Functional programming, Seminar No. 3

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Today

We will study

Motivation

Let us recall the example of a higher order function from the previous seminar:

```
changeTwiceBy :: (Int -> Int) -> Int -> Int
changeTwiceBy operation value = operation (operation value)
```

One may implement the ’same’ functions for Boolean values and strings:

```
changeTwiceBy :: (Bool -> Bool) -> Bool -> Bool
changeTwiceByBool operation value =
  operation (operation value)

changeTwiceBy :: (String -> String) -> String -> String
changeTwiceBy operation value =
  operation (operation value)
```

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```

Too much boilerplate.

Parametric polymorphism

The key idea of parametric polymorphism that the same function might be called on distinct data types. Here are the first polymorphic examples:

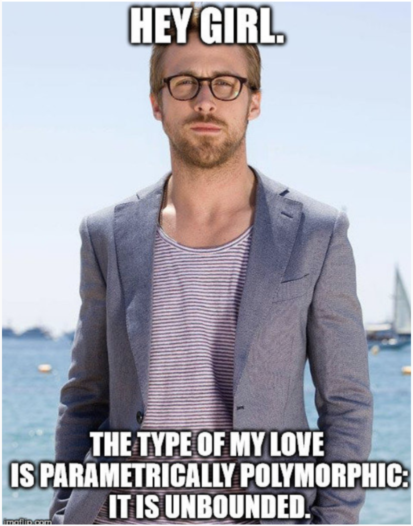
```
id :: a -> a
id x = x

const :: a -> b -> a
const a b = a

fst :: (a, b) -> a
fst (a, b) = a

swap :: (a, b) -> (b, a)
swap (a, b) = (b, a)
```

Meme time


HEY GIRL.
THE TYPE OF MY LOVE
IS PARAMETRICALLY POLYMORPHIC:
IT IS UNBOUNDED.
imgflip.com

Example

GHCi session

```
> id 6
6
> id 6.0
6.0
> const True 6
True
> const 6 True
6
> fst ('5', 5)
'5'
> fst (5, '5')
5
```

A brief clarification

- In such signatures as $a \to b \to a$, a, b are type variables that range over arbitrary data types. In fact, a, b are bounded by universal quantifier hidden under the carpet.
- In fact, the functions from the previous slide have the following signatures:

```haskell
id :: forall a. a -> a
id x = x

const :: forall a b. a -> b -> a
const a b = a

fst :: forall a b. (a, b) -> a
fst (a, b) = a

swap :: forall a b. (a, b) -> (b, a)
swap (a, b) = (b, a)
```

Higher order functions and parametric polymorpism

More examples

```
infixr 9 .
(.) :: (b -> c) -> (a -> b) -> a -> c
f . g = \x -> f (g x)

flip :: (a -> b -> c) -> b -> a -> c
flip f b a = f a b

fix :: (a -> a) -> a
fix f = f (fix f)

curry :: ((a, b) -> c) -> a -> b -> c
curry f x y = f (x, y)

uncurry :: (a -> b -> c) -> ((a, b) -> c)
uncurry f p = f (fst p) (snd p)
```

Examples with composition

More examples

```
incNegate :: Int -> Int
incNegate x = negate (x + 1)
incNegate x = negate $ x + 1
incNegate x = (negate . (+1)) x
incNegate x = negate . (+1) $ x
incNegate   = negate . (+1)
```

`curry` and `uncurry`

More examples

```
> uncurry (*) (3,4)
12
> curry fst 3 4
3
> curry id 3 4
(3,4)
> uncurry const (3,4)
3
> uncurry (flip const) (3,4)
4
```

Examples with `flip`

More examples

```haskell
show2 :: Int -> Int -> String
show2 x y = show x ++ " and " ++ show y

showSnd, showFst, showFst' :: Int -> String
showSnd  = show2 1
showFst  = flip show2 2
showFst' = (`show2` 2)
```

GHCi session

```
> showSnd 10
"1 and 10"
> showFst 10
"10 and 2"
> showFst' 42
"42 and 2"
```

Bye-bye boilerplate!

All those functions such as the following one

changeTwiceBy with Int

```haskell
changeTwiceBy :: (Int -> Int) -> Int -> Int
changeTwiceBy operation value =
  operation (operation value)
```

might be replaced by one of the following ones:

applyTwice

```haskell
applyTwice :: (a -> a) -> a -> a
applyTwice f a = f (f a)
applyTwice f a = f . f $ a
applyTwice f = f . f
```

HOF, polymorpism, and lists

Examples

```haskell
map     :: (a -> b)      -> [a] -> [b]

filter  :: (a -> Bool)   -> [a] -> [a]

zipWith :: (a -> b -> c) -> [a] -> [b] -> [c]

length :: [a] -> Int
```

We discuss their implementations closely on the next seminar. Here we just discuss them briefly.

The composition examples + list functions

More examples

```haskell
foo, bar :: [Int] -> Int
foo patak =
  length $ filter odd $
  map (div 2) $ filter even $ map (div 7) patak
bar =
  length . filter odd .
  map (div 2) . filter even . map (div 7)

zip :: [a] -> [b] -> [(a, b)]
zip = zipWith (,)
```

The composition examples + list functions

More examples

```
stringsTransform :: [String] -> [String]
stringsTransform l =
  map (\s -> map toUpper s)
   (filter (\s -> length s == 5) l)

stringsTransform l =
  map (\s -> map toUpper s) $
  filter (\s -> length s == 5) l

stringsTransform l =
  map (map toUpper) $ filter ((== 5) . length) l

stringsTransform =
  map (map toUpper) . filter ((== 5) . length)
```

Bounded polymorphism and type classes

Bounded polymorphism and type classes

The idea of bounded (ad hoc) polymorphism is that one has a general interface with instances for each concrete data type.

More examples

```
> :t 9
9 :: Num p => p
> 9 :: Int
9
> 9 :: Double
9.0
> 9 :: Rational
9 % 1
> 9 :: Char
<interactive>:6:1: error:
* No instance for (Num Char) arising from the literal '9'
```

- Let us take a look a the following function

```
elem' :: a -> [a] -> Bool
elem' _ [] = False
elem' x (y:ys) = x == y || elem' x ys
```

- `a` is an arbitrary type for which equality is defined as usual.

Type classes. Motivation

Type variables in polymorphic function are bounded with universal quantifier. In ad hoc polymorphism, type variables are also bounded with $\forall$ but with the additional condition. This kind of quantification is called *bounded*.

```
elem :: forall a. Eq a => a -> [a] -> Bool
elem _ [] = False
elem x (y:ys) = x == y || elem x ys
```

The notion of a type class

- *A type class* is a collection of functions with type signatures with a common type parameter. An example given:

```
class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool
  x /= y = neg (x == y)
```

- A type class name introduce a constraint called *context*:

```
elem :: Eq a => a -> [a] -> Bool
```

- The definition above without a context is not well-defined:

```
<interactive>:4:19: error:
* No instance for (Eq a) arising from a use of '=='
Possible fix: add (Eq a) to the context of
the type signature for: elem' :: forall a. a -> [a] -> Bool
```

Instance declarations

A given data type a has the *instance* of a type class if every function of that class is implemented for a. An example:

Example

```
instance Eq Bool where
  True == True   = True
  False == False = True
  _ == _         = False
```

Polymorphism + instance declarations

- A type parameter in an instance declaration might be polymorphic as well:

Example

```
instance Eq a => Eq [a] where
  []        == []        = True
  (x : xs) == (y : ys) = x == y && xs == ys
  _         == _         = False
```

- Without the context `Eq a =>`, this definition yields type error.

Some the Eq instances

- The Eq type class has the following instances (some of them)

Eq instances

```
instance Eq a => Eq [a]
instance Eq Ordering
instance Eq Int
instance Eq Float
instance Eq Double
instance Eq Char
instance Eq Bool
```

- See the standard library source code to have a look at the implementation.

The `Show` type class

- The `Show` type class allows one to represent a value as a string:

Some `Eq` instances

```
class Show a where
  show :: a -> String
```

- One needs to have a `Show` instance to show a value of a given type.

Some of the Show instances

Here are some of the `Show` instances:

Some `Show` instances

```
instance Show Integer
instance Show Int
instance Show Char
instance Show Bool
instance (Show a, Show b) => Show (a, b)
```

Ordering. Motivation

- Let us take a look at the `quicksort` function:

Some `quicksort` instances

```haskell
quicksort :: [a] -> [a]
quicksort [] = []
quicksort (x:xs) =
  quicksort small ++ (x : quicksort large)
  where
    small = [y | y <- xs, y <= x]
    large = [y | y <- xs, y > x]
```

Ordering. Motivation

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Some `quicksort` instances

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  quicksort small ++ (x : quicksort large)
  where
    small = [y | y <- xs, y <= x]
    large = [y | y <- xs, y > x]
```

- Here we have the same situation. The definition of `quicksort` as above is wrong. There exist types elements of which are incomparable, complex numbers, e.g.

The Ord type class

The full definition of `Ord` is the following one:

Ord

```haskell
data Ordering = LT | EQ | GT
class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (<=), (>), (>=) :: a -> a -> Bool
  max, min :: a -> a -> a
  compare x y = if x == y then EQ
                else if x <= y then LT
                else GT
  x <= y =
    case compare x y of
           GT -> False
           _ -> True
  {-# MINIMAL compare | (<=) #-}
```

`Ord` instances

`Ord` instances

```
instance Ord Int
instance Ord Float
instance Ord Double
instance Ord Char
instance Ord Bool
```

The `Num` type class

`Num` is a type class with the general interface of usual arithmetic operations.

Num

```
class Num a where
  (+), (-), (*) :: a -> a -> a
  negate, abs, signum :: a -> a
  fromInteger :: Integer -> a
  {-# MINIMAL (+), (*), abs, signum,
      fromInteger, (negate | (-)) #-}
```

Some `Num` instances

```
instance Num Integer
instance Num Int
instance Num Float
instance Num Double
```

The `Enum` type class

`Enum` is a type class for sequentially ordered types.

Some `Enum` instances

```
class Enum a where
  succ, pred :: a -> a
  toEnum     :: Int -> a
  fromEnum   :: a -> Int

  enumFrom       :: a -> [a]                -- [n..]
  enumFromThen   :: a -> a -> [a]           -- [n,m..]
  enumFromTo     :: a -> a -> [a]           -- [n..m]
  enumFromThenTo :: a -> a -> a -> [a]      -- [n,m..p]
  {-# MINIMAL toEnum, fromEnum #-}
```

`Enum` instances

Some `Num` instances

```
instance Enum Integer
instance Enum Int
instance Enum Char
instance Enum Bool
instance Enum Float
instance Enum Double
```

The `Fractional` type class

- The `Fractional` type class is a general interface for numeric division

Some `Num` instances

```
class Num a => Fractional a where
  (/) :: a -> a -> a
  recip :: a -> a
  fromRational :: Rational -> a
  {-# MINIMAL fromRational, (recip | (/)) #-}
```

- We require the `Num a` restriction.
- The `Fractional` instances are `Float` and `Double`.

Summary

On this seminar, we

- took a look at parametric polymorphism to see how to avoid boilerplate
- discussed type classes and ad hoc polymorphism
- studied such basic type classes as `Eq`, `Show`, etc

Summary

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On the next seminar, we will

- delve into the variety of Haskell data types: algebraic data types, newtypes, type synonyms, etc
- feel the power of pattern matching
- discuss folds
- see how to enforce evaluation in Haskell