Тесты

Тест 1

```
format long;
x_0 = -1;
x_n = 1;
t = sym('t');
q =@(t)(-(1+t/2).*(t-3));
r =@(t)(exp(t/2).*(t-3));
f =@(t)(-(2-t).*(t-3));
alpha = [1 0 0];
beta = [1 0 0];
syms u(t);
Du = diff(u);
```

Решаем следующий дифур:

```
diff(u,2) + q*diff(u) - r*u == f
```

```
ans(t) =
```

$$\frac{\partial^2}{\partial t^2}\,u(t) - \left(\frac{t}{2}+1\right)\,(t-3)\,\frac{\partial}{\partial t}\,u(t) - \mathrm{e}^{t/2}\,u(t)\;\;(t-3) = (t-2)\;\;(t-3)$$

Краевые условия:

```
alpha(1)*u(-1)-alpha(2)*Du(-1) == alpha(3)
```

```
ans = 
```

$$u(-1) = 0$$

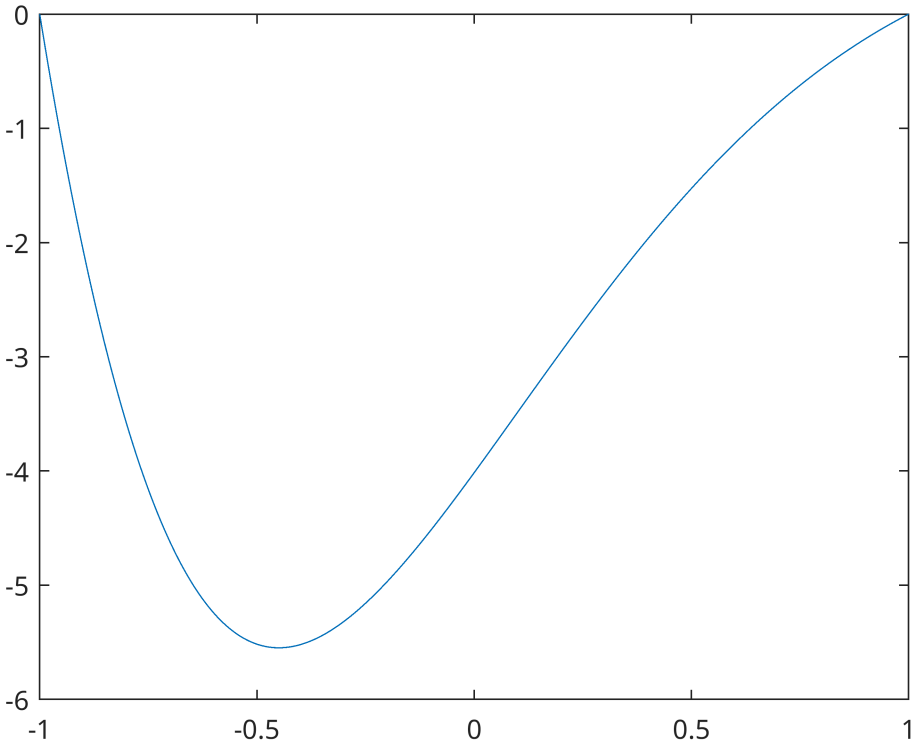
```
beta(1)*u(1)+beta(2)*Du(1) == beta(3)
```

```
ans = 
```

$$u(1) = 0$$

Точное решение:

```
fun = @(x,y) [y(2); (1+x/2)*(x-3)*y(2)+(exp(x/2)*(x-3))*y(1)-(2-x)*(x-3)];
bc = @(ya, yb) [ya(1); yb(1)];
xmesh = linspace(-1, 1, 1000);
solinit = bvpinit(xmesh, [0 1]);
sol = bvp4c(fun, bc, solinit);
plot(sol.x,sol.y(1,:));
```



Численное решение:

```
epsilon = 1e-6;
[u_ch, x_ch, d] = Richardson(x_0, x_n, q, r, f, alpha, beta,epsilon);
```

```
Количество итераций:
    11
```

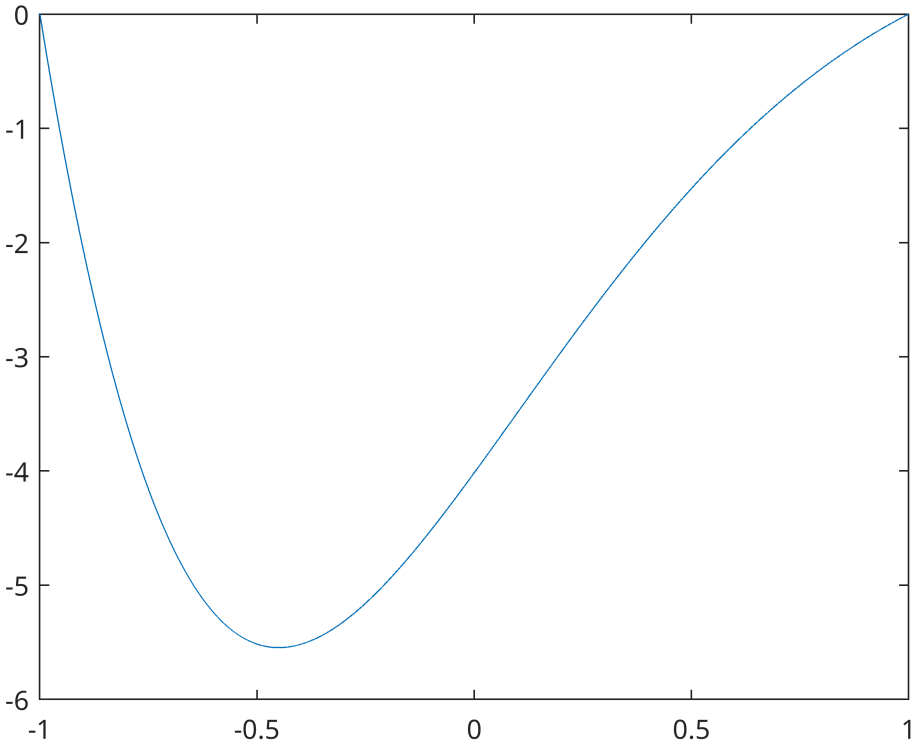
```
%h = x(2) - x(1);
```

Погрешность:

```
d
```

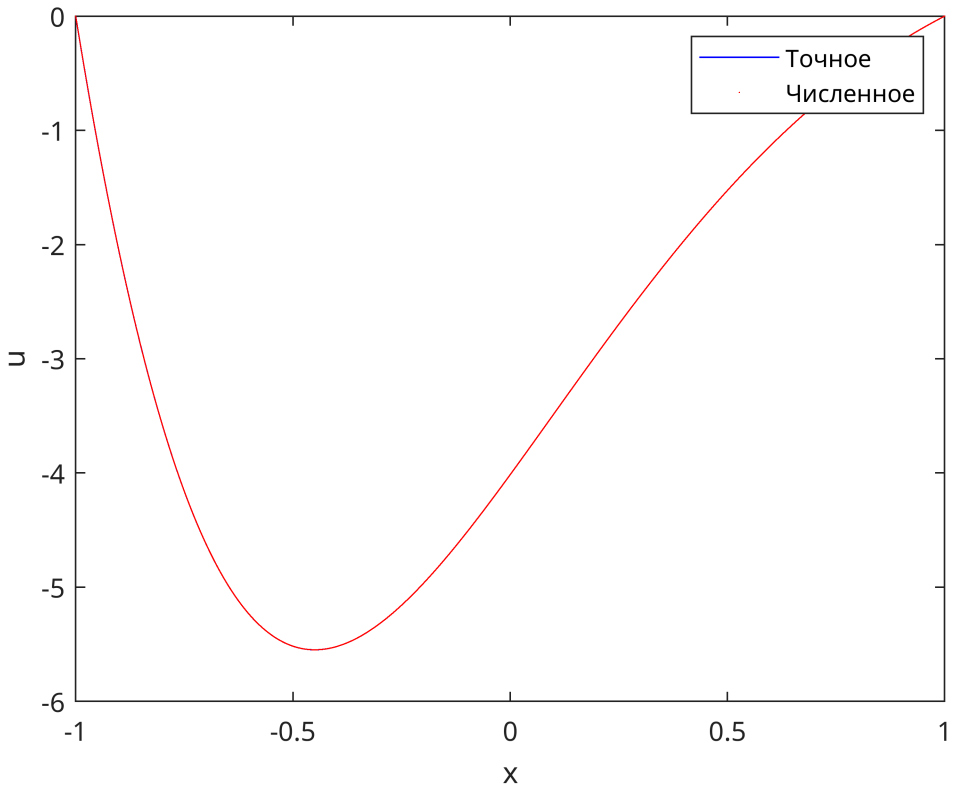
```
d =
     2.925574461206300e-07
```

```
%[u_ch,x_ch] = net_met(x_0, h, x_n, q, r, f, alpha, beta);
plot(x_ch,u_ch);
```



На одном графике:

```
plot(sol.x,sol.y(1,:),'-b',x_ch,u_ch,'r.','MarkerSize',1);
ylabel('u');
xlabel('x');
legend('Точное', 'Численное');
```



Тест **2**

```
format long;
x_0 = -1;
x_n = 1;
t = sym('t');
q =@(t)(-(1+t/2).*(5+2*t)./(4 + t));
r =@(t)((1 + exp(t/2)).*(5+2*t)./(4 + t));
f =@(t)(-(2+t).*(5+2*t)./(4 + t));
alpha = [0.5 1 -0.2];
beta = [0.3 1 -0.3];
syms u(t);
Du = diff(u);
```

Решаем следующий дифур:

```
diff(u,2) + q*diff(u) - r*u == f
```

```
ans(t) =
```

$$\frac{\partial^2}{\partial t^2}\,u(t) - \frac{u(t)\,(2\,t+5)\,(\mathrm{e}^{t/2}+1)}{t+4} - \frac{\left(\frac{t}{2}+1\right)\,(2\,t+5)\,\frac{\partial}{\partial t}\,u(t)}{t+4} = -\frac{(2\,t+5)\,(t+2)}{t+4}$$

Краевые условия:

```
alpha(1)*u(-1)-alpha(2)*Du(-1) == alpha(3)
```

```
ans =
```

$$\frac{u(-1)}{2} - \left(\left(\frac{\partial}{\partial t}\, u(t)\right)\Big|_{t=-1}\right) = -\frac{1}{5}$$

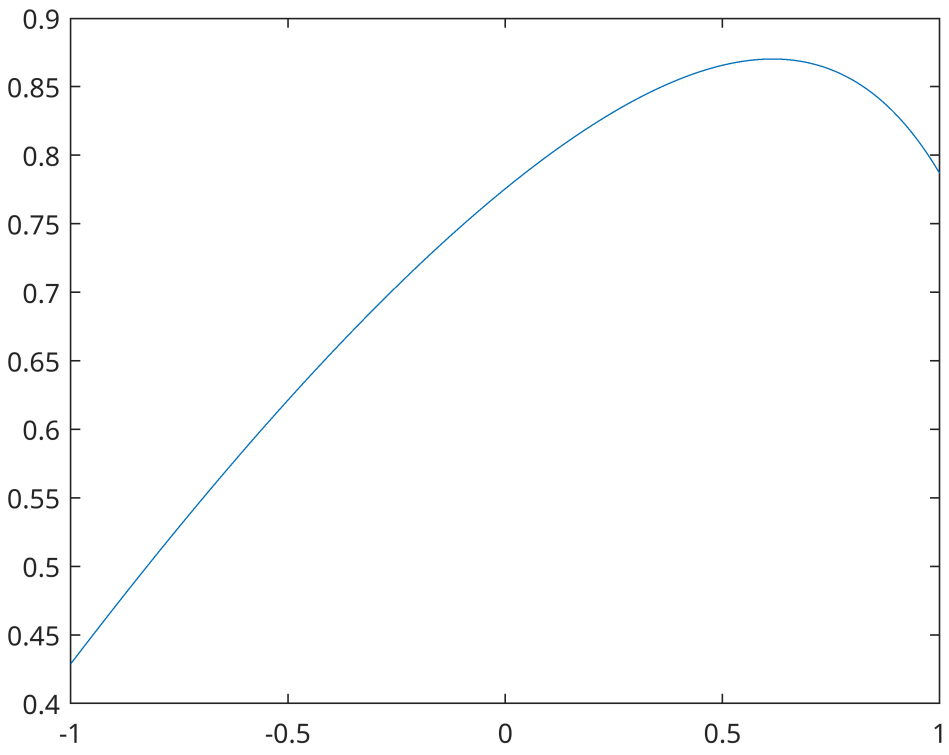
```
beta(1)*u(1)+beta(2)*Du(1) == beta(3)
```

```
ans =
```

$$\frac{3\,u(1)}{10} + \left(\left(\frac{\partial}{\partial t}\, u(t)\right)\Big|_{t=1}\right) = -\frac{3}{10}$$

Точное решение:

```
fun = @(x,y) [y(2);
    (1+x/2)*(5+2*x)/(4 + x)*y(2)+((1 + exp(x/2))*(5+2*x)/(4 + x))*y(1)-(2+x)*(5+2*x)/(4
bc = @(ya, yb) [0.5*ya(1)-ya(2) + 0.2; 0.3*yb(1) + yb(2) + 0.3];
xmesh = linspace(-1, 1, 1000);
solinit = bvpinit(xmesh, [0 1]);
sol = bvp4c(fun, bc, solinit);
plot(sol.x,sol.y(1,:));
```



Численное решение:

```
epsilon = 1e-6;
[u_ch, x_ch, d] = Richardson(x_0, x_n, q, r, f, alpha, beta,epsilon);
```

```
Количество  итераций:
     9
```

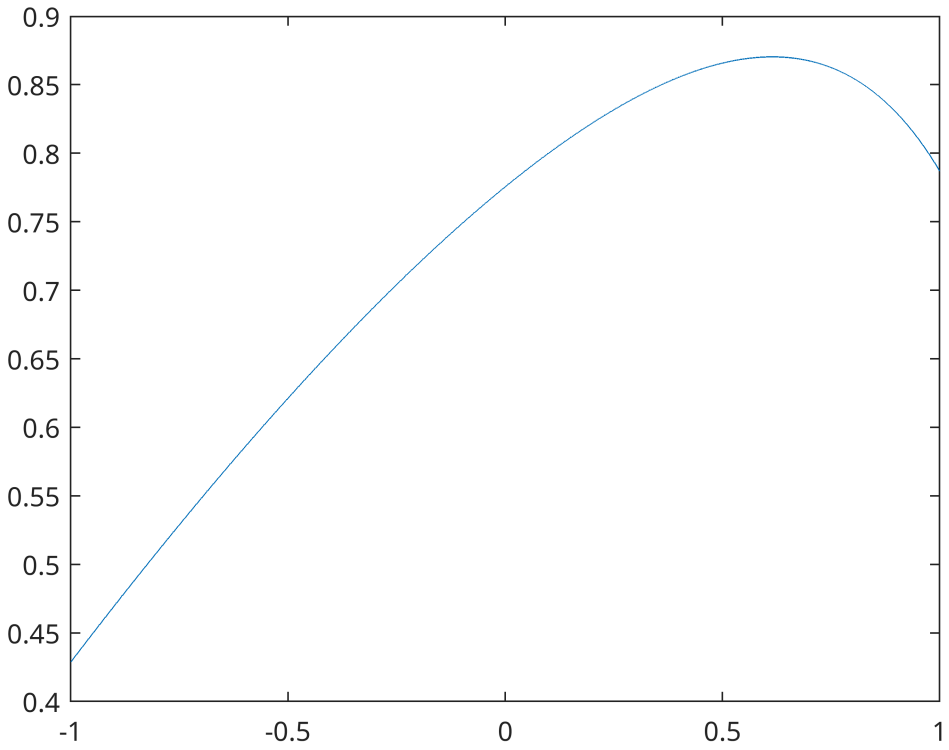
```
%h = x(2) - x(1);
```

Погрешность:

```
d
```

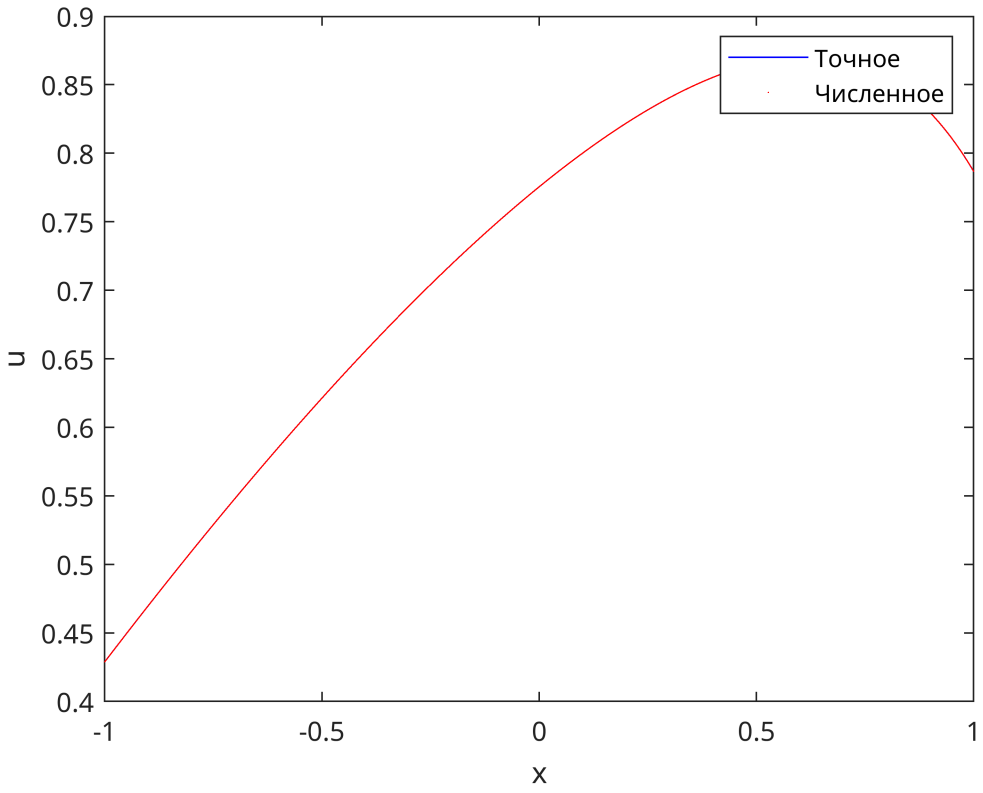
```
d =
     4.924936665891740e-07
```

```
%[u_ch,x_ch] = net_met(x_0, h, x_n, q, r, f, alpha, beta);
plot(x_ch,u_ch);
```



На одном графике:

```
plot(sol.x,sol.y(1,:),'-b',x_ch,u_ch,'r.','MarkerSize',1);
ylabel('u');
xlabel('x');
legend('Точное', 'Численное');
```



Тест 3

```
format long;
x_0 = -1;
x_n = 1;
t = sym('t');
q =@(t)((1-t/2).*(7+3*t)./(6+t));
r =@(t)((1+cos(t)/2).*(7+3*t)./(6+t));
f =@(t)(-(1-t/3).*(7+3*t)./(6+t));
alpha = [-2 -1 0];
beta = [0 1 0];
syms u(t);
Du = diff(u);
```

Решаем следующий дифур:

```
diff(u,2) + q*diff(u) - r*u == f
```

```
ans(t) =
```

$$\frac{\partial^2}{\partial t^2}\,u(t) - \frac{u(t)\;(3\,t+7)\;\left(\frac{\cos(t)}{2}+1\right)}{t+6} - \frac{\left(\frac{t}{2}-1\right)\;(3\,t+7)\,\frac{\partial}{\partial t}\,u(t)}{t+6} = \frac{\left(\frac{t}{3}-1\right)\;(3\,t+7)}{t+6}$$

Краевые условия:

```
alpha(1)*u(-1)-alpha(2)*Du(-1) == alpha(3)
```

```
ans =
```

$$\left(\left(\frac{\partial}{\partial t}\,u(t)\right)\Big|_{t=-1}\right) - 2\,u(-1) = 0$$

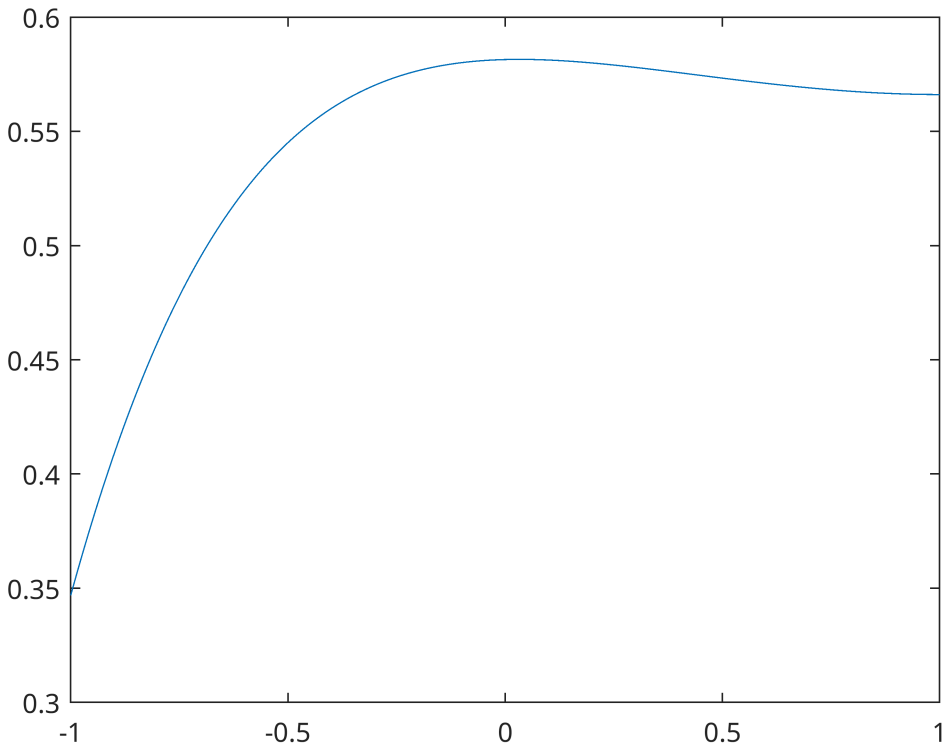
```
beta(1)*u(1)+beta(2)*Du(1) == beta(3)
```

```
ans =
```

$$\left(\left(\frac{\partial}{\partial t}\,u(t)\right)\Big|_{t=1}\right) = 0$$

Точное решение:

```
fun = @(x,y) [y(2); ...
    -(1-x/2)*(7+3*x)/(6 + x)*y(2)+((1 + cos(x)/2)*(7+3*x)/(6 + x))*y(1)-(1-x/3)*(7+3*x)
bc = @(ya, yb) [-2*ya(1)+ya(2); yb(2)];
xmesh = linspace(-1, 1, 1000);
solinit = bvpinit(xmesh, [0 1]);
sol = bvp4c(fun, bc, solinit);
plot(sol.x,sol.y(1,:));
```



Численное решение:

```
epsilon = 1e-6;
[u_ch, x_ch, d] = Richardson(x_0, x_n, q, r, f, alpha, beta,epsilon);
```

```
Количество  итераций:
     8
```

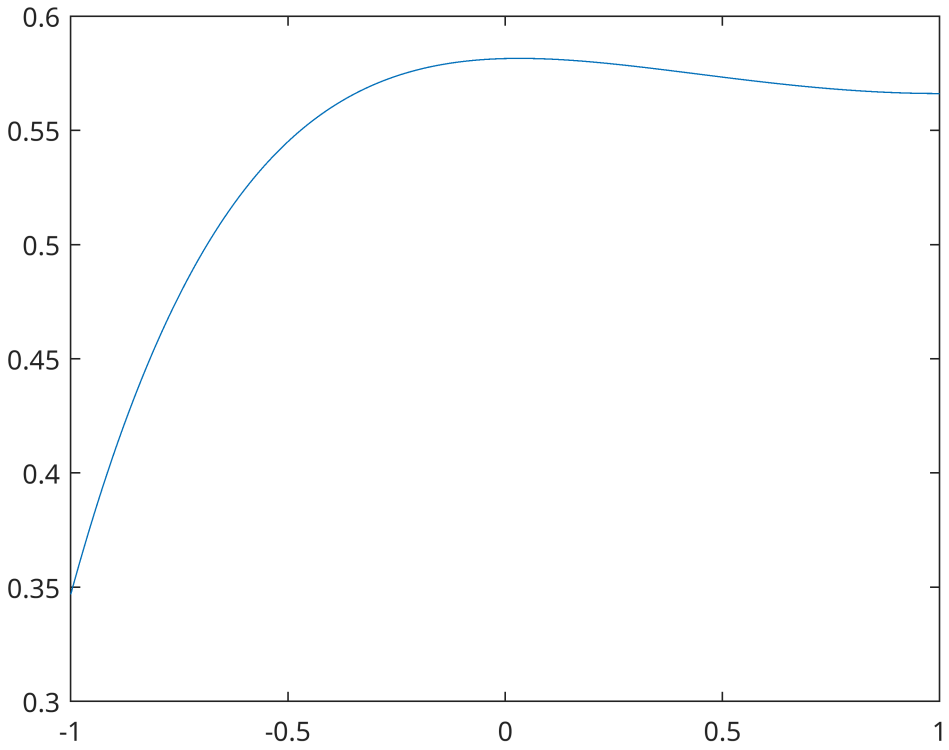
```
%h = x(2) - x(1);
```

Погрешность:

```
d
```

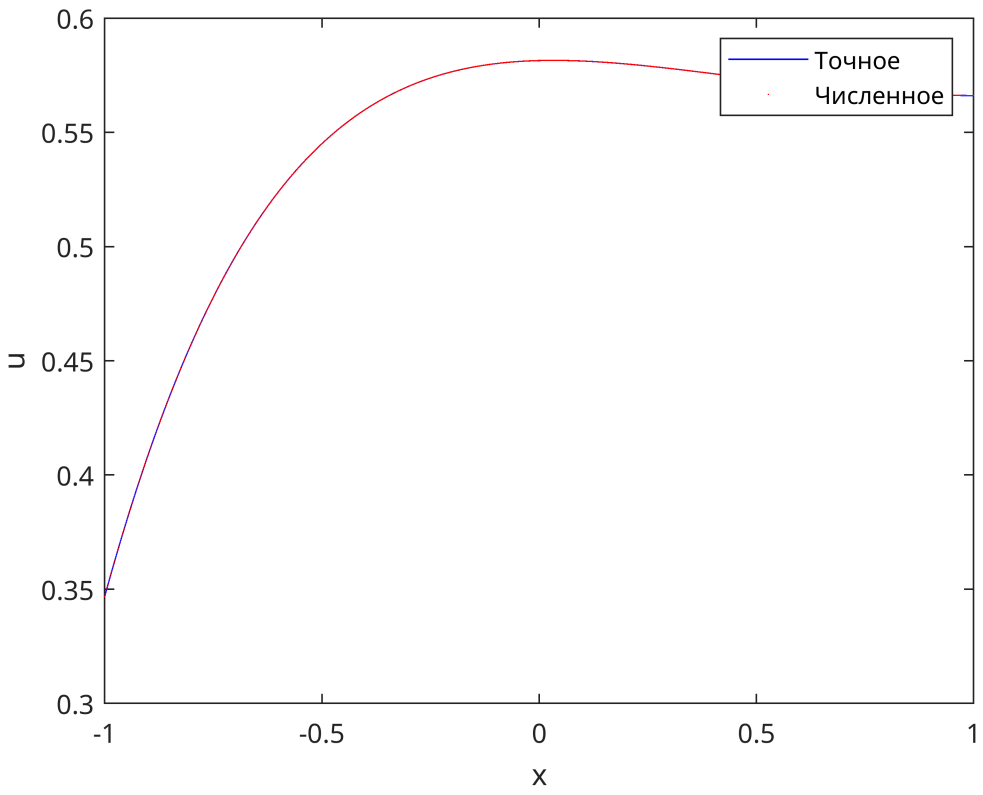
```
d =
     4.150550874104653e-07
```

```
%[u_ch,x_ch] = net_met(x_0, h, x_n, q, r, f, alpha, beta);
plot(x_ch,u_ch);
```



На одном графике:

```
plot(sol.x,sol.y(1,:),'-b',x_ch,u_ch,'r.','MarkerSize',1);
ylabel('u');
xlabel('x');
legend('Точное', 'Численное');
```


0.6
0.55
0.5
0.45
0.4
0.35
0.3
u
-1
-0.5
0
0.5
1
x
Точное
Численное