Meteor(oid?) models and density estimation

Martin Baláž[1] Juraj Tóth, PhD.[1] Peter Vereš, PhD.[2]

[1]Comenius University
[2]Minor Planet Center, Harvard-Smithsonian Center for Astrophysics

DAA colloquium

2020–02–26

Overview

Motivation

All work and no play makes Jack a dull boy...

English proverb

Simulation

Parametric estimation

Introduction

Sometimes we know what to expect...

- establish the values of σ and μ.

Algorithm

- define the class of distributions

Histograms

Bin width

- usually we set bin width manually

Bin width

- usually we set bin width manually
- there are ±rigorous methods of determination of optimal bin width

Case study I: heights of people

Case study II: height by sex

- we can extract information from the distribution

Summary

- advantages
  - simple
  - can extract information
- disadvantages
  - we have to know the distribution
  - dependent on binning
  - there must be many data points

Kernel density estimation

No parameters

Sometimes this all is not applicable...

No parameters

Sometimes this all is not applicable...

- we cannot use parametric methods
- the distribution is not **known**
- there are **too many** parameters or minima
-

One dimension

Bandwidth

- similar to bin width in histograms

$$\hat{F}(\mathbf{x}) = \frac{1}{nh} \sum_{i=1}^{n} K\left(h\left(x - x_i\right)\right)$$

Multiple dimensions

Correlation

$$\hat{F}(\mathbf{x}) = \frac{1}{n\left|\mathbf{H}\right|} \sum_{i=1}^{n} K\left(\mathbf{H}^{-1}\left(\mathbf{x} - \mathbf{x}_i\right)\right)$$

Convolution

KDE can be defined as a convolution

Adaptive KDE

Bandwidth need not be constant

- sometimes it is beneficial to vary h
- generally
  - narrow bandwidth where data are abundant
  - wide bandwidth where data are sparse

Summary

References

- **Hwang, J.-N. – Lay, S.-R. and Lippman, A.**: Nonparametric multivariate density estimation: a case study. IEEE Transactions on Signal Processing 42, 1994.
- **Vida, D. – Brown, P. – Campbell-Brown, M.**: Modeling the measurement accuracy of pre-atmosphere velocities of meteoroids. MNRAS 479, 2018