,3}+f_{3,8}\left(-1-\frac{p_{3,4}}{p_{3,8}}\right)+\frac{f_{5,9}\,p_{5,3}}{p_{5,9}}$$

$$-x_{4,3}-x_{4,6}-x_{4,7}==\frac{f_{3,8}\,p_{3,4}}{p_{3,8}}+\frac{f_{6,9}\,p_{6,4}}{p_{6,9}}+\frac{f_{7,8}\,p_{7,4}}{p_{7,8}}$$

$$x_{4,7}+x_{\xi,7}==f_{8,7}+\frac{f_{6,9}\,p_{6,7}}{p_{6,9}}+f_{7,8}\left(-1-\frac{p_{7,4}}{p_{7,8}}\right)$$

$$x_{4,6}==f_{9,6}+f_{6,9}\left(-1-\frac{p_{6,7}}{p_{6,9}}\right)+\frac{f_{7,8}\,p_{7,6}}{p_{7,8}}$$

$$-x_{\xi,2}-x_{\xi,3}-x_{\xi,7}==f_{3,8}+f_{5,9}+f_{6,9}+f_{7,8}-f_{8,2}-f_{8,3}-f_{8,7}-f_{9,2}-f_{9,5}-f_{9,6}+f_{2,8}\left(1-\frac{p_{2,5}}{p_{2,8}}\right)+f_{2,9}\left($$

In[894]:=
```
ps = partSolve[g, -b, t, x̃];
ps // forma
```

8

Out[895]//TableForm=

$$\tilde{x}_{1,2}\to -f_{9,5}-\frac{f_{2,8}\,p_{2,1}}{p_{2,8}}-\frac{f_{2,8}\,p_{2,5}}{p_{2,8}}-\frac{f_{2,9}\,p_{2,1}}{p_{2,9}}-\frac{f_{2,9}\,p_{2,5}}{p_{2,9}}-\frac{f_{3,8}\,p_{3,5}}{p_{3,8}}-f_{5,9}\left(-1-\frac{p_{5,1}}{p_{5,9}}\right)-\frac{f_{5,9}\,p_{5,1}}{p_{5,9}}$$

$$\tilde{x}_{1,5}\to f_{9,5}+\frac{f_{2,8}\,p_{2,5}}{p_{2,8}}+\frac{f_{2,9}\,p_{2,5}}{p_{2,9}}+\frac{f_{3,8}\,p_{3,5}}{p_{3,8}}+f_{5,9}\left(-1-\frac{p_{5,1}}{p_{5,9}}\right)$$

$$\tilde{x}_{4,3}\to -f_{9,6}-\frac{f_{3,8}\,p_{3,4}}{p_{3,8}}-f_{6,9}\left(-1-\frac{p_{6,7}}{p_{6,9}}\right)-\frac{f_{6,9}\,p_{6,4}}{p_{6,9}}-\frac{f_{7,8}\,p_{7,4}}{p_{7,8}}-\frac{f_{7,8}\,p_{7,6}}{p_{7,8}}$$

$$\tilde{x}_{4,6}\to f_{9,6}+f_{6,9}\left(-1-\frac{p_{6,7}}{p_{6,9}}\right)+\frac{f_{7,8}\,p_{7,6}}{p_{7,8}}$$

$$\tilde{x}_{4,7}\to 0$$

$$\tilde{x}_{8,2}\to f_{8,2}+f_{9,2}+f_{9,5}+f_{2,8}\left(-1-\frac{p_{2,1}}{p_{2,8}}\right)+\frac{f_{2,8}\,p_{2,1}}{p_{2,8}}+\frac{f_{2,8}\,p_{2,5}}{p_{2,8}}+f_{2,9}\left(-1-\frac{p_{2,1}}{p_{2,9}}\right)+\frac{f_{2,9}\,p_{2,1}}{p_{2,9}}+\frac{f_{2,9}\,p_{2,5}}{p_{2,9}}+\frac{f_{3,8}\,p_{3,5}}{p_{3,8}}+$$

$$\tilde{x}_{8,3}\to f_{8,3}+f_{9,6}+f_{3,8}\left(-1-\frac{p_{3,4}}{p_{3,8}}\right)+\frac{f_{3,8}\,p_{3,4}}{p_{3,8}}+\frac{f_{5,9}\,p_{5,3}}{p_{5,9}}+f_{6,9}\left(-1-\frac{p_{6,7}}{p_{6,9}}\right)+\frac{f_{6,9}\,p_{6,4}}{p_{6,9}}+\frac{f_{7,8}\,p_{7,4}}{p_{7,8}}+\frac{f_{7,8}\,p_{7,6}}{p_{7,8}}$$

$$\tilde{x}_{8,7}\to f_{8,7}+\frac{f_{6,9}\,p_{6,7}}{p_{6,9}}+f_{7,8}\left(-1-\frac{p_{7,4}}{p_{7,8}}\right)$$

In[896]:=
```
Simplify[(balanceEqs /. {x → x̃, ξ → root[t]}) /. ps]
```

Out[896]= {True, True, True, True, True, True, True, True}

In[897]:=
```
matrt = Timing[δMatr = δ1[g, t]];
rootT = VertexCount[g];
TableForm[δMatr, TableHeadings → {uNb[g, t], δ[#[[2]] #[[1]] == rootT & /@ EdgeList[g]}]] // forma
```

Out[899]//TableForm=

	$\delta_{1,2}$	$\delta_{1,5}$	$\delta_{4,3}$	$\delta_{4,7}$	$\delta_{4,6}$	δ_{2}	δ_{3}	δ_{7}
$4\leftrightarrow 7$	0	0	−1	1	0	0	1	−1

In[900]:=
```
λ = SparseArray[λ, {Length[λ], Length[λ[[1]]] + Length[II*]}];
(*λ=λ[[;;-2]]*)
```

In[901]:=
```
dopEq = # == 0 & /@ Flatten[λ.{x_# & /@ EdgeList[g]}^T];
dopEq // forma
```

Out[902]//TableForm=

$$x_{1,2}-\frac{p_{1,5}\,x_{1,5}}{p_{1,2}}==0$$

$$x_{4,3}-\frac{p_{4,7}\,x_{4,7}}{p_{4,3}}==0$$

$$x_{4,3}-\frac{p_{4,6}\,x_{4,6}}{p_{4,3}}==0$$

In[903]:=
```
Λ = λ. (δMatr)^T;
"cicle det's:"
Λ // forma
```

Out[904]= cicle det's:

Out[905]//TableForm=

$$\begin{array}{c} 0 \\ -1 - \frac{p_{4\leftrightarrow 7}}{p_{4\leftrightarrow 3}} \\ -1 \end{array}$$

In[823]:=
```
(*сделать нормально*)
```

In[906]:=
```
"Uc="
Uc = {1}
"Unc="
Unc = {}
```

Out[906]= $U_c =$

Out[907]= $\{1\}$

Out[908]= $U_{nc} =$

Out[909]= $\{\}$

In[914]:=
```
Λc = Λ[[{3}, Uc]];
Λnc = Λ[[{3}, Unc]];
"Λc="
Λc // MatrixForm
```

Out[916]= $\Lambda_c =$

Out[917]//MatrixForm=

$$\begin{pmatrix} -1 \end{pmatrix}$$

In[918]:=
```
"det(Λc)="
Simplify[det = Det[Λc]] // forma
```

Out[918]= $\det(\Lambda_c) =$

Out[919]//TableForm=

$$-1$$

In[920]:=
```
MatrixRank[Λc]
```

Out[920]= 1

In[921]:=
```
"UT="
utind = Cases[t[[6]], ξ_ /; ξ ≠ 0];
UT = EdgeList[g][[utind]]
```

Out[921]= $U_T =$

Out[923]= $\{1 \leftrightarrow 2, 8 \leftrightarrow 2, 8 \leftrightarrow 3, 4 \leftrightarrow 3, 1 \leftrightarrow 5, 4 \leftrightarrow 6, 8 \leftrightarrow 7\}$

In[924]:=
```
"UNb="
UNb = uNb[g, t]
```

Out[924]= $U_{Nb} =$

Out[925]= $\{4 \leftrightarrow 7\}$

In[926]:=
```
A = -λ.{x̃# & /@ EdgeList[g]}^T /. ps;
"A="
A // MatrixForm
```

Out[927]= A=

Out[928]//MatrixForm=

$$\begin{pmatrix} f_{9,\bullet\to 5}+\frac{f_{2,\bullet\to 8}\,p_{2,\bullet\to 1}}{p_{2,\bullet\to 8}}+\frac{f_{2,\bullet\to 8}\,p_{2,\bullet\to 5}}{p_{2,\bullet\to 8}}+\frac{f_{2,\bullet\to 9}\,p_{2,\bullet\to 1}}{p_{2,\bullet\to 9}}+\frac{f_{2,\bullet\to 9}\,p_{2,\bullet\to 5}}{p_{2,\bullet\to 9}}+\frac{f_{3,\bullet\to 8}\,p_{3,\bullet\to 5}}{p_{3,\bullet\to 8}}+\frac{p_{1,\bullet\to 5}\left(f_{9,\bullet\to 5}+\frac{f_{2,\bullet\to 8}\,p_{2,\bullet\to 5}}{p_{2,\bullet\to 8}}+\frac{f_{2,\bullet\to 9}\,p_{2,\bullet\to 5}}{p_{2,\bullet\to 9}}+\frac{f_{3,\bullet\to 8}\,p_{3,\bullet\to 5}}{p_{3,\bullet\to 8}}+f_{5,\bullet\to 9}\left(-1-\frac{p_{5,\bullet\to 1}}{p_{5,\bullet\to 9}}\right)\right)}{p_{1,\bullet\to 2}}+f_{5,} \\ f_{9,\bullet\to 6}+\frac{f_{3,\bullet\to 8}\,p_{3,\bullet\to 4}}{p_{3,\bullet\to 8}}+f_{6,\bullet\to 9}\left(-1-\frac{p_{6,\bullet\to 7}}{p_{6,\bullet\to 9}}\right)+\frac{f_{6,\bullet\to 9}\,p_{6,\bullet\to 4}}{p_{6,\bullet\to 9}}+\frac{f_{7,\bullet\to 8}\,p_{7,\bullet\to 4}}{p_{7,\bullet\to 8}}+\frac{f_{7,\bullet\to 8}\,p_{7,\bullet\to 6}}{p_{7,\bullet\to 8}} \\ f_{9,\bullet\to 6}+\frac{f_{3,\bullet\to 8}\,p_{3,\bullet\to 4}}{p_{3,\bullet\to 8}}+f_{6,\bullet\to 9}\left(-1-\frac{p_{6,\bullet\to 7}}{p_{6,\bullet\to 9}}\right)+\frac{f_{6,\bullet\to 9}\,p_{6,\bullet\to 4}}{p_{6,\bullet\to 9}}+\frac{p_{4,\bullet\to 6}\left(f_{9,\bullet\to 6}+f_{6,\bullet\to 9}\left(-1-\frac{p_{6,\bullet\to 7}}{p_{6,\bullet\to 9}}\right)+\frac{f_{7,\bullet\to 8}\,p_{7,\bullet\to 6}}{p_{7,\bullet\to 8}}\right)}{p_{4,\bullet\to 3}}+\frac{f_{7,\bullet\to 8}\,p_{7,\bullet\to 4}}{p_{7,\bullet\to 8}}+ \end{pmatrix}$$

In[940]:=
```
β = A[[3]] - Λnc.{x# & /@ U_Nb[[U_nc]]}^T;
"β="
β // forma
```

Out[941]= β=

Out[942]//TableForm=

$$f_{9,6}+\frac{f_{3,8}\,p_{3,4}}{p_{3,8}}+f_{6,9}\left(-1-\frac{p_{6,7}}{p_{6,9}}\right)+\frac{f_{6,9}\,p_{6,4}}{p_{6,9}}+\frac{p_{4,6}\left(f_{9,6}+f_{6,9}\left(-1-\frac{p_{6,7}}{p_{6,9}}\right)+\frac{f_{7,8}\,p_{7,6}}{p_{7,8}}\right)}{p_{4,3}}+\frac{f_{7,8}\,p_{7,4}}{p_{7,8}}+\frac{f_{7,8}\,p_{7,6}}{p_{7,8}}$$

In[943]:=
```
"решаем уравнение Λcxc=β:"
xc = LinearSolve[Λc, β]
```

Out[943]= решаем уравнение $\Lambda_c x_c=\beta$:

Out[944]=
$$\left\{f_{6,\bullet\to 9}-f_{9,\bullet\to 6}-\frac{f_{3,\bullet\to 8}\,p_{3,\bullet\to 4}}{p_{3,\bullet\to 8}}+\frac{f_{6,\bullet\to 9}\,p_{4,\bullet\to 6}}{p_{4,\bullet\to 3}}-\frac{f_{9,\bullet\to 6}\,p_{4,\bullet\to 6}}{p_{4,\bullet\to 3}}-\frac{f_{6,\bullet\to 9}\,p_{6,\bullet\to 4}}{p_{6,\bullet\to 9}}+\right.$$
$$\left.\frac{f_{6,\bullet\to 9}\,p_{6,\bullet\to 7}}{p_{6,\bullet\to 9}}+\frac{f_{6,\bullet\to 9}\,p_{4,\bullet\to 6}\,p_{6,\bullet\to 7}}{p_{4,\bullet\to 3}\,p_{6,\bullet\to 9}}-\frac{f_{7,\bullet\to 8}\,p_{7,\bullet\to 4}}{p_{7,\bullet\to 8}}-\frac{f_{7,\bullet\to 8}\,p_{7,\bullet\to 6}}{p_{7,\bullet\to 8}}-\frac{f_{7,\bullet\to 8}\,p_{4,\bullet\to 6}\,p_{7,\bullet\to 6}}{p_{4,\bullet\to 3}\,p_{7,\bullet\to 8}}\right\}$$

In[945]:=
```
xcp = MapThread[x_#1 → #2 &, {U_Nb[[U_c]], Flatten[xc]}];
xcp // TableForm
```

Out[946]//TableForm=

$$x_{4,\bullet\to 7}\to f_{6,\bullet\to 9}-f_{9,\bullet\to 6}-\frac{f_{3,\bullet\to 8}\,p_{3,\bullet\to 4}}{p_{3,\bullet\to 8}}+\frac{f_{6,\bullet\to 9}\,p_{4,\bullet\to 6}}{p_{4,\bullet\to 3}}-\frac{f_{9,\bullet\to 6}\,p_{4,\bullet\to 6}}{p_{4,\bullet\to 3}}-\frac{f_{6,\bullet\to 9}\,p_{6,\bullet\to 4}}{p_{6,\bullet\to 9}}+\frac{f_{6,\bullet\to 9}\,p_{6,\bullet\to 7}}{p_{6,\bullet\to 9}}+\frac{f_{6,\bullet\to 9}\,p_{4,\bullet\to 6}\,p_{6,\bullet\to 7}}{p_{4,\bullet\to 3}\,p_{6,\bullet\to 9}}-\frac{f_{7,\bullet\to 8}\,p_{7,\bullet\to 4}}{p_{7,\bullet\to 8}}-\frac{f_{7,\bullet\to 8}\,p_{7,\bullet\to 6}}{p_{7,\bullet\to 8}}-$$

In[947]:=
```
s = solveAll[g, t];
s // TableForm
```

Out[948]//TableForm=

$$x_{1,\bullet\to 2}\to -f_{9,\bullet\to 5}-\frac{f_{2,\bullet\to 8}\,p_{2,\bullet\to 1}}{p_{2,\bullet\to 8}}-\frac{f_{2,\bullet\to 8}\,p_{2,\bullet\to 5}}{p_{2,\bullet\to 8}}-\frac{f_{2,\bullet\to 9}\,p_{2,\bullet\to 1}}{p_{2,\bullet\to 9}}-\frac{f_{2,\bullet\to 9}\,p_{2,\bullet\to 5}}{p_{2,\bullet\to 9}}-\frac{f_{3,\bullet\to 8}\,p_{3,\bullet\to 5}}{p_{3,\bullet\to 8}}-f_{5,\bullet\to 9}\left(-1-\frac{p_{5,\bullet\to 1}}{p_{5,\bullet\to 9}}\right)-\frac{f_{5,\bullet\to 9}\,p_{5,\bullet\to 1}}{p_{5,\bullet\to 9}}$$

$$x_{1,\bullet\to 5}\to f_{9,\bullet\to 5}+\frac{f_{2,\bullet\to 8}\,p_{2,\bullet\to 5}}{p_{2,\bullet\to 8}}+\frac{f_{2,\bullet\to 9}\,p_{2,\bullet\to 5}}{p_{2,\bullet\to 9}}+\frac{f_{3,\bullet\to 8}\,p_{3,\bullet\to 5}}{p_{3,\bullet\to 8}}+f_{5,\bullet\to 9}\left(-1-\frac{p_{5,\bullet\to 1}}{p_{5,\bullet\to 9}}\right)$$

$$x_{4,\bullet\to 3}\to -f_{9,\bullet\to 6}-\frac{f_{3,\bullet\to 8}\,p_{3,\bullet\to 4}}{p_{3,\bullet\to 8}}-f_{6,\bullet\to 9}\left(-1-\frac{p_{6,\bullet\to 7}}{p_{6,\bullet\to 9}}\right)-\frac{f_{6,\bullet\to 9}\,p_{6,\bullet\to 4}}{p_{6,\bullet\to 9}}-\frac{f_{7,\bullet\to 8}\,p_{7,\bullet\to 4}}{p_{7,\bullet\to 8}}-\frac{f_{7,\bullet\to 8}\,p_{7,\bullet\to 6}}{p_{7,\bullet\to 8}}-x_{4,\bullet\to 7}$$

$$x_{4,\bullet\to 6}\to f_{9,\bullet\to 6}+f_{6,\bullet\to 9}\left(-1-\frac{p_{6,\bullet\to 7}}{p_{6,\bullet\to 9}}\right)+\frac{f_{7,\bullet\to 8}\,p_{7,\bullet\to 6}}{p_{7,\bullet\to 8}}$$

$$x_{8,\bullet\to 2}\to f_{8,\bullet\to 2}+f_{9,\bullet\to 2}+f_{9,\bullet\to 5}+f_{2,\bullet\to 8}\left(-1-\frac{p_{2,\bullet\to 1}}{p_{2,\bullet\to 8}}\right)+\frac{f_{2,\bullet\to 8}\,p_{2,\bullet\to 1}}{p_{2,\bullet\to 8}}+\frac{f_{2,\bullet\to 8}\,p_{2,\bullet\to 5}}{p_{2,\bullet\to 8}}+f_{2,\bullet\to 9}\left(-1-\frac{p_{2,\bullet\to 1}}{p_{2,\bullet\to 9}}\right)+\frac{f_{2,\bullet\to 9}\,p_{2,\bullet\to 1}}{p_{2,\bullet\to 9}}+\frac{f_{2,\bullet\to 9}\,p_{2,\bullet\to 5}}{p_{2,\bullet\to 9}}-$$

$$x_{8,\bullet\to 3}\to f_{8,\bullet\to 3}+f_{9,\bullet\to 6}+f_{3,\bullet\to 8}\left(-1-\frac{p_{3,\bullet\to 4}}{p_{3,\bullet\to 8}}\right)+\frac{f_{3,\bullet\to 8}\,p_{3,\bullet\to 4}}{p_{3,\bullet\to 8}}+\frac{f_{5,\bullet\to 9}\,p_{5,\bullet\to 3}}{p_{5,\bullet\to 9}}+f_{6,\bullet\to 9}\left(-1-\frac{p_{6,\bullet\to 7}}{p_{6,\bullet\to 9}}\right)+\frac{f_{6,\bullet\to 9}\,p_{6,\bullet\to 4}}{p_{6,\bullet\to 9}}+\frac{f_{7,\bullet\to 8}\,p_{7,\bullet\to 4}}{p_{7,\bullet\to 8}}+\frac{f_{7,\bullet\to 8}\,p_{7,}}{p_{7,\bullet\to 8}}$$

$$x_{8,\bullet\to 7}\to f_{8,\bullet\to 7}+\frac{f_{6,\bullet\to 9}\,p_{6,\bullet\to 7}}{p_{6,\bullet\to 9}}+f_{7,\bullet\to 8}\left(-1-\frac{p_{7,\bullet\to 4}}{p_{7,\bullet\to 8}}\right)-x_{4,\bullet\to 7}$$

In[949]:=
```
"общее решение:"
xsol = ((s /. xcp) ~Join~ xcp);
xsol /. {ξ_u_↦v_ → ξu,v} // Simplify // TableForm
```

Out[949]= общее решение:

Out[951]//TableForm=

$$x_{1,2} \to f_{5,9} - f_{9,5} - \frac{f_{2,8}\,(p_{2,1}+p_{2,5})}{p_{2,8}} - \frac{f_{2,9}\,(p_{2,1}+p_{2,5})}{p_{2,9}} - \frac{f_{3,8}\,p_{3,5}}{p_{3,8}}$$

$$x_{1,5} \to f_{9,5} + \frac{f_{2,8}\,p_{2,5}}{p_{2,8}} + \frac{f_{2,9}\,p_{2,5}}{p_{2,9}} + \frac{f_{3,8}\,p_{3,5}}{p_{3,8}} + f_{5,9}\left(-1 - \frac{p_{5,1}}{p_{5,9}}\right)$$

$$x_{4,3} \to \frac{p_{4,6}\,(f_{7,8}\,p_{6,9}\,p_{7,6} - (-f_{9,6}\,p_{6,9} + f_{6,9}\,(p_{6,7}+p_{6,9}))\,p_{7,8})}{p_{4,3}\,p_{6,9}\,p_{7,8}}$$

$$x_{4,6} \to f_{9,6} + f_{6,9}\left(-1 - \frac{p_{6,7}}{p_{6,9}}\right) + \frac{f_{7,8}\,p_{7,6}}{p_{7,8}}$$

$$x_{8,2} \to -f_{5,9} + f_{8,2} + f_{9,2} + f_{9,5} + f_{2,8}\left(-1 + \frac{p_{2,5}}{p_{2,8}}\right) + f_{2,9}\left(-1 + \frac{p_{2,5}}{p_{2,9}}\right) + \frac{f_{3,8}\,p_{3,5}}{p_{3,8}} + \frac{f_{5,9}\,p_{5,2}}{p_{5,9}}$$

$$x_{8,3} \to f_{8,3} + f_{3,8}\left(-1 - \frac{p_{3,4}}{p_{3,8}}\right) + \frac{f_{6,9}\,p_{4,6}}{p_{4,3}} - \frac{f_{9,6}\,p_{4,6}}{p_{4,3}} + \frac{f_{5,9}\,p_{5,3}}{p_{5,9}} + \frac{f_{6,9}\,p_{4,6}\,p_{6,7}}{p_{4,3}\,p_{6,9}} - \frac{f_{7,8}\,p_{4,6}\,p_{7,6}}{p_{4,3}\,p_{7,8}}$$

$$x_{8,7} \to f_{8,7} + f_{9,6} + \frac{f_{3,8}\,p_{3,4}}{p_{3,8}} + \frac{f_{9,6}\,p_{4,6}}{p_{4,3}} + \frac{f_{6,9}\,(p_{4,3}\,(p_{6,4}-p_{6,9}) - p_{4,6}\,(p_{6,7}+p_{6,9}))}{p_{4,3}\,p_{6,9}} + f_{7,8}\left(-1 + \frac{(p_{4,3}+p_{4,6})\,p_{7,6}}{p_{4,3}\,p_{7,8}}\right)$$

$$x_{4,7} \to -\frac{\frac{f_{3,8}\,p_{3,4}\,p_{4,3}}{p_{3,8}} + f_{9,6}\,(p_{4,3}+p_{4,6}) - \frac{f_{6,9}\,(p_{4,6}\,(p_{6,7}+p_{6,9}) + p_{4,3}\,(-p_{6,4}+p_{6,7}+p_{6,9}))}{p_{6,9}} + \frac{f_{7,8}\,(p_{4,6}\,p_{7,6} + p_{4,3}\,(p_{7,4}+p_{7,6}))}{p_{7,8}}}{p_{4,3}}$$

In[952]:=
```
"eq test:"
Simplify[balanceEqs /. ξ → root[t] /. s /. xcp]
Simplify[(dopEq /. s) /. xcp]
```

Out[952]= eq test:

Out[953]= {True, True, True, True, True, True, True, True}

Out[954]=
$$\left\{\frac{1}{p_{1,\bullet\to 2}}\left(f_{9,\bullet\to 5}\,(p_{1,\bullet\to 2} + p_{1,\bullet\to 5}) + \frac{f_{2,\bullet\to 8}\,(p_{1,\bullet\to 5}\,p_{2,\bullet\to 5} + p_{1,\bullet\to 2}\,(p_{2,\bullet\to 1} + p_{2,\bullet\to 5}))}{p_{2,\bullet\to 8}} + \frac{f_{2,\bullet\to 9}\,(p_{1,\bullet\to 5}\,p_{2,\bullet\to 5} + p_{1,\bullet\to 2}\,(p_{2,\bullet\to 1} + p_{2,\bullet\to 5}))}{p_{2,\bullet\to 9}} + \frac{f_{3,\bullet\to 8}\,(p_{1,\bullet\to 2} + p_{1,\bullet\to 5})\,p_{3,\bullet\to 5}}{p_{3,\bullet\to 8}} - f_{5,\bullet\to 9}\left(p_{1,\bullet\to 2} + p_{1,\bullet\to 5}\left(1 + \frac{p_{5,\bullet\to 1}}{p_{5,\bullet\to 9}}\right)\right)\right) == 0,\right.$$

$$\left.\frac{1}{p_{4,\bullet\to 3}}\left(\frac{f_{3,\bullet\to 8}\,p_{3,\bullet\to 4}\,p_{4,\bullet\to 3}\,p_{4,\bullet\to 7}}{p_{3,\bullet\to 8}} + f_{9,\bullet\to 6}\,(p_{4,\bullet\to 6}\,p_{4,\bullet\to 7} + p_{4,\bullet\to 3}\,(p_{4,\bullet\to 6} + p_{4,\bullet\to 7})) - \frac{f_{6,\bullet\to 9}\,(p_{4,\bullet\to 6}\,p_{4,\bullet\to 7}\,(p_{6,\bullet\to 7} + p_{6,\bullet\to 9}) + p_{4,\bullet\to 3}\,(p_{4,\bullet\to 6}\,(p_{6,\bullet\to 7} + p_{6,\bullet\to 9}) + p_{4,\bullet\to 7}\,(-p_{6,\bullet\to 4} + p_{6,\bullet\to 7} + p_{6,\bullet\to 9})))}{p_{6,\bullet\to 9}} + \frac{f_{7,\bullet\to 8}\,(p_{4,\bullet\to 6}\,p_{4,\bullet\to 7}\,p_{7,\bullet\to 6} + p_{4,\bullet\to 3}\,(p_{4,\bullet\to 6}\,p_{7,\bullet\to 6} + p_{4,\bullet\to 7}\,(p_{7,\bullet\to 4} + p_{7,\bullet\to 6})))}{p_{7,\bullet\to 8}}\right) == 0,\ \text{True}\right\}$$