

Листинг 2

```

In[ ]:= ClearAll["Global`*"]
SetDirectory[NotebookDirectory[]];
Needs["FlowSolver`"]

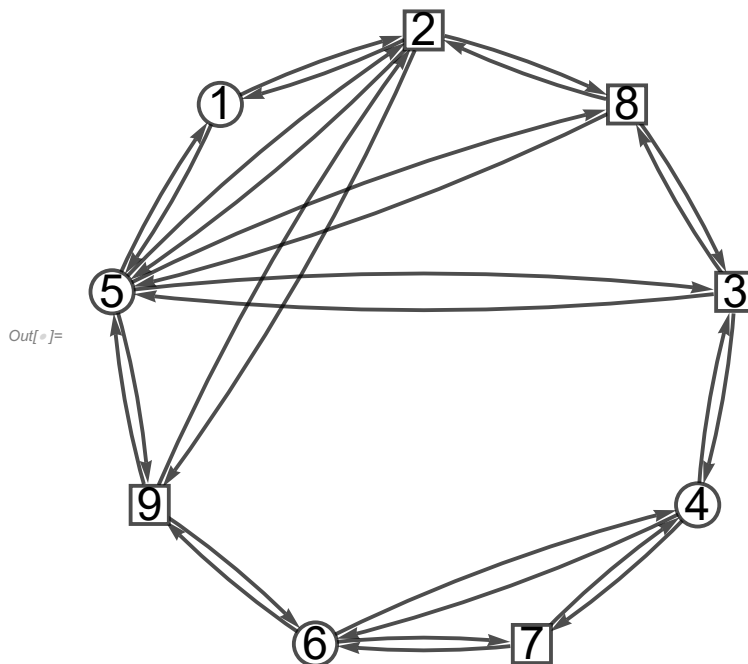
In[ ]:= readGraph2[file_, dir_] := Module[{
    fn = FileNameJoin[{dir, file}],
    stream, imod, umod, u, b
},
    stream = OpenRead[fn];
    imod = Read[stream, {Word, Number}][[2]];
    umod = Read[stream, {Word, Number}][[2]];

    u = ({#[[1]] ↔ #[[2]], #[[2]] ↔ #[[1]]} & /@ ReadList[stream, Expression, umod]) // Flatten;
    b = ConstantArray[0, imod];
    (b[[Read[StringToStream[StringTake[#, {5, -3}]], Number]]] = #2) & @@@
    ReadList[stream, {Word, Expression}, imod];
    {Graph[u, VertexSize -> Medium, VertexLabels -> Placed["Name", Center],
        VertexStyle -> Directive[White],
        VertexShapeFunction -> {xx_ -> If[SameQ[b[[xx]], x], "Square", "Circle"]},
        VertexLabelStyle -> Directive[Black, 24], GraphLayout -> "CircularEmbedding"], b}]

In[ ]:= forma[ff_] := ({ff /. {ξu→v -> ξu,v}} // TableForm)

In[ ]:=
{g, b} = readGraph2["grDET0.txt", NotebookDirectory[]];
GraphPlot[g, EdgeStyle -> Directive[Black, Thick],
    VertexStyle -> Directive[EdgeForm[Thick], White], MultiedgeStyle -> .05]

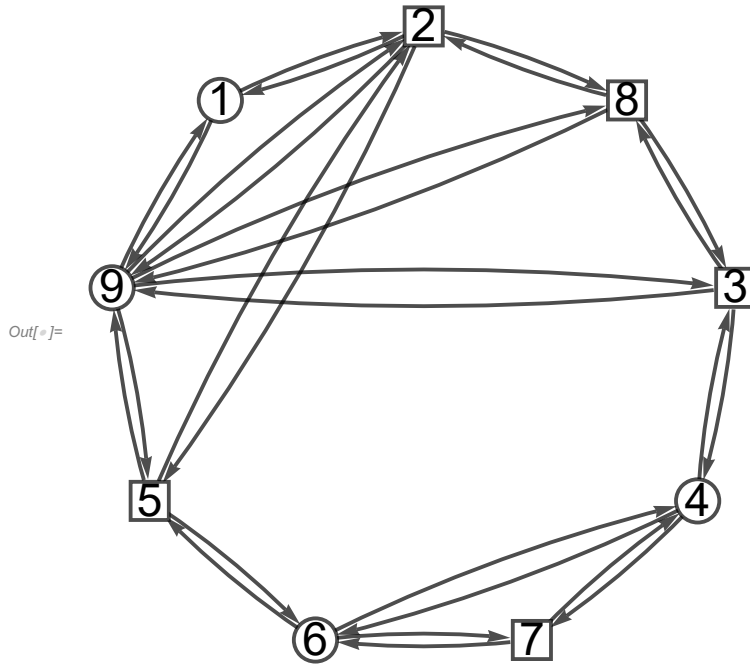
```



```

In[ ]:= g = VertexReplace[g, {5 → 9, 9 → 5}];
GraphPlot[g, EdgeStyle → Directive[Black, Thick],
  VertexStyle → Directive[EdgeForm[Thick], White], MultiedgeStyle → .05]

```



```

In[ ]:= balanceEqs = ((Total[x_# & /@ EdgeList[g, _ → #]] - Total[x_# & /@ EdgeList[g, # → _]]) ==
  MapIndexed[#1 /. x → x_#2[[1]] &, b][[#]] & /@ VertexList[g];
balanceEqs //
forma

```

Out[]:=//TableForm=

$$\begin{aligned}
 & -x_{1,2} - x_{1,9} + x_{2,1} + x_{9,1} = 0 \\
 & x_{1,2} - x_{2,1} - x_{2,5} - x_{2,8} - x_{2,9} + x_{5,2} + x_{8,2} + x_{9,2} = x_2 \\
 & x_{1,9} + x_{2,9} + x_{3,9} + x_{5,9} + x_{8,9} - x_{9,1} - x_{9,2} - x_{9,3} - x_{9,5} - x_{9,8} = x_9 \\
 & x_{2,8} + x_{3,8} - x_{8,2} - x_{8,3} - x_{8,9} + x_{9,8} = x_8 \\
 & -x_{3,4} - x_{3,8} - x_{3,9} + x_{4,3} + x_{8,3} + x_{9,3} = x_3 \\
 & x_{3,4} - x_{4,3} - x_{4,6} - x_{4,7} + x_{6,4} + x_{7,4} = 0 \\
 & x_{4,7} + x_{6,7} - x_{7,4} - x_{7,6} = x_7 \\
 & x_{4,6} + x_{5,6} - x_{6,4} - x_{6,5} - x_{6,7} + x_{7,6} = 0 \\
 & x_{2,5} - x_{5,2} - x_{5,6} - x_{5,9} + x_{6,5} + x_{9,5} = 0
 \end{aligned}$$

```

In[ ]:= M = {9};
Print["M = ", M];

M = {9}

```

```

In[ ]:= (*Do[inclist=EdgeList[g,u→_];
  Do[p_v=1/Length[inclist];,{v,inclist}];,{u,VertexList[g]})*)

```

```

In[ ]:= (*p_#&/@EdgeList[g]*)

```

```

In[ ]:= (*incl=
  DeleteCases[DeleteDuplicates[Cases[IncidenceList[g,#],i_→j_→{i,j}]]//Flatten],
  v_/;v==#]&/@M*)
incl = (IncidenceList[g, #] & /@ M) // Flatten

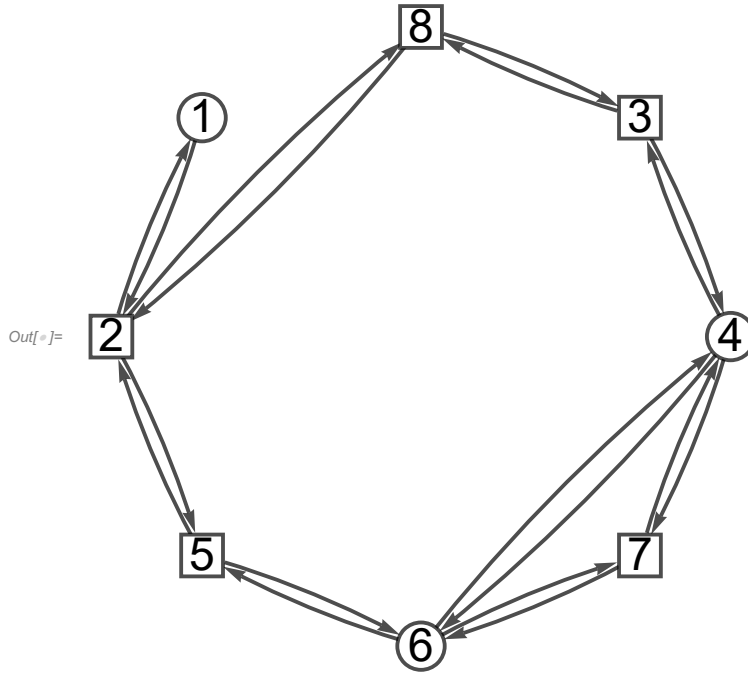
```

Out[]:= {1 → 9, 9 → 1, 8 → 9, 9 → 8, 2 → 9, 9 → 2, 5 → 9, 9 → 5, 3 → 9, 9 → 3}

```

In[*]:= (*Do[If[MemberQ[M, j[[1]]], b[[j[[2]]]] += f_j, b[[j[[1]]]] -= f_j], {j, incL}]) *)
b = Fold[If[MemberQ[M, #2[[1]]], ReplacePart[#, #2[[2]] -> #[[#2[[2]]]] - f_#2],
  ReplacePart[#, #2[[1]] -> #[[#2[[1]]]] + f_#2]] &, b, incL];
b = b[[Range[g // VertexCount] ~ Complement ~ M]];
ng = VertexDelete[g, M];
GraphPlot[ng, EdgeStyle -> Directive[Black, Thick],
  VertexStyle -> Directive[EdgeForm[Thick], White], MultiedgeStyle -> .05]
b

```



```

Out[*]= {f1→9 - f9→1, x + f2→9 - f9→2, x + f3→9 - f9→3, 0, f5→9 - f9→5, 0, x, x + f8→9 - f9→8}

```

```

In[*]:= CC[g_, M_] :=
  (DeleteDuplicates[Cases[IncidenceList[g, #], i_ -> j_ /; j == #]] & /@ M) // Flatten

```

```

iii+[g_] := Cases[IncidenceList[g, i], u_ -> v_ /; u == i -> v]

```

```

In[*]:= M+ = CC[g, M]

```

```

Out[*]= {1 -> 9, 8 -> 9, 2 -> 9, 5 -> 9, 3 -> 9}

```

```

In[ ]:=  $\overline{b1} = \text{Fold}[$ 
       $\text{Module}[\{bb = \#1, i = \#2_{[[1]]}, k = \#2_{[[2]]}\}, \left( \text{Fold}[\text{Module}[\{bbb = \#1, jj = \#2\}, \text{ReplacePart}[\right.$ 
       $bbb, \left( \left( \left( \{jj \rightarrow bbb_{[[jj]]} - \frac{p_{i \rightarrow jj}}{p_{i \rightarrow k}} f_{i \rightarrow k}, i \rightarrow bbb_{[[i]]} + \frac{p_{i \rightarrow jj}}{p_{i \rightarrow k}} f_{i \rightarrow k} \right) \right) \right) //$ 
       $\text{Flatten}]] \&, bb, ii_i[\overline{ng}]] \left. \right) \&, \overline{b}, M^+]$ 

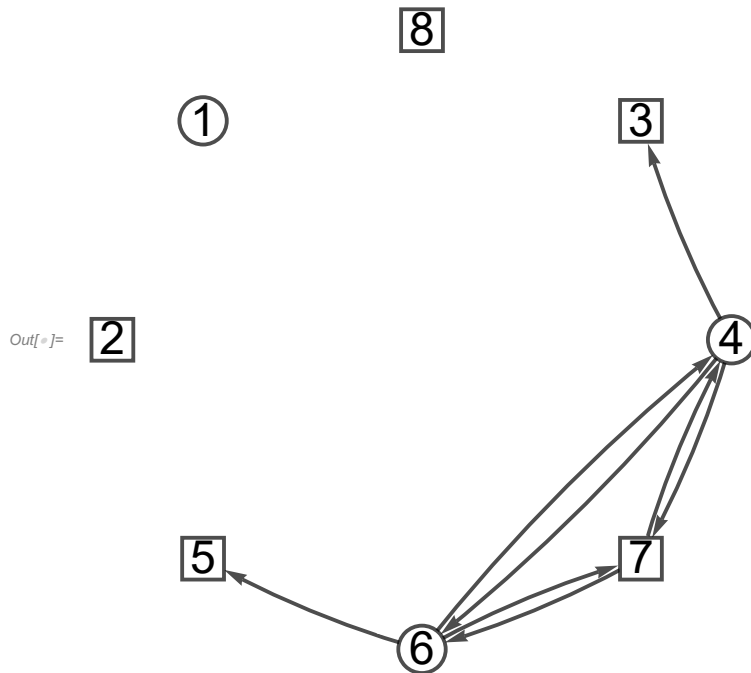
```

$$\begin{aligned}
\text{Out[]} = & \left\{ f_{1 \rightarrow 9} - f_{9 \rightarrow 1} + \frac{f_{1 \rightarrow 9} p_{1 \rightarrow 2}}{p_{1 \rightarrow 9}} - \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 1}}{p_{2 \rightarrow 9}}, \right. \\
& x + f_{2 \rightarrow 9} - f_{9 \rightarrow 2} - \frac{f_{1 \rightarrow 9} p_{1 \rightarrow 2}}{p_{1 \rightarrow 9}} + \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 1}}{p_{2 \rightarrow 9}} + \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 5}}{p_{2 \rightarrow 9}} + \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 8}}{p_{2 \rightarrow 9}} - \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 2}}{p_{5 \rightarrow 9}} - \frac{f_{8 \rightarrow 9} p_{8 \rightarrow 2}}{p_{8 \rightarrow 9}}, \\
& x + f_{3 \rightarrow 9} - f_{9 \rightarrow 3} + \frac{f_{3 \rightarrow 9} p_{3 \rightarrow 4}}{p_{3 \rightarrow 9}} + \frac{f_{3 \rightarrow 9} p_{3 \rightarrow 8}}{p_{3 \rightarrow 9}} - \frac{f_{8 \rightarrow 9} p_{8 \rightarrow 3}}{p_{8 \rightarrow 9}}, - \frac{f_{3 \rightarrow 9} p_{3 \rightarrow 4}}{p_{3 \rightarrow 9}}, \\
& f_{5 \rightarrow 9} - f_{9 \rightarrow 5} - \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 5}}{p_{2 \rightarrow 9}} + \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 2}}{p_{5 \rightarrow 9}} + \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 6}}{p_{5 \rightarrow 9}}, - \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 6}}{p_{5 \rightarrow 9}}, x, \\
& \left. x + f_{8 \rightarrow 9} - f_{9 \rightarrow 8} - \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 8}}{p_{2 \rightarrow 9}} - \frac{f_{3 \rightarrow 9} p_{3 \rightarrow 8}}{p_{3 \rightarrow 9}} + \frac{f_{8 \rightarrow 9} p_{8 \rightarrow 2}}{p_{8 \rightarrow 9}} + \frac{f_{8 \rightarrow 9} p_{8 \rightarrow 3}}{p_{8 \rightarrow 9}} \right\}
\end{aligned}$$

```

In[ ]:= GraphPlot[Fold[HighlightGraph[#1, u_ -> v_ /; u == #2, GraphHighlightStyle -> "White"] &,
   $\overline{ng}$ , #_{[[1]]} & /@ M^+], EdgeStyle -> Directive[Black, Thick],
  VertexStyle -> Directive[EdgeForm[Thick], White], MultiedgeStyle -> .05]

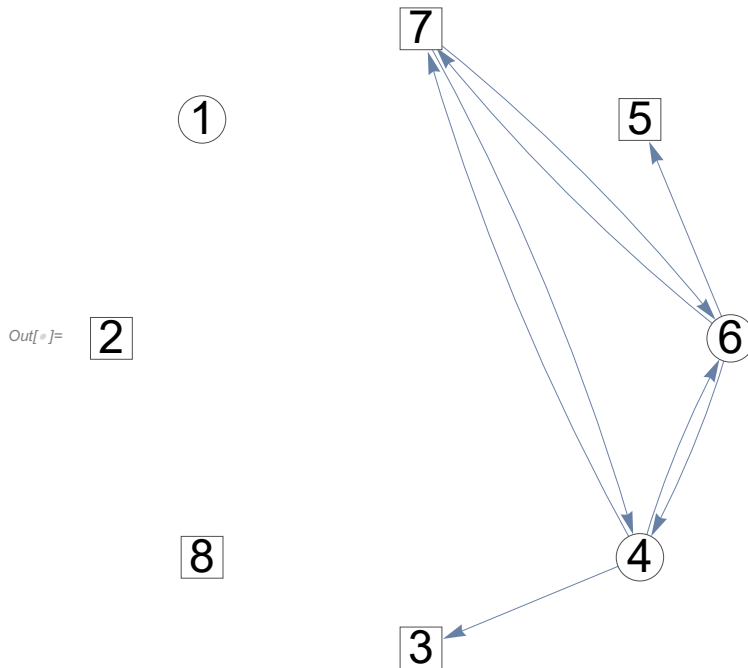
```



```

In[ ]:=  $\overline{g1}$  = Fold[EdgeDelete[#1, u_  $\leftrightarrow$  v_ /; u == #2] &,  $\overline{ng}$ , #[[1]] & /@ M+];
GraphPlot[ $\overline{g1}$ , MultiedgeStyle  $\rightarrow$  .05]

```



```

In[ ]:= IIrem = VertexList[ $\overline{g1}$ ] ~ Complement ~ (M+[[All, 1]])
Out[ ]:= {4, 6, 7}


```

```

In[ ]:=  $\lambda$  = SparseArray[
  Replace[
    EdgeList[ $\overline{g1}$ ] /. # & /@ Flatten[Module[{i = #, jf, Icur}, {Icur = iii+[ $\overline{g1}$ ];
      jf = First[Icur];
      {((i  $\rightarrow$  jf)  $\rightarrow$  1, (i  $\leftrightarrow$  #)  $\rightarrow$  -  $\frac{p_{i \rightarrow \#}}{p_{i \rightarrow jf}}$ )} & /@ Icur[[2 ;;]]] & /@ IIrem, 1]],
    _  $\leftrightarrow$  _  $\rightarrow$  0, 2]]

```

```

Out[ ]:= SparseArray[
   Specified elements: 10
  Dimensions: {5, 8}
]

```

```

In[ ]:= Grid[ $\lambda$ ]

```

```

Out[ ]:=


|   |                                                     |   |                                                     |   |                                                     |                                                     |                                                     |
|---|-----------------------------------------------------|---|-----------------------------------------------------|---|-----------------------------------------------------|-----------------------------------------------------|-----------------------------------------------------|
| 1 | - $\frac{p_{4 \rightarrow 7}}{p_{4 \rightarrow 3}}$ | 0 | 0                                                   | 0 | 0                                                   | 0                                                   | 0                                                   |
| 1 | 0                                                   | 0 | 0                                                   | 0 | 0                                                   | - $\frac{p_{4 \rightarrow 6}}{p_{4 \rightarrow 3}}$ | 0                                                   |
| 0 | 0                                                   | 0 | 0                                                   | 1 | - $\frac{p_{6 \rightarrow 4}}{p_{6 \rightarrow 7}}$ | 0                                                   | 0                                                   |
| 0 | 0                                                   | 0 | 0                                                   | 1 | 0                                                   | 0                                                   | - $\frac{p_{6 \rightarrow 5}}{p_{6 \rightarrow 7}}$ |
| 0 | 0                                                   | 1 | - $\frac{p_{7 \rightarrow 6}}{p_{7 \rightarrow 4}}$ | 0 | 0                                                   | 0                                                   | 0                                                   |


```

```

In[ ]:=  $\mathbf{g} = \overline{g1}$ ;
 $\mathbf{b} = \mathbf{b1}$ ;

```

```

In[ ]:= II* = Cases[MapIndexed[{#1, #2} &,  $\mathbf{b}$ ],
  {el_, i_] /; MemberQ[el, x] || SameQ[el, x]  $\Rightarrow$  i] // Flatten
Out[ ]:= {2, 3, 7, 8}

```

```

In[ ]:= buildt = Timing[{t, g} = buildTree[g, II*];][[1]]
TableForm[t[[1 ;; 4]],
  TableHeadings -> {"pred", "dir", "depth", "d"}, t // pred // Length // Range]]

```

Out[]:= 0.

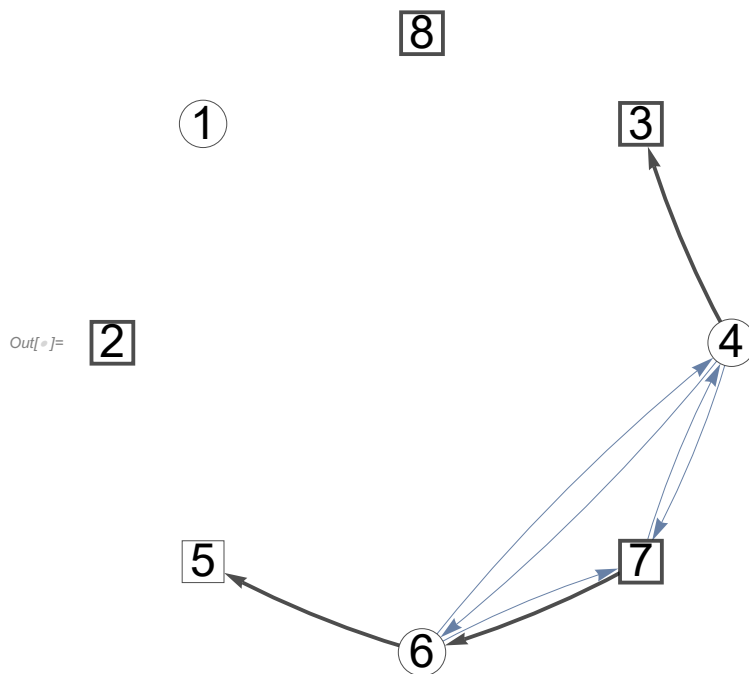
Out[]//TableForm=

	1	2	3	4	5	6	7	8	9
pred	0	9	9	3	6	7	9	9	0
dir	0	1	1	-1	1	1	1	1	0
depth	0	1	1	2	3	2	1	1	0
d	9	8	4	7	9	5	6	3	2

```

In[ ]:= GraphPlot[HighlightGraph[
  Fold[HighlightGraph[#1, Style[u_ -> v_ /; u == #2, White]] &, n, #[[1]] & /@ M],
  {Style[u_ /; VertexQ[g, u] && pred[t][[u]] == root[t], EdgeForm[Thick]],
  Style[u_ -> v_ /; (pred[t][[u]] == v && dir[t][[u]] == -1) ||
    (pred[t][[v]] == u && dir[t][[v]] == 1), Directive[Black, Thick]}],
  GraphHighlightStyle -> None], MultiedgeStyle -> .05]

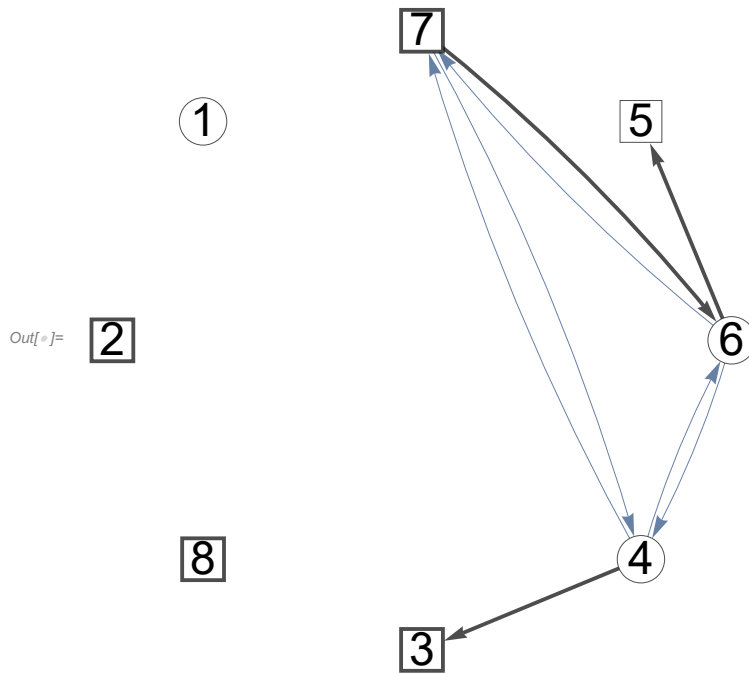
```



```

In[ ]:= GraphPlot[HighlightGraph[g1,
  {Style[u_ /; VertexQ[g, u] && pred[t][[u]] == root[t], EdgeForm[Thick]],
  Style[u_ -> v_ /; (pred[t][[u]] == v && dir[t][[u]] == -1) ||
  (pred[t][[v]] == u && dir[t][[v]] == 1), Directive[Black, Thick]}],
  GraphHighlightStyle -> None], MultiedgeStyle -> .05]

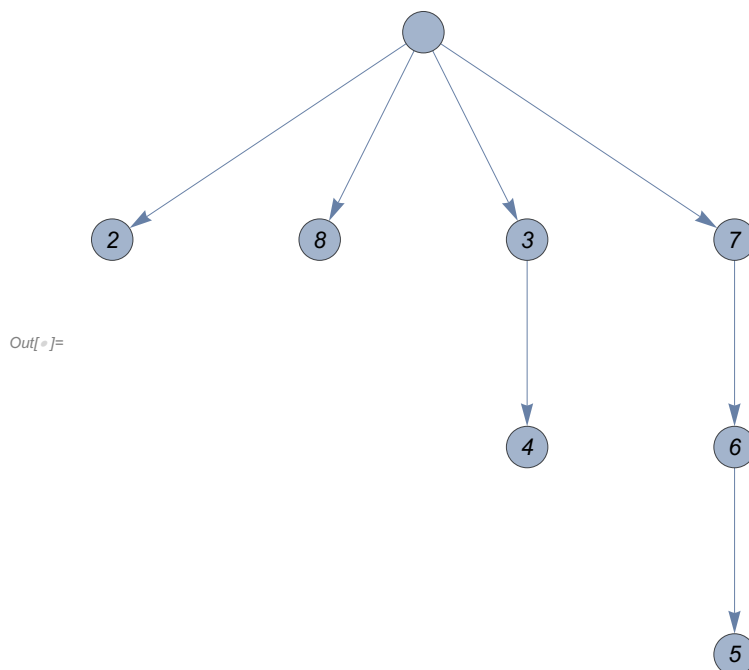
```



```

In[ ]:= t[[7]] (*пометить на графе*)

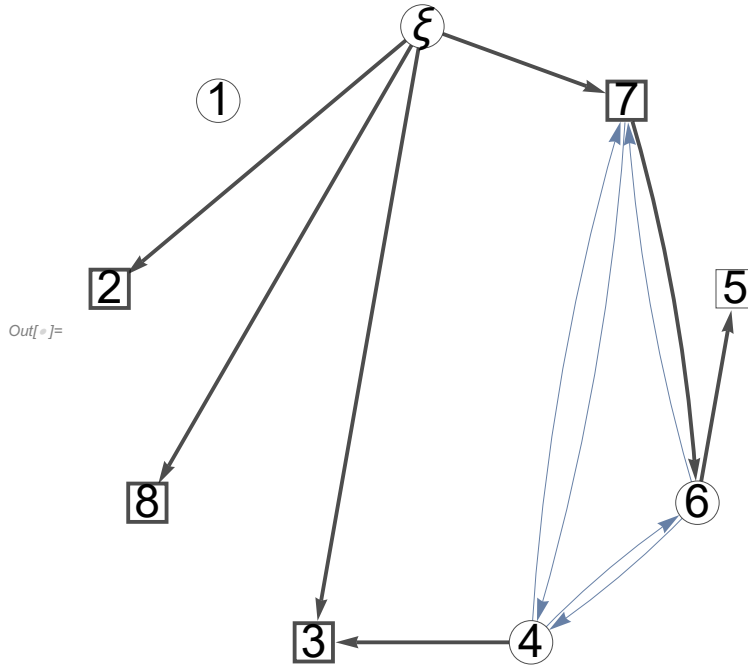
```



```

In[ ]:= (*GraphPlot[g,MultiedgeStyle->.05]*)
GraphPlot[HighlightGraph[g,
{Style[u_ /; VertexQ[g, u] && pred[t][[u]] == root[t], EdgeForm[Thick]],
Style[u_ -> v_ /; (pred[t][[u]] == v && dir[t][[u]] == -1) ||
(pred[t][[v]] == u && dir[t][[v]] == 1), Directive[Black, Thick]}],
GraphHighlightStyle -> None], MultiedgeStyle -> .05]

```



```

In[ ]:= AppendTo[b, -Total[b]];
b = Simplify[b /. x -> 0]

```

$$\begin{aligned}
\text{Out[]} = & \left\{ -f_{9 \rightarrow 1} + f_{1 \rightarrow 9} \left(1 + \frac{p_{1 \rightarrow 2}}{p_{1 \rightarrow 9}} \right) - \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 1}}{p_{2 \rightarrow 9}}, \right. \\
& -f_{9 \rightarrow 2} - \frac{f_{1 \rightarrow 9} p_{1 \rightarrow 2}}{p_{1 \rightarrow 9}} + \frac{f_{2 \rightarrow 9} (p_{2 \rightarrow 1} + p_{2 \rightarrow 5} + p_{2 \rightarrow 8} + p_{2 \rightarrow 9})}{p_{2 \rightarrow 9}} - \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 2}}{p_{5 \rightarrow 9}} - \frac{f_{8 \rightarrow 9} p_{8 \rightarrow 2}}{p_{8 \rightarrow 9}}, \\
& -f_{9 \rightarrow 3} + \frac{f_{3 \rightarrow 9} (p_{3 \rightarrow 4} + p_{3 \rightarrow 8} + p_{3 \rightarrow 9})}{p_{3 \rightarrow 9}} - \frac{f_{8 \rightarrow 9} p_{8 \rightarrow 3}}{p_{8 \rightarrow 9}}, -\frac{f_{3 \rightarrow 9} p_{3 \rightarrow 4}}{p_{3 \rightarrow 9}}, \\
& -f_{9 \rightarrow 5} - \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 5}}{p_{2 \rightarrow 9}} + \frac{f_{5 \rightarrow 9} (p_{5 \rightarrow 2} + p_{5 \rightarrow 6} + p_{5 \rightarrow 9})}{p_{5 \rightarrow 9}}, -\frac{f_{5 \rightarrow 9} p_{5 \rightarrow 6}}{p_{5 \rightarrow 9}}, \\
& 0, -f_{9 \rightarrow 8} - \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 8}}{p_{2 \rightarrow 9}} - \frac{f_{3 \rightarrow 9} p_{3 \rightarrow 8}}{p_{3 \rightarrow 9}} + \frac{f_{8 \rightarrow 9} (p_{8 \rightarrow 2} + p_{8 \rightarrow 3} + p_{8 \rightarrow 9})}{p_{8 \rightarrow 9}}, \\
& \left. -f_{1 \rightarrow 9} - f_{2 \rightarrow 9} - f_{3 \rightarrow 9} - f_{5 \rightarrow 9} - f_{8 \rightarrow 9} + f_{9 \rightarrow 1} + f_{9 \rightarrow 2} + f_{9 \rightarrow 3} + f_{9 \rightarrow 5} + f_{9 \rightarrow 8} \right\}
\end{aligned}$$


```
In[ ]:= balanceEqs = (Total[x# & /@ Edgelist[g, _ -> #]] - Total[x# & /@ Edgelist[g, # -> _]]) /.
      root[t] -> xi) == b[#]] & /@ VertexList[g];
balanceEqs //
forma
```

Out[]//TableForm=

$$\begin{aligned}
0 &= -f_{9,1} + f_{1,9} \left(1 + \frac{p_{1,2}}{p_{1,9}} \right) - \frac{f_{2,9} p_{2,1}}{p_{2,9}} \\
x_{\xi,2} &= -f_{9,2} - \frac{f_{1,9} p_{1,2}}{p_{1,9}} + \frac{f_{2,9} (p_{2,1} + p_{2,5} + p_{2,8} + p_{2,9})}{p_{2,9}} - \frac{f_{5,9} p_{5,2}}{p_{5,9}} - \frac{f_{8,9} p_{8,2}}{p_{8,9}} \\
x_{\xi,8} &= -f_{9,8} - \frac{f_{2,9} p_{2,8}}{p_{2,9}} - \frac{f_{3,9} p_{3,8}}{p_{3,9}} + \frac{f_{8,9} (p_{8,2} + p_{8,3} + p_{8,9})}{p_{8,9}} \\
x_{4,3} + x_{\xi,3} &= -f_{9,3} + \frac{f_{3,9} (p_{3,4} + p_{3,8} + p_{3,9})}{p_{3,9}} - \frac{f_{8,9} p_{8,3}}{p_{8,9}} \\
-x_{4,3} - x_{4,6} - x_{4,7} + x_{6,4} + x_{7,4} &= -\frac{f_{3,9} p_{3,4}}{p_{3,9}} \\
x_{4,7} + x_{6,7} - x_{7,4} - x_{7,6} + x_{\xi,7} &= 0 \\
x_{4,6} - x_{6,4} - x_{6,5} - x_{6,7} + x_{7,6} &= -\frac{f_{5,9} p_{5,6}}{p_{5,9}} \\
x_{6,5} &= -f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} + \frac{f_{5,9} (p_{5,2} + p_{5,6} + p_{5,9})}{p_{5,9}} \\
-x_{\xi,2} - x_{\xi,3} - x_{\xi,7} - x_{\xi,8} &= -f_{1,9} - f_{2,9} - f_{3,9} - f_{5,9} - f_{8,9} + f_{9,1} + f_{9,2} + f_{9,3} + f_{9,5} + f_{9,8}
\end{aligned}$$

```
In[ ]:= ps = partSolve[g, -b, t, x];
ps // forma
```

Out[]//TableForm=

$$\begin{aligned}
\tilde{x}_{4,3} &\rightarrow \frac{f_{3,9} p_{3,4}}{p_{3,9}} \\
\tilde{x}_{4,6} &\rightarrow 0 \\
\tilde{x}_{4,7} &\rightarrow 0 \\
\tilde{x}_{6,4} &\rightarrow 0 \\
\tilde{x}_{6,5} &\rightarrow -f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} + \frac{f_{5,9} (p_{5,2} + p_{5,6} + p_{5,9})}{p_{5,9}} \\
\tilde{x}_{6,7} &\rightarrow 0 \\
\tilde{x}_{7,4} &\rightarrow 0 \\
\tilde{x}_{7,6} &\rightarrow -f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} - \frac{f_{5,9} p_{5,6}}{p_{5,9}} + \frac{f_{5,9} (p_{5,2} + p_{5,6} + p_{5,9})}{p_{5,9}} \\
\tilde{x}_{9,2} &\rightarrow -f_{9,2} - \frac{f_{1,9} p_{1,2}}{p_{1,9}} + \frac{f_{2,9} (p_{2,1} + p_{2,5} + p_{2,8} + p_{2,9})}{p_{2,9}} - \frac{f_{5,9} p_{5,2}}{p_{5,9}} - \frac{f_{8,9} p_{8,2}}{p_{8,9}} \\
\tilde{x}_{9,3} &\rightarrow -f_{9,3} - \frac{f_{3,9} p_{3,4}}{p_{3,9}} + \frac{f_{3,9} (p_{3,4} + p_{3,8} + p_{3,9})}{p_{3,9}} - \frac{f_{8,9} p_{8,3}}{p_{8,9}} \\
\tilde{x}_{9,7} &\rightarrow -f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} - \frac{f_{5,9} p_{5,6}}{p_{5,9}} + \frac{f_{5,9} (p_{5,2} + p_{5,6} + p_{5,9})}{p_{5,9}} \\
\tilde{x}_{9,8} &\rightarrow -f_{9,8} - \frac{f_{2,9} p_{2,8}}{p_{2,9}} - \frac{f_{3,9} p_{3,8}}{p_{3,9}} + \frac{f_{8,9} (p_{8,2} + p_{8,3} + p_{8,9})}{p_{8,9}} \\
\tilde{x}_{1 \rightarrow 0} &\rightarrow 0
\end{aligned}$$

```
In[ ]:= Simplify[(balanceEqs /. {x -> x, xi -> root[t]}) /. ps]
```

$$\begin{aligned}
\text{Out[]} &= \left\{ f_{9 \rightarrow 1} + \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 1}}{p_{2 \rightarrow 9}} == f_{1 \rightarrow 9} \left(1 + \frac{p_{1 \rightarrow 2}}{p_{1 \rightarrow 9}} \right), \text{True}, \text{True}, \text{True}, \right. \\
&\quad \left. \text{True}, \text{True}, \text{True}, \text{True}, f_{9 \rightarrow 1} + f_{1 \rightarrow 9} \left(-1 - \frac{p_{1 \rightarrow 2}}{p_{1 \rightarrow 9}} \right) + \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 1}}{p_{2 \rightarrow 9}} == 0 \right\}
\end{aligned}$$

```
In[ ]:= matrt = Timing[deltaMatr = delta1[g, t]];
roott = VertexCount[g];
TableForm[deltaMatr, TableHeadings -> {unb[g, t], delta[#, #1] == roott & /@ Edgelist[g]}] // forma
      {#, #1] == roott
      # True
```

Out[]//TableForm=

	$\delta_{4,3}$	$\delta_{4,7}$	$\delta_{7,4}$	$\delta_{7,6}$	$\delta_{6,7}$	$\delta_{6,4}$	$\delta_{4,6}$	$\delta_{6,5}$	δ_2	δ_3	δ_7	δ_8
$4 \rightarrow 7$	-1	1	0	0	0	0	0	0	0	1	-1	0
$7 \rightarrow 4$	1	0	1	0	0	0	0	0	0	-1	1	0
$6 \rightarrow 7$	0	0	0	1	1	0	0	0	0	0	0	0
$6 \rightarrow 4$	1	0	0	1	0	1	0	0	0	-1	1	0
$4 \rightarrow 6$	-1	0	0	-1	0	0	1	0	0	1	-1	0

```
In[ ]:= λ = SparseArray[λ, {Length[λ], Length[λ[[1]]] + Length[II*] }];
(*λ=λ[[;;-2]]*)
```

```
In[ ]:= dopEq = # == 0 & /@ Flatten[λ.{x_# & /@ EdgeList[g]}]^T];
dopEq // forma
```

Out[]//TableForm=

$$\begin{aligned} x_{4,3} - \frac{p_{4,7} x_{4,7}}{p_{4,3}} &= 0 \\ x_{4,3} - \frac{p_{4,6} x_{4,6}}{p_{4,3}} &= 0 \\ -\frac{p_{6,4} x_{6,4}}{p_{6,7}} + x_{6,7} &= 0 \\ -\frac{p_{6,5} x_{6,5}}{p_{6,7}} + x_{6,7} &= 0 \\ x_{7,4} - \frac{p_{7,6} x_{7,6}}{p_{7,4}} &= 0 \end{aligned}$$

```
In[ ]:= Δ = λ.(δMatr)^T;
"cicle det's:"
Δ // forma
```

Out[]= cicle det's:

Out[]//TableForm=

$$\begin{array}{cccccc} -1 - \frac{p_{4 \rightarrow 7}}{p_{4 \rightarrow 3}} & 1 & 0 & 1 & -1 & \\ -1 & 1 & 0 & 1 & -1 - \frac{p_{4 \rightarrow 6}}{p_{4 \rightarrow 3}} & \\ 0 & 0 & 1 & -\frac{p_{6 \rightarrow 4}}{p_{6 \rightarrow 7}} & 0 & \\ 0 & 0 & 1 & 0 & 0 & \\ 0 & 1 & -\frac{p_{7 \rightarrow 6}}{p_{7 \rightarrow 4}} & -\frac{p_{7 \rightarrow 6}}{p_{7 \rightarrow 4}} & \frac{p_{7 \rightarrow 6}}{p_{7 \rightarrow 4}} & \end{array}$$

```
In[ ]:= rank = MatrixRank[Δ]
```

Out[]= 5

```
In[ ]:= "Uc="
Uc = Range[rank]
"Unc="
Unc = Range[Length[Δ[[1]]]] ~ Complement ~ Uc
```

Out[]= Uc=

Out[]= {1, 2, 3, 4, 5}

Out[]= Unc=

Out[]= { }

```
In[ ]:= Δc = Δ[[All, Uc]];
Δnc = Δ[[All, Unc]];
"Δc="
Δc // MatrixForm
```

Out[]= Δc=

Out[]//MatrixForm=

$$\left(\begin{array}{cccccc} -1 - \frac{p_{4 \rightarrow 7}}{p_{4 \rightarrow 3}} & 1 & 0 & 1 & -1 & \\ -1 & 1 & 0 & 1 & -1 - \frac{p_{4 \rightarrow 6}}{p_{4 \rightarrow 3}} & \\ 0 & 0 & 1 & -\frac{p_{6 \rightarrow 4}}{p_{6 \rightarrow 7}} & 0 & \\ 0 & 0 & 1 & 0 & 0 & \\ 0 & 1 & -\frac{p_{7 \rightarrow 6}}{p_{7 \rightarrow 4}} & -\frac{p_{7 \rightarrow 6}}{p_{7 \rightarrow 4}} & \frac{p_{7 \rightarrow 6}}{p_{7 \rightarrow 4}} & \end{array} \right)$$

```
In[*]:= "det (Λc) ="
Simplify[det = Det[Λc]] // forma
```

```
Out[*]:= det (Λc) =
```

```
Out[*]//TableForm=
```

$$\frac{p_{6,4} (p_{4,6} p_{4,7} p_{7,4} + p_{4,3} (p_{4,6} p_{7,4} + p_{4,7} (p_{7,4} + p_{7,6})))}{p_{4,3}^2 p_{6,7} p_{7,4}}$$

```
In[*]:= "UT="
utind = Cases[t[[6]], ξ_ /; ξ ≠ 0];
UT = EdgeList[g][[utind]]
```

```
Out[*]:= UT=
```

```
Out[*]:= {9 ↔ 2, 9 ↔ 3, 4 ↔ 3, 6 ↔ 5, 7 ↔ 6, 9 ↔ 7, 9 ↔ 8}
```

```
In[*]:= "UNb="
UNb = uNb[g, t]
```

```
Out[*]:= UNb=
```

```
Out[*]:= {4 ↔ 7, 7 ↔ 4, 6 ↔ 7, 6 ↔ 4, 4 ↔ 6}
```

```
In[*]:= A = -λ. {x# & /@ EdgeList[g]}T /. ps;
"A="
A // MatrixForm
```

```
Out[*]:= A=
```

```
Out[*]//MatrixForm=
```

$$\begin{pmatrix} -\frac{f_{3 \rightarrow 9} p_{3 \rightarrow 4}}{p_{3 \rightarrow 9}} & -\frac{f_{3 \rightarrow 9} p_{3 \rightarrow 4}}{p_{3 \rightarrow 9}} & 0 & \left(-f_{9 \rightarrow 5} - \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 5}}{p_{2 \rightarrow 9}} + \frac{f_{5 \rightarrow 9} (p_{5 \rightarrow 2} + p_{5 \rightarrow 6} + p_{5 \rightarrow 9})}{p_{5 \rightarrow 9}}\right) p_{6 \rightarrow 5} & \left(-f_{9 \rightarrow 5} - \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 5}}{p_{2 \rightarrow 9}} - \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 6}}{p_{5 \rightarrow 9}} + \frac{f_{5 \rightarrow 9} (p_{5 \rightarrow 2} + p_{5 \rightarrow 6} + p_{5 \rightarrow 9})}{p_{5 \rightarrow 9}}\right) p_{7 \rightarrow 6} \\ -\frac{f_{3 \rightarrow 9} p_{3 \rightarrow 4}}{p_{3 \rightarrow 9}} & -\frac{f_{3 \rightarrow 9} p_{3 \rightarrow 4}}{p_{3 \rightarrow 9}} & 0 & \left(-f_{9 \rightarrow 5} - \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 5}}{p_{2 \rightarrow 9}} + \frac{f_{5 \rightarrow 9} (p_{5 \rightarrow 2} + p_{5 \rightarrow 6} + p_{5 \rightarrow 9})}{p_{5 \rightarrow 9}}\right) p_{6 \rightarrow 5} & \left(-f_{9 \rightarrow 5} - \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 5}}{p_{2 \rightarrow 9}} - \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 6}}{p_{5 \rightarrow 9}} + \frac{f_{5 \rightarrow 9} (p_{5 \rightarrow 2} + p_{5 \rightarrow 6} + p_{5 \rightarrow 9})}{p_{5 \rightarrow 9}}\right) p_{7 \rightarrow 6} \\ 0 & 0 & 0 & p_{6 \rightarrow 5} & p_{7 \rightarrow 6} \\ \left(-f_{9 \rightarrow 5} - \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 5}}{p_{2 \rightarrow 9}} + \frac{f_{5 \rightarrow 9} (p_{5 \rightarrow 2} + p_{5 \rightarrow 6} + p_{5 \rightarrow 9})}{p_{5 \rightarrow 9}}\right) p_{6 \rightarrow 5} & \left(-f_{9 \rightarrow 5} - \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 5}}{p_{2 \rightarrow 9}} - \frac{f_{5 \rightarrow 9} p_{5 \rightarrow 6}}{p_{5 \rightarrow 9}} + \frac{f_{5 \rightarrow 9} (p_{5 \rightarrow 2} + p_{5 \rightarrow 6} + p_{5 \rightarrow 9})}{p_{5 \rightarrow 9}}\right) p_{7 \rightarrow 6} & p_{6 \rightarrow 5} & p_{7 \rightarrow 6} & p_{7 \rightarrow 4} \end{pmatrix}$$

```
In[*]:= β = A (* -Λnc. {x# & /@ UNb[[Unc]]})T *);
"β="
β // forma
```

```
Out[*]:= β=
```

```
Out[*]//TableForm=
```

$$\begin{pmatrix} -\frac{f_{3,9} p_{3,4}}{p_{3,9}} & -\frac{f_{3,9} p_{3,4}}{p_{3,9}} & 0 & \left(-f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} + \frac{f_{5,9} (p_{5,2} + p_{5,6} + p_{5,9})}{p_{5,9}}\right) p_{6,5} & \left(-f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} - \frac{f_{5,9} p_{5,6}}{p_{5,9}} + \frac{f_{5,9} (p_{5,2} + p_{5,6} + p_{5,9})}{p_{5,9}}\right) p_{7,6} \\ -\frac{f_{3,9} p_{3,4}}{p_{3,9}} & -\frac{f_{3,9} p_{3,4}}{p_{3,9}} & 0 & \left(-f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} + \frac{f_{5,9} (p_{5,2} + p_{5,6} + p_{5,9})}{p_{5,9}}\right) p_{6,5} & \left(-f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} - \frac{f_{5,9} p_{5,6}}{p_{5,9}} + \frac{f_{5,9} (p_{5,2} + p_{5,6} + p_{5,9})}{p_{5,9}}\right) p_{7,6} \\ 0 & 0 & 0 & p_{6,5} & p_{7,6} \\ \left(-f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} + \frac{f_{5,9} (p_{5,2} + p_{5,6} + p_{5,9})}{p_{5,9}}\right) p_{6,5} & \left(-f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} - \frac{f_{5,9} p_{5,6}}{p_{5,9}} + \frac{f_{5,9} (p_{5,2} + p_{5,6} + p_{5,9})}{p_{5,9}}\right) p_{7,6} & p_{6,5} & p_{7,6} & p_{7,4} \end{pmatrix}$$

```
In[*]:= "решаем уравнение Λcxc=β:"
xc = LinearSolve[Λc, β[[1]]]
```

```
Out[*]:= решаем уравнение Λcxc=β:
```

[illegible]

Out[•]//TableForm=

Out[•]//TableForm=

Out[•]//TableForm=

$$\begin{aligned}
X_{4,3} &\rightarrow \frac{p_{4,6} p_{4,7} (f_{3,9} p_{2,9} p_{3,4} p_{5,9} p_{6,4} p_{6,7} p_{7,4} + p_{3,9} (-f_{2,9} p_{2,5} + f_{9,5} p_{2,9}) p_{5,9} (p_{6,4} p_{6,7} p_{7,6} + p_{6,5} (p_{6,4} p_{7,6} + p_{6,7} (p_{7,4} + p_{7,6}))) + f_{5,9} p_{2,9} (p_{2,9} p_{3,9} p_{5,9} p_{6,4} p_{6,7} (p_{4,6} p_{7,6} + p_{4,7} (p_{7,4} + p_{7,6})))}{p_{2,9} p_{3,9} p_{5,9} p_{6,4} p_{6,7} (p_{4,6} p_{7,6} + p_{4,7} (p_{7,4} + p_{7,6})))} \\
X_{7,6} &\rightarrow -f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} - \frac{f_{5,9} p_{5,6}}{p_{5,9}} + \frac{f_{5,9} (p_{5,2} + p_{5,6} + p_{5,9})}{p_{5,9}} + \frac{(-f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} + \frac{f_{5,9} (p_{5,2} + p_{5,6} + p_{5,9})}{p_{5,9}}) p_{6,5}}{p_{6,4}} + \frac{(-f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} + \frac{f_{5,9} (p_{5,2} + p_{5,6} + p_{5,9})}{p_{5,9}}) p_{6,5}}{p_{6,7}} \\
X_{6,5} &\rightarrow -f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} + \frac{f_{5,9} (p_{5,2} + p_{5,6} + p_{5,9})}{p_{5,9}} \\
X_{9,2} &\rightarrow -f_{9,2} - \frac{f_{1,9} p_{1,2}}{p_{1,9}} + \frac{f_{2,9} (p_{2,1} + p_{2,5} + p_{2,8} + p_{2,9})}{p_{2,9}} - \frac{f_{5,9} p_{5,2}}{p_{5,9}} - \frac{f_{8,9} p_{8,2}}{p_{8,9}} \\
X_{9,3} &\rightarrow \frac{f_{3,9} p_{2,9} p_{5,9} p_{6,4} p_{6,7} (p_{3,4} p_{4,3} (p_{4,6} p_{7,4} + p_{4,7} (p_{7,4} + p_{7,6})) + (p_{3,8} + p_{3,9}) (p_{4,6} p_{4,7} p_{7,4} + p_{4,3} (p_{4,6} p_{7,4} + p_{4,7} (p_{7,4} + p_{7,6})))) - p_{3,9} (f_{9,3} p_{2,9} p_{5,9} p_{6,4} p_{6,7} (p_{4,6} p_{7,4} + p_{4,7} (p_{7,4} + p_{7,6})))}{p_{2,9} p_{3,9} p_{5,9} p_{6,4} p_{6,7} (p_{4,6} p_{7,4} + p_{4,7} (p_{7,4} + p_{7,6})))} \\
X_{9,7} &\rightarrow \frac{-p_{5,9} (f_{3,9} p_{2,9} p_{3,4} p_{4,3} p_{6,4} p_{6,7} (p_{4,6} p_{7,4} + p_{4,7} (p_{7,4} + p_{7,6})) + (f_{2,9} p_{2,5} + f_{9,5} p_{2,9}) p_{3,9} (p_{4,3} p_{6,4} p_{6,7} (p_{4,6} p_{7,4} + p_{4,7} (p_{7,4} + p_{7,6}))) + p_{4,6} p_{7,4} p_{4,7} (p_{7,4} + p_{7,6}))}{p_{2,9} p_{3,9} p_{5,9} p_{6,4} p_{6,7} (p_{4,6} p_{7,4} + p_{4,7} (p_{7,4} + p_{7,6})))} \\
X_{9,8} &\rightarrow -f_{9,8} - \frac{f_{2,9} p_{2,8}}{p_{2,9}} - \frac{f_{3,9} p_{3,8}}{p_{3,9}} + \frac{f_{8,9} (p_{8,2} + p_{8,3} + p_{8,9})}{p_{8,9}} \\
X_{4,7} &\rightarrow \frac{p_{4,3} p_{4,6} (f_{3,9} p_{2,9} p_{3,4} p_{5,9} p_{6,4} p_{6,7} p_{7,4} + p_{3,9} (-f_{2,9} p_{2,5} + f_{9,5} p_{2,9}) p_{5,9} (p_{6,4} p_{6,7} p_{7,6} + p_{6,5} (p_{6,4} p_{7,6} + p_{6,7} (p_{7,4} + p_{7,6}))) + f_{5,9} p_{2,9} (p_{2,9} p_{3,9} p_{5,9} p_{6,4} p_{6,7} (p_{4,6} p_{7,6} + p_{4,7} (p_{7,4} + p_{7,6}))))}{p_{2,9} p_{3,9} p_{5,9} p_{6,4} p_{6,7} (p_{4,6} p_{7,6} + p_{4,7} (p_{7,4} + p_{7,6})))} \\
X_{7,4} &\rightarrow \frac{(f_{5,9} p_{2,9} p_{3,9} (p_{4,6} p_{4,7} (p_{5,6} p_{6,5} (p_{6,4} + p_{6,7})) + p_{5,2} (p_{6,5} p_{6,7} + p_{6,4} (p_{6,5} + p_{6,7}))) + p_{5,9} (p_{6,5} p_{6,7} + p_{6,4} (p_{6,5} + p_{6,7}))) + p_{4,3} (p_{4,7} p_{6,4} (p_{5,6} p_{6,5} (p_{6,4} + p_{6,7})) + p_{5,2} (p_{6,5} p_{6,7} + p_{6,4} (p_{6,5} + p_{6,7})))}{p_{2,9} p_{3,9} p_{5,9} p_{6,4} p_{6,7} (p_{4,6} p_{7,6} + p_{4,7} (p_{7,4} + p_{7,6})))} \\
X_{6,7} &\rightarrow \frac{(-f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} + \frac{f_{5,9} (p_{5,2} + p_{5,6} + p_{5,9})}{p_{5,9}}) p_{6,5}}{p_{6,7}} \\
X_{6,4} &\rightarrow \frac{(-f_{9,5} - \frac{f_{2,9} p_{2,5}}{p_{2,9}} + \frac{f_{5,9} (p_{5,2} + p_{5,6} + p_{5,9})}{p_{5,9}}) p_{6,5}}{p_{6,4}} \\
X_{4,6} &\rightarrow -\frac{p_{6,4} (f_{3,9} p_{2,9} p_{3,4} p_{5,9} p_{7,4} + p_{3,9} (-f_{2,9} p_{2,5} + f_{9,5} p_{2,9}) p_{5,9} + f_{5,9} p_{2,9} (p_{5,2} + p_{5,9})) p_{7,6}}{p_{3,9}} - \frac{(-f_{2,9} p_{2,5} + f_{9,5} p_{2,9}) p_{5,9} + f_{5,9} p_{2,9} (p_{5,2} + p_{5,9})}{p_{6,7}} p_{6,5} (p_{6,4} p_{7,6} + p_{6,7} (p_{7,4} + p_{7,6})))}
\end{aligned}$$

```

In[ ]:= "eq test:"
Simplify[balanceEqs /.  $\xi \rightarrow \text{root}[t]$  /. s /. xcp]
Simplify[(dopEq /. s) /. xcp]
Out[ ]:= eq test:

Out[ ]:=  $\left\{ f_{9 \rightarrow 1} + \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 1}}{p_{2 \rightarrow 9}} == f_{1 \rightarrow 9} \left( 1 + \frac{p_{1 \rightarrow 2}}{p_{1 \rightarrow 9}} \right), \text{True}, \text{True}, \text{True}, \right.$ 
 $\left. \text{True}, \text{True}, \text{True}, \text{True}, f_{9 \rightarrow 1} + f_{1 \rightarrow 9} \left( -1 - \frac{p_{1 \rightarrow 2}}{p_{1 \rightarrow 9}} \right) + \frac{f_{2 \rightarrow 9} p_{2 \rightarrow 1}}{p_{2 \rightarrow 9}} == 0 \right\}$ 

Out[ ]:= {True, True, True, True, True}

```