Mon. Not. R. Astron. Soc. **320**, 417–423 (2001)

Effects of excitation and ionization in meteor trains

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Accepted 2000 June 1. Received 2000 June 1; in original form 1999 December 2

ABSTRACT

In a previous paper we examined the problem of the determination of the ionization coefficient β, and we now extend our analysis to the luminous efficiency τ, relating this to an excitation coefficient ζ, essentially the number of times an atom is excited during the thermalization process. We discuss the scattering processes involved and develop integral equations expressing ζ in terms of the relevant scattering cross-sections. The data available in applying these equations are (i) direct measurements of β and ζ for iron particles moving through an ionization chamber, with velocities between 18 and $46\,\mathrm{km\,s^{-1}}$; (ii) scattering cross-sections σ_i and σ_e, for ionization and excitation on collision between Fe atoms and air molecules, measured under controlled single-atom collision in the laboratory for relative velocities between 60 and $120\,\mathrm{km\,s^{-1}}$; (iii) theoretical calculations of the diffusion cross-section σ_d of iron atoms in air over the relevant velocity range. Employing an approximation in which the scattering is isotropic in the centre-of-mass frame (the random scattering approximation, RSA), we take the laboratory simulation results for the ionization and luminosity and invert the equations for β and ζ to obtain values of β_0 and ζ_0, the ionization and excitation probabilities at the first collision. We find that for velocities above $42\,\mathrm{km\,s^{-1}}$ the value of ζ_0 becomes greater than unity. As ζ_0 is a probability, this must be incorrect, leading to the conclusion that the contribution from subsequent collisions is underestimated. This is possible if small-angle scattering is underestimated in the RSA. To investigate this we have taken the extreme case in which the trajectories of atoms are supposed to undergo no deviation at all on ionization or excitation. This now enables us to derive the ratios σ_i/σ_d and σ_e/σ_d and hence, using the theoretical values of the diffusion cross-section, values of σ_i and σ_e. The values so obtained appear unacceptably high and inconsistent with the experimental cross-sections. A possible reason for the failure of the equations is the assumption that ionization permanently removes atoms available for excitation, which will not be true if charge transfer takes place. No cross-sections are available for this process, but we can evaluate ζ in the limiting case when the loss of velocity during transfer may be neglected. To compare our results for the ionization and excitation cross-sections with the experimental ones, we have extrapolated them to $60\,\mathrm{km\,s^{-1}}$ and find quite satisfactory agreement. We have accordingly extended the results for the excitation and ionization coefficients to velocities above $60\,\mathrm{km\,s^{-1}}$ utilizing the experimental cross-sections for this region. We consider the values of ζ and the corresponding luminous efficiencies to be the best available in the present circumstances.

Key words: atomic processes – meteors, meteoroids.

1 INTRODUCTION

The ionization coefficient and luminous efficiency of meteors are of paramount importance for meteor physics, knowledge of them being necessary to evaluate densities, masses, height distributions and fluxes. In a previous paper (Jones 1997), henceforth referred to as Paper I, we have investigated the determination of the ionization coefficient β and examined the best estimates that may currently be made. After a discussion of earlier theoretical work, we proposed a new approach in which an integral equation for β was set up in terms of $\beta_0(v)$, the probability of ionization when a meteoric atom collides with an air molecule at velocity v. In the present paper we consider further the foundations of this approach and also extend our considerations to the luminous efficiency.

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In terms of the luminous intensity I (the power radiated by the meteor in a specified frequency range), velocity v and rate of change of mass dm/dt, the luminous efficiency τ may be defined through

$$I = \frac{1}{2}\tau v^2 \frac{\mathrm{d}m}{\mathrm{d}t}, \tag{1}$$

so that it is the proportion of the kinetic energy of the meteoroid converted into radiant energy in the visual range and as such dimensionless. One may attempt to determine τ by a number of methods.

(i) *Direct observational determination*. The most detailed observational study has been made by Verniani (1965) who obtained the luminous efficiency as a function of velocity by photographic determinations of the deceleration and the luminosity, the former giving the mass m (with some definite assumption as to the density) and the latter the product $m\tau$. He found, for the photographic luminous intensity,

$$\tau_{\mathrm{p}} = 5.25 \times 10^{-5} v. \tag{2}$$

Here, as throughout the paper, v is measured in km s^{-1}. The linear dependence on velocity was previously suggested by Whipple (1938). However, no reliance can be placed on equation (2), as it appears that the method used by Verniani to determine the mass of the meteoroid is invalidated by an inadequate account of the effect of fragmentation.

(ii) *Simulation in the laboratory, minute metallic particles being accelerated electrostatically and injected at meteor velocities into an ionization chamber.* Numerous investigations show that almost all cometary meteoroids contain a significant amount of iron and that this metal dominates the radiation in the spectral region of interest (Verniani 1965; Tagliaferri & Slattery 1969; Savage & Boitnott 1971; Becker & Slattery 1973; Bronshten 1983). Friichtenicht & Becker (1973) have obtained the luminous efficiency of microparticles of iron moving at velocities from 11–46 km s^{-1} through air in an ionization chamber. In contrast to the behaviour shown in equation (2), they find that τ_{p} increases rapidly to a maximum at $v \simeq 18$ km s^{-1}. Following Bronshten (1983) we shall refer to this general dependence as Type B while that of equation (2) will be referred to as Type A (Bronshten actually refers to Models A and B but this is somewhat misleading as no particular mathematical or physical models are implied here).

(iii) *Calculation in terms of the scattering cross-sections for excitation and momentum loss.* We should mention here the work of Öpik (1955, 1958). In principle the luminous efficiency may be found by computations of the populations of all possible excited states and the probabilities for transitions between levels. As remarked by Friichtenicht & Becker, this is a heroic task, attempted by Öpik using arc intensities to obtain the transition probabilities. However he was hampered by having almost no information on the actual processes by which the levels are populated by collisions and the collision cross-sections, so that, other than the fact that he did suggest the possibility of Type B behaviour, Öpik's work is now largely of historical interest.

More recently Boitnott & Savage (1972) have determined the emission probability under controlled single-collision conditions in the laboratory and integrated over all collisions to obtain the total radiation, with values for τ much smaller than those given by Friichtenicht & Becker (1973). The integration over collisions was performed using the expression proposed by Sida (1969), the direct analogue of the Massey–Sida formula for the ionization coefficient. In Paper I we showed this formula to be incorrect and we propose an alternative expression below. However, Boitnott & Savage's results are consistent with Type B, which, indeed will result under fairly general conditions. It is convenient to define an excitation coefficient ζ by writing

$$\tau m v^2/2 = \varepsilon\zeta, \tag{3}$$

where ε is a mean excitation energy and m an atomic mass. It will be seen that ζ is a sum of excitation probabilities over collisions. Now once v becomes greater than the threshold value for excitation, we may expect the efficiency τ to increase from zero with velocity, in which case ζ must increase more rapidly than v^2. However if ζ increases less rapidly than this above some velocity v_{m} then τ must have a maximum at $v = v_{\mathrm{m}}$.

In the following we shall focus attention on method (iii), the theoretical determination of the excitation coefficient, in relation to method (ii), simulation in the laboratory. We shall begin by examining the general form of the equations in terms of the scattering cross-sections. Two approaches may be considered. In the first we set up Boltzmann equations for the rate of change of particle distribution functions with time, as we intend to discuss elsewhere. However a second much more direct approach is possible, without the introduction of distribution functions as intermediate auxiliary quantities. In the following, instead of discussing how the relevant quantities change with time, we consider the discrete changes at each collision and obtain integral equations for β and ζ, in terms of the probabilities β_0 and ζ_0 of ionization and excitation on an individual collision. We shall take the results of method (ii) above and invert the equations for β and ζ to obtain β_0 and ζ_0, comparing the cross-sections thus obtained with laboratory determinations. Our results are consistent with the possibility that the luminosity is influenced by charge transfer from the meteoric ions to the molecules of the air, as strongly suggested by the evidence considered in Paper I. To extend the luminosity values obtained from method (ii) to higher velocities, we calculate ζ under the assumption that charge transfer is dominant, using the ionization cross-sections determined by Boitnott & Savage (1972).

2 INTEGRAL EQUATIONS

We may obtain an integral equation for the luminosity coefficient by reference to the derivation given in Paper I for the equation for the ionization coefficient in the absence of charge transfer, though our development here will be more general than that given in the previous paper. We assume that the atoms ablating off the surface and the air molecules are in free molecular flow, the presence of the unablated part of the meteoroid not affecting their motion. The meteor atoms will all have essentially the same initial velocity V, and since V is much higher than the thermal velocities of the molecules of the air these are taken to be initially at rest.

The luminosity coefficient ζ is given by

$$\zeta = \sum_j \zeta_j. \tag{4}$$

Here ζ_j is the mean number of times an atom is excited into level j as it is brought from its initial velocity V (the velocity of the meteoroid) to thermal velocities by collisions with the air molecules, it being understood that the excitation energy ε_j corresponds to radiation in the visual range.

To obtain an integral equation for ζ, let $\beta_0(v)$ and $\zeta_0(v)$ be the respective probabilities that an atom of velocity v will be ionized or excited on collision with a molecule of the air. Thus

$$\zeta_0 = \sigma_e(v)/\sigma_t(v) \tag{5}$$

in terms of the total cross-section σ_t and the total excitation cross-section, given, in terms of the cross-sections for excitation into the individual levels, by

$$\sigma_e = \sum_j \sigma_j. \tag{6}$$

To ensure the total collision cross-section is precisely defined, we shall assume that ψ_p, the angle through which the relative direction of motion is turned on collision, is rigorously zero if the impact parameter p (the length of the perpendicular from the centre of the scattering field to the initial line of motion of the incoming particle) is greater than some critical value a. The total cross-section will thus be πa^2. The classical total scattering cross-section is strictly speaking arbitrary in that it can be set to any value greater than πa^2, in which case there are particles with impact parameter greater than a, which are said to be scattered but which suffer no deviation ψ. The definitions of β_0 and ζ_0 are then altered, but it may readily be shown that the equations derived below, in particular equations (7), (8) and (11), remain valid if a component is added to represent the particles that are said to suffer elastic scattering but in fact suffer no change in their trajectory.

ζ will be ζ_0 plus the contribution from all subsequent collisions. We assume that an atom has lost the energy ε_j by radiation before further collision, and also note that Fe ions contribute very little to the radiation. Then at the second collision the fraction of the number of original atoms available for excitation will be $1 - \beta_0$, that is, the original ones apart from those that have been ionized. In the absence of charge transfer we now have

$$\zeta = \zeta_0 + (1 - \beta_0)\langle\zeta\rangle_\varepsilon, \tag{7}$$

where $\langle\zeta\rangle_\varepsilon$ is the average of $\zeta(\varepsilon)$ over all possible final energies ε‘ after a non-ionizing collision with an initial particle of kinetic energy ε. Equation (7) corresponds to the expression, derived in Paper I, for the ionization coefficient:

$$\beta = \beta_0 + (1 - \beta_0)\langle\beta\rangle_\varepsilon. \tag{8}$$

Formally,

$$\langle\zeta\rangle_\varepsilon = \int_0^\varepsilon \zeta(\varepsilon')p(\varepsilon, \varepsilon')\,\mathrm{d}\varepsilon', \tag{9}$$

where $p(\varepsilon, \varepsilon')\,\mathrm{d}\varepsilon'$ is the probability per unit length of its trajectory that a particle of kinetic energy ε is scattered into the range $(\varepsilon', \varepsilon' + \mathrm{d}\varepsilon')$ given by

$$p(\varepsilon, \varepsilon') = \frac{\sigma(\varepsilon, \varepsilon')}{\sigma_t(\varepsilon)}. \tag{10}$$

The generalized cross-section $\sigma(\varepsilon, \varepsilon')$ is defined such that $\sigma(\varepsilon, \varepsilon')\,\mathrm{d}\varepsilon'$ is the probability per unit length of its trajectory that a particle of velocity v is scattered into the range $(\varepsilon', \varepsilon' + \mathrm{d}\varepsilon')$, and on integration over ε' one obtains the total cross-section σ_t.

Although it had been assumed that once ionization has taken place the atom in question is no longer available for excitation, there is evidence that charge transfer to the molecules of the air takes place. The smallness of the observational β of visual meteors compared with that of artificial meteors in an ionization chamber indicates that charge transfer and subsequent recombination removes most of the ionization (Jones 1997). We have no data on the cross-sections involved, except that they appear to be large. We can however set up equations for ζ and β for high velocities under the assumption that charge transfer is dominant, so that the atom can be re-ionized. In that case equation (7) is replaced by

$$\zeta = \zeta_0 + \langle\zeta\rangle_\varepsilon. \tag{11}$$

3 ELASTIC AND INELASTIC SCATTERING CROSS-SECTIONS

During the collisions of the comparatively slow atoms and molecules, one normally expects the electrons to follow the motions of the nuclei adiabatically. That is, at every point in the motion the nuclei may be taken to be stationary in calculating the energy of the electrons, which are in their ground state for each position. The energy of the ground state plus the Coulombic interaction of the nuclei can be taken to be the potential energy of the system and the collision is elastic. Taking the duration of a collision as $\Delta t = a/v$, where v is the relative particle velocity and a the order of an atomic or molecular radius, we would expect the uncertainty principle to imply transitions of energy ΔE from the ground state to be rare unless $\Delta E \Delta t'' h$, Planck's constant. Here $\Delta t \sim 10^{-12}$ s while ΔE is the order of eV. Since $h = 4.135 \times 10^{-15}$ eV s, the above condition is certainly not obeyed and on this basis we would expect the probability of ionization in the meteor train to be small. However, as discussed by Bates & Massey (1954) the high ionization probability which evidently obtains in the train in actuality may be understood on the basis of curve crossing. Similar considerations apply to excitation processes. Fig. 1 shows the total electronic energy of the quasi-molecule (meteor atom plus air molecule) plotted against separation. The lower curve $S_1S'_1$ represents the energy for the state $A + M$, air molecule plus meteoric atom, while $S_2S'_2$ represents the energy of $A + M^+ + \mathrm{e}$, air molecule plus meteoric ion plus free electron. It is supposed that if the matrix elements of the Hamiltonian between the two states were zero then at some separation R the energy curves would cross, as indicated by the dotted line. In practice, elementary perturbation theory shows a ‘repulsion’ to take place between the two states, to give the state of affairs as shown. Imagine now a collision to take place between A and M, so that the point indicating the energy of the combined system moves fromS_1 towards R. In the vicinity of R the two curves are close together and there is a finite probability P of the system making a transition

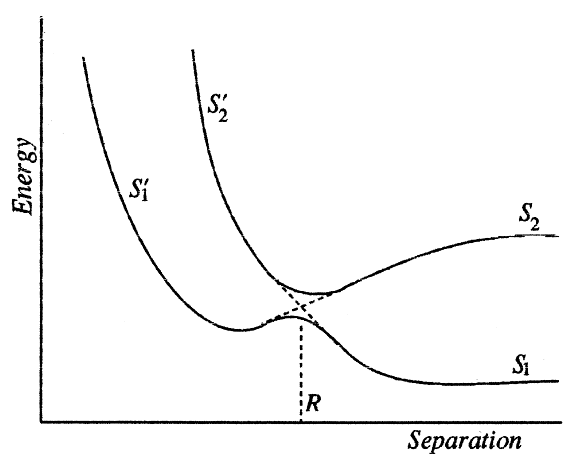

Figure 1. Illustration of ionization by curve crossing.

to the upper curve, now moving towards S'_2 until at this point the total electronic energy becomes equal to the original kinetic energy of the atom and molecule at large separation. At this point, therefore, the system reverses its motion. When it again reaches the vicinity of R there is no possibility of transition back to the lower curve, as the free electron will have left the quasi-molecule and the system moves along to S_2, where the collision is at an end. There is a second possible path, whereby the system remains on the lower curve as R is passed (the probability of this being $1 - P$), moving on to S'_1 before reversing its motion, and then making a transition to the upper curve as R is again passed (the probability of this being P). One sees that the probability of ionization by these processes is

$$P_{\mathrm{i}} = P + (1 - P)P. \tag{12}$$

This expression may not give the total probability. The actual case may well be very much more complicated, with many energy levels and transitions involved, but Fig. 1 does illustrate the basic principles of ionization and charge transfer by curve crossing.

Although the electrons will not follow the nuclei completely adiabatically if transitions such as those discussed above take place, at velocities for which ionization is appreciable the ionization energy will be much less than the kinetic energy so that the ionization process will have little effect on the dynamics of the collision. The collisional process is characterized by the differential scattering cross-section I, defined such that the probability of scattering between the angles ψ and $\psi + \mathrm{d}\psi$ is $2\pi I(\psi)\sin\psi\,\mathrm{d}\psi$, where ψ is the angle through which the direction of motion is turned in the centre-of-mass frame. Let us suppose that we are to calculate I under the assumption that the electrons do follow the nuclear motions adiabatically, so that the interaction potential governing the collision is the Coulombic nuclear interaction plus the ground-state energy of the electrons. We are therefore calculating the dynamics of the elastic collisions of hypothetical particles interacting through $V(r)$. To avoid confusion with the true elastic cross-section we shall term that cross-section the adiabatic cross-section. Similar remarks hold for the excitation processes. For reasons we have given, the adiabatic cross-section of the hypothetical particles will therefore be a good approximation to the true total cross-section.

Referring to Paper I we see that if μ is the ratio of the mass of a meteoric atom to the mass of a molecule of the air, the relationship between the initial and final kinetic energies ε and ε' in the rest frame of the atmosphere is, for an elastic collision,

$$\varepsilon' = \frac{\varepsilon}{(1+\mu)^2}(1 + \mu^2 + 2\mu\cos\psi) \tag{13}$$

and it follows that the adiabatic generalized cross-section is

$$\sigma_{\mathrm{a}}(\varepsilon,\varepsilon') = \begin{cases} \dfrac{(1+\mu)^2 2\pi I(\psi)}{2\mu\varepsilon}, & \varepsilon\left(\dfrac{1-\mu}{1+\mu}\right)^2 < \varepsilon' < \varepsilon, \\ 0, & \text{otherwise,} \end{cases} \tag{14}$$

where ψ has been defined generally earlier and specifically in this expression is the angle through which the relative direction is turned as the energy decreases from ε to ε'.

It will readily verified that on integration of $\sigma(\varepsilon,\varepsilon')$ over ε' one obtains the elastic cross-section

$$\sigma_{\mathrm{a}} = 2\pi\int_0^{\pi} I(\psi)\sin\psi\,\mathrm{d}\psi. \tag{15}$$

We might note that the classical differential scattering cross-section is given by

$$I(\psi) = -\frac{p}{\sin\psi}\frac{\mathrm{d}p}{\mathrm{d}\psi} \tag{16}$$

(Mott & Massey 1965). Hence if ψ is identically zero for $p > a$ then equation (15) may be rewritten as

$$\sigma_{\mathrm{a}} = \pi\int_0^{a^2}\mathrm{d}p^2. \tag{17}$$

We wish to set up approximate forms for $\sigma(\varepsilon,\varepsilon')$ and $p(\varepsilon,\varepsilon')$. The only information we have available concerning the elastic scattering relates to the diffusion cross-section

$$\sigma_{\mathrm{d}} = 2\pi\int_0^{\pi}(1-\cos\psi)I(\psi)\sin\psi\,\mathrm{d}\psi. \tag{18}$$

This is always well defined, the small angle scattering (which can lead to the divergence of σ_{a} in classical theory) receiving much less weight than in equation (15). It is to be noted from equation (13) that the energy loss on elastic collision of a particle of initial energy ε is

$$\varepsilon - \varepsilon' = \frac{\varepsilon}{(1+\mu)^2}(1-\cos\psi), \tag{19}$$

and we therefore see that the mean energy loss per unit path length is

$$\Delta\varepsilon = \frac{2\varepsilon\mu\sigma_{\mathrm{d}}}{(1+\mu)^2}. \tag{20}$$

In the present context an important aspect of this result is that over most of the range of meteoric atom energies ε is much greater than the ionization and excitation energies, so that if σ_{d} is comparable with σ_{i} and σ_{e} the energy loss on collision is principally the result of elastic collisions.

To set up an approximation for $\sigma_{\mathrm{a}}(\varepsilon,\varepsilon')$ in terms of σ_{d}, we follow Paper I in assuming the random scattering approximation (RSA), in which I is independent of ψ, and write

$$\sigma_{\mathrm{a}}(\varepsilon,\varepsilon') = \begin{cases} \dfrac{(1+\mu)^2 2\pi\sigma_{\mathrm{d}}}{2\mu\varepsilon}, & \varepsilon\left(\dfrac{1-\mu}{1+\mu}\right)^2 < \varepsilon' < \varepsilon, \\ 0, & \text{otherwise.} \end{cases} \tag{21}$$

It may be verified that this approximation has the virtue of giving the correct mean energy loss (equation (20)). On integration over ε', the elastic scattering cross-section is given as σ_{d}. It is in fact readily seen that the right-hand sides of equations (15) and (18) reduce to one another when I is set to a constant.

One now obtains

$$p(\varepsilon,\varepsilon') = \begin{cases} \dfrac{(1+\mu)^2}{4\mu\varepsilon}, & \varepsilon\left(\dfrac{1-\mu}{1+\mu}\right)^2 < \varepsilon' < \varepsilon, \\ 0, & \text{otherwise.} \end{cases} \tag{22}$$

and it will be seen that it is consistent with this approximation to set the total cross-section equal to σ_{d}, the denominators of equation (5), for ζ_0 and the corresponding equation for β_0.

This approximation should certainly be adequate when v is sufficiently small, when the elastic cross-section will be much greater than the excitation and ionization cross-sections, in which case the total cross-section may be replaced by the elastic cross-section in the denominator of equation (5). However, we shall find the inelastic cross-sections to be greater than the diffusion cross-section at higher velocities, in which case the approximation breaks down. We note that although the diffusion cross-section gives the correct mean energy loss (apart from the comparatively small ionization or excitation energy), it is weighted towards higher scattering angles. This leads us to suppose that at high energies much of the ionization tales place at small scattering angles for which the energy loss is small. To take account of this we add to $p(\varepsilon,\varepsilon')$ a delta function term representing inelastic scattering with no energy change apart from the ionization or excitation threshold energy, and therefore omitted from the term involving σ_d. To obtain an approximation to σ_t at high energies, we again note that at high velocities the deviation on ionizing or exciting collision will be small and approximate the total cross-section by

$$\sigma_t = \sigma_d + \sigma_i + \sigma_e. \tag{23}$$

Equation (7) is replaced by

$$\zeta(v) = \zeta_0(v)[1 + \zeta(v)] + [1 - \zeta_0(v) - \beta_0(v)]\langle\zeta\rangle_v. \tag{24}$$

Together with equation (23), equation (24) may be rearranged to read

$$\zeta = \frac{\sigma_e}{\sigma_d + \sigma_i} + \frac{\sigma_d}{\sigma_d + \sigma_i}\langle\zeta\rangle_v. \tag{25}$$

This approximation will be accurate in the limits of small and large velocities.

We may similarly use equation (23) in equation (19) of Paper I for the ionization coefficient, to obtain

$$\beta = \frac{\sigma_i}{\sigma_d + \sigma_i} + \frac{\sigma_d}{\sigma_d + \sigma_i}\langle\beta\rangle_v. \tag{26}$$

These equations have been derived assuming there is no charge transfer. Although there is evidence that charge transfer does take place, we have no direct knowledge of the relevant cross-sections and here discuss only the limiting situation where the loss in velocity during the transfer process may be neglected. In this case we do not have to assume that all inelastic collisions take place with zero deviation of the trajectory. We approximate the total generalized cross-section by

$$\sigma(\varepsilon,\varepsilon') = (\sigma_t - \sigma_d)\delta(\varepsilon - \varepsilon') + \sigma_a(\varepsilon,\varepsilon'), \tag{27}$$

with $\sigma_a(\varepsilon,\varepsilon')$ given by equation (21). This approximation ensures that the correct mean energy loss (equation 20) is obtained in terms of σ_d. Combining equations (9), (10), (11) and (27), we now reach the result

$$\zeta = \frac{\sigma_e}{\sigma_d} + \langle\zeta\rangle_v, \tag{28}$$

while the corresponding result for the ionization coefficient is

$$\beta = \frac{\sigma_i}{\sigma_d} + \langle\beta\rangle_v. \tag{29}$$

This limit will be exact if the transfer takes place during the same collision as the ionization – in other words, if it is the air molecule, rather than the meteoric atom, that is ionized.

4 APPLICATION OF THE INTEGRAL EQUATIONS

We have now set up integral equations for the ionization coefficient β and luminosity coefficient ζ in terms of the relevant scattering cross-sections. As input data for these equations we have available the following.

(i) Direct measurements of β and ζ for iron particles moving through an ionization chamber, with velocities between 18 and 46 km s^{-1}.

(ii) Scattering cross-sections for ionization on collision between Fe atoms and air molecules measured under controlled single-atom collision in the laboratory for relative velocities between 60 and 120 km s^{-1}.

(iii) Theoretical calculations of the diffusion cross-section of iron atoms in air over the relevant velocity range.

The results obtained by Friichtenicht & Becker (1973) for the luminous efficiency of iron may be approximated to good accuracy by

$$\tau = 0.151v^{-0.75}, \tag{30}$$

for velocities between 20 and 46 km s^{-1}. Using the relative intensities for the Fe multiplets given by Bointnott & Savage (1972), we can then obtain for the luminosity coefficient

$$\zeta = 0.01333v^{1.25}, \qquad (20 \leqslant v \leqslant 46). \tag{31}$$

The simulation of Slattery & Friichtenicht (1967) gives, for the ionization coefficient of iron,

$$\beta = 5.96\times10^{-6}v^{3.12}, \qquad (20 \leqslant v \leqslant 45). \tag{32}$$

The exact integral equations will involve differential scattering cross-sections. These are unavailable and so we have used an approximation in which the elastic differential scattering cross-section is taken to be a constant. Assuming that the iron atoms are ionized and there is no subsequent charge transfer to the oxygen or nitrogen molecules, the relevant equations are then (7), (8), (9) and (22). Given the laboratory determinations of β and ζ, we are able to invert (7) and (8) to obtain values of β_0 and ζ_0, the ionization and excitation probabilities on collision. The results are shown in Fig. 2 and it will be seen that for velocities above 42 km s^{-1} the value obtained for ζ_0 exceeds unity. As ζ_0 is a probability, this must be incorrect. This implies that the second

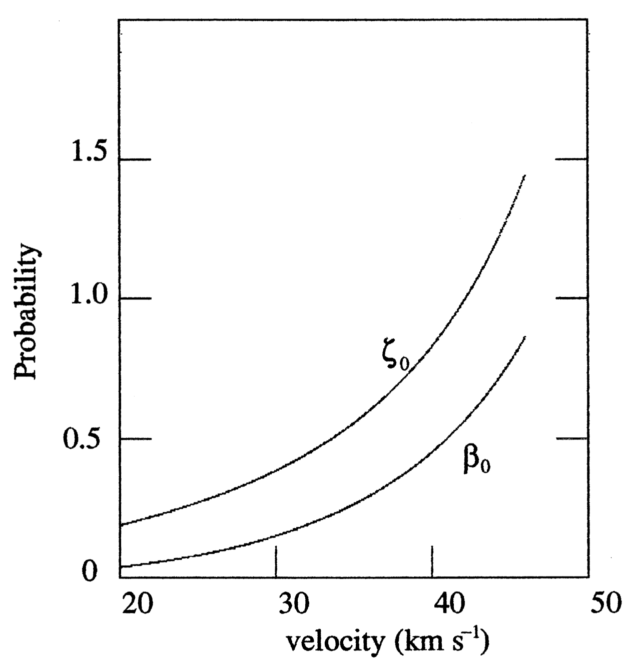

Figure 2. Probabilities of excitation (ζ_0) and ionization (β_0) on a single collision, on application of equations (7) and (8) to the results of simulation in the laboratory.

term on the right-hand side of equation (7), the contribution from subsequent collisions is underestimated. This is possible if small-angle scattering is underestimated in the RSA. To investigate this possibility, we have taken the extreme case in which the trajectories of atoms are supposed to undergo no deviation at all on ionization or excitation, when equations (25) and (26) are obtained. Using the simulation results of equations (31) and (32) now enables us to derive the ratios σ_i/σ_d and σ_e/σ_d, as shown in Fig. 3. It will be seen that for the higher velocities the ratios become very large: at $46\,\mathrm{km\,s^{-1}}$ they are 6.31 and 10.54 respectively. We take the diffusion cross-section as given by Bronshten (1983), that is

$$\sigma_d = 5.6\times10^{-15}v^{-0.8}, \qquad (33)$$

where the velocity is in $\mathrm{km\,s^{-1}}$ and the units of the cross-section are cm^2. At $46\,\mathrm{km\,s^{-1}}$ we hereby find $\sigma_d = 2.61\times10^{-16}\,\mathrm{cm}^2$, giving $\sigma_i = 1.65\times10^{-15}\,\mathrm{cm}^2$ and $\sigma_e = 2.75\times10^{-15}\,\mathrm{cm}^2$. These values appear unacceptably high. Boitnott & Savage (1972) have measured the ionization and excitation cross-sections under single-collision conditions, where incoming N_2 and O_2 molecules with velocities between 60 and $120\,\mathrm{km\,s^{-1}}$ impinge upon iron atoms. They find the cross-sections to be very approximately constant at $\sigma_i = 2.75\times10^{-16}\,\mathrm{cm}^2$ and $\sigma_e = 1.5\times10^{-16}\,\mathrm{cm}^2$.

It seems clear that equations (7) and (8) cannot be correct as they stand, as even with low-angle scattering dominant the values they require for the ionization and excitation cross-sections are far too great. The reason for the failure of the equations would appear to be the assumption that ionization permanently removes atoms available for excitation. However, this will not be true if charge transfer takes place. As we have commented already, unfortunately no cross-sections are available for this process, but we can discuss the limiting case when the loss of velocity during transfer may be neglected. ζ is now given by equation (28) and with $p(\varepsilon,\varepsilon')$ given by equation (22), one finds $\sigma_e/\sigma_d = 0.36\zeta$, or

$$\sigma_e = 4.08\times10^{-3}v^{1.25}\sigma_d. \qquad (34)$$

To make a comparison with the results of Boitnott & Savage, we have extrapolated this equation to $60\,\mathrm{km\,s^{-1}}$ and taking σ_d to be given by equation (33) we obtain the cross-section shown in Fig. 4. This is in quite satisfactory agreement with the single-collision results, encouraging us to extend the results for the excitation coefficient to velocities above $60\,\mathrm{km\,s^{-1}}$, utilizing the

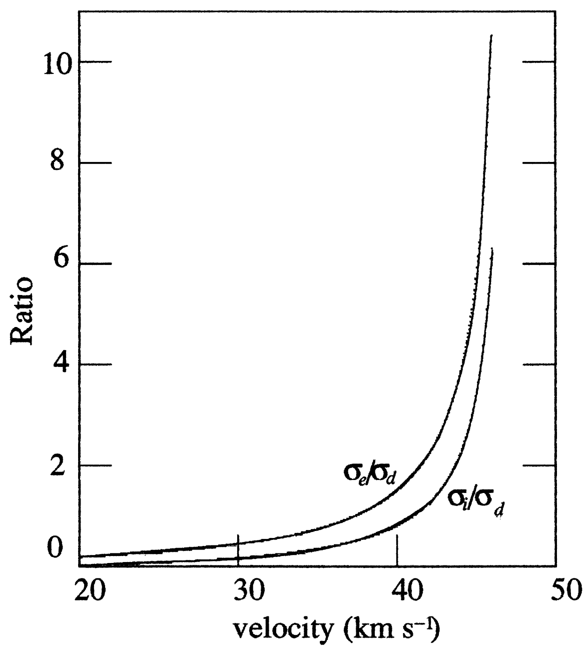

Figure 3. Ratios of excitation and ionization cross-sections to diffusion cross-section found by application of equations (25) and (26) to laboratory simulation results for β and ζ.

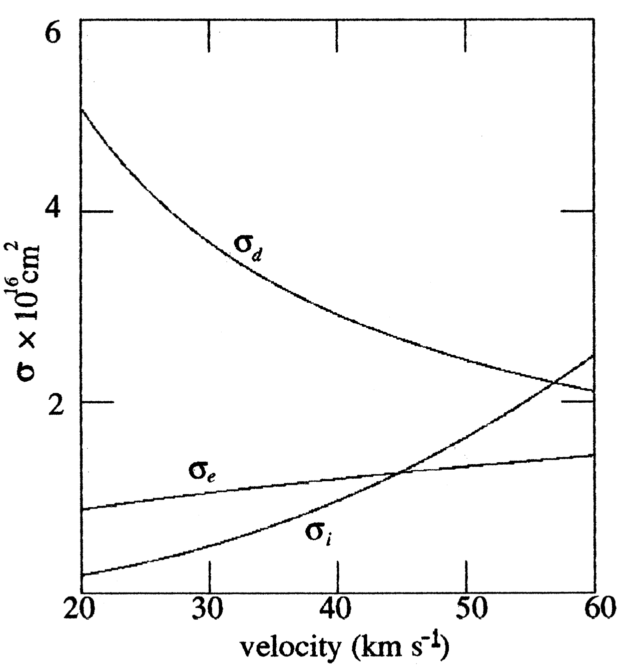

Figure 4. Excitation and ionization cross-sections found by application of equations (28) and (29) to laboratory simulation results for β and ζ.

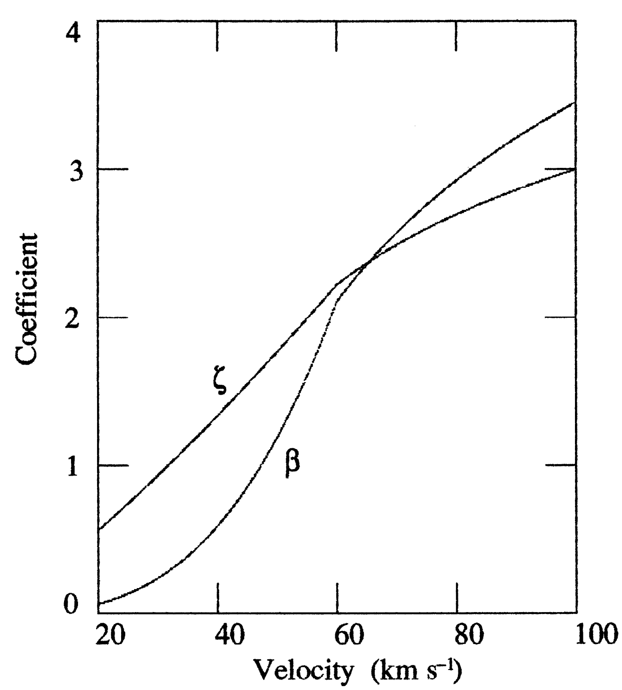

Figure 5. Ionization and excitation coefficients found for velocities less than $60\,\mathrm{km\,s^{-1}}$ from equations (31) and (32) and for higher velocities from equations (28) and (29) using excitation and ionization cross-sections measured in the laboratory.

Boitnott–Savage cross-sections for this region, with the diffusion cross-section again given by equation (33). The results are shown in Fig. 5, as are those for a similar treatment of the ionization.

5 DISCUSSION

We have taken values of the luminosity and ionization coefficients of iron, directly determined in the laboratory, and inverted integral equations (equations 7 and 8) for these coefficients to obtain values of the probabilities β_0 and ζ_0 of ionization and excitation on individual collision. Above $42\,\mathrm{km\,s^{-1}}$ the value given for ζ_0 is found to be above unity, which must be incorrect, by definition of ζ_0 as a probability. The application of the integral equations requires the distribution of change in velocity on collision. To test the possibility that the values of ζ_0 were too high because of a bias of our approximation to this distribution towards the lower velocities, we modified the integral equation so that it was assumed that there was no change in velocity on ionization or excitation. However the resulting values for the ionization and excitation cross-sections were found to be very large and inconsistent with experimental values.

Equations (7) and (8) were derived under the assumption that there is no charge transfer from the Fe ions to the molecules of the air. As we have remarked, there is in fact evidence that such transfer does take place and that the net ionization of the Fe atoms is small. In the absence of any direct knowledge of the relevant cross-sections, we have discussed only the limit that the loss in velocity during the transfer process may be neglected; the equations are also applicable if it is the air molecules that are ionized and the iron atoms do not suffer ionization at any stage. ζ is now given by equation (28), from which the excitation cross-section σ_e may be obtained if we assume the diffusion cross-section given by Bronshten and an extrapolation to give σ_e at $60\,\mathrm{km\,s^{-1}}$ yields satisfactory agreement with the single-collision result of Boitnott & Savage. Accordingly, we have extended the results for the excitation coefficient and also the ionization coefficient to velocities above $60\,\mathrm{km\,s^{-1}}$ utilizing the Boitnott–Savage cross-sections for this region. We should stress that the values for β apply only to a meteoroid composed of iron alone. The corresponding data are unavailable for the other constituents of a typical meteoroid, but if the respective contributions can be taken to be the same as at low velocities then for a cometary meteoroid with composition as given in Paper I the ionization coefficient will be approximately 0.2β where, as before, β refers to iron alone.

As we have seen, the observational determination of the luminous efficiency is vitiated by fragmentation, and we conclude the values of ζ shown in Fig. 5 (obtained assuming charge transfer or direct ionization of the air molecules to take place) to be the best available in the present circumstances. However, the considerations of this paper highlight the paucity of our knowledge of the essential components of a theoretical calculation of the ionization and luminosity coefficients. We require, and at the present time lack, differential scattering cross-sections for both elastic and inelastic scattering processes, including that of charge transfer from the ionized meteoric atoms to the molecules of the air.

We finally comment that since equations (7) and (8) are coupled equations for β and ζ, we have discussed only the velocity region above $20\,\mathrm{km\,s^{-1}}$, (where data for both b and z exist), with the specific aim of relating to Boitnott & Savage's (1972) high-velocity measurement of scattering cross-sections. With reference to lower velocities, Friichtenicht & Becker (1973) tabulate values of the luminous efficiency of iron down to the minimum possible velocity of a meteor ($11.2\,\mathrm{km\,s^{-1}}$, the escape velocity). Values for less than the escape velocity have possible relevance to the luminosity of space debris and we might note that the minimum transition energy for an appreciable contribution to the luminosity is 3.06 eV. On taking account of conservation of momentum, this gives a threshold velocity of $3.04\,\mathrm{km\,s^{-1}}$ for excitation on individual collision. The corresponding velocity for ionization is $9\,\mathrm{km\,s^{-1}}$, indicating the ionization of space debris by the mechanism discussed in this paper to be small. The work function for metallic iron is 4.48 eV, so that the minimum velocity of impact of an air molecule, for ejection of a conduction electron to become part of an external plasma, is approximately $5.9\,\mathrm{km\,s^{-1}}$.

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