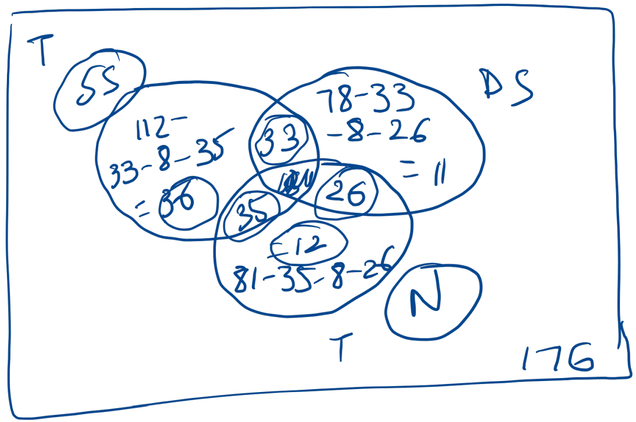
2d.



$$P(SS \cap DS \cap N \mid T)$$

$$= P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{8}{176}}{\frac{150}{176}} = \frac{8}{150} = \underline{\underline{0.0533}}$$

3 e) P(atleast one not born in E) = 1 - P(all born in E)

$= 1 - (0.75)^6 = 0.822$

f) Probability of atleast 1 not born in England out of n ppl

$\Rightarrow 1 - (0.75)^n > 0.80$ solve for n;

$-(0.75)^n > -0.2$

$(0.75)^n < 0.2$

$n \ln(0.75) < \ln(0.2)$

$n = \frac{\ln(0.2)}{\ln(0.75)} = 5.59 \approx \underline{\underline{6}}$

5. $F(t) = \dfrac{(t+k)^2}{144}$; $t = 8, 9, 10$;

$F(10) = P(t \leq 10) = 1$;

$\dfrac{(10+k)^2}{144} = 1$; $\Rightarrow (10+k)^2 = 144 \Rightarrow 10+k = \pm 12$; $k = 2$, $\cancel{-22}$

So, $F(t) = \dfrac{(t+2)^2}{144}$.

$P(t \leq 8) = F(8) = \dfrac{(8+2)^2}{144} = \dfrac{100}{144}$

$P(t \leq 9) = F(9) = \dfrac{(9+2)^2}{144} = \dfrac{121}{144}$

$P(t \le 10) = F(10) = 1;$

$$F(t) = \frac{(t+2)^2}{144}$$

x	8	9	10
$P(x)$	$\frac{100}{144}$	$\frac{21}{144}$	$\frac{23}{144}$

$P(x=8) = F(8) = \frac{100}{144}$

$P(x=9) = F(9) - F(8)$

$= \frac{121}{144} - \frac{100}{144} = \frac{21}{144}$

$P(x=10) = F(10) - F(9)$

$= 1 - \frac{121}{144} = \frac{23}{144}$

6. $f(x) = kx^2$; $x = 2, 3, 4, 5, 6$; $k = \frac{1}{90}$;

x	2	3	4	5	6
$f(x)$	$4k$	$9k$	$16k$	$25k$	$36k$

$E(x) = \sum x p(x) = \sum x f(x) = (2)(4k) + (3)(9k) + (4)(16k) + (5)(25k) + (6)(36k)$

$= 8k + 27k + 64k + 125k + 216k$

$= 440k = \frac{440}{90} = \frac{44}{9}$.

$$\sum x^2 f(x) = 4(4k) + 9(9k) + (16)(16k) + (25)(25k)$$

$$+ (36)(36k) = 16k + 81k + 256k + 625k + 1296k$$

$$= 2274k =$$

$$Var(X) = E(X^2) - (E(X))^2 = \sum x^2 f(x) - \left(\sum x f(x)\right)^2$$

$$= 2274k - (440k)^2 \; ; \quad k = \frac{1}{90}$$

$$= 25.26 - 23.90$$

$$= 1.358 \; ; \qquad SD = \sqrt{Var(X)} = 1.165$$

6.a) $f(x) = kx^2 \quad x = 2, 3, 4, 5, 6.$

x	2	3	4	5	6
$P(X=x)\ f(x)$	$4k$	$9k$	$16k$	$25k$	$36k$.

Total prob $= 1 \Rightarrow \sum f(x) = 1 \Rightarrow 4k + 9k + 16k + 25k + 36k = 1$

$\Rightarrow 90k = 1 \Rightarrow k = \frac{1}{90}.$

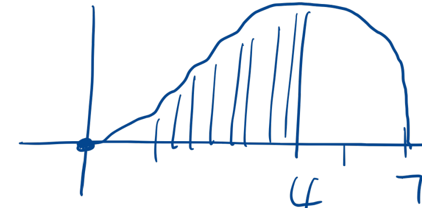
6d) $E(3X-7) = E(3X) - E(7) = 3E(X) - 7$

$= 3\left(\frac{44}{9}\right) - 7 = \frac{44}{3} - 7 = \frac{44-21}{3} = \frac{23}{3};$

$V(3X-7) = V(3X) - V(7) = 3^2 V(X) - 0 = 9V(X) = 9(1.358)$

$= 12.222.$

8\. a) $f(x) = \begin{cases} kx^2(7-x) & ; \ 0 \le x \le 7 \\ 0 & ; \ \text{otherwise}; \end{cases}$



Continuous distribution.

Total prob = $P(x \le 7) = \int_0^7 kx^2(7-x)\,dx = 1;$

$\Rightarrow \int_0^7 (7kx^2 - kx^3)\,dx = 1$

$\Rightarrow \int_0^7 7kx^2\,dx - \int_0^7 kx^3\,dx = 1$

$\Rightarrow \frac{7}{3}k\, x^3 \Big|_0^7 - \frac{k}{4}x^4 \Big|_0^7 = 1.$

$\frac{7^4 k}{3} - \frac{7^4 k}{4} = 1;$

$\frac{7^4 k}{12} = 1;$

$k = \frac{12}{7^4} = \frac{12}{2401}.$

b) $E(X) = \int_0^7 x f(x)\,dx = \int_0^7 x\left(7kx^2 - kx^3\right)dx$

$$= \int_0^7 \left(7kx^3 - kx^4\right)dx = \int_0^7 7kx^3\,dx - \int_0^7 kx^4\,dx$$

$$= \frac{7}{4}kx^4 - \frac{k}{5}x^5 \Big|_0^7 = \frac{k}{4}7^5 - \frac{k}{5}7^5 = \frac{7^5 k}{20}$$

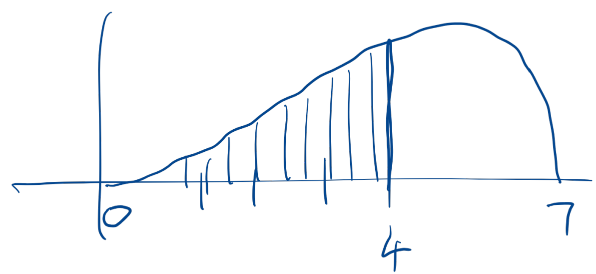
$$= \frac{7^5}{20}\cdot\frac{12}{7^4} = \frac{7\cdot\cancel{12}\,3}{\cancel{20}\,5} = \frac{21}{5}$$

8c) $P(X<4) = F(4) = \int_0^4 f(x)\,dx = \int_0^4 (7kx^2 - kx^3)\,dx$

$$= \frac{7}{3}k\,x^3 \Big|_0^4 - \frac{k}{4}x^4 \Big|_0^4$$

$$= \frac{448k}{3} - \frac{256k}{4} = \frac{1024k}{12} = \frac{1024}{12} \cdot \frac{12}{7^4}$$

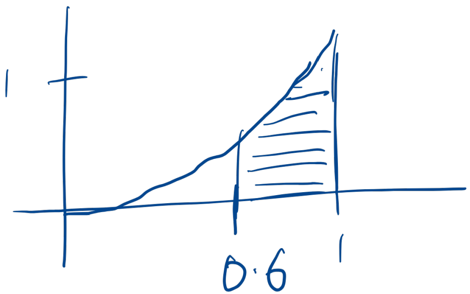
$$= \frac{1024}{2401}$$



10. $P(x>0.6) = $ blue area $= 1 - P(x \leq 0.6)$

$= 1 - F(0.6)$

$= 1 - \int_0^{0.6} \left(\frac{10}{4}x - x^3\right) dx$



or $1 - \left[\frac{1}{4}\left\{5(0.6)^2 - (0.6)^4\right\}\right]$

$$F(x) = \begin{cases} 0, & x<0 \\ \frac{1}{4}(5x^2 - x^4), & 0 \leq x \leq 1 \\ 1, & x>1 \end{cases}$$

CDF,

$$\text{PDF} = F'(x) = f(x) = \begin{cases} 0, & x<0 \\ \frac{1}{4}(10x - 4x^3), & 0 \leq x \leq 1 \\ 0, & x>1; \end{cases}$$

7. b) X = # of wins;

c) $n = 24$; probability of a win $= p$;

probability of a loss $= 1-p$

$$P(X=7) = P(X=6)$$

$$\binom{24}{7} p^7 (1-p)^{24-7} = \binom{24}{6} p^6 (1-p)^{24-6}$$

$$\frac{\cancel{24!}}{\cancel{17!}\,\cancel{7!}} \cancel{p^7}\,\cancel{(1-p)^{17}} = \frac{\cancel{24!}}{\cancel{18!}\,\cancel{6!}} \cancel{p^6}\,(1-p)^{\cancel{18}}$$

$$\frac{p}{7} = \frac{1-p}{18} \Rightarrow 18p = 7 - 7p \Rightarrow 25p = 7 \Rightarrow p = \frac{7}{25}$$

$P(x=7)$ = probability of winning 7 games in 24 games.

$= \text{PDF}$

$= \binom{n}{r} p^r (1-p)^{n-r}$ $\qquad n = 24$

$r = 7$

$17! = 1 \cdot 2 \cdot 3 \cdot \ldots \ldots 16 \cdot 17$

$18! = 1 \cdot 2 \cdot 3 \ldots \ldots 16 \cdot 17 \cdot 18$

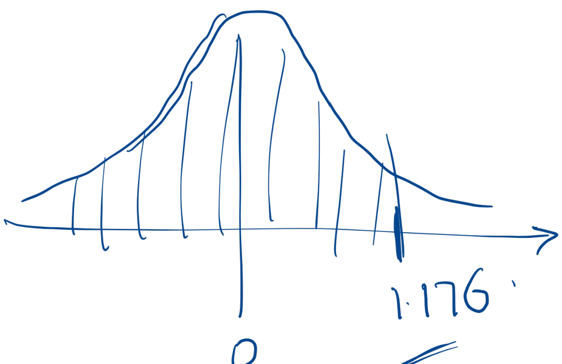
12. $\mu = 4.3$ $\sigma = 0.17$

a) $P(X < 4.5)$

S1) $Z = \dfrac{X-\mu}{\sigma} = \dfrac{4.5 - 4.3}{0.17} = \dfrac{0.2}{0.17}$

$Z = 1.176$

$P(X < 4.5) = P(Z < 1.176) = KDF(Z < 1.176)$



$= \text{normalcdf}(-999, 1.176)$

$= \displaystyle\int_{-\infty}^{1.176} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$

$= 0.8802 \simeq 88.02\%$

b) $P(x>4)$: $\quad z = \dfrac{x-\mu}{\sigma} = \dfrac{4-4.3}{0.17} = \dfrac{-0.3}{0.17} = -1.764$

$$P(x>4) = 1 - P(x<4)$$
$$= 1 - \int_{-\infty}^{4} f(x)\,dx$$
$$= 1 - \text{normalcdf}(-999, -1.764) \qquad \text{(TI 84 Calc)}$$
$$= 1 - \underline{\underline{0.0388}} \cong 0.9611 \cong 96.11\%$$

c) $P(4 < X < 4.5) = P(X < 4.5) - P(X > 4)$

$$= P(X < 4.5) - (1 - P(X < 4))$$

$$= (a) - (b)$$

$$= 0.8802 - (1 - 0.9611)$$

$$= 0.8413 \cong 84.13\%$$

13 a) $S_{xx} = \sum x^2 - \dfrac{(\sum x)^2}{n}$

$= 211 - \dfrac{(45)^2}{10} = 8.5$

$S_{xy} = \sum xy - \dfrac{(\sum x)(\sum y)}{n}$

$= 1460 - \dfrac{(45)(321)}{10} = 1460 - 1444.5 = \underline{\underline{15.5}}$

b) $y = a + \underline{b}x$; $b = \dfrac{S_{xy}}{S_{xx}} = \dfrac{15.5}{8.5} = 1.82$

$a = y - bx \cong \bar{y} - b\bar{x} = \dfrac{321}{10} - (1.82)\dfrac{45}{10} = 23.91$;

$$y = 23.91 + 1.82x$$

c) $b = 1.82 = \frac{1.82}{1}$

when 1 year passes by, the mileage increases by 1820 miles; (1.82×1000)

d) $y = 23.91 + 1.82x$

$x = 45$; $y = 23.91 + (1.82)(45) = 105.81.$

$= (105.81 \times 1000)$

$= 105810$ miles;

$$\rho = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{Cov}{\sigma_x \sigma_y} = \frac{15.5}{\sqrt{(8.5)(572.9)}}$$

$$\rho = 0.222$$

$$S_{yy} = \sum y^2 - \frac{(\sum y)^2}{n} = (10877) - \frac{(321)^2}{10}$$

$$= 10877 - 10304.1$$

$$= 572.9$$

$$\sum (x-\bar{x})^2 = \sum (x^2 - 2x\bar{x} + \bar{x}^2)$$

$$= \sum x^2 - 2\bar{x} \sum x + \sum \bar{x}^2$$

$$= \sum x^2 - 2\bar{x}\, n\bar{x} + n\bar{x}^2$$

$$= \sum x^2 - 2n\bar{x}^2 + n\bar{x}^2$$

$$= \sum x^2 - n\bar{x}^2$$

$$= \sum x^2 - n \cdot \left(\frac{\sum x}{n}\right)^2 = \sum x^2 - \frac{(\sum x)^2}{n} = S_{xx}$$

So $\sum (y-\bar{y})^2 = \sum y^2 - \frac{(\sum y)^2}{n}$; $= S_{yy}$

$$\sum (x-\bar{x})(y-\bar{y}) = S_{xy} = \sum xy - \frac{(\sum x)(\sum y)}{n}$$