ELECTROMAGNETIC AND WEAK INTERACTIONS

A. SALAM and J. C. WARD *
Imperial College, London

Received 24 September 1964

One of the recurrent dreams in elementary particles physics is that of a possible fundamental synthesis between electro-magnetism and weak interactions [1]. The idea has its origin in the following shared characteristics:

1) Both forces affect equally all forms of matter-leptons as well as hadrons.
2) Both are vector in character.
3) Both (individually) possess universal coupling strengths. Since universality and vector character are features of a gauge-theory these shared characteristics suggest that weak forces just like the electromagnetic forces arise from a gauge principle.

There of course also are profound differences:

1) Electromagnetic coupling strength is vastly different from the weak. Quantitatively one may state it thus: if weak forces are assumed to have been mediated by intermediate bosons (W), the boson mass would have to equal 137 M_p, in order that the (dimensionless) weak coupling constant $g_W{}^2/4\pi$ equals $e^2/4\pi$.

In the sequel we assume just this. For the outrageous mass value itself ($M_W \approx 137\, M_p$) we can offer no explanation. We seek however for a synthesis in terms of a group structure such that the remaining differences, viz:

2) Contrasting space-time behaviour (V for electromagnetic versus V and A for weak).
3) And contrasting ΔS and ΔI behaviours both appear as aspects of the same fundamental symmetry. Naturally for hadrons at least the group structure must be compatible with SU_3.

Lepton interactions define both the unit of the electric charge and (from μ-decay) the (bare) value of weak coupling constant. Leptons therefore must be treated first.

There is only one genuine lepton multiplet (in the limit $m_e = m_\mu = 0$) which really treats the neutrino field on the same footing ** as μ and e. This is the Konopinski-Mahmoud multiplet.

$$L = \begin{pmatrix} \nu \\ e^- \\ \mu^+ \end{pmatrix} \tag{1}$$

In terms of SU_3 generators ***, the electric charge clearly equals:

$$Q_l = \begin{pmatrix} 0 & & \\ & -1 & \\ & & +1 \end{pmatrix} = -2U_3 = -\tfrac{2}{3}\sqrt{3}\,(I_0 - V_0) = 2(I_3 - V_3), \tag{2}$$

while the weak interaction (with no neutral currents) has the unique form †

* Permanent address, John Hopkins University, Baltimore.

** There are other schemes where one postulates multiplets consisting of a two-component neutrino field together with a four-component electron or muon. These do not satisfy even the most elementary requirement of a genuine group-structure, i.e. that in some limit at least, the particles concerned should be transformable, one into the other.

***
$$[T^i, T^j] = \mathrm{i} f^{ijk} T^k$$
$$\{T^i, T^j\} = \tfrac{1}{3}\delta^{ij}\,\mathbf{1} + d^{ijk}\, T^k$$

$$I_3 = T^3,\ I^{\pm} = \tfrac{1}{2}\sqrt{2}\,(T^1 \mp \mathrm{i}T^2),\ I_0 = T^8,\ [I_0, \boldsymbol{I}] = 0$$
$$U_3 = \tfrac{1}{2}\sqrt{3}\,T^8 - \tfrac{1}{2}T^3,\ U^{\pm} = \tfrac{1}{2}\sqrt{2}\,(T^6 \mp \mathrm{i}T^7),$$
$$U_0 = \tfrac{1}{2}\sqrt{3}\,T^3 + \tfrac{1}{2}T^8$$
$$V_3 = \tfrac{1}{2}\sqrt{3}\,T^8 + \tfrac{1}{2}T^3,\ V^{\pm} = \tfrac{1}{2}\sqrt{2}\,(T^4 \mp \mathrm{i}T^5),$$
$$V_0 = \tfrac{1}{2}\sqrt{3}\,T^3 - \tfrac{1}{2}T^8$$

Note

$$Q_h = T^3 + \tfrac{1}{3}\sqrt{3}\,T^8 = \tfrac{2}{3}\sqrt{3}\quad U_0 = \tfrac{2}{3}\sqrt{3}\,(I_0 + V_0) = \tfrac{2}{3}(I_3 + V_3)$$
$$Q_l = T^3 - \sqrt{3}\,T^8 = -2U_3 = -\tfrac{2}{3}\sqrt{3}\,(I_0 - V_0) = +2\,(I_3 - V_3)$$

Explicitly,

$$Q_h = \begin{pmatrix} \frac{2}{3} & & \\ & -\frac{1}{3} & \\ & & -\frac{1}{3} \end{pmatrix},\ Q_l = \begin{pmatrix} 0 & & \\ & -1 & \\ & & +1 \end{pmatrix}$$

Q_h is the conventional hadron charge operator, Q_l gives lepton-charge.

† Define

$$\psi_L = \tfrac{1}{2}(1 + \gamma_5)\,\psi \qquad \psi_R = \tfrac{1}{2}(1 - \gamma_5)\,\psi$$
$$(A^+B)_L = \tfrac{1}{2}A^+\gamma_4\gamma_\mu(1 + \gamma_5)B$$

$$\mathcal{L}_{\text{weak}} = [((e^-)^\dagger \nu)_L + (\nu^\dagger \mu)_R]\, W^- + \text{h.c.}$$

$$= (I_{1L} + V_{1R})\, W_1 + (I_{2L} - V_{2R})\, W_2. \quad (3)$$

Here $W^\pm = \frac{1}{2}\sqrt{2}\,(W_1 + iW_2)$ and $I_{1L} = (\psi^+ I_1 \psi)_L$ etc. Now $I_{1L} + V_{1R}$, $I_{2L} - V_{2R}$, $I_{3L} - V_{3R}$ generate an SU_2 sub-group. Since two of the group generators $I_{1L} + V_{1R}$, $V_{2L} - V_{2R}$ give the weak currents, ideally one would have liked the neutral component [2] $(I_{3L} - V_{3R})$ to represent electromagnetism. This unfortunately is not the case. $I_{3L} - V_{3R}$ equals $\frac{1}{4}(Q_l + 3\gamma_5 Q_h)$ so that in addition to the "correct' electromagnetic current $\frac{1}{2}Q_l$ there appears also unwanted parity violating term $\gamma_5 Q_h$. We are therefore forced to extend the group structure as follows:

The free Lagrangian $(\psi^+ \partial \psi)$ is invariant for the following $SU_2 \otimes U_1$ transformation

$$L' = UL$$

where

$$U = \exp ie[(I_L + V_R')\varepsilon - \sqrt{3}\,l\,(I_{0L} - V_{0R})\epsilon_0]. \quad (4)$$

Here l is an arbitrary constant determining the relative strength of the Abelian gauge $(I_{0L} - V_{0R})$ compared to the gauge $(I_L + V_R')$ $(V' = V_1, -V_2, -V_3)$. Following the well-known procedure, construct $\mathcal{L}_{\text{int}}$ by replacing $(\partial\psi)$ in $\mathcal{L}_f$ with the co-variant derivative

$$\mathcal{D}L = \partial L + ie[(I_L + V_R')\cdot \mathbf{W} - \sqrt{3}\,l\,(I_{0L} - V_{0R})W_0]L \quad (5)$$

In terms of the two orthogonal combinations of fields,

$$A^0 = W_0 \cos\theta + W_3 \sin\theta$$

$$X^0 = -W_0 \sin\theta + W_3 \cos\theta$$

$$(l = \tan\theta)$$

one can write the neutral component of (4) in the form

$$(\cos\theta\, Q_x X^0 + \sin\theta\, Q_l A^0) \quad (6)$$

where $Q_x = \frac{1}{4}\sec^2\theta\,(3\gamma_5 Q_h + (2\cos 2\theta - 1)Q_l)$. The full interaction Lagrangian equals

$$\mathcal{L}_{\text{int}} = -ieL^+ \gamma_4 \gamma_\mu [(I_L^- + V'^+)\, W^+ + \text{h.c.} +$$

$$+ \sin\theta\, Q_l A^0 + \cos\theta\, Q_x X^0]L\,. \quad (7)$$

This Lagrangian has the following characteristics
1. The electromagnetic interaction $ie\,\sin\theta\,(\bar{\mu}^+ \mu^+ - \bar{e}\, e)$ is necessarily accompanied by a neutral current term $-ie\cos\theta\, Q_x X^0$. For the special case $\theta = 30^\circ$ $(3l^2 = 1)$ this neutral current conserves parity and is pure axial-vector, with the same sign of coupling for e^- and μ^+. For $\theta = 45^\circ$ $(l^2 = 1)$ the orthogonal fields A^0 and X^0 equal $\frac{1}{2}\sqrt{2}\,(W_3 \pm W_0)$ and the full neutral Lagrangian reads

$$\tfrac{1}{2}\sqrt{2}\, ie\,(\mu^+\mu - e^+ e)A_0 \quad (8)$$

$$-\tfrac{1}{2}\sqrt{2}\, ie\,[\nu^+\gamma_5\nu - \tfrac{1}{2}e^+(1+\gamma_5)\,l + \tfrac{1}{2}\mu^+(1-\gamma_5)\mu]X_0\,.$$

Within Lepton physics the only assumption one need make is that X^0 is not mass-less (in order that the neutral X^0 current does not contribute to what is experimentally called the electric charge). When we come to consider hadrons, the absence of neutral leptonic currents interacting with heavy particles however requires that we assume X^0 particles are at least as massive as W^+ or W^-. The appearance of the X^0 current is about the minimum price one must pay to achieve the synthesis of weak and electromagnetic interactions we are seeking.

Since the interaction of W_3 with W^+, W^- (including the coupling strength) is completely determined by the gauge principle, it is crucial to check that the electric charge carried by W^+, W^- identically equals the charge on μ^+ end e^-. Group-theoretically this is equivalent to making sure that the W's belong to the *same* group representation $(SU_2 \times U_1)$ as L itself.

Concretely, in accordance with the transformation properties of the W's, one may define the field strengths

$$\mathbf{W}_{\mu\nu} = \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu - e\, \mathbf{W}_\mu \times \mathbf{W}_\nu \quad (9)$$

$$W_{0\mu\nu} = \partial_\mu W_{0\nu} - \partial_\nu W_{0\mu}\,. \quad (10)$$

The gauge-Lagrangian for the W fields equals

$$\mathcal{L}(W) = \tfrac{1}{4}(\mathbf{W}_{\mu\nu}\cdot \mathbf{W}_{\mu\nu} + W_{0\mu\nu}W_{0\mu\nu})\,. \quad (11)$$

In terms of A^0 and X^0, rewrite (9) and (10) in the form

$$W^\pm_{\mu\nu} = \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu \mp ieW^\pm_\mu\,(\cos\theta\, X^0 + \sin\theta\, A^0)_\nu$$

$$\pm ie\, W^\pm_\nu\,(\cos\theta\, X^0 + \sin\theta\, A^0)_\mu$$

$$W^0_{\mu\nu} = \partial_\mu X^0 - \partial_\nu X^0_\mu - ie\cos\theta\,(W^-_\mu W^+_\nu - W^+_\mu W^-_\nu)$$

$$A^0_{\mu\nu} = \partial_\mu A^0_\nu - \partial_\nu A^0_\mu - ie\sin\theta\,(W^-_\mu W^+_\nu - W^+_\mu W^-_\nu).$$

Clearly,

$$\mathcal{L}(W) = \tfrac{1}{4}(A^0_{\mu\nu}A^0_{\mu\nu} + X^0_{\mu\nu}X^0_{\mu\nu} + W^+_{\mu\nu}W^-_{\mu\nu} + W^-_{\mu\nu}W^+_{\mu\nu}) \quad (12)$$

Comparing the coupling constants in (7) and (9), it is obvious that the dynamical charge carried by W^- and W^+ equals respectively the charge on e^- and μ^+ both in sign and magnitude.

Note that the magnetic moment of $W^\pm$ equals 2 Bohr magnetons irrespective of the value of θ.

For leptons

$$Q_l = T^3 - \sqrt{3}\, T^8$$

while for hadrons the Gell-Mann - Nishijima formula gives

$$Q_h = T^3 + \tfrac{1}{3}\sqrt{3}\, T^8 .$$

To correspond to this fundamental difference, clearly the appropriate infinitesimal unitary transformation for hadrons equals

$$U = 1 + i\,(\varepsilon \cdot \boldsymbol{I} + \tfrac{1}{3}\sqrt{3}\, l\, I_0)$$

instead of the leptonic transformation

$$U = 1 + i\,(\varepsilon \cdot \boldsymbol{I} - \sqrt{3}\, l\, I_0).$$

Now weak interactions for hadrons experimentally appear to exhibit the pattern

$$F^V + D^A$$

It was pointed out [3,4] in an earlier paper that this precisely is the consequence of assuming that the 9-fold of baryons B transforms as a representation of the group [5] structure $(SU_3)_L \otimes (SU_3)_R$. Specialising for weak and electromagnetic interactions to the sub-group $(SU_2 \times U_1)$ in place of the full SU_3 group, consider the double gauge transformation

$$B' = [\exp i\,(I_L + V_R)]\, B\, [\exp{-i}\,(I_R + V_L)]$$

where I, stands for,

$$I = I_1\epsilon_1 + I_2\epsilon_2 + I_3\epsilon_3 + \tfrac{1}{3}\sqrt{3}\, l\, I_0\epsilon_0$$

and likewise for V.

With the standard gauge procedure, this gives rise to the currents

$$- \tfrac{1}{2}B^+\, \gamma_4\gamma_\mu\, [I + V, B]$$

$$- \tfrac{1}{2}B^+\, \gamma_4\gamma_\mu\gamma_5\, \{I - V, B\}$$

The neutral components of $\mathcal{L}_{int}$ are

$$\mathcal{L}_{int} = -ie\ \sin\theta\, A^o\ \mathrm{Tr}\ B^+[Q_h, B]A^o_\mu - ie\ \cos\theta\, X^o J_x \tag{13}$$

where

$$J_x = \tfrac{1}{4}\sec^2\theta\ \mathrm{Tr}\, B^+((1 + 2\cos 2\theta)[Q_h, B] - \gamma_5\{Q, B\}). \tag{14}$$

Once again, note that for $\theta = 60^o$ ($l^2 = 3$) the new neutral current is purely axial vector *. It is gratifying that relative electric (as well as weak) charges on baryons are the same in magnitude as on leptons. This is not true in any obvious manner for the new neutral charge appearing in J_x.

Summarising the full Lagrangian equals

$$\mathcal{L}_{weak} = -ie\, W^- \Big(((e^-)^+\nu)_L + (\nu^+_\mu)_R + \tfrac{1}{2}\mathrm{Tr}\, B^+[I^+ + V^+, B] + \tfrac{1}{2}\mathrm{Tr}\ B^+\gamma_5\{I^+ - V^+, B\} + \text{meson terms}\Big) \tag{15}$$

$$\mathcal{L}_{em} = -ie\ \sin\theta\, A^o(p^+p + \ldots + \mu\ \mu + \ldots) \tag{16}$$

$$\mathcal{L}_{neutral} = -ie\ \cos\theta\, X^o\, (L^+Q_x L + J_x) \tag{17}$$

A number of problems remain
1) Even though the neutral X^o current conserves strangeness and even though it is only just the lepton pairs e^+e^-, $\mu^+\mu^-$, $\nu^-\nu$ which make their appearance.

The upper limit established in the CERN neutrino experiment on $\nu + p \to \nu + p$ (1% of σ_{weak}) implies that the effective coupling of X^o particles must be much smaller than $g_{W^\pm}$. We speculate that there possibly may exist a geometrical relation among the effective coupling constants like

$$e^2 g^2_{x^o} = (g^2_{W^\pm})^2 .$$

We have no convincing reasons for this relation. This however appears to be the most plausible conjecture for g_x^2, once we do accept that $g^2_{W^\pm} \neq e^2$.
2) The weak hadron Lagrangian $[I^+ + V^+]_F + \gamma_5\{I^+ - V^+\}_D$ shows no sign of the Cabibbo suppression for the strangeness-changing currents $V^\pm$; it also predicts the combination $(V+A)$ in $\epsilon^- \to$ n + leptons compared to $(V-A)$ for β-decay.

There are two distinct views about Cabibbo's phenomenological Lagrangian $[I^\pm \cos\theta + V^\pm \sin\theta]_F + \gamma_5(a[I^\pm \cos\theta + V^\pm \sin\theta]_F + b\{I^\pm \cos\theta + V^\pm \sin\theta\}_D)$. One is that the Cabibbo suprression is a consequence of the symmetry-breaking mechanism operating in strong-interaction physics. The second view postulates that the suppression is intrinsic and results from a rotation of the weak relative to the strong i-spin and hypercharge axes. Now the most general charge and CP-conserving rotations one may consider are the following

* To incorporate the mesons, the natural assumption [3] appears to be to consider the P.S. particles M, together with a set of yet undiscovered scalar mesons M_2, to form a 9-fold $M = M_1 + iM_2$. The appropriate gauge transformation is

$$M' = \exp(iI)\, M \exp(-iV)$$

giving the gauge Lagrangian (see ref. 5, eq. (33) -

$$\mathcal{L} = -\tfrac{1}{2}\mathrm{Tr}\,[(\partial M_1 + \tfrac{1}{2}[I+V, M_1] - \tfrac{1}{2}\{I - V, M_2\})^2 + (\delta M_2 + \tfrac{1}{2}[I + V, M_2)] + \tfrac{1}{2}\{I - V, M_2\})^2$$

$$B'_L = X(\theta_1)\, B_L X^{-1}(\theta_2)$$
$$B'_R = X(\theta_3)\, B_R X^{-1}(\theta_4)$$

where

$$X = \exp\,(2i\theta\, T^7).$$

The net-effect of these is to replace I and V in (18) by the appropriate combinations of $I(\theta) = X^{-1}(\theta)\, I(\theta)$ and $V(\theta)$. For $I^{\pm}$, $V^{\pm}$, this may indeed give the Cabibbo suppression but for neutral currents (since $Q(\theta) = Q \cos 2\theta + \frac{1}{2} \sin 2\theta\, T^6$) any value of θ other than 0^o or 90^o inevitably gives rise to neutral strangeness-changing currents ($K^0 \to \mu^+ + \mu^-$). For such currents there appears to be no experimental evidence. The "rotation" view is therefore incompatible with the present theory.

In a subsequent paper we consider the problem of starting with $[I+V]_F + \gamma_5 \{I - V\}_D$, to show that the strong symmetry-breaking mechanisms do lead to a relative suppression of $[V^{\pm}]_F$ and $\gamma_5 \{V^{\pm}\}_D$ currents, as well as the appearance of $\gamma_5 [V^{\pm}]_F$ terms.

The authors would like to thank Prof. J. Prentki for a discussion.

References

1. A. Salam and J. C. Ward, Il Nuovo Cimento 11 (1959) 568;
S. Glashow, Nuclear Phys. 10 (1959) 103;
J. Schwinger, Ann. of Phys. 2 (1957) 407.
2. The group structure $I_L + V'_R$ for weak interactions was studied by R. Gatto (Il Nuovo Cimento 28 (1963) 567; and Y. Neeman, Il Nuovo Cimento 27 (1963) 567.
3. A. Salam and J. C. Ward, A gauge theory of elementary interactions, Phys, Rev., to be published
4. Y. Nambu and P. Freund, Phys. Rev. Letters 12 (1964) 714;
M. Gell-Mann, Physics Letters 12 (1964) 63;
M. Gell-Mann and Y. Neeman, to be published.
R. Makunda, R. E. Marshak and S. Okubu, to be published.
5. This group was introduced in elementary particle physics by A. Salam and J. C. Ward, Il Nuovo Cimento 27 (1961) 922;
M. Gell-Mann, Phys. Rev. 125 (1962) 1067;
and in slightly different form by R. E. Marshak and S. Okubo, 19 (1961) 1226.

* * * * *

ON THE RATIO BETWEEN $\Delta S = -\Delta Q$ AND $\Delta S = 2$ DECAY AMPLITUDES

D. HORN
Department of Physics, Tel-Aviv University, Tel-Aviv, Israel

Received 23 September 1964

It has already been noted [1] that a classification of the weak currents on the basis of unitary symmetry which takes into account the $\Delta S = -\Delta Q$ amplitudes must also involve $\Delta S = 2$ transitions. The reason is simple. In order to account for $\Delta T_z = \frac{3}{2}$ terms one must consider representations larger than $\underline{8}$ in the product $\underline{8} \otimes \underline{8}$. The possibilities are $\underline{10} \oplus \overline{\underline{10}}$ or $\underline{27}$. Both include $T_z = \frac{3}{2}$ and $S = 2$ terms. This effect was discussed in detail in ref. 1.

The present experimental upper limit of the magnitude of $\Delta S = -\Delta Q$ decay rates compared with $\Delta S = \Delta Q$ ones is about 10-15% [2]. According to a recent experimental estimate on the basis of an observed $\Sigma^+ \to n + e^+ + \nu_e$ event [3], the branching ratio for this decay is 250 times smaller than that predicted by a V-A theory with the Fermi coupling constant. Using this result we want to find an estimate for the magnitude of the $\Delta S = 2$ amplitude.

We have now to introduce some theoretical assumptions. We use here an extension of Cabibbo's theory [4]. Let us assume for simplicity that the representations responsible for these decays are $\underline{10} \oplus \overline{\underline{10}}$ [5]. We now make Cabibbo's assumption that the weak interactions conserve Y', the hypercharge defined in the system rotated by $\exp\,(2i\theta F_7)$. This means that the weak current is given by the term $T' = 1$, $Y' = 0$ in the decuplet. This is a small addition to the usual octet current which is responsible for most decays. The rotation causes a specific mixture of the elements 4, 7, 9 and 10 ($\Delta T_z = \frac{3}{2}, 1, \frac{1}{2}, 0$ and $\Delta Y = -1, 0, 1, 2$ respectively) of the decuplet [6]. Because of the smallness of θ we can expand this expression in a power series in θ and keep the first terms only.