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An affine macro-finance term structure model for the euro area

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Abstract

A joint model of macroeconomic and term structure dynamics is specified and estimated for the euro area. The model comprises a backward-looking Phillips curve, a dynamic IS equation, a monetary policy rule as well as a specification of the dynamics of trend growth and the natural real interest rate. Under the condition of no arbitrage, yields of all maturities are affine functions of the macroeconomic driving forces. With the exception of a shock to potential output growth, the response of short-term yields to macroeconomic shocks is generally stronger than that of long-term yields. Impulse responses of all bond yields are fairly persistent, which reflects the persistence of their macroeconomic driving forces. Across the whole maturity spectrum, about ninety percent of the variation in yields is explained jointly by monetary policy shocks and shocks to the natural real rate of interest; the relative contribution of the latter shock increases with time to maturity. Cost-push shocks explain at most eight percent, while shocks to the output gap play an even less important role.

JEL classification: E43; E44; E52

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1. Introduction

Starting from the seminal contributions of Vasiček (1977) and Cox, Ingersoll, and Ross (1985), there is a large and growing literature that explores the dynamics of the term structure of interest rates in an arbitrage-free framework. Within this literature, the class of models

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in which bond yields are affine functions of a vector of state variables has become particularly prominent.[1]

In the empirical finance literature, the state vector usually consists of (latent) factors, which are interpreted as level, slope or curvature according to their impact on different maturity ranges of the term structure. In these models, bond yields are essentially explained by bond yields themselves.[2] From an economic perspective, however, the macroeconomic factors that stand behind the dynamics of short and long-term rates are of vital interest. In order to establish this nexus, a recent strand of the literature combines the principle of arbitrage-free valuation with elements from dynamic macro models. Most of these combined approaches are nested within the class of affine multifactor models. In contrast to the finance literature, however, some or all of the factors are no longer unspecified, but rather identified as macroeconomic variables such as inflation or real activity. These macro-finance models make it possible to assess the impact of macroeconomic shocks on bond yields of any maturity.

Term structure models in the macro-finance literature differ from each other primarily with respect to the way the macroeconomy is modelled. For instance, in Ang and Piazzesi (2003), Fendel (2004) or Ang, Dong, and Piazzesi (2005) a reduced-form VAR represents macroeconomic dynamics. The VAR is linked to the term structure by a Taylor-type monetary policy rule: movements in the short-term interest rate are traced back to movements in inflation, a real activity component, and some unobservable components. Dewachter and Lyrio (2006) and Dewachter, Lyrio, and Maes (2006) augment their model with long-run macroeconomic attractors for inflation, the output gap and the real interest rate. Other papers such as Bekaert, Cho, and Moreno (2005), Hördahl, Tristani, and Vestin (2006), Hördahl and Tristani (2007) or Rudebusch and Wu (2004), utilize a more structural macroeconomic framework, some of them incorporating elements of equilibrium models with rational expectations.

In this paper, the macroeconomic model underlying the term structure dynamics follows the lines of Laubach and Williams (2003) and Mesonnier and Renne (2006).[3] Its core elements are a 'backward-looking' Phillips curve and aggregate demand (IS) equation. Monetary policy is represented by a Taylor-type rule that allows for interest-rate smoothing and persistent policy shocks. The model also incorporates a specification of the dynamics of potential output growth and the natural real rate of interest. This allows to analyze the impact of shocks to these real driving forces, which are not accounted for in most other papers of the macro-finance literature.

The model is estimated using quarterly macroeconomic data (short-term interest rate, inflation, growth rate of gross domestic product) for the euro area for the period from 1981 to 2006. The data set for the time before 1999 relates to a hypothetical euro area. Bond yields enter the econometric model as of 1998 only. To my knowledge, the only other paper that explores the joint dynamics of the macroeconomy and the arbitrage-free term structure in the euro area is Hördahl and Tristani (2007), which uses monthly data for 1999 – 2006. Their model comprises both forward- and backward-looking elements, features a time-varying inflation target (which I treat as constant) but does not explicitly account for movements in the natural real interest rate (which I do).

[1] See Duffie and Kan (1996) and Dai and Singleton (2000).

[2] See, e.g., Babbs and Nowman (1998), Cassola and Luis (2003), Duan and Simonato (1999) or de Jong (2000) for empirical applications that estimate the latent factor process from a panel of observed bond yields.

[3] Note that these papers do not consider term structure implications.

The fit of the model to observed yields and macro variables turns out to be satisfactory, so it can be used for policy analysis. The high persistence of the macroeconomic variables is mirrored in the impulse responses of bond yields to macroeconomic shocks. This is particularly noticeable for a shock to the natural real rate of interest which has a strong and long-lasting effect on all yields. For this shock, it is long-term rates that react most strongly on impact. The other shocks (inflation, output gap, monetary policy), in contrast, affect short-term rates more strongly than long-term yields. However, since the initial response at the short end of the yield curve may be quite dynamic, longer-term yields can react more strongly than the one-year rate during the first few quarters after the shock.

A forecast-error variance decomposition of the model-implied yields shows that the three main driving forces of bond yields are cost-push shocks, shocks to the natural rate of interest, and monetary policy shocks. The cost-push shocks, i.e., idiosyncratic shocks to the inflation rate, never explain more than 17% of the variation of bond yields for any maturity and any forecast horizon. Thus, the bulk of variation stems from the other two shocks, where in general monetary policy shocks are dominant for shorter-term yields and shorter forecast horizons. Real shocks, in contrast, matter for variations in long-term bond yields and increase in importance as the forecast horizon increases. Concerning unconditional variances, monetary policy shocks and shocks to the natural real rate together explain about 90% of the variation for all yields. The contribution of cost-push shocks never exceeds 8%, and shocks to the output gap play an even smaller role.

The remainder of the paper is structured as follows. Section 2 outlines the set up of the macro model and – based on that – derives arbitrage-free term structure dynamics. Section 3 describes the estimation approach as well as the data. Parameter estimates, the fit of the model, impulse responses and the variance decomposition are discussed in Section 4, the last section concludes and gives an outlook on possible extensions and refinements.

2. The model

2.1. The macroeconomic module

This subsection introduces a small structural macroeconomic model, that explains the joint dynamics of inflation, the output gap, the one-period nominal and real interest rate, the natural real rate of interest, and potential output growth. The next subsection will establish the connection between these macroeconomic variables and the term structure of interest rates. The macroeconomic module is based on Mesonnier and Renne (2006) (MR), who employ it for estimating the natural real rate of interest in the euro area. Their specification can in turn be interpreted as a modification of the models by Rudebusch and Svensson (1999) and Laubach and Williams (2003). The MR model consists of a dynamic supply schedule (backward-looking Phillips curve), a dynamic demand specification (backward-looking IS equation), and a specification of the joint dynamics of potential output growth and the natural real rate of interest. These are represented by the following equations, the time frequency is quarterly:

$$\pi_{t+1} = c_\pi + \alpha_1\pi_t + \alpha_2\pi_{t-1} + \alpha_3\pi_{t-2} + \beta z_t + \epsilon^\pi_{t+1} \quad (1)$$

$$z_{t+1} = \psi_z z_t + (1+L)\gamma(i_t - \pi_{t+1|t} - r^*_t) + \epsilon^z_{t+1} \quad (2)$$

$$r^*_t = c_r + \theta_r a_t \quad (3)$$

$$\Delta y^*_t = c_y + \theta_y a_t + \epsilon^y_t \quad (4)$$

$$a_{t+1} = \psi_a a_t + \epsilon^a_{t+1} \tag{5}$$

$$y_t = y^*_t + z_t \tag{6}$$

The Phillips curve Eq. (1) relates current inflation π to its own lags and the previous period's output gap z. The latter is defined in (6) as the difference between log actual output y and log potential output y^*. Inflation can also be affected by idiosyncratic, serially uncorrelated cost-push shocks ϵ^π. Unlike MR, it will not be assumed that the α_i in (1) sum to unity, but rather that their sum is smaller than one. Thus, since the output gap z should be zero on average, I have to include the constant c_π to allow the unconditional expectation of inflation to differ from zero.

The IS Eq. (2) describes the dynamics of the output gap. Besides depending on the last quarter's output gap and idiosyncratic demand shocks ϵ^z, it is linked to $(i_t - \pi_{t+1|t} - r^*_t)$ and its lag.[4] The expression $i_t - \pi_{t+1|t}$ represents the model-consistent (ex-ante) real interest rate, i.e., the difference between the nominal one-quarter interest rate i_t and the one-step-ahead expectation of inflation $\pi_{t+1|t} \equiv E_t(\pi_{t+1})$. The variable r^*_t is the natural, neutral or equilibrium real interest rate (NRI). The notion of a natural real interest rate goes back to Wicksell (1898) and has gained revived prominence in the literature of New-Keynesian models.[5] In these models, that are characterized by nominal rigidities, the NRI represents the real rate in the hypothetical equilibrium with perfectly flexible prices. The NRI is a function of real shocks and represents an important benchmark for monetary policy. Real rates exceeding the NRI represent a contractionary monetary policy stance, whereas a real interest rate below the NRI stands for an expansionary stance. This property carries over to the – not explicitly microfounded – model considered here. When the real rate is below (above) the NRI, the negative (positive) real-rate gap $(i_t - \pi_{t+1|t} - r^*_t)$ stimulates (decreases) demand[6] and – ceteris paribus – increases (decreases) inflation via the Phillips curve.

In a hypothetical world without additional demand and cost-push shocks, monetary policy could steer nominal rates in a way that equalizes the actual real rate to its natural counterpart and would thus permanently stabilize output-gap and inflation fluctuations. However, the presence of idiosyncratic shocks implies that the task of monetary policy is not that trivial. Shocks to the NRI and idiosyncratic supply or demand shocks occur simultaneously, all exerting pressures on inflation and the output gap, that may differ in size, direction and persistence, thereby creating a trade-off for monetary policy.

In line with its definition, the NRI is assumed to share a common trend with potential output. Moreover, consistent with a standard Ramsey-type growth model, the steady state of the NRI should be a function of the steady state of potential-output growth (as well as the intertemporal elasticity of substitution in consumption and the time preference of households). This is reflected in Eqs. (3) and (4). The NRI r^*_t and potential output growth Δy^*_t share a common persistent component a_t, the dynamics of which is given by (5). In the following, a_t will be referred to as the trend growth rate. The additional transitory shock ϵ^y is specific to potential output growth; NRI-specific shocks are also conceivable, but I will follow MR and abstract from those: as a_t, r^*_t, Δy^*_t are all unobservable, with specification (3) and (4) it is already hard to distinguish statistically between the persistent component a_t and the transitory ϵ^y_t. The problem would be aggravated by

[4] L is the lag-operator. Thus, the real rate gap and its lag have the same impact, governed by γ, on the output gap. Relaxing this assumption does not lead to a significant change of results.

[5] See Woodford (2003). See, e.g., Amato (2005) for a discussion of the concept of the NRI.

[6] Note that the parameter γ is typically negative.

including an additional NRI-shock.[7] Finally, the steady state values[8] of the NRI and potential output growth are given by c_r and c_y, respectively.

Unlike MR who treat the short-term nominal interest rate as exogenous, I close the model with a monetary policy rule of the following form:

$$i_t = \phi_i i_{t-1} + (1 - \phi_i)(c_i + \phi_\pi \pi_t + \phi_g \Delta y_t) + \nu_t. \tag{7}$$

The form of this reaction function is fairly common in the literature. The current policy rate is a convex combination of a target interest rate

$$i_t^* = c_i + \phi_\pi \pi_t + \phi_g \Delta y_t$$

and the previous period's rate i_{t-1}. The monetary policy shock ν_t captures influences on the short rate that are independent of the systematic components i_{t-1} and i_t^*.

The target interest rate i_t^* is a linear function of contemporaneous inflation π_t and output growth Δy_t. This particular measure of real activity is also used in the monetary policy rules in Ang et al. (2005). However, in most specifications in the literature some sort of output *gap* is employed instead. For taking a similar approach in a model-consistent way, I would either have to assume that the policy maker in fact observes z_t or that he uses an estimate of it. For instance, if one supposes that the central bank knows the true model (1)–(6), it could compute the conditional expectation of z_t based on observed current and past inflation, interest rates and output growth. In order to keep the model simple, however, I abstract from those considerations and will stick to the specification (7) which has the advantage that the central bank reacts to observable variables only.

The monetary policy shock in (7) is allowed to be persistent as well,

$$\nu_t = \psi_\nu \nu_{t-1} + \epsilon_t^\nu. \tag{8}$$

This is motivated by the observation that the level of the short-term interest rate i_t is highly persistent,[9] and the persistence inherited from inflation and real activity is not sufficient to fully capture that: regressing i_t on π_t and Δy_t would generate residuals with strong remaining serial correlation. However, it is a priori not clear how to appropriately account for the high persistence. Setting ϕ_i in (7) equal to zero, all persistence would have to be captured by ψ_ν in (8), implying that it is monetary policy shocks themselves that are persistent. Constraining instead ν_t to be white noise, persistence would have to be attributed fully to interest-rate smoothing by the central bank. The question of how to 'distribute' persistence of i_t to interest-rate smoothing and policy shocks lies at the heart of the discussion about 'monetary policy gradualism'.[10] I try to be as agnostic as possible about it and let the data decide. It will turn out that both ψ_ν and ϕ_i can be estimated with satisfying precision.

[7] The main thing to note is that the current specification is sufficient to make sure that while sharing the common trend a_t, the NRI and potential output growth are not perfectly correlated with each other. The variance of ϵ^y determines the covariance of the two variables. Moreover, one can show that there is an observationally equivalent specification that allows the NRI to have an idiosyncratic component, while potential output growth features none.

[8] Here and in the following, the notion of a steady state refers to the situation in which all shocks are zero. Since the considered model is linear, the steady state of a variable coincides with its unconditional expectation.

[9] The first-order autocorrelation is about 0.97.

[10] See, e.g., Rudebusch (2002), Gerlach-Kristen (2004) or Rudebusch (2005).

As it stands, (7) implicitly assumes a constant inflation and growth objective as one may rewrite (7) as

$$i_t = \phi_i i_{t-1} + (1-\phi_i)[\tilde{c}_i + \phi_\pi(\pi_t - \pi^*) + \phi_g(\Delta y_t - (\Delta y)^*)] + \nu_t$$

where π^* and $(\Delta y)^*$ represent the inflation and output growth target. In principle, it is preferable to have both objectives to be time-varying. However, with the term structure application in view, this would require to formulate a complete law of motion of these time-varying objectives. Under the no-arbitrage condition, any long-term bond yield is a risk-adjusted expectation of the average of future short rates. Thus, in order to compute this expectation consistent with the model, the dynamics of the short rate have to be fully specified. Since these depend – via the monetary policy rule – on the inflation and the growth target, one would have to specify the dynamics of those as well. As in Hördahl et al. (2006) I have tried to model the inflation target as a (near-)random walk, which, however did not lead to satisfactory results.[11] Hence, I will stick to the rule (7) and (8) that abstracts from time-varying targets. That this might be a reasonable choice is confirmed by the residuals of the estimated policy rule that shows no signs of misspecification. However, I cannot rule out that time variation in the inflation objective – that I do not explicitly account for – is picked up by monetary policy shocks, which in turn drives up their estimated persistence.

The model is completed by stipulating that the five shocks are contemporaneously uncorrelated. Moreover, for pricing bonds and for estimating the model, it will be assumed that they are all normally distributed. Hence for the vector $\epsilon_t = (\epsilon_t^\pi, \epsilon_t^a, \epsilon_t^z, \epsilon_t^y, \epsilon_t^\nu)$,

$$\epsilon_t \sim N(0, Q), \quad \text{with} \quad Q = \mathrm{diag}(\sigma_\pi^2, \sigma_a^2, \sigma_z^2, \sigma_y^2, \sigma_\nu^2), \tag{9}$$

where the σ_i denote the standard deviations of the respective shocks, and diag$x$ denotes a square matrix with the vector x building the main diagonal and zeros elsewhere.

The structure of the system (1)–(8) allows for a convenient Markovian representation of the model, that will be useful when employing it below for pricing bonds. Define the 12×1-vector X_t as

$$X_t = (\pi_t, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, g_t, i_t, i_{t-1}, a_t, a_{t-1}, z_t, z_{t-1}, \nu_t)'$$

where here and in the following $g_t \equiv \Delta y_t$ for notational convenience. Then one can write (1)–(8) as

$$\mathcal{K}_0 X_t = c_0 + \mathcal{K}_1 X_{t-1} + R_0 \epsilon_t,$$

where $\mathcal{K}_0$ and $\mathcal{K}_1$ are 12×12, c is 12×1, and R_0 is 12×5. The matrix $\mathcal{K}_0$ is not diagonal, since the monetary policy rule implies contemporaneous relationships between the elements of X_t. However, the equation can be multiplied through by the inverse of $\mathcal{K}_0$ to obtain

$$X_t = c + \mathcal{K} X_{t-1} + R\epsilon_t, \tag{10}$$

with $\mathcal{K} = \mathcal{K}_0^{-1}\mathcal{K}_1$, $c = \mathcal{K}_0^{-1} c_0$ and $R = \mathcal{K}_0^{-1} R_0$.

[11] Maybe this could be attributed to the particular dynamics of inflation within the relatively short period since 1981, with a distinct downward trend at the beginning and a rather ‘flat’ evolution since about 1999, see Fig. 1.

2.2. Pricing long-term bonds

Taking the structural macroeconomic model, compactly represented by the SVAR(1) (10), as a basis, I will now derive arbitrage-free prices of nominal n-period bonds. Let P_t^n denote the time t price of a pure discount bond paying one unit of account at time $t+n$ with certainty. Then the family of bond price processes is arbitrage-free if and only if there exists a sequence of strictly positive random variables $\{M_t\}$ such that

$$P_t^n = E_t(M_{t+1} P_{t+1}^{n-1}), \tag{11}$$

for all t and n.[12] The random variable M_t is called the stochastic discount factor (SDF) or pricing kernel. Bond prices are related to yields y_t^n via

$$y_t^n = -\frac{1}{n} \ln P_t^n. \tag{12}$$

The joint macro-finance model will belong to the affine class of term structure models.[13] Discrete-time models from this family are characterized by four components: first, the short-term interest rate is an affine function of factors; second, the evolution of the factor vector is a linear autoregressive process; third, market prices of risk are affine functions of the factors; and fourth, there is a pricing kernel which is an exponentially-affine function of the short rate and ‘priced’ factor innovations.

Here, the factor vector is given by X_t and the short rate is a particularly simple transformation, namely

$$i_t = \delta' X_t, \tag{13}$$

where δ is a 12×1-vector with a one on the sixth position, that picks i_t from X_t, and zeros elsewhere. The factor process is given by (10) which is rewritten here slightly using a normalization of shock variances

$$X_t = c + \mathcal{K} X_{t-1} + \Sigma v_t, \quad v_t \sim N(0, I_5) \tag{14}$$

i.e., $\Sigma = RQ^{0.5}$, and I_5 denotes the 5×5-identity matrix.

The market price of risk vector λ_t is also an affine function of X_t,

$$\lambda_t = \lambda_0 + \lambda_1 X_t, \tag{15}$$

where λ_0 and λ_1 are a vector and a matrix of appropriate dimensions.

Finally, the pricing kernel is an exponential-affine function of the vector of factors and its innovations,

$$M_{t+1} = \exp(-0.5\lambda_t'\lambda_t - i_t - \lambda_t' v_{t+1}). \tag{16}$$

Solving (11) given the specified dynamics of the pricing kernel, leads to a solution function mapping the factor vector into bond prices,

$$P_t^n = \exp(\tilde{A}_n + \tilde{B}_n' X_t), \tag{17}$$

12 See Irle (1998) for a more rigorous statement and a proof of the equivalence.

13 Cf. Duffie and Kan (1996) and Dai and Singleton (2000). See Backus, Foresi, and Telmer (1998) for an introduction to the discrete-time version.

where $\tilde{A}_n$ and $\tilde{B}_n$ satisfy the difference equations[14]

$$\tilde{A}_{n+1} = \tilde{A}_n + \tilde{B}_n'(c - \Sigma\lambda_0) + \frac{1}{2}\tilde{B}_n'\Sigma\Sigma'\tilde{B}_n \tag{18}$$

$$\tilde{B}_{n+1}' = \tilde{B}_n'(\mathcal{K} - \Sigma\lambda_1) - \delta', \tag{19}$$

with initial condition $\tilde{A}_0 = 0$ and $\tilde{B}_0 = 0_{12\times 1}$.

The exponential-affine form for bond prices in (17) implies that continuously compounded yields are affine functions of the state vector X_t,

$$y_t^n = A_n + B_n' X_t \tag{20}$$

with $A_n = -\tilde{A}_n/n$ and $B_n = -\tilde{B}_n/n$. Note that this implies for the one-period interest rate y_t^1

$$y_t^1 = \delta' X_t = i_t \tag{21}$$

as expected.

3. Data and estimation approach

3.1. Macroeconomic and bond yield data

Since the beginning of stage three of European Monetary Union (EMU) in 1999, 30 quarters have elapsed until 2006Q2. Hence, estimating models for the euro area with quarterly data still requires compromises of some sort. One may either stick to a relatively short sample period by not taking too many data points before 1999 into account, or one has to rely on artificial euro area data. The approach chosen here will be a mixture of these two possibilities.

As macroeconomic data, I will employ inflation, output growth and the short-term interest rate. An empirical proxy for the output gap will not be used, instead z_t is kept as a latent variable in the model. The data are quarterly, cover the period 1981Q2–2006Q2 and come from the database of the Area Wide Model (AWM).[15] These are artificial euro area data that have by now been utilized in several empirical studies. The data set is updated until 2006Q2 by Bundesbank staff. Inflation, π_t, is 100 times the annualized quarter-to-quarter change of the seasonally adjusted log HICP, output growth Δy_t is 100 times the quarter-to-quarter change (not annualized) of seasonally adjusted log real GDP. The interest rate i_t is a monthly average of the 3-month money market rate.

By using artificial euro area data for the time before 1999, the resulting estimate of the monetary policy reaction function can be interpreted as an artificial monetary policy maker setting a joint 3-month rate for the whole artificial euro area before 1999. While fully recognizing the conceptual problems implicit in this procedure, we are in line with several other studies from the literature that choose a similar approach.[16]

For bond yields, one could likewise use artificial rates for the time before 1999. In fact, the Statistical Data Warehouse of the ECB provides such data for the euro area. However, using those would not really be consistent with the model set up. The artificial yields are weighted averages

[14] See, e.g., Ang and Piazzesi (2003).

[15] See Fagan, Henry, and Mestre (2001).

[16] See, e.g., Gerlach and Schnabel (2000), Peersman and Smets (2003), Gerlach-Kristen (2003), or Gerdesmeier and Roffia (2004).

of the euro area member country yields, thus, the postulated no-arbitrage relation is unlikely to hold between those yields. Consequently, yield data will only be employed as of 1998. From 1999 on, these are zero-coupon swap rates from Bloomberg with maturities of one, two, three, five, seven, and ten years. For the year 1998 for which these data had not been available, I use the corresponding yields for Germany. Alternatively to the strategy chosen here, one could have used German yields for a more extended period before 1999, which would have generated a less thin basis for estimating the parameters related to the term structure module. In fact, German yields may have played a benchmark role in the euro-area group of countries before 1999. However, by employing these yields for estimating the model one would implicitly presume that German bond yields were reacting to aggregate 'euro-area' state variables as opposed to German ones in the time before 1998. Hence, in order to avoid this inconsistency, I decided to use the shorter sample of yields.

All data are shown in Fig. 1. The different sample periods for macro- and yield-data can be adequately accounted for within the state space framework as explained in the following.

3.2. Estimation approach

In total, there are 26 free parameters to be quantified. Given the relatively short period of time, and the fact that bond yields enter as of 1998Q1 only, it is not feasible to estimate all parameters simultaneously. Hence, I will make use of a three-step approach that starts with a calibration of two intercepts and two parameter ratios. Second, I will estimate the parameters of the macro module, and finally – given the latter and the calibrated parameters – estimate the parameters corresponding to the term structure module.

3.2.1. Step 0: Calibration

First, I set $c_y = 0.49$ and $c_r = 2.71$, which corresponds to an (annualized) potential output growth of 1.96%, and a long-run natural real interest rate of 2.71%, respectively. These values have been obtained by estimating the macro-module with the interest rate specification switched off, they are also similar in magnitude to those obtained by Mesonnier and Renne (2006) for the sample until 2002Q4.[17] The remaining constants c_π and c_i cannot be chosen independently. Having calibrated c_r and c_y, I include the Phillips-curve constant c_π in the set of parameters to estimated. Assuming that the output gap is zero on average, $E(z_t) = 0$, Eqs. (1)–(6) fully determine the unconditional expectations of π_t, Δy_t, and i_t as functions of the parameters. Hence, by taking unconditional expectations of (7), the constant c_i results as a function of these steady-state values. Second, the variance of σ_a^2 is normalized to unity in order to achieve identification. Finally, the calibration of Mesonnier and Renne (2006) is used who fix the variance ratio $\sigma_y/\sigma_z = 0.5$ and the ratio $\theta_r/\theta_y = 16$.[18]

For the next steps I collect the remaining parameters in two vectors,

$$\psi_{\text{mac}} = (c_\pi, \alpha_1, \alpha_2, \alpha_3, \beta, \sigma_\pi, \psi_z, \gamma, \sigma_z, \psi_a, \theta_y, \phi_i, \phi_\pi, \phi_g, \sigma_v, \psi_v)'$$

containing the parameters of the macro module and

$$\psi_{ts} = (\lambda_{0,1}, \ldots, \lambda_{0,5}, h)'$$

[17] They obtain $c_y = 0.52$ and $c_r = 3.1$.

[18] See their paper for justifications of these values and robustness analyses.

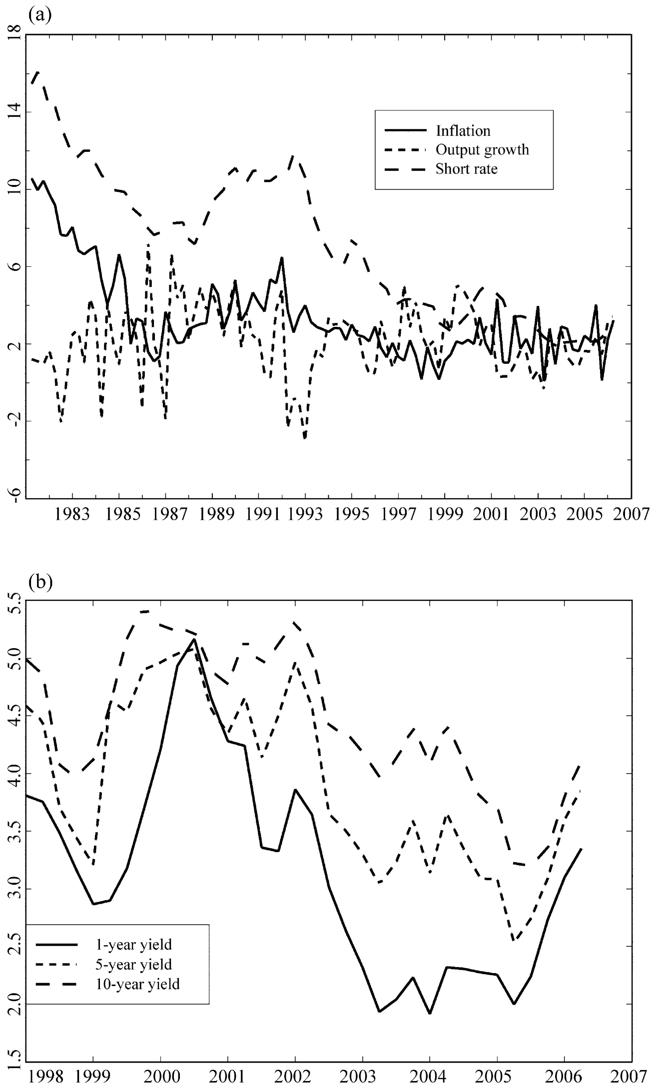

Fig. 1. Euro area data. (a) Inflation, output growth, short rate. (b) One-, five-, and ten-year yields. Inflation is 100 times the annualized quarter-to-quarter change of the seasonally adjusted log HICP. Output growth is 100 times the quarter-to-quarter change (here annualized) of seasonally adjusted log real GDP. The short rate is a monthly average of the 3-month money market rate. Yields are zero-coupon swap rates.

consisting of the market-price-of-risk parameters and a measurement-error variance that will be defined below.

Concerning the market-price-of-risk parameters, it is usually assumed that λ_1 in (15) is different from zero, i.e., some of the market prices – the components of λ_t – are in fact time-varying. However, since the time series of yields included in the estimation process is relatively short, it turned out that time-varying market prices of risk cannot be estimated with satisfactory precision. Allowing some of the elements in λ_1 to be different from zero improves the fit of bond yields for all maturities. In general, time-varying risk parameters would also imply more interesting impulse responses as term premia are also affected by changes in macroeconomic state variables. However, it turns out that estimating elements of λ_1 – based on the short sample of available yields – implies impulse response patterns which are not reliable. Thus, as Fendel (2004) and Cassola and Luis (2003), who use a much longer sample in their studies for Germany, I treat market prices of risk as constant. Of course, this implies that bond-yield dynamics will satisfy a version of the expectations hypothesis (with non-zero but constant term premia). It should be noted, though, that the more general no-arbitrage approach chosen here parameterizes term premia in a parsimonious way, which implies a consistent set of premia for all maturities, i.e., even for those that are not included in the data set used for estimation.

3.2.2. Step 1: Estimating ψ_{mac}

For estimating the macroeconomic parameters, ψ_{mac}, I construct the likelihood for the observed time series of inflation, output growth and the short-term interest rate. To this end, I construct the state space model capturing the dynamics of these variables.[19] The state vector is X_t, the transition equation is (10). The measurement vector is

$$Y_t^{\text{mac}} = (\pi_t, g_t, i_t)',$$

hence the measurement equation for $t = 1, \ldots, T$, where $T = 2006Q2$, is given by

$$Y_t^{\text{mac}} = Z_{\text{mac}} X_t, \tag{22}$$

where Z_{mac} is a 3×12 matrix that selects π_t, g_t and i_t from the state vector X_t. Note that the measurement equation contains no error term. The Kalman filter is used to construct the likelihood

$$\mathcal{L}^{\text{mac}}(\psi_{\text{mac}}) = p(Y_1^{\text{mac}}, \ldots, Y_T^{\text{mac}}; \psi_{\text{mac}})$$

which is then maximized to obtain $\hat{\psi}^{\text{mac}}$.

3.2.3. Step 2: Estimating ψ_{ts}

In this step, I take $\hat{\psi}_{\text{mac}}$ as given and estimate ψ_{ts}. This estimation utilizes observations of bond yields $y_t^{n_j}$ with maturities $(n_1, n_2, \ldots, n_6) = (4, 8, 12, 20, 28, 40)$, measured in quarters for the period $t = T^* + 1, \ldots, T$, $(T^* = 1997Q4)$. Bond yields are related to the state vector via (20). Stacking these relations, one obtains

$$\begin{pmatrix} y_t^{n_1} \\ \vdots \\ y_t^{n_6} \end{pmatrix} = \begin{pmatrix} A_{n_1} \\ \vdots \\ A_{n_6} \end{pmatrix} + \begin{pmatrix} B'_{n_1} \\ \vdots \\ B'_{n_6} \end{pmatrix} X_t. \tag{23}$$

The right-hand side contains the model solution, i.e., arbitrage-free yields. However, since the macroeconomic factors will not be able to price bonds of all maturities perfectly, a vector of measurement errors is added to the latter relation. Written in compact notation,

$$Y_t^{ts} = d_{ts} + Z_{ts} X_t + \xi_t, \tag{24}$$

[19] See Hamilton (1994) for state space models and the Kalman filter in general, and Lemke (2006) for estimating term structure models in a state space framework. Estimation and numerical computations have been conducted using GAUSS employing also its TSM and MAXLIK package.

i.e., d_{ts} contains the A_{n_i} and Z_{ts} takes the B_{n_i}. For the distribution of the vector ξ_t of measurement errors I choose the simple specification

$$\xi_t \sim N(0, h^2 I_6). \tag{25}$$

This is not an innocuous assumption since it implies that the difference between theoretical and observed yields has the same variance for all maturities. Alternatively, one may specify a different error variance for each maturity, which, however, would come at the cost of additional free parameters that would have to be estimated.

Thus, for $t = T^* + 1, \ldots, T$ the joint dynamics of macroeconomic variables and bond yields are described by the combined measurement equation

$$\begin{pmatrix} Y_t^{\mathrm{mac}} \\ Y_t^{ts} \end{pmatrix} = \begin{pmatrix} 0 \\ d_{ts} \end{pmatrix} + \begin{pmatrix} Z_{\mathrm{mac}} \\ Z_{ts} \end{pmatrix} X_t + \begin{pmatrix} 0 \\ \xi_t \end{pmatrix} \tag{26}$$

The measurement Eqs. (22) and (26) together with the transition Eq. (10) define a state space model in which the measurement vector changes its dimension: up to T^* it comprises only macro variables (dimension 3), from then on it contains both macro variables and bond yields (dimension 9). However, for this system, it is still straightforward to apply the Kalman filter and obtain the joint likelihood

$$\mathcal{L}(\psi_{\mathrm{mac}}, \psi_{ts}) = p(Y_1^{\mathrm{mac}}, \ldots, Y_{T^*}^{\mathrm{mac}}, Y_{T^*+1}, \ldots Y_T; \psi_{\mathrm{mac}}, \psi_{ts}) \tag{27}$$

where $Y_t = (Y_t^{\mathrm{mac}}, Y_t^{ts})$. The estimate of ψ_{ts} is obtained as

$$\hat{\psi}_{ts} = arg\max_{\psi_{ts}} \mathcal{L}(\hat{\psi}_{\mathrm{mac}}, \psi_{ts}) \tag{28}$$

where $\hat{\psi}^{\mathrm{mac}}$ is the estimate obtained from step 1.

One may wonder why the observations before $T^* + 1$ (no bond yields in that period) are needed for estimating the term structure parameters ψ_{ts}. This becomes clear if one considers the following factorization of the joint density:

$$\begin{aligned} & p(Y_1^{\mathrm{mac}}, \ldots, Y_{T^*}^{\mathrm{mac}}, Y_{T^*+1}, \ldots Y_T; \psi_{\mathrm{mac}}, \psi_{ts}) \\ & \quad = p(Y_1^{\mathrm{mac}}, \ldots, Y_{T^*}^{\mathrm{mac}}; \psi_{\mathrm{mac}}) \cdot p(Y_{T^*+1}, \ldots, Y_T | Y_1^{\mathrm{mac}}, \ldots, Y_{T^*}^{\mathrm{mac}}; \psi_{\mathrm{mac}}, \psi_{ts}) \end{aligned} \tag{29}$$

The first factor does in fact not depend on ψ_{ts} and will not affect the estimate of ψ_{ts}. The second factor depending on ψ_{ts}, however, is a conditional density which can only be computed correctly if the conditioning information, i.e., the evolution of Y_t^{mac} before T^* is properly taken into account.

The results of the second step are estimates of market prices of risk, $\lambda_{0,1} \ldots, \lambda_{0,5}$, and the standard deviation h of the measurement error ξ in (24). Estimating all five elements in λ_0 yielded insignificant estimates, a result that is common in the literature.[20] Thus, I only estimate the parameters corresponding to inflation (ϵ^{π}), trend-growth (ϵ^{a}), and monetary-policy (ϵ^{ν}) shocks, since these turn out to be the most relevant sources of variation in yields, as the variance decomposition in the next section will show.

[20] Ang and Piazzesi (2003) and Hördahl et al. (2006), for instance, use a heuristic iterative procedure to restrict some market-price-of-risk parameters to zero based on t-statistics.

Table 1
Parameter estimates

c_π	α_1	α_2	α_3	β	σ_π	ψ_z
0.627	0.309	0.119	0.269	0.177	1.037	0.872
(0.27)	(0.10)	(0.10)	(0.09)	(0.09)	(0.07)	(0.06)
γ	σ_z	ψ_a	σ_y	c_y	θ_y	c_r
−0.070	0.349	0.967	0.175	0.490	0.036	2.710
(0.03)	(0.03)	(0.03)	–	–	(0.016)	–
θ_r	c_i	ϕ_i	ϕ_π	ϕ_g	σ_v	ψ_v
0.580	1.670	0.931	1.020	2.036	0.455	0.333
–	–	(0.02)	(0.36)	(1.75)	(0.03)	(0.10)
$\lambda_{0,\pi}$	$\lambda_{0,a}$	$\lambda_{0,z}$	$\lambda_{0,y}$	$\lambda_{0,v}$	h	
−0.836	0.213	0.0	0.0	0.236	0.288	
(0.36)	(0.17)	–	–	(0.08)	(0.02)	

ML-estimates of parameters of the macroeconomic module (first three rows of parameters) based on sample 1981Q2–2006Q2, term structure parameters based on sample 1998Q1–2006Q2 (fourth row). Asymptotic standard errors in parentheses, based on inverse Hessian. Parameters without standard errors are calibrated or functions of other estimated parameters, see main text for details.

4. Results

4.1. Estimation results

The parameter estimates of the two-step estimation procedure are given in Table 1. Standard errors are based on the inverse Hessian of the likelihood. First of all, all of the estimates appear reasonable with respect to sign and size. For those parameters that have also been estimated by Mesonnier and Renne (2006), the results can be compared. However, one has to be aware of three differences between their estimation and the one conducted here: first, they assume that the α_i coefficients of lagged inflation in the Phillips curve (1) sum to one, while I estimate them without that restriction and add a constant to that equation. Second, they treat the short-term interest rate as exogenous, while here it is endogenized. Third, their sample is from 1979Q1 to 2002Q4, while the one considered here dates from 1981Q2 to 2006Q2.

The lag parameters of inflation sum to 0.7, thus the decision to relax the unit root assumption appears reasonable.[21] The autoregressive parameters of trend growth a_t and the output gap z_t are higher than in the study by MR. The estimates of the key transmission parameters β (impact of the output gap in the Phillips curve) and γ (impact of the real interest rate gap in the IS equation)[22] are very similar to those of Mesonnier and Renne in terms of size and estimation precision. This differs from the results by Hördahl et al. (2006) who find the respective parameters in their model to be insignificantly different from zero. However, they use monthly instead of quarterly data and the model mixes backward- and forward-looking elements, which prevents a direct comparison of the results.

[21] Also, all tests reject a unit root in inflation for the estimation period.
[22] The γ here corresponds to λ in MR.

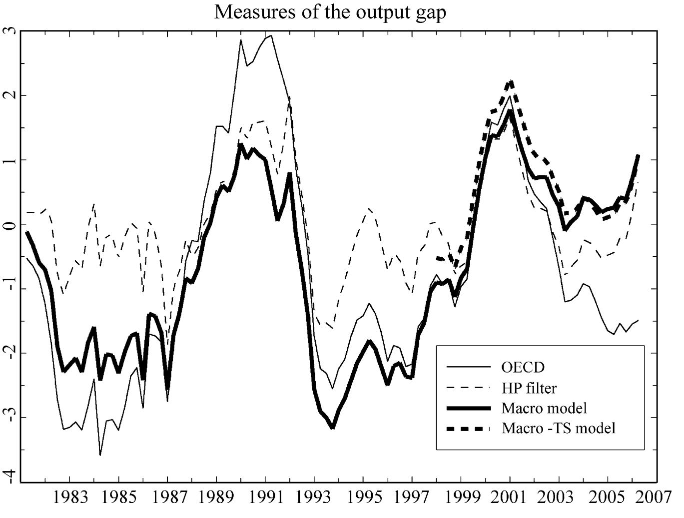

Fig. 2. Different measures of the output gap. 'Macro model' refers to the smoothed output gap, based on observations of inflation, output growth and the short rate only. 'Macro-TS model' refers to the smoothed output gap, when bond yields are included in the measurement vector as well (as of 1998).

The reaction parameter on inflation in the monetary policy rule is slightly exceeding unity and significant. The parameter governing the reaction to output growth is slightly greater than 2 (i.e., corresponding to about 0.5 for annualized growth) but is estimated fairly imprecisely. There is a distinct degree of interest rate smoothing indicated by an estimated ϕ_i of 0.93. It is also possible to estimate the persistence of monetary policy shocks quite precisely, finding the autoregressive parameter ψ_ν to be about 0.33. As a plausibility check we estimated the policy rule also as a single equation by nonlinear least squares, specifying the error to be an AR(1). This yielded very similar results, in terms of size and precision of the estimated parameters. Point estimates of ϕ_π and ϕ_g are 1.39 and 2.20, respectively, i.e., slightly higher than the system estimates. The autoregressive parameters ψ_ν and ϕ_i are estimated as 0.93 and 0.33, respectively.

For a further heuristic check of the plausibility of the estimates, Fig. 2 shows the Kalman-smoothed estimate of z_t, the model-implied output gap, together with an output gap measure resulting from HP-filtering and that provided by the OECD. As already mentioned, no proxy for the output gap has been used within the estimation process. Against this background, the estimated z_t process tracks the dynamics of the two empirical measures quite well. However, there are distinct differences in levels during certain episodes; but the OECD gap and the HP-implied gap – both widely used in empirical studies – also differ from each other significantly from time to time. While the solid bold line ('Macro model') is based on Kalman smoothing that only uses the state space model with the macroeconomic variables in the measurement equation, the dashed bold line ('Macro TS model') additionally uses term structure information from 1998 on. Compared to the pure-macro case, it implies a slightly higher gap most of the time. However, the dynamics of the estimated gap do hardly change. While one may have expected a priori that the latent factor z_t may change in a peculiar fashion in order to fit long-term bond yields, the results show that its estimated evolution is not very much affected by the inclusion of long-term interest rates in the measurement vector.

As to the term structure parameters, two of the three market-price-of-risk parameters that are estimated are significant. These parameters govern the size and maturity structure of risk premia.

For the short sample since 1998, yield risk premia turn out to be very small and even slightly negative at the short end of the maturity spectrum. For instance, for maturities of 1, 5 and 10 years, I obtain yield risk premia[23], of −8, −7, and 7 basis points respectively.[24] Risk premia of such a small magnitude raise the question whether bonds should be rather priced under the assumption of market prices of risk being equal to zero, i.e., $\lambda_1 = 0,\ \lambda_0 = 0$ in (15). Using this specification, however, would markedly deteriorate the fit of bond yields. Thus, for the following analyses, λ_0 is set as provided by the ML estimates in Table 1.

The standard deviation of the measurement error for bond yields is precisely estimated and amounts to about 29 basis points. This is comparable to the results of Hördahl et al. (2006), who allow for maturity-dependent measurement errors that exhibit standard deviations of between 23 and 28 basis points. As an additional measure of fit, Fig. 3 plots the actual yields versus model-implied yields $\hat{y}^n_{t|T}$ for selected maturities, where

$$\hat{y}^n_{t|T} = \hat{A}_n + \hat{B}'_n \hat{X}_{t|T}. \tag{30}$$

That is, A_n and B_n in (20) are replaced by their estimates (which are in turn based on the ML estimates of structural parameters) and $\hat{X}_{t|T}$ is the Kalman-smoothed estimated of the state vector.[25] It is worthwhile emphasizing that unlike, e.g., Fendel (2004) or Ang and Piazzesi (2003), the specification in this paper does not use additional latent ‘term structure factors’. Rather, bond prices are functions only of those variables that play a well-defined role within the macroeconomic model. Fig. 3 shows that the dynamics of the yields are traced quite well by the macroeconomic factors. However, the result for the maturity of one year, in particular, suggests that an additional term structure factor or a change in the specification of the macro-module may be required to improve the model’s fit.[26] The results of Fendel (2004) employing such a latent factor, however, show that there are also episodes of persistent deviations of model-implied yield from observed ones. Unfortunately, most other macro-finance papers on the term structure do not show comparable graphs. Finally, the model’s fit of bond yields has been compared to that implied by simple univariate AR(p) models and regression models with observable macro variables, their lags, and lags of bond yields as explanatory covariates. It turns out that for maturities around 1 year, the model’s fit is comparable to that of a univariate AR model, and the unrestricted regression model yields a better fit. For longer maturities, however, the model’s fit is markedly superior compared to both univariate AR and the considered regression models.

Fig. 4 shows the mean yield curve implied by the model (line) and the average of the corresponding yields from the data (circles). The model-implied mean yield curve is the average of the yields as computed in (30). The figure reveals that average yields are fitted well along the whole maturity spectrum.[27]

[23] These approximately correspond to the difference between actual bond yields and their hypothetical counterparts that would prevail under the pure expectations hypothesis. See the appendix in Hördahl et al. (2006) that shows how to compute forward premia and yield risk premia in affine models.

[24] Experimenting with time-varying market prices of risk showed that the ten-year premium fluctuates between −8 and 20 basis points.

[25] The smoothing sets those elements of the state vector which are observable – i.e., inflation, output growth, the interest rate and their lags – automatically equal to their observed values.

[26] This is also reflected in one-step-ahead forecast errors which show some remaining autocorrelation.

[27] Note that one advantage of the arbitrage-free approach to term structure modeling is the possibility to compute yields for any maturity, and not only for those maturities that have been included in the estimation process.

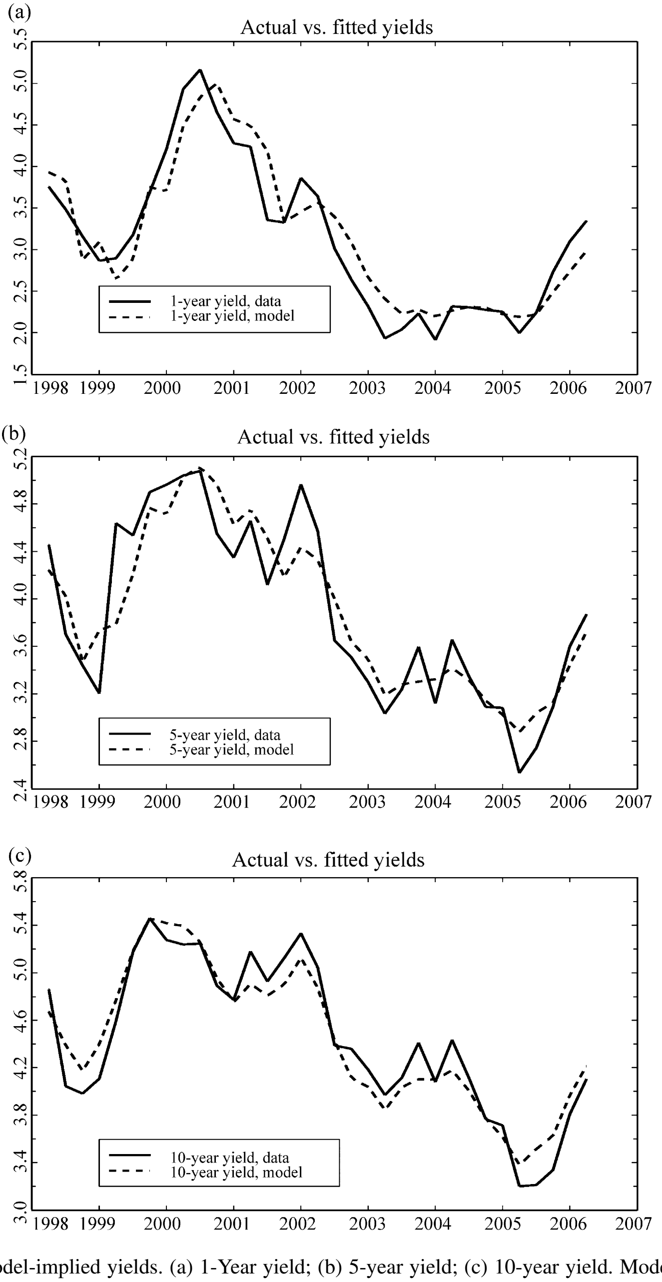

Fig. 3. Actual vs. model-implied yields. (a) 1-Year yield; (b) 5-year yield; (c) 10-year yield. Model-implied yields are based on smoothed states.

4.2. Impulse response analysis

The estimated macro-term-structure model can be used for various policy experiments. In the following I will show impulse responses of key macroeconomic variables and selected bond yields to the shocks of the model. Compared to a simple VAR analysis, the model framework considered

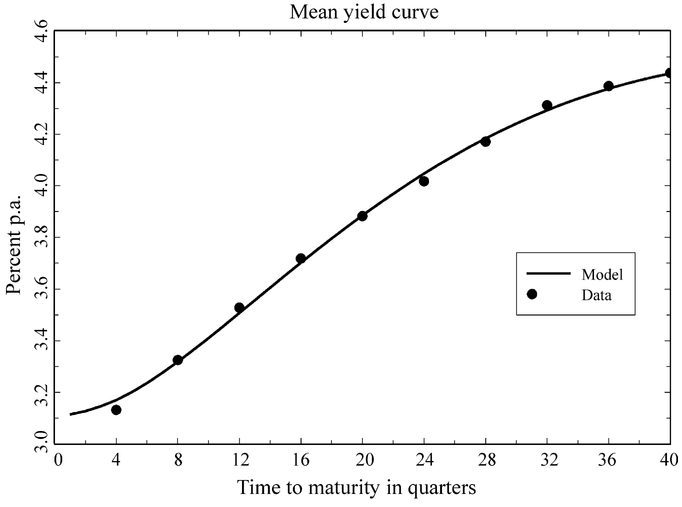

Fig. 4. Actual and model-implied mean yield curve. For each time to maturity, the solid circle represents the average of the corresponding yield over the period 1998Q1–2006Q2. The model counterpart is the average of the fitted yields (based on smoothed states).

here offers two advantages. First, since the equations of the macroeconomic module have a structural interpretation, the respective shocks are directly identified through the specification of the model. Moreover, unlike for an unconstrained VAR, the model allows to trace the impulse response to a latent variable, the trend rate of output growth. Second, unlike an unrestricted VAR, the model implies a consistent 'term structure of impulse responses' for bond yields: as the loadings of yields on macroeconomic factors are parsimoniously parameterized functions of the structural parameters, impulse responses can in principle be computed for yields of any maturity. Moreover, the impulse responses of bond yields obey the no-arbitrage condition.

Given these value-addeds of the macro-finance model with respect to a simple VAR, a comparison of impulse responses with the latter could still serve useful as a broad plausibility check of the results. However, such a comparison is hampered here by the different sample periods of macro and yield data. For instance, if one fits a simple VAR to inflation, a directly observable measure of the output gap and the short-term interest rates over the sample period that I use for estimating the macro module, the resulting impulse responses can be compared with that of the structural model used here.[28] For estimating a joint VAR of these macroeconomic variables and selected yields, however, one would have to rely on the shorter sample beginning in 1998. Using this short sample for estimating the complete VAR of macro variables and yields leads to fairly wide confidence bands, especially for the reaction of yields to macro variables. As another problem, the impulse responses of macro variables to macro variables differ markedly from those obtained over the longer sample. This, in turn, provides another argument against comparing the results from the combined model of this paper (long sample for macro variables, shorter sample for yields) to that of a simple VAR that is based solely on information from the short sample.

Before considering the results, it is useful to know that the estimated structural parameters constitute a $\mathcal{K}$ matrix in (10) that contains only stable roots, but some of them come in complex

[28] It turns out that they are broadly similar, with the exception of the monetary policy shock: it leads to a decline of the output gap and inflation in the model used here, but it generates an increase of these variables in the unrestricted VAR.

conjugate pairs. This implies the familiar result in the dynamic macroeconomics literature that some of the impulse responses will not take a direct way back to zero but will rather cross the zero line once before dying out.

I will consider responses to an inflation shock ϵ^{π}, a shock to the persistent component of potential output growth ϵ^{a}, an output gap shock ϵ^{z}, and a monetary policy shock ϵ^{ν}.[29] Shocks via ϵ^{y} will not be considered, since this idiosyncratic component of potential output growth does not have a very useful interpretation: as discussed above, it mainly serves to govern the strength of the comovement of the natural rate of interest (NRI) and potential output growth.

The size of all shocks will be one percentage point, which helps to facilitate the visual inspection of the different responses to a specific shock. However, for each figure, I supply the estimated standard deviation of the respective shock which is meant to give a hint on the 'typical' magnitude of that shock. The exception is ϵ^{a} which is set to 3.472 rather than to unity, which corresponds to a shock to annualized potential output growth of one half percentage point.[30] Moreover, a shock of ϵ_t^{a} that affects a_t in (5) will be synonymously referred to either as a 'shock to the persistent component of potential output growth' (or trend growth for short), see (4), or as a 'shock to the natural real rate of interest (NRI)', see (3).

In order to assess the significance of the impulse responses, I compute 90% confidence bands based on the following Monte-Carlo approach. Denote by $\hat{\psi}' = (\hat{\psi}'_{\text{mac}}, \hat{\psi}'_{ts})$ the vector of ML estimates of the parameters (see Section 3.2) and denote their estimated asymptotic variance-covariance matrix by $\hat{C}$. I draw 2000 realizations of the parameter vector from $N(\hat{\psi}, \hat{C})$, and for each of the draws the associated set of impulse responses is computed. From the 2000 sets of impulse responses the associated 90% confidence bands are obtained.[31]

In order to have a compact and clearly arranged representation of impulse responses, I abstain from adding the confidence bands themselves to the impulse responses in Figs. 5–8. I rather represent significance by using a black-filled symbol whenever the confidence band of a particular impulse response at a given horizon does not include zero, and non-filled symbols for insignificant impulse responses.

4.2.1. Cost-push shock

Starting with a shock to inflation, Fig. 5, this has the initial effect of raising current inflation π_0 but also expected inflation $\pi_{1|0}$ for the next period. Abstracting for a moment from changes in the policy rate i, this decreases the real interest rate in (2). Since the NRI r^* is not affected by the shock, this leads to a negative real rate gap and – as γ is negative – to an increase of the output gap in the next period. As potential output growth is unaffected, actual output growth changes one-to-one with changes in the output gap.[32] Thus, monetary policy will increase i as a response to both higher inflation and output growth. However, the interest rate response is subdued due to the strong interest rate smoothing. For the following periods, inflation will remain elevated due to its own persistence and due to positive impulses from the output gap which are themselves persistent. The latter feedback mechanism is also the reason for the lively responses of inflation in the first five quarters.

[29] To be precise, one would have to distinguish in terminology between the monetary policy shock ν_t in (7) and the shock ϵ^{ν} to that shock in (8).

[30] See Eqs. (5) and (4) above and note that θ_y is estimated as 0.036. Then $3.472 \cdot 0.036 \cdot 4 = 0.5$.

[31] Strictly speaking, for a given response and a given horizon, I construct the intervals, of which the lower and upper limit correspond to the 5- and 95-percentile of the bootstrapped impulse responses, respectively.

[32] From (6), $\Delta z_t = \Delta y_t - \Delta y_t^*$.

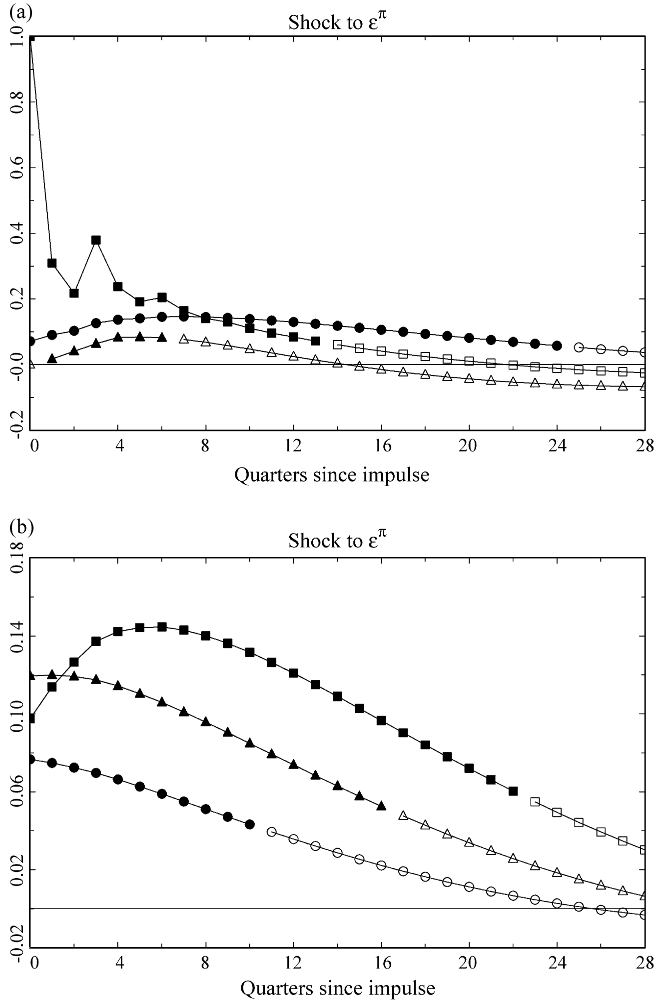

Fig. 5. Impulse response to an inflation shock. (a) Inflation (□), output gap (△), short rate (◯). (b) One- (□), five- (△), ten-year yield (◯). Response to a one-time shock to ϵ^{π} of 1%. (Stdd. dev of that shock is 1.04.) All responses in percentage points. Black-filled symbols denote significance at the 10% level.

For interpreting the responses of long-term interest rates, it is simplest to think in terms of the expectations hypothesis.[33] In general, the response to the inflation shock is smaller, the longer the time to maturity. However, since the one-year rate mirrors the hump-shaped response of the short rate while the longer-term rates do not, the one-year yield does not react the strongest on impact. Corresponding to the muted response of the short rate, the responses of long-term yields are also relatively small, the maximum of about 15 basis points is exhibited by the one-year rate

[33] This is a particularly good approximation in the case considered here as risk premia are estimated as time-invariant and small. However, impulse responses have to be interpreted with care as the time-constancy of term premia may be to some extent a consequence of the short estimation horizon.

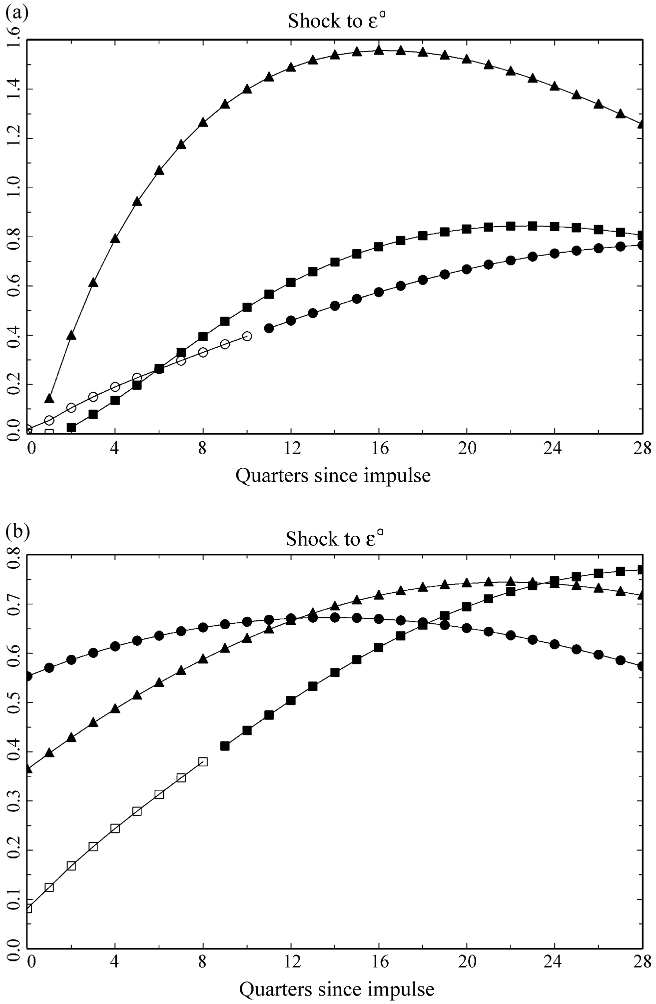

Fig. 6. Impulse response to a trend-growth-rate shock. (a) Inflation (□), output gap (△), short rate (◯). (b) One- (□), five- (△), ten-year yield (◯). Response to a one-time shock to ϵ^a of 3.472. (Stdd. dev of that shock is 1.00.) Note, the loading of a_t on potential output growth Δy_t^* is $\theta_y = 0.036$. Thus, the shock increases *annualized* potential output growth on impact by 0.5 percentage points.) All responses in percentage points. Black-filled symbols denote significance at the 10% level.

after six quarters. It is important to note that in this and the following scenarios, a response that increases short rates more than long rates does not necessarily imply an inverted yield curve in the respective period after the shock. Rather, the yield spreads implied by the impulse responses have to be interpreted as deviations from the average yield curve, which is – as Fig. 4 shows – upward-sloping.

4.2.2. Shock to trend growth

The shock to trend growth, Fig. 6, has a very persistent effect on the economy as ψ_a in (5) is estimated as 0.97. First of all, the shock increases actual output growth on impact by as much as potential output growth. Due to the lag structure of the model, the output gap does not react immediately. Moreover, the shock increases the NRI r_t^* and thus generates a negative real-interest-rate gap in the IS equation. This in turn raises the output gap in the next period, which then feeds through to inflation, providing in turn an additional stimulus to the output gap via inflation expectations. Due to both channels that have an impact on the real-rate gap – an elevated NRI that goes back to steady state very slowly and an increase in inflation expectations – there is a strong pressure driving the output gap upwards, which in turn fuels inflation further.[34] In order to counterbalance this process, monetary policy has to raise interest rates strongly. However, it is constrained by the high smoothing parameter in the policy rule. Thus, interest rates rise quite slowly but for a fairly prolonged time.

As the reaction coefficient ϕ_g is not estimated precisely, it may be asked in how far the latter result depends on the specification of monetary policy. Experimenting with a stronger monetary policy reaction function (results not shown), i.e., ceteris paribus increasing the reaction parameters ϕ_π or ϕ_g, or decreasing the smoothing coefficient ϕ_i in (7), leads to a weaker reaction of the output gap and inflation, which is due to a stronger narrowing of the real interest rate gap. Hence, the model mechanics do still imply that a persistent increase in potential output growth causes a boom, but this would be the less distinct, the stronger monetary policy reacts.

The considered shock on a_t in (5) will raise both potential output growth Δy_t^* and the natural rate of interest r_t^*. The observed behavior of he impulse response functions mainly stems from the effect on the natural real rate of interest and the described widening of the real rate gap. In fact, a shock to the component ϵ_t^y that provides a one-time impulse on potential output growth but not on the natural rate of interest would lead to a small and negative effect on the output gap and inflation.[35]

The slow but very persistent increase in the short rate is reflected in the reaction of longer-term yields. The lifetime of the one-year bond in period 0 only covers periods within which the short rate will not have been increased by much yet. For longer maturities, however, the expected high short rates in the future are incorporated in the bond yield. This implies that the initial effect of the shock increases with time to maturity. As time goes by, the yield spread becomes smaller, and the transition back to steady state will eventually be characterized by a yield-spread response which is negative.

4.2.3. Output-gap shock

As a response to an output gap shock, Fig. 7, actual output growth also increases, inducing the central bank to raise the short rate by $(1 - \phi_i) \cdot \phi_g$. Due to the relatively high ψ_z in the IS Eq. (2), the output gap is quite persistent and goes back to zero quite slowly. Simultaneously, an elevated output gap has its usual impact on inflation which gives again rise to an additional stimulus to the output gap via the real interest rate. In order to reduce inflation, the monetary authority increases the policy rate. However, following the prescribed rule (7), there is a counterbalancing effect resulting from actual output growth being slightly negative as the output gap goes back down to steady state.

[34] It should be noted that impulse responses of inflation would not be significant at the 5% level.

[35] As already noted, I do not explore effects of ϵ_t^y more deeply as these shocks turn out to be of minor importance, quantitatively. In a variance decomposition of bond yields, variation stemming from ϵ_t^y contributes less than 0.1%. See footnote for a discussion of the role of ϵ_t^y in the model.

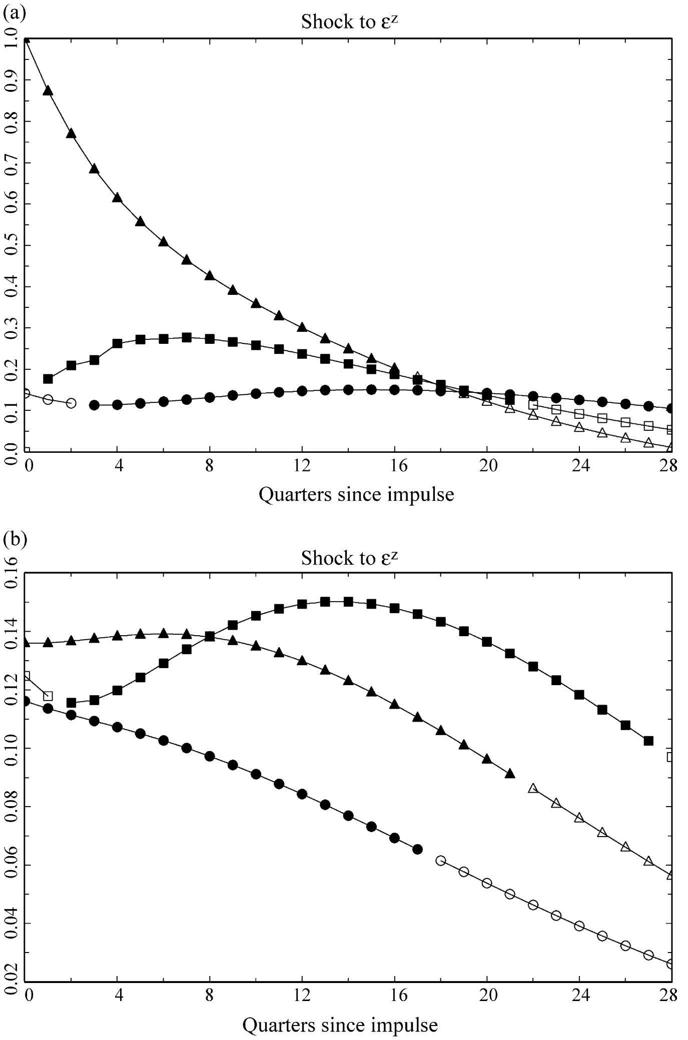

Fig. 7. Impulse response to an output-gap shock. (a) Inflation (□), output gap (△), short rate (◯). (b) One- (□), five- (△), ten-year yield (◯). Response to a one-time shock to ϵ^z of 1%. (Note: stdd. dev of that shock is 0.35.) All responses in percentage points. Black-filled symbols denote significance at the 10% level.

The response of interest rates is similar to the inflation-shock case. Again, the relative magnitude of the response is quite small.[36] The 'S-shaped' movement in the one-year rate reflects the slight 'S-shaped' response of the short rate (which is just less clearly visible due to the different scaling).

4.2.4. Monetary-policy shock

Finally, consider the effects of a contractionary monetary policy shock in Fig. 8. The output gap decreases via the real-rate channel, inflation only reacts in the second period after the shock

[36] At the 5% level, the impulse responses of five- and ten-year yields would only be significant within the first year after the impulse, the one-year-yield response only for quarters 3–7 after the shock.

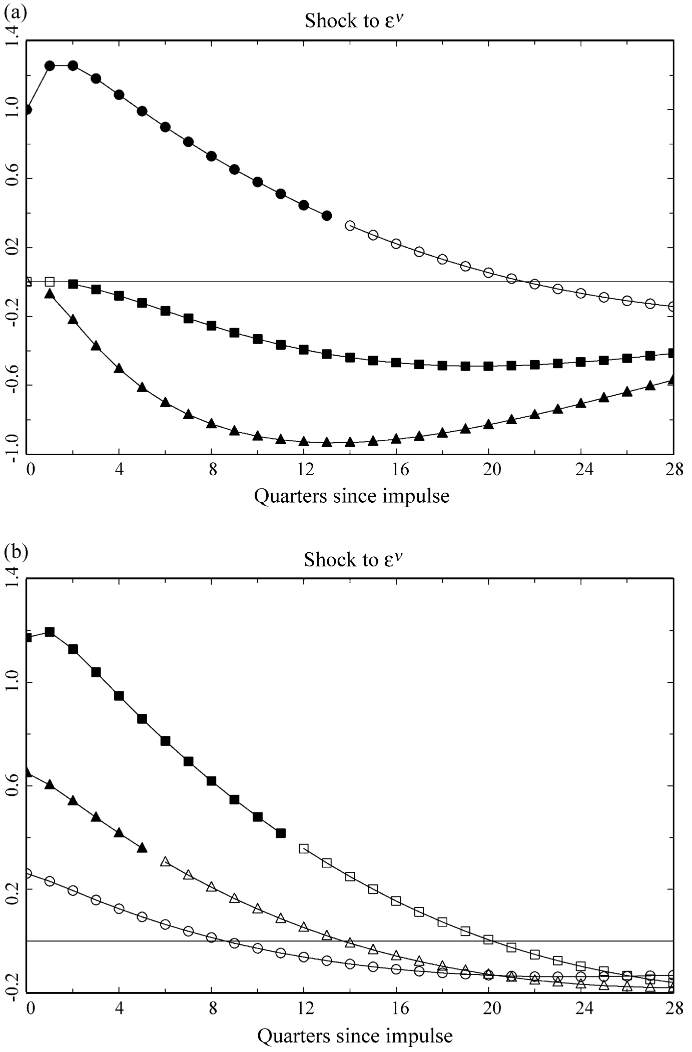

Fig. 8. Impulse response to a monetary-policy shock. (a) Inflation (□), output gap (△), short rate (◯). (b) One- (□), five- (△), ten-year yield (◯). Response to a one-time shock to ϵ^{ν} of 1%. (Stdd. dev of that shock is 0.46.) All responses in percentage points. Black-filled symbols denote significance at the 10% level.

due to its reaction to the negative output gap. The fact that the interest rate increases further for two periods after the shock can be explained as follows. First, inflation has not yet reacted and does not call for an interest rate reduction. The output gap has decreased implying a decrease in actual output growth, in turn requiring a decrease in the interest rate. However, this effect on the interest rate is very small. The important impact on the short rate comes via the smoothing channel combined with the persistence of the policy shock itself: the value of slightly more than 1.2 percentage points observed for the first period after the shock is the sum of ϕ_i and ψ_ν showing up in (7) and (8), respectively.

Table 2
Forecast-error-variance decomposition

Horizon (quarters)	4	10	20	30	40	∞
1-Year yield						
Inflation	5.4	9.3	11.9	10.8	9.2	7.9
Trend growth	0.7	3.1	12.8	25.2	33.9	42.5
Output gap	0.6	1.0	1.7	2.0	1.8	1.6
Monetary policy	93.2	86.5	73.4	62.0	55.0	48.0
3-Year yield						
Inflation	11.1	14.8	14.9	11.8	9.7	8.3
Trend growth	4.2	11.1	28.9	43.1	50.4	56.8
Output gap	1.2	1.9	2.7	2.6	2.2	1.9
Monetary policy	83.4	72.1	53.4	42.5	37.6	33.0
7-Year yield						
Inflation	16.5	16.1	11.0	7.6	6.3	5.5
Trend growth	32.6	50.4	68.3	74.2	76.5	78.7
Output gap	3.3	3.7	3.3	2.5	2.1	1.8
Monetary policy	47.5	29.7	17.5	15.6	15.1	13.9
10-Year yield						
Inflation	13.0	10.6	6.5	4.6	3.9	3.5
Trend growth	62.2	75.4	83.3	84.9	85.5	86.6
Output gap	3.5	3.3	2.5	1.9	1.6	1.4
Monetary policy	21.4	10.6	7.6	8.6	8.9	8.4

The table entries show the proportions (in percent) of the h-period forecast error variances of the respective yield that can be attributed to cost-push shocks, ϵ_t^{π}, shocks to trend growth, ϵ_t^{a}, shocks to the output gap, ϵ_t^{z}, and monetary-policy shocks, ϵ_t^{ν}, respectively. Shocks to potential output growth that do not affect the natural real rate of interest, i.e., ϵ_t^{y} in (4), are negligible (variance proportion < 0.1 percent for all yields and horizons) and thus not shown.

After an immediate rise, long-term rates are monotonically decreasing. The impact of the shock is bigger for short-term than for long-term yields. The response of the ten-year yield is not significant at any horizon. For the first three quarters after the shock, the one-year yield exhibits an increase of more than one percentage point. Again, this is a direct consequence of the described temporary upward move of the short rate.

4.3. Forecast-error-variance decomposition

In order to explore the main driving forces of yields of different maturities I conduct a forecast-error-variance decomposition.[37] Table 2 shows results for yields of 1, 3, 7 and 10 years to maturity and for different forecast horizons.[38] Overall, it is monetary policy shocks, ϵ_t^{ν}, and shocks to the natural real rate of interest, ϵ_t^{a}, that account for the bulk of variation in bond yields of all maturities over any horizon.

Regarding unconditional variances, these two shocks together explain at least 90% of the variation for all yields considered. For a maturity of one-year, monetary policy contributes slightly

[37] See Appendix A for computational details.

[38] Just like impulse response functions, the forecast-error-variance decomposition is a function of the structural parameters of the model and can be computed for any time to maturity of interest, not only for those yields that have been utilized for estimation.

more to the overall variation, whereas for increasing time to maturity, the proportion explained by the real shock monotonically increases, reaching around 86% for the ten-year bond. The contribution of cost-push shocks, ϵ_t^{π}, attains its maximum (8.4%) for a time to maturity of 10 quarters[39] and then decreases in importance for longer-term bonds. The contribution of idiosyncratic shocks to the dynamic IS equation, ϵ_t^{z}, is small as it never exceeds 2%.

Comparing the contributions across different forecast horizons (i.e., reading the table from left to right), it turns out that for all yields monetary policy shocks – contributing a maximum of 93.2% for the one-year yield at the one-year horizon – decrease in importance with increasing horizon. In contrast, shocks to trend growth become more important the longer the forecast horizon. The horizons at which output-gap and inflation shock provide their highest contribution changes with time to maturity. For instance, for the one-year rate, inflation contributes most for the five-year horizon while for the seven-or ten-year yield, inflation is most important for one-quarter forecast errors.

Considering the results across yields (i.e., reading the table from top to bottom), the proportion explained by monetary policy shocks decreases with time to maturity, while trend-growth shocks become more relevant. This holds for all horizons. Inflation always provides its highest contribution somewhere in the middle of the maturity spectrum.

5. Summary and outlook

In this paper, a structural model has been presented that intends to capture the joint dynamics of key macroeconomic variables and the term structure of interest rates for the euro area. The macroeconomic module has been estimated using quarterly data from 1981 to 2006. Parameter estimates of the term structure module (market prices of risk and variance of the measurement error) have been based on bond yield observations from 1998 to 2006. Owing to this fairly thin data basis, the estimation approach has been a mixture of using calibration or zero restrictions for quantifying some of the parameters, and estimating the others. Although such mixed approaches are not unusual in both the literature estimating macroeconomic models like the one used here and the empirical term structure literature, some general caution is warranted in interpreting the quantitative results based on the relatively short sample that I have used.

Nevertheless, parameter estimates are reasonable and comparable to those obtained for similar models of the literature. The estimated dynamics of inflation, the output gap and trend growth exhibit considerable persistence. The Taylor-type monetary policy rule is characterized by strong interest rate smoothing and monetary policy shocks which are also serially correlated.

Contrary to the majority of the literature, I have not used additional abstract latent factors for improving the model's explanatory power. However, the macroeconomic state variables alone turn out to provide an adequate fit of bond yields for the period 1998–2006. Yield risk premia are estimated to be quite small (below 10 basis points). This result may be partly owed to the fact that I have assumed constant market prices of risk and also due to the fact that the average yield curve over the estimation period has been relatively flat. However, experimenting with time-varying risk parameters showed similar term premia on average.

The estimated model is well suited for policy analyses as it can trace out the effects of nominal and real macroeconomic shocks on both macroeconomic variables and the whole maturity spectrum of bond yields. The impulse responses of macroeconomic variables are reasonable. The

[39] Six basis points higher than for the 3-year yield shown in the table.

persistence of macroeconomic dynamics is mirrored in the reaction of bond yields to the macroeconomic impulses. Shocks to inflation, the output gap and the short rate affect short-term rates more than long-term yields. However, this ordering can be different in the first few periods after the shock. The response to a shock to the natural real rate of interest is different in nature. For the first three years after the shock, the response is the stronger the longer the time to maturity. Thereafter, the 'term structure of impulse responses' eventually becomes inverted before the impact of the shock dies out.

Across the whole maturity spectrum, around 90% of variation in yields is explained jointly by monetary policy shocks and shocks to the natural real rate of interest. Regarding the relative contributions of these two shocks, the longer the time to maturity the more is explained by variations in the natural real rate (equivalently by variations in the persistent component of potential output growth). Idiosyncratic inflation shocks explain at most 8%, while shocks to the output gap play an even less important role. However, it is important to keep in mind that these results refer to the theoretical forecast-error-variance decomposition implied by the model. If additional latent factors were introduced to increase the empirical fit, they would presumably account for some of the variation of yields that is now captured by the interpretable macroeconomic factors. Moreover, even with respect to the latter, some care has to be taken when interpreting the results: it cannot be fully ruled out that the interpretable – via their roles in the structural model – but nevertheless empirically unobservable variables 'monetary policy shock' and 'natural real rate of interest' capture some residual variation. This disclaimer, however, would apply to all macro-finance models from the literature that work with a comparable set up.

There is a number of possible modifications and extensions to the presented approach. First, experiments with estimating jointly the monetary policy rule and term structure parameters[40] show that this implies a better fit of the yield dynamics, a much larger reaction coefficient on output growth in the monetary policy rule and a somewhat higher degree of persistence of the policy shock. However, this comes at the cost of a deteriorating fit of the macro variables.

Second, it would be interesting to examine different specifications of the monetary policy rule. That might include rules that are forward-looking and rules that react to estimates of the output gap or the natural real rate of interest. Moreover, a time-varying inflation objective may be incorporated. Finally, given a standard objective function of monetary policy, the optimal interest rate rule within a certain class of reaction functions may be derived. All these variations may potentially lead to a better fit of the term structure and would also yield important insights about how the reactions of long-term bond rates depend on different characteristics of monetary policy behavior.

Third, in this paper I decided to explore the impact of macro variables on the yield curve without relying on additional latent state variables. However, in order to improve the fit of the yield dynamics and in particular for using the model for forecasting purposes, the model may be augmented by one or two abstract latent factors. As already mentioned, this would also allow to conduct variance decompositions as in Ang and Piazzesi (2003) quantifying which fraction of yields can be attributed to interpretable macroeconomic shocks and which is left for other sources left unexplained.

Fourth, just like the majority of the macro-finance models in the literature, the model presented here captures the unidirectional link from macroeconomic driving forces to the yield curve.

[40] That is, in the second stage of the estimation procedure, all other macro-parameters are fixed but the parameters in (7) are estimated jointly with the market prices of risk and the measurement error variance.

However, it is conceivable that there exists a feedback in the other direction, motivated for example by the presence of long-term interest rate in the IS curve.[41] But this would probably imply a serious complication when it comes to solving for arbitrage-free yields: as usual, the mapping from state variables to yields will depend on the state dynamics; in models with feedback, however, state dynamics itself will be affected by arbitrage-free yields. This will require a different solution method, a problem which will be picked up in another paper.

Fifth, and finally, it is likely that the euro-area yield curve is to some extent affected not only by euro-area macroeconomic factors but also by US fundamentals. For capturing such impacts in a no-arbitrage framework, one would have to specify two pricing kernels for the two countries, potentially allowing them to share common factors, as done in Backus, Foresi, and Telmer (2001) and Dewachter and Maes (2001). That approach would identify common as well as country-specific driving forces for the two term structures, and it would also imply a description of the dynamics of the exchange rate.

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Appendix A. Forecast-error-variance decomposition

Recall that the state process is given by (14),

$$X_t = c + \mathcal{K}X_{t-1} + \Sigma v_t, \qquad v_t \sim N(0, I_5)$$

and yields depend on states via (20),

$$y_t^n = A_n + B_n' X_t$$

Hence, the h-period-ahead forecast of future yields $\hat{y}_{t+h}^n$ is given by the conditional expectation

$$\hat{y}_{t+h}^n \equiv E_t y_{t+h}^n = A_n + B_n' E_t X_{t+h},$$

and for the forecast error one obtains

$$y_{t+h}^n - \hat{y}_{t+h}^n = B_n' \mathcal{K}^{h-1} \Sigma v_{t+1} + B_n' \mathcal{K}^{h-2} \Sigma v_{t+2} + \cdots + B_n' \mathcal{K}^1 \Sigma v_{t+h-1} + B_n' \mathcal{K}^0 \Sigma v_{t+h}.$$

Defining the 1×5 row vector $\psi_i^n \equiv B_n' \mathcal{K}^{h-i} \Sigma$, we get

$$\begin{aligned} y_{t+h}^n - \hat{y}_{t+h}^n &= \psi_1^n v_{t+1} + \cdots + \psi_h^n v_{t+h} \\ &= \psi_{1,1}^n v_{t+1}^1 + \cdots + \psi_{h,1}^n v_{t+h}^1 + \psi_{1,2}^n v_{t+1}^2 + \cdots + \psi_{h,2}^n v_{t+h}^2 + \cdots \\ &\quad + \psi_{1,5}^n v_{t+1}^5 + \cdots + \psi_{h,5}^n v_{t+h}^5 \end{aligned}$$

[41] See, e.g., Goodfriend (1998) and Svensson (1997).

where the scalars $\psi^n_{i,k}$ and v^k_{t+i} denote the k th elements of ψ^n_i and v_{t+i}, respectively. Since the different v^k_{t+i} are all pairwise uncorrelated, the total forecast-error variance is given by

$$FV(y^n_{t+h}) = \psi^n_1 \psi^{n'}_1 + \cdots + \psi^n_h \psi^{n'}_h$$

and the contribution to this variance which stems from the k th shock is

$$FV_k(y^n_{t+h}) = (\psi^n_{1,k})^2 + \cdots + (\psi^n_{h,k})^2.$$

Accordingly, the proportion of the h-period forecast-error variance attributable to the k th shock is given by the ratio $[FV_k(y^n_{t+h})]/[FV(y^n_{t+h})]$.

References

Amato, J. D. (2005). *The role of the natural rate of interest in monetary policy. Working paper 171*. BIS.

Ang, A., Dong, S., & Piazzesi, M. (2005). *No-arbitrage Taylor rules*. Mimeo.

Ang, A., & Piazzesi, M. (2003). A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. *Journal of Monetary Economics*, *50*, 745–787.

Babbs, S. H., & Nowman, K. B. (1998). An application of generalized vasicek term structure models to the UK Gilt-edged market: A Kalman filter analysis. *Applied Financial Economics*, *8*, 637–644.

Backus, D., Foresi, S., & Telmer, C. (1998). *Discrete-time models of bond pricing. Working paper 6736*. NBER.

Backus, D. K., Foresi, S., & Telmer, C. I. (2001). Affine term structure models and the forward premium anomaly. *Journal of Finance*, *56*, 279–304.

Bekaert, G., Cho, S., & Moreno, A. (2005). *New-Keynesian macroeconomics and the term structure. Working paper 11340*. NBER.

Cassola, N., & Luis, J. B. (2003). A two-factor model of the german term structure of interest rates. *Applied Financial Economics*, *13*, 783–806.

Cox, J. C., Ingersoll, J. E., & Ross, S. A. (1985). A theory of the term structure of interest rates. *Econometrica*, *53*, 385–407.

Dai, Q., & Singleton, K. J. (2000). Specification analysis of affine term structure models. *Journal of Finance*, *55*, 1943–1978.

de Jong, F. (2000). Time Series and cross-section information in affine term-structure models. *Journal of Business and Economic Statistics*, *18*, 300–314.

Dewachter, H., & Lyrio, M. (2006). Macro factors and the term structure of interest rates. *Journal of Money, Credit and Banking*, *38*, 119–140.

Dewachter, H., Lyrio, M., & Maes, K. (2006). A joint model for the term structure of interest rates and the macroeconomy. *Journal of Applied Econometrics*, *21*, 439–462.

Dewachter, H., & Maes, K. (2001). *An admissible affine model for joint term structure dynamics of interest rates. Working paper*. CES, Catholic University of Leuven.

Duan, J., & Simonato, J. (1999). Estimating and testing exponential-affine term structure models by Kalman filter. *Review of Quantitative Finance and Accounting*, *13*, 111–135.

Duffie, D., & Kan, R. (1996). A yield-factor model of interest rates. *Mathematical Finance*, *6*, 379–406.

Fagan, G., Henry, J., & Mestre, R. (2001). *An area-wide model (AWM) for the euro area. Working paper 42*. European Central Bank.

Fendel, R. (2004). *Towards a joint characterization of monetary policy and the dynamics of the term structure of interest rates. Discussion paper 24*. Deutsche Bundesbank.

Gerdesmeier, D., & Roffia, B. (2004). Empirical estimates of reaction functions for the euro area. *Swiss Journal of Economics and Statistics*, *140*, 37–66.

Gerlach, S., & Schnabel, G. (2000). The Taylor rule and interest rates in the EMU area. *Economics Letters*, *67*, 165–171.

Gerlach-Kristen, P. (2003). *Interest rate reaction functions and the Taylor rule in the euro area. Working paper 258*. European Central Bank.

Gerlach-Kristen, P. (2004). Interest-rate smoothing: Monetary policy inertia or unobserved variables. *Contributions to Macroeconomics*, 4.

Goodfriend, M. (1998). Using the term structure of interest rates for monetary policy. *Federal Reserve Bank of Richmond Economic Quarterly*, *84*, 13–30.

Hamilton, J. (1994). *Time series analysis*. Princeton, NJ: Princeton University Press.

Hördahl, P., & Tristani, O. (2007). *Inflation risk premia in the term structure of interest rates. Working paper 734*. European Central Bank.

Hördahl, P., Tristani, O., & Vestin, D. (2006). A joint econometric model of macroeconomic and term structure dynamics. *Journal of Econometrics*, *131*, 405–444.

Irle, A. (1998). *Finanzmathematik—Die Bewertung von Derivaten*. Teubner, Stuttgart.

Laubach, T., & Williams, J. C. (2003). Measuring the natural rate of interest. *Review of Economics and Statistics*, *85*, 1063–1070.

Lemke, W. (2006). *Term structure modeling and estimation in a state space framework*. Berlin: Springer.

Mesonnier, J. S., & Renne, J. P. (2006). A time-varying "Natural Rate" of interest for the euro area. *European Economic Review*, *51*, 1768–1784.

Peersman, G., & Smets, F. (2003). The monetary transmission mechanism in the euro area: More evidence from VAR analysis. In J. Angeloni, A. Kashyap, & B. Mojon (Eds.), *Monetary policy transmission in the euro area*. Cambridge University Press.

Rudebusch, G. D. (2002). Term structure evidence on interest rate smoothing and monetary policy inertia. *Journal of Monetary Economics*, *49*, 1161–1187.

Rudebusch, G. D. (2005). *Monetary policy inertia: Fact or fiction? Working paper 2005–19*. Federal Reserve Bank of San Francisco.

Rudebusch, G. D., & Svensson, L. E. O. (1999). Policy rules for inflation targeting. In J. B. Taylor (Ed.), *Monetary policy rules*. The University of Chicago Press.

Rudebusch, G. D., & Wu, T. (2004). *A Macro-finance model of the term structure, monetary policy, and the economy. Working paper*. Federal Reserve Bank of San Francisco.

Svensson, L. (1997). Inflation forecast targeting: Implementing and monitoring inflation targets. *European Economic Review*, *41*, 1111–1146.

Vasiček, O. A. (1977). An equilibrium characterisation of the term structure. *Journal of Financial Economics*, *5*, 177–188.

Wicksell, K. (1898). *Geldzins und Gueterpreise*. Gustav Fischer.

Woodford, M. (2003). *Interest and prices*. Princeton, NJ: Princeton University Press.