

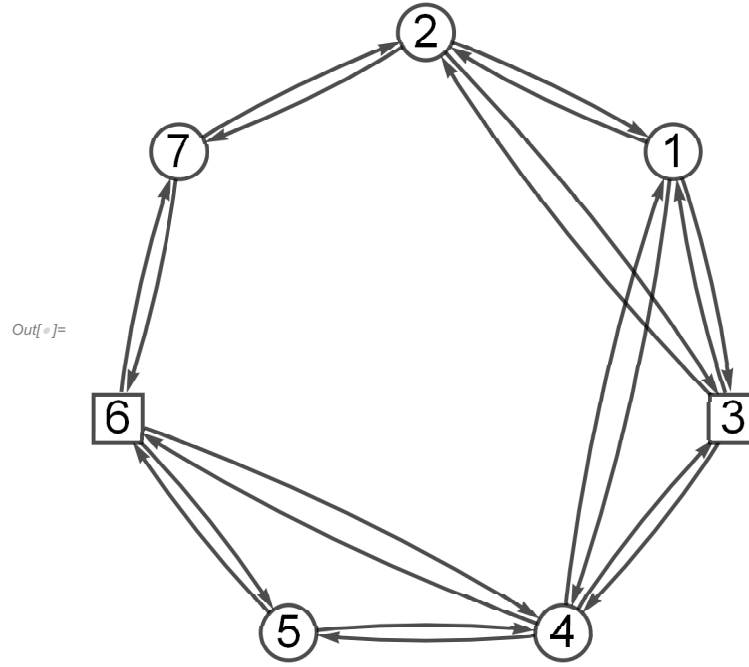
```

ClearAll["Global*"]
SetDirectory[NotebookDirectory[]];
Needs["FlowSolver"]

readGraph2[file_, dir_] := Module[{
  fn = FileNameJoin[{dir, file}],
  stream, imod, umod, u, b
},
  stream = OpenRead[fn];
  imod = Read[stream, {Word, Number}][[2]];
  umod = Read[stream, {Word, Number}][[2]];
  u = ({#[[1]]#[[2]], #[[2]]#[[1]]} &/@ReadList[stream, Expression, umod]) // Flatten;
  b = ConstantArray[0, imod];
  (b[[Read[StringToStream[StringTake[#1, {5, -3}]], Number]]] = #2) & @@@ ReadList[stream, {Word, Expression}, {
    Graph[u, VertexSize -> Medium, VertexLabels -> Placed["Name", Center], VertexStyle -> Directive[White],
    VertexShapeFunction -> {xx_ -> If[SameQ[b[[xx]], x], "Square", "Circle"]}, VertexLabelStyle -> Directive[Black,
    b]}]
  forma[ff_] := ((ff /. { $\xi_{-u.-v.} \rightarrow \xi_{u,v}$ })) // TableForm)

{g, b} = readGraph2["gr.txt", NotebookDirectory[]];
GraphPlot[g, EdgeStyle -> Directive[Black, Thick], VertexStyle -> Directive[EdgeForm[Thick], White], Multiedge

```



```
balanceEqs = ((Total[x_#&/@EdgeList[g, -#]] - Total[x_#&/@EdgeList[g, #-]]) == MapIndexed[#1/.x -> x,
```

```
balanceEqs//forma
```

$$x_{2,7} + x_{6,7} - x_{7,2} - x_{7,6} == 0$$

$$x_{1,2} - x_{2,1} - x_{2,3} - x_{2,7} + x_{3,2} + x_{7,2} == 0$$

$$x_{4,6} + x_{5,6} - x_{6,4} - x_{6,5} - x_{6,7} + x_{7,6} == x_6$$

$$-x_{1,2} - x_{1,3} - x_{1,4} + x_{2,1} + x_{3,1} + x_{4,1} == 0$$

$$x_{1,3} + x_{2,3} - x_{3,1} - x_{3,2} - x_{3,4} + x_{4,3} == x_3$$

$$x_{1,4} + x_{3,4} - x_{4,1} - x_{4,3} - x_{4,5} - x_{4,6} + x_{5,4} + x_{6,4} == 0$$

$$x_{4,5} - x_{5,4} - x_{5,6} + x_{6,5} == 0$$

```
M = {7};
```

```
Print["M = ", M];
```

```
M = {7}
```

```
(*incL = DeleteCases[DeleteDuplicates[Cases[IncidenceList[g, #], i.j_ -> {i, j}]]//Flatten], v._;v == #]&/@M*
```

```
incL = (IncidenceList[g, #]&/@M)//Flatten
```

$\{72, 27, 76, 67\}$

(*Do [If [MemberQ [M, j[[1]]], b[[j[[2]]]] += f_j, b[[j[[1]]]] -= f_j], {j, incL}]*)

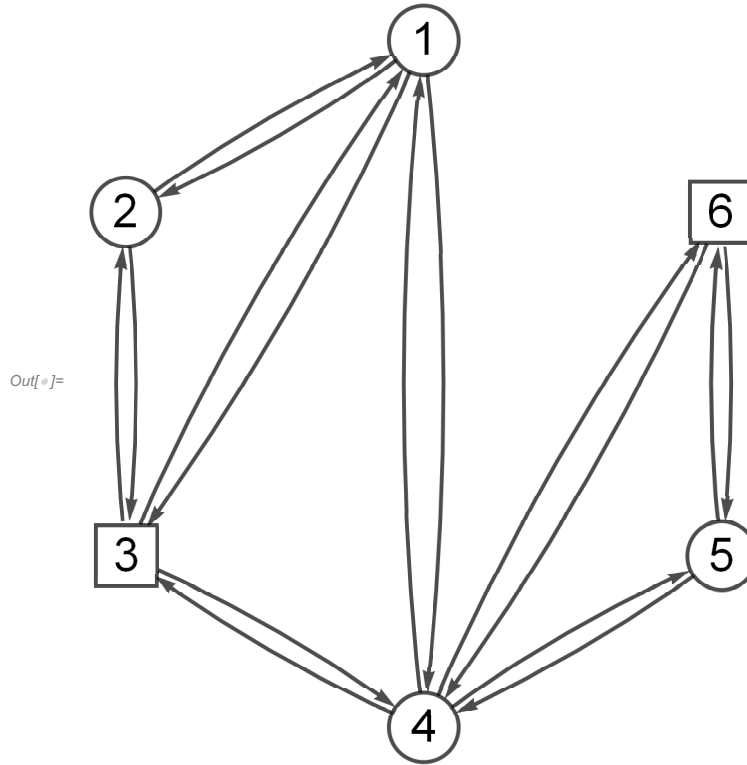
$\bar{b} = \text{Fold} \left[\text{If} \left[\text{MemberQ} \left[M, \#2_{[[1]]} \right], \text{ReplacePart} \left[\#, \#2_{[[2]]} \rightarrow \#_{[[\#2_{[[2]]}]]} + f_{\#2} \right], \text{ReplacePart} \left[\#, \#2_{[[1]]} \rightarrow \#_{[[\#2_{[[1]]}]]} - f_{\#2} \right] \right], \bar{b}, \{j, \text{incL}\} \right]$

$\bar{b} = \text{Delete} [\bar{b}, \#] \&@@M;$

$\overline{\text{ng}} = \text{VertexDelete}[g, M];$

$\text{GraphPlot}[\overline{\text{ng}}, \text{EdgeStyle} \rightarrow \text{Directive}[\text{Black}, \text{Thick}], \text{VertexStyle} \rightarrow \text{Directive}[\text{EdgeForm}[\text{Thick}], \text{White}], \text{Multiedge} \rightarrow \text{True}]$

\bar{b}



$\{0, -f_{27} + f_{72}, x, 0, 0, x - f_{67} + f_{76}\}$

$\text{CC}[g, M] := (\text{DeleteDuplicates}[\text{Cases}[\text{IncidenceList}[g, \#], i \rightarrow j; j == \#]] \&@@M) // \text{Flatten}$

$\text{ii}_L^+[g] := \text{Cases}[\text{IncidenceList}[g, i], u \rightarrow v; u == i \rightarrow v]$

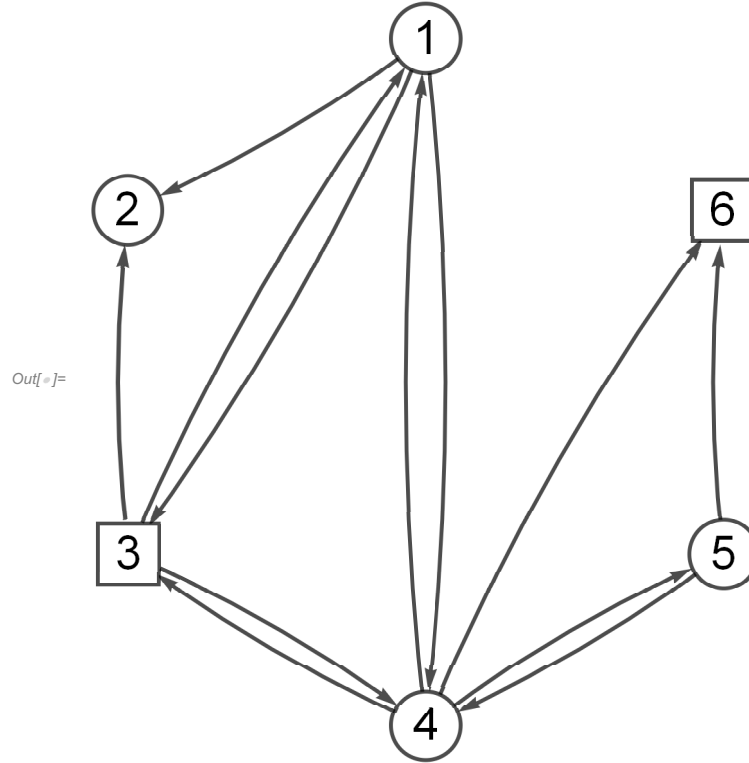
$$M^+ = \text{CC}[g, M]$$

$$\{27, 67\}$$

$$\overline{\mathbf{b1}} = \text{Fold} \left[\text{Module} \left[\left\{ \mathbf{bb} = \#1, i = \#2_{[[1]]}, k = \#2_{[[2]]} \right\}, \left(\text{ReplacePart} \left[\mathbf{bb}, \left(\left(\left\{ \# \rightarrow \mathbf{bb}_{[[\#]]} + \frac{p_{i\#}}{p_{ik}} f_{ik}, i \rightarrow \mathbf{bb}_{[[\#]]} \right\} \right) \right] \right) \right] \right]$$

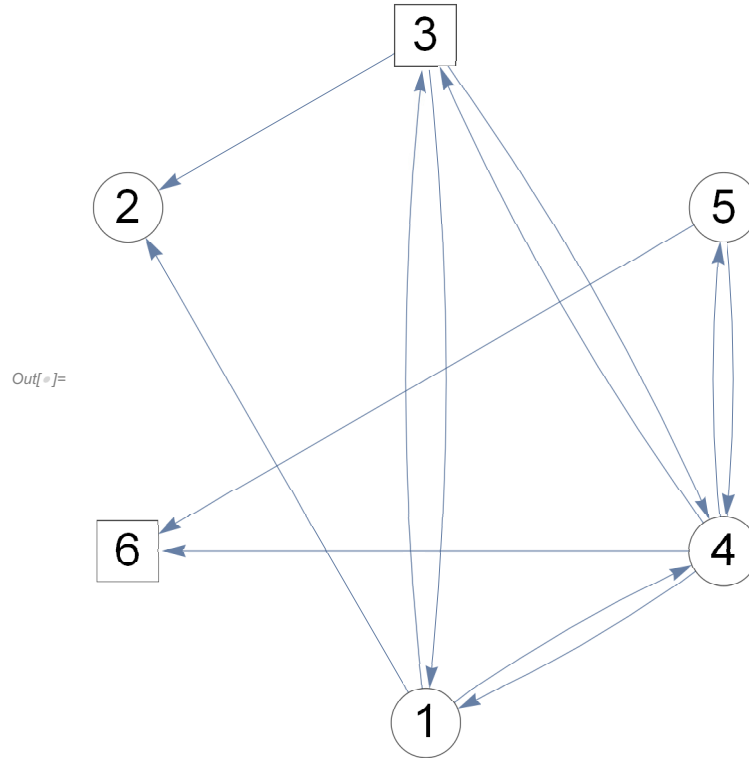
$$\left\{ \frac{f_{27p_{21}}}{p_{27}}, -f_{27} + f_{72} - \frac{f_{27p_{21}}}{p_{27}}, x + \frac{f_{27p_{23}}}{p_{27}}, \frac{f_{67p_{64}}}{p_{67}}, \frac{f_{67p_{65}}}{p_{67}}, x - f_{67} + f_{76} - \frac{f_{67p_{65}}}{p_{67}} \right\}$$

$$\text{GraphPlot} \left[\text{Fold} \left[\text{HighlightGraph}[\#1, \mathbf{u.v.}/; \mathbf{u} == \#2, \text{GraphHighlightStyle} \rightarrow \text{"White"}] \&, \overline{\mathbf{ng}}, \#_{[[1]]} \& / @ M^+ \right], \text{VertexStyle} \rightarrow \text{Directive}[\text{EdgeForm}[\text{Thick}], \text{White}], \text{MultiedgeStyle} \rightarrow .05]$$



$$\overline{\mathbf{g1}} = \text{Fold} \left[\text{EdgeDelete}[\#1, \mathbf{u.v.}/; \mathbf{u} == \#2] \&, \overline{\mathbf{ng}}, \#_{[[1]]} \& / @ M^+ \right];$$

$$\text{GraphPlot} \left[\overline{\mathbf{g1}}, \text{MultiedgeStyle} \rightarrow .05 \right]$$



$\Pi_{\text{rem}} = \text{VertexList}[\overline{g1}] \sim \text{Complement} \sim (M^+[[\text{All}, 1]])$

$\{1, 3, 4, 5\}$

$\lambda = \text{SparseArray}[\text{Replace}[(\text{EdgeList}[\overline{g1}] /. \# \& / @ \text{Flatten}[\text{Module}[\{i = \#, jf, \text{Icur}\}, (\text{Icur} = \text{ii}_i^+[\overline{g1}];$

$jf = \text{First}[\text{Icur}];$

$\left(\left\{(ijf) \rightarrow 1, (i\#) \rightarrow -\frac{p_{i\#}}{p_{ijf}}\right\} \& / @ \text{Icur}[[2;;]]\right) \& / @ \Pi_{\text{rem}}, 1]] , -- \rightarrow 0, 2]]$

$\text{SparseArray}[\square]$

$\text{Grid}[\lambda]$

1	0	$-\frac{p_{13}}{p_{12}}$	0	0	0	0	0	0	0	0	0
1	0	0	0	$-\frac{p_{14}}{p_{12}}$	0	0	0	0	0	0	0
0	1	0	$-\frac{p_{31}}{p_{32}}$	0	0	0	0	0	0	0	0
0	1	0	0	0	0	$-\frac{p_{34}}{p_{32}}$	0	0	0	0	0
0	0	0	0	0	1	0	$-\frac{p_{43}}{p_{41}}$	0	0	0	0
0	0	0	0	0	1	0	0	$-\frac{p_{45}}{p_{41}}$	0	0	0
0	0	0	0	0	1	0	0	0	0	0	$-\frac{p_{46}}{p_{41}}$
0	0	0	0	0	0	0	0	0	1	$-\frac{p_{56}}{p_{54}}$	0

$g = \overline{g1};$

$b = \overline{b1};$

$\Pi^* = \text{Cases}[\text{MapIndexed}[\{\#1, \#2\} \& b], \{\text{el}, i\} /; \text{MemberQ}[\text{el}, x] : \rightarrow i] // \text{Flatten}$

$\{3, 6\}$

$\text{buildt} = \text{Timing}[\{t, g\} = \text{buildTree}[g, \Pi^*];] [[1]]$

$\text{TableForm}[t[[1;;4]], \text{TableHeadings} \rightarrow \{\{\text{"pred"}, \text{"dir"}, \text{"depth"}, d\}, t // \text{pred} // \text{Length} // \text{Range}\}]$

0.015625

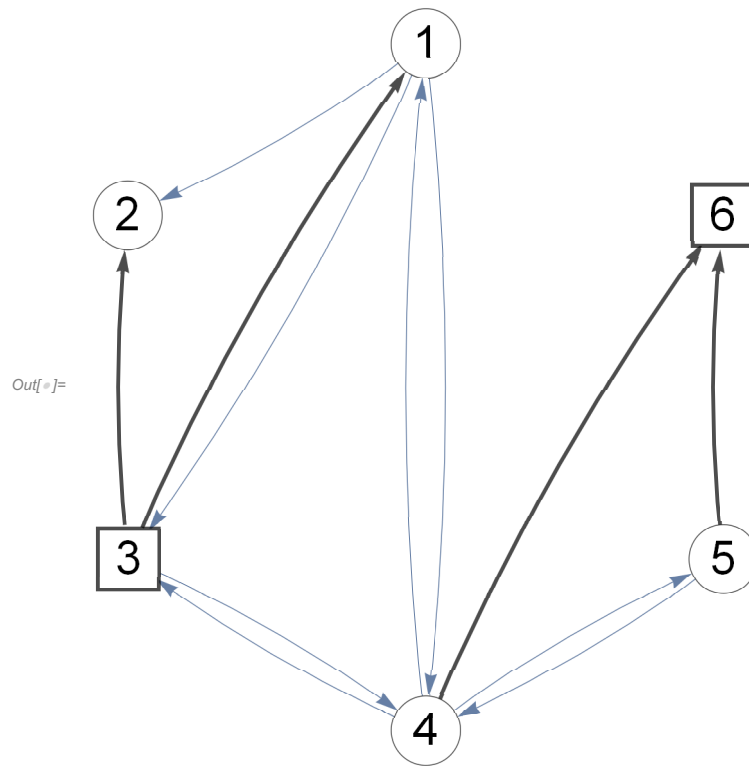
	1	2	3	4	5	6	7
pred	3	3	7	6	6	7	0
dir	1	1	1	-1	-1	1	0
depth	2	2	1	2	2	1	0
d	2	7	1	5	3	4	6

$\text{GraphPlot}[\text{HighlightGraph}[\text{Fold}[\text{HighlightGraph}[\#1, \text{Style}[\text{u}_v /; u == \#2, \text{White}]] \& \overline{\text{ng}}, \#_{[[1]]} \& @M^+],$

$\{\text{Style}[\text{u}_v /; \text{VertexQ}[g, u] \& \& \text{pred}[t][[u]] == 7, \text{EdgeForm}[\text{Thick}],$

$\text{Style}[\text{u}_v /; (\text{pred}[t][[u]] == v \& \& \text{dir}[t][[u]] == -1) \|(\text{pred}[t][[v]] == u \& \& \text{dir}[t][[v]] == 1), \text{Directive}[\text{Black}, \text{Thick}]$

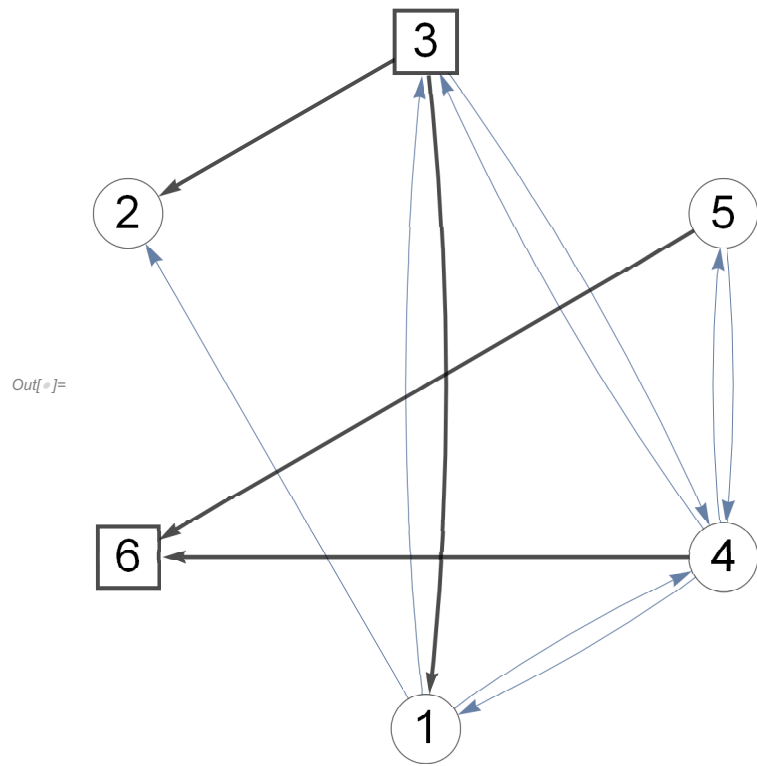
$\text{MultiedgeStyle} \rightarrow .05]$



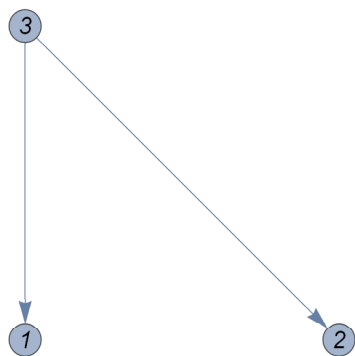
```

GraphPlot[
HighlightGraph [g1, {Style[u_;/VertexQ[g,u]&&pred[t][[u]] == 7, EdgeForm[Thick]},
Style[u_v_/(pred[t][[u]] == v&&dir[t][[u]] == -1)||(pred[t][[v]] == u&&dir[t][[v]] == 1), Directive[Black, Thick]
MultiedgeStyle -> .05]

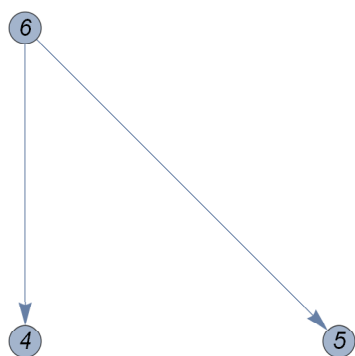
```



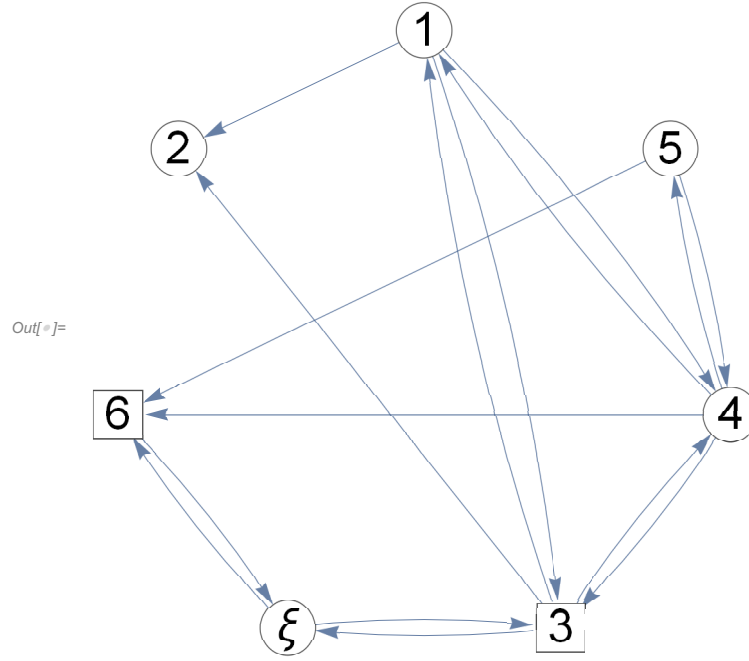
VertexDelete[t[[7]], 7](* *)



*Out[*n*]=*



GraphPlot[*g*, MultiedgeStyle → .05]



AppendTo[b, -Total[b]];

b = Simplify[b/.x → 0]

$$\left\{ \frac{f_{27}p_{21}}{p_{27}}, f_{72} + f_{27} \left(-1 - \frac{p_{21}}{p_{27}} \right), \frac{f_{27}p_{23}}{p_{27}}, \frac{f_{67}p_{64}}{p_{67}}, \frac{f_{67}p_{65}}{p_{67}}, f_{76} + f_{67} \left(-1 - \frac{p_{65}}{p_{67}} \right), -f_{72} - f_{76} + f_{27} \left(1 - \frac{p_{23}}{p_{27}} \right) + f_{67} \left(1 - \frac{p_{64}}{p_{67}} \right) \right\}$$

balanceEqs = ((Total[x_#&/@EdgeList[g, -#]] - Total[x_#&/@EdgeList[g, #-]]) /. 7 → ξ) == b[[#]]&/@VertexList[g];

balanceEqs//forma

$$\begin{aligned} x_{1,2} + x_{3,2} &== f_{7,2} + f_{2,7} \left(-1 - \frac{p_{2,1}}{p_{2,7}} \right) \\ x_{4,6} + x_{5,6} - x_{6,\xi} + x_{\xi,6} &== f_{7,6} + f_{6,7} \left(-1 - \frac{p_{6,5}}{p_{6,7}} \right) \\ -x_{1,2} - x_{1,3} - x_{1,4} + x_{3,1} + x_{4,1} &== \frac{f_{2,7}p_{2,1}}{p_{2,7}} \\ x_{1,3} - x_{3,1} - x_{3,2} - x_{3,4} - x_{3,\xi} + x_{4,3} + x_{\xi,3} &== \frac{f_{2,7}p_{2,3}}{p_{2,7}} \\ x_{1,4} + x_{3,4} - x_{4,1} - x_{4,3} - x_{4,5} - x_{4,6} + x_{5,4} &== \frac{f_{6,7}p_{6,4}}{p_{6,7}} \\ x_{4,5} - x_{5,4} - x_{5,6} &== \frac{f_{6,7}p_{6,5}}{p_{6,7}} \\ x_{3,\xi} + x_{6,\xi} - x_{\xi,3} - x_{\xi,6} &== -f_{7,2} - f_{7,6} + f_{2,7} \left(1 - \frac{p_{2,3}}{p_{2,7}} \right) + f_{6,7} \left(1 - \frac{p_{6,4}}{p_{6,7}} \right) \end{aligned}$$

ps = partSolve[g, -b, t, x̃];

ps//forma

7

$$\tilde{x}_{1,2} \rightarrow 0$$

$$\tilde{x}_{1,3} \rightarrow 0$$

$$\tilde{x}_{1,4} \rightarrow 0$$

$$\tilde{x}_{3,1} \rightarrow \frac{f_{2,7} p_{2,1}}{p_{2,7}}$$

$$\tilde{x}_{3,2} \rightarrow f_{7,2} + f_{2,7} \left(-1 - \frac{p_{2,1}}{p_{2,7}} \right)$$

$$\tilde{x}_{3,4} \rightarrow 0$$

$$\tilde{x}_{3,7} \rightarrow 0$$

$$\tilde{x}_{4,1} \rightarrow 0$$

$$\tilde{x}_{4,3} \rightarrow 0$$

$$\tilde{x}_{4,5} \rightarrow 0$$

$$\tilde{x}_{4,6} \rightarrow -\frac{f_{6,7} p_{6,4}}{p_{6,7}}$$

$$\tilde{x}_{5,4} \rightarrow 0$$

$$\tilde{x}_{5,6} \rightarrow -\frac{f_{6,7} p_{6,5}}{p_{6,7}}$$

$$\tilde{x}_{6,7} \rightarrow 0$$

$$\tilde{x}_{7,3} \rightarrow f_{7,2} + f_{2,7} \left(-1 - \frac{p_{2,1}}{p_{2,7}} \right) + \frac{f_{2,7} p_{2,1}}{p_{2,7}} + \frac{f_{2,7} p_{2,3}}{p_{2,7}}$$

$$\tilde{x}_{7,6} \rightarrow f_{7,6} + f_{6,7} \left(-1 - \frac{p_{6,5}}{p_{6,7}} \right) + \frac{f_{6,7} p_{6,4}}{p_{6,7}} + \frac{f_{6,7} p_{6,5}}{p_{6,7}}$$

Simplify [(balanceEqs/. {x → \tilde{x} , ξ → 7}) /.ps]

{True, True, True, True, True, True, True}

matrt = Timing[δ Matr = $\delta 1[g, t]$];

roott = VertexCount[g];

$$\text{TableForm} \left[\delta\text{Matr}, \text{TableHeadings} \rightarrow \left\{ \text{uNb}[g, t], \delta \left\{ \begin{array}{ll} \#_{[[2]]} & \#_{[[1]]} == \text{roott} \\ \#_{[[1]]} & \#_{[[2]]} == \text{roott} \\ \# & \text{True} \end{array} \right. \&/\text{@EdgeList}[g] \right\} \right] // \text{forma}$$

	$\delta_{1,2}$	$\delta_{3,2}$	$\delta_{1,3}$	$\delta_{3,1}$	$\delta_{1,4}$	$\delta_{4,1}$	$\delta_{3,4}$	$\delta_{4,3}$	$\delta_{4,5}$	$\delta_{5,4}$	$\delta_{5,6}$	$\delta_{4,6}$	δ_3	δ_6	δ_3	δ_6
12	1	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	1	1	0	0	0	0	0	0	1	1	-1	0	0
41	0	0	0	-1	0	1	0	0	0	0	0	-1	-1	1	0	0
34	0	0	0	0	0	0	1	0	0	0	0	1	1	-1	0	0
43	0	0	0	0	0	0	0	1	0	0	0	-1	-1	1	0	0
45	0	0	0	0	0	0	0	0	1	0	1	-1	0	0	0	0
54	0	0	0	0	0	0	0	0	0	1	-1	1	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
67	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1

$\lambda = \text{SparseArray}[\lambda, \{\text{Length}[\lambda], \text{Length}[\lambda[[1]]] + 4\}];$

$(*\lambda = \lambda[[;; - 2]]*)$

$\text{dopEq} = \# == 0 \&/\text{@Flatten} [\lambda. \{x_{\#} \&/\text{@EdgeList}[g]\}^T];$

$\text{dopEq} // \text{forma}$

$$\begin{aligned}
x_{1,2} - \frac{p_{1,3}x_{1,3}}{p_{1,2}} &== 0 \\
x_{1,2} - \frac{p_{1,4}x_{1,4}}{p_{1,2}} &== 0 \\
-\frac{p_{3,1}x_{3,1}}{p_{3,2}} + x_{3,2} &== 0 \\
x_{3,2} - \frac{p_{3,4}x_{3,4}}{p_{3,2}} &== 0 \\
x_{4,1} - \frac{p_{4,3}x_{4,3}}{p_{4,1}} &== 0 \\
x_{4,1} - \frac{p_{4,5}x_{4,5}}{p_{4,1}} &== 0 \\
x_{4,1} - \frac{p_{4,6}x_{4,6}}{p_{4,1}} &== 0 \\
x_{5,4} - \frac{p_{5,6}x_{5,6}}{p_{5,4}} &== 0
\end{aligned}$$

$$\Lambda = \lambda.(\delta\text{Matr})^\text{T};$$

“cicle det’s:”

$\Lambda//\text{forma}$

cicle det’s:

$$\begin{array}{cccccccccc}
1 & -\frac{p_{13}}{p_{12}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & -\frac{p_{14}}{p_{12}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 - \frac{p_{31}}{p_{32}} & -\frac{p_{31}}{p_{32}} & -\frac{p_{31}}{p_{32}} & \frac{p_{31}}{p_{32}} & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & -\frac{p_{34}}{p_{32}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -\frac{p_{43}}{p_{41}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -\frac{p_{45}}{p_{41}} & 0 & 0 & 0 \\
0 & 0 & -\frac{p_{46}}{p_{41}} & 1 + \frac{p_{46}}{p_{41}} & -\frac{p_{46}}{p_{41}} & \frac{p_{46}}{p_{41}} & \frac{p_{46}}{p_{41}} & -\frac{p_{46}}{p_{41}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{p_{56}}{p_{54}} & 1 + \frac{p_{56}}{p_{54}} & 0 & 0
\end{array}$$

" U_c ="

$U_c = \text{Range}[8]$

" U_{nc} ="

$U_{nc} = \{9, 10\}$

$U_c =$

$\{1, 2, 3, 4, 5, 6, 7, 8\}$

$$U_{\text{nc}} =$$

$$\{9, 10\}$$

$$\Lambda_{\text{c}} = \Lambda \left[[\text{All}, U_{\text{c}}] \right];$$

$$\Lambda_{\text{nc}} = \Lambda \left[[\text{All}, U_{\text{nc}}] \right];$$

$$\text{"}\Lambda_{\text{c}}\text{"}$$

$$\Lambda_{\text{c}} // \text{MatrixForm}$$

$$\Lambda_c =$$

$$\left(\begin{array}{cccccccccc} 1 & -\frac{p_{13}}{p_{12}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -\frac{p_{14}}{p_{12}} & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 - \frac{p_{31}}{p_{32}} & -\frac{p_{31}}{p_{32}} & -\frac{p_{31}}{p_{32}} & \frac{p_{31}}{p_{32}} & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & -\frac{p_{34}}{p_{32}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -\frac{p_{43}}{p_{41}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -\frac{p_{45}}{p_{41}} & 0 & 0 \\ 0 & 0 & -\frac{p_{46}}{p_{41}} & 1 + \frac{p_{46}}{p_{41}} & -\frac{p_{46}}{p_{41}} & \frac{p_{46}}{p_{41}} & \frac{p_{46}}{p_{41}} & -\frac{p_{46}}{p_{41}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\frac{p_{56}}{p_{54}} & 1 + \frac{p_{56}}{p_{54}} & 0 \end{array} \right)$$

$$\text{"}\det(\Lambda_c)\text{"}$$

$$\text{Simplify}[\det = \text{Det}[\Lambda_c]] // \text{forma}$$

$$\det(\Lambda_c) =$$

$$-\frac{1}{p_{1,2}^2 p_{3,2}^2 p_{4,1}^3 p_{5,4}} (p_{1,2} p_{3,1} p_{3,4} (p_{1,3} p_{4,1} (p_{4,5} p_{4,6} (p_{5,4} + p_{5,6}) + p_{4,3} (p_{4,6} p_{5,4} + p_{4,5} (p_{5,4} + p_{5,6}))) + p_{1,4} (p_{4,3} p_{4,5} p_{4,6} + p_{4,5} p_{4,6} p_{5,4} + p_{4,3} p_{4,6} p_{5,6})))$$

$$\text{"}U_T\text{"}$$

$$\text{utind} = \text{Cases}[t[[6]], \xi _ /; \xi \neq 0];$$

$$U_T = \text{EdgeList}[g][[\text{utind}]]$$

$$U_T =$$

$$\{31, 32, 73, 46, 56, 76\}$$

" U_{Nb} ="

$U_{\text{Nb}} = \text{uNb}[g, t]$

$U_{\text{Nb}} =$

$\{12, 13, 14, 41, 34, 43, 45, 54, 37, 67\}$

$A = -\lambda. \{ \tilde{x}_{\#} \&/@ \text{EdgeList}[g] \}^{\text{T}}/.ps;$

" $A=$ "

$A//\text{MatrixForm}$

$A=$

$$\left(\begin{array}{c} 0 \\ 0 \\ -f_{72} - f_{27} \left(-1 - \frac{p_{21}}{p_{27}} \right) + \frac{f_{27} p_{21} p_{31}}{p_{27} p_{32}} \\ -f_{72} - f_{27} \left(-1 - \frac{p_{21}}{p_{27}} \right) \\ 0 \\ 0 \\ -\frac{f_{67} p_{46} p_{64}}{p_{41} p_{67}} \\ -\frac{f_{67} p_{56} p_{65}}{p_{54} p_{67}} \end{array} \right)$$

$\beta = A - \Lambda_{\text{nc.}} \{ x_{\#} \&/@ U_{\text{Nb}} [[U_{\text{nc}}]] \}^{\text{T}};$

" $\beta=$ "

$\beta//\text{forma}$

$\beta=$

0

$$-f_{7,2} - f_{2,7} \left(-1 - \frac{p_{2,1}}{p_{2,7}} \right)$$

0

$$\frac{f_{6,7}p_{5,6}p_{6,5}}{p_{5,4}p_{6,7}}$$

$$\mathbf{x}c = \text{LinearSolve}[\Lambda c, \beta]$$

xcp//TableForm

s//TableForm

$$x_{76} \rightarrow f_{76} + f_{67} \left(-1 - \frac{p_{65}}{p_{67}} \right) + \frac{f_{67} p_{64}}{p_{67}} + \frac{f_{67} p_{65}}{p_{67}} - x_{14} - x_{34} + x_{41} + x_{43} + x_{67}$$

