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## SPECIAL TOPICS IN BUSINESS ECONOMICS

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Second assignment

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# 1 Abstract

In the above assignment, we will use the Datasetworld.xlsx file, which is an Excel file containing various data points such as GDP per capita for each country, energy consumption, and others. Using the data available through the R tool, we will attempt to apply various methods such as Technical Efficiency under Constant Returns to Scale (CRS) and Variable Returns to Scale (VRS), Scale Efficiency, Malmquist Productivity Index and its Components, and Stochastic Production Function Estimation. As we understand, the overall framework of the entire assignment will revolve around assessing how efficiently countries utilize their resources to generate economic output. More specific the project falls within the domain of efficiency and productivity analysis in the field of applied economics.

## 2 The technical efficiency under CRS and VRS for each regions over the periods 2007-2014,the corresponding scale efficiency measure.

In this particular question, we are tasked with calculating the technical efficiency under Constant Returns to Scale (CRS) and Variable Returns to Scale (VRS). Technical efficiency refers to the relationship between inputs and outputs. We state that a position is technically efficient when the maximum output is achieved with a given set of inputs, that is when a firm operates at the boundaries of its production technology. In the context of efficiency analysis, Constant Returns to Scale (CRS) represent a theoretical limit indicating the highest efficiency in utilizing inputs for output generation, assuming that there is no change in efficiency with a variation in the scale of production. Conversely, Variable Returns to Scale (VRS) is a similar theoretical limit that considers the prospect that changes in the production scale(input) may result in increased or decreased efficiency(output). To calculate technical efficiency, we will use the non-parametric method, best known as Data Envelopment Analysis (DEA). This is a comparative method that identifies performance. In this process, each unit is compared with its best-performing neighbor, and thus, its performance is calculated in relation to others. Lastly, it should be noted that the DEA was conducted in an input-oriented manner, which means that inputs were used to evaluate the efficiency of each respective unit.

Table 1: East Asia & Pacific

NO.	DMU name	CRS efficiency	VRS efficiency	Scale efficiency
1	Australia	1	1	1
2	Cambodia	0.428	1	0.428
3	China	0.387	0.546	0.709
4	Egypt, Arab Rep.	0.397	0.663	0.599
5	Gabon	1	1	1
6	Honduras	0.523	1	0.523
7	Indonesia	0.426	0.691	0.617
8	Japan	1	1	1
9	Korea, Rep.	0.524	0.54	0.969
10	Malaysia	0.477	0.552	0.864
11	Mongolia	1	1	1
12	New Zealand	1	1	1
13	Philippines	0.56	1	0.560
14	Singapore	1	1	1
15	Tajikistan	0.611	1	0.611
16	Tanzania	0.203	0.795	0.255
17	Thailand	0.42	0.552	0.761
18	Tunisia	0.69	0.939	0.735
19	Vietnam	1	1	1
20	Average efficiency	0.666	0.857	

Table 2: Europe &amp; Central Asia

NO.	DMU name	CRS efficiency	VRS efficiency	Scale efficiency
1	Albania	0.502	1	0.502
2	Angola	0.334	0.664	0.502
3	Armenia	0.333	0.973	0.342
4	Austria	0.698	0.707	0.988
5	Azerbaijan	0.34	0.449	0.758
6	Belarus	0.172	0.297	0.580
7	Belgium	0.489	0.5	0.978
8	Bosnia and Herzegovina	1	1	1
9	Bulgaria	0.22	0.381	0.578
10	Croatia	0.503	0.653	0.770
11	Cyprus	1	1	1
12	Czech Republic	0.313	0.355	0.880
13	Denmark	1	1	1
14	Dominican Republic	0.517	0.663	0.779
15	Eritrea	0.277	1	0.277
16	Estonia	0.379	0.99	0.382
17	Ethiopia	1	1	1
18	Finland	0.512	0.519	0.987
19	France	0.496	0.511	0.970
20	Georgia	1	1	1
21	Germany	0.543	0.557	0.974
22	Ghana	0.43	0.812	0.530
23	Greece	0.653	0.693	0.943
24	Guatemala	0.35	0.562	0.622
25	Haiti	1	1	1
26	Hong Kong SAR, China	0.994	1	0.994
27	Hungary	0.36	0.434	0.830
28	Iceland	1	1	1
29	India	0.157	0.601	0.262
30	Ireland	1	1	1
31	Italy	0.656	0.696	0.943
32	Kazakhstan	0.181	0.226	0.802
33	Kyrgyz Republic	0.156	0.777	0.200
34	Latvia	0.525	0.829	0.633
35	Lithuania	0.505	0.649	0.777
36	Luxembourg	1	1	1
37	Macedonia, FYR	0.282	1	0.282
38	Moldova	0.178	1	0.178
39	Netherlands	0.509	0.518	0.983
40	Norway	1	1	1
41	Poland	0.268	0.307	0.873
42	Portugal	0.557	0.598	0.932
43	Romania	0.277	0.363	0.764
44	Russian Federation	0.119	0.144	0.824
45	Slovak Republic	0.41	0.488	0.841
46	Slovenia	0.525	0.642	0.817
47	Spain	0.574	0.619	0.926
48	Sweden	0.607	0.611	0.994
49	Switzerland	1	1	1
50	Turkmenistan	0.118	0.579	0.203
51	Ukraine	0.094	0.143	0.656
52	United Kingdom	0.606	0.657	0.921
53	Uzbekistan	0.054	0.194	0.277
54	Average efficiency	0.504	0.653	

We observe that in Table 1 and 2, we have included an additional metric, scale efficiency, which is used to assess how closely a decision-making unit operates at its most productive size. It takes values from 0 to 1. If it is below 1, it means that the decision-making unit utilizes its productive resources inefficiently relative to its size. If it equals 1, it indicates that it uses its productive resources optimally in relation to its size.

The above perspective highlighting the results, presentation in Tables 1 and 2 is derived from the following study published in The Asian Journal of Shipping and Logistics. In summary, the study aims to showcase the technical efficiency of ports in South Asia and the Middle East, providing insights for their management improvement. The study utilizes Data Envelopment Analysis (DEA) models for the respective units.

In this section, we compare and analyze technical efficiency, i.e., how effectively countries utilize their inputs Energy consumption, Gross fixed capital formation and Labor force to generate output, GDP per capita in a regional context. In Table 1, assuming Constant Returns to Scale (CRS), we observe that only 7 out of the 19 countries, or 37%, are deemed efficient ( $\theta = 1$ ). In contrast, in the adjacent column assuming Variable Returns to Scale (VRS), 11 out of the 19 countries, or 58%, are considered efficient ( $\theta = 1$ ). The higher percentage of efficiency under the assumption of Variable Returns to Scale (VRS) indicates that scale is a significant factor influencing the efficiency of countries. Specifically, under the VRS assumption, it may suggest that flexibility in production scale contributes to the overall improvement in efficiency. Similar conclusions can be drawn regarding technical efficiency in Table 2. Whereas under the assumption of Constant Returns to Scale (CRS), only 21% of countries are observed to be efficient, meaning that under the assumption of Variable Returns to Scale (VRS), the percentage of efficient countries increases to 30%. Countries with a value of 1 in both technical and scale efficiency can be considered the most effective. This is because they not only use their inputs efficiently to generate output but also operate at the most effective size. Countries with a value of 1 in both technical and scale efficiency, based on Table 1, include Australia, Gabon, Japan, Mongolia, New Zealand, Singapore, and Vietnam. According to Table 2, such countries are Bosnia and Herzegovina, Cyprus, Denmark, Ethiopia, Georgia, Haiti, Iceland, Ireland, Luxembourg, Norway, and Switzerland. Finally, the average efficiency under the assumption of CRS seems to be lower for both regions, for reasons previously mentioned. Additionally, on average, the efficiency of countries belonging to the East Asia & Pacific region is higher than that of countries in the Europe & Central Asia region, under both assumptions.

The observed differentiation in both regions among countries in Tables 1 and 2 may be attributed to energy consumption, where energy efficiency holds greater significance in developed countries compared to developing ones. Specifically, through various methodological approaches in the Energy Economics article, it was revealed that countries classified as developed have multiplicative effects on their productivity through energy efficiency. It emphasizes that technical efficiency is influenced by various factors, including energy development, depending on the level of economic development. The above results and conclusions should be approached with caution due to the issue of heterogeneity among countries.

It is worth noting that, for the sake of simplicity in the presentation, only two out of the seven regions were shown. The selection was based on the number of observations. The remaining tables can be found in the R tool for those who desire more detailed information regarding the differentiation in technical efficiency across regions. Additionally, one ob-

servation was removed due to technical difficulties in implementing the DEA. It should be acknowledged that this certainly influenced the results for the respective region. Specifically, one of the inputs from Zimbabwe was excluded.

### 3 The malmquist productivity index and its components (scale,technical and efficiency change) for the period 1990-2014.

Table 3: Decomposition of average annual changes,1990-2014.

Begin of Table			
Country	M	MLEFFCH	MTECH
Albania	1.042	1.041	1.009
Algeria	0.989	1.009	0.981
Angola	0.989	1.010	0.981
Argentina	0.974	0.994	0.981
Armenia	0.974	0.998	0.992
Australia	0.983	1.002	0.982
Austria	0.986	1.005	0.981
Azerbaijan	1.055	1.035	1.024
Bangladesh	0.966	0.986	0.981
Belarus	0.989	0.992	1.000
Belgium	0.988	1.007	0.981
Benin	1.014	1.060	0.995
Bolivia	1.010	1.019	1.012
Bosnia and Herzegovina		1.462	
Botswana	1.007	1.007	1.018
Brazil	0.984	1.002	0.983
Bulgaria	1.014	1.011	1.006
Cambodia	0.991	1.032	0.986
Cameroon	1.023	1.026	1.010
Canada	0.987	1.005	0.983
Chile	0.965	0.985	0.981
China	0.916	0.933	0.983
Colombia	0.979	0.999	0.981
Congo, Dem. Rep.	1.049	1.054	1.008
Congo, Rep.	1.037	1.055	1.006
Costa Rica	1.010	0.999	1.016
Cote d'Ivoire	1.022	1.375	0.965
Croatia	0.983	1.003	0.981
Cuba	0.999	1.002	1.003
Cyprus	1.001	1.007	1.000
Czech Republic	0.985	1.006	0.981
Denmark	0.989	1.009	0.981
Dominican Republic	0.975	0.986	0.990
Ecuador	0.987	1.006	0.982
Egypt, Arab Rep.	0.979	0.999	0.981



Continuation of Table 3			
Country	M	MLEFFCH	MTECH
El Salvador	0.996	1.009	1.009
Eritrea	0.994	1.174	0.965
Estonia	1.005	0.995	1.024
Ethiopia	0.971	1.187	0.950
Finland	0.988	1.008	0.981
France	0.991	1.008	0.983
Gabon	1.026	1.036	1.004
Georgia	1.012	1.431	0.962
Germany	0.987	1.004	0.983
Ghana	1.007	1.021	1.006
Greece	0.995	1.014	0.981
Guatemala	1.015	1.010	1.010
Haiti	1.009	1.528	0.965
Honduras	1.012	1.011	1.012
Hong Kong SAR, China	0.973	0.993	0.981
Hungary	0.993	1.014	0.981
Iceland		0.984	
India	0.955	0.974	0.982
Indonesia	0.968	0.987	0.982
Iran, Islamic Rep.	0.981	1.000	0.981
Iraq	1.058	1.414	0.965
Ireland	0.932	0.967	0.952
Israel	0.981	1.001	0.981
Italy	0.997	1.014	0.983
Jamaica	1.028	1.039	1.011
Japan	0.965	0.999	0.959
Jordan	1.016	1.007	1.012
Kazakhstan	0.978	0.998	0.981
Kenya	1.041	1.024	1.023
Korea, Rep.	0.959	0.977	0.982
Kuwait	1.004	1.026	0.978
Kyrgyz Republic	1.003	1.055	0.994
Latvia	1.148	1.124	1.029
Lebanon	0.998	1.000	0.999
Libya	1.055	1.286	0.954
Lithuania	0.992	0.981	1.015
Luxembourg	0.996	1.000	0.996
Macedonia, FYR	1.002	1.023	1.006
Malaysia	0.966	0.986	0.981
Malta	0.985	1.032	1.002
Mauritius	0.988	1.007	1.008
Mexico	0.990	1.009	0.982
Moldova	1.026	1.074	0.995

Continuation of Table 3			
Country	M	MLEFFCH	MTECH
Mongolia	1.039	1.095	0.986
Morocco	0.976	0.996	0.981
Mozambique	0.998	1.051	0.982
Namibia	1.024	1.039	1.004
Nepal	1.069	1.071	1.005
Netherlands	0.986	1.005	0.982
New Zealand	0.986	1.006	0.981
Nicaragua	0.997	1.032	0.995
Niger	1.024	1.054	1.004
Nigeria	0.976	0.996	0.981
Norway	0.984	1.004	0.981
Pakistan	0.983	1.004	0.981
Panama	1.038	1.012	1.027
Paraguay	0.997	1.015	1.001
Peru	0.969	0.989	0.981
Philippines	0.979	1.000	0.981
Poland	0.965	0.985	0.981
Portugal	0.990	1.009	0.981
Romania	0.979	1.000	0.981
Russian Federation	0.995	1.012	0.983
Saudi Arabia	0.986	1.004	0.983
Senegal	1.024	1.032	1.013
Singapore	0.966	0.986	0.981
Slovak Republic	0.967	0.986	0.981
Slovenia	0.989	1.004	0.985
South Africa	0.993	1.013	0.981
Spain	0.989	1.007	0.983
Sri Lanka	0.976	0.980	0.999
Sudan	1.029	1.002	1.033
Sweden	0.986	1.005	0.981
Switzerland	0.961	1.001	0.953
Tajikistan	0.980	1.079	0.971
Tanzania	1.018	1.007	1.015
Thailand	0.967	0.987	0.981
Togo	1.000	1.129	0.970
Trinidad and Tobago	0.980	1.004	0.995
Tunisia	1.000	0.992	1.011
Turkey	0.978	0.998	0.981
Turkmenistan	0.983	1.002	0.982
Ukraine	1.015	1.035	0.981
United Arab Emirates	1.026	1.044	0.983
United Kingdom	0.956	0.991	0.959
United States	1.005	1.013	0.993

Continuation of Table 3			
Country	M	MLEFFCH	MTECH
Uruguay	1.005	0.997	1.012
Uzbekistan	0.978	0.999	0.981
Venezuela, RB	0.996	1.016	0.981
Vietnam	0.951	1.160	0.954
Yemen, Rep.	1.001	1.247	0.945
Zambia	0.992	1.335	0.959
End of Table			

*Note: M: Malmquist Index, MEFFCH: Malmquist Efficiency Change, MTECH: Malmquist Technical Change.*

To measure the productivity performance of an economic unit over different time periods, we use the Malmquist Productivity Index. Essentially, this index aids in assessing changes in productivity across various time periods for a given economic unit. Generally, we know that the TFP index, defined as the ratio of the output to input for two different time periods ( $\text{Productivity} = \text{Output} / \text{Input}$ ), corresponds to the simplest cases of productivity calculation. The Productivity (Growth) Index corresponds to calculating the output to input ratio for two different time periods to evaluate productivity. The Malmquist Index not only measures changes in productivity but also takes into account variations in efficiency and technology. Analyzing the Malmquist Index with respect to its components provides a more detailed analysis of productivity dynamics.

In addition to the Malmquist index in Table 3, we also observe some of its components, which will be briefly analyzed below. It is worth noting that all the indices are output-oriented. The Malmquist Efficiency Change (MEFFCH) index refers to efficiency based on the output of the economic unit in relation to the use of resources. In other words, an increase in production using the same or fewer resources, or a reduction in the use of inputs for the same output, is considered an improvement in the overall efficiency of the economic unit. Finally, the Malmquist Technical Change (MTECH) index pertains to how an economic unit can generate more output with the same or fewer resources. This means that a reduction in resource use or an increase in production by adjusting technology is considered an improvement in efficiency.

The calculation method is mentioned below:

$$\begin{aligned}
\bullet \quad tc &= \sqrt{\frac{e_{10}}{e_{11}} \cdot \frac{e_{00}}{e_{01}}} \\
\bullet \quad ec &= \frac{e_{11}}{e_{00}} \\
\bullet \quad mq &= tc * ec
\end{aligned}$$

**tc** represents the technology change index, **ec** represents the efficiency change index, and **mq** represents the Malmquist index. The efficiencies are denoted as follows:  $e_{00}$  is the efficiencies for period 0 with reference technology from period 0.  $e_{10}$  is the efficiencies for period 1 with reference technology from period 0.  $e_{11}$  is the efficiencies for period 1 with

reference technology from period 1. Finally,  $e_{01}$  is the efficiencies for period 0 with reference technology from period 1.

It is worth noting that, before delving into the commentary on the table of the above results, the R tool was used through the package `benchmarking(malmq)` for the analysis. We were not able to obtain values for all the indices either due to error-solving reasons or because a value or a formula led to a numerical operation that the computer could not perform.

Observing Table 3, Latvia appears to experience the highest growth in the Total Factor Productivity (TFP) index with an annual rate of 14.8%, followed by Nepal with an annual rate of 6.9%. In both cases, this increase is mainly attributed to the improvement in the efficiency and performance of the economic units, rather than changes in the technology they use. In contrast, Ireland experienced the largest decline in the Total Factor Productivity (TFP) Index with an annual rate of decrease equal to 6.8%, followed by Vietnam. This decrease is mainly attributed to technological change. This means that the reduction in overall productivity is not primarily due to efficiency or performance issues but rather to changes in the technology used by the economic units in these two countries.

It is observed that several developing countries have a Malmquist index above 1, and some even have a measurable difference from 1. In contrast, developed countries, few of them have an index above 1 without a significant deviation from unity. Based on the findings of this article (Environmentally sensitive productivity growth A global analysis using Malmquist–Luenberger index). This could be attributed either to the convergence hypothesis or be related to environmental sensitivity, where in developed countries, it tends to be higher (based on GDP). During the existence of convergence, economies starting with a lower GDP per capita experience a higher rate of increase in their productivity compared to countries that begin with a higher GDP per capita.

4 Estimate a stochastic production function implementing a Cobb-Douglas and a Translog production function for all countries in our sample. Which of the abovementioned production functions estimate best the relationship between GDP per capita (constant 2010 US\$) and Energy consumption (kg of oil equivalent per capita), Gross fixed capital formation (constant 2010 US\$) and Labor force, total. How can you estimate the effect on countries' inefficiency of CO2 emissions (metric tons per capita) and Renewable energy consumption (% of total final energy consumption).

**Cobb-Douglas:** The Cobb-Douglas function is a production function commonly used in the context of businesses to describe how production depends on inputs. Specifically, the Cobb-Douglas function is a tool for calculating production and how production factors combine to produce goods or services.

**Translog:** The Translog production function is a more general form of a production function compared to the Cobb-Douglas.

**Comparison** The Translog function is more complex and can capture non-linear relationships compared to a Cobb-Douglas. Specifically, it allows for the existence of non-linear relationships among production factors. It has a more flexible functional form and makes fewer assumptions about production elasticities compared to Cobb-Douglas.

## 5 References

Energy Economics

The Asian Journal of Shipping and Logistics

Environmentally sensitive productivity growth: A global analysis using Malmquist–Luenberger index