PARADOXES OF PROBABILITY

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ABSTRACT

Probability paradoxes present a challenge to knowledge reasoning within probability principles and knowledge updating. Approaches for solving self-location problems such as Sleeping Beauty and Judy Benjamin have been discussed within logic separately. However, considering the initial scenario of symmetry that can be found in both these paradoxes, a shared solution was proposed. Expanding on previous modeling attempts, we applied Possible World Models to the Sleeping Beauty Paradox, reflecting on its limitations.

1. INTRODUCTION

Probability paradoxes is introduced when intuitive reasoning leads to conflictive answer and goes against the standard probability theory principles. It challenges us on how to update our probabilities due to uncertainty and limited information. There are several paradoxes surrounding probability theory. In this report, we will talk about two paradoxes.

1. Sleeping Beauty Paradox
2. Judy Benjamin Paradox

Sleeping beauty paradox dives into how temporal self-location complicates the belief update. The key dilemma here is, her first answer for probability is $\frac{1}{2}$ for all scenarios she wakes up, and after many awakenings her answers changes to $\frac{1}{3}$ for Heads and $\frac{2}{3}$ for Tails without any external evidence. This points to classical Bayesian and Frequency-based reasoning.

Judy Benjamin Paradox dives into how conditional probabilities complicates the global belief adjustments. The setup for this paradox is a soldier who is dropped into enemy territory. The territory is divided into red territory (R) and blue territory (B). Each territory is further split into Second Company Area (S) and the Headquarters Company Area ($\neg S$). Based on a radio message Judy updates the probability of each area. The message was: "If you are in R (Red territory), the odds are 3:1 that you are in Headquarters ($\neg S$)".

The paradox in this case is based on standard Bayesian reasoning which is incomplete and doesn't answer probabilities in a broader scope. More detailed discussion is presented in their respective section such as Discussion and Modeling on how the probabilities are updated. Some solutions will also be discussed, such as Elga's proposal that awakening counts as an evidence in the Sleeping Beauty paradox; and Douven's and Romeijn's proposal of Jeffrey's rule to solve the paradox of Judy Benjamin. Both paradoxes introduce new forms of uncertainty and how the standard principles of probabilities are affected by these paradoxes.

2. SLEEPING BEAUTY PARADOX

We will talk about Adam Elga's paper [1] in this section, named *Self-locating belief and the Sleeping beauty problem* explaining how the temporal self-locating belief changes the probabilities. First we will explain the problem setup of the sleeping beauty. A fair coin is tossed, in case of Heads, sleeping beauty is awakened on Monday but only once. In case of Tails, sleeping beauty is awakened on twice, Monday and Tuesday. So, in this setup sleeping beauty considers three scenarios:

1. H_1: Heads on Monday
2. T_1: Tails on Monday
3. T_2: Tails on Tuesday

2.1. Probabilities

Before sleeping beauty is put to sleep, she has the following probabilities for Heads and Tails:

$$P(\text{Heads}) = \frac{1}{2},\ \ P(\text{Tails}) = \frac{1}{2} \tag{1}$$

After the sleeping beauty is awakened, it is indistinguishable for her to differentiate between Monday and Tuesday if the coin lands on tail, so she assigns both of their probabilities equal to each other:

$$P(T_1) = P(T_2) \tag{2}$$

Since, the total probability constraint is:

$$P(T_1) + P(T_2) + P(H_1) = 1 \tag{3}$$

Considering (2), we can write this equation as:

$$P(H_1) + 2x = 1 \tag{4}$$

$$x + 2x = 1 \tag{5}$$

$$x = 1/3 \tag{6}$$

Here, sleeping beauty's probability assignment is likely equal for all three scenarios. But, since Tails is more likely to be the scenarios if she is awakened many times, that makes Tails having the probability $P(\text{Tails}) = \frac{2}{3}$ and Heads with a probability $P(\text{Heads}) = \frac{1}{3}$.

2.2. Analysis of the paradox

Let's analyze the paradox in this situation.

Before sleeping, $\frac{1}{2}$ is assigned to both Heads and Tails. After waking up, sleeping beauty updates the probabilities for both Heads and Tails, even though she has no new evidence. The additional awakening corresponds to Tails so she updates the probability to $\frac{2}{3}$ for Tails and $\frac{1}{3}$ for Heads. So,

$$Heads < Tails \tag{7}$$

Violation of Reflection Principle:

In the paper, there is also a mention of Reflection Principle by Bas van Fraassen [2]: Any agent who is certain that she will tomorrow believe in x in proposition R (though she will neither receive new information nor suffer any cognitive mishaps in the intervening time), ought now to believe x in R.

This probability change from $P(\text{Heads}) = \frac{1}{2}$ to $P(\text{Heads}) = \frac{1}{3}$ violates this principle.

2.3. Proposed Solutions

In this paper, Elga proposed a solution for this paradox by treating awakening as evidence which is proportional to their frequency.

Since the number of awakening differs for both heads and tails, so Elga suggests that that sleeping beauty should assign the following probabilities:

$$P(H_1) = \frac{1}{3}, P(T_1) = \frac{1}{3}, P(T_2) = \frac{1}{3} \tag{8}$$

In the paper, Elga also pointed out David Lewis concept about temporal uncertainty in centered possible worlds and points out how belief does change decision theory if we replace the space of possible world by the space of centered possible world.

3. JUDY BENJAMIN PARADOX

In this section, we will talk about the focus and the solution that the authors Igor Douven and Jan-Willem Romeijn gave to the Judy Benjamin problem in the paper *A new resolution of the Judy Benjamin problem* [3].

To begin, we will explain the Judy Benjamin problem, as well as Bayes' and Jeffreys rules in this context, and why those approach can't be used in this problem.

The problem consists on the following: A soldier named Judy Benjamin is dropped into an area divided in 4 quadrants. The column quadrants are the Red territory (R) and the Blue territory ($\neg R$), and the row quadrants are the Second Company Area (S), and the Headquarters Company Area ($\neg S$).

	$B = \neg R$	R
$Q = \neg S$		
S		

Judy Benjamin has the same probability of landing in each of the quadrants. That is,

$$P_0(R \cap S) = P_0(R \cap \neg S) = P_0(\neg R \cap S) = P_0(\neg R \cap \neg S) = \frac{1}{4} \tag{9}$$

Note that P_0 refers to the probability before any update.

After landing, she hears on her radio the following sentence:

"We are not sure where you are. If you are in the Red territory, then the odds for being at the Headquarters area versus the Second Company area are 3:1"

The problem is now stated as the following: "How does she adjust her degrees of belief?"

Now, let's introduce Bayes' rule and Jeffrey's rule, and see why it doesn't apply in this scenario.

3.1. Bayes' Rule

Bayes' rule refers as the following:

$$P_1(A|B) = \frac{P_1(B|A) \cdot P_1(A)}{P_1(B)} \tag{10}$$

where A and B are events and $P_1(B) \neq 0$. Note that P_1 is the probability after update.

Let's give an example. Let's say we want to update her belief on the event $E = \neg R \cap S|S$. Then, by Bayes' rule, we need to calculate

$$P_1(\neg R \cap S|S) = \frac{P_1(S|\neg R \cap S) \cdot P_1(\neg R \cap S)}{P_1(S)} \tag{11}$$

In this case, we don't know anything about $P_1(\neg R \cap S)$. The only notice we have is the condition $P_1(\neg S|R)$ and $P_1(S|R)$, so to figure $P_1(\neg R \cap S)$, we would have to make assumptions or get more information which we don't have. That is why Bayes' rules doesn't apply here. It could only be applied if we knew all the results of all the conditions.

3.2. Jeffrey's rule

Jeffrey's rule refers as the following: Let $\{A_1, \ldots, A_n\}$ be a partition of the space. Then, we have that, for any event H,

$$P_1(H) = \sum_{i=1}^{n} P_0(H|A_i) \cdot P_1(A_i) \tag{12}$$

To show that Jeffrey's rule does not apply in this scenario, we will consider the partition $\{R \cap S, R \cap \neg S, \neg R \cap S, \neg R \cap \neg S\}$ (the quadrants of the space). If we wanted to apply the formula from above, we will have to figure $P_1(\neg R \cap S)$ and $P_1(\neg R \cap \neg S)$. As mentioned with the Bayes' rule, we can't figure these values without more assumptions or information. That is why, Jeffrey's rule does not apply in this scenario.

3.3. Updated probability

We will adopt the following 3 results with the information we have right now:

1. Given the notice of the radio, we have that the probability that Judy is in the Red territory and in Headquarters is three times bigger that being in Red territory and in Second Company area. As this conditionals need to sum 1 (because Judy within R can only be on S or $\neg S$), we have that
$$P_0(\neg S|R) = \frac{3}{4}, P_0(S|R) = \frac{1}{4} \tag{13}$$
2. Our second result is that all the other conditions belief does not change after any update, as the radio information does not contain any information about them. That is
$$P_1(A|X) = P_0(A|X) \tag{14}$$
being A any event and $X \in \{\neg R, R \cap S, R \cap \neg S\}$.
3. Since the new information does not have any update in the probability of being in the red or blue territory, we have that
$$P_1(R) = P_0(R) = \frac{1}{2} \tag{15}$$

3.4. New approach

The authors of the paper mentioned [3], give a new approach to this problem, modifying the Jeffrey's rule for this case.

First, we will define the so called Adam conditioning.

Definition: Let $\{\neg A, A \cap X_1, \ldots, A \cap X_n\}$ be a partition of the space available, let C be any event and let $P_0(X_i|A) > 0$ for all $i \in \{1, \ldots, n\}$. Then, the **Adam Conditioning** update consist of

$$\begin{aligned} P_1(C) &= P_0(C|\neg A) \cdot P_0(\neg A) \\ &\quad + \sum_{i=1}^{n} P_0(C|A \cap X_i) \cdot P_1(X_i|A) \cdot P_0(A) \end{aligned} \tag{16}$$

Now, we will introduce Bradley's theorem.

Theorem (Bradley): Given a partition $\{\neg A, A \cap X_1, \ldots, A \cap X_n\}$ of the space, and given $P_0(X_i|A) > 0$ for all $i \in \{1, \ldots, n\}$, we can update the conditional beliefs P_1 using Adam Conditioning iff the all following properties are satisfied for any event C:

1. $P_1(A) = P_0(A)$
2. $P_1(C|A \cap X_i) = P_0(C|A \cap X_i)$ for all $i \in \{1, \ldots, n\}$
3. $P_1(C|A \cap \neg X_i) = P_0(C|A \cap \neg X_i)$ for all $i \in \{1, \ldots, n\}$
4. $P_1(C|\neg A) = P_0(C|\neg A)$

See [3] for a proof of Bradley's theorem.

In our scenario, we have that the partition is $X = \{\neg R, R \cap S, R \cap \neg S\}$ and the Adam Conditioning update is

$$\begin{aligned} P_1(C) &= P_0(C|\neg R) \cdot P_0(\neg R) \\ &\quad + P_0(C|R \cap S) \cdot P_1(S|R) \cdot P_0(R) \\ &\quad + P_0(C|R \cap \neg S) \cdot P_1(\neg S|R) \cdot P_0(R) \end{aligned} \tag{17}$$

Note that, given the update probabilities we have made, we have that:

$$P_0(\neg R) = \frac{1}{2}, P_1(S|R) = \frac{1}{4}, P_1(\neg S|R) = \frac{3}{4} \tag{18}$$

then, applying these changes, the equation results in:

$$\begin{aligned} P_1(C) &= P_0(C|\neg R) \cdot \frac{1}{2} + P_0(C|R \cap S) \cdot \frac{1}{4} \cdot \frac{1}{2} \\ &\quad + P_0(C|R \cap \neg S) \cdot \frac{3}{4} \cdot \frac{1}{2} \end{aligned} \tag{19}$$

and so we have that:

$$\begin{aligned} P_1(C) &= \frac{1}{2} P_0(C|\neg R) \\ &\quad + \frac{1}{8}\left[P_0(C|R \cap S) + 3 \cdot P_0(C|R \cap \neg S)\right] \end{aligned} \tag{20}$$

Let see if the properties of Bradley's Theorem are satisfied with the partition X.

1. $P_1(R) = P_0(R)$, by (15)

2. $P_1(C|R \cap S) = P_0(C|R \cap S)$, and $P_1(C|R \cap \neg S) = P_0(C|R \cap \neg S)$, by (14)
3. $P_1(C|R \cap \neg S) = P_0(C|R \cap \neg S)$ and $P_1(C|R \cap \neg\neg S) = P_1(C|R \cap S) = P_0(C|R \cap S)$, by (14)
4. $P_1(C|\neg R) = P_0(C|\neg R)$, by (14)

We have proof that Bradley's theorem applies for this scenario.

Then, we also have proof that the following formula updates the beliefs P_1 for any event C:

$$P_1(C) = \frac{1}{2}P_0(C|\neg R) + \frac{1}{8}\left[P_0(C|R \cap S) + 3P_0(C|R \cap \neg S)\right] \tag{21}$$

This equation is very similar to the Jeffrey's equation. We remember that Jeffrey's equation with X being the partition is:

$$P_1(C) = P_0(C|\neg R) \cdot P_1(\neg R) + P_0(C|R \cap S) \cdot P_1(R \cap S) + P_0(C|R \cap \neg S) \cdot P_1(R \cap \neg S) \tag{22}$$

4. DISCUSSION AND COMPARATIVE ANALYSIS OF THE SLEEPING BEAUTY AND JUDY BENJAMIN PARADOXES

The Sleeping Beauty (SB) and Judy Benjamin (JB) are paradoxes because they are interpreted as being thought-provoking problems in philosophy and probability. They challenge how we think about updating beliefs when information is incomplete or unusual. Both revolve around conditional probability and self-locating beliefs, but they differ in their setups, the type of uncertainty involved, and the way evidence is introduced.

4.1. Nature of the Uncertainty

The SB problem centers around temporal uncertainty. Beauty must update her beliefs about the outcome of a coin toss based on when she is awake, without knowing which day it is [1]. This makes the problem cyclical: Beauty is put to sleep, awakened, and her memory is erased after the first awakening if the coin lands Tails, repeating the process. She has to reason about her position in time and how this connects to the outcome of the coin toss.

In contrast, the JB problem deals with spatial uncertainty. Judy Benjamin is dropped into an unknown location and starts with equal probabilities for being in any of the four quadrants (Red or Blue, Headquarters or Second Company). When she receives a radio message about conditional probabilities, she must update her beliefs about her location. While SB is about reasoning under repeated temporal awakenings, JB is about reasoning under static spatial constraints [3].

Despite these differences, both problems share the feature of self-locating uncertainty: Beauty must figure out where she is in time, and Judy must determine where she is in space. Both involve an agent reasoning about their position in a world structured by probabilities, starting from a point of symmetry.

4.2. Initial Symmetry of Beliefs

Another important notice is the way information is presented in each problem. In SB, Beauty does not receive any new external information when she wakes up. She already knew the experimental setup beforehand: that she would be awakened once or twice depending on the coin toss. The problem for her is to interpret the fact that she is awake and whether this provides indirect evidence about the coin's outcome.

By contrast, JB involves direct and external information. Judy receives a message from a reliable source, which tells her about a conditional probability (e.g., "If you are in Red territory, there is a 75% chance you are in Headquarters"). This message gives her something new to incorporate into her reasoning. While SB challenges how we handle uncertainty when no new evidence is introduced, JB focuses on how to update beliefs with partial information.

The similarity, however, is that in both problems, the new information—whether implicit (waking up) or explicit (the radio message)—forces the agent to adjust their initial symmetrical probabilities to account for an asymmetrical reality.

4.3. Evidence and Belief Revision

A related difference is in how probabilities are updated. In SB, the question is whether Beauty should change her belief in Heads (the coin toss result) to 1/3, as argued by thirders, or keep it at 1/2, as argued by halfers. The discussion revolves around whether Beauty's awakening itself counts as new information or merely confirms something she already expected. The thirders argument is that waking up provides indirect evidence that favors Tails, as there are more awakenings in that scenario. The halfers counter-argument is that waking up was guaranteed to happen, so it should not change her belief in Heads [1].

In JB, the belief update is more straightforward because Judy receives a clear conditional probability that directly shifts her beliefs. However, the challenge is how to properly adjust her probabilities for all quadrants while keeping the relationships implied by the new information.

Despite the differences in the nature of the updates, both involve agents dealing with partial knowledge and needing to rationally adjust their previous beliefs based on what they know—or have just realized—about their situation.

4.4. A Unified Framework for Sleeping Beauty and Judy Benjamin

While the Sleeping Beauty (SB) and Judy Benjamin (JB) paradoxes differ in their setups, they share a similar structure on interpreting probabilistic belief revision when an agent is uncertain about their self-location. Luc Bovens [4], in connecting these two problems, highlights their shared structural similarities under a unified framework.

4.5. The Symmetry Argument

Bovens applies a symmetry argument to solve the Sleeping Beauty paradox. The symmetry comes from the fact that, if Beauty is awakened, she is equally likely to be in either the Heads scenario (Monday) or the Tails scenario (Monday or Tuesday).

Initial Probabilities: Beauty has no information about whether it is Monday or Tuesday, so the symmetry suggests that before she is awakened, all four outcomes:
$(H|Mo)$, $(H|Tu)$, $(T|Mo)$, $(T|Tu)$ are equally probable, with
$P(H|Mo) = P(H|Tu) = P(T|Mo) = P(T|Tu) = \frac{1}{4}$.

Conditional Update: When Beauty is awakened, she must revise her beliefs about the coin toss since Beauty learns that: $P(H|Mo) = 1$ This excludes the possibility of Tuesday if Heads but leaves $P(T|Mo)$ and $P(Tails|Tu)$ symmetrical. Bovens' resolution is that, using symmetry, Beauty should assign a probability of 1/3 to Heads and 2/3 to Tails because the scenario in which Tails occurred is twice as likely as the scenario in which Heads occurred.

Similarly, Bovens applies a symmetry argument to the Judy Benjamin paradox. The central claim is that the symmetry between the quadrants (or possible locations) should be preserved in the belief revision process.

Initial Probabilities: At the start, the rows (Blue and Red) and columns (Headquarters and Second Company) are symmetrical in their prior probabilities. Excluding the southeast quadrant (Red-Second Company) affects the remaining column (Blue) and row (Headquarters) equally, preserving symmetry. Thus: $P(B) = P(Q)$

Conditional Update: When Judy is told that if she is in Red territory, the probability of being in Headquarters (Q) is $q \to 1$.

She can use this information to revise her belief about her location. However, this new information does not affect her belief about Blue territory (because the information is only about Red territory). $P(Q|B) = \frac{1}{2}$

The law of total probability can then be applied [4]:

$$P(Q) = P(Q|B)P(B) + P(Q|R)(1 - P(B)) \tag{23}$$

and because $P(B) = P(Q)$, then $P(B) = \frac{2}{3}$.

4.5.1. *A Shared Solution*

Bovens proposes a common solution to both problems [4], which draws on the symmetry argument used in both the Judy Benjamin and Sleeping Beauty problems.

For the Judy Benjamin Problem: The symmetry between the quadrants in JB suggests that Judy's belief in Blue territory should remain unchanged, even after updating her beliefs about Red territory. The conditional probability regarding Red territory (that there's a 3:1 chance of being in Headquarters if in Red) updates her belief about the Red territories but does not provide any information that would alter her belief about the Blue territories.

For the Sleeping Beauty Problem: Similarly, the symmetry between the possible days of awakening (Monday and Tuesday) suggests that Beauty should update her beliefs such that Tails is twice as likely as Heads. Hence, Beauty should assign a 2/3 probability to Tails and a 1/3 probability to Heads upon awakening, based on the symmetry of the setup.

Bovens' theory highlights how understanding symmetry and conditional probabilities can provide solutions to these paradoxical problems; and since the Judy Benjamin and Sleeping Beauty problems share a similar structural framework, they should be solved using the same reasoning.

5. APPLYING POSSIBLE WORLDS MODEL TO SLEEPING BEAUTY

As formerly mentioned, both SB and JB paradoxes can be interpreted under the same initial probability symmetry principle [4], as both paradoxes involve self-locating symmetric problems. For simplicity, we will now introduce a possible novel representation of the SB paradox.

In epistemic logic, the notion of possible worlds model conforms a way to represent and reason on knowledge [5].

In this particular scenario, a possible world model can be defined by three notions. The temporal location can be either Monday (M) or Tuesday ($\neg M$). Regarding the coin toss, it either lands on Heads (H) or it lands on Tails ($\neg H$). Lastly, the sleeping beauty can either be sleeping (S) or it can be woken up and thus not sleeping ($\neg S$).

Following the symmetric scenarios introduced in the previous section, we represent the set of possible worlds as a cubic model. However, considering the paradox's constraint, a corner must be removed as it is not possible to have a world where the coin has landed Heads, it is Tuesday and the sleeping beauty is woken up ($H, \neg M, \neg S$). See Figure 1 for the described graphical representation. For simplicity of reading, we do not include the reflexive self-loops on the possible worlds model representation.

Note also, that instead of assigning agents to the edges' labels, the edges are labeled by propositions or "events" inspired by Van Benthem and Pacuit epistemic temporal logic frames [6, 7], to capture temporal and causal relations. Thus,

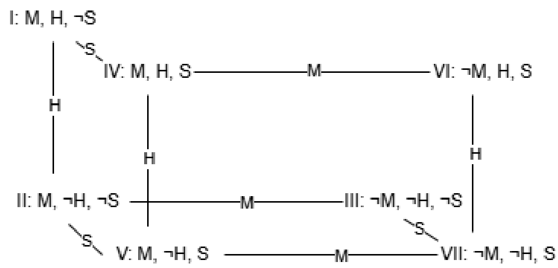

Fig. 1. Possible Worlds Model while sleeping

the model and reachability relations will be updated based on the knowledge gain -or uncertainty loss- around any of the three notions M, H or S.

As per the paradox's narrative, the sleeping beauty is woken up eventually. Either once or twice, but we are certain that she will be woken up at least one time. As a result, the epistemic representation in Figure 1 is updated and the sleeping beauty can reason and consider all possible scenarios where she is awake ($\neg S$), discarding the sleeping ones (S).

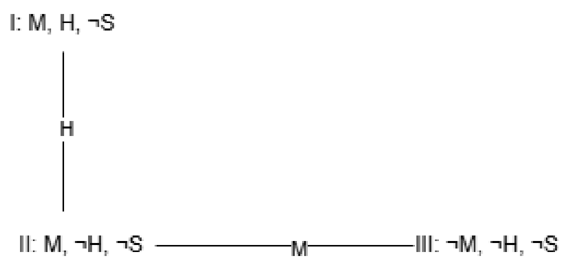

Fig. 2. Possible Worlds Model while awake

The diagram included in Figure 2 is aligned with the shared solution included on the previous section, as world models where the coin has landed on Tails ($\neg H$) are twice as frequent (or likely as for the shared solution) as that where the coin lands on Heads (H).

By reducing complexity and simultaneously considering scenarios where its temporal location differ, we can reason about probabilities –although not directly related– while abstracting and ignoring whether the day is Monday or Tuesday. Such limitations are also the case for the sleeping beauty while reasoning, as those facts remain unknown to her.

However, when trying to apply to the Judy Benjamin problem, the explicit introduction of Probability measures as new knowledge presents a challenge within this scenario representation.

6. CONCLUSION

In this paper, we explored two well-known probability paradoxes: the Sleeping Beauty (SB) paradox and the Judy Benjamin (JB) paradox. These paradoxes challenge traditional approaches to belief updates under uncertainty, offering insights into how conditional probabilities and self-locating beliefs influence rational decision-making.

Beyond their theoretical concepts, these paradoxes also connect to real-world decision-making and how personal belief affect our decisions and probabilities of different scenarios. This can be seen in many practical applications such as forecasting, interpreting anthropic principles in cosmology, etc. For example, SB paradox provides an insightful way to think about anthropic principle and how self-locating beliefs plays a big role in understanding the universe.

Our examination of the SB paradox focused on its emphasis on temporal uncertainty and the implications of repeated awakenings on belief revision. It could be questioned whether the act of awakening itself should be treated as evidence to justify updating the probability of the coin toss outcome. Conversely, the analysis of the JB paradox centered on spatial uncertainty, where belief updates are driven by explicit external information.

Both paradoxes points out the limitation to the Bayesian theorem and how it may require some modification with the approaches we used for handling certain scenarios such as the temporal and spatial uncertainty. Despite the apparent differences in their setups, they share a structural similarity in how both paradoxes deal with self-locating uncertainty and conditional probability updates.

Through a comparative analysis, it was discussed how the role of symmetry could play a role in resolving these paradoxes. Drawing on Bovens' symmetry arguments, we highlighted how this perspective provides a unified framework for addressing both the SB and JB problems.

Additionally, we demonstrated how possible world models can provide a novel approach to understanding the SB paradox, focusing on the constraints and the symmetrical relationships in its setup. For example, the constraints put by coin toss if it is Heads or Tails and the symmetry in her awakening, if it is Monday under Heads or both Monday or Tuesday under Tails. The symmetry also helps with understanding the difference between probabilities $\frac{1}{2}$ and $\frac{1}{3}$.

By examining these problems by using both existing and new approaches, this report analyzed the complexities of belief revision in uncertain environments and the value of symmetry and structured reasoning in resolving probability paradoxes.

6.1. Future Work and Limitations

Although different approaches have been presented around self-locating problem paradoxes, modeling its solutions is as challenging and our possible worlds model scenario is limited by both, the need to represent temporal properties and probability notions over each world.

As it was introduced in [1] and further analyzed on [8], the application of Lewis' theory on Centered World Models [9], to both these paradoxes should be further expanded. Centered Worlds theory positions itself as a relevant line of work to expand on, as the introduction of individuals and especially times, applies to these paradoxes [1] and the Worlds Model temporal representation limitations encountered.

Since in the case of Judy Benjamin, external information is additionally considered, the study of Public Announcement Logic [10] and its different ways of being represented would be relevant as well as extending Kripke Models with Probability Distributions [11], as to update the representation with probability notions, particularly relevant after the radio announcement.

7. REFERENCES

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A. WORK DISTRIBUTION

All members of the group were involved in literature research as well as reading through the research papers given by the professor. The work was afterwards distributed equally. Everyone wrote and reviewed the report. Below the sections to which each student was the main responsible for:

1. Arooj Chaudhry (s204759) : Introduction (Sec. 1), Sleeping Beauty Paradox (Sec. 2) and Conclusion (Sec. 6).

2. Cecilie Mai Do (s185394) : Discussion and Comparative analysis of the sleeping beauty and Judy Benjamin paradoxes (Sec. 4) and Conclusion (Sec. 6).

3. Laura Pascual Hebrero (s233668) : Abstract, Applying possible world models to sleeping beauty (Sec. 5) and Future works and limitations (Sec. 6.1).

4. Sergio Monzon (s232515) : Judy Benjamin Paradox (Sec. 3)