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This paper presents an essentially affine model of the term structure of interest rates making use of macroeconomic factors and their long-run expectations. The model extends the approach pioneered by Kozicki and Tinsley (2001) by modeling *consistently* long-run inflation expectations *simultaneously* with the term structure. Application to the U.S. economy shows the importance of long-run inflation expectations in the modeling of long-term bond yields. The paper also provides a macroeconomic interpretation for the latent factors found in standard finance models of the yield curve: the level factor represents the long-run inflation expectation of agents; the slope factor captures business cycle conditions; and the curvature factor expresses a clear independent monetary policy factor.

JEL codes: E43, E44, E52
Keywords: essentially affine term structure model, macroeconomic factors, long-run market expectation, monetary policy rule.

THE MODELING OF THE term structure dynamics is evolving in a clear direction. It is no longer sufficient to capture the movements in the yield curve within a no-arbitrage framework based on unobservable or latent factors, as it is by now standard in the finance literature. Term structure models should also be able to identify the economic forces behind these movements. In this context,

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one of the remaining challenges lies in finding a proper economic explanation for the variation in long-term bond yields (see, for instance, Gürkaynak, Sack, and Swanson 2003). We address this issue by making use of an affine term structure model based solely on factors with a well-defined macroeconomic interpretation. We solve the empirical misfit of long-term yields with the inclusion of filtered long-run inflation expectations. We also provide a macroeconomic interpretation for the latent factors found in standard finance models.

The framework for (latent) affine term structure models is set out in Duffie and Kan (1996) and summarized in Dai and Singleton (2000). The latent factors derived from this line of research have no direct economic meaning and are usually labeled according to their effect on the yield curve, i.e. as a level, a slope, and a curvature factor. Although this framework has been extended with the inclusion of observable macroeconomic variables, there still seems to be a missing factor necessary to fit the long end of the term structure. For instance, Ang and Piazzesi (2003) find that even though macroeconomic factors clearly affect the short end of the yield curve, its long end is not accounted for. They close their model with the inclusion of a latent factor for this purpose.[1] Kozicki and Tinsley (2001, 2002) suggest that the missing factor may be related to the long-run inflation expectation of agents (endpoints). This finding clearly paves the way for a full macroeconomic interpretation of the term structure dynamics.

The model proposed in this paper extends the approach pioneered by Kozicki and Tinsley (2001, 2002) by modeling *consistently* long-run inflation expectations *simultaneously* with the term structure. The method improves on the approach taken in the literature to use long-run expectations of macroeconomic variables in order to fit the yield curve. Currently, a two-step approach is used where long-run expectations are first filtered from the data using some statistical procedure, and then subsequently used to fit the term structure. A drawback of this method is that not all available information is used to filter the long-run expectations since only a subset of the data series is used. Another disadvantage is that these filtered expectations are not necessarily consistent with the notion of the expected long-run value. A variable representing long-run expectations should follow a (possibly degenerate) martingale model under the empirical probability measure.

Our approach is related to other papers. In line with Ang and Piazzesi (2003), Bekaert, Cho, and Moreno (2003), Hördahl, Tristani, and Vestin (2003), Kozicki and Tinsley (2001, 2002), and Rudebusch and Wu (2003), we focus on explaining the term structure by means of well-defined macroeconomic variables. Our paper, however, deviates from the approach taken by Hördahl, Tristani, and Vestin (2003) and Rudebusch and Wu (2003) in that we impose the martingale assumption on the latent macroeconomic variables, which allows us to interpret these factors as long-run (limiting) expectations of observable macroeconomic factors. This martingale

1. Dewachter, Lyrio, and Maes (2001), using Ang and Piazzesi's framework, also show that the misfit of the long end of the term structure can be quite substantial. Large and highly persistent pricing errors (up to 6% p.a.) are found and clearly point to the existence of a possible additional factor.

assumption is in accordance with the models of Kozicki and Tinsley (2001, 2002). Unlike these authors, however, we use a one-step filtering approach by modeling simultaneously the macroeconomic and term structure dynamics within an essentially affine term structure model.

We apply the proposed model to the U.S. economy and show the importance of long-run inflation expectations in the modeling of long-term bonds. We also relate the set of macroeconomic variables estimated in our model to the latent factors typically found in the finance literature. We find that the level effect can be linked to long-run inflation expectations, that the slope factor correlates well with predictable inflation and business cycle components, and that the curvature effect is related to the current stance of monetary policy, i.e. to real interest rate movements not related to standard macroeconomic conditions.

The remainder of the paper is organized as follows. Section 1 presents a continuous-time model for the macroeconomy and the term structure. Section 2 implements the model by estimating the macroeconomic dynamics and filtering the long-run expectations by means of the Kalman filter. The implied interest rate policy rule is also discussed. Section 3 analyses the influence of each macroeconomic state variable on the yield curve and relates these variables to those of a latent factor Vasicek model. Section 4 contrasts the filtered long-run expectations with survey expectations and with the expectations obtained from a structural break analysis. Section 5 concludes.

1. AN AFFINE TERM STRUCTURE MODEL WITH MACRO FACTORS

We present a continuous-time model of the term structure of interest rates that incorporates both macroeconomic factors and their long-run expectations. We first set out the model under the empirical probability measure and introduce the definition of long-run expectation of the macroeconomic variables. The basis is a non-stationary model for the output gap, inflation, and the real interest rate. Next, we model the dynamics under the risk-neutral measure and derive the implications for the term structure of interest rates.

1.1 Incorporating Long-Run Macroeconomic Expectations

The difficulty of the purely macroeconomic approach to model the long end of the yield curve suggests some misspecification in the modeling of the long-run expectation of the short-run interest rate process. As shown by Kozicki and Tinsley (2001, 2002), this misspecification can be attributed to the absence of time-varying attractors (endpoints) for the short-run interest rate. In this paper, we construct such attractors from the macroeconomic and term structure dynamics. We start by assuming the following dynamics for the output gap, $y(t)$, inflation, $\pi(t)$, and the instantaneous real interest rate, $\rho(t)$:

$$dy(t) = [\kappa_{yy}y(t) + \kappa_{y\pi}(\pi(t) - \pi^*(t)) + \kappa_{y\rho}(\rho(t) - \rho^*(t))dt + \sigma_y\, dW_y(t)\,, \quad (1)$$

$$d\pi(t) = [\kappa_{\pi y}y(t) + \kappa_{\pi\pi}(\pi(t) - \pi^*(t)) + \kappa_{\pi\rho}(\rho(t) - \rho^*(t))]dt + \sigma_\pi\, dW_\pi(t)\,, \quad (2)$$

$$d\rho(t) = [\kappa_{\rho y}y(t) + \kappa_{\rho\pi}(\pi(t) - \pi^*(t)) + \kappa_{\rho\rho}(\rho(t) - \rho^*(t))]dt + \sigma_\rho\, dW_\rho(t)\,, \quad (3)$$

where $W_j(t)$, $j = \{y, \pi, \rho\}$, denote independent standard Brownian motions defined on the probability space (Ω, F, P) with filtration F_t and time set $[0, T]$, $0 \leq T < \infty$.[2] The variables $\pi^*(t)$ and $\rho^*(t)$ can be interpreted as long-run macroeconomic attractors (central tendencies) if two conditions are satisfied: (1) the market does not expect any change in these variables ($E_t\, d\pi^*(t) = 0$ and $E_t\, d\rho^*(t) = 0$); and (2) the macroeconomic variables y, π, and ρ converge to their respective central tendencies. The central tendency of the output gap (y) is, by construction, equal to zero. Formally, we introduce the interpretation of $\pi^*(t)$ and $\rho^*(t)$ into our model by Condition (1) assuming a standard Brownian motion for the stochastic trend $\pi^*(t)$:

$$d\pi^*(t) = \sigma_{\pi*}\, dW_{\pi*}(t)\,, \quad (4)$$

where $W_{\pi*}(t)$ denotes an independent standard Brownian motion and Condition (2) by imposing negative eigenvalues on the System (1)–(3). The latter condition amounts to restricting this system to a continuous-time vector error correction model (VECM) system with cointegrating relations of $\pi(t)$ with $\pi^*(t)$ and $\rho(t)$ with $\rho^*(t)$, where

$$\rho^*(\mathrm{t}) = \gamma_0 + \gamma_{\pi*}\pi^*(t)\,. \quad (5)$$

Imposing stability of the System (1)–(3) in terms of deviations from these central tendencies, we ensure that they act as long-run attractors in this system:

$$\lim_{s\to\infty} E_t(y(s)) = 0\,,$$

$$\lim_{s\to\infty} E_t(\pi(s) \,|\, \pi^*(t)) = \pi^*(t)\,,$$

$$\lim_{s\to\infty} E_t(\rho(s) \,|\, \rho^*(t)) = \rho^*(t)\,,$$

where E_t denotes the time t expectation under the empirical probability measure.

The dynamics of the above system conform well to the standard macroeconomic view. Each of the observable economic variables (output gap and inflation) is affected by three channels: the instantaneous real interest rate (ρ), the other economic variable (output gap or inflation), and a mean reverting component modeling the possible inertia in the adjustment process. Following the recent literature, we assume that the central

2. The adoption of a Gaussian (Vasicek 1977) type of model reflects our intention to offer complete flexibility with respect to the magnitudes and sizes of the conditional and unconditional correlations among the factors. This specification fulfills the admissibility conditions specified in Dai and Singleton (2000). The costs associated with this choice are twofold: (1) the lack of flexibility in fitting the interest rate volatility, since we assume constant conditional variances for the factors; and (2) the possibility of negative interest rates.

bank uses a linear policy rule for the real interest rate according to Equation (3). The model is closed with the following definition for the instantaneous interest rate $i(t)$:

$$i(t) \equiv \pi(t) + \rho(t)\,. \tag{6}$$

In matrix notation, the dynamics of the System (1)–(5) can be rewritten as:

$$df(t) = (\psi + Kf(t))dt + S\,dW(t)\,, \tag{7}$$

where $f(t)$ and $dW(t)$ are n-dimensional vectors of factors and shocks given by

$$f(t) \equiv \begin{pmatrix} y(t) \\ \pi(t) \\ \rho(t) \\ \pi^*(t) \end{pmatrix}, \quad dW \equiv \begin{pmatrix} dW_y(t) \\ dW_\pi(t) \\ dW_\rho(t) \\ dW_{\pi^*}(t) \end{pmatrix}.$$

The parameters to be estimated are present in the n-dimensional vector of constants ψ and in the $n \times n$ matrices K and S:

$$\psi = (-\kappa_{y\rho}\gamma_0, -\kappa_{\pi\rho}\gamma_0, -\kappa_{\rho\rho}\gamma_0, 0)',$$

$$\mathrm{K} = \begin{pmatrix} \kappa_{yy} & \kappa_{y\pi} & \kappa_{y\rho} & -\kappa_{y\pi} - \kappa_{y\rho}\gamma_{\pi^*} \\ \kappa_{\pi y} & \kappa_{\pi\pi} & \kappa_{\pi\rho} & -\kappa_{\pi\pi} - \kappa_{\pi\rho}\gamma_{\pi^*} \\ \kappa_{\rho y} & \kappa_{\rho\pi} & \kappa_{\rho\rho} & -\kappa_{\rho\pi} - \kappa_{\rho\rho}\gamma_{\pi^*} \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad S \equiv \mathrm{diag}(\sigma_y, \sigma_\pi, \sigma_\rho, \sigma_{\pi^*})\,.$$

1.2 The Term Structure of Interest Rates

Equations (6) and (7) specify the instantaneous interest rate and the dynamics of the macroeconomic variables. This system must, therefore, also determine (up to some risk premium component) the dynamics of the term structure of interest rates. Absence of arbitrage opportunities implies that the price at time t of a zero-coupon default-free bond maturing at time $t + \tau$ is defined as[3]:

$$p(t,\tau) = E_t^Q\left(\exp\left(-\int_t^{t+\tau} i(u)du\right)\right),$$

where Q denotes the risk-neutral probability measure. This probability measure is unknown and can only be determined by assuming some specification for the market price of risk. Following the recent literature (e.g., Duffee, 2002, Duarte, 2003), we allow for time variability in the price of risk, $\xi(t)$, by specifying it as an affine function of the factors:

3. The study is restricted to bonds with a finite maturity. This is an important restriction; otherwise, non-stationarity under the risk-neutral measure could lead to an ill-defined term structure model (see Campbell, Lo, and MacKinlay 1997, p. 433).

$$\xi(t) = S\Lambda + S^{-1}\,\Xi f(t)\,,$$

where $\Lambda \equiv (\lambda_y, \lambda_\pi, \lambda_\rho, \lambda_{\pi^*})'$ and Ξ is an $n \times n$ matrix containing the sensitivities of the prices of risk to the level of the factors. Changing probability measures is done by means of Girsanov's theorem[4]:

$$dW(t) = d\tilde{W}(t) - \xi(t)dt\,,$$

where $\tilde{W}_i(t)$ denotes independent standard Brownian motions under the risk-neutral measure. In terms of this measure, the macroeconomic dynamics can be restated as:

$$df(t) = (\tilde{\psi} + \tilde{K}f(t))dt + S\,d\tilde{W}(t)\,,$$

where $\tilde{K} = K - \Xi$ and $\tilde{\psi} = \psi - S^2\Lambda$. It is well known that given the essentially affine dynamics under Q, bond prices can be expressed as an exponentially affine function of the factors $f(t)$:

$$p(t,T) = p(f(t),\tau) = \exp(-a(\tau) - b(\tau)'f(t))\,, \tag{8}$$

where $\tau = T - t$ denotes the time to maturity of the bond and $b(\tau)$ is a $n \times 1$ vector. The values for $a(\tau)$ and $b(\tau)$ are determined by the no-arbitrage condition in the bond markets:

$$D^Q p(f(t),\tau) = i(t)p(f(t),\tau)\,, \tag{9}$$

where D^Q denotes the Dynkin operator under the risk-neutral measure. The intuitive meaning of the latter condition is that, once transformed to a risk-neutral world, instantaneous holding returns for all bonds are equal to the instantaneous riskless interest rate.

Equations (8) and (9) determine the solution for the functions $a(\tau)$ and $b(\tau)$ in terms of the following system of ordinary differential equations (ODEs), which, in the general case, can be solved numerically by Runge–Kutta methods:

$$\frac{\partial a(\tau)}{\partial \tau} = b(\tau)'\tilde{\psi} - \frac{1}{2}b(\tau)'S^2b(\tau)\,,$$
$$\frac{\partial b(\tau)}{\partial \tau} = b_0 + \tilde{K}'b(\tau)\,.$$

A particular solution to this system of ODEs is obtained by specifying a set of initial conditions for a and b. From Equation (8), the relevant initial conditions are: $a(0) = 0$ and $b(0) = 0$. The vector of constants b_0 (0 1 1 0)$'$ is defined by the interest rate definition in Equation (6).

Notice that the introduction of long-run expectations solves the problems faced by standard macro models in fitting the long end of the term structure. These long-run expectations basically do not affect the short end of the yield curve, with loadings starting at zero for the instantaneous interest rate ($\tau = 0$). Nevertheless, they do

4. For a proof that Girsanov's Theorem can be applied in this framework, see Dai and Singleton (2002, Appendix B).

affect the long maturities in a crucial way since they serve as attractors for the observable macroeconomic variables.

2. ESTIMATION

2.1 Data

We base our analysis on data from McCulloch and Kwon (1993) and Bliss (1997) provided by Duffee (2002).[5] This data set consists of end-of-the-month yields of zero-coupon U.S. Treasury bonds with maturities of 3 and 6 months and 1, 2, 5, and 10 years. We use a quarterly frequency in order to incorporate the output gap series resulting in a data set with 140 observations (1964:Q1–1998:Q4).[6] The output (GDP) and inflation series are from the International Financial Statistics (IFS) database provided by the International Monetary Fund (IMF). A proxy for the output gap is obtained by using a Hodrick–Prescott (HP) filter on the GDP series.[7] By construction, the long-run expectation of the resulting output gap is equal to zero. Inflation is constructed by taking the yearly percentage change in the CPI index, that is $\pi_t = \ln \text{CPI}_t - \ln \text{CPI}_{t-4}$.

Table 1 gives some descriptive statistics of the sample series. The main characteristic to be observed is the correlation matrix showing the extreme correlation among the various bonds and significant but more moderate correlations between bonds on the one side and output gap or inflation on the other side. The output gap is positively correlated with the term structure up to 2-year yields and negatively correlated afterwards, while inflation is positively correlated with the entire term structure. The output gap and inflation are positively correlated with each other.

2.2 The Kalman Filter

The model is estimated by means of a Kalman filter. This allows us to filter the unobservable long-run inflation expectation of agents. Due to its Gaussian structure, the model is estimated in a consistent way. To apply this procedure, we first derive the discrete-time dynamics implied by the continuous-time model so as to match the observation frequency of the data.[8]

The measurement equation used in the estimation includes both the observed macroeconomic variables, output gap and inflation, and the observed yield curve. In this way, we ensure that the filtered central tendency of inflation is consistent with both the macroeconomic dynamics and the term structure of interest rates. Consider a set of observed yields, $\hat{y}(t,\tau_i)$, $i = 1, \ldots, m$. Given that the yield is defined as $-\ln p(t,\tau_i)/\tau_i$, the model implies a linear relation between the theoretical yields

5. We thank Gregory Duffee for making the data available on his website.

6. Although the original data set starts in 1958:Q1, we decide not to use the observations before 1964 due to its unreliability for long-term bonds (Fama and Bliss 1987).

7. We use a standard "lambda" in the filtering procedure equal to 1600.

8. For details on the Kalman filter procedure, see Dewachter, Lyrio, and Maes (2002).

TABLE 1

SUMMARY STATISTICS FOR THE DATA USED (1964:Q1–1998:Q4)

	Yield_{1q}	Yield_{2q}	Yield_{1yr}	Yield_{2yr}	Yield_{5yr}	Yield_{10yr}	y	π
Mean (%)	6.522	6.778	7.009	7.252	7.564	7.770	0.034	4.776
Std (%)	2.624	2.660	2.615	2.525	2.405	2.324	1.642	2.844
Min (%)	2.780	2.878	3.090	3.822	4.055	4.157	−4.689	1.128
Max (%)	15.241	15.924	15.911	16.107	15.696	15.065	3.879	13.502
Auto	0.986	0.987	0.989	0.992	0.995	0.997	0.869	0.993
Skew	1.333	1.316	1.201	1.175	1.098	0.917	−0.331	1.199
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.110)	(0.000)
Kurt	4.745	4.764	4.382	4.247	3.950	3.543	3.413	3.769
	(0.000)	(0.000)	(0.001)	(0.003)	(0.022)	(0.189)	(0.318)	(0.063)
JB	59.188	58.550	44.782	41.284	33.365	21.351	3.555	36.971
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.169)	(0.000)
				Correlations				
Yield_{1q}	1.000							
Yield_{2q}	0.995	1.000						
Yield_{1yr}	0.982	0.994	1.000					
Yield_{2yr}	0.959	0.973	0.990	1.000				
Yield_{5yr}	0.900	0.914	0.943	0.979	1.000			
Yield_{10yr}	0.852	0.866	0.899	0.947	0.991	1.000		
y	0.190	0.177	0.144	0.068	−0.033	−0.078	1.000	
π	0.680	0.683	0.661	0.617	0.558	0.533	0.078	1.000

NOTES: The bond yield data are based on spliced data from McCulloch and Kwon (1993) and Bliss (1997) provided by Duffee (2002) and concern U.S. Treasury bonds with maturities of 3 and 6 months and 1, 2, 5, and 10 years. Output gap (y) and inflation (π) data are constructed as mentioned in the text. The data series cover the period from 1964:Q1 until 1998:Q4, totalling 140 quarterly time series observations. Mean denotes the sample arithmetic average, expressed as p.a. percentage, Std standard deviation, Min minimum, Max maximum, Auto the first order quarterly autocorrelation, Skew and Kurt stand for skewness and kurtosis, respectively, while underneath these statistics are the significance levels at which the null of no skewness and the null of no excess kurtosis may be rejected. JB stands for the Jarque–Bera normality test statistic with the significance level at which the null of normality may be rejected underneath it.

and the factors, i.e. $\hat{y}(t,\tau_i) = a(\tau_i)/\tau_i + (b'(\tau_i)/\tau_i)f(t)$. The measurement equation can then be written as:

$$\begin{pmatrix} \hat{y}_1(t,\tau_1) \\ \vdots \\ \hat{y}_m(t,\tau_m) \\ y(t) \\ \pi(t) \end{pmatrix} = \begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} B \\ e'_1 \\ e'_2 \end{pmatrix} \begin{pmatrix} y(t) \\ \pi(t) \\ \rho(t) \\ \pi^*(t) \end{pmatrix} + \varepsilon_t \,, \tag{10}$$

where e_i is a $n \times 1$ column vector of zeros with a one on the i-th row, ε_t is an $(m + 2) \times 1$ vector of measurement errors with $E_t(\varepsilon(t)\varepsilon'(t)) = R$ and

$$a = (a(\tau_1)/\tau_1,\ldots,a(\tau_m)/\tau_m)' \,,$$

$$B = \begin{pmatrix} b(\tau_1)'/\tau_1 \\ \vdots \\ b(\tau_m)'/\tau_m \end{pmatrix} .$$

Finally, we impose zero measurement errors for the output gap and inflation series. This ensures that the actual values for these factors are in the information set of the agents. In other words, at each point in time, agents update their assessments

TABLE 2

Maximum Likelihood Estimates (1964:Q1–1998:Q4)

	y	π	ρ	π^*		
$\kappa_{y,\bullet}$	−0.989	−0.450	0.084			
	(0.135)	(0.065)	(0.049)			
$\kappa_{\pi,\bullet}$	1.131	−0.365	−0.389			
	(0.106)	(0.096)	(0.124)			
$\kappa_{\rho,\bullet}$	0.274	−1.171	−1.911			
	(0.150)	(0.250)	(0.291)			
γ_0			0.004			
			(0.004)			
$\gamma_\bullet$				0.325		
				(0.137)		
$\sigma^2_{g\bullet}$	0.00034	0.00011	0.00085	0.00004		
	(0.00006)	(0.00001)	(0.00012)	(0.00002)		
$\lambda_\bullet$	71.274	−242.629	40.263			
	(26.502)	(147.569)	(18.262)			
$\Xi_{\bullet\pi*}$	1.304	−3.147	4.416	0.104		
	(0.534)	(1.010)	(0.933)	(0.043)		
$\Xi_{\rho,\bullet}$	1.526	−1.124	−2.046			
	(0.193)	(0.260)	(0.316)			
R_{1q}	13.734					
R_{2q}	14.548	22.834				
R_{1yr}	5.898	18.004	19.409			
R_{2yr}	−2.402	9.156	12.753	10.038		
R_{5yr}	−11.143	−2.357	2.156	2.321	0.001	
R_{10yr}	−7.412	−0.975	2.178	2.242	0.076	0.000

Notes: Maximum likelihood estimates with robust standard errors between brackets. The values in the measurement error covariance matrix (R) are multiplied by 10^6. Total likelihood is equal to 5929.443 or 42.353 on average (excluding constant in the likelihood).

about these variables to the observed ones.[9] This perfect updating is obtained by imposing zero variance-covariance structure on the $m + 1$-th and $m - 2$-th rows and columns of the variance-covariance matrix R. Given the measurement equation and the conditional normality of the factors, we can apply the standard Kalman filter algorithm.[10]

2.3 Parameter Estimates

The estimation results of the model are presented in Table 2.[11] The most important feature of the estimates is the finding that all of the macroeconomic variables (y, π, and ρ) exhibit statistically significant reversion towards their central tendencies (equal to zero for y). This can be inferred from the statistically significant

9. Orphanides (2003a) stresses the importance of measurement errors in output gap and inflation for the implementation of monetary policy. Although we could allow for these measurement errors within the Kalman filter approach, there would be a remaining problem. Agents would not reset their inferences about the output gap and inflation to the revised published values for these variables. In order to keep the output gap and inflation factors as close as possible to the observed ones, we opt for a perfect updating in the Kalman filter procedure.

10. In practice, since the matrix K is in general not diagonal, the computation of the conditional mean and variance of the factors is not trivial. Dewachter, Lyrio, and Maes (2001) and Fackler (2000) provide equivalent procedures to compute these moments.

11. The full model is estimated in a single step procedure. Optimization was performed using the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm with a convergence tolerance for the gradient of the estimated coefficients equal to 1E-04. The robustness of the "optimum" reported is verified by checking convergence from an array of starting points.

negative diagonal elements of the K matrix. This finding corroborates the idea that each of the macroeconomic series has a trend–cycle decomposition.

Since we impose a Brownian motion model on the long-run inflation expectation under the empirical probability measure, the system is non-stationary under this measure. Under the risk-neutral measure, however, the state space dynamics is stable, i.e. the real parts of the eigenvalues of the matrix $\tilde{K}$ are all negative. Moreover, K has imaginary eigenvalues, indicating an oscillating impulse response for the dynamic system. Figure 1 presents the filtered time series for the four factors involved: two observable ones (output gap, y, and inflation, π) and two non-observable ones (the real interest rate, ρ, and the long-run inflation expectation, π^*).

Table 2 also provides important information with respect to the estimated market prices of risk. Due to identification issues, we only estimate a subset of the prices of risk.[12] Note that they are estimated with a relatively high precision. Only one of the parameter estimates is not significant at the 5% level. More striking, however, is the observation that the inflation expectations enter significantly in the prices of risk attached to any of the state variables. Inflation expectations thus tend to function as a general factor of the size of all of the risk premia, indicating that market prices of risk attached to any of these factors tend to increase with the level of the expected long-run inflation.

2.4 The Interest Rate Policy Rule

The above economic framework estimates the continuous-time nominal interest rate policy rule adopted by the central bank. For ease of comparison with the current literature, however, we express the discrete-time rule implied by the continuous-time macro model in a way similar to Rudebusch (2002) and Clarida, Galí, and Gertler (1998):

$$i_t = 0.37 i_t^{\mathrm{CB}} + 0.63 i_{t-1} + \upsilon_t \,,$$

where υ_t is an i.i.d. shock and i_t^{CB} represents the central bank target for the nominal interest rate which is given by[13]

$$i_t^{\mathrm{CB}} = [i_t^* + \underset{(0.11)}{0.06} y_t + \underset{(0.37)}{2.70}(\pi_t - \pi_t^*) - \underset{(0.42)}{2.30}(\pi_{t-1} - \pi_{t-1}^*)] \,.$$

The variable $i_t^* = \pi_t^* + \rho_t^*$ represents the long-run interest rate expectation. Standard errors are reported in parentheses. As in Rudebusch (2002) and Clarida, Galí, and Gertler (1998), the policy rule is a weighted average of the desired level in

12. We face identification problems when we allow for the estimation of all market prices of risk. The chosen set of market prices of risk avoids identification and gives enough flexibility in the determination of the time-varying risk premia. Duffee (2002) and Dai and Singleton (2002) also estimate restricted sets of the market prices of risk due to identification problems.

13. The interest rate target can also be written as $i_t^{CB} = [i_t^* + 0.06 y_t + 0.4(\pi_t - \pi_t^*) + 2.3\Delta(\pi_t - \pi_t^*)]$. In this representation, the central bank target responds positively to the output gap, the deviation from inflation from long-run inflation expectations, and the change in the inflation deviation. The last term suggests that if the deviation of inflation from long-run inflation expectations is increasing, the policy will be tightened more aggressively than if the deviation is constant or declining. We thank the referee for suggesting this interpretation.

quarter t, i_t^{CB} , and the actual interest rate level in the previous quarter. This is an extension of the initial rule proposed by Taylor (1993) and reflects the central bank's desire to smooth interest rates. Our estimates point to a significantly lower role for the smoothing component (0.63) than the ones found in the literature (usually above 0.8 for quarterly data). This corroborates recent findings by Rudebusch (2002) that the adjustment of interest rates is much higher than frequently reported in the literature. In line with Rudebusch (2002), we find that the interest rate inertia decreases significantly once additional factors (in our case long-run inflation expectations) are taken into account.

In line with the literature, we also observe a strong reaction in the central bank target to the inflation gap (i.e. inflation above the steady state inflation rate), leading to a tightening in the monetary policy. Our estimate is significantly above 1 (2.70) and implies an inflation stabilizing monetary policy. The point estimate for the coefficient on lagged inflation is also significant but has the wrong sign. This fact is also found by Clarida, Galí, and Gertler (1998) using a similar model.[14] We also find that a temporary demand shock (i.e. positive output gap) induces a small (0.06) and insignificant increase in the central bank interest rate target. The right-bottom panel of Figure 1 shows the estimated central bank target in comparison with actual short-term (3-month) nominal interest rate.

3. A MACROECONOMIC INTERPRETATION OF THE YIELD CURVE

Based on the empirical estimates of the previous section, we now proceed to explain the term structure from a macroeconomic perspective. We first explain the roles played by each of the macroeconomic variables in shaping the yield curve. We then compare the term structure fit from our macro model with that from a latent factor model. Finally, we relate the estimated macro factors to the latent ones found in traditional finance models.

3.1 Decomposing the Yield Curve

Figure 2 shows the factor loadings for the yield curve on each of the macroeconomic factors. Unlike the standard latent factor literature, we do not find evidence of a direct macroeconomic level effect. Instead, we find that inflation and the real interest rate (in accordance with Equation (6)) determine primarily the range of short maturities. The loadings of these factors typically dampen quite fast and are relatively small for the longer end of the yield curve. The long-run inflation expectation affects mainly the long end of the term structure without influencing its short end. Finally, the loadings on the output gap increase in magnitude (become more negative) between low and intermediate maturities.

As an alternative, we track the responses across the yield curve of orthogonalized components of the macroeconomic factors. We orthogonalize the macroeconomic

14. We refer to the third line of results (adding lagged inflation) in Table 3 of Clarida, Galí, and Gertler (1998). These authors use a forward-looking value for inflation in the central bank target whereas we use the current one. Their study uses U.S. monthly data for the period 1979:10-199412.

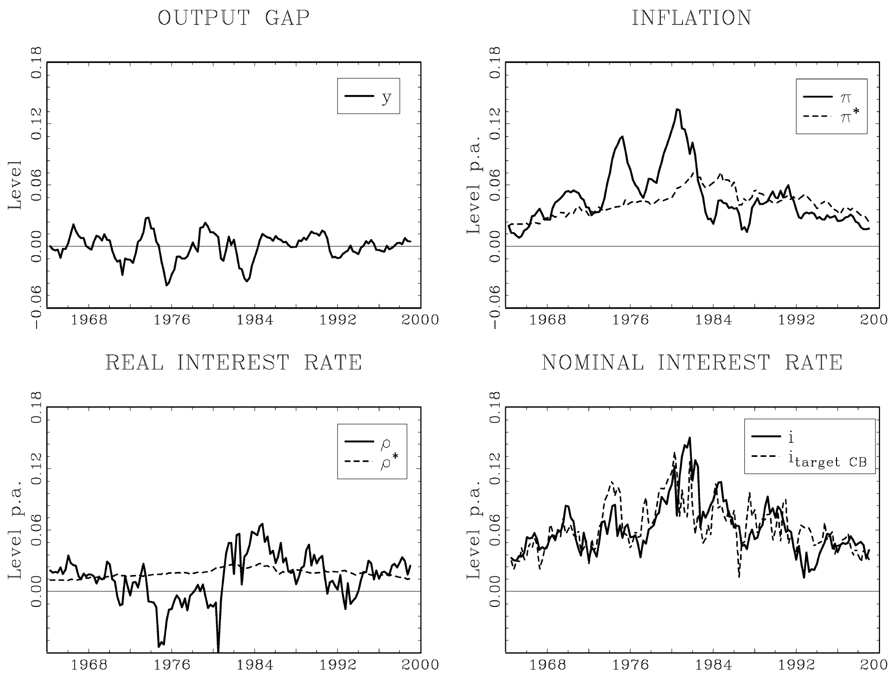

FIG. 1. Macro Variables and Their Estimated Central Tendencies

factors by means of a Cholesky factorization based on the following ordering: π^*, $\pi - \pi^*$, y, and $\rho - \rho^*$.[15] We then transform the above loadings on the macro factors to those relevant for the Cholesky factors. The following transformation is used:

$$B_{\text{chol}} = BZ^{-1}L\,.$$

with $(\pi^*(t), \pi(t) - \pi^*(t), y(t), \rho(t) - \rho*(t))' = Zf(t)$ and $LL' = ZE(f(t)f'(t))Z'$. Figure 3 presents these new factor loadings, B_{chol}. As can be seen, the first Cholesky factor (a linear function of π^*) exerts an important level effect. A shock to inflation expectations is transmitted through the entire yield curve. Interestingly, we find that interest rates respond more than one-to-one to long-run inflation expectations. We recover a sensitivity of the short-term interest rates to long-run inflation expectations equal to 1.44.[16] The second and third factors are basically important for the short end of the term structure, as expected from independent mean-reverting effects. The fourth Cholesky factor loadings show a much smaller decay and also exerts some effect on the intermediate maturities.

15. Note that this factorization separates the permanent factors, i.e. the stochastic trend $\pi^*(t)$, from the temporary disequilibria, i.e. the vector error correction terms $\pi(t) - \pi^*(t), y(t)$ and $\rho(t) - \rho^*(t)$.

16. This value is found at the intersection of the vertical axis with the curve “cholfac1,” which represents movements in the long-run inflation expectation.

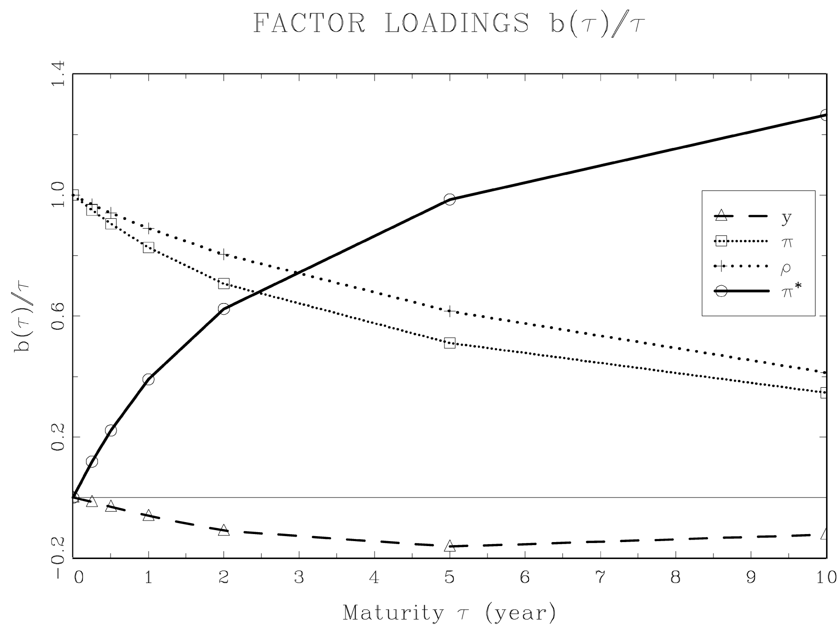

FIG. 2. Estimated Factor Loadings

3.2 Performance Relative to a Latent Factor Model

The macroeconomic model performs reasonably well when compared to a standard latent factor model.[17] Table 3 gives some diagnostic statistics that allow us to compare the performance of the macro model with the benchmark of a latent three-factor Vasicek model. Keeping in mind that the three-factor latent model is sufficiently flexible to fit accurately the term structure, we still find the macro model to be competitive. Looking at the mean and autocorrelation of the fitting errors, the latent factor model seems to be modeling the short end of the term structure better while the macro model seems to do better in the range of longer maturities. The lack of flexibility of the macro model becomes clear by the size and standard deviation of the measurement errors, which are especially larger for the short end of the yield curve. Nevertheless, when compared to the latent factor model, the macro model displays somewhat lower autocorrelation in the measurement errors for maturities above two quarters. Figure 4 displays the fit of the macro model for the term structure of interest rates. It becomes clear from this figure that the misfit of long-term yields present in standard macroeconomic models is resolved.

An alternative method to test the adequacy of the model is to run regressions of the actual yields (or changes in yields) on the implied yields based on the macro model. Model adequacy is then tested by the null hypothesis that the implied yields are unbiased predictors of the actual yields. Running a regression in levels, we are

17. The parameter estimates for the latent factor model are available upon request.

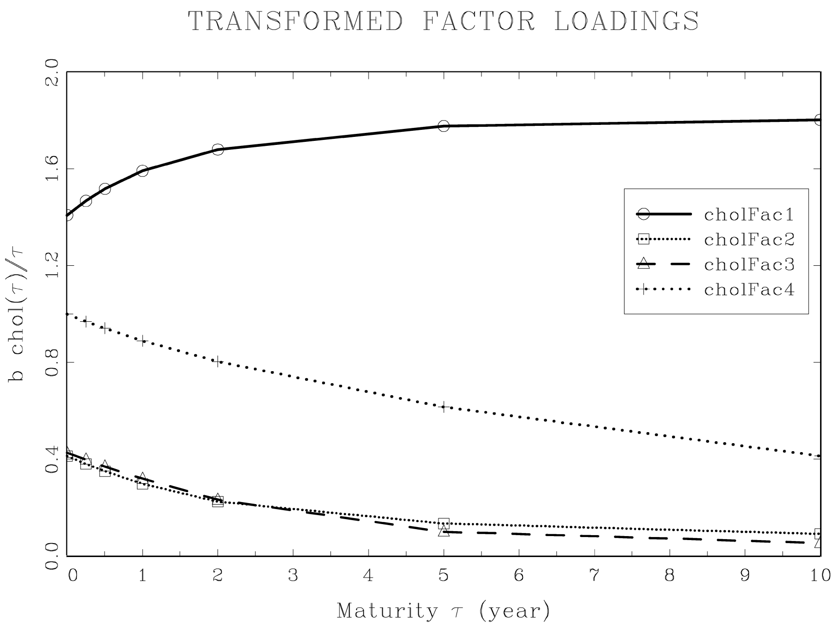

FIG. 3. Transformed Factor Loadings

not able to reject the null hypothesis of unbiased implied yields (see Table A1 in the Appendix). For all maturities, more than 93% of the variation of the yields is explained. So, in general, the macro model gives a reasonable description of the yield curve. Performing the regression on yield changes, however, reveals some biases. The regression coefficients tend to be significantly different from 1. Nevertheless, the fit is still reasonable since for all maturities more than 60% of the variation in the yield changes is explained. In short, although we have a good macroeconomic description of the yield curve, the restriction to fit the yield curve only by means of macro factors naturally limits the flexibility of the model.

TABLE 3

TERM STRUCTURE FITTING ERRORS

	Macro model			Three-factor latent model		
	Mean (%)	Std.dev. (%)	Autocorr.	Mean (%)	Std.dev. (%)	Autocorr.
Yield_{1q}	0.13	0.62	0.32	−0.0002	0.34	0.15
Yield_{2q}	0.21	0.67	0.35	0.14	0.32	0.25
Yield_{1yr}	0.18	0.64	0.38	0.17	0.29	0.45
Yield_{2yr}	0.15	0.53	0.37	0.15	0.28	0.44
Yield_{5yr}	0.09	0.35	0.31	0.10	0.20	0.41
Yield_{10yr}	0.07	0.26	0.31	0.08	0.17	0.46

NOTE: Autocorr. stands for one-lag autocorrelation.

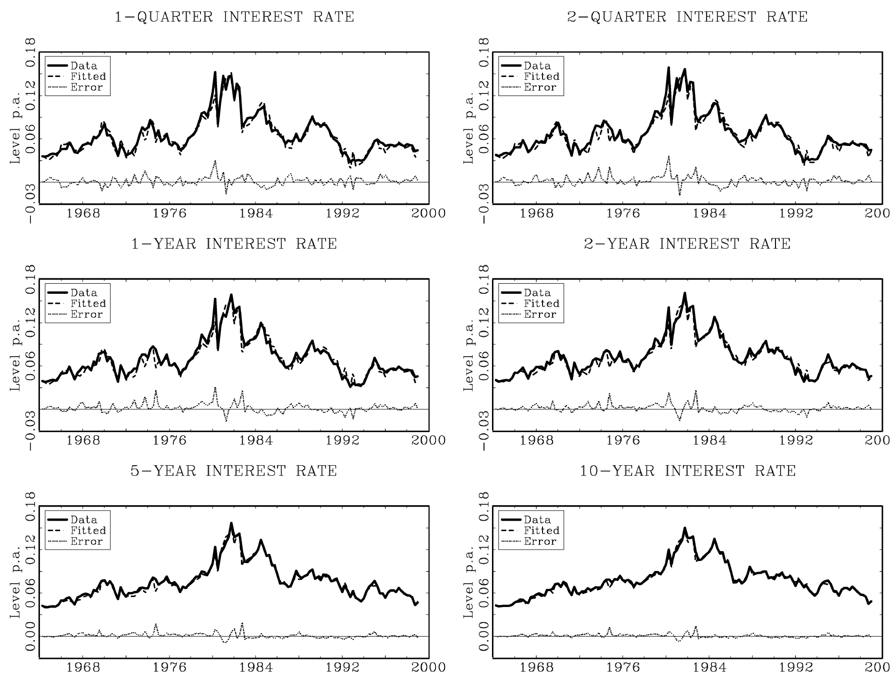

FIG. 4. Model Fit (Errors) of the Term Structure of Interest Rates

3.3 A Macroeconomic Interpretation of the Latent Factors

Even though the macroeconomic interpretation derived from the macro model is by definition more informative than the results obtained from a standard latent factor model, it is important to find out the link between the two representations. This would allow us to give the latent factors an economic interpretation beyond the standard level, slope, and curvature labels traditionally attached to them. Here we perform such an exercise and show that there is a clear link between the latent and macro factors.

For this purpose, we perform an ordinary least squares (OLS) regression of each of the latent factors on the orthogonalized components of the macroeconomic factors (the Cholesky factors of Section 3.1). This allows us to assess the contribution of the independent components of the macroeconomic variables to each of the latent factors. As mentioned before, we use the following ordering for the Cholesky decomposition: π^*, $\pi - \pi^*$, y, and $\rho - \rho^*$.[18] Note that this amounts to a permanent-temporary decomposition with one independent stochastic trend (π^*) and three independent temporary components. The regression results of the latent factors on the orthogonalized factors are displayed in Table 4. The results for the latent level factor show that it can be seen as a *long-run inflation expectation factor.* The filtered long-run inflation expectation explains about 99% of the total variation of the level

18. Alternative orderings in the Cholesky factorization do not change the results in a significant way.

TABLE 4

Regression of Latent Factors upon Orthogonalized Macro Factors

	Level factor			Slope factor			Curvature factor		
	Coef.	$\bar{R}^2$	$\Delta\bar{R}^2$	Coef.	$\bar{R}^2$	$\Delta\bar{R}^2$	Coef.	$\bar{R}^2$	$\Delta\bar{R}^2$
Cte	−0.028			0.012			0.014		
	(0.001)			(0.003)			(0.002)		
Cholfac1	2.048	0.989	0.989	−0.383	0.064	0.064	−0.226	0.034	0.034
	(0.014)			(0.072)			(0.051)		
Cholfac1	0.068	0.994	0.005	0.428	0.438	0.374	−0.020	0.028	−0.006
	(0.007)			(0.035)			(0.025)		
Cholfac1	0.007	0.994	0.000	0.505	0.645	0.207	−0.048	0.024	−0.004
	(0.011)			(0.055)			(0.039)		
Cholfac1	0.019	0.994	0.000	0.141	0.656	0.011	0.770	0.715	0.691
	(0.011)			(0.060)			(0.042)		

Notes: The three latent factors are regressed upon orthogonalized macro factors. The Cholesky decomposition was done with the macro factors in the following order: π^*, $\pi-\pi^*$, y, and $\rho-\rho^*$. Standard errors between brackets.

factor. The latent slope factor is mainly explained by the second and third Cholesky factors, respectively, inflation shocks orthogonal to long-run inflation expectations, and output gap effects not attributable to inflation components. More than 88% of the explained variability of the slope factor is due to these two factors. Finally, given the positive regression coefficients for both the second and third Cholesky factors, we can interpret the slope factor as a *business cycle factor*. That is, it tends to

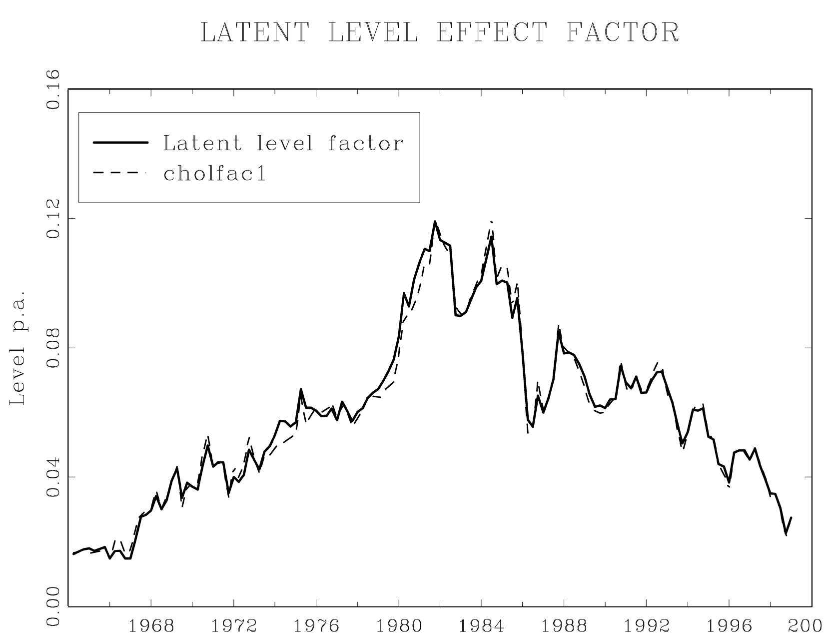

Fig. 5. Latent Level Effect Factor

correlate positively with both demand inflation and the output gap shocks, both indicating a clear link to the business cycle. The weights on these factors (0.428 and 0.505, respectively) are, furthermore, similar to the coefficients suggested by Taylor (1993). Finally, the latent curvature factor is mainly explained by the last Cholesky factor. More than 96% of the total explained variation of this factor is due to the fourth Cholesky factor. By construction, this factor represents real interest movements orthogonal to (independent from) all other macroeconomic variables. In particular, this factor captures policy actions beyond the endogenous responses to inflation deviations and the output gap (and summarized by the business cycle factor). As such, this factor can be interpreted as an independent monetary policy factor that we label the *monetary stance factor*. The Cholesky factor regression coefficient is positive, meaning that an increase in the curvature is positively related to a tougher monetary policy.

Figures 5–7 give a graphical display of the latent factors together with the fit of the retained Cholesky factors. As can be seen in Figure 5, the fit is especially significant for the latent level effect factor when compared with the first Cholesky factor, which is based solely on π^*. Note also that the latent slope effect factor (Figure 6) has a strong business cycle component and that the fit based on the second and third Cholesky factors track these cycles quite well. Interestingly, this latent factor suggests deeper economic downturns than the fitted macro factor (Cholesky factors). This is consistent with the evidence in Orphanides (2003b, p. 1003), which suggests

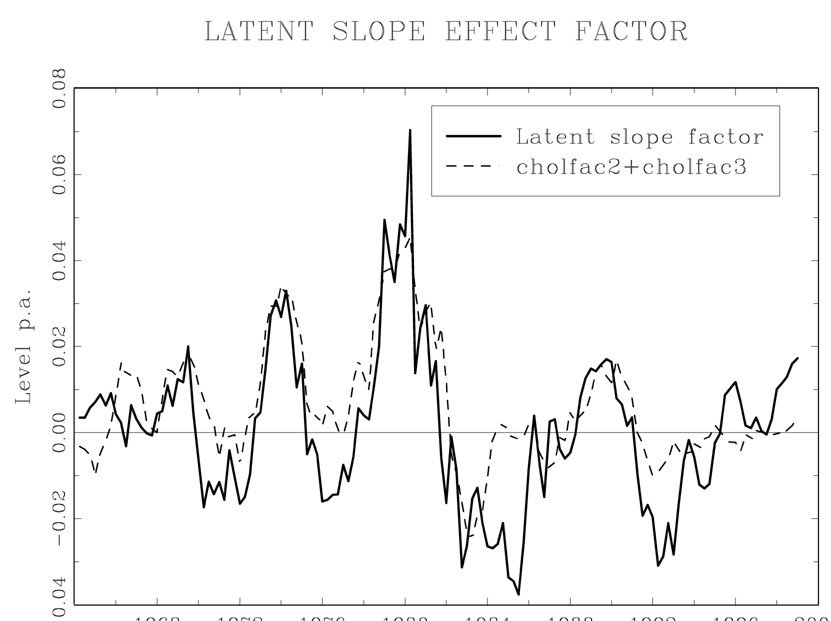

FIG. 6. Latent Slope Effect Factor

that our real-time perceptions of the output gap in past recessions were more pessimistic than our current views of the experience in those periods.[19] Finally, we assess the latent curvature effect factor (Figure 7) that we interpret as a monetary stance variable. According to this interpretation, the stance of monetary policy was rather loose in the seventies and then abruptly changed in the beginning of the eighties when the monetary policy became particularly strong. The historical switches in inflation fighting policies corroborate this interpretation of the latent factor in terms of toughness of the monetary policy stance.

Summarizing, we find a clear interpretation of the latent factors in macroeconomic terms within the limitations of the fit for the latent slope and curvature factors. The standard latent level factor reflects a macroeconomic expectations effect, clearly related to long-run inflation expectations. The slope factor represents business cycle conditions while the curvature factor is related to the monetary policy stance held by the central bank.

4. CROSS-VALIDATION OF LONG-RUN INFLATION EXPECTATIONS

The introduction of long-run inflation expectations plays a crucial role in the above analysis. Despite the fact that these expectations are formally modeled as

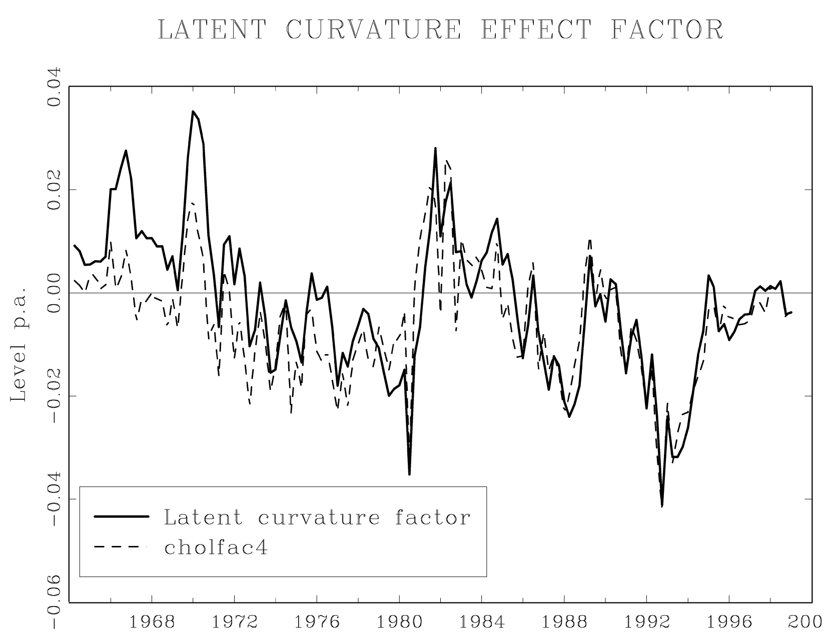

FIG. 7. Latent Curvature Effect Factor

19. We thank the referee for pointing out this fact to us.

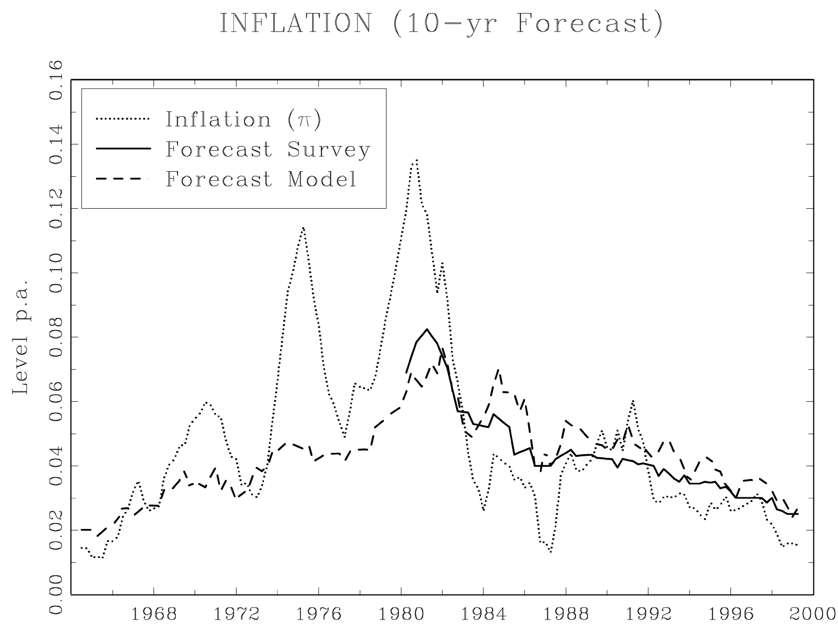

FIG. 8. Comparison of Average 10-Year Inflation Forecast—Model vs. Survey of Professional Forecasters (Federal Reserve Bank of Philadelphia)

long-run attractors and that the mean reversion towards them is statistically significant, they still remain a *filtered* representation of the true long-run inflation expectation of the agents. In order to assess the plausibility of this filtered expectation series, we conduct two experiments. First, we relate the filtered inflation expectations to survey data. We then relate them to long-run inflation expectations obtained from a univariate structural break analysis, using the methodology of Bai and Perron (1998), as implemented by Kozicki and Tinsley (2001).

Figure 8 contrasts the 10-year average inflation forecast implied by the macro model with the market long-run inflation expectation as recorded in the Survey of Professional Forecasters provided by the Federal Reserve Bank of Philadelphia. This figure provides supporting evidence for the filtered long-run expectations as the two series are highly correlated (0.90). In fact, regression results do not allow us to reject the one-to-one relation between the survey and the filtered expectation.[20] Note that these results are particularly strong since the survey inflation expectation series was not used whatsoever in the filtering procedure.

A second method to assess the filtered long-run inflation expectation consists of a break analysis for the long-run endpoints, as proposed by Kozicki and Tinsley (2001).[21] We follow these authors in assuming that market expectations are an

20. The regression results are available upon request.

21. Unlike these authors, we use the methodology proposed by Bai and Perron (1998).

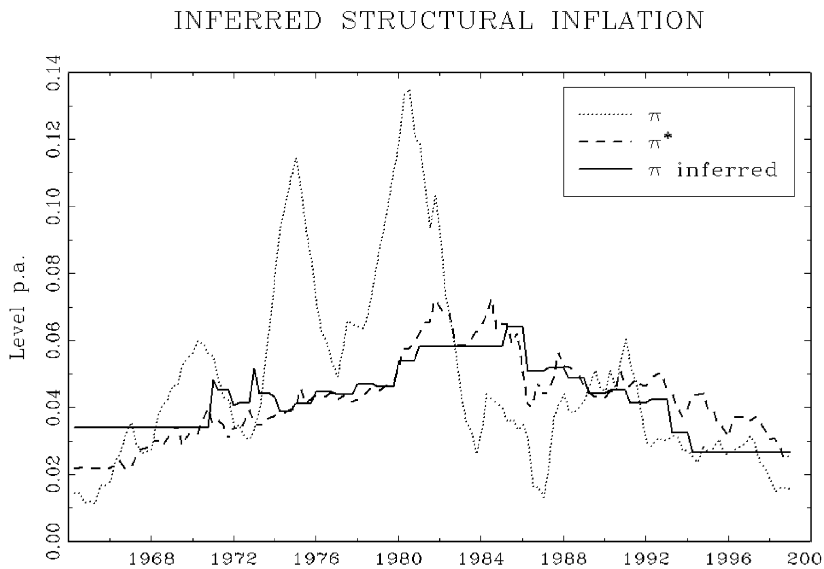

FIG. 9. Inferred Structural Long-Run Inflation Expectation

average of heterogeneous, agent-specific, inflation expectations. These expectations are built on an agent-specific information lag used to update their long-run expectations. Figure 9 contrasts the optimal fit based on the mean squared error (MSE) criteria (curve π_{inferred}) with the long-run inflation expectation obtained from the macro model (curve π^*). As can be seen from this figure, the fit is reasonable, with a correlation between the two series equal to 0.92. In short, also from a structural break point of view we can account for the filtered long-run inflation expectation.

5. CONCLUSIONS

In this paper, we have proposed a macro model that consistently and simultaneously models the rate of inflation, the output gap, and the term structure of interest rates. This approach extends and corroborates the research by Kozicki and Tinsley (2001). We extend their approach by modeling long-run inflation expectations simultaneously with the term structure and by including time-varying prices of risk specifications. Our results reinforce the basic message that inflation expectations are important in modeling the term structure dynamics from a macroeconomic point of view.

We find, moreover, that four macroeconomic factors model both the term structure as well as the macroeconomic dynamics rather well. While inflation expectations play a crucial role for long-term maturities, actual macro variables such as inflation and the real interest rate are of primary importance for short-term maturities. This macroeconomic decomposition allows us to interpret the standard level, slope, and curvature effect factors typically found in the finance literature. We find the level factor to be closely linked to the long-run inflation expectation, the slope factor to be an aggregate series for the business cycle condition, and the curvature factor to be related to the monetary stance of the central bank.

Next to filtering long-run inflation expectations, we also allow for time-varying prices of risk. The time variability is assumed to be captured by the macroeconomic variables. An interesting result is that long-run inflation expectations determine to a large extent the level of the prices of risk of all sources of risk. For instance, the price of risk attached to business cycle conditions changes with the level of the long-run inflation expectation. Long-run inflation expectations are, therefore, also a prime determinant of the level of risk premia.

APPENDIX A: TERM STRUCTURE FIT

TABLE A1

Regression of Yield Data on Model Implied Yields

	Levels			First differences		
	Cte.	Coef.	$\bar{R}^2$	Cte.	Coef.	$\bar{R}^2$
Yield_{1q}	0.002545	0.9813	0.945	0.000026	0.7717	0.665
	(0.001390)	(0.0202)		(0.000582)	(0.0465)	
Yield_{2q}	0.002267	0.9973	0.937	0.000014	0.7728	0.609
	(0.001553)	(0.0220)		(0.000639)	(0.0526)	
Yield_{1yr}	0.002191	0.9944	0.940	0.000012	0.7770	0.623
	(0.001548)	(0.0213)		(0.000589)	(0.0513)	
Yield_{2yr}	0.002360	0.9878	0.956	0.000003	0.7866	0.667
	(0.001359)	(0.0181)		(0.000492)	(0.0472)	
Yield_{5yr}	0.002138	0.9836	0.979	−0.000003	0.7957	0.744
	(0.000969)	(0.0123)		(0.000336)	(0.0397)	
Yield_{10yr}	0.001544	0.9890	0.987	−0.000001	0.8354	0.781
	(0.000772)	(0.0096)		(0.000257)	(0.0377)	

Note: Regression of yield data on model implied yields and a constant, both in levels and in first differences. Standard errors between brackets.

LITERATURE CITED

Ang, Andrew, and Monika Piazzesi (2003). “A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables.” *Journal of Monetary Economics* 50, 745–787.

Bai, Jushan, and Pierre Perron (1998). “Estimating and Testing Linear Models with Multiple Structural Changes.” *Econometrica* 66, 47–78.

Bekaert, Geert, Senghoon Cho, and Antonio Moreno (2003). “New-Keynesian Macroeconomics and the Term Structure.” Columbia University, manuscript.

Bliss, Robert R. (1997). “Testing Term Structure Estimation Methods.” *Advances in Futures and Options Research* 9, 197–231.

Campbell, John Y., Andrew W. Lo, and A. Craig MacKinlay (1997). *The Econometrics of Financial Markets*. Princeton, NJ: Princeton University Press.

Clarida, Richard, Jordi Galí, and Mark Gertler (1998). "Monetary Policy Rules in Practice. Some International Evidence." *European Economic Review* 42, 1033–1067.

Dai, Qiang, and Kenneth J. Singleton (2000). "Specification Analysis of Affine Term Structure Models." *Journal of Finance* 55, 1943–1978.

Dai, Qiang, and Kenneth J. Singleton (2002). "Expectation Puzzles, Time-Varying Risk Premia, and Affine Models of the Term Structure." *Journal of Financial Economics* 63, 415–441.

Dewachter, Hans, Marco Lyrio, and Konstantijn Maes (2001). "Estimation of a Joint Model for the Term Structure of Interest Rates and the Macroeconomy." CES Discussion Paper 01.18, KULeuven.

Dewachter, Hans, Marco Lyrio, and Konstantijn Maes (2002). "The Effect of Monetary Unification on German Bond Markets." *European Financial Management*, forthcoming.

Duarte, Jefferson (2003). "Evaluating an Alternative Risk Preference in Affine Term Structure Models." *Review of Financial Studies*, forthcoming.

Duffee, Gregory R. (2002). "Term Premia and Interest Rate Forecasts in Affine Models." *Journal of Finance* 57, 405–443.

Duffie, Darrell, and Rui Kan (1996). "A Yield-factor Model of Interest Rates." *Mathematical Finance* 6, 379–406.

Fackler, Paul L. (2000). "Moments of Affine Diffusions." North Carolina State University, manuscript.

Fama, Eugene F., and Robert R. Bliss (1987). "The Information in Long-Maturity Forward Rates." *American Economic Review* 77, 680–692.

Gürkaynak, Refet S., Brian Sack, and Eric Swanson (2003). "The Excess Sensitivity of Long-Term Interest Rates: Evidence and Implications for Macroeconomic Models." Board of Governors of the Federal Reserve System, Finance and Economics Discussion Series 2003-50.

Hördahl, Peter, Oreste Tristani, and David Vestin (2003). "A Joint Econometric Model of Macroeconomic and Term Structure Dynamics." European Central Bank, manuscript.

Kozicki, Sharon, and Peter A. Tinsley (2001). "Shifting Endpoints in the Term Structure of Interest Rates." *Journal of Monetary Economics* 47, 613–652.

Kozicki, Sharon, and Peter A. Tinsley (2002). "Dynamic Specifications in Optimizing Trend-Deviation Macro Models." *Journal of Economic Dynamics and Control* 26, 1585–1611.

McCulloch, J. Huston, and Heon-Chul Kwon (1993). "U.S. Term Structure Data, 1947–1991." Ohio State University Working Paper No. 93-96.

Orphanides, Athanasios (2003a). "Monetary Policy Evaluation with Noisy Information." *Journal of Monetary Economics* 50, 605–631.

Orphanides, Athanasios (2003b). "Historical Monetary Policy Analysis and the Taylor Rule." *Journal of Monetary Economics* 50, 983–1022.

Rudebusch, Glenn D. (2002). "Term Structure Evidence on Interest Rate Smoothing and Monetary Policy Inertia." *Journal of Monetary Economics* 49, 1161–1187.

Rudebusch, Glenn D., and Tao Wu (2003). "A Macro-Finance Model of the Term Structure, Monetary Policy, and the Economy." Federal Reserve Bank of San Francisco, Working Paper No. 2003-17.

Taylor, John B. (1993). "Discretion versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy* 39, 195–214.

Vasicek, Oldrich (1977). "An Equilibrium Characterization of the Term Structure." *Journal of Financial Economics* 5, 177–188.