Hi Sergey, Here is a simple way to give the main idea. Best, Marc

Proof technique - Basic Steps

For simplicity, assume $f_* = 0$ and f smooth

Key assumptions

1 Sufficient Decrease in function values

$\exists\, C > 0$ s.t $f(x_{k+1}) - f(x_k) \leq -C\, |x_{k+1} - x_k|^2$

2 Gradient bound for iterates gap

$\exists\, D > 0$ s.t. $D\, |\nabla f(x_k)| \leq |x_{k+1} - x_k|$

3 f is a KL-function

$\exists$ a concave desingularizing function φ s.t.

$$\varphi'(f(x_k)) \cdot |\nabla f(x_k)| \geq 1$$

Equipped with 1 → 3 we can then show that $\{x_k\}$ is a Cauchy sequence as follows.

Basic Proof steps

Using 1 + 2 $\Longrightarrow$

$$f(x_{k+1}) \overset{1}{\leq} f(x_k) - C\,|x_{k+1}-x_k|\cdot|x_{k+1}-x_k|$$

$$\overset{2}{\leq} f(x_k) - CD\,|x_{k+1}-x_k|\cdot|\nabla f(x_k)|$$

Now since φ is concave :

$$\varphi(u) \leq \varphi(\bar{u}) + (u-\bar{u})\,\varphi'(\bar{u}) \Longrightarrow$$

$$\varphi(f(x_{k+1})) \leq \varphi(f(x_k)) + (f(x_{k+1})-f(x_k))\,\varphi'(f(x_k))$$

Therefore, using 1 and 3 KL $\Longrightarrow$

$$\leq \varphi(f(x_k)) - CD\,\varphi'(f(x_k))\,|x^{k+1}-x^k|\,|\nabla f(x_k)|$$

$$\overset{3}{\leq} \varphi(f(x_k)) - CD\,|x^{k+1}-x^k|$$

$$\Rightarrow CD\,|x_{k+1}-x_k| \leq \varphi(f(x_k)) - \varphi(f(x_{k+1}))$$

Hence, telescoping $\Longrightarrow \sum |x_{k+1}-x_k| < \infty$ $\square$