

Листинг 2

```

In[1]:= ClearAll["Global`*"]
SetDirectory[NotebookDirectory[]];
Needs["FlowSolver`"]

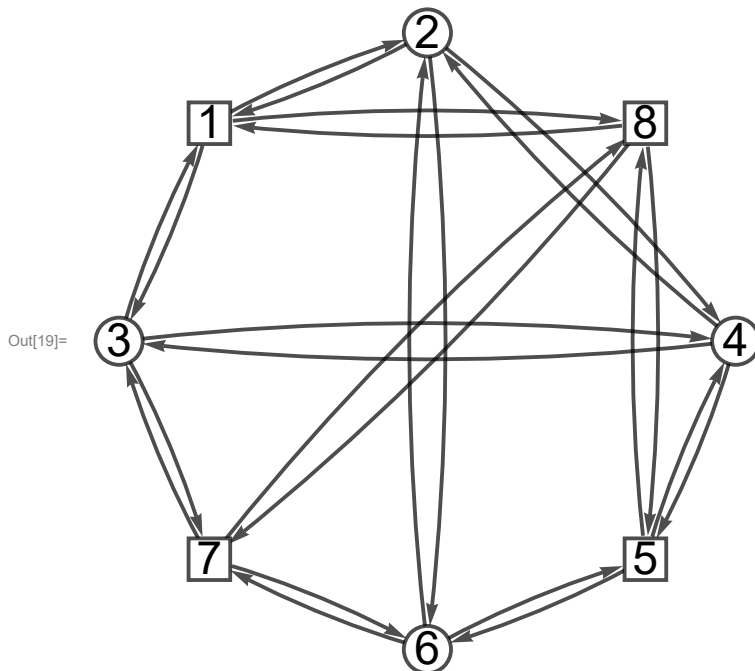
In[4]:= readGraph2[file_, dir_] := Module[{
    fn = FileNameJoin[{dir, file}],
    stream, imod, umod, u, b
},
    stream = OpenRead[fn];
    imod = Read[stream, {Word, Number}][[2]];
    umod = Read[stream, {Word, Number}][[2]];

    u = ({#[[1]] ↔ #[[2]], #[[2]] ↔ #[[1]]} & /@ ReadList[stream, Expression, umod]) // Flatten;
    b = ConstantArray[0, imod];
    (b[[Read[StringToStream[StringTake[#1, {5, -3}]]], Number]] = #2) & @@@
    ReadList[stream, {Word, Expression}, imod];
    {Graph[u, VertexSize -> Medium, VertexLabels -> Placed["Name", Center],
        VertexStyle -> Directive[White],
        VertexShapeFunction -> {xx_ -> If[SameQ[b[[xx]], x], "Square", "Circle"]},
        VertexLabelStyle -> Directive[Black, 24], GraphLayout -> "CircularEmbedding"], b]}

In[5]:= forma[ff_] := ({ff /. {ξu→v -> ξu,v}} // TableForm)

In[18]:= {g, b} = readGraph2["gr.txt", NotebookDirectory[]];
GraphPlot[g, EdgeStyle -> Directive[Black, Thick],
    VertexStyle -> Directive[EdgeForm[Thick], White], MultiedgeStyle -> .05]

```



```
In[20]:= balanceEqs = (Total[x_# & /@ EdgeList[g, _ -> #]] - Total[x_# & /@ EdgeList[g, # -> _]]) ==
      MapIndexed[#1 /. x -> x_#2[[1]] &, b][[#]] & /@ VertexList[g];
balanceEqs //
forma
```

Out[21]/TableForm=

```
- X1,2 - X1,3 - X1,8 + X2,1 + X3,1 + X8,1 == X1
X1,2 - X2,1 - X2,4 - X2,6 + X4,2 + X6,2 == 0
X1,8 + X5,8 + X7,8 - X8,1 - X8,5 - X8,7 == X8
X1,3 - X3,1 - X3,4 - X3,7 + X4,3 + X7,3 == 0
X2,4 + X3,4 - X4,2 - X4,3 - X4,5 + X5,4 == 0
X2,6 + X5,6 - X6,2 - X6,5 - X6,7 + X7,6 == 0
X4,5 - X5,4 - X5,6 - X5,8 + X6,5 + X8,5 == X5
X3,7 + X6,7 - X7,3 - X7,6 - X7,8 + X8,7 == X7
```

```
In[22]:= {p1->2 = 1/4, p2->1 = 2/5,
      p1->8 = 1/2, p8->1 = 1/6,
      p1->3 = 1/4, p3->1 = 2/7,
      p2->4 = 1/5, p4->2 = 3/5,
      p2->6 = 2/5, p6->2 = 1/8,
      p8->5 = 1/2, p5->8 = 1/5,
      p8->7 = 1/3, p7->8 = 1/3,
      p4->5 = 1/5, p5->4 = 3/5,
      p4->3 = 1/5, p3->4 = 2/7,
      p5->6 = 1/5, p6->5 = 3/8,
      p6->7 = 1/2, p7->6 = 1/3,
      p7->3 = 1/3, p3->7 = 3/7}
```

```
Out[22]= {1/4, 2/5, 1/2, 1/6, 1/4, 2/7, 1/5, 3/5, 2/5, 1/8, 1/2, 1/5, 1/3, 1/3, 1/5, 3/5, 1/5, 2/7, 1/5, 3/8, 1/2, 1/3, 1/3, 3/7}
```

```
In[23]:= M = {8};
Print["M = ", M];

M = {8}
```

```
In[25]:= (*incl=
      DeleteCases[DeleteDuplicates[Cases[IncidenceList[g,#],i->j->{i,j}]]//Flatten],
      v_/;v==#]&/@M*)
incl = (IncidenceList[g, #] & /@ M) // Flatten
```

```
Out[25]= {1 -> 8, 8 -> 1, 8 -> 5, 5 -> 8, 8 -> 7, 7 -> 8}
```

```
In[27]:= (f_# & /@ incl)
```

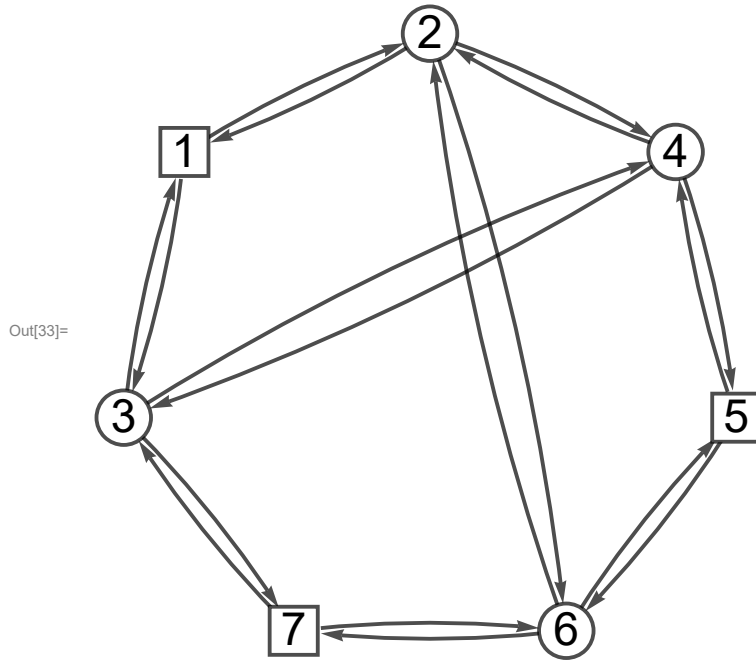
```
In[29]:= {f1->8, f8->1, f8->5, f5->8, f8->7, f7->8} = {1, 2, 1, 3, 1, 2}
```

```
Out[29]= {1, 2, 1, 3, 1, 2}
```

```

In[30]:= (*Do[If[MemberQ[M, j[[1]]], b[[j[[2]]]] += f_j, b[[j[[1]]]] -= f_j], {j, incL}]*
 $\bar{b}$  = Fold[If[MemberQ[M, #2[[1]]], ReplacePart[#, #2[[2]] → #[[#2[[2]]]] + f_#2],
  ReplacePart[#, #2[[1]] → #[[#2[[1]]]] - f_#2]] &, b, incL];
 $\bar{b}$  = Delete[ $\bar{b}$ , #] &@@M;
 $\bar{ng}$  = VertexDelete[g, M];
GraphPlot[ $\bar{ng}$ , EdgeStyle → Directive[Black, Thick],
  VertexStyle → Directive[EdgeForm[Thick], White], MultiedgeStyle → .05]
 $\bar{b}$ 

```



Out[34]= {1 + x, 0, 0, 0, -2 + x, 0, -1 + x}

```

In[35]:= CC[g_, M_] :=
  (DeleteDuplicates[Cases[IncidenceList[g, #], i_ ↔ j_ /; j == #]] & /@ M) // Flatten

```

```

ii_i_ [g_] := Cases[IncidenceList[g, i], u_ ↔ v_ /; u == i ↔ v]

```

```

In[37]:= M* = CC[g, M]

```

Out[37]= {1 ↔ 8, 5 ↔ 8, 7 ↔ 8}

```

In[38]:=  $\bar{b1}$  = Fold[Module[{bb = #1, i = #2[[1]], k = #2[[2]]},
  (ReplacePart[bb, ((({# → bb[[#]] +  $\frac{p_{i \rightarrow \#}}{p_{i \rightarrow k}} f_{i \rightarrow k}$ , i → bb[[i]] -  $\frac{p_{i \rightarrow \#}}{p_{i \rightarrow k}} f_{i \rightarrow k}$ }) &) /@ ii_i_ [ $\bar{ng}$ ]) //
  Flatten]]], &,  $\bar{b}$ , M*]

```

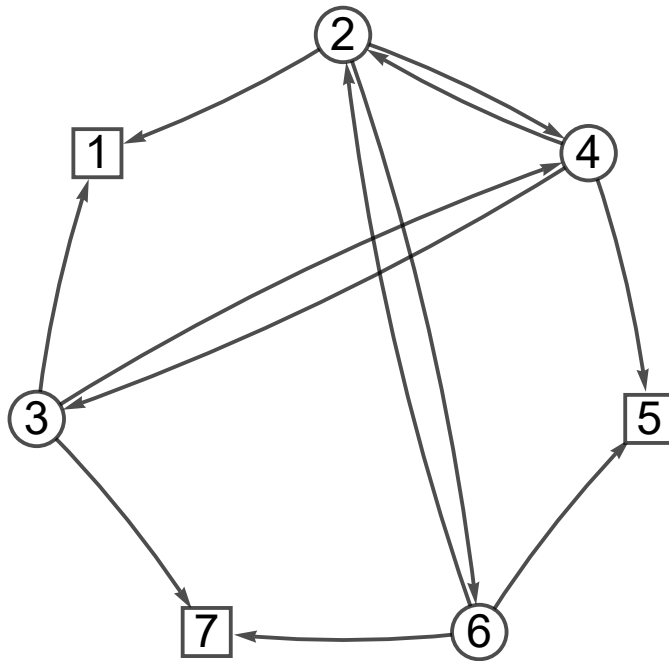
Out[38]= { $\frac{1}{2} + x$, $\frac{1}{2}$, $\frac{5}{2}$, 9, -11 + x, 5, -3 + x}

```

In[39]:= GraphPlot[Fold[HighlightGraph[#1, u_ -> v_ /; u == #2, GraphHighlightStyle -> "White"] &,
   $\overline{n\overline{g}}$ , #[[1]] & /@  $M^+$ ], EdgeStyle -> Directive[Black, Thick],
  VertexStyle -> Directive[EdgeForm[Thick], White], MultiedgeStyle -> .05]

```

Out[39]=

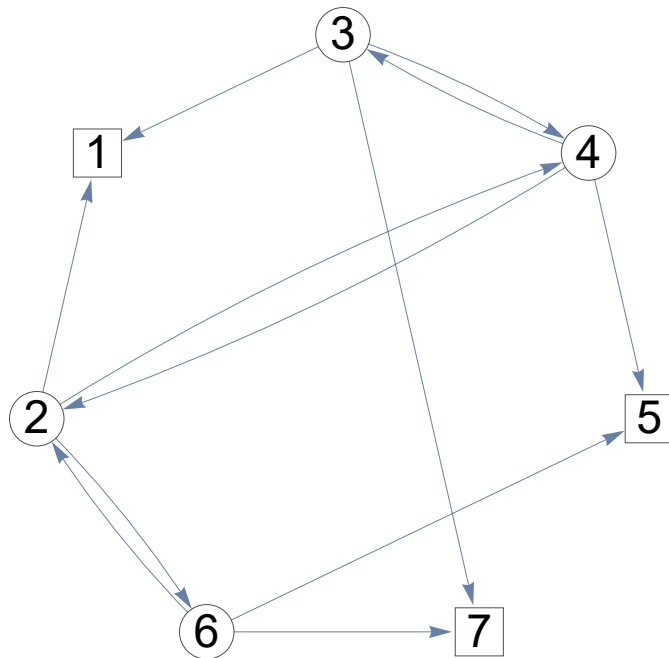


```

In[40]:=  $\overline{g1}$  = Fold[EdgeDelete[#1, u_ -> v_ /; u == #2] &,  $\overline{n\overline{g}}$ , #[[1]] & /@  $M^+$ ];
GraphPlot[ $\overline{g1}$ , MultiedgeStyle -> .05]

```

Out[41]=




```

In[42]:=  $II_{rem}$  = VertexList[ $\overline{g1}$ ] ~ Complement ~ ( $M^+$ [[All, 1]])

```

Out[42]= {2, 3, 4, 6}

```
In[67]:= λ = SparseArray[
  Replace[EdgeList[g1] /. # & /@ Flatten[Module[{i = #, jf, Icur}, {Icur = iii†[g1];
    jf = First[Icur];
    {({(i ↔ jf) → 1, (i ↔ #) → - $\frac{p_{i \leftrightarrow \#}}{p_{i \leftrightarrow jf}}$ }) & /@ Icur[[2 ;;]]}] & /@ IIrem, 1]],
    _ ↔ _ → 0, 2]]
```

```
Out[67]= SparseArray[  Specified elements: 16  
Dimensions: {8, 12} ]
```

```
In[44]:= Grid[λ]
```

```
Out[44]=
1 0 -1/2 0 0 0 0 0 0 0 0 0
1 0 0 0 -1 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 -1 0 0 0
0 1 0 0 0 0 0 0 0 0 0 -3/2
0 0 0 1 0 0 -1/3 0 0 0 0 0
0 0 0 1 0 0 0 -1/3 0 0 0 0
0 0 0 0 0 1 0 0 0 -3 0 0
0 0 0 0 0 1 0 0 0 0 -4 0
```

```
In[45]:= g = g1;
b = b1;
```

```
In[47]:= II* = Cases[MapIndexed[{#1, #2} &, b], {el_, i_} /; MemberQ[el, x] &:= i] // Flatten
```

```
Out[47]= {1, 5, 7}
```

```
In[48]:= buildt = Timing[{t, g} = buildTree[g, II*];][[1]]
TableForm[t[[1 ;; 4]],
  TableHeadings → {"pred", "dir", "depth", "d"}, t // pred // Length // Range}]
```

```
Out[48]= 0.
```

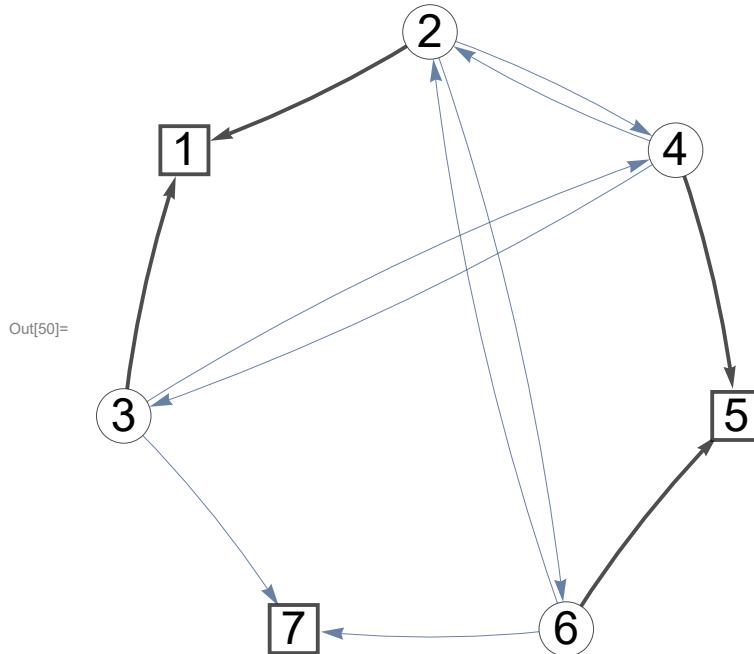
```
Out[49]//TableForm=
```

	1	2	3	4	5	6	7	8
pred	8	1	1	5	8	5	8	0
dir	1	-1	-1	-1	1	-1	1	0
depth	1	2	2	2	1	2	1	0
d	2	3	5	6	4	7	8	1

```

In[50]:= GraphPlot[HighlightGraph[
  Fold[HighlightGraph[#1, Style[u_ → v_ /; u == #2, White]] &, n̄g, #[[1]] & /@ M+],
  {Style[u_ /; VertexQ[g, u] && pred[t][[u]] == root[t], EdgeForm[Thick]],
  Style[u_ → v_ /; (pred[t][[u]] == v && dir[t][[u]] == -1) ||
    (pred[t][[v]] == u && dir[t][[v]] == 1), Directive[Black, Thick]}],
  GraphHighlightStyle → None], MultiedgeStyle → .05]

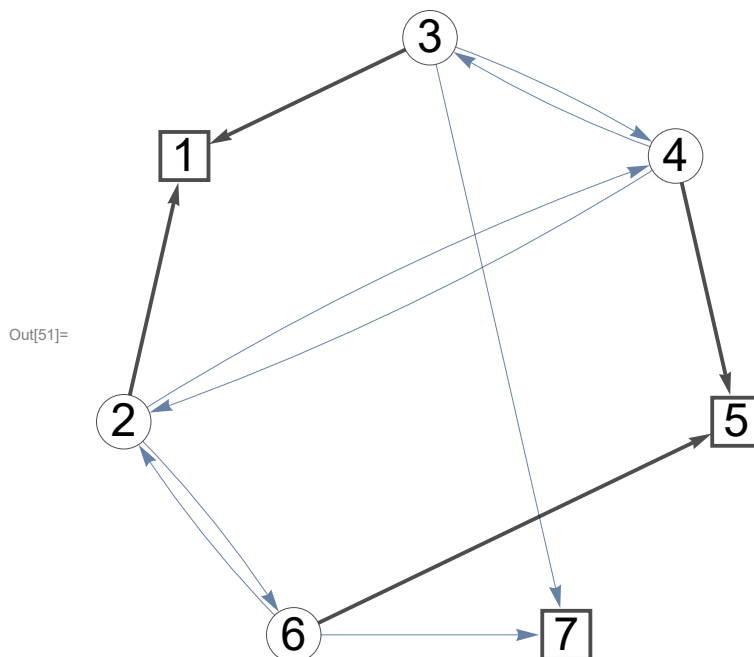
```



```

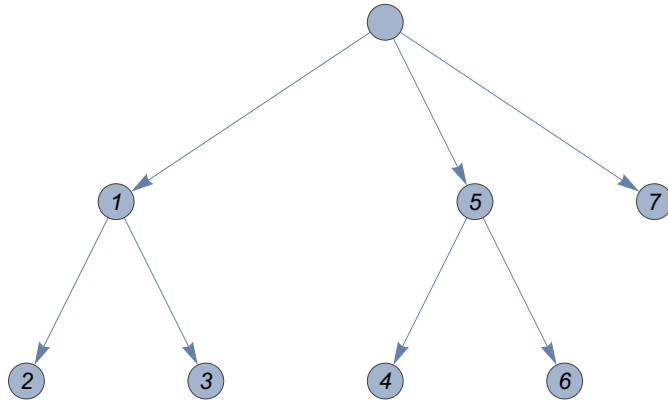
In[51]:= GraphPlot[HighlightGraph[g1,
  {Style[u_ /; VertexQ[g, u] && pred[t][[u]] == root[t], EdgeForm[Thick]],
  Style[u_ → v_ /; (pred[t][[u]] == v && dir[t][[u]] == -1) ||
    (pred[t][[v]] == u && dir[t][[v]] == 1), Directive[Black, Thick]}],
  GraphHighlightStyle → None], MultiedgeStyle → .05]

```



In[52]:= **t[[7]]** (*пометить на графе*)

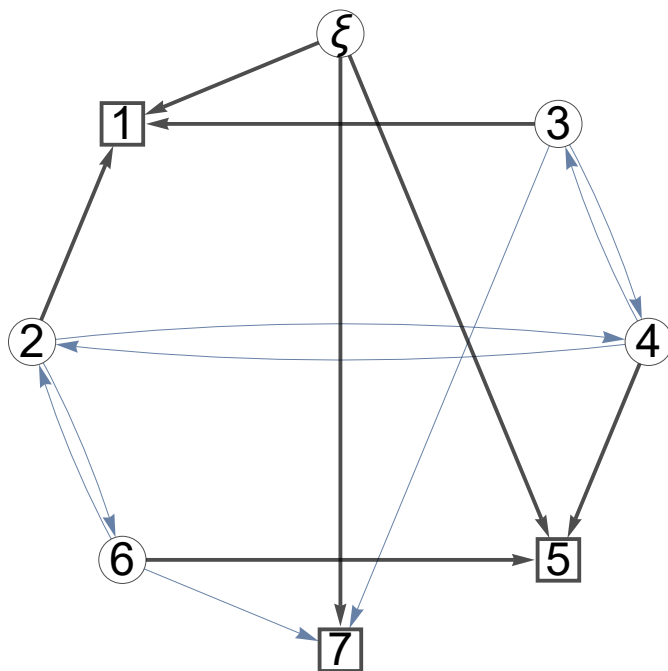
Out[52]=



In[53]:= **(*GraphPlot[g,MultiedgeStyle→.05]*)**

GraphPlot[HighlightGraph[g,
{Style[u_ /; VertexQ[g, u] && pred[t][[u]] == root[t], EdgeForm[Thick]],
Style[u_ ↔ v_ /; (pred[t][[u]] == v && dir[t][[u]] == -1) ||
(pred[t][[v]] == u && dir[t][[v]] == 1), Directive[Black, Thick]}],
GraphHighlightStyle → None], MultiedgeStyle → .05]

Out[53]=



In[54]:= **AppendTo[b, -Total[b]];**
b = Simplify[b /. x → 0]

Out[55]= $\left\{\frac{1}{2}, \frac{1}{2}, \frac{5}{2}, 9, -11, 5, -3, -\frac{7}{2}\right\}$

```
In[56]:= balanceEqs = (Total[x# & /@ EdgeList[g, _ -> #]] - Total[x# & /@ EdgeList[g, # -> _]]) /.
      root[t] -> ξ) == b[#] & /@ VertexList[g];
balanceEqs //
forma
```

```
Out[57]//TableForm=

$$\begin{aligned} x_{2,1} + x_{3,1} + x_{\xi,1} &= \frac{1}{2} \\ -x_{2,1} - x_{2,4} - x_{2,6} + x_{4,2} + x_{6,2} &= \frac{1}{2} \\ -x_{3,1} - x_{3,4} - x_{3,7} + x_{4,3} &= \frac{5}{2} \\ x_{2,4} + x_{3,4} - x_{4,2} - x_{4,3} - x_{4,5} &= 9 \\ x_{2,6} - x_{6,2} - x_{6,5} - x_{6,7} &= 5 \\ x_{4,5} + x_{6,5} + x_{\xi,5} &= -11 \\ x_{3,7} + x_{6,7} + x_{\xi,7} &= -3 \\ -x_{\xi,1} - x_{\xi,5} - x_{\xi,7} &= -\frac{7}{2} \end{aligned}$$

```

```
In[58]:= ps = partSolve[g, -b, t, x̃];
ps // forma
```

```
Out[59]//TableForm=

$$\begin{aligned} \tilde{x}_{2,1} &\rightarrow -\frac{1}{2} \\ \tilde{x}_{2,4} &\rightarrow 0 \\ \tilde{x}_{2,6} &\rightarrow 0 \\ \tilde{x}_{3,1} &\rightarrow -\frac{5}{2} \\ \tilde{x}_{3,4} &\rightarrow 0 \\ \tilde{x}_{3,7} &\rightarrow 0 \\ \tilde{x}_{4,2} &\rightarrow 0 \\ \tilde{x}_{4,3} &\rightarrow 0 \\ \tilde{x}_{4,5} &\rightarrow -9 \\ \tilde{x}_{6,2} &\rightarrow 0 \\ \tilde{x}_{6,5} &\rightarrow -5 \\ \tilde{x}_{6,7} &\rightarrow 0 \\ \tilde{x}_{8,1} &\rightarrow \frac{7}{2} \\ \tilde{x}_{8,5} &\rightarrow 3 \\ \tilde{x}_{8,7} &\rightarrow -3 \end{aligned}$$

```

```
In[60]:= Simplify[(balanceEqs /. {x -> x̃, ξ -> root[t]}) /. ps]
```

```
Out[60]= {True, True, True, True, True, True, True, True}
```

```
In[61]:= matrt = Timing[δMatr = δ1[g, t]];
roott = VertexCount[g];
TableForm[δMatr, TableHeadings -> {uNb[g, t], δ

$$\begin{cases} \# [2] & \# [1] == \text{roott} \text{ \& } /@ \text{EdgeList}[g] \\ \# [1] & \# [2] == \text{roott} \\ \# & \text{True} \end{cases}$$
] // forma
```

```
Out[63]//TableForm=
```

	$\delta_{2,1}$	$\delta_{3,1}$	$\delta_{2,4}$	$\delta_{4,2}$	$\delta_{2,6}$	$\delta_{6,2}$	$\delta_{4,5}$	$\delta_{4,3}$	$\delta_{3,4}$	$\delta_{6,5}$	$\delta_{6,7}$
$2 \leftrightarrow 4$	-1	0	1	0	0	0	1	0	0	0	0
$4 \leftrightarrow 2$	1	0	0	1	0	0	-1	0	0	0	0
$2 \leftrightarrow 6$	-1	0	0	0	1	0	0	0	0	1	0
$6 \leftrightarrow 2$	1	0	0	0	0	1	0	0	0	-1	0
$4 \leftrightarrow 3$	0	1	0	0	0	0	-1	1	0	0	0
$3 \leftrightarrow 4$	0	-1	0	0	0	0	1	0	1	0	0
$6 \leftrightarrow 7$	0	0	0	0	0	0	0	0	0	-1	1
$3 \leftrightarrow 7$	0	-1	0	0	0	0	0	0	0	0	0

```
In[68]:= λ = SparseArray[λ, {Length[λ], Length[λ[[1]]] + 3}];
(*λ=λ[[;;-2]]*)
```



```
In[69]:= dopEq = # == 0 & /@ Flatten[λ.{x# & /@ EdgeList[g]}];
dopEq // forma
```

Out[70]/TableForm=

$$\begin{aligned}x_{2,1} - \frac{x_{2,4}}{2} &== 0 \\x_{2,1} - x_{2,6} &== 0 \\x_{3,1} - x_{3,4} &== 0 \\x_{3,1} - \frac{3x_{3,7}}{2} &== 0 \\x_{4,2} - \frac{x_{4,5}}{3} &== 0 \\x_{4,2} - \frac{x_{4,3}}{3} &== 0 \\x_{6,2} - 3x_{6,5} &== 0 \\x_{6,2} - 4x_{6,7} &== 0\end{aligned}$$

```
In[71]:= Δ = λ.(δMatr)ᵀ;
"cicle det's:"
Δ // forma
```

Out[72]= cicle det's:

Out[73]/TableForm=

$$\begin{array}{cccccccc} -\frac{3}{2} & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & -\frac{5}{2} \\ -\frac{1}{3} & \frac{4}{3} & 0 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & -3 & 4 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 0 \end{array}$$

```
In[78]:= "Uc="
Uc = Range[8]
"Unc="
Unc = {}
```

Out[78]= Uc=

Out[79]= {1, 2, 3, 4, 5, 6, 7, 8}

Out[80]= Unc=

Out[81]= {}

```
In[82]:= Δc = Δ[All, Uc];
Δnc = Δ[All, Unc];
"Δc="
Δc // MatrixForm
```

Out[84]= Δc=

Out[85]/MatrixForm=

$$\left(\begin{array}{cccccccc} -\frac{3}{2} & 1 & -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & -\frac{5}{2} \\ -\frac{1}{3} & \frac{4}{3} & 0 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & -3 & 4 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -4 & 0 \end{array} \right)$$

```
In[86]:= "det ( $\Delta_c$ ) ="
Simplify[det = Det [ $\Delta_c$ ] ] // forma
```

```
Out[86]= det ( $\Delta_c$ ) =
```

```
Out[87]/TableForm=
-  $\frac{676}{9}$ 
```

```
In[88]:= "U_T="
utind = Cases[t[[6]],  $\xi_-$  /;  $\xi \neq 0$ ];
U_T = EdgeList[g][[utind]]
```

```
Out[88]= U_T=
```

```
Out[90]= { 8  $\leftrightarrow$  1, 2  $\leftrightarrow$  1, 3  $\leftrightarrow$  1, 4  $\leftrightarrow$  5, 8  $\leftrightarrow$  5, 6  $\leftrightarrow$  5, 8  $\leftrightarrow$  7 }
```

```
In[91]:= "U_Nb="
U_Nb = uNb[g, t]
```

```
Out[91]= U_Nb=
```

```
Out[92]= { 2  $\leftrightarrow$  4, 4  $\leftrightarrow$  2, 2  $\leftrightarrow$  6, 6  $\leftrightarrow$  2, 4  $\leftrightarrow$  3, 3  $\leftrightarrow$  4, 6  $\leftrightarrow$  7, 3  $\leftrightarrow$  7 }
```

```
In[93]:= A = - $\lambda$ .{x_# & /@ EdgeList[g]}^T /. ps;
"A="
A // MatrixForm
```

```
Out[94]= A=
```

```
Out[95]/MatrixForm=

$$\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{5}{2} \\ \frac{5}{2} \\ \frac{5}{2} \\ -3 \\ 0 \\ -15 \\ 0 \end{pmatrix}$$

```

```
In[96]:=  $\beta$  = A -  $\Delta_{nc}$ .{x_# & /@ U_Nb[[U_nc]]}^T;
" $\beta$ ="
 $\beta$  // forma
```

```
Out[97]=  $\beta$ =
```

```
Out[98]/TableForm=

$$\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{5}{2} \\ \frac{5}{2} \\ \frac{5}{2} \\ -3 \\ 0 \\ -15 \\ 0 \end{pmatrix}$$

```

```
In[99]:= "решаем уравнение  $\Delta_c x_c = \beta$ :"
xc = LinearSolve[ $\Delta_c$ ,  $\beta$ ]
```

```
Out[99]= решаем уравнение  $\Delta_c x_c = \beta$ :
```

```
Out[100]=  $\left\{ \left\{ -\frac{4777}{1352} \right\}, \left\{ -\frac{775}{338} \right\}, \left\{ -\frac{4777}{2704} \right\}, \left\{ -\frac{2889}{676} \right\}, \left\{ -\frac{2325}{338} \right\}, \left\{ -\frac{4755}{1352} \right\}, \left\{ -\frac{2889}{2704} \right\}, \left\{ -\frac{1585}{676} \right\} \right\}$ 
```

```
In[101]:= xcp = MapThread[x#1 → #2 &, {UNb[[Uc]], Flatten[xc]}];
xcp // TableForm
```

```
Out[102]//TableForm=
```

```
x2→4 →  $-\frac{4777}{1352}$ 
x4→2 →  $-\frac{775}{338}$ 
x2→6 →  $-\frac{4777}{2704}$ 
x6→2 →  $-\frac{2889}{676}$ 
x4→3 →  $-\frac{2325}{338}$ 
x3→4 →  $-\frac{4755}{1352}$ 
x6→7 →  $-\frac{2889}{2704}$ 
x3→7 →  $-\frac{1585}{676}$ 
```

```
In[103]:= s = solveAll[g, t];
s // TableForm
```

```
Out[104]//TableForm=
```

```
x2→1 →  $-\frac{1}{2} - x_{2→4} - x_{2→6} + x_{4→2} + x_{6→2}$ 
x3→1 →  $-\frac{5}{2} - x_{3→4} - x_{3→7} + x_{4→3}$ 
x4→5 →  $-9 + x_{2→4} + x_{3→4} - x_{4→2} - x_{4→3}$ 
x6→5 →  $-5 + x_{2→6} - x_{6→2} - x_{6→7}$ 
x8→1 →  $\frac{7}{2} + x_{2→4} + x_{2→6} + x_{3→4} + x_{3→7} - x_{4→2} - x_{4→3} - x_{6→2}$ 
x8→5 →  $3 - x_{2→4} - x_{2→6} - x_{3→4} + x_{4→2} + x_{4→3} + x_{6→2} + x_{6→7}$ 
x8→7 →  $-3 - x_{3→7} - x_{6→7}$ 
```

```
In[105]:= "общее решение:"
xsol = (s /. xcp) ~Join~ xcp);
xsol /. {xi_u_v_ -> v_ -> xi_u_v} // Simplify // TableForm
```

```
Out[105]= общее решение:
```

```
Out[107]//TableForm=
```

```

X2,1 -> - 4777
          2704
X3,1 -> - 4755
          1352
X4,5 -> - 2325
          338
X6,5 -> - 963
          676
X8,1 -> 1203
          208
X8,5 -> - 1823
          676
X8,7 -> 1117
          2704
X2,4 -> - 4777
          1352
X4,2 -> - 775
          338
X2,6 -> - 4777
          2704
X6,2 -> - 2889
          676
X4,3 -> - 2325
          338
X3,4 -> - 4755
          1352
X6,7 -> - 2889
          2704
X3,7 -> - 1585
          676
```

```
In[108]:= "eq test:"
Simplify[balanceEqs /. xi -> root[t] /. s /. xcp]
Simplify[(dopEq /. s) /. xcp]
```

```
Out[108]= eq test:
```

```
Out[109]= {True, True, True, True, True, True, True, True}
```

```
Out[110]= {True, True, True, True, True, True, True, True}
```