① Necesario son las variables de estado del sistema

① $A_1\dot{h}_1 + \frac{1}{R_{12}}(h_1-h_2) = F_1$

$\llcorner\ A_1\dot{h}_1 = F_1 - \frac{h_1}{R_{12}} + \frac{h_2}{R_{12}}$

$\llcorner\ \dot{h}_1 = \frac{F_1}{A_1} - \frac{h_1}{A_1R_{12}} + \frac{h_2}{A_1R_{12}}$ Ⓐ

la 1ra ec $\dot{h}_1$ despejada

② $A_2\dot{h}_2 + \frac{1}{R_2}h_2 = \frac{1}{R_{12}}(h_1-h_2) + F_2$

$\llcorner\ A_2\dot{h}_2 = \frac{h_1}{R_{12}} - \frac{h_2}{R_{12}} + F_2 - \frac{1}{R_2}h_2$

$\llcorner\ \dot{h}_2 = \frac{h_1}{A_2R_{12}} - \frac{h_2}{A_2R_{12}} + \frac{F_2}{A_2} - \frac{h_2}{A_2R_2}$

$\llcorner\ \dot{h}_2 = \frac{h_1}{A_2R_{12}} - \frac{h_2}{A_2}\left(\frac{1}{R_{12}} + \frac{1}{R_2}\right) + \frac{F_2}{A_2}$ Ⓒ

la 2da ecuación $\dot{h}_2$ despejada

① la representación del modelo como sistema de var de estados

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{A_1R_{12}} & \frac{1}{A_1R_{12}} \\ \frac{1}{A_2R_{12}} & -\frac{1}{A_2}\left(\frac{1}{R_{12}} + \frac{1}{R_2}\right) \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} & 0 \\ 0 & \frac{1}{A_2} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}$$

$$V_T = [A_1 \ A_2] \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

②

2. teniendo en cuenta el modelo de la función de transferencia y $D=0$, tenemos que

$$G(s) = \frac{C(sI-A)^{d}B + \det[sI-A]D}{\det[sI-A]}$$

$$\therefore G(s) = \frac{C(sI-A)^{d}B + \cancel{\det[sI-A]D}}{\det[sI-A]} \Rightarrow G(s) = \frac{C(sI-A)^{d}B}{\det[sI-A]} \quad ①$$

Ahora, mediante lo obtenido previamente, se halla los otros elementos para $G(s)$

$$(sI-A) = \left[s\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \underbrace{\begin{bmatrix} -\frac{1}{A_1R_{12}} & \frac{1}{A_1R_{12}} \\ \frac{1}{A_2R_{12}} & -\frac{1}{A_2}\left(\frac{1}{R_{12}}+\frac{1}{R_2}\right)\end{bmatrix}}_{X} \right]$$

$$(sI-A) = \left[\begin{bmatrix}s & 0\\ 0 & s\end{bmatrix} - X\right] = \begin{bmatrix} s+\frac{1}{A_1R_{12}} & -\frac{1}{A_1R_{12}} \\ -\frac{1}{A_2R_{12}} & s+\frac{1}{A_2}\left(\frac{1}{R_{12}}+\frac{1}{R_2}\right)\end{bmatrix}$$

④ llamemos a estos 4 elementos respectivamente A B / C D

$$\det[sI-A] = AD - BC = \left(\overset{A}{s} + \overbrace{\frac{1}{A_1R_{12}}}^{B}\right)\left(\overset{A'}{s} + \overbrace{\frac{1}{A_2}\left(\frac{1}{R_{12}}+\frac{1}{R_2}\right)}^{B'}\right) - \left(\left(\overbrace{-\frac{1}{A_1R_{12}}}^{C}\right)\left(\overbrace{-\frac{1}{A_2R_{12}}}^{D}\right)\right) = AA' + AB' + BA' + BB' - (CD)$$

$$\det[sI-A] = s^2 + s\left(\frac{1}{A_2}\left(\frac{1}{R_{12}}+\frac{1}{R_2}\right)\right) + s\left(\frac{1}{A_1R_{12}}\right) + \left(\frac{1}{A_1R_{12}}\left(\frac{1}{A_2}\left(\frac{1}{R_{12}}+\frac{1}{R_2}\right)\right)\right) - \left(\frac{1}{A_1R_{12}}\right)\left(\frac{1}{A_2R_{12}}\right)$$

$$\det[sI-A] = s^2 + s\left(\frac{1}{A_2R_{12}} + \frac{1}{A_2R_2} + \frac{1}{A_1R_{12}}\right) + \underbrace{\left(\frac{1}{A_1R_{12}}\right)\left(\cancel{\frac{1}{A_2R_{12}}} + \frac{1}{A_2R_2} - \cancel{\frac{1}{A_2R_{12}}}\right)}_{F} \Rightarrow F = \left(\frac{1}{A_1R_{12}A_2R_2}\right)$$

③

$$\det[sI-A] = s^2 + s\left(\frac{1}{A_2R_{12}} + \frac{1}{A_2R_2} + \frac{1}{A_1R_{12}}\right) + \frac{1}{A_1R_{12}A_2R_2} \quad *$$

$$(sI-A)^{d} = \begin{bmatrix} \overbrace{s + \frac{1}{A^2}\left(\frac{1}{R_{12}} + \frac{1}{R_2}\right) \quad \frac{1}{A_2R_{12}}}^{X} \\ \frac{1}{A_1R_{12}} \quad s + \frac{1}{A_1R_{12}} \end{bmatrix}$$

$$C(sI-A)B = [A_1 \ \ A_2] \ X \ \underbrace{\begin{bmatrix} \frac{1}{A_1} & 0 \\ 0 & \frac{1}{A_2} \end{bmatrix}}_{F} = \left[A_1s + \frac{A_1}{A_2}\left(\frac{1}{R_{12}} + \frac{1}{R_2}\right) + \frac{A_2}{A_1R_{12}} \quad \frac{A_1}{A_2R_{12}} + A_2s + \frac{A_2}{A_1R_{12}}\right] F$$

$$C(sI-A)B = \left[s + \frac{1}{A_2}\left(\frac{1}{R_{12}} + \frac{1}{R_2}\right) + \frac{A_2}{A_1^2R_{12}} \quad \frac{A_1}{A_2^2R_{12}} + s + \frac{1}{A_1R_{12}}\right]$$

Reemplazando en $G(s) = \dfrac{C(sI-A)^{d}B}{\det[sI-A]}$

$$G(s) = \frac{1}{s^2 + s\left(\frac{1}{A_2R_{12}} + \frac{1}{A_2R_2} + \frac{1}{A_1R_{12}}\right) + \frac{1}{A_2A_1R_{12}R_2}}\left[s + \frac{1}{A_2R_{12}} + \frac{1}{A_2R_2} + \frac{A_2}{A_1^2R_{12}} \quad \frac{A_1}{A_2^2R_{12}} + s + \frac{1}{A_1R_{12}}\right]$$

$$= \left[\frac{s + \frac{1}{A_2R_{12}} + \frac{1}{A_2R_2} + \frac{A_2}{A_1^2R_{12}}}{s^2 + \frac{s}{A_2R_{12}} + \frac{s}{A_2R_2} + \frac{s}{A_1R_{12}} + \frac{1}{A_2A_1R_{12}R_2}} \quad \frac{\frac{A_1}{A_2^2R_{12}} + s + \frac{1}{A_1R_{12}}}{s^2 + \frac{s}{A_2R_{12}} + \frac{s}{A_2R_2} + \frac{s}{A_1R_{12}} + \frac{1}{A_2R_2A_1R_{12}}}\right]$$

$A_1 = 150 \quad A_2 = 50 \quad R_{12} = 9 \quad R_2 = 12$

continuando

$$G(s) = \left[\frac{s + \frac{1}{A_2 R_{12}} + \frac{1}{A_2 R_2} + \frac{A_2}{A_1^2 R_{12}}}{s^2 + \frac{s}{A_2 R_{12}} + \frac{s}{A_2 R_2} + \frac{s}{A_1 R_{12}} + \frac{1}{A_2 A_1 R_{12} R_2}} \quad \frac{\frac{A_1}{A_2^2 R_{12}} + s + \frac{1}{A_1 R_{12}}}{s^2 + \frac{s}{A_2 R_{12}} + \frac{s}{A_2 R_2} + \frac{s}{A_1 R_{12}} + \frac{1}{A_2 A_1 R_{12} R_2}} \right]$$

con $R_2 = 12$

$$G(s) = \left[\frac{810000(s) + 3350}{810000(s)^2 + 3750(s) + 1} \quad \frac{810000(s) + 6900}{810000(s)^2 + 3750(s) + 1} \right]$$

con $R = 50$

$$G(s) = \left[\frac{10125000(s) + 29050}{10125000(s)^2 + 34050(s) + 3} \quad \frac{3375000(s) + 25000}{3375000(s)^2 + 11350(s) + 1} \right]$$

con $R = 100$

$$G(s) = \left[\frac{20250000\, s + 54050}{20250000(s)^2 + 64050(s) + 3} \quad \frac{6750000(s) + 50000}{6750000(s)^2 + 21350(s) + 1} \right]$$

con $R = 200$

$$G(s) = \left[\frac{40500000(s) + 104050}{40500000(s)^2 + 124050(s) + 3} \quad \frac{13500000(s) + 100000}{13500000(s)^2 + 41350(s) + 1} \right]$$