Листинг 2

In[1]:=
```
ClearAll["Global`*"]
SetDirectory[NotebookDirectory[]];
Needs["FlowSolver`"]
```

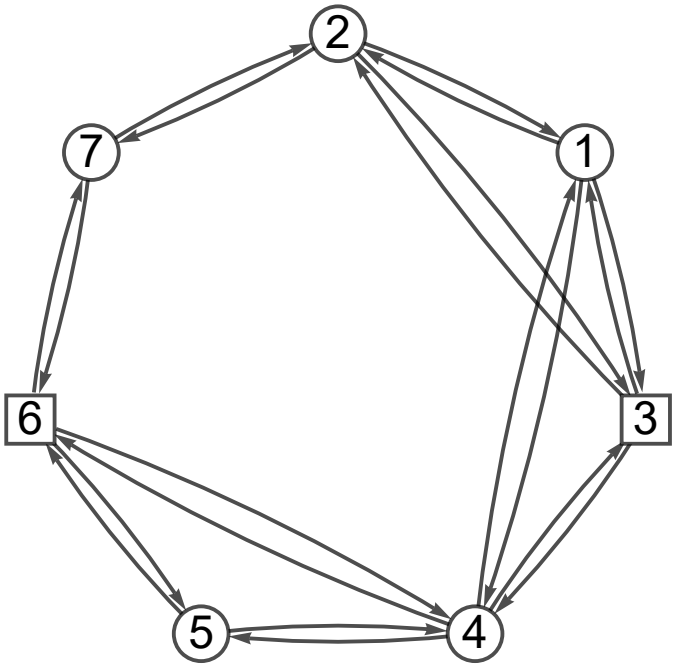
In[4]:=
```
readGraph2[file_, dir_] := Module[{
    fn = FileNameJoin[{dir, file}],
    stream, imod, umod, u, b
    },
    stream = OpenRead[fn];
    imod = Read[stream, {Word, Number}][[2]];
    umod = Read[stream, {Word, Number}][[2]];

  u = ({#[[1]] ↔ #[[2]], #[[2]] ↔ #[[1]]} & /@ ReadList[stream, Expression, umod]) // Flatten;
  b = ConstantArray[0, imod];
    (b[[Read[StringToStream[StringTake[#1, {5, -3}]], Number]]] = #2) &@@@
   ReadList[stream, {Word, Expression}, imod];
  {Graph[u, VertexSize -> Medium, VertexLabels → Placed["Name", Center],
    VertexStyle → Directive[White],
    VertexShapeFunction → {xx_ :> If[SameQ[b[[xx]], x], "Square", "Circle"]},
    VertexLabelStyle -> Directive[Black, 24], GraphLayout -> "CircularEmbedding"], b}]
```

In[5]:=
```
forma[ff_] := ((ff /. {ξ_u_↔v_ → ξu,v}) // TableForm)
```

In[6]:=
```
{g, b} = readGraph2["gr.txt", NotebookDirectory[]];
GraphPlot[g, EdgeStyle → Directive[Black, Thick],
 VertexStyle → Directive[EdgeForm[Thick], White], MultiedgeStyle → .05]
```

Out[7]=



In[8]:=
```
balanceEqs = ((Total[x_# & /@ EdgeList[g, _ ↔ #]] - Total[x_# & /@ EdgeList[g, # ↔ _]])) ==
      MapIndexed[#1 /. x → x_#2[[1]] &, b][[#]] & /@ VertexList[g];
balanceEqs //
 forma
```

Out[9]//TableForm=

$$x_{2,7} + x_{6,7} - x_{7,2} - x_{7,6} == 0$$
$$x_{1,2} - x_{2,1} - x_{2,3} - x_{2,7} + x_{3,2} + x_{7,2} == 0$$
$$x_{4,6} + x_{5,6} - x_{6,4} - x_{6,5} - x_{6,7} + x_{7,6} == x_6$$
$$-x_{1,2} - x_{1,3} - x_{1,4} + x_{2,1} + x_{3,1} + x_{4,1} == 0$$
$$x_{1,3} + x_{2,3} - x_{3,1} - x_{3,2} - x_{3,4} + x_{4,3} == x_3$$
$$x_{1,4} + x_{3,4} - x_{4,1} - x_{4,3} - x_{4,5} - x_{4,6} + x_{5,4} + x_{6,4} == 0$$
$$x_{4,5} - x_{5,4} - x_{5,6} + x_{6,5} == 0$$

In[10]:=
```
M = {7};
Print["M = ", M];
```

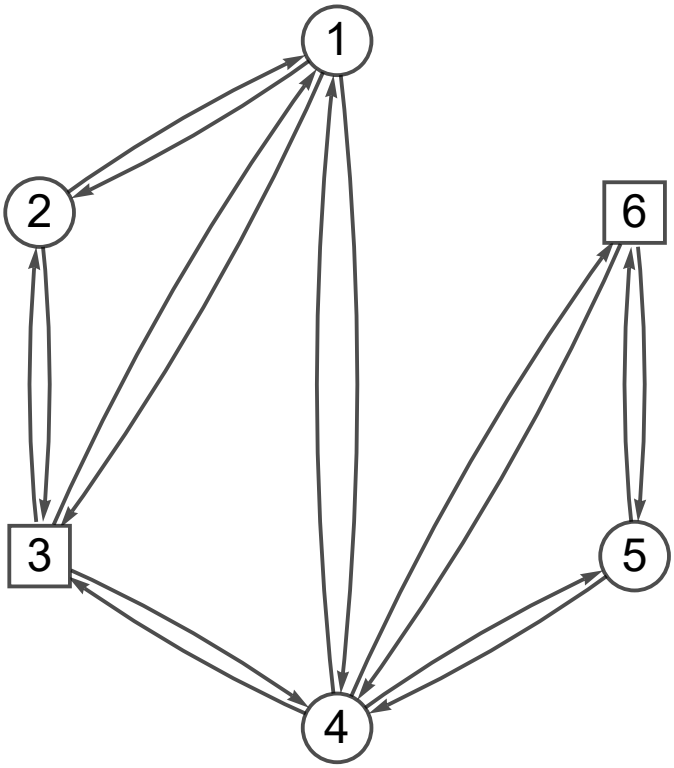
```
M = {7}
```

In[12]:=
```
(*incL=
 DeleteCases[DeleteDuplicates[Cases[IncidenceList[g,#],i_↔j_:>{i,j}]//Flatten],
    v_/;v==#]&/@M*)
incL = (IncidenceList[g, #] & /@ M) // Flatten
```

Out[12]= $\{7 \leftrightarrow 2, 2 \leftrightarrow 7, 7 \leftrightarrow 6, 6 \leftrightarrow 7\}$

In[13]:=
```
(*Do[If[MemberQ[M,j[[1]]],b[[j[[2]]]]+=f_j,b[[j[[1]]]]-=f_j],{j,incL}]*)
b̄ = Fold[If[MemberQ[M, #2[[1]]], ReplacePart[#, #2[[2]] → #[[#2[[2]]]] + f_#2],
      ReplacePart[#, #2[[1]] → #[[#2[[1]]]] - f_#2]] &, b, incL];
b̄ = Delete[b̄, #] &@@ M;
n̄ḡ = VertexDelete[g, M];
GraphPlot[n̄ḡ, EdgeStyle → Directive[Black, Thick],
 VertexStyle → Directive[EdgeForm[Thick], White], MultiedgeStyle → .05]
b̄
```

Out[16]=



Out[17]=
$$\{0,\ -f_{2\bullet\leftrightarrow 7}+f_{7\bullet\leftrightarrow 2},\ x,\ 0,\ 0,\ x-f_{6\bullet\leftrightarrow 7}+f_{7\bullet\leftrightarrow 6}\}$$

In[18]:=
```
CC[g_, M_] :=
  (DeleteDuplicates[Cases[IncidenceList[g, #], i_ ↔ j_ /; j == #]] &/@ M) // Flatten

ii_i_^+[g_] := Cases[IncidenceList[g, i], u_ ↔ v_ /; u == i :> v]
```

In[20]:=
```
M⁺ = CC[g, M]
```

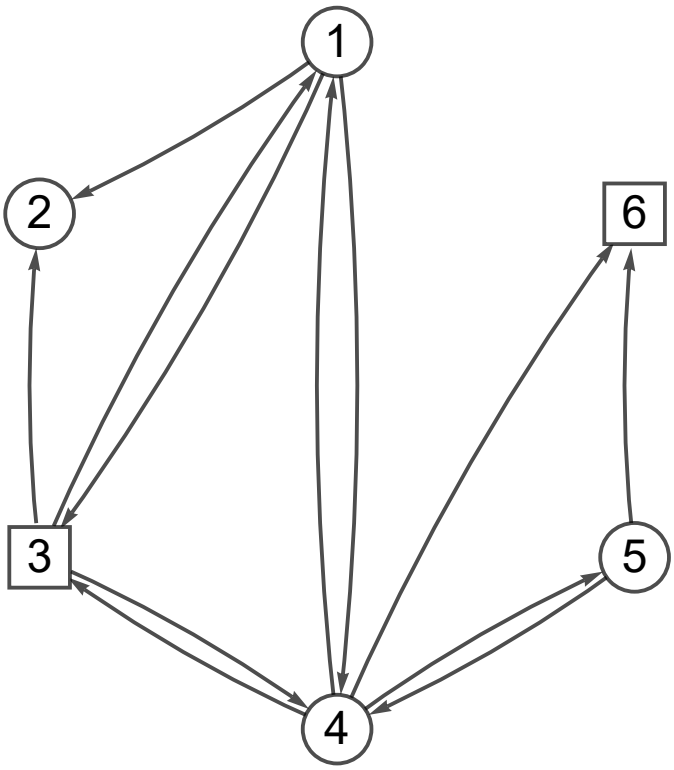
Out[20]=
$$\{2 \bullet\!\!\leftrightarrow 7,\ 6 \bullet\!\!\leftrightarrow 7\}$$

In[21]:=
```
b1̄ = Fold[Module[{bb = #1, i = #2[[1]], k = #2[[2]]},
     (ReplacePart[bb, ((({# → bb[[#]] + p_{i↔#}/p_{i↔k} f_{i↔k}, i → bb[[i]] - p_{i↔#}/p_{i↔k} f_{i↔k}}) &)
      /@ ii_i^+[n̄ḡ]) // Flatten]) &, b̄, M⁺]
```

Out[21]=
$$\left\{\frac{f_{2\bullet\to 7}\,p_{2\bullet\to 1}}{p_{2\bullet\to 7}},\ -f_{2\bullet\to 7}+f_{7\bullet\to 2}-\frac{f_{2\bullet\to 7}\,p_{2\bullet\to 1}}{p_{2\bullet\to 7}},\right.$$
$$\left.x+\frac{f_{2\bullet\to 7}\,p_{2\bullet\to 3}}{p_{2\bullet\to 7}},\ \frac{f_{6\bullet\to 7}\,p_{6\bullet\to 4}}{p_{6\bullet\to 7}},\ \frac{f_{6\bullet\to 7}\,p_{6\bullet\to 5}}{p_{6\bullet\to 7}},\ x-f_{6\bullet\to 7}+f_{7\bullet\to 6}-\frac{f_{6\bullet\to 7}\,p_{6\bullet\to 5}}{p_{6\bullet\to 7}}\right\}$$

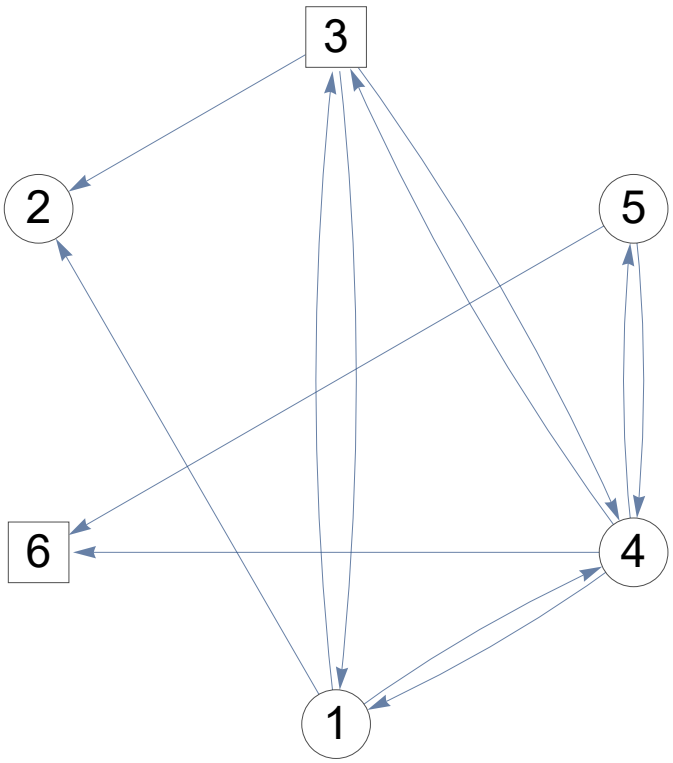
In[22]:=
```
GraphPlot[Fold[HighlightGraph[#1, u_ ↔ v_ /; u == #2, GraphHighlightStyle → "White"] &,
  ng, #[[1]] & /@ M+], EdgeStyle → Directive[Black, Thick],
 VertexStyle → Directive[EdgeForm[Thick], White], MultiedgeStyle → .05]
```

Out[22]=



In[23]:=
```
g1 = Fold[EdgeDelete[#1, u_ ↔ v_ /; u == #2] &, ng, #[[1]] & /@ M+];
GraphPlot[g1, MultiedgeStyle → .05]
```

Out[24]=



In[25]:=
$$\text{II}_{\text{rem}} = \text{VertexList}[\overline{\text{g1}}] \sim \text{Complement} \sim \left(\text{M}^{+}[\![\text{All}, 1]\!]\right)$$

Out[25]=
$$\{1, 3, 4, 5\}$$

In[26]:=
$$\lambda = \text{SparseArray}\Big[\text{Replace}\Big[\Big(\text{EdgeList}[\overline{\text{g1}}] /. \# \& /@ \text{Flatten}\Big[\text{Module}\Big[\{\text{i} = \#, \text{jf}, \text{Icur}\}, \Big(\text{Icur} = \text{ii}_{\text{i}}^{\dagger}[\overline{\text{g1}}];\ \text{jf} = \text{First}[\text{Icur}];\ \Big(\Big\{(\text{i} \leftrightarrow \text{jf}) \to 1, (\text{i} \leftrightarrow \#) \to -\frac{p_{\text{i}\leftrightarrow\#}}{p_{\text{i}\leftrightarrow\text{jf}}}\Big\} \& /@ \text{Icur}[\![2\ ;;]\!]\Big)\Big] \& /@ \text{II}_{\text{rem}}, 1\Big]\Big), \_ \leftrightarrow \_ \to 0, 2\Big]\Big]$$

Out[26]=
SparseArray[ Specified elements: 16
Dimensions: {8, 12}]

In[27]:=
$$\text{Grid}[\lambda]$$

Out[27]=
$$\begin{array}{cccccccccccc}
1 & 0 & -\frac{p_{1\leftrightarrow3}}{p_{1\leftrightarrow2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & -\frac{p_{1\leftrightarrow4}}{p_{1\leftrightarrow2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -\frac{p_{3\leftrightarrow1}}{p_{3\leftrightarrow2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & -\frac{p_{3\leftrightarrow4}}{p_{3\leftrightarrow2}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & -\frac{p_{4\leftrightarrow3}}{p_{4\leftrightarrow1}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -\frac{p_{4\leftrightarrow5}}{p_{4\leftrightarrow1}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\frac{p_{4\leftrightarrow6}}{p_{4\leftrightarrow1}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{p_{5\leftrightarrow6}}{p_{5\leftrightarrow4}} & 0
\end{array}$$

In[29]:=
$$\text{g} = \overline{\text{g1}};$$
$$\text{b} = \overline{\text{b1}};$$

In[31]:=
$$\text{II}^{*} = \text{Cases}[\text{MapIndexed}[\{\#1, \#2\} \&, \text{b}], \{\text{el\_}, \text{i\_}\} /; \text{MemberQ}[\text{el}, \text{x}] :\to \text{i}] // \text{Flatten}$$

Out[31]=
$$\{3, 6\}$$

In[32]:=
```
buildt = Timing[{t, g} = buildTree[g, II*];][[1]]
TableForm[t[[1 ;; 4]],
 TableHeadings → {{"pred", "dir", "depth", "d"}, t // pred // Length // Range}]
```

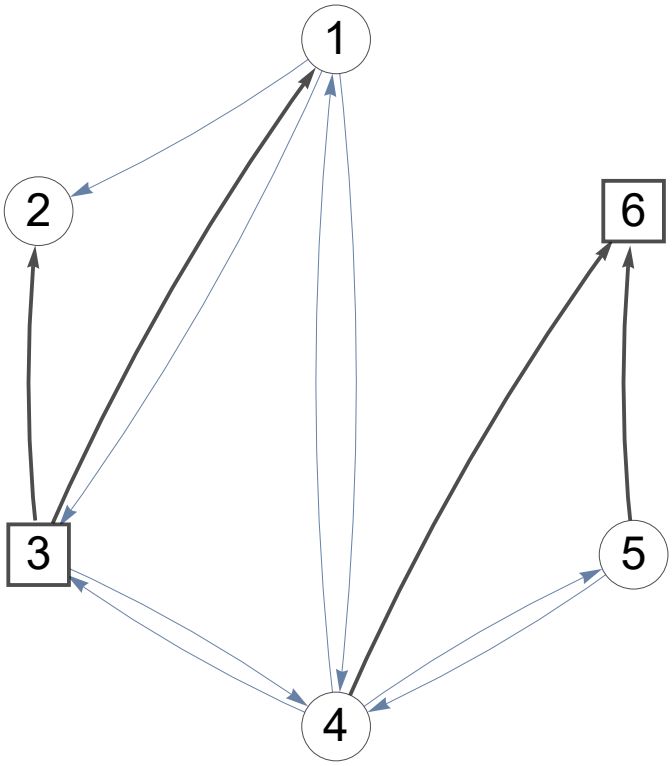
Out[32]=
$$0.015625$$

Out[33]//TableForm=

	1	2	3	4	5	6	7
pred	3	3	7	6	6	7	0
dir	1	1	1	−1	−1	1	0
depth	2	2	1	2	2	1	0
d	2	7	1	5	3	4	6

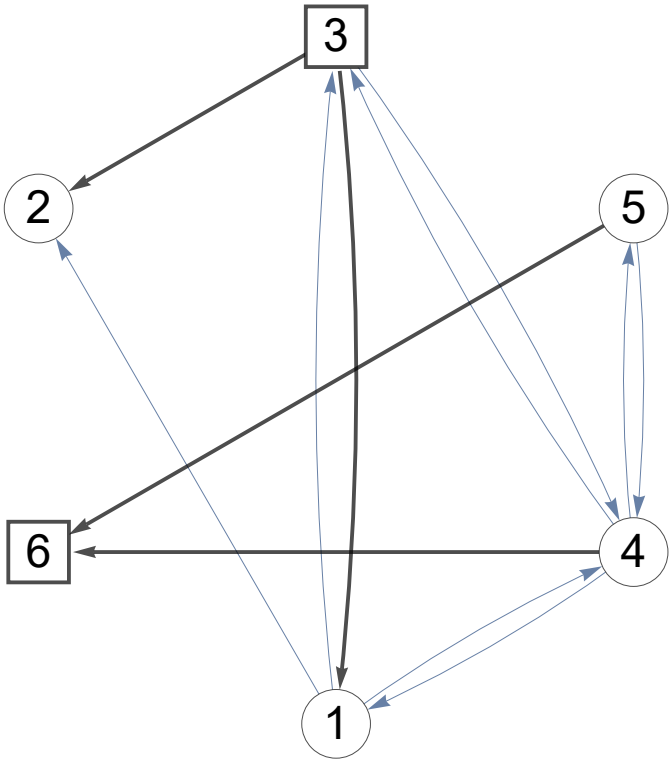
In[34]:=
```
GraphPlot[HighlightGraph[
  Fold[HighlightGraph[#1, Style[u_ ↦ v_ /; u == #2, White]] &, ng, #[[1]] & /@ M^+],
  {Style[u_ /; VertexQ[g, u] && pred[t][[u]] == 7, EdgeForm[Thick]], Style[u_ ↦ v_ /;
     (pred[t][[u]] == v && dir[t][[u]] == -1) || (pred[t][[v]] == u && dir[t][[v]] == 1),
   Directive[Black, Thick]]}, GraphHighlightStyle → None], MultiedgeStyle → .05]
```

Out[34]=

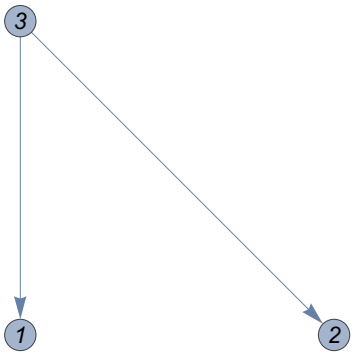


In[35]:=
```
GraphPlot[
 HighlightGraph[g1, {Style[u_ /; VertexQ[g, u] && pred[t][[u]] == 7, EdgeForm[Thick]],
   Style[u_ ↔ v_ /; (pred[t][[u]] == v && dir[t][[u]] == -1) ||
      (pred[t][[v]] == u && dir[t][[v]] == 1), Directive[Black, Thick]]},
  GraphHighlightStyle → None], MultiedgeStyle → .05]
```

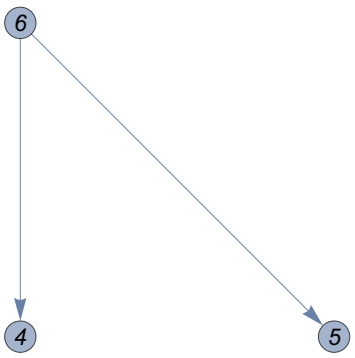
Out[35]=



In[36]:=
```
VertexDelete[t[[7]], 7](*пометить на графе*)
```

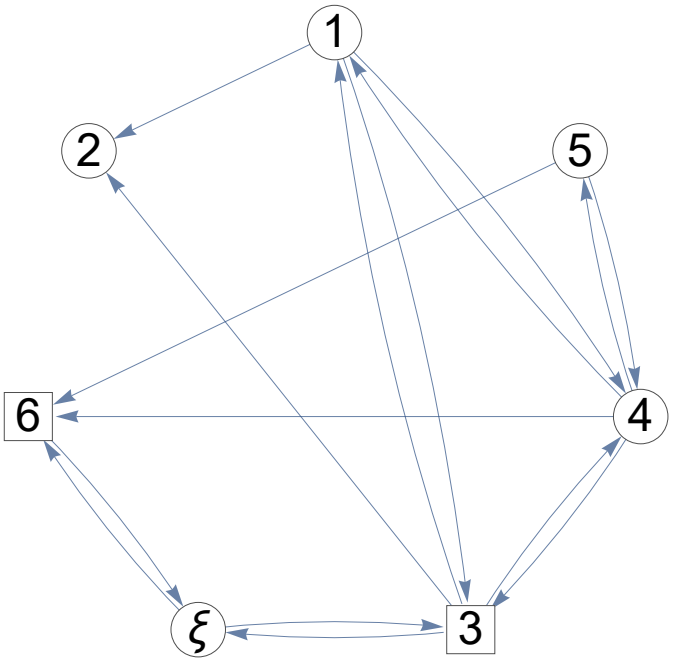


Out[36]=



In[37]:=
```
GraphPlot[g, MultiedgeStyle → .05]
```

Out[37]=



In[38]:=
```
AppendTo[b, -Total[b]];
b = Simplify[b /. x → 0]
```

Out[39]=

$$\left\{\frac{f_{2\bullet\to 7}\,p_{2\bullet\to 1}}{p_{2\bullet\to 7}},\ f_{7\bullet\to 2}+f_{2\bullet\to 7}\left(-1-\frac{p_{2\bullet\to 1}}{p_{2\bullet\to 7}}\right),\ \frac{f_{2\bullet\to 7}\,p_{2\bullet\to 3}}{p_{2\bullet\to 7}},\ \frac{f_{6\bullet\to 7}\,p_{6\bullet\to 4}}{p_{6\bullet\to 7}},\ \frac{f_{6\bullet\to 7}\,p_{6\bullet\to 5}}{p_{6\bullet\to 7}},\right.$$
$$\left.f_{7\bullet\to 6}+f_{6\bullet\to 7}\left(-1-\frac{p_{6\bullet\to 5}}{p_{6\bullet\to 7}}\right),\ -f_{7\bullet\to 2}-f_{7\bullet\to 6}+f_{2\bullet\to 7}\left(1-\frac{p_{2\bullet\to 3}}{p_{2\bullet\to 7}}\right)+f_{6\bullet\to 7}\left(1-\frac{p_{6\bullet\to 4}}{p_{6\bullet\to 7}}\right)\right\}$$

In[40]:=
```
balanceEqs =
  ((Total[x_# & /@ EdgeList[g, _ ↔ #]] - Total[x_# & /@ EdgeList[g, # ↔ _]]) /. 7 → ξ) ==
    b[[#]] & /@ VertexList[g];
balanceEqs //
 forma
```

Out[41]//TableForm=

$$x_{1,2}+x_{3,2}==f_{7,2}+f_{2,7}\left(-1-\frac{p_{2,1}}{p_{2,7}}\right)$$
$$x_{4,6}+x_{5,6}-x_{6,\xi}+x_{\xi,6}==f_{7,6}+f_{6,7}\left(-1-\frac{p_{6,5}}{p_{6,7}}\right)$$
$$-x_{1,2}-x_{1,3}-x_{1,4}+x_{3,1}+x_{4,1}==\frac{f_{2,7}\,p_{2,1}}{p_{2,7}}$$
$$x_{1,3}-x_{3,1}-x_{3,2}-x_{3,4}-x_{3,\xi}+x_{4,3}+x_{\xi,3}==\frac{f_{2,7}\,p_{2,3}}{p_{2,7}}$$
$$x_{1,4}+x_{3,4}-x_{4,1}-x_{4,3}-x_{4,5}-x_{4,6}+x_{5,4}==\frac{f_{6,7}\,p_{6,4}}{p_{6,7}}$$
$$x_{4,5}-x_{5,4}-x_{5,6}==\frac{f_{6,7}\,p_{6,5}}{p_{6,7}}$$
$$x_{3,\xi}+x_{6,\xi}-x_{\xi,3}-x_{\xi,6}==-f_{7,2}-f_{7,6}+f_{2,7}\left(1-\frac{p_{2,3}}{p_{2,7}}\right)+f_{6,7}\left(1-\frac{p_{6,4}}{p_{6,7}}\right)$$

In[42]:=
```
ps = partSolve[g, -b, t, x̃];
ps // forma
```

7

Out[43]//TableForm=

$$\tilde{x}_{1,2} \to 0$$
$$\tilde{x}_{1,3} \to 0$$
$$\tilde{x}_{1,4} \to 0$$
$$\tilde{x}_{3,1} \to \frac{f_{2,7}\, p_{2,1}}{p_{2,7}}$$
$$\tilde{x}_{3,2} \to f_{7,2} + f_{2,7}\left(-1 - \frac{p_{2,1}}{p_{2,7}}\right)$$
$$\tilde{x}_{3,4} \to 0$$
$$\tilde{x}_{3,7} \to 0$$
$$\tilde{x}_{4,1} \to 0$$
$$\tilde{x}_{4,3} \to 0$$
$$\tilde{x}_{4,5} \to 0$$
$$\tilde{x}_{4,6} \to -\frac{f_{6,7}\, p_{6,4}}{p_{6,7}}$$
$$\tilde{x}_{5,4} \to 0$$
$$\tilde{x}_{5,6} \to -\frac{f_{6,7}\, p_{6,5}}{p_{6,7}}$$
$$\tilde{x}_{6,7} \to 0$$
$$\tilde{x}_{7,3} \to f_{7,2} + f_{2,7}\left(-1 - \frac{p_{2,1}}{p_{2,7}}\right) + \frac{f_{2,7}\, p_{2,1}}{p_{2,7}} + \frac{f_{2,7}\, p_{2,3}}{p_{2,7}}$$
$$\tilde{x}_{7,6} \to f_{7,6} + f_{6,7}\left(-1 - \frac{p_{6,5}}{p_{6,7}}\right) + \frac{f_{6,7}\, p_{6,4}}{p_{6,7}} + \frac{f_{6,7}\, p_{6,5}}{p_{6,7}}$$

In[44]:=
```
Simplify[(balanceEqs /. {x → x̃, ξ → 7}) /. ps]
```

Out[44]= {True, True, True, True, True, True, True}

In[45]:=
```
matrt = Timing[δMatr = δ1[g, t]];
rootT = VertexCount[g];
TableForm[δMatr, TableHeadings → {uNb[g, t], δ[# [[2]] [[1]] == rootT, # [[1]] [[2]] == rootT, # True] & /@ EdgeList[g]}] // forma
```

Out[47]//TableForm=

	$\delta_{1,2}$	$\delta_{3,2}$	$\delta_{1,3}$	$\delta_{3,1}$	$\delta_{1,4}$	$\delta_{4,1}$	$\delta_{3,4}$	$\delta_{4,3}$	$\delta_{4,5}$	$\delta_{5,4}$	$\delta_{5,6}$
$1 \leftrightarrow 2$	1	−1	0	1	0	0	0	0	0	0	0
$1 \leftrightarrow 3$	0	0	1	1	0	0	0	0	0	0	0
$1 \leftrightarrow 4$	0	0	0	1	1	0	0	0	0	0	0
$4 \leftrightarrow 1$	0	0	0	−1	0	1	0	0	0	0	0
$3 \leftrightarrow 4$	0	0	0	0	0	0	1	0	0	0	0
$4 \leftrightarrow 3$	0	0	0	0	0	0	0	1	0	0	0
$4 \leftrightarrow 5$	0	0	0	0	0	0	0	0	1	0	1
$5 \leftrightarrow 4$	0	0	0	0	0	0	0	0	0	1	−1
$3 \leftrightarrow 7$	0	0	0	0	0	0	0	0	0	0	0
$6 \leftrightarrow 7$	0	0	0	0	0	0	0	0	0	0	0

In[48]:=
```
λ = SparseArray[λ, {Length[λ], Length[λ[[1]]] + 4}];
(*λ=λ[[;;-2]]*)
```

In[49]:=
```
dopEq = # == 0 & /@ Flatten[λ.{x_# & /@ EdgeList[g]}^T];
dopEq // forma
```

Out[50]//TableForm=

$$x_{1,2} - \frac{p_{1,3}\, x_{1,3}}{p_{1,2}} == 0$$
$$x_{1,2} - \frac{p_{1,4}\, x_{1,4}}{p_{1,2}} == 0$$
$$-\frac{p_{3,1}\, x_{3,1}}{p_{3,2}} + x_{3,2} == 0$$
$$x_{3,2} - \frac{p_{3,4}\, x_{3,4}}{p_{3,2}} == 0$$
$$x_{4,1} - \frac{p_{4,3}\, x_{4,3}}{p_{4,1}} == 0$$
$$x_{4,1} - \frac{p_{4,5}\, x_{4,5}}{p_{4,1}} == 0$$
$$x_{4,1} - \frac{p_{4,6}\, x_{4,6}}{p_{4,1}} == 0$$
$$x_{5,4} - \frac{p_{5,6}\, x_{5,6}}{p_{5,4}} == 0$$

In[51]:=
```
Λ = λ.(δMatr)^T;
"cicle det's:"
Λ // forma
```

Out[52]= cicle det's:

Out[53]//TableForm=

$$\begin{array}{cccccccccc}
1 & -\frac{p_{1\to 3}}{p_{1\to 2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & -\frac{p_{1\to 4}}{p_{1\to 2}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 - \frac{p_{3\to 1}}{p_{3\to 2}} & -\frac{p_{3\to 1}}{p_{3\to 2}} & -\frac{p_{3\to 1}}{p_{3\to 2}} & \frac{p_{3\to 1}}{p_{3\to 2}} & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & -\frac{p_{3\to 4}}{p_{3\to 2}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -\frac{p_{4\to 3}}{p_{4\to 1}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -\frac{p_{4\to 5}}{p_{4\to 1}} & 0 & 0 & 0 \\
0 & 0 & -\frac{p_{4\to 6}}{p_{4\to 1}} & 1 + \frac{p_{4\to 6}}{p_{4\to 1}} & -\frac{p_{4\to 6}}{p_{4\to 1}} & \frac{p_{4\to 6}}{p_{4\to 1}} & \frac{p_{4\to 6}}{p_{4\to 1}} & -\frac{p_{4\to 6}}{p_{4\to 1}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{p_{5\to 6}}{p_{5\to 4}} & 1 + \frac{p_{5\to 6}}{p_{5\to 4}} & 0 & 0
\end{array}$$

In[58]:=
```
"U_c="
U_c = Range[8]
"U_nc="
U_nc = {9, 10}
```

Out[58]= $U_c =$

Out[59]= $\{1, 2, 3, 4, 5, 6, 7, 8\}$

Out[60]= $U_{nc} =$

Out[61]= $\{9, 10\}$

In[62]:=
```
Λc = Λ[[All, Uc]];
Λnc = Λ[[All, Unc]];
"Λc="
Λc // MatrixForm
```

Out[64]= $\Lambda_c =$

Out[65]//MatrixForm=

$$\begin{pmatrix}
1 & -\frac{p_{1\leftrightarrow 3}}{p_{1\leftrightarrow 2}} & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & -\frac{p_{1\leftrightarrow 4}}{p_{1\leftrightarrow 2}} & 0 & 0 & 0 & 0 & 0 \\
-1 & -\frac{p_{3\leftrightarrow 1}}{p_{3\leftrightarrow 2}} & -\frac{p_{3\leftrightarrow 1}}{p_{3\leftrightarrow 2}} & -\frac{p_{3\leftrightarrow 1}}{p_{3\leftrightarrow 2}} & \frac{p_{3\leftrightarrow 1}}{p_{3\leftrightarrow 2}} & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & -\frac{p_{3\leftrightarrow 4}}{p_{3\leftrightarrow 2}} & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -\frac{p_{4\leftrightarrow 3}}{p_{4\leftrightarrow 1}} & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -\frac{p_{4\leftrightarrow 5}}{p_{4\leftrightarrow 1}} & 0 \\
0 & 0 & -\frac{p_{4\leftrightarrow 6}}{p_{4\leftrightarrow 1}} & 1+\frac{p_{4\leftrightarrow 6}}{p_{4\leftrightarrow 1}} & -\frac{p_{4\leftrightarrow 6}}{p_{4\leftrightarrow 1}} & \frac{p_{4\leftrightarrow 6}}{p_{4\leftrightarrow 1}} & \frac{p_{4\leftrightarrow 6}}{p_{4\leftrightarrow 1}} & -\frac{p_{4\leftrightarrow 6}}{p_{4\leftrightarrow 1}} \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{p_{5\leftrightarrow 6}}{p_{5\leftrightarrow 4}} & 1+\frac{p_{5\leftrightarrow 6}}{p_{5\leftrightarrow 4}}
\end{pmatrix}$$

In[91]:=
```
"det(Λc)="
Simplify[det = Det[Λc]] // forma
```

Out[91]= $\det(\Lambda_c) =$

Out[92]//TableForm=

$$-\frac{1}{p_{1,2}^2\, p_{3,2}^2\, p_{4,1}^3\, p_{5,4}}$$

$$\begin{aligned}
&(p_{1,2}\, p_{3,1}\, p_{3,4}\, (p_{1,3}\, p_{4,1}\, (p_{4,5}\, p_{4,6}\, (p_{5,4}+p_{5,6}) + p_{4,3}\, (p_{4,6}\, p_{5,4} + p_{4,5}\, (p_{5,4}+p_{5,6}))) + \\
&\quad p_{1,4}\, (p_{4,3}\, p_{4,5}\, p_{4,6}\, (p_{5,4}+p_{5,6}) + \\
&\quad\quad p_{4,1}\, (p_{4,5}\, p_{4,6}\, (p_{5,4}+p_{5,6}) + p_{4,3}\, (p_{4,6}\, p_{5,4} + p_{4,5}\, (p_{5,4}+p_{5,6}))))) + \\
&\quad p_{1,3}\, p_{1,4}\, (p_{3,2}\, p_{3,4}\, (p_{4,3}\, p_{4,5}\, p_{4,6}\, (p_{5,4}+p_{5,6}) + \\
&\quad\quad p_{4,1}\, (p_{4,5}\, p_{4,6}\, (p_{5,4}+p_{5,6}) + p_{4,3}\, (p_{4,6}\, p_{5,4} + p_{4,5}\, (p_{5,4}+p_{5,6}))))) + \\
&\quad p_{3,1}\, (p_{3,2}\, p_{4,3}\, p_{4,5}\, p_{4,6}\, (p_{5,4}+p_{5,6}) + p_{3,4}\, (p_{4,3}\, p_{4,5}\, p_{4,6}\, (p_{5,4}+p_{5,6}) + \\
&\quad\quad p_{4,1}\, (p_{4,5}\, p_{4,6}\, (p_{5,4}+p_{5,6}) + p_{4,3}\, (p_{4,6}\, p_{5,4} + p_{4,5}\, (p_{5,4}+p_{5,6})))))))
\end{aligned}$$

In[68]:=
```
"UT="
utind = Cases[t[[6]], ξ_ /; ξ ≠ 0];
UT = EdgeList[g][[utind]]
```

Out[68]= $U_T =$

Out[70]= $\{3 \leftrightarrow 1,\ 3 \leftrightarrow 2,\ 7 \leftrightarrow 3,\ 4 \leftrightarrow 6,\ 5 \leftrightarrow 6,\ 7 \leftrightarrow 6\}$

In[71]:=
```
"UNb="
UNb = uNb[g, t]
```

Out[71]= $U_{Nb} =$

Out[72]= $\{1 \leftrightarrow 2,\ 1 \leftrightarrow 3,\ 1 \leftrightarrow 4,\ 4 \leftrightarrow 1,\ 3 \leftrightarrow 4,\ 4 \leftrightarrow 3,\ 4 \leftrightarrow 5,\ 5 \leftrightarrow 4,\ 3 \leftrightarrow 7,\ 6 \leftrightarrow 7\}$

In[73]:=
```
A = -λ.{x̃# & /@ EdgeList[g]}^T /. ps;
"A="
A // MatrixForm
```

Out[74]= A=

Out[75]//MatrixForm=

$$\begin{pmatrix} 0 \\ 0 \\ -f_{7,2} - f_{2,7}\left(-1 - \frac{p_{2,1}}{p_{2,7}}\right) + \frac{f_{2,7}\, p_{2,1}\, p_{3,1}}{p_{2,7}\, p_{3,2}} \\ -f_{7,2} - f_{2,7}\left(-1 - \frac{p_{2,1}}{p_{2,7}}\right) \\ 0 \\ 0 \\ -\frac{f_{6,7}\, p_{4,6}\, p_{6,4}}{p_{4,1}\, p_{6,7}} \\ -\frac{f_{6,7}\, p_{5,6}\, p_{6,5}}{p_{5,4}\, p_{6,7}} \end{pmatrix}$$

In[76]:=
```
β = A - Λnc.{x# & /@ U_Nb[[U_nc]]}^T;
"β="
β // forma
```

Out[77]= β=

Out[78]//TableForm=

$$\begin{array}{l} 0 \\ 0 \\ -f_{7,2} - f_{2,7}\left(-1 - \frac{p_{2,1}}{p_{2,7}}\right) + \frac{f_{2,7}\, p_{2,1}\, p_{3,1}}{p_{2,7}\, p_{3,2}} \\ -f_{7,2} - f_{2,7}\left(-1 - \frac{p_{2,1}}{p_{2,7}}\right) \\ 0 \\ 0 \\ -\frac{f_{6,7}\, p_{4,6}\, p_{6,4}}{p_{4,1}\, p_{6,7}} \\ -\frac{f_{6,7}\, p_{5,6}\, p_{6,5}}{p_{5,4}\, p_{6,7}} \end{array}$$

In[79]:=
```
"решаем уравнение Λc xc=β:"
xc = LinearSolve[Λc, β]
```

Out[79]= решаем уравнение $\Lambda_c x_c = \beta$:

Out[80]=

$$\left\{\left\{\left(\cdots 63 \cdots\right) + f_{7,2}\, p_{1,3}\, p_{1,4}\, p_{2,7}\, p_{3,2}\, p_{3,4}\, p_{4,3}\, p_{4,5}\, p_{4,6}\, p_{5,6}\, p_{6,7}\right) \Big/ \left(p_{2,7}\left(p_{1,2}\, p_{1,3}\, p_{3,1}\, p_{3,4}\, p_{4,1}\, p_{4,3}\, p_{4,5}\, p_{5,4} + p_{1,2}\, p_{1,4}\, p_{3,1}\, p_{3,4}\, p_{4,1}\, p_{4,3}\, p_{4,5}\, p_{5,4} + \cdots 24 \cdots + p_{1,3}\, p_{1,4}\, p_{3,1}\, p_{3,4}\, p_{4,3}\, p_{4,5}\, p_{4,6}\, p_{5,6} + p_{1,3}\, p_{1,4}\, p_{3,2}\, p_{3,4}\, p_{4,3}\, p_{4,5}\, p_{4,6}\, p_{5,6}\right) p_{6,7}\right)\right\},$$

$$\left\{-\frac{\cdots 1 \cdots}{\cdots 1 \cdots}\right\},\ \cdots 4 \cdots,\ \left\{\cdots 1 \cdots\right\},\ \left\{\frac{\frac{p_{4,3}\, p_{\cdots 1}\left(\cdots 1 \cdots - \cdots 1 \cdots\right)}{p_{4,1}\, p_{5\ \cdots 1\ \cdots 1}} + \cdots 1 \cdots}{-\frac{p_{1,4\ \cdots 4 \cdots}\, p_{\cdots 1}}{p_{1,2\ \cdots 2 \cdots}\, p_{5\ \cdots 1\ \cdots 1}} - \cdots 1 \cdots}\right\}\Big\}$$

large output | show less | show more | show all | set size limit...

In[81]:=
```
xcp = MapThread[x_#1 → #2 &, {U_Nb[[U_c]], Flatten[xc]}];
xcp // TableForm
```

Out[82]//TableForm=

$$\cdots 1 \cdots$$

large output | show less | show more | show all | set size limit...

In[83]:=
```
s = solveAll[g, t];
s // TableForm
```

Out[84]//TableForm=

$$x_{3\bullet\to 2} \to f_{7\bullet\to 2} + f_{2\bullet\to 7}\left(-1 - \frac{p_{2\bullet\to 1}}{p_{2\bullet\to 7}}\right) - x_{1\bullet\to 2}$$

$$x_{3\bullet\to 1} \to \frac{f_{2\bullet\to 7}\, p_{2\bullet\to 1}}{p_{2\bullet\to 7}} + x_{1\bullet\to 2} + x_{1\bullet\to 3} + x_{1\bullet\to 4} - x_{4\bullet\to 1}$$

$$x_{5\bullet\to 6} \to -\frac{f_{6\bullet\to 7}\, p_{6\bullet\to 5}}{p_{6\bullet\to 7}} + x_{4\bullet\to 5} - x_{5\bullet\to 4}$$

$$x_{4\bullet\to 6} \to -\frac{f_{6\bullet\to 7}\, p_{6\bullet\to 4}}{p_{6\bullet\to 7}} + x_{1\bullet\to 4} + x_{3\bullet\to 4} - x_{4\bullet\to 1} - x_{4\bullet\to 3} - x_{4\bullet\to 5} + x_{5\bullet\to 4}$$

$$x_{7\bullet\to 3} \to f_{7\bullet\to 2} + f_{2\bullet\to 7}\left(-1 - \frac{p_{2\bullet\to 1}}{p_{2\bullet\to 7}}\right) + \frac{f_{2\bullet\to 7}\, p_{2\bullet\to 1}}{p_{2\bullet\to 7}} + \frac{f_{2\bullet\to 7}\, p_{2\bullet\to 3}}{p_{2\bullet\to 7}} + x_{1\bullet\to 4} + x_{3\bullet\to 4} + x_{3\bullet\to 7} - x_{4\bullet\to 1} - x_{4\bullet\to 3}$$

$$x_{7\bullet\to 6} \to f_{7\bullet\to 6} + f_{6\bullet\to 7}\left(-1 - \frac{p_{6\bullet\to 5}}{p_{6\bullet\to 7}}\right) + \frac{f_{6\bullet\to 7}\, p_{6\bullet\to 4}}{p_{6\bullet\to 7}} + \frac{f_{6\bullet\to 7}\, p_{6\bullet\to 5}}{p_{6\bullet\to 7}} - x_{1\bullet\to 4} - x_{3\bullet\to 4} + x_{4\bullet\to 1} + x_{4\bullet\to 3} + x_{6\bullet\to 7}$$

In[85]:=
```
"общее решение:"
xsol = (s ~Join~ xcp);
xsol /. {ξ_u_↦v_ → ξu,v} // Simplify // TableForm
```

Out[85]= общее решение:

Out[87]//TableForm=

$$x_{3,2} \to f_{7,2} + f_{2,7}\left(-1 - \frac{p_{2,1}}{p_{2,7}}\right) - x_{1,2}$$

$$x_{3,1} \to \frac{f_{2,7}\, p_{2,1}}{p_{2,7}} + x_{1,2} + x_{1,3} + x_{1,4} - x_{4,1}$$

$$x_{5,6} \to -\frac{f_{6,7}\, p_{6,5}}{p_{6,7}} + x_{4,5} - x_{5,4}$$

$$x_{4,6} \to -\frac{f_{6,7}\, p_{6,4}}{p_{6,7}} + x_{1,4} + x_{3,4} - x_{4,1} - x_{4,3} - x_{4,5} + x_{5,4}$$

$$x_{7,3} \to f_{7,2} + f_{2,7}\left(-1 + \frac{p_{2,3}}{p_{2,7}}\right) + x_{1,4} + x_{3,4} + x_{3,7} - x_{4,1} - x_{4,3}$$

$$x_{7,6} \to f_{7,6} + f_{6,7}\left(-1 + \frac{p_{6,4}}{p_{6,7}}\right) - x_{1,4} - x_{3,4} + x_{4,1} + x_{4,3} + x_{6,7}$$

$$x_{1,2} \to -\frac{p_{1,3}\, p_{1,4}\,(f_{6,7}\, p_{2,7}\, p_{3,1}\, p_{3,4}\, p_{4,3}\, p_{4,5}\, p_{4,6}\,(p_{5,4}\, p_{6,4}+p_{5,6}\,(p_{6,4}+p_{6,5}))+(-f_{7,2}\, p_{2,7}\, p_{3,2}\,(p_{3,1}\, p_{4,3}\, p_{4,5}\, p_{4,6}\,(p_{5,4}+p_{5,6})+p_{3,4}\,(p_{4,3}\, p_{4,5}\, p}{p_{2,7}\,(p_{1,2}\, p_{3,1}\, p_{3,4}\,(p_{1,3}\, p_{4,1}\,(p_{4,5}\, p_{4,6}\,(p_{5,4}+p_{5}}$$

$$x_{1,3} \to -\frac{p_{1,2}\, p_{1,4}\,(f_{6,7}\, p_{2,7}\, p_{3,1}\, p_{3,4}\, p_{4,3}\, p_{4,5}\, p_{4,6}\,(p_{5,4}\, p_{6,4}+p_{5,6}\,(p_{6,4}+p_{6,5}))+(-f_{7,2}\, p_{2,7}\, p_{3,2}\,(p_{3,1}\, p_{4,3}\, p_{4,5}\, p_{4,6}\,(p_{5,4}+p_{5,6})+p_{3,4}\,(p_{4,3}\, p_{4,5}\, p}{p_{2,7}\,(p_{1,2}\, p_{3,1}\, p_{3,4}\,(p_{1,3}\, p_{4,1}\,(p_{4,5}\, p_{4,6}\,(p_{5,4}+p_{5}}$$

$$x_{1,4} \to -\frac{p_{1,2}\, p_{1,3}\,(f_{6,7}\, p_{2,7}\, p_{3,1}\, p_{3,4}\, p_{4,3}\, p_{4,5}\, p_{4,6}\,(p_{5,4}\, p_{6,4}+p_{5,6}\,(p_{6,4}+p_{6,5}))+(-f_{7,2}\, p_{2,7}\, p_{3,2}\,(p_{3,1}\, p_{4,3}\, p_{4,5}\, p_{4,6}\,(p_{5,4}+p_{5,6})+p_{3,4}\,(p_{4,3}\, p_{4,5}\, p}{p_{2,7}\,(p_{1,2}\, p_{3,1}\, p_{3,4}\,(p_{1,3}\, p_{4,1}\,(p_{4,5}\, p_{4,6}\,(p_{5,4}+p_{5}}$$

$$x_{4,1} \to -\frac{p_{4,3}\, p_{4,5}\, p_{4,6}\,(f_{6,7}\, p_{2,7}\,(p_{1,2}\,(p_{1,3}+p_{1,4})\, p_{3,1}+p_{1,3}\, p_{1,4}\,(p_{3,1}+p_{3,2}))\, p_{3,4}}{p_{2,7}\,(p_{1,2}\, p_{3,1}\, p_{3,4}\,(p_{1,3}\, p_{4,1}\,(p_{4,5}\, p_{4,6}\,(p_{5,4}+p_{5,6})+p_{4,3}\,(p_{4,6}\, p_{5,4}+p_{4,5}\,(p_{5,4}+p_{5,6})))+p_{1,4}\,(p_{4,3}\, p_{4,5}\, p_{4,6}\,(p_{5,4}+p_{5,6})+p_{4,1}\,(p_{4,5}\, p_{4,6}$$

$$x_{3,4} \to -\frac{p_{3,1}\, p_{3,2}\,(-f_{6,7}\, p_{1,3}\, p_{1,4}\, p_{2,7}\, p_{4,3}\, p_{4,5}\, p_{4,6}\,(p_{5,4}\, p_{6,4}+p_{5,6}\,(p_{6,4}+p_{6,5}))+(-f_{7,2}\, p_{2,7}\,(p_{1,3}\, p_{1,4}\,(p_{4,3}\, p_{4,5}\, p_{4,6}\,(p_{5,4}+p_{5,6})+p_{4,1}\,(p_{4,5}\, p_{4,}}{p_{2,7}\,(p_{1,2}\, p_{3,1}\, p_{3,4}\,(p_{1,}}$$

$$x_{4,3} \to -\frac{p_{4,1}\, p_{4,5}\, p_{4,6}\,(f_{6,7}\, p_{2,7}\,(p_{1,2}\,(p_{1,3}+p_{1,4})\, p_{3,1}+p_{1,3}\, p_{1,4}\,(p_{3,1}+p_{3,2}))\, p_{3,4}}{p_{2,7}\,(p_{1,2}\, p_{3,1}\, p_{3,4}\,(p_{1,3}\, p_{4,1}\,(p_{4,5}\, p_{4,6}\,(p_{5,4}+p_{5,6})+p_{4,3}\,(p_{4,6}\, p_{5,4}+p_{4,5}\,(p_{5,4}+p_{5,6})))+p_{1,4}\,(p_{4,3}\, p_{4,5}\, p_{4,6}\,(p_{5,4}+p_{5,6})+p_{4,1}\,(p_{4,5}\, p_{4,6}$$

$$x_{4,5} \to -\frac{p_{4,1}\, p_{4,3}\, p_{4,6}\,(f_{6,7}\, p_{2,7}\,(p_{1,2}\,(p_{1,3}+p_{1,4})\, p_{3,1}+p_{1,3}\, p_{1,4}\,(p_{3,1}+p_{3,2}))\, p_{3,4}}{p_{2,7}\,(p_{1,2}\, p_{3,1}\, p_{3,4}\,(p_{1,3}\, p_{4,1}\,(p_{4,5}\, p_{4,6}\,(p_{5,4}+p_{5,6})+p_{4,3}\,(p_{4,6}\, p_{5,4}+p_{4,5}\,(p_{5,4}+p_{5,6})))+p_{1,4}\,(p_{4,3}\, p_{4,5}\, p_{4,6}\,(p_{5,4}+p_{5,6})+p_{4,1}\,(p_{4,5}\, p_{4,6}$$

$$x_{5,4} \to -\frac{p_{5,6}\,(f_{6,7}\, p_{2,7}\,(p_{1,2}\, p_{3,1}\, p_{3,4}\,(p_{1,3}\, p_{4,1}\,(p_{4,5}\, p_{4,6}\, p_{6,5}+p_{4,3}\,(p_{4,5}\, p_{6,5}+p_{4,6}\,(p_{6,4}+p_{6,5})))+p_{1,4}\,(p_{4,3}\, p_{4,5}\, p_{4,6}\, p_{6,5}+p_{4,1}\,(p_{4,5}\, p_{4,6}\, p_{6,5}+|}{p_{2,7}\,(p_{1,2}\, p_{3,1}\, p_{3,4}\,(p_{1,3}\, p_{4,1}\,(p_{4,5}\, p_{4,6}\,(p_{5,4}+p_{5,6})+p_{4,3}$$

In[88]:=
```
"eq test:"
Simplify[balanceEqs /. ξ → 7 /. s /. xcp]
Simplify[(dopEq /. s) /. xcp]
```

Out[88]= eq test:

Out[89]= {True, True, True, True, True, True, True}

Out[90]= {True, True, True, True, True, True, True}

In[ ]:=