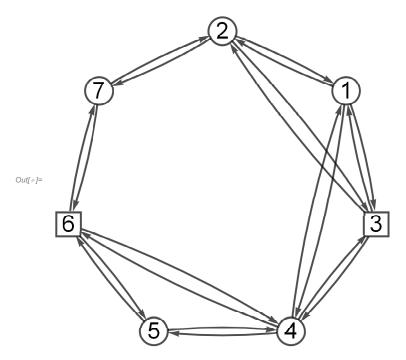
```
\mathbf{2}
 ClearAll["Global*"]
 SetDirectory[NotebookDirectory[]];
 Needs["FlowSolver"]
 readGraph2[file_, dir_]:=Module[{
                     fn = FileNameJoin[{dir, file}],
                     stream, imod, umod, u, b
                     },
                     stream = OpenRead[fn];
                     imod = Read[stream, \{Word, Number\}][[2]];
                     umod = Read[stream, \{Word, Number\}][[2]];
                   u = \left(\left\{\#_{[[1]]}\#_{[[2]]},\#_{[[2]]}\#_{[[1]]}\right\} \&/@\text{ReadList[stream, Expression, umod]}\right) //\text{Flatten};
 b = \text{ConstantArray}[0, \text{imod}];
                     (b[[\mathit{Read}[\mathsf{StringToStream}[\mathsf{StringTake}[\#1,\{5,-3\}]], \mathsf{Number}]]] = \#2) \& @@@\mathsf{ReadList}[\mathsf{stream},\{\mathsf{Word},\mathsf{Express}]] = \#2) \& @@@\mathsf{ReadList}[\mathsf{stream},[\mathsf{word},\mathsf{word}]] = \#2) \& @@@\mathsf{ReadList}[\mathsf{stream},[\mathsf{word},\mathsf{word}]] = \#2) \& @@@\mathsf{ReadList}[\mathsf{stream},[\mathsf{word},\mathsf{word}]] = \#2) \& @@@\mathsf{ReadList}[\mathsf{stream},[\mathsf{word},\mathsf{word}]] = \#2) \& @@\mathsf{word}[\mathsf{stream},[\mathsf{word},\mathsf{word}]] = \#2) \& @@\mathsf{word}[\mathsf{word},[\mathsf{word},\mathsf{word}]] = \#2) \& @@\mathsf{word}[\mathsf{word},[\mathsf{word},\mathsf{word}]] = \#2) \& @\mathsf{word}[\mathsf{word},[\mathsf{word},\mathsf{word}]] = \#2) \& @\mathsf{w
 \{ \operatorname{Graph}[u,\operatorname{VertexSize->Medium},\operatorname{VertexLabels} \rightarrow \operatorname{Placed}[\text{``Name''},\operatorname{Center}],\operatorname{VertexStyle} \rightarrow \operatorname{Directive}[\operatorname{White}],
 \mbox{VertexShapeFunction} \rightarrow \{\mbox{xx}_- : \rightarrow \mbox{If}[\mbox{SameQ}[b[[\mbox{xx}]], x], \mbox{``Square"}, \mbox{``Circle"}]\}, \mbox{VertexLabelStyle-} > \mbox{Directive}[\mbox{Black}, \mbox{Black}, \mbox{``Circle"}]\}, \mbox{VertexLabelStyle-} > \mbox{Directive}[\mbox{Black}, \mbox{``Circle"}], \mbox{``Circle"}] = \mbox{VertexLabelStyle-} > \mbox{Directive}[\mbox{Black}, \mbox{``Circle"}], \mbox{``Circle"}], \mbox{``Circle"}] = \mb
 b}]
\text{forma[ff\_]:=} \left(\left(\text{ff/.}\left\{\xi_{\texttt{-u\_v\_}} \to \xi_{u,v}\right\}\right) / / \text{TableForm}\right)
 \{g,b\} = \text{readGraph2}[\text{"gr.txt"}, \text{NotebookDirectory}]];
GraphPlot[\textit{g}, EdgeStyle \rightarrow Directive[Black, Thick], VertexStyle \rightarrow Directive[EdgeForm[Thick], White], MultiedgeTorm[Thick], White] and the property of the p
```



 $balance Eqs = ((Total [x_\# \& / @Edge List[g, \_\#]] - Total [x_\# \& / @Edge List[g, \#\_]])) == MapIndexed [\#1/.x \rightarrow x_3] + MapIndexed [\#1/.x \rightarrow x_4] + MapIndexed [\#1/.x \rightarrow x_3] + MapIndexed [\#1/.x \rightarrow x_4] + MapIndexe$ 

$$\begin{aligned} x_{2,7} + x_{6,7} - x_{7,2} - x_{7,6} &== 0 \\ x_{1,2} - x_{2,1} - x_{2,3} - x_{2,7} + x_{3,2} + x_{7,2} &== 0 \\ x_{4,6} + x_{5,6} - x_{6,4} - x_{6,5} - x_{6,7} + x_{7,6} &== x_6 \\ -x_{1,2} - x_{1,3} - x_{1,4} + x_{2,1} + x_{3,1} + x_{4,1} &== 0 \\ x_{1,3} + x_{2,3} - x_{3,1} - x_{3,2} - x_{3,4} + x_{4,3} &== x_3 \\ x_{1,4} + x_{3,4} - x_{4,1} - x_{4,3} - x_{4,5} - x_{4,6} + x_{5,4} + x_{6,4} &== 0 \\ x_{4,5} - x_{5,4} - x_{5,6} + x_{6,5} &== 0 \end{aligned}$$

 $M = \{7\};$ 

Print["M = ", M];

 $M = \{7\}$ 

 $(*incL = DeleteCases[DeleteDuplicates[Cases[IncidenceList[g,\#],i\_j_-: \rightarrow \{i,j\}]//Flatten], v\_/;v == \#]\&/@M* \\ incL = (IncidenceList[g,\#]\&/@M)//Flatten$ 

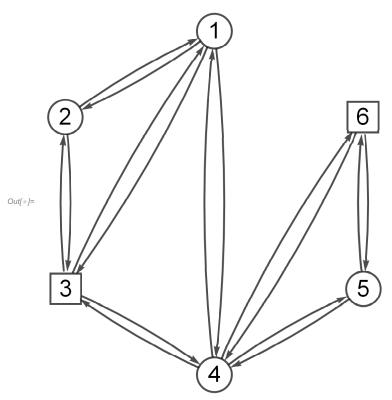
 $\{72, 27, 76, 67\}$ 

 $(*\text{Do}\left[\text{If}\left[\text{MemberQ}\left[M,j_{[[1]]}\right],b_{\left[\left[j\left[[2]\right]\right]\right]}+=f_{j},b_{\left[\left[j\left[[1]\right]\right]\right]}-=f_{j}\right],\left\{j,\text{incL}\right\}\right]*)$ 

$$\begin{split} \bar{b} &= \text{Fold}\left[\text{If}\left[\text{MemberQ}\left[M, \#2_{[[1]]}\right], \text{ReplacePart}\left[\#, \#2_{[[2]]} \to \#_{[[\#2[[2]]]]} + f_{\#2}\right], \text{ReplacePart}\left[\#, \#2_{[[1]]} \to \#_{[B]}\right] \right] \\ \bar{b} &= \text{Delete}\left[\bar{b}, \#\right] \&@@M; \end{split}$$

 $\overline{\text{ng}} = \text{VertexDelete}[g, M];$ 

 $GraphPlot \ [\overline{ng}, EdgeStyle \rightarrow Directive [Black, Thick], VertexStyle \rightarrow Directive [EdgeForm [Thick], White], Multiever \ \overline{b}$ 



 $\{0, -f_{27} + f_{72}, x, 0, 0, x - f_{67} + f_{76}\}$ 

 $\label{eq:cc} \textbf{CC[g\_,M\_]:=(DeleteDuplicates[Cases[IncidenceList[g,\#],i\_j\_/;j==\#]]\&/@M)//Flatten}$ 

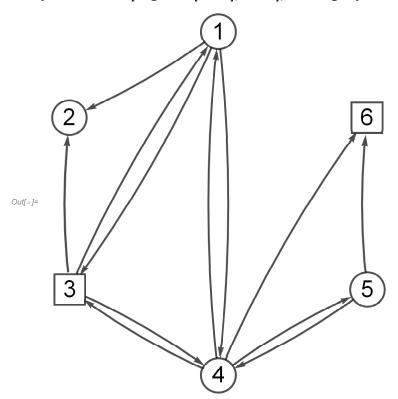
 $\mathrm{ii}^+_{\mathsf{i}_-}[\mathsf{g}_-]{:=}\mathsf{Cases}[\mathsf{IncidenceList}[g,i], \mathsf{u}_-\mathsf{v}_-/; u == i :\to v]$ 

$$M^+ = CC[g, M]$$

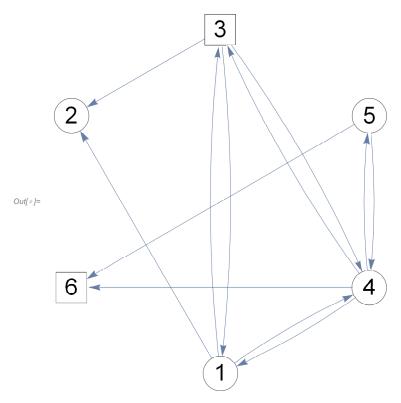
 $\{27, 67\}$ 

$$\overline{\text{b1}} = \text{Fold} \left[ \text{Module} \left[ \left\{ \text{bb} = \#1, i = \#2_{[[1]]}, k = \#2_{[[2]]} \right\}, \left( \text{ReplacePart} \left[ \text{bb}, \left( \left( \left\{ \# \to \text{bb}_{[[\#]]} + \frac{p_{i\#}}{p_{ik}} f_{ik}, i \to \text{bb}_{[[\#]]} + \frac{p_{i\#}}{p_{i\#}} f_{ik}, i \to \text{bb}_{[[\#]]} + \frac{p_{i\#}}{p_{i\#}} f_{ik}, i \to \text{bb}_{[[\#]]} + \frac{p_{i\#}}{p_{i\#}} f_{ik}, i \to \text{bb}_{[\#]} + \frac{p$$

 $\begin{aligned} & \text{GraphPlot}\left[\text{Fold}\left[\text{HighlightGraph}[\#1, u\_v\_/; u == \#2, \text{GraphHighlightStyle} \rightarrow \text{``White''}]\&, \overline{\text{ng}}, \#_{[[1]]}\&/@M^+\right], \\ & \text{VertexStyle} \rightarrow \text{Directive}\left[\text{EdgeForm}\left[\text{Thick}\right], \text{White}\right], \\ & \text{MultiedgeStyle} \rightarrow .05\right] \end{aligned}$ 



$$\label{eq:continuity} \begin{split} \overline{\mathbf{g1}} &= \mathrm{Fold}\left[\mathrm{EdgeDelete}[\#1, \mathbf{u}_{-}\mathbf{v}_{-}/; u == \#2]\&, \overline{\mathbf{ng}}, \#_{[[1]]}\&/@M^{+}\right]; \\ &\mathrm{GraphPlot}\left[\overline{\mathbf{g1}}, \mathrm{MultiedgeStyle} \rightarrow .05\right] \end{split}$$



 $\mathrm{II}_{\mathrm{rem}} = \mathrm{VertexList}\left[\overline{\mathrm{g1}}\right] \sim \mathrm{Complement} \sim (M^+[[\mathrm{All},1]])$ 

 $\{1, 3, 4, 5\}$ 

 $\lambda = \text{SparseArray} \left[ \text{Replace} \left[ \left( \text{EdgeList} \left[ \overline{\text{g1}} \right] \right. \right. \right. \right. \\ \left. \left. \text{Module} \left[ \left\{ i = \#, \text{jf, Icur} \right\}, \left( \text{Icur} = \text{ii}_i^+ \left[ \overline{\text{g1}} \right] \right. \right] \right] \right] \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\} \\ \left. \text{if } = \text{First} \left[ \text{Icur} \right] \right\}$ 

$$\left(\left\{(i\mathrm{jf})\rightarrow1,(i\#)\rightarrow-\frac{p_{i\#}}{p_{i\mathrm{jf}}}\right\}\right)\&/@\mathrm{Icur}[[2;;]]\right)\right]\&/@\mathrm{II}_{\mathrm{rem}},1\right]\right),{}_{--}\rightarrow0,2\right]\right]$$

 $\operatorname{SparseArray}\left[ _{\square}\right]$ 

 $\operatorname{Grid}[\lambda]$ 

 $g = \overline{\mathbf{g1}};$ 

 $b = \overline{\mathbf{b1}};$ 

$$\label{eq:interpolation} \begin{split} \text{II*} &= \text{Cases}[\text{MapIndexed}[\{\#1,\#2\}\&,b], \{\text{el.,i.}\}/; \text{MemberQ[el},x]: \to i]//\text{Flatten} \\ &\{3,6\} \end{split}$$

 $\mathrm{buildt} = \mathrm{Timing}\left[\left\{t, g\right\} = \mathrm{buildTree}\left[g, \mathrm{II}^*\right];\right]\left[\left[1\right]\right]$ 

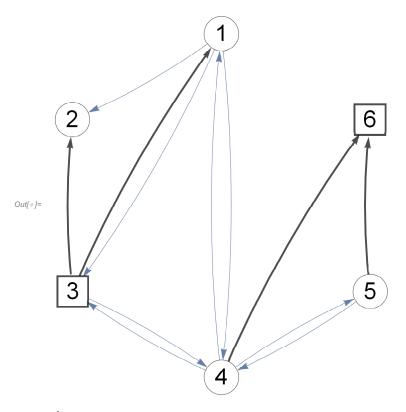
 $\textbf{TableForm}[t[[1;;4]], \textbf{TableHeadings} \rightarrow \{\{\texttt{"pred"}, \texttt{"dir"}, \texttt{"depth"}, d\}, t//\texttt{pred}//\texttt{Length}//\texttt{Range}\}]$ 

0.015625

	1	2	3	4	5	6	7
pred				6		7	0
dir depth	1	1	1	-1	-1	1	0
depth	2	2	1	2	2	1	0
d	2	7	1	5	3	4	6

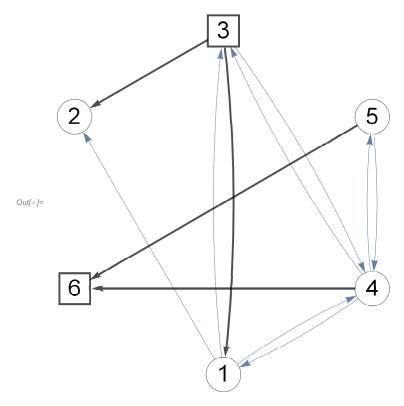
 $\begin{aligned} & \text{GraphPlot}\left[\text{HighlightGraph}\left[\text{Fold}\left[\text{HighlightGraph}[\#1,\text{Style}[\textbf{u}\_\textbf{v}\_/;u==\#2,\text{White}]}\right]\&,\overline{\textbf{ng}},\#_{[[1]]}\&/@M^+\right],\\ & \text{Style}[\textbf{u}\_/;\text{VertexQ}[g,u]\&\&\text{pred}[t][[u]]==7,\text{EdgeForm}[\text{Thick}]], \end{aligned}$ 

 $Style[u\_v\_/;(pred[t][[u]] == v\&\&dir[t][[u]] == -1) \|(pred[t][[v]] == u\&\&dir[t][[v]] == 1), Directive[Black, Thick MultiedgeStyle <math>\rightarrow .05]$ 

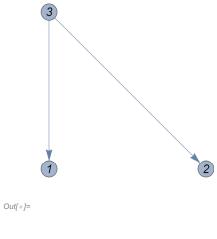


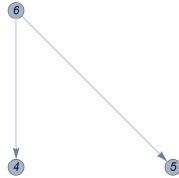
# GraphPlot[

 $\begin{aligned} & \text{HighlightGraph} \ [\overline{\textbf{g1}}, \{\text{Style}[\textbf{u}\_/; \text{VertexQ}[\textbf{g}, \textbf{u}]\&\&\text{pred}[t][[\textbf{u}]] == 7, \text{EdgeForm}[\text{Thick}]], \\ & \text{Style}[\textbf{u}\_\textbf{v}\_/; (\text{pred}[t][[\textbf{u}]] == v\&\&\text{dir}[t][[\textbf{u}]] == -1) \| (\text{pred}[t][[\textbf{v}]] == u\&\&\text{dir}[t][[\textbf{v}]] == 1), \text{Directive}[\text{Black}, \text{Thick}], \\ & \text{MultiedgeStyle} \rightarrow .05] \end{aligned}$ 

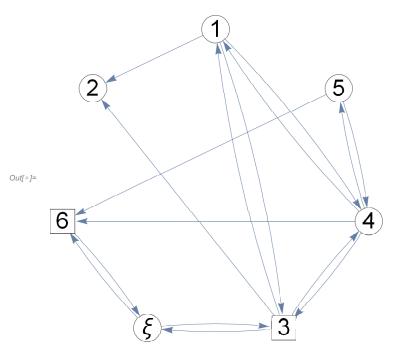


 ${\bf VertexDelete}[t[[7]],7](*\ *)$ 





 ${\tt GraphPlot}[g, {\tt MultiedgeStyle} \rightarrow .05]$ 



AppendTo[b, -Total[b]];

 $b = \operatorname{Simplify}[b/.x \to 0]$ 

$$\left\{\frac{f_{27}p_{21}}{p_{27}},f_{72}+f_{27}\left(-1-\frac{p_{21}}{p_{27}}\right),\frac{f_{27}p_{23}}{p_{27}},\frac{f_{67}p_{64}}{p_{67}},\frac{f_{67}p_{65}}{p_{67}},f_{76}+f_{67}\left(-1-\frac{p_{65}}{p_{67}}\right),-f_{72}-f_{76}+f_{27}\left(1-\frac{p_{23}}{p_{27}}\right)+f_{67}\left(1-\frac{p_{23}}{p_{27}}\right)\right\}$$

 $balance Eqs = \left( \left( \text{Total} \left[ x_\# \& / @ \text{EdgeList}[g, \_\#] \right] - \text{Total} \left[ x_\# \& / @ \text{EdgeList}[g, \#\_] \right] \right) / .7 \rightarrow \xi \right) == b[[\#]] \& / @ \text{VertexInterval}$  balance Eqs / forma

$$\begin{split} x_{1,2} + x_{3,2} &== f_{7,2} + f_{2,7} \left( -1 - \frac{p_{2,1}}{p_{2,7}} \right) \\ x_{4,6} + x_{5,6} - x_{6,\xi} + x_{\xi,6} &== f_{7,6} + f_{6,7} \left( -1 - \frac{p_{6,5}}{p_{6,7}} \right) \\ -x_{1,2} - x_{1,3} - x_{1,4} + x_{3,1} + x_{4,1} &== \frac{f_{2,7}p_{2,1}}{p_{2,7}} \\ x_{1,3} - x_{3,1} - x_{3,2} - x_{3,4} - x_{3,\xi} + x_{4,3} + x_{\xi,3} &== \frac{f_{2,7}p_{2,3}}{p_{2,7}} \\ x_{1,4} + x_{3,4} - x_{4,1} - x_{4,3} - x_{4,5} - x_{4,6} + x_{5,4} &== \frac{f_{6,7}p_{6,4}}{p_{6,7}} \\ x_{4,5} - x_{5,4} - x_{5,6} &== \frac{f_{6,7}p_{6,5}}{p_{6,7}} \\ x_{3,\xi} + x_{6,\xi} - x_{\xi,3} - x_{\xi,6} &== -f_{7,2} - f_{7,6} + f_{2,7} \left( 1 - \frac{p_{2,3}}{p_{2,7}} \right) + f_{6,7} \left( 1 - \frac{p_{6,4}}{p_{6,7}} \right) \end{split}$$

 $ps = partSolve[g, -b, t, \tilde{x}];$ 

### ps//forma

$$\tilde{x}_{1,2} \to 0$$

$$\tilde{x}_{1,3} \to 0$$

$$\tilde{x}_{1,4} \to 0$$

$$\tilde{x}_{3,1} \to \frac{f_{2,7}p_{2,1}}{p_{2,7}}$$

$$\tilde{x}_{3,2} \to f_{7,2} + f_{2,7} \left( -1 - \frac{p_{2,1}}{p_{2,7}} \right)$$

$$\tilde{x}_{3,4} \to 0$$

$$\tilde{x}_{3,7} \to 0$$

$$\tilde{x}_{4,1} \to 0$$

$$\tilde{x}_{4,3} \to 0$$

$$\tilde{x}_{4,5} \to 0$$

$$\tilde{x}_{4,6} \rightarrow -\frac{f_{6,7}p_{6,4}}{p_{6,7}}$$

$$\tilde{x}_{5,4} \to 0$$

$$\tilde{x}_{5,6} o -rac{f_{6,7}p_{6,5}}{p_{6,7}}$$

$$\tilde{x}_{6,7} \rightarrow 0$$

$$\begin{split} \tilde{x}_{7,3} &\to f_{7,2} + f_{2,7} \left( -1 - \frac{p_{2,1}}{p_{2,7}} \right) + \frac{f_{2,7}p_{2,1}}{p_{2,7}} + \frac{f_{2,7}p_{2,3}}{p_{2,7}} \\ \tilde{x}_{7,6} &\to f_{7,6} + f_{6,7} \left( -1 - \frac{p_{6,5}}{p_{6,7}} \right) + \frac{f_{6,7}p_{6,4}}{p_{6,7}} + \frac{f_{6,7}p_{6,5}}{p_{6,7}} \end{split}$$

#### Simplify [(balance Eqs/. $\{x \to \tilde{x}, \xi \to 7\})\,/.\mathrm{ps}]$

{True, True, True, True, True, True}

 $matrt = Timing[\delta Matr = \delta 1[g, t]];$ 

roott = VertexCount[g];

$$\text{TableForm} \left[ \delta \text{Matr, TableHeadings} \rightarrow \left\{ \begin{aligned} \text{uNb}[g,t], \delta & & \&/@\text{EdgeList}[g] \\ \#_{[[2]]} & \#_{[[1]]} == \text{roott} \\ \#_{[[1]]} & \#[[2]] == \text{roott} \\ \# & \text{True} \end{aligned} \right\} \right] / / \text{forma}$$

	$\delta_{1,2}$	$\delta_{3,2}$	$\delta_{1,3}$	$\delta_{3,1}$	$\delta_{1,4}$	$\delta_{4,1}$	$\delta_{3,4}$	$\delta_{4,3}$	$\delta_{4,5}$	$\delta_{5,4}$	$\delta_{5,6}$	$\delta_{4,6}$	$\delta_3$	$\delta_6$	$\delta_3$	$\delta_6$
12	1	-1	0	1	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	1	1	0	0	0	0	0	0	1	1	-1	0	0
41	0	0	0	-1	0	1	0	0	0	0	0	-1	-1	1	0	0
34	0	0	0	0	0	0	1	0	0	0	0	1	1	-1	0	0
43	0	0	0	0	0	0	0	1	0	0	0	-1	-1	1	0	0
45	0	0	0	0	0	0	0	0	1	0	1	-1	0	0	0	0
54	0	0	0	0	0	0	0	0	0	1	-1	1	0	0	0	0
37	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
67	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1

 $\lambda = \text{SparseArray}[\lambda, \{\text{Length}[\lambda], \text{Length}[\lambda[[1]]] + 4\}];$ 

$$(*\lambda = \lambda[[;;-2]]*)$$

$$\label{eq:dopEq} \begin{split} \mathrm{dopEq} &= \# == 0 \& / @ \mathrm{Flatten} \left[ \lambda. \left\{ x_\# \& / @ \mathrm{EdgeList}[g] \right\}^\mathsf{T} \right]; \\ \mathrm{dopEq} / / \mathrm{forma} \end{split}$$

$$x_{1,2} - \frac{p_{1,3}x_{1,3}}{p_{1,2}} == 0$$

$$x_{1,2} - \frac{p_{1,4}x_{1,4}}{p_{1,2}} == 0$$

$$-\frac{p_{3,1}x_{3,1}}{p_{3,2}} + x_{3,2} == 0$$

$$x_{3,2} - \frac{p_{3,4}x_{3,4}}{p_{3,2}} == 0$$

$$x_{4,1} - \frac{p_{4,3}x_{4,3}}{p_{4,1}} == 0$$

$$x_{4,1} - \frac{p_{4,5}x_{4,5}}{p_{4,1}} == 0$$

$$x_{4,1} - \frac{p_{4,6}x_{4,6}}{p_{4,1}} == 0$$

$$x_{5,4} - \frac{p_{5,6}x_{5,6}}{p_{5,4}} == 0$$

# $\Lambda = \lambda.(\delta \mathrm{Matr})^\mathsf{T};$

### "cicle det's:"

### $\Lambda//forma$

cicle det's:

1	$-\tfrac{p_{13}}{p_{12}}$	0	0	0	0	0	0	0	0
1	0	$-\tfrac{p_{14}}{p_{12}}$	0	0	0	0	0	0	0
$-1 - \frac{p_{31}}{p_{32}}$	$-\frac{p_{31}}{p_{32}}$	$-\frac{p_{31}}{p_{32}}$	$\frac{p_{31}}{p_{32}}$	0	0	0	0	0	0
-1	0	0	0	$-\frac{p_{34}}{p_{32}}$	0	0	0	0	0
0	0	0	1	0	$-\tfrac{p_{43}}{p_{41}}$	0	0	0	0
0	0	0	1	0	0	$-\tfrac{p_{45}}{p_{41}}$	0	0	0
0	0	$-\frac{p_{46}}{p_{41}}$	$1 + \frac{p_{46}}{p_{41}}$	$-\tfrac{p_{46}}{p_{41}}$	$\frac{p_{46}}{p_{41}}$	$\frac{p_{46}}{p_{41}}$	$-rac{p_{46}}{p_{41}}$	0	0
0	0	0	0	0	0	$-rac{p_{56}}{p_{54}}$	$1 + \frac{p_{56}}{p_{54}}$	0	0

" $U_c$ ="

 $U_c = \text{Range}[8]$ 

" $U_{\rm nc}$ ="

 $U_{\rm nc}=\{9,10\}$ 

 $U_c =$ 

 $\{1, 2, 3, 4, 5, 6, 7, 8\}$ 

$$\begin{split} &U_{\rm nc} = \\ &\{9, 10\} \\ &\mathbf{\Lambda c} = \mathbf{\Lambda} \left[ \left[ \mathbf{All}, U_{c} \right] \right]; \\ &\mathbf{\Lambda nc} = \mathbf{\Lambda} \left[ \left[ \mathbf{All}, U_{\rm nc} \right] \right]; \\ &\mathbf{"} \mathbf{\Lambda}_{c} = \mathbf{"} \end{split}$$

 $\Lambda c//MatrixForm$ 

 $\det(\Lambda_c) =$ 

 $Simplify[det = Det[\Lambda c]]/forma$ 

$$\det(\Lambda_c) =$$

$$-\frac{1}{p_{1,2}^2p_{3,2}^3p_{4,1}^3p_{5,4}}\left(p_{1,2}p_{3,1}p_{3,4}\left(p_{1,3}p_{4,1}\left(p_{4,5}p_{4,6}\left(p_{5,4}+p_{5,6}\right)+p_{4,3}\left(p_{4,6}p_{5,4}+p_{4,5}\left(p_{5,4}+p_{5,6}\right)\right)\right)+p_{1,4}\left(p_{4,3}p_{4,5}p_{4,6}\left(p_{5,4}+p_{5,6}\right)+p_{4,3}\left(p_{4,6}p_{5,4}+p_{4,5}\left(p_{5,4}+p_{5,6}\right)\right)\right)+p_{1,4}\left(p_{4,3}p_{4,5}p_{4,6}\left(p_{5,4}+p_{5,6}\right)+p_{4,3}\left(p_{4,6}p_{5,4}+p_{4,5}\left(p_{5,4}+p_{5,6}\right)\right)\right)+p_{1,4}\left(p_{4,3}p_{4,5}p_{4,6}\left(p_{5,4}+p_{5,6}\right)+p_{4,6}\left(p_{5,4}+p_{$$

" $U_T$ ="

utind = Cases[ $t[[6]], \xi_-/; \xi \neq 0$ ];

 $U_T = \text{EdgeList}[g][[\text{utind}]]$ 

 $U_T =$ 

 $\{31, 32, 73, 46, 56, 76\}$ 

"
$$U_{\mathrm{Nb}}$$
="

$$U_{\mathrm{Nb}} = \mathrm{uNb}[g,t]$$

$$U_{\mathrm{Nb}} =$$

 $\{12, 13, 14, 41, 34, 43, 45, 54, 37, 67\}$ 

$$A = -\lambda.\left\{\tilde{x}_\# \&/@\text{EdgeList}[g]\right\}^\mathsf{T}/.\text{ps};$$

### $A//{ m MatrixForm}$

A=

$$\begin{pmatrix}
0 \\
0 \\
-f_{72} - f_{27} \left(-1 - \frac{p_{21}}{p_{27}}\right) + \frac{f_{27}p_{21}p_{31}}{p_{27}p_{32}} \\
-f_{72} - f_{27} \left(-1 - \frac{p_{21}}{p_{27}}\right) \\
0 \\
0 \\
-\frac{f_{67}p_{46}p_{64}}{p_{41}p_{67}} \\
-\frac{f_{67}p_{56}p_{65}}{p_{54}p_{67}}
\end{pmatrix}$$

$$\beta = A - \Lambda \text{nc.} \left\{ x_\# \& / @U_{\text{Nb}} \left[ \left[ U_{\text{nc}} \right] \right] \right\}^\intercal;$$

 $\beta$ //forma

 $\beta =$ 

0

$$-f_{7,2} - f_{2,7} \left(-1 - \frac{p_{2,1}}{p_{2,7}}\right) + \frac{f_{2,7}p_{2,1}p_{3,1}}{p_{2,7}p_{3,2}}$$
$$-f_{7,2} - f_{2,7} \left(-1 - \frac{p_{2,1}}{p_{2,7}}\right)$$

0

0

$$-\frac{f_{6,7}p_{4,6}p_{6,4}}{p_{4,1}p_{6,7}}$$
$$-\frac{f_{6,7}p_{5,6}p_{6,5}}{p_{5,4}p_{6,7}}$$

#### " $\Lambda_c x_c = \beta$ :"

### $xc = LinearSolve[\Lambda c, \beta]$

 $\Lambda_c x_c = \beta$ :



 $\texttt{xcp} = \texttt{MapThread}\left[x_{\#1} \rightarrow \#2\&, \left\{U_{\texttt{Nb}}\left[\left[U_{c}\right]\right], \texttt{Flatten}[\texttt{xc}]\right\}\right];$ 

xcp//TableForm



s = solveAll[g, t];

 $s//{
m TableForm}$ 

$$x_{32} \to f_{72} + f_{27} \left( -1 - \frac{p_{21}}{p_{27}} \right) - x_{12}$$

$$x_{31} \to \frac{f_{27}p_{21}}{p_{27}} + x_{12} + x_{13} + x_{14} - x_{41}$$

$$x_{56} \to -\frac{f_{67}p_{65}}{p_{67}} + x_{45} - x_{54}$$

$$x_{46} \to -\frac{f_{67}p_{64}}{p_{67}} + x_{14} + x_{34} - x_{41} - x_{43} - x_{45} + x_{54}$$

$$x_{73} \to f_{72} + f_{27} \left( -1 - \frac{p_{21}}{p_{27}} \right) + \frac{f_{27}p_{21}}{p_{27}} + \frac{f_{27}p_{23}}{p_{27}} + x_{14} + x_{34} + x_{37} - x_{41} - x_{43}$$

$$x_{76} \to f_{76} + f_{67} \left( -1 - \frac{p_{65}}{p_{67}} \right) + \frac{f_{67}p_{64}}{p_{67}} + \frac{f_{67}p_{65}}{p_{67}} - x_{14} - x_{34} + x_{41} + x_{43} + x_{67}$$

```
" ."
xsol = (s \sim Join \sim xcp);
xsol/. \{\xi_{-\mathbf{u},\mathbf{v}} \to \xi_{\mathbf{u},\mathbf{v}}\} //Simplify//TableForm
          x_{3,2} \to f_{7,2} + f_{2,7} \left( -1 - \frac{p_{2,1}}{p_{2,7}} \right) - x_{1,2}
          x_{3,1} \to \frac{f_{2,7}p_{2,1}}{p_{2,7}} + x_{1,2} + x_{1,3} + x_{1,4} - x_{4,1}
          x_{5,6} \rightarrow -\frac{f_{6,7}p_{6,5}}{p_{6,7}} + x_{4,5} - x_{5,4}
          x_{4,6} \rightarrow -\frac{f_{6,7}p_{6,4}}{p_{6,7}} + x_{1,4} + x_{3,4} - x_{4,1} - x_{4,3} - x_{4,5} + x_{5,4}
          x_{7,3} \to f_{7,2} + f_{2,7} \left( -1 + \frac{p_{2,3}}{p_{2,7}} \right) + x_{1,4} + x_{3,4} + x_{3,7} - x_{4,1} - x_{4,3}
            x_{7,6} \to f_{7,6} + f_{6,7} \left( -1 + \frac{p_{6,4}}{p_{6,7}} \right) - x_{1,4} - x_{3,4} + x_{4,1} + x_{4,3} + x_{6,7}
               x_{1,2} \rightarrow -\frac{p_{1,3}p_{1,4}(f_{6,7}p_{2,7}p_{3,1}p_{3,4}p_{4,3}p_{4,5}p_{4,6}(p_{5,4}p_{6,4}+p_{5,6}(p_{6,4}+p_{6,5})) + (-f_{7,2}p_{2,7}p_{3,2}(p_{3,1}p_{4,3}p_{4,5}p_{4,6}(p_{5,4}+p_{5,6}) + p_{3,4}(p_{4,3}p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})) + (-f_{7,2}p_{2,7}p_{3,2}(p_{5,4}+p_{5,6}) + p_{3,4}(p_{5,4}+p_{5,6}) + (-f_{7,2}p_{2,7}p_{3,2}(p_{5,4}+p_{5,6}) + p_{3,4}(p_{5,4}+p_{5,6}) + (-f_{7,2}p_{2,7}p_{5,4}+p_{5,6}) + (-f_
                  x_{1:3} \rightarrow -\frac{p_{1,2}p_{1,4}(f_{6,7}p_{2,7}p_{3,1}p_{3,4}p_{4,3}p_{4,5}p_{4,6}(p_{5,4}p_{6,4}+p_{5,6}(p_{6,4}+p_{6,5})) + (-f_{7,2}p_{2,7}p_{3,2}(p_{3,1}p_{4,3}p_{4,5}p_{4,6}(p_{5,4}+p_{5,6}) + p_{3,4}(p_{4,3}p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})) + (-f_{7,2}p_{2,7}p_{3,2}(p_{5,4}+p_{5,6}) + p_{3,4}(p_{4,3}p_{4,5}p_{4,6}) + (-f_{7,2}p_{2,7}p_{3,2}(p_{5,4}+p_{5,6}) + p_{3,4}(p_{4,3}p_{4,5}p_{4,6}(p_{5,4}+p_{5,6}) + p_{3,4}(p_{4,3}p_{4,5}p_{4,6}) + (-f_{7,2}p_{2,7}p_{3,2}(p_{4,3}p_{4,5}p_{4,6}(p_{5,4}+p_{5,6}) + p_{3,4}(p_{4,3}p_{4,5}p_{4,6}(p_{5,4}+p_{5,6}) + p_{3,4}(p_{4,3}p_{4,5}p_{4,6}(p_{5,4}+p_{5,6}) + p_{3,4}(p_{4,3}p_{4,5}p_{4,6}(p_{5,4}+p_{5,6}) + p_{3,4}(p_{4,3}p_{4,5}p_{4,6}(p_{5,4}+p_{5,6}) + p_{3,4}(p_{4,3}p_{4,5}p_{4,6}(p_{5,4}+p_{5,6}) + p_{3,4}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6}) + p_{3,4}(p_{5,4}+p_{5,6}) + p
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        p_{2,7}(p_{1,2}p_{3,1}p_{3,4}(p_{1,3}p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6}
               x_{1,4} \rightarrow -\frac{p_{1,2}p_{1,3}(f_{6,7}p_{2,7}p_{3,1}p_{3,4}p_{4,3}p_{4,5}p_{4,6}(p_{5,4}p_{6,4}+p_{5,6}(p_{6,4}+p_{6,5})) + (-f_{7,2}p_{2,7}p_{3,2}(p_{3,1}p_{4,3}p_{4,5}p_{4,6}(p_{5,4}+p_{5,6}) + p_{3,4}(p_{4,3}p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})) + (-f_{7,2}p_{2,7}p_{3,2}(p_{5,4}+p_{5,6}) + p_{3,4}(p_{5,4}+p_{5,6}) + (-f_{7,2}p_{2,7}p_{3,2}(p_{5,4}+p_{5,6}) + p_{3,4}(p_{5,4}+p_{5,6}) + (-f_{7,2}p_{2,7}p_{5,4}+p_{5,6}) + (-f_
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        p_{2,7}(p_{1,2}p_{3,1}p_{3,4}(p_{1,3}p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               p_{4,3}p_{4,5}p_{4,6}(f_{6,7}p_{2,7}(p_{1,2}(p_{1,3}+p_{1,4})p_{3,1}+p_{1,3}p_{1,4}(p_{3,1}+p_{3,2}))p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2}
               x_{4,1} \rightarrow -\frac{r_{4,31} - r_{4,6} - r_{5,6} - 
               x_{3,4} \rightarrow -\frac{p_{3,1}p_{3,2}(-f_{6,7}p_{1,3}p_{1,4}p_{2,7}p_{4,3}p_{4,5}p_{4,6}(p_{5,4}p_{6,4}+p_{5,6}(p_{6,4}+p_{6,5})) + (-f_{7,2}p_{2,7}(p_{1,3}p_{1,4}(p_{4,3}p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{6,5})))}{(-f_{7,2}p_{2,7}(p_{1,3}p_{1,4}(p_{4,3}p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{6,5}))))}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       p_{4,1}p_{4,5}p_{4,6}(f_{6,7}p_{2,7}(p_{1,2}(p_{1,3}+p_{1,4})p_{3,1}+p_{1,3}p_{1,4}(p_{3,1}+p_{3,2}))p_{3,4}(p_{3,1}+p_{3,2}))p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2
               x_{4,3} \rightarrow -\frac{r_{4,1}r_{4,5}r_{4,6}(p_{5,4}+p_{5,6})+p_{4,3}(p_{4,6}p_{5,4}+p_{4,5}(p_{5,4}+p_{4,5}(p_{5,4}+p_{4,5}(p_{5,4}+p_{4,5}(p_{5,4}+p_{5,6})))+p_{1,4}(p_{4,3}p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            p_{4,1}p_{4,3}p_{4,6}(f_{6,7}p_{2,7}(p_{1,2}(p_{1,3}+p_{1,4})p_{3,1}+p_{1,3}p_{1,4}(p_{3,1}+p_{3,2}))p_{3,4}(p_{3,1}+p_{3,2}))p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2}))p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2}))p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2}))p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{3,2})p_{3,4}(p_{3,1}+p_{
               x_{4,5} \rightarrow -\frac{r_{4,1}r_{3,1}r_{3,4}(p_{1,2}p_{3,1}p_{3,4}(p_{1,3}p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,3}(p_{4,6}p_{5,4}+p_{4,5}(p_{5,4}+p_{5,6})))+p_{1,4}(p_{4,3}p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})))+p_{1,4}(p_{4,3}p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{
               x_{5,4} \rightarrow - \frac{p_{5,6}(f_{6,7}p_{2,7}(p_{1,2}p_{3,1}p_{3,4}(p_{1,3}p_{4,1}(p_{4,5}p_{4,6}p_{6,5}+p_{4,3}(p_{4,5}p_{6,5}+p_{4,6}(p_{6,4}+p_{6,5}))) + p_{1,4}(p_{4,3}p_{4,5}p_{4,6}p_{6,5}+p_{4,1}(p_{4,5}p_{4,6}p_{6,5}+p_{4,6}(p_{6,4}+p_{6,5}))) + p_{1,4}(p_{4,3}p_{4,5}p_{4,6}p_{6,5}+p_{4,1}(p_{4,5}p_{4,6}p_{6,5}+p_{4,6}(p_{6,4}+p_{6,5}))) + p_{1,4}(p_{4,3}p_{4,5}p_{4,6}p_{6,5}+p_{4,1}(p_{4,5}p_{4,6}p_{6,5}+p_{4,6}(p_{6,4}+p_{6,5}))) + p_{1,4}(p_{4,3}p_{4,5}p_{4,6}p_{6,5}+p_{4,1}(p_{4,5}p_{4,6}p_{6,5}+p_{4,6}(p_{6,4}+p_{6,5}))) + p_{1,4}(p_{4,3}p_{4,5}p_{4,6}p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,4}+p_{6,5}))) + p_{1,4}(p_{4,3}p_{4,5}p_{4,6}p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{4,6}(p_{6,5}+p_{6,5}+p_{4,6}(p_{6,5}+p_{6,5}+p_{4,6}(p_{6,5}+p_{6,5}+p_{4,6}(p
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            p_{2,7}(p_{1,2}p_{3,1}p_{3,4}(p_{1,3}p_{4,1}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,3}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,3}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6}(p_{5,4}+p_{5,6})+p_{4,5}(p_{4,5}p_{4,6})+p_{4,5}(p_{4,5}p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{4,5}(p_{5,4}+p_{5,6})+p_{5,5}(p_{5,4}+p_{5,6})+p_{5,5}(p_{5,5}+p_{5,5})+p_{5,5}(p_{5,5}+p_{5,5})+p_{5,5}(
     "eq test:"
Simplify[balanceEqs/.\xi \rightarrow 7/.s/.xcp]
Simplify[(dopEq/.s)/.xcp]
eq test:
```

{True, True, True, True, True, True, True}

{True, True, True, True, True, True, True, True}