

④ By valid I suppose we mean unbiased.

①  $\theta_{FL} = m_1$

$$E \theta_{FL} = E m_1 = \frac{\theta}{1} = \theta \Rightarrow \text{unbiased}$$

②  $\theta_{w_1} = \frac{1}{\sum_{i=1}^{n-1} \frac{1}{i}} \sum_{i=1}^{n-1} m_i$

$$E \theta_w = \frac{1}{\sum_{i=1}^{n-1} \frac{1}{i}} \cdot \sum_{i=1}^{n-1} \frac{\theta}{i} = \theta \Rightarrow \text{unbiased}$$

③  $\theta_L = \frac{1}{n-1} \sum_{i=1}^{n-1} i m_i$

$$E \theta_L = \frac{1}{n-1} \sum_{i=1}^{n-1} i \frac{\theta}{i} = \theta \Rightarrow \text{unbiased}$$

④  $\theta_{\pi} = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} i(n-i) m_i$

$$E \theta_{\pi} = \frac{2}{n(n-1)} \cdot \underbrace{\sum_{i=1}^{n-1} i(n-i)}_{\frac{n(n-1)}{2}} \cdot \frac{\theta}{i} = \theta \Rightarrow \text{unbiased}$$

⑤  $\theta_H = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} i^2 m_i$

$$E \theta_H = \frac{2}{n(n-1)} \cdot \underbrace{\sum_{i=1}^{n-1} i^2}_{\binom{n}{2}} \cdot \frac{\theta}{i} = \theta \Rightarrow \text{unbiased}$$

⑤ I suppose that we have set of reads with <sup>(nearly)</sup> uniform coverage across genome and the same sequencing depth for all individuals. ~~+ reads are the~~

We can build the "SNP matrix" but for reads (sort of...)

The Tajima estimate could be calculated between reads in such a way:

where  $l_g$  is the len of genome and  $l_r$  is the len of reads

$$\theta_F = \frac{1}{\binom{n}{2}} \sum_{i,j} \pi_{ij} \frac{l_g}{l_r}$$

#reads

hamming distance between reads

$$\theta_{\pi} = \frac{2}{n(n-1)} \sum_{i=1}^{n-1} i(n-i) m_i \quad \text{where } n = \text{\# of reads / coverage depth per individual} \quad \text{and } m_i = \text{\# mutations in } i\text{th indiv. / coverage depth per indiv.}$$