**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Given: *μ* = 45 and *σ* = 8

Let X be time need for servicing the customer’s car

To find: Pr(service manager cannot meet his commitment) = Pr(X>50)

Because the manager begins work 10 minutes after the car dop-off, hence 60 – 10 = 50 mins remaining for the work to be done on time.

Convert 50 to z-score.

In z-table, 0.625 value = 0.73237

Pr(X<=50) = 0.73237

∴ Pr(X>50) = 1 – 0.73237 = 0.2676

∴ the answer is B. 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Given: *μ* = 38 and *σ* =6

Total number of employees N = 400

z-score formula is

X = 44 ∴ 1.0 value = 0.84134 in the z-table for normal distribution.

Pr(X>44) = 1 – 0.84134 = 0.15866 = 15.86%

Pr(38 < X < 44) = 0.84134 – 0.5 = 0.34134 = 34.13%

15.86 < 34.13 Hence, the statement is FALSE.

X = 30 ∴ -1.333 value = 0.09176 in the z-table for normal

Pr(X < 30) = 0.09176 = 9.176%

N = 400 the number of employees with probability 0.917 of them being under age 30 = 0.0917\*400=36.68 or approximately 36 employees.

1. FALSE statement
2. TRUE statement
3. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

ANSWER:

If X and Y are independent, and X ∼ Normal(µ1, σ12 ), Y ∼ Normal(µ2, σ22 ),

then X + Y ∼ Normal ( µ1 + µ2, σ12 + σ22 )

2 X1  ∼ N ( 2\*µ,2\*σ2 ) ∼ N ( 2µ, 2 σ2 )

X1 + X2 ∼ N (µ+ µ, σ2 + σ2 ) ∼ N ( 2 µ, 2 σ2 )

Therefore, both are same distribution and parameters.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

ANSWER:

We can work this problem in reverse order.

The Probability of getting value between a and b should be 0.99.

So the Probability of going wrong, or the Probability outside the a and b area is 0.01 (ie. 1-0.99).

The Probability towards left from a = -0.005 (ie. 0.01/2).

The Probability towards right from b = +0.005 (ie. 0.01/2).

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

For Probability 0.005 the Z Value is -2.57 (from Z Table).

Zvalue(-0.005)\*20+100 = -(-2.57)\*20+100 = 151.4

Zvalue(+0.005)\*20+100 = (-2.57)\*20+100 = 48.6

So, option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year? P1

ANSWER:

We can work this problem in reverse order.

The Probability of getting PROFIT should be 0.95.

So the Probability of going wrong, or the Probability of LOSS is 0.05 (ie. 1-0.95).

The Probability towards left = -0.025 (ie. 0.05/2).

The Probability towards right = +0.025 (ie. 0.05/2).

So since we have the probabilities of LEFT and RIGHT, we need to calculate X, the random variable which has got these probabilities.

By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.

For Probability 0.025 the Z Value is -1.96 (from Z Table).

GIVEN $1 = 45 rs.

Hence, the range would be above X values multiplied by 45.

1.719\*45 = 77.355 rs

25.72\*45 = 1157.4 rs.

A. The rupee range is 77.355 rs to 1157.4 rs

PROFIT for the company as a whole has normal distribution with (12,72)

Prwhole  ∼ (12,72)

Most likely division 2 with the P2 profit distribution will make a loss in a given year.