Columbia University
IEOR4703 – Monte Carlo Simulation Methods (Hirsa)
Term Project 1
`VaR`, `CVaR`, and Stress Test via Simulation

Data

For this study, use only the provided data set (50 mutual funds) spanning from December 31, 2005, to December 31, 2024.

Short Description

The objective of this study is to develop a Jupyter Notebook intended to calculate the value-at-risk (`VaR`), conditional value-at-risk (`CVaR`), and stress test a mutual fund portfolio via simulation.

Consider a scenario where a portfolio manager is tasked with investing \$1,000,000 on behalf of a client, distributing the capital equally across d mutual funds selected from a pool of 50. Here, d is a user-defined number, constrained by the condition $1 \leq d \leq 50$. The manager's goal is to assess the investment risk by analyzing the `VaR` and `CVaR`, while exploring how changes in the number of mutual funds affect the risk profile over different investment horizons.

As a quantitative analyst, your task is to create a straightforward tool that enables the portfolio manager to assess risks associated with selecting any d mutual funds from the pool, over a chosen investment horizon, τ. This horizon may be one, three, or five years, though you will provide flexibility for the manager to define it. The tool should accommodate any start date after December 31, 2005, and any end date before December 31, 2024.

To develop this tool for the portfolio manager, follow these steps:

- Construct a historical distribution of returns for each mutual fund over the given investment horizon, τ.
- For the selected d mutual funds, create a combined historical distribution of returns for the entire portfolio.
- Utilizing the historical distribution data of the portfolio, apply simulation techniques to calculate the value-at-risk (`VaR`) and conditional value-at-risk (`CVaR`) at a specified confidence level, α.

Constructing historical distribution of returns for a given horizon τ

For each mutual fund, you are given $p_t^{(i)}$ for $t = 1, \ldots, T$ where $p_t^{(i)}$ is the value of the i^{th} mutual fund at time t and T the total number of available days. There are many techniques for efficient

construction of long horizon returns from daily returns. In this case study, we do it as follows: to build the historical distribution of returns for a given τ for each mutual fund, we first calculate the return for time horizon τ, that is

$$r_{t,\tau}^{(i)} = \frac{p_{t+\tau}^{(i)} - p_t^{(i)}}{p_t^{(i)}} \quad \text{for } i = 1, \ldots, 50, \; t = 1, \ldots, T-\tau$$

To construct the time series for these returns

$$\{r_{t,\tau}^{(i)}\}_{t=1,\ldots,T-\tau}$$

There are few options. Note that there would not be enough data to make these returns non-overlapping. We can roll the returns, $r_{t,\tau}^{(i)}$, daily, weekly, monthly, or quarterly. You make the rolling period as a hyper-parameter for the portfolio manager to be able to assess `VaR` and `CVaR` as she/he tweaking it, let's write is as

$$\{r_{(j-1)\delta+1,\tau}^{(i)}\}_{j=1,\ldots,L}$$

For daily rolling, $\delta = 1$, for weekly rolling $\delta = 5$, for monthly rolling $\delta = 22$, and for quarterly rolling $\delta = 66$. L is implied depending on the start date, the end date, time horizon τ, and rolling period, δ. For example if you are using the entire data, L would be

$$L = \left\lfloor \frac{T - \tau + 1}{\delta} \right\rfloor$$

where $\lfloor . \rfloor$ implies floor. Naturally for starting date, t_s and end date t_e we get

$$L = \left\lfloor \frac{(t_e - t_s + 1) - \tau + 1}{\delta} \right\rfloor$$

Calculating `VaR` and `CVaR`

Once you have the historical distribution you can form the historical `CDF` and for any given α you can calculate `VaR` and `CVaR` via simulation. In calculation of `VaR` and `CVaR`, does it matter if returns are annualized or not?

Stress Test

Having holdings[1] allows us to stress test the portfolio, focusing on one or more sectors. The fund manager should be able to perform these stress tests on the portfolio.

User-defined Parameters

- t_s: start date
- t_e: end date

[1]Refers to the individual securities (such as stocks and bonds) in which a mutual fund manager invests, representing the underlying assets that make up the fund's portfolio. These assets are collectively owned by investors when they purchase shares in the mutual fund.

- d: number of mutual funds in the portfolio
- δ: rolling period e.g. 30 for monthly rolling
- τ: time horizon, e.g. 365 for a year
- α: confidence level