

Листинг 2

```

In[*]:= ClearAll["Global`*"]
SetDirectory[NotebookDirectory[]];
Needs["FlowSolver`"]

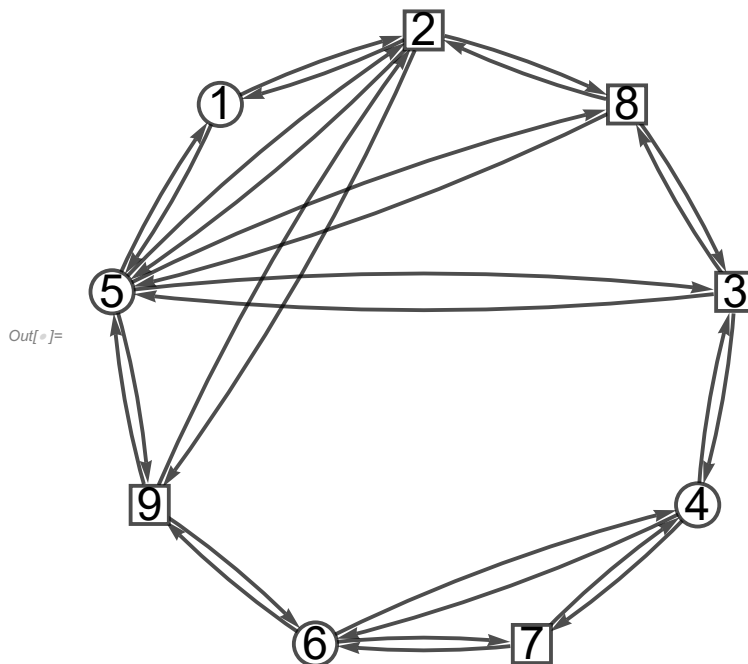
In[*]:= readGraph2[file_, dir_] := Module[{
    fn = FileNameJoin[{dir, file}],
    stream, imod, umod, u, b
},
    stream = OpenRead[fn];
    imod = Read[stream, {Word, Number}][[2]];
    umod = Read[stream, {Word, Number}][[2]];

    u = ({#[[1]] ↔ #[[2]], #[[2]] ↔ #[[1]]} & /@ ReadList[stream, Expression, umod]) // Flatten;
    b = ConstantArray[0, imod];
    (b[[Read[StringToStream[StringTake[#1, {5, -3}]], Number]]] = #2) & @@@
    ReadList[stream, {Word, Expression}, imod];
    {Graph[u, VertexSize -> Medium, VertexLabels -> Placed["Name", Center],
        VertexStyle -> Directive[White],
        VertexShapeFunction -> {xx_ -> If[SameQ[b[[xx]], x], "Square", "Circle"]},
        VertexLabelStyle -> Directive[Black, 24], GraphLayout -> "CircularEmbedding"], b]}

In[*]:= forma[ff_] := ({ff /. {ξu→v → ξu,v} } // TableForm)

In[*]:=
{g, b} = readGraph2["grDET0.txt", NotebookDirectory[]];
GraphPlot[g, EdgeStyle -> Directive[Black, Thick],
    VertexStyle -> Directive[EdgeForm[Thick], White], MultiedgeStyle -> .05]

```



```

In[ ]:= balanceEqs = (Total[x_# & /@ EdgeList[g, _ -> #]] - Total[x_# & /@ EdgeList[g, # -> _]]) ==
      MapIndexed[#1 /. x -> x_#2[[1]] &, b][[#]] & /@ VertexList[g];
balanceEqs //
      forma

```

Out[]:=TableForm=

```

- X1,2 - X1,5 + X2,1 + X5,1 == 0
X1,2 - X2,1 - X2,5 - X2,8 - X2,9 + X5,2 + X8,2 + X9,2 == X2
X1,5 + X2,5 + X3,5 - X5,1 - X5,2 - X5,3 - X5,8 - X5,9 + X8,5 + X9,5 == 0
X2,8 + X3,8 + X5,8 - X8,2 - X8,3 - X8,5 == X8
- X3,4 - X3,5 - X3,8 + X4,3 + X5,3 + X8,3 == X3
X3,4 - X4,3 - X4,6 - X4,7 + X6,4 + X7,4 == 0
X4,7 + X6,7 - X7,4 - X7,6 == X7
X4,6 - X6,4 - X6,7 - X6,9 + X7,6 + X9,6 == 0
X2,9 + X5,9 + X6,9 - X9,2 - X9,5 - X9,6 == X9

```

```

In[ ]:= M = {9};
Print["M = ", M];

M = {9}

```

```

In[ ]:= (*Do[inclist=EdgeList[g,u->_];
      Do[p_v=1/Length[inclist];,{v,inclist}];,{u,VertexList[g]}]*)

```

```

In[ ]:= (*p_#&/@EdgeList[g]*)

```

```

In[ ]:= (*incl=
      DeleteCases[DeleteDuplicates[Cases[IncidenceList[g,#],i_>j_>{i,j}]]//Flatten],
      v_/{v==#}&/@M*)
incl = (IncidenceList[g, #] & /@ M) // Flatten

```

```

Out[ ]:= {9 -> 6, 6 -> 9, 9 -> 5, 5 -> 9, 9 -> 2, 2 -> 9}

```

```

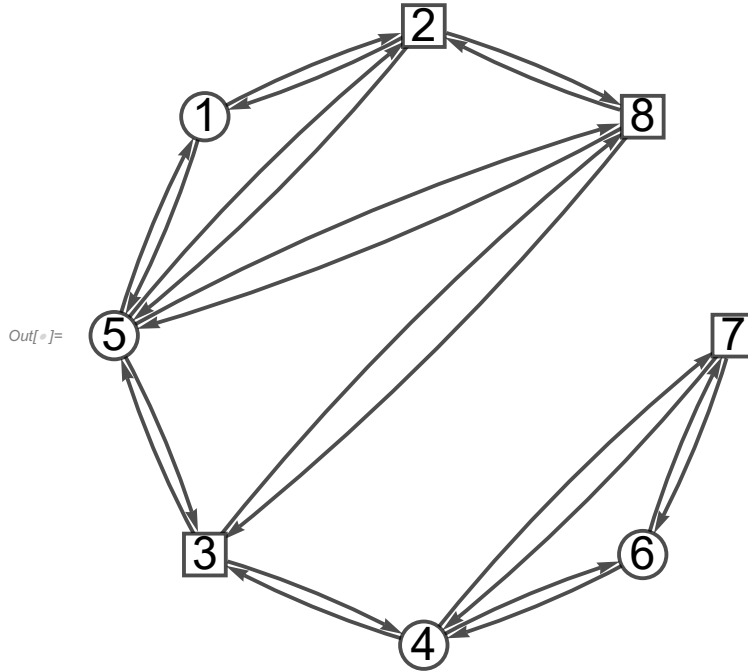
In[*]:= (*Do[If[MemberQ[M, j[[1]]], b[[j[[2]]]] += f_j, b[[j[[1]]]] -= f_j], {j, incL}]) *)
b = Fold[If[MemberQ[M, #2[[1]]], ReplacePart[#, #2[[2]] → #[[#2[[2]]]] - f_#2],
  ReplacePart[#, #2[[1]] → #[[#2[[1]]]] + f_#2]] &, b, incL];
b = b[Range[VertexCount[g]] ~ Complement ~ M];
ng = VertexDelete[g, M];
GraphPlot[ng, EdgeStyle → Directive[Black, Thick],
  VertexStyle → Directive[EdgeForm[Thick], White], MultiedgeStyle → .05]
b

```

```

Out[*]:= {0, x + f2→9 - f9→2, x, 0, f5→9 - f9→5, f6→9 - f9→6, x, x}

```



```

Out[*]:= {0, x + f2→9 - f9→2, x, 0, f5→9 - f9→5, f6→9 - f9→6, x, x}

```

```

In[*]:= CC[g_, M_] :=
  (DeleteDuplicates[Cases[IncidenceList[g, #], i_ ↔ j_ /; j == #]] & /@ M) // Flatten

```

```

iii+[g_] := Cases[IncidenceList[g, i], u_ ↔ v_ /; u == i ↔ v]

```

```

In[*]:= M+ = CC[g, M]

```

```

Out[*]:= {6 ↔ 9, 5 ↔ 9, 2 ↔ 9}

```

```

In[ ]:=  $\overline{b1} = \text{Fold}[$ 
       $\text{Module}[\{bb = \#1, i = \#2_{[[1]]}, k = \#2_{[[2]]}\}, \left( \text{Fold}[\text{Module}[\{bbb = \#1, jj = \#2\}, \text{ReplacePart}[\right.$ 
         $bbb, \left( \left( \left( \{jj \rightarrow bbb_{[[jj]]} - \frac{p_{i \rightarrow jj}}{p_{i \rightarrow k}} f_{i \rightarrow k}, i \rightarrow bbb_{[[i]]} + \frac{p_{i \rightarrow jj}}{p_{i \rightarrow k}} f_{i \rightarrow k} \right) \right) \right) //$ 
         $\text{Flatten}]] \&, bb, ii_i[\overline{ng}]] \left. \right) \&, \overline{b}, M^+]$ 

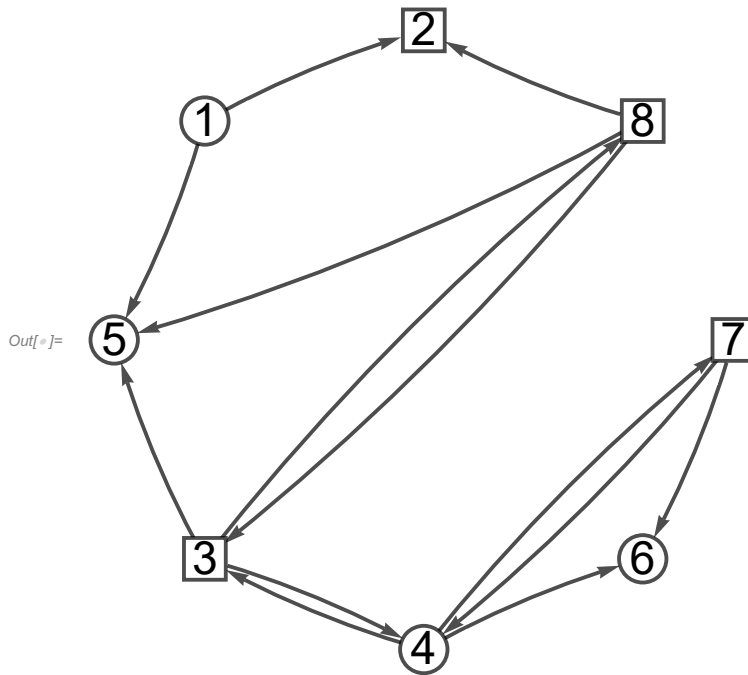
```

$$\begin{aligned}
\text{Out[]} = & \left\{ -\frac{f_{2 \rightarrow 9} p_{2 \rightarrow 1}}{p_{2 \rightarrow 9}} - \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 1}}{p_{5 \rightarrow 9}}, \right. \\
& x + f_{2 \rightarrow 9} - f_{9 \rightarrow 2} + \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 1}}{p_{2 \rightarrow 9}} + \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 5}}{p_{2 \rightarrow 9}} + \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 8}}{p_{2 \rightarrow 9}} - \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 2}}{p_{5 \rightarrow 9}}, x - \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 3}}{p_{5 \rightarrow 9}}, \\
& - \frac{f_{6 \rightarrow 9} p_{6 \rightarrow 4}}{p_{6 \rightarrow 9}}, f_{5 \rightarrow 9} - f_{9 \rightarrow 5} - \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 5}}{p_{2 \rightarrow 9}} + \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 1}}{p_{5 \rightarrow 9}} + \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 2}}{p_{5 \rightarrow 9}} + \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 3}}{p_{5 \rightarrow 9}} + \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 8}}{p_{5 \rightarrow 9}}, \\
& \left. f_{6 \rightarrow 9} - f_{9 \rightarrow 6} + \frac{f_{6 \rightarrow 9} p_{6 \rightarrow 4}}{p_{6 \rightarrow 9}} + \frac{f_{6 \rightarrow 9} p_{6 \rightarrow 7}}{p_{6 \rightarrow 9}}, x - \frac{f_{6 \rightarrow 9} p_{6 \rightarrow 7}}{p_{6 \rightarrow 9}}, x - \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 8}}{p_{2 \rightarrow 9}} - \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 8}}{p_{5 \rightarrow 9}} \right\}
\end{aligned}$$

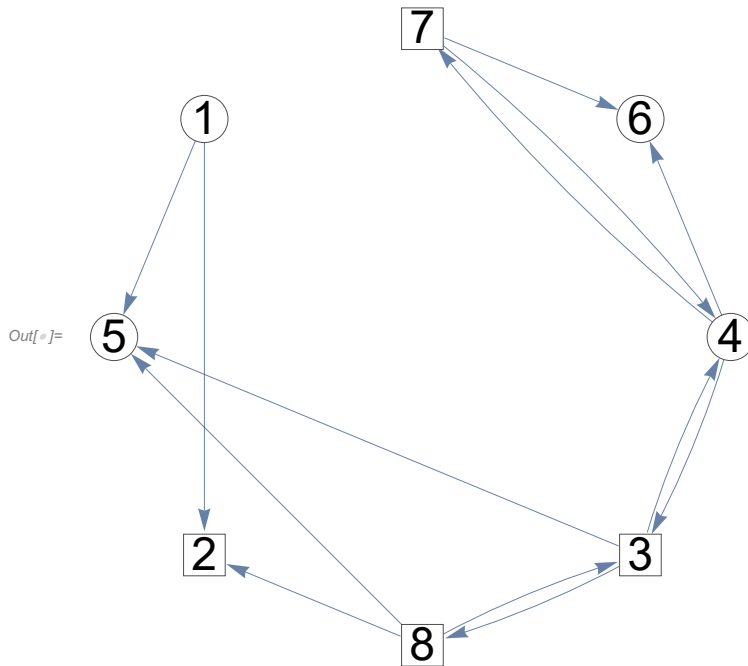
```

In[ ]:=  $\text{GraphPlot}[\text{Fold}[\text{HighlightGraph}[\#1, u_ \rightarrow v_ /; u == \#2, \text{GraphHighlightStyle} \rightarrow \text{"White"}] \&, \overline{ng}, \#_{[[1]]} \& /@M^+], \text{EdgeStyle} \rightarrow \text{Directive}[\text{Black}, \text{Thick}], \text{VertexStyle} \rightarrow \text{Directive}[\text{EdgeForm}[\text{Thick}], \text{White}], \text{MultiedgeStyle} \rightarrow .05]$ 

```



```
In[ ]:=  $\overline{g1}$  = Fold[EdgeDelete[#1, u_  $\leftrightarrow$  v_ /; u == #2] &,  $\overline{ng}$ , #[[1]] & /@ M+];
GraphPlot[ $\overline{g1}$ , MultiedgeStyle -> .05]
```



```
In[ ]:= IIrem = VertexList[ $\overline{g1}$ ] ~ Complement ~ (M+[[All, 1]])
Out[ ]:= {1, 3, 4, 7, 8}
```

```
In[ ]:=  $\lambda$  = SparseArray[
  Replace[ (EdgeList[ $\overline{g1}$ ] /. # & /@ Flatten[Module[{i = #, jf, Icur}, {Icur = iii+[ $\overline{g1}$ ];
    jf = First[Icur];
    { (i  $\rightarrow$  jf)  $\rightarrow$  1, (i  $\leftrightarrow$  #)  $\rightarrow$  -  $\frac{p_{i \rightarrow \#}}{p_{i \rightarrow jf}}$  } & /@ Icur[[2 ;;]] ] & /@ IIrem, 1] ),
    _  $\leftrightarrow$  _  $\rightarrow$  0, 2]]
```

Out[]:= SparseArray[ Specified elements: 16
Dimensions: {8, 13}]

```
In[ ]:= Grid[ $\lambda$ ]
```

Out[]:=

1	- $\frac{p_{1 \rightarrow 5}}{p_{1 \rightarrow 2}}$	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	- $\frac{p_{3 \rightarrow 8}}{p_{3 \rightarrow 4}}$	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	- $\frac{p_{3 \rightarrow 5}}{p_{3 \rightarrow 4}}$	0
0	0	0	0	1	0	0	- $\frac{p_{4 \rightarrow 7}}{p_{4 \rightarrow 3}}$	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	- $\frac{p_{4 \rightarrow 6}}{p_{4 \rightarrow 3}}$	0
0	0	0	0	0	0	0	0	1	0	- $\frac{p_{7 \rightarrow 6}}{p_{7 \rightarrow 4}}$	0	0
0	0	1	0	0	0	- $\frac{p_{8 \rightarrow 3}}{p_{8 \rightarrow 2}}$	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	- $\frac{p_{8 \rightarrow 5}}{p_{8 \rightarrow 2}}$	0	0	0

```
In[ ]:= g =  $\overline{g1}$ ;
b =  $\overline{b1}$ ;
```

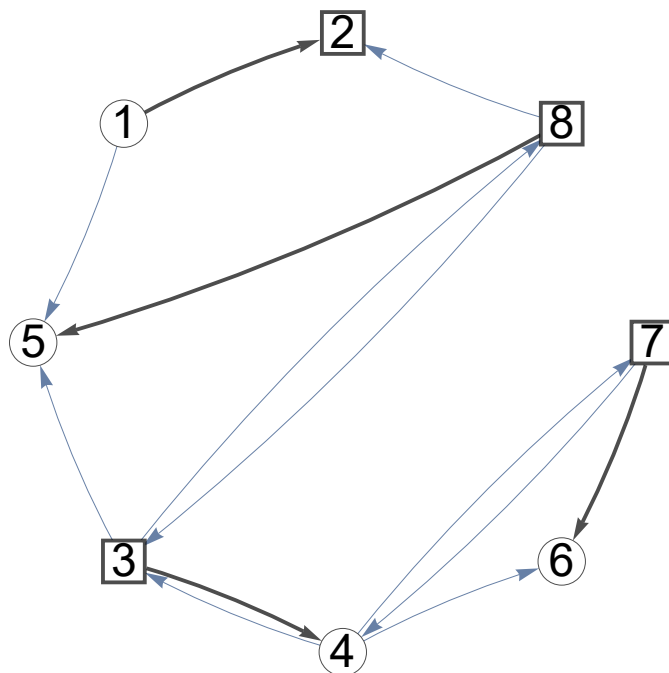
`Out[•]= {2, 3, 7, 8}`

$$Out[\bullet] = 0.$$

Out[•]//TableForm=

	1	2	3	4	5	6	7	8	9
pred	2	9	9	3	8	7	9	9	0
dir	-1	1	1	1	1	1	1	1	0
depth	2	1	1	2	2	2	1	1	0
d	8	1	4	7	3	9	6	5	2

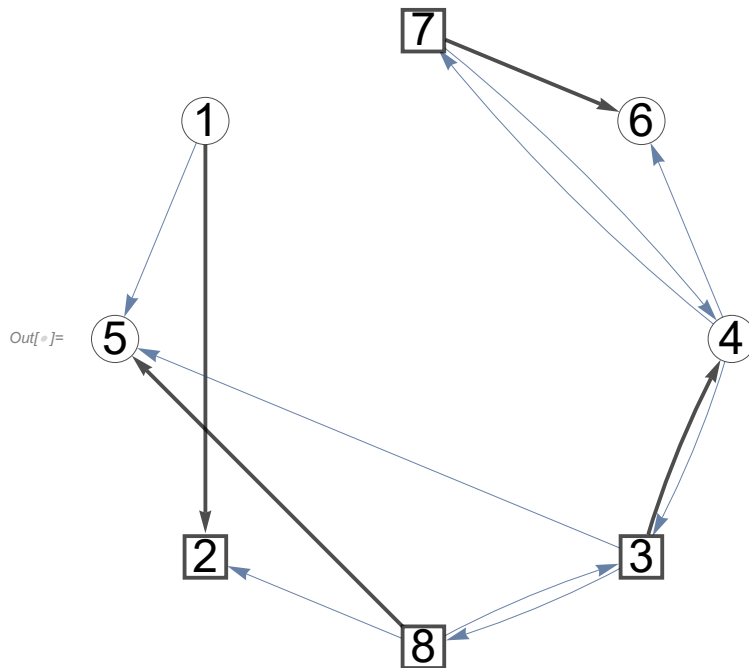
Out[•]=



```

In[ ]:= GraphPlot[HighlightGraph[g1,
  {Style[u_ /; VertexQ[g, u] && pred[t][u] == root[t], EdgeForm[Thick]],
  Style[u_ -> v_ /; (pred[t][u] == v && dir[t][u] == -1) ||
    (pred[t][v] == u && dir[t][v] == 1), Directive[Black, Thick]}],
  GraphHighlightStyle -> None], MultiedgeStyle -> .05]

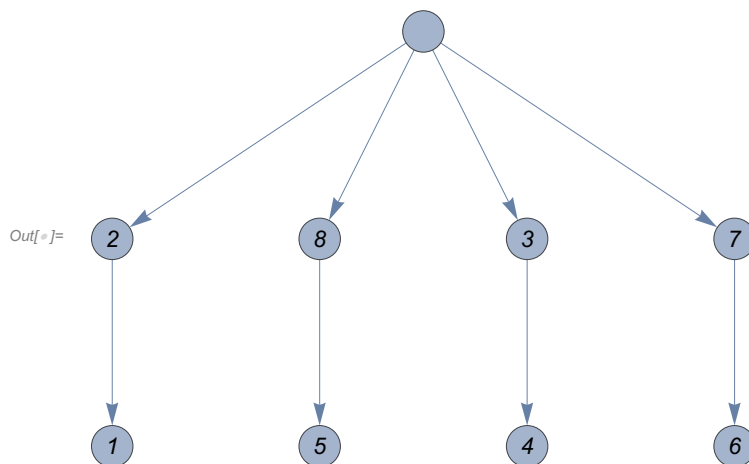
```



```

In[ ]:= t[[7]] (*пометить на графе*)

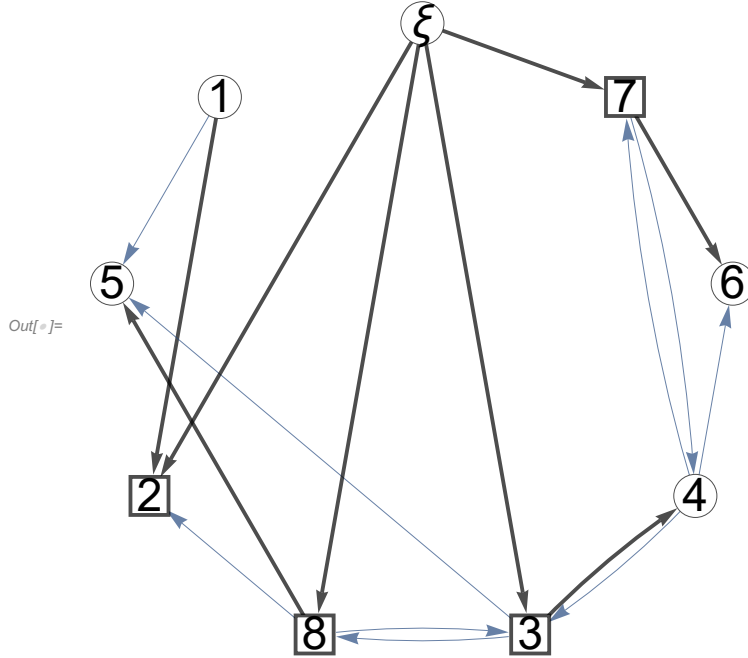
```



```

In[ ]:= (*GraphPlot[g,MultiedgeStyle->.05]*)
GraphPlot[HighlightGraph[g,
{Style[u_ /; VertexQ[g, u] && pred[t][[u]] == root[t], EdgeForm[Thick]],
Style[u_ -> v_ /; (pred[t][[u]] == v && dir[t][[u]] == -1) ||
(pred[t][[v]] == u && dir[t][[v]] == 1), Directive[Black, Thick]}],
GraphHighlightStyle -> None], MultiedgeStyle -> .05]

```



```

In[ ]:= AppendTo[b, -Total[b]];
b = Simplify[b /. x -> 0]

```

$$\begin{aligned}
\text{Out[]} = \left\{ -\frac{f_{2 \rightarrow 9} p_{2 \rightarrow 1}}{p_{2 \rightarrow 9}} - \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 1}}{p_{5 \rightarrow 9}}, -f_{9 \rightarrow 2} + \frac{f_{2 \rightarrow 9} (p_{2 \rightarrow 1} + p_{2 \rightarrow 5} + p_{2 \rightarrow 8} + p_{2 \rightarrow 9})}{p_{2 \rightarrow 9}} - \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 2}}{p_{5 \rightarrow 9}}, \right. \\
-\frac{f_{5 \rightarrow 9} p_{5 \rightarrow 3}}{p_{5 \rightarrow 9}}, -\frac{f_{6 \rightarrow 9} p_{6 \rightarrow 4}}{p_{6 \rightarrow 9}}, -f_{9 \rightarrow 5} - \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 5}}{p_{2 \rightarrow 9}} + \frac{f_{5 \rightarrow 9} (p_{5 \rightarrow 1} + p_{5 \rightarrow 2} + p_{5 \rightarrow 3} + p_{5 \rightarrow 8} + p_{5 \rightarrow 9})}{p_{5 \rightarrow 9}}, \\
-f_{9 \rightarrow 6} + \frac{f_{6 \rightarrow 9} (p_{6 \rightarrow 4} + p_{6 \rightarrow 7} + p_{6 \rightarrow 9})}{p_{6 \rightarrow 9}}, -\frac{f_{6 \rightarrow 9} p_{6 \rightarrow 7}}{p_{6 \rightarrow 9}}, \\
\left. -\frac{f_{2 \rightarrow 9} p_{2 \rightarrow 8}}{p_{2 \rightarrow 9}} - \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 8}}{p_{5 \rightarrow 9}}, -f_{2 \rightarrow 9} - f_{5 \rightarrow 9} - f_{6 \rightarrow 9} + f_{9 \rightarrow 2} + f_{9 \rightarrow 5} + f_{9 \rightarrow 6} \right\}
\end{aligned}$$


```

In[ ]:= balanceEqs = (Total[x# & /@ EdgeList[g, _ -> #]] - Total[x# & /@ EdgeList[g, # -> _]]) /.
      root[t] -> ξ) == b[#]] & /@ VertexList[g];
balanceEqs //
forma

```

```

Out[ ]//TableForm=

$$\begin{aligned}
-X_{1,2} - X_{1,5} &= -\frac{f_{2,9} p_{2,1}}{p_{2,9}} - \frac{f_{5,9} p_{5,1}}{p_{5,9}} \\
X_{1,2} + X_{8,2} + X_{\xi,2} &= -f_{9,2} + \frac{f_{2,9} (p_{2,1} + p_{2,5} + p_{2,8} + p_{2,9})}{p_{2,9}} - \frac{f_{5,9} p_{5,2}}{p_{5,9}} \\
X_{1,5} + X_{3,5} + X_{8,5} &= -f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} + \frac{f_{5,9} (p_{5,1} + p_{5,2} + p_{5,3} + p_{5,8} + p_{5,9})}{p_{5,9}} \\
X_{3,8} - X_{8,2} - X_{8,3} - X_{8,5} + X_{\xi,8} &= -\frac{f_{2,9} p_{2,8}}{p_{2,9}} - \frac{f_{5,9} p_{5,8}}{p_{5,9}} \\
-X_{3,4} - X_{3,5} - X_{3,8} + X_{4,3} + X_{8,3} + X_{\xi,3} &= -\frac{f_{5,9} p_{5,3}}{p_{5,9}} \\
X_{3,4} - X_{4,3} - X_{4,6} - X_{4,7} + X_{7,4} &= -\frac{f_{6,9} p_{6,4}}{p_{6,9}} \\
X_{4,7} - X_{7,4} - X_{7,6} + X_{\xi,7} &= -\frac{f_{6,9} p_{6,7}}{p_{6,9}} \\
X_{4,6} + X_{7,6} &= -f_{9,6} + \frac{f_{6,9} (p_{6,4} + p_{6,7} + p_{6,9})}{p_{6,9}} \\
-X_{\xi,2} - X_{\xi,3} - X_{\xi,7} - X_{\xi,8} &= -f_{2,9} - f_{5,9} - f_{6,9} + f_{9,2} + f_{9,5} + f_{9,6}
\end{aligned}$$


```

```

In[ ]:= ps = partSolve[g, -b, t, ξ];
ps // forma

```

```

Out[ ]//TableForm=

$$\begin{aligned}
\tilde{X}_{1,2} &\rightarrow \frac{f_{2,9} p_{2,1}}{p_{2,9}} + \frac{f_{5,9} p_{5,1}}{p_{5,9}} \\
\tilde{X}_{1,5} &\rightarrow 0 \\
\tilde{X}_{3,4} &\rightarrow -\frac{f_{6,9} p_{6,4}}{p_{6,9}} \\
\tilde{X}_{3,5} &\rightarrow 0 \\
\tilde{X}_{3,8} &\rightarrow 0 \\
\tilde{X}_{4,3} &\rightarrow 0 \\
\tilde{X}_{4,6} &\rightarrow 0 \\
\tilde{X}_{4,7} &\rightarrow 0 \\
\tilde{X}_{7,4} &\rightarrow 0 \\
\tilde{X}_{7,6} &\rightarrow -f_{9,6} + \frac{f_{6,9} (p_{6,4} + p_{6,7} + p_{6,9})}{p_{6,9}} \\
\tilde{X}_{8,2} &\rightarrow 0 \\
\tilde{X}_{8,3} &\rightarrow 0 \\
\tilde{X}_{8,5} &\rightarrow -f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} + \frac{f_{5,9} (p_{5,1} + p_{5,2} + p_{5,3} + p_{5,8} + p_{5,9})}{p_{5,9}} \\
\tilde{X}_{9,2} &\rightarrow -f_{9,2} - \frac{f_{2,9} p_{2,1}}{p_{2,9}} + \frac{f_{2,9} (p_{2,1} + p_{2,5} + p_{2,8} + p_{2,9})}{p_{2,9}} - \frac{f_{5,9} p_{5,1}}{p_{5,9}} - \frac{f_{5,9} p_{5,2}}{p_{5,9}} \\
\tilde{X}_{9,3} &\rightarrow -\frac{f_{5,9} p_{5,3}}{p_{5,9}} - \frac{f_{6,9} p_{6,4}}{p_{6,9}} \\
\tilde{X}_{9,7} &\rightarrow -f_{9,6} - \frac{f_{6,9} p_{6,7}}{p_{6,9}} + \frac{f_{6,9} (p_{6,4} + p_{6,7} + p_{6,9})}{p_{6,9}} \\
\tilde{X}_{9,8} &\rightarrow -f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} - \frac{f_{2,9} p_{2,8}}{p_{2,9}} - \frac{f_{5,9} p_{5,8}}{p_{5,9}} + \frac{f_{5,9} (p_{5,1} + p_{5,2} + p_{5,3} + p_{5,8} + p_{5,9})}{p_{5,9}}
\end{aligned}$$


```

```

In[ ]:= Simplify[(balanceEqs /. {x -> ξ, ξ -> root[t]}) /. ps]

```

```

Out[ ]:= {True, True, True, True, True, True, True, True, True}

```

```

In[ ]:= matrt = Timing[ $\delta$ Matr =  $\delta$ 1[g, t]];
roott = VertexCount[g];
TableForm[ $\delta$ Matr, TableHeadings  $\rightarrow$  {unb[g, t],  $\delta$ [#2] [#1]==roott & /@ EdgeList[g]} // forma
 $\left\{ \begin{array}{l} \# \\ \# \end{array} \right.$  True

```

Out[]//TableForm=

	$\delta_{1,2}$	$\delta_{1,5}$	$\delta_{8,2}$	$\delta_{3,4}$	$\delta_{4,3}$	$\delta_{3,8}$	$\delta_{8,3}$	$\delta_{4,7}$	$\delta_{7,4}$	$\delta_{8,5}$	$\delta_{7,6}$
1 \leftrightarrow 5	-1	1	0	0	0	0	0	0	0	-1	0
8 \leftrightarrow 2	0	0	1	0	0	0	0	0	0	0	0
4 \leftrightarrow 3	0	0	0	1	1	0	0	0	0	0	0
3 \leftrightarrow 8	0	0	0	0	0	1	0	0	0	0	0
8 \leftrightarrow 3	0	0	0	0	0	0	1	0	0	0	0
4 \leftrightarrow 7	0	0	0	1	0	0	0	1	0	0	0
7 \leftrightarrow 4	0	0	0	-1	0	0	0	0	1	0	0
4 \leftrightarrow 6	0	0	0	1	0	0	0	0	0	0	-1
3 \leftrightarrow 5	0	0	0	0	0	0	0	0	0	-1	0

```

In[ ]:=  $\lambda$  = SparseArray[ $\lambda$ , {Length[ $\lambda$ ], Length[ $\lambda$ [[1]]] + Length[II*]}];
(* $\lambda$ = $\lambda$ [[;;-2]]*)

```

```

In[ ]:= dopEq = # == 0 & /@ Flatten[ $\lambda$ .{x# & /@ EdgeList[g]}T];
dopEq // forma

```

Out[]//TableForm=

```

x1,2 -  $\frac{p_{1,5} x_{1,5}}{p_{1,2}}$  == 0
x3,4 -  $\frac{p_{3,8} x_{3,8}}{p_{3,4}}$  == 0
x3,4 -  $\frac{p_{3,5} x_{3,5}}{p_{3,4}}$  == 0
x4,3 -  $\frac{p_{4,7} x_{4,7}}{p_{4,3}}$  == 0
x4,3 -  $\frac{p_{4,6} x_{4,6}}{p_{4,3}}$  == 0
x7,4 -  $\frac{p_{7,6} x_{7,6}}{p_{7,4}}$  == 0
x8,2 -  $\frac{p_{8,3} x_{8,3}}{p_{8,2}}$  == 0
x8,2 -  $\frac{p_{8,5} x_{8,5}}{p_{8,2}}$  == 0

```

```

In[ ]:=  $\Delta$  =  $\lambda$ .( $\delta$ Matr)T;
"cycle det's:"
 $\Delta$  // forma

```

Out[]:= cycle det's:

Out[]//TableForm=

```

-1 -  $\frac{p_{1 \leftrightarrow 5}}{p_{1 \leftrightarrow 2}}$  0 0 0 0 0 0 0 0
0 0 1 -  $\frac{p_{3 \leftrightarrow 8}}{p_{3 \leftrightarrow 4}}$  0 1 -1 1 0
0 0 1 0 0 1 -1 1 -  $\frac{p_{3 \leftrightarrow 5}}{p_{3 \leftrightarrow 4}}$ 
0 0 1 0 0 -  $\frac{p_{4 \leftrightarrow 7}}{p_{4 \leftrightarrow 3}}$  0 0 0
0 0 1 0 0 0 -  $\frac{p_{4 \leftrightarrow 6}}{p_{4 \leftrightarrow 3}}$  0
0 0 0 0 0 0 1  $\frac{p_{7 \leftrightarrow 6}}{p_{7 \leftrightarrow 4}}$  0
0 1 0 0 -  $\frac{p_{8 \leftrightarrow 3}}{p_{8 \leftrightarrow 2}}$  0 0 0 0
 $\frac{p_{8 \leftrightarrow 5}}{p_{8 \leftrightarrow 2}}$  1 0 0 0 0 0 0  $\frac{p_{8 \leftrightarrow 5}}{p_{8 \leftrightarrow 2}}$ 

```

```

In[ ]:= MatrixRank[ $\Delta$ ]

```

Out[]:= 8

```
In[*]:= "U_c="
      U_c = {1, 2, 3, 4, 5, 6, 7, 8}
      "U_nc="
      U_nc = {9}
```

```
Out[*]:= U_c=
```

```
Out[*]:= {1, 2, 3, 4, 5, 6, 7, 8}
```

```
Out[*]:= U_nc=
```

```
Out[*]:= {9}
```

```
In[*]:= Δc = Δ[All, U_c];
      Δnc = Δ[All, U_nc];
      "Δ_c="
      Δc // MatrixForm
```

```
Out[*]:= Δ_c=
```

$$\text{Out[*]//MatrixForm} = \begin{pmatrix} -1 - \frac{p_{1 \leftrightarrow 5}}{p_{1 \leftrightarrow 2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 - \frac{p_{3 \leftrightarrow 8}}{p_{3 \leftrightarrow 4}} & 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 & -\frac{p_{4 \leftrightarrow 7}}{p_{4 \leftrightarrow 3}} & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -\frac{p_{4 \leftrightarrow 6}}{p_{4 \leftrightarrow 3}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{p_{7 \leftrightarrow 6}}{p_{7 \leftrightarrow 4}} \\ 0 & 1 & 0 & 0 & -\frac{p_{8 \leftrightarrow 3}}{p_{8 \leftrightarrow 2}} & 0 & 0 & 0 \\ \frac{p_{8 \leftrightarrow 5}}{p_{8 \leftrightarrow 2}} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
In[*]:= "det(Δ_c) ="
      Simplify[det = Det[Δc]] // forma
```

```
Out[*]:= det(Δ_c) =
```

$$\text{Out[*]//TableForm} = \frac{(p_{1,2} + p_{1,5}) p_{3,8} (p_{4,6} p_{4,7} p_{7,4} + p_{4,3} (p_{4,6} p_{7,4} + p_{4,7} (p_{7,4} + p_{7,6}))) p_{8,3}}{p_{1,2} p_{3,4} p_{4,3}^2 p_{7,4} p_{8,2}}$$

```
In[*]:= "U_T="
      utind = Cases[t[[6]], ξ_ /; ξ ≠ 0];
      U_T = EdgeList[g][[utind]]
```

```
Out[*]:= U_T=
```

```
Out[*]:= {1 ↔ 2, 9 ↔ 2, 9 ↔ 3, 3 ↔ 4, 8 ↔ 5, 7 ↔ 6, 9 ↔ 7, 9 ↔ 8}
```

```
In[*]:= "U_Nb="
      U_Nb = uNb[g, t]
```

```
Out[*]:= U_Nb=
```

```
Out[*]:= {1 ↔ 5, 8 ↔ 2, 4 ↔ 3, 3 ↔ 8, 8 ↔ 3, 4 ↔ 7, 7 ↔ 4, 4 ↔ 6, 3 ↔ 5}
```

```
In[ ]:= A = -λ. {x̃_# & /@ EdgeList[g]}^T /. ps;
"A="
A // MatrixForm
```

```
Out[ ]:= A=
```

```
Out[ ]//MatrixForm=
```

$$\begin{pmatrix} -\frac{f_{2 \rightarrow 9} p_{2 \rightarrow 1}}{p_{2 \rightarrow 9}} - \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 1}}{p_{5 \rightarrow 9}} & & & & & & & & \\ & \frac{f_{6 \rightarrow 9} p_{6 \rightarrow 4}}{p_{6 \rightarrow 9}} & & & & & & & \\ & & \frac{f_{6 \rightarrow 9} p_{6 \rightarrow 4}}{p_{6 \rightarrow 9}} & & & & & & \\ & & & \frac{f_{6 \rightarrow 9} p_{6 \rightarrow 4}}{p_{6 \rightarrow 9}} & & & & & \\ & & & & 0 & & & & \\ & & & & 0 & & & & \\ & & & & & \frac{\left(-f_{9 \rightarrow 6} + \frac{f_{6 \rightarrow 9} (p_{6 \rightarrow 4} + p_{6 \rightarrow 7} + p_{6 \rightarrow 9})}{p_{6 \rightarrow 9}}\right)}{p_{7 \rightarrow 4}} p_{7 \rightarrow 6} & & & \\ & & & & & & 0 & & \\ & & & & & & & \frac{\left(-f_{9 \rightarrow 5} - \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 5}}{p_{2 \rightarrow 9}} + \frac{f_{5 \rightarrow 9} (p_{5 \rightarrow 1} + p_{5 \rightarrow 2} + p_{5 \rightarrow 3} + p_{5 \rightarrow 8} + p_{5 \rightarrow 9})}{p_{5 \rightarrow 9}}\right)}{p_{8 \rightarrow 2}} p_{8 \rightarrow 5} & & \end{pmatrix}$$

```
In[ ]:= β = A - Δnc. {x_# & /@ U_Nb[[U_nc]]}^T;
"β="
β // forma
```

```
Out[ ]:= β=
```

```
Out[ ]//TableForm=
```

$$\begin{pmatrix} -\frac{f_{2,9} p_{2,1}}{p_{2,9}} - \frac{f_{5,9} p_{5,1}}{p_{5,9}} & & & & & & & & \\ & \frac{f_{6,9} p_{6,4}}{p_{6,9}} & & & & & & & \\ & & \frac{f_{6,9} p_{6,4}}{p_{6,9}} + \frac{p_{3,5} x_{3,5}}{p_{3,4}} & & & & & & \\ & & & 0 & & & & & \\ & & & 0 & & & & & \\ & & & & \frac{\left(-f_{9,6} + \frac{f_{6,9} (p_{6,4} + p_{6,7} + p_{6,9})}{p_{6,9}}\right)}{p_{7,4}} p_{7,6} & & & \\ & & & & & 0 & & & \\ & & & & & & \frac{\left(-f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} + \frac{f_{5,9} (p_{5,1} + p_{5,2} + p_{5,3} + p_{5,8} + p_{5,9})}{p_{5,9}}\right)}{p_{8,2}} p_{8,5} & & - \frac{p_{8,5} x_{3,5}}{p_{8,2}} \end{pmatrix}$$

$$\begin{aligned}
X_{1 \rightarrow 5} &\rightarrow \frac{f_{5 \rightarrow 9} p_{1 \rightarrow 5} p_{2 \rightarrow 9} p_{5 \rightarrow 1} + f_{5 \rightarrow 9} p_{5 \rightarrow 1} p_{2 \rightarrow 1} p_{5 \rightarrow 9}}{(p_{1 \rightarrow 2} + p_{1 \rightarrow 5}) p_{2 \rightarrow 9} p_{5 \rightarrow 9}} \\
X_{8 \rightarrow 2} &\rightarrow \frac{f_{5 \rightarrow 9} p_{1 \rightarrow 5} p_{2 \rightarrow 9} p_{5 \rightarrow 1} p_{8 \rightarrow 5} + f_{5 \rightarrow 9} p_{1 \rightarrow 2} p_{2 \rightarrow 9} p_{5 \rightarrow 2} p_{8 \rightarrow 5} + f_{5 \rightarrow 9} p_{1 \rightarrow 5} p_{2 \rightarrow 9} p_{5 \rightarrow 2} p_{8 \rightarrow 5} + f_{5 \rightarrow 9} p_{1 \rightarrow 2} p_{2 \rightarrow 9} p_{5 \rightarrow 3} p_{8 \rightarrow 5} + f_{5 \rightarrow 9} p_{1 \rightarrow 5} p_{2 \rightarrow 9} p_{5 \rightarrow 3} p_{8 \rightarrow 5} + f_{5 \rightarrow 9} p_{1 \rightarrow 2} p_{2 \rightarrow 9} p_{5 \rightarrow 3} p_{8 \rightarrow 5}}{p_{3 \rightarrow 4} p_{6 \rightarrow 9} (p_{4 \rightarrow 3} p_{4 \rightarrow 6} p_{7 \rightarrow 4} + p_{4 \rightarrow 3} p_{4 \rightarrow 7} p_{7 \rightarrow 4} + p_{4 \rightarrow 6} p_{4 \rightarrow 7} p_{7 \rightarrow 4} + p_{4 \rightarrow 3} p_{4 \rightarrow 7} p_{7 \rightarrow 6})} \\
X_{3 \rightarrow 8} &\rightarrow \frac{p_{3 \rightarrow 8} X_{3 \rightarrow 5}}{p_{3 \rightarrow 8}} \\
X_{8 \rightarrow 3} &\rightarrow \frac{p_{8 \rightarrow 2} \left(\frac{\left(\frac{f_{2 \rightarrow 9} p_{2 \rightarrow 1} - f_{5 \rightarrow 9} p_{5 \rightarrow 1}}{p_{2 \rightarrow 9} p_{5 \rightarrow 9}} \right) p_{8 \rightarrow 5}}{p_{8 \rightarrow 2}} - \left(-1 \frac{p_{1 \rightarrow 5}}{p_{1 \rightarrow 2}} \right) \left(\frac{\left(-f_{9 \rightarrow 5} \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 5} + f_{5 \rightarrow 9} (p_{5 \rightarrow 1} p_{5 \rightarrow 2} p_{5 \rightarrow 3} p_{5 \rightarrow 8} p_{5 \rightarrow 9}) \right)}{p_{2 \rightarrow 9} p_{5 \rightarrow 9}} p_{8 \rightarrow 5} \right)}{p_{8 \rightarrow 2}} - \frac{p_{8 \rightarrow 5} X_{3 \rightarrow 5}}{p_{8 \rightarrow 2}} \right)}{\left(1 + \frac{p_{1 \rightarrow 5}}{p_{1 \rightarrow 2}} \right) p_{8 \rightarrow 3}} \\
X_{4 \rightarrow 7} &\rightarrow \frac{p_{4 \rightarrow 3} p_{4 \rightarrow 6} (f_{6 \rightarrow 9} p_{3 \rightarrow 4} p_{6 \rightarrow 4} p_{7 \rightarrow 4} + f_{6 \rightarrow 9} p_{3 \rightarrow 4} p_{6 \rightarrow 4} p_{7 \rightarrow 6} + f_{6 \rightarrow 9} p_{3 \rightarrow 4} p_{6 \rightarrow 7} p_{7 \rightarrow 6} + f_{6 \rightarrow 9} p_{3 \rightarrow 4} p_{6 \rightarrow 9} p_{7 \rightarrow 6} - f_{9 \rightarrow 6} p_{3 \rightarrow 4} p_{6 \rightarrow 9} p_{7 \rightarrow 6} + p_{3 \rightarrow 5} p_{6 \rightarrow 9} p_{7 \rightarrow 4} X_3)}{p_{3 \rightarrow 4} p_{6 \rightarrow 9} (p_{4 \rightarrow 3} p_{4 \rightarrow 6} p_{7 \rightarrow 4} + p_{4 \rightarrow 3} p_{4 \rightarrow 7} p_{7 \rightarrow 4} + p_{4 \rightarrow 6} p_{4 \rightarrow 7} p_{7 \rightarrow 4} + p_{4 \rightarrow 3} p_{4 \rightarrow 7} p_{7 \rightarrow 6})} \\
X_{7 \rightarrow 4} &\rightarrow - \frac{p_{7 \rightarrow 6} (-f_{6 \rightarrow 9} p_{3 \rightarrow 4} p_{4 \rightarrow 3} p_{4 \rightarrow 6} p_{6 \rightarrow 4} - f_{6 \rightarrow 9} p_{3 \rightarrow 4} p_{4 \rightarrow 6} p_{4 \rightarrow 7} p_{6 \rightarrow 4} - f_{6 \rightarrow 9} p_{3 \rightarrow 4} p_{4 \rightarrow 3} p_{4 \rightarrow 6} p_{6 \rightarrow 7} - f_{6 \rightarrow 9} p_{3 \rightarrow 4} p_{4 \rightarrow 3} p_{4 \rightarrow 7} p_{6 \rightarrow 7} - f_{6 \rightarrow 9} p_{3 \rightarrow 4} p_{4 \rightarrow 6} p_{4 \rightarrow 7}}{p_{4 \rightarrow 3} p_{6 \rightarrow 9}} \\
X_{4 \rightarrow 6} &\rightarrow \frac{-p_{4 \rightarrow 7} \left(-f_{9 \rightarrow 6} + \frac{f_{6 \rightarrow 9} (p_{6 \rightarrow 4} p_{6 \rightarrow 7} + p_{6 \rightarrow 9})}{p_{6 \rightarrow 9}} \right) p_{7 \rightarrow 6} - p_{4 \rightarrow 7} \left(\frac{f_{6 \rightarrow 9} p_{6 \rightarrow 4} + p_{3 \rightarrow 5} X_{3 \rightarrow 5}}{p_{6 \rightarrow 9} p_{3 \rightarrow 4}} \right)}{\frac{p_{4 \rightarrow 3} p_{7 \rightarrow 4}}{p_{4 \rightarrow 3}} - \frac{p_{4 \rightarrow 7} \left(1 + \frac{p_{4 \rightarrow 6}}{p_{4 \rightarrow 3}} \right) p_{4 \rightarrow 7}}{p_{4 \rightarrow 3} p_{7 \rightarrow 4}}}
\end{aligned}$$

```
In[ ]:= s = solveAll[g, t];
s // TableForm
```

Out[]//TableForm=

$$\begin{aligned}
X_{1 \leftrightarrow 2} &\rightarrow \frac{f_{2 \leftrightarrow 9} p_{2 \leftrightarrow 1}}{p_{2 \leftrightarrow 9}} + \frac{f_{5 \leftrightarrow 9} p_{5 \leftrightarrow 1}}{p_{5 \leftrightarrow 9}} - X_{1 \leftrightarrow 5} \\
X_{3 \leftrightarrow 4} &\rightarrow -\frac{f_{6 \leftrightarrow 9} p_{6 \leftrightarrow 4}}{p_{6 \leftrightarrow 9}} + X_{4 \leftrightarrow 3} + X_{4 \leftrightarrow 6} + X_{4 \leftrightarrow 7} - X_{7 \leftrightarrow 4} \\
X_{8 \leftrightarrow 5} &\rightarrow -f_{9 \leftrightarrow 5} - \frac{f_{2 \leftrightarrow 9} p_{2 \leftrightarrow 5}}{p_{2 \leftrightarrow 9}} + \frac{f_{5 \leftrightarrow 9} (p_{5 \leftrightarrow 1} + p_{5 \leftrightarrow 2} + p_{5 \leftrightarrow 3} + p_{5 \leftrightarrow 8} + p_{5 \leftrightarrow 9})}{p_{5 \leftrightarrow 9}} - X_{1 \leftrightarrow 5} - X_{3 \leftrightarrow 5} \\
X_{7 \leftrightarrow 6} &\rightarrow -f_{9 \leftrightarrow 6} + \frac{f_{6 \leftrightarrow 9} (p_{6 \leftrightarrow 4} + p_{6 \leftrightarrow 7} + p_{6 \leftrightarrow 9})}{p_{6 \leftrightarrow 9}} - X_{4 \leftrightarrow 6} \\
X_{9 \leftrightarrow 2} &\rightarrow -f_{9 \leftrightarrow 2} - \frac{f_{2 \leftrightarrow 9} p_{2 \leftrightarrow 1}}{p_{2 \leftrightarrow 9}} + \frac{f_{2 \leftrightarrow 9} (p_{2 \leftrightarrow 1} + p_{2 \leftrightarrow 5} + p_{2 \leftrightarrow 8} + p_{2 \leftrightarrow 9})}{p_{2 \leftrightarrow 9}} - \frac{f_{5 \leftrightarrow 9} p_{5 \leftrightarrow 1}}{p_{5 \leftrightarrow 9}} - \frac{f_{5 \leftrightarrow 9} p_{5 \leftrightarrow 2}}{p_{5 \leftrightarrow 9}} + X_{1 \leftrightarrow 5} - X_{8 \leftrightarrow 2} \\
X_{9 \leftrightarrow 3} &\rightarrow -\frac{f_{5 \leftrightarrow 9} p_{5 \leftrightarrow 3}}{p_{5 \leftrightarrow 9}} - \frac{f_{6 \leftrightarrow 9} p_{6 \leftrightarrow 4}}{p_{6 \leftrightarrow 9}} + X_{3 \leftrightarrow 5} + X_{3 \leftrightarrow 8} + X_{4 \leftrightarrow 6} + X_{4 \leftrightarrow 7} - X_{7 \leftrightarrow 4} - X_{8 \leftrightarrow 3} \\
X_{9 \leftrightarrow 7} &\rightarrow -f_{9 \leftrightarrow 6} - \frac{f_{6 \leftrightarrow 9} p_{6 \leftrightarrow 7}}{p_{6 \leftrightarrow 9}} + \frac{f_{6 \leftrightarrow 9} (p_{6 \leftrightarrow 4} + p_{6 \leftrightarrow 7} + p_{6 \leftrightarrow 9})}{p_{6 \leftrightarrow 9}} - X_{4 \leftrightarrow 6} - X_{4 \leftrightarrow 7} + X_{7 \leftrightarrow 4} \\
X_{9 \leftrightarrow 8} &\rightarrow -f_{9 \leftrightarrow 5} - \frac{f_{2 \leftrightarrow 9} p_{2 \leftrightarrow 5}}{p_{2 \leftrightarrow 9}} - \frac{f_{2 \leftrightarrow 9} p_{2 \leftrightarrow 8}}{p_{2 \leftrightarrow 9}} - \frac{f_{5 \leftrightarrow 9} p_{5 \leftrightarrow 8}}{p_{5 \leftrightarrow 9}} + \frac{f_{5 \leftrightarrow 9} (p_{5 \leftrightarrow 1} + p_{5 \leftrightarrow 2} + p_{5 \leftrightarrow 3} + p_{5 \leftrightarrow 8} + p_{5 \leftrightarrow 9})}{p_{5 \leftrightarrow 9}} - X_{1 \leftrightarrow 5} - X_{3 \leftrightarrow 5} - X_{3 \leftrightarrow 8} + X_{8 \leftrightarrow 2} + X_{8 \leftrightarrow 5}
\end{aligned}$$

```
xsol = ((s /. xcp) ~Join~ xcp);
xsol /. { $\xi_{u \rightarrow v} \rightarrow \xi_{u,v}$ } // TableForm
```

Out[•]//TableForm=

[illegible]

```
Simplify[balanceEqs /.  $\xi \rightarrow \text{root}[t]$  /. s /. xcp]
Simplify[(dopEq /. s) /. xcp]
```

```
Out[8]= {True, True, True, True, True, True, True, True}
```