Листинг 2

In[1]:=
```
ClearAll["Global`*"]
SetDirectory[NotebookDirectory[]];
Needs["FlowSolver`"]
```

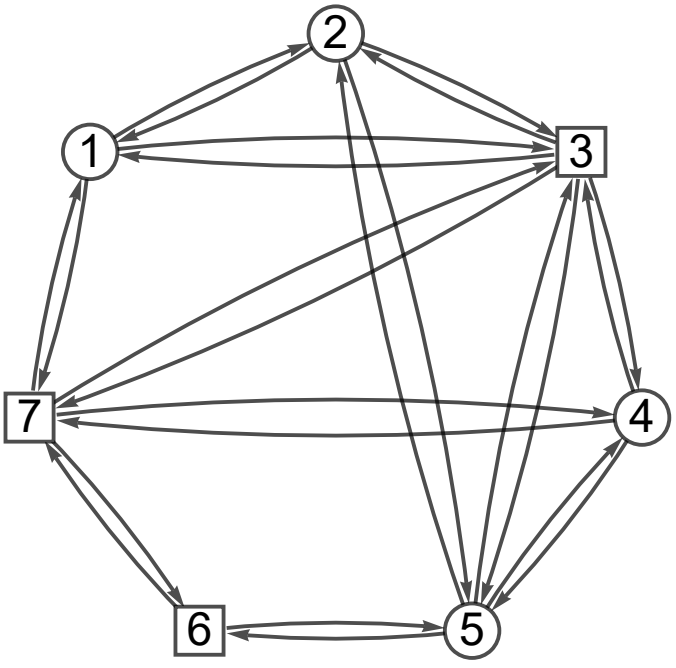
In[4]:=
```
readGraph2[file_, dir_] := Module[{
    fn = FileNameJoin[{dir, file}],
    stream, imod, umod, u, b
    },
    stream = OpenRead[fn];
    imod = Read[stream, {Word, Number}][[2]];
    umod = Read[stream, {Word, Number}][[2]];

  u = ({#[[1]] ↔ #[[2]], #[[2]] ↔ #[[1]]} & /@ ReadList[stream, Expression, umod]) // Flatten;
  b = ConstantArray[0, imod];
    (b[[Read[StringToStream[StringTake[#1, {5, -3}]], Number]]] = #2) &@@@
   ReadList[stream, {Word, Expression}, imod];
  {Graph[u, VertexSize -> Medium, VertexLabels → Placed["Name", Center],
    VertexStyle → Directive[White],
    VertexShapeFunction → {xx_ :> If[SameQ[b[[xx]], x], "Square", "Circle"]},
    VertexLabelStyle -> Directive[Black, 24], GraphLayout -> "CircularEmbedding"], b}]
```

In[5]:=
```
forma[ff_] := ((ff /. {ς_u_↔v_ → ς_u,v}) // TableForm)
```

In[133]:=
```
{g, b} = readGraph2["gr.txt", NotebookDirectory[]];
GraphPlot[g, EdgeStyle → Directive[Black, Thick],
 VertexStyle → Directive[EdgeForm[Thick], White], MultiedgeStyle → .05]
```

Out[134]=



In[135]:=
```
balanceEqs = ((Total[x_# & /@ EdgeList[g, _ ↔ #]] - Total[x_# & /@ EdgeList[g, # ↔ _]])) ==
      MapIndexed[#1 /. x → x_#2[[1]] &, b][[#]] & /@ VertexList[g];
balanceEqs //
 forma
```

Out[136]//TableForm=

$$-x_{1,2} - x_{1,3} - x_{1,7} + x_{2,1} + x_{3,1} + x_{7,1} == 0$$
$$x_{1,2} - x_{2,1} - x_{2,3} - x_{2,5} + x_{3,2} + x_{5,2} == 0$$
$$x_{1,3} + x_{2,3} - x_{3,1} - x_{3,2} - x_{3,4} - x_{3,5} - x_{3,7} + x_{4,3} + x_{5,3} + x_{7,3} == x_3$$
$$x_{1,7} + x_{3,7} + x_{4,7} + x_{6,7} - x_{7,1} - x_{7,3} - x_{7,4} - x_{7,6} == x_7$$
$$x_{2,5} + x_{3,5} + x_{4,5} - x_{5,2} - x_{5,3} - x_{5,4} - x_{5,6} + x_{6,5} == 0$$
$$x_{3,4} - x_{4,3} - x_{4,5} - x_{4,7} + x_{5,4} + x_{7,4} == 0$$
$$x_{5,6} - x_{6,5} - x_{6,7} + x_{7,6} == x_6$$

In[137]:=
```
M = {7};
Print["M = ", M];
```

M = {7}

In[139]:=
```
p_# & /@ EdgeList[g]
```

Out[139]= $\left\{\frac{1}{6}, \frac{1}{4}, \frac{1}{2}, \frac{1}{9}, \frac{1}{3}, \frac{1}{10}, \frac{1}{4}, \frac{2}{9}, \frac{1}{2}, \frac{1}{5}, \frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{2}, \frac{2}{9}, \frac{2}{5}, \frac{1}{6}, \frac{1}{10}, \frac{2}{3}, \frac{3}{10}, \frac{1}{5}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}\right\}$

In[140]:=
```
{p_1↔2 = 1/6, p_2↔1 = 1/4,
 p_1↔3 = 1/2, p_3↔1 = 1/9,
 p_1↔7 = 1/3, p_7↔1 = 1/10,
 p_2↔3 = 1/4, p_3↔2 = 2/9,
 p_2↔5 = 1/2, p_5↔2 = 1/5,
 p_3↔4 = 1/3, p_4↔3 = 1/6,
 p_3↔5 = 1/9, p_5↔3 = 1/2,
 p_3↔7 = 2/9, p_7↔3 = 2/5,
 p_4↔5 = 1/6, p_5↔4 = 1/10,
 p_4↔7 = 2/3, p_7↔4 = 3/10,
 p_5↔6 = 1/5, p_6↔5 = 1/4,
 p_6↔7 = 3/4, p_7↔6 = 1/5}
```

Out[140]= $\left\{\frac{1}{6}, \frac{1}{4}, \frac{1}{2}, \frac{1}{9}, \frac{1}{3}, \frac{1}{10}, \frac{1}{4}, \frac{2}{9}, \frac{1}{2}, \frac{1}{5}, \frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{2}, \frac{2}{9}, \frac{2}{5}, \frac{1}{6}, \frac{1}{10}, \frac{2}{3}, \frac{3}{10}, \frac{1}{5}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}\right\}$

In[141]:=
```
(*incL=
 DeleteCases[DeleteDuplicates[Cases[IncidenceList[g,#],i_↔j_:>{i,j}]//Flatten],
    v_/;v==#]&/@M*)
incL = (IncidenceList[g, #] & /@ M) // Flatten
```

Out[141]= {1 ↔ 7, 7 ↔ 1, 3 ↔ 7, 7 ↔ 3, 4 ↔ 7, 7 ↔ 4, 6 ↔ 7, 7 ↔ 6}

In[85]:=
```
f_# & /@ incL
```

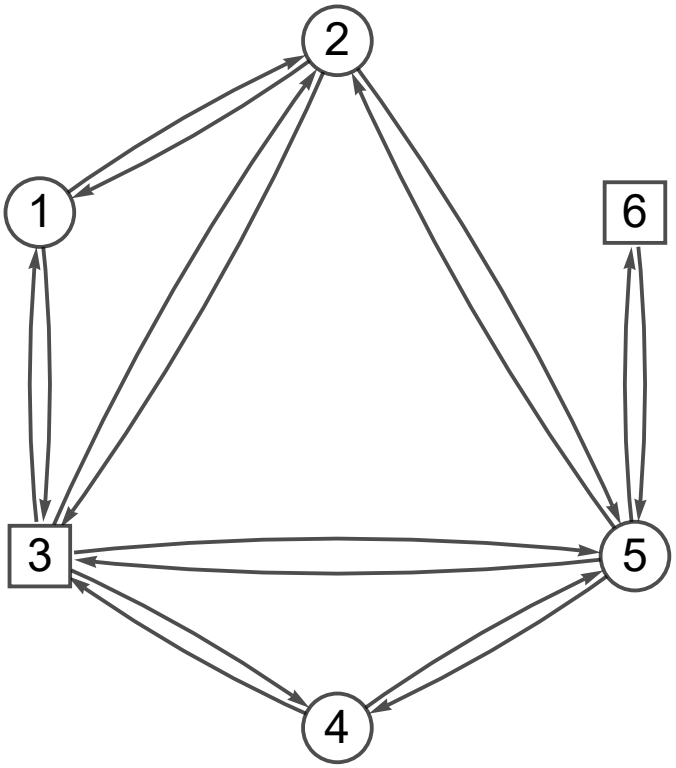
Out[85]= {3, 2, 2, 2, 2, 4, 8, 6}

In[142]:=
```
{f_1↔7, f_7↔1, f_3↔7, f_7↔3, f_4↔7, f_7↔4, f_6↔7, f_7↔6} = {
  2, 3,
  2, 2,
  4, 2,
  6, 8}
```

Out[142]= {2, 3, 2, 2, 4, 2, 6, 8}

In[143]:=
```
(*Do[If[MemberQ[M,j[[1]]],b[[j[[2]]]]+=f_j,b[[j[[1]]]]-=f_j],{j,incL}]*)
b̄ = Fold[If[MemberQ[M, #2[[1]]], ReplacePart[#, #2[[2]] → #[[#2[[2]]]] - f_#2],
      ReplacePart[#, #2[[1]] → #[[#2[[1]]]] + f_#2]] &, b, incL];
b̄ = Delete[b̄, #] & @@ M;
n̄ḡ = VertexDelete[g, M];
GraphPlot[n̄ḡ, EdgeStyle → Directive[Black, Thick],
 VertexStyle → Directive[EdgeForm[Thick], White], MultiedgeStyle → .05]
b̄
```

Out[146]=



Out[147]= $\{-1, 0, x, 2, 0, -2 + x\}$

In[148]:=
```
CC[g_, M_] :=
  (DeleteDuplicates[Cases[IncidenceList[g, #], i_ ↔ j_ /; j == #]] & /@ M) // Flatten

ii_i^+[g_] := Cases[IncidenceList[g, i], u_ ↔ v_ /; u == i :↔ v]
```

In[150]:=
```
M⁺ = CC[g, M]
```

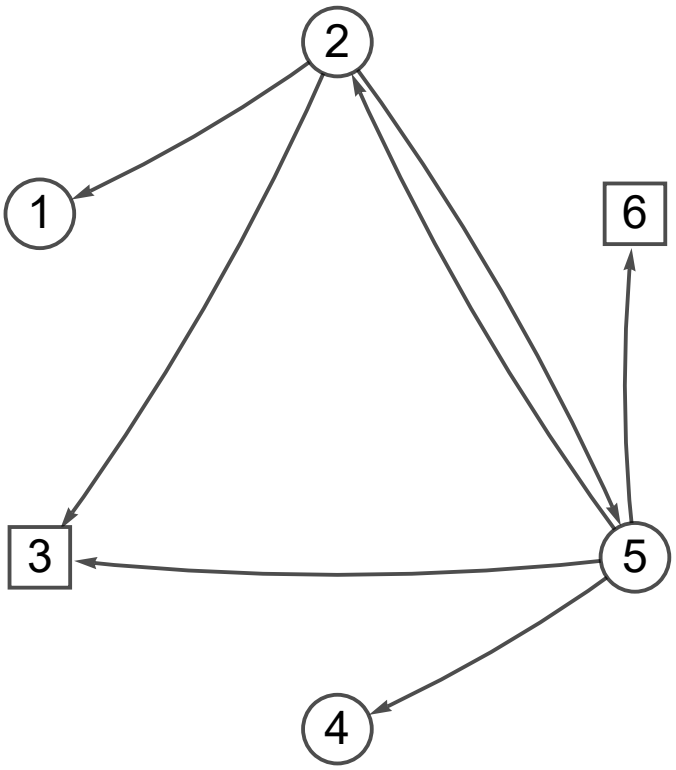
Out[150]= $\{1 \leftrightarrow 7, 3 \leftrightarrow 7, 4 \leftrightarrow 7, 6 \leftrightarrow 7\}$

In[151]:=
```
b1̄ = Fold[
  Module[{bb = #1, i = #2[[1]], k = #2[[2]]}, (Fold[Module[{bbb = #1, jj = #2}, ReplacePart[
      bbb, ((({jj → bbb[[jj]] - (p_{i↔jj}/p_{i↔k}) f_{i↔k}, i → bbb[[i]] + (p_{i↔jj}/p_{i↔k}) f_{i↔k}}))) //
       Flatten]] &, bb, ii_i^+[n̄ḡ]])] &, b̄, M⁺]
```

Out[151]= $\{2, -3, 3 + x, 1, -4, x\}$

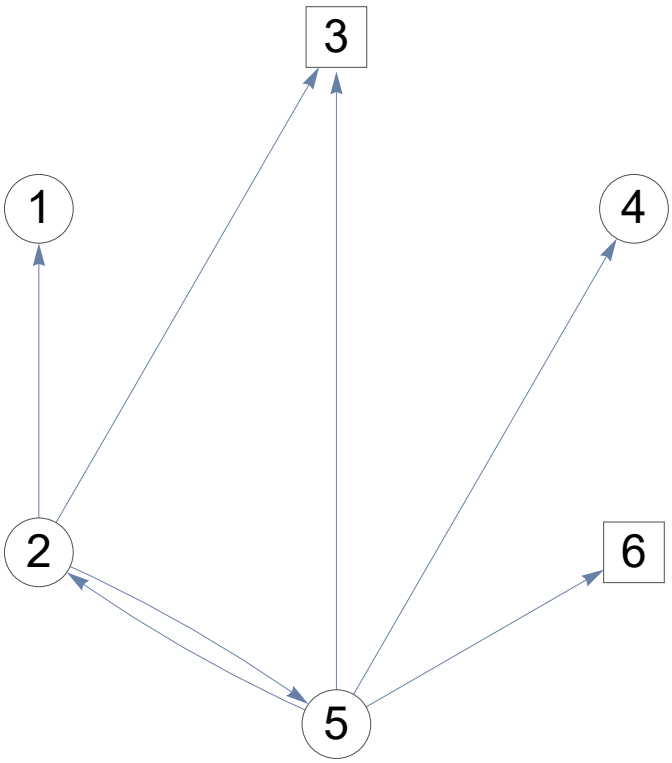
In[152]:=
```
GraphPlot[Fold[HighlightGraph[#1, u_ ↔ v_ /; u == #2, GraphHighlightStyle → "White"] &,
  ng, #[[1]] & /@ M+], EdgeStyle → Directive[Black, Thick],
 VertexStyle → Directive[EdgeForm[Thick], White], MultiedgeStyle → .05]
```

Out[152]=



In[153]:=
```
g1 = Fold[EdgeDelete[#1, u_ ↔ v_ /; u == #2] &, ng, #[[1]] & /@ M+];
GraphPlot[g1, MultiedgeStyle → .05]
```

Out[154]=



In[155]:=
```
IIrem = VertexList[g1] ~Complement~ (M+[[All, 1]])
```

Out[155]=
```
{2, 5}
```

In[180]:=
```
λ = SparseArray[
  Replace[(EdgeList[g1] /. # & /@ Flatten[Module[{i = #, jf, Icur},
      (Icur = ii[g1];
       jf = First[Icur];
       ({(i ↔ jf) → 1, (i ↔ #) → -p(i↔#)/p(i↔jf)}) & /@ Icur[[2 ;;]])] & /@ IIrem, 1]),
   _ ↔ _ → 0, 2]]
```

Out[180]=
```
SparseArray[  Specified elements: 10
Dimensions: {5, 7} ]
```

In[157]:=
```
Grid[λ]
```

Out[157]=

1	−1	0	0	0	0	0
1	0	−2	0	0	0	0
0	0	0	1	$-\frac{5}{2}$	0	0
0	0	0	1	0	$-\frac{1}{2}$	0
0	0	0	1	0	0	−1

In[158]:=
```
g = g1;
b = b1;
```

In[160]:=
```
II* = Cases[MapIndexed[{#1, #2} &, b],
    {el_, i_} /; MemberQ[el, x] || SameQ[el, x] :> i] // Flatten
```

Out[160]=
```
{3, 6}
```

In[161]:=
```
buildt = Timing[{t, g} = buildTree[g, II*];][[1]]
TableForm[t[[1 ;; 4]],
 TableHeadings → {{"pred", "dir", "depth", "d"}, t // pred // Length // Range}]
```

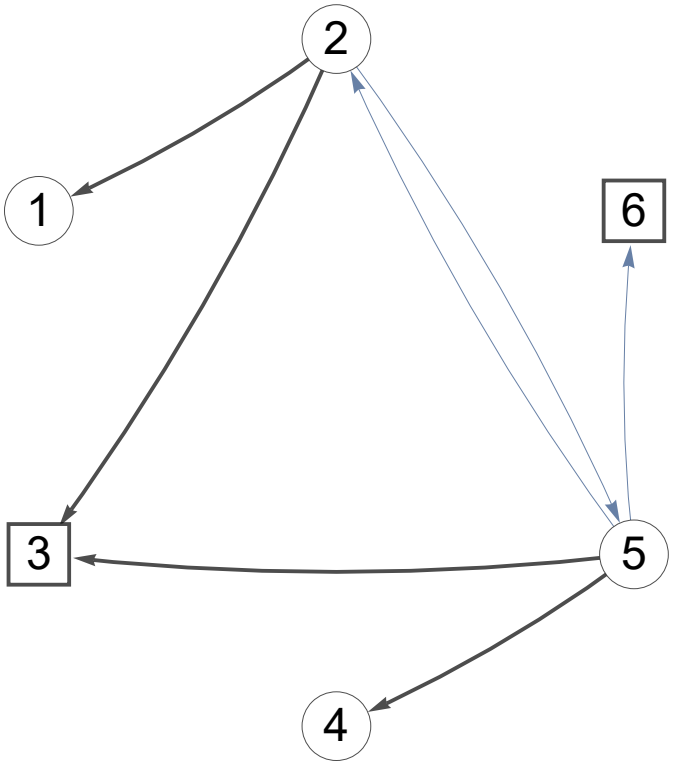
Out[161]=
```
0.
```

Out[162]//TableForm=

	1	2	3	4	5	6	7
pred	2	3	7	5	3	7	0
dir	1	−1	1	1	−1	1	0
depth	3	2	1	3	2	1	0
d	5	1	2	6	4	7	3

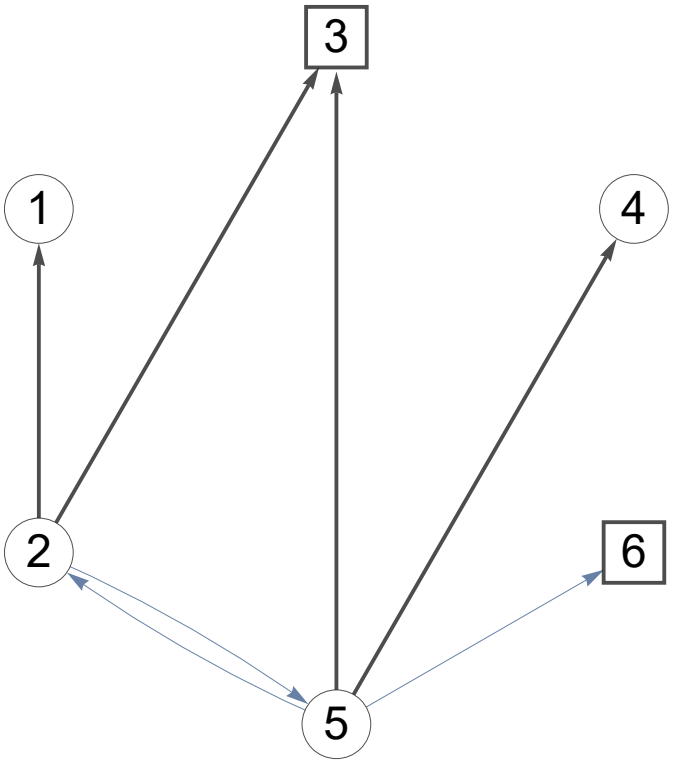
In[163]:=
```
GraphPlot[HighlightGraph[
  Fold[HighlightGraph[#1, Style[u_ ↦ v_ /; u == #2, White]] &, ng, #[[1]] & /@ M+],
  {Style[u_ /; VertexQ[g, u] && pred[t][[u]] == root[t], EdgeForm[Thick]],
   Style[u_ ↦ v_ /; (pred[t][[u]] == v && dir[t][[u]] == -1) ||
       (pred[t][[v]] == u && dir[t][[v]] == 1), Directive[Black, Thick]]},
  GraphHighlightStyle → None], MultiedgeStyle → .05]
```

Out[163]=



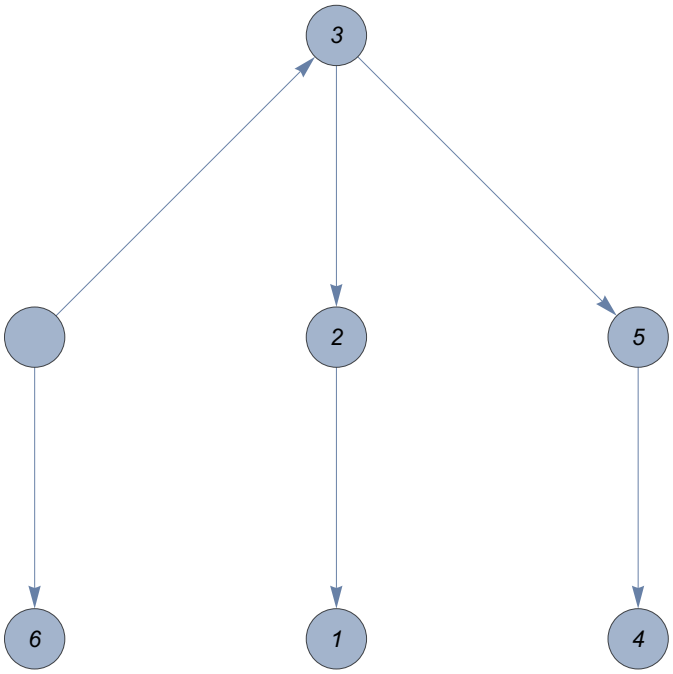
In[164]:=
```
GraphPlot[HighlightGraph[g1,
  {Style[u_ /; VertexQ[g, u] && pred[t][[u]] == root[t], EdgeForm[Thick]],
   Style[u_ ↔ v_ /; (pred[t][[u]] == v && dir[t][[u]] == -1) ||
       (pred[t][[v]] == u && dir[t][[v]] == 1), Directive[Black, Thick]]},
  GraphHighlightStyle → None], MultiedgeStyle → .05]
```

Out[164]=



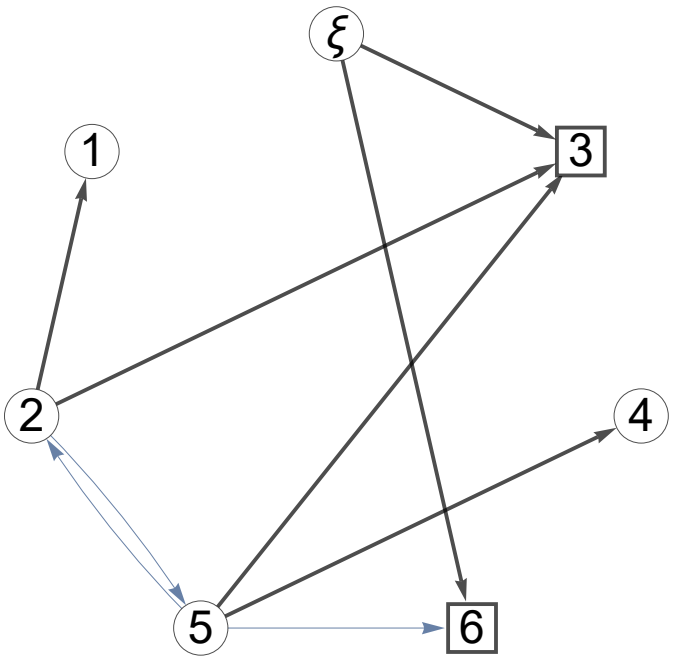
In[165]:=
```
t[[7]](*пометить на графе*)
```

Out[165]=



In[166]:=
```
(*GraphPlot[g,MultiedgeStyle→.05]*)
GraphPlot[HighlightGraph[g,
  {Style[u_ /; VertexQ[g, u] && pred[t][[u]] == root[t], EdgeForm[Thick]],
   Style[u_ ↔ v_ /; (pred[t][[u]] == v && dir[t][[u]] == -1) ||
       (pred[t][[v]] == u && dir[t][[v]] == 1), Directive[Black, Thick]]},
  GraphHighlightStyle → None], MultiedgeStyle → .05]
```

Out[166]=



In[167]:=
```
AppendTo[b, -Total[b]];
b = Simplify[b /. x → 0]
```

Out[168]=
```
{2, -3, 3, 1, -4, 0, 1}
```

In[169]:=
```
balanceEqs = ((Total[x_# & /@ EdgeList[g, _ ↔ #]] - Total[x_# & /@ EdgeList[g, # ↔ _]]) /.
      root[t] → ξ) == b[[#]] & /@ VertexList[g];
balanceEqs //
 forma
```

Out[170]//TableForm=

$$x_{2,1} == 2$$
$$-x_{2,1} - x_{2,3} - x_{2,5} + x_{5,2} == -3$$
$$x_{2,3} + x_{5,3} + x_{\xi,3} == 3$$
$$x_{2,5} - x_{5,2} - x_{5,3} - x_{5,4} - x_{5,6} == -4$$
$$x_{5,4} == 1$$
$$x_{5,6} + x_{\xi,6} == 0$$
$$-x_{\xi,3} - x_{\xi,6} == 1$$

In[171]:=
```
ps = partSolve[g, -b, t, x̃];
ps // forma
```

Out[172]//TableForm=

$$\tilde{x}_{2,1} \to 2$$
$$\tilde{x}_{2,3} \to 1$$
$$\tilde{x}_{2,5} \to 0$$
$$\tilde{x}_{5,2} \to 0$$
$$\tilde{x}_{5,3} \to 3$$
$$\tilde{x}_{5,4} \to 1$$
$$\tilde{x}_{5,6} \to 0$$
$$\tilde{x}_{7,3} \to -1$$
$$\tilde{x}_{7,6} \to 0$$

In[173]:=
```
Simplify[(balanceEqs /. {x → x̃, ξ → root[t]}) /. ps]
```

Out[173]=
```
{True, True, True, True, True, True, True}
```

In[174]:=
```
matrt = Timing[δMatr = δ1[g, t]];
rootr = VertexCount[g];
TableForm[δMatr, TableHeadings → {uNb[g, t], δ[#[[2]] & /@ Select[EdgeList[g], #[[1]] == rootr &]]}] // forma
```

Out[176]//TableForm=

	$\delta_{2,1}$	$\delta_{2,3}$	$\delta_{2,5}$	$\delta_{5,2}$	$\delta_{5,3}$	$\delta_{5,4}$	$\delta_{5,6}$	δ_{3}	δ_{6}
$2 \bullet\!\!-\!\!\bullet 5$	0	−1	1	0	1	0	0	0	0
$5 \bullet\!\!-\!\!\bullet 2$	0	1	0	1	−1	0	0	0	0
$5 \bullet\!\!-\!\!\bullet 6$	0	0	0	0	−1	0	1	1	−1

```
λ = SparseArray[λ, {Length[λ], Length[λ[[1]]] + Length[II*]}];
(*λ=λ[[;;-2]]*)
```

In[182]:=
```
dopEq = # == 0 & /@ Flatten[λ.{x_# & /@ EdgeList[g]}^T];
dopEq // forma
```

Out[183]//TableForm=

$$x_{2,1} - x_{2,3} == 0$$
$$x_{2,1} - 2\,x_{2,5} == 0$$
$$x_{5,2} - \frac{5\,x_{5,3}}{2} == 0$$
$$x_{5,2} - \frac{x_{5,4}}{2} == 0$$
$$x_{5,2} - x_{5,6} == 0$$

In[184]:=
```
Λ = λ.(δMatr)^T;
"cicle det's:"
Λ // forma
```

Out[185]=
```
cicle det's:
```

Out[186]//TableForm=

$$\begin{array}{ccc} 1 & -1 & 0 \\ -2 & 0 & 0 \\ -\frac{5}{2} & \frac{7}{2} & \frac{5}{2} \\ 0 & 1 & 0 \\ 0 & 1 & -1 \end{array}$$

In[187]:=
```
"Uc="
Uc = {1, 2, 3}
"Unc="
Unc = {}
```

Out[187]= $U_c =$

Out[188]= $\{1, 2, 3\}$

Out[189]= $U_{nc} =$

Out[190]= $\{\}$

In[191]:=
```
MatrixRank[Λ]
```

Out[191]= 3

In[192]:=
```
Λc = Λ[[{1, 2, 3}, Uc]];
Λnc = Λ[[{1, 2, 3}, Unc]];
"Λc="
Λc // MatrixForm
```

Out[194]= $\Lambda_c =$

Out[195]//MatrixForm=

$$\begin{pmatrix} 1 & -1 & 0 \\ -2 & 0 & 0 \\ -\frac{5}{2} & \frac{7}{2} & \frac{5}{2} \end{pmatrix}$$

In[196]:=
```
"det(Λc)="
Simplify[det = Det[Λc]] // forma
```

Out[196]= $\det(\Lambda_c) =$

Out[197]//TableForm=

-5

In[198]:=
```
"UT="
utind = Cases[t[[6]], ξ_ /; ξ ≠ 0];
UT = EdgeList[g][[utind]]
```

Out[198]= $U_T =$

Out[200]= $\{2 \leftrightarrow 1, 2 \leftrightarrow 3, 7 \leftrightarrow 3, 5 \leftrightarrow 4, 5 \leftrightarrow 3, 7 \leftrightarrow 6\}$

In[201]:=
```
"UNb="
UNb = uNb[g, t]
```

Out[201]= $U_{Nb} =$

Out[202]= $\{2 \leftrightarrow 5, 5 \leftrightarrow 2, 5 \leftrightarrow 6\}$

In[203]:=
```
A = -λ. {x̃# & /@ EdgeList[g]}^T /. ps;
"A="
A // MatrixForm
```

Out[204]= $A =$

Out[205]//MatrixForm=

$$\begin{pmatrix} -1 \\ -2 \\ \frac{15}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$$

In[209]:=
```
β = A(∗-Λnc.{x#&/@U_Nb[[U_nc]]}^T∗);
"β="
β // forma
```

Out[210]= $\beta=$

Out[211]//TableForm=

-1
-2
$\frac{15}{2}$
$\frac{1}{2}$
0

In[212]:=
```
"решаем уравнение Λ_c x_c=β:"
xc = LinearSolve[Λc, β[[1 ;; 3]]]
```

Out[212]= решаем уравнение $\Lambda_c x_c=\beta$:

Out[213]= $\left\{\{1\}, \{2\}, \left\{\frac{6}{5}\right\}\right\}$

In[214]:=
```
xcp = MapThread[x_#1 → #2 &, {U_Nb[[U_c]], Flatten[xc]}];
xcp // TableForm
```

Out[215]//TableForm=

$x_{2\bullet\to 5} \to 1$
$x_{5\bullet\to 2} \to 2$
$x_{5\bullet\to 6} \to \frac{6}{5}$

In[216]:=
```
s = solveAll[g, t];
s // TableForm
```

Out[217]//TableForm=

$x_{2\bullet\to 1} \to 2$
$x_{2\bullet\to 3} \to 1 - x_{2\bullet\to 5} + x_{5\bullet\to 2}$
$x_{5\bullet\to 3} \to 3 + x_{2\bullet\to 5} - x_{5\bullet\to 2} - x_{5\bullet\to 6}$
$x_{5\bullet\to 4} \to 1$
$x_{7\bullet\to 3} \to -1 + x_{5\bullet\to 6}$
$x_{7\bullet\to 6} \to -x_{5\bullet\to 6}$

In[218]:=
```
"общее решение:"
xsol = ((s /. xcp) ~Join~ xcp);
xsol /. {ξ_u_↦v_ → ξ_u,v} // Simplify // TableForm
```

Out[218]= общее решение:

Out[220]//TableForm=

$x_{2,1} \to 2$
$x_{2,3} \to 2$
$x_{5,3} \to \frac{4}{5}$
$x_{5,4} \to 1$
$x_{7,3} \to \frac{1}{5}$
$x_{7,6} \to -\frac{6}{5}$
$x_{2,5} \to 1$
$x_{5,2} \to 2$
$x_{5,6} \to \frac{6}{5}$

In[221]:=
```
"eq test:"
Simplify[balanceEqs /. ξ → root[t] /. s /. xcp]
Simplify[(dopEq /. s) /. xcp]
```

Out[221]= eq test:

Out[222]= {True, True, True, True, True, True, True}

Out[223]= {True, True, True, False, False}