CSE 280A

Assignment ~~2~~

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List 8. || (3)

(3) I. The hint:

$$p_{ki} = \frac{\binom{n-i-1}{k-2}}{\binom{n-1}{k-1}}$$

the same stars and bars. We put aside any i elements to put to one specific tree. All others trees: n−i are put in k−1 But S&B theorem: $\binom{n-i-1}{k-2}$

stars and bars

Place n (n indiv.) obj between k bars so at least 1 indiv in each bar (k trees) (in each tree)

By stars & bars theorem: $\binom{n-1}{k-1}$.

Note: we don't take into account the topology of trees. Only placing vertices to various trees

II.

$$p_{ki} = \frac{\binom{n-i-1}{k-2}}{\binom{n-1}{k-1}} = \frac{(n-i-1)!}{\cancel{(k-2)!}\,(n-i-k+1)!} \cdot \frac{\overset{k-1}{\cancel{(k-1)!}}\,(n-k)!}{(n-1)!} =$$

$$= \frac{(n-k)!}{(n-k-i+1)!\,(i-1)!} \cdot \underbrace{\frac{(i-1)!\,(n-i-1)!}{(n-1)!}}_{=\frac{1}{\binom{n-1}{i}}\cdot i} \cdot (k-1) = \boxed{\binom{n-k}{i-1} \Big/ \binom{n-1}{i} \cdot \frac{k-1}{i}}$$

III. To get $\mathbb{E} m_i = \frac{\theta}{i}$ we need to write m_i in an indicator fashion. Let ξ_{kl} be the size of l-th lineage at epoch k. Also let n_{kl} be # of muts occured in l-th lineage at epoch k.

$$\Rightarrow m_i = \underbrace{\sum_{k\ge 2}^{n}}_{\text{all epochs}} \sum_{l=1}^{k} \underbrace{\mathbb{1}[\xi_{kl}=i]}_{\text{we need the epoch to be the size } i} \cdot n_{kl}$$

$$\Rightarrow \mathbb{E} m_i = \sum_{k\ge 2}^{n} \sum_{l=1}^{k} \underbrace{P(\xi_{kl}=i)}_{=p_{ki}} \underbrace{\mathbb{E}(n_{kl} \mid \xi_{kl}=i)}_{=\frac{\theta}{k}} = \frac{1}{i}\sum_{k=2}^{n} \frac{k(k-1)}{\binom{n-1}{i}} \binom{n-k}{i-1}\,\theta = \frac{\theta}{i}$$