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Volatility spillovers between oil prices and the stock market under structural breaks

Bradley T. Ewing [a,1], Farooq Malik [b,*]

[a] Rawls College of Business, Texas Tech University, Lubbock, TX 79409-2101, United States
[b] College of Business, Zayed University, P. O. Box 19282, Dubai, United Arab Emirates

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ABSTRACT

This paper employs univariate and bivariate GARCH models to examine the volatility of oil prices and US stock market prices incorporating structural breaks using daily data from July 1, 1996 to June 30, 2013. We endogenously detect structural breaks using an iterated algorithm and incorporate this information in GARCH models to correctly estimate the volatility dynamics. We find no volatility spillover between oil prices and US stock market when structural breaks in variance are ignored in the model. However, after accounting for structural breaks in the model, we find strong volatility spillover between the two markets. We compute optimal portfolio weights and dynamic risk minimizing hedge ratios to highlight the significance of our empirical results which underscores the serious consequences of ignoring these structural breaks. Our findings are consistent with the notion of cross-market hedging and sharing of common information by financial market participants in these markets.

1. Introduction

Both oil prices and stock market prices are intrinsically linked with the economy. There is robust evidence in the literature documenting a strong relationship between oil prices and the economy (see Hamilton, 2003). Moreover, since stock prices are the present discounted value of future net earnings which are dependent on the economy, one should expect to find a significant relationship between changes in the prices of oil and the stock market (see Jones & Kaul, 1996). It is therefore natural to expect the prices and/or volatilities of these two series to be linked in asset pricing models. Ross (1989) shows that volatility in asset returns depends

* Corresponding author. Tel.: +971 4 4021545; fax: +971 4 4021010.
E-mail addresses: bradley.ewing@ttu.edu (B.T. Ewing), farooq.malik@zu.ac.ae (F. Malik).
[1] Tel.: +1 806 834 3939.

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upon the rate of information flow, suggesting that the flow of information from one market can be incorporated into the volatility generating process of another related market. However, these dynamics may change over time due to structural changes in the underlying economy or fundamentals that drive these two markets. Thus, it is important to take into account the possible existence of sudden changes, or breaks, in the time series behaviors of these prices or their respective volatilities. This paper specifically examines the volatility linkage that may exist between oil and stock prices allowing for structural breaks in volatility. An accurate understanding of the time series relationship between the two markets will be useful for financial market participants and policy makers. The importance of this relationship is evident from the fact that changes in both of these series are widely followed by popular news media.

The present paper studies the volatility dynamics of oil prices and US stock market prices using daily returns from July 1, 1996 to June 30, 2013. We find significant structural breaks in volatility (i.e., volatility shifts) in both of these series using modified iterated cumulative sums of squares (ICSS) algorithm. The findings are consistent with the recent evidence that there are structural breaks in variance of oil prices (see Ewing & Malik, 2010) and stock market returns (see Perron & Qu, 2010; Starica & Granger, 2005). We introduce these detected structural breaks into our univariate GARCH models to accurately capture the impact of news on volatility in each individual market and then into our bivariate GARCH models to accurately estimate the volatility spillovers across markets. We find strong evidence of significant spillovers of volatility between oil prices and the stock market after structural breaks are incorporated into the model. We further show that these spillovers would not be empirically captured if structural breaks are ignored in the model. We compute optimal portfolio weights and dynamic risk minimizing hedge ratios to highlight the serious consequences of ignoring these structural breaks in volatility.

Volatility in oil prices affects consumer behavior which directly impacts the performance of the overall economy. Stocks constitute an important asset in a standard portfolio and stock market prices are considered as a useful indicator of future economic performance. Changes in the volatility of oil and stock prices can potentially alter the respective investments in these markets. Lamoureux and Lastrapes (1990) show that pricing of contingent claims depends on the underlying volatility dynamics. Thus, correctly estimating volatility dynamics in oil and stock prices is important for building accurate pricing models, forecasting future price volatility, and for a better understanding of broader financial markets and the overall economy.

2. Literature review

A growing body of research has emerged on the relationship between oil prices and stock market prices. Jones and Kaul (1996) show that the reaction of the US stock market to oil shocks can be completely accounted for by the impact of these shocks on real cash flows. Using a vector autoregression (VAR) framework, Sadorsky (1999) has shown that both oil prices and a univariate GARCH measure of oil price volatility play significant roles in affecting stock market returns. Basher and Sadorsky (2006) document the impact of oil price changes on emerging stock markets. In a recent study, Driesprong, Jacobsen, and Maat (2008) show economically and statistically significant predictability of stock returns when incorporating oil price changes in their model using data from both developed and emerging markets.

Although the interaction between oil prices and stock market prices in level form is well documented, the recent focus of studies is to examine their interactions at the volatility level. This is primarily because volatility in the prices of oil and the stock market is an important input in modern macro-econometric models, financial market risk assessment calculations, and asset pricing formulas.[2] Sadorsky (2003) shows that the conditional volatility in oil prices, among other variables, has a significant impact on the conditional volatility of technology stock prices. Using a multivariate GARCH model, Malik and Hammoudeh (2007) find significant transmission of volatility and shocks among US equity, Gulf equity and global crude oil markets. Malik and Ewing (2009) provide evidence of significant transmission of shocks and volatility between oil prices and US equity sector returns. Arouri, Jouini, and Nguyen (2011) take a recent generalized VAR-GARCH approach to examine the extent of volatility transmission between oil prices and stock markets in Europe and the United States at the sector-level with corresponding implications for portfolio management.

[2] For example, Haigh and Holt (2002) find that modeling the time-varying hedge ratios via multivariate GARCH methodology, which takes into account volatility spillovers across markets, results in significant reductions in uncertainty which benefits an energy trader.

However, in most studies in the existing literature, there is a general assumption that the unconditional variance of the underlying series is constant implying that volatility is generated by a stable GARCH process. But markets often experience structural breaks in the unconditional variance which causes breaks in the GARCH parameters. There is recent evidence that there are structural breaks in variance in oil prices (see Ewing & Malik, 2010) and stock returns (see Perron & Qu, 2010; Starica & Granger, 2005). These structural breaks in volatility could be caused by political, social, economic or natural events. In a recent paper, Mensi, Hammoudeh, and Yoon (2014) detect structural breaks in the crude oil market and highlight the implications this has for policy makers and financial market participants [see also Arouri, Lahiani, Levy, and Nguyen (2012)]. Lamoureux and Lastrapes (1990) show that volatility persistence is overestimated when standard GARCH models are applied to a series with underlying structural breaks in variance. Mikosch and Starica (2004) give a detailed theoretical explanation supported with evidence from simulations and stock market data that ignoring structural breaks in variance results in higher volatility persistence within a GARCH model. Starica and Granger (2005) using daily stock market returns found most of the time series dynamics to be concentrated in shifts of the unconditional variance. They report that forecasts based on their non-stationary unconditional model were superior to those provided by the stationary GARCH model. Recently, Rapach and Strauss (2008) show that forecasts generated from models that incorporate structural breaks, detected with modified ICSS algorithm, improve the forecasts in the case of exchange rate volatility. Thus, there is robust evidence to suggest that a properly specified GARCH model should account for structural breaks, if such breaks exist.

Interestingly, there is no study which examines the volatility between crude oil prices and stock market prices under structural breaks. The present paper fills a void in the existing literature by explicitly modeling the volatility and shock transmission mechanism between oil and stock market returns using recent daily data allowing for the possibility of structural breaks in volatility. This point is particularly important given the evidence on political unrest/regime changes, geo-political events, financial and economic crises, that can potentially alter the inter-market relationships.

3. Empirical methodology

This section documents how we identify structural breaks in variance. We also describe our univariate and bivariate GARCH models, and discuss how we incorporate structural breaks into our models to accurately capture the underlying volatility dynamics.

3.1. Detecting structural breaks

A structural break in the unconditional variance will result in a structural break in the GARCH process (see Hillebrand, 2005). Inclan and Tiao (1994) provide a cumulative sums of squares (*IT*) statistic to test the null hypothesis of a constant unconditional variance against the alternative hypothesis of a break in the unconditional variance. Andreou and Ghysels (2002) and Sanso, Arrago, and Carrion (2004) show that the *IT* statistic is significantly oversized when used on a dependent process like GARCH. Fortunately, a nonparametric modification can be made to the *IT* statistic which makes it appropriate for a dependent process like GARCH (see Lee & Park, 2001; Sanso et al., 2004).

Inclan and Tiao (1994) propose an iterated cumulative sums of squares (ICSS) algorithm which is based on the *IT* statistic for testing multiple breaks in the unconditional variance. Their algorithm can be applied to the modified *IT* statistic with the nonparametric adjustment to avoid the problems that occur when the standard *IT* statistic is applied to a dependent process. In the present paper, we apply the ICSS algorithm to the modified *IT* statistic for detecting structural breaks in the unconditional variance which is referred in the literature as the "modified ICSS algorithm." We use the standard 5% significance level to test for multiple breaks in the unconditional variance of our return series.[3]

[3] Interested readers are referred to Rapach and Strauss (2008) who provide a comprehensive description as they use this exact methodology to detect structural breaks in the variance of exchange rates.

Table 1
Descriptive statistics.

	Oil returns	Stock returns
Mean	0.00035	0.00020
Std. dev.	0.02510	0.01297
Skewness	−0.17018	−0.22198
Maximum	0.16413	0.10957
Minimum	−0.17091	−0.09469
Kurtosis	7.73681	10.1982
Jarque–Bera	4003 (0.00)	9232 (0.00)
Q(16)	44.12 (0.00)	85.46 (0.00)

Notes: The sample of daily returns is from July 1, 1996 to June 30, 2013. The number of usable observations is 4260. Q(16) is the Ljung–Box statistic for serial correlation. Jarque–Bera statistic is used to test whether or not the series resembles normal distribution. Actual probability values in parentheses. The correlation between returns of oil and stocks is 0.155.

3.2. Univariate GARCH model

We use the benchmark GARCH (1,1) model given as:

$$R_t = \mu + \rho R_{t-1} + \varepsilon_t \tag{1}$$

$$h_t = \omega + \alpha\varepsilon^2_{t-1} + \beta h_{t-1} \tag{2}$$

where R_t represents the corresponding oil or stock market return series and ε_t is normally distributed with a zero mean. h_t represents the conditional variance which depends upon the mean volatility level (ω), the news from previous period (ε^2_{t-1}), and the conditional variance from the previous period (h_{t-1}). The sum of α and β measures the volatility persistence for a given shock. In our analyses, the Q-statistic detected significant autocorrelation in the oil and stock return series and thus an AR(1) specification was used in Eq. (1). The modified ICSS algorithm is applied to the residual series (ε_t) obtained from Eq. (1) to detect structural breaks in the variance.

3.3. Bivariate GARCH model

Here we use the same mean equation as the univariate model but use the popular BEKK parameterization given by Engle and Kroner (1995) for the bivariate GARCH (1,1) model which is given as:

$$H_{t+1} = C'C + B'H_tB + A'\varepsilon_t\varepsilon_t{}'A \tag{3}$$

note that for our bivariate case, C is a 2×2 lower triangular matrix with three parameters and B is a 2×2 square matrix of parameters which relates current levels of conditional variances to past conditional variances. A is a 2×2 square matrix of parameters which measures how conditional variances are correlated with past squared errors. For our bivariate case, the total number of estimated parameters is eleven.

Expanding the conditional variance for each equation in the bivariate GARCH (1,1) model gives:

$$h_{11,t+1} = c^2_{11} + b^2_{11}h_{11,t} + 2b_{11}b_{21}h_{12,t} + b^2_{21}h_{22,t} + a^2_{11}\varepsilon^2_{1,t} + 2a_{11}a_{21}\varepsilon_{1,t}\varepsilon_{2,t} + a^2_{21}\varepsilon^2_{2,t} \tag{4}$$

$$h_{22,t+1} = c^2_{12} + c^2_{22} + b^2_{12}h_{11,t} + 2b_{12}b_{22}h_{12,t} + b^2_{22}h_{22,t} + a^2_{12}\varepsilon^2_{1,t} + 2a_{12}a_{22}\varepsilon_{1,t}\varepsilon_{2,t} + a^2_{22}\varepsilon^2_{2,t}. \tag{5}$$

Eqs. (4) and (5) illustrate how shocks and volatility are transmitted across the two series over time.[4] We use quasi-maximum likelihood estimation with robust standard errors calculated by the method given by Bollerslev and Wooldridge (1992).

[4] The coefficient terms in Eqs. (4) and (5) are a non-linear function of the estimated elements from Eq. (3). Following Ewing and Malik (2005), a first-order Taylor expansion around the mean is used to calculate the standard errors for these coefficient terms.

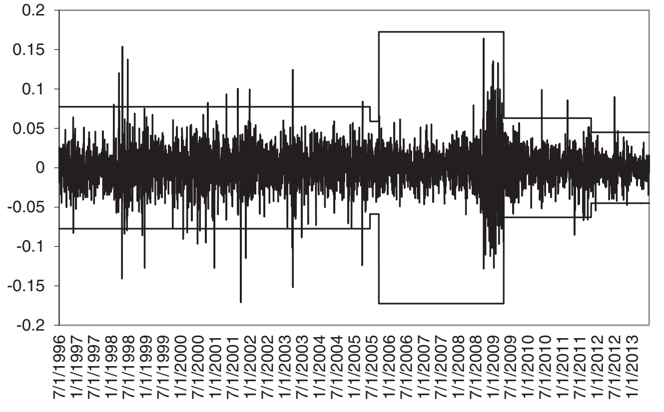


Fig. 1. Daily oil returns. Note: Bands at ± 3 standard deviations, change points estimated using modified ICSS algorithm.

3.4. GARCH models with structural breaks

Lamoureux and Lastrapes (1990) document that standard GARCH models overestimate volatility persistence as they ignore structural breaks and these breaks should be incorporated into a GARCH model to obtain accurate parameter estimates. Consequently, we extend our univariate GARCH model with structural breaks as:

$$R_t = \mu + \rho R_{t-1} + \varepsilon_t \tag{6}$$

$$h_t = \omega + d_1 D_1 + \ldots + d_n D_n + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \tag{7}$$

where, following Lamoureux and Lastrapes (1990), and Aggarwal, Inclan, and Leal (1999), $D_1 \ldots, D_n$, are a set of dummy variables taking a value of one from each structural break point onwards and zero elsewhere.

For our bivariate GARCH model, we follow Ewing and Malik (2005) and add a set of dummy variables to the model given in (3) such that:

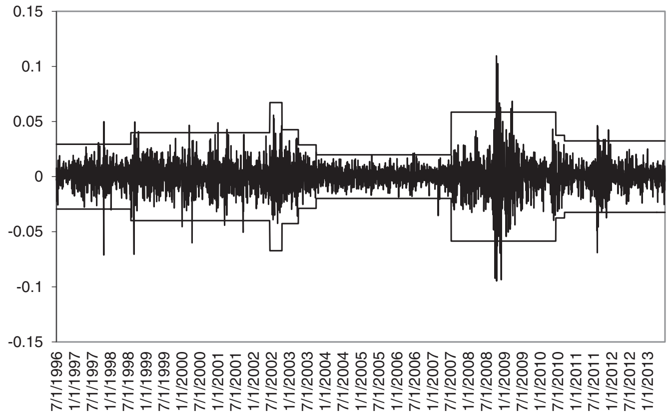
$$H_{t+1} = C'C + B'H_t B + A'\varepsilon_t \varepsilon_t{}' A + \sum_{i=1}^{n} D_i{}' X_i{}' X_i D_i. \tag{8}$$

where D_i is a 2×2 square diagonal matrix of parameters, X_i is a 1×2 row vector of dummy variables and n is the number of detected structural breaks. First (second) element in X_i row vector represents the dummy for first (second) series. If the first series undergoes a volatility break at time t, then the first element will take a value of zero before time t and a value of one from time t onwards.

4. Data

We use daily price data for crude oil and the US stock market from July 1, 1996 to June 30, 2013.[5] Oil data is the daily spot price for West Texas Intermediate, a primary crude stream traded on the domestic market at Cushing, Oklahoma. The data was obtained from the U.S. *Department of Energy*. To be consistent with most studies, the benchmark *S&P 500 Index* is used to track the US stock market and the data was obtained from the *Wall Street Journal*. Consistent with earlier research, returns are used as both series in level form contained

[5] Our sample period starts in 1996 to facilitate comparison with the relevant literature as most of the recent studies on this topic (including most of the cited literature in our paper) use the last 10–15 years of recent data mostly because the global economy and the data generating process of oil and stock market activity was fundamentally different before the 1990s.



Note: Bands at ±3 standard deviations, change points estimated using modified ICSS algorithm.

Fig. 2. Daily stock market returns. Note: Bands at ± 3 standard deviations, change points estimated using modified ICSS algorithm.

a unit root. Table 1 gives descriptive statistics for both return series and shows excess kurtosis which indicates that a GARCH type model is appropriate. The correlation between the return series in our sample is 0.155. A plot of oil and stock returns is shown in Figs. 1 and 2, respectively.

5. Empirical results

The modified ICSS algorithm detected four structural breaks for the oil series and eight break points for the stock return series (see Table 2) and the corresponding volatility regimes (with bands at ± 3 standard deviations) are identified in Figs. 1 and 2. Not surprisingly, we see shifts in variance during the period of the recent financial crisis. In September 2008, the oil series experienced a significant increase in volatility possibly due to turmoil in financial markets. Interestingly, the stock market moved into a high volatility regime in July 2007 as signs of stress in the US real estate market surfaced at that time. Political, economic, social or environmental events may coincide with our detected break points. Most recently, for example, the Great Recession, technological advances in horizontal drilling and hydraulic fracturing, or any of the various international political events. However, markets may anticipate some events in advance or may take extra time to respond to other events, so we do not expect breaks points reported here to precisely coincide with actual real world

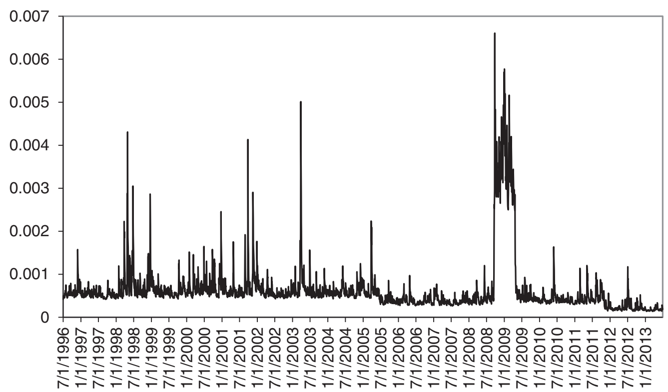


Fig. 3. Conditional variance for the Oil GARCH model.

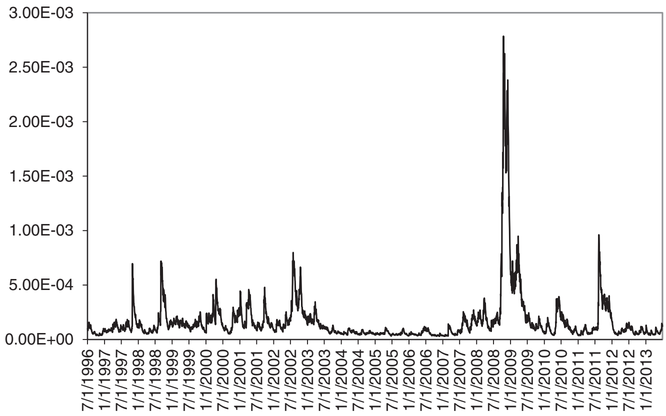


Fig. 4. Conditional variance for the oil GARCH model with breaks.

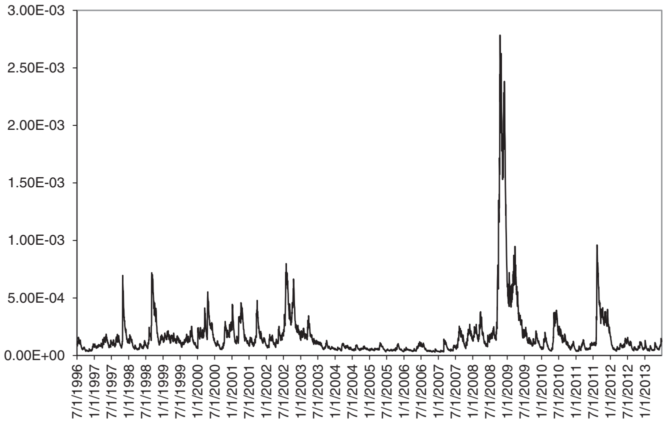


Fig. 5. Conditional variance for the stocks GARCH model.

events. In this paper, we do not purport to identify the causes of the structural breaks but rather our focus is on how these breaks affect volatility dynamics.[6]

Results from our baseline univariate GARCH model are provided in Table 3. We found all parameters to be highly significant with a volatility persistence of 0.985 for the oil series and a volatility persistence of 0.989 for the stock return series, if structural breaks are ignored. This high level of volatility persistence is consistent with earlier studies. We then incorporate the detected structural breaks into our univariate GARCH model by including a set of dummy variables in the variance equation. As can be seen from Table 3, the volatility persistence drops substantially for both the oil and stock markets after accounting for structural breaks, a finding which is consistent with previous literature. The estimated half-life of shocks changes dramatically from about 45 days to about 3 days for the oil market and from 62 days to 16 days for the stock market. This implies that after accounting for breaks a shock is expected to lose half of its original impact in few days.

The significance of structural breaks is further supported by the likelihood ratio statistic (LR). The likelihood ratio statistic is calculated as $LR = 2[L(\Theta_1) - L(\Theta_0)]$ where $L(\Theta_1)$ and $L(\Theta_0)$ are the maximum log likelihood values obtained from the GARCH models with and without structural breaks, respectively. This statistic is asymptotically χ^2 distributed with degrees of freedom equal to the number of restrictions from the more

[6] One should be cautious when looking at news reports for events surrounding these break points as there is a natural bias in media to always cite reasons for sudden market volatility even in cases when markets are adjusting to some previous news event.

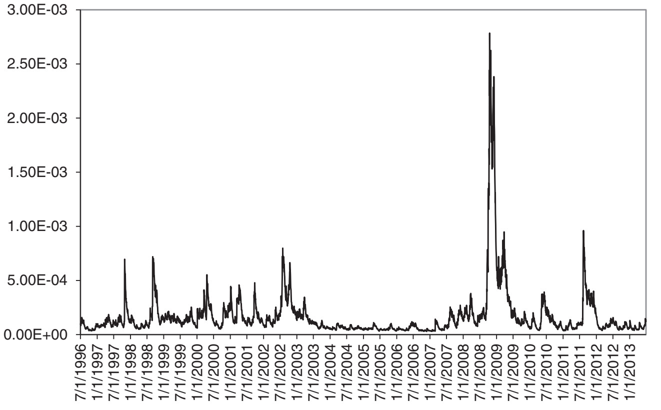


Fig. 6. Conditional variance for the stocks GARCH model with breaks.

general model (with breaks) to the more parsimonious model (without breaks). We reject the null of no change even at the 1% significance level for both oil and stocks models.

The standard residual diagnostics were examined. The correlogram of the standardized residuals was used to test for remaining serial correlation in the mean equation. In all cases, we found that the Q-statistics were not significant implying that the mean equation was correctly specified. Engle (2001) recommends looking at the squared standardized residuals ($\varepsilon_t / \sqrt{h_t}$) to check if the model has captured all the ARCH effects. Thus we look at the Ljung–Box Q-statistics for all estimated models and in all cases (except oil GARCH model without breaks) this commonly used diagnostic test reveals no problems with the model's performance. The kurtosis and skewness in the standardized residuals went down for models after structural breaks are incorporated. However, Jarque–Bera rejects all models for the null of normality at the conventional level of significance. [7]

While our intention is to model the volatility and shock transmission between oil and stock return series allowing for structural breaks, it is helpful to first examine the baseline case of the bivariate GARCH model without structural breaks, which we report in Table 4. Consistent with our univariate GARCH models, we find that both oil and stock return volatility is significantly affected (i.e., caused) by news and volatility in its own respective market. However, it is interesting to note that volatility in either oil or the stock market is not directly affected by news and volatility from the other market (i.e., in the first (second) equation the coefficients for h_{22} (h_{11}) and ε_2^2 (ε_1^2) are not statistically significant). We also find that the indirect impact of volatility in the other market is not significant (i.e., in both the first and second equations, the coefficients for h_{12} and $\varepsilon_1\varepsilon_2$ are statistically insignificant).

The results for the bivariate GARCH model after incorporating structural breaks are presented in Table 5. We still find that both the oil and stock return volatility is significantly affected by news and volatility in its own market. However, what is interesting is that we now find that volatility in both oil and the stock market is significantly affected by the volatility from the other market (i.e., in the first (second) equation the coefficient for h_{22} (h_{11}) is statistically significant). The coefficients which capture the direct volatility transmission across markets are not only statistically significant but these coefficients are quite a bit larger in magnitude than before. It is important to note that each market is affected more strongly by other markets' volatility than its own volatility. Interestingly, the oil market is affected more strongly by the stock market than vice versa. A possible explanation for this is that increased equity price volatility makes financing oil field development more costly, and as oil reserves are a depleting asset, this would translate into a relatively greater degree of oil market risk. We also find that the indirect impact of volatility across markets is now significant (i.e., in both the first and second equation the coefficient for h_{12} is statistically significant). The results further indicate that own volatility impact in each market is smaller in size, consistent with our univariate GARCH results (see smaller coefficient for h_{22} (h_{11}) in Eq. (2) (1)).[8]

[7] Fitted conditional variance plots for all four estimated univariate GARCH models are presented in Figs. 3–6.

[8] For multivariate GARCH models, the overall volatility persistence is calculated by summing all the ARCH and GARCH terms. We do not calculate and report the volatility persistence as some of the coefficients are insignificant and thus interpretation of volatility persistence by summation is not meaningful.

Table 2
Structural breaks in volatility.

Series	Break points	Time period	Standard deviation
Oil return	4	July 1, 1996–June 13, 2005	0.0258
		June 14, 2005–September 14, 2008	0.0196
		September 15, 2008–April 20, 2009	0.0575
		April 21, 2009–October 27, 2011	0.0210
		October 28, 2011–June 30, 2013	0.0150
Stock return	8	July 1, 1996–July 29, 1998	0.0098
		July 30, 1998–June 16, 2002	0.0133
		June 17, 2002–October 17, 2002	0.0224
		October 18, 2002–April 2, 2003	0.0142
		April 3, 2003–October 1, 2003	0.0096
		October 2, 2003–July 9, 2007	0.0066
		July 10, 2007–June 10, 2010	0.0195
		June 11, 2010–September 7, 2010	0.0125
		September 8, 2010–June 30, 2013	0.0108

Notes: Time periods detected by modified ICSS algorithm. Sample period is from July 1, 1996 to June 30, 2013.

The volatility transmission across markets is usually attributed to cross-market hedging and changes in shared information which simultaneously changes expectations across markets as argued by Fleming, Kirby, and Ostdiek (1998). Thus our significant volatility spillover results could be interpreted as an outcome of cross-market hedging.

Here we briefly report the findings from several residual diagnostics. The Ljung–Box test for serial correlation in the cross product between standardized residuals was computed for both estimated models. This statistic will capture serial correlation in the second moments and is popular as a diagnostic test for misspecification in the variance equation. Both of these test statistic values were insignificant at the 10% level implying that no autocorrelation remains in the residuals of the estimated models. It is interesting to note that the skewness and kurtosis was reduced in the cross product between standardized residuals for the models after structural breaks are incorporated. Finally, the Jarque–Bera test rejects both models for the null of normality at conventional levels of significance.

6. Economic implications

Our results have important economic implications because decisions regarding asset pricing, risk management and portfolio allocation require accurate estimation of conditional volatility. In order to understand the

Table 3
Estimation results for univariate GARCH models.

Model	ω	α	β	α + β	Half-life (days)	Log likelihood
Panel A: Oil						
Breaks ignored	9.0E−06 (0.00)	0.061 (0.01)	0.924 (0.01)	0.985	45.86	10,050.28
Breaks accounted for	1.0E−04 (0.00)	0.114 (0.00)	0.695 (0.00)	0.809	3.27	10,092.47
Panel B: Stocks						
Breaks ignored	1.7E−06 (0.00)	0.086 (0.00)	0.903 (0.00)	0.989	62.66	13,285.59
Breaks accounted for	4.3E−06 (0.00)	0.081 (0.00)	0.877 (0.00)	0.958	16.15	13,313.91

Notes: P-values in parenthesis are based on robust standard errors calculated from the method given by Bollerslev and Wooldridge (1992). α + β measures the volatility persistence. Half-life gives the point estimate of half-life (j) in days given as $(\alpha+\beta)^j = ½$. Estimated variance equation without structural breaks for GARCH model is $h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$. Four dummy variables were used for the oil GARCH model with breaks, the coefficient (p-value) were −4.85E−05 (0.0004), 0.000623 (0.0003), −0.000613 (0.0003) and −4.64E−05 (0.0022). Eight dummy variables were used for the stocks GARCH model with breaks, the coefficient (p-value) were 3.93E−06 (0.0215), 1.75E−05 (0.0310), −1.57E−05 (0.0742), −6.07E−06 (0.1531), −1.60E−06 (0.2632), 5.68E−06 (0.0003), −3.91E−08 (0.9941) and −4.54E−06 (0.3921).

Table 4
Results of bivariate GARCH model ignoring structural breaks.

Oil conditional variance equation:

$$\underset{}{h_{11,t+1}} = \underset{(3.65)}{4.41 \times 10^{-6}} + \underset{(99.50)}{0.950h_{11,t}} \underset{(-0.67)}{-0.027h_{12,t}} + \underset{(0.33)}{1.92 \times 10^{-4}h_{22,t}} + \underset{(4.54)}{0.044\varepsilon^2_{1,t}} + \underset{(0.35)}{0.010\varepsilon_{1,t}\varepsilon_{2,t}} + \underset{(0.17)}{5.85 \times 10^{-4}\varepsilon^2_{2,t}}$$

Stocks conditional variance equation:

$$h_{22,t+1} = \underset{(1.20)}{1.59 \times 10^{-6}} + \underset{(0.37)}{3.9 \times 10^{-6}h_{11,t}} + \underset{(0.74)}{0.003h_{12,t}} + \underset{(81.96)}{0.907h_{22,t}} + \underset{(0.62)}{1.1 \times 10^{-4}\varepsilon^2_{1,t}} \underset{(-1.23)}{-0.006\varepsilon_{1,t}\varepsilon_{2,t}} + \underset{(7.74)}{0.084\varepsilon^2_{2,t}}$$

Notes: h_{11} is the conditional variance for the oil return series and h_{22} is the conditional variance for the stock return series. Directly below the estimated coefficients (in parentheses) are the corresponding t-values. The mean equations included a constant term and a lagged return term. Results for the mean equations are not reported for the sake of brevity but are available upon request.

importance of volatility concerning the above financial decisions, we follow the applications provided by Kroner and Ng (1998).

First, let us consider a problem often encountered by portfolio managers which is to compute the optimal fully invested portfolio holding subject to a no-shorting constraint. Assuming zero expected returns and a mean-variance utility function, the risk minimizing portfolio weight is given as $w_t = (h_{22t} - h_{12t})/(h_{11t} - 2h_{12t} + h_{22t})$. The optimal portfolio holding of the stock market portfolio is given as w_t if $0 \le w_t \le 1$, 1 if $w_t > 1$ and 0 if $w_t < 0$. Consequently, the optimal holding of the oil portfolio is given as $1 - w_t$. Based on our results, the model that ignores structural breaks gives an average optimal weight of 0.24 while the model that incorporates structural breaks gives an average of 0.02 as shown in Table 6. A portfolio weight of 0.24 implies that an investor willing to invest \$100 will get minimum risk from a portfolio comprised of oil and stocks if the investor holds \$24 in oil and \$76 in stocks. The correlation between the optimal weight series generated from the two models had a low value of 0.64 underscoring the fact that the model choice matters.

As another example, let us consider the problem of estimating the dynamic risk minimizing hedge ratio using both specifications of our bivariate GARCH model. Kroner and Sultan (1993) show that an investor should short \$$\beta$ of the stock portfolio that is \$1 long in the oil portfolio to minimize the risk of a portfolio, where the 'risk minimizing hedge ratio' β is given as $\beta_t = (h_{12,t}/h_{22,t})$, where $h_{12,t}$ is the conditional covariance between oil and stock returns, and $h_{22,t}$ is the conditional variance of the stock returns. We found the average estimated value of the risk minimizing hedge ratio in the bivariate GARCH model without structural breaks to be 0.34 compared to 0.94 for the model that accounts for structural breaks as shown in Table 6. For example, investors will minimize their potential risk exposure while holding a long position for \$100 in the oil portfolio if they short sell stocks for \$34 using the model without structural breaks and short sell stocks for \$94 for the model with structural breaks. One can also see from this example that the choice of the model affects the estimated hedge ratio and ignoring structural breaks will lead to non-optimal hedging decisions. The correlation between the hedge ratio series generated from the two models had a low value of 0.81 showing again that model choice matters.

Table 5
Results of bivariate GARCH model incorporating structural breaks.

Oil conditional variance equation:

$$h_{11,t+1} = \underset{(3.17)}{6.92 \times 10^{-6}} + \underset{(18.95)}{0.766h_{11,t}} \underset{(-19.78)}{-2.10h_{12,t}} + \underset{(7.05)}{1.44h_{22,t}} + \underset{(3.63)}{0.030\varepsilon^2_{1,t}} \underset{(-0.78)}{-0.08\varepsilon_{1,t}\varepsilon_{2,t}} + \underset{(0.42)}{6.4 \times 10^{-4}\varepsilon^2_{2,t}}$$

Stocks conditional variance equation:

$$h_{22,t+1} = \underset{(0.55)}{1.21 \times 10^{-6}} + \underset{(3.67)}{0.024h_{11,t}} + \underset{(8.87)}{0.268h_{12,t}} + \underset{(19.80)}{0.727h_{22,t}} + \underset{(0.18)}{7.5 \times 10^{-6}\varepsilon^2_{1,t}} \underset{(-0.36)}{-0.001\varepsilon_{1,t}\varepsilon_{2,t}} + \underset{(8.18)}{0.077\varepsilon^2_{2,t}}$$

Notes: h_{11} is the conditional variance for the oil return series and h_{22} is the conditional variance for the stocks return series. Directly below the estimated coefficients (in parentheses) are the corresponding t-values. The mean equations included a constant term and a lagged return term. Results for the mean equations are not reported for the sake of brevity but are available upon request.

Table 6
Summary statistics for portfolio weights and hedge ratios.

	Model ignoring breaks	Model incorporating breaks
Panel A: Portfolio weights		
Mean	0.2412	0.0241
Median	0.1970	0.0481
Maximum	6.5700	3.8900
Minimum	−4.3500	−4.3600
Std. dev.	0.5702	0.6893
Skewness	0.2747	0.1711
Kurtosis	13.3377	5.2794
Correlation	0.64	
Panel B: Hedge ratios		
Mean	0.3396	0.9402
Median	0.4410	0.8560
Maximum	12.300	13.100
Minimum	−10.500	−8.4000
Std. dev.	1.4367	1.5404
Skewness	−0.2847	0.1594
Kurtosis	9.0687	6.2883
Correlation	0.81	

7. Summary and concluding remarks

This paper employs univariate and bivariate GARCH models to examine volatility dynamics of oil and the stock market return series using daily data from July 1, 1996 to June 30, 2013. We detect structural breaks in volatility of oil and stock market returns endogenously using an iterated algorithm. We find significant direct and indirect transmission of volatility between oil and the stock market if structural breaks are incorporated into the model. However, if we (erroneously) ignore structural breaks in variance, then we do not find any direct or indirect volatility spillover effects between these two important markets. This paper makes a timely and essential contribution by accurately estimating the volatility dynamics of oil and the stock market.

Understanding the behavior of volatility in oil and stock prices is not only important for derivative valuation and hedging decisions but also has significant consequences for broader financial markets, the oil industry, and the overall economy. Since many different financial assets are traded based on these series, it is important for financial market participants to understand the volatility transmission mechanism across these series over time in order to make appropriate decisions. We compute optimal portfolio weights and dynamic risk minimizing hedge ratios to highlight the significance of our findings. Our results support the idea of cross-market hedging and sharing of common information by market participants.

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