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Time Series Estimation of the Bond Default Risk Premium

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The bond default risk premium, measured by the spread between higher and lower grade bond returns, is often estimated with univariate time series procedures and used as an input in financial models. In this paper, time series properties of the historical default risk premium are analyzed and forecasting results from univariate time series models are compared. An autoregressive model with an overreaction component provides the best statistical fit for the bond default risk premium series. A random walk model exhibits the worst fit. The findings are robust over a variety of model specifications and measurement choices. For all forms of the time series process the univariate time series models explain a small percentage of the variation in the default risk premium, raising questions about traditional approaches to estimating the expected default risk premium.

I. INTRODUCTION

Practical and theoretical financial models both use the expected bond default risk premium, measured by the forecasted spread between higher and lower grade bond returns, as a measure of market risk aversion. Univariate time series models are used, either explicitly or implicitly, to forecast the expected bond default risk premium. Nevertheless, a number of contradictions exist in the literature with respect to the premium's assumed time series properties. Some authors assume that the historic series follows a random process with a stationary mean while others assume that an autoregressive model is at work. For example, Berry, Burmeister, and McElroy (1988) demonstrate the Arbitrage Pricing Model with an expected default risk premium equal to the historic mean of the series, implicitly assuming a random variable with a stationary mean. In another application, Fama and French (1989, 1993) use a first-order autoregressive model to estimate the expected default risk premium. Weak form market efficiency applied to default risk calls for a random walk series where the expected bond default risk premium is the current premium.

In this paper, time series properties of the default risk premium are analyzed to determine whether the series is best described as an autoregressive process, random variable, or random walk. Empirical tests are conducted to identify the time series process for historic default risk premiums. One-step ahead forecasts for each of the alternative models are also evaluated. The paper's findings are relevant for a number of default premium applications.[1]

II. AUTOREGRESSIVE, RANDOM VARIABLE, AND RANDOM WALK TIME SERIES MODELS

The bond default risk premium (hereafter, noted by *DRP*) is modeled as an autoregressive time series, following an approach used for parameter stability tests by Bos and Newbold (1984):

$$DRP_{t+1} = \alpha DRP_t + (1 - \alpha) DRP_{\text{avg.}} + e_{t+1} \tag{1}$$

where $(1 - \alpha)$ is the rate of autoregression and e_{t+1} is the random error. The model in Equation 1 implies that the current bond default risk premium regresses back to the long run underlying mean $DRP_{\text{avg.}}$ at a rate of $(1 - \alpha)$. The value of α determines the strength of the autoregression. If the default premium takes more than one period to return to the underlying mean, an autoregressive process describes the time series and α will be significantly different from zero with a value between one and negative one. If the value of α is one, the model reduces to a random walk:

$$DRP_{t+1} = DRP_t + e_{t+1} \tag{2}$$

where the default risk premium varies randomly with no tendency to revert to the underlying mean. In this case, the best proxy for the default risk premium is the last observed value (DRP_t). If the value of α is zero, the model reduces to the following random variable process:

$$DRP_{t+1} = DRP_{\text{avg.}} + e_{t+1} \tag{3}$$

where deviations from the underlying mean are limited to single periods and are serially uncorrelated. For this model the underlying mean is the best proxy for the default risk premium. The appropriate proxy for the expected default risk premium depends on which model, Equation 1, Equation 2, or Equation 3, best describes the time series of historical default risk premiums. The following empirical model is used to identify the default risk premium time series process:

$$DRP_{t+1} = b_0 + b_1 DRP_t + e_{t+1} \tag{4}$$

where

DRP_{t+1} = the return differential between the total return on long-term corporate bonds and the total return on long-term government bonds in period $t + 1$;
DRP_t = the return differential between the total return on long-term corporate bonds and the total return on long-term government bonds in period t;
b_0 = the rate of reversion to the mean times the underlying mean of the series: $(1 - \alpha)DRP_{\text{avg.}}$;
b_1 = the parameter α in Equation 1;
e_{t+1} = a random error term with a mean of zero and a variance of σ_e^2.

If b_1 is significant and equal to one the empirical model reduces to a random walk. If b_1 is not statistically different from zero, the model reduces to a random variable with an underlying mean of b_0. If b_1 is significantly different from zero and has a value between a negative one and a positive one, the default risk premium follows an autoregressive process.

III. DEFAULT RISK PREMIUMS, DATA, AND HOLDING PERIODS

Total rate of return data for long term corporate bonds and government bonds are taken from the major source of time series data on market returns, Ibbotson's (1991) Stock Bond and Bill Index Yearbook.[2] Ibbotson derives corporate bond total rate of return data from index returns provided by Salomon Brothers'.[3] For the period from 1968 through 1991 Ibbotson uses the High-Grade Long-Term Corporate Bond Index. For the period from 1926 through 1968 Ibbotson constructed total rates of return on long term corporate bonds by using yield indexes and calculated capital gains. Ibbotson's yield index data are taken from the Salomon Brothers' monthly yield index from 1946 through 1968 and from the Standard and Poor's monthly High-Grade Corporate Composite yield data for the period from 1925 through 1945. Ibbotson calculated monthly capital gains for the corporate bond index. Monthly total rates of return for the period from 1926 through 1968 are equal to the calculated capital gains plus income.

Ibbotson obtained total returns on long-term U.S. Government bonds over the 1926-1976 period from the Government Bond file at the Center for Research in Security Prices at the University of Chicago. Total returns are calculated for selected government bonds whose returns do not reflect potential tax benefits, impaired negotiability, or special redemption or call privileges. From 1977 to 1991, Ibbotson employed the same procedure with data from the *Wall Street Journal*. Total returns are calculated as the change in the flat price, defined as the average of the bid and ask prices plus the accrued coupon. Prior to January 1985, the number of half-months instead of the number of days is used in the calculation.[4]

Estimation of Equation 4 requires the default risk premium to be measured over a holding period. The shortest period available in the Ibbotson return file is monthly. A monthly holding period is also consistent with most applications of default risk premium estimates. Longer holding periods result in a loss of degrees of freedom, but an annual holding period is also used to allow for a comparison. Equation 4 is estimated for monthly and annual holding periods from February 1926 through December 1975. January 1976 through December 1990 is used as a holdout sample for one-step-ahead forecasts.

IV. PARAMETER STABILITY TESTS

Fama and French (1989) suggest eliminating the Great Depression period from empirical studies of risk premiums. Kim, Nelson and Startz (1991) find that distortions surrounding the Great Depression and World War II cause a much different time series for financial market measures, although they do not study default risk premiums. Tinic and West (1984) point out the importance of January effects in the studies of risk premiums. To address these concerns the autoregressive model of Equation 4 is extended to allow empirical tests of the stability of the intercept and slope (rate of autoregression) in January and key subperiods. Dummy variables are created for January months ($DJ = 1$ if the month is January), the depression subperiod ($DD = 1$ if the observation is occurs during October 1929 through December 1940), and World War II ($DW = 1$ if the observation occurs during December 1941 through August 1945). The extended empirical model with dummy variables and interactions between dummy variables and DRP is specified as follows:

$$\begin{aligned} DRP_{t+1} = \alpha_0 + \alpha_1 DRP_t + \beta_0 DJ_t + \beta_1 (DJ_t) DRP_t + \pi_0 DD_t \\ + \pi_1 (DD_t) DRP_t + \gamma_0 DW_t + \gamma_1 (DW_t) DRP_t + \varepsilon_{t+1} \end{aligned} \tag{5}$$

Student t-statistics for the coefficients of the dummy variables and dummy variable interactions with DRP provide tests for stability of the intercept and slope coefficients. For example, when the period is October 1929 through December 1940 ($DD = 1$) the intercept is equal to $\alpha_0 + \pi_0$. A positive (negative) and statistically significant t-statistic for the coefficient π_0 is interpreted as a higher (lower) intercept for the October 1929 to December 1940 period. When $DD = 1$ the slope of the empirical model is $\alpha_1 + \pi_1$. A positive (negative) and statistically significant t-statistic for the coefficient π_1 is interpreted as a higher (lower) slope (rate of autoregression) for the depression era.

The dummy variable model in Equation 5 provides statistical tests for stability of the parameters of the autoregressive time series model for specific subperiods. A more general test of stability of the parameters of the basic model over the entire estimation period is provided by tests conducted on a time varying coefficient model. The model appearing in equation (4) does not have time subscripts on the coefficients, reflecting an implicit assumption that the parameters are stable. A more general model is specified below:

$$DRP_{t+1} = b_{0,t} + b_{1,t}DRP_t + e_{t+1} \tag{6}$$

Two state equations are then expressed as follows:

$$b_{0,t} = x_0 b_{0,t-1} + (1 - x_0)b_{0,\text{avg.}} + u_{0t} \tag{6'}$$

$$b_{1,t} = x_1 b_{1,t-1} + (1 - x_1)b_{1,\text{avg.}} + u_{1t} \tag{6''}$$

If the standard error for a state equation is zero (variance of $u_i = 0$) then $b_{it} = b_{i,t-1}$ and the coefficient is constant. If the standard errors of both state equations are zero the empirical model collapses to the constant coefficient model of Equation 4. A likelihood ratio test with a chi-square distribution is used to test the hypothesis of a constant coefficient model.[5]

V. RESULTS

Time series regression results for the basic autoregressive model of Equation (4) and the dummy variable model of Equation 5 are reported in Table 1. The slope coefficient (b_1) is negative and significant for both annual and monthly holding periods in the basic model and the dummy variable model. The negative value of b_1, which is significantly different from zero ($\mathbf{H_0}$: $b_1 = 0$) and minus one ($\mathbf{H_0}$: $b_1 = -1$), implies an autoregressive process where the underlying mean is more important than the most recent observation. Unlike the usual autoregressive process where disturbances adjust back toward the underlying mean, the negative value of b_1 indicates a process of alternating overreactions after a disturbance. A disturbance higher than the underlying mean results in a correction process that tends to overcorrect, resulting in the next observation having a value lower than the mean. The process of smaller and smaller overcorrection continues until the observations converge on the underlying mean.

Student-t statistics for the dummy variable coefficients and the dummy variable interaction coefficients are not statistically significant for any of the coefficients except the January intercept term. The bond default risk premium has a significantly higher intercept in January, but the rate of autoregression is

Table 1. **Empirical Results from Time Series Estimation of Autoregressive Models of the Default Risk Premium (*DRP*)**
(February 1926 through December 1975 with Dummy Variable Tests for January Effects (*DJ*), Depression Era Effects (*DD*), and World War II Effects (*DW*))

Independent Variable	Monthly Data: Basic AR Model	Monthly Data: Dummy Variable Model	Annual Data: Basic AR Model	Annual Data: Dummy Variable Model
Intercept	0.0082	0.0033	0.0099	0.6199
	(1.6478)*	(0.0705)	(2.2087)*	(1.5130)
DRP_t	−0.2215	−0.2014	−0.3256	−0.3296
	(−5.5510)***	(−4.6586)***	(−2.3149)**	(−2.2319)**
DJ		0.6996		
		(4.7194)***		
$(DJ)DRP_t$		0.0193		
		(0.1706)		
DD		0.1028		1.3868
		(0.9443)		(1.3462)
$(DD)DRP_t$		−0.0977		0.0586
		(−1.3509)		(0.2168)
DW		0.0592		0.1713
		(0.3344)		(0.0839)
$(DW)DRP_t$		−0.1740		−1.5703
		(−0.5172)		(−1.0731)
F-Statistic	30.8138***	10.0655***	5.3589***	2.0994*
Adjusted R^2	.0475	.0716	.0833	.0769
Box-Pierce q-Statistic	7.4586		1.6912	
Durbin-Watson	2.0468	2.033	2.040	2.145

Notes: The t-statistic for each estimated coefficient is in parentheses ( ). Levels of significance for the statistics are identified as follows:
* significant at the .10 level
** significant at the .05 level
*** significant at the .01 level
The 95% confidence interval for the t-test of the *DRP* coefficient does not include either 0 or −1.

not affected by the January effect. The insignificant t-statistics on the dummy variable and dummy variable interaction variables suggests insignificant differences in the intercept and slope coefficients of the empirical model for the depression and World War II periods. The results from the dummy variable model support a stable time series process.

The F-statistics for both the basic model and the dummy variable model are significant, but the variation in the default premium explained by the models is very low, as indicated by the small R^2. Several tests on the residuals of the empirical models are conducted. The Durbin-Watson test statistics for all the models in Table 1 are well within the upper and lower limits for the model, indicating an absence of higher order autoregressive lags. The Box-Pierce test statistic for the null hypothesis that the model's errors are independent is approximately distributed as a Chi-square statistic. The insignificant values of the Box-Pierce statistic, constructed for lags up to a 5th order model, in Table 1 suggesting an absence of higher order lags. Finally, Box-Jenkins analysis is also conducted to test for higher order lags but the results are lengthy and are not reported to conserve space. The analysis does not reveal a significant reduction in the model's residual sum of squares for higher order lags of the *DRP* variable, confirming the first-order autoregressive specifications.[6]

Several additional test procedures are used to check model specifications. First, estimates of the variances of the error terms in the state equations of the time varying parameter model of equation (6') and (6") are found to be zero. The zero values of the variance estimates and the resulting zero value of the Chi-square test statistic for time variation support a constant coefficient model. Second, potential time variation in the variance of the error term of the model is tested with GARCH analysis. The variances of the residuals for the models in Table 1 are found to be constant, confirming the assumption of homoskedastic errors.[7] Overall, the results of the basic model are remarkably robust over a wide range of time periods and measurement alternatives.[8]

VI. ONE-STEP-AHEAD FORECASTING

In this section, forecasting accuracy of the AR1 model is evaluated. One-step-ahead forecasting results of random variable, random walk, and the estimated autoregressive models are presented for the holdout period from January 1976 through December 1990. Mean squared error (MSE) and mean absolute error measures are constructed for both annual and monthly holding periods based on differences between estimated and observed risk premiums. The MSE is a good measure of forecasting accuracy since it is equivalent to the variance of the prediction errors and penalizes large forecast errors more than small errors. Forecasting results are reported in Table 2.

Table 2. **Measures of One-Step-Ahead Forecasting Accuracy for a Holdout Period** (January 1976–December, 1990)

Model	Mean Squared Error	Mean Absolute Error
Monthly	1/76-12/90*	
Auto-Regressive	.000114	.007292
Auto-Regressive with January Effect	.000124	.007442
Random Walk	.000293	.012089
Random Variable	.000120	.007593
Annual	1976-1990**	
Auto-Regressive	.000535	.018625
Random Walk	.001162	.030133
Random Variable	.000490	.017057

Notes: * The sample size is 180.
** The sample size is 15.

There is minimal difference in forecasting accuracy between the autoregressive and random variable models. The autoregressive model with a January effect does not improve the forecasting accuracy of the basic autoregressive model. The autoregressive model is slightly more accurate for monthly holding periods, while the random variable model has less error for annual holding periods. This result is not surprising given the weight placed on the underlying mean relative to the most recent observation in the autoregressive findings. The overreaction patterns have time to cancel out over longer periods, leaving the mean value. The random walk model has very poor performance compared to the other models. These results are robust with respect to the holding period and the measure of forecasting accuracy.

VII. CONCLUSION

This paper examines time series properties of the default risk premium for different holding periods over the period January 1926 through December 1990. The analysis indicates that the default risk premium time series follows an autoregressive pattern. The finding is robust over a variety of alternative specifications and is stable over time. While the autoregressive model offers the best statistical fit for the bond default risk premium series, one-step ahead forecasts suggest that either an autoregressive or random variable model may be used with little loss in forecasting accuracy. However, the random walk model exhibits significantly more error and is a poor model for forecasting. This risk premium inefficiency is a topic for future research. The significant January effect finding for the intercept coefficient suggests a higher default premium proxy for January months. This is an important consideration since spurious January effects will be found in applications of the premium to pricing models if the same proxy is used for January as for other months of the year.

Several extensions are beyond the scope of this paper but are worthy of future research. The autoregressive nature of the default risk premium time series suggests market overreaction in bond markets. Further research is needed to determine if the overreaction can be exploited. Also, traditional univariate approaches to finding a proxy for expected default risk premiums (such as using the historic mean value) do not explain much of the variation in the premium. All univariate models have a low R-squared value. An analysis of multivariate time series models is needed to build better forecasting procedures. Finally, an analysis of the sensitivity of pricing models to risk premium forecast procedure is needed.

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NOTES

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1. Practical applications of bond default risk premiums are abundant. For example, see discussions of active intermarket swaps by Strong and Hall (1986). Implied in these trading strategies is an autoregressive default premium series with an underlying normal level of the default premium and a predictable correction process.

2. Realized total rates of return reflect changes in the yield spread and show the market's actual reward for risk. The total rate of return is used in this study since it is most commonly used in academic studies to measure the bond default risk premium.

3. Data provided by a major dealer is appropriate because most large corporate bond transactions occur in the over-the-counter market.

4. The adjustment for number of days is a minor issue since most bond coupons are paid on the 15th of the month.

5. A Kalman filter approach is used to obtain maximum likelihood estimates of the state-space parameters in a time varying coefficient model. The test for time varying coefficients is provided by a likelihood ratio test. For an in-depth discussion of the estimation process and test statistics for the state-space model see Kahl and Ledolter (1983).

6. For a complete discussion of the Box-Jenkins time series analysis procedure see Box and Jenkins (1976). The basic procedure tests for significant reductions in the residual sum of squares of the model when a higher order lag structure is introduced. The results of this analysis are lengthy and are not reported here to conserve space.

7. A generalized autoregressive conditional heteroskedastic estimation technique (GARCH) is employed to test for the presence of heteroskedastic errors. The hypothesis of constant variance of the residuals (homoskedasticity) can not be rejected. Complete test results are not reported to conserve space. For a more detailed reference to GARCH analysis see Bollerslev (1986).

8. Log transformations of the *DRP* variable are used to allow for potential sensitivity of the analysis to the time series of percentage changes rather than levels of the default risk premium. Test results for coefficients of the models in Table 1 were not affected by this transformation. The results of the estimation procedures using the transformation of *DRP* are not reported since they are not significantly different from the findings in Table 1.

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