

# Exploratory Data Analysis and Regression Modelling with Iowa Housing Data

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## Introduction

The Ames dataset that we will be using for our analysis consists of detailed housing information in Iowa. The dataset includes the data regarding the house characteristics, such as style, condition, room counts, areas and other multiple interiors and exterior design elements, and information on properties of the lot it is built on, mechanical systems. The exploratory data analysis (EDA) we will conduct will determine the most key predictor variables within the batch of predictor variables we aim to look into that are most likely to have a strong positive or negative correlation with our response variable.

## The Dataset

As the initial step, we study the dataset's properties to better understand the data we currently hold. Our Ames dataset consists of 1,135 observations and 56 variables. From these 56 variables, there is only one response variable and 55 predictor variables that we are looking to downsize for a better and more efficient regression model. For our EDA as well as our regression model, we will be focusing on the variables in Table 1 below:

Variable	Definition	Response or Predictor	Data Type
<i>SalePrice</i>	Sale price of the house	Response	Integer
<i>TotalSqftCalc</i>	Total area of the house in square feet	Predictor	Integer
<i>TotalBathCalc</i>	Total number of baths in the house	Predictor	Numeric
<i>TotRmsAbvGrd</i>	Total number of rooms above ground	Predictor	Integer
<i>QualityIndex</i>	Quality index	Predictor	Integer
<i>OverallQual</i>	Score for overall quality	Predictor	Integer
<i>OverallCond</i>	Score for overall condition	Predictor	Integer

Table 1: Variable Types and Definitions

We observe no missing values within any of the variables, yet we see a possible outlier in our response variable, *SalePrice*, due to the broad range and high variance. This indicates that we may need to conduct a logarithmic transformation on our response variable to normalize and prevent our analysis from producing skewed, inaccurate results.

Variable	Obs.	Miss.	Min	1st Qtr.	Median	Mean	3rd Qtr.	Max	Range	Variance	Std.	Coef. Var
<i>SalePrice</i>	1,135	0	62,383	142,188	177,500	197,212	229,900	755,000	692,617	5,979,050,377	77,324	0
<i>TotalSqftCalc</i>	1,135	0	825	1,630	1,955	2,122	2,486	5,771	4,946	502,987	709	0
<i>TotalBathCalc</i>	1,135	0	1	2	3	2	3	5	4	1	1	0
<i>TotRmsAbvGrd</i>	1,135	0	4	30	6	7	40	12	8	2	1	0
<i>QualityIndex</i>	1,135	0	12	6	35	34	7	72	60	52	7	0
<i>OverallQual</i>	1,135	0	3	5	6	6	7	10	7	2	1	0
<i>OverallCond</i>	1,135	0	3	5	5	6	6	9	6	1	1	0

Table 2: Descriptive Summary Statistics

## Exploratory Data Analysis

The correlation analysis between the predictor variables and response variable is done by having our response variable in a standard form and in a logarithmic transformed form to compare and observe any change in results. Any correlation between 0 and 1 is considered positive, indicating that the variables are

positive linear related, whereas the correlation between 0 and -1 is the indicator of a negative linear relationship. The higher negative or positive correlation score a predictor variable receives when analyzed against the response variable indicates a strong linear relationship that needs to be further studied. Below is the SalePrice's, our response variable's correlation with the six predictor variables we have chosen to utilize for our study.

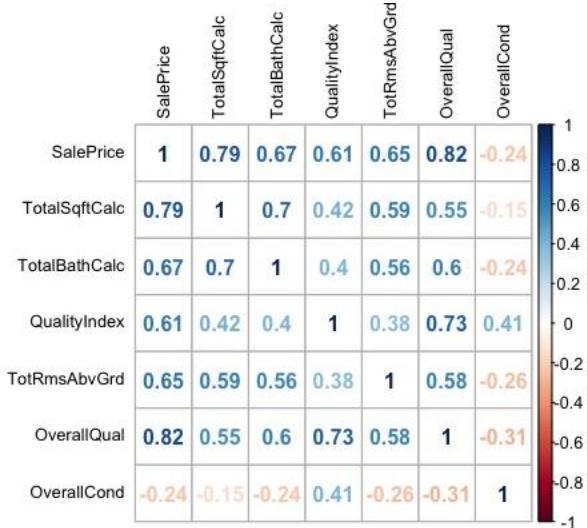


Figure 1: Correlation Plot of Selected Predictor Variables and Sale Price

TotalSqftCalc	TotalBathCalc	QualityIndex	TotRmsAbvGrd	OverallQual	OverallCond
0.786	0.671	0.611	0.647	0.825	-0.238

Table 3: Predictor Variable Correlation to Response Variable SalePrice

When the correlation between the selected predictor variables, TotalSqftCalc (Total Square Footage), TotalBathCalc (Total Number of Bathrooms), QualityIndex (Quality Index), TotRmsAbvGrd (Total Number of Rooms Above Grade), Overall Qual (Overall Quality), and finally OverallCond (Overall Condition), and response variable SalePrice modeled we see that five out of six predictor variables have a noticeable positive correlation with the response variable. The predictor variables with the highest positive correlation with SalePrice are OverallQual with 0.825, followed by TotalSqftCalc, TotalBathCalc, TotRmsAbvGrd, and QualityIndex with 0.786, 0.671, 0.647, and 0.611, respectively. OverallCond is the only predictor variable that is negatively correlated with the SalePrice, with a correlation of -0.238, presented in Table 3.

The scatterplots of selected predictor variables plotted against the response variable SalePrice in Figure 2 presents the clear and distinct correlation also was shown in Table 3. The LOESS line in each scatterplot is straight, which means the correlations we have obtained through the correlation analysis and scatter plots of predictor variables against the response variable are distinct and viable.

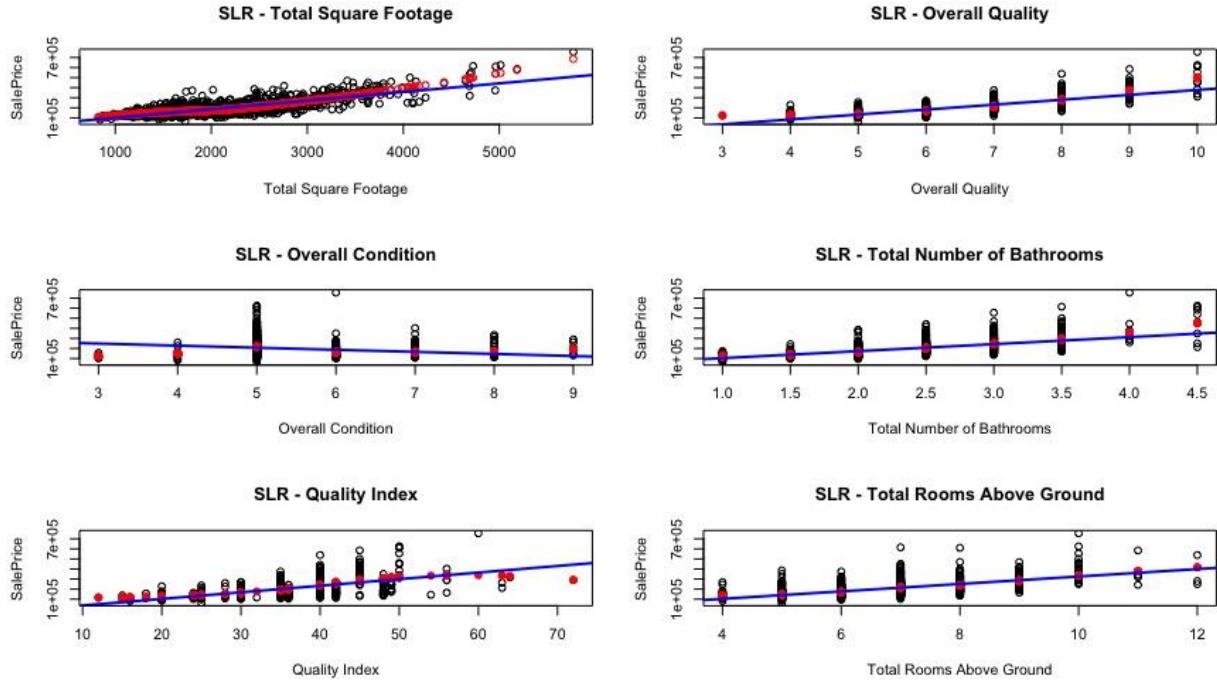


Figure 2: Scatterplots of Selected Variables against Response Variable SalePrice

A secondary step was taken to transform the response variable SalePrice into a logarithmic form to observe improvements in correlation values between predictor variables and log(SalePrice). Table 4 presented the outcome of the correlation matrix ran for predictor variables vs. logarithm of SalePrice. This approach's outcome resulted in a general improvement in correlations between the response variable and all predictors except for TotalSqftCalc. As the decrease in correlation between TotalSqftCal and SalePrice when SalePrice is transformed is not significant, we can conclude that converting SalePrice to logarithmic form has improved our results.

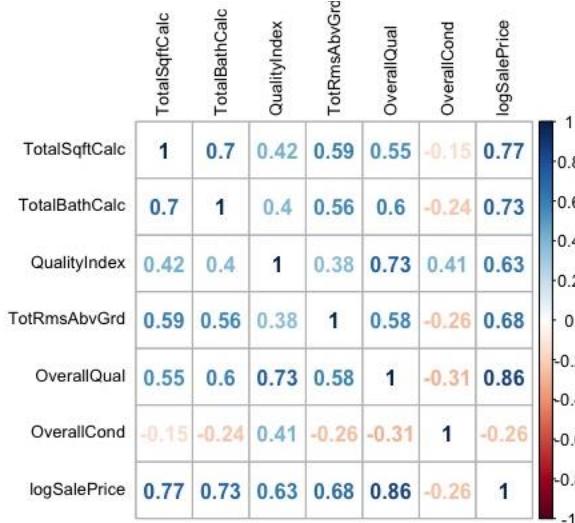
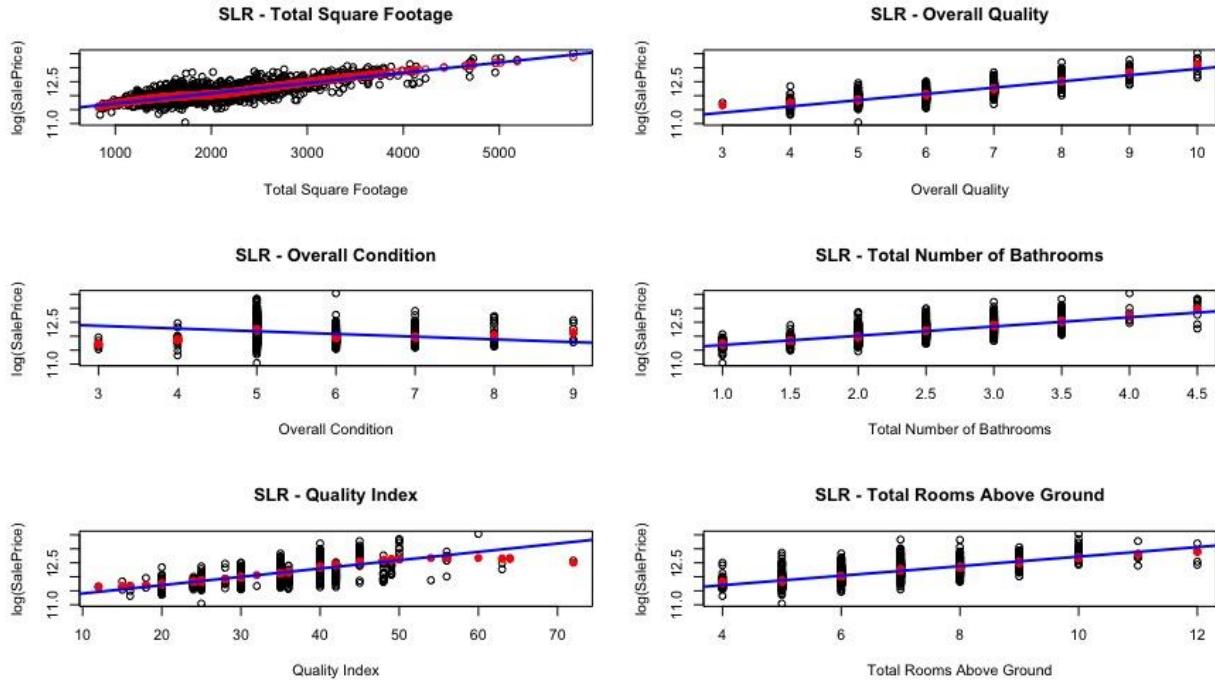


Figure 3: Correlation Plot of Selected Predictor Variables and Logarithmic Sale Price

TotalSqftCalc	TotalBathCalc	QualityIndex	TotRmsAbvGrd	OverallQual	OverallCond
0.767	0.728	0.634	0.679	0.860	-0.256

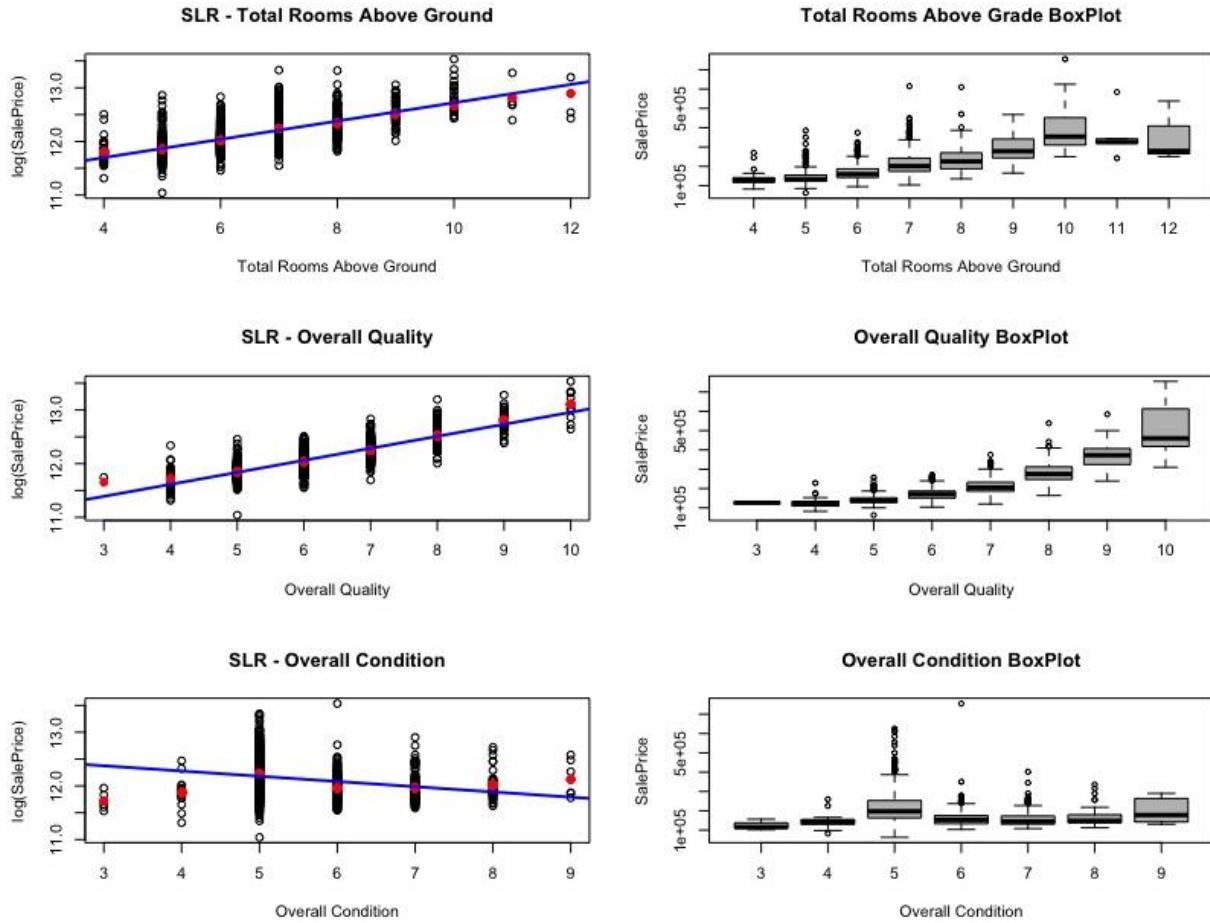
Table 4: Predictor Variable Correlation to Response Variable log(SalePrice)

Figure 4 is showing the scatterplots for logarithmic form SalePrice, and predictor variables present a very similar outcome with the SalePrice scatterplots before response variable transformation yet with a slight improvement in relationships. The LOESS line remains straight, as the correlation between variables is in a feasible condition.



**Figure 4: Scatterplots of Selected Variables against Response Variable log(SalePrice)**

Upon reviewing our correlation matrices and scatterplots for both SalePrice and log(SalePrice), we can identify that the highest positive correlations the response variables have is with OverallQual, TotalSqftCalc and TotalBathCalc. These three variables are the most critical indicators of the SalePrice of a house in Iowa state. On the other hand, in both matrices and plots, OverallCond was the only predictor variable that was negatively correlated with the response variable. While the negative correlation was slight, we have gained the insight that the increase in OverallCond of the house does not positively impact SalePrice and vice versa.



**Figure 5: Scatterplots and Boxplots of Three Selected Variables against Response Variable  $\log(\text{SalePrice})$**

Continuous data is more desired for the type and size of analysis conducted with the Ames dataset. As an example, we can look at the TotalSqftCalc variable, which is the only continuous predictor variable where we will see the LOESS line is a much better fit over the continuous data, therefore producing more accurate and viable results. The scatter and box plots in Figure 5 shows us that OverallQual (Overall Quality) and TotRmsAbvGrd (Total Rooms Above Grade) perform well within the analysis even though they are discrete variables, yet OverallCond (Overall Condition) variable could benefit from being transformed from being a discrete variable to continuous variable. Therefore, in order to improve our results and better understand the relationship between the SalePrice and Overall Condition, it would be crucial to change the predictor variable from discrete to continuous.

### Simple Linear Regression Modelling

Per our EDA we concluded that TotalSqftCalc (Total Square Footage) is one of the most solid and well fit predictor variables that also is a continuous variable which we will use in our regression models. In addition to TotalSqftCalc, we will bring in another continuous predictor variable, GarageArea in order to run two separate Simple Linear Regression (SLR) models and assess the performance of each model through comparison. For the remainder of the report we will refer to SLR model with TotalSqftCalc as SLR Model #1 and SLR model with GarageArea as SLR Model #2. We started off with plotting the same scatterplots we have used in EDA phase for initial understanding and visualization of TotalSqftCalc and GarageArea's relation to our response variable, SalePrice.

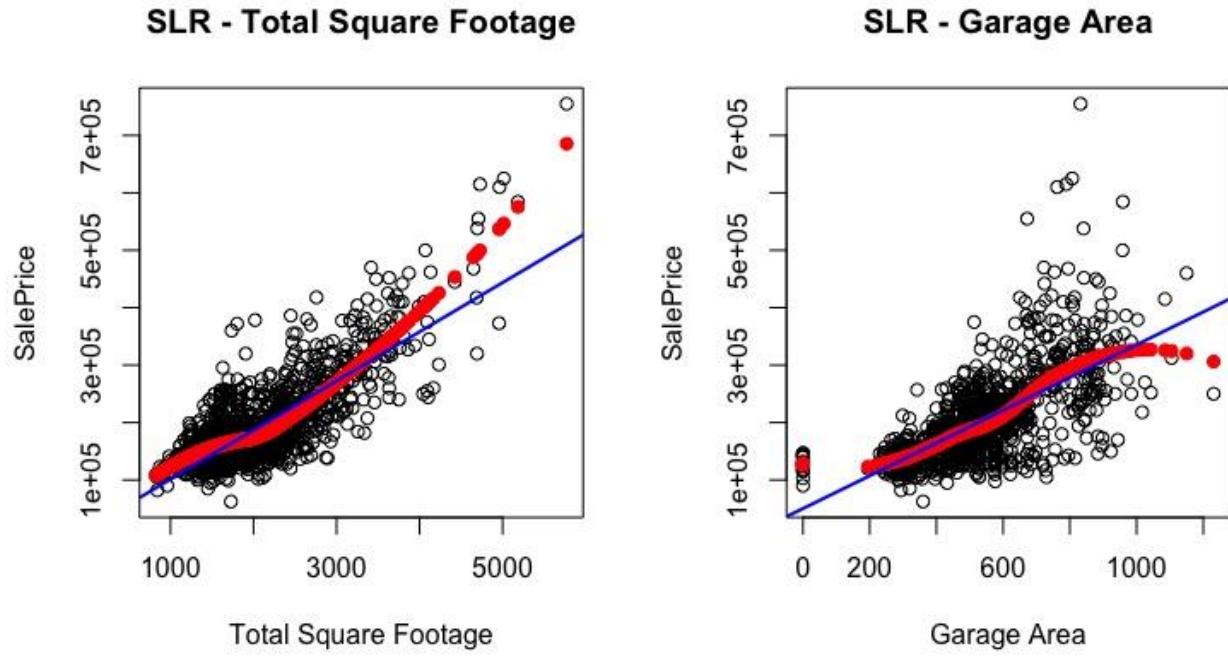


Figure 6: Scatterplot of TotalSqftCalc and GarageArea against SalePrice

The positive correlation between TotalSqftCalc and SalePrice is also observed in the scatterplot where we plotted GarageArea against SalePrice (See Figure 6). The both scatterplots presented above indicate that an increase in TotalSqftCalc and/or GarageArea is highly likely to indicate an increase in SalePrice of the house. If we look at Table 5 we can see that for a 1 unit increase in TotalSqftCalc SalePrice increases by 85.66 and for a 1 unit increase in Garage Area SalePrice increases by 285.36.

$$SalePrice = 15,455.70 + 85.66 * TotalSqftCalc + \varepsilon, \text{ where } \varepsilon \sim \mathcal{N}(0, 47,860.172)$$

$$SalePrice = 50,191.22 + 285.36 * GarageArea + \varepsilon, \text{ where } \varepsilon \sim \mathcal{N}(0, 57103.032)$$

Both models have a p-value of less than 0.01 which indicated that both variables are statistically significant. R<sup>2</sup> of 0.62 in SLR Model #1 (TotalSqftCalc) and 0.46 in SLR Model #2 (GarageArea) suggest a modest explanation of the variance in response variable SalePrice through TotalSqftCalc and GarageArea. When we compare the two R<sup>2</sup> output of each model, we can see that SLR Model #1 is a better fit than SLR Model #2.

### Comparison of SLR Model #1 and Model #2

	<i>Dependent variable:</i>	
	SalePrice	
	SLR Model #1	SLR Model #2
	(1)	(2)
Constant	15,455.70*** (4,482.66)	50,191.22*** (5,070.83)
TotalSqftCalc	85.66*** (2.00)	
GarageArea		285.36*** (9.28)
Observations	1,135	1,135
R <sup>2</sup>	0.62	0.46
Adjusted R <sup>2</sup>	0.62	0.45
Residual Std. Error (df = 1133)	47,856.17	57,103.03
F Statistic (df = 1; 1133)	1,827.53***	946.35***

*Note:*

\*p<0.1; \*\*p<0.05; \*\*\* p<0.01

**Table 5: SLR Summary Comparison Table for TotalSqftCalc and GarageArea**

The Diagnostic Plots for both Model #1 and Model #2 show parallel outputs with the results we obtained from SLR summary table for each in Table 5. With neither of the Model #1 and #2 diagnostics plots we see a pattern in the residuals vs. fitted values plot yet we see that the points are clustered and linear around the 0 line which suggest homoscedasticity and that the variances of the error terms are equal.

The Quantile-Quantile Plot (Q-Q Plot) of both models are lightly tailed. This suggests that while we can possibly see some fluctuation at the extreme ends of the distribution and the Q-Q plot does not suggest precise normality, the distribution of Sample Quantiles and Theoretical Quantiles fall into a relatively straight line. Some positive skew was observed yet neither of the Q-Q plots suggest any extreme abnormalities.

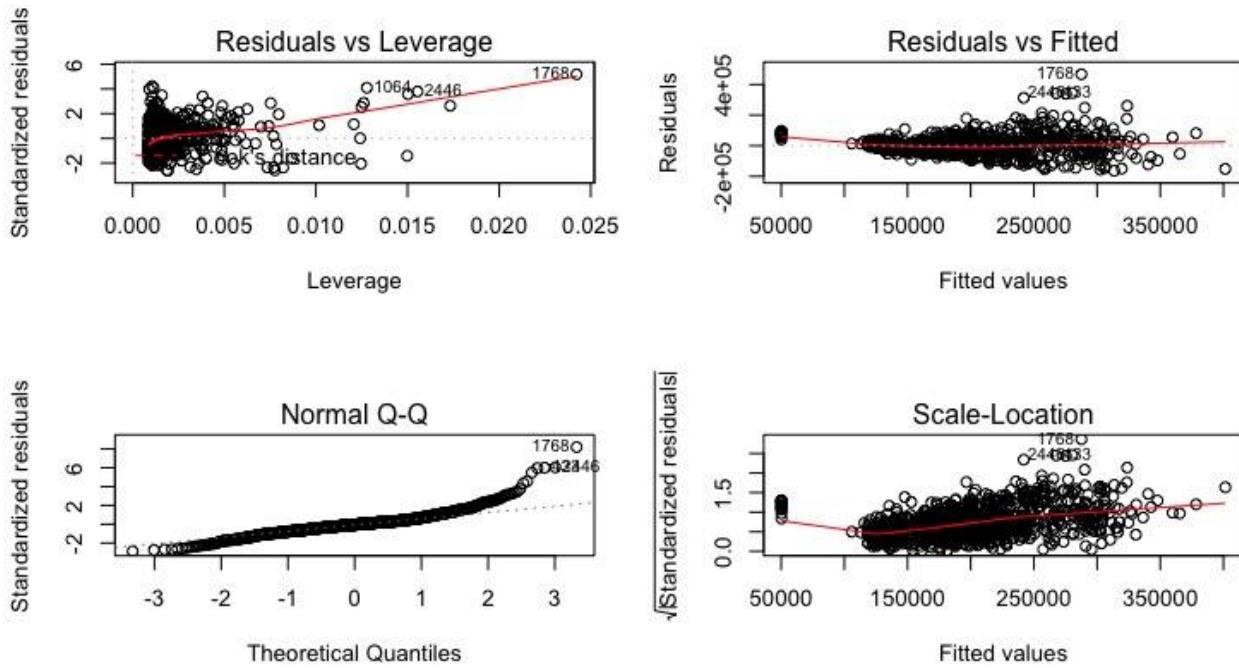


Figure 8: Diagnostic Plots for SLR Model #1

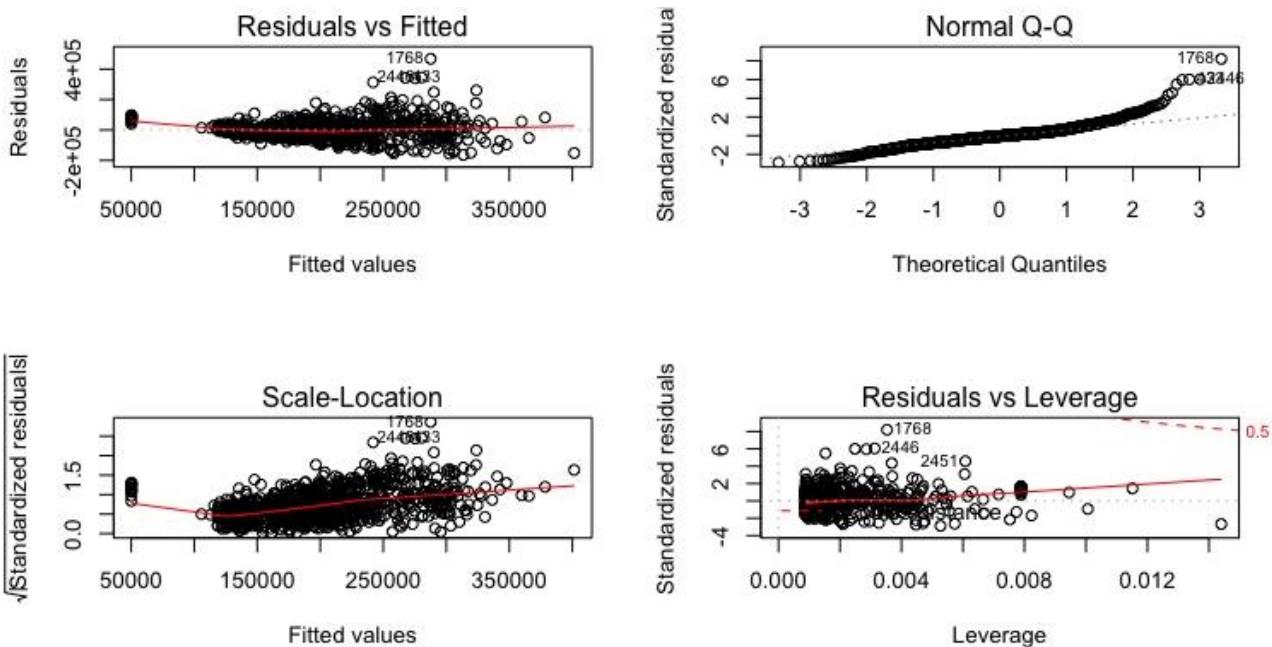


Figure 8: Diagnostic Plots for SLR Model #2

## Multiple Linear Regression Modelling

In order to expand our Linear Regression Model, next we are running a Multiple Linear Regression (MLR) Model with predictor variables that we used in our SLRs and in addition, include two new variables, TotRmsAbvGrd (Total Rooms Above Ground) and LotArea.

MLR Model #1 Results & Comparison			
	Dependent variable:		
	MLR Model #1 (1)	SalePrice SLR Model #1 (2)	SLR Model #2 (3)
Constant	-72,964.50*** (5,683.40)	15,455.70*** (4,482.66)	50,191.22*** (5,070.83)
TotalSqftCalc	51.91*** (2.12)	85.66*** (2.00)	
GarageArea	144.52*** (7.22)		285.36*** (9.28)
TotRmsAbvGrd	11,667.50*** (1,056.29)		
LotArea	0.85*** (0.16)		
Observations	1,135	1,135	1,135
R <sup>2</sup>	0.76	0.62	0.46
Adjusted R <sup>2</sup>	0.76	0.62	0.45
Residual Std. Error	37,913.41 (df = 1130)	47,856.17 (df = 1133)	57,103.03 (df = 1133)
F Statistic	896.73*** (df = 4; 1130)	1,827.53*** (df = 1; 1133)	946.35*** (df = 1; 1133)
Note:		*p<0.1; **p<0.05; ***p<0.01	

Table 6: MLR Summary Table for SalePrice and MLR Selected Predictor Variables

Multiple Linear Regression model with the 4 selected variables (aka Model #1) have the p-value of less than 0.01 as the SLR models, and show statistical significance. The MLR model performed better in regard to R<sup>2</sup> compared to both SLR Model #1 and #2 with 0.76 (see Table 6). This indicates that 76% of the variance can be explained and determined through predictor variables in our MLR model. Also, the MLR model summary gives us the below equation for SalePrice:

$$\text{SalePrice} = -72,964.50 + 51.91 * \text{TotalSqftCalc} + 144.52 * \text{GarageArea} + 11,667.50 * \text{TotRmsAbvGrd} + 0.85 * \text{LotArea} + \varepsilon,$$

The diagnostic plots for MLR Model #1 present extremely similar Residuals vs. Fitted Values and Q-Q plot graphs as both our SLR models. The change of output between SLR models and MLR Model #1 is unrecognizable through the plots in Figure 8.

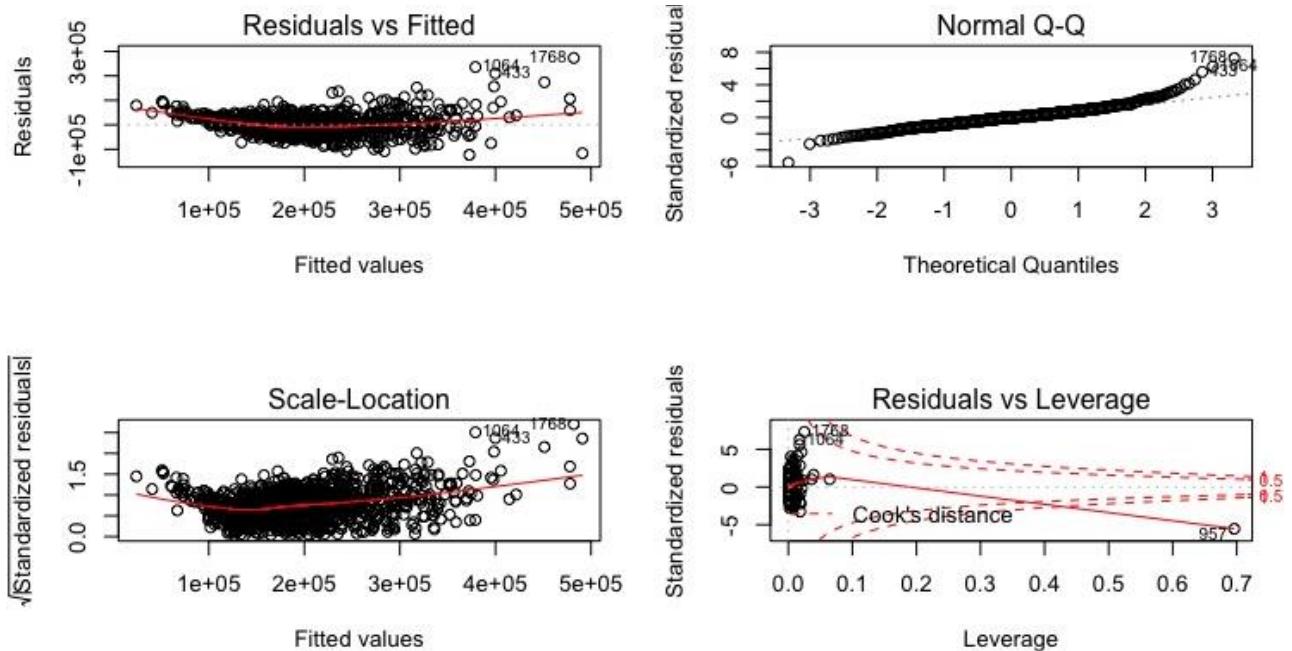


Figure 8: Diagnostic Plots for MLR Model #1

The same steps taken with a transformed response variable, Logarithm of SalePrice, as MLR Model #2. Despite not seeing any change in p-value the  $R^2$  has slightly increased. This suggests that MLR Model #2 is slightly better with  $R^2$  equal to 0.78 than MLR Model #1 which had a  $R^2$  value of 0.76.

Comparison of MLR Model #1 and Model #2

	Dependent variable:	
	SalePrice	log(SalePrice)
	MLR Model #1	MLR Model #2
	(1)	(2)
Constant	-72,964.50*** (5,683.40)	10.89*** (0.02)
TotalSqftCalc	51.91*** (2.12)	0.0002*** (0.0000)
GarageArea	144.52*** (7.22)	0.001*** (0.0000)
TotRmsAbvGrd	11,667.50*** (1,056.29)	0.07*** (0.004)
LotArea	0.85*** (0.16)	0.0000*** (0.0000)
Observations	1,135	1,135
R <sup>2</sup>	0.76	0.78
Adjusted R <sup>2</sup>	0.76	0.77
Residual Std. Error (df = 1130)	37,913.41	0.16
F Statistic (df = 4; 1130)	896.73***	976.99***

Note: \*p<0.1; \*\* p<0.05; \*\*\* p<0.01

Table 7: MLR #1 and MLR #2 Model Summary Statistics Comparison

The Residuals vs Fitted Values plot for log(SalePrice) presents a more dense and better distributed graph. The Q-Q Plot diagnostic plot shows the most significant improvement compared to SLR models and MLR

Model #1. We see that transforming the response variable to logarithmic form has given a Normal Q-Q Plot that symmetric distribution. The points being on a straight diagonal line which indicates that both Residuals and Theoretical Quantiles are distributed precisely also presented in Figure 9.

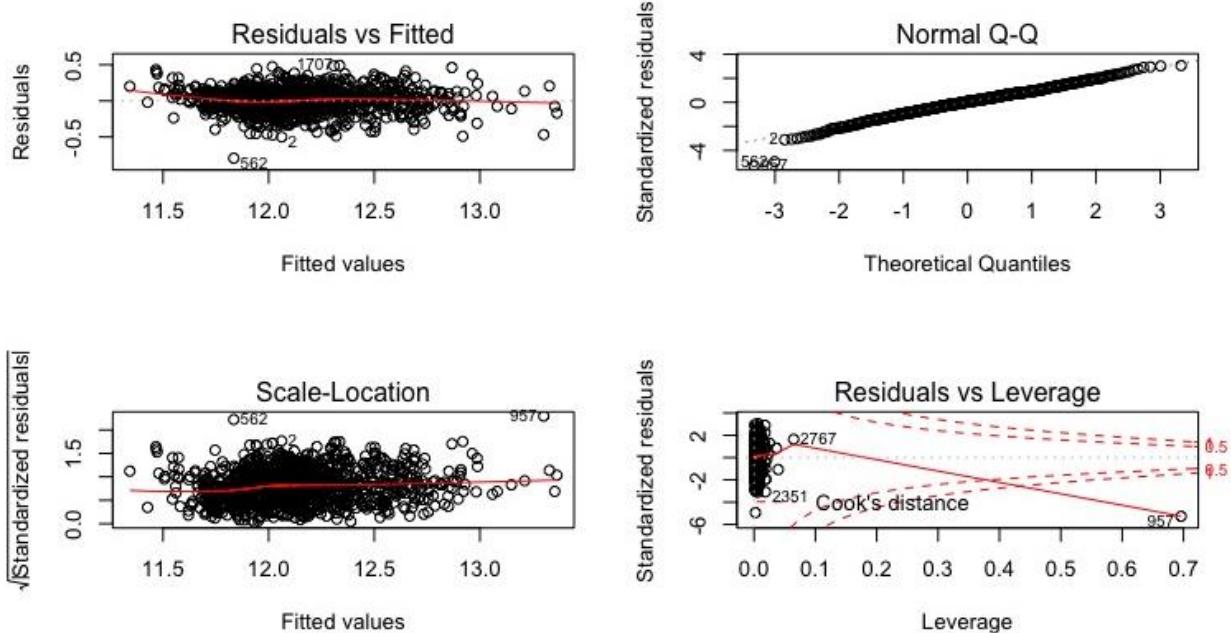


Figure 9: Diagnostic Plots for MLR Model #2

Finally, when we compare the computed Mean Square Error (MSE) and Mean Absolute Error (MAE) values of each MLR Model in Table 8 we see that both MSE and MAE has decreased moving from MLR Model #1 to MLR Model #2. This suggests that the transformation of SalePrice in MLR Model #2 has resulted in better and more accurate regression model outcome.

	MLR Model #1	MLR Model #2
Mean Standard Error (MSE)	1421094578	1258260254
Mean Absolute Error (MAE)	27414.76	25204.51

## Conclusion

In conclusion, our exploratory data and correlation analysis for the response variable, SalePrice, and selected predictor variables from Ames dataset in general present a significant positive correlation. In addition to analyzing SalePrice, we also investigated the impact of response variable logarithmic transportation on the established correlations. Our analysis showed a slight improvement in correlations as we transformed the response variable, while our top indicators remained as OverallQual, TotalSqtCalc, and TotalBathCalc. Finally, we have determined that the OverallCond predictor variable, which is also the only variable that is negatively correlated to SalePrice, is the predictor variable that could benefit from a transformation from being discrete to continuous.

The two Simple Linear Regression (SLR) Model that was ran following the EDA has shown that TotalSqtCalc, compared to GarageArea, is the better predictor variables in predicting SalePrice with a higher R-square. Both TotalSqtCalc and GarageArea SLR models plotted lightly tailed Normal Q-Q Plot and Residuals vs. Fitted Values Plot that suggested homoscedasticity. The model performance and accuracy has improved noticeably when both variables we used for SLR models as well as two additional predictor variables, TotRmsAbvGrd and LotArea, were used in a Multiple Linear Regression model (MLR Model #1). The

diagnostic plots have not shown significant improvement between SLR and MLR models, yet we observed improvement in summary statistics of both linear models. Finally, the MLR model (MLR Model #2) where we transformed SalePrice response variable to logarithmic form and used the same predictor variables as MLR Model #1, proved to have the most accuracy in predicting SalePrice. The Normal Q-Q plot points were in a almost straight diagonal line and the  $R^2$  increase to 0.78 from 0.76 in MLR Model #1. The comparison of MSE and MAE of MLR Model #1 and #2 has re-iterated the model results, showing a decrease in errors moving from Model #1 to #2.

## The R code for Assignment #2

```
# Serra Uzun
# MSDS_410 Supervised Learning Methods_FALL 2020
# 09.27.2020
# Assignment_02

#Import dataset
ames.df <- readRDS('/Users/serrauzun/Desktop/MSDS_410_Supervised/ames_sample.Rdata');

# columns and the types using the structure
str(ames.df)

## 'data.frame': 1135 obs. of 56 variables:
## $ SID : int 1 2 3 4 5 6 10 11 13 14 ...
## $ PID : int 526301100 526350040 526351010 526353030 527105010
527105030 527162130 527163010 527166040 527180040 ...
## $ LotFrontage : int 141 80 81 93 74 78 60 75 63 85 ...
## $ LotArea : int 31770 11622 14267 11160 13830 9978 7500 10000 840
2 10176 ...
## $ LotConfig : chr "Corner" "Inside" "Corner" "Corner" ...
## $ Neighborhood : chr "NAmes" "NAmes" "NAmes" "NAmes" ...
## $ HouseStyle : chr "1Story" "1Story" "1Story" "1Story" ...
## $ OverallQual : int 6 5 6 7 5 6 7 6 6 7 ...
## $ OverallCond : int 5 6 6 5 5 6 5 5 5 5 ...
## $ YearBuilt : int 1960 1961 1958 1968 1997 1998 1999 1993 1998 1990
...
## $ YearRemodel : int 1960 1961 1958 1968 1998 1998 1999 1994 1998 1990
...
## $ Exterior1 : chr "BrkFace" "VinylSd" "Wd Sdng" "BrkFace" ...
## $ BsmtFinSF1 : int 639 468 923 1065 791 602 0 0 0 637 ...
## $ BsmtFinSF2 : int 0 144 0 0 0 0 0 0 0 0 ...
## $ CentralAir : chr "Y" "Y" "Y" "Y" ...
## $ GrLivArea : int 1656 896 1329 2110 1629 1604 1804 1655 1465 1341
...
## $ BsmtFullBath : int 1 0 0 1 0 0 0 0 0 1 ...
## $ BsmtHalfBath : int 0 0 0 0 0 0 0 0 0 0 ...
## $ FullBath : int 1 1 1 2 2 2 2 2 2 1 ...
## $ HalfBath : int 0 0 1 1 1 1 1 1 1 1 ...
## $ BedroomAbvGr : int 3 2 3 3 3 3 3 3 3 2 ...
## $ TotRmsAbvGrd : int 7 5 6 8 6 7 7 7 7 5 ...
## $ Fireplaces : int 2 0 0 2 1 1 1 1 1 1 ...
## $ GarageCars : int 2 1 1 2 2 2 2 2 2 2 ...
## $ GarageArea : int 528 730 312 522 482 470 442 440 393 506 ...
## $ WoodDeckSF : int 210 140 393 0 212 360 140 157 0 192 ...
## $ OpenPorchSF : int 62 0 36 0 34 36 60 84 75 0 ...
## $ EnclosedPorch : int 0 0 0 0 0 0 0 0 0 0 ...
## $ ThreeSsnPorch : int 0 0 0 0 0 0 0 0 0 0 ...
```

```

## $ ScreenPorch      : int  0 120 0 0 0 0 0 0 0 0 0 ...
## $ PoolArea         : int  0 0 0 0 0 0 0 0 0 0 ...
## $ MoSold           : int  5 6 6 4 3 6 6 4 5 2 ...
## $ YrSold           : int  2010 2010 2010 2010 2010 2010 2010 2010 2010 2010
...
## $ SaleCondition    : chr  "Normal" "Normal" "Normal" "Normal" ...
## $ SalePrice        : int  215000 105000 172000 244000 189900 195500 189000
175900 180400 171500 ...
## $ TotalSqftCalc   : int  2295 1508 2252 3175 2420 2206 1804 1655 1465 1978
...
## $ TotalBathCalc   : num  2 1 1.5 3.5 2.5 2.5 2.5 2.5 2.5 2.5 ...
## $ CornerLotInd    : num  1 0 1 1 0 0 0 1 0 0 ...
## $ FireplaceInd1   : num  0 0 0 0 1 1 1 1 1 1 ...
## $ FireplaceInd2   : num  1 0 0 1 0 0 0 0 0 0 ...
## $ FireplaceAdder1: num  1 0 0 1 1 1 1 1 1 1 ...
## $ FireplaceAdder2: num  1 0 0 1 0 0 0 0 0 0 ...
## $ CentralAirInd   : num  1 1 1 1 1 1 1 1 1 1 ...
## $ BrickInd         : num  1 0 0 1 0 0 0 0 0 0 ...
## $ VinylSidingInd  : num  0 1 0 0 1 1 1 0 1 0 ...
## $ PoolInd          : num  0 0 0 0 0 0 0 0 0 0 ...
## $ WoodDeckInd     : num  1 1 1 0 1 1 1 1 0 1 ...
## $ PorchInd         : num  1 1 1 1 1 1 1 1 1 1 ...
## $ QualityIndex    : int  30 30 36 35 25 36 35 30 30 35 ...
## $ I2006            : num  0 0 0 0 0 0 0 0 0 0 ...
## $ I2007            : num  0 0 0 0 0 0 0 0 0 0 ...
## $ I2008            : num  0 0 0 0 0 0 0 0 0 0 ...
## $ I2009            : num  0 0 0 0 0 0 0 0 0 0 ...
## $ I2010            : num  1 1 1 1 1 1 1 1 1 1 ...
## $ u                : num  0.74166 0.88833 0.00677 0.23496 0.50085 ...
## $ train            : num  0 0 1 1 1 1 1 1 1 1 ...
## - attr(*, "na.action")= 'omit' Named int  9 17 18 19 31 33 34 49 51 56 ...
.
## ... - attr(*, "names")= chr  "12" "23" "24" "25" ...

```

**colnames(ames.df)**

## [1] "SID"	"PID"	"LotFrontage"	"LotArea"
## [5] "LotConfig"	"Neighborhood"	"HouseStyle"	"OverallQual"
## [9] "OverallCond"	"YearBuilt"	"YearRemodel"	"Exterior1"
## [13] "BsmtFinSF1"	"BsmtFinSF2"	"CentralAir"	"GrLivArea"
## [17] "BsmtFullBath"	"BsmtHalfBath"	"FullBath"	"HalfBath"
## [21] "BedroomAbvGr"	"TotRmsAbvGrd"	"Fireplaces"	"GarageCars"
## [25] "GarageArea"	"WoodDeckSF"	"OpenPorchSF"	"EnclosedPorch"
## [29] "ThreeSsnPorch"	"ScreenPorch"	"PoolArea"	"MoSold"
## [33] "YrSold"	"SaleCondition"	"SalePrice"	"TotalSqftCalc"
## [37] "TotalBathCalc"	"CornerLotInd"	"FireplaceInd1"	"FireplaceInd2"
## [41] "FireplaceAdder1"	"FireplaceAdder2"	"CentralAirInd"	"BrickInd"
## [45] "VinylSidingInd"	"PoolInd"	"WoodDeckInd"	"PorchInd"
## [49] "QualityIndex"	"I2006"	"I2007"	"I2008"
## [53] "I2009"	"I2010"	"u"	"train"

```

# subset of predictor variables;
small.df <- ames.df[,c('SalePrice','TotalSqftCalc','TotalBathCalc','QualityIndex',
                      'TotRmsAbvGrd','OverallQual','OverallCond')];
str(small.df)

## 'data.frame':    1135 obs. of  7 variables:
## $ SalePrice     : int  215000 105000 172000 244000 189900 195500 189000 17
5900 180400 171500 ...
## $ TotalSqftCalc: int  2295 1508 2252 3175 2420 2206 1804 1655 1465 1978 .
..
## $ TotalBathCalc: num  2 1 1.5 3.5 2.5 2.5 2.5 2.5 2.5 ...
## $ QualityIndex  : int  30 30 36 35 25 36 35 30 30 35 ...
## $ TotRmsAbvGrd : int  7 5 6 8 6 7 7 7 7 5 ...
## $ OverallQual   : int  6 5 6 7 5 6 7 6 6 7 ...
## $ OverallCond   : int  5 6 6 5 5 6 5 5 5 5 ...

# count of missing values for each variable
sapply(small.df, function(x) sum(is.na(x)))

##      SalePrice TotalSqftCalc TotalBathCalc  QualityIndex  TotRmsAbvGrd
##                 0             0             0             0             0
##      OverallQual  OverallCond
##                 0             0

#summary statistics of small.df
summary(small.df)

##      SalePrice      TotalSqftCalc  TotalBathCalc  QualityIndex  TotRmsAbv
## Grd
##  Min.   : 62383   Min.   : 825   Min.   :1.000   Min.   :12.0   Min.   : 4
## .00
##  1st Qu.:142188   1st Qu.:1630   1st Qu.:2.000   1st Qu.:30.0   1st Qu.: 6
## .00
##  Median :177500   Median :1955   Median :2.500   Median :35.0   Median : 6
## .00
##  Mean   :197212   Mean   :2122   Mean   :2.338   Mean   :34.4   Mean   : 6
## .53
##  3rd Qu.:229900   3rd Qu.:2486   3rd Qu.:3.000   3rd Qu.:40.0   3rd Qu.: 7
## .00
##  Max.   :755000   Max.   :5771   Max.   :4.500   Max.   :72.0   Max.   :12
## .00
##      OverallQual  OverallCond
##  Min.   : 3.000   Min.   :3.000
##  1st Qu.: 5.000   1st Qu.:5.000
##  Median : 6.000   Median :5.000
##  Mean   : 6.302   Mean   :5.515
##  3rd Qu.: 7.000   3rd Qu.:6.000
##  Max.   :10.000  Max.   :9.000

```

```

# a more detailed descriptive summary with stat.desc
install.packages("pastecs", repos = "http://cran.us.r-project.org")

## Installing package into '/Users/serrauzun/Library/R/3.5/library'
## (as 'lib' is unspecified)

##
## The downloaded binary packages are in
## /var/folders/7x/6nv1vk957xl_frh93z69k4cm0000gn/T//RtmpKIHNxN downloaded_p
ackages

library(pastecs)
small.df.summary <- as.data.frame(t(round(stat.desc(small.df), 1)))
small.df.summary

##          nbr.val nbr.null nbr.na   min     max    range      sum
## SalePrice       1135         0      0 62383 755000.0 692617.0 223835036.0
## TotalSqftCalc   1135         0      0   825   5771.0   4946.0  2408234.0
## TotalBathCalc   1135         0      0     1     4.5     3.5   2653.5
## QualityIndex    1135         0      0     0     12     72.0    60.0  39044.0
## TotRmsAbvGrd    1135         0      0     0     4     12.0     8.0  7411.0
## OverallQual     1135         0      0     0     3     10.0     7.0  7153.0
## OverallCond     1135         0      0     0     3     9.0      6.0  6260.0
##             median      mean  SE.mean CI.mean.0.95      var std.dev
## SalePrice     177500.0 197211.5  2295.2      4503.3 5979050376.5 77324.3
## TotalSqftCalc 1955.0   2121.8   21.1      41.3   502986.5   709.2
## TotalBathCalc   2.5     2.3     0.0      0.0      0.6     0.7
## QualityIndex    35.0    34.4    0.2      0.4      52.0    7.2
## TotRmsAbvGrd    6.0     6.5     0.0      0.1      1.8    1.4
## OverallQual     6.0     6.3     0.0      0.1      1.7    1.3
## OverallCond     5.0     5.5     0.0      0.1      0.8    0.9
##             coef.var
## SalePrice        0.4
## TotalSqftCalc   0.3
## TotalBathCalc   0.3
## QualityIndex    0.2
## TotRmsAbvGrd    0.2
## OverallQual    0.2
## OverallCond    0.2

#####
#####

# Correlation Plots
#####

#####

# Install and Load the corrplot package
#install.packages('corrplot', dependencies=TRUE)
library(corrplot)

## corrplot 0.84 loaded

```

```

# Correlations with SalePrice;
cor(small.df)

##                                     SalePrice TotalSqftCalc TotalBathCalc QualityIndex TotRmsAbvGrd
## SalePrice      1.0000000   0.7856833   0.6714113   0.6107868   0.647
## TotalSqftCalc 0.7856833   1.0000000   0.7032977   0.4222650   0.590
## TotalBathCalc  0.6714113   0.7032977   1.0000000   0.4028544   0.564
## QualityIndex   0.6107868   0.4222650   0.4028544   1.0000000   0.375
## TotRmsAbvGrd  0.6472545   0.5900370   0.5649834   0.3750965   1.000
## OverallQual    0.8245066   0.5465695   0.5954692   0.7325319   0.584
## OverallCond   -0.2384085  -0.1485253  -0.2368500   0.4102262  -0.260
##                                     OverallQual OverallCond
## SalePrice        0.8245066  -0.2384085
## TotalSqftCalc   0.5465695  -0.1485253
## TotalBathCalc   0.5954692  -0.2368500
## QualityIndex    0.7325319  0.4102262
## TotRmsAbvGrd   0.5843620  -0.2609945
## OverallQual     1.0000000  -0.3095300
## OverallCond    -0.3095300  1.0000000

# Correlations with Log(SalePrice);
# Need to drop SalePrice and add logSalePrice;
log.df <- subset(small.df, select=-c(SalePrice));
head(log.df)

##   TotalSqftCalc TotalBathCalc QualityIndex TotRmsAbvGrd OverallQual OverallCond
## 1          2295            2.0         30             7            6
## 2          1508            1.0         30             5            5
## 3          2252            1.5         36             6            6
## 4          3175            3.5         35             8            7
## 5          2420            2.5         25             6            5
## 6          2206            2.5         36             7            6

log.df$logSalePrice <- log(small.df$SalePrice);
head(log.df)

```

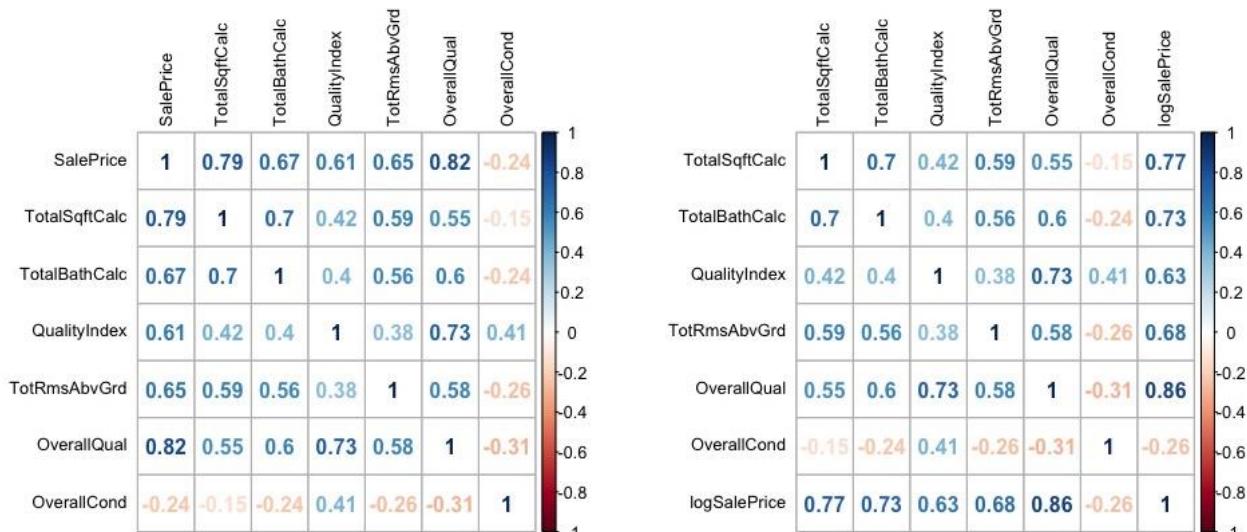
```

##   TotalSqftCalc TotalBathCalc QualityIndex TotRmsAbvGrd OverallQual Overall
1Cond
## 1      2295        2.0          30            7           6
5
## 2      1508        1.0          30            5           5
6
## 3      2252        1.5          36            6           6
6
## 4      3175        3.5          35            8           7
5
## 5      2420        2.5          25            6           5
5
## 6      2206        2.5          36            7           6
6
##   logSalePrice
## 1      12.27839
## 2      11.56172
## 3      12.05525
## 4      12.40492
## 5      12.15425
## 6      12.18332

sales_cor <- as.data.frame(cor(small.df))
sales_log_cor <- as.data.frame(cor(log.df))

par(mfrow = c(1,2))
corrplot(cor(small.df),method='number',tl.cex = 0.8, tl.col = "black")
corrplot(cor(log.df),method='number', tl.cex = 0.8, tl.col = "black")

```



```
#####
#####  
# Scatterplots, Scatterplot Smoothers, and Simple Linear Regression  
# Response Variable: SalePrice  
#####
```

```

#####
par(mfrow = c(3,2))

### TotalSfqtCalc
loess.1 <- loess(SalePrice ~ TotalSqftCalc,data=small.df);
lm.1 <- lm(SalePrice ~ TotalSqftCalc,data=small.df);

plot(small.df$TotalSqftCalc, small.df$SalePrice,xlab='Total Square Footage',y
lab='SalePrice')
points(loess.1$x,loess.1$fitted,type='p',col='red')
abline(coef=lm.1$coef,col='blue',lwd=2)
title('SLR - Total Square Footage')

### OverallQual
loess.2 <- loess(SalePrice ~ OverallQual,data=small.df);

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : pseudoinverse used at 6

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : neighborhood radius 1

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : reciprocal condition number 0

lm.2 <- lm(SalePrice ~ OverallQual,data=small.df);

plot(small.df$SalePrice ~ small.df$OverallQual,xlab='Overall Quality',ylab='S
alePrice')
points(loess.2$x,loess.2$fitted,type='p',col='red',pch=19)
abline(coef=lm.2$coef,col='blue',lwd=2)
title('SLR - Overall Quality')

# Note that Loess() outputs warning messages. Can we guess why?

### OverallCond
loess.3 <- loess(SalePrice ~ OverallCond,data=small.df);

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : pseudoinverse used at 5

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : neighborhood radius 1

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : reciprocal condition number 0

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : There are other near singularities as well. 1

lm.3 <- lm(SalePrice ~ OverallCond,data=small.df);

```

```

plot(small.df$SalePrice ~ small.df$OverallCond,xlab='Overall Condition',ylab=
'SalePrice')
points(loess.3$x,loess.3$fitted,type='p',col='red',pch=19)
abline(coef=lm.3$coef,col='blue',lwd=2)
title('SLR - Overall Condition')

### TotalBathCalc
loess.4 <- loess(SalePrice ~ TotalBathCalc ,data=small.df);
lm.4 <- lm(SalePrice ~ TotalBathCalc,data=small.df);

plot(small.df$SalePrice ~ small.df$TotalBathCalc,xlab='Total Number of Bathro
oms',ylab='SalePrice')
points(loess.4$x,loess.4$fitted,type='p',col='red',pch=19)
abline(coef=lm.4$coef,col='blue',lwd=2)
title('SLR - Total Number of Bathrooms')

### QualityIndex
loess.5 <- loess(SalePrice ~ QualityIndex ,data=small.df);
lm.5 <- lm(SalePrice ~ QualityIndex,data=small.df);

plot(small.df$SalePrice ~ small.df$QualityIndex,xlab='Quality Index',ylab='Sa
lePrice')
points(loess.5$x,loess.5$fitted,type='p',col='red',pch=19)
abline(coef=lm.5$coef,col='blue',lwd=2)
title('SLR - Quality Index')

### TotRmsAbvGrd
loess.6 <- loess(SalePrice ~ TotRmsAbvGrd ,data=small.df);

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : pseudoinverse used at 6

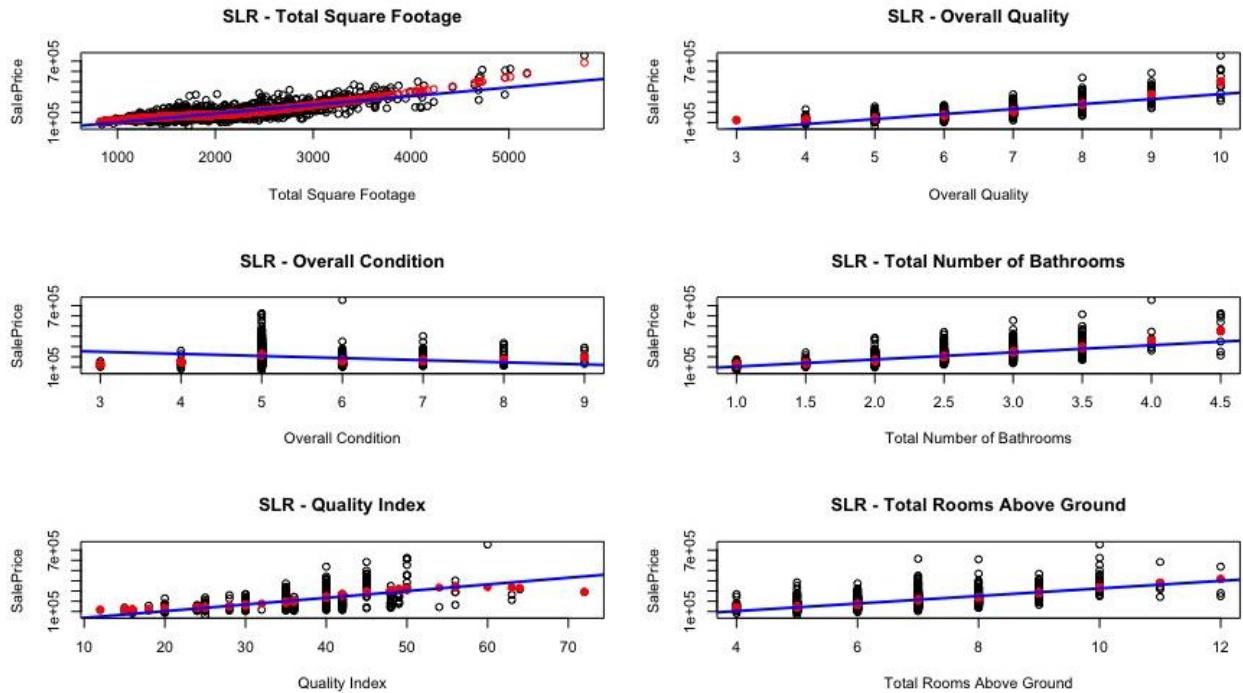
## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : neighborhood radius 1

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : reciprocal condition number 0

lm.6 <- lm(SalePrice ~ TotRmsAbvGrd,data=small.df);

plot(small.df$SalePrice ~ small.df$TotRmsAbvGrd,xlab='Total Rooms Above Groun
d',ylab='SalePrice')
points(loess.6$x,loess.6$fitted,type='p',col='red',pch=19)
abline(coef=lm.6$coef,col='blue',lwd=2)
title('SLR - Total Rooms Above Ground')

```



```
#####
##### # Scatterplots, Scatterplot Smoothers, and Simple Linear Regression
##### # Response Variable: log(SalePrice)
#####

par(mfrow = c(3,2))

### TotalSfqtCalc
loess.1 <- loess(log(SalePrice) ~ TotalSfqtCalc, data=small.df);
lm.1 <- lm(log(SalePrice) ~ TotalSfqtCalc, data=small.df);

plot(small.df$TotalSfqtCalc, log(small.df$SalePrice), xlab='Total Square Footage', ylab='log(SalePrice)')
points(loess.1$x, loess.1$fitted, type='p', col='red')
abline(coef=lm.1$coef, col='blue', lwd=2)
title('SLR - Total Square Footage')

### OverallQual
loess.2 <- loess(log(SalePrice) ~ OverallQual, data=small.df);

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : pseudoinverse used at 6

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : neighborhood radius 1

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : reciprocal condition number 0
```

```

lm.2 <- lm(log(SalePrice) ~ OverallQual,data=small.df);

plot(log(small.df$SalePrice) ~ small.df$OverallQual,xlab='Overall Quality',ylab='log(SalePrice)')
points(loess.2$x,loess.2$fitted,type='p',col='red',pch=19)
abline(coef=lm.2$coef,col='blue',lwd=2)
title('SLR - Overall Quality')

# Note that Loess() outputs warning messages. Can we guess why?

### OverallCond
loess.3 <- loess(log(SalePrice) ~ OverallCond,data=small.df);

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : pseudoinverse used at 5

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : neighborhood radius 1

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : reciprocal condition number 0

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : There are other near singularities as well. 1

lm.3 <- lm(log(SalePrice) ~ OverallCond,data=small.df);

plot(log(small.df$SalePrice) ~ small.df$OverallCond,xlab='Overall Condition',ylab='log(SalePrice)')
points(loess.3$x,loess.3$fitted,type='p',col='red',pch=19)
abline(coef=lm.3$coef,col='blue',lwd=2)
title('SLR - Overall Condition')

### TotalBathCalc
loess.4 <- loess(log(SalePrice) ~ TotalBathCalc,data=small.df);
lm.4 <- lm(log(SalePrice) ~ TotalBathCalc,data=small.df);

plot(log(small.df$SalePrice) ~ small.df$TotalBathCalc,xlab='Total Number of Bathrooms',ylab='log(SalePrice)')
points(loess.4$x,loess.4$fitted,type='p',col='red',pch=19)
abline(coef=lm.4$coef,col='blue',lwd=2)
title('SLR - Total Number of Bathrooms')

### QualityIndex
loess.5 <- loess(log(SalePrice) ~ QualityIndex,data=small.df);
lm.5 <- lm(log(SalePrice) ~ QualityIndex,data=small.df);

plot(log(small.df$SalePrice) ~ small.df$QualityIndex,xlab='Quality Index',ylab='log(SalePrice)')
points(loess.5$x,loess.5$fitted,type='p',col='red',pch=19)
abline(coef=lm.5$coef,col='blue',lwd=2)

```

```

title('SLR - Quality Index')

### TotRmsAbvGrd
loess.6 <- loess(log(SalePrice) ~ TotRmsAbvGrd,data=small.df);

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : pseudoinverse used at 6

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : neighborhood radius 1

## Warning in simpleLoess(y, x, w, span, degree = degree, parametric =
## parametric, : reciprocal condition number 0

lm.6 <- lm(log(SalePrice) ~ TotRmsAbvGrd,data=small.df);

plot(log(small.df$SalePrice) ~ small.df$TotRmsAbvGrd,xlab='Total Rooms Above
Ground',ylab='log(SalePrice)')
points(loess.6$x,loess.6$fitted,type='p',col='red',pch=19)
abline(coef=lm.6$coef,col='blue',lwd=2)
title('SLR - Total Rooms Above Ground')



```

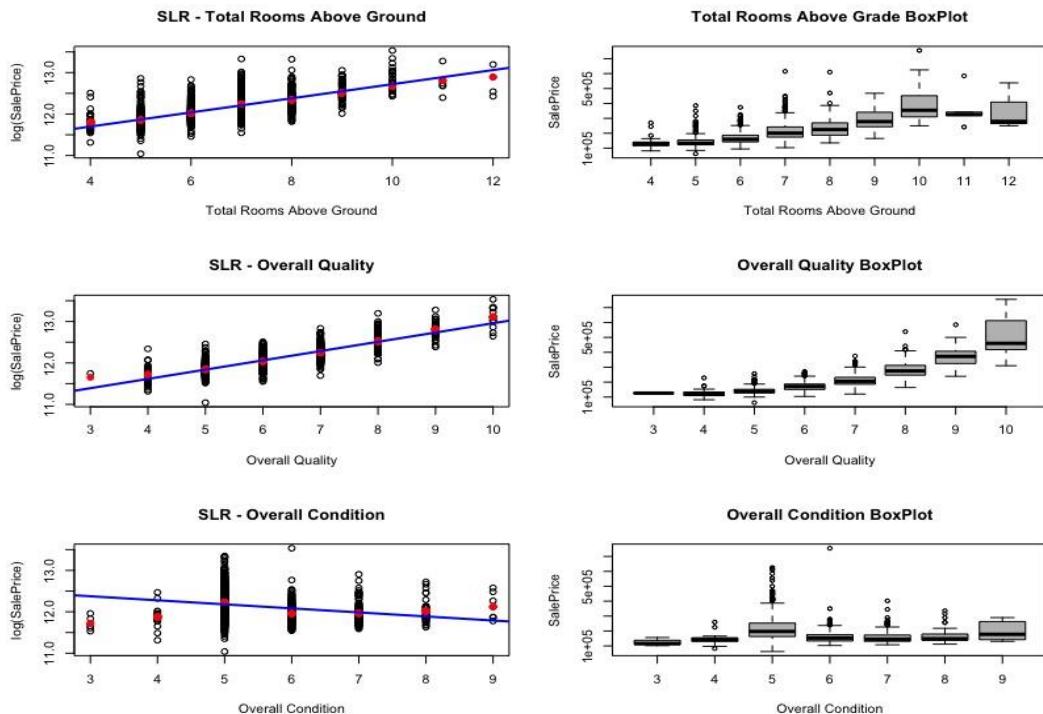
```

#Total Rooms Above Grade
plot(log(small.df$SalePrice) ~ small.df$TotRmsAbvGrd,xlab='Total Rooms Above
Ground',ylab='log(SalePrice)')
points(loess.6$x,loess.6$fitted,type='p',col='red',pch=19)
abline(coef=lm.6$coef,col='blue',lwd=2)
title('SLR - Total Rooms Above Ground')
boxplot(small.df$SalePrice ~ small.df$TotRmsAbvGrd,col = "gray",xlab='Total R
ooms Above Ground',ylab='SalePrice', main = "Total Rooms Above Grade BoxPlot"
)

#Overall Quality
plot(log(small.df$SalePrice) ~ small.df$OverallQual,xlab='Overall Quality',yl
ab='log(SalePrice)')
points(loess.2$x,loess.2$fitted,type='p',col='red',pch=19)
abline(coef=lm.2$coef,col='blue',lwd=2)
title('SLR - Overall Quality')
boxplot(small.df$SalePrice ~ small.df$OverallQual,col = "gray", xlab='Overall
Quality',ylab='SalePrice', main = "Overall Quality BoxPlot")

#Overall Condition
plot(log(small.df$SalePrice) ~ small.df$OverallCond,xlab='Overall Condition',
ylab='log(SalePrice)')
points(loess.3$x,loess.3$fitted,type='p',col='red',pch=19)
abline(coef=lm.3$coef,col='blue',lwd=2)
title('SLR - Overall Condition')
boxplot(small.df$SalePrice ~ small.df$OverallCond, col = "gray",xlab='Overall
Condition',ylab='SalePrice', main = "Overall Condition BoxPlot")

```



```

####Knitr
install.packages("knitr",repos = "http://cran.us.r-project.org")

## Installing package into '/Users/serrauzun/Library/R/3.5/library'
## (as 'lib' is unspecified)

##
## There is a binary version available but the source version is later:
##       binary source needs_compilation
## knitr   1.28   1.29          FALSE

## installing the source package 'knitr'

library(knitr)

install.packages("rmarkdown",repos = "http://cran.us.r-project.org")

## Installing package into '/Users/serrauzun/Library/R/3.5/library'
## (as 'lib' is unspecified)

##
## There is a binary version available but the source version is later:
##       binary source needs_compilation
## rmarkdown   2.1    2.3          FALSE

## installing the source package 'rmarkdown'

library(rmarkdown)

#####
#####
# Assignment #2
#####
#####

#Question 1
par(mfrow = c(1,2))

#SLR with Total SF
SLR_loess_1a <- loess(SalePrice ~ TotalSqftCalc ,data=ames.df)
SLR_model_1a <- lm(SalePrice ~ TotalSqftCalc,data=ames.df)

plot(ames.df$SalePrice ~ ames.df$TotalSqftCalc,xlab='Total Square Footage', y
lab='SalePrice')
points(SLR_loess_1a$x, SLR_loess_1a$fitted,type='p',col='red',pch=19)
abline(coef=SLR_model_1a$coef,col='blue',lwd=2)
title('SLR - Total Square Footage')

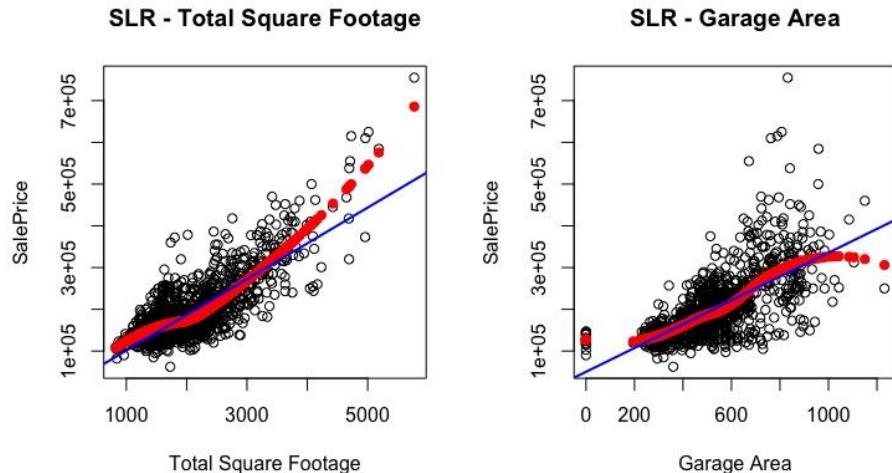
#SLR with Garage Area
SLR_loess_1b <- loess(SalePrice ~ GarageArea ,data=ames.df)
SLR_model_1b <- lm(SalePrice ~ GarageArea, data=ames.df)

```

```

plot(ames.df$SalePrice ~ ames.df$GarageArea,xlab='Garage Area', ylab='SalePrice')
points(SLR_loess_1b$x, SLR_loess_1b$fitted,type='p',col='red',pch=19)
abline(coef=SLR_model_1b$coef,col='blue',lwd=2)
title('SLR - Garage Area')

```



```

#Plot model summaries with stargazer
install.packages('stargazer',dependencies=TRUE, repos = "http://cran.us.r-project.org")

## Installing package into '/Users/serrauzun/Library/R/3.5/library'
## (as 'lib' is unspecified)

##
## The downloaded binary packages are in
## /var/folders/7x/6nv1vk957xl_frh93z69k4cm0000gn/T//RtmpARqicL downloaded_packages

library(MASS)

## Warning: package 'MASS' was built under R version 3.5.2

library(stargazer)

##
## Please cite as:

## Hlavac, Marek (2018). stargazer: Well-Formatted Regression and Summary Statistics Tables.

## R package version 5.2.2. https://CRAN.R-project.org/package=stargazer

out.path <- '/Users/serrauzun/Desktop/MSDS_410_Supervised/Assignment #2/';

summary(SLR_model_1a)

##
## Call:

```

```

## lm(formula = SalePrice ~ TotalSqftCalc, data = ames.df)
##
## Residuals:
##   Min     1Q   Median     3Q    Max
## -125289 -30405   -7691   28057  245192
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept) 15455.698   4482.657   3.448 0.000586 ***
## TotalSqftCalc  85.661      2.004  42.750 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 47860 on 1133 degrees of freedom
## Multiple R-squared:  0.6173, Adjusted R-squared:  0.617
## F-statistic: 1828 on 1 and 1133 DF,  p-value: < 2.2e-16

summary(SLR_model_1b)

##
## Call:
## lm(formula = SalePrice ~ GarageArea, data = ames.df)
##
## Residuals:
##   Min     1Q   Median     3Q    Max
## -163880 -30450   -2889   20582  467388
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept) 50191.216   5070.832   9.898 <2e-16 ***
## GarageArea   285.362      9.276  30.763 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 57100 on 1133 degrees of freedom
## Multiple R-squared:  0.4551, Adjusted R-squared:  0.4546
## F-statistic: 946.3 on 1 and 1133 DF,  p-value: < 2.2e-16

file.name_1 <- 'SLR Model Comparison.html';
stargazer(SLR_model_1a, SLR_model_1b, type=c('html'), out=paste(out.path,file.name_1,sep=''),
           title=c('Comparison of SLR Model #1 and Model #2'),
           align=TRUE, digits=2, digits.extra=2, initial.zero=TRUE,
           column.labels=c('SLR Model #1','SLR Model #2'), intercept.bottom=FALSE )
LSE )

#Coefficients
summary(SLR_model_1a)

##
## Call:

```

```

## lm(formula = SalePrice ~ TotalSqftCalc, data = ames.df)
##
## Residuals:
##    Min     1Q   Median     3Q    Max
## -125289 -30405   -7691   28057  245192
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 15455.698   4482.657   3.448 0.000586 ***
## TotalSqftCalc     85.661      2.004  42.750 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 47860 on 1133 degrees of freedom
## Multiple R-squared:  0.6173, Adjusted R-squared:  0.617
## F-statistic: 1828 on 1 and 1133 DF,  p-value: < 2.2e-16

SLR_model_1a$coefficients

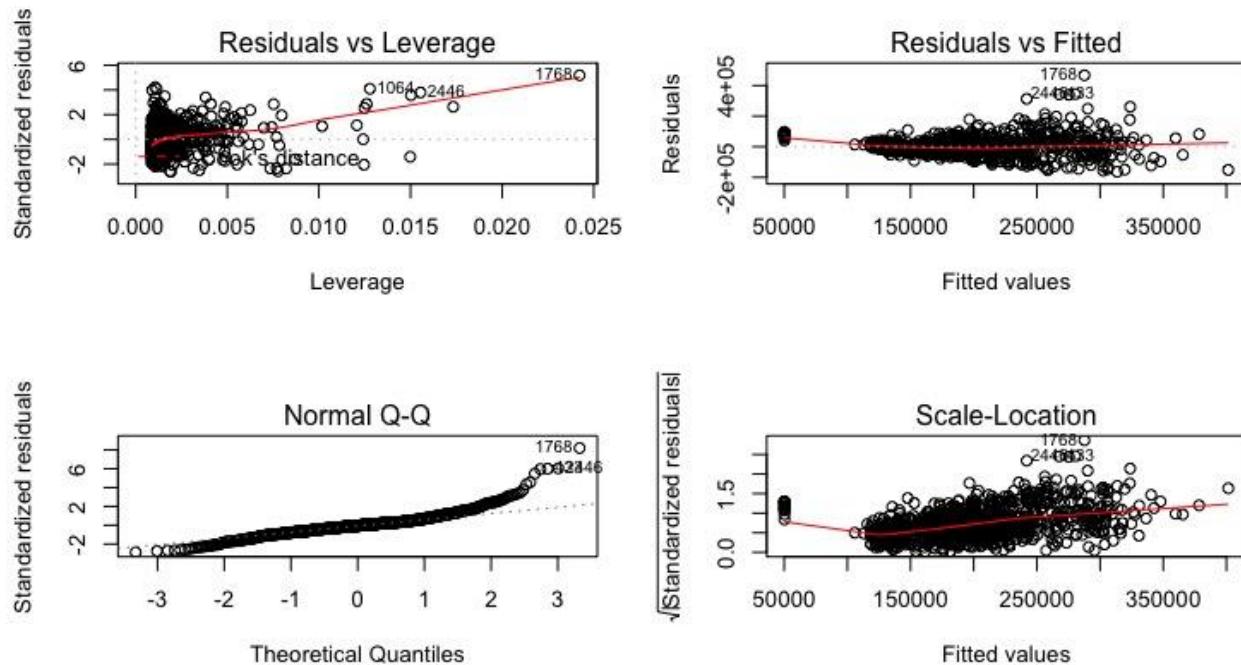
## (Intercept) TotalSqftCalc
## 15455.69802     85.66145

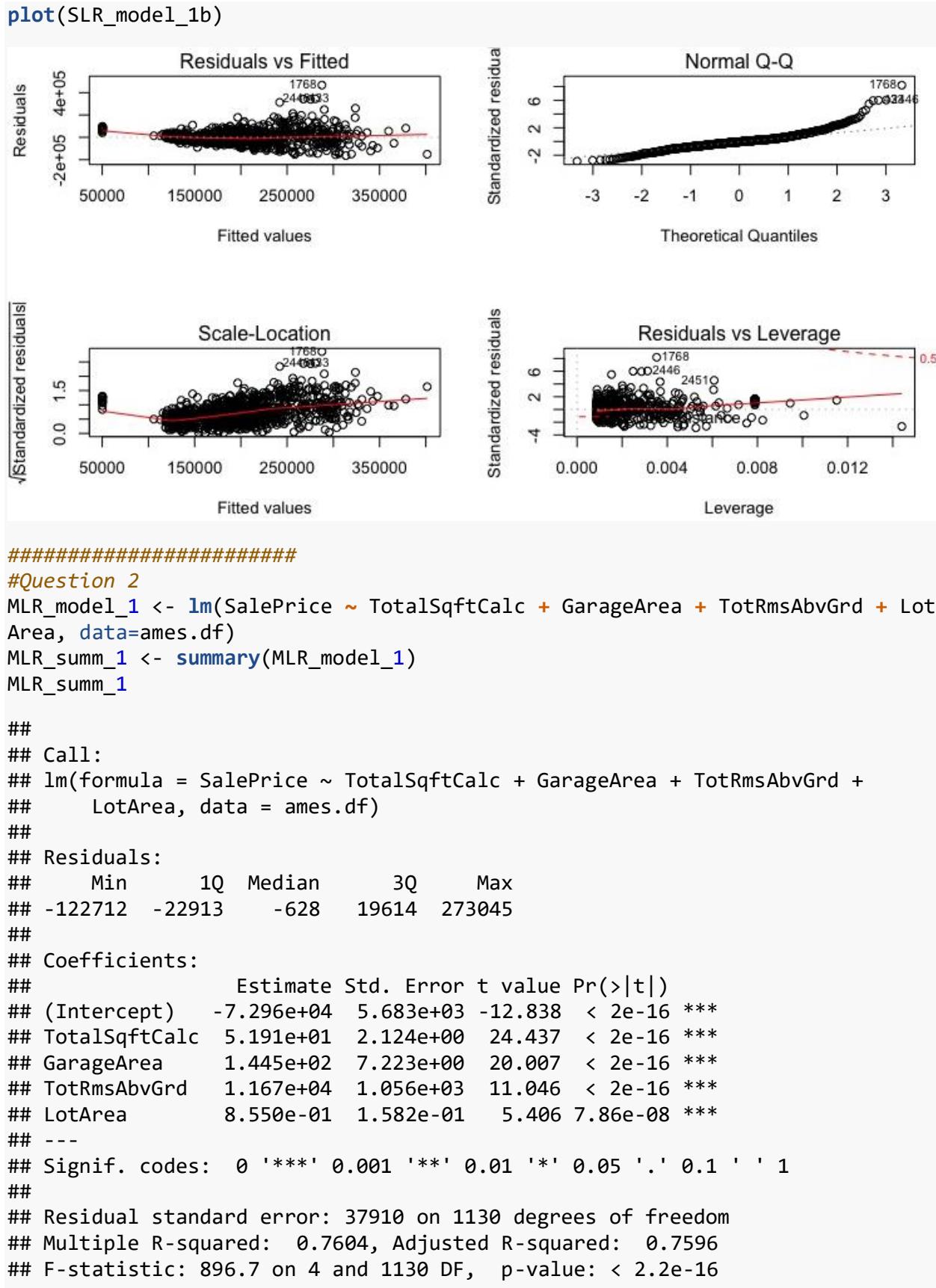
SLR_model_1b$coefficients

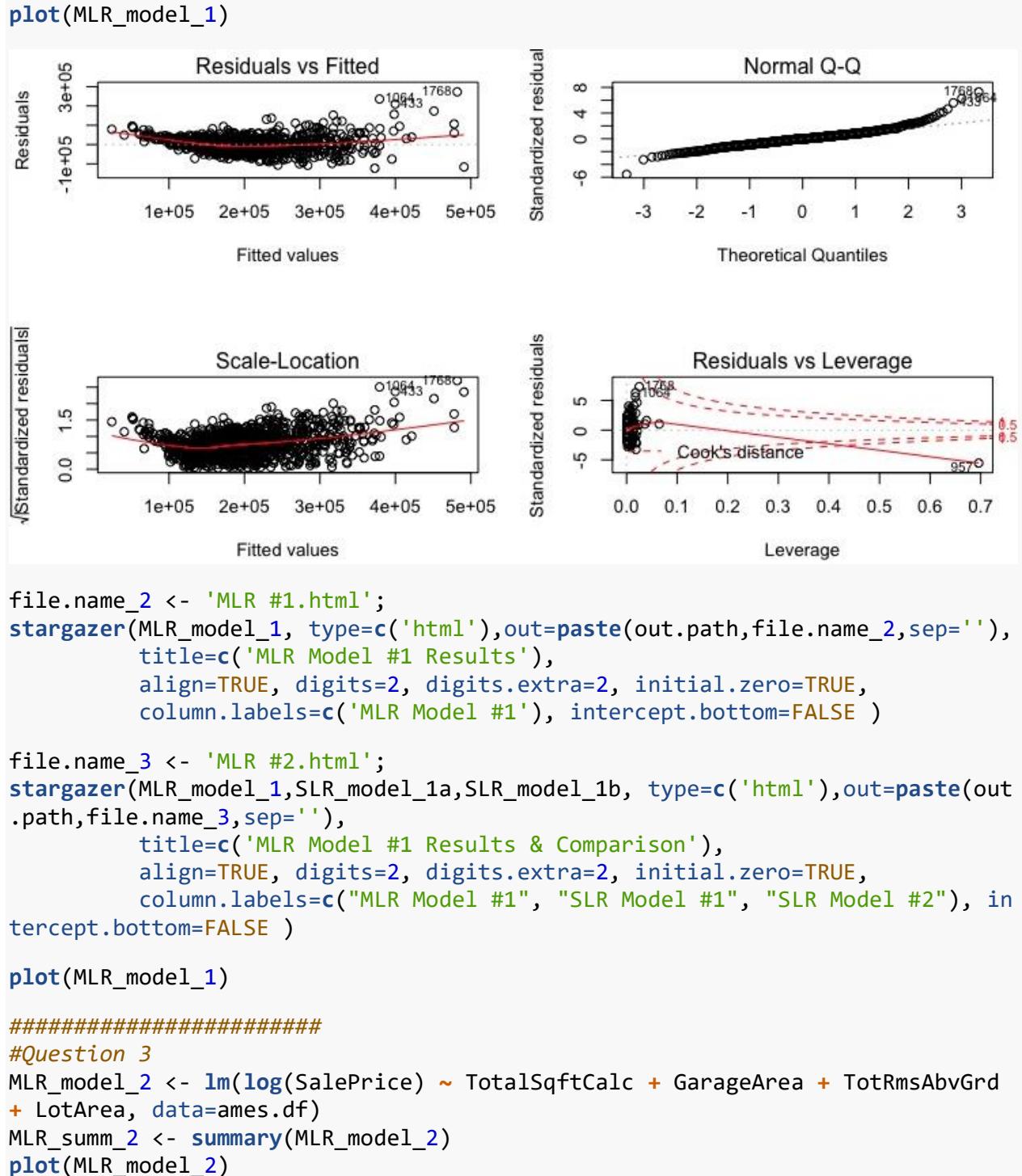
## (Intercept) GarageArea
## 50191.2164     285.3615

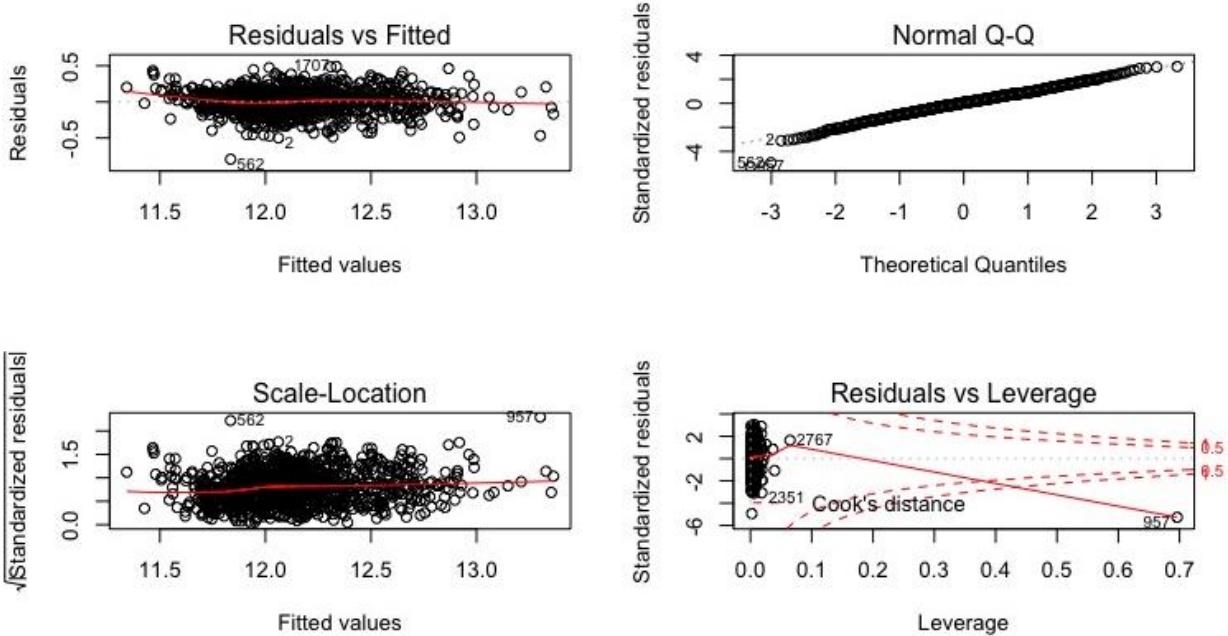
#Diagnostic Plots
plot(SLR_model_1a)

```









```

file.name_4 <- 'MLR Question Results Comp.html';
stargazer(MLR_model_1, MLR_model_2, type=c('html'), out=paste(out.path,file.name_4,sep='')),
           title=c('Comparison of MLR Model #1 and Model #2'),
           align=TRUE, digits=2, digits.extra=2, initial.zero=TRUE,
           column.labels=c('MLR Model #1','MLR Model #2'), intercept.bottom=FALSE )

```

*#MSE for both MLR*

```

mse.1 <- mean(MLR_model_1$residuals^2)
mae.1 <- mean(abs(MLR_model_1$residuals))

mse.2 <- mean((ames.df$SalePrice-exp(MLR_model_2$fitted.values))^2)
mae.2 <- mean(abs(ames.df$SalePrice-exp(MLR_model_2$fitted.values)))

```

```

mse.1
## [1] 1431094578

mae.1
## [1] 27414.76

mse.2
## [1] 1258260254

mae.2
## [1] 25204.51

```

```
mse.1/mse.2  
## [1] 1.13736  
mae.1/mae.2  
## [1] 1.087693
```