Листинг 2

In[5]:=
```
ClearAll["Global`*"]
SetDirectory[NotebookDirectory[]];
Needs["FlowSolver`"]
```

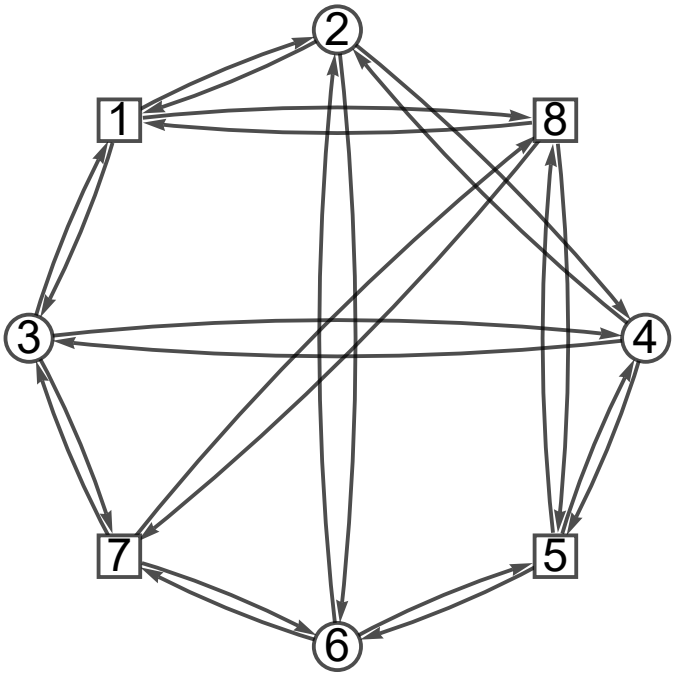
In[8]:=
```
readGraph2[file_, dir_] := Module[{
    fn = FileNameJoin[{dir, file}],
    stream, imod, umod, u, b
    },
    stream = OpenRead[fn];
    imod = Read[stream, {Word, Number}][[2]];
    umod = Read[stream, {Word, Number}][[2]];

  u = ({#[[1]] ↔ #[[2]], #[[2]] ↔ #[[1]]} & /@ ReadList[stream, Expression, umod]) // Flatten;
  b = ConstantArray[0, imod];
    (b[[Read[StringToStream[StringTake[#1, {5, -3}]], Number]]] = #2) &@@@
   ReadList[stream, {Word, Expression}, imod];
  {Graph[u, VertexSize -> Medium, VertexLabels → Placed["Name", Center],
    VertexStyle → Directive[White],
    VertexShapeFunction → {xx_ :> If[SameQ[b[[xx]], x], "Square", "Circle"]},
    VertexLabelStyle -> Directive[Black, 24], GraphLayout -> "CircularEmbedding"], b}]
```

In[9]:=
```
forma[ff_] := ((ff /. {ξ_u_↔v_ → ξu,v}) // TableForm)
```

In[92]:=
```
{g, b} = readGraph2["gr.txt", NotebookDirectory[]];
GraphPlot[g, EdgeStyle → Directive[Black, Thick],
 VertexStyle → Directive[EdgeForm[Thick], White], MultiedgeStyle → .05]
```

Out[93]=



In[94]:=
```
balanceEqs = ((Total[x_# & /@ EdgeList[g, _ ↔ #]] - Total[x_# & /@ EdgeList[g, # ↔ _]])) ==
      MapIndexed[#1 /. x → x_#2[[1]] &, b][[#]] & /@ VertexList[g];
balanceEqs //
 forma
```

Out[95]//TableForm=

$$-x_{1,2}-x_{1,3}-x_{1,8}+x_{2,1}+x_{3,1}+x_{8,1}==x_1$$
$$x_{1,2}-x_{2,1}-x_{2,4}-x_{2,6}+x_{4,2}+x_{6,2}==0$$
$$x_{1,8}+x_{5,8}+x_{7,8}-x_{8,1}-x_{8,5}-x_{8,7}==x_8$$
$$x_{1,3}-x_{3,1}-x_{3,4}-x_{3,7}+x_{4,3}+x_{7,3}==0$$
$$x_{2,4}+x_{3,4}-x_{4,2}-x_{4,3}-x_{4,5}+x_{5,4}==0$$
$$x_{2,6}+x_{5,6}-x_{6,2}-x_{6,5}-x_{6,7}+x_{7,6}==0$$
$$x_{4,5}-x_{5,4}-x_{5,6}-x_{5,8}+x_{6,5}+x_{8,5}==x_5$$
$$x_{3,7}+x_{6,7}-x_{7,3}-x_{7,6}-x_{7,8}+x_{8,7}==x_7$$

In[14]:=
```
M = {8};
Print["M = ", M];
```

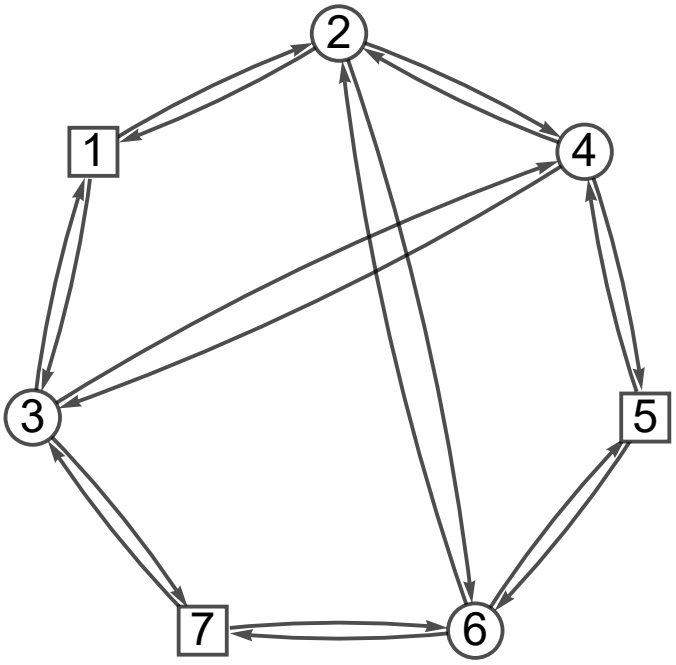
```
M = {8}
```

In[96]:=
```
(*incL=
 DeleteCases[DeleteDuplicates[Cases[IncidenceList[g,#],i_↔j_:>{i,j}]//Flatten],
    v_/;v==#]&/@M*)
incL = (IncidenceList[g, #] & /@ M) // Flatten
```

Out[96]= $\{1 \leftrightarrow 8,\ 8 \leftrightarrow 1,\ 8 \leftrightarrow 5,\ 5 \leftrightarrow 8,\ 8 \leftrightarrow 7,\ 7 \leftrightarrow 8\}$

In[97]:=
```
(*Do[If[MemberQ[M,j[[1]]],b[[j[[2]]]]+=f_j,b[[j[[1]]]]-=f_j],{j,incL}]*)
b̄ = Fold[If[MemberQ[M, #2[[1]]], ReplacePart[#, #2[[2]] → #[[#2[[2]]]] + f_#2],
      ReplacePart[#, #2[[1]] → #[[#2[[1]]]] - f_#2]] &, b, incL];
b̄ = Delete[b̄, #] & @@ M;
n̄ḡ = VertexDelete[g, M];
GraphPlot[n̄ḡ, EdgeStyle → Directive[Black, Thick],
 VertexStyle → Directive[EdgeForm[Thick], White], MultiedgeStyle → .05]
b̄
```

Out[100]=



Out[101]= $\{x - f_{1\leftrightarrow 8} + f_{8\leftrightarrow 1},\ 0,\ 0,\ 0,\ x - f_{5\leftrightarrow 8} + f_{8\leftrightarrow 5},\ 0,\ x - f_{7\leftrightarrow 8} + f_{8\leftrightarrow 7}\}$

In[102]:=
```
CC[g_, M_] :=
 (DeleteDuplicates[Cases[IncidenceList[g, #], i_ ↔ j_ /; j == #]] & /@ M) // Flatten

ii_i^† [g_] := Cases[IncidenceList[g, i], u_ ↔ v_ /; u == i :> v]
```

In[104]:=
```
M⁺ = CC[g, M]
```

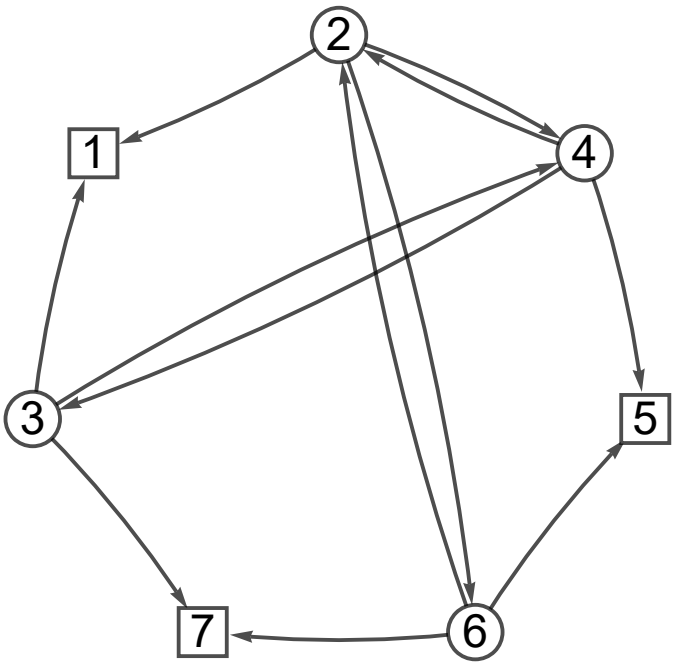
Out[104]=
$$\{1 \leftrightarrow 8,\ 5 \leftrightarrow 8,\ 7 \leftrightarrow 8\}$$

In[105]:=
```
b1 = Fold[Module[{bb = #1, i = #2[[1]], k = #2[[2]]},
    (ReplacePart[bb, ((({# → bb[[#]] + p_{i↔#}/p_{i↔k} f_{i↔k}, i → bb[[i]] - p_{i↔#}/p_{i↔k} f_{i↔k}}) &) /@ ii_i^†[ng]) //
      Flatten]) ] &, b, M⁺]
```

Out[105]=
$$\left\{x - f_{1\leftrightarrow 8} + f_{8\leftrightarrow 1} - \frac{f_{1\leftrightarrow 8}\, p_{1\leftrightarrow 2}}{p_{1\leftrightarrow 8}},\ \frac{f_{1\leftrightarrow 8}\, p_{1\leftrightarrow 2}}{p_{1\leftrightarrow 8}},\ \frac{f_{1\leftrightarrow 8}\, p_{1\leftrightarrow 3}}{p_{1\leftrightarrow 8}} + \frac{f_{7\leftrightarrow 8}\, p_{7\leftrightarrow 3}}{p_{7\leftrightarrow 8}},\ \frac{f_{5\leftrightarrow 8}\, p_{5\leftrightarrow 4}}{p_{5\leftrightarrow 8}},\right.$$
$$\left. x - f_{5\leftrightarrow 8} + f_{8\leftrightarrow 5} - \frac{f_{5\leftrightarrow 8}\, p_{5\leftrightarrow 4}}{p_{5\leftrightarrow 8}},\ \frac{f_{5\leftrightarrow 8}\, p_{5\leftrightarrow 6}}{p_{5\leftrightarrow 8}} + \frac{f_{7\leftrightarrow 8}\, p_{7\leftrightarrow 6}}{p_{7\leftrightarrow 8}},\ x - f_{7\leftrightarrow 8} + f_{8\leftrightarrow 7} - \frac{f_{7\leftrightarrow 8}\, p_{7\leftrightarrow 6}}{p_{7\leftrightarrow 8}}\right\}$$

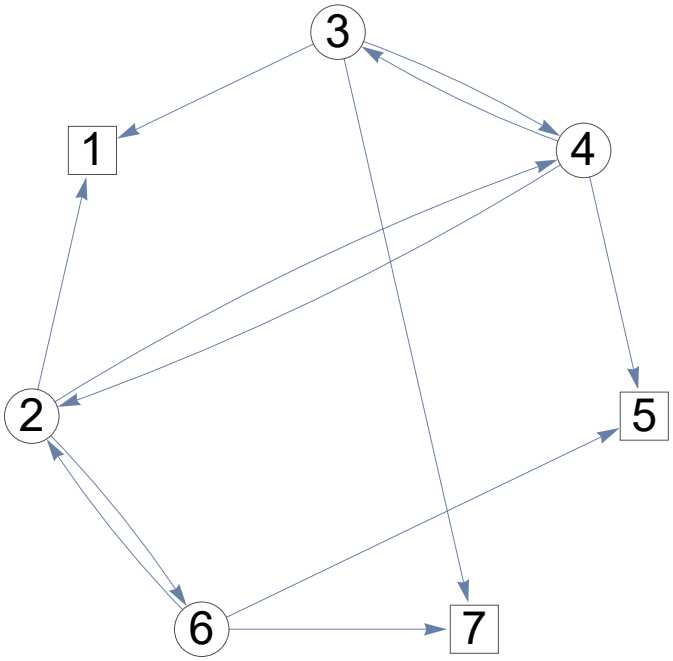
In[106]:=
```
GraphPlot[Fold[HighlightGraph[#1, u_ ↔ v_ /; u == #2, GraphHighlightStyle → "White"] &,
   ng, #[[1]] & /@ M⁺], EdgeStyle → Directive[Black, Thick],
  VertexStyle → Directive[EdgeForm[Thick], White], MultiedgeStyle → .05]
```

Out[106]=



In[107]:=
```
g1 = Fold[EdgeDelete[#1, u_ ↔ v_ /; u == #2] &, ng, #[[1]] & /@ M+];
GraphPlot[g1, MultiedgeStyle → .05]
```

Out[108]=



In[109]:=
```
IIrem = VertexList[g1] ~Complement~ (M+[[All, 1]])
```

Out[109]=
```
{2, 3, 4, 6}
```

In[132]:=
```
λ = SparseArray[
  Replace[(EdgeList[g1] /. # & /@ Flatten[Module[{i = #, jf, Icur},
      (Icur = ii[g1];
       jf = First[Icur];
       ({(i ↔ jf) → 1, (i ↔ #) → -pi↔#/pi↔jf}) & /@ Icur[[2 ;;]])] & /@ IIrem, 1]),
   _ ↔ _ → 0, 2]]
```

Out[132]=
```
SparseArray[ <image> ]
```
Specified elements: 16
Dimensions: {8, 12}

In[111]:=
```
Grid[λ]
```

Out[111]=

$$\begin{array}{cccccccccccc}
1 & 0 & -\frac{p_{2\leftrightarrow 4}}{p_{2\leftrightarrow 1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & -\frac{p_{2\leftrightarrow 6}}{p_{2\leftrightarrow 1}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{p_{3\leftrightarrow 4}}{p_{3\leftrightarrow 1}} & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{p_{3\leftrightarrow 7}}{p_{3\leftrightarrow 1}} \\
0 & 0 & 0 & 1 & 0 & 0 & -\frac{p_{4\leftrightarrow 5}}{p_{4\leftrightarrow 2}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & -\frac{p_{4\leftrightarrow 3}}{p_{4\leftrightarrow 2}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -\frac{p_{6\leftrightarrow 5}}{p_{6\leftrightarrow 2}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -\frac{p_{6\leftrightarrow 7}}{p_{6\leftrightarrow 2}} & 0
\end{array}$$

In[112]:=
```
g = g1;
b = b1;
```

In[114]:=
```
II* = Cases[MapIndexed[{#1, #2} &, b], {el_, i_} /; MemberQ[el, x] :> i] // Flatten
```

Out[114]=
```
{1, 5, 7}
```

In[115]:=
```
buildt = Timing[{t, g} = buildTree[g, II*];][[1]]
TableForm[t[[1 ;; 4]],
 TableHeadings → {{"pred", "dir", "depth", "d"}, t // pred // Length // Range}]
```

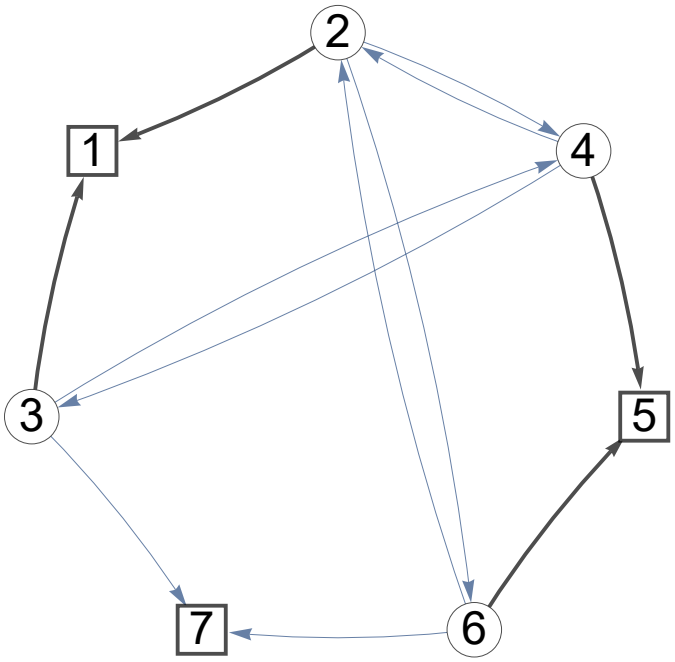
Out[115]=
```
0.
```

Out[116]//TableForm=

	1	2	3	4	5	6	7	8
pred	8	1	1	5	8	5	8	0
dir	1	−1	−1	−1	1	−1	1	0
depth	1	2	2	2	1	2	1	0
d	2	3	5	6	4	7	8	1

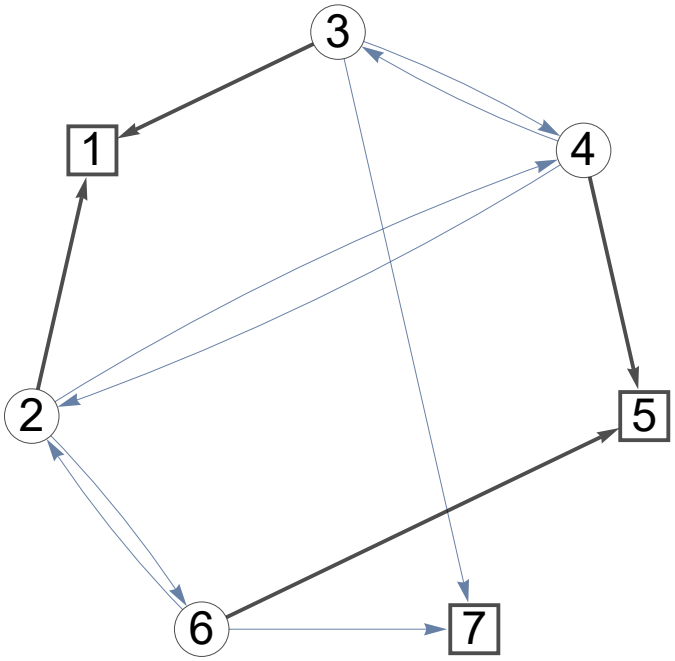
In[117]:=
```
GraphPlot[HighlightGraph[
  Fold[HighlightGraph[#1, Style[u_ ↔ v_ /; u == #2, White]] &, ng, #[[1]] & /@ M+],
  {Style[u_ /; VertexQ[g, u] && pred[t][[u]] == root[t], EdgeForm[Thick]],
   Style[u_ ↔ v_ /; (pred[t][[u]] == v && dir[t][[u]] == -1) ||
      (pred[t][[v]] == u && dir[t][[v]] == 1), Directive[Black, Thick]]},
  GraphHighlightStyle → None], MultiedgeStyle → .05]
```

Out[117]=



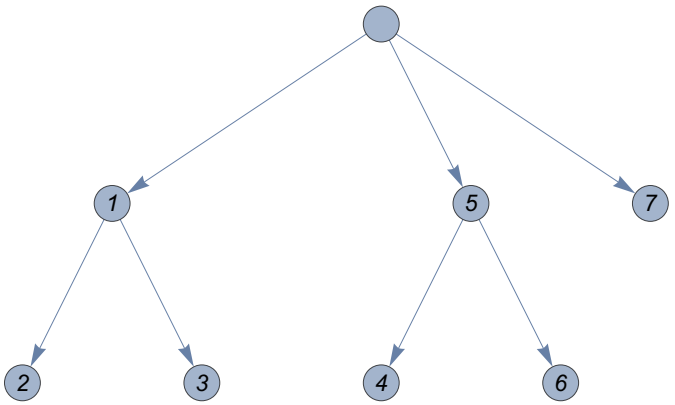
In[118]:=
```
GraphPlot[HighlightGraph[g1,
  {Style[u_ /; VertexQ[g, u] && pred[t][[u]] == root[t], EdgeForm[Thick]],
   Style[u_ ↔ v_ /; (pred[t][[u]] == v && dir[t][[u]] == -1) ||
       (pred[t][[v]] == u && dir[t][[v]] == 1), Directive[Black, Thick]]},
  GraphHighlightStyle → None], MultiedgeStyle → .05]
```

Out[118]=



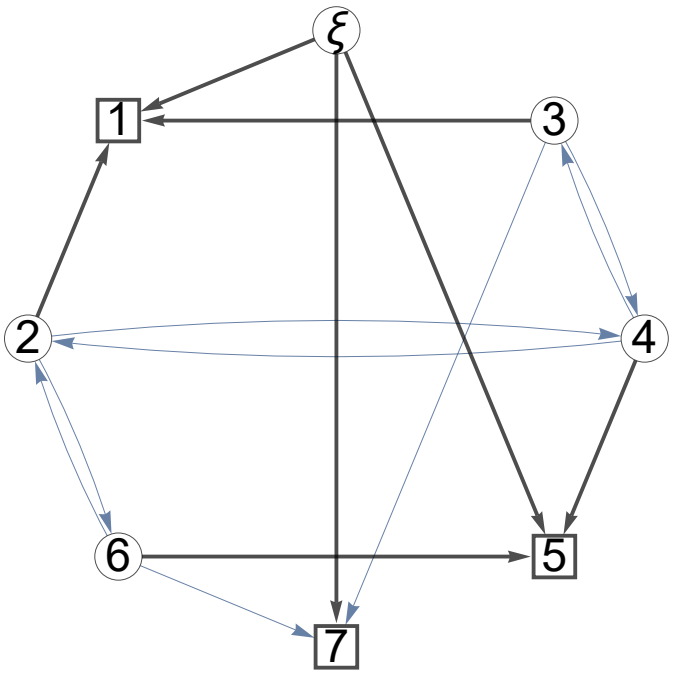
In[119]:=
```
t[[7]](*пометить на графе*)
```

Out[119]=



In[120]:=
```
(*GraphPlot[g,MultiedgeStyle→.05]*)
GraphPlot[HighlightGraph[g,
  {Style[u_ /; VertexQ[g, u] && pred[t][[u]] == root[t], EdgeForm[Thick]],
   Style[u_ ↔ v_ /; (pred[t][[u]] == v && dir[t][[u]] == -1) ||
       (pred[t][[v]] == u && dir[t][[v]] == 1), Directive[Black, Thick]]},
  GraphHighlightStyle → None], MultiedgeStyle → .05]
```

Out[120]=



In[121]:=
```
AppendTo[b, -Total[b]];
b = Simplify[b /. x → 0]
```

Out[122]=

$$\left\{f_{8\leftrightarrow 1}+f_{1\leftrightarrow 8}\left(-1-\frac{p_{1\leftrightarrow 2}}{p_{1\leftrightarrow 8}}\right),\ \frac{f_{1\leftrightarrow 8}\,p_{1\leftrightarrow 2}}{p_{1\leftrightarrow 8}},\ \frac{f_{1\leftrightarrow 8}\,p_{1\leftrightarrow 3}}{p_{1\leftrightarrow 8}}+\frac{f_{7\leftrightarrow 8}\,p_{7\leftrightarrow 3}}{p_{7\leftrightarrow 8}},\ \frac{f_{5\leftrightarrow 8}\,p_{5\leftrightarrow 4}}{p_{5\leftrightarrow 8}},\right.$$
$$f_{8\leftrightarrow 5}+f_{5\leftrightarrow 8}\left(-1-\frac{p_{5\leftrightarrow 4}}{p_{5\leftrightarrow 8}}\right),\ \frac{f_{5\leftrightarrow 8}\,p_{5\leftrightarrow 6}}{p_{5\leftrightarrow 8}}+\frac{f_{7\leftrightarrow 8}\,p_{7\leftrightarrow 6}}{p_{7\leftrightarrow 8}},\ f_{8\leftrightarrow 7}+f_{7\leftrightarrow 8}\left(-1-\frac{p_{7\leftrightarrow 6}}{p_{7\leftrightarrow 8}}\right),$$
$$\left.f_{7\leftrightarrow 8}-f_{8\leftrightarrow 1}-f_{8\leftrightarrow 5}-f_{8\leftrightarrow 7}+f_{1\leftrightarrow 8}\left(1-\frac{p_{1\leftrightarrow 3}}{p_{1\leftrightarrow 8}}\right)+f_{5\leftrightarrow 8}\left(1-\frac{p_{5\leftrightarrow 6}}{p_{5\leftrightarrow 8}}\right)-\frac{f_{7\leftrightarrow 8}\,p_{7\leftrightarrow 3}}{p_{7\leftrightarrow 8}}\right\}$$

In[123]:=
```
balanceEqs = ((Total[x_# & /@ EdgeList[g, _ ↔ #]] - Total[x_# & /@ EdgeList[g, # ↔ _]]) /.
      root[t] → ξ) == b[[#]] & /@ VertexList[g];
balanceEqs //
 forma
```

Out[124]//TableForm=

$$x_{2,1}+x_{3,1}+x_{\xi,1}==f_{8,1}+f_{1,8}\left(-1-\frac{p_{1,2}}{p_{1,8}}\right)$$
$$-x_{2,1}-x_{2,4}-x_{2,6}+x_{4,2}+x_{6,2}==\frac{f_{1,8}\,p_{1,2}}{p_{1,8}}$$
$$-x_{3,1}-x_{3,4}-x_{3,7}+x_{4,3}==\frac{f_{1,8}\,p_{1,3}}{p_{1,8}}+\frac{f_{7,8}\,p_{7,3}}{p_{7,8}}$$
$$x_{2,4}+x_{3,4}-x_{4,2}-x_{4,3}-x_{4,5}==\frac{f_{5,8}\,p_{5,4}}{p_{5,8}}$$
$$x_{2,6}-x_{6,2}-x_{6,5}-x_{6,7}==\frac{f_{5,8}\,p_{5,6}}{p_{5,8}}+\frac{f_{7,8}\,p_{7,6}}{p_{7,8}}$$
$$x_{4,5}+x_{6,5}+x_{\xi,5}==f_{8,5}+f_{5,8}\left(-1-\frac{p_{5,4}}{p_{5,8}}\right)$$
$$x_{3,7}+x_{6,7}+x_{\xi,7}==f_{8,7}+f_{7,8}\left(-1-\frac{p_{7,6}}{p_{7,8}}\right)$$
$$-x_{\xi,1}-x_{\xi,5}-x_{\xi,7}==f_{7,8}-f_{8,1}-f_{8,5}-f_{8,7}+f_{1,8}\left(1-\frac{p_{1,3}}{p_{1,8}}\right)+f_{5,8}\left(1-\frac{p_{5,6}}{p_{5,8}}\right)-\frac{f_{7,8}\,p_{7,3}}{p_{7,8}}$$

In[125]:=
```
ps = partSolve[g, -b, t, x̃];
ps // forma
```

8

Out[126]//TableForm=

$$\tilde{x}_{2,1} \to -\frac{f_{1,8}\, p_{1,2}}{p_{1,8}}$$
$$\tilde{x}_{2,4} \to 0$$
$$\tilde{x}_{2,6} \to 0$$
$$\tilde{x}_{3,1} \to -\frac{f_{1,8}\, p_{1,3}}{p_{1,8}} - \frac{f_{7,8}\, p_{7,3}}{p_{7,8}}$$
$$\tilde{x}_{3,4} \to 0$$
$$\tilde{x}_{3,7} \to 0$$
$$\tilde{x}_{4,2} \to 0$$
$$\tilde{x}_{4,3} \to 0$$
$$\tilde{x}_{4,5} \to -\frac{f_{5,8}\, p_{5,4}}{p_{5,8}}$$
$$\tilde{x}_{6,2} \to 0$$
$$\tilde{x}_{6,5} \to -\frac{f_{5,8}\, p_{5,6}}{p_{5,8}} - \frac{f_{7,8}\, p_{7,6}}{p_{7,8}}$$
$$\tilde{x}_{6,7} \to 0$$
$$\tilde{x}_{8,1} \to f_{8,1} + f_{1,8}\left(-1 - \frac{p_{1,2}}{p_{1,8}}\right) + \frac{f_{1,8}\, p_{1,2}}{p_{1,8}} + \frac{f_{1,8}\, p_{1,3}}{p_{1,8}} + \frac{f_{7,8}\, p_{7,3}}{p_{7,8}}$$
$$\tilde{x}_{8,5} \to f_{8,5} + f_{5,8}\left(-1 - \frac{p_{5,4}}{p_{5,8}}\right) + \frac{f_{5,8}\, p_{5,4}}{p_{5,8}} + \frac{f_{5,8}\, p_{5,6}}{p_{5,8}} + \frac{f_{7,8}\, p_{7,6}}{p_{7,8}}$$
$$\tilde{x}_{8,7} \to f_{8,7} + f_{7,8}\left(-1 - \frac{p_{7,6}}{p_{7,8}}\right)$$

In[127]:=
```
Simplify[(balanceEqs /. {x → x̃, ξ → root[t]}) /. ps]
```

Out[127]= {True, True, True, True, True, True, True, True}

In[128]:=
```
matrt = Timing[δMatr = δ1[g, t]];
rootT = VertexCount[g];
TableForm[δMatr, TableHeadings → {uNb[g, t], δ[# & /@ EdgeList[g]]}] // forma
```

Out[130]//TableForm=

	$\delta_{2,1}$	$\delta_{3,1}$	$\delta_{2,4}$	$\delta_{4,2}$	$\delta_{2,6}$	$\delta_{6,2}$	$\delta_{4,5}$	$\delta_{4,3}$	$\delta_{3,4}$	$\delta_{6,5}$	$\delta_{6,7}$
$2 \leftrightarrow 4$	-1	0	1	0	0	0	1	0	0	0	0
$4 \leftrightarrow 2$	1	0	0	1	0	0	-1	0	0	0	0
$2 \leftrightarrow 6$	-1	0	0	0	1	0	0	0	0	1	0
$6 \leftrightarrow 2$	1	0	0	0	0	1	0	0	0	-1	0
$4 \leftrightarrow 3$	0	1	0	0	0	0	-1	1	0	0	0
$3 \leftrightarrow 4$	0	-1	0	0	0	0	1	0	1	0	0
$6 \leftrightarrow 7$	0	0	0	0	0	0	0	0	0	-1	1
$3 \leftrightarrow 7$	0	-1	0	0	0	0	0	0	0	0	0

In[133]:=
```
λ = SparseArray[λ, {Length[λ], Length[λ[[1]]] + 3}];
(*λ=λ[[;;-2]]*)
```

In[134]:=
```
dopEq = # == 0 & /@ Flatten[λ.{x# & /@ EdgeList[g]}^T];
dopEq // forma
```

Out[135]//TableForm=

$$x_{2,1} - \frac{p_{2,4}\, x_{2,4}}{p_{2,1}} == 0$$
$$x_{2,1} - \frac{p_{2,6}\, x_{2,6}}{p_{2,1}} == 0$$
$$x_{3,1} - \frac{p_{3,4}\, x_{3,4}}{p_{3,1}} == 0$$
$$x_{3,1} - \frac{p_{3,7}\, x_{3,7}}{p_{3,1}} == 0$$
$$x_{4,2} - \frac{p_{4,5}\, x_{4,5}}{p_{4,2}} == 0$$
$$x_{4,2} - \frac{p_{4,3}\, x_{4,3}}{p_{4,2}} == 0$$
$$x_{6,2} - \frac{p_{6,5}\, x_{6,5}}{p_{6,2}} == 0$$
$$x_{6,2} - \frac{p_{6,7}\, x_{6,7}}{p_{6,2}} == 0$$

In[136]:=
```
Λ = λ.(δMatr)^T;
"cicle det's:"
Λ // forma
```

Out[137]= cicle det's:

Out[138]//TableForm=

$$\begin{array}{cccccccc}
-1-\frac{p_{2\leftarrow 4}}{p_{2\leftarrow 1}} & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\
-1 & 1 & -1-\frac{p_{2\leftarrow 6}}{p_{2\leftarrow 1}} & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1-\frac{p_{3\leftarrow 4}}{p_{3\leftarrow 1}} & 0 & -1 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & -1-\frac{p_{3\leftarrow 7}}{p_{3\leftarrow 1}} \\
-\frac{p_{4\leftarrow 5}}{p_{4\leftarrow 2}} & 1+\frac{p_{4\leftarrow 5}}{p_{4\leftarrow 2}} & 0 & 0 & \frac{p_{4\leftarrow 5}}{p_{4\leftarrow 2}} & -\frac{p_{4\leftarrow 5}}{p_{4\leftarrow 2}} & 0 & 0 \\
0 & 1 & 0 & 0 & -\frac{p_{4\leftarrow 3}}{p_{4\leftarrow 2}} & 0 & 0 & 0 \\
0 & 0 & -\frac{p_{6\leftarrow 5}}{p_{6\leftarrow 2}} & 1+\frac{p_{6\leftarrow 5}}{p_{6\leftarrow 2}} & 0 & 0 & \frac{p_{6\leftarrow 5}}{p_{6\leftarrow 2}} & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -\frac{p_{6\leftarrow 7}}{p_{6\leftarrow 2}} & 0
\end{array}$$

In[139]:=
```
"U_c="
U_c = Range[1, 8]
"U_nc="
U_nc = {}
```

Out[139]= $U_c =$

Out[140]= {1, 2, 3, 4, 5, 6, 7, 8}

Out[141]= $U_{nc} =$

Out[142]= { }

In[143]:=
```
Λc = Λ[[All, Uc]];
Λnc = Λ[[All, Unc]];
"Λc="
Λc // MatrixForm
```

Out[145]= $\Lambda_c =$

Out[146]//MatrixForm=

$$\begin{pmatrix}
-1-\frac{p_{2\leftrightarrow 4}}{p_{2\leftrightarrow 1}} & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\
-1 & 1 & -1-\frac{p_{2\leftrightarrow 6}}{p_{2\leftrightarrow 1}} & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1-\frac{p_{3\leftrightarrow 4}}{p_{3\leftrightarrow 1}} & 0 & -1 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & -1-\frac{p_{3\leftrightarrow 7}}{p_{3\leftrightarrow 1}} \\
-\frac{p_{4\leftrightarrow 5}}{p_{4\leftrightarrow 2}} & 1+\frac{p_{4\leftrightarrow 5}}{p_{4\leftrightarrow 2}} & 0 & 0 & \frac{p_{4\leftrightarrow 5}}{p_{4\leftrightarrow 2}} & -\frac{p_{4\leftrightarrow 5}}{p_{4\leftrightarrow 2}} & 0 & 0 \\
0 & 1 & 0 & 0 & -\frac{p_{4\leftrightarrow 3}}{p_{4\leftrightarrow 2}} & 0 & 0 & 0 \\
0 & 0 & -\frac{p_{6\leftrightarrow 5}}{p_{6\leftrightarrow 2}} & 1+\frac{p_{6\leftrightarrow 5}}{p_{6\leftrightarrow 2}} & 0 & 0 & \frac{p_{6\leftrightarrow 5}}{p_{6\leftrightarrow 2}} & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -\frac{p_{6\leftrightarrow 7}}{p_{6\leftrightarrow 2}} & 0
\end{pmatrix}$$

In[147]:=
```
"det(Λc)="
Simplify[det = Det[Λc]] // forma
```

Out[147]= $\det(\Lambda_c) =$

Out[148]//TableForm=

$$\frac{1}{p_{2,1}^2\, p_{3,1}^2\, p_{4,2}^2\, p_{6,2}^2}$$

$$\begin{aligned}
&(-((p_{2,4}\,p_{2,6}+p_{2,1}\,(p_{2,4}+p_{2,6}))\,p_{3,4}\,(p_{3,1}+p_{3,7})\,p_{4,2}\,p_{4,5}+(p_{3,4}\,p_{3,7}+p_{3,1}\,(p_{3,4}+p_{3,7}))\\
&\qquad p_{4,3}\,(p_{2,4}\,p_{2,6}\,(p_{4,2}+p_{4,5})+p_{2,1}\,(p_{2,6}\,p_{4,2}+p_{2,4}\,(p_{4,2}+p_{4,5}))))\\
&\quad p_{6,2}\,p_{6,5}+(p_{2,1}\,p_{2,4}\,(p_{3,4}\,p_{3,7}\,(p_{4,3}\,p_{4,5}+p_{4,2}\,(p_{4,3}+p_{4,5}))+\\
&\qquad p_{3,1}\,(p_{3,7}\,p_{4,3}\,(p_{4,2}+p_{4,5})+p_{3,4}\,(p_{4,3}\,p_{4,5}+p_{4,2}\,(p_{4,3}+p_{4,5}))))\,p_{6,5}-\\
&\quad((p_{2,4}\,p_{2,6}+p_{2,1}\,(p_{2,4}+p_{2,6}))\,p_{3,4}\,(p_{3,1}+p_{3,7})\,p_{4,2}\,p_{4,5}+(p_{3,4}\,p_{3,7}+p_{3,1}\,(p_{3,4}+p_{3,7}))\\
&\qquad p_{4,3}\,(p_{2,4}\,p_{2,6}\,(p_{4,2}+p_{4,5})+p_{2,1}\,(p_{2,6}\,p_{4,2}+p_{2,4}\,(p_{4,2}+p_{4,5}))))\,(p_{6,2}+p_{6,5}))\,p_{6,7})
\end{aligned}$$

In[149]:=
```
"UT="
utind = Cases[t[[6]], ξ_ /; ξ ≠ 0];
UT = EdgeList[g][[utind]]
```

Out[149]= $U_T =$

Out[151]= $\{8 \leftrightarrow 1,\ 2 \leftrightarrow 1,\ 3 \leftrightarrow 1,\ 4 \leftrightarrow 5,\ 8 \leftrightarrow 5,\ 6 \leftrightarrow 5,\ 8 \leftrightarrow 7\}$

In[152]:=
```
"UNb="
UNb = uNb[g, t]
```

Out[152]= $U_{Nb} =$

Out[153]= $\{2 \leftrightarrow 4,\ 4 \leftrightarrow 2,\ 2 \leftrightarrow 6,\ 6 \leftrightarrow 2,\ 4 \leftrightarrow 3,\ 3 \leftrightarrow 4,\ 6 \leftrightarrow 7,\ 3 \leftrightarrow 7\}$

In[154]:=
```
A = -λ.{x̃_# & /@ EdgeList[g]}^T /. ps;
"A="
A // MatrixForm
```

Out[155]= A=

Out[156]//MatrixForm=

$$\begin{pmatrix} \frac{f_{1\leftrightarrow 8}\, p_{1\leftrightarrow 2}}{p_{1\bullet\to 8}} \\ \frac{f_{1\leftrightarrow 8}\, p_{1\leftrightarrow 2}}{p_{1\bullet\to 8}} \\ \frac{f_{1\leftrightarrow 8}\, p_{1\leftrightarrow 3}}{p_{1\bullet\to 8}} + \frac{f_{7\leftrightarrow 8}\, p_{7\leftrightarrow 3}}{p_{7\bullet\to 8}} \\ \frac{f_{1\leftrightarrow 8}\, p_{1\leftrightarrow 3}}{p_{1\bullet\to 8}} + \frac{f_{7\leftrightarrow 8}\, p_{7\leftrightarrow 3}}{p_{7\bullet\to 8}} \\ -\frac{f_{5\bullet\to 8}\, p_{4\bullet\to 5}\, p_{5\bullet\to 4}}{p_{4\bullet\to 2}\, p_{5\bullet\to 8}} \\ 0 \\ \frac{p_{6\bullet\to 5}\left(-\frac{f_{5\bullet\to 8}\, p_{5\bullet\to 6}}{p_{5\bullet\to 8}} - \frac{f_{7\bullet\to 8}\, p_{7\bullet\to 6}}{p_{7\bullet\to 8}}\right)}{p_{6\bullet\to 2}} \\ 0 \end{pmatrix}$$

In[157]:=
```
β = A - Λnc.{x_# & /@ U_Nb[[U_nc]]}^T;
"β="
β // forma
```

Out[158]= β=

Out[159]//TableForm=

$$\begin{array}{l} \frac{f_{1,8}\, p_{1,2}}{p_{1,8}} \\ \frac{f_{1,8}\, p_{1,2}}{p_{1,8}} \\ \frac{f_{1,8}\, p_{1,3}}{p_{1,8}} + \frac{f_{7,8}\, p_{7,3}}{p_{7,8}} \\ \frac{f_{1,8}\, p_{1,3}}{p_{1,8}} + \frac{f_{7,8}\, p_{7,3}}{p_{7,8}} \\ -\frac{f_{5,8}\, p_{4,5}\, p_{5,4}}{p_{4,2}\, p_{5,8}} \\ 0 \\ \frac{p_{6,5}\left(-\frac{f_{5,8}\, p_{5,6}}{p_{5,8}} - \frac{f_{7,8}\, p_{7,6}}{p_{7,8}}\right)}{p_{6,2}} \\ 0 \end{array}$$

In[160]:=
```
"решаем уравнение Λc xc=β:"
xc = LinearSolve[Λc, β]
```

Out[160]= решаем уравнение $\Lambda_c x_c = \beta$:

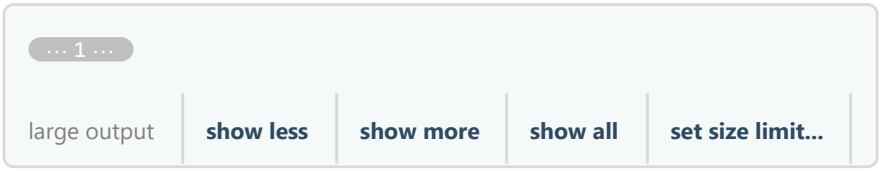
Out[161]=

$$\left\{\left\{\frac{\cdots 1 \cdots}{\left(p_{1\bullet\to 8}\, p_{5\bullet\to 8}\left(p_{2\bullet\to 1}\, p_{2\bullet\to 4}\, p_{3\bullet\to 1}\, p_{3\bullet\to 4}\, p_{4\bullet\to 2}\, p_{4\bullet\to 3}\, p_{6\bullet\to 2}\, p_{6\bullet\to 5} + p_{2\bullet\to 1}\, p_{2\bullet\to 6}\, p_{3\bullet\to 1}\, p_{3\bullet\to 4}\, p_{4\bullet\to 2}\, p_{4\bullet\to 3}\, p_{6\bullet\to 2}\, p_{6\bullet\to 5} + \cdots 52 \cdots + p_{2\bullet\to 4}\, p_{2\bullet\to 6}\, p_{3\bullet\to 4}\, p_{3\bullet\to 7}\, p_{4\bullet\to 3}\, p_{4\bullet\to 5}\, p_{6\bullet\to 5}\, p_{6\bullet\to 7}\right)\, p_{7\bullet\to 8}\right)\right\},\right.$$
$$\left\{-\frac{p_{4\bullet\to 3}\left(\cdots 1 \cdots\right)}{p_{1\bullet\to 8}\, p_{5\,\cdots 1 \cdots\,\cdots 1 \cdots\,\cdots 1 \cdots\, p_{7\bullet\to 8}}}\right\},\ \cdots 4 \cdots,\ \left\{-\frac{\cdots 1 \cdots}{\cdots 1 \cdots}\right\},$$
$$\left.\left\{\frac{\frac{p_{3\bullet\to 4}^2 \cdots 1 \cdots \left(-\frac{\cdots 1 \cdots}{p_{\cdots 1 \cdots}} + \cdots 1 \cdots - \cdots 1 \cdots\right)}{p_{3\bullet\to 1}^2} - \frac{\cdots 1 \cdots}{p^2_{\cdots 1 \cdots}}}{\frac{p_{3\bullet\to 4}\left(\cdots 1 \cdots\right)\left(-\frac{\cdots 1 \cdots}{p_{\cdots 1 \cdots}} + \cdots 1 \cdots\right)}{p_{3\bullet\to 1}} - \frac{p_{\cdots 1 \cdots}\,\cdots 1 \cdots\left(\cdots 1 \cdots\right)}{p_{3\bullet\to 1}}}\right\}\right\}$$

large output | show less | show more | show all | set size limit...

In[162]:=
```
xcp = MapThread[x_#1 → #2 &, {U_Nb[[U_c]], Flatten[xc]}];
xcp // TableForm
```

Out[163]//TableForm=



In[164]:=
```
s = solveAll[g, t];
s // TableForm
```

Out[165]//TableForm=

$$x_{2\bullet\to1} \to -\frac{f_{1\bullet\to8}\, p_{1\bullet\to2}}{p_{1\bullet\to8}} - x_{2\bullet\to4} - x_{2\bullet\to6} + x_{4\bullet\to2} + x_{6\bullet\to2}$$

$$x_{3\bullet\to1} \to -\frac{f_{1\bullet\to8}\, p_{1\bullet\to3}}{p_{1\bullet\to8}} - \frac{f_{7\bullet\to8}\, p_{7\bullet\to3}}{p_{7\bullet\to8}} - x_{3\bullet\to4} - x_{3\bullet\to7} + x_{4\bullet\to3}$$

$$x_{4\bullet\to5} \to -\frac{f_{5\bullet\to8}\, p_{5\bullet\to4}}{p_{5\bullet\to8}} + x_{2\bullet\to4} + x_{3\bullet\to4} - x_{4\bullet\to2} - x_{4\bullet\to3}$$

$$x_{6\bullet\to5} \to -\frac{f_{5\bullet\to8}\, p_{5\bullet\to6}}{p_{5\bullet\to8}} - \frac{f_{7\bullet\to8}\, p_{7\bullet\to6}}{p_{7\bullet\to8}} + x_{2\bullet\to6} - x_{6\bullet\to2} - x_{6\bullet\to7}$$

$$x_{8\bullet\to1} \to f_{8\bullet\to1} + f_{1\bullet\to8}\left(-1 - \frac{p_{1\bullet\to2}}{p_{1\bullet\to8}}\right) + \frac{f_{1\bullet\to8}\, p_{1\bullet\to2}}{p_{1\bullet\to8}} + \frac{f_{1\bullet\to8}\, p_{1\bullet\to3}}{p_{1\bullet\to8}} + \frac{f_{7\bullet\to8}\, p_{7\bullet\to3}}{p_{7\bullet\to8}} + x_{2\bullet\to4} + x_{2\bullet\to6} + x_{3\bullet\to4} + x_{3\bullet\to7} - x_{4\bullet\to2} - x_{4\bullet\to3} - x$$

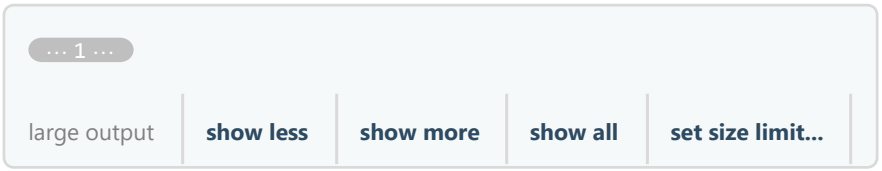
$$x_{8\bullet\to5} \to f_{8\bullet\to5} + f_{5\bullet\to8}\left(-1 - \frac{p_{5\bullet\to4}}{p_{5\bullet\to8}}\right) + \frac{f_{5\bullet\to8}\, p_{5\bullet\to4}}{p_{5\bullet\to8}} + \frac{f_{5\bullet\to8}\, p_{5\bullet\to6}}{p_{5\bullet\to8}} + \frac{f_{7\bullet\to8}\, p_{7\bullet\to6}}{p_{7\bullet\to8}} - x_{2\bullet\to4} - x_{2\bullet\to6} - x_{3\bullet\to4} + x_{4\bullet\to2} + x_{4\bullet\to3} + x_{6\bullet\to2} + x$$

$$x_{8\bullet\to7} \to f_{8\bullet\to7} + f_{7\bullet\to8}\left(-1 - \frac{p_{7\bullet\to6}}{p_{7\bullet\to8}}\right) - x_{3\bullet\to7} - x_{6\bullet\to7}$$

In[169]:=
```
"общее решение:"
xsol = ((s /. xcp) ~Join~ xcp);
xsol /. {ξ_u_↔v_ → ξ_u,v} // TableForm
```

Out[169]= общее решение:

Out[171]//TableForm=



In[172]:=
```
"eq test:"
Simplify[balanceEqs /. ξ → root[t] /. s /. xcp]
Simplify[(dopEq /. s) /. xcp]
```

Out[172]= eq test:

Out[173]= {True, True, True, True, True, True, True, True}

Out[174]= {True, True, True, True, True, True, True, True}